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Two Stages to the Current Recession

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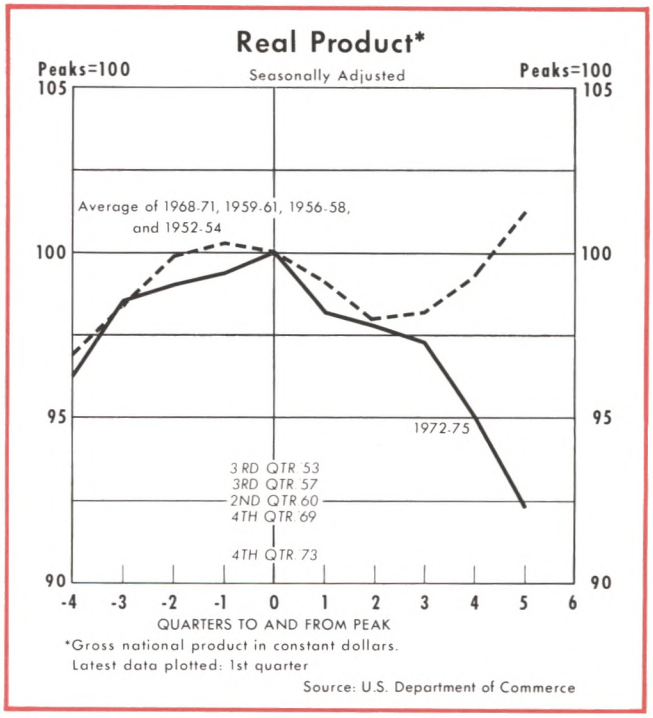
THE pattern of economic developments since the fall of 1973 has seemed baffling to many. It has been a period of prolonged and deep economic recession combined with intensified inflation for a year. The recession has had substantially different characteristics from the four previous recessions since 1950, including two distinct periods within the current contraction.

In terms of causal forces, the recent downturn also differs from earlier contractions. All previous recessions in the past twenty-five years were preceded by periods of pronounced monetary restraint. The current downturn, by contrast, was preceded, and accompanied for a time, by stimulative policy actions. As the downturn progressed, however, the growth of the money stock did slow markedly. In addition, developments which have hampered production — such as the higher price of oil, unfavorable weather conditions, and the effects of price controls — have played a much greater role in the current downturn than in earlier recessions.

This article reviews economic developments since the fall of 1973 and finds that some of the confusion concerning the current recession is eliminated if the period is divided into two stages. The first stage, which began in the late fall of 1973, was largely a response to constraints placed on aggregate supply. The second stage, which began in the early fall of 1974, reflected, in addition, a reduction in the growth of demand for goods and services.

For this study, November 1973 is used as a tentative peak for the past expansion.[1] It was the month of greatest industrial production, and in the following three months production declined. The second stage of the recession is postulated to have begun after September 1974. Industrial production and employment, which had risen, on balance, in the previous seven months, declined sharply after September. Comparisons are made with averages of periods just before and after the four preceding cyclical peaks, as selected by the National Bureau of Economic Research. They are November 1969, May 1960, July 1957, and July 1953.



COURSE OF THE RECESSION

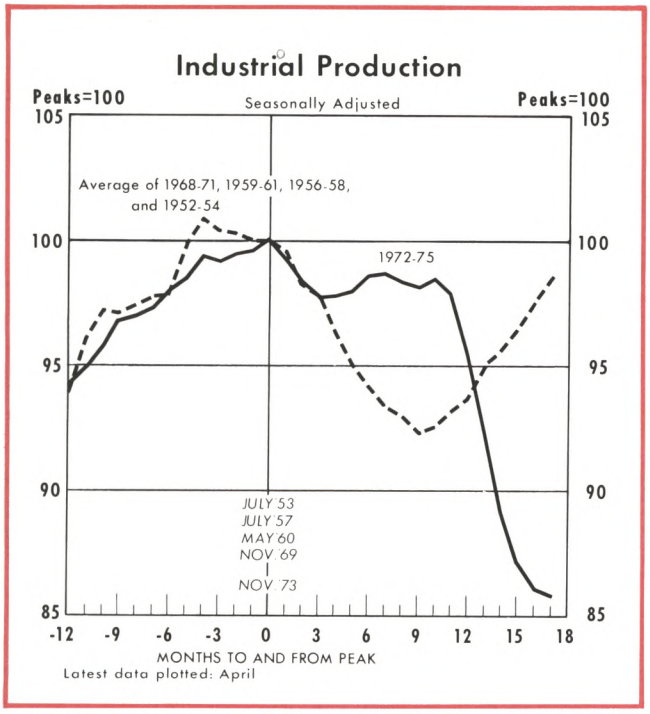
Production

Total real output of goods and services has declined markedly throughout the recent economic contraction. In the first five quarters following the peak (ending first quarter 1975), real output decreased at a 6 percent annual rate, and available data indicate that

[1]There are, of course, differences of opinion with respect to the dating of the cyclical peak.

the contraction probably has continued into the current quarter. By comparison, in the four previous recessions, real output declined for only two quarters, on average, and by the fifth quarter output typically exceeded the previous cyclical high.

Industrial production declined for three months after the 1973 cyclical peak, then increased slightly for the seven months from February to September 1974. During this period, production was bolstered by increased inventory demand as a result of anticipated shortages of many items and by anticipations of inventory profits as prices jumped. However, from September 1974 to April 1975 industrial production again fell sharply at a 21 percent annual rate.

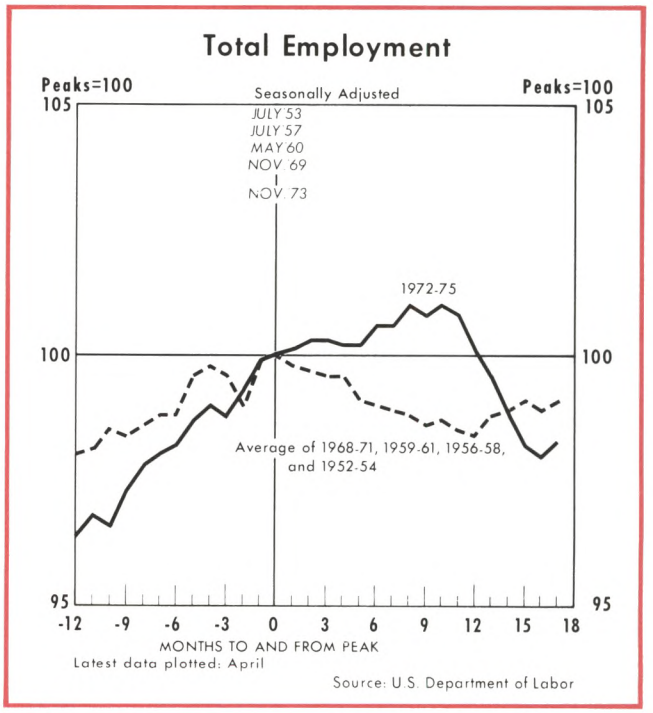


In the four previous recessions the pattern was for industrial production to contract for about nine or ten months. Thereafter, production turned up and rose, on average, at an 11 percent annual rate from the tenth to the seventeenth month, the stage comparable to the period of marked drop in the current cycle.

Employment

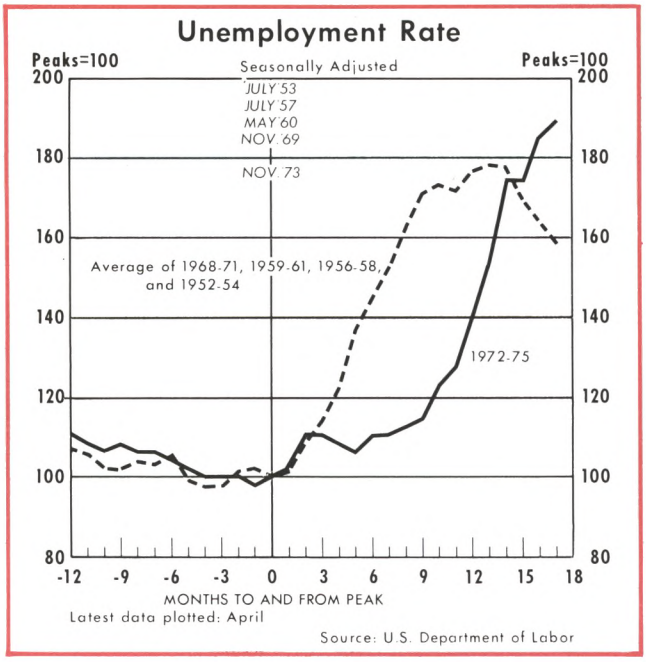
A distinctive feature of the current cycle has been the relatively high level of employment throughout much of the contraction. Total employment, which had risen a rapid 4 percent in the year preceding November 1973, continued to increase, but at a slower 1 percent annual rate during the first stage of the economic downturn. By contrast, in the corresponding ten-month period of earlier recessions, employment declined at an average 2 percent rate.

