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Some Considerations in the Use of Monetary Aggregates for the Implementation of Monetary Policy

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OVER the past several years, the Federal Reserve System has paid increased attention to monetary aggregates as a means to achieve the ultimate national employment and price goals. Evaluation and further development of this process has been hindered, however, by a continuing controversy about which monetary aggregate is appropriate. Specifically, the question concerning the efficacy of monetary actions often becomes lost in discussions about whether M1 or M2 or some other monetary aggregate is giving the best information about the monetary influences being transmitted to the economy.

The Federal Open-Market Committee (FOMC) currently specifies desired ranges for three monetary aggregates in terms of average growth rates over four quarters from a recent base period.[1] Adoption of this strategy represents a compromise in the controversy regarding the appropriate monetary aggregate for the implementation of monetary policy.

Two questions are at issue in this controversy. (1) Which monetary aggregate projects future patterns of economic activity with the smallest error? (2) Which monetary aggregate can the Federal Reserve control with the smallest error? While considerable attention has been given to answering the first question, much less effort has been directed toward resolving the second, but equally important, issue.[2]

[1]The aggregates are currency plus demand deposits held by the nonbank public (M1), M1 plus time deposits at commercial banks other than large marketable certificates of deposit (M2), and M2 plus saving accounts at savings and loan associations and mutual savings banks (M3).

[2]One study, for example, found that in predictions over four quarter periods from 1962 to 1974, using numerous measures of monetary aggregates, the smallest mean and variance of *projection* errors was associated with the monetary base. See Leonall C. Andersen, "Selection of a Monetary Aggregate for Economic Stabilization," this *Review* (October 1975), pp. 9-15. The study presented here is an elaboration on the evaluation of such tests.

The purpose of this article is to address the issue of the appropriate monetary aggregate for the *implementation* of monetary policy. In this light, the overall question at hand involves how the Federal Reserve can transmit its direct actions through some monetary aggregate (such as M1 or M2) and ultimately to the pattern of economic activity with a minimum of slippage or error.

SOURCES OF ERROR

The appropriate monetary aggregate for achieving a desired pattern of economic activity would be the one with the smallest probability of error — *projection error* plus *control error*. The controversy can only be settled by taking into consideration both types of error. While a particular monetary aggregate might give very good projections of the likely pattern of economic activity, that information is not very useful to the monetary authorities for achieving a desired pattern if they have virtually no control over that aggregate.

The ultimate concern of monetary policymakers is the general pattern of economic activity, such as aggregate output, employment, and prices. Evaluation of the appropriate monetary aggregate, for policy purposes, therefore requires a choice among the various measures of economic activity. Nominal GNP is the candidate adopted here, since it incorporates, in a general way, the major variables addressed in policy deliberations. While nominal GNP is not the explicit goal of monetary policy, its use here avoids such debatable issues as the weight that inflation is given in policy discussions relative to output, employment, and other considerations.[3] Also, since the FOMC has

[3]The use of nominal GNP also casts the issue in terms of the recent discussions about the *velocities* of various monetary aggregates in recent years and their implications for the

usually placed its main emphasis on M1 and M2, attention is focused on those aggregates, along with the monetary base.

Control Errors

Achievement of the specified ranges of growth rates of the monetary aggregates is implemented mainly through open-market purchases and sales of Government securities by the Federal Reserve System. These day-to-day activities are the means by which the Federal Reserve attempts to achieve its longer term monetary growth targets. These activities can be summarized by changes in the monetary base.

The monetary base is derived from the consolidated monetary accounts of the Treasury and the Federal Reserve System.[4] It is an asset held by the public in the form of currency and by commercial banks in the form of reserves. The major and dominant source of change in the monetary base is the open-market transactions of the Federal Reserve. Since these transactions can generally be used to offset changes in other sources, changes in the monetary base are, for all practical purposes, under the direct control of the Federal Reserve.[5]

Thus, while the Federal Reserve has not decided *explicitly* to control the monetary base, all of their actions can be subsumed into changes in the base. Even though the Federal funds rate currently is used as the day-to-day operating target in the implementation of monetary policy, open-market operations to achieve desired changes in that rate result in changes in the monetary base. The monetary base, therefore, serves well as a summary measure of the monetary actions of the Federal Reserve.

Money (M), however defined, is related at a point in time to the monetary base (B) by a money multiplier (m) in the following identity:

$$(1)\ M = mB.$$

There are, of course, different multipliers for M1 and M2. This framework offers a concise method of accounting for public and commercial bank preference, which in turn can affect the amount of money which exists in the economy. The multipliers reflect the public's desired holdings of currency and time deposits relative to private demand deposits, commercial banks' desired holdings of excess reserves relative to private demand deposits, Federal government's holdings of demand deposits relative to private demand deposits, and the distribution of deposits among classes of banks.[6]

Since the money identity (equation (1) above) is multiplicative, the percent change in money (M) can be separated into two components — the percent change in the multiplier (m) and the percent change in the monetary base (B). Using changes in natural logarithms to approximate percentage changes gives:

$$(2)\ \Delta \ln M = \Delta \ln m + \Delta \ln B$$

Identity (2) separates a change in money ($\Delta \ln M$) into the effects caused by the change in the component ($\Delta \ln B$) which is under the direct control of the Federal Reserve from those effects caused by the change in the component ($\Delta \ln m$) which is not under the Federal Reserve's direct control.[7] In order to achieve a desired change in money, it would be necessary for the Federal Reserve to forecast (at least implicitly) changes in the multiplier. These multipliers are not constant, nor do they change at a constant rate. Instead, they move in response to changes in the public's monetary preferences and management of Government's demand deposits.

These events, if not properly anticipated by the Federal Reserve, would be the source of control error in the implementation of monetary policy. Actions of the public and the Government can either dampen or exaggerate the effect of any Federal Reserve action on a particular monetary aggregate. The predictability of these changes in the multiplier, even if considered only implicitly, is an important consideration in determining appropriate policy actions.

Changes in the multiplier can be divided into two components — the predicted element ($\Delta \ln \hat{m}$), which is not necessarily constant and may perhaps be

conduct (and evaluation) of monetary actions. See, for example, "The Fifth Report on the Conduct of Monetary Policy," U.S., Senate, 95th Congress, Committee on Banking, Housing, and Urban Affairs, August 5, 1977.

[4]For a detailed discussion of the monetary base, see Anatol B. Balbach and Albert E. Burger, "Derivation of the Monetary Base," this *Review* (November 1976), pp. 2-8.

[5]See Jack L. Rutner, "The Federal Reserve's Impact on Several Reserve Aggregates," Federal Reserve Bank of Kansas City *Monthly Review* (May 1977), pp. 14-22.

[6]For a detailed analysis, see Jerry L. Jordan, "Elements of Money Stock Determination," this *Review* (October 1969), pp. 10-19.

[7]Identity (2) does not imply that the multiplier is independent of changes in the base. Factors affecting the multiplier are considered later. Forecasts of the multipliers can consist of: projections of the multipliers *per se*, the components of the multipliers, or variables which make up the various components or ratios. These forecasts are, in effect, projections of the demand for a particular monetary aggregate.

related to changes in the base, and the unpredicted element (ε).

$$(3)\ \Delta \ln m = \Delta \ln \hat{m} + \varepsilon$$

Over any period of time, the percent change of a particular monetary aggregate is equal to the percent change in the monetary base ($\Delta \ln B$), the *predicted* portion of the percent change of the multiplier ($\Delta \ln \hat{m}$) and the nonpredicted portion of the percent change of the multiplier (ε). The term (ε) represents the *control error* over a particular period.

$$(4)\ \Delta \ln M = \Delta \ln B + \Delta \ln \hat{m} + \varepsilon$$

Projection Errors

Addressing the problem of projection errors is much more complicated than the control error problem. The framework of the latter problem is relatively straightforward, involving simple accounting relationships between the balance sheets of the public, commercial banks, and the monetary authorities. The underlying accounting relationships are identities and all the complexities of changes in public, commercial banks, and Government behavior can be lumped together in one term — the money multiplier.

The mechanism which links the various monetary aggregates to economic activity is more obscure, more complex, and a point of dispute among economic analysts. In general, however, the percent change in nominal GNP ($\Delta \ln Y$) is related to the percent change in money, changes in other exogenous variables, and random disturbances. The question is, of course, what specific form does this relationship take? The presumption adopted here is that an equation which relates the percent change of nominal GNP to the percent change of money only is sufficient for the empirical comparisons made in this article.[8]

$$(5)\ \Delta \ln Y = a_0 + a_1 \Delta \ln M + \mu$$

In this equation, the constant term (a_o) embodies the average influence of changes in the omitted exogenous variables. The term (μ) embodies the systematic influence of changes in the omitted variables and the random disturbances.[9] The *projection error* for a particular period is (μ).

Total Errors

Substituting equation (4) into (5) yields the following equations for the percent change in nominal GNP, where money is defined as M1 in the first equation and M2 in the second.

$$(5a)\ \Delta \ln Y = a_0 + a_1 \Delta \ln \hat{m}_1 + a_1 \Delta \ln B + (a_1\varepsilon_1 + \mu_1)$$

$$(5b)\ \Delta \ln Y = b_0 + b_1 \Delta \ln \hat{m}_2 + b_1 \Delta \ln B + (b_1\varepsilon_2 + \mu_2)$$

The terms ($a_1\varepsilon_1 + \mu_1$) and ($b_1\varepsilon_2 + \mu_2$) are the measured *total errors* in achieving the desired percent change in nominal GNP by using each concept of money — errors in achieving a desired change in money plus errors in projecting nominal GNP from the actual change in money.

CRITERION FOR SELECTING AN APPROPRIATE MONETARY AGGREGATE

The criterion for selecting among monetary aggregates is based on the mean and variance of the total error in achieving a desired percent change in nominal GNP, using equations (5a) and (5b), for a given set of Federal Reserve actions.[10] The total errors in achieving desired changes in nominal GNP for the two concepts of money are given by the following equations:

$$\gamma_1 = a_1 \varepsilon_1 + \mu_1, \text{ for M1, and}$$

$$\gamma_2 = b_1 \varepsilon_2 + \mu_2, \text{ for M2.}$$

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[8]There is considerable evidence that the response of nominal GNP to a change in money is distributed over time. For the sake of simplicity, lagged money terms are not included in equation (5), but they are considered in the empirical analysis later in this article.

[9]In general, the exclusion of important variables from a relationship will bias estimates of the remaining coefficients and the distribution of the error term. Estimates of the mean and variance of the errors will be biased according to the degree of correlation between the included and excluded variables. If the true relationship between nominal income and money is

$$\Delta \ln Y = a_0' + a_1' \Delta \ln M + a_2 \Delta \ln Z + \mu'$$

the estimated variance of μ' will be overstated by the estimated variance of μ. The greater the correlation between $\Delta \ln M$ and $\Delta \ln Z$, the larger this effect will be. Although some of the effect of $a_2 \Delta \ln Z$ will be captured in the constant of equation (5), the estimated variance of μ will probably understate the variance of non-monetary influences on nominal GNP ($a_2 \Delta \ln Z + \mu'$). This underestimate also will depend on the degree of correlation between $\Delta \ln M$ and $\Delta \ln Z$. However, one study which included other exogenous variables such as government spending, export demand and strike dummies, gave results which suggest strongly that while equation (5) might give biased estimates of the projection error, the *relative* errors between various monetary aggregates are not adversely affected. See Andersen, "Selection of a Monetary Aggregate," p. 14.

[10]There is no firm rule for choosing between mean and variance as a criterion. Instead, it is a decision appropriate for the policymakers, based on their preferences. While monetary policy is made over one year time horizons, short-run developments often have been important. Thus, an *average* error of near zero, but with relatively large variance might not be preferable to a somewhat larger average error, but with significantly smaller variance. In essence, the question is whether the policymakers prefer infrequent but large errors to more regular but relatively small errors.

