# R E V I E W

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The Federal Reserve Board, which has the authority to regulate bank holding companies (BHCs), has stated on many occasions that it expects BHCs to act as sources of strength for their bank subsidiaries. As Board statements indicate, acting as a source of strength includes injecting capital into bank subsidiaries that have capital ratios below the required levels. In the first article in this issue, "Do Bank Holding Companies Act as 'Sources of Strength' for Their Bank Subsidiaries?" R. Alton Gilbert tests whether affiliation with BHCs affects the amount of capital injected into troubled banks.

The author estimates a model designed to explain capital injections into troubled banks—those banks that must receive capital injections or reduce their assets to meet the minimum capital requirements. Gilbert finds that affiliation with BHCs does affect capital injections, but the effect depends on the size of a BHC relative to the size of its troubled bank subsidiary. The larger a BHC relative to the size of its troubled bank subsidiary, the larger the capital injection ratio.

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Regional economic multipliers are commonly used to estimate the effects of a major change in a region's economy, such as the construction of a convention center or the closing of a manufacturing facility. These measures estimate the changes in output, income and employment resulting from an initial change in spending. In the second article in this *Review*, "A Consumer's Guide to Regional Economic Multipliers," Cletus C. Coughlin and Thomas B. Mandelbaum provide an elementary discussion of regional multipliers. Their discussion identifies numerous theoretical and measurement problems, which suggest that any analyses using these multipliers should be viewed with caution and skepticism.

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The American public has become increasingly skeptical about the future of the Social Security system. This skepticism primarily reflects concern about the financing of the retirement and medical needs of the baby-boom generation. In the third article in this *Review*, "The Future of Social Security: An Update," Keith M. Carlson examines the current financial status of the Social Security system and addresses the concern about its long-run viability.

Carlson focuses on the 1990 annual reports of the Social Security trust funds and compares the projected surpluses with those prepared following the enactment of the 1983 Social Security amendments. The projected surpluses are now smaller than forecast in 1983. The financial condition of Social Security is affected particularly by real wage and

productivity growth. A review of the assumptions underlying the revised outlook indicates that the near-term surpluses will be smaller than generally expected and the far-term deficits will be larger.

\* \* \*

In the fourth article in this issue, "Learning, Rational Expectations and Policy: A Summary of Recent Research," James B. Bullard presents a synopsis of the recent work on the microfoundations of rational expectations, with special emphasis on the implications for macroeconomic policy. The theory of rational expectations does not specify a method of expectations formation, sometimes called a learning mechanism. Bullard demonstrates that, as a result, the application of the theory can be ambiguous. This is an important fact about modern macroeconomic literature because the treatment of expectations plays a crucial role in deriving policy implications in standard macromodels.

#### R. Alton Gilbert

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# Do Bank Holding Companies Act as "Sources of Strength" for Their Bank Subsidiaries?

HOSE WHO WISH to determine why banks fail typically focus on the characteristics of banks and their local markets that make them vulnerable to losses.¹ A key factor often overlooked, however, is capital injections by shareholders. A bank whose losses exceed its capital need not fail, if its shareholders (existing or new) inject sufficient additional funds to restore its capital ratio to a level acceptable to the bank's supervisory agency. Likewise, the shareholders of a bank with a positive, but relatively low, capital ratio can always remove it from its "problem bank" status by injecting sufficient capital.

The 1980s were marked by large numbers of bank failures and large shares of commercial bank assets written off as losses compared with the previous four decades. Taking advantage of this opportunity for comparison, this paper focuses on the incentives of bank holding companies (BHCs) relative to those of other investors to inject additional capital into troubled banks.

We test the hypothesis that capital injections into troubled bank subsidiaries of BHCs are larger than the capital injections into troubled independents.

The hypothesis underlying this study is that BHCs have strong incentives to maintain "favorable" reputations in the financial markets and with the Federal Reserve Board (Board). In particular, BHCs want to maintain favorable reputations with the Board because, under general criteria specified in the Bank Holding Company Act, the Board has the authority to approve or deny applications by BHCs to acquire additional subsidiaries. If BHCs inject relatively more capital into their troubled banks, then it is argued that, at least compared to owners of independent banks, BHCs are "sources of strength" for their bank subsidiaries. If BHC affiliation has no influence on the amount of capital injected into troubled banks, then BHCs provide no more support in this form for their subsidiary banks than other bank owners.2

considered in this paper. Thus, BHCs may be sources of strength in other ways, even if they do not inject more capital into their troubled banks than other bank owners.

<sup>&</sup>lt;sup>1</sup>Bovenzi, Marino and McFadden (1983) and Demirgüc-Kunt (1989).

<sup>&</sup>lt;sup>2</sup>Other types of BHC actions to aid troubled bank subsidiaries, such as providing new management, are not

# THE "SOURCE OF STRENGTH" PRINCIPLE: A GENERAL DISCUSSION

The Federal Reserve Board issued a statement on April 22, 1987, about the obligations of BHCs toward their troubled bank subsidiaries. In summary, it said:

The statement reaffirmed a long-standing Board policy that holding companies should use their available resources to provide adequate capital funds to their subsidiary banks during periods of financial stress or adversity. The Board issued the policy statement to remind holding companies of its expectation that they provide financial and managerial strength to their subsidiary banks.<sup>3</sup>

BHCs may take various types of actions to aid their troubled bank subsidiaries besides capital injections. This statement, however, indicates that the Board expects BHCs to employ this method of assistance if losses reduce the banks' capital ratios below acceptable levels.

The Board also has cited the principle of source of strength in denying some applications for the formation of BHCs. In these cases, the Board projected that the dividends from bank subsidiaries necessary to sevice the debt of the new BHCs would reduce the capital ratios of the acquired banks to unacceptably low levels.<sup>4</sup>

The Financial Institutions Reform, Recovery and Enforcement Act of 1989 (FIRREA) makes a weaker version of the source-of-strength principle a legal requirement for multi-bank holding companies that own both solvent and insolvent subsidiary banks. FIRREA requires that multi-bank holding companies use the capital of their subsidiary banks to cover the losses of each individual bank subsidiary. Thus, it has eliminated the option of a multi-bank holding company to

abandon an individual subsidiary bank, thereby leaving its negative net worth to be absorbed by uninsured depositors or by the Federal Deposit Insurance Corporation while retaining its solvent bank subsidiaries. FIRREA does not require BHCs to use the capital of the parent organization or nonbank subsidiaries to cover negative net worth of their bank subsidiaries. The Federal Reserve Board's standard for "source of strength," although not a legal requirement, covers a broader range of circumstances, including those in which the capital of a BHC's subsidiary bank, after absorbing losses, is low but still positive. Also, the Board expects BHCs to use all of their resources to aid their troubled bank subsidiaries, not just the capital of their bank subsidiaries.

## THE GENERAL HYPOTHESIS AND ITS IMPLICATIONS

This paper investigates banks whose recent losses are so large that they must either receive capital injections or reduce their assets to meet the minimum capital standards. During 1985-88, the years covered by this study, the minimum capital requirements of commercial banks were specified in terms of the ratio of their primary capital to their total assets. The major components of primary capital are (1) equity capital (investment by shareholders plus retained earnings) and (2) loan loss reserve.

The minimum requirements set by the federal supervisory agencies for all banks in 1985 called for primary capital to be greater than or equal to 5.5 percent of total assets. The supervisory authorities indicated that even higher capital ratios would be required for banks whose assets were of relatively poor quality. Since the banks in this study incurred large losses relative

<sup>3</sup>Board of Governors (1987), pp. 71-72. See Duncan (1987) and Bureau of National Affairs (1987) for legal interpretations of the Board's statement. The Board also attempted to force MCorp to act as a source of strength for its bankrupt subsidiaries. See Bureau of National Affairs (1989). In mid-May 1990, the Fifth Circuit U.S. Court of Appeals ruled against the Federal Reserve in its suit against MCorp. See Quint (1990). Despite this ruling, BHCs still have incentives to maintain their reputations in financial markets, and the Federal Reserve still has authority to approve or deny any subsequent BHC applications.

<sup>4</sup>These denials are based on projections of the payments necessary to service the acquisition debt and the earnings of the banks to be acquired, not on the records of the BHCs in acting as sources of strength in the past. For a recent case in which the Board applied this principle in denying an application by a BHC, see the *Federal Reserve Bulletin* (April 1990), pp. 257-58.

<sup>5</sup>Gilbert, Stone and Trebing (1985). The bank supervisory authorities also specified a minimum ratio of total capital to total assets of 6 percent. Total capital includes primary capital plus long-term debt of the bank that is subordinated to its deposits. This paper does not consider the adequacy of the total capital ratio or the incentives for banks to issue subordinated debt to meet the total capital ratio.

to their capital, their minimum primary capital ratio is assumed to be 6 percent.6

Capital injections into independent banks generally involve the sale of additional bank stock to existing or new shareholders. BHCs that own their bank subsidiaries outright, however, often inject capital into them directly, leaving the number of shares outstanding unchanged.7 When a BHC injects capital directly, the transaction has the following effects on the balance sheet of the BHC: a reduction in "cash" and an increase in another category of assets called "investment in subsidiaries." On the subsidiary's balance sheet, the transaction involves an increase in both its cash and its "equity capital." In this paper, capital injections are measured as the sum of funds raised by issuing additional shares and injecting capital directly.

# BHCs and the Source of Strength Hypothesis

BHCs have reasons to inject capital into their troubled subsidiaries over and above those that apply to individuals who own shares in troubled independent banks. These reasons differ somewhat depending on whether the subsidiary would actually fail or simply have low but positive capital ratios without a capital injection.

First, consider the incentives of BHCs. BHCs wish to convince financial market participants that they are strong, reliable organizations, in order to reduce their borrowing costs and possibly boost the price of their stocks. In several cases, BHCs have covered the losses of their subsidiaries' creditors just to maintain

their reputations in the financial markets.<sup>8</sup> As previously discussed, BHCs also are concerned about their reputations with the Federal Reserve Board. Their ability to acquire subsidiaries could be jeopardized if the Board views their failure to support subsidiaries as a lack of willingness to act as sources of strength.

Of course, individuals who own banks also are concerned about their reputations with investors and with bank supervisors. The failure of their banks will make it more costly for them to raise funds in the future to buy other banks. In fact, individuals with poor reputations with bank supervisors may be barred from buying banks. Thus, whether BHCs have greater incentives to recapitalize their troubled banks than individuals who own troubled independents is an empirical issue.

Up to now, the discussion has focused on banks that would fail without capital injections. Most banks in this study, however, had positive primary capital after absorbing their losses, even without capital injections. These banks can raise their capital ratios by reducing their assets or receiving capital injections. BHCs also have reasons for injecting capital into these banks that may not apply to individuals who own stock in troubled independents. Increases in the capital ratios of their subsidiary banks achieved through reductions in assets, rather than by injections of capital, may not demonstrate the BHCs' ability and willingness to act as sources of strength.10 Thus, if BHCs do not inject additional capital, they may be denied permission to acquire additional subsidiaries in the future.11

<sup>&</sup>lt;sup>6</sup>Losses recognized in the current year reduce primary capital, since they reduce either the loan loss reserve or equity capital. Loans that are not collectible are declared loan losses by bank management and charged against the loss reserve, thereby reducing that component of primary capital. A bank increases its loan loss reserve in the current year though an expense item that is charged against current income. Negative earnings in the current year, perhaps because of a relatively large provision for loan losses, reduce equity capital.

<sup>&</sup>lt;sup>7</sup>A BHC would inject capital into a bank subsidiary rather than buy additional shares only if it owned all of the bank's stock. Otherwise, a direct injection of capital is a gift to other shareholders that increases the value of their shares even though they have not increased their investment in the bank.

<sup>&</sup>lt;sup>8</sup>For a description of cases in which BHCs covered the losses of their subsidiaries, rather than forcing the creditors of their subsidiaries to absorb them, see Cornyn, et al (1986), pp. 187-91. The authors interpret these actions as attempts by BHCs to portray themselves as strong, reliable organizations that honor their obligations.

Individuals must apply to the supervisory authorities for permission to buy controlling interest in banks. If individuals refuse to inject capital into their failing banks, the supervisory authorities may block their acquisitions of banks in the future. See Spong (1985), pp. 94-95.

<sup>10</sup>The Board's record in acting on BHC applications to acquire subsidiaries makes it difficult to provide a more definitive description of the Board's standards for source of strength. In most cases in which the Board has stated the principle of source of strength as the basis for denials, the basic issue is excessive debt in the formation of one-bank holding companies. Few denials have been based on the failure of BHCs to act as sources of strength for their bank subsidiaries.

<sup>11</sup>Note that the Board statement quoted above refers to the use of "available resources to provide adequate capital funds to their subsidiary banks during periods of financial stress or adversity." This statement could be interpreted as indicating that a BHC would not be considered a source of strength if it fails to inject capital into a subsidiary bank with a low but positive capital ratio.

#### The Issue of Size

BHCs differ greatly in terms of the magnitude of their assets and the number of their subsidiaries. Many BHCs, for example, have only a single subsidiary. The incentives for the owners of such a one-bank holding company to inject capital into their troubled subsidiary are likely to be similar to those of individuals who own a troubled independent. The failure of a BHC's sole subsidiary would have no adverse effects on the BHC's cost of funds because it would be out of business as well. Moreover, a BHC with only one subsidiary has certainly not shown that it considers permission to acquire additional subsidiaries to be a valuable privilege.

For multi-bank holding companies, the incentive to inject capital is presumably greater, the larger are the total assets of all subsidiaries in the BHC to the assets of its troubled subsidiary. The prospective penalties imposed on a BHC by both the financial markets and the Board for a failure to inject capital into a troubled subsidiary are likely to be positively related to the total assets of the BHC.<sup>12</sup> Thus, in modeling the determinants of capital injections into troubled banks, variables designed to reflect affiliation with BHCs reflect the assets of BHCs relative to the assets of their troubled bank subsidiaries.

## Assessing the Financial Strength of BHCs

Capital injections by BHCs also are assumed to be influenced by their own financial conditions. If primary capital ratios at the other banks in a BHC exceed the levels required by bank supervisors, the BHC can channel capital from them, via dividends, to the troubled subsidiary. Alternatively, a BHC with most of its subsidiaries in strong financial condition could raise funds in the financial markets at lower interest rates (reflecting lower risk premiums) than BHCs that have larger numbers of troubled banks.

In contrast, if the primary capital ratios of the banks in a BHC are generally below required levels, banks have little or no excess capital to pay out as dividends to the BHC to channel to its more troubled subsidiaries. Moreover, such a BHC would have less incentive to promote its

reputation with the Board as a source of strength by attempting to come to the aid of one of its bank subsidiaries; the relatively low capital ratios of its other subsidiaries clearly indicate that the BHC is not in a position to act as a source of financial strength.

### A FORMAL SPECIFICATION OF THE BANK CAPITAL INJECTION MODEL

Equation 1 presents the model of the determinants of capital injections into troubled banks used in this study.

The variables used are defined below: the signs above the variables in equation 1 show the expected signs of their estimated coefficients.

INJ = the ratio of the capital injected into a bank to the total assets of the bank at the end of the prior year.

RHCS10 = dummy variable with a value of unity if the ratio of total banking assets of a BHC to the assets of its troubled bank subsidiary (RHCS) is greater than unity but less than 10, zero otherwise.

RHCS50 = dummy variable with a value of unity if RHCS is greater than or equal to 10 but less than 50, zero otherwise.

RHCS100 = dummy variable with a value of unity if RHCS is greater than or equal to 50 but less than 100, zero otherwise.

RHCS100+ = dummy variable with a value of unity if RHCS is greater than or equal to 100, zero otherwise.

FSHC = the ratio of the sum of primary capital of banks in the BHC, other than the troubled banks included in this study, to the sum of their total assets.

NINJ = the ratio of the capital injection necessary to make primary capital equal to 6 percent

have made fewer acquisitions. Under this assumption, the penalty for not investing in a troubled subsidiary in the form of foregone opportunities for future acquisitions is proportional to the total assets of the BHC.

<sup>12</sup>While it is impossible to determine the value of future acquisitions to a BHC without knowing its plans, a reasonable guess might be that BHCs that have grown large through acquisitions would place higher value on the privilege to make additional acquisitions than BHCs that

of total assets as of the end of the prior year, to their total assets at the end of the prior year.

ROA = the ratio of net income of the banks in the county of a troubled bank, other than the troubled bank itself, to their total assets. Income is measured over the calendar year prior to the year in which the bank becomes a troubled bank, and total assets are measured at the end of that year.

## Identification of Troubled Banks and the Measure of Capital Injections

Equation 1 is designed to explain capital injections into a specific group of banks — those whose primary capital ratios initially exceeded 6 percent but who had losses in the current year that drove their primary capital below 6 percent. To raise their capital ratios up to 6 percent, these banks must receive capital injections or reduce their assets. The capital injected into each bank is measured as the sum of capital injections over four quarters, beginning in the quarter in which the losses reduced their primary capital below 6 percent of their total assets in the initial period.

Each bank included in the study had, at the end of the prior year, primary capital that exceeded 6 percent of its total assets, as illustrated in inequality 2.

(2)  $0.06A_0 < C_0$ 

where

 $A_0$  = total assets in the last quarter of the prior year,

 $C_0$  = primary capital in the last quarter of the prior year.

Then, during some quarter of the current year, each bank in the study had losses sufficient to reduce its primary capital (net of any capital injection in that quarter) below 6 percent of its total assets at the end of the prior year  $(A_0)$ , as illustrated in inequality 3.

(3)  $0.06A_0 > [C_t - I_t],$ 

where

C<sub>t</sub> = primary capital in the first quarter of the current year (quarter t) in which this inequality holds,

I, = capital injections in the first quarter of

the current year (quarter t) in which this inequality holds.

Capital injections in quarter t are subtracted from the right side of inequality 3 because some banks inject capital immediately to cover at least part of their losses; in fact, if they inject enough capital, their primary capital would not actually fall below 6 percent of  $A_{\rm o}$ . Yet these banks should be included in the analysis because they received large capital injections to offset large losses.

In deriving values of the dependent variable, capital injections are summed over four quarters, primarily because of the typical timing of the capital injections. Most capital injections occurred in the fourth quarter of the year in which losses reduced the primary capital of a bank enough to satisfy inequality 3. Of the 256 banks in this study that received capital injections, 238 received capital injections during this quarter, regardless of the actual quarter in which the inequality first held. Moreover, 221 of these received their only capital injection in that fourth quarter. Given this timing, a four-quarter period starting with the quarter in which each bank satisfies inequality 3 is adequate for examining capital injections into troubled banks. Thus, capital injections are measured as the sum of capital injections in quarter t through t+3. The equation for calculating the dependent variable, INJ, is specified in equation 4:

(4) INJ = 
$$\sum_{j=0}^{3} I_{t+j}/A_0$$

Because the capital injections are summed over four quarters, an additional constraint is imposed on the balance sheets of the banks included in this study: their primary capital in quarter t+3 (net of capital injections in quarters t through t+3) must be less than 6 percent of A<sub>0</sub>, their total assets at the end of the year before their large losses. This is because banks may have increased their primary capital in quarters t+1 through t+3 in ways that do not involve explicit capital injections; among these are positive earnings that are retained, recoveries on loans previously charged off as losses and changes in accounting practices. Such changes in primary capital after quarter t will affect the amount of capital injections needed to meet their capital requirements. This condition is

stated in inequality 5: (5)  $0.06A_0 > [C_{t+3} - \sum_{j=0}^{3} I_{t+j}].$  Finally, some banks that satisfy inequalities 3 and 5 are excluded from this study because they were involved in mergers or were purchased by new owners around the time of their losses. <sup>13</sup> Including these banks would have potentially biased the results of this study. Suppose, for instance, that troubled banks sold to new owners are acquired by BHCs that injected additional capital into their new bank subsidiaries. Including these banks in the study would exaggerate the significance of BHCs as sources of strength for their existing bank subsidiaries.

In deriving the dependent variable, INJ, the dollar value of capital injections is divided by  $A_{\rm o}$  to create a measure of capital injections that is unaffected by contemporaneous changes in total assets. Using the total assets existing when the capital injections were made could bias the estimate of the effects of affiliation with BHCs on the size of capital injections.

Suppose, for example, that BHCs inject enough capital into their troubled subsidiaries to meet capital requirements without reducing their assets, while shareholders of independent banks choose combinations of capital injections and reductions in assets. Deflating capital injections by the total assets when such injections are made would bias downward the estimate of the effect of affiliation with BHCs on the size of capital injections. Deflating by  ${\rm A}_{\rm 0}$  avoids this potential bias.  $^{\rm 14}$ 

#### Relative Size of BHCs and Their Troubled Bank Subsidiaries

Equation 1 includes dummy variables for different levels of RHCS — the ratio of the total banking assets of the BHC to the assets of its troubled subsidiary. These dummy variables apply only to multi-bank holding companies. No subsidiaries of one-bank holding companies are included in the study because all that met the inequality conditions were sold around the time they became troubled banks.