Reflecting a marked rise in employment since mid-1971, 65.2 percent of the non-institutional population of working force age (16 through 64) were employed in the third quarter of the current contraction (III/1974). On average, 62.7 percent held jobs at the four earlier cyclical highs. As the recession intensified in the period from last September to March, total employment decreased at a rapid 6 percent rate. Nevertheless, 63 percent of the noninstitutional population of working force age were employed in the first quarter of 1975, or slightly more than had been employed at the *peaks* of the four earlier cycles.

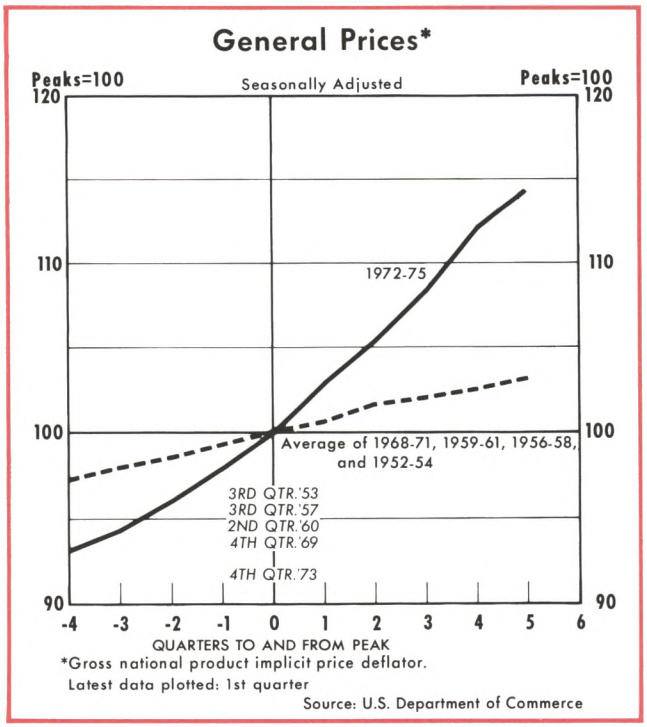


The unemployment rate remained relatively low until early last fall, but has risen abruptly since then. In the first two months after the peak in economic activity, the unemployment rate rose. Then, from the second to the ninth month after the turn (January to August 1974) the unemployment rate changed little, in marked contrast to pronounced increases in comparable periods of the earlier recessions. Since the ninth month, the trends have been reversed; the rate has risen sharply in recent months at a time when it usually remained on a high plateau. At the seventeenth month after the turning point (April 1975), the unemployment rate, at 8.9 percent, was higher than in any of the four previous recessions.

The combination of high unemployment and a relatively large percentage of the working age population



employed reflects a rise in the participation rate in the labor force. A larger share of women have entered the labor force in the last several years, and more recently, second and third members of many households have sought jobs to meet the higher cost of living or to maintain income when another member loses a job.



Prices

The rate of inflation has typically moderated after about two quarters of economic sluggishness. On average, during the final year of the four earlier economic expansions and the first two quarters of retrenchment, general prices, as measured by the implicit GNP deflator, rose at a 3 percent annual rate. From the second to the fifth quarters after the earlier cyclical peaks, overall prices increased at a slower 2 percent average rate.

Despite the sharp and prolonged contraction, the rate of inflation in the most recent downturn did not behave as in previous recessions. For the period from four quarters before to two quarters after the most recent peak, general prices advanced at an 8.5 percent annual rate. From the second to the fifth quarter of the recession, the pace of inflation accelerated to 12 percent. In the fifth quarter, however, prices rose at a slower 8.5 percent pace, and preliminary indications are that the slowing has continued into the sixth quarter (II/1975).

FIRST STAGE – NOVEMBER 1973 TO SEPTEMBER 1974

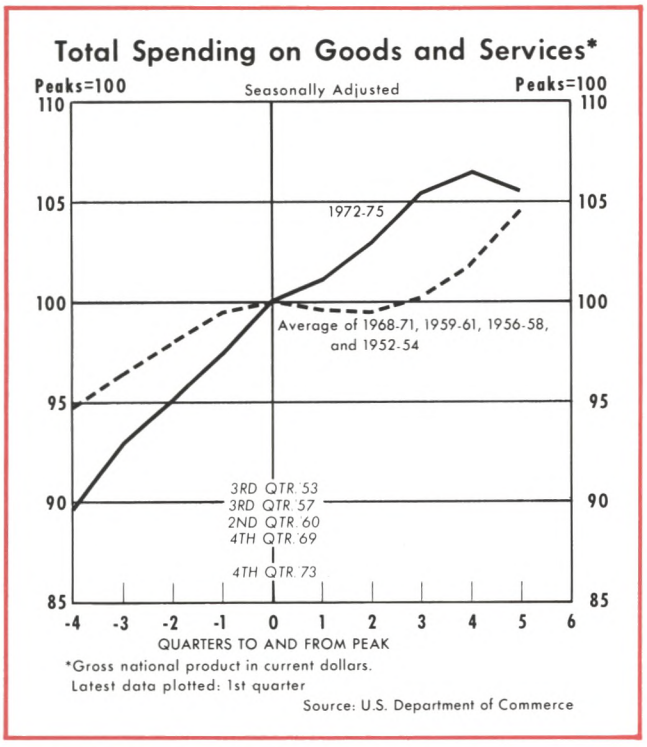
The contraction in production in the current recession, as noted above, has come in two distinct waves — the first from the November 1973 peak to September 1974 and the second from September 1974 to the present. The two periods of decline displayed substantially different characteristics and resulted from different causes. Therefore, in examining the recession it is useful to analyze the two periods separately, even though there is obviously an overlap of causes and effects.

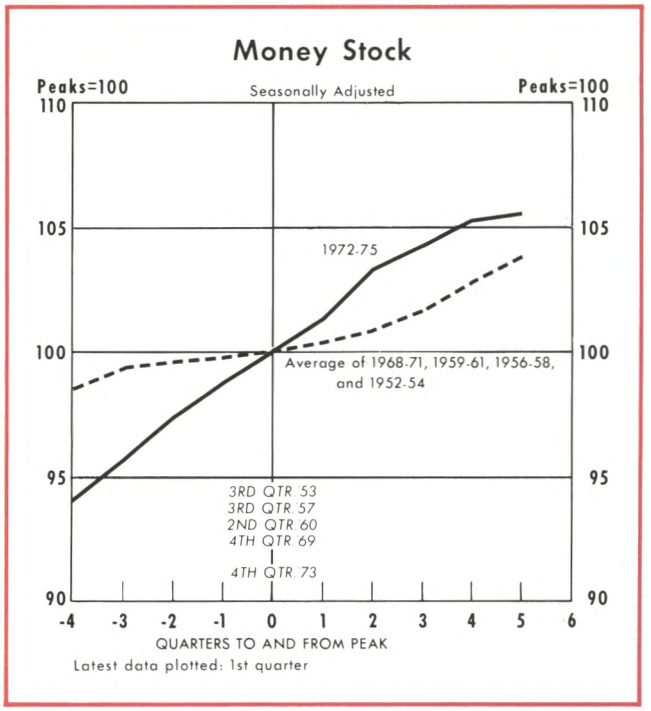
Demand

Growth in demand for goods and services, as indicated by nominal gross national product, moderated after the 1973 peak, but remained rapid by historical standards for the first three quarters of the recession. From the fourth quarter of 1973 to the third quarter of 1974, total spending rose at a 7 percent annual rate. By comparison, even in the final year of economic expansion preceding the four earlier economic contractions, total spending rose at only a 5.5 percent rate on average; then, in the first three quarters of recession the rise in spending slowed to a 0.3 percent rate.

Policy Actions

The unusual demand growth during the first three quarters of the current recession is partially explained by monetary developments. Monetary actions, as





measured by the growth rate of the money stock, were restrictive prior to each of the four earlier business cycle peaks. In each case the rate of money growth fell substantially below its trend rate of growth. This has been cited as a prominent causal factor in the cyclical changes in demand and subsequent recessions. In the year preceding the current recession, the rate of monetary expansion slowed, but fell only slightly below its pre-1973 trend rate of 6 percent in the second half of the year. From the fourth quarter of 1972 to the fourth quarter of 1973, the money stock rose 6.3 percent.

In the last two quarters of the most recent economic expansion (II/1973 - IV/1973) money growth did slow to a 5.3 percent annual rate. This break in the pace of money expansion probably contributed to the moderation in the growth rate of total spending after the fourth quarter of 1973.

After the recession began, demand growth continued for a time to be stimulated by monetary expansion. In the first two quarters of the current recession (IV/1973 - II/1974), the money stock grew at a 6.7 percent annual rate, slightly above the trend rate of growth.

Fiscal developments were also more expansive in the first stage of the current recession than in the corresponding periods of earlier recessions. In the year ending third quarter 1974, Federal Government expenditures, on a national income accounts basis, jumped 16 percent. In the corresponding spans of the four earlier cycles, Federal spending rose an average of 4 percent.

Even though tax receipts, bolstered by rising nominal incomes, rose sharply, Government expenditures exceeded receipts by a substantial margin in 1973 and 1974. The deficit in the national income accounts budget was $14 billion in these two years, compared with an average deficit of $6 billion in the corresponding periods of the earlier cycles.

Production

Despite the relatively rapid growth in demand for goods and services in the first stage of the current recession, output declined. Total real output decreased at a 3.5 percent annual rate from the fourth quarter of 1973 to the third quarter of 1974. In the corresponding periods of the four earlier recessions, output declined at a 2.4 percent rate.

The apparent paradox of substantial production cutbacks in the face of rapidly expanding demand during the first three quarters of the current recession reflects a number of supply constraints which sharply increased production costs. In 1973, rates of production were very high in many industries, and marginal costs rose rapidly with increased output. Then, the economy was struck by a number of adverse factors which reduced resource availability and raised costs.

Through monopolistic actions of producing nations and domestic price (and, therefore, production) controls, oil supplies were cut back and energy costs increased sharply. Adverse weather conditions, both here and abroad, resulted in widespread crop failures and increased food prices. Depreciation of the dollar relative to other currencies raised domestic production costs by increasing the dollar prices of imported goods, and caused significant shifts in demand for resources. Compliance with environmental and safety laws consumed resources and added further to costs, while making adjustment to all the other changing forces much more difficult and costly.

In addition, output was inhibited by marked changes in consumer spending and by price controls. The rapidly rising prices of certain items prompted consumers to shift consumption patterns, causing abrupt contractions in the demand for some items (such as autos) while intensifying upward pressures on prices of other commodities. Price controls, still in effect in early 1974, prevented firms from adjusting to increasing production costs. With fixed selling prices, production of many goods was no longer profitable; some marginal facilities were closed, and plant expansions were postponed. As a result, production in related industries was hampered further by shortages for a time, and later by higher prices for inputs.

These interferences in the productive process made the country poorer by reducing output capabilities. The adverse effects of these constraints were distributed through the economy by higher prices, a chief factor in the intensification of inflation during a period of recession.

SECOND STAGE – SEPTEMBER 1974 TO DATE

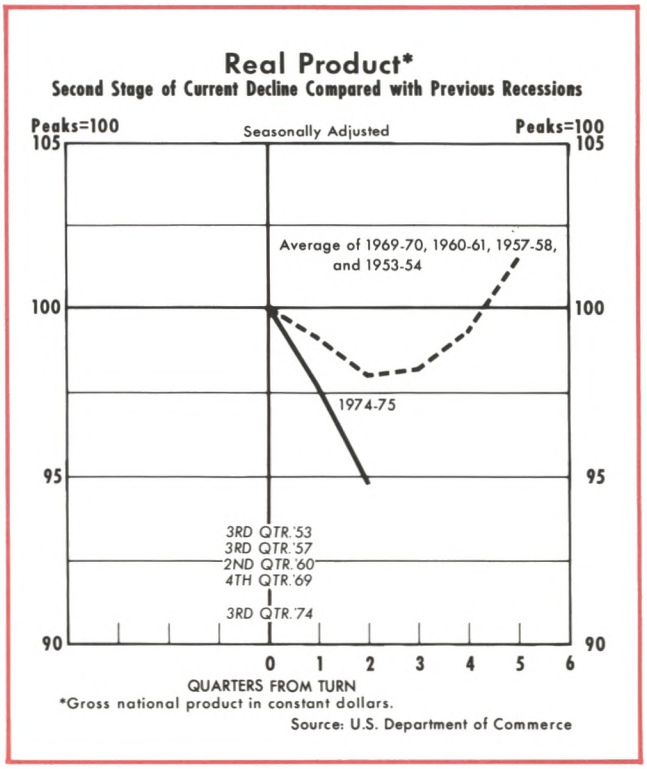
Demand

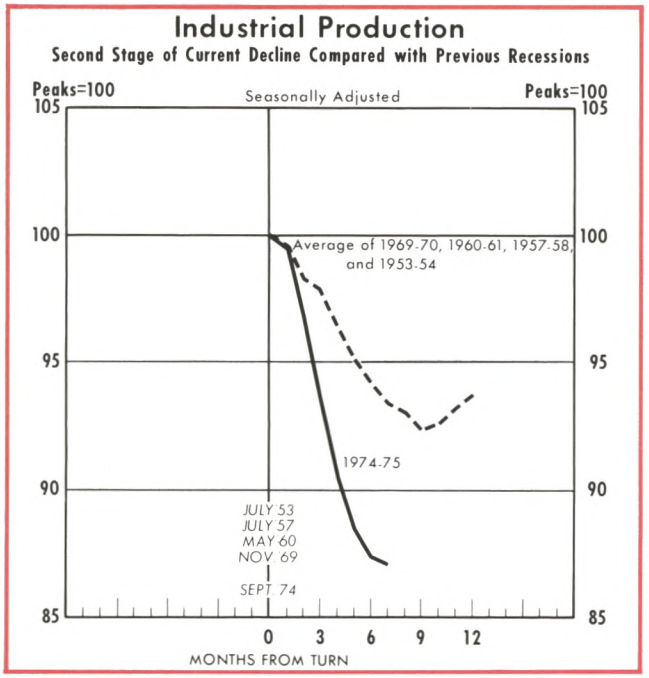
As the effects of some of the supply interferences were diminishing last fall, production was severely hampered a second time by a marked slowing in demand for goods and services. Total spending, which had risen at a 7 percent annual rate in the first three quarters of the recession, decelerated sharply last fall and declined in the first quarter of this year. By comparison, in the four previous recessions spending rose at an average 0.3 percent rate in the first three quarters, but at an accelerated 9 percent rate in the fourth and fifth quarters. With the reduction in demand growth, it was not feasible to maintain existing production levels, and sharp cutbacks occurred.

The money stock, after rising at a 6.7 percent annual rate in the first two quarters of the recession, slowed abruptly to a 3 percent pace in the following three quarters (ending March 1975). In the earlier recessions money typically increased at an accelerated rate in the third through fifth quarters. A few months after money began accelerating in previous recessions, growth in demand also accelerated. A few months after money growth began decelerating during the current recession, growth in demand also decelerated.

The first stage of the current recession, being caused in large part by constraints on supply, was unlike any of the earlier recessions, which were largely demand induced. However, the second stage, which has been fostered by a slowing in the growth of demand, is similar to the earlier experiences. Hence, for certain studies, comparisons of the period since last September with the periods immediately following the previous cyclical peaks have more relevance than comparisons which use the entire span of the current recession.

At this second stage turning point, the growth rate in total demand for goods and services fell precipitously from the aforementioned 7 percent rate in the first stage to the virtual plateau in the first two quarters of the second stage. Between the comparable periods before and after the four earlier cyclical



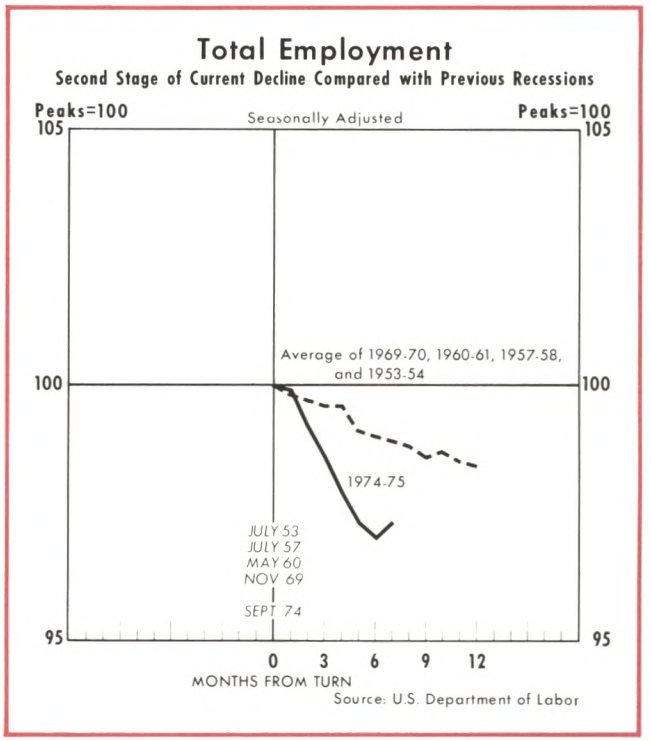


peaks, demand growth decelerated nearly the same amount, averaging a drop of 6 percentage points.

Production, Employment, and Prices

Total real output of goods and services fell at a 10 percent annual rate from the third quarter of 1974 to the first quarter of 1975. In the first two quarters of the four earlier recessions, real output declined at an average 4 percent rate. Industrial production dropped at a 21 percent rate from September 1974 to April 1975, compared with an average 11 percent rate in the first seven months of the earlier recessions. Total employment declined at a 6 percent rate from September to April, whereas it decreased at an average 2 percent rate in the comparable earlier periods.

Prices continued to rise rapidly for a time after the second stage of the recession began, the rate of increase has now slowed. From September to November 1974, consumer prices rose at a rapid 12 percent annual rate, about the same pace as in the first stage. Then, from November to April 1975 prices rose at a reduced 7 percent rate. In the earlier recessions, increases in consumer prices decelerated less than two percentage points at the seventh month, on average. The greater and quicker reduction in the rate of inflation in the second stage than in earlier recessions reflects the greater severity of the current economic decline, some dampening price effects from the first stage recession, and a dissipation of the short-run price effects resulting from removal of wage-price



controls, the dollar devaluation, and the markup of oil prices.

SUMMARY AND CONCLUSIONS

The current recession has been severe by post World War II standards, with output contracting by a greater magnitude and for a longer period than in any of the four previous recessions experienced since 1950. Not only has the current contraction been deep and prolonged, but it has been, in effect, two recessions. The first, induced largely by constraints on supply, had characteristics which differ strikingly from prior experience.

Previous recessions were preceded, and accompanied for a time, by a slow (relative to trend) rate of money growth. By contrast, money expansion in the current cycle continued to be rapid, except for a brief period, through the first two quarters of the recession. However, from the second quarter of 1974 to the first quarter of this year, the rate of money growth slowed markedly. Fiscal actions, on the other hand, have been expansive since a quarter before the current cyclical peak.

Total spending for goods and services rose substantially during the first three quarters of the current recession, pausing only moderately after the cyclical peak. However, spending growth slowed significantly

after the third quarter of the contraction, causing the recession to enter the second stage.

Until last fall, the chief cause of the downturn came from the supply side. The nation's ability to produce was reduced by increased energy costs, unfavorable weather, costs of environmental and safety programs, the impact of dollar devaluation, and the effects of price controls. The quantity of goods and services available for consumption thus declined. Much of the current recession and the persistence of inflation have reflected the process of adjustment that the economy has been making to the constraints placed on production.

Recovery from the current recession depends on the overcoming or removal of constraints on supply. Elimination of wage and price controls was a significant step in attaining greater output, since production tends to expand when profits are enhanced. With normal weather, agricultural production should increase, placing downward pressures on the price of food. While some adjustment to the higher cost of fuel has taken place, a full adjustment will take additional time.

Economic recovery is also dependent on a pick-up in demand growth. In view of the projected sizable Federal deficits, and the probable monetary creation that will occur in financing them, total demand is likely to receive a substantial boost from fiscal and monetary developments in the near future. Since January the money stock has again risen sharply. Expansionary developments are now welcome, because demand is inadequate. Yet, a stimulation of demand in excess of the ability of the economy to produce would likely result in a re-intensification of inflationary pressures at a later date.

A Monetary Model of Nominal Income Determination

LEONALL C. ANDERSEN

DURING the past several years the Federal Reserve Bank of St. Louis has presented a number of empirical studies demonstrating a strong and predictable response of nominal gross national product (GNP) to changes in the nation's money stock. These studies found that changes in the secular trend of the money stock are the prime determinant of changes in the secular trend of GNP. They also found that short-run changes in GNP are related to similar changes in the money stock. On the other hand, changes in Government spending were found to have only a temporary short-run influence on changes in GNP. A number of other studies, using variations of the approach of the St. Louis studies, have yielded similar empirical relationships.

The St. Louis studies related changes in nominal GNP directly to current and past changes in the money stock and in high-employment Government expenditures. No underlying set of relationships was specified. Instead, the empirical relationship was presented as the reduced-form of an unspecified structure of the economy.

This article sets forth a theoretical model consisting of a set of postulated relationships which form a basis for the relationship of nominal income (GNP) to the money stock and Government expenditures. The theoretical properties of this model are similar to the empirical relationships found in the St. Louis studies. The parameters of the model are estimated for the period from first quarter 1955 to fourth quarter 1973. Empirical tests fail to reject the theory as an explanation of nominal income determination in the sample period.

THEORETICAL MODEL

This section develops a model of nominal income determination which is based primarily on a prominent theory of the influence of changes in the money stock on income.[1] At times, however, assertions different from those in the prominent theory are made. It is also asserted that the responses of holders of money balances to changes in the dependent variables are in relative terms; that is, rates of change or percent discrepancies. The model, therefore, is expressed in log-linear form.

Desired Nominal Money Balances

The theory underlying this study is based on the assertion that households and business firms desire nominal money balances to conduct market transactions and to hold as a store of value. Specifically, the desired amount of nominal money balances, in the aggregate, depends on the perceived price of newly produced final goods and services, perceived nominal income, the expected rate of inflation, yields on alternative financial assets, and the technical efficiency of the economy's system of making money payments.[2]

Since holders of money balances do not have perfect information regarding market opportunities, and since there exist costs of both collecting information and conducting market transactions, the desire to hold money balances depends on the perceived, instead

NOTE: This presentation was given as part of the Lawrence H. Seltzer Memorial Lecture Series at Wayne State University, Detroit, Michigan, May 21, 1975.

[1]For statements of this theory, see Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results," *Journal of Political Economy* (August 1959), pp. 327-351; "A Theoretical Framework for Monetary Analysis," *Journal of Political Economy* (March/April 1970), pp. 193-238; and "A Monetary Theory of Nominal Income," *Journal of Political Economy* (March/April 1971), pp. 323-337.

[2]For various asserted forms of the money demand function, see David E. W. Laidler, *The Demand For Money: Theories and Evidence* (Scranton: International Textbook Company, 1969). For an excellent statement of the "conventional view" of the money demand function, see Stephen M. Goldfeld, "The Demand For Money Revisited," *Brookings Papers on Economic Activity* — 3 (1973), pp. 577-638. Also see John T. Boorman, "The Evidence on the Demand for Money: Theoretical Formulations and Empirical Results," in John T. Boorman and Thomas M. Havrilesky, *Money Supply, Money Demand, and Macroeconomic Models* (Boston: Allyn and Bacon, Inc., 1972), pp. 248-291, and Stephen Rousseas, *Monetary Theory* (New York: Alfred A. Knopf, 1972).

of the actual, price of goods and services and nominal income. Given the influences of the other factors, the amount of nominal money balances demanded is positively related to the perceived price of newly produced final goods and services. Given the influences of both perceived price and the other factors, the amount of money balances demanded is positively related to the amount of goods and services that perceived nominal income can purchase.

It is asserted that the response of holders of money balances is of the same magnitude with regard to both perceived price and the amount of goods and services that perceived nominal income can purchase. The desire for nominal money balances, therefore, is formulated in terms of perceived nominal income only (Equation 1).[3]

A continuing rise in the price of goods and services results in a given amount of nominal money balances held as a store of value commanding, over time, a decreasing amount of goods and services. Given the influences of the other factors, the amount of money demanded is negatively related to the rate of inflation expected to prevail in the future (Equation 1).[4]

Other financial assets are substitutes for money balances as a store of value, and they yield a rate of return. These rates of return, with constant, perceived prices for goods and services, are measured in terms of real rates of interest. To the holder of a financial asset, the real rate of interest is the annual percent increase in the amount of goods and services, at current prices, that a given dollar value of a financial asset will command over the term to maturity. With the influence of other factors held constant, the amount of money demanded is negatively related to real rates of return on alternative financial assets.

It is asserted that the dominant substitutes for money as a store of value are short-term financial assets and that the time horizon upon which expectations of the rate of inflation are based is the same for money balances as for these assets. It is generally accepted that the nominal rate of interest on a financial asset is the sum of the real rate of return and the rate of inflation expected over the term of the financial asset. The nominal short-term interest rate, therefore, is presumed to capture the negative influence on the amount of money demanded of both the real short-term interest rate and the expected rate of inflation (Equation 1).

The technical efficiency of the system of making money payments is defined as the average amount of money balances required to be held to conduct a given nominal volume of transactions. With the influences of other factors held constant, the amount of money demanded is negatively related to the technical efficiency of the system of making money payments (Equation 1). In other words, the smaller the amount of money balances technically required to be held to conduct a given dollar volume of transactions, the smaller is the amount of money demanded.

(1) $\ln M^{\bullet}(t) = \alpha_0 \ln E(t) + \alpha_1 \ln Y^{P}(t) + \alpha_2 \ln r(t)$.

$\alpha_0 < 0$, $\alpha_1 > 0$, and $\alpha_2 < 0$.

$M^{\bullet}$ = desired nominal money balances.

E = technical efficiency of the payments system.

Y^{P} = perceived level of nominal income.

r = nominal short-term interest rate.

Adjustment of Spending by Households and Business Firms

The nominal stock of money is assumed to be exogenous, determined by the monetary authorities. Therefore, holders of money balances, in the aggregate, cannot adjust their holdings when a discrepancy occurs between actual and desired money balances. Instead, it is asserted that individual holders of money balances attempt to add to (reduce) money balances by reducing (increasing) their rate of spending on goods and services. Such an adjustment continues until desired money balances are brought into equality with actual money balances.

It is assumed that the rate of change in the dollar volume of spending by households and firms on newly produced final goods and services from both domestic and foreign sources is proportional to the percent discrepancy between actual and desired money balances.[5] It is also assumed that the response of spending is distributed over time (Equation 2).

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[3]Ignoring the other factors,

$$\ln M^{\bullet} = \alpha_1 \ln P^{P} + \alpha_2 \ln \frac{Y^{P}}{P^{P}} = \alpha_1 \ln P^{P} + \alpha_2 \ln Y^{P} - \alpha_2 \ln P^{P}.$$

By assertion $\alpha_1 = \alpha_2$; therefore, $\ln M^{\bullet} = \alpha_1 \ln Y^{P}$. Friedman in "A Monetary Theory of Nominal Income" asserts that for all practical purposes $\alpha_1 = \alpha_2 = 1$. In this study the only assertion made is that $\alpha_1 = \alpha_2$. The validity of this assertion is tested later in the study and the hypothesis that the two responses are identical cannot be rejected. Friedman's assertion that they both equal unity is also tested and is rejected.

[4]It is important to note that there are two different influences on the amount of money balances demanded related to the price of newly produced final goods and services. One is the positive influence of the perceived price level for the current period, and the other is the negative influence of the expected rate of change in the price level over the relevant future.

[5]This specification of the adjustment process is a marked departure from that specified in other studies. Some specify

$$(2)\ \frac{d \ln Y^d}{dt} = \lambda [\ln M(t) - \ln M^*(t)].$$

M = actual nominal money balances.

$\frac{d \ln Y^d}{dt}$ = rate of change in spending by households and business firms for newly produced, final goods and services.

λ = speed of adjustment. $0 < \lambda \leq \infty$. The larger is λ, the faster is the speed of adjustment. $\lambda = \infty$ is instantaneous adjustment.

Combining Desired Money Balances and Adjustment Process

The preceding assertions regarding the demand for nominal money balances and the adjustment process are now combined by substituting equation (1) into equation (2). The result is that the rate of change of nominal spending by households and business firms is related to the level of actual nominal money balances, the technical efficiency of the payments system, the level of the nominal short-term interest rate, and the perceived level of nominal income.

$$(3)\ \frac{d \ln Y^d}{dt} = \lambda [\ln M(t) - \alpha_0 \ln E(t) - \alpha_1 \ln Y^P(t) - \alpha_2 \ln r(t)].$$

Definition of Nominal Income

Nominal income is defined as total value added (that is, factor payments) in the domestic production of new final goods and services. Total nominal spending on domestic product is also equal, by definition, to value added. Nominal income, accordingly, is equal to nominal spending for both domestic and foreign product by domestic households, business firms, and all units of government, plus nominal spending by foreigners for domestic product, less domestic nominal spending for foreign produced goods and services.

$$(4)\ Y(t) = Y^d(t) + G(t) + X(t) - IM(t).$$

Y = nominal income.

Y^d = spending by household and business firms for both domestic and foreign product.

G = spending by all units of government for both domestic and foreign product.

X = spending by foreigners for domestic product.

IM = domestic spending for foreign product.

Domestic Spending for Foreign Product

Domestic nominal spending for foreign product is assumed to be a constant proportion of total nominal spending in the economy.

$$(5)\ \delta(t) = \frac{IM(t)}{Y^d(t) + G(t) + X(t)} = \delta.$$

Dynamic Form of the Model

The model, up to this point, involves the change in the *level* of income in response to changes *in the level* of the money stock, or in Government spending, or in foreign spending on domestic product, or in the short-term interest rate, or in the technical efficiency of the payments system. All of these variables, however, are continuously changing over time. The model is differentiated with respect to time to reflect the response of households and business firms to those variables which are continuously changing.

In its dynamic form, the rate of change in desired money balances is related to the rates of change in the technical efficiency of the payments system, in the short-term interest rate, and in perceived income.

$$(1')\ \frac{d \ln M^*}{dt} = \alpha_0 \frac{d \ln E}{dt} + \alpha_1 \frac{d \ln Y^P}{dt} + \alpha_2 \frac{d \ln r}{dt}.$$

The *change* in the rate of change in nominal spending by households and business firms is proportional to the discrepancy between the rate of change in actual money balances and the rate of change in desired money balances.

$$(2')\ \frac{d^2 \ln Y^d}{dt^2} = \lambda [\frac{d \ln M}{dt} - \frac{d \ln M^*}{dt}].$$

The statement combining the adjustment process and the factors influencing desired money balances is accordingly modified.

$$(3')\ \frac{d^2 \ln Y^d}{dt^2} = \lambda [\frac{d \ln M}{dt} - \alpha_0 \frac{d \ln E}{dt} - \alpha_1 \frac{d \ln Y^P}{dt} - \alpha_2 \frac{d \ln r}{dt}].$$

Substituting equation (5) into (4) and differentiating with respect to time, the rate of change in nominal income is equal to the weighted sum of the rates of change of spending for product by domestic households and business firms, domestic units of government, and foreigners.

$$(4')\ \frac{d \ln Y}{dt} = W_1(t) \frac{d \ln Y^d}{dt} + W_2(t) \frac{d \ln G}{dt} + W_3(t) \frac{d \ln X}{dt}.$$

$$W_1(t) = [1-\delta] \frac{Y^d(t)}{Y(t)};\ W_2(t) = [1-\delta] \frac{G(t)}{Y(t)};$$

$$\text{and } W_3(t) = [1-\delta] \frac{X(t)}{Y(t)}.$$

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that the actual money stock changes in response to a discrepancy between actual and desired money balances; others specify that "real" money balances change; and others specify that the short-term interest rate changes. See references in fn. 2 for specific examples of these other specifications. For discussion of the importance of the specification of the adjustment process in empirical research, see: A. A. Walters, "The Demand for Money — The Dynamic Properties of the Multiplier," *Journal of Political Economy* (June 1967), pp. 293-298, and D. R. Starleaf, "The Specification of Money Demand — Supply Models Which Involve the Use of Distributed Lags," *Journal of Finance* (September 1970), pp. 743-760.

Summary of Variables

The endogenous variables of the model are the change in the rate of change in nominal spending by households and business firms, the rate of change in desired money balances, and the rate of change in nominal income. It will also be developed in subsequent analyses that the rate of change in the perceived level of nominal income is an endogenous variable related to past rates of change in nominal income. The exogenous variables are the rates of change in actual money balances, in the short-term interest rate, in the technical efficiency of the payments system, in Government spending on goods and services, and in foreign spending on domestic product.

The model is one of partial analysis, inasmuch as the nominal short-term interest rate is an exogenous variable. There is no provision for feed-back effects on the rate of change in nominal income. These effects would emanate from changes in exogenous variables which change the rate of change in the short-term interest rate, either by induced changes in the real interest rate or in the expected rate of inflation.

Conditions for Dynamic Equilibrium

Dynamic equilibrium is defined as the state in which all of the variables are changing at constant rates.[6] This occurs when the rates of change in actual and desired money balances are equal. Three conditions are required for this equality to be fulfilled — *changes* in the rates of change in nominal spending by households and business firms and in nominal income equal zero (E-1 and E-2), and the rates of change in nominal income and in the perceived level of nominal income are equal (E-3).

$$\text{(E-1)}\quad \frac{d^2 \ln Y^d}{dt^2} = 0.$$

$$\text{(E-2)}\quad \frac{d^2 \ln Y}{dt^2} = 0.$$

$$\text{(E-3)}\quad \frac{d \ln Y^P}{dt} = \frac{d \ln Y}{dt}.$$

[6]The definition of equilibrium as *constant rates of change* is a marked departure from the standard literature in monetary economics, other than some growth models. The standard analysis defines equilibrium as *constant levels* of the endogenous variables. The equilibrium conditions for the standard analysis would be that desired and actual levels of money balances are equal, that the rates of change in spending by households and business firms and in nominal income equal zero, and that the perceived level of nominal income equals the actual level. Thus, in the standard analysis there is both stock and flow equilibrium. In the dynamic form in this study, however, equilibrium is defined as constant rates of change in both stock and flow variables.

Dynamic Equilibrium State

The relationships of the model (Equations 3′, 4′, and the weights) together with the three dynamic equilibrium conditions (Equations E-1 through E-3) are used to solve for the dynamic equilibrium state of the endogenous variables.[7] In the dynamic equilibrium state, taking into consideration the postulated signs of the coefficients, the rate of change in nominal spending by households and business firms is positively related to the rates of change in actual money balances, in the technical efficiency of the payments system, and in the nominal short-term interest rate, and is negatively related to the rates of change in government spending and exports (Equation ES-1). The rate of change in nominal income is positively related to the rates of change in actual money balances, in the technical efficiency of the payments system, and in the nominal short-term interest rate (Equation ES-2).

$$\text{(ES-1)}\quad \frac{d \ln Y^d}{dt} = \frac{1}{W_1(t)\alpha_1}\left[\frac{d \ln M}{dt} - \alpha_0 \frac{d \ln E}{dt} - \alpha_2 \frac{d \ln r}{dt} - W_2(t) \frac{d \ln G}{dt} - W_3(t) \frac{d \ln X}{dt}\right].$$

$$\text{(ES-2)}\quad \frac{d \ln Y}{dt} = \frac{1}{\alpha_1}\left[\frac{d \ln M}{dt} - \alpha_0 \frac{d \ln E}{dt} - \alpha_2 \frac{d \ln r}{dt}\right].$$

A Change in Dynamic Equilibrium State

An existing dynamic equilibrium state is disturbed, in this model, by two types of events. One type is the initial creation of a discrepancy between the rates of change in actual and desired money balances. Factors initiating such a discrepancy are changes in the rate of change in actual money balances or changes in the rate of change in desired money balances. This latter change can result from changes in the rate of change in either the technical efficiency of the payments system or in the short-term interest rate. The other type of equilibrium disturbance is a change in the rate of change in government spending or in foreign spending for domestic product, which initiates a change in the rate of change in nominal income. Following both types of disturbance, a new equilibrium state is achieved when the rate of change in nominal income has changed to the extent that the rate of change in desired money balances is brought into equality with the rate of change in actual money balances.

[7]This analysis and the one immediately following is for partial equilibrium only inasmuch as the interest rate is exogenous. The model only considers the influence of exogenous shocks on the choice between money and new production and not on the choice between money and existing assets.

For example, starting from dynamic equilibrium (that is, all variables changing at constant rates) assume there is a maintained increase in the rate of change in actual money balances, with the rates of change in the other independent variables remaining as they were. First, there occurs a positive discrepancy between the rates of change in actual and desired money balances, resulting in an increase in the rate of change in nominal spending by households and business firms. As a result, there is an increase in the rate of change in nominal income. This latter increase, in turn, results in an increase in the rate of change in perceived nominal income and thereby in the rate of change in desired money balances.

As a result, the discrepancy between the rates of change in actual and desired money balances is narrowed and the rate of change in spending by households and business firms decreases. This decrease reduces the rate of change in nominal income from what it was initially. This process continues until there is equality between the rates of change in actual and desired money balances. In the new dynamic equilibrium state, the rates of change in spending by households and business firms and in nominal income are higher than their starting equilibrium values (ES-1 and ES-2).

In another example, starting from equilibrium, assume there is a maintained increase in the rate of change in government spending, with rates of change in the other independent variables remaining the same. The first response is an increase in the rate of change in nominal income which produces an increase in the rate of change in perceived nominal income. This results in an increase in the rate of change in desired money balances.

As a consequence, there is a negative discrepancy between the rates of change in actual and desired money balances, resulting in a subsequent decrease in the rate of change in spending by households and business firms. This is accompanied by a decrease in the rates of change in nominal income and in perceived nominal income, which, in turn decreases the rate of change in desired money balances. This process continues until the rate of change in desired money balances equals that of actual money balances.

In the new dynamic equilibrium state the rate of change in nominal income is the same as it was before the increase in the rate of change in government spending (ES-2). The rate of change in spending by households and business firms has been permanently decreased (ES-1), offsetting fully the initial increase in the rate of change in nominal income generated by the increase in the rate of change in government spending.

The length of time required for achieving the new equilibrium state depends on (1) the speed of response of the rate of change in spending by households and business firms to a discrepancy between the rates of change in actual and desired money balances and (2) on the length of time required for the rate of change in perceived nominal income to respond fully to past rates of change in actual nominal income. The time path to a new equilibrium — whether it oscillates or moves only in one direction — depends on the response characteristics of the relationships comprising the model.

EMPIRICAL FORM OF THE THEORETICAL MODEL

The theoretical model consists of three relationships. One is for the change in the rate of change in spending by households and business firms (Equation 3′). The other two are the identity for the rate of change in nominal income (Equation 4′), and the weights involved in this identity. The theoretical model is next given an empirical form which provides the basis for testing whether or not the theory can be accepted. This section presents the empirical model and the following section uses it to test the theory.

Data Problems

A major data problem is involved in developing the empirical form of the model — the theory is in terms of changes in time which are infinitesimally small while data on the variables are available only for discrete points in time separated by a month or a quarter of a year. A linear approximation in discrete time is used — the first difference of natural logarithms of a variable between two discrete points in time is presumed to approximate its rate of change (Appendix).

Another data problem exists inasmuch as there are no measurements of the technical efficiency of the payments system or of the perceived level of nominal income. It is assumed that, on average, the technical efficiency of the payments system increases at a constant rate (Appendix).[8] The second problem is solved

[8]Examples of developments which are usually presumed to have increased the efficiency of the payments system (a reduced amount of money balances required to carry out a given volume of money payments) over the sample period

by employing a procedure presented by Philip Cagan.[9] According to this procedure, the rate of change in perceived nominal income can be approximated by a weighted average of past rates of change in actual nominal income (Appendix). In this procedure, the weights sum to unity.

Estimated Parameters of the Model

Only the parameters of the relationship explaining the change in the rate of change in spending for product by households and business firms are estimated; the other relationships are identities. Two versions of this relationship are estimated. One is based on the assertion that desired money balances are positively related to perceived price and the amount of goods and services that perceived nominal income can purchase, with no assertions regarding the magnitudes of response. The other one is based on the assertion that the magnitude of response of desired money balances is the same with regard to perceived price as with regard to the amount of goods and services that perceived nominal income can purchase — the perceived income formulation used in the theoretical discussion. The general forms of these two relationships are as follows:

$$(A)\ \Delta\ln Y^d_t - \Delta\ln Y^d_{t-1} = a_0 + a_1 \Delta\ln M_t + a_2 \Delta\ln r_t + a_3 \sum_{i=1}^{n} w_i \Delta\ln P_{t-i} + a_4 \sum_{i=1}^{n} w_i \Delta\ln Q_{t-i} + \varepsilon_t.$$

$$(B)\ \Delta\ln Y^d_t - \Delta\ln Y^d_{t-1} = b_0 + b_1 \Delta\ln M_t + b_2 \Delta\ln r_t + b_3 \sum_{i=1}^{n} w_i \Delta\ln Y_{t-i} + \varepsilon_t.$$

$\Delta\ln Y^d_t - \Delta\ln Y^d_{t-1}$ = *change* in the rate of change in spending by households and business firms for product.

a_0 and b_0 = response to the average rate of change in the technical efficiency of the payments system.

$\Delta\ln M_t$ = rate of change in nominal money balances.

$\Delta\ln r_t$ = rate of change in nominal short-term interest rate.

$\sum_{i=1}^{n} w_i \Delta\ln P_{t-i}$ = weighted sum of past rates of change in price.

$\sum_{i=1}^{n} w_i \Delta\ln Q_{t-i}$ = weighted sum of past rates of change in the amount of goods and services that nominal income can purchase.

$\sum_{i=1}^{n} w_i \Delta\ln Y_{t-i}$ = weighted sum of past rates of change in nominal income.

ε_t = a random error term.

Ordinary least-squares regressions, using quarterly data from first quarter 1955 to fourth quarter 1973, are used to estimate the parameters of equations A and B. Spending by households and business firms is measured by the sum of consumption plus investment in the national income accounts. Nominal income is measured by nominal GNP.[10] The price level is measured by the GNP deflator, and the amount of goods and services that nominal income can purchase is measured by nominal GNP divided by the price deflator. Two definitions of money are used. One (M_1) is the sum of demand deposits and currency held by the nonbank public. The other one (M_2) is M_1 plus time and savings deposits at commercial banks, other than large, negotiable certificates of deposit. The short-term interest rate is measured by the 4- to 6- month commercial paper rate. Two zero-one dummy variables are included for the average influence of major strikes on income, with D_1 equaling one for the quarter of a strike and D_2 equaling one for the quarter following a strike.[11]

A test was performed for a structural change in the model (Appendix). For M_1 such a change is alleged by some analysts to have occurred after the fourth quarter of 1966 when a change in the trend growth of the ratio of GNP to M_1 is observed. A similar change in the trend growth of the ratio of GNP to M_2 occurred after the fourth quarter of 1961. In both cases the structural change hypothesis was rejected. A statistical procedure (Appendix) was next used to select the appropriate number of lagged changes in the price level, nominal income, and in the amount of goods and services that nominal income can purchase; four lagged terms were deemed appropriate. The estimated parameters of equations A and B are reported in Tables I and II.

EMPIRICAL MODEL AS A TEST OF THE THEORY

The empirical model is used to test whether or not the theory of nominal income determination advanced earlier can be accepted. The part of this theory regarding the determination of nominal spending by households and business firms carries specific implica-

are the introduction of faster means of transferring funds and the adoption of improved methods of managing money balances. See George Garvy and Martin R. Blyn, *The Velocity of Money* (New York: Federal Reserve Bank of New York, 1969), pp. 207-218, for further discussion of this point.

[9]Phillip Cagan, "The Monetary Dynamics of Hyperinflation," *Studies in the Quantity Theory of Money*, ed. Milton Friedman (Chicago: University of Chicago Press, 1956), pp. 31-41.

[10]In national income terms, the definition of nominal income used in this study is the sum of consumption, investment, government expenditures, and exports less imports. This sum should be adjusted for depreciation and indirect business taxes, to be identical to value added. It is assumed that variations in these two magnitudes will have little influence on the outcome of this study.

[11]The major strikes in the sample period were: steel III/1959, autos IV/1964, and autos IV/1970.

Table I

Regression Results*
Equation (A)

Independent Variable	M_1	M_2
D_1	−1.996	−1.947
	(−3.765)	(−3.637)
D_2	2.259	1.921
	(3.978)	(3.321)
$\Delta \ln M_t$	.685	.534
	(4.003)	(3.814)
$\Delta \ln r_t$	.022	.032
	(2.257)	(3.096)
$\Delta \ln Q_{t-1}$	− .797	− .799
	(−5.250)	(−5.182)
$\Delta \ln Q_{t-2}$	− .249	− .208
	(−1.905)	(−1.580)
$\Delta \ln Q_{t-3}$	.263	.238
	(2.051)	(1.842)
$\Delta \ln Q_{t-4}$	− .278	− .301
	(−2.341)	(−2.518)
$\Delta \ln P_{t-1}$	− .918	− .811
	(−2.105)	(−1.855)
$\Delta \ln P_{t-2}$	− .558	− .592
	(−1.193)	(−1.254)
$\Delta \ln P_{t-3}$	.143	.232
	(.298)	(.479)
$\Delta \ln P_{t-4}$	.251	.274
	(.606)	(.657)
Constant	1.074	.772
	(3.078)	(2.135)
$\bar{R}^2$	.569	.561
SEE	.875	.883
DW	2.009	2.037

*Numbers in parentheses are t-values.

Table II

Regression Results*
Equation (B)

Independent Variable	M_1	M_2
D_1	−1.930	−1.886
	(−3.744)	(−3.616)
D_2	2.380	2.041
	(4.326)	(3.630)
$\Delta \ln M_t$	.701	.572
	(4.527)	(4.309)
$\Delta \ln r_t$	.020	.030
	(2.183)	(2.974)
$\Delta \ln Y_{t-1}$	− .782	− .781
	(−5.242)	(−5.157)
$\Delta \ln Y_{t-2}$	− .274	− .225
	(−2.164)	(−1.759)
$\Delta \ln Y_{t-3}$	.226	.197
	(1.846)	(1.592)
$\Delta \ln Y_{t-4}$	− .309	− .315
	(−2.739)	(−2.765)
Constant	1.152	.911
	(3.785)	(2.792)
$\bar{R}^2$	.580	.570
SEE	.864	.874
DW	2.036	2.049

*Numbers in parentheses are t-values.

tions regarding the estimated coefficients reported in Tables I and II. If the estimated coefficients are consistent with the implied coefficients, at the five percent level of statistical significance, this part of the theory is accepted.

The model as a whole constitutes the theory of nominal income determination. This theory is tested by the ability of the empirical model to forecast nominal income.[12] Relatively small root mean squared errors in forecasting nominal income are taken as evidence that the theory of nominal income determination can be accepted.

Testing Implications of Theory Regarding the Responses of Households and Business Firms

It was asserted in the theoretical section that the response of holders of money balances is of the same magnitude with regard to perceived price as with regard to the amount of goods and services that perceived nominal income can purchase. Therefore, the theoretical formulation was in terms of perceived nominal income. This assertion implies in Equations A and B that for each lag the coefficients for the rates of change in price, in the amount of goods and services that nominal income can purchase, and in nominal income are equal. This implication was tested and could not be rejected at the five percent level of statistical significance (Appendix).[13] As a result of this test, estimates of the coefficients of Equation B are used for the balance of this study (Table II).

The theory of determination of nominal spending by households and business firms implies that all of the coefficients in Equation B are statistically significant from zero at the five percent level and that they have the following signs: b_0, b_1, and b_2 are positive and b_3 is negative.[14] All of the estimated coefficients in Table II are statistically significant from zero at the five percent level and have the implied signs. The

[12]A more appropriate test would be to solve for the reduced-form of the model, and then to test whether the estimated coefficients of this equation are consistent with those implied by the theory. The reduced-form is a nonlinear equation with variable coefficients, which poses some very difficult problems of estimation. The forecasting test was therefore selected.

[13]The proposition that the elasticity of desired money balances with respect to the perceived level of nominal income equals unity was tested and rejected (Appendix). The proposition that this elasticity is greater than unity was accepted.

[14]These implied signs are derived by applying the postulated signs for Equations 1′ and 2′ to Equation 3′. The magnitude of b_3 is derived by adding the coefficients on the lagged income terms.

empirical evidence is consistent with the implications of this part of the theory, which is, therefore, accepted.

Testing Theory of Nominal Income Determination

A test of the theory of nominal income determination is the ability of the empirical form of the model to forecast nominal income with relatively small root mean squared errors. This forecasting ability of the model is determined by dynamic simulations. In a dynamic simulation actual values of the quarterly rates of change in money balances, in government spending plus exports, and in the nominal short-term interest rate are used. At the start of the simulation, actual lagged rates of change in nominal income are used, but, subsequently, simulated rates of change are incorporated. Two types of simulations are performed — *ex post* for the sample period and a number of *ex ante* simulations beyond various sample periods.

The simulation model consists of Equation B (Table II), Equation (4′), and the definition of the weights. Using the discrete-time linear approximation to rates of change in Equation (4′):

$$\Delta \ln Y_t = W_t \, \Delta \ln Y_t^d + (1-W_t) \, \Delta \ln Z_t.$$

$$W_t = (1-\delta) \frac{Y_{t-1}^d}{Y_{t-1}}.$$

Z = government spending plus exports.

δ = ratio of imports to total spending on product. The ratio is held constant at its average value in the sample period.

Ex post simulations are used to ascertain the ability of the model to capture movements in nominal income over the sample period. The root-mean-squared-error (RMSE) for the quarterly rates of change of nominal income (expressed as annual rates) is 2.85 percentage points using M_1 and 2.95 using M_2. Although the errors in the quarterly rates of change (expressed as annual rates) are quite large, the errors tend to be offsetting over the sample period. In the fourth quarter of 1973, the error in the level of nominal income was 0.36 percent using M_1 and 0.33 percent using M_2. Over the sample period, the RMSE for the quarterly levels of nominal income is 1.57 percent using M_1 and 2.07 percent using M_2.

Ex ante simulations are used to determine the ability of the model to forecast nominal income beyond a sample period, with known values of the exogenous variables. The coefficients of Equation B are estimated for the sample period from first quarter 1955 to fourth quarter 1961. The coefficients are then re-estimated for the sample period from first quarter 1955 to fourth quarter 1962. This procedure of lengthening the sample period by four quarters is continued until fourth quarter 1973, the terminal date.

Dynamic simulations are then performed for the eight quarters following each sample period. The simulated annual rate of change in nominal income over the first four quarters and over the entire eight quarters are calculated. For the set of first four quarters, using all the post sample periods, the RMSE's of the annual rate of change in nominal income are 1.59 percentage points using M_1 and 1.40 using M_2. For the set of eight quarter periods, the RMSE's are 0.99 percentage points using M_1 and 0.82 using M_2.

For each post sample period, the simulated level of nominal income is calculated for the fourth quarter and the eighth quarter. The RMSE's are then calculated for the set of all post sample periods. Using M_1, the RMSE in the fourth quarter level of nominal income is 1.59 percent and using M_2 is 1.40 percent. For the eighth quarter level, the RMSE is 1.98 percent using M_1 and 1.64 percent using M_2.

The *ex ante* simulation results indicate that the empirical model forecasts the level of nominal income with relatively small RMSE's, compared with forecasts from nine major econometric models of the U.S. economy. Carl F. Christ, in reviewing the *ex ante* forecasting performance of these models, concluded, "All have RMSE's for real and nominal GNP that are 1 percent or less for one quarter ahead, and 3 percent or less for five or six quarters ahead."[15] The model presented in this study, which is based on a markedly different theory than eight of these nine models, forecasts nominal GNP with smaller RMSE's than several of the large scale econometric models of the U.S. This forecasting evidence thus leads to acceptance of the theoretical model of nominal income determination.

CONCLUSIONS

A theoretical model of nominal income determination was developed, based on a set of postulates regarding the behavior of households and business firms. The central postulate is that the rate of change in nominal spending by households and business firms for newly produced final goods and services responds to a discrepancy between the rates of change in actual and desired nominal money balances.

On the basis of the empirical tests, the theoretical model was accepted as representing the determination

[15]Carl F. Christ, "Judging the Performance of Econometric Models of the U.S. Economy," *International Economic Review* (February 1975), p. 64.

of nominal income in the period 1955 - 1973. The estimated coefficients of the empirical model have the signs implied by the theory. In addition, *ex post* and *ex ante* dynamic simulations indicate that it forecasts nominal income with relatively small errors.

The empirical results mentioned in the introduction of the St. Louis reduced-form equation relating changes in GNP to changes in money and government spending are consistent with the theoretical properties of the model presented in this study. These theoretical properties are developed in the section which discusses the changes in the dynamic equilibrium state.

A theoretical property of the model is that a change in the trend growth of nominal money balances changes the trend growth of nominal income. Another theoretical property is that short-run changes in the growth of nominal balances result in short-run changes in the growth of nominal income. A third theoretical property (based on partial analysis) is that changes in the growth of government spending exert a short-run, but not a long-run, influence on growth of nominal income. Thus, there is "crowding-out" of spending by households and business firms, but only in partial equilibrium analysis. Since the desire to hold money balances is negatively related to the short-term interest rate, the financing of the growth of government expenditures by issuing debt may have a positive influence on growth on nominal income. The estimated interest elasticity of desired money balances is very small (about .03); therefore, for debt financing to have much of an influence on growth of nominal income, the elasticity of the interest rate with respect to government debt must be very large or the increase in debt outstanding must be exceedingly large.

APPENDIX

Data Problems

The linear approximation of rates of change in discrete time is given by the following:

$$\frac{d \ln Y}{dt} = \frac{1}{Y}\frac{dY}{dt} \simeq \ln Y_t - \ln Y_{t-1}.$$

The technical efficiency of the payments system is assumed, on average, to increase at a constant rate given by the following exponential growth function:

$E(t) = ae^{bt}$; or $\ln E(t) = \ln a + bt$.

e = the base of natural logarithms.

b = constant rate of growth.

a = beginning level of E.

t = an index of time.

Differentiating with regard to time yields:

$$\frac{d \ln E}{dt} = b.$$

A variation of Phillip Cagan's procedure is used for approximating the rate of change in the perceived level of nominal income. It is assumed that changes in the rate of change in the perceived level of nominal income are proportional to the discrepancy between the actual rate of change in nominal income and the rate of change in the perceived level of nominal income.

$$\frac{d^2 \ln Y^P}{dt^2} = \beta\left[\frac{d \ln Y}{dt} - \frac{d \ln Y^P}{dt}\right].$$

$\frac{d^2 \ln Y^P}{dt^2}$ = the change in the rate of change in the perceived level of nominal income.

β = the adjustment coefficient. $O < \beta \leq \infty$. The larger is λ the faster is the speed of adjustment.

According to Cagan's procedure, the rate of change in the perceived level of nominal income can be approximated in discrete time by the following:

$$\triangle \ln Y_t^P = (1-e^{-\beta}) \sum_{i=0}^{T} \triangle \ln Y_{t-i} e^{-\beta i}.$$

$$(1-e^{-\beta}) \sum_{i=o}^{T} e^{-\beta i} = 1.$$

In the discrete time form of the model, it is asserted that the rate of change in perceived nominal income is that at the beginning of the period. The index (i), therefore, runs from 1 to T. Instead of assuming, as Cagan did, various values of β and then using their implied values for $\Delta \ln Y_t^P$ directly in the regression equation and selecting the one which has the largest $\bar{R}^2$, the coefficients for each lagged income term were estimated directly. Four lagged changes in nominal income (the lag structure used in this study) implies that $\beta = 1.15$, with the

implied weights(0.68, 0.22, 0.07, and 0.02) summing to 0.99. The estimated coefficients for the four lagged income terms, given their standard errors of estimate, are approximately consistent with those implied by these weights.

Test for Structural Change

A test of the hypothesis of a structural change in the coefficients of the model is performed by introducing a zero-one dummy variable into equation (A) of the text. This equation is used rather than Equation B, which incorporates a special assumption regarding the parameters on perceived price and the amount of goods and services that perceived nominal income can purchase. The dummy variable has the value of zero in the M_1 equation from first quarter 1955 to fourth quarter 1966, and then a value of one to fourth quarter 1973. It has a zero value in the M_2 equation from first quarter 1955 to fourth quarter 1961, and then a value of one to fourth quarter 1973.

Two forms of equation (A) are estimated. One is the original specification. The second one has all the original variables, but adds the dummy variable itself (to measure a change in the constant) and the product of the dummy and each of the original variables (to measure a change in the regression coefficients).[1]

If the estimated coefficients for these added variables are not statistically significant from zero, the structural change hypothesis is rejected. An F test is conducted to test the null hypothesis that all the regression coefficients of the added set of variables are equal to zero. If the calculated F value is less than the critical F value, at the five percent level of significance, the null hypothesis is not rejected.

Since the number of lagged ΔlnQ and ΔlnP terms have not been specified at this point, the test for structural change was conducted for lag specifications from one quarter to ten quarters (Table A). For every length of lag, the calculated F value is less than the critical value. As a result, the hypothesis of a structural change in both the M_1 and M_2 equations is rejected for each of the lag structures examined.

Table A

Test For Structural Change
Equation (A)

Number of Lags	Calculated F M_1	Calculated F M_2	Critical F(.05)
1	1.64	1.67	2.36
2	1.45	1.34	2.17
3	1.58	1.54	2.05
4	1.05	1.42	1.98
5	.99	1.17	1.93
6	.87	.91	1.90
7	.82	.93	1.89
8	.75	.98	1.89
9	.75	1.21	1.91
10	.83	1.12	1.91

[1]Except D_1 and D_2.

Determination of Lag Structure

A statistical procedure is used to determine the appropriate number of lagged ΔlnQ and ΔlnP terms in equation (A). Each equation (M_1 and M_2) is estimated eleven times, first with no lags and then increasing the length of lag for ΔlnQ and ΔlnP by one quarter until ten sets of the two lagged terms are included. The F test is used to test the null hypothesis that the two estimated coefficients for each *added* lag are zero. If the calculated F value is greater than the critical value, at the five percent level of significance, the null hypothesis is rejected. Table B shows that the null hypothesis is rejected when lags one and four are added, but not when any of the other lags are added.

Table B

Test of Lag Structure
Equation (A)

Each Added Lag	Calculated F M_1	Calculated F M_2	Critical F(.05)
1	16.88	16.92	4.00
2	1.19	.89	4.00
3	.45	.16	4.00
4	4.11	4.83	4.00
5	.20	.20	4.00
6	1.02	1.00	4.00
7	.24	.26	4.00
8	1.24	1.41	4.00
9	.39	.30	4.04
10	2.30	2.78	4.04
Lags			
1-4	6.37	6.09	2.53
5-10	.91	1.01	2.29

The F test is next used to test the null hypothesis that as a set all the coefficients for lags one to four are equal to zero. This test, at the five percent level, rejects the null hypothesis (Table B). A similar test is conducted for the coefficients as a set for lags five to ten, when lags one to four are included in the regression. In this second test the null hypothesis cannot be rejected (Table B). On the basis of these results, four lagged ΔlnQ and ΔlnP terms are considered to be appropriate. Table I in the text presents the estimated coefficients for this specification, equation (A).

Test of Equal Response Hypothesis

The hypothesis that the response of desired nominal money balances is the same with regard to both perceived price and the amount of goods and services that nominal income can purchase is tested by imposing a linear constraint on equation (A). An F test is used to test this constraint. If the calculated F value for the constraint is less than the critical value, at the five percent level of significance, the proposition is accepted.

Assuming that the weights in forming $\Delta \ln Q^p$ and $\Delta \ln P^p$ are equal for each lag, the equal response hypothesis implies that for each lag the estimated coefficients for ΔlnQ and ΔlnP are equal. Since by definition

Table C

Test of Unitary Elasticity Hypothesis Regression Results

	M_1	M_2
D_1	−2.035	−2.022
	(−3.864)	(−3.708)
D_2	2.486	2.095
	(4.424)	(3.550)
$\Delta \ln r_t$	.015	.025
	(1.672)	(2.438)
$\Delta \ln Y_{t-1} - \Delta \ln M_t$	− .696	− .645
	(−4.723)	(−4.276)
$\Delta \ln Y_{t-2} - \Delta \ln M_t$	− .230	− .148
	(−1.797)	(−1.129)
$\Delta \ln Y_{t-3} - \Delta \ln M_t$	.264	.240
	(2.118)	(1.859)
$\Delta \ln Y_{t-4} - \Delta \ln M_t$	− .213	− .184
	(−2.007)	(−1.680)
Constant	.571	.040
	(4.063)	(.356)
$\bar{R}^2$	.558	.526
SEE	.887	.918
DW	2.045	2.027

*Numbers in parentheses are t-values.

$\Delta \ln Q_t + \Delta \ln P_t = \Delta \ln Y_t$, the coefficients on each set of lagged $\Delta \ln Q$ and $\Delta \ln P$ terms are constrained to be equal by specifying $\Delta \ln Y$ terms (Equation B). Table II of the text presents the estimated coefficients for equation B. Regression results for this equation are tested against those for equation (A). The calculated F value are .58 for M_1 and .64 for M_2. The critical F value is 2.53, therefore, the equal response hypothesis is accepted in both cases. These results also lead to the acceptance of the assumptions made regarding the equality of weights in forming $\Delta \ln Q^p$ and $\Delta \ln P^p$.

A frequent hypothesis in monetary economics is that the elasticity of desired money balances with regard to nominal income is unity. Since the weights sum to unity, the hypothesis implies that the coefficient on money in equation (B) is equal to, but opposite in sign, the sum of the coefficients on the lagged $\Delta \ln Y$ terms. This constraint is imposed on equation (B) by dropping the $\Delta \ln M_t$ term and subtracting $\Delta \ln M_t$ from each of the lagged $\Delta \ln Y$ terms. The regression results are reported in Table C and are tested against the results for equation (B). The calculated F value for M_1 is 4.57 and for M_2 is 7.99. The critical value of F is 4.00; therefore, the proposition that the elasticity equals unity is rejected in both cases. The estimated elasticity of desired money balances with regard to perceived nominal income — 1.62 for M_1 and 1.97 for M_2, — is derived by dividing the sum of the coefficients on the lagged income terms by the coefficient on money and changing the sign.