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In This Issue . . .

In the first article in this *Review*, "The Discount Rate and Market Interest Rates: Theory and Evidence," Daniel L. Thornton discusses the theoretical links between the Federal Reserve's discount rate and market interest rates and presents some empirical evidence on the extent of this link. He finds that, both in theory and practice, the direct relationship between the discount rate and money market rates is extremely weak. Consequently, any observed relationship between these rates must be due to an expectations effect or to a change in Federal Reserve behavior. If the latter is correct, however, there should have been a stronger association between the discount rate and money market rates following the Federal Reserve's change in operating procedure in the fall of 1982. Thornton finds, however, that, if anything, this relationship has become weaker.

* * *

In the second article in this *Review*, "A Microeconomic System-Wide Approach to the Estimation of the Demand for Money," Salam K. Fayyad describes how the microeconomic system-wide approach to money demand differs from the usual money demand specifications. Using a neoclassical utility function defined over five expenditure categories, two of which are presumed to capture the flow of "monetary services," he demonstrates how the microeconomic system-wide approach can be implemented. Fayyad then examines in-sample predictions of budget shares for expenditures on monetary services and the other expenditure categories estimated by this approach over the I/1969–I/1985 period. His results indicate that the microeconomic system-wide approach to money demand estimation yields predictions that closely track the actual behavior of the flow of monetary services over this period.

The Discount Rate and Market Interest Rates: Theory and Evidence

Daniel L. Thornton

THE relationship between the Federal Reserve's discount rate and money market interest rates continues to be a topic of much interest and even more confusion. A significant number of money market analysts and some in public service believe that the discount rate is an important tool through which the Federal Reserve exerts its influence over the economy — particularly market interest rates. This view appears to have gathered strength from recent evidence that discount rate changes have a statistically significant effect on market interest rates and from the presumed effects of a 1982 change in the Federal Reserve's operating procedure.[1] Consequently, the long-standing discrepancy between what economic theory says about the relationship between the discount rate and market interest rates and the view among many money market analysts appears to have become larger. The purpose of this article is to narrow the gap by pointing out that, both in theory and in practice, changes in the Federal Reserve's discount rate, *per se*, have essentially no effect on market interest rates. At best they "signal" changes in the Federal Reserve's use of other more powerful tools of policy. Any impact of a discount rate change on market interest rates is due to changing expectations or to a change in Federal Reserve operations following the discount rate change.

THE MARKET ANALYST'S VIEW

Figure 1 illustrates a commonly held view of the relationship between a cut in the discount rate and the response of market interest rates; it shows the hypothetical time path of market interest rates before and after a hypothetical cut in the Federal Reserve discount rate at time t_0, and it reflects the perception that a cut in the discount rate *causes* market interest rates to be permanently lower than they otherwise would have been. This cause-and-effect relationship is purely qualitative. It is not clear whether a 1 percentage-point cut in the discount rate will lower market rates by 1 percentage point or only a few basis points. It merely is asserted that market rates will be lower.

The view that the discount rate is preeminent in the money market contrasts sharply with economic theory and the perception of many economists that the discount rate is the least powerful of the Federal Reserve's tools for influencing the money stock and interest rates. Before turning to this analysis in detail, it is instructive to consider some casual evidence against the idea that the discount rate is preeminent in the money market. Chart 1 shows the three-month Treasury bill, federal funds and discount rates weekly for the period from October 1982 to June 1986. What do these data show about the effect of a discount rate change on market interest rates? First, in a number of instances, discount rate changes are followed closely by a leveling off of market interest rates or by a movement in the opposite direction. While this does not rule out the possibility that market rates would have been higher (lower) if the discount rate had not been cut (raised), it does suggest that the market analyst

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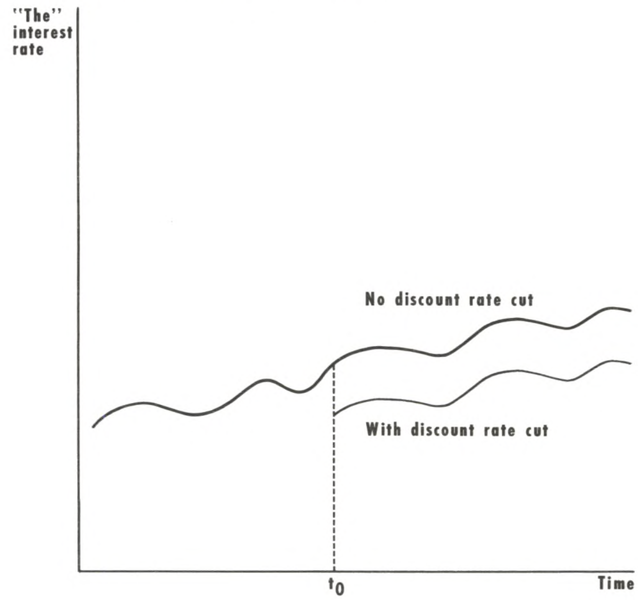
[1]See Thornton (1982) for a summary of some of the usual sources of confusion; Thornton (1982), Sellon and Seibert (1982) and Smirlock and Yawitz (1985) for empirical estimates of a change in the discount rate on market interest rates; and Batten and Thornton (1984, 1985) and Hakkio and Pearce (1986) for empirical estimates of an impact of a discount rate change on the foreign exchange market.

view is not supported by a simple analysis of interest rate behavior.

Second, nearly all discount rate changes *follow*, rather than *lead*, movements in market interest rates in the same direction.[2] It would seem that changes in market interest rates motivate discount rate changes rather than the reverse. Furthermore, even when market rates declined (increased) following a discount rate cut (increase), it is particularly difficult to determine whether market rates would have moved in the same or similar fashion in the absence of a change in the discount rate. While all of this is inconclusive, it provides weak and often contrary evidence of a discount rate/market interest rate line of causation, and provides little comfort to those who believe the view illustrated by figure 1.

Figure 1
Hypothetical Response to a Discount Rate Cut



THE DISCOUNT RATE AND MARKET RATES IN THEORY

Because the interest rate is the price of credit, any impact of discount rate changes on market interest rates must come via their effect on the supply of or the demand for credit. In this regard, three distinct — though not necessarily mutually exclusive — effects of a discount rate change can be identified. These are illustrated in figure 2. Prior to the discount rate cut, the credit market is in equilibrium at an interest rate of R_0, corresponding to the intersection of the initial supply and demand curves, S_0 and D_0, respectively.

The Direct Effect

The first effect, called the direct (or substitution) effect, causes a shift in the supply of credit. Discount window borrowing is one method depository institutions use to adjust their reserve position. Alternatively, they can buy federal funds or sell government securities directly in the money market.[3] Since these alternatives are close substitutes, the demand for borrowed reserves depends on the spread between market interest rates, especially the federal funds rate, and the discount rate. As the federal funds-discount rate spread increases, borrowings from the Federal Reserve tend to increase and vice versa. Thus, the level of discount window borrowings usually is expressed as:

$$(1)\quad Borr = \alpha(R_f - R_d), \qquad \alpha > 0,$$

where Borr denotes the aggregate level of indebtedness of depository institutions to the Federal Reserve and R_f and R_d denote the federal funds and discount rate, respectively.

To illustrate the direct effect of a change in the discount rate on market interest rates, assume that the discount rate is cut. In response, depository institutions increase their borrowings and reduce their use of alternative sources of reserves. The increase in borrowings produces an increase in the monetary base and, in turn, the supply of credit — illustrated in figure 2 by a shift from S_0 to S_1. Thus, a discount rate cut has a direct effect, causing market interest rates to decline from R_0 to R_1. The effect of an increase in the discount rate would be symmetric.

The Announcement Effect

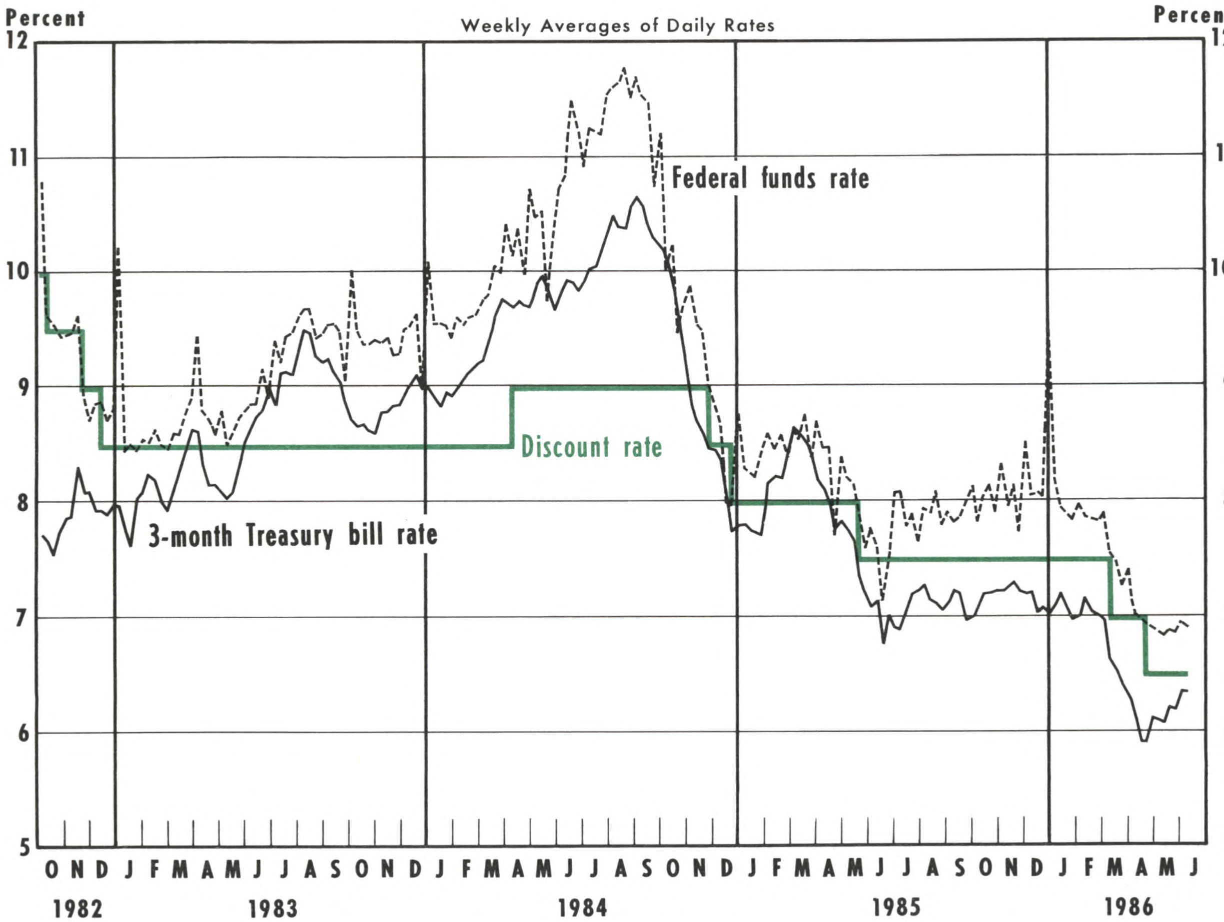
Additionally, discount rate changes can have an "announcement effect." If a change in the discount rate is interpreted as a "signal" that the Federal Reserve will alter its policy with respect to the growth of reserves and the money stock, the market may react in anticipation of a policy change. A cut in the discount rate usually is thought to be a signal that the Federal Reserve is going to pursue an easier monetary policy so the market reacts in anticipation of Federal Reserve

[2]This is true of other periods as well; see Thornton (1982), p. 14.

[3]Depository institutions also can call in loans or carry the deficiency over into the next reserve period. They rarely, if ever, use these alternatives, however.

Chart 1

Selected Interest Rates



open market operations that will increase the supply of credit.[4] Consequently, there is an immediate shift in the supply of credit, relative to demand, in anticipation of further monetary ease. If the announcement effect occurs, it is over and above the direct effect of a discount rate change, and is illustrated by the shift from S_1 to S_2 in figure 2.[5]

The Policy Effect

Finally, there could be a "policy effect" if the Federal Reserve actually changes its policy and increases the growth rate of reserves. This also can be illustrated by the shift from S_0 to S_1. If the market correctly anticipates the direction and magnitude of the policy effect, market interest rates will remain permanently lower at R_2. Of course, this requires that the market's expectations be correct, both in terms of the actual change in Federal Reserve policy and in terms of the impact of that policy change on the market.[6] As the Federal Reserve purchases more securities, speculators sell off those acquired in anticipation of the policy change. If the market overanticipates Federal Reserve actions, however, market rates first will fall below and then

[4]This is not the only possible interpretation for the market. See Batten and Thornton (1984) and Smith (1963) for a discussion of this point.

[5]This also could have been illustrated by a reduction in the demand for credit, but was illustrated as a shift in supply to keep the figure simple.

[6]This brief discussion gives rise to several issues not analyzed in this paper, such as the effectiveness of policy and the credibility of the central bank. For a general discussion of the credibility issue, see Cukierman (1986).

subsequently rise to their long-run equilibrium. Furthermore, if the market's expectations are incorrect and Federal Reserve policy remains unchanged, interest rates will rise back to R_1 — the only impact of a discount rate change would be the direct effect.

DISCOUNT WINDOW BORROWINGS AND THE FED'S OPERATING PROCEDURE

Some have argued that the policy effect has become more important since the October 1982 change in the Federal Reserve's operating procedure. At that time, the Board switched from a nonborrowed reserve to a borrowed reserve operating procedure. It is now widely believed that the Federal Reserve operates to achieve a certain average level of borrowed reserves (called the initial borrowing assumption) over a given time period.[7] The mechanics of this operating procedure can be illustrated by tracing the reaction of the Federal Reserve to an unexpected increase in the demand for reserves. Other things unchanged, an increase in the demand for reserves tends to cause both borrowings and the funds rate to rise, as depository institutions attempt to satisfy their demand for reserves in the money market and at the Federal Reserve discount window. As borrowings increase relative to the borrowing assumption, the Fed increases the supply of nonborrowed reserves via open market purchases of government securities; in response, both borrowing and the federal funds rate fall.

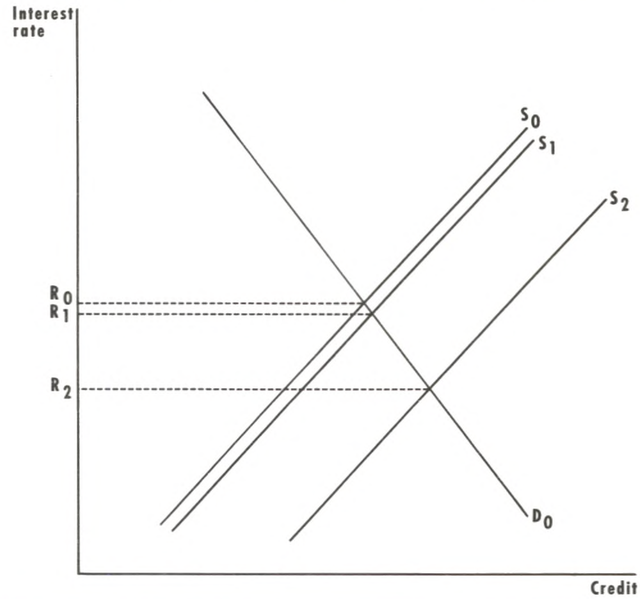
A cut in the discount rate, not accompanied by a change in the initial borrowing assumption, works analogously. If the Federal Reserve cuts the discount rate, the demand for borrowed reserves will increase at all levels of the federal funds rate, causing borrowings to increase relative to the initial borrowing assumption. If the initial borrowing assumption is unchanged, the Fed must increase the supply of nonborrowed reserves through open market operations until the federal funds rate has declined by enough to return borrowings to the level of the borrowing assumption.

The above implies that equation 1 can be written as:

$$(2)\quad Borr^* = \alpha(R_f - R_d),$$

where Borr* denotes the Federal Reserve's initial borrowing assumption. Equation 2 implies a constant spread between the federal funds and discount rates.

Figure 2
Three Possible Effects of a Discount Rate Cut on Market Interest Rates



Any change in the discount rate will be matched by an equal change in the federal funds rate, providing there is no compensatory change in the borrowing assumption.

It should be emphasized that it is not the discount rate change *per se* that affects market interest rates, but the subsequent policy effect if the Federal Reserve strictly adheres to an operating procedure that attempts to maintain the level of borrowings assumed by its current policy directive. If the market perceives this behavior, it could also strengthen any announcement effect.

The Importance of the Liquidity Effect

All of the potential effects of a change in the discount rate on market interest rates (but, in particular, the policy effect) depend on the so-called "liquidity effect" — the change in interest rates associated with an unanticipated increase in the growth rate of the money supply. While such an effect is widely touted in theoretical discussions, there is little empirical evidence to support it. Yet, without a liquidity effect or at least the expectation of a liquidity effect, changes in the discount rate could not have an impact on a broad spectrum of market interest rates.[8]

[7]For a discussion of this, see Roley (1986), Wallich (1984) and Federal Reserve Bank of New York (1986).

[8]This, of course, ignores the possible effect of changes in expectations of inflation on interest rates. See Brown and Santoni (1983), Cagan and Gandolfi (1969) and Melvin (1983) for a review of the direct evidence on the liquidity effect.

Which Market Interest Rates?

Much of the discussion thus far has been carried out in terms of the federal funds rate. In reality, there are a large number of different rates: the rates on federal funds, Treasury bills, notes and bonds, commercial bank loans, mortgages, etc. Hence, the array of credit market assets should be divided into those that are closely related to the discount rate and those that are less closely related to it.

The market for federal funds is one segment of the credit market that is particularly sensitive to discount rate changes and to changes in Federal Reserve operations. Federal funds are simply the reserve assets of one depository institution that are sold (lent) to another for the purpose of achieving both institutions' desired reserve positions. Because such funds are close substitutes for reserves supplied by the Federal Reserve, including those supplied through the discount window, changes in the discount rate or Federal Reserve policy should initially affect the federal funds rate and subsequently other market rates. (See page 10 for a discussion of the relationship between the discount rate and the prime rate.)

Borrowings and the Rate Spread

The relationship between the discount rate and market interest rates rests, in one way or another, on the strength of the relationship between borrowings and the rate spread. Equations 1 and 2, however, imply that borrowings depend on more than the spread between the market and discount rates. To see this, assume that there are no impediments to borrowing so that depository institutions can borrow any amount they desire at the discount window. If this were the case, borrowings would rise whenever market rates were above the discount rate and fall whenever the discount rate is above the market rate. If we abstract from problems of inflation and inflationary expectations, the market rate would always equal the discount rate.[9] But if $R_f = R_d$, however, equation 1 implies that borrowings would be zero.

The data in chart 2, which show weekly adjustment borrowings and the federal funds rate/discount rate spread from October 1982 to June 1986, indicate that the discount and federal funds rates are seldom equal.[10] Moreover, when the rates are equal, borrowings are not zero. This is *prima facie* evidence that borrowing is not explained solely by the interest rate spread. Indeed, Federal Reserve regulations, which set forth the conditions under which depository institutions may use the discount window, make it clear that borrowing is a privilege and explicitly state that it is inappropriate to borrow "to take advantage of a differential between the discount rate and the rate on alternative sources of funds."[11]

A visual inspection of chart 2 shows that there is usually a positive relationship between borrowings and the rate spread, that is, that increases in borrowings tend to be associated with increases in the spread and vice versa. There are, however, some marked departures from this relationship. The most obvious of these occurred with the sharp increase in borrowings in May–June 1984 and November 1985. Both of these events were accompanied by special circumstances. The former is associated with heavy discount window borrowings by Continental Bank of Illinois and the latter with the largest single-day borrowing from the Federal Reserve when the Bank of New York (BONY) experienced a computer failure on November 21, 1985.[12] Even when these outliers are ignored, however, there are instances when borrowings and the spread move in opposite directions. Moreover, there is considerable variation in the relationship between the average level of borrowings and the average level of the spread. The most obvious of these is the period from June 13, 1984, through October 3, 1984, when the spread averaged over 200 basis points and borrowings averaged less than a billion dollars, as compared to an average spread of 70 basis points and average borrowings of $.7 billion over the entire period.[13]

The strength of the relationship between borrow-

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[9]Under this arrangement, one can envision the Federal Reserve pushing down interest rates by lowering the discount rate. As this is done, however, money growth will accelerate and so will inflation. As a result, nominal interest rates will rise and money will grow even faster. Hence, even if the discount window were completely "open," the Federal Reserve would be unable to control interest rates with the discount rate in anything but the short run.

[10]Borrowing from the Federal Reserve is divided into three categories: adjustment borrowing, seasonal borrowing and extended credit borrowing. The borrowing assumption, however, pertains only to adjustment and seasonal borrowings; see Partian, Hamdani and Camilli (1986).

[11]This is called the "reluctance of banks to borrow from the Federal Reserve," and at one time there was considerable discussion over whether this reluctance was "inherent" or "induced."

[12]See Federal Reserve Bank of New York (1986) for a discussion of the BONY borrowings.

[13]It could be that depository institutions became more reluctant to borrow from the Federal Reserve in light of the large borrowings by Continental Bank.

The Discount Rate and the Prime Rate

One possible reason for the hypothesized strong effects of discount rate changes on interest rates is the fact that discount rate changes and changes in the commercial bank prime rate often occur together and are usually accompanied by a great deal of publicity. Both of these rates are administered rates that do not change daily with market forces, but change less frequently and by fairly large amounts.

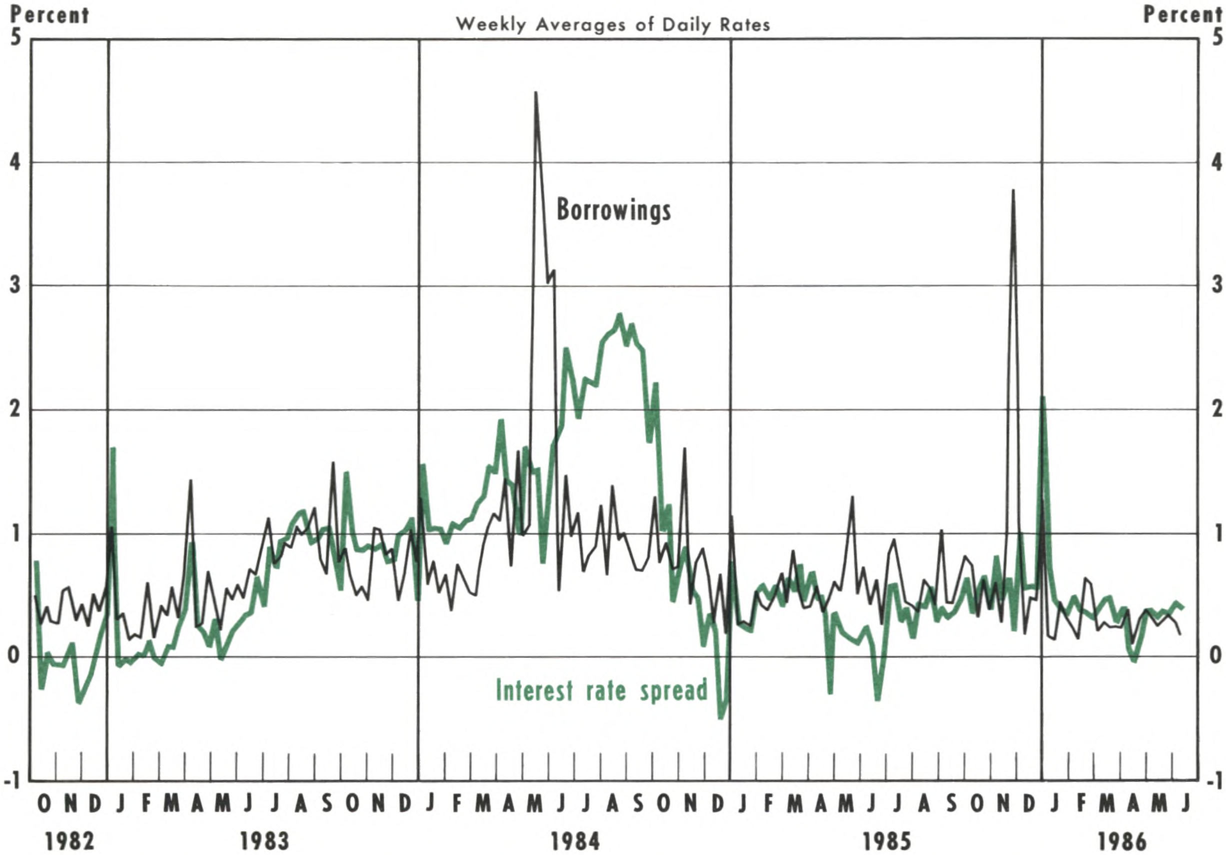
Because changes in the prime rate often follow on the heels of changes in the discount rate, it may lead some to conclude incorrectly the latter caused the former. Because both are administered rates, however, they are likely to respond similarly but not precisely coterminously, to market rates. For example, as market interest rates fall relative to these administered rates, these rates become increasingly out of line with the market. Hence, there is an incentive for the Federal Reserve to cut the discount rate *and* for commercial banks to cut their prime rate. If the Federal Reserve cuts the discount rate first, banks may feel additional pressure to cut their prime rate, but this does not imply that the former caused the latter. Rather both rates are merely responding to market forces.

The table above shows that on four occasions since October 1982 discount rate and prime rate changes were effective on the same day. In each instance, the announcement of a cut in the prime rate followed the announcement of the discount rate change. For the remaining five changes in the discount rate, changes in the prime rate followed discount rate changes by a week or more. Also, there were a number of changes in the prime rate that were not even remotely associated with changes in the discount rate. It would appear that changes in market interest rates are primarily responsible for changes in both of these administered rates.

Prime rate		Discount rate	
Date effective	Change	Date effective	Change
October 7, 1982	13.5% to 13%		
		October 12, 1982	10% to 9.5%
October 13, 1982	13% to 12%		
November 22, 1982	12% to 11.5%	November 22, 1982	9.5% to 9%
		December 14, 1982	9% to 8.5%
January 11, 1983	11.5% to 11%		
February 25, 1983	11% to 10.5%		
August 8, 1983	10.5% to 11%		
March 19, 1984	11% to 11.5%		
April 5, 1984	11.5% to 12%		
		April 9, 1984	8.5% to 9%
May 8, 1984	12% to 12.5%		
June 25, 1984	12.5% to 13%		
September 27, 1984	13% to 12.75%		
October 16, 1984	12.75% to 12.5%		
October 29, 1984	12.5% to 12%		
November 8, 1984	12% to 11.75%		
		November 21, 1984	9% to 8.5%
November 28, 1984	11.75% to 11.25%		
December 19, 1984	11.25% to 10.75%		
		December 24, 1984	8.5% to 8%
January 15, 1985	10.75% to 10.5%		
May 20, 1985	10.5% to 10%	May 20, 1985	8% to 7.5%
June 18, 1985	10% to 9.5%		
March 7, 1986	9.5% to 9%	March 7, 1986	7.5% to 7%
April 21, 1986	9% to 8.5%	April 21, 1986	7% to 6.5%

Chart 2

Adjustment plus Seasonal Borrowings from Federal Reserve and Federal Funds-Discount Rate Spread



ings and the spread can be estimated statistically by considering the equation:

(3) $\text{Borr}_t = \alpha_0 + \alpha_1(R_f - R_d) + u_t$.

The term u_t is a random disturbance that can be thought of as capturing the effect of all factors other than the rate spread that determine deviations in borrowing from its average level. From a statistical point of view, the variation in borrowings can be decomposed into two sources: the proportion explained by the rate spread and that explained by all other factors. (Since the factors that go into u_t are not explicitly identified, this is called "unexplained variation.")

Equation 3 is estimated with ordinary least squares, using the weekly data shown in chart 2. The outliers for the weeks ending May 16 to June 6, 1984, and November 27, 1985, were deleted.[14] The results are presented in the first row of table 1. The coefficient of determination, denoted $\bar{R}^2$, measures the proportion of the variation in borrowings explained by the rate spread, and $1-\bar{R}^2$ is the proportion of variation explained by all other factors. The $\bar{R}^2$ indicates that only 35 percent of the variation in borrowings is accounted for by the spread, leaving 65 percent to be accounted for by other factors.

The fit can be improved by putting in a dummy variable that takes on the value one for the period from the week ending June 13, 1984, to October 3, 1984, when the spread was unusually high, and zero elsewhere. The results of including a dummy variable are shown in the second row of table 1. While including the dummy variable boosts the $\bar{R}^2$ somewhat, it does not explain this anomaly. Nevertheless, even after accounting for this apparent shift in the borrowing function, the spread and the dummy variable explain only

[14]If these outliers are not removed, the $\bar{R}^2$ falls to about .15.

Table 1
Estimates of Equation 3

Intercept	Dummy variable	Spread	$\bar{R}^2$	SE
.420*		.291*	.35	.28
(14.74)		(10.04)		
.368*	−.410*	.419*	.40	.27
(12.21)	(4.03)	(9.94)		

*Indicates the variable is statistically significant at the 5 percent level.

40 percent of the total variation in borrowings, leaving the bulk of the variation to be explained by other factors.[15]

IN SEARCH OF THE DIRECT EFFECT: SOME EMPIRICAL ESTIMATES

Separating the three possible effects of discount rate changes on market interest rates — the direct, policy and announcement effects — is difficult. The results in table 1, however, provide a basis for estimating the likely direct effect of a discount rate change on interest rates. From the second row of table 1, we see that a 1 percentage-point (100 basis-point) decline in the discount rate will cause borrowings to increase by $.419 billion. All other things the same, this will increase the monetary base (in the form of borrowed reserves) by the same amount. Given an M1-monetary base multiplier of 2.7, this will produce a $1.13 billion increase in M1.[16] Such changes in the money stock shift the supply of credit to the right, causing market interest rates to fall. The effect of this on market rates depends on the extent of the shift in the supply of credit and the interest sensitivity of the demand for credit, so it is possible, in principle, to determine the effect of an exogenous change in the money stock on interest rates.

The largest estimates of this liquidity effect come from estimated short-run money demand equations. For example, usual estimates suggest that a $1.13 billion change in M1 would produce a 67 basis-point initial change in the three-month Treasury bill rate, but only a six basis-point effect in the long-run equilibrium rate.[17] It is well known, however, that such equations have unreasonably large estimates of the liquidity effect.[18] Other studies, which attempt to estimate the liquidity effect directly, show only small and transient effects of unanticipated changes in money on interest rates. Using these estimates, a $1.13 billion change in the money stock would produce about a one basis-point change in the T-bill rate initially, with no long-run effect whatsoever.[19]

Put into another perspective, since October 1982 the average, *absolute* weekly change in M1 has been $1.77 billion, more than one and one-half times the estimated $1.13 billion change in M1 associated with a full 1 percentage-point change in the discount rate. Thus, the direct effect of a change in the discount rate on market interest rates, all other things constant, is likely to be small.

Technical Vs. Nontechnical Changes in the Discount Rate

Alternatively, estimates of the magnitude of the direct effect can be obtained by classifying discount rate changes according to the reason they were made. Some discount rate changes are made solely as technical adjustments, designed to align the discount rate with market interest rates. Others are made for policy-related reasons. These are called nontechnical changes.

[15]Because borrowings fluctuate with market interest rates, they can be a source of cyclical variation in the money stock. Because of this, some have suggested that the discount rate be tied to some market interest rate. Opponents of this view have argued that no single interest rate adequately represents the appropriate opportunity cost for all institutions. If this were true, rates other than the federal funds rate might explain borrowings. To test this, the second equation on table 1 was reestimated with the difference between the three-month Treasury bill and federal funds rates added as a separate regressor. The coefficient on the difference between these rates was not statistically significantly different from zero at the 5 percent level (t-ratio = 1.26). Hence, it appears that the federal funds rate is the primary interest rate on which borrowing depends.

[16]The M1 multiplier averaged much less than this during all of the period under consideration, i.e., 2.7 is approximately its current level.

[17]These estimates are based on current levels of M1 and interest rates. Using a short-run interest elasticity estimate from the "nominal-adjustment" specification of the short-run demand for money of −.015 and a money stock of $670 billion, the percentage change in the interest rate would be about 11 percent. A T-bill rate of 6 percent translates into a 67 basis-point change in market interest rates. The long-run effect was calculated under the assumption of a long-run elasticity of about −.14 (−.015/.11). These estimates are in line with the results from Thornton (1985).

[18]See Carr and Darby (1981).

[19]See Brown and Santoni (1983). Similar estimates would be obtained from Cagan and Gandolfi (1969) and Melvin (1983).

Since the response of borrowings to a discount rate change should be the same regardless of the reason for the change, *ceteris paribus*, the direct effect of a discount rate change on market interest rates should be the same for all changes in the discount rate.[20] Furthermore, there should be no change in the market's perception of policy when discount rate changes are purely technical adjustments. For nontechnical changes, however, not only is there a direct effect due to the impact on borrowings and the supply of credit, but a potential announcement effect, which may or may not be validated by subsequent Federal Reserve actions. If the discount rate changes that are made purely as technical adjustments do not affect market interest rates, this is further evidence that there is essentially no direct effect of discount rate changes. Any interest rate effects come through an announcement effect or subsequent policy changes.

It should be noted that the fact that the Federal Reserve changes the discount rate from time to time solely to bring it in line with market interest rates is itself *prima facie* evidence that the link between borrowings and the federal funds/discount rate spread is not the sole determinant of depository institution borrowing. If it were, the Federal Reserve should never have to make such technical adjustments, but this is not the case. Of the nine discount rate changes from October 1982 to June 1986 listed in table 2, three were stated to have been made solely for technical reasons and three of the remaining six mentioned technical concerns as one of the reasons for the change.[21]

Recent empirical work provides strong evidence that *only* discount rate changes made for policy reasons affect market interest rates.[22] This work is updated here by estimating the equation:

$$(4)\quad \Delta R_t = \alpha_0 + \sum_{i=1}^{10} \alpha_i \Delta R_{t-i} + \beta \Delta DR_t + u_t,$$

where ΔR denotes the one-day change in a market interest rate, and ΔDR denotes the change in the discount rate.[23] This equation was estimated using daily data from October 1, 1982, to June 11, 1986, using both the federal funds and three-month Treasury bill rates. The T-bill rate was selected to represent market interest rates in general. Estimates of the coefficient on ΔDR and some summary statistics are presented in table 3.[24] The results indicate that a change in the discount rate has a positive, significant effect on both the federal funds and T-bill rates on the next market day. The effect on the federal funds rate is roughly 2.5 times that on the T-bill rate.

When the discount rate changes are partitioned into those made for technical reasons (ΔDRT) and those made for nontechnical reasons ($\Delta DRNT$), the results indicate that discount rate changes made solely for technical reasons had no significant effect on the federal funds rate. The results for the T-bill rate are less clear. The coefficient on discount rate changes made solely for technical reasons is smaller than that for policy-related reasons, but is statistically significant at the 5 percent level. A closer look, however, reveals that only one of the three discount rate changes made solely for technical reasons is associated with movement in the T-bill rate in the expected direction. The half-percent decline in the discount rate on October 12, 1982, is associated with a 37 basis-point decline in the T-bill rate. In contrast, the half-percent increase on April 9, 1984, is associated with a 9 basis-point decline in the T-bill rate and the half-percent decrease on April 21, 1986, is associated with no change in the T-bill rate.

When discount rate changes made for purely technical reasons are partitioned into the one made on October 12, 1982 ($\Delta DRTO$), and the other two (ΔDRT), the results indicate that significance of technical changes on the three-month Treasury bill rate is due to the change on October 12. Furthermore, the effect on the federal funds rate is significant at the 10 per-

[20]This discussion assumes that the Federal Reserve is not trying to control the money stock, and in particular, it is not using a monetary base or total reserves target. If it were, any change in the discount rate would have no direct effect on interest rates because the effect of such a change would be neutralized by compensatory open market operations.

[21]The classification used is based upon the Federal Reserve's announced statement of intentions as used by Thornton (1982) and Batten and Thornton (1984, 1985). Smirlock and Yawitz (1985) investigate alternative schemes, but find that the one employed here works best. Their results are supported by Hakkio and Pearce (1986).

[22]See Thornton (1982), Batten and Thornton (1984, 1985), Smirlock and Yawitz (1985) and Hakkio and Pearce (1986).

[23]ΔDR takes on the value of the discount rate change on the day that the change became effective. The one exception is the change that was announced on November 21, 1984, effective immediately. Since the announcement was made at 4:15 p.m. EST after the market closed, the ΔDR takes on a value on November 23. (The federal funds rate declined by 35 basis points between November 21 and 23 and *increased* by 4 basis points between November 20 and 21).

[24]The coefficients on the distributed lag of the dependent variable are not reported because they are intended only to capture the effect of all previously known information on these interest rates and are not of importance themselves.

Table 2

Discount Rate Changes, October 1982 to June 1986

Date effective	Change	Classification	Reason
October 12, 1982	10% to 9.5%	T	Action taken to bring the discount rate into closer alignment with short-term market interest rates
November 22, 1982	9.5% to 9%	P	Action taken against the background of continued progress toward greater price stability and indications of continued sluggishness in business activity and relatively strong demand for liquidity
December 14, 1982	9% to 8.5%	P	Action taken in light of current business conditions, strong competitive pressures on prices and further moderation of cost increases, a slowing of private credit demands and present indications of some tapering off in growth of the broader monetary aggregates
April 9, 1984	8.5% to 9%	T	Action taken to bring discount rate into closer alignment with short-term interest rates
November 21, 1984	9% to 8.5%	P	Action taken in view of slow growth of M1 and M2 and the moderate pace of business expansion, relatively stable prices and a continued strong dollar internationally
December 24, 1984	8.5% to 8%	P	Essentially the same as before plus to bring the discount rate into more appropriate alignment with short-term market interest rates
May 20, 1985	8% to 7.5%	P	Action taken in the light of relatively unchanged output in the industry sector stemming from rising imports and a strong dollar. Rate reduction is consistent with declining trend in market interest rates
March 7, 1986	7.5% to 7%	P	Action taken in context of similar action by other important industrial countries and for closer alignment with market interest rates. A further consideration was a sharp decline in oil prices
April 21, 1986	7% to 6.5%	T	Action taken to bring discount rate into closer alignment with prevailing levels of market rates

P = policy related
T = technical
Source: *Federal Reserve Bulletin,* paraphrased from statements in various issues, and the *Wall Street Journal.*

cent level when these data are partitioned in this way. This is the only instance when a technical change in the discount rate had a significant effect on market rates.[25] The preponderance of evidence suggests that discount rate changes made solely for technical reasons have no statistically significant effect on market interest rates.[26] This result is consistent with our previous finding that there is little, if any, direct effect of a discount rate change on market interest rates.

It could be, however, that discount rate changes made solely for technical reasons are more readily anticipated than those made for policy reasons.[27] If this were the case, and if the market perceived the effect of the corresponding change in the money supply on interest rates, market rates would change prior to the change in the discount rate so there would be no statistically significant effect following the announcement of a discount rate change. Hakkio and Pearce (1986) report that discount rate changes made for technical reasons are no more readily forecasted than those made for nontechnical reasons. Hence, this

[25]This change was announced two days after the Federal Reserve de-emphasized M1 as a monetary target. (See Thornton (1983) for a discussion of this period.) While there was no immediate announcement of the decision to de-emphasize M1, there were leaks to this effect, so the market may have interpreted the October 12 decrease in the discount rate as an indication that the Federal Reserve would move toward an easier policy. There were leaks to the press on October 7 that the Federal Reserve would pay more attention to interest rates and less to M1 growth. See BNA's *Daily Report for Executives,* October 8, 1982.

[26]This finding has been reiterated by Thornton (1982), Smirlock and Yawitz (1985) and the results presented in table 5 for the money market, and by Batten and Thornton (1984, 1985) and Hakkio and Pearce (1986) for the foreign exchange market.

[27]This conjecture is offered by Batten and Thornton (1984).

Table 3

Estimates of Equation 4 for Technical and Nontechnical Discount Rate Changes

Constant	ΔDR	ΔDRNT	ΔDRT	ΔDRT0	$\bar{R}^2$	SE
		Federal funds rate				
−.011	.690*				.20	.35
(0.94)	(2.95)					
−.010		.827*	.412		.20	.35
(0.91)		(2.90)	(1.02)			
−.010		.829*	−.009	1.289	.20	.35
(0.87)		(2.91)	(0.02)	(1.81)		
		Treasury bill rate				
−.000	.267*				.03	.08
(0.12)	(4.74)					
−.000		.299*	.204*		.03	.08
(0.10)		(4.32)	(2.08)			
.000		.297*	−.066	.789*	.04	.08
(0.02)		(4.33)	(0.56)	(4.55)		

*Indicates statistical significance at the 5 percent level.

alternative interpretation appears to have little merit.[28]

The Discount Rate As a Penalty Rate

Another way of estimating the direct effect of a discount rate change on market interest rates comes from noting that depository institutions have little incentive to borrow from the Federal Reserve when the discount rate is a "penalty rate," that is, when it is above the federal funds rate. Depository institutions that borrow from the Federal Reserve when the discount rate is a penalty rate are assumed to do so for reasons other than to minimize the explicit cost of obtaining reserve-adjustment funds. Changes in the discount rate that come when the discount rate is a penalty rate — especially changes that leave the discount rate at penalty levels — should have no effect on borrowing and, hence, no direct effect on market interest rates.[29] If estimates indicate that discount rate changes made when the discount rate is not a penalty rate do not have an effect on market rates, while those made when the discount rate is a penalty rate do have a significant effect, this would be further evidence that there is no direct effect of a discount rate change on market interest rates.

To test this hypothesis, discount rate changes were partitioned into those when the discount rate was a penalty rate (ΔDRP) prior to the announcement and those when the discount rate was not a penalty rate (ΔDRNP).[30] The results, reported in table 4, indicate that changes made when the discount rate was a penalty rate are statistically significant.[31] Furthermore,

[28]Their "forecasts," however, are based on in-sample results and are not true *ex ante* forecasts.

[29]While this idea is common in the literature, e.g., Broaddus and Cook (1983) and Sellon and Seibert (1982), it is sometimes presented in such a way that it appears that the only effect is the direct effect. In this case, any finding of a significant effect of a discount rate change on market interest rates implies that it is produced via the direct effect. We have shown, however, this is not the case.

[30]The partition used was based upon whether the discount rate was a penalty rate with respect to the federal funds rate. There was only one instance when the discount rate was a penalty with respect to the T-bill rate. Such a partition is of little interest, however, since the evidence in footnote 15 indicates that the federal funds rate is the relevant opportunity cost variable.

[31]Sellon and Seibert (1982) performed a similar analysis on data for the period from February 1980 to August 1982 and found that discount rate changes made when the discount rate was a penalty rate had no statistically significant effect on market interest rates or borrowings. During this period, however, such discount rate changes were primarily those made for technical reasons; thus it appears that the Sellon and Seibert result is due to this fact and not to the fact that the discount rate was at a penalty level at the time of the change. See Thornton (1982) for the technical vs. nontechnical results over a similar period.

changes made when the discount rate was not a penalty rate were not statistically significant. These results are precisely the opposite of those that should have been obtained if the effect of a discount rate change, reported in table 3, were due to a direct effect.

Evidence on the Announcement and Policy Effects

The evidence indicates that discount rate changes do not directly affect market interest rates. Consequently, the effect on market rates indicated in table 3 must be due to an announcement effect, a policy effect or both. Because the effect measured in table 3 occurs on the day following the announcement of a change in the discount rate and changes made for technical reasons have no effect on market rates, this strongly suggests that it is, at least in part, an announcement effect. It is impossible to determine, however, whether the expectations were subsequently validated by changes in the rate at which the Federal Reserve supplied reserves.[32]

Attempts made to test directly for a policy response following a discount rate change were inconclusive.[33] Nevertheless, some evidence bears on the policy effect, at least in terms of its implications for the period following the October 1982 change in the Federal Reserve's operating procedure. First, if the Fed's new operating procedure attempts to maintain a constant spread between the federal funds and discount rate, borrowings always should be close to their assumed level. Chart 3 plots the actual level of adjustment plus seasonal borrowings and their assumed level for weekly data from October 6, 1982, through December 1985. As the chart shows, the actual level of borrowing often deviates from the initial borrowing assumption, sometimes by a considerable magnitude. Two of the most notable deviations, of course, occurred in mid-1984 and November 1985. Even when these unusual periods are ignored, the average absolute deviation of borrowings from the initial borrowing assumption is $226 million, over 40 percent of the average level of the initial borrowing assumption during the period.

Table 4

Estimates of Equation 4 for Penalty and Non-Penalty Discount Rate Changes

Constant	ΔDRP	ΔDRNP	$\bar{R}^2$	SE
		Federal funds rate		
−.010	.741*	.588	.20	.35
(0.93)	(2.58)	(1.46)		
		Treasury bill rate		
−.000	.372*	.060	.03	.08
(0.04)	(5.41)	(0.62)		

*Indicates statistical significance at the 5 percent level.

Furthermore, there is a tendency for the initial borrowing assumption to follow, rather than lead, changes in actual borrowings. It appears that the federal funds/discount rate spread is maintained when the borrowing assumption changes; the demand for borrowed reserves is not forced to conform to the borrowing assumption.

Second, if the policy effect is strong, the response of market interest rates, especially the federal funds rate, to a change in the discount rate should be larger since the October 1982 change in the operating procedure. To test this, equation 4 was reestimated for the period from October 1, 1979, to June 11, 1986, and the response of market interest rates to nontechnical changes in the discount rate was allowed to be different for the periods October 1, 1979, to October 5, 1982, and October 6, 1982, to June 11, 1986. The results are reported in table 5 with the coefficients for the pre- and post-October 1982 periods denoted by ΔDRNTPRE82 and ΔDRNTPOST82, respectively.[34]

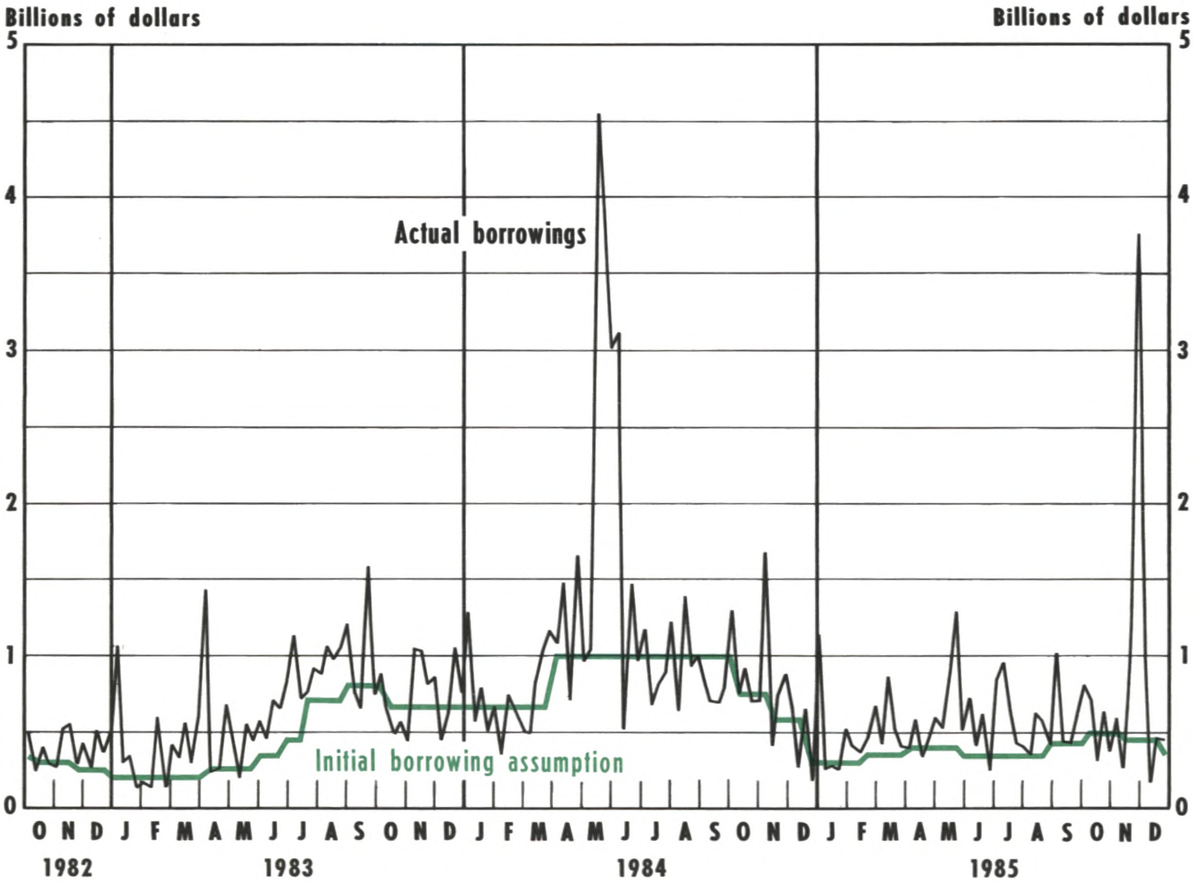
[32]Alternatively, Smirlock and Yawitz (1985) allow for the change in the discount rate to impact market interest rates with a lag of up to five days. Because they cannot reject the hypothesis that effects past the initial day are significant, they conclude that the rapid adjustment is consistent with market efficiency. Because the market rates nearly always return to levels prior to discount rate changes, however, it is possible to find no statistically significant long-run effect simply by making the lag "long enough" or a permanent effect (as they found) by making it "short enough."

[33]Several attempts to directly test various hypotheses were conducted, but the results were unsatisfactory. For example, discount rate changes that indicate a change in policy — regardless of the reason given for the change — should be followed by a sharp change in the growth of nonborrowed reserves. Hence, statistical tests of nonborrowed reserve growth before and after discount rate changes were undertaken. Because the nonborrowed reserve data only are available biweekly, the tests were also done using weekly M1 data. The results indicated no statistically significant change in the growth rate of either nonborrowed reserves or M1; however, the data were highly variable and the observations few. Hence, these tests should be considered inconclusive.

[34]Because of the differences in the variation of the dependent variables between the periods, the equation was estimated adjusting for heteroskedasticity. Also, the pre-October 1982 period includes a surcharge variable because Thornton (1982) has shown the results are sensitive to this modification. While not reported here, the surcharge coefficient is nearly identical to that reported by Thornton. The coefficient on ΔDRNTPRE82 differs from that reported by Thornton primarily because of a difference in the sample period; however, all of the qualitative conclusions are the same.

Chart 3

Adjustment plus Seasonal Borrowings from Federal Reserve and Initial Borrowing Assumption



The results show that the responsiveness of the federal funds rate to changes in the discount rate was essentially the same during the two periods. Indeed, an F-test of the equality of the two coefficients does not reject the hypothesis that the response was the same. There is a statistically significant difference in the responsiveness of the T-bill rate; however, it has become less, not more, responsive to changes in the discount rate. The evidence suggests that the shift in the Fed's operating procedure has not increased the initial response of market interest rates to discount rate changes; if anything it appears to have lowered it.

Finally, there is one additional piece of evidence on the announcement vs. policy effect of a discount rate change. The effect of the discount rate on market interest rates, especially the policy effect, implies causality running from the federal funds rate to other market interest rates. In order to investigate this, tests of "Granger causality" were conducted using both daily and weekly data for the federal funds and three-month Treasury bill rates. These tests are designed to determine whether changes in one rate precede or follow changes in the other. (Details and results are presented in the appendix.) The results using the daily data indicate that changes in the T-bill rate *precede* changes in the federal funds, the reverse of what the policy-effect hypothesis would most strongly imply. The results using weekly data are less definitive, indicating that at times either rate precedes the other. While this result is not particularly surprising, the fact that the stronger (most statistically significant) effect is from the T-bill rate to the federal funds rate is inconsistent with a strong policy effect.

While these results are disquieting to those who support the policy effect, they are not conclusive. The importance one assigns to the announcement or pol-

Table 5

Estimates of Equation 4 with the Discount Rate Partitioned into the Pre- and Post-October 1982 Periods

Dependent Variable	Constant	ΔDRT	ΔDRNTPRE82	ΔDRNTPOST82	F-Test[1]	$\bar{R}^2$	SE
Federal funds rate	−.006	.382	.824*	.779*	.013	.14	1.01
	(0.54)	(1.39)	(2.85)	(2.71)			
Treasury bill rate	−.001	.129	.686*	.292*	10.206*	.04	1.00
	(0.23)	(1.68)	(6.57)	(4.43)			

*Indicates statistical significance at the 5 percent level.
[1]Test of the hypothesis that the coefficients on ΔDRNTPRE82 and ΔDRNTPOST82 are equal.

icy effects depends on the interpretation of a discount rate change. If it is believed that discount rate changes are primarily signals that the Federal Reserve is going to continue its present policy of ease or restraint, the policy effect should be nil. If, on the other hand, discount rate changes typically signal a change in the rate at which the Federal Reserve is going to supply reserves to the system, the extent to which one believes this change will affect market interest rates depends on one's view of the liquidity effect. If the liquidity effect is believed to be weak and transient — as most empirical work suggests — the response of the market to such changes is essentially noise, with no real significance for the future course (or level) of market interest rates. In such instances, discount rate cuts that are followed by more expansionary monetary policy ultimately might be followed by higher, not lower, interest rates if such a policy change gives rise to expectations of higher inflation. On the other hand, if one believes that the liquidity effect is strong and lasting, changes in the discount rate will be thought to have permanent effects on market interest rates, but only if followed by a change in Federal Reserve policy.

CONCLUSIONS

This article was intended to clarify the relationship between the Federal Reserve's discount rate and market interest rates. Three distinct, though not mutually exclusive, potential effects of a discount rate change on market interest rates were outlined: (1) the "direct, *ceteris paribus*, effect," which abstracts from market reactions to the discount rate change and any subsequent change in Federal Reserve operations; (2) the "announcement effect," which reflects the changing expectations of the Federal Reserve's activity based on the announced change in the discount rate; and (3) the "policy effect," the impact of a subsequent change in Federal Reserve activity on the market. Special attention was given to the hypothesis that the impact of discount rate changes on market interest rates became stronger following the Federal Reserve's switch from a nonborrowed reserve to a borrowed reserve operating procedure in October 1982.

The evidence showed a statistically significant effect of a change in the discount rate on both the federal funds and Treasury bill rates immediately following the discount rate change. A series of tests provided evidence, consistent with the theory, that the direct effect of a discount rate change is nil. Consequently, the impact of a discount rate change on market rates is due to an announcement effect, a policy effect or both. The rapidity with which market rates respond to the discount rate change suggests that the announcement effect is operative. Furthermore, some indirect tests of the policy effect produced results that are inconsistent with it, suggesting that discount rate changes have had no permanent effect on market interest rates.

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(See appendix on next page.)

Appendix

Tests of Granger Causality

Tests of "Granger causality" are really tests of temporal ordering of time series. The test of causality running from the federal funds rate to the Treasury bill rate is performed by estimating, using ordinary least squares (OLS), the equation

$$\Delta R_{Tt} = \alpha_0 + \sum_{i=1}^{K} \delta_i \Delta R_{Tt-i} + \sum_{i=1}^{K} \mu_i \Delta R_{ft-i},$$

where Δ denotes the first difference operator, i.e., $\Delta R_{ft} = R_{ft} - R_{ft-1}$, and R_f and R_T denote the federal funds and three-month Treasury bill rates, respectively. The procedure consists of testing the hypothesis that $\mu_1 = \mu_2 = \ldots = \mu_K = 0$. If this hypothesis is rejected, it is said the "causality" runs from the federal funds rate (R_f) to the three-month Treasury bill rate (R_T). To test for causality running from the Treasury bill rate to the federal funds rate, the equation

$$\Delta R_{ft} = \beta_0 + \sum_{i=1}^{K} \lambda_i \Delta R_{ft-i} + \sum_{i=1}^{K} \varepsilon_i \Delta R_{Tt-i}$$

Table A.1

Granger Causality Results for ΔR_f and ΔR_T: Daily Data

Tests of μ's = 0

Lags of ΔR_f	Lags of ΔR_T: 1	2	3	4	5	6	7	8	9	10	11	12
1	.342	.355	.358	.382	.377	.378	.346	.346	.338	.342	.342	.341
2	.624	.610	.616	.646	.646	.647	.583	.585	.575	.586	.587	.585
3	.570	.524	.481	.519	.526	.527	.486	.497	.498	.512	.513	.512
4	.678	.647	.617	.677	.681	.682	.651	.662	.660	.675	.675	.675
5	.775	.739	.707	.741	.715	.716	.672	.685	.683	.675	.676	.675
6	.707	.682	.666	.713	.709	.704	.695	.704	.694	.686	.686	.684
7	.718	.706	.696	.751	.754	.746	.650	.653	.634	.645	.646	.639
8	.494	.480	.457	.492	.473	.471	.462	.436	.439	.428	.429	.423
9	.339	.325	.305	.311	.291	.292	.317	.284	.246	.218	.218	.219
10	.267	.242	.223	.223	.197	.198	.250	.216	.171	.185	.184	.186
11	.238	.227	.217	.220	.198	.199	.237	.213	.178	.178	.172	.172
12	.211	.208	.205	.218	.199	.200	.217	.199	.176	.168	.159	.163

Tests of ε's = 0

Lags of ΔR_T	Lags of ΔR_f: 1	2	3	4	5	6	7	8	9	10	11	12
1	.681	.379	.304	.195	.173	.107	.102	.098	.059	.061	.057	.060
2	.837	.581	.419	.249	.167	.097	.054	.050*	.031*	.032*	.026*	.026*
3	.597	.372	.469	.386	.300	.198	.117	.106	.068	.070	.058	.055
4	.640	.409	.462	.540	.453	.310	.166	.147	.084	.087	.071	.064
5	.524	.288	.302	.293	.437	.397	.256	.235	.145	.149	.125	.114
6	.625	.382	.385	.360	.474	.524	.338	.303	.174	.178	.146	.132
7	.673	.476	.480	.466	.590	.639	.377	.315	.166	.168	.135	.116
8	.686	.526	.552	.540	.676	.733	.482	.408	.220	.222	.177	.152
9	.770	.620	.648	.641	.765	.809	.563	.477	.299	.301	.246	.213
10	.792	.634	.654	.659	.799	.835	.638	.560	.381	.382	.317	.276
11	.850	.714	.734	.740	.859	.878	.707	.627	.435	.434	.381	.343
12	.787	.633	.649	.628	.745	.777	.658	.579	.424	.425	.384	.319

*Indicates significance at the 5 percent level.

Table A.2
Granger Causality Results for ΔR_f and ΔR_T: Weekly Data

Tests of μ's = 0

Lags of ΔR_f	Lags of ΔR_T: 1	2	3	4	5	6	7	8	9	10	11	12
1	.269	.258	.242	.314	.319	.312	.361	.415	.352	.339	.341	.434
2	.538	.505	.493	.580	.570	.564	.615	.673	.649	.632	.635	.736
3	.291	.337	.423	.466	.477	.453	.501	.536	.567	.482	.485	.525
4	.325	.374	.512	.602	.613	.584	.617	.648	.677	.607	.606	.632
5	.209	.248	.352	.501	.535	.531	.549	.590	.648	.593	.596	.544
6	.025*	.031*	.053	.086	.086	.051	.066	.077	.058	.056	.057	.058
7	.028*	.034*	.056	.084	.085	.066	.092	.108	.085	.085	.087	.087
8	.038*	.047*	.077	.113	.112	.088	.108	.131	.117	.116	.118	.118
9	.061	.074	.115	.162	.161	.130	.158	.187	.162	.164	.166	.161
10	.074	.088	.136	.169	.170	.147	.185	.216	.225	.225	.228	.221
11	.092	.109	.161	.205	.210	.192	.241	.275	.293	.297	.299	.292
12	.099	.117	.170	.215	.209	.207	.265	.294	.346	.356	.361	.370

Tests of ε's = 0

Lags of ΔR_T	Lags of ΔR_f: 1	2	3	4	5	6	7	8	9	10	11	12
1	.073	.031*	.022*	.039*	.041*	.038*	.046*	.041*	.046*	.045*	.042*	.054
2	.181	.097	.070	.115	.123	.115	.134	.123	.137	.136	.128	.157
3	.043*	.029*	.008*	.021*	.024*	.020*	.024*	.029*	.080	.078	.077	.087
4	.027*	.016*	.005*	.015*	.018*	.015*	.017*	.022*	.069	.071	.065	.071
5	.045*	.024*	.007*	.018*	.021*	.015*	.017*	.022*	.063	.065	.066	.111
6	.040*	.022*	.009*	.021*	.024*	.021*	.024*	.030*	.071	.073	.067	.167
7	.045*	.027*	.012*	.027*	.031*	.028*	.032*	.044*	.103	.107	.103	.161
8	.027*	.017*	.007*	.017*	.018*	.016*	.019*	.029*	.101	.104	.107	.103
9	.044*	.027*	.013*	.028*	.030*	.027*	.030*	.045*	.149	.153	.158	.138
10	.062	.036*	.014*	.030*	.033*	.030*	.034*	.049*	.144	.141	.157	.108
11	.044*	.028*	.013*	.026*	.027*	.025*	.028*	.041*	.115	.109	.110	.146
12	.063	.041*	.020*	.038*	.041*	.037*	.042*	.059	.150	.143	.146	.183

*Indicates significance at the 5 percent level.

is estimated and the hypothesis that $\varepsilon_1 = \varepsilon_2 = \ldots = \varepsilon_K = 0$ is tested. If the hypothesis is rejected, the causality runs from the Treasury bill rate to the federal funds rate. If the hypotheses concerning the μ's and the ε's are both rejected, there is said to be bidirectional causality between the rates. If neither is rejected, the series are said to be independent.

The tests were performed using both daily and weekly data. Because the test results are quite sensitive to the order of the lag, K, the tests were performed on all orders up to K=12.[1] The significance levels corresponding to the F-statistics for all orders are presented in tables A.1 and A.2 for the daily and weekly data, respectively.

The tests using daily data show unidirectional causality from R_T to R_f, the opposite of what is required for policy actions to be transmitted from the federal funds rate to other market interest rates. It should be noted that the daily federal funds rate series exhibits considerably more variability than the T-bill rate series. When these data are smoothed by averaging over a week, the tests indicate bidirectional causality; however, the stronger relationship appears to be running from the T-bill rate to the federal funds rate.

[1]For a discussion of this procedure, see Thornton and Batten (1985).

A Microeconomic System-Wide Approach to the Estimation of the Demand for Money

Salam K. Fayyad

A STABLE demand-for-money relationship is a necessary condition for the viability of a monetary policy based on the use of monetary aggregates as intermediate policy targets. In recent years, standard money-demand formulations have exhibited large shifts that remain largely unexplained today despite extensive research efforts devoted to determining the reasons for these shifts.

This paper presents an alternative to the standard single-equation method of estimating the demand for money. The alternative, called the microeconomic system-wide approach to demand analysis, differs in several fundamental ways from the usual money-demand specification. The purpose of this article is to show how the system-wide approach can be applied to estimating the demand for money. The results indicate that in-sample predictions made using this approach closely track the actual data over the 1969–85 period.

THE STANDARD MONEY DEMAND FORMULATION: A BRIEF REVIEW

Over the past three decades, most demand-for-money studies have employed similar specifications. Typically they use income (as a transaction variable) and one or more (typically two) interest rates (to capture the effect of the opportunity cost of holding money) as explanatory variables; the dependent variable is generally the stock of real M1 balances.

The wide acceptance of the standard money demand specification is understandable. It embodies a proposition, which, since Keynes' *General Theory*, has constituted a key tenet of the received wisdom on the demand for money: the desire to hold money balances is directly related to the need to conduct transactions and inversely related to the opportunity cost of holding money balances. In addition, it performed remarkably well in a statistical sense. The coefficients of the explanatory variables had "sensible" signs and magnitudes, and the estimated model fit the data very well.

The disquietude accompanying Goldfeld's (1976) discovery that his standard formulation of the demand-for-money function began in 1973 to systematically overpredict the real money balances underscores the importance that has been attached to the stability, and hence predictability, of the demand for money. It is not surprising that the reported shift in Goldfeld's specification, or what, after 1976, became generally known as the "case of the missing money," instigated a seemingly tireless search for a verifiable explanation of what happened.[1]

A review of the vast literature devoted to finding the reasons for the shift in money demand reveals that these studies are largely unsuccessful in accounting

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[1]A recent study suggests that the demand-for-money function has undergone shifts in the periods III/1962, IV/1973, IV/1979, and I/1980. See Mizrach and Santomero (1986).

for it. For example, Laidler has commented that:

> The first thing to be said . . . is whatever else they do, they do not rescue the demand for M_1 function from the suspicion of instability. . . . [T]he often unsatisfactory results . . . indicate that further work is required, rather than that the line of inquiry that they represent should be abandoned.[2]

The inconclusiveness of the evidence on what caused the standard money-demand specification to shift in 1973 can be viewed as an indication that examining alternative approaches to the demand-for-money formulation might be useful. The alternative offered here is derived from a microeconomic system-wide approach to demand analysis.

A NEW APPROACH TO MODELING MONEY DEMAND: THE MICROECONOMIC SYSTEM-WIDE APPROACH

The basic premise which underlies any micro theoretic approach to consumer demand analysis is that the consumer maximizes a neoclassical utility function subject to a budget constraint.[3] A model consistent with both the principles of microeconomic theory and aggregation theory yields specific behavioral implications which can be tested using available data aggregated over goods and consumers. (Some criticisms of including money in this approach appear on page 24.)

This study uses a neoclassical utility function defined over five expenditure categories, two of which are presumed to capture the "monetary services" in the U.S. economy. By restricting the analysis to five expenditure categories, this study assumes the existence of a macro utility function that is weakly separable in these categories.[4]

The solution to the consumer choice problem when the utility function is defined over five goods is a system of five demand equations. In each equation, the quantity demanded of a specific good is expressed as a function of the total amount available for spending on all five goods, and (in the general case) of their prices. Naturally, the exact specification of these demand equations will depend on the specific form of the utility function chosen. This study, however, uses a general demand system consistent with the maximization of an arbitrary neoclassical utility function. Thus, while the system is subject to all the restrictions that economic theory implies, the results are invariant to the functional form of the utility function being maximized. This choice avoids the loss of generality which may result if a particular functional form is specified and permits testing of hypotheses about the structure of the utility function itself.

The microeconomic system-wide approach to demand analysis deals with the allocation of total spending among the individual goods considered. Thus, for the specific set of goods chosen, the explanatory variables in the demand system are the amount available for spending and the prices of these goods. This approach provides a convenient means for acquiring detailed information about utility-based attributes of goods; this information is readily available by inspecting the signs (and, of course, the statistical and economic significance) of estimated income, own- and cross-price parameters.

The demand model used in this study, the absolute price version of the Rotterdam model, was chosen primarily because the theoretical restrictions are readily expressed in terms of the model's parameters. This makes it relatively easy to impose and to test the validity of these restrictions.[5] Another attractive feature of the Rotterdam model is that it can be used to provide predictions of the value (budget) shares of the goods included in the analysis. These predictions can be used to compute measures of informational inaccuracies useful in assessing the performance of the demand system as a whole and the individual demand equations as well.

The monetary variables used are the real flow of monetary services provided by various monetary assets, not the simple sum of the real stocks of monetary aggregates generally used in standard money-demand analysis. A measure of the monetary-service flows was obtained by evaluating the stocks of monetary assets at their corresponding user-cost prices (see the discussion of the data below). The user-cost price of each monetary asset is the difference between the interest

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[2]See Judd and Scadding (1982), p. 1014.

[3]A neoclassical utility function is one that is continuously twice differentiable and quasiconcave with positive marginal utility everywhere.

[4]A neoclassical utility function is weakly separable in a block of goods if and only if the marginal rate of substitution between any two goods inside the block is independent of consumption outside that block. While this separability assumption may seem overly restrictive, it is actually less restrictive than that maintained by studies in which money is considered to be the sole argument in the utility function. See, for example, Ewis and Fisher (1984).

[5]In the aggregated-over-consumers version of the model derived by Barnett (1979), the macro parameters are subject to the same restrictions as their micro counterparts. (See footnote 9.)

Can Money Be Included in a Microeconomic System-Wide Demand Model?

The applicability of the theoretical restrictions in the general case of demand for money and goods has been questioned. In fact, if, as in the Samuelsonian tradition, the utility-based analysis of the demand for money is handled by putting money and prices in the utility function, then, by the strong results produced by Samuelson and Sato (1984), the restrictions in question, *as they pertain to the demand for goods,* are simply unattainable. While potentially disquieting, the Samuelson-Sato results are not unqualifiedly binding. Indeed, these results are founded in the view, long espoused by Samuelson, that, in connection with the inclusion of money in the preference structures, money is wanted solely for the purposes of facilitating transactions. As Samuelson (1983), p. 117, states:

> In this connection, I have reference to none of the tenuous concepts of money, as a numeraire commodity, or as a composite commodity, but to money proper, the distinguishing features of which are its indirect usefulness, not for its own sake but for what it can buy, its acceptability, its not being "used up" by use, etc., etc.

This is the rationale behind Samuelson's inclusion of prices and money in the utility function specified to be homogeneous of degree zero in both money and prices. It is precisely this formulation to which the Samuelson-Sato results pertain.

One could argue that Samuelson's view of what money is wanted for is unduly restrictive. In fact, of the assets currently regarded as potential sources of monetary services in the U.S. economy (see table 1), only a few are "generally acceptable in exchange."[1] Furthermore, the supposition, based on Samuelson's view, that money cannot properly be treated like other commodities can also be questioned. Households consume the services provided by various expenditure categories ostensibly because of the utility they derive from these expenditure categories: in general, little, if any, effort is directed toward deciphering the nature of the utility involved in those cases. By the same token, it can be maintained that money is held because of the utility it provides, without having to speculate as to whether that utility derives from money's "general acceptability in exchange," the serenity its holders experience by holding it, or from any other knowable or even unknowable attribute.[2] Indeed, if money can be treated like other goods, then it can be included in the utility function in precisely the same manner as any other good. In that case, the Samuelson-Sato results would not apply, and one could thus impose or test for any of the restrictions implied by economic theory.

Interestingly, even within the Samuelson-Sato framework, the theoretical restrictions would not be unattainable if the utility function were weakly separable in the block of goods (see Samuelson and Sato (1984), pp. 592–95). Hence, it is legitimate to impose or test for any of the restrictions implied by theory if one assumes, or, even better, tests for and (where applicable) imposes blockwise weak separability in goods. The latter was done in this study, since weak separability in the block of goods could not be statistically rejected.[3]

[1] In this study, the Federal Reserve's definition of monetary assets was taken as given. The use of the Fed's definitions does not mean that the list of assets which appears in table 1 includes all assets that provide monetary services in the U.S. economy or, for that matter, that all assets included in the list provide such services.

[2] Of course, this amounts to suggesting that money is held for the "moneyness" of it. While tautological, this statement can be made operational by hypothesizing that, on the margin, the extent to which income is forgone when monetary assets are held is a measure of the moneyness that these assets possess. The gain that is realized by adopting this hypothesis is considerable; not only does it play a key role in the measurement of the flow of monetary services in terms of readily observable data, it also inherently captures the various degrees of moneyness provided by various monetary assets.

[3] The manner in which weak separability was tested for is discussed at length in Fayyad (1986), chapter 4. A preliminary draft of a paper on this subject is available on request.

rate paid on that asset and the maximum available holding-period yield.[6]

THE MODEL

For n goods, the (discrete-time) absolute price version of the Rotterdam model is given by

$$(1)\quad w^*_{it}\, Dx_{it} = \mu_i\, Dm^*_t + \sum_{j=1}^{n} \pi_{ij}\, Dp_{jt} + \varepsilon_{it} \qquad i=1, ..., n.$$ [7]

The x's and p's denote the quantities and prices of the various goods, respectively, and the subscript t indexes time. D is the log-change operator; thus $Dx_{it} = \Delta(\log x_{it})$. w_i denotes the expenditure (value) share of the ith good. $w^*_{it} = (w_{i,t-1} + w_{it})/2$ is that good's average value share over two successive time periods. Thus, the dependent variables in the model are the (average) share-weighted growth rates of the quantities of goods. The explanatory variables are the growth rates of real income (m^*_t) and prices.[8] The last term in the system of demand equation 1, ε_t, denotes the demand disturbance. The properties of this term are discussed in appendix A in conjunction with the estimation procedure employed in this study.

The parameters of the model are μ_i and π_{ij}. μ_i $(= p_i \frac{\partial x_i}{\partial m})$ is the *marginal* budget share of the ith good. π_{ij} $(= \frac{p_i p_j}{m} \frac{\partial x_i}{\partial p_j}, (j = 1, ..., n))$ is the ith good's price coefficient. Under the standard assumptions of economic theory, if the consumer maximizes a neoclassical utility function subject to a budget constraint, then the above parameters satisfy the following constraints:

$$(2)\quad \sum_i \mu_i = 1,$$

$$(3)\quad \sum_j \pi_{ij} = 0,$$

(4) $[\pi_{ij}]$ is symmetric and negative semidefinite.[9]

In the estimation procedure employed in this study, the constraints $\Sigma\mu_i = 1$, $\Sigma_j\pi_{ij} = 0$, and $[\pi_{ij}] = [\pi_{ji}]$ were imposed. The negative semidefiniteness of the $[\pi_{ij}]$ matrix was not imposed; however, it was checked for.

THE DATA

The data consist of U.S. quarterly time series of expenditures on, and prices of, food, nondurables, services and two blocks of monetary assets for the period I/1969–I/1985. Together, the two blocks of monetary assets, M1 and ABM1, comprise the 27 assets that the Federal Reserve Board currently recognizes as potential sources of monetary services in the U.S. economy. M1 is the narrow monetary aggregate, consisting of currency and total checkable deposits. ABM1 consists of the non-M1 monetary assets shown in table 1.

Data on the first three commodity groups (food, nondurables and services) were obtained as follows: A time series on the price of each commodity group (p_{it}) was generated from available time series on current-dollar and (1972) constant-dollar consumption expenditures ($p_{it}\, q_{it}$ and $p_{i,72}\, q_{it}$, respectively) and the identity $(p_{it}/p_{i,72}) = (p_{it}\, q_{it}/p_{i,72}\, q_{it})$. Per-capita constant-dollar expenditures in each quarter were then obtained by dividing the aggregate constant-dollar expenditures by the corresponding mid-quarter population size (N_t). Thus, in terms of the variables which appear in the estimated system, $x_{it} = \frac{1}{N_t} p_{i,72}\, q_{it}$.

One can generate the data on the quantities and prices of M1 and ABM1 monetary services as follows:

(1) Convert the nominal balances of monetary assets into real balances by deflating the former by the "true cost-of-living index." In this study, this index was the geometric mean of the Consumer Price Index and the Commerce Department's implicit

[6]The maximum available holding-period yield is the highest yield of those available either on the monetary assets or on Baa-rated bonds.

[7]A detailed discussion of the model's derivation and applications can be found in Theil (1971, 1975, 1976, 1980), Barten (1969), and Barnett (1979, 1981).

[8]In this study, the terms "expenditure" and "income" are used interchangeably. When the latter is used, however, it means "full income," that is, income augmented by expenditure on the monetary assets that are included in this study. In the estimation procedure employed in this study, Dm^*_t is replaced by Dx_t, where $Dx_t = \sum_i w^*_{it} Dx_{it}$. See Theil (1971), pp. 331–32.

[9]A question may arise as to whether these restrictions are applicable, given that the data are aggregated over both goods and consumers. Insofar as goods are concerned, Hicks' composite commodity theorem can be used, assuming that each of the commodity groups is an elementary good. Resolving the more formidable issue of aggregation over consumers requires using the aggregation results produced by Barnett (1979). In his aggregated-over-consumers absolute price version of the Rotterdam model, Barnett treats the macrocoefficients, μ and π, as population versions of weighted average microcoefficients, with the weights proportional to corresponding incomes. He then shows that the macrocoefficients have the same properties as their micro counterparts, μ_i and π_{ij}.

Table 1

Potential Monetary Assets

Component	Asset Description
1	Currency and traveler's checks
2	Demand deposits held by households
3	Demand deposits held by business firms
4	Other checkable deposits less Super NOW accounts
5	Super NOW accounts at commercial banks
6	Super NOW accounts at thrifts
7	Overnight repurchase agreements
8	Overnight Eurodollars
9	Money market mutual fund shares
10	Money market demand deposit accounts at commercial banks
11	Money market demand deposit accounts at thrifts
12	Savings deposits less MMDAs at commercial banks
13	Savings deposits less MMDAs at savings and loans
14	Savings deposits less MMDAs at mutual savings banks
15	Savings deposits less MMDAs at credit unions
16	Small time deposits and retail repurchase agreements at commercial banks
17	Small time deposits and retail repurchase agreements at thrifts
18	Small time deposits at credit unions
19	Large time deposits at commercial banks
20	Large time deposits at thrifts
21	Institutional money market mutual funds
22	Term repurchase agreements at commercial banks and thrifts
23	Term Eurodollars
24	Savings bonds
25	Short-term Treasury securities
26	Banker's acceptances
27	Commercial paper

price deflator for personal consumption expenditures.

(2) Evaluate the real balances of each monetary asset in the base period at its real user-cost price to obtain the real expenditure on that asset during the base period.[10]

(3) Sum the expenditures thus obtained over the components of M1 and ABM1.

(4) Compute the Törnqvist-Theil Divisia quantity indexes for M1 and ABM1 for the entire sample period.

(5) Set the base-period expenditures obtained in (3) equal to the respective quantity indexes computed in (4) and interpolate to acquire complete series on the real expenditures on the monetary services provided by M1 and ABM1.

(6) Construct Törnqvist-Theil Divisia price indexes of the user-cost prices of the respective components of M1 and ABM1.

THE ESTIMATION RESULTS

The maximum likelihood estimates of the parameters of the absolute price version of the Rotterdam model and the associated income and price elasticities are reported in table 2.[11] Estimates of the income coefficients (μ_i) are all positive and statistically significant at usual significance levels, indicating that the five commodity groups included in this study are normal goods.

The income elasticity of demand for M1 shown in table 2 (0.53) is similar to those reported in other studies. Moreover, it corresponds closely to its theoretical value of 0.50 implied by the Baumol (1952)–Tobin (1956) inventory — theoretic model of the transactions demand for money.

The own- and cross-price coefficients (π_{ij}) are generally estimated with less precision than the income coefficients. Consistent with the standard assumptions of economic theory, estimates of the (Slutsky) own-price coefficients are negative, although not all are statistically significantly different from zero at usual significance levels.[12]

[10]The user-cost price of money was derived by Barnett (1978). See also Barnett (1986) and (1981), chapter 7.

[11]The income elasticity of demand for the ith commodity group, μ_i, is given by $\mu_i = \frac{\mu_i}{w_i}$. This result can be verified by a simple manipulation of the definition $\mu_i = \frac{\partial x_i}{\partial m}$. On the other hand, the Hicks-Allen price elasticity of demand for the ith group, μ_{ij}, is given by $\mu_{ij} = \frac{\pi_{ij}}{w_i}$, which can also be verified by a simple manipulation of the definition $\pi_{ij} = \frac{p_i\, p_j}{m} \frac{\partial x_j}{\partial p_j}$.

[12]Negativity of the own-price coefficients, the diagonal elements in the $[\pi_{ij}]$ matrix, is a necessary, but not sufficient, condition for it to be negative semidefinite; a matrix is negative semidefinite if and only if all of its characteristic roots are nonpositive, and at least one root is zero. This property, which was not imposed in this study, was examined by computing the characteristic roots of the estimates of the $[\pi_{ij}]$ matrix in table 2. The computed characteristic roots are (0.0000, 0.0000, −0.0022, −0.0097, −0.1743); thus, the negativity condition is satisfied.

Table 2

Maximum Likelihood Estimates of the Absolute Price Version of the Rotterdam Model

Equation	μ_i	π_{ij}				
		Food	Nondurables	Services	ABM1	M1
Food	0.187083	−0.116241	0.058199	0.057593	0.000352	0.000097
	(6.682878)	(−7.380222)	(4.867328)	(3.167629)	(2.913345)	(1.779196)
Nondurables	0.291246	0.058199	−0.034326	−0.024572	0.000549	0.000150
	(9.685506)	(4.867328)	(−1.918331)	(−1.228305)	(2.129760)	(1.319010)
Services	0.467938	0.057593	−0.024572	−0.034144	0.000882	0.000242
	(10.304847)	(3.167629)	(−1.228305)	(−1.135707)	(1.596066)	(1.686502)
ABM1	0.042172	0.000352	0.000549	0.000882	−0.001399	−0.000384
	(2.090867)	(2.913345)	(2.129760)	(1.596066)	(−2.558969)	(−1.832747)
M1	0.011561	0.000097	0.000150	0.000242	−0.000384	−0.000105
	(3.043011)	(1.779196)	(1.319010)	(1.686502)	(−1.832747)	(−0.276950)

Note: t-ratios are in parentheses.

Average Income Elasticities

Food	Nondurables	Services	ABM1	M1
0.828044	1.390072	0.942341	1.027321	0.525813

Average Hicks-Allen Elasticities

	Food	Nondurables	Services	ABM1	M1
Food	−0.514493	0.257394	0.254911	0.001560	0.000428
Nondurables	0.277775	−0.163834	−0.117278	0.002619	0.000718
Services	0.115982	−0.049483	−0.068760	0.001775	0.000487
ABM1	0.008585	0.013365	0.021474	−0.034071	−0.009354
M1	0.004394	0.006841	0.010991	−0.017464	−0.004763

The Model's In-Sample Predictive Performance

As stated earlier, the Rotterdam model can be used to provide predictions of the value (budget) shares. The model's implied prediction of the share of the ith good at time t is given by

$$\hat{w}_{i,t+1} = w_{it} - e_{it},$$

where w_{it} is the actual value share of the ith good, and e_{it} is the residual of the ith demand equation at time t.

The in-sample predictive performance of the Rotterdam model can be evaluated in terms of its information-theory results; a general discussion of this method of assessing prediction accuracy is presented in appendix B. Computed measures of information (prediction) inaccuracies from the model are reported in table 3, along with information-inaccuracy measures for a naive (no-change) extrapolation of the value shares. The reported measures show substantial reductions in information inaccuracies when the model results are compared with the naive predictions.[13]

Further insight into the model's in-sample predictions may be gained by plotting the actual and predicted shares; this is done in charts 1–5. An inspection of these charts reveals that the model's in-sample predictions track the data extremely well; this is especially true for the M1 equation despite considerable variability in the actual shares of M1. These results suggest that the demand for M1, as derived in this

[13]In fact, in view of the greater variability of the shares of M1 and ABM1 relative to the shares of the other goods and services shown in charts 1–5, it is not surprising that predictions from the money equations "beat" the naive model by a larger margin than predictions from the other equations. In the presence of high period-to-period variation in the actual shares, the no-change naive model will always perform poorly.

Table 3

Average Information Inaccuracies[1]

	Rotterdam Model	Naive
System Results		
Uncorrected information inaccuracy	14.40	452.60
Information inaccuracy with d.f. correction	14.82	
Percent reduction from naive	96.73%	
Single Equation Results		
Food information inaccuracy	3.505	12.18
Percent reduction from naive	71.22%	
Nondurables information inaccuracy	4.141	11.95
Percent reduction from naive	65.35%	
Services information inaccuracy	5.943	34.54
Percent reduction from naive	82.79%	
ABM1 information inaccuracy	5.083	381.30
Percent reduction from naive	98.67%	
M1 information inaccuracy	0.646	51.30
Percent reduction from naive	98.74%	

[1]The information inaccuracies are to be multiplied by 10^{-4}.

Chart 1

Actual vs. Predicted Value Shares of Food

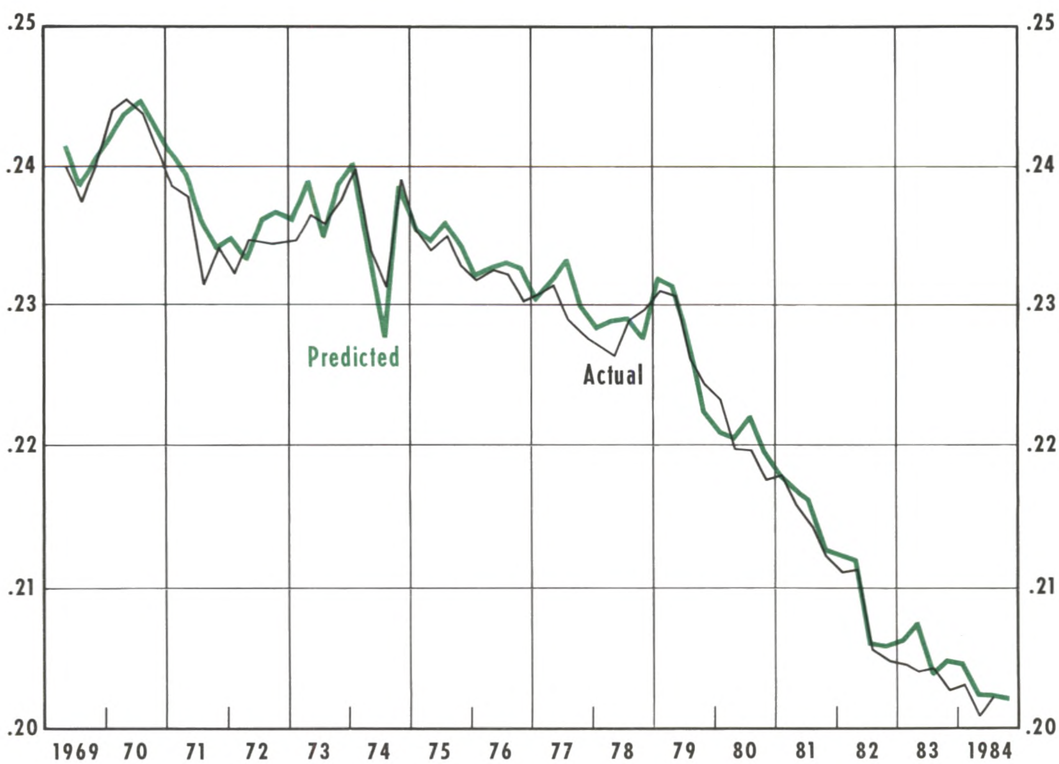


Chart 2

Actual vs. Predicted Value Shares of Nondurables

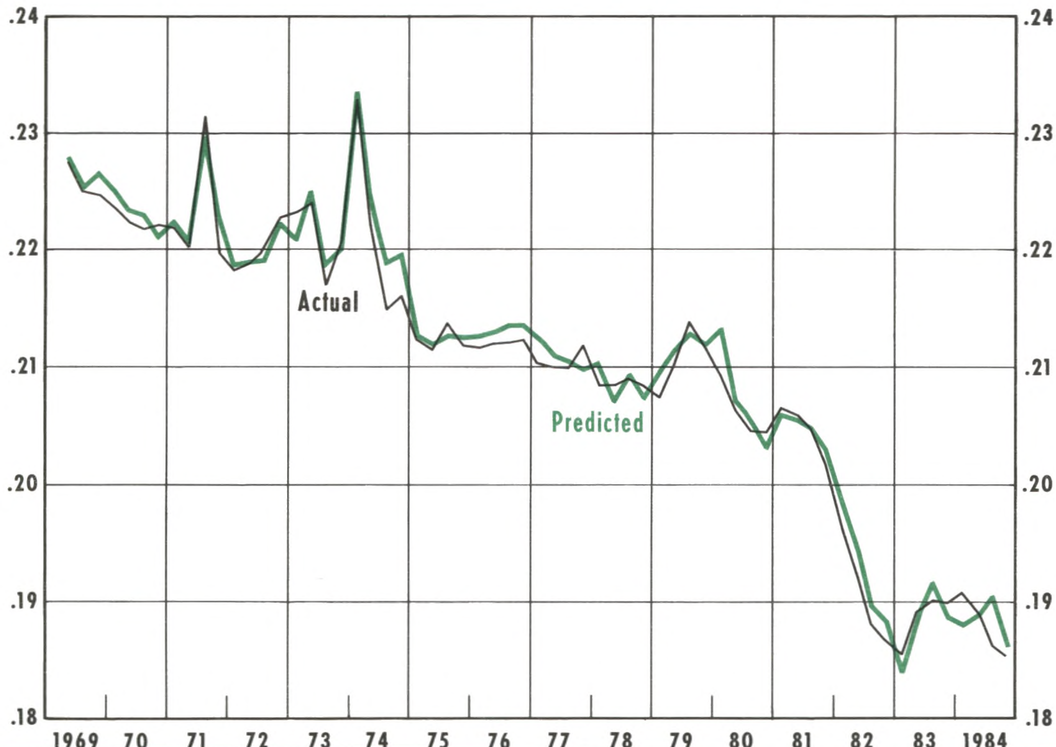


Chart 3

Actual vs. Predicted Value Shares of Services

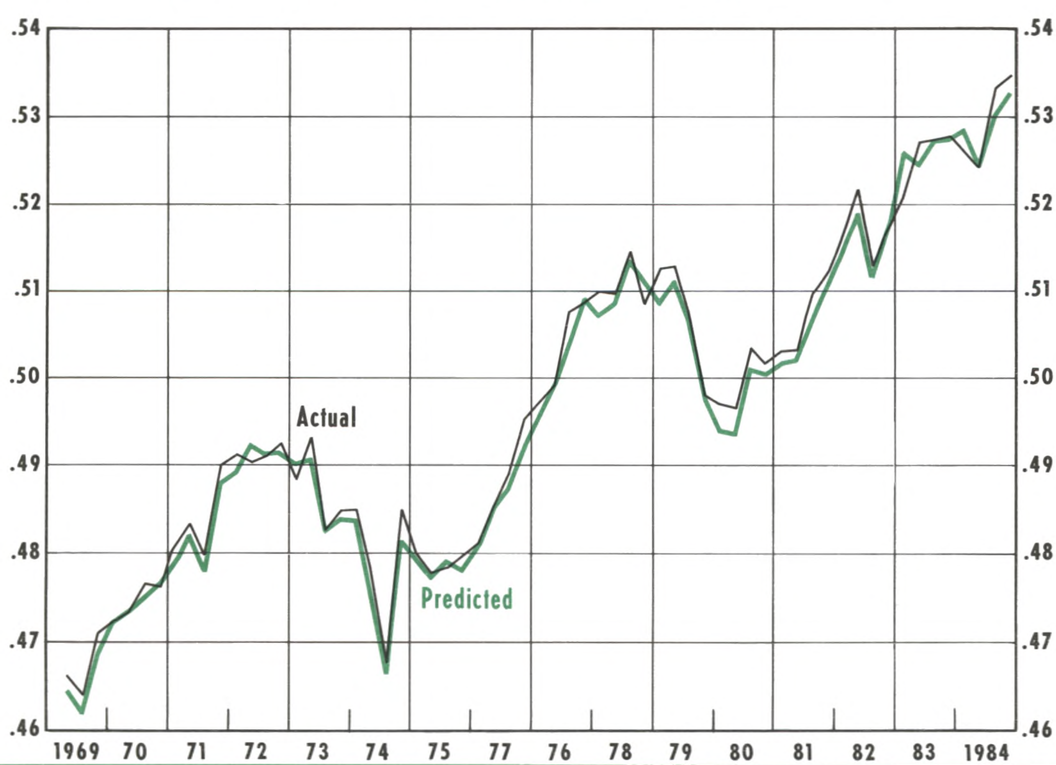


Chart 4

Actual vs. Predicted Value Shares of ABM1

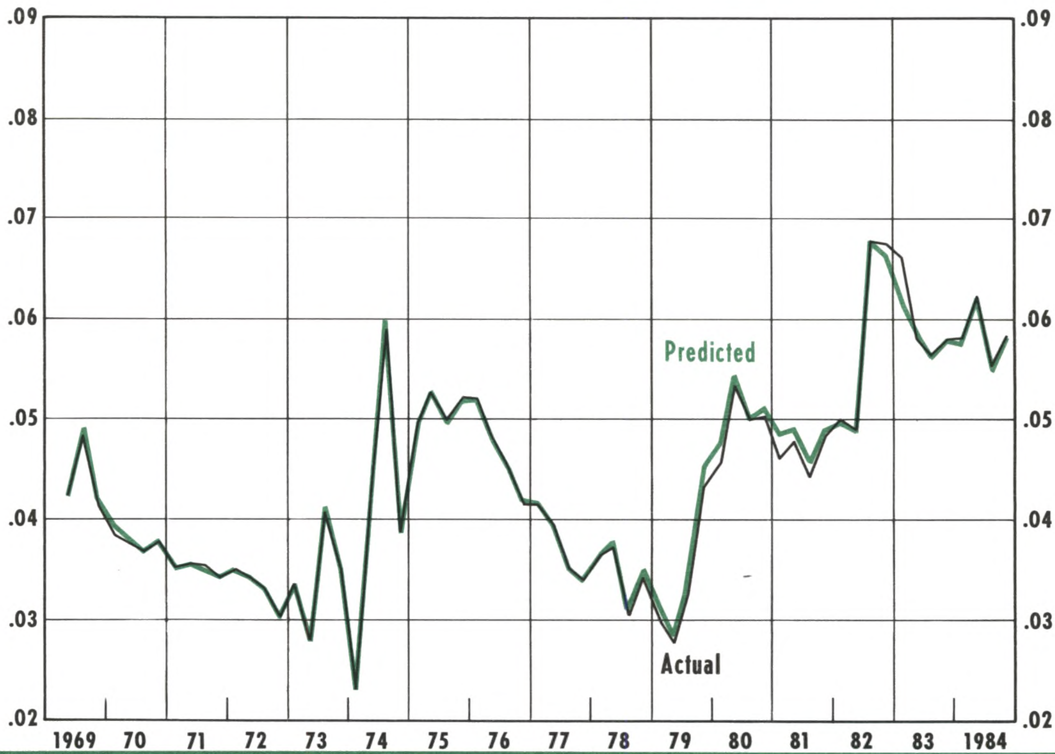
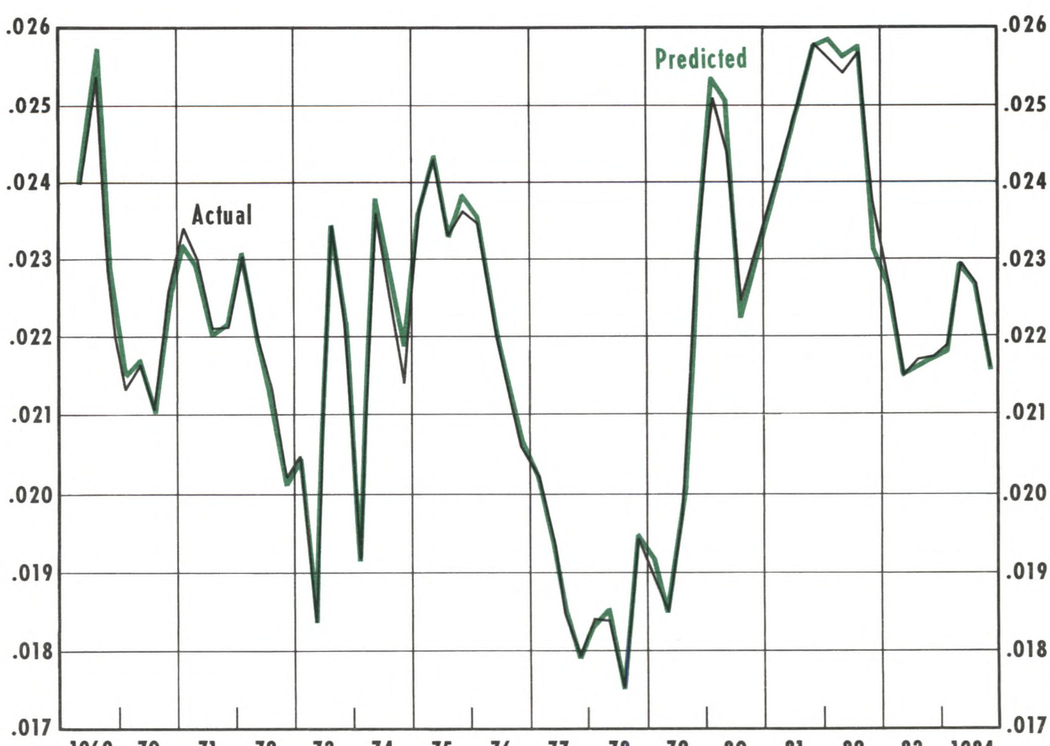


Chart 5

Actual vs. Predicted Value Shares of M1



study, was more stable than the demand for other goods, services and financial assets over the sample period.

CONCLUDING REMARKS

This study has discussed an approach to the estimation of the demand for money that relies on a methodology markedly different from that employed in the conventional money demand analysis. The approach is explicitly derived from the principles of microeconomic theory and emphasizes the importance of interaction among goods. The modeling process is not influenced by a search for "goodness of fit"; instead the emphasis is placed on the model's consistency with explicit utility-maximizing conditions.

The empirical results produced in this study show that it is possible to specify a model of money demand that closely tracks the actual behavior of the flow of M1's monetary services despite its considerable variability over this period. Thus, there seems to be nothing mysterious about that variability; it can be explained in terms of changes in relevant economic variables. These results indicate that money demand has been considerably more stable over the past two decades than standard money demand analysis has suggested.

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(See appendixes A and B on following pages)

Appendix A

In order to estimate the functional form of the demand equations, a stochastic version of that form should be specified and the disturbance terms interpreted. The system of demand equations can be written as follows,

$$(1)\quad X_{it} = \mu_i X_t + \sum_{j=1}^{5} \pi_{ij} P_{jt} + \varepsilon_{it}, \qquad i=1, ..., 5, \; t=1, ..., T,$$

where X_{it} $(= w^*_{it} Dx_{it})$ is a T-dimensional vector of observations on the left-hand-side variables of the ith commodity group, $P_{jt}(=Dp_{jt})$ is a T-dimensional vector of the log-change in the price of the ith commodity group, μ_i is the marginal budget share of the ith commodity group, $[\pi_{ij}]$ is a 5 x 5 matrix of the price coefficients, and X_t $(= \sum_{i=t}^{5} w^*_{it} Dx_{it})$ is a T-dimensional vector of the (budget-share) weighted sum of the log-change in expenditures on the five commodity groups.

The last term in equation 1, ε_{it}, is the disturbance term of the ith demand equation. The disturbance terms, $[\varepsilon_{it}]$, are assumed to capture the random effects of all variables other than income and all prices. The disturbance terms are further assumed to be normally distributed with mean zero and a variance-covariance matrix $\Sigma \otimes I_T$, such that

$$(2)\quad E(\varepsilon_{is}, \varepsilon_{jt}) = \sigma_{ij} \qquad \text{for } s=t,$$

$$(3)\quad \text{and } E(\varepsilon_{is}, \varepsilon_{jt}) = 0 \qquad \text{for } s \neq t,$$

where $\otimes$ is the Kronecker product, I_T is a T x T identity matrix, and σ_{ij} is the i, jth element of the 5 x 5 matrix, Σ.

Another property of the demand-disturbance terms is that their sum vanishes with unit probability (see Barten (1969), p. 16 and Theil (1971), p. 333). A potentially troublesome implication of this property is that

$$\sum_j \sigma_{ij} = E(\varepsilon_{it}(\varepsilon_{1t} + ... + \varepsilon_{st})) = 0.$$

Thus, the covariance matrix, Σ, is singular, and as such, cannot have a rank that is larger than n − 1. In what follows, it is assumed that the rank of Σ is exactly n − 1. In order to circumvent the complications posed by this singularity problem, one equation of the system (1) is deleted. The legitimacy of this procedure can be verified easily by summing over i any four of the five equations of the system and using the properties of that system in order to recover the deleted equation. In fact, a major advantage of the estimation method used in this study, full information maximum likelihood (FIML), is that the parameter estimates it produces are invariant to the equation deleted (see Barten (1969), pp. 25–27).

Formulation of the Likelihood Function

For notational convenience, the system of demand equations (1) may be written as follows,

$$(4)\quad y_t = g(x_t, \Theta) + \varepsilon_t,$$

where y_t are the vectors of the left-hand-side variables of (1), x_t are the vectors of the right-hand-side variables, ε_t are the vectors of the demand-disturbance terms, and Θ is the vector of the parameters μ_i and π_{ij}. Since the additive disturbance vectors $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{4t})$, t = 1, ..., T, are assumed to be independently normally distributed with mean 0 and variance-covariance matrix Σ, it follows that the vectors y_t must also be independently normally distributed with mean $g(x_t, \Theta)$ and variance-covariance matrix Σ. In arriving at the vector-valued function g, it is assumed for notational convenience that prior restriction on the parameters has already been eliminated by substitution.

Given the observed data on $y = (y_1, ..., y_T)$ and $x = (x_1, ..., x_T)$, the log-likelihood function on Θ and Σ is given by

$$(5)\quad L(\Theta, \Sigma; y, x) = -(T(n-1)/2) \log 2\pi \; |\Sigma| - 1/2 \sum_{t=1}^{T} [(y_t - g(x_t, \Theta))' \; \Sigma^{-1} \; (y_t - g(x, \Sigma))].$$

This function is to be maximized with respect to the elements of the parameter vector Θ and the elements of the variance-covariance matrix, Σ. For computational convenience, however, and since the asymptotic distribution of Σ is not at all needed, a stepwise-optimization procedure is used. This procedure involves first maximizing the log-likelihood function (5) with respect to the elements of Σ, for a given value of Θ, to obtain an expression for Σ in terms of the elements of Θ. Thus, for $\Theta = \Theta^*$, the value of Σ that maximizes (5) is given by

$$(6)\quad \Sigma^*(\Theta^*; y, x) = \frac{1}{T} \sum_{t=1}^{T} (y_t - g(x_t, \Theta))(y_t - g(x_t, \Theta))'.$$

Substitution of Σ^* into (5) yields

$$(7)\quad L(\Theta; y, x) = -(T(n-1)/2) \log 2\pi |\Sigma(\Theta; y, x)| - \frac{1}{2}(T(n-1)),$$

which is the concentrated likelihood function.

The second step in the optimization procedure becomes immediately clear when one recognizes that maximizing the log-likelihood function (5) is equivalent to minimizing the determinant of Σ in (7). The latter is accomplished by searching the feasible parameter space for the value of Θ at which $|\Sigma|$ is minimized. The values of the elements of Θ thus obtained, $\hat{\Theta}$, are the maximum likelihood estimates of the system (1). The asymptotic covariance matrix of $\hat{\Theta}$ is obtained by inverting the matrix $-[\frac{\partial^2 L}{\partial\Theta\partial\Theta'}]$, which is numerically evaluated at $\Theta = \hat{\Theta}$. Naturally, the elements of that matrix pertain only to the estimated parameters. The asymptotic covariance matrix of the entire vector of (estimated as well as computed) parameter estimates is derived in Fayyad (1986).

Appendix B

Available results from information theory can be used to develop measures by which the performance of each of the estimated equations as well as that of the system as a whole can be gauged.[1] Consider an infinitesimal change in the budget share of the ith commodity ($w_i = p_i x_i/m$):

$$dw_i = \frac{x_i}{m} dp_i + \frac{p_i}{m} dx_i - \frac{p_i x_i}{m^2} dm,$$

from which it follows that

$$(1)\quad dw_i = w_i \, d \log p_i + w_i \, d \log x_i - w_i \, d \log m.$$

The finite-change analog of equation 1 is given by

$$(2)\quad \Delta w_{it} = w^*_{it} Dp_{it} + w^*_{it} Dx_{it} - w^*_{it} Dm_t.$$

Since $Dm_t = Dx_t + Dp_t$, it follows that equation 2 can be rewritten as

$$(3)\quad \Delta w_{it} = w^*_{it} Dx_{it} + w^*_{it} (Dp_{it} - Dp_t) - w^*_{it} Dx_t.$$

Observe that the first term on the right-hand-side of equation 3, to be interpreted as the quantity component of the change in the budget share of the ith good, is the dependent variable of the ith demand equation of the estimates system. Thus, given the log-changes in real income and relative prices, the Rotterdam model can be used to provide conditional forecasts of $w^*_{it} Dx_{it}$ and, through equation 3 of Δw_{it}. Since the prediction of $w^*_{it} Dx_{it}$ is equal to the right-hand-side of the ith demand equation with the disturbance term deleted, it follows that

$$\hat{w}_{i,t+1} = w_{it} - e_{it},$$

where $\hat{w}_{i,t+1}$ is the implied prediction of w_{it}, and e_{it} is the residual of the ith demand equation in period t.

In view of the fact that the budget shares are positive and add up to unity, they may be viewed as probabilities. A measure of the model fit can be acquired by determining the expected gain in information from the actual shares, which can be viewed as posterior probabilities, when the implied predictions (or the fitted values) of these shares are viewed as prior probabilities. That measure is given by

$$(4)\quad I_t = \sum_{i=1}^{n} w_{it} \log \frac{w_{it}}{\hat{w}_{it}},$$

where I_t is the information inaccuracy of the predictions provided by the system of demand equations. It is to be noted that not only is this measure of information inaccuracy additive over goods, as is indicated by the expression in (4), but it is also additive over time. Thus, it is possible to construct an average index of information inaccuracy, $\bar{I}$, over the period from t_1 to t_2 by using the following formula

$$(5)\quad \bar{I} = \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} I_t .$$

Observe that the information-inaccuracy measures presented above pertain to the predictions which are provided by the system of demand equations as a whole. It is possible to acquire a single-equation measure of information inaccuracy by using the formula

$$(6)\quad I_{it} = w_{it} \log \frac{w_{it}}{\hat{w}_{it}} + (1 - w_{it}) \log \frac{1 - w_{it}}{1 - \hat{w}_{it}},$$

where $1 - w_{it}$ is the combined budget share of all commodities other than the ith. As before, the average (over time) index of a single-equation information inaccuracy can be obtained by using formula 5.

In order to provide comparability of the information inaccuracy across models, a correction (adjustment) factor should be applied to these measures (see Theil (1971), pp. 651–52, and Barnett (1981), p. 150). Adjustment was achieved in this study by multiplying the information-inaccuracy measure in each case by a factor of ML/(ML-K), where M is the number of jointly estimated equations, L is the number of time periods (quarters), and K is the number of unrestricted parameters. Clearly, this procedure is closely akin to the degrees-of-freedom adjustment of the correlation coefficient.

[1]See Theil (1967), pp. 1–48; (1971), pp. 646–50, and Barnett (1981), pp. 149–54.