If BHCs as a group inject more capital into their troubled bank subsidiaries than other bank owners, the coefficients on the dummy variables for RHCS will be positive and statistically significant. Moreover, as explained previously, the coefficients are expected to be larger for higher values of RHCS.

## Financial Strength of the BHC (FSHC)

FSHC reflects the financial strength of BHCs; it is measured as the ratio of the primary capital of the banking subsidiaries of a BHC — other than that of their troubled bank subsidiaries in this study — to their total assets. The primary capital and total assets values used are those in the fourth quarter of the year prior to the year in which the subsidiary first satisfies inequality 3. For troubled banks that are not subsidiaries of BHCs, FSHC has a value of zero.

<sup>13</sup>The following rules are designed to exclude banks that were sold to new owners around the time they became troubled banks. Each bank that was a subsidiary of a BHC in 1985 must have been a subsidiary of the same BHC in December 1983. Similarly, banks that were not affiliates of BHCs in 1985 must also have been independent banks in December 1983. In a similar fashion, subsidiaries of BHCs included in the study for 1986 were affiliates of the same BHCs as of December 1984 or were independent banks in both periods. The rules for including banks in the study for 1987 and 1988 have similar timing.

Troubled independent banks that were sold to new owners are not excluded from the study because it is more difficult to obtain information on their ownership than on that of BHCs. There are 113 banks that met the other criteria for inclusion in the study that were excluded because of changes in their ownership around the time they became troubled banks. These 113 include 24 in which multibank holding companies bought troubled independent banks. The multibank holding companies injected capital into 19 of these banks.

14The possible distortions that would result from scaling capital injections by total assets as of the time of the capital injections can be illustrated by considering two banks that are identical in every way other than the response of their shareholders to losses. In December 1984, Bank A and Bank B had primary capital of \$6 and total assets of \$100. In the first quarter of 1985, each had a loss of \$2, reducing primary capital to \$4 and total assets to \$98. For the rest of 1985, loan losses and net income were zero for each bank. During 1985, shareholders inject \$2 into Bank A, returning primary capital to \$6 and total assets to \$100. Bank B receives a capital injection of \$1 during 1985 and reduces its assets by \$15, raising its primary capital ratio to approximately 6 percent [5/(100-2+1-15)]

Since the capital injection into Bank A is twice that for Bank B, the value of the dependent variable should be twice as large for Bank A. If capital injections were deflated by total assets as of the time of the capital injections, however, the ratio would be 0.02 for Bank A (2/100) and 0.0119 for Bank B (1/84).

### Capital Injection Necessary to Maintain the Prior Level of Total Assets (NIN.I)

NINJ is defined and calculated as the capital injection necessary to raise the troubled bank's primary capital to 6 percent of  $A_0$  divided by  $A_0$ . The calculation of the values of NINJ is illustrated in equation 6:

(6) NINJ = 0.06 - 
$$[C_{t+3} - \sum_{j=0}^{3} I_{t+j}]/A_0$$
.

NINJ is included as an independent variable in the equation for capital injections to avoid possible biases in estimating the effect of BHC affiliation on capital injections. For instance, suppose that BHC subsidiaries require capital injections of 4 percent of total assets  $(A_0)$  to meet their capital requirements without reducing their assets, while independent banks need only 1 percent. Including the independent variable NINJ controls for such differences.

#### **Expected Future Profits**

Bank losses may cause shareholders to lower the future profits they expect their banks to earn for a given level of assets, making them reluctant to add capital into these banks. Alternatively, their expected profits may be unaffected by current losses; in these cases, current shareholders may add capital to offset part or all of the reduction in primary capital.

The average rates of return on assets (ROA) of other banks in the counties of the troubled banks are used to indicate the prospects for future profits. ROA is included to determine whether shareholders of troubled banks located in areas in which other banks have achieved relatively high ROA are more likely to inject capital into their banks.

#### THE DATA

## Location and Size of the Banks

The study includes all banks located in the 20 states in table 1 in the years 1985-88 that meet

Table 1

Distribution of Troubled Banks by Year and Location

		Other	
Year <sup>1</sup>	Texas	states <sup>2</sup>	Total
1985	35	80	115
1986	115	93	208
1987	134	67	201
1988	90	44	134
Total	374	284	658

1Year in which bank first satisfied inequality 3 in text.

<sup>2</sup>The other states are Arkansas, Colorado, Georgia, Illinois, Indiana, Iowa, Kentucky, Michigan, Minnesota, Missouri, Montana, New Mexico, North Dakota, Ohio, Pennsylvania, Tennessee, West Virginia, Wisconsin and Wyoming.

the various criteria. These 20 states permit multibank holding companies but do not permit statewide branching. The restriction on statewide branching is important for the construction of ROA, since data on the profit rates of individual branches are not available.

Information in table 1 highlights the deteriorating condition of banks in Texas over the period. In 1985, about 30 percent of the banks in the study were Texas banks; by 1987 and 1988, about two-thirds of them were located in Texas.

Table 2 shows the asset size of the banks in the study. While the vast majority had total assets under \$100 million, a few had total assets in excess of \$1 billion.

### Distribution of Banks by Capital Injections Necessary to Meet Capital Requirements (NINJ)

The distribution of banks by values of NINJ, the capital injection ratio necessary to raise the primary capital ratio to 6 percent without reducing total assets, is shown in table 3. Of the 658 banks, 76 had such large losses that their

explaining changes in equity capital. Wall and Peterson (1987, 1988) reported that changes in the equity capital ratios of banks were influenced predominantly by capital requirements of bank supervisors. These studies are consistent with the hypothesis that capital requirements influence capital injections by bank shareholders.

<sup>15</sup>Evidence in other studies is consistent with the hypothesis that capital requirements influence capital injections. Dahl and Shrieves (1990) found that banks with capital ratios below required levels were more likely to receive capital injections than other banks. Mingo (1975) estimated changes in bank equity capital as a function of several independent variables, including the capital requirements of bank supervisors. Capital requirements were significant in

Table 2

Distribution of Troubled Banks by Asset Size

Assets		Texas	Other States		
(millions of dollars)	Number	Percentage	Number	Percentage	
Assets < \$25	119	31.8%	140	49.3%	
\$25 ≤ Assets < \$50	117	31.3	72	25.4	
\$50 ≤ Assets < \$75	54	14.4	29	10.2	
\$75 ≤ Assets < \$100	27	7.2	11	3.9	
\$100 ≤ Assets < \$1,000	49	13.1	31	10.9	
\$1,000 ≤ Assets	8	2.1	1	0.4	
Total	374		284		

Table 3

Distribution of Banks by Capital Injection Ratios Necessary to Meet Capital Requirements without Reducing Assets

	1	Texas	Other States		
NINJ	Number	Percentage	Number	Percentage	
0.01 ≥ NINJ > 0	68	18.2%	77	27.1%	
0.02 ≥ NINJ > 0.01	66	17.6	77	27.1	
0.03 ≥ NINJ > 0.02	72	19.3	40	14.1	
0.04 ≥ NINJ > 0.03	55	14.7	29	10.2	
0.05 ≥ NINJ > 0.04	34	9.1	25	8.8	
0.06 ≥ NINJ > 0.05	23	6.1	16	5.6	
NINJ > 0.06	56	15.0	20	7.0	
Total	374		284		

primary capital would have been negative without capital injections. At the other extreme, 145 banks required capital injections that were less than 1 percent of their total assets. Table 3 shows that values of NINJ are not clustered at the extreme values.

## Affiliation with Bank Holding Companies

Table 4 indicates that about half of the Texas banks and about 56 percent of the banks located in the other states are subsidiaries of multibank holding companies. The rest were independent.

#### THE RESULTS

Table 4 provides a comparison of the incidence of capital injections into independent banks and subsidiaries of BHCs, ignoring other determinants. The patterns of capital injection frequency over time were different for banks in

Texas than for those in other states. For independent banks and subsidiaries of BHCs in Texas, the percentages of banks receiving capital injections declined over time. In each year, however, a higher percentage of BHC subsidiaries than independent banks received capital injections, and the percentages were significantly higher in 1986, 1987 and for the 1985-88 period as a whole.

For banks in other states, there was no consistent pattern over time in the incidence of capital injections. As with the Texas banks, the percentage of banks that received capital injections was higher each year for BHC subsidiaries than for independent banks. The percentages are significantly higher in 1985, 1987 and for the 1985-88 period as a whole. Thus, these direct comparisons show that, in general, a larger proportion of BHC-owned troubled banks received capital injections than did troubled independents.

Table 4

Frequency of Capital Injections into Troubled Banks

			Year <sup>1</sup>		
	1985	1986	1987	1988	1985-88
BANKS IN TEXAS:					
Independent banks					
No. with INJ>0	10	10	10	7	37
Others	10	41	54	44	149
Percentage with INJ>0	50.0%	19.6%	15.6%	13.7%	19.9%
BHC subsidiaries					
No. with INJ>0	10	31	27	9	77
Others	5	33	43	30	111
Percentage with INJ>0	66.7%	48.4%	38.6%	23.1%	41.0%
Percentage significantly higher for BHC					
subsidiaries <sup>2</sup>	NO	YES	YES	NO	YES
(t-statistic for equality of					
proportions in parentheses)	(1.01)	(3.45)	(3.11)	(1.13)	(4.55)
BANKS IN OTHER STATES: Independent banks					
No. with INJ>0	13	14	11	11	49
Others	25	15	25	12	77
Percentage with INJ>0	34.2%	48.3%	30.6%	47.8%	38.9%
BHC subsidiaries					
No. with INJ>0	30	33	19	11	93
Others	12	31	12	10	65
Percentage with INJ>0	71.4%	51.6%	61.3%	52.4%	58.9%
Percentage significantly higher for BHC					
subsidiaries <sup>2</sup>	YES	NO	YES	NO	YES
(t-statistic for equality of					
proportions in parentheses)	(3.58)	(0.29)	(2.64)	(0.30)	(3.42)
BANKS IN ALL 20 STATES: Percentage significantly higher for BHC					
subsidiaries <sup>2</sup>	YES	YES	YES	NO	YES
(t-statistic for equality of	IES	120	TES	NO	150
proportions in parentheses)	(3.46)	(2.96)	(3.83)	(1.14)	(5.84)

<sup>1</sup>Year in which banks first satisfied inequality 3 in the text.

# Estimation of Equation for Capital Injections

The statistically significant differences in table 4 may indicate that BHCs acted as sources of strength for their subsidiaries; on the other hand, these differences may be attributed to other factors not explicitly shown in the table. Equation 1 is designed to indicate whether other determinants of capital injections, in fact, are

responsible for the BHC effect on capital injections suggested by table 4.

Equation 1 is estimated using Tobit regression analysis, instead of OLS regression methods, because a large number of the troubled banks had zero capital injections. In the sample used, only 256 of the 658 banks had positive capital injections. The shaded insert discusses the nature of Tobit analysis and explains how to interpret the estimated coefficients.

<sup>&</sup>lt;sup>2</sup>Percentages are significantly different at the 5 percent level. See Wonnacott and Wonnacott (1990), pp. 273-75.

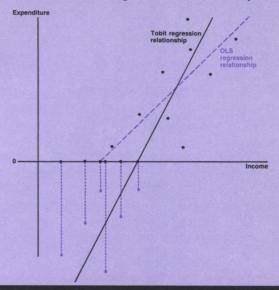
## **Tobit Regression Analysis**

Tobit analysis is appropriate in estimating coefficients using data sets in which large proportions of the observations for the dependent variable are at some limiting value, such as zero. Tobin (1958), who pioneered this type of analysis, estimated the influence of income and other variables on household expenditure on durable goods. A large percentage of households made no durable purchases in the year of the survey. Such a data set is illustrated in figure 1. A possible explanation for the zero expenditures is that, since expenditures on durable goods is not continuous, purchases are not made until the "desire" to buy the goods exceeds a certain level. We cannot observe desires, however: we observe only expenditures. "Negative" expenditures, corresponding to various levels of desire below the threshold level, cannot be observed.

Tobit analysis is used in estimating the determinants of capital injections because a majority of the banks in the study have zero capital injections. This truncation of the dependent variable at zero reflects the effects of regulation. While desired capital injections into many of the banks in the study may be negative, government supervisors restrict dividends (negative capital injections) of banks with negative earnings and capital ratios below the minimum required level.

Estimation of the relationship between expenditures and income with the data in figure 1 using OLS techniques will yield a biased estimate of the relationship between desired expenditures and income. The points represented by circles are the unobserved desired expenditures below the threshold. Those observations are recorded as zero expenditures and positive income, as indicated by the dashed lines. The problem with OLS estimation is illustrated using the following notation: let E, be desired levels of expenditure, E, the observed levels of expenditure, and I, household income for the ith household. Equation A1 is the relationship between desired expenditures and income.

Figure 1
Tobit and OLS Regression Relationships



(A1) 
$$E_i^* = \alpha + \beta I_i + \epsilon_i^* (i = 1, 2, ..., n)$$
.

The expected value of  $\varepsilon_i^*$  is assumed to be zero with variance of  $\sigma^2$ . Equation A2 presents the relationship between  $E_i$  and  $E_i^*$ 

(A2) 
$$E_i = E_i^* \text{ if } E_i^* > 0$$
  
= 0 if  $E_i^* \leq 0$ .

Equation A3 is that relationship estimated using OLS techniques:

(A3) 
$$E_i = \hat{\alpha} + \hat{\beta} I_i + \varepsilon_i$$
.

In this equation,  $\beta$  is a biased estimate of  $\beta$  in equation A1; thus,  $\varepsilon_i$  would not have an expected value of zero with variance of  $\sigma^2$ .

In Tobit analysis, parameters are estimated using maximum likelihood analysis. The likelihood function has two parts, one for observations with values of the dependent variable at the limiting value, the other for the remaining observations. The computer programs search through several iterations for the values of the regression coefficients that maximize the value of a likelihood function.<sup>1</sup>

The effects of unit changes in independent variables on the dependent variable are es-

<sup>&</sup>lt;sup>1</sup>See Kmenta (1986), pp. 560-63.

timated using the regression coefficients derived from Tobit regression analysis. Estimates derived from these coefficients alone, however, would overstate the size of the effects. The coefficients must be multiplied by the expected probability that an observation will have a value of the dependent variable other than the limiting value. To illustrate this point, consider the Tobit regression equations estimated in table 7 of the text, which indicate that affiliation with a relatively large BHC has a positive and statistically significant effect on the desired capital injection ratio of a bank. Whether such affiliation has positive effects on observed capital injections depends on the probability that the desired capital injection

ratio of a bank will exceed zero. For many of the banks in the study, desired capital injections are assumed to be negative, since their observed capital injections are zero and their dividends are restricted by supervisors. The effects of affiliation with relatively large BHCs on desired capital injections are weighted by the expected probability that capital injections will be positive. This weighting involves multiplying the Tobit regression coefficients by the expected probability that the capital injection ratio is positive. The expected probability of a positive capital injection ratio is derived from the Tobit regression equation with the independent variables set equal to their mean values.

Results for the estimation of equation 1 for 1985 through 1988 are presented in table 5. Separate equations are estimated for Texas banks because the substantial deterioration of earnings and capital adequacy of the entire Texas industry in recent years may have influenced Texas shareholders' incentive to inject capital into troubled banks.

The results indicate that the effects of affiliation with BHCs are qualitatively similar for troubled banks whether in Texas or elsewhere. BHC-owned troubled banks do not receive larger capital injections than independent banks, holding constant the other determinants of capital injections, when the total banking assets of the BHCs are less than 10 times the total assets of their troubled bank subsidiaries (RHCS10). In both equations, the coefficients on RHCS100 and RHCS100+ are positive and statistically significant, indicating that if the BHC's total banking assets are at least 50 times as large as the assets of its troubled subsidiary, it injects more capital into the subsidiary than do the owners of independent banks. Outside Texas, this condition also holds for BHCs with total banking assets at least 10 times the assets of their troubled banks' subsidiaries. Thus, the decision to inject capital depends on the size of BHCs relative to the size of their troubled subsidiaries.

The coefficients on the variable NINJ are positive and significant in each equation, as the hypothesis suggested. However, the variable FSHC — the weighted average capital ratio of the banking subsidiaries of BHCs, other than those included in the study — does not help ex-

Table	5		
<b>Tobit</b>	Regressi	on Re	sults
(t-stat	istics in	paren	theses)

Dependent variable: INJ

Independent variables	Texas	Other states
Constant	-0.0589** (7.73)	-0.0217** (4.66)
RHCS10	0.0142 (1.49)	-0.0055 (0.88)
RHCS50	0.0109 (0.93)	0.0175** (2.78)
RHCS100	0.0468** (4.07)	0.0275** (3.91)
RHCS100+	0.0192* (2.36)	0.0114* (2.28)
NINJ	0.5867** (5.57)	0.6835** (7.46)
ROA	0.7722* (2.42)	- 0.2550 (0.85)
No. of observations	374	284
No. with INJ>0	114	142
Likelihood ratio test statistic	49.94**	79.67**
Predicted probability of		

<sup>\*</sup>statistically significant at the 5 percent level

0.2889

INJ>0 with independent

variables at mean values

0.5137

<sup>\*\*</sup>statistically significant at the 1 percent level

Table 6								
<b>ROA</b> of	Other	Banks	in	Same	County	as	<b>Troubled</b>	Bank

	19851	19861	19871	1988²
Banks in Texas				
Mean ROA	0.0056	0.0054	-0.0012	-0.0097
SD	0.0141	0.0039	0.0071	0.0139
N	35	115	134	90
Banks in other states				
Mean ROA	0.0077	0.0058	0.0047	0.0039
SD	0.0051	0.0072	0.0063	0.0062
N <sup>2</sup>	79	89	65	44

<sup>&</sup>lt;sup>1</sup>Year in which banks first satisfy inequality 3. SD — standard deviation

plain differences in capital injections among troubled banks. When FSHC was added as an independent variable to the equations in table 5, its estimated coefficient was not significantly different from zero in either equation.

# ROA as a Determinant of Capital Injections

The coefficient on ROA is significant for the Texas regression but insignificant in the other. An examination of the ROA variable suggests that this reflects the sharp drop in profitability of Texas banks relative to that at non-Texas banks over the years covered by this study. The decline in profitability among Texas banks (table 6) induced a corresponding decline in capital injections by Texas bank owners (table 4).<sup>16</sup>

### **Alternative Specification**

Table 5 represents one way to assess the impact of BHC affiliation on capital injections into troubled banks. It is possible, however, that the coefficients on dummy variables for the ratio RHCS in fact may reflect more than simply BHC affiliation. Capital injections may actually be influenced by the absolute size of BHCs and, in-

Equation 7 examines this possibility by incorporating the asset size of troubled banks and BHCs into the equation:

(7) INJ = f(S50·D1, S75·D1, S75+·D1, S50·D2, S75·D2, S75+·D2, BHC100, BHC1000, BHC1000+, NINJ, ROA).

The additional independent variables are:

S50 — dummy variable with a value of unity if the total assets of the troubled bank are greater than \$25 million but less than or equal to \$50 million, zero otherwise

S75 — dummy variable with a value of unity if the total assets of the troubled bank are greater than \$50 million but less than or equal to \$75 million, zero otherwise

S75+ — dummy variable with a value of unity if the total assets of the troubled bank are greater than \$75 million, zero otherwise

D1 — dummy variable with a value of unity if a bank is an independent bank, zero otherwise

coefficient on ROA is insignificant for the Texas equation and remains insignificant in the other equation. Thus, after allowing for different capital injection ratios in different years, differences in ROA across the counties in which the troubled banks were located did not help explain differences in capital injection ratios across troubled banks.

N — number of banks

<sup>&</sup>lt;sup>2</sup>Data for ROA are missing for a few banks.

dependent of BHC affiliation, by the size of troubled banks themselves. Such separate size effects might well confound the interpretation of the RHCS dummy variables.

<sup>&</sup>lt;sup>16</sup>One way to separate the effects on capital injections of variation in ROA across counties from the effects of changes in average ROA over time is to add dummy variables for individual years as independent variables. In regressions not reported here, dummy variables for individual years are added as independent variables to the equations estimated in table 5. In those regressions, the

D2 — dummy variable with a value of unity if a bank is a BHC subsidiary, zero otherwise

BHC100 — dummy variable with a value of unity if the troubled bank is a subsidiary of a BHC with total banking assets of \$100 million or less, zero otherwise

BHC1000 — dummy variable with a value of unity if the troubled bank is a subsidiary of a BHC with total banking assets greater than \$100 million but less than or equal to \$1 billion, zero otherwise

BHC1000+ — dummy variable with a value of unity if the troubled bank is a subsidiary of a BHC with total banking assets greater than \$1 billion, zero otherwise.

Table 7 indicates that capital injection ratios are larger for subsidiaries of the larger BHCs. Among Texas banks, capital injections increase if they are subsidiaries of BHCs with total assets over \$100 million. For banks in the other states, capital injections are larger if banks are in subsidiaries with total assets in excess of \$1 billion.

The BHC effect on capital injections, however, is not just a matter of BHC size. Table 7 also indicates that, holding constant the size of the BHC, the larger BHC subsidiaries have lower capital injections ratios. Coefficients on dummy variables for the size of independent banks are not statistically significant. Thus, tables 5 and 7 yield the same implications for the effect of BHC affiliation on capital injections: it is the size of BHCs relative to the size of their troubled bank subsidiaries that influences capital injection ratios.

### Economic Significance of Affiliation with BHCs for Capital Injections

The empirical results support the hypothesis that BHC subsidiaries receive larger capital injections than other troubled banks. But are the effects of affiliation with BHCs on capital injections large enough to be economically significant? If the BHC effect is estimated to yield only a few extra dollars for a typical troubled bank, the statistically significant effects could be dismissed easily as being economically irrelevant.

Economic significance can be gauged by estimating the difference between the capital injection ratios of a BHC subsidiary and an independent bank with identical values of the other

independent variables. These effects could be estimated alternatively using the results in tables 5 or 7. Table 7 is used because its maximum likelihood test statistics are larger, indicating greater explanatory power of the equations.

Estimation of the size of the effect of affiliation with BHCs involves an analysis of the size of the regression coefficient. As explained in the insert on Tobit analysis, the regression coefficients must be multiplied by the probability of the dependent variable being greater than zero (the fraction at the bottom of the table) to estimate the effects of a unit change in an independent variable on the size of the dependent variable. In table 7, the fraction is 0.2817 for the second equation (Texas banks) and 0.5147 for the fourth equation (banks located elsewhere). These equations are used in the analysis of the economic significance of affiliation with BHCs on capital injections.

The estimates of BHC effects on the size of capital injections are presented in table 8. These effects are assumed to be zero unless the regression coefficients are significantly different from zero at the 5 percent level. They are calculated as follows: Suppose a troubled bank in Texas, with total assets of less than \$25 million, is a subsidiary of a BHC whose total assets are greater than \$100 million but less than or equal to \$1 billion. Compared with an independent bank of the same size and characteristics other than BHC affiliation, the capital injection is estimated to be higher at the BHC subsidiary by 1.67 percent of its total assets, which is calculated as 0.0592 (the coefficient on BHC1000) times 0.2817, the adjustment to regression coefficients appropriate for Tobit analysis. The effect for a subsidiary of a BHC of similar size that has total assets between \$25 million and \$50 million is estimated as follows:

(0.0592 - 0.0399)0.2817 = 0.0054.

These estimates have a consistent pattern: in a given size range, capital injection ratios are larger for banks that are subsidiaries of larger BHCs, and smaller for larger bank subsidiaries.

The effects of BHC affiliation can be stated in dollar terms by assuming certain asset sizes for the representative BHC subsidiaries. For instance, suppose the troubled bank has total assets of \$15 million and is a subsidiary of a BHC with total assets between \$100 million and \$1 billion. Its capital injection is estimated to be \$250,500 larger than that into a similar sized in-

Table 7

Alternative Tobit Regression Results: 1985-88
(t-statistics in parentheses)

Dependent variable: INJ

Independent variables	Te	xas	Other	states
Constant	-0.0498**	-0.0604**	-0.0217**	-0.0249**
	(6.69)	(9.15)	(4.77)	(5.72)
S50-D1	-0.0179 (1.74)		-0.0094 (1.22)	
S75-D1	-0.2314 (0.01)		- 0.1385 (0.03)	
S75 + ·D1	-0.2129 (0.01)		-0.1404 (0.02)	
S50-D2	-0.0395*	-0.0399*	- 0.0058	-0.0057
	(2.45)	(2.44)	(0.86)	(0.85)
S75-D2	-0.0405*	-0.0409*	-0.0150	-0.0147
	(2.32)	(2.30)	(1.77)	(1.72)
S75 + ·D2	-0.0467**	-0.0470**	-0.0153*	-0.0153*
	(2.90)	(2.88)	(2.07)	(2.04)
BHC100	0.0046	0.0136	- 0.0037	-0.0005
	(0.24)	(0.72)	(0.49)	(0.06)
BHC1000	0.0496**	0.0592**	0.0040	0.0073
	(3.19)	(3.85)	(0.49)	(0.92)
BHC1000+	0.0659**	0.0757**	0.0289**	0.0321**
	(3.91)	(4.51)	(4.22)	(4.77)
NINJ	0.5880**	0.6130**	0.7489**	0.7613**
	(5.73)	(5.97)	(8.18)	(8.27)
ROA	0.8513*	0.9090**	- 0.1078	-0.2021
	(2.73)	(2.90)	(0.36)	(0.68)
No. of observations	374	374	284	284
No. with INJ>0	114	114	142	142
Likelihood ratio test statistic	78.58**	67.37**	97.71**	89.88**
Predicted probability of INJ > 0 with independent variables at mean				
values	0.2129	0.2817	0.4768	0.5147

<sup>\*</sup> statistically significant at the 5 percent level.

<sup>\*\*</sup> statistically significant at the 1 percent level.

Table 8

Estimated Effects of Affiliation with BHCs on the Size of Capital Injection Ratios

	Size of BHCs (millions of dollars)					
Size of subsidiary bank (millions of dollars)	\$1,000 ≥ Assets > \$100	Assets	> \$1,000			
	Texas	Texas	Other			
\$25 ≥ Assets > 0	0.0167	0.0213	0.0165			
\$50 ≥ Assets > \$25	0.0054	0.0101	0.0165			
\$75 ≥ Assets > \$50	0.0052	0.0098	0.0165			
Assets > \$75	0.0034	0.0081	0.0086			

dependent bank.<sup>17</sup> While this paper presents no criterion for economic significance, these effects are too large to be dismissed as economically insignificant.

### Holding Company Affiliation's Influence on the Incidence of Bank Failure

The influence of RHCS on capital injections affects how one interprets some recent studies of the determinants of bank failure. Belongia and Gilbert (1990) find that increases in RHCS reduce the probability of bank failure. Gajewski (1989) reports a similar finding: the larger the number of bank subsidiaries in a BHC, the lower the probability of failure by its subsidiary banks. This study indicates that the lower probability of failure by bank subsidiaries of large, multibank holding companies results, in part, from the larger capital injections by multibank holding companies into their troubled banks.

#### CONCLUSION

The Federal Reserve Board expects BHCs to serve as sources of strength for their bank subsidiaries. BHCs can do so by injecting capital into their troubled subsidiaries when losses reduce their capital ratios below levels acceptable to bank supervisors.

This paper examines whether BHCs have injected relatively more capital into their troubled subsidiaries than the owners of similarly troubled independent banks, holding constant the values of other determinants of capital injections. The empirical results indicate that the BHC effect depends on the total banking assets of the BHCs as well as the assets of their troubled bank subsidiaries. BHCs with total assets that are at least 50 times larger than those of their troubled subsidiaries tend to inject more capital into those subsidiaries than other bank owners inject into their banks.

These results are consistent with the view that BHCs weigh the effects of capital injections on their reputations in the financial markets, and with the Board, against the opportunity cost of such injections. The results are also consistent with recent studies which found that the subsidiaries of relatively large BHCs have a lower probability of failure than other banks with similar characteristics—their lower failure rate reflects large capital injections into such banks.

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dicate that this Texas bank subsidiary would receive an extra capital injection of \$202,808 because of its affiliation with the BHC. This amount equals 0.0468 (coefficient on RHCS 100) times 0.2889 (predicted probability of INJ>0) times \$15 million.

<sup>17</sup>The estimate of the size of the extra capital injection due to affiliation with a BHC in this case also can be calculated using the results in table 5. The bank subsidiary has total assets of \$15 million. Suppose the BHC has total assets of \$750 million. Results from table 5 in-

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# A Consumer's Guide to Regional Economic Multipliers

ROPONENTS of major construction projects, such as a stadium, airport or convention center, point to their potentially large and widespread benefits. Since these projects are costly and may require public funds, estimates of their economic benefits are used by the community to assess their desirability. Similarly, the closing of a major manufacturing facility-or a large cutback in its production—is of interest throughout the community because of its anticipated adverse consequences. In a hypothetical, but realistic, example discussed later in this paper, a \$50 million decline in aircraft sales by a St. Louis manufacturer is estimated to cause a \$132 million reduction in output in the St. Louis economy and the elimination of 1,130 jobs. Upon hearing such a prediction, some basic questions come to mind. Where do numbers such as these come from? How accurate are they likely to be?

Often, such numbers are obtained using "regional economic multipliers," which is a standard way to identify the potential effects of a major change in a region's economy. These measures estimate the changes in output, income and employment resulting from an initial change in spending. In this article, we provide an

elementary discussion of regional multipliers and explain why they should be viewed with both caution and skepticism.

Our consumer's guide begins by discussing the basics of an input-output model; such a model identifies the relationships among different sectors in an economy and, thus, is used to calculate regional multipliers. We then describe different kinds of regional multipliers and discuss their major shortcomings.

# A NON-MATHEMATICAL LOOK AT MATHEMATICAL INPUT-OUTPUT MODELS

An input-output model is a mathematical description of how all sectors of an economy are related. In lieu of a technical discussion of this topic, we describe the structure of input-output models using two approaches.¹ In the text, we present an intuitive discussion; in the appendix, the discussion has more detail, but remains accessible to readers comfortable with elementary algebra.

<sup>&</sup>lt;sup>1</sup>See Miller and Blair (1985) for a technical discussion of input-output models.

In constructing an input-output model, one begins by separating economic activity in a region (any geographic area, such as a country, state or metropolis) into a number of producing sectors. These sectors may be highly aggregated for example, manufacturing, services, mining and construction-or fairly disaggregated-automobile production, hospitals, coal mining and new office construction. The value of products flowing from each sector as a producer to each sector as a purchaser provides the essential information for a model. A model with steel and automobile sectors, for example, would include the dollar value of the steel produced in the region that is sold to regional auto manufacturers, as well as the dollar value of the automobiles produced in the region that are sold to the region's steel industry.

An industry's demand for inputs from other industries is related closely to its own level of output. For example, the automobile industry's demand for steel is related closely to the number of automobiles produced. Thus, steel sales are related to automobile production. Indeed, inputoutput models assume that, for each industry in the region, there is a constant relationship between the value of its output and the value of inputs it purchases from all other industries in the region. Suppose that the value of automobile production in a given year is \$1 million and the auto industry purchased \$200,000 of steel. If automobile production were to double to \$2 million, the input-output model then assumes that the auto industry's steel purchases would double as well, in this case, to \$400,000.

In contrast, these models make no explicit assumption about the relationship between the value of the output and purchases of inputs of groups other than the region's industrial firms. The demand of these external units is referred to as final demand because the outputs are leaving the region's processing sectors. Final demand includes purchases by government, purchases by households and firms from other regions and, in some cases, purchases by households in the specific region under consideration. Thus, the value of total output of each industry in the region is divided into that which is used in the production of other goods

in the same region (called interindustry sales) and that which is purchased by final demanders.

An input-output model, however, goes beyond describing the flows of goods and services between sectors and to final demand. It also allows the user to determine the values for the gross output of each industry necessary to meet these final demands. This information allows the calculation of regional multipliers.

#### REGIONAL MULTIPLIERS

One primary use for an input-output model is the estimation of the total effect on an economy of changes in the components of final demand for the goods and services produced within the region. The term "impact analysis" is used to characterize such a study, particularly when the change is due to a single event that occurs within a relatively short period of time.

A change in final demand, like a change in federal government demand for aircraft, sets a region's economy in motion, as productive sectors buy and sell goods and services from one another. These relationships cause the total effect to exceed its initial change in final demand. The ratio of the total economic effect on a regional economy to the initial change is called a regional multiplier.<sup>2</sup> The total effect is measured in terms of output, income or employment, giving rise to output, income and employment multipliers.

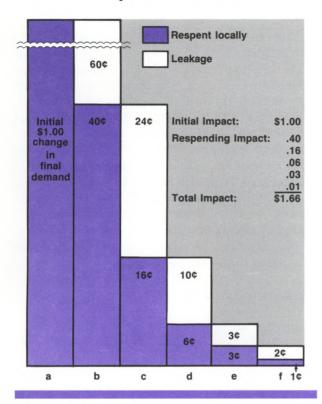
An output multiplier of 1.66, for example, indicates that, if a firm's sales in one region to buyers in another region increase by \$100 million, total sales throughout the region are expected ultimately to increase by \$166 million. The additional \$66 million in regional economic activity is generated because the \$100 million change in spending, by affecting production, income and employment in the region, stimulates additional changes in spending that cause further changes in production, income and employment in the region.

Suppose the initial change in spending in a regional economy occurs as an apparel manufacturer receives payment for sales to a wholesaler

<sup>&</sup>lt;sup>2</sup>The regional multiplier is analogous to the standard Keynesian multiplier used in macroeconomics: an initial increase in demand leads to an even greater expansion of regional income, as the income received from this demand

is spent, creating income for others who spend and create additional income for still others. This process of respending comes to an end when the demand increase is offset by "leakages" through saving, taxation and imports.

# Figure 1 The Multiplier Process



in another region. As figure 1 shows, for every such dollar of spending that enters this hypothetical region (column a), 40 cents is respent within the region (column b). In our example, this respending includes payments to other firms within the region for inputs such as cloth, buttons and electricity and for services such as legal and janitorial services.

The remaining 60 cents of the initial dollar is considered a "leakage", as it is spent outside of the regional economy. Leakages include payments for inputs the manufacturer buys from other regions, tax payments to all levels of government and dividends distributed to stockholders in other regions. In the second round of respending (column c), 16 cents of the 40 cents is respent within the region, while the remaining 24 cents is leakage. One example of local respending would be purchases of regionally

produced goods by the suppliers of the apparel manufacturer using the payments they receive from the apparel manufacturer.

This process continues until any additional spending within the region is inconsequential. The change in total business activity can be calculated by adding the initial dollar to the total respending within the region. In our example, this would total \$1.66 (\$1 + \$0.40 + \$0.16 + \$0.06 + \$0.03 + \$0.01). Thus, the apparel sector's multiplier indicates that \$1.66 of total business activity is generated for each dollar of additional external sales by the region's producers.<sup>3</sup> Conversely, for each dollar reduction in final demand, total regional business activity is expected to decline by \$1.66.

Input-output models enable researchers to calculate multipliers for various sectors of the economy. Each sector has a unique multiplier because each has a different pattern of purchases from firms in and outside the region.

#### **Open and Closed Models**

In practice, the value of multipliers depends on whether households are considered part of the interrelated processing sectors or a component of final demand outside the regional economy. The separation of households from processing sectors is arbitrary; households as consumers and workers are enmeshed with the industrial sectors. Households earn their incomes by providing their labor services and spend their incomes as consumers. Naturally, the amount that consumers spend is a function of their incomes, which depend on the receipts of the sectors.

If an input-output model classifies households as part of final demand rather than part of the region's productive economy, then income received by households is considered a leakage, and household respending does not contribute to the multiplier effect. If households are included in the interrelated processing sectors, the spending of income received by households within the region adds to the total effect. Thus, multipliers derived from so-called "closed models," in which regional households are included, are larger than those derived from "open mod-

because, relative to the nation, a region tends to purchase a higher proportion of its inputs from other regions. In other words, leakages are greater.

<sup>&</sup>lt;sup>3</sup>A given sector's multiplier is smaller for a region than for the nation of which it is a part. This reflects the greater interindustry linkages at the national level. A region's multiplier for a specific sector tends to be smaller

els," which treat households as part of final demand.4

#### **Output Multipliers**

An output multiplier for a given sector is the total value of sales by all sectors of the regional economy necessary to satisfy a dollar's worth of final demand for that sector's output. An important point is that this value of total business activity is larger than the market value of currently produced goods and services because some of the respending in input-output models is for the purchase of intermediate goods and services. The value of these inputs are counted again when the final goods they are used to produce are sold. Thus, Stevens and Lahr (1988) conclude that output multipliers are almost always misleading because of this double-counting.

A hypothetical example may clarify how output multipliers are used. Table 1 presents multipliers for the St. Louis Metropolitan Statistical Area (MSA) derived from one frequently used input-output system, the Regional Input-Output Modeling System (RIMS II) developed by the U.S. Department of Commerce.<sup>5</sup> Suppose the federal government cuts orders for military fighter jets produced in the St. Louis area by \$50 million. To estimate the total output effect, the \$50 million reduction of sales is multiplied by the total output multiplier for aircraft, 2.64, resulting in a total change of -\$132 million.6 Thus, total output in the St. Louis MSA would be expected to fall by \$132 million because of the initial reduction in aircraft production.

This total multiplier effect is the sum of the output effects on each of the sectors of the St. Louis economy. Not surprisingly, the transportation equipment (except motor vehicles) industry, which includes aircraft production, is affected the most. For every dollar change in final demand for aircraft, output of the transportation equipment industry changes in the same direction by \$1.09. As table 1 shows, the effects go

beyond this industry. For example, for every dollar change in final demand for aircraft, output of the real estate sector changes by \$0.09 and the output of the retail trade sector changes by \$0.07.

The output multiplier for households (industry 39) indicates a \$0.62 change in household earnings resulting from every dollar change in final demand for aircraft. The household earnings effect reflects the sum of earnings paid to households employed in the other 38 industries listed in the table. In other words, some portion of the \$1.09 multiplier effect listed for transportation equipment is distributed as payments to households. A similar statement can be made about the other industries. An estimate of exactly how much is distributed to households is listed in the column for the earnings multiplier, which is related closely to the income multiplier discussed in the next section.

#### **Income Multipliers**

Income multipliers translate the effects of changes in final demand into changes in household income. Two kinds of income multipliers can be constructed: those that indicate regional income changes associated with an initial change in *output* and those that indicate income changes associated with an initial change in *income*.<sup>7</sup>

The first approach converts an initial \$1 final-demand change into the total change in income. This conversion is straightforward. The \$1 final-demand change in a specific sector causes changes in that sector and all other related sectors. These final-demand changes lead to changes in household income in each of these sectors. Summing the household income changes over all sectors yields the total change in household income. The income multiplier is simply the ratio of the total change in household income to the \$1 change in final demand.

<sup>&</sup>lt;sup>4</sup>Multipliers derived from open models are referred to as Type I or simple multipliers, while those derived from closed models are called Type II or total multipliers. The same distinction applies to income and employment multipliers. In addition to households, other sectors can be shifted from final demand to the region's productive sectors. Bourque (1969), for example, creates multipliers for the Washington economy that account for the induced effects of state and local government spending.

<sup>&</sup>lt;sup>5</sup>See U.S. Department of Commerce (1986) for a description of RIMS II. The St. Louis MSA includes the City of St.

Louis and Franklin, Jefferson, St. Charles and St. Louis counties in Missouri as well as Clinton, Jersey, Madison, Monroe and St. Clair counties in Illinois.

<sup>&</sup>lt;sup>6</sup>Prof. Fredrick Raines of Washington University provided valuable assistance on the use of the RIMS II model for this example.

Income multipliers can also be distinguished on the basis of whether households are considered part of the interrelated processing sectors or a component of final demand.

Table 1

Multipliers for the St. Louis Area Aircraft Industry

Industry Aggregation	Output <sup>1</sup>	Earnings <sup>1</sup>	Employment
1. Agricultural products and agricultural,	\$0.0034	\$0.0010	0.1
forestry and fishery services			
2. Forestry and fishery products	0.0000	0.0000	0.0
3. Coal mining	0.0043	0.0013	0.0
4. Crude petroleum and natural gas	0.0002	0.0000	0.0
5. Miscellaneous mining	0.0004	0.0001	0.0
6. New construction	0.0000	0.0000	0.0
7. Maintenance and repair construction	0.0203	0.0089	0.3
8. Food and kindred products and tobacco	0.0377	0.0054	0.2
9. Textile mill products	0.0004	0.0001	0.0
10. Apparel	0.0087	0.0023	0.1
11. Paper and allied products	0.0061	0.0013	0.0
12. Printing and publishing	0.0126	0.0040	0.2
13. Chemicals and petroleum refining	0.0512	0.0041	0.1
14. Rubber and leather products	0.0102	0.0024	0.1
15. Lumber and wood products and furniture	0.0019	0.0005	0.0
16. Stone, clay, and glass products	0.0027	0.0008	0.0
17. Primary metal industries	0.0438	0.0083	0.2
18. Fabricated metal products	0.0279	0.0075	0.3
19. Machinery, except electrical	0.0243	0.0090	0.3
20. Electric and electronic equipment	0.0231	0.0081	0.3
21. Motor vehicles and equipment	0.0208	0.0028	0.1
22. Transportation equipment, except motor vehicles	1.0935	0.3485	9.0
23. Instruments and related products	0.0092	0.0031	0.1
24. Miscellaneous manufacturing industries	0.0030	0.0008	0.0
25. Transportation	0.0554	0.0232	0.8
26. Communication	0.0247	0.0064	0.2
27. Electric, gas, water, and sanitary services	0.0414	0.0040	0.1
28. Wholesale trade	0.0666	0.0253	0.9
29. Retail trade	0.0715	0.0345	2.4
30. Finance	0.0273	0.0087	0.3
31. Insurance	0.0240	0.0083	0.3
32. Real estate	0.0914	0.0024	0.3
33. Hotels and lodging places and amusements	0.0169	0.0056	0.5
34. Personal services	0.0121	0.0055	0.6
35. Business services	0.0564	0.0282	1.2
36. Eating and drinking places	0.0507	0.0154	1.7
37. Health services	0.0354	0.0199	0.8
38. Miscellaneous services	0.0346	0.0129	0.7
39. Households	0.6231	0.0024	0.4
Total	\$2.6374 <sup>3</sup>	\$0.6231	22.6

<sup>&</sup>lt;sup>1</sup>Dollars per dollar of final sales

<sup>&</sup>lt;sup>2</sup>Number of jobs per \$1 million of final sales (1987 dollars)

<sup>&</sup>lt;sup>3</sup>The output multiplier is 2.0143 for open models, that is, when the induced effects of the household sector are excluded.

The second approach uses a different denominator in the calculation of the income multiplier. A \$1 change in final demand for a specific sector's output initially becomes a \$1 change in output by that sector. In the first approach, this dollar's worth of output is used as the denominator in the income multiplier. The second approach replaces the denominator with the initial additional income received by workers in the sector. The resulting multiplier indicates the total change in income resulting from the initial change in income.

Although the RIMS II model does not provide income multipliers, it does furnish closely related earnings multipliers that can be applied to the current example. (Income generally includes transfer payments, dividends, interest and rent in addition to earnings.) Table 1 shows that the earnings multiplier for aircraft manufacturing is 0.6231, indicating that for each \$1 change in output from the aircraft industry in St. Louis, earnings throughout St. Louis will change by approximately \$0.62, other things equal. Approximately \$0.35 of this change occurs in the transportation equipment (except motor vehicles) industry. Multiplying the hypothesized \$50 million reduction by 0.623 yields an estimated decline in regional earnings of \$31.15 million.

### **Employment Multipliers**

In many cases, citizens are as interested in employment as in output or income, that is, the number of regional jobs a particular economic change is expected to generate or eliminate. If the relationship between the value of a sector's output and its employment level can be estimated, then employment multipliers can be calculated.

As with the discussion of income multipliers, there are employment multipliers that translate initial *output* changes into regional employment changes and, using a different approach, employment multipliers that translate initial *employment* changes into regional employment changes.

As Stevens and Lahr (1988) note, most employment multipliers are estimated in terms of jobs rather than "full-time equivalent" employees. Unfortunately, the relationship between jobs and full-time equivalent employees is not the same, either across economic sectors or across

regions for the same sector. Thus, not only are all comparisons of employment multipliers suspect, but employment multipliers do not identify the mix of full- and part-time workers.

Returning to our example, if the employment multiplier for St. Louis' aircraft sector of 22.6 jobs per \$1 million of final sales, shown in table 1, is applied to the \$50 million reduction in aircraft sales, the estimated regional employment effect will be a loss of 1,130 jobs.8 In addition to the reduction of employment in the transportation equipment (except motor vehicles) industry of 450 (9 times 50), there are noteworthy employment effects in retail trade (120 jobs) and restaurant services (85 jobs).

To summarize, the estimated effects of the hypothetical \$50 million spending cut for aircraft would be: total spending (output) declines by \$132 million; earnings, by \$31.15 million; and employment, by 1,130 jobs. From a different perspective, these results suggest that, for each job lost, regional earnings will fall by \$27,566 and total spending by \$116,814. While the earnings per job figure seems reasonable, the spending decline of \$116,814 per job is of little practical value because this figure is not the loss in actual output per worker. The region's output will fall by substantially less than total sales losses.

# HOW ACCURATE ARE REGIONAL MULTIPLIERS? MORE REASONS TO BE CAUTIOUS

The widespread use of input-output multipliers in regional impact analysis suggests that many economists, government officials, firms and others have found them useful. Yet, multiplier analysis has important limitations stemming from both the theoretical basis of input-output models and measurement problems. A few of these problems have already been identified; more are discussed below. An awareness of these limitations is necessary to accurately interpret such multipliers.

## Transitory Effects

Multipliers estimate short-term economic changes; they do not take into account a regional economy's long-term adjustments. Thus, many

<sup>&</sup>lt;sup>8</sup>To be consistent with the St. Louis RIMS II model, the \$50 million reduction is measured in 1987 dollars.

of the economic effects identified by multipliers are likely transitory. For example, while a reduction in federal government purchases of aircraft will cause a reduction in regional employment, at least some of this reduction will be temporary as workers whose jobs were eliminated find new jobs in the region. Thus, multipliers may overstate the loss of jobs, if one looks beyond the immediate effects. To

Recent research by Taylor (1990) stresses this transitory effect. Taylor estimates that all 50 states would experience at least a small reduction in employment in the short run if real federal defense spending was cut by 10 percent. In all but 17 states, however, these losses would be more than offset by the creation of new jobs.<sup>11</sup>

#### **Supply Constraints**

The input coefficients measuring the interindustry flows between sectors are "fixed" in input-output models; in other words, at any level of output, an industry's relative pattern of purchases from other sectors is unchanged. When the initial change considered is an increase, this assumption requires that excess productive capacity exists in regions in which inputs are purchased. In addition, a sufficient surplus of labor with adequate skills is also assumed to be available at unchanged wages.<sup>12</sup>

If these assumptions are invalid, producers will have to alter their purchasing patterns to increase production. For example, suppose a region's auto assembly plant plans to increase its production and sales to other regions by 50 percent. If the plant's suppliers within the region were operating at, or near, full capacity, the assembly plant would have to buy a larger proportion of its inputs from firms outside the

region, at least until local suppliers could expand their production. In this case, the input coefficients expressing the flows from each regional industry to the assembly plant would overstate the true short-run relationship, and multipliers derived from the model would also be too large. If the supply of unemployed, adequately skilled workers were insufficient, the assembly plant might substitute capital for labor in its production or hire workers from other firms in the region. In this case, estimated impacts based on earlier multiplier relationships could overstate the actual consequences.

Of course, the magnitude of this problem varies from case to case. It is more severe when considering a substantial increase in output, especially in a small nonmetropolitan region; it is less important when analyzing larger, more complex economies. Supply constraints in most industries also are a bigger problem closer to business cycle peaks than during recessions.

#### **Ignoring Interregional Feedbacks**

Multipliers exist because the expansion of one sector's production causes the region's other sectors to expand as well; this, in turn, may induce further expansion in the first sector. These feedback effects operate not only between related sectors within a region, but also between the economies of individual regions. Suppose, for example, as a Missouri aircraft plant expands, it purchases tires produced in Tennessee. This could cause the tire plant to buy more industrial chemicals from Missouri firms. In addition, some of the increased income that Tennessee tire plant workers receive might be spent purchasing consumer goods and services produced in Missouri. Thus, the interrelations of the two state economies would cause a

<sup>9</sup>Not all displaced workers, of course, will find jobs within the region, even in the long run. The loss of such workers who leave for jobs in other regions will tend to retard the region's growth. Yet, such restructuring ultimately enhances the nation's economic performance, by redistributing labor resources to the industries and regions where they are most needed.

<sup>&</sup>lt;sup>10</sup>Some of the problems inherent in input-output models, including the measurement of long-run impacts and changes in multipliers over time, are addressed by linking input-output models to econometric models. Such models, however, are available for few regions. See Conway (1990) and Israilevich and Mahidhara (1990).

<sup>11</sup>Taylor (1990) uses a U.S. input-output model to estimate the effects of the defense spending cuts on each of the nation's industries. A state's short-run employment effects

were estimated by allocating each U.S. industry's loss to a state based on its industrial composition. A state's long-run reabsorption of labor was based on the assumption that its employment continued to grow at the historical rate.

<sup>&</sup>lt;sup>12</sup>The fixed relationship implies that additional material and labor inputs are available at existing prices and wage rates, respectively. That is, the increase in the quantity demanded would not bid up input prices and wages. Otherwise, an increase in production and employment in one sector may result in an employment decrease in another sector, and the multiplier will overstate the actual effect.

<sup>&</sup>lt;sup>13</sup>This suggests another use of multiplier analysis: evaluating the feasibility of an expansion in light of existing regional labor pools.

larger multiplier effect on the Missouri economy than if the interregional effects were ignored.

Although they are not included in multipliers calculated from single-region models, these interregional feedbacks are captured by multipliers derived from interregional and multiregional input-output models. <sup>14</sup> Such models are rarely developed, however, because of their extensive data requirements. <sup>15</sup>

How important is this omission for singleregion models? While the importance varies from region to region, evidence suggests that ignoring such feedback effects does not substantially affect aggregate multiplier values. <sup>16</sup> For individual industries, however, interregional feedbacks could be quite important.

#### Do Multipliers Change Over Time?

The multipliers used in impact studies are often generated using data based on transactions for a period that occurred five or more years earlier. In many cases, a full survey to develop new input-output coefficients is costly, so existing models, often several years old, are used. Even if a new survey is conducted, it may take several years to complete the input-output table. If the structural relationships indicated by a regional input-output model change significantly over these years, the reliability of impact analyses using the older model is diminished.<sup>17</sup>

There are several reasons why input-output relationships change considerably over time. First, technological changes and inventions of new products alter manufacturers' input purchasing patterns. Partly because of technological changes, for example, auto manufacturers purchase more plastic and robots and less steel and labor per unit of output than they did 10 years ago.

Second, if producers in a sector have significantly expanded their output since the inputoutput table was constructed, the assumed fixed relationship between the ratio of the value of input to output for each input will no longer hold. For example, the costs of inputs that are relatively insensitive to output levels, say, purchases of accounting services, could be spread over additional units of output as expansion occurs, lowering some input coefficients.

Furthermore, relative price changes across commodities induce substitution of the relatively cheaper inputs for the more costly ones. If oil prices rose relative to coal prices, for example, some manufacturers might purchase more coal and less oil. Finally, changes in interregional or international trade patterns—perhaps due to changes in transportation costs or exchange rates—could substantially alter patterns of trade and, thus, multiplier values.

If obsolete values are used as input-output coefficients, the multiplier values and impact estimates derived from them will be inaccurate. Conway (1977) provided some insights into the importance of changing multiplier values by examining three survey-based input-output models for the state of Washington. He found that, on average, simple output multipliers changed by approximately 5 percent in each of the 1963-67, 1967-72 and 1963-72 periods. Output multipliers for individual sectors changed by as much as 17.7 percent in the 1963-72 period. The change in total income multipliers was somewhat greater than the change in output multipliers; on average, they changed by roughly 10 percent in the three periods. One sector's income multiplier fell by 33.3 percent between 1963 and 1972, while another's rose by 14.1 percent.

Although these results are based on only one region, they suggest that the use of multipliers based on an older model may be misleading, especially given the dramatic changes that have influenced the way goods and services have been produced in recent decades. These changes include substantial fluctuations in oil

<sup>14</sup>Examples include the nine-region interregional model of Japan (Japanese Government, 1974), the three-region Dutch interregional model (Oosterhaven 1981) and Polenske's (1980) multiregional model of the 50 U.S. states and District of Columbia.

<sup>&</sup>lt;sup>15</sup>Multipliers, as shown by Stevens and Lahr (1988), can also be derived from economic base models. Such models generally divide regional economic activity into two sectors—production for export and production for local use—rather than the many sectors found in input-output models. This extreme aggregation is the source of one of several limitations of economic base multipliers. See Richardson (1985) for a critical review of these multipliers.

<sup>&</sup>lt;sup>16</sup>Miller and Blair (1985), pp. 127-28, found that the average error when interregional feedbacks were ignored (averaged over all sectors in all regions) was less than 3 percent in five models, 7 percent in one model and 14.4 percent in another. Also see Guccione, et al. (1988).

<sup>&</sup>lt;sup>17</sup>A related problem arises if one attempts to use inputoutput models for long-run regional forecasting. Such cases require so-called dynamic input-output models as described in Miller and Blair (1985), pp. 340-51, or combination econometric-input-output models such as the one described by Conway (1990).

prices, widespread technological innovation, increased use of services in production processes and heightened foreign trade.

Nobel Prize winner Wassily Leontief has raised serious doubts about the accuracy of models based on the federal government's input-output table of the U.S. economy. The frequently used RIMS II model, for example, was derived from the 1977 U.S. input-output table, so the underlying structural relationships have had more than a decade to change. For the same reason, Leontief suggests that, when the 1982 benchmark U.S. input-output table becomes available in early 1991, it will be of only historical interest.

## The Construction of Regional Input-Output Tables

Stevens and Lahr (1990) stress an even more fundamental problem: the lack of information on the relationships among producers within a region makes constructing an accurate regional input-output model very difficult. Since the appropriate coefficients are unknown, an analyst must estimate them. One way to do this is to assume that regional input-output coefficients are the same as the available, but dated, national coefficients. This method is not used, however, because regional production processes are seldom identical to national production processes and, even if they were, firms within the region are more dependent on inputs from outside the region than national input-output coefficients would indicate. Another method is to conduct a comprehensive survey of the region's firms; this, however, is rarely undertaken because of its high cost.

In most cases, regional input-output coefficients are estimated using methods that alter existing U.S. input-output models to better reflect regional transactions. <sup>19</sup> This alteration is accomplished by surveying key regional sectors or by using recently published regional data. These methods are imperfect, however, and the lack of accurate information remains a major obstacle in calculating regional multipliers. <sup>20</sup> The consumer of such multipliers is thus justified in wondering whether the value of a multiplier is accurate.

## Calculating the Change in Final Demand

Multipliers are used to estimate the total effect on a region's economy of an initial change in final demand; however, estimating the initial change is not a simple exercise. Changes in final demand have financial consequences. Ignoring this fact can result in estimates of final demand changes larger than are actually warranted.

To continue with our earlier example, assume that the reduction in defense expenditures for aircraft produced in St. Louis is associated with a reduction in federal income taxes. As a result, St. Louis taxpayers will pay less in taxes and spend more on goods and services, including those produced locally, which would offset some of the negative effects associated with the decline in aircraft expenditures.

These offsetting effects are likely to be more pronounced in cases of locally funded projects. For example, suppose that a city was considering building a sports stadium and financing it by raising local taxes. In general, the larger the burden of these taxes on local residents, the larger the reduction in demand for local production. Thus, an analysis that looks at the demand effects associated with spending on the stadium alone would overstate the multiplier effects of the project.

Moreover, determining the initial change in final demand stemming from a spending change is not always straightforward. In describing the use of RIMS II, for example, the U.S. Department of Commerce (1986) illustrates the importance of determining the actual final demand change in the context of tourist expenditures in Louisiana. A first step was to estimate the anticipated increase in tourist expenditures, an exercise subject to considerable error because of the difficulty of predicting the number of additional tourists as well as the changes in their expenditure levels. In the Department of Commerce example, for instance, increased expenditures on retail goods of \$280 million were predicted.

Given this figure, it was necessary to determine how much of this amount reflected final

ings, Israilevich and Martins (1990) have recently developed a method that might prove an improvement over past approaches.

<sup>&</sup>lt;sup>18</sup>See Leontief (1990).

<sup>&</sup>lt;sup>19</sup>For a description of five such models, see Brucker, et al

<sup>&</sup>lt;sup>20</sup>For a critical review of nonsurvey and partial survey methods, see Miller and Blair (1985), pp. 266-316. Hew-

demand for goods and services produced in Louisiana. The expenditure for retail goods is unlikely to be only for goods and services produced in Louisiana because this expenditure reflects changes for the output of wholesalers, transporters, manufacturers and retailers. Assumptions must be made about the distribution of changes among these producers and how much of the output of these producers takes place in Louisiana. Thus, estimating final demand is subject to substantial error.

# Estimating the Regional Impact of a New Industry

If a regional input-output model is available and the magnitude of the initial change in final demand is known, estimating the regional impact of a change in the final demand of an existing sector is straightforward. It is more difficult, however, to estimate the regional impact of a firm in an entirely new sector. Since the existing interindustry flows do not indicate such a firm's relationships with other sectors, the existing table cannot be applied directly; additional information is needed. Detailed information is required on the new firm's sales to, and purchases from, other regional producing sectors and other suppliers of inputs.

Additional information also may be needed if a new firm produces goods or services that are substitutes for those previously produced outside the region. Suppose, for example, that a new firm produces an input that many regional firms previously purchased from other regions. The enhanced availability of the input from the new source will cause some firms to purchase more of it from within the region. The change could alter purchases from firms outside of the region and input coefficients to an extent that more survey information or new estimates of interindustry linkages would be required. A related situation would be the closing of a firm that is the sole producer of a product. If the firm sells much of its output to other regional sectors, the firm's elimination would force them to import the good, altering their external and interindustry linkages.

#### CONCLUSION

Economic multipliers are often used to estimate the total regional effect associated with an initial change in a regional economy, after accounting for all the relationships among the economy's sectors. Multipliers used in regional impact studies are usually based on single-region inputoutput models, which mathematically describe the industrial structure and linkages of the economy at one point in time. Numerous theoretical and measurement problems, however, suggest that estimates based on these multipliers should be viewed with caution.

The accuracy of multipliers depends on how accurately the model describes the current regional economy. If the structure of the economy has changed since the model was developed or if the new development itself will substantially alter the regional economy's structure, multipliers will tend to be inaccurate. In such cases, their accuracy can be improved by using additional information.

In addition, identifying the precise magnitude of an initial change in final demand is not always straightforward. The financing of a particular expenditure can offset an increase or decrease in demand for locally produced goods and services. Further complications can arise in determining how much of a change in expenditures is a change in final demand.

Despite their limitations, economic multipliers based on input-output models are popular and will continue to be used in economic impact studies. Meaningful analysis using multipliers is not clear-cut and mechanistic, but rather requires the exercise of careful judgment by an experienced and knowledgeable analyst. Such analysis allows for a degree of subjectivity and therefore, possible bias to enter the analysis, however. The consumer should be aware that estimates based on regional multipliers may be subject to considerable error.

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# **Appendix Foundations of Input-Output Models**

An input-output model is a mathematical description of the relationships among all sectors of an economy. To construct such a model, economic activity in a region must be separated into a number of producing sectors. These sectors may be highly aggregated, or they may be identified in considerably more detail. For example, a regional input-output model developed by the Department of Commerce contains more than 500 industries, 370 of which are specific manufacturing industries.1 The manufacture of household furniture contains six separate categories: 1) wood household furniture; 2) wood television and radio cabinets; 3) upholstered household furniture; 4) metal household furniture; 5) mattresses and bedsprings; and 6) household furniture, not classified elsewhere.

The necessary data for a specific time period are the flows of products in monetary terms from each of the sectors as producers to each of the sectors as purchasers. A model with steel and automobile sectors, for example, requires

the dollar value of steel produced in the region that is sold to regional auto manufacturers and the dollar value of automobiles produced in the region sold to the region's steel industry. An industry's demand for inputs from other industries is related closely to its own output. For example, steel sales are related to automobile production.

In contrast, no explicit assumption is made regarding the relationship between the value of sales to buyers outside the industrial sectors (such as households, government and buyers in other regions) and the quantity they produce. Thus, the sales of a specific industry are made to other producers within the region, which are called interindustry sales, and to external units, which is termed final demand. Assuming the economy consists of n sectors, the total output of sector i,  $x_i$ , can be written as:

(1) 
$$x_i = z_{i1} + z_{i2} + \ldots + z_{ii} + \ldots + z_{in} + y_{i'}$$

<sup>&</sup>lt;sup>1</sup>See U.S. Department of Commerce (1986) for additional details.

where the z terms represent the interindustry sales by sector i to each of the n sectors (including itself) and  $y_i$  is the final demand for sector i's output. There will be an equation similar to (1) for each sector of the regional economy.

The interindustry flows of goods are summarized in table 1A. The entries in each row are the sales of a sector, such as sector 1, to itself  $(z_{11})$  and to each other sector in the economy  $(z_{12} \ldots z_{1n})$ . These interindustry flows can also be considered from the perspective of any sector's purchases: each column in table 1A expresses one sector's purchases of the products of the various producing sectors in the region. For example, the first column in table 1A indicates sector 1's purchases from itself  $(z_{11})$  and from each of the other sectors  $(z_{21} \ldots z_{n1})$ .

A sector's purchases of inputs, however, are not restricted to purchases from itself and other sectors. For example, labor is an input that must be purchased. A purchase of labor is an example of what is termed value-added. In addition, a sector may purchase raw materials and other material inputs from other regions.

#### **Income and Product Accounts**

The transactions in table 1A are a subset of a complete set of income and product accounts for an economy. To illustrate a "complete" set, table 2A is a flow table for an economy with two industries. The z's are simply the interindustry sales between the two sectors. Final demand for the two sectors  $(Y_1 \text{ and } Y_2)$  consists of three "domestic" components and one "foreign" component. In the regional context, domestic and foreign refer to within and outside of the

Table 1A Input-Output Table of Interindustry Flows						
			Purchasi	ng sect	or	
Selling sector	1	2		i		n
1	Z <sub>11</sub>	Z <sub>12</sub>		Z <sub>1i</sub>		Z <sub>1n</sub>
2	Z <sub>21</sub>	Z <sub>22</sub>		Z <sub>2i</sub>		Z <sub>2n</sub>
i	Z <sub>i1</sub>	Z <sub>i2</sub>		Z <sub>ii</sub>		Z <sub>in</sub>

Regional economy. The domestic demand components are: C, consumer (household) purchases; I, purchases for private investment purchases; and G, purchases by government within the region. The foreign demand component is export sales (E) that include sales of finished goods to all consumers, investors and governments outside of the region as well as sales of intermediate goods to producers in other regions. Thus, final demand for industry 1's output can be expressed as  $Y_1 = C_1 + I_1 + G_1 + E_1$  and for industry 2's output as  $Y_2 = C_2 + I_2 + G_2 + E_2$ .

Zn2

The payments sector of table 2A identifies three categories of payments, two of which may be termed "value-added" payments. These payments equal the value of the sector's output

		Processing sectors		Final demand				Total output
		1	2		(	Y)		(X)
Processing sectors	1 2	Z <sub>11</sub> Z <sub>21</sub>	Z <sub>12</sub> Z <sub>22</sub>	C <sub>1</sub> C <sub>2</sub>		G <sub>1</sub> G <sub>2</sub>		X <sub>1</sub> X <sub>2</sub>
Payments sector		L <sub>1</sub> N <sub>1</sub> M <sub>1</sub>	L <sub>2</sub> N <sub>2</sub> M <sub>2</sub>	L <sub>c</sub> N <sub>c</sub> M <sub>c</sub>	L <sub>i</sub> N <sub>i</sub> M <sub>i</sub>	N <sub>G</sub>		L N M
Total outlays (X)		X <sub>1</sub>	X <sub>2</sub>	С	1	G	Е	x

minus the value of the inputs it purchases from other sectors. Value-added payments by the two processing sectors consist of payments for labor services ( $L_1$  and  $L_2$ ) and for all other value-added items, such as tax payments for government services, interest payments for capital services and rental payments for land services ( $N_1$  and  $N_2$ ). The final component of the payments sector is for purchases of imported inputs ( $M_1$  and  $M_2$ ). Regional imports include all purchases from beyond the region's borders.

The entries in the intersection of the value-added rows and the final demand columns of table 2A represent payments by final consumers for labor and other services. For example,  $L_C$  includes household payments for domestic help,  $L_G$  represents payments to government workers and  $N_C$  includes interest payments by households. Lastly, the entries in the intersection of the imports row and final demand columns represent purchases of imported items by households ( $M_C$ ), private businesses ( $M_I$ ) and government ( $M_G$ ).  $M_E$  represents imported items that are re-exported.

Summing the entries in the total output column yields the total gross output (X) of the economy

(2) 
$$X = X_1 + X_2 + L + N + M$$
.

Total gross output may also be found by summing across the total outlays row. Thus,

(3) 
$$X = X_1 + X_2 + C + I + G + E$$
.

Equating the two expressions for X, subtracting  $X_1$  and  $X_2$  from both sides and rearranging terms leaves

(4) 
$$L + N = C + I + G + (E-M)$$
.

The left-hand side, total factor payments, is gross regional income, while the right-hand side, the total spent on consumption and investment goods, total government purchases and net exports, is gross regional product. Thus, the equation simply expresses the equality of regional income and output.

$$X_j = a_{1j} X_j + a_{2j} X_j ... + a_{jj} X_j + ... + a_{nj} X_j + W_j,$$

where Wi is the sum of Li, Ni and Mi. The element, ani,

#### An Input-Output Model

The preceding income and product accounts can be transformed into an input-output model rather easily. Given the flow of inputs from sector i to sector j  $(z_{ij})$  and the total output of j  $(X_j)$ , the ratio of the value of the input to the output (that is,  $z_{ij}$  / $X_j$ ) is known as an input coefficient or an input-output coefficient and is denoted by  $a_{ij}$ . Input coefficients can be calculated for each interindustry flow and are viewed as measuring a fixed relationship between each of a regional sector's inputs and its total output. For example, if one-quarter of sector 2's output was purchased from sector 1, then  $a_{12} = 0.25$ . This relationship is calculated from the flows of inputs and output for a given period, often one year.

The fixed nature of the relationship implies that the ratio of the value of each input to each output remains unchanged irrespective of the level of output, reflecting an assumption of constant returns to scale in production. This means that an identical percentage change in all inputs will cause an identical change in output. For instance, doubling all inputs will cause a doubling of output.

The fixed input coefficients allow the total output of sector i, X<sub>i</sub>, to be written as:

(5) 
$$X_i = a_{i1}X_1 + a_{i2}X_2 + \ldots + a_{ii}X_i + \ldots + a_{in}X_n + Y_i$$

The input coefficients in the preceding equation state how much of sector i's output is needed per dollar of output in each of the other sectors. Multiplying the input coefficients by the output of these sectors provides the dollar value of sector i's output that is directed to these sectors. As before,  $Y_i$  states how much of regional sector i's output is used to satisfy final demand.<sup>2</sup>

For each sector of the economy, there will be an equation similar to (5). These equations can be solved to yield an answer to the following question: if the final demands for each of the n sectors were known, how much output from each of the sectors would be necessary to supply them? Thus, values for the gross output of each

does not necessarily represent sector's j's total purchases of input n, but only that portion purchased within the region. Production in j may also require importing inputs from other regions or nations, which are included in the payments sector.

<sup>&</sup>lt;sup>2</sup>A sector's output, say j, can also be expressed as the sum of each sector's purchases from other regional sectors and payments for value-added components and imported inputs:

sector can be determined given the input coefficients and the final demands.

#### Open and Closed Models

The described input-output model depends on the existence of a sector distinct from the interrelated processing sectors. These final demands consist of consumption purchases by households, sales to government and gross private investment within the region as well as sales to the rest of the world. The separation of households from the processing sectors is arbitrary; households as consumers and workers are enmeshed with the industrial sectors. Households earn incomes by providing their labor services and, as consumers, spend their income in a predictable fashion. Although households tend to purchase goods for final consumption, the amount of their purchases is related directly to their income, which depends on the outputs of the sectors.

In some input-output models, the household sector is shifted from final demand to the interrelated processing sectors. Such a model is known as a closed model with respect to households, while an open model keeps the household sector as part of final demand. In the context of income and product accounts, household consumption ( $C_1$  and  $C_2$  in table 2A) is shifted to the processing sectors in a closed model.

A closed model requires a separate row and column in table 1A for the household sector. The row would show how the "output" of the household sector, its labor services, is used as an input by the various sectors and the column would show how the household sector's consumption purchases are distributed among the sectors.

Household input coefficients are calculated as other input coefficients are. The household sector's row entries are the value of labor services sold to each sector for a given period divided by the value of the sector's total output for the same period. The elements of the household purchases column are the value of household purchases from each sector for a given period divided by the total output of the household sector.

#### Keith M. Carlson

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# The Future of Social Security: An Update

SINCE ITS CREATION in 1935, the Social Security system has become an integral part of the U.S. economic and social structure. With the composition of the population currently undergoing substantial change, however, skepticism abounds among U.S. citizens (particularly those between 25 and 45 years old) as to whether the Social Security system, as currently structured, can survive into the middle of the 21st century.<sup>1</sup>

In a recent issue of the *Wall Street Journal*, a 1990 Gallup poll indicated that "47% of Americans do not believe that Social Security will be able to pay them *any* benefit when they retire." These results suggest that the public's thinking about the viability of Social Security has changed little over the last 10 years. Myers (1985) reports similar results for polls taken in 1980. This skepticism continues despite the 1983 enactment of the recommendations of President Reagan's National Commission on Social Security Reform. This legislation was designed to put the Social

Security system on sound financial footing until 2056.4

This article provides an update on the financial status of the Social Security system and addresses concern about its long-run financial viability. The fundamental economic and demographic assumptions that underlie estimates of the system's future costs and income are examined to determine the system's financial viability. This evaluation provides the basis for determining whether the skepticism about Social Security's future is justified.

## THE 1983 SOCIAL SECURITY AMENDMENTS

The Social Security Act has been amended many times over the years, with the amendments changing coverage, insured status, benefit formulas and financing provisions. One of the most sweeping changes took place in 1984,

<sup>&</sup>lt;sup>1</sup>The challenges of an aging population are common in most industrial countries. See Halter and Hemming (1987).

<sup>&</sup>lt;sup>2</sup>Rasmussen (1990).

<sup>&</sup>lt;sup>3</sup>These were the 1983 amendments to the Social Security Act, which will be discussed further below.

<sup>&</sup>lt;sup>4</sup>In February 1984, President Reagan, in his annual budget message, declared: "The Social Security system has been rescued from the threat of insolvency raised by rampant

inflation, excessive liberalizations, and lagging growth of its tax base." Office of Management and Budget (1984).

<sup>&</sup>lt;sup>5</sup>Social Security is defined as Old-Age and Survivors Insurance (OASI), Disability Insurance (DI) and Hospital Insurance (HI), also referred to as Medicare, Part A. OASI + DI = OASDI and OASDI + HI = OASDHI. Supplementary Medical Insurance (SMI), also referred to as Medicare, Part B, is discussed only briefly since it is a voluntary program and not financed with the payroll tax.

following the passage of the 1983 amendments, which were based on the Report of the National Commission on Social Security Reform.<sup>6</sup>

The 1983 amendments produced the following changes:<sup>7</sup>

- Mandatory coverage was extended to all newly hired federal employees and employees of nonprofit organizations.
- (2) The cost-of-living adjustment (COLA) was delayed for six months. Also, a provision was introduced that based the COLA on the lower of wage increases or consumer prices when the trust fund is low.
- (3) A phased-in increase in the normal retirement age was enacted.
- (4) Scheduled increases in the Old-Age and Survivors and Disability Insurance (OASDI) payroll tax rate were accelerated.
- (5) Payroll taxes were raised for self-employed individuals with a partial offset in the form of a tax credit.
- (6) A portion of OASDI benefits became subject to income taxes with such taxes returned to the trust fund.

These amendments increased the balance in the OASDI trust funds almost immediately. Despite the magnitude of this legislation and its surplus-generating effects, however, the public apparently remains unconvinced about Social Security's long-run viability.<sup>8</sup>

## THE 1990 SOCIAL SECURITY REPORTS

Each of the Social Security trust funds is governed by a Board of Trustees, which is required to submit an annual report to Congress. Though the Old-Age and Survivors Insurance (OASI) Trust Fund and the Disability Insurance (DI) Fund are separate, one report covers both funds; a separate report is prepared for the Hospital

Insurance (HI) Fund. These documents report the status of the trust funds only; they make no recommendations for legislation.<sup>9</sup>

#### **Terms and Procedures**

These reports summarize recent developments in the trust funds' financial operations and, more importantly, assess the funds' actuarial status. This assessment is made by developing various assumptions about economic growth, productivity, inflation, unemployment and demographics (fertility, mortality and immigration), then examining the program's projected income and outgo. These projections provide the basis for assessing both the short-run and long-run status of the funds.

The short-run assessment focuses on the adequacy of reserves to pay benefits for the next five years. The principal measure used to assess the short-run "liquidity" of the fund is the contingency fund ratio. This ratio, which is defined as the amount in the trust fund at the beginning of the year divided by anticipated expenditures in that year, reflects the accumulation of interest earnings in the fund.

The long-run assessment focuses on income and outgo for a 75-year projection period. Several measures are used to assess the system's long-run "solvency." The most commonly used measure is the comparison of average annual income (excluding interest earnings on trust funds) with average annual cost, both expressed as a percent of "taxable payroll." The income rate is the combined payroll contribution rate as scheduled in the law plus taxation of benefits, expressed as a percentage of taxable payroll. The cost rate is the annual outgo (benefits plus administrative expense) expressed as a percentage of taxable payroll. The difference between these rates can be averaged over a period of years to determine the fund's "actuarial balance."

Another measure of actuarial balance over the long run is the present value of future income (excluding interest), outgo and taxable payroll,

<sup>&</sup>lt;sup>6</sup>This is commonly referred to as "the Greenspan Commission," after Alan Greenspan, its chairman. The commission was created by President Reagan in 1981 as a response to the continuing deterioration of the OASI trust fund. Its report was published in January 1983.

<sup>&</sup>lt;sup>7</sup>For further discussion, see Svahn and Ross (1983) and Myers (1985), pp. 282-99.

<sup>&</sup>lt;sup>8</sup>For a critical review of the Commission's report, see Robertson (1985).

<sup>&</sup>lt;sup>9</sup>This is left to the Advisory Council on Social Security, which is appointed every four years.

<sup>10&</sup>quot;Taxable payroll" is defined as the earnings on which employees, employers and self-employed persons make contributions to the OASDI program.

Table 1

Demographic Assumptions (Ultimate Values): 1990 Social Security Reports

Demographic measure	1989	Optimistic assumption Alternative I	Intermediate assumption Alternatives II-A & II-B	Pessimistic assumption Alternative III	
Fertility rate (lifetime births per 1,000 women)	1,930	2,200	1,900	1,600	
Mortality rate (life expectancy at birth in years)	expectancy at birth		80.6	84.4	
nnual net immigration 585 (thousands of persons)		750	600	450	

which includes the trust fund balance at the beginning of the period. This measure was introduced in 1988. For the OASDI funds, a long-range (75-year) income rate between 95 percent and 105 percent of the long-range cost rate traditionally indicates the program is adequately financed.

A series of projected contingency fund ratios also can be calculated over a long-run period. If a trust fund becomes depleted during the projection period, the year in which this occurs provides a summary measure of the financial condition of the fund.

#### **Assumptions**

Projections of Social Security's future income and outgo reflect a host of assumptions about economic conditions and demographic factors. <sup>11</sup> The factors influencing income are those that affect the size and composition of the working population and the general level of earnings. The factors influencing outgo are those that affect the size and composition of the beneficiary population and the general level of benefits.

In its annual Social Security reports, the Board of Trustees presents four sets of economic and demographic assumptions. Alternative I presents the most optimistic outlook, including the fastest economic growth and the lowest inflation. Alternatives II-A and II-B are intermediate ones that share the same demographic assumptions; II-A has faster growth and lower inflation than II-B. Alternative III provides the most pessimistic outlook, with the slowest growth, highest infla-

tion and least favorable demographic assumptions. The Board of Trustees typically focuses on alternative II-B.

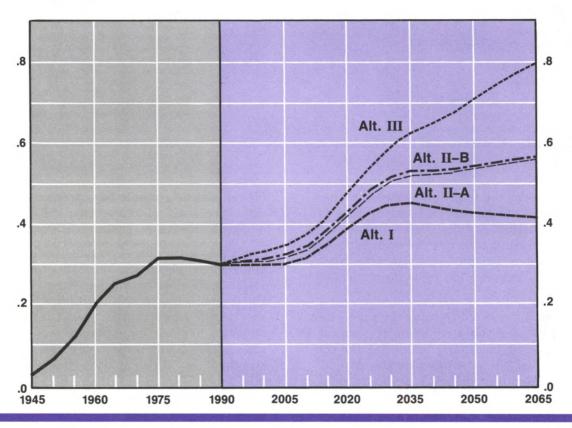
Whether a variable is classified as part of the optimistic or pessimistic alternatives depends solely on its effect on the Social Security system. For example, the highest death rate (and, thus, shortest life expectancy) appears in the optimistic alternative. For the economic factors, the optimistic case assumes the fastest real growth and the lowest inflation rate. These projections "are intended to be indicators of the trend and range of future income and outgo, under a variety of plausible economic and demographic conditions." The analysis is intended to present a range of possible outcomes, rather than make an exact prediction.

Demographic Assumptions—The key demographic factors used to derive these projections are the fertility rate, the death rate and net immigration. Table 1 summarizes the assumptions contained in the 1990 Social Security reports. The optimistic alternative contains a combination of demographic factors, each of which has the effect of raising the proportion of workers to beneficiaries. The pessimistic alternative reflects just the opposite: a combination of factors, each of which lowers the worker-beneficiary ratio. The demographic assumptions for the intermediate alternatives are about halfway between these two. Each variable reaches its ultimate value at different times: the fertility rate reaches its ultimate value in 2015; life expectancy changes slowly reaching this value in

<sup>&</sup>lt;sup>11</sup>See Myers (1985), pp. 387-90.

<sup>12</sup>Board of Trustees, Federal Old-Age (1990), p. 33.

Figure 1
Ratio of OASDI Beneficiaries to Covered
Employment



2065; the immigration numbers presumably apply beginning in 1995.<sup>13</sup>

Figure 1 summarizes the beneficiary-worker ratio implied by these demographic assumptions. Alternative III, the pessimistic projection, shows a generally rapid rise in the ratio throughout the projection period. The intermediate projections indicate that the problems inherent in a rising ratio will be most serious from about 2010 to 2030. This corresponds "roughly" to the period dated 65 years from the beginning and end of the baby boom—1946-64. The optimistic projection shows an increase from 2010 to 2030, then a slight decline from 2030 to 2065.

**Economic Assumptions**—The key economic assumptions are summarized in table 2. The optimistic alternative includes the most favorable of each economic assumption, in combination

with the most favorable of demographic assumptions. In other words, it includes the fastest GNP growth, the lowest inflation, the lowest unemployment rate and the fastest growth in real wages. The pessimistic alternative reflects recurring business cycles and generally weak growth with high inflation. Of the two intermediate projections, A reflects more favorable economic assumptions than B.

The key economic assumption is the relationship between wages and prices, that is, the growth in real wages. This measure captures two important facets of the Social Security system: the nominal wage is the basis for estimating Social Security income, and, with benefits tied to movements in the price level, projected consumer price index (CPI) movements provide an indication of changes in Social Security costs.

period. This constant rate is termed the ultimate value (or level).

<sup>13</sup>The Social Security reports assume that most variables level off at a constant rate after an initial adjustment

Table 2

Economic Assumptions (Ultimate Values): 1990 Social Security Reports

1948-		Optimistic	Intern	nediate	Pessimistic	
Indicator 1989	1989	Alternative I	Alternative II-A	Alternative II-B	Alternative III	
Real GNP growth rate	3.3%	2.7%	1.8%	1.5%	0.5%	
Average wage growth rate	5.3	4.2	4.7	5.3	5.8	
CPI-W rate of change	4.0	2.0	3.0	4.0	5.0	
Real wage growth rate	1.2	2.2	1.7	1.3	0.8	
Unemployment rate	5.6	5.0	5.5	6.0	7.0	
Labor force growth rate	1.6	0.5	0.0	0.0	-0.6	

The Board of Trustees, Federal Old-Age (1990) report indicates that Social Security income, expressed as a percent of taxable payroll, is little affected by different assumptions about the growth of real wages. With the maximum amount of earnings that are taxable tied automatically to the increase in average wages, the income rate is essentially the OASDI contribution rate scheduled in the law.<sup>14</sup>

Social Security outgo relative to taxable payroll, however, is sensitive to the growth of real wages. In particular, an increase in the real wage decreases benefit costs as a percent of taxable payroll. For example, if wages increase faster, while inflation is unchanged, taxable payroll will increase with little change in benefit costs. 15

### Status of the Trust Funds

**OASDI**—The status of the OASI fund and the DI funds are presented both separately and together in the Board of Trustees, Federal Old-Age report. For purposes of this summary, only the combined status is presented.

Figure 2 shows the contingency fund ratio for the OASDI funds, both historically and for the four alternative sets of projections through 2065. The Board is required to assess the short-run status (the next five years) of the trust funds. It is clear from the figure that the funds are liquid for this horizon. Even under the most pessimistic alternative, the funds are not depleted until 2023. For the immediate future (until almost 2010), the number of OASDI beneficiaries relative to the number of covered workers does not rise sharply (see figure 1). Thus, the current schedule for the payroll tax indicates rising trust fund balances until the 2010s, regardless of which set of projections is used.

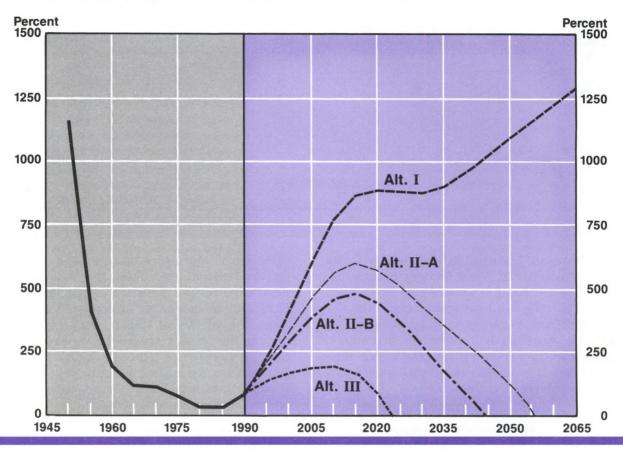
For the long run, however, conclusions about the viability of the funds vary, depending on the set of assumptions. According to figure 2, without further legislation, the funds will be depleted somewhere between 2023 and 2056, unless the optimistic economic-demographic projections occur.

Figure 3 shows another method that the Board uses to determine the actuarial status of the trust funds. It is a chart of the income rates and cost rates (both expressed as a percentage of taxable payroll). The income rates exclude interest income on assets and are essentially the same for each set of assumptions, varying only to the extent that taxation of benefits varies with the assumptions. The projected departure from past patterns shown in figure 3 is quite dramatic. The variation in the annual surplus or deficit relative to taxable payroll is very small

<sup>14</sup>There is a slight effect reflecting credit for income taxes attributable to partial taxation of benefits. Because the income thresholds are constant according to present law, the credits to the trust funds will be a smaller proportion of taxable payroll for the more optimistic projections.

<sup>&</sup>lt;sup>15</sup>With the denominator increasing faster than the numerator, the ratio falls. See Board of Trustees, Federal Old-Age (1990), p. 106.

Figure 2
Contingency Fund Ratios for OASDI



from 1940 to 1985 compared with what is scheduled to come. The years in which the programs start running a deficit occur somewhat earlier than when the trust funds are depleted (see figure 2).

HI—The Board of Trustees, Federal Hospital (1990) report indicates that, even though it is currently in surplus and will continue so for a few years, the fund will be depleted by about 2005 under the intermediate assumptions. <sup>16</sup> With the pessimistic set of assumptions, the report indicates the fund will be depleted by 1999; under the optimistic assumptions it will be depleted by 2018.

Figure 4 shows the long-term projections of income and outgo for the hospital insurance fund until 2060. This figure shows that the surpluses currently being experienced are likely

to be short-lived. The switch to growing deficits will occur even before the number of beneficiaries for the OASDI system increases, primarily reflecting a much faster rise in the cost of hospital care per beneficiary than the rise in taxable earnings.

The Board of Trustees of the HI fund reports that

[n]ot only are the anticipated reserves and financing of the HI program inadequate to offset this demographic change [the decline in covered workers per HI enrollee], but under all but the most optimistic assumptions, the trust fund is projected to become exhausted even before the major demographic shift begins to occur.

The Board, therefore, urges that the Congress take early remedial measures to bring future HI program costs and financing into balance, and to

Medicare, Part A, to \$125,000. In addition, the deductible for hospital insurance was raised.

¹6Board of Trustees, Federal Hospital (1990). This report was prepared before legislation was recently enacted increasing the maximum for earnings taxable under

Figure 3

OASDI Income and Outgo Relative to Taxable Payroll

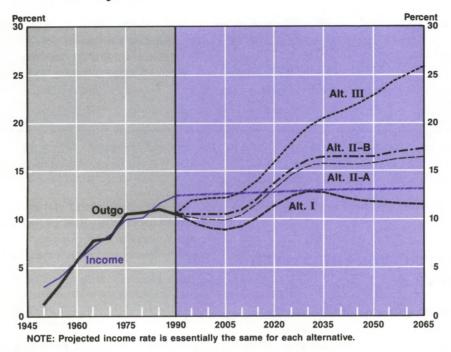


Figure 4
Hospital Insurance Income and Outgo
Relative to Taxable Payroll

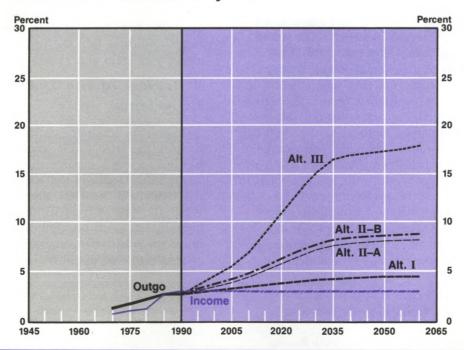


Table 3

Year in Which OASDI Trust Funds Are Estimated to be Depleted

	1990 Report			19	983 Repo	ort	1982 Report		
Alternative	OASI	DI	Total	OASI	DI	Total	OASI	DI	Total
1	NA	NA	NA	NA	NA	NA	1983	NA	1983
II-A	2060	2025	2056	NA	NA	NA	1983	NA	1983
II-B	2046	2020	2043	NA	NA	NA	1983	NA	1983
III	2027	1998	2023	2028	2021	2027	1983	NA	1983

<sup>&</sup>quot;NA" indicates that the fund is not estimated to be depleted during the projection period.

maintain an adequate trust fund against contingencies. 17

### 1990 REPORT COMPARED WITH 1983 REPORT

The 1983 amendments supposedly put the Social Security system on firm financial ground. As noted above, the amendments affected mainly the OASDI program rather than Medicare. Has there been any erosion in this footing since 1983? If so, why?

### Status of the Trust Funds

**OASDI**—Table 3 compares the most recent report of the status of the OASDI funds from the Board of Trustees, Federal Old-Age (1990) with those from 1982 and 1983. The 1983 amendments were a response to the short-run crisis facing the system at that time. A comparison of the 1982 and 1983 reports indicates that the amendments appeared to put the system on sound financial footing. As the 1990 column in table 3 indicates, however, substantial deterioration has occurred since 1983. On a combined basis, the trust funds are projected to be depleted during the projection period for all alternatives except the most optimistic, Alternative I. The seriousness of this projection change is difficult to discern; the direction of movement, however, is cause for concern.

**HI**—Comparing the status of the HI trust fund in 1990 with that of 1983 indicates that not all surprises are bad news. In its 1983 report, the Board concluded that

[t]ax rates currently specified in the law . . . are sufficient, along with interest earnings and assets in the fund, to support program expenditures only over the next six to seven years. 18

More specifically, the fund was projected to be depleted by no later than 1996 with the optimistic projections, by either 1990 or 1991 with the intermediate assumptions and as early as 1988 with the pessimistic projections. Clearly, these projections were wrong, chiefly because of errors in projecting the costs of the program. The schedule of tax rates in effect in 1983 was not changed as of 1990. The 1990 report indicates that the fund will be depleted as early as 1999 and as late as 2019.

# A Review of the 1983-89 Experience

Despite the gloomier outlook suggested in table 3, the trust funds' experience since 1983 has been more favorable than was projected in 1983. Table 4 shows the status of the funds in the 1980s and the 1983 alternative projections. While the OASDI trust fund balance moved quite closely with the optimistic set of projections, the health insurance fund balance far exceeded the most optimistic projections. These

<sup>&</sup>lt;sup>17</sup>Board of Trustees, Federal Hospital (1990), pp. 11-12.

<sup>&</sup>lt;sup>18</sup>Board of Trustees, Federal Hospital (1983), p. 49.

<sup>&</sup>lt;sup>19</sup>For a summary of the changes in the provisions in the administration of Medicare, Part A, see *Social Security Bulletin* (1989), pp. 54-55.

Table 4

Status of Social Security Trust Funds—1983-89: Actual vs. 
Projected (billions of dollars)

				Control of the latest and the latest			
	1983	1984	1985	1986	1987	1988	1989
OASDI FUND							
Actual	\$24.9	\$31.1	\$42.2	\$46.9	\$68.8	\$109.8	\$163.0
1983 projections							
Alternative I	27.9	30.2	39.6	46.2	67.4	111.4	170.2
Alternative II-A	27.6	28.8	35.6	39.5	45.3	76.8	117.6
Alternative II-B	27.5	27.6	32.5	36.1	39.7	53.9	82.4
Alternative III	26.2	21.9	24.2	32.2	27.2	51.1	80.0
HEALTH INSURANCE FUND							
Actual	12.9	15.7	20.5	40.0	53.7	69.6	85.6
1983 projections							
Alternative I	11.8	12.6	15.2	27.6	31.4	33.5	34.6
Alternative II-A	11.7	11.6	11.2	17.1	23.2	20.3	14.3
Alternative II-B	11.7	11.2	10.2	11.8	12.6	16.1	7.8
Alternative III	11.4	9.1	5.5	1.8	6.3	-	

outcomes suggest that there is much greater uncertainty connected with the health insurance trust funds because of the difficulty in forecasting costs of medical care. Estimates of the viability of the health insurance trust funds have tended to be too pessimistic.

To isolate whether the 1983-89 outcome for the trust funds was attributable to demographic or economic factors, tables 5 and 6 summarize the actual experience over this period and compares it with the 1983 projections. The demographic projections were quite accurate except for net immigration, which was higher than estimated. The economic experience was generally consistent with the more optimistic projections made in 1983. The actual growth of real wages was midway between alternatives I and II-A. Given the uncertainty that prevailed in 1983 about the strength and longevity of the expansion, it is not surprising that the experience during the long expansion matches more closely the optimistic projections.

# How Projections Have Changed from 1983 to 1990

The 1983 report received considerable attention because of the surpluses it projected, reflecting the 1983 amendments. For the Alternative II-B projections, the balance in the OASDI fund was projected to be nearly \$21 trillion by 2045.20 In the 1990 report, the largest balance for the II-B projection is now estimated at \$9.2 trillion in 2025. Figure 5 summarizes the II-B projections for both the 1983 and the 1990 reports. The source of the revisions requires further investigation, especially since the economic-demographic experience during the 1983-89 period was more favorable than projected.

Table 7 summarizes the demographic assumptions for the 1983 and 1990 reports.<sup>21</sup> The fertility rate generally has been revised downward, and projected life expectancy and net annual immigration have been revised upward. One way to summarize these demographic effects is

for the 1990 report. These are to be distinguished from the numbers in table 5 which were averages for the 1983-89 period from the 1983 report.

<sup>&</sup>lt;sup>20</sup>In 1983, the HI funds were projected for only 25 years and were projected to deplete quickly.

<sup>21</sup>The numbers in this table are values for the last year of the projection period—2060 for the 1983 report and 2065

Table 5

Accuracy of 1983 Demographic Assumptions for 1983-89 Period

Demographic measure	Actual	Alternative I	Alternative II-A	Alternative II-B	Alternative III
Fertility rate (lifetime births per 1,000 women)	1,860	1,940	1,880	1,880	1,800
Mortality rate (life expectancy at birth in years)	74.8	74.6	75.3	75.3	76.0
Net annual immigration (thousands of persons)	612	450	400	400	350

Table 6
Accuracy of 1983 Economic Assumptions for 1983-89 Period

Indicator	Actual	Alternative I	Alternative II-A	Alternative II-B	Alternative III
Real GNP growth rate	3.9%	4.6%	4.0%	3.2%	2.5%
Annual wage growth rate	5.3	5.2	5.0	5.3	6.1
CPI-W rate of change	3.4	3.0	3.3	4.3	5.9
Real wage growth rate	1.9	2.3	1.7	1.0	0.2
Unemployment rate—end of period, 1989	5.3	5.6	5.8	6.9	7.8

to look at the projected ratio of beneficiaries to employment. Figure 6 shows that there has been a substantial upward revision in the ratio of beneficiaries to covered employment since 1983. This indicates that OASDI expenditures are revised upward relative to income. Revision of this ratio reflects mainly a downward revision of projected employment.

Table 8 shows the economic assumptions for the 1983 and 1990 reports.<sup>22</sup> Despite an economic performance that conformed most closely with the 1983 optimistic projection, the economic projections were revised downward from 1983 to 1990. The nature of the revision can be summarized most simply by looking at the real GNP growth rate. These projections were revised downward by 0.3 percent to 0.5 percent. The ultimate unemployment rate was revised upward from 0.5 percent to 1 percent, and the growth rate of real wages was revised downward by 0.2 percent to 0.3 percent. While these differences might appear small, they have significantly different implications about the future of the Social Security system when carried through the 75-year projection period.

<sup>&</sup>lt;sup>22</sup>Like table 7, these ultimate values are for the last year of the projection period, and have to be distinguished from the numbers in table 6, which cover the 1983-89 period.

Figure 5
OASDI Trust Fund Balance
(Alternative II-B), 1990 vs. 1983

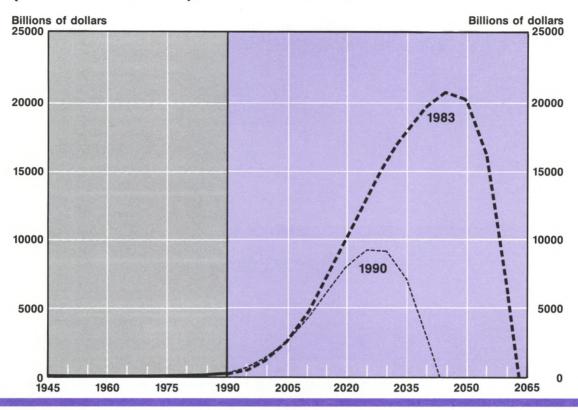


Table 7

Demographic Assumptions (Ultimate Values): 1990 vs. 1983

Social Security Reports

	Altern	ative I	Alternatives II-A and II-B Alte			ternative III	
Demographic measure	1990	1983	1990	1983	1990	1983	
Fertility rate (lifetime							
births per 1,000 women)	2,200	2,300	1,900	2,000	1,600	1,600	
Mortality rate (life expectancy at birth							
in years)	77.8	77.3	80.6	80.4	84.4	85.5	
Annual net immigration							
(thousands of persons)	750	450	600	400	450	350	

Figure 6
Ratio of OASDI Beneficiaries to Covered
Employment (Alternative II-B), 1990 vs. 1983

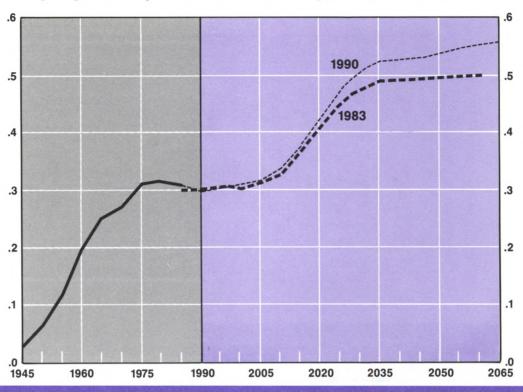


Table 8

Economic Assumptions (Ultimate Values): 1990 vs. 1983 Social Security Reports

	Altern	ative I	Alterna	Alternative II-A		Alternative II-B		Alternative III	
Indicator	1990	1983	1990	1983	1990	1983	1990	1983	
Real GNP growth rate	2.7%	3.2%	1.8%	2.3%	1.5%	1.9%	0.5%	0.8%	
Average wage growth rate	4.2	4.5	4.7	5.0	5.3	5.5	5.8	6.0	
CPI-W rate of change	2.0	2.0	3.0	3.0	4.0	4.0	5.0	5.0	
Real wage growth rate	2.2	2.5	1.7	2.0	1.3	1.5	0.8	1.0	
Unemployment rate	5.0	4.0	5.5	5.0	6.0	5.5	7.0	6.5	
Labor force growth rate	0.5	0.6	0.0	0.1	0.0	0.1	-0.6	-0.6	

Figure 7
OASDI Trust Fund Balance
(Alternative II-B), 1990 vs. 1983

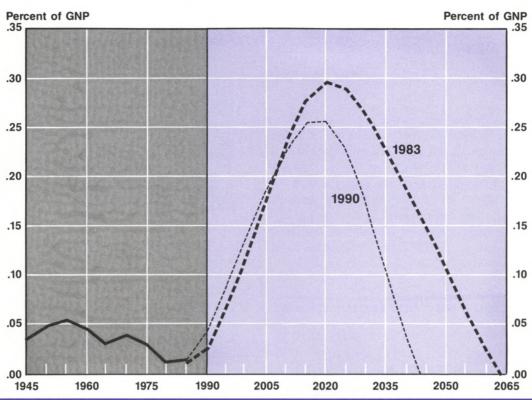


Figure 7 summarizes the trust fund balances for Alternative II-B from the two reports as a percent of GNP. Once again, the difference is striking. As a percent of GNP, the OASDI trust fund balances are now projected to be higher until 2015, before peaking at 25 percent, and then falling to zero by 2043, instead of 2063, as projected in 1983.

# THE CONSISTENCY OF PROJECTION ASSUMPTIONS

The point of the alternative projections is to "present a range of generally consistent sets of economic assumptions which have been designed to encompass most of the possibilities that might be encountered."<sup>23</sup> The intermediate alternatives are considered the most probable.

In fact, the official statement of opinion by the chief actuary refers only to the II-B alternative.<sup>24</sup>

While the projected values of the other economic variables are presumably consistent with the assumed rates of real growth, their consistency is not actually examined in these reports. In this section, we examine the projections' internal consistency by looking at trends in productivity and real wages, capital formation and immigration.

### **Productivity and Real Wages**

Productivity growth is a key determinant of the growth in real wages.<sup>25</sup> Table 9 summarizes U.S. productivity and real wage trends since 1959. From 1959 to 1988, real earnings grew at

<sup>&</sup>lt;sup>23</sup>Board of Trustees, Federal Old-Age (1990), p. 35.

<sup>&</sup>lt;sup>24</sup>lbid., p. 149.

<sup>25</sup>This relationship is discussed at length in U.S. Department of Health and Human Services (1988).

a 0.9 percent average annual rate, while productivity grew at a 1.6 percent rate. Most of the growth in earnings and productivity occurred before 1968. Since then, productivity has expanded at a 1.2 percent annual rate and real earnings growth has been essentially zero. While the 1970s were buffeted by supply-side shocks, the 1980s seem to have been characterized by stagnant productivity and earnings growth. Thus, in light of the experience of the last two decades, all four alternatives appear relatively optimistic.

Another way to evaluate the real wage assumption is to examine forecasts based on information contained in its own past movement. Figure 8 gives one such forecast for the 1990s.<sup>26</sup> The average rate of change for the real wage forecast for the 1989-99 period is -0.3 percent, which falls below the pessimistic alternative of 0.2 percent. Indeed, except for Alternative III, all the projections of real wage growth lie outside the forecasted 95 percent confidence bands.<sup>27</sup>

### **Production Function Implications**

Another way to evaluate these alternatives is to examine the growth in the capital stock that would be required to provide the projected GNP, given the assumption about employment growth.<sup>28</sup> Table 10 summarizes the results of these calculations and shows the 1948-89 period for comparison.

The optimistic Alternative I, with a growth rate of real GNP of 2.7 percent for the next 75 years, appears unattainable, given past U.S. experience. Based on relationships that prevailed over the last 40 years, the capital stock would have to grow 7 percent per year for the next 75 years. The fastest rate that the capital stock grew in any single year during the 1948-89 period was 5.4 percent in 1966.<sup>29</sup>

The II-A alternative implies a growth of per capita GNP of 1.62 percent per year. With employment projected to grow at a 0.18 percent rate, the capital stock would have to grow at a 4.66 percent annual rate over the 75-year projection period. From 1948 to 1989, the capital stock grew that fast only four times.

Table 9

Growth Rates of Productivity and Real Wages

	Productivity	Annual real wages
Historical (1959-88)	1.6%	0.9%
1959-68	2.5	2.6
1969-78	1.4	0.1
1979-88	0.9	0.0
Projected <sup>1</sup>		
Alternative I	2.2	2.2
Alternative II-A	1.9	1.7
Alternative II-B	1.7	1.3
Alternative III	1.4	0.8

<sup>&</sup>lt;sup>1</sup>Rates of change are ultimate values.

The annual reports suggest that Alternative II-B is the "most realistic" one. As noted earlier, it is based on the same demographic assumptions as II-A, but its economic assumptions are more pessimistic. The implied growth of the capital stock of 3.12 percent seems reasonable, given the historical record.

Finally, Alternative III presumes very slow growth in the capital stock. Such a growth rate is easily achievable, given the postwar experience.

Although the alternative projections in the Social Security annual reports appear to be consistent with past experience, there is little indication of the probability that could be assigned to their occurrence. A closer analysis of these alternatives indicates that, based on historical relationships between capital, labor and output, the optimistic alternative is quite unlikely to occur. Furthermore, the intermediate II-A projection also appears overly optimistic. As a result, to assess the future of Social Security, attention should be focused on the more probable alternatives, II-B and III.

### **Immigration**

Another way to examine the alternative assumptions is to consider to what extent immigration, rather than the capital stock, could be changed so as to achieve the different alterna-

<sup>&</sup>lt;sup>26</sup>This is derived by fitting a Box-Jenkins model (ARMA) to the percent change in the real wage (adjusted for trend).

<sup>&</sup>lt;sup>27</sup>If real wages continue to move as they have historically, there is a 95 percent probability that the rate of change of real wages will average between -1.6 percent and 0.9 per

cent during the 1989-99 period. These confidence bands are not shown on figure 8.

<sup>&</sup>lt;sup>28</sup>For a description of the methodology underlying these calculations, see Carlson (1990).

<sup>&</sup>lt;sup>29</sup>Ibid., p. 11.

Figure 8
Real Wage

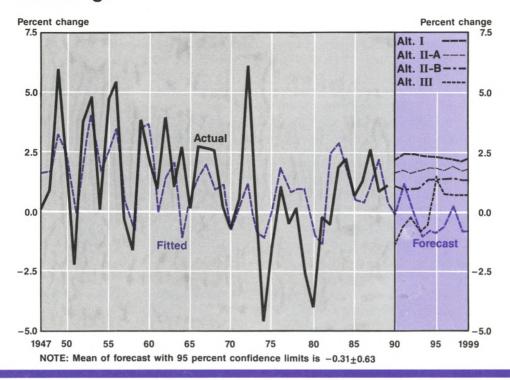


Table 10

Capital Stock Growth Required to Achieve Social Security's Projected Alternatives<sup>1</sup>

	Historical		Projected:	1990-2065	
	1948-89	1	II-A	II-B	III
Real GNP growth	3.27%	2.68%	1.93%	1.60%	0.81%
Minus: population growth	1.30	0.58	0.31	0.31	0.05
Equals: per capita GNP growth Or	1.97	2.10	1.62	1.29	0.76
Real GNP growth	3.27	2.68	1.93	1.60	0.81
Technical progress	0.81	0.81	0.81	0.81	0.81
Plus: employment contribution	1.36	0.40	0.14	0.13	-0.15
Employment growth	1.72	0.50	0.18	0.17	-0.19
Times: elasticity	(0.79)	(0.79)	(0.79)	(0.79)	(0.79)
Plus: capital stock contribution	0.76	1.47	0.98	0.66	0.15
Capital stock growth	3.60	7.02	4.66	3.12	0.71
Times: elasticity	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)
Minus: employment growth	1.72	0.50	0.18	0.17	-0.19
Plus: change in employment-population ratio	0.42	-0.08	-0.13	-0.14	-0.24
Employment growth	1.72	0.50	0.18	0.17	-0.19
Minus: population growth	1.30	0.58	0.31	0.31	0.05
Equals: per capita GNP growth	1.97	2.10	1.62	1.29	0.76

<sup>1</sup>Some components do not add to total because of rounding or production function error.

tives.<sup>30</sup> Once again, the feasibility of alternatives I and II-A appears doubtful.

Real GNP growth in Alternative II-A could be achieved with a growth rate of employment of 0.46 percent per year if the capital stock grew at its 1948-89 trend rate of 3.60 percent. Based on Census Bureau estimates of the impact of immigration on total population, this employment growth could be achieved with immigration of about one million persons per year from 1990 to 2065.<sup>31</sup> This is 200,000 more than the Census Bureau's high-immigration assumption.

Applying the same methodology to Alternative I indicates the limitations of immigration policy in boosting GNP growth. Achieving the 2.68 percent growth rate in real GNP would require immigration of almost four million persons per year over the 1990-2065 period.

#### CONCLUSION

The U.S. Social Security system was created in 1935 and has withstood 50-plus years of substantial social, economic, demographic and political change. Its evolution has not been smooth, however, and several financing crises have occurred.

Partly as a result of these periodic crises, the American public has become increasingly skeptical about Social Security's future. This skepticism reflects primarily concern about the financing of the retirement and medical needs of the baby-boom generation, those persons born between 1946 and 1964. Coupled with a low birth rate in the 1970s and 1980s, the evolving demographics indicate a sharp drop in the labor force relative to the total population beginning in about 2010-15.

One of the most important pieces of Social Security legislation was the amendments of 1983, which resulted from the recommendations of the National Commission on Social Security Reform. As a result of these amendments, the system, many thought, was placed on a sound financial footing. By accelerating the scheduled increase in payroll taxes and designing a plan of phasing in increases in retirement age, the amendments were intended to expand the system's trust funds sharply in the 1990s and early 2000s, thereby easing the burden on

future workers of providing for the needs of the retiring baby-boom generation.

This article focused on the 1990 annual reports of the Social Security trust funds to determine if the status of those funds has changed since 1983. It showed that the projected surpluses are now smaller than initially forecast, even though the 1983-89 economic experience was more favorable than had been expected.

The OASDI fund is not in short-run danger of being depleted; however, it is now projected to be depleted in 2043. Moreover, the hospital insurance fund is headed for depletion in 2003. These dates of depletion are based on the intermediate II-B projection, which the system's Board of Trustees considers the most probable. In addition, the system's own actuary concludes that the trust funds, based on its own guidelines, are not actuarially sound.

A review of the assumptions that underlie this revised outlook showed that the downward revisions chiefly reflected forecasts of slower real economic growth. The financial condition of the Social Security system is affected particularly by a downward revision in real wage and productivity growth.

Using production function relationships derived from the 1948-89 experience, an analysis of the projections presented in the 1990 Social Security report indicated that attention is properly focused on the II-B assumptions. Recent economic experience suggests, however, that the pessimistic projections cannot be ignored. An examination of immigration possibilities and a time series analysis of real wages also suggested that pessimistic alternatives II-B and III are more likely to occur than alternatives I and II-A, which are more optimistic. Recent productivity trends suggest that the near-term Social Security surpluses will be smaller than generally expected and the far-term deficits will be much larger.

The key role of economic assumptions in projecting income and outgo for the Social Security system suggests that the nation's macroeconomic policy is vital to its long-run viability, especially to the extent that such policy can influence long-term growth. The country's underlying demographics have set the stage for potential

<sup>&</sup>lt;sup>30</sup>For an extensive discussion of the economic effects of immigration, see Simon (1989).

<sup>31</sup>Bureau of the Census (1989).

problems in the 21st century; barring further major changes in immigration policy, there is little that can be done to alter these trends.<sup>32</sup>

The future of Social Security is clouded by the changing demographics. While Social Security trust funds are currently in surplus and balances are building, projections of future surpluses are being revised downward. Nonetheless, these surpluses are projected to continue for the next 20-25 years. Looking 75 years into the future, however, these surpluses, even if realized, would be relatively short-lived.

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<sup>32</sup>The recently enacted immigration law is expected to increase the immigration rate to almost 700,000 persons per year when fully effective in 1995.

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# Learning, Rational Expectations and Policy: A Summary of Recent Research

N THE THREE DECADES since the publication of the seminal work on rational expectations, a steely consensus has been forged in the economics profession regarding acceptable modeling procedures.1 Simply stated, the consensus is that economic actors do not persist in making foolish mistakes in forecasting over time. People are presumed to be able to both detect past patterns in their prediction errors and base their behavior on the "best possible" forecast of future economic conditions. In the classic phrase of Robert Lucas, the predictions of individuals must be "... free of systematic and easily correctable biases."2 The current wide acceptability of this notion is testament to the success of the rational expectations revolution.

Unfortunately, the consensus that in equilibrium systematic forecast errors should be eliminated has been insufficient to end the debate in macroeconomics over expectational assumptions. In particular, some current research examines the idea that *how* systematic forecast errors are

eliminated may have important implications for macroeconomic policy. Researchers who focus on this question are studying what is called "learning," because any method of expectations formation is known as a learning mechanism.<sup>3</sup>

This paper provides a synopsis of some of the recent research on learning. Three important points are emphasized within the context of the survey. The most salient point is the close relationship between learning issues and macroeconomic policy. In fact, the topic attracts attention precisely because of its perceived policy implications. The second point is more subtle: learning is implicitly an integral part of rational expectations models, and current research only makes this fact explicit. There is little prospect that one can avoid the study of learning by assuming rational expectations. The third point is that including learning in macroeconomic models is unlikely to either confirm or overturn completely the results from rational expectations macromodels. Instead, the concept of rational expectations equilibrium seems to provide the appropri-

for different readers. This terminology developed because rational expectations is justified by the notion that people eliminate systematic forecast errors over time, and the dynamic elimination of errors is a definition of learning.

<sup>&</sup>lt;sup>1</sup>The seminal work is Muth (1960, 1961).

<sup>&</sup>lt;sup>2</sup>Lucas (1977), p. 224.

<sup>&</sup>lt;sup>3</sup>The phrase "learning mechanism" is used in this paper in a broad way, even though it conjures up different images

ate benchmark for the study of learning in that, when systems converge under learning, they typically converge to stationary rational expectations equilibria.

The next section provides a non-technical introduction to learning and rational expectations. The subsequent section looks at the effects of introducing learning through a simple example attributable to Albert Marcet and Thomas Sargent. Some interpretation and discussion of the example is offered in the third section, along with a review of other attempts to introduce learning into macroeconomic models. The final section provides summary comments.

# THE LEARNING IMPLICIT IN RATIONAL EXPECTATIONS

### Why Study Learning?

Since decisions made today affecting productive behavior are presumed to be based, in part, on individual assessments of the future, macroeconomic theories and models generally have provided a role for expectations. Around 1970, however, researchers began to realize that the policy implications of their models were often quite sensitive to the choice of expectational assumptions. This failure of robustness has been increasingly apparent in the last 20 years and has drawn increased attention to the problem of how expectations are formed. This line of research constitutes what has been called the rational expectations revolution.

A capsule characterization of rational expectations contains the following themes: (1) In equilibrium (a steady state in dynamic terms), expectations are "correct" in the sense that individuals make no systematic forecast errors; (2) Individuals use all available information (as defined by the researcher) in forming forecasts; (3) Expectations vary with changes in government policy; and (4) Individuals know "the

model" and thus can predict as well as the economist manipulating the model.<sup>5</sup> These are the tenets of the theory and are often espoused by its advocates.

The first tenet, the heuristic notion that individuals eliminate systematic forecast errors, is the one most responsible for the rise of the rational expectations hypothesis. In a deterministic environment, this idea implies that, once learning is complete, people have perfect foresight. In a stochastic environment, it means that the remaining forecast errors are white noise.

Since the consensus is that the elimination of systematic forecast errors is a sensible postulate, all reasonable long-run equilibria must be rational expectations equilibria. Macroeconomists generally are interested in the effects of changes in policy parameters at these steady-state equilibrium points.<sup>6</sup> Given such equilibria, there is at least one reason why the explicit specification of learning could matter: in a model with multiple rational expectations equilibria, the learning mechanism may serve to select the actual outcome.<sup>7</sup> The next section contains an example of learning as a selection mechanism.

# Representing Learning Via Econometric Techniques

Macroeconomists generally have avoided specifying explicitly the optimal learning mechanism underlying rational expectations for a number of reasons. First and foremost, they considered rational expectations a shortcut in expectations modeling that made explicit specification unnecessary. Further, ascertaining the full implications of the rational expectations hypothesis turned out to be a difficult problem; presumably, these implications must be understood before the issue of learning can be investigated.

There was, however, at least one additional reason why learning was essentially ignored—explicit specifications of how agents form expec-

<sup>&</sup>lt;sup>4</sup>Sargent (1987) has documented some of this early research in rational expectations.

<sup>&</sup>lt;sup>5</sup>A more formal approach to defining rational expectations is pursued in the next section.

<sup>&</sup>lt;sup>6</sup>For a discussion of policy along the transition path, see Taylor (1975).

This is the same as saying that learning provides a stability theory for rational expectations equilibria. In the best case, given initial conditions and some parameter values, statements claiming that the dynamic evolution of the economy always leads to a particular stationary equilibrium

can be made. This is called global stability. Alternatively, one can define local stability, where a particular stationary equilibrium is stable only if the initial conditions are near that equilibrium.

<sup>&</sup>lt;sup>8</sup>Lucas' comment, '... take the rational expectations equilibrium ... as the model to be tested and view [learning] as ... an adjunct to the theory that serves to lend it plausibility," hints at this pragmatism. See Lucas (1987a), p. 231.

tations have often been attacked. In particular, one key criticism of the adaptive expectations formulation was that it allowed systematic errors to persist over time.<sup>9</sup>

The notion of adaptive expectations is essentially that people predict the future value of a variable via a geometric distributed lag (or Koyck lag) of its past values. One of Muth's results was that adaptive expectations is an optimal prediction method if the variable being forecast follows a random walk. 10 Muth's result makes it clear that, from the beginning, rational expectations theorists had econometric techniques in mind when thinking about ways to model optimal forecasting.

The idea of looking to econometrics to model learning obviously can be extended, since the Koyck lag is only one of many available econometric techniques. In principle, it should be possible to take advantage of the developments in econometric theory to shed light on the problem of how expectations are formed. Furthermore, if econometric methods are to be applied to solve the inference problem faced by individuals, a good place to start would be some variant of least squares regression. This is the logic behind the recent work on least squares learning, an example of which is presented below.

The application of econometric theory to the expectations problem was vigorously pursued recently because the mathematical knowledge to analyze the problem became available.<sup>11</sup> This technology was developed in the engineering literature and has been extended to economics by Albert Marcet and Thomas Sargent.<sup>12</sup> Economists can now determine, therefore, whether it makes any difference for policy implications if the learning mechanism behind rational expectations is explicitly specified. Viewed this way, the work on learning can be considered part of a continuing effort to understand the microfoundations of expectations formation.

The impetus for a detailed analysis of learning has come from both advances in research technology and one pressing problem: many rational expectations models are characterized by multiple equilibria. Moreover, these different equilibria can have different policy implications, as the next section illustrates. 13

### LEARNING AND THE UNPLEAS-ANT MONETARIST ARITHMETIC

This section examines a simple expository example of Marcet and Sargent;<sup>14</sup> it is a simplified version of the "unpleasant monetarist arithmetic" of Sargent and Neil Wallace.<sup>15</sup> The example is meant only to illustrate the types of issues that can arise; it is not intended as a definitive statement of the effects of including learning in economic models. As the next section will discuss, these effects are still uncertain.

The model's most important feature is that there are two steady states (high inflation and low inflation) with differing policy implications. At the low-inflation steady state, a permanent increase in the government deficit implies a permanent increase in the stationary inflation rate. However, increasing the government deficit leads to lower steady-state inflation if the economy is in the high-inflation stationary equilibrium. Because the policy implications of the model hinge on the choice, it is desirable to name one of the stationary states as the more likely outcome. The policy conclusions turn out to be different under learning as opposed to perfect foresight, even though systematic forecast errors are eventually eliminated under learning and the only possible limit points of the system are the two perfect foresight stationary states. This occurs because the stability properties of the two steady states are altered under learning as opposed to perfect foresight.

In Marcet and Sargent's model, fiat currency is the only money. The model consists of two

<sup>&</sup>lt;sup>9</sup>See Cagan (1956) and Nerlove (1958) for expositions of the adaptive expectations hypothesis.

<sup>10</sup>See Muth (1960).

<sup>11</sup>Formally, the problem is one of stability in a recursive stochastic environment. This means that the economist wants to take uncertainty, time and feedback (beliefs affect outcomes and outcomes affect beliefs) into account in the same model to find out the conditions under which convergence will occur.

<sup>12</sup>See Marcet and Sargent (1988, 1989a,b,c).

<sup>13</sup>The selection of an actual outcome among these potential outcomes is the subject of a spirited debate in the

macroeconomics literature which is not considered in detail here. A number of authors have suggested methods of choosing a unique equilibrium through deeper theorizing, that is, by applying certain restrictions to the model to eliminate unsavory equilibria. See, for instance, McCallum (1983), Taylor (1977) and Evans (1985, 1986). Some recent evaluations of these selection criteria suggest they are less than satisfactory. See, for instance, Pesaran (1988) and Boyd and Dotsey (1990).

<sup>&</sup>lt;sup>14</sup>See Marcet and Sargent (1989b).

<sup>&</sup>lt;sup>15</sup>See Sargent and Wallace (1981, 1987).

equations relating the price level to the stock of currency. Let  $p_t$  be the price level and  $c_t$  be the stock of currency per capita at time t. The model is given by

$$(1) p_{t} = \lambda F_{t} p_{t+1} + \gamma c_{t}$$

(2) 
$$c_t = c_{t-1} + \delta p_t$$

(3) 
$$F_t p_{t+1} = \beta_t p_t$$

where  $0 < \lambda < 1$ ;  $\gamma, \delta > 0$ ;  $p_t, c_t > 0$ ; and  $c_0 > 0$ is given. The term  $F_{t}p_{t+1}$  is the forecast at time t of the price at time t+1, where  $\beta$ , is the expected (gross) inflation rate at time t. Rearranging equation 1 with c, on the left-hand side yields a linear money demand equation. Equation 2 can be interpreted as a government budget constraint, where the government chooses c, to maintain a fixed real deficit  $\delta$ . The model has some microfoundations, since it can be derived in an overlapping generations framework with utility-maximizing individuals. The model generates time paths for  $c_t$  and  $p_t$  given an additional assumption on how expectations are formed. The remainder of the section will develop and compare the results under two alternative assumptions: rational expectations (or perfect foresight) and learning as described by a least squares autoregression.

### **Perfect Foresight Dynamics**

First, close the model by assuming individuals have rational expectations, which in this case is equivalent to perfect foresight because the model is deterministic. Then

(4) 
$$\beta_{t} = \frac{p_{t+1}}{p_{t}}$$
.

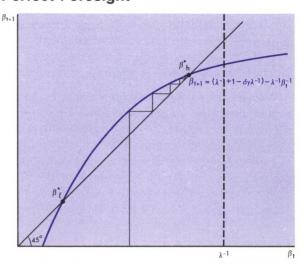
Equations 1-4 can be rearranged to yield:16

(5) 
$$\beta_{t+1} = [\lambda^{-1} + 1 - \delta \gamma \lambda^{-1}] - \lambda^{-1} \beta_t^{-1}$$
.

A rational expectations equilibrium is a sequence  $\{\beta_t\}_{t=0}^{\infty}$  that satisfies (5), and it determines the equilibrium sequences for  $c_t$  and  $p_t$ . The difference equation 5 has two roots,  $\beta_\ell^*$  and  $\beta_h^*$ , where  $1 < \beta_\ell^* < \beta_h^* < \lambda^{-1}$ ; these roots are the stationary states of the model. The real government deficit  $\delta$  is a shift parameter in equation 5.

The differing policy implications at the two steady states are illustrated in figure 1, which is borrowed from Marcet and Sargent's paper. A

Figure 1
Perfect Foresight



permanent increase in the government deficit  $\delta$  shifts the entire curve down in the figure. At the low-inflation steady state,  $\beta_\ell^*$ , this parameter change raises the stationary inflation rate. But at the high-inflation steady state,  $\beta_h^*$ , a permanent increase in  $\delta$  lowers the stationary inflation rate. In the first case, the comparative statics are "classical," while in the second they are "perverse." Any policy advice about inflation based on this model differs depending on which asymptotic outcome is considered more likely to be observed.

The high-inflation stationary state,  $\beta_{\rm h}^*$ , is the attractor for all initial values  $\beta_{\rm 0}$  between  $\beta_{\ell}^*$  and  $\lambda^{-1}.^{17}$  The low-inflation stationary state is attainable only if  $\beta_{\rm 0}=\beta_{\ell}^*$ . If the initial conditions are between zero and  $\beta_{\ell}^*$  no equilibrium sequence exists. Altogether, there are many possible equilibrium sequences in the model, and they can be indexed by  $\beta_{\rm 0}$ . Of these, all but one converge to the high-inflation stationary state  $\beta_{\rm h}^*$ , where the comparative statics are perverse.

In related work, Sargent and Wallace have used similar arguments to claim that monetarist models can yield "unpleasant" results. Under rational expectations, the "bad" stationary state with the perverse comparative statics is the

<sup>16</sup>See the appendix for the detailed derivation of this and subsequent statements in this section.

<sup>&</sup>lt;sup>17</sup>The appendix also describes how to determine stability properties in figures 1 and 2. Only initial values between zero and  $\lambda^{-1}$  are feasible.

eventual outcome for virtually all initial conditions, provided an equilibrium exists.

Perfect foresight is a strong assumption, but it can be made palatable by arguing that people eliminate systematic errors in their forecasts over time. Therefore, perfect foresight may provide a good approximation once the steady state has been attained and learning is complete.

Marcet and Sargent have developed techniques to analyze this argument in detail. Operationally speaking, people can be viewed as using some type of statistical technique to infer a future price from available data. One widely known technique is ordinary least squares (OLS). The next portion of the paper analyzes the model using the assumption of least squares learning.

### Least Squares Learning Dynamics

To develop results analogous to the perfect foresight case under least squares learning, Marcet and Sargent replace equation 4 with

(4') 
$$\beta_t = \left[\sum_{s=1}^{t-1} p_{s-1}^2\right]^{-1} \left[\sum_{s=1}^{t-1} p_s p_{s-1}\right].$$

People form their forecasts of future inflation by calculating  $\beta_t$ , which is found via a first order autoregression using available data through time t-1. The difference equation that describes the evolution of  $\beta_t$  is in this case given by:

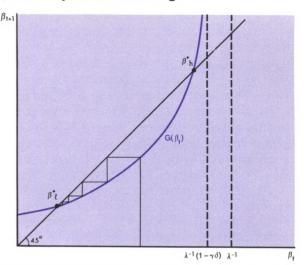
(5') 
$$\beta_{t} = \beta_{t-1} + \frac{p_{t-2}^{2}}{\sum_{t=1}^{t-1} p_{t-1}^{2}} \left[ \frac{(1-\lambda \beta_{t-2})}{(1-\lambda \beta_{t-1})} G(\beta_{t-1}) - \beta_{t-1} \right],$$

where

(5") 
$$G(\beta_{t-1}) = \frac{1 - \lambda \beta_{t-1}}{1 - \lambda \beta_{t-1} - \gamma \delta}$$
.

While equation 5' is quite complicated, it can be interpreted without too much difficulty. Near a stationary equilibrium, the term  $(1-\lambda\beta_{t-2})/(1-\lambda\beta_{t-1})$  is close to unity, while the term multiplying the brackets is always between zero and one. Therefore, near a steady state, equation 5' states that  $\beta_t$  is approximately a convex combination of  $\beta_{t-1}$  and  $G(\beta_{t-1})$ . That is, near a stationary equilibrium, the projected gross inflation rate is a weighted average of last period's projected gross inflation rate and a certain function

Figure 2 **Least Squares Learning** 



 $G(\beta_{t-1})$ . Alternatively, one can view (5') as last period's prediction "updated" by a certain small amount.

The possible steady states under learning,  $\beta_\ell^*$  and  $\beta_h^*$ , are the same steady states possible under perfect foresight. This captures the notion that systematic forecast errors are eliminated over time and suggests that stationary rational expectations equilibria provide a good benchmark for the study of learning. Marcet and Sargent claim, in fact, that when least squares learning mechanisms converge, they always converge to rational expectations equilibria. 18

Now that a method of expectations formation has been added to the model, however, some aspects have changed. Marcet and Sargent have completed an analysis of the complicated difference equation given by (5'). They show that equation 5' describing the dynamics under least squares learning is closely related to the following simpler equation:

(6) 
$$\beta_t = G(\beta_{t-1})$$
.

For the purposes of this exposition, it will suffice to analyze (6).<sup>19</sup> The graph that Marcet and Sargent use to describe the dynamics of (6) is given in figure 2.

<sup>&</sup>lt;sup>18</sup>See Marcet and Sargent (1989a,b,c).

<sup>&</sup>lt;sup>19</sup>Readers interested in the actual dynamics under least squares learning are referred to Marcet and Sargent (1989b).

The approximate dynamics under least squares learning as described by (6) indicate that the low-inflation steady state  $\beta_\ell^*$  is the attractor for all initial conditions  $\beta_0$  between zero and  $\beta_h^*$ . The high-inflation stationary state is attained only if  $\beta_0 = \beta_h^*$ . If  $\beta_0$  is between  $\beta_h^*$  and  $\lambda^{-1}$ , no equilibrium exists under least squares learning. The policy advice emanating from this analysis is therefore approximately the opposite from that offered in the perfect foresight case. Under least squares learning, the low-inflation stationary equilibrium with the classical comparative statics is the asymptotic outcome for virtually all initial conditions, provided that an equilibrium exists.

It is important to emphasize at this point that no generic result links learning with "good" equilibria and rational expectations with "bad" equilibria. In some other model, rational expectations may be associated with good outcomes and learning with bad outcomes. Nevertheless, some may take comfort in the fact that, at least for the present case, the comparative statics are once again "classical" under a plausible assumption about how expectations are formed.

### **Experimental Evidence**

So far, the example has illustrated that, while the potential stationary equilibrium inflation values are exactly the same under perfect foresight and least squares learning, the stability properties are approximately reversed, leading to opposite policy implications. Nothing has been said about whether perfect foresight or least squares learning is the better description of human behavior in this context. Recently, however, Ramon Marimon and Shyam Sunder have gathered some experimental evidence that bears on this issue.<sup>20</sup>

Marimon and Sunder summarize the results from a set of controlled experiments that use students as subjects. Their model is an overlapping generations version of the one used by Marcet and Sargent. The students were oriented to the context of the model and asked to forecast inflation, with monetary rewards for more accurate predictions. The authors were especially interested in characterizing the actual outcomes for inflation in the model where the decisions are made by humans.

The results of these experiments indicate that actual inflation tends to cluster around the low-

inflation stationary equilibrium. Marimon and Sunder never observed a tendency of the inflation rate to converge to the high inflation stationary equilibrium in any of their experiments. Based on the forecasts made by their subjects, the authors conclude that least squares learning provides a good approximation to observed behavior. While the work has some caveats and is open to interpretation, it at least provides preliminary evidence on the viability of assuming least squares learning.

# LEARNING IN OTHER MACROECONOMIC CONTEXTS

The notion of a rational expectations revolution stresses the much greater emphasis that macroeconomists have placed on the plans of individuals in macroeconomic models since 1970. One way to think about current research in the area is to categorize models according to their treatment of expectations formation. While most authors want to analyze models where expectations are "rational," several different approaches have been taken. These different approaches within rational expectations macroeconomics reflect unresolved issues in the theory of expectations formation. Various views have been espoused about the specific information that individuals take into account when they are planning for the future; such issues become salient partially because rational expectations theory does not specify a method of learning. Thus, at least three different categories of macromodels that attempt to analyze individuals with rational expectations can be identified.

### Forecast Functions That Use Only Historical Data

The first group contains models in which individuals forecast using only the history of the economy as a guide. In the previous example, for instance, learning was modeled as being based on the historical price sequence that people observed. The approach could be extended relatively easily to include historical time series on other variables in the individuals' forecast functions. While authors assuming rational expectations normally do not make statements about the learning mechanism implicitly underlying their models, it seems that an appropriate

<sup>20</sup>See Marimon and Sunder (1990).

use of available history is what many have in mind.

Considering the use of historical data only, one might be tempted to conclude (say, from looking at the results of the model from the last section) that the only possible long-run outcomes are stationary states. In fact, most discussion of rational expectations applies to dynamic steady states. This notion of long-run equilibrium as a stationary state, which dominates most macroeconomic thought, was challenged recently by an argument that the dynamic equilibrium of an economy might be periodic or chaotic, even under rational expectations and perfect competition.21 Therefore, the artificial economy may not converge to any of the multiple rational expectations steady states, instead remaining in a permanent periodic equilibrium. Cyclical equilibria are important because, presumably, policy implications are altered if long-run equilibria are periodic.22

# Forecast Functions That Include the Beliefs of Others

A second group of macromodels contains forecast functions where, in addition to historical time series, the beliefs of others also play a role.<sup>23</sup> Consider, for instance, the view of Cass and Shell: "In seeking to optimize his own actions, an economic actor must attempt to predict the moves of all other economic actors."<sup>24</sup> Such consideration adds a new element to the inference process.<sup>25</sup> Of course, the notion of interaction among individuals (especially individual firms) has a long history in economics; it is eliminated in the Arrow-Debreu competitive equilibrium framework by the assumption of a large population.

Still, a participant contemplating a forecast may well be concerned with the aggregate expectations of the remaining players, or merely with the beliefs of one other player, such as the government in a monetary policy game.<sup>26</sup> The

game—theoretic macromodels that take this latter view are structurally very different from the traditional models, even though they are both based on rational expectations. Most importantly, they are different in terms of policy implications: these models often yield "... an equilibrium that is extremely sensitive to the public's beliefs about the monetary authority's preferences."<sup>27</sup> This conclusion does not hold in a model in which individual forecasts are a function of historical data only, such as the model of section two, because it excludes the preferences of the government and the public's beliefs about them.

# Forecast Functions That Include Frivolous Variables

When individuals forecast based only on relevant past history, they are sometimes said to be basing expectations on "fundamentals." When deciding what is relevant, however, some people may rationally take into account what seems to an observer to be irrelevant information in the form of a frivolous variable (typically called the sunspot variable). This variable acquires importance in determining the actual outcomes of the economy only because people think it is important. The frivolous variable serves only to signal changes in expectations. Once some individuals take the frivolous variable into account in forming expectations, it becomes rational for all others to do so, since the variable actually does influence outcomes.28

Predictions in models with this type of individual forecast function are based not only on relevant and objectively irrelevant historical information, but also on the expectations of others, since the beliefs of others are taken into account when the frivolous variable is assessed. The literature on sunspot equilibria provides a third but distinctly different strain of rational expectations macroeconomics, with distinctly different policy implications.<sup>29</sup> In particular, ". . . a con-

<sup>&</sup>lt;sup>21</sup>See, for instance, Grandmont (1985). Chaos means that the equilibrium sequence is aperiodic but bounded and displays sensitive dependence on initial conditions. For a general discussion of chaos, see Butler (1990).

<sup>&</sup>lt;sup>22</sup>In the overlapping generations model, for example, the existence of periodic equilibria is disturbing because it implies that welfare varies from generation to generation.

<sup>&</sup>lt;sup>23</sup>John Maynard Keynes, for instance, discussed this type of forecast function in some detail. See Keynes (1936), p. 156.

<sup>&</sup>lt;sup>24</sup>Cass and Shell (1989).

<sup>&</sup>lt;sup>25</sup>But not necessarily a strategic element. See Rogoff (1989).

<sup>&</sup>lt;sup>26</sup>This theme is outlined in detail in the volume edited by Frydman and Phelps (1983).

<sup>&</sup>lt;sup>27</sup>Rogoff (1989).

<sup>&</sup>lt;sup>28</sup>See, for instance, Azariadis (1981).

<sup>&</sup>lt;sup>29</sup>See Cass and Shell (1989).

sideration of the complete set of possible equilibria, [including the sunspot equilibria], associated with a given policy regime may alter one's evaluation of the relative desirability of alternative policies, relative to the conclusion that one might reach if one considered only a single possible equilibrium . . ."<sup>30</sup>

The existence of sunspot equilibria raises the question of whether models with learning have dynamics converging to them. One author, Michael Woodford, has shown that exactly this sort of dynamics is possible.<sup>31</sup> Also, in general, the extent of the sunspot phenomenon is wideranging since there are no limits to the number of possible frivolous variables.

These three approaches to macroeconomics differ according to their alternative assumptions about what it means to assume rational expectations. Since the theory provides no method of expectations formation (that is, no learning process), researchers are free to provide their own: perhaps individuals base their expectations on sunspots, or the expectations of others, or a straightforward application of classical or Bayesian inference to historical time series. As part of the legacy of the rational expectations revolution, all three approaches place heavy emphasis on the role of individuals' views of the future in influencing current macroeconomic equilibria.

Unfortunately, there is little prospect that econometric analysis will decide which version of rational expectations is correct. Because the theory is not well-defined, the empirical tests are unconvincing.<sup>32</sup> While economists want to assume that expectations are rational, the implications of this consensus for modeling and for policy are in doubt.

# SUMMARY: THE DIFFICULTY OF DEFINING OPTIMAL BELIEFS

Several lines of research, each in its own way, are attempting to extend the rational expectations hypothesis to include learning. The first and most obvious is a direct attempt to find mild assumptions showing that most reasonable methods of expectations formation converge to particular types of rational expectations equilibria. If this can be done, the concept of rational expectations equilibrium can be said to provide

a good approximation to the concept of long-run competitive equilibrium. This literature, however, generally ignores the problem of the expectations of others and of frivolous variables.

Even simple learning mechanisms often have not yielded the outcomes that intuition suggests might have occurred. The notion that a reasonable method of expectations formation must give rise to a dynamic system that always converges to a certain a priori plausible rational expectations equilibrium has been gradually eroded. The general conclusion so far seems to be that explicitly introducing learning into macroeconomic models is unlikely to provide a widely applicable selection criterion for rational expectations equilibria. That is, in a rational expectations model with multiple equilibria, introducing a learning mechanism does not appear to reduce the set of potential outcomes in any meaningful way.

One reason for this disappointing result is that it is difficult to define optimal learning. Not only is the class of plausible mechanisms quite large, it is also hard to limit the learning techniques under study to one that can be justified by some optimality argument. One of the biggest problems is that the usual statistical techniques are, strictly speaking, inapplicable to the problem of individual inference in the context of macroeconomic models.33 The source of difficulty is that, in models with expectations, there is an aspect of simultaneity in the sense that beliefs affect outcomes and outcomes affect beliefs. In order to apply standard inference techniques, people must be unaware of the effects of beliefs on outcomes. Making this assumption is unsatisfactory, however, because it means that individuals ignore relevant and potentially useful information when forming their forecasts.

Nevertheless, work continues on ways to explicitly model learning; Marcet and Sargent provide one example. Some attempt is made to choose the learning mechanism via an optimization criterion, and the asymptotic properties of the implied systems are then analyzed. This research agenda is difficult and relies to a large extent on mathematical machinery only recently developed to study such systems.

The policy implications of including learning have been emphasized in this summary of the recent research. In models with multiple ra-

<sup>30</sup>Woodford (1988).

<sup>31</sup>See Woodford (1990).

<sup>32</sup>Webb (1988).

<sup>33</sup>See Marcet and Sargent (1989a,b,c).

tional expectations equilibria, the consensus opinion that people eliminate systematic forecast errors is generally not enough to determine the actual outcome. Moreover, because different rational expectations equilibria have different implications for policy, merely stating that the economy will converge to one of the possible stationary equilibria, without saying which one, is insufficient for useful policy advice. Work on how expectations are formed has only just begun; one hopes that it will lead economists eventually to understand how equilibrium is achieved and what their policy advice is likely to produce.

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### Appendix Details of the Marcet and Sargent Model

This appendix provides the mathematical details for Marcet and Sargent's model outlined in section two. For convenience, the model is reproduced here; it consists of the three equations:

(1) 
$$p_t = \lambda F_t p_{t+1} + \gamma c_t$$

(2) 
$$c_{t} = c_{t-1} + \delta p_{t}$$

(3) 
$$F_{*}p_{*+} = \beta_{*}p_{*}$$

with  $0 < \lambda < 1$ ;  $\gamma, \delta > 0$ ;  $p_{\nu}, c_{\tau} > 0$ ; and  $c_{\nu} > 0$  given. First close the model under perfect foresight:

(4) 
$$\beta_{t} = \frac{p_{t+1}}{p_{t}}$$
.

Substituting (4) in (1) and rearranging shows that  $\beta_0 < \lambda^{-1}$  is required to be compatible with  $c_0 > 0$ . Substitute (3) in (1) to obtain

$$(A.1) p_t = \lambda \beta_t p_t + \gamma c_t.$$

(A.2) 
$$p_{t-1} = \lambda \beta_{t-1} p_{t-1} + \gamma c_{t-1}$$
.

Rearranging (A.2),

(A.3) 
$$c_{t-1} = \gamma^{-1}[p_{t-1} - \lambda \beta_{t-1} p_{t-1}]$$

Substituting (2) into (A.1) gives

$$(A.4) p_1 = \lambda \beta_1 p_1 + \gamma [c_{1-1} + \delta p_1].$$

Now substitute (A.3) into (A.4):

(A.5) 
$$p_t = \lambda \beta_t p_t + \gamma [\gamma^{-1}(p_{t-1} - \lambda \beta_{t-1} p_{t-1}) + \delta p_t]$$

$$(A.6) \lambda \beta_t = 1 - \beta_{t-1}^{-1} + \lambda - \gamma \delta$$

or, iterating forward and rearranging,

(5) 
$$\beta_{1+1} = (1 + \lambda^{-1} - \gamma \delta \lambda^{-1}) - \lambda^{-1} \beta_{1}^{-1}$$

as in the text. A rational expectations equilibrium is defined as a sequence  $\{\beta_t\}_{t=1}^{\infty}$  that solves (5). Proceed to analyze (5) as follows:

$$(A.7) \frac{\mathrm{d}\beta_{t+1}}{\mathrm{d}\beta_t} = \lambda^{-1}\beta_t^{-2} > 0$$

$$(A.8) \frac{d^2 \beta_{t+1}}{d \beta_t^2} = -2 \lambda^{-1} \beta_t^{-3} < 0.$$

The roots of (5) are found by setting  $\beta_{t+1} = \beta_t$  and applying the quadratic formula:

$$(A.9) \beta_{\ell}^*, \beta_h^* =$$

$$\frac{(1 + \lambda^{-1} - \gamma \delta \lambda^{-1}) \pm \sqrt{(1 + \lambda^{-1} - \gamma \delta \lambda^{-1})^2 - 4\lambda^{-1}}}{2}.$$

The level of the real deficit  $\delta > 0$  must be chosen such that these roots are real. This requires

(A.10) 
$$\delta < \lambda \gamma^{-1} [\lambda^{-1} + 1 - (4\lambda^{-1})^{1/2}] = \delta_{max}$$

The roots also satisfy

(A.11) 
$$1 < \beta_{\ell_1}^* \beta_{k}^* < \lambda^{-1}$$
.

These facts provide the basis for the qualitative graph of figure 1.

To analyze the dynamics of the model using the graph, consider some initial condition  $\beta_0$  on the horizontal axis. Find the value  $\beta_1$  by tracing up from  $\beta_0$  to the plotted function. The value of  $\beta_1$  now serves as the input for the next period. To transfer the  $\beta_1$  value to the horizontal axis, trace horizontally from the plotted function to the 45 degree line, and from the 45 degree line down to the horizontal axis. Now repeat the procedure as though  $\beta_1$  is the initial condition.

Next, solve the model under least squares learning. Use

(4') 
$$\beta_t = \left[\sum_{s=1}^{t-1} p_{s-1}^2\right]^{-1} \left[\sum_{s=1}^{t-1} p_s p_{s-1}\right].$$

In order to get to equation 5' in the text, (4') must be written in recursive form. To do this, define temporarily two vectors

(A.12) 
$$P_{t-1} = [p_0, ..., p_{t-1}]'$$

(A.13) 
$$P_{t-2} = [p_1, ..., p_{t-2}]'$$
.

Then

$$(A.14) \beta_{t} = [P'_{t-2}P_{t-2}]^{-1} P'_{t-2}P_{t-1}$$

and

(A.15) 
$$\beta_{t-1} = [P'_{t-3}P_{t-3}]^{-1} P'_{t-3}P_{t-2}$$

where now

$$(A.16) P_{t-2} = [p_1, ..., p_{t-2}]'$$

(A.17) 
$$P_{t-3} = [p_0, ..., p_{t-3}]'$$
.

The additional information in  $\beta_t$  is the observation  $p_{t-1}$ . The relationship between  $\beta_t$  and  $\beta_{t-1}$  is given by a well-known recursive least squares formula<sup>1</sup> which can be applied as follows:

(A.18) 
$$\beta_{t} = \beta_{t-1} + [P'_{t-3}P_{t-3}]^{-1} p_{t-2}(p_{t-1} - p_{t-2}\beta_{t-1})f_{t}^{-1}$$

where

(A.19) 
$$f_t = 1 + p_{t-2} [P'_{t-3}P_{t-3}]^{-1} p_{t-2}$$

is a scalar. Since

(A.20) 
$$[P'_{t-3}P_{t-3}]^{-1} = \left[\sum_{s=1}^{t-2} p_{s-1}^2\right]^{-1}$$
,

the scalar  $f_t$  can be written as

(A.21) 
$$f_{t} = \left[\sum_{s=1}^{t-2} p_{s-1}^{2}\right]^{-1} \left[\sum_{s=1}^{t-2} p_{s-1}^{2} + p_{t-2}^{2}\right].$$

Substituting into (A.18) yields

(A.22) 
$$\beta_{t} = \beta_{t-1} + \frac{p_{t-2}}{\sum_{s=1}^{t-1} p_{s-1}^{2}} [p_{t-1} - p_{t-2} \beta_{t-1}].$$

To obtain equation (5'), use (1), (2) and (3) to find

(A.23) 
$$p_{t-1} = G(\beta_{t-1}) \left[ \frac{1 - \lambda \beta_{t-2}}{1 - \lambda \beta_{t-1}} \right] p_{t-2}$$

Substituting (A.23) into (A.22) gives the equation in the text:

(5') 
$$\beta_{t} = \beta_{t-1} + \frac{p_{t-2}^{2}}{\sum_{t=1}^{t-1} p_{t-1}^{2}} \left[ \frac{(1-\lambda\beta_{t-2})}{(1-\lambda\beta_{t-1})} G(\beta_{t-1}) - \beta_{t-1} \right].$$

A set of positive sequences  $\{\beta_t, c_t, p_t\}_{t=0}^{\infty}$  satisfying (1)-(3) and (4') is an equilibrium under least squares learning. To approximate the dynamics under least squares learning, consider the simpler but closely related difference equation:

(6) 
$$\beta_{t} = G(\beta_{t-1}) = \frac{1 - \lambda \beta_{t-1}}{1 - \lambda \beta_{t-1} - \gamma \delta}$$

The derivatives are given by

$$(A.24) \frac{dG}{d\beta_{t-1}} = \lambda \gamma \delta (1 - \lambda \beta_{t-1} - \gamma \delta)^{-2} > 0$$

$$(A.25) \frac{d^{2}G}{d\beta_{t-1}^{2}} = 2\lambda^{2}\gamma\delta(1 - \lambda\beta_{t-1} - \gamma\delta)^{-3} > 0$$

provided  $\beta < \lambda^{-1}$   $(1 - \gamma \delta)$ . This provides the information for the second qualitative graph, given by figure 2.

<sup>&</sup>lt;sup>1</sup>See Harvey (1981), p. 54.

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