This basis for a criterion can be cumbersome to apply, however, requiring estimates of the errors in predicting the M1 and M2 multipliers.[11] A criterion can be developed, however, on a less straightforward, but still rigorous, basis. To facilitate development of this criterion, it is convenient to consider a hypothetical and unspecified monetary aggregate (M^*). This aggregate may be M1 or M2, or it may be one of the numerous other monetary aggregates that have been mentioned in the controversy. For the moment, its exact composition is not important. This variable is *defined* as that monetary aggregate which gives the smallest total error — control plus prediction.[12] Using this variable, equation (5) is written as

$$(5c)\ \Delta \ln Y = c_0 + c_1 \Delta \ln \hat{m}^* + c_1 \Delta \ln B + (c_1\varepsilon^* + \mu^*)$$

Also,

$$\gamma^* = c_1\varepsilon^* + \mu^*.$$

Since γ^* is defined as having the smallest total error, the errors for M1 and M2 must be at least as large as the error using M^*. That is

$$\gamma_1 \geq \gamma^* \leq \gamma_2.$$

The specification of equation (5) in terms of M^* allows comparison of the total errors of using M1 and M2 to the errors that would be made using the monetary base directly as the monetary target in the implementation of monetary policy. Thus, a joint problem can be addressed. First, if the Federal Reserve is interested in pursuing a monetary aggregate (as opposed to interest rate) policy, is an *intermediate* target, like M1 or M2, required? Second, if an intermediate target seems to work better than using the monetary base directly, which aggregate serves better?

Two further assumptions are made in developing the criterion. The first assumption is that, without considering the available evidence, the errors in forecasting the multipliers for M1 and M2 are zero ($\varepsilon_1 = \varepsilon_2 = 0$). The second assumption is that no forecasts are made regarding changes in the unspecified aggregate's multiplier, and thus, $\varepsilon^* = \Delta \ln m^*$.

In other words, the Federal Reserve is presumed to be able to predict perfectly the M1 and M2 multipliers, and thus all of the total error in using either of these variables is due only to projection errors ($\gamma_1 = \mu_1$ and $\gamma_2 = \mu_2$). Also the Federal Reserve is presumed to have no knowledge about the future pattern of m^* (that is, $\gamma^* = c_1 \Delta \ln m^* + \mu^*$). This latter assumption is equivalent to a situation where the Federal Reserve acts to control the monetary base *only*, with no regard for probable effects on the monetary aggregates. With these assumptions, equations (5 a-b-c) can be rewritten as:

$$(6a)\ \Delta \ln Y = a_0 + a_1 \Delta \ln M1 + \mu_1$$

$$(6b)\ \Delta \ln Y = b_0 + b_1 \Delta \ln M2 + \mu_2$$

$$(6c)\ \Delta \ln Y = c_0 + c_1 \Delta \ln B + (c_1 \Delta \ln m^* + \mu^*)$$

With these assumptions, the test consists of comparing the means and variances of errors made in simulating the percent change of nominal GNP using each of these equations. If the mean of γ^* is found to be smaller than the mean of both μ_1 and μ_2, the monetary base is unambiguously superior to either M1 or M2 as a monetary policy tool, in terms of achieving, *on average*, the desired percent change of nominal GNP over a period of a year. Relative variances of the errors give an indication of how much confidence the Federal Reserve can have in hitting each target. Knowing that the mean error is zero, for example, is not very comforting if the error is +50 percentage points in one year and —50 percentage points in the next.

The results would be ambiguous if the tests revealed that the average total error from using the base (γ^*) exceeded either of the errors found for the other monetary aggregates (μ_1 or μ_2). There would also be a problem if the results using the monetary base showed a significantly larger variance relative to the M1 or M2 results. Such findings would then require investigation of the assumptions about the errors in predicting the M1 or M2 multipliers. For example, if the average value of γ^* were found to be 1.0 percent and the mean of μ_1 were estimated to be 0.5 percent, with equal variances, the case for the monetary base would require that the average error in predicting the M1 multiplier be at least greater than 0.5 percent or exhibit extreme variance.[13]

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[11]See Albert E. Burger, "The Relationship Between Monetary Base and Money: How Close?," this *Review* (October 1975), pp. 5-7.

[12]Again, this "small" error must be defined in terms of policymakers preferences as to possible trade-offs between mean and variance.

[13]Another issue is the covariance of the control errors and prediction errors generated by the use of M1 and M2. The variance of the total errors are

$$\mathrm{var}(\gamma_1) = \mathrm{var}(a_1\varepsilon_1) + \mathrm{var}(\mu_1) + 2\mathrm{cov}(a_1\varepsilon_1\mu_1)$$

$$\mathrm{var}(\gamma_2) = \mathrm{var}(b_1\varepsilon_2) + \mathrm{var}(\mu_2) + 2\mathrm{cov}(b_1\varepsilon_2\mu_2)$$

$$\mathrm{var}(\gamma^*) = \mathrm{var}(c_1\Delta \mathrm{lnm}^*) + \mathrm{var}(\mu^*) + 2\mathrm{cov}(c_1\Delta \mathrm{lnm}^*\mu^*)$$

Thus, even though the variance of γ^* is approximately equal to the variance of both μ_1 and μ_2, the variance of the

If the means and variances of the errors are not significantly different from each other, then another criterion is relevant. That criterion is to select the monetary aggregate which would be the simplest to use in the implementation of monetary policy. The monetary base best meets this criterion, since its use requires less information. The multipliers do not have to be projected.

In light of the preceding discussion, the criterion adopted here is $\mu_1 \geq \gamma^* \leq \mu_2$, in terms of *both* their means and variances.[14] Given the difference in the sources of the measurable errors, the criterion "stacks the deck" against the monetary base. There is only one measurable type of error present for both M1 and M2 (μ_1 and μ_2), while the measurable error for the base includes two types ($c_1 \Delta\ln m^* + \mu^*$). If $\mu_1 > \gamma^* < \mu_2$, using the monetary base directly in the implementation of monetary policy would produce more certain achievement of a desired change in nominal GNP than would using either M1 and M2 as intermediate targets. If $\gamma^* = \mu_1 = \mu_2$, then using the monetary base has the same errors as using M1 and M2. Consequently, since using the monetary base requires less information than M1 and M2, it would be the preferred aggregate for the implementation of monetary policy.

EMPIRICAL TESTS

Empirical tests are conducted to determine whether or not $\mu_1 \geq \gamma^* \leq \mu_2$, in terms of their means and variances. Since the FOMC presently considers setting monetary policy over four quarters from a recent base period, the relevant error is that associated with such a time interval. First, appropriate empirical forms of equations (6 a-b-c) are estimated. Then, errors in post-sample simulations over four quarters of the average percent changes in nominal GNP are estimated for each empirical relationship.

Empirical Form of Relationships

The parameters of equations (6 a-b-c) are first estimated by ordinary least squares, using quarterly data for the period I/1952 to IV/1975 in order to determine the "best" specification for each equation. A stepwise procedure is used to determine the lag specification of each equation to be used in conducting the test. Regressions are first run using only the contemporaneous observation of each independent variable and two dummy variables, one for the quarter of a major strike and one for the following quarter. Then, the number of lags is increased by one until the final regressions include 10 lagged quarters. The number of lags is selected on the basis of F tests for the significance of each added lag. These tests lead to the acceptance of 5 lagged quarters for changes in M1 and B and 3 lagged quarters for changes in M2.[15] Also, the constant is not statistically significant from zero in the M2 equation, so its parameters are estimated suppressing the constant.

Simulation Errors

The beyond-sample-period percentage point errors of the simulated percent change in nominal GNP are measured by making simulations over four subsequent quarters. For each aggregate, the equation is first estimated for the sample period I/1952 to IV/1961 and then reestimated for successive 4 quarter extensions of the sample period (using the specification found to be appropriate for the period I/1952-IV/1975). Using known values of the independent variables, simulations are made of the average quarter-to-quarter percent change in nominal GNP (annual rate) over the four quarters beyond each sample period. The accompanying table presents percentage point errors of the simulations for each post-sample period. The means of the errors and their variances (in both arithmetic and absolute values) are presented at the bottom of the table.

The mean errors are not significantly different from zero (at the 5 percent level) in all three cases, and the variance of the errors using changes in the monetary base is not significantly different from those using changes in either M1 or M2.[16] These results show that the error in using the monetary base directly as the monetary target in the implementation of mone-

total errors in the M1 and M2 cases might be less than the variance of γ^*. Thus, even though the average errors might be equal, the precision of projections using M1 or M2 might be better than that from using the base. This is true if

(1) either cov ($a_1\varepsilon_1\mu_1$) or cov ($b_1\varepsilon_2\mu_2$) is negative and, if so,

(2) that covariance is greater than (in absolute terms) one-half of the variance of the control error appropriate for that variable.

[14]If the criterion is met, the problem of making a trade-off decision between means and variances does not arise.

[15]F-tests were run to determine whether there was any structural change in the equations after the II/1971 period. No evidence of structural change was found, thus, it is presumed that the lag specification, selected on the basis of the I/1952-IV/1975 regressions, can be used in estimating the equations for shorter periods in that interval.

[16]See Bernard Ostle, *Statistics in Research* (Ames: The State University of Iowa Press), pp. 119 and 123 for the tests used. The tests indicate that the three distributions of errors are identical.

Table I

Beyond-Sample-Period Errors in Projecting Average Rate of Change in Nominal GNP

Sample Period from First Quarter 1952 to Fourth Quarter:	Percentage Point Error in Average Quarterly Rate of Change Over Four Subsequent Quarters		
	M1	M2	B
1961	.42%	—2.25%	—2.18%
1962	— .94	—1.95	— .31
1963	— .65	— .07	.17
1964	1.54	.43	3.90
1965	— .75	—2.60	— .03
1966	—3.04	—2.64	— .53
1967	—3.64	—1.05	.25
1968	—1.73	—2.00	.47
1969	— .28	2.10	—1.98
1970	—1.59	—3.79	—1.33
1971	1.61	.30	4.21
1972	.94	.62	.16
1973	— .35	—1.85	—1.94
1974	3.23	1.97	1.72
1975	1.00	— .78	— .24
Mean of Signed Errors	— .28	— .90	.16
Variance of Signed Errors	3.31	3.06	3.61
Mean of Absolute Errors	1.45	1.63	1.29
Variance of Absolute Errors	1.15	1.10	1.84

tary policy is *at least* as small as those resulting from use of M1 or M2, over a one year period, even if the Federal Reserve has perfect knowledge of the future patterns of the public's monetary preferences. To the extent that errors are likely in predicting movements in the multipliers, the case for the monetary base becomes stronger.[17]

CONCLUSIONS

The purpose of this exercise is to cast some light on the relevant considerations in the problem of choosing the appropriate aggregate for monetary policy purposes. The main considerations are projection and control errors in achieving a desired pattern of economic activity. Both types of error must be taken into consideration in determining the appropriate monetary aggregate for the implementation of monetary policy.

The empirical results presented here suggest that, even if the M1 and M2 multipliers could be forecast with virtually no error, the total errors in achieving a desired pattern of economic activity as measured by change in nominal GNP, in terms of their means and variances, probably would not be less than those found for the monetary base. Also, using the monetary base directly in the implementation of monetary policy would be simpler, requiring no estimates of the M1 or M2 multipliers. Consequently, serious consideration should be given to using the monetary base directly as the monetary target in the implementation of monetary policy in place of such intermediate targets as M1 and M2.

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[17]Burger, for example, has found variances for errors in predicting the rate of change of the M1 multiplier of .0576 for annual periods over the interval IV/1953-IV/1973. See Burger, "The Relationship Between Monetary Base and Money," p. 6.

Given that the simulation period is four quarters ahead, the variance in the total error for each aggregate would be influenced by the four quarterly variances in the projection errors and the control errors and the numerous covariance terms. As noted in footnote 2, an earlier study found evidence that projection errors did not significantly influence the relative rankings of the measured variances in the errors which are used in the test. Also, the relative sizes of the measured variances in projection errors are influenced by the correlation between the included monetary aggregate and excluded variables which have an important influence on nominal GNP. One important excluded variable is Government expenditures. A test of the correlation between current and lagged percent changes in each aggregate and high-employment Government expenditures could not reject the null hypothesis that the correlations are equal.

Debt-Management Policy and the Own Price Elasticity of Demand for U.S. Government Notes and Bonds

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DEBT-management policies of the U.S. Government are actions which affect the composition of the publicly held Federal debt. Such actions include operations of both the U.S. Treasury and the Federal Reserve. As a macroeconomic policy tool, discretionary debt-management policy attempts to affect economic activity in a specific way by altering the maturity structure of the Government's debt. The effectiveness of such a policy depends upon the extent to which changes in the composition of the debt affect the structure of interest rates, and the extent to which changes in the structure of interest rates affect economic activity.

The effectiveness of discretionary debt-management policy has been debated for a long time, both on a theoretical and an empirical level. A major attempt at discretionary debt-management policy, called "Operation Twist," occurred in the early 1960s. The Treasury, in coordination with the Federal Reserve, attempted to twist the structure of interest rates in order to lower long-term interest rates to promote investment and economic growth, while raising short-term rates to improve the balance-of-payments deficit. Empirical studies of "Operation Twist" have not conclusively determined whether such debt-management policies are effective.[1]

On the theoretical level, there are two major approaches to the term structure of interest rates which have conflicting implications for the effectiveness of debt-management policy. The pure expectations theory implies that debt-management operations have no lasting impact on the structure of interest rates.[2] The preferred-habitat theory, on the other hand, implies that changes in the quantity of short-term relative to long-term debt can have significant effects on the term structure of interest rates.[3] A large amount of empirical work on both theories has accumulated, but with inconclusive results. At the present time, the preferred-habitat theory cannot be rejected, so that it is not clear whether changes in the relative quantities of debt affect the structure of interest rates. However, if such effects exist, their magnitude may be quite small.

This paper investigates the effect of debt-management operations on the structure of interest rates. It is shown that even if the maturity structure of the debt

[1]See, for example, Franco Modigliani and Richard Sutch, "Innovations in Interest Rate Policy," *The American Economic Review* (May 1966), pp. 178-97.

[2]David Meiselman, *The Term Structure of Interest Rates* (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962); Burton Gordon Malkiel, *The Term Structure of Interest Rates: Expectations and Behavior Patterns* (Princeton: Princeton University Press, 1966).

[3]Modigliani and Sutch, "Innovations in Interest Rate Policy," and "Debt Management and the Term Structure of Interest Rates: An Empirical Analysis," *The Journal of Political Economy* (August 1967), pp. 569-89; Charles R. Nelson, *The Term Structure of Interest Rates* (New York: Basic Books, Inc., 1972).

is admitted as a variable which affects the structure of interest rates, there are reasons to expect that such an effect is small. This conclusion helps to explain the inability of researchers to identify empirically such debt-management effects on the term structure of interest rates. It also implies that only massive changes in the composition of the debt could significantly alter the differential between long- and short-term interest rates.

To derive these results, demand curves for short- and long-term debt are used to formulate a term-structure equation similar to that of other researchers. This equation relates the long-term rate to the short-term rate, expected future short-term rates, and the stocks of short- and long-term debt.[4] In this framework, the effects of the debt variables on the long-term rate depend upon the elasticity of demand for long-term debt. The own price elasticities of demand for forty-seven Treasury issues marketed between 1952 and 1976 are measured, and the demands for both short- and long-term securities are found to be very elastic. These large elasticities of demand imply that debt-management operations have little effect on the term structure of interest rates.

THE PRICE ELASTICITY OF DEMAND FOR TREASURY NOTES AND BONDS

It is relatively easy to measure the own price elasticity of demand for a commodity in introductory economics courses. Two points on the demand curve are chosen, and then a simple formula is used to obtain the price elasticity. However, in actual empirical work this technique is generally not operational, and a more involved approach must be employed. Both demand and supply functions for the commodity must be appropriately specified, time series data on the relevant variables must be collected, and simultaneous equation estimation techniques must be employed that control for the variables that shift the demand and supply curves. Using this approach, the measurement of the own price elasticity of demand for a financial asset is especially difficult because of the problems of specifying the asset's supply curve, and because of high correlations among prices of alternative assets.

The simpler method of using two points on an asset's demand curve can be employed, however, in the measurement of the own price elasticity of demand for U.S. Treasury notes and bonds. This approach is made possible by the Treasury's past use of the "subscription sale" technique for marketing such securities.

Subscription Sales and Demand Curves for Treasury Notes and Bonds

Prior to November 1970, and on three occasions during 1976, the U.S. Treasury sold Treasury notes and bonds on a subscription basis, in contrast to the auction method that is used for Treasury bills.[5] When the Treasury offers debt issues on a subscription basis, it announces the maturity date, coupon rate, and price at which it will issue debt, and invites tenders for the issue.[6] The Treasury also announces the approximate amount of debt which it plans to issue as a result of the subscription sale. In the event that the volume of tenders is greater than the amount of debt which the Treasury wishes to sell, subscriptions are filled on a partial basis known as allotments. The allotment procedures, which have varied frequently from issue to issue, are published in the announcement of the offering. However, the fraction of the order which will be

[4]See Modigliani and Sutch, "Innovations in Interest Rate Policy," and "Debt Management and the Term Structure of Interest Rates."

[5]This auction method has also been used in marketing Treasury notes and bonds since November 1970, with the exception of the three issues in 1976.

[6]For example, in April 1976, the Treasury announced: "The Department of the Treasury will offer to sell $3.5 billion of 10-year notes as one of three securities to be issued for the purpose of refunding debt maturing May 15 and raising new cash. The amount of the offering may be increased by a reasonable amount to the extent that the total amount of subscriptions for $500,000 or less accompanied by 20% deposit so warrants. . ."

"The notes now being offered will be 7⅞% Treasury Notes of Series A-1986 dated May 17, 1976, due May 15, 1986 (Cusip No. 912827 FP 2). They will be sold at par. Interest will be payable on a semiannual basis on November 15, 1976, and thereafter on May 15 and November 15. . ."

"Subscriptions will be received through Wednesday, May 5, 1976, at any Federal Reserve Bank or Branch and at the Bureau of the Public Debt, Washington, D. C. 20226; provided, however, that subscriptions up to $500,000 accompanied by a 20% deposit will be considered timely received if they are mailed to any such agency under a postmark no later than Tuesday, May 4, 1976. . ."

"The Secretary of the Treasury expressly reserves the right to accept or reject any or all subscriptions, in whole or in part, and his action in any such respect shall be final. Subject to these reservations, subscriptions for $500,000, or less, will be allotted in full provided that 20% of the face value of the securities for each subscriber is submitted as a deposit. . ."

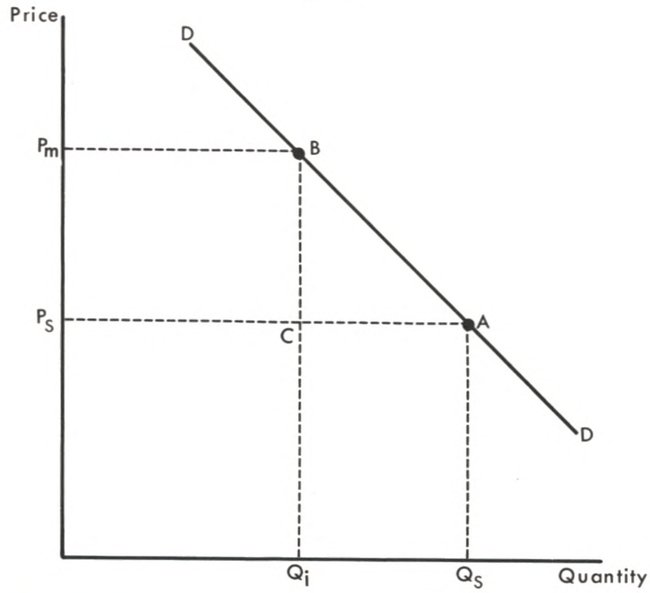
"Subscriptions not accompanied by the 20% deposit will be received subject to a percentage allotment irrespective of the size of the subscription. No allotment will be made of these subscriptions until and unless the subscriptions accompanied by 20% deposit pursuant to the preceding paragraph have been allotted in full. . ."

filled, the allotment ratio, is not known until after all offers to buy have been submitted.

Subscription sales of Government securities offer a unique opportunity to observe two points on the market demand curve for the particular security being offered. First, the Treasury announces a price, usually par, and invites the private sector to make offers for the amount that they wish to purchase at that price (P_s in Figure I). Once the volume of subscriptions has been counted, a point on the demand curve, such as A in Figure I, can be located. After the subscription books are closed, but before the date of issue of the security, the Treasury announces allotment fractions and the total amount of the security which will be issued, represented by Q_i in Figure I. When the quantity which the Treasury issues is less than the amount of subscriptions submitted, the issue is said to have been oversubscribed. Once the amount to be issued has been determined by the Treasury, a second point on the demand curve for this issue can be observed. This point is determined by the amount issued and the price at which the issue sells in the Government securities market, P_m in Figure I.[7]

These two points can be safely regarded as approximations to two points on the same demand curve. First, the time which elapses from the close of the subscription books to the date of issue of the security is quite short.[8] Second, the securities are usually traded by Government securities dealers in the intervening period on a 'when issued' basis once the allocation has become known. Therefore, very little information that would shift the demand curve for the particular issue would become available between the time the volume of subscriptions, Q_s, is submitted and the time the market price, P_m, for the issued volume, Q_i, is observed. Third, small shifts in the demand curve would result in only small changes in the position or shape of the demand curve, so that various measures of points A and B in Figure I are still close approximations of two points on the same

Figure I



[7]It might be argued that the quantity Q_s, associated with point A in Figure I, is an overestimate of the true quantity demanded at the announced price, on the grounds that the economic units which submit bids which are subject to partial allocation inflate those bids based on their expectations of the allocation ratio (the percent of their bid which will be filled). The allocation ratio has been quite variable from issue to issue, ranging from a low of 5 percent to a high of 70 percent. The mean of the allocation ratios is 27.4 percent, and the standard deviation is 17.2 percent. Thus, it would seem to be quite difficult to guess the allocation ratio on any particular issue with great confidence.

It might also be argued that Q_s is an overestimate of the true quantity demanded at price P_s on the grounds that market participants submit bids with the expectation that P_m exceeds P_s. Thus, Q_s includes some speculative demand by traders who, knowing the prices of outstanding securities which are close substitutes and knowing (or knowing approximately) the amount to be issued, inflate their bids with the intention of purchasing for resale. According to this argument, the larger the expected price differential, $P_m - P_s$, the larger would be the quantity differential, $Q_s - Q_i$. However, such behavior, although possible, does not apparently characterize a large portion of the demand by market participants for these issues. Using the data in Tables I and II, with P_m the price on the first day of trading, the simple correlation between $P_m - P_s$ and $Q_s - Q_i$ is very low (0.19), as is the simple correlation between the percentage price change and the percentage quantity change (0.08). (This assumes, of course, that market traders expect the market price to be P_m. Considering that information on close substitutes is readily available, this assumption does not seem overly tenuous.) Consequently, even though there may be some speculative demand for these issues at price P_s which leads to Q_s being an overestimate of the true quantity demanded, the above correlations indicate that the problem is not very severe. In this regard, see footnote 15 below.

[8]In the case of the 10 year note issued in May 1976, which is cited in footnote 6, the subscription books closed on May 5, 1976 and the security was issued on May 17, 1976. Only eight trading days elapsed between these two dates. This is a typical lag for subscriptions issued since the 1950s.

The possibility of the demand curve being shifted because of monetary policy actions which affect short-term rates is minimized because of the 'even-keel' commitment. ". . . even-keel has meant that, for a period encompassing the announcement and settlement dates of a large new security offering or refunding by the Treasury, the Federal Reserve has not made new monetary policy decisions (as contained in announcements from the Board of Governors or as specified in the second paragraph of the policy directives of the Federal Open Market Committee) that would impede the orderly marketing of Treasury securities and significantly increase risks of market disruption from sharp changes in market attitudes in the course of a financing." Stephen H. Axilrod, "The FOMC Directive as Structured in the Late 1960's: Theory and Appraisal," in *Open Market Policies and Operating Procedures — Staff Studies*, Board of Governors of the Federal Reserve System (July 1971), p. 28.

demand curve.[9] Thus, we can assume, without danger of large measurement error, that points A and B in Figure I approximate two points on the same demand curve.

Data have been obtained from various issues of the *Treasury Bulletin* on fifty-one subscription issues which were offered during the period from June 1952 through August 1976. Issues exchanged exclusively in advance refunding operations — not exchanged for cash — were excluded from the sample.[10] Two issues which were auctioned in 1963 were also excluded. Data for the fifty-one issues are given in Table I, including the offering date, maturity date, coupon, term-to-maturity, offering price by the Treasury (P_s), volume of subscriptions tendered (Q_s — excluding subscriptions tendered by Government trust accounts and the Federal Reserve System), and the volume of subscriptions filled (Q_i). All of these issues were oversubscribed.

The additional data which is required to calculate the price elasticity of demand for each security is the market price, P_m. Data which were used to construct measures of this variable were obtained from closing quotations published daily in *The New York Times.* Table II contains daily market quotations from the first quotation subsequent to the opening of the subscription books, through the date of issue of the security.[11] From these, four measures of the market price quotation were constructed: 1) the market price on the first day of trading subsequent to the opening of the subscription books (P_1); 2) the average of the prices on the first five days of trading (P_2); 3) the average of the prices of all trading days from the first day of trading through the day of issue (P_3); and 4) the market price on the day of issue (P_4).

These prices can be compared with the issue prices set by the Treasury. There are only four cases in which the market price fails to rise above the Treasury issue price using at least one of the four measures of the market price.[12] For these four issues, no meaningful negatively sloped demand curve can be constructed. Thus, our sample is reduced to forty-seven issues for which a negatively sloped demand curve was observed using at least one of the four measures of market price.

Given two points on a demand curve, the appropriate measure of the price elasticity, $\varepsilon(Q,P)$, depends on the functional form assumed for the demand curve. In elementary texts, where the emphasis is on linear demand curves, the distinction is frequently made between arc and point elasticities, and several formulas are typically suggested for computing arc elasticities.[13] If the demand curve is log-linear, then it is appropriate to construct the arc elasticity estimate as the ratio of the difference of the logarithms of the two quantities to the difference of the logarithms of the two prices, since the elasticity is constant along the entire range of the demand curve.

An alternative case, which is of interest in the later discussion of the term structure of interest rates, is a semi-logarithmic demand curve, in which the logarithm of the quantity demanded is a function of the level of the price or interest rate. In this case, it is appropriate to compute the arc elasticity as the ratio of the difference in the logarithms of the two quantities to the percentage change in the price or interest rate, where the latter can be measured in the various ways typically suggested for a linear demand function. In our sample, however, the differences in the two price or interest rate observations are so small that insignificant measurement errors are introduced in the semi-logarithmic case if the elasticity is measured by the ratio of the difference of the logarithms of the quantities to the difference of the logarithms of the prices.[14]

Table III contains the measured price and interest rate elasticities (in absolute values) for each of the securities in the sample, based on the four measures of the market price and the corresponding yields to maturity. The securities have been arranged in order of increasing maturity rather than by date of issue, so that the elasticities of issues with similar maturities can be compared.

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[9]In fact, various measures of the price, P_m, associated with point B are used in the analysis below without substantively affecting our conclusions.

[10]Advance refunding consists of offering holders of an existing security the option of exchanging it, prior to its maturity, for a newly issued security.

[11]The market quotations as published in *The New York Times* give fractional prices in 32nds of a point. In Table II the price quotations have been converted to a decimal basis and rounded to the second decimal place.

[12]In Table II, these issues are those for which the subscription books opened on: 1/12/59j, 4/04/60, 10/30/67b, and 5/08/68.

[13]For four alternative formulas for computing arc elasticities with linear demand curves see, Kenneth E. Boulding, *Economic Analysis: Microeconomics*, 4th ed. (New York: Harper & Row, 1966), p. 194.

[14] Using the notation of Figure I:

$$\varepsilon(Q,P) = \frac{dQ}{Q}\frac{P}{dP} = \frac{d \ln Q}{d \ln P} = \frac{\ln Q_s - \ln Q_i}{\ln P_s - \ln P_m}$$

Table I

SUBSCRIPTION ISSUES: 1952-1976

Date Subscription Books Open	Issue Date	Coupon	Maturity Date	Term-to-Maturity Years	Term-to-Maturity Months	Subscriptions Tendered*	Subscriptions Filled* (Public)	Treasury's Price at Issue
8/04/76	8/16/76	8.00	8/15/86	10	0	24,426	8,039	100.00
5/05/76	5/17/76	7.875	5/15/86	10	0	9,000	4,747	100.00
2/03/76	2/17/76	8.00	2/15/83	7	0	29,211	6,019	100.00
8/05/70	8/17/70	7.50	2/15/72	1	6	18,629	3,172	99.95
8/05/68	8/15/68	5.625	8/15/74	6	0	23,557	5,473	99.62
5/08/68	5/15/68	6.00	8/15/69	1	3	10,160	3,242	100.00
2/13/68	2/21/68	5.625	5/15/69	1	3	9,734	4,138	100.00
10/30/67	11/15/67	5.625	2/15/69	1	3	8,159	3,251	100.00
10/30/67	11/15/67	5.75	11/15/74	7	0	14,055	1,575	100.00
8/22/67	8/30/67	5.375	2/15/71	3	5.5	5,952	2,457	99.92
8/01/67	8/15/67	5.25	11/15/68	1	3	9,594	3,847	99.94
1/30/67	2/15/67	4.75	5/15/68	1	3	16,427	2,099	99.875
1/30/67	2/15/67	4.75	2/15/72	5	0	21,996	1,866	99.625
11/01/66	11/15/66	5.625	2/15/68	1	3	5,017	1,791	100.00
11/01/66	11/15/66	5.375	11/15/71	5	0	14,029	1,734	100.00
11/01/65	11/15/65	4.25	5/15/67	1	6	6,030	3,171	99.83
2/01/65	2/15/65	4.00	11/15/66	1	9	10,149	1,766	99.85
11/02/64	11/15/64	4.00	5/15/66	1	6	15,458	3,077	100.00
8/03/64	8/15/64	3.875	2/15/66	1	6	12,985	2,173	100.00
3/31/64	4/08/64 (R)	3.875	8/13/65	1	4	10,227	1,066	99.70
10/28/63	11/15/63	3.875	5/15/65	1	6	16,064	3,972	100.00
6/11/63	6/20/63	4.00	8/15/70	7	2	16,262	1,906	100.00
7/30/62	8/15/62	4.00	2/15/69	6	6	6,643	1,744	100.00
4/09/62	4/18/62	3.75	8/15/68	6	4	6,727	1,158	100.00
1/15/62	1/24/62 (R)	4.00	10/01/69	7	8.5	1,519	1,014	99.75
10/02/61	10/11/61 (R)	3.25	5/15/63	1	7	5,587	2,195	99.875
5/01/61	5/15/61	3.25	5/15/63	2	0	12,110	1,916	100.00
2/06/61	2/15/61	3.25	8/15/62	1	6	15,375	3,720	100.00
8/01/60	8/15/60 (R)	3.875	5/15/68	7	9	5,158	1,045	100.00
4/04/60	4/14/60	4.00	5/15/62	2	1	6,688	2,184	100.00
10/06/59	10/15/59	5.00	8/15/64	4	10	11,025	2,216	100.00
3/23/59	4/01/59	4.00	10/01/69	10	6	1,452	569	100.00
3/23/59	4/01/59 (R)	4.00	5/15/63	4	1.5	2,952	1,643	100.00
1/12/59	1/23/59	4.00	2/15/80	21	1	1,750	834	99.00
1/12/59	1/21/59	3.25	5/15/60	1	4	5,508	2,738	99.75
9/29/58	10/10/58	3.50	11/15/59	1	1	2,581	1,079	100.00
6/03/58	6/03/58	3.25	5/15/85	26	11	2,470	1,035	100.50
4/07/58	4/15/58	2.625	2/15/63	4	10	15,639	3,869	100.00
2/28/58	3/10/58	3.00	8/15/66	8	5.5	6,615	1,384	100.00
11/20/57	11/29/57	3.75	11/15/62	4	11.5	7,686	1,043	100.00
11/20/57	12/02/57	3.875	11/15/74	16	11.5	3,717	554	100.00
9/16/57	9/26/57	4.00	8/15/62	4	11	6,021	1,900	100.00
9/16/57	10/01/57	4.00	10/01/69	12	0	4,548	557	100.00
3/18/57	3/28/57 (R)	3.50	5/15/60	3	1.5	5,768	842	100.00
7/11/55	7/20/55 (R)	3.00	2/15/95	39	7	1,695	796	100.00
5/03/55	5/17/55	2.00	8/15/56	1	3	5,477	4,020	100.00
9/23/54	10/04/54	1.625	5/15/57	2	7.5	8,178	4,143	100.00
5/04/54	5/17/54	1.875	2/15/59	4	9	12,621	5,076	100.00
10/28/53	11/09/53	2.75	9/15/61	7	10	12,493	2,189	100.00
4/13/53	5/01/53	3.25	6/15/78-83	30	1.5	5,549	1,487	100.00
6/16/52	7/01/52	2.375	6/15/58	5	11.5	11,593	4,145	100.00

(R) = Reopened issue

*Millions of dollars

Source: Table PDO-4, "Offerings of Public Marketable Securities Other than Weekly Treasury Bills," and Table PDO-6, "Allotments by Investor Classes on Subscriptions for Public Marketable Securities," selected issues of the *Treasury Bulletin*.

Table II

MARKET PRICES — CLOSING ASKED

Date Subscription Books Opened	Trading Days Since Closing Of Subscription Books													
	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13	+14
8/04/76	101.03	101.09	101.09	101.06	101.25	101.25	101.53	101.78*						
5/05/76	100.50	100.03	99.72	99.69	99.53	99.72	99.56	99.50*						
2/03/76	100.53	100.94	100.94	101.03	101.00	101.19	101.44	101.69*						
8/05/70	100.25	100.25	100.19	100.19	100.19	100.25	100.22	100.13*						
8/05/68	n.t.	100.13	99.97	99.94	99.78	99.88	99.84	99.84*						
5/08/68	n.t.	99.94	99.97	99.94	99.94*									
2/13/68	n.t.	100.03	100.03	100.03	100.03	100.03*								
10/30/67[a]	n.t.	n.t.	99.97	99.94	99.84	99.88	99.88	99.88	99.84	99.94	100.03*			
10/30/67[b]	n.t.	99.97	100.03	99.97	99.88	99.97	99.94	99.81	99.75	99.84	100.00*			
8/22/67	n.t.	99.88	99.88	99.91	99.94	99.94*								
8/01/67	99.97	99.97	99.97	99.91	99.91	99.91	99.94	99.97	99.97	99.97*				
1/30/67[c]	n.t.	n.t.	100.19	100.19	100.16	100.09	100.09	100.13	100.06	100.09	100.00*			
1/30/67[d]	n.t.	n.t.	100.25	100.28	100.22	100.16	100.19	100.19	100.03	100.00	99.88*			
11/01/66[e]	n.t.	100.00	100.00	100.00	100.03	100.00	100.00	100.03*						
11/01/66[f]	n.t.	100.16	100.06	100.03	100.06	99.97	99.94	100.00*						
11/01/65	n.t.	n.t.	99.84	99.84	99.81	n.a.	99.84	99.88	99.88*					
2/01/65	99.91	99.88	99.84	99.88	99.88	99.88	99.88	99.88	99.88*					
11/02/64	n.t.	100.06	100.09	100.09	100.09	100.09	100.09	100.09	100.09*					
8/03/64	100.09	100.06	100.06	100.06	100.06	100.03	100.03	100.03	100.06	100.06*				
3/31/64	99.72	99.72	99.72	99.72	99.75	99.78	99.78	99.75	99.78	99.78*				
10/28/63	100.09	100.06	100.06	100.01	100.00	100.00	100.00	100.00	n.a.	n.a.	n.a.	100.03*		
6/11/63	100.31	100.31	100.31	100.28	100.28	100.34	100.34*							
7/30/62	n.t.	100.09	100.28	100.25	100.28	100.25	100.34	100.53	100.53	100.63	100.66*			
4/09/62	100.25	100.06	100.06	100.13	100.16	100.13	100.16*							
1/15/62	n.t.	99.81	99.78	99.78	99.78	99.78	99.78*							
10/02/61	99.94	99.91	99.94	99.97	99.91	99.94*								
5/01/61	n.t.	100.44	100.41	100.44	100.38	100.38	100.44	100.47	100.50	100.47*				
2/06/61	100.28	100.28	100.28	100.25	100.28	100.31*								
8/01/60	101.25	100.69	101.00	101.81	100.69	100.88	100.75	100.75	101.06	101.06*				
4/04/60	n.t.	99.97	99.88	99.84	99.69	99.63	99.63	99.50*						
10/06/59	100.88	101.13	100.88	100.88	100.88	101.00*								
3/23/59[g]	100.13	100.19	100.06	100.00	100.00	100.03*								
3/23/59[h]	n.t.	99.94	99.88	99.91	99.94	100.03*								
1/12/59[i]	n.t.	99.09	99.31	99.06	98.50	98.56	98.75	98.68	98.56*					
1/12/59[j]	99.69	99.66	99.66	99.66	99.56	99.53	99.53*							
9/29/58	100.00	100.00	100.00	100.03	100.06	100.19	100.25	100.28	100.34*					
6/03/58	101.25*													
4/07/58	100.53	100.56	100.63	100.63	100.75	100.88*								
2/28/58	100.63	100.56	100.59	100.66	100.66	100.59*								
11/20/57[k]	100.94	101.06	100.94	100.75	101.13	101.31*								
11/20/57[l]	102.00	102.00	102.63	102.63	102.13	102.25	102.81*							
9/16/57[m]	100.19	100.19	100.06	100.03	100.03	100.03	100.00	100.00*						
9/16/57[n]	100.38	100.38	100.09	100.06	100.03	100.03	99.97	100.00	100.00	100.00	100.03*			
3/18/57	100.06	100.16	100.16	100.16	100.16	100.13	100.16	100.13*						
7/11/55	100.16	100.25	100.28	100.22	100.16	100.16	100.06*							
5/03/55	100.00	100.03	100.03	100.03	100.03	100.03	100.03	100.03	100.03	100.03*				
9/23/54	100.09	100.06	100.06	100.06	100.06	100.06	100.06*							
5/04/54	100.53	100.53	100.47	100.38	100.44	100.31	100.31	100.38	100.31*					
10/28/53	100.94	100.94	101.09	100.97	100.84	100.94	100.81*							
4/13/53	n.t.	100.38	100.19	100.25	100.44	100.31	100.34	100.38	100.28	99.88	100.00	100.00	100.05	100.00*
6/16/52	n.t.	100.47	100.44	100.50	100.44	100.41	100.38	100.44	100.53	100.50	100.53	100.53*		

[a]matures 2/15/69
[b]matures 11/15/74
[c]matures 5/15/68
[d]matures 2/15/72
[e]matures 2/15/68
[f]matures 11/15/71
[g]matures 10/01/69
[h]matures 5/15/63
[i]matures 2/15/80
[j]matures 5/15/60
[k]matures 11/15/62
[l]matures 11/15/74
[m]matures 8/15/62
[n]matures 10/01/69

n.t. = not traded
n.a. = not available; microfilms of price quotations for these days were not available.
* = day of issue

Source: Selected issues of *The New York Times*.

The choice of the measure of the market price does not seem to be a significant factor in affecting the major conclusions to be drawn from Table III. In all cases, the price and interest elasticities are large.[15] The mean price and interest elasticities (shown at the bottom of Table III) using the price on the first trading day are not generally as large as the elasticities using other price measures, with one exception, but the larger elasticity values using the alternative price measures are also more variable across issues. The price elasticities for longer maturities (five years and over) seem to be considerably smaller on average than those for the shorter maturities (one to five years). Since a given price elasticity, $\varepsilon(Q,P)$, produces a larger yield elasticity, $\varepsilon(Q,R)$, the longer the term-to-maturity, this difference between the average price elasticities of the different maturities is offset, with the result that the average yield elasticities for the two maturity groupings are not significantly different from each other.[16]

For all of the measured series, whether price elasticities or yield elasticities, the values computed for the individual securities tend to exhibit considerable variance across issues, as indicated by the series' standard deviations (bottom of Table III). The large variance among issues produces a standard deviation which is large relative to the mean elasticity. However, the computed means on all elasticity measures, for both maturity groupings, are significantly different from zero at the 2½ percent level. In thirteen of the sixteen cases, the mean price and yield elasticities are significantly different from zero at the 0.5 percent level.[17]

Given the large elasticities in Table III, the question arises as to whether these results can be generalized to conclude that the price and interest elasticities of demand for other Treasury securities are also large. Treating the elasticities in Table III as sample observations drawn from a population of elasticities for all Treasury securities, the probability that the own price or interest elasticity is larger than a specified value for any security can be computed.[18] If the probability is high that the elasticity of demand is large for any given security, then we have greater confidence that the large elasticities in Table III are representative of the elasticities of demand for other Treasury issues. Under the assumption that the individual elasticity estimates are drawn from a normal distribution, the probabilities that the elasticities are larger than 1, 5, 10, 25, and 50 are computed in Table IV. From these results it is seen that the probability is very high that the Government debt, both long- and short-term, is very elastic with respect to its own price or yield, all other factors held constant.

THE ROLE OF DEMAND ELASTICITIES IN THE ASSESSMENT OF DEBT-MANAGEMENT POLICY

Discretionary debt-management policy, as usually defined, deals with the manipulation of the relative

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[15]We have adjusted several of the elasticity computations under the assumption that the total bid, Q_s, is inflated (see footnote 7). In one case, it is assumed that the true value of Q_s, called Q_s^*, exceeds Q_i by half of the amount by which Q_s exceeds Q_i; that is, $Q_s^* = Q_i + 0.5\ (Q_s - Q_i)$. In the second case, Q_s^* is assumed to exceed Q_i by only one-fourth of the amount by which Q_s exceeds Q_i; that is, $Q_s^* = Q_i + 0.25\ (Q_s - Q_i)$. Under the former assumption, the elasticities reported in Table III would be multiplied by a correction factor averaging 0.65, while under the latter assumption the correction factor averages 0.4. Biases of this magnitude in our computations do not substantively alter our conclusions.

[16]To be precise, a given price elasticity produces a larger yield elasticity the longer the *duration* of the bonds. Duration and term-to-maturity are identical measures of the time structure of bonds for non-coupon bonds, such as Treasury bills. But for coupon bonds, such as the Treasury notes and bonds discussed in this paper, duration and term-to-maturity are not equivalent. However, for coupon bonds selling at par or premiums, duration increases with term-to-maturity, so that the stated relationship holds for almost all the issues listed in Tables I and II. For coupon bonds selling at discounts, duration increases with term-to-maturity up to a maximum, and then decreases as term-to-maturity increases. This case, although possible, does not appear to be of significant importance in the results reported here.

For a discussion of duration, see Michael H. Hopewell and George G. Kaufman, "Bond Price Volatility and Term to Maturity: A Generalized Respecification," *The American Economic Review* (September 1973), pp. 749-53; and Roman L. Weil, "Macaulay's Duration: An Appreciation," *The Journal of Business* (October 1973), pp. 589-92.

The formula relating price and yield elasticities is of the form:

$$\varepsilon(Q,P) = \frac{dQ}{Q}\frac{P}{dP} = \frac{dQ}{Q}\left(-\frac{1}{D}\ \frac{1+R}{R}\right)\frac{R}{dR}$$

$$= -\frac{1}{D}\ \frac{1+R}{R}\ \varepsilon(Q,R)$$

where D = duration

$$= \frac{1+R}{R} - \frac{F}{P}\ \frac{[1+R-n(R-c)]}{R\ (1+R)^n}$$

and F = face value of bonds
P = price of bonds
R = yield on bonds
c = coupon rate on bonds
Q = quantity of bonds

Tests for the equality of the average yield elasticities for the two maturity groupings were performed using t-tests at the 5 percent level (two-tailed test).

[17]A one-tailed test was applied in both cases.

[18]In this case, the "population of elasticities" is more specifically the elasticities of demand for Treasury securities over the range of the market demand curve in which the Treasury operates.

Table III

ESTIMATED PRICE AND INTEREST ELASTICITIES FOR VARIOUS SUBSCRIPTION ISSUES: 1952-1976*

Date	Term-to-Maturity Years	Months	$\hat{\varepsilon}(Q,P_1)$	$\hat{\varepsilon}(Q,P_2)$	$\hat{\varepsilon}(Q,P_3)$	$\hat{\varepsilon}(Q,P_4)$	$\hat{\varepsilon}(Q,R_1)$	$\hat{\varepsilon}(Q,R_2)$	$\hat{\varepsilon}(Q,R_3)$	$\hat{\varepsilon}(Q,R_4)$
9/29/58	1	1	—	4361.15	671.31	256.95	—	160.13	24.36	9.15
5/08/68	1	3	—	—	—	—	—	—	—	—
2/13/68	1	3	2851.80	2851.80	2851.80	2851.80	191.73	191.73	191.73	191.73
10/30/67	1	3	—	—	—	3067.66	—	—	—	206.24
8/01/67	1	3	3044.78	9133.42	9133.42	3044.78	193.02	536.98	536.98	193.02
1/30/67	1	3	653.38	776.46	875.45	1644.94	37.03	44.10	49.85	94.87
11/01/66	1	3	—	10301.10	10301.10	3434.04	—	722.59	722.59	230.88
5/03/55	1	3	—	1546.53	1031.07	1031.07	—	36.21	24.57	24.57
1/12/59	1	4	—	—	—	—	—	—	—	—
3/31/64	1	4	11272.80	7515.58	4509.80	2819.05	661.38	420.46	242.95	150.92
8/05/70	1	6	590.71	681.45	681.45	983.92	60.88	70.46	70.46	101.74
11/01/65	1	6	6416.39	6416.39	3208.36	1283.54	400.72	400.72	200.20	79.89
11/02/64	1	6	2691.09	2018.52	1794.33	1794.33	152.92	114.49	101.68	101.68
8/03/64	1	6	1987.21	2554.73	3576.27	2980.37	110.83	140.48	197.03	164.04
10/28/63	1	6	1553.27	3493.98	4658.40	4658.40	86.63	192.68	257.14	257.14
2/06/61	1	6	507.50	526.27	507.50	458.46	23.18	24.08	23.18	20.93
10/02/61	1	7	1435.99	1581.98	1555.62	1435.99	71.88	85.95	71.88	71.88
2/01/65	1	9	2910.93	5820.99	5820.99	5820.99	197.65	396.17	396.17	396.17
5/01/61	2	0	419.96	450.63	419.96	393.22	25.35	27.20	25.35	23.62
4/04/60	2	1	—	—	—	—	—	—	—	—
9/23/54	2	7.5	755.93	971.81	1133.72	1133.72	31.23	40.58	45.70	45.70
3/18/57	3	1.5	3208.13	1375.46	1375.46	1481.19	335.69	142.29	142.29	152.06
8/22/67	3	5.5	—	—	—	4420.83	—	—	—	795.86
3/23/59	4	1.5	—	—	—	1953.49	—	—	—	292.61
5/04/54	4	9	172.31	194.25	228.16	294.27	14.14	15.96	18.95	24.66
10/06/59	4	10	183.13	173.32	171.49	161.25	38.13	35.98	35.65	33.47
4/07/58	4	10	264.24	225.98	212.33	159.42	30.63	26.06	24.41	18.19
9/16/57	4	11	607.63	1153.98	1648.29	—	106.69	209.08	306.92	—
11/20/57	4	11.5	213.48	209.05	196.81	153.46	35.00	34.16	32.13	24.82
1/30/67	5	0	394.48	414.31	487.93	965.08	82.78	86.49	102.51	204.47
11/01/66	5	0	1307.73	3485.54	6970.03	—	302.67	801.63	1604.31	—
6/16/52	5	11.5	219.34	229.07	219.34	194.57	28.22	29.64	28.22	25.19
8/05/68	6	0	285.84	455.12	502.13	661.66	80.85	129.29	142.74	188.39
4/09/62	6	4	704.65	1354.29	1257.62	1100.53	145.74	274.03	263.03	226.63
7/30/62	6	6	1486.65	582.14	352.61	203.30	333.68	133.07	80.38	45.44
2/03/76	7	0	298.83	178.27	144.39	94.26	125.58	74.88	60.26	39.07
10/30/67	7	0	—	—	—	—	—	—	—	—
6/11/63	7	2	692.63	715.68	692.65	631.61	170.43	173.93	170.43	154.84
1/15/62	7	8.5	672.10	1008.05	1008.05	1343.99	181.13	271.79	271.79	326.19
8/01/60	7	9	128.52	147.27	162.06	151.41	32.28	37.15	40.72	38.11
10/28/53	7	10	186.16	182.30	188.15	215.90	35.14	34.34	35.41	40.77
2/28/58	8	5.5	249.09	253.10	253.10	265.93	54.43	55.76	55.76	58.62
5/05/76	10	0	130.39	—	—	—	68.69	—	—	—
8/04/76	10	0	108.45	101.58	88.75	62.99	58.71	54.66	47.76	33.77
3/23/59	10	6	721.10	1171.49	1338.78	3123.19	249.35	415.89	467.94	1248.62
9/16/57	12	0	553.65	1106.25	2334.25	7000.64	208.94	418.92	932.23	2798.79
11/20/57	16	11.5	96.12	84.44	81.95	68.69	45.14	39.57	38.48	32.12
1/12/59	21	1	815.62	—	—	—	502.49	—	—	—
6/03/58	26	11	116.99	116.99	116.99	116.99	67.92	67.92	67.92	67.92
4/13/53	30	1.5	347.20	425.45	693.74	—	193.88	237.11	388.41	—
7/11/55	39	7	472.78	360.30	420.29	1260.11	323.55	251.57	323.55	1133.38
Maturities		$\bar{x}$	2087.03	2797.17	2459.26	1908.69	140.24	176.89	162.70	148.23
1 to 5		s	2668.49	3029.01	2799.71	1568.46	162.41	191.10	183.96	168.71
years		N	20	23	23	25	20	23	23	25
		t: $H_o\mu=0$	3.50	4.43	4.21	6.08	3.86	4.44	4.24	4.39
Maturities		$\bar{x}$	475.63	651.14	911.20	1027.11	156.74	188.82	269.57	391.90
Over 5		s	385.55	792.84	1574.45	1722.16	126.63	193.73	391.93	722.08
years		N	21	19	19	17	21	19	19	17
		t: $H_o\mu=0$	5.65	3.58	2.52	2.46	5.67	4.25	3.00	2.24

*Elasticities in absolute value

$\bar{x}$ = sample mean.

s = sample standard deviation

N = number of observations in sample

t: $H_o\mu=0$ is the t-value for testing the hypothesis (H_o) that the population mean (μ) is equal to zero ($\mu=0$).

$\hat{\varepsilon}(Q,P_i)$ = the estimated price elasticity of demand for a security, using price P_i as the measure of the market price.

$\hat{\varepsilon}(Q,R_i)$ = the estimated interest rate elasticity of demand for a security, using yield R_i as the measure of the market interest rate (R_i is the yield corresponding to price P_i).

P_1, P_2, P_3, P_4: Measures of market prices as defined in the text.

R_1, R_2, R_3, R_4: Measures of market yields corresponding to the price measures.

Table IV

PROBABILITIES THAT OWN PRICE AND INTEREST ELASTICITIES ARE GREATER THAN SPECIFIED VALUES

	$Pr[\varepsilon(Q,P_i) > X]$				$Pr[\varepsilon(Q,R_i) > X]$			
Value (X)	$\hat{\varepsilon}(Q,P_1)$	$\hat{\varepsilon}(Q,P_2)$	$\hat{\varepsilon}(Q,P_3)$	$\hat{\varepsilon}(Q,P_4)$	$\hat{\varepsilon}(Q,R_1)$	$\hat{\varepsilon}(Q,R_2)$	$\hat{\varepsilon}(Q,R_3)$	$\hat{\varepsilon}(Q,R_4)$
Maturity 1 - 5 Years								
1.0	.78	.82	.81	.89	.80	.82	.81	.81
5.0	.78	.82	.81	.89	.80	.82	.80	.80
10.0	.78	.82	.81	.89	.79	.81	.80	.79
25.0	.78	.82	.81	.89	.76	.79	.77	.77
50.0	.77	.82	.81	.88	.71	.75	.73	.72
Maturity Over 5 Years								
1.0	.89	.79	.72	.72	.89	.83	.75	.71
5.0	.89	.79	.72	.72	.88	.83	.75	.70
10.0	.89	.79	.72	.72	.88	.82	.75	.70
25.0	.88	.79	.71	.72	.85	.80	.73	.69
50.0	.87	.78	.71	.71	.80	.76	.71	.68

maturity composition of a given stock of interest-bearing Government debt to accomplish a desired change in the term structure of interest rates. Two major hypotheses exist in the term-structure literature which have conflicting implications for the effectiveness of such debt-management policies. The first hypothesis in its purest form is known as the "expectations hypothesis" of the term structure. This hypothesis maintains that interest rates on long-term securities are determined as a geometric average of current short-term interest rates, and the expectations of future short-term interest rates that will prevail over the life of the long-term security.[19] Given short-term rates and expectations regarding future short-term rates, the long-term rate is determined independently of the maturity structure of the outstanding debt.

The second hypothesis was originally formulated as a "segmented markets" theory, but in recent years has been revised and has come to be known as a "preferred habitat" theory.[20] In this latter form, the theory holds that different classes of lenders (and in the case of private debt, borrowers) have a preference for different maturity segments of the debt market. These preferred maturities, or preferred habitats, are assumed to be well-defined for different groups of market participants, but they are not mutually exclusive across groups as the proponents of the "segmented markets" hypothesis maintained.[21] Thus, for the market as a whole, arbitrage will occur across the maturity spectrum, and the short-term rate and expectations of future short-term rates should be relevant in determining the long-term rate. However, since individual groups of market participants are hypothesized to have well-defined maturity preferences, demand and supply imbalances in a particular maturity segment cannot be completely arbitraged away. Consequently, the theory maintains that substantial changes in the maturity composition of the outstanding debt should also have an influence on the long-term rate, given the short-term rate and expectations of future short-term rates.

In two articles published in 1966 and 1967, Modigliani and Sutch investigated the effects of various measures of the maturity composition of the Federal debt on the average yields on long-term Treasury securities.[22] They found very little empirical evidence that debt variables significantly affect the long-term rate. Current and lagged values of the short-term rate, which can be considered as proxy measures for expected future short-term rates, accounted for almost all the variation in long-term rates. Modigliani and Sutch concluded that debt-management effects, if they exist, have only a small impact on the long-term rate.

[19]Meiselman, *The Term Structure of Interest Rates;* Malkiel, *The Term Structure of Interest Rates.*

[20]John M. Culbertson, "The Term Structure of Interest Rates," *The Quarterly Journal of Economics* (November 1957), pp. 485-517; Modigliani and Sutch, "Innovations in Interest Rate Policy," and "Debt Management and the Term Structure of Interest Rates;" and Nelson, *The Term Structure of Interest Rates.*

[21]Modigliani and Sutch, "Innovations in Interest Rate Policy;" Nelson, *The Term Structure of Interest Rates.*

[22]Modigliani and Sutch, "Innovations in Interest Rate Policy," and "Debt Management and the Term Structure of Interest Rates."

Other researchers who have done similar empirical work have also found that such debt-management effects are small.[23] Some have maintained that problems of measuring the various debt variables, and especially the inability to accurately measure a debt variable which includes all debt and not only Treasury debt, may bias these empirical tests.[24] Thus, it is argued that debt-management policies may have a significantly larger effect on long-term rates than has been reported, but that measurement problems prevent its empirical identification. Discretionary debt-management policies, according to this line of argument, may yet be found to be very effective in changing the structure of interest rates.

Utilizing the information reported in the first section of this paper on the elasticities of demand for Treasury securities, it can be shown that there are other reasons to conclude that even if debt-management variables affect long-term rates, the effect is small. This can be demonstrated by deriving an equation similar to that investigated by Modigliani and Sutch, but starting from demand functions for Government securities rather than the preferred-habitat theory.

Consider the following market demand functions for long- and short-term Government debt:

$$\ln(Q_s^d/W) = \alpha_0 + \alpha_1 R_s + \alpha_2 R_s^e - \alpha_3 R_l + Z_s\lambda_s \qquad (1)$$

$$\ln(Q_l^d/W) = \beta_0 - \beta_1 R_s - \beta_2 R_s^e + \beta_3 R_l + Z_l\lambda_l \qquad (2)$$

where:

- Q_s^d is the quantity demanded of short-term debt
- Q_l^d is the quantity demanded of long-term debt
- R_s is the current interest rate on short-term debt
- R_l is the current interest rate on long-term debt
- R_s^e is the expected future interest rate on short-term debt
- W is total wealth
- Z_s, Z_l are vectors of other variables affecting Q_s and Q_l, respectively, including rates of return on other assets

and $\alpha_i > O$, $\beta_i > O$, λ_s and λ_l are coefficients. Since the demand functions are expressed in terms of interest rates rather than prices, the own elasticities of demand are positive and the cross elasticities are negative. The functional form indicated in equations (1) and (2) has been chosen primarily for expositional convenience. However, this form has been used in recent studies of asset demand functions, and recent theoretical work suggests that it is preferred to the more traditional linear and log-linear specifications.[25] The restriction of wealth elasticities to unity is maintained to eliminate detail which is not relevant to this discussion. None of the conclusions of the subsequent analysis is affected by this constraint.[26] By subtracting equation (2) from equation (1) the following expression can be obtained:[27]

$$\ln(Q_s/W) - \ln(Q_l/W) = \ln(Q_s/Q_l) = (\alpha_0 - \beta_0) + (\alpha_1 + \beta_1) R_s + (\alpha_2 + \beta_2) R_s^e - (\alpha_3 + \beta_3) R_l + Z_s\lambda_s - Z_l\lambda_l \qquad (3)$$

This equation, in turn, can be solved for the long-term rate to obtain:

$$R_l = \left(\frac{\alpha_0 - \beta_0}{\alpha_3 + \beta_3}\right) + \left(\frac{\alpha_1 + \beta_1}{\alpha_3 + \beta_3}\right) R_s + \left(\frac{\alpha_2 + \beta_2}{\alpha_3 + \beta_3}\right) R_s^e - \left(\frac{1}{\alpha_3 + \beta_3}\right) \ln(Q_s/Q_l) + \frac{Z_s\lambda_s}{\alpha_3 + \beta_3} - \frac{Z_l\lambda_l}{\alpha_3 + \beta_3} \qquad (4)$$

By appropriate manipulation, this equation can be rewritten as:

$$R_l = \left(\frac{\alpha_0 - \beta_0}{\alpha_3 + \beta_3}\right) + \left(\frac{\alpha_1 + \beta_1}{\alpha_3 + \beta_3}\right) R_s + \left(\frac{\alpha_2 + \beta_2}{\alpha_3 + \beta_3}\right) R_s^e + \left(\frac{1}{\alpha_3 + \beta_3}\right)\left(\frac{Q_l}{DEBT}\right) - \left(\frac{1}{\alpha_3 + \beta_3}\right)\left(\frac{Q_s}{DEBT}\right) + \varepsilon \qquad (5)$$

where DEBT is the quantity of debt outstanding at all maturities, say short (Q_s), intermediate (Q_n) and long (Q_l), and where the influence of the (unspecified) variables in the vectors Z_s and Z_l have been impounded in the error term ε.[28]

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[23]For example, Frank de Leeuw, "A Model of Financial Behavior," in *The Brookings Quarterly Econometric Model of the United States,* ed. James S. Duesenberry et al. (Chicago: Rand McNally & Company, 1965); Neil Wallace, "The Term Structure of Interest Rates and The Maturity Composition of the Federal Debt" (Ph.D. dissertation, University of Chicago, December 1964).

[24]See the "Discussions" and "Comments" to Modigliani and Sutch, "Innovations in Interest Rate Policy," and "Debt Management and the Term Structure of Interest Rates."

[25]Phillip Cagan and Anna J. Schwartz, "Has the Growth of Money Substitutes Hindered Monetary Policy?" *Journal of Money, Credit and Banking* (May 1975), pp. 137-59; J. B. Ramsey and R. H. Rasche, "The Velocity of M2 and of Its Components," *Workshop Paper No. 7504* (Michigan State University, June 1976); Ramsey, "Limiting Functional Forms for Market Demand Curves," *Econometrica* (March 1972), pp. 327-41.

[26]Equations (1) and (2), with the constrained wealth elasticities, are consistent with the general asset demand specifications suggested by James Tobin, "An Essay on Principles of Debt Management," in *Fiscal and Debt Management Policies,* by William Fellner et al. (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963), p. 216, and "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit and Banking* (February 1969), p. 24.

[27]It is implicitly assumed here that the supplies of Q_s and Q_l are exogenously determined by the Treasury, so that the superscripts on these two variables are dropped.

[28]$\ln(Q_s/Q_l) = \ln[(Q_s/DEBT)\left(\frac{DEBT}{Q_l}\right)] = \ln(Q_s/DEBT) - \ln(Q_l/DEBT)$ where DEBT is the quantity of debt outstanding at all maturities, say short (Q_s), intermediate (Q_n)

Equation (5) is one form of the equation which Modigliani and Sutch (1966) proposed and tested as the "preferred habitat" model.[29] It can be seen from equation (5) that the magnitude of the parameters α_3 and β_3 will be crucial in determining whether one can find sizable impacts of the maturity composition of the debt on the long-term rate. If either of these parameters is very large, then the true coefficients of the maturity-composition variables are very small. In addition, since the variation of the maturity structure of the debt is quite limited in any sample period, the precision of the estimates of these coefficients will not be very high. Consequently, it is quite likely that if either α_3 or β_3 is large, it will be possible to reject the hypothesis that changes in the maturity structure of the debt have a significant impact on the long-term rate, for a given short-term rate.

The parameters in equation (5), such as β_3 and α_3, are associated with the elasticities and cross-elasticities of demand for short- and long-term Government securities, which can be derived from equations (1) and (2). The interest rate elasticity of demand for long-term debt (Q_l) is equal to $\beta_3 R_l$, while the interest elasticity of demand for short-term debt (Q_s) is equal to $\alpha_1 R_s$.[30] The cross-elasticity of demand for short-term debt with respect to the long-term interest rate is given by $-\alpha_3 R_l$.[31] Although there is insufficient information to estimate α_3, estimates of β_3 and α_1 for individual Treasury securities can be obtained from the elasticities of demand in Table III and measures of the interest rates which correspond to the prices in Table II. Estimates of α_1 and β_3 are given in Table V using the four measures of the market yields corresponding to the price measures discussed earlier. Very few of the values of α_1 and β_3 in Table V are below ten, and many are larger than twenty-five. The probability that β_3 is larger than a specified value can be computed in the same manner as the computations for the elasticities presented in Table IV. The probabilities that β_3 is greater than 10 and 25 are presented in Table VI. These probabilities are based on the data in Table V with maturities greater than 5 years.

From Table VI it can be seen that the data from the subscription sales suggest that it is highly probable that β_3 is larger than 10. If this is the case for long-term debt as a whole, then the coefficients of the debt-composition variables in the "term structure" equation (5) are even more likely to be less than 0.1, since the denominator of this coefficient is the sum of α_3 and β_3 (and both are positive).

To illustrate the implication of such a parameter value, assume that 10 percent of the outstanding Government debt is switched from long-term to short-term debt by an advance refunding operation. This would be a very large debt-management operation relative to the advance refunding operations which were attempted in the early 1960s as part of "Operation Twist." With $\frac{1}{\alpha_3+\beta_3} = 0.1$, an operation of such a magnitude would imply a change in the long-term rate of two basis points, according to equation (5).[32] With this information, it is not surprising that attempts to estimate maturity-structure effects in specifications such as equation (5) have been notably unsuccessful. The evidence presented here suggests that even large changes in the maturity composition of the Government debt will have very minor impacts on the long-term rates on Government securities, and supports the position that debt management can be dismissed as a useful tool of stabilization policy.

The effects discussed above are not merely a function of the linear approximation of the debt vari-

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and long (Q_l). Then $\ln(Q_s/\text{DEBT}) = \ln\left(1 - \frac{Q_l + Q_n}{\text{DEBT}}\right)$.

But $\ln\left(1 - \frac{Q_l + Q_n}{\text{DEBT}}\right) \simeq -\left(\frac{Q_n + Q_l}{\text{DEBT}}\right)$. Similarly $\ln\left(\frac{Q_l}{\text{DEBT}}\right) \simeq -\frac{Q_s + Q_n}{\text{DEBT}}$. Therefore, $\ln(Q_s/Q_l) \simeq$

$$-\frac{(Q_n + Q_l)}{\text{DEBT}} + \frac{(Q_s + Q_n)}{\text{DEBT}} = \frac{Q_s}{\text{DEBT}} - \frac{Q_l}{\text{DEBT}}.$$

This approximation $[\ln(1 - X) \simeq -X]$ is accurate only for values of X between -0.3 and $+0.3$; that is, when the ratio of the type of debt to total debt is less than 1/3. However, its use here does not alter the conclusions drawn below, as will be shown later using the original term: $\ln(Q_s/Q_l)$. The approximation is employed here in order to compare equation (5) with the work of Modigliani and Sutch.

[29]Modigliani and Sutch assumed three maturity classes of debt — short, intermediate and long maturities — and approximated the expected future short-term rate by a distributed lag on past short-term rates. See Modigliani and Sutch, "Innovations in Interest Rate Policy."

[30]$\varepsilon(Q_l,R_l) = \frac{d \ln Q_l}{d R_l} R_l = \beta_3 R_l$ from equation (2);

and $\varepsilon(Q_s,R_s) = \frac{d \ln Q_s}{d R_s} R_s = \alpha_1 R_s$ from equation (1)

[31]$\varepsilon(Q_s,R_l) = \frac{d \ln Q_s}{d R_l} R_l = -\alpha_3 R_l$ from equation (1)

[32]

$$\frac{1}{\alpha_3 + \beta_3} \frac{Q_l - 0.1 \text{ DEBT}}{\text{DEBT}} - \frac{1}{\alpha_3 + \beta_3} \frac{Q_s + 0.1 \text{ DEBT}}{\text{DEBT}}$$

$$= (0.1)\left[\frac{Q_l}{\text{DEBT}} - 0.1\right] - (0.1)\left[\frac{Q_s}{\text{DEBT}} + 0.1\right]$$

$$= (0.1)\frac{Q_l}{\text{DEBT}} - (0.1)\frac{Q_s}{\text{DEBT}} - 0.02$$

Table V

ESTIMATES OF α_1 AND β_3 USING VARIOUS MEASURES OF THE MARKET YIELD*

Date	Term-to-Maturity Years	Months	$\hat{\alpha}_1^1$ or $\hat{\beta}_3^1$	$\hat{\alpha}_1^2$ or $\hat{\beta}_3^2$	$\hat{\alpha}_1^3$ or $\hat{\beta}_3^3$	$\hat{\alpha}_1^4$ or $\hat{\beta}_3^4$
9/29/58	1	1	—	46.03	7.22	2.88
5/08/68	1	3	—	—	—	—
2/13/68	1	3	34.29	34.29	34.29	34.29
10/30/67	1	3	—	—	—	36.89
8/01/67	1	3	36.64	101.62	101.62	36.64
1/30/67	1	3	8.08	9.53	10.72	19.99
11/01/66	1	3	—	128.85	128.85	41.29
5/03/55	1	3	—	18.27	12.45	12.45
1/12/59	1	4	—	—	—	—
3/31/64	1	4	161.78	103.05	59.78	37.35
8/05/70	1	6	8.32	9.59	9.59	13.74
11/01/65	1	6	91.89	91.89	45.98	18.44
11/02/64	1	6	38.64	29.03	25.83	25.83
8/03/64	1	6	29.07	36.72	51.31	42.80
10/28/63	1	6	22.72	50.09	66.72	66.72
2/06/61	1	6	7.58	7.86	7.58	6.89
10/02/61	1	7	21.87	26.09	21.87	21.87
2/01/65	1	9	48.79	97.36	97.36	97.36
5/01/61	2	0	8.39	8.96	8.39	7.86
4/04/60	2	1	—	—	—	—
9/23/54	2	7.5	19.64	25.40	28.55	28.55
3/18/57	3	1.5	96.49	41.22	41.22	44.01
8/22/67	3	5.5	—	—	—	147.55
3/23/59	4	1.5	—	—	—	73.32
5/04/54	4	9	8.04	9.01	10.60	13.65
10/06/59	4	10	7.95	7.53	7.46	7.02
4/07/58	4	10	12.21	10.47	9.85	7.48
9/16/57	4	11	26.97	52.57	77.04	—
11/20/57	4	11.5	9.88	9.66	9.12	7.17
			$\hat{\beta}_3^1$	$\hat{\beta}_3^2$	$\hat{\beta}_3^3$	$\hat{\beta}_3^4$
1/30/67	5	0	17.64	18.40	21.71	42.79
11/01/66	5	0	56.70	149.53	298.87	—
6/16/52	5	11.5	12.32	12.92	12.32	11.05
8/05/68	6	0	14.44	22.94	25.29	33.30
4/09/62	6	4	39.34	73.55	70.61	60.91
7/30/62	6	6	83.75	33.60	20.43	11.70
2/03/76	7	0	15.90	9.56	7.73	5.09
10/30/67	7	0	—	—	—	—
6/11/63	7	2	43.15	44.02	43.15	39.25
1/15/62	7	8.5	44.96	67.41	67.41	80.88
8/01/60	7	9	8.75	10.01	10.93	10.25
10/28/53	7	10	13.43	13.14	13.52	15.47
2/28/58	8	5.5	18.67	19.11	19.11	20.07
5/05/76	10	0	8.80	—	—	—
8/04/76	10	0	7.48	6.97	6.11	4.36
3/23/59	10	6	62.57	104.21	117.22	312.39
9/16/57	12	0	52.76	105.26	233.58	700.22
11/20/57	16	11.5	12.15	10.71	10.44	8.79
1/12/59	21	1	123.61	—	—	—
6/03/58	26	11	21.35	21.35	21.35	21.35
4/13/53	30	1.5	60.06	73.36	119.92	—
7/11/55	39	7	108.10	84.11	108.10	378.05
Maturities		$\bar{x}$	34.96	41.53	37.97	34.08
1 to 5		s	39.38	37.29	35.47	33.10
years		N	20	23	23	25
		t: $H_0\mu=0$	3.97	5.34	5.13	5.15
Maturities		$\bar{x}$	39.33	46.32	64.62	103.29
Over 5		s	33.70	41.76	81.22	188.04
years		N	21	19	19	17
		t: $H_0\mu=0$	5.35	4.83	3.47	2.26

*$\beta_3 = \frac{\varepsilon(Q_L,R_L)}{R_L}$ and $\alpha_1 = \frac{\varepsilon(Q_s,R_s)}{R_s}$.

$\hat{\beta}_3^i = \beta_3$ using R_i as the market yield, as defined in Table III.

$\bar{x}$, s, N, and t: $H_0\mu=0$ are defined at the bottom of Table III.

Table VI

PROBABILITIES THAT DEBT COEFFICIENT β_3 EXCEEDS SPECIFIED VALUES [Pr (β_3 > X)]*

Value (X)	$\hat{\beta}_3^1$	$\hat{\beta}_3^2$	$\hat{\beta}_3^3$	$\hat{\beta}_3^4$
10	.81	.81	.75	.69
25	.66	.70	.69	.66

*Based on data in Table V with maturities greater than 5 years.

ables.[33] To show this, the original term for the debt variables in equation (4), $\ln(Q_s/Q_l)$, has been calculated for the fiscal years 1967-1976 (Table VII). In Table VII, short- and long-term debt are defined in the conventional manner: short-term debt includes securities with one year or less to maturity; long-term debt includes securities with 10 or more years to maturity. If the coefficient on $\ln(Q_s/Q_l)$ in equation (4) is 0.1 (that is, if $1/(\alpha_3 + \beta_3) = 0.1$, as given in the example above), then the impact of the debt variable on the long-term interest rate, given the short-term rate, has been less than 25 basis points over the period 1967-1976.[34] Furthermore, debt-management operations can again be shown to have relatively small effects on the long-term rate for given short-term rates.

Table VIII presents the effects of two debt-management operations based on the data in Table VII: switching 10 percent of the outstanding Government debt from short-term to long-term debt, and switching 5 percent of the outstanding debt from long-term to short-term debt.[35] In the former case, the long-term rate is raised by less than 15 basis points (0.15 percent) in each of the years. Thus, a shifting of 10 percent of the debt from the short- to long-term end of the maturity spectrum, which again is a large debt-management operation, results in a relatively small change in the long-term rate, given the short-term rate.

In the latter case, the shifting of 5 percent of the debt from the long- to short-term end of the maturity spectrum results in the long-term rate declining by less than 17 basis points (0.17 percent) in all but one

[33]See footnote 28 and equations (4) and (5).

[34]Note that the assumption that the coefficient is 0.1 is a liberal one for assessing the effect of the debt variables. As noted earlier, it is very likely that the coefficient is smaller than 0.1, which implies even smaller debt-management effects.

[35]The debt variable, $\ln(Q_s/Q_l)$, would now be:

$\ln[(Q_s - 0.1\ \text{DEBT})/(Q_l + 0.1\ \text{DEBT})]$,

or $\ln[(Q_s + 0.05\ \text{DEBT})/(Q_l - 0.05\ \text{DEBT})]$.

Note that if 10 percent of the debt were switched from long- to short-term, the long-term debt would be wiped out in most years. Since such an operation is not very likely, a switch of 5 percent of the debt was used in Table VIII instead.

Table VII

EFFECT OF DEBT VARIABLE $\ln(Q_s/Q_l)$ ON LONG-TERM RATE IN EQUATION (4) ASSUMING $1/(\alpha_3 + \beta_3) = .1$

End of Fiscal Year	Total Amount of Outstanding Debt Privately Held (Debt)*	Debt Maturing Within 1 Year (Q_s)*	Debt Maturing in 10 Years or More (Q_l)*	Q_s/Q_l	$\ln(Q_s/Q_l)$	Effect of Debt Variable on Long-Term Rate** $(.1)\ln(Q_s/Q_l)$
1967	150,321	56,561	19,121	2.958	1.085	.109
1968	159,671	66,746	18,780	3.554	1.268	.127
1969	156,008	69,311	18,434	3.760	1.324	.132
1970	157,910	76,443	16,148	4.734	1.555	.156
1971	161,863	74,803	14,002	5.342	1.676	.168
1972	165,978	79,509	13,280	5.987	1.790	.179
1973	167,869	84,041	13,305	6.317	1.843	.184
1974	164,862	87,150	13,411	6.498	1.871	.187
1975	210,382	115,677	13,468	8.589	2.150	.215
1976	279,782	150,296	14,739	10.197	2.322	.232

*In millions of dollars

**In percentage points; 1.00 = 1%; .20 = 20 Basis Points

Source: Table FD-4, "Maturity Distribution and Average Length of Marketable Interest-Bearing Public Debt Held by Private Investors," selected issues of the *Treasury Bulletin.*

Table VIII

EFFECT OF DEBT OPERATIONS ON THE LONG-TERM RATE USING EQUATION (4) WITH $1/(\alpha_3 + \beta_3) = .1$

	Shifting 10% of Total Debt from Short-Term to Long-Term					Shifting 5% of Total Debt from Long-Term to Short-Term				
End of Fiscal Year	$Q_s - (.1)$ Debt (Q'_s)*	$Q_l + (.1)$ Debt (Q'_l)*	$\ln(Q'_s/Q'_l)$	$(.1)\ln(Q'_s/Q'_l)$	Effect on Long-Term Rate (ΔR_l)**	$Q_s + (.05)$ Debt (Q''_s)*	$Q_l - (.05)$ Debt (Q''_l)*	$\ln(Q''_s/Q''_l)$	$(.1)\ln(Q''_s/Q''_l)$	Effect on Long-Term Rate (ΔR_l)**
1967	41,528.9	34,153.1	.196	.020	+.089	64,077.1	11,605.0	1.709	.171	−.062
1968	50,778.9	34,747.1	.379	.038	+.089	74,729.6	10,796.5	1.935	.194	−.067
1969	53,710.2	34,034.8	.456	.046	+.086	77,111.4	10,633.6	1.981	.198	−.066
1970	60,652.0	31,939.0	.641	.064	+.092	84,338.5	8,252.5	2.324	.232	−.076
1971	58,616.7	30,188.3	.664	.066	+.102	82,896.2	5,908.9	2.641	.264	−.096
1972	62,911.2	29,877.8	.745	.075	+.104	87,807.9	4,981.1	2.869	.287	−.108
1973	67,254.1	30,091.9	.804	.080	+.104	92,434.5	4,911.6	2.935	.294	−.110
1974	70,663.8	29,897.2	.860	.086	+.101	95,393.1	5,167.9	2.916	.292	−.105
1975	94,638.8	34,506.2	1.009	.101	+.114	126,196.1	2,948.9	3.756	.376	−.161
1976	122,317.8	42,717.2	1.052	.105	+.127	164,285.1	749.9	5.389	.539	−.307

*In millions of dollars. Q_s, Q_l and Debt are given in Table VII.

**In percentage points, 1.00 = 1 percent; .20 = 20 Basis Points.

$\Delta R_l = (.1)\ln(Q_s/Q_l) - (.1)\ln(Q'_s/Q'_l)$

or $= (.1)\ln(Q_s/Q_l) - (.1)\ln(Q''_s/Q''_l)$

case. The exception is for fiscal year 1976, when the long-term rate would decrease by about 31 basis points under this debt-management operation. This change is still not very large, and is accounted for by the fact that shifting 5 percent of the debt from long- to short-term in fiscal 1976 reduces the amount of long-term debt outstanding to only $750 million. It is to be expected from equation (4) that if the amount of long-term debt outstanding were virtually eliminated by a debt-management operation, the long-term rate would fall considerably more than would otherwise be the case.

From the examples given in Table VIII, we again find that, even using actual ratios of short- to long-term debt, the large elasticities of demand for Treasury securities imply that the debt variable has a relatively small impact on the long-term rate. Only massive changes in the maturity composition of the debt will have very large effects.[36]

CONCLUSIONS

Measures of the own price (and interest rate) elasticity of demand for Treasury securities, derived from data on Treasury subscription sales, indicate that the demands for both long- and short-term Government debt are very elastic. Market demand functions for long- and short-term debt were used to obtain a Modigliani-Sutch equation of the term structure of interest rates. The large interest rate elasticities of demand imply that the coefficients of the maturity composition of the debt in this equation are expected to be quite small. Based on these estimates, even large changes in the maturity composition of the debt will have little effect on long-term interest rates on Treasury securities. These results are consistent with, and help to explain, the empirical results found by Modigliani and Sutch and other researchers, and support the position that discretionary debt-management operations have little usefulness as a policy tool.

[36]The above discussion implicitly assumes that the stocks of debt of differing maturities can be taken as exogenous variables (see footnote 27). This may not be an appropriate representation of the behavior of the Treasury. However, the introduction of the simultaneous determination of the supply and demand for Government debt, by introducing a debt-service minimization policy, prevents estimation of any maturity-composition effects using this term-structure framework (see Appendix).

Appendix follows on next page.

APPENDIX

This appendix considers the case of endogenous supplies of Government debt. There has been a great deal of discussion of Treasury policies which suggests that the goal of the Treasury, at least throughout the 1950s, was to manage the maturity structure of the debt so as to minimize the cost of the debt service.[1] If this is the case, we can characterize the behavior of the Treasury by the following supply equations:

$$\ln(Q_s/DEBT) = \gamma_0 - \gamma_1 R_s + \gamma_2 R_l \quad (\gamma_i > 0) \qquad (A.1)$$

$$\ln(Q_l/DEBT) = \delta_0 + \delta_1 R_s - \delta_2 R_l \quad (\delta_i > 0) \qquad (A.2)$$

Equations (A.1) and (A.2) imply that as the long-term rate goes up or the short-term rate goes down, the Treasury shortens the average maturity of the debt, and vice-versa for lengthening the maturity of the debt. By subtracting (A.2) from (A.1) we obtain:

$$\ln(Q_s/DEBT) - \ln(Q_l/DEBT) = \ln(Q_s/Q_l) = (\gamma_0 - \delta_0) - (\gamma_1 + \delta_1)R_s + (\gamma_2 + \delta_2)R_l \qquad (A.3)$$

When this is substituted into equation (3), the resulting solution for R_l is:[2]

$$R_l = \frac{\kappa_0 + (\alpha_1 + \beta_1 + \gamma_1 + \delta_1) R_s + (\alpha_2 + \beta_2) R_s^e}{(\gamma_2 + \delta_2 + \alpha_3 + \beta_3)}$$

$$\text{where } \kappa_0 = (\alpha_0 - \beta_0 - \gamma_0 + \delta_0) \qquad (A.4)$$

Equation (A.4) has a form similar to that of the estimated Modigliani-Sutch equation, but implies that the maturity-composition terms do not appear in the equation. Consequently, the introduction of the simultaneous determination of the supply and demand for Government debt, as a result of a debt-service minimization policy, prevents estimation of any maturity-composition effects using this term-structure framework.

[1]See U.S., Congress, Joint Economic Committee, *Employment, Growth, and Price Levels,* Study Paper No. 19, Warren L. Smith, "Debt Management in the United States," 86th Cong., 2nd sess., 1960. In the late 1960s the ability of the Treasury to pursue any policies with respect to the maturity structure of the Government debt was severely limited by legal restrictions on the maximum coupon which could be placed on new bonds. Since this coupon was substantially below prevailing market rates for long-term issues, the Treasury was effectively prohibited from issuing new bonds.

[2]Ignoring the terms in Z_s and Z_l.

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