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Frederick T. Furlong

This paper shows that the rise in nominal interest rates boosted the income-tax related incentives for corporate leverage in the 1980s, but market-value leverage among nonfinancial corporations in the latter half of the 1980s still was higher than would be expected given the estimated tax incentives.

Research Officer, Federal Reserve Bank of San Francisco. The author thanks the editorial committee, Reuven Glick, Bharat Trehan, and Mark Levonian for their helpful comments, and Michael Weiss for his valuable research assistance.

The 1980s were marked by a greater emphasis on debt financing by corporations. This shift away from equity financing is apparent in the rise in the aggregate, book-value, debt-to-equity ratio of nonfinancial corporations. As shown in Chart 1, aggregate, book-value leverage began rising in 1984, corresponding with an unprecedented surge in the net retirement of equities that many attribute to the increase in corporate restructuring in the 1980s.

The decade also was punctuated by two key tax reform laws that brought about major changes in marginal income tax rates. The 1981 tax reform act, for example, reduced the maximum marginal tax rate on ordinary, personal income from 70 percent to 50 percent. The 1986 tax reform act further reduced the maximum rate on ordinary, personal income, lowered the maximum tax rate on corporate profits, and raised the maximum marginal tax rate capital gains. 1

With this combination of developments, it is only natural to look for a link between the income-tax rate changes and the shift away from equity and toward debt financing by nonfinancial corporations during the 1980s. This paper examines this connection. It differs from previous studies in two ways. First, it considers the effects on corporate leverage of changes in nominal interest rates working through tax incentives as well as the direct effects of changes in income-tax rates. The analysis in this paper suggests that tax-related incentives toward leverage increase with nominal interest rates, and that this interest rate link had a pronounced influence on income-tax incentives for corporate leverage in the 1980s. Moreover, changes in income-tax rates, in theory, cause the nominal interest rate to change, thereby partly offsetting the direct effects of income-tax rate changes.

This paper also differs from previous studies in that it evaluates the relationship between income-tax incentives and aggregate, market-value leverage among nonfinancial corporations. The empirical evidence indicates that market-value leverage among nonfinancial corporations in the latter part of the 1980s was greater than can be accounted for by income-tax incentives alone. This finding is consistent with the predominant view in the literature that factors such as financial innovation and deregulation,
relaxed antitrust standards, improvements in “takeover technology,” and higher levels of free cash flow, rather than income-tax incentives, contributed to higher leverage in the 1980s.2

This latter result is of particular interest in that the supposed boost to corporate leverage in 1980s is not apparent in the level of market-value leverage among non-financial corporations. This point is illustrated in Chart 2, which traces the market-value debt-to-equity ratio (D/E) for nonfinancial corporations. The estimates of leverage in the chart are based on Flow of Funds and National Income Accounts data.3 The chart shows that market-value corporate leverage has tended to increase since the early 1950s. The most apparent run-up in leverage, however, occurred in the early 1970s, not the 1980s. In fact, the average level of market-value leverage for the 1980s was about the same as that for the second half of the 1970s.4

The paper presents a model relating the marginal benefit of corporate leverage to income tax rates and the nominal interest rate. The theoretical framework is used to examine how and why income-tax incentives for leverage changed over time. The estimated empirical relationship between income-tax incentives and corporate leverage then is used to determine the contribution of income-tax incentives to higher market-value, nonfinancial corporate leverage in the 1980s.

Based on seasonally-adjusted quarterly data for nonfinancial corporations.

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I. The Model

Income Taxes and Leverage

To illustrate how income tax considerations can affect a firm’s choice regarding market-value leverage, the value of a firm (project) financed only with equity is compared with the value of the same firm financed also with some debt. Assuming two time periods, let $I$ be the initial investment, $Y$ be the net nominal return from the project in the second period, and $p$ be the inflation rate from Period 1 to Period 2. For simplicity, it is assumed that all investors have perfect foresight.

All investors are assumed to face flat tax rates on ordinary, personal income ($t_p$), corporate profits ($t_c$), and personal, equity income ($t_e$). Furthermore, profits are paid both in the form of dividends and capital gains in proportions $w$ and $(1-w)$, respectively, where $0 \leq w \leq 1$. The marginal tax rate on personal, equity income is defined as the weighted average of an individual’s marginal tax rate on ordinary, personal income and the marginal tax rate on capital gains, such that $t_e = wt_p + (1-w)t_k$, where $t_k$ is the tax rate on capital gains.5

With 100 percent equity financing, the value of the firm in Period 1 is the discounted value of the gross, after-tax, real return in Period 2:

$$V_E = \frac{1 + \{(1-t_e)[w(1-t_p)+(1-w)(1-t_k)]\}-p}{1+r},$$

where $r$ is the real after-tax required return. The required real after-tax return is exogenous and applies to all investors.

To incorporate the effects of leverage, the initial investor is assumed to issue debt to other (outside) investors in Period 1 in some proportion, $a$, of the initial investment, where $0 < a < 1$.6 The nominal rate-of-return on the debt, $R$, is the sum of the real required rate-of-return and the rate of inflation adjusted for taxes on interest income, so that

$$R = \frac{r+p}{1-t_p}. \tag{2}$$

This expression is the Darby (1975) respecification of the Fisher equation, and it implies that an increase (decrease) in the marginal tax rate on ordinary, personal income will raise (lower) the before-tax, nominal interest rate on debt.7

Given these assumptions, the value of debt in Period 1 can be expressed as

$$D = al = al\left[\frac{R(1-t_p)-p}{1+r}\right], \tag{3}$$

where $D$ is both market-value and book-value of debt.8

The value of the firm with debt financing, then, can be derived from (1) and (3). This is accomplished by adjusting the before-tax claims of the equity holder in (1) by the before-tax claims of the debtholders and adding the after-tax value of debt. The value of the firm with debt financing is:

$$V_D = \frac{Y-D+(Y-DR)[(1-t_e)[w(1-t_p)+(1-w)(1-t_k)]]}{1+r} - p(I-D)$$

or

$$V_D = \frac{Y-D}{1+r} + \frac{D}{1+r} + \frac{DR[1-(1-t_c)\{w(1-t_p)+(1-w)(1-t_k)\}]}{1+r} \tag{4}$$

From (4), it can be seen that the initial investor would have an incentive to use debt financing as long as the tax rate on interest income is less than the effective rate on equity income—that is, as long as $t_p < [t_e + t_c(1-t_c)]$. The tax rate on equity income reflects the double taxation of corporate profits—first when the corporation pays taxes on earnings and again when personal taxes are paid on dividends or capital gains. Interest on debt, on the other hand, is tax-deductible for corporations, and, thus, is taxed only once, as ordinary, personal income.

When interest income is taxed at a lower rate than equity income, then, the value of the firm is positively related to the amount of debt financing. For the case in which the initial investor issues debt to finance the project, the marginal benefit of debt versus equity financing is

$$\frac{\partial V_D}{\partial D} = \frac{R[(1-t_e)[w(1-t_p)+(1-w)(1-t_k)]]}{1+r} > 0. \tag{5}$$

For an existing corporation, (5) is the marginal tax benefit from using debt (rather than equity) to finance new investment.9 The expression shows that the income-tax incentives for leveraging are a function of the marginal tax rates as well as the nominal interest rate.

The reason the nominal interest rate has an effect is the presence of the inflation premium.10 From (5), the effect of inflation, and, thus, of the nominal interest rate, on the incentives for leveraging, holding taxes constant, is:

$$\frac{\partial g}{\partial p} = \frac{1-(1-t_e)[w(1-t_p)+(1-w)(1-t_k)]}{1+r} > 0. \tag{6}$$
An increase in inflation (rise in the nominal rate of return) reinforces the positive effect on the value of the firm from issuing debt, assuming that the tax rate on interest income is less than the effective rate on equity income. The reason for this is that the higher nominal income due to higher inflation is taxed at a lower rate when it is taken as interest income rather than as equity income.

The unambiguous sign in (6) in part stems from the absence of "bracket creep," which is assumed away by having flat tax rates. With a progressive tax rate structure and no inflation indexation, \( t_p \) would rise with inflation due to bracket creep. If the marginal tax rate on ordinary income rises due to inflation, the theoretical effects of inflation on the incentives for leveraging are ambiguous. In the U.S., the 1981 tax reform act introduced inflation indexation (effective in 1985), but in prior years the marginal tax rates for individuals increased with inflation. In any case, the empirical evidence in the next section indicates that the bracket creep effect has not dominated.

The effect of a change in the tax rate on ordinary, personal income on the incentives for a firm to leverage can be shown formally by differentiating (5) with respect to \( t_p \). This yields

\[
\frac{\partial g}{\partial t_p} = \frac{\partial R}{\partial t_p} \frac{[(1-t_p) - (1-t_k)w(1-t_p) + (1-w)(1-t_k)]}{1 + r} + R(1-t_p)w - 1
\]

or

\[
\frac{\partial g}{\partial t_p} = R \left\{ 1 - \frac{(1-t_c)[w(1-t_p) + (1-w)(1-t_k)]}{1-t_p} \right\} \frac{1}{1 + r} + \frac{R[(1-t_c)w - 1]}{1 + r}.
\]

As background to the discussion in the empirical section on the effects of interest rates and tax rates on income-tax incentives for leveraging, it is useful to consider the two sets of terms on the right-hand side of (7). The second set of terms on the right-hand side of (7) represents the direct effect of a change in \( t_p \) on the marginal benefit from an increase in leverage. This term is negative for all allowable values of \( w \)—that is, \( 0 \leq w \leq 1 \). This direct effect generally is what analysts have in mind when arguing that lower marginal tax rates on ordinary, personal income favor greater corporate leverage.

When debt is issued to outside investors, the overall effect of a change in \( t_p \) on the incentives for leveraging, however, will be less negative than that suggested by the direct effect. This is true since a change in the personal tax rate alters the before-tax, nominal rate of return. The effect of the change in the nominal interest rate is represented by the first set of terms on the right-hand side of (7). This set of terms is positive for allowable values of \( w \), and increases with \( w \). Ignoring the feedback of tax rates on the nominal interest rate, then, would lead to an overstatement of the effect of a change in \( t_p \).

Thus, the sign of the derivative in (7) is negative as long as some corporate profits are realized in the form of capital gains \((w<1)\). A higher marginal tax rate on ordinary income, then, would lead to less debt and lower leverage. Likewise, a lower tax rate would lead to more debt and higher leverage.

However, if profits are paid out only in dividends \((w=1)\), then, changes in a flat marginal tax rate on ordinary, personal income would not affect the marginal benefit form leveraging. From (5), the marginal benefit from leveraging would be

\[
g = \frac{R(1-t_p)t_c}{1 + r}.
\]

or, in the more familiar form,

\[
g = \frac{(r+p)t_c}{1 + r}.
\]

For a given after-tax rate of return, the marginal benefit of leverage depends on the corporate tax rate, but not on the other tax rates, when \( w=1 \).

From (5), it also follows that the incentives for issuing debt versus equity to finance new investment are positively related to the tax rates on corporate profits and capital gains—that is,

\[
\frac{\partial g}{\partial t_c} = \frac{R[w(1-t_p) + (1-w)(1-t_k)]}{1 + r} > 0
\]

and

\[
\frac{\partial g}{\partial t_k} = \frac{R(1-t_p)(1-w)}{1 + r} > 0.
\]

A higher corporate tax rate or higher personal tax rate on capital gains would lead to an increase in debt and leverage. The reason is the higher tax rates lower the return on equity income relative to that on interest income.

**Determination of Corporate Leverage**

When financing new investment an investor would choose all debt when the tax rate on equity income is higher
than that on interest income, if the tax treatment of interest and equity income were the only consideration. Pure debt (or pure equity) is not the observed pattern of corporate financing, however, so other factors must affect the choice of equity financing versus debt financing. Corporate leverage decisions, for example, can be affected by non-debt tax shields associated with depreciation deductions and investment tax credits. DeAngelo and Masulis (1980) point out that non-debt tax shields offset the income-tax advantage of leverage and could be influential enough to determine D/E ratios for individual firms.17

Non-tax considerations also can affect leverage; many of these make leverage more costly, and work to offset income-tax incentives favoring debt financing. An oftencited non-tax impediment to debt financing is the cost of bankruptcy. The argument is that dead weight losses are associated with a firm becoming insolvent and not meeting its debt obligations.18 Everything else equal, at some degree of leverage, further increases in debt financing will raise the probability of bankruptcy and the expected cost of bankruptcy. Hence, bankruptcy costs would bias a firm toward equity financing, and changes in expected bankruptcy cost would be negatively related to changes in D/E ratios.

Costs associated with information asymmetries and agency problems also can be affected by, and in turn affect, the degree of corporate leverage.19 In the case of an owner-managed firm, the manager (agent), who has more information about the firm than do outside investors, has incentives to increase the firm’s risk to the detriment of the debtholders (principals). Ex post, such incentives for risk-taking will increase with leverage.20 In Jensen and Meckling (1976), the monitoring and other agency costs associated with outside financing will be borne by the owner and reduce the value of the firm relative to its value with 100 percent inside financing. To the extent that inside financing is identified with equity and outside financing with debt, information asymmetries and agency costs would serve to offset the tax shield advantages of debt, and, thus, limit D/E ratios.

For many corporations, of course, agency problems exist between managers and non-manager stockholders. For such firms, much of the equity as well as the debt can be viewed as outside financing. With agency costs associated both with outside equity and debt, such costs would not necessarily increase monotonically with leverage. Jensen and Meckling argue that, for a given volume of inside financing and firm size, total agency costs should fall and then rise as the fraction financed through outside equity rises.21 In this context, a firm’s D/E mix, in principle, could be determined uniquely without tax effects.

Even so, the income tax effects discussed above can be important influences on firms’ debt and equity choices.22 The optimal D/E ratio for a corporation should balance the marginal effects from leveraging related to income-tax factors and other tax and non-tax factors. With uncertainty, there would be an expected marginal benefit from leveraging associated with income tax considerations comparable to (5). Given that the expected marginal benefit from leverage is positive, in equilibrium the expected net marginal effect of all other factors on leverage must just offset that benefit.

Assuming that the expected net marginal cost of other factors is some function \( f() \) of the level of leverage, represented by the D/E ratio, and a vector of other variables, \( X \), the long-run level of leverage would satisfy the condition,

\[
g - f(D/E, X) = 0,
\]

where \( g \) in this context is the expected marginal income-tax benefit from leveraging. If this equality does not hold at a given point in time, a corporation could be expected to adjust its leverage over time to eliminate the difference between the expected marginal benefit and the marginal cost.

II. Empirical Results

In this section the theoretical constructs developed above are used to evaluate empirically how income-tax considerations for corporate leverage have behaved and how these incentives have affected aggregate leverage among nonfinancial corporations in the 1980s. The analysis proceeds first by evaluating how income-tax incentives per se changed over time and then by relating the changes in aggregate, corporate leverage to the estimated income-tax incentives.

Estimated Income-Tax Incentives

To evaluate quantitatively how and why income-tax incentives have changed over time, estimates of the marginal value of issuing corporate debt can be derived by using (5). Using the undiscounted value, the marginal gain from leveraging is defined as:

\[
G = R \left[ (1-t_p) - (1-t_c) [ w(1-t_p) + (1-w)(1-t_k) ] \right].
\]
Using (11) requires choosing an appropriate before-tax interest rate and estimating the relevant tax rates. The nominal interest rate selected is the 10-year Treasury bond rate. Using a Treasury security rate, rather than a corporate bond rate, tends to understate the tax effect since expected rates-of-returns should be positively related to risk. On the other hand, using a corporate interest rate would overstate the tax effect since it would be the promised rather than the expected rate-of-return. In any case, the empirical results are not very sensitive to the use of either an interest rate on corporate bonds or one on a longer-term Treasury instrument.

The estimated tax rates should reflect the marginal tax rates of the investors that would hold the additional debt or equity issued. With regard to the stock of outstanding securities, we observed that individual investors hold both equity and debt (apparently for diversification motives), which means that, for estimating the average value of the income-tax incentive, the appropriate tax-rates for personal income (both interest and equity) are weighted averages of the tax rates for the investors holding corporate securities. If it is further assumed that new debt and equity is acquired by investors in different tax brackets in the same proportion as the outstanding stocks, the average marginal tax rates also are appropriate for evaluating the effects of taxes on the marginal value of leverage. In this section, then, (11) is evaluated using estimates of the weighted average marginal tax rates for personal income—interest, dividends, and capital gains, along with the maximum tax rate on corporate profits.

For ordinary, personal income, separate estimates were made for tax rates on interest income and for those on dividend income. This is necessary because debt and equity instruments are not held in the same proportions among investors subject to different marginal income-tax rates. Equities tend to be held by investors with higher incomes. The weighted-average marginal tax rates were derived through 1986 based on data from Individual Income Tax Returns for the appropriate years. The average marginal rate on interest income is based on the distribution of interest income across adjusted gross income categories. This assumes that the distribution of corporate debt holdings is proportional to the distribution of all debt. The average marginal tax rate on dividends is based on the distribution of dividend income across adjusted gross income categories. The estimates after 1986 were derived by applying the weights based on 1986 income data to the marginal tax rates for the different income categories for each year.

The tax rate on capital gains is based on estimates of the average marginal rate from the Congressional Budget Office (CBO). The CBO estimates represent tax rates on realized capital gains. The common assumption is that the effective tax rate is considerably lower than the rate on realized gains because of the general deferral of taxes, the selective realization of losses and gains, and the increase of basis at death. The usual convention is to set the effective capital gains tax rate equal to one-fourth the rate on realized capital gains.

In estimating the average marginal personal tax rate on equity income, w usually is set equal to one-half based on the observation that, historically, corporate profits have been distributed about equally through dividends and capital gains. Over the period from 1950 through 1988, for example, the ratio of dividends to after-tax profits among nonfinancial corporations averaged just about 50 percent.

Chart 3
Dividends to After-Tax Profits

Based on seasonally-adjusted quarterly data for nonfinancial corporations.
Chart 3, however, indicates that using a fixed value for \( w \) may not be appropriate. The dividend to profits ratio jumped in the 1980s, averaging 72 percent after 1981 and 44 percent from 1950 through 1981. The significance of this change depends on whether the higher ratio is permanent or temporary. The higher ratio could reflect a permanent endogenous response to the shift in tax rates in the 1980s, which narrowed the spread between the rate on ordinary income and that on capital gains.

Alternatively, the change in the ratio could be temporary. First, corporations may have increased dividends as a way of adjusting leverage in response to developments in the 1980s that are argued to have encouraged debt financing. Second, the rapid appreciation in stock prices in the 1980s are indicative of higher expected profits. If dividends are related to long-run profits, the higher ratios of dividends to current income observed in the 1980s could decline as higher levels of profits are realized in the future.

Based on these considerations, two sets of weights are considered, one with a value of \( w \) fixed at 0.44 and the second with a value of \( w \) set equal to 0.44 for the period through 1981 and equal to 0.58 after 1981. The choice of 0.58 for the more recent years assumes that the increase in the share of long-run profits paid out in dividends is equal to half of the observed rise in the aggregate, dividends-to-profits ratio.

Chart 4 shows the estimates of \( G \), which are affected by income tax rates as well as by interest rates. The dark line traces the estimated values of \( G \) when \( w \) is allowed to change, while the light line traces the estimates when \( w \) is held constant. The chart shows that the tax advantage of debt over equity financing increased, on balance, over the last three decades. The incentives were greatest in 1982 and remained relatively high through 1984. After declining markedly through 1986, they rebounded some through 1989. The estimates of the tax incentives for leveraging in 1989 were a bit lower than at the start of the decade and about equal to the level prevailing in the mid-1970s.

To identify the relative importance income-tax rates and the nominal interest rate in determining movements in \( G \), it is useful to separate the two effects. To isolate the tax rate effects, the term in braces in (11) commonly is used. This approach amounts to measuring the effect of income taxes holding the before-tax nominal interest rate constant. Doing this, however, ignores the theoretical feedback from tax rates to the before-tax nominal interest rate.

The discussion in the previous section suggests that, in theory, the more appropriate approach would be to evaluate the tax rate effects holding the after-tax nominal interest rate constant. This says that the marginal effect of debt financing should be expressed in terms of the tax rates and the after-tax nominal interest rate. Using (2) and (11), the undiscounted marginal value of leveraging can be expressed as:

\[
G = (r + p) \left\{ 1 - \frac{(1-t_p)[w(1-t_p) + (1-w)(1-t_k)]}{1-t_p} \right\} \tag{12}
\]

where \((r + p)\) is the after-tax nominal interest rate on debt. In this expression for \( G \), the term in braces, in principle, captures the effects of changes in tax rates on the incentives for leveraging, including those due to changes in the before-tax nominal interest rate that are related to income-tax rate changes.
Chart 5 shows that accounting for the effects of taxes on the nominal interest rate alters the perspective on how recent tax law changes have affected incentives for corporations to leverage. The black line is the value of the term in braces from (11), multiplied by the average value of the ten-year Treasury bond rate for 1978 and 1979. The green line is the value of the term in braces from (12), multiplied by the average of the after-tax ten-year Treasury rate for 1978 and 1979.

Both series in Chart 5, however, show that the bias in the income tax rates toward debt financing has declined since about the mid-1960s. The upward trend in $G$, shown in the previous chart, then, is due to the rise in nominal interest rates. That is, based on these estimates, higher interest rates, rather than tax policy per se, have increased the relative attractiveness of debt financing.

With respect to the recent tax law changes, the series in Chart 5 indicate that the changes in income-tax rates following the 1981 tax reform act boosted the incentives for leveraging. This would be expected, given that the major income-tax changes in the 1981 act lowered marginal tax rates on ordinary income, with the maximum rate reduced from 70 percent to 50 percent. The increase in the bias toward debt financing from this act, however, did not do much more than offset the decline in the bias inherent in U.S. income tax policy during the second half of the 1970s.

The relatively strong incentives for leveraging in the early 1980s primarily reflect the higher nominal interest rates that prevailed in that period rather than changes in marginal tax rates. Moreover, the subsequent decline in these incentives from 1984 through 1986 was due to the drop in nominal interest rates, which essentially offset the effects of the 1981 tax act. By 1986, the tax advantage of debt versus equity financing was only a little above the levels prevailing in the 1970s (see Chart 4).

The income tax rate changes following the 1986 tax act reduced the bias toward debt financing, as indicated by the decline in the series plotted in Chart 5. Although the 1986 tax act lowered marginal tax rates on ordinary income and raised them on capital gains, which, according to the discussion above, should have favored debt financing, it also lowered the marginal tax rate on corporate profits, which should have reduced the tax bias toward debt financing. The estimates in Chart 5, showing a net decline after 1986, suggest that the change in the corporate tax rate simply dominated. However, the effect of the law is more complicated. The reduction in the maximum marginal tax rate on ordinary, personal income from 50 percent to 33 percent (28 percent for the highest tax brackets) lowered the average marginal tax rate for individuals earning dividend income by much more than the average marginal tax rate for individuals earning interest income. As a result, the estimated tax incentives for leveraging were not boosted much by the lower tax rates on ordinary, personal income. In fact, in the case of the green line in Chart 5, which takes into account the effects of tax rates on nominal interest rates, the net effect of the changes in personal tax rates was to reduce the incentives for leveraging, and to reinforce the effect of the lower corporate tax rate. This is not a result that would have been anticipated based on the model presented above, in which marginal tax rates on interest and dividend income are equal and move together.
Tax Incentives and Leverage

The discussion in this section turns to the empirical evidence on the relationship between income-tax incentives and the aggregate, market-value, debt-to-equity ratio for nonfinancial corporations. The analysis starts with (10), and the assumption that expected values are based on lagged observations, except for the marginal tax rates. For the empirical analysis, the marginal benefit from leveraging due to income taxes is represented by $G$. It is further assumed that $f()$ takes the form $B_1(D/E)_{t-1}^2$, with the marginal cost of leveraging hypothesized to be positively related to the level of leverage. The leverage ratio $(D/E)$ is the market-value, debt-to-equity ratio plotted in Chart 2.

When the equality in (10) does not hold, corporations are assumed to adjust (at a cost) to the difference. Using the log-linear change in leverage, the adjustment process can be expressed as:

$$\Delta \log(D/E)_t = b_0 \{\log G_{t-1} - [b_1 + b_2 \log(D/E)_{t-1}]\} + \epsilon_t,$$

or

$$\Delta \log(D/E)_t = b_0 \log G_{t-1} + c_1 + c_2 \log(D/E)_{t-1} + \epsilon_t, \quad (13)$$

where $G_{t-1}$ is based on the ten-year Treasury bond rate at $t-1$ and the tax rates prevailing at $t$. In the expressions, $b_0$ is expected to have a positive sign. That coefficient should reflect the average cost of adjusting leverage, which is assumed to be constant over time. The coefficient $b_1$ is equal to $\log(B_1)$, so the sign of $b_1$ depends on whether $0 < B_1 < 1$, $B_1 = 1$ or $B_1 > 1$. This means that the sign of the constant term in (13), $c_1 = b_0 b_1$, could be positive, negative or zero. The expected sign of the coefficient on lagged leverage, $c_2 = b_0 b_2$, is negative. The term $\epsilon_t$ is a random disturbance term.

One problem estimating (13) is that ex post changes in aggregate corporate leverage reflect not only decisions regarding debt and equity financing, but also exogenous shocks to equity prices. If corporations take their share prices to be random walks and do not react to contemporaneous changes in these prices, the change in corporate leverage in period $t$ that would be related to income-tax incentives and the marginal cost of leverage could be expressed as:

$$\Delta \log(D/E)_t + b_3 \Delta \log SP_t,$$

where SP represents aggregate stock prices, and $b_3$ would be expected to be equal to 1.\(^\text{32}\) On the other hand, if changes in stock prices were exogenous and there were offsetting adjustments to the effects of changes in stock prices on leverage, $b_3$ could be greater than 1.

Allowing for stock price shocks, the leverage adjustment equation can be rewritten as:

$$\Delta \log(D/E)_t = b_0 \log G_{t-1} + c_1 + c_2 \log(D/E)_{t-1} + b_4 \Delta \log SP_t + \epsilon_t, \quad (14)$$

where the change in stock prices is the log difference of the S&P500 index.\(^\text{33}\) The coefficient, $b_4$, is expected to be negative and of the same magnitude as $b_3$.

To allow for more flexibility in the short-run dynamics of the adjustment in corporate leverage, lagged values for the log changes in $G$ and in $D/E$ were included in (14). Lagged changes in leverage were significant, but lagged values of the change in tax incentives were not. The regression results in the table were derived by including the first and second lagged values for the change in leverage.

The results in the first column of that table show that the coefficients have the expected signs. The coefficient for $G$ is positive and statistically significant, while the one for lagged leverage is negative and significant. The positive sign on the constant term indicates that $B_1$ is estimated to be less than one. The coefficient on the change in stock

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**Regression Results**

Sample period: 1956:1 to 1989:2

<table>
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<tr>
<th>Independent Var.</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<td></td>
<td>(1.86)</td>
<td>(2.90)</td>
<td></td>
<td>(3.52)</td>
</tr>
<tr>
<td>$\Delta \log(D/E)_{t-2}$</td>
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<td>-.304</td>
<td>-.339</td>
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<td></td>
<td>(4.94)</td>
<td>(4.99)</td>
<td>(5.57)</td>
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<tr>
<td>$\Delta \log(D/E)_{t-1}$</td>
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<td>-.109</td>
<td>-.138</td>
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<tr>
<td></td>
<td>(2.13)</td>
<td>(2.80)</td>
<td>(2.50)</td>
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<tr>
<td>$\Delta \log G_{t-1}$</td>
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<td>-.127</td>
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<td></td>
<td>(12.97)</td>
<td>(13.45)</td>
<td>(13.70)</td>
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<td>$d85$</td>
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<td>.051</td>
<td>.031</td>
<td>.039</td>
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<tr>
<td></td>
<td></td>
<td>(2.80)</td>
<td>(1.82)</td>
<td>(1.41)</td>
</tr>
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$R^2$ .60 .63 .60 .07
SEE .061 .060 .061 .093

Figures in parentheses are the absolute values of the $t$-statistics.
prices is significantly different from zero, and its absolute value is greater than one, which suggests that corporations may attempt to offset some of the effect of stock price changes that occur during a quarter.\textsuperscript{34} The empirical results are very similar whether \( G \) is defined using the fixed value of \( w \) or allowing \( w \) to change after 1981. The statistics in the table are derived assuming the weight, \( w \), changes.

These results, then, are consistent with the hypothesis that market-value leverage among non-financial corporations is affected by the difference between the leverage gains related to income taxes and the net cost of other factors. Of central interest to this paper is whether that relationship shifted during the 1980s. Such a shift should be reflected in the values of the coefficients in (14). For example, a larger estimated constant term for more recent years would be consistent with developments not directly related to income-tax factors in the 1980s, on balance, favoring more debt financing relative to equity financing than was the case in earlier years.

Data on the net issuance of equity by nonfinancial corporations in Chart I suggests that a shift in the relationship might have occurred around 1984. The Quandt (1958) likelihood method also was employed to help identify the most likely date for a shift in the leverage relationship. The test indicates that a likely break in the 1980s occurred in the latter part of 1985.

To evaluate the statistical significance of the break in the relationship, the results from the Quandt test were used. Accordingly, a bivariate dummy variable was used to test for a change in the constant term after the third quarter of 1985. The coefficient on the dummy variable, \( d_{85} \), in Column 2 of the table is statistically significant. The estimated increase in the constant term indicates that, even on a market-value basis, changes in corporate leverage have been larger in recent years than would be expected given stock price movements and income tax incentives for leveraging.

To evaluate the extent to which controlling for the effects of income-tax incentives for leveraging makes a difference to this results, the leverage equation was estimated without \( G \) and lagged leverage. A comparison of the statistics for the dummy variable in Columns 2 and 3 shows that the estimated shift is smaller and only marginally significant when only changes in stock prices are taken into account. At the same time, the results in Column 4 indicate that controlling for the effects of changes in stock prices is important. When the change in stock prices is not included, the estimated coefficient for \( d_{85} \) is not statistically significant.\textsuperscript{35} As also can be seen from the results in Column 4, income-tax incentives explain a fairly small portion of the quarterly change in aggregated, market-value leverage among nonfinancial corporations.

The regression results, then, suggest that changes in market-value corporate leverage did increase significantly in the latter part of the 1980s, and that influences beyond income-tax incentives contributed to the increase. This result combined with the data on the estimated income-tax advantage of debt shown in Chart 4 suggest that changes in income-tax incentives for leveraging were not the impetus for the rise in corporate restructuring in the second half of the 1980s. As shown in Chart 4, in the latter part of the 1980s the estimated income-tax incentives for corporate leveraging were low relative to the first part of the decade and a bit lower on average than in the latter part of the 1970s. The other influences that contributed to the higher leverage could be those discussed in the introduction and identified in other studies as contributing to the surge in corporate restructuring in the second part of the 1980s.

While changes in income-tax incentives may not have spurred the much discussed rise in corporate restructuring in the second part of the 1980s, the relatively high estimated income-tax advantage of debt over equity financing in the first half of the 1980s may have contributed to a higher average level of leverage over the decade. The strong tax-incentives in the first part of the decade should have resulted in higher leverage than if the incentives had remained at the levels prevailing in 1978 and 1979.

To estimate how much the tax incentives might have affected corporate leverage during the 1980s, two dynamic simulations were conducted using the historical relationship of the change in aggregate, market-value, nonfinancial corporate leverage to income-tax incentives and lagged leverage. The simulations were run beginning in 1980. For one simulation \( G \) took on its historical values and in the other \( G \) was set equal to its average value over the 1978-79 period. The simulation results show an average level of market-value leverage for the 1980s that is about five percentage points higher with the historical movement in income-tax incentives than is the case when the income-tax incentives are held at the levels prevailing in the latter part of the 1970s.
III. Conclusion

Income-tax incentives for corporate leverage are a function of nominal interest rates as well as income-tax rates. The estimates of income-tax incentives for leveraging indicate that nominal interest rates have been important. Over the past 25 years, the rise in interest rates has accounted for the estimated net increase in the income-tax bias favoring debt over equity financing.

Even during the 1980s, which were punctuated by major changes in income-tax rates, the swings in nominal interest rates had a significant impact on the estimated income-tax advantage of debt financing. In the first half of the 1980s, high nominal interest rates raised the income-tax advantage of debt versus equity financing for corporations relative to the levels prevailing in the second part of the 1970s. The subsequent net drop in interest rates reduced the income-tax advantage in the second half of the 1980s to levels that generally were not much different from those in the latter part of the 1970s. This pattern suggests that income-tax incentives per se were not the catalysts for the sizeable net reductions in equity associated with corporate restructuring beginning in 1984. Nevertheless, the relatively high income-tax incentives for leveraging in the first part of the decade should have encouraged more debt financing relative to equity financing and should have contributed to a measurably higher average level of leverage over the decade than would have been the case if those incentives had remained at their lower pre-1980s level.

While income-tax incentives may not have provided the impetus for corporate restructuring in the second part of the 1980s, accounting for their effect does help to reconcile to some extent the difference between the pictures presented by the data on book-value and market-value leverage. It is somewhat surprising that a marked shift toward debt financing in the 1980s is not obvious when looking at aggregate, market-value leverage for nonfinancial corporations. However, evidence for such a shift is found when the change in market-value corporate leverage is weighed against the changes in the benefits and costs of leverage. When the effects of income-tax incentives are taken into account, along with the effects of changes in stock prices, changes in market-value corporate leverage are significantly larger in the second half of the 1980s. This result is consistent with a shift to debt financing that is related to developments other than changes in income-tax incentives. While the regression analysis does not identify the factors that have boosted leverage, other studies suggest that financial innovation and deregulation, an easing of antitrust standards, as well as an increase in free cash flow may have been important influences.
NOTES

1. The 1986 act provided for a reduction in the maximum marginal tax rate on ordinary, personal income from 50 percent to 33 percent, a reduction in the maximum corporate tax rate from 46 percent to 34 percent, and an increase in the maximum tax rate on capital gains from 20 percent to 33 percent.

2. Gertler and Hubbard (1989) and Summers (1989), for example, argue that financial innovations like the rise in junk bonds, which facilitated corporate restructuring, probably were more important than tax rate changes to the rise in corporate debt. Auerbach (1989a,b) also discounts the importance of changes in tax rates to the rise in corporate borrowing. Jensen (1987) discusses the other factors mentioned in the text, with an emphasis on the role of free cash flow. Also see Jensen (1988). Free cash flow is defined here as that portion of cash flow (profits plus depreciation) that cannot be reinvested in the firm profitably.

3. The estimate of the market value of nonfinancial corporate equity is taken from the Flow of Funds Accounts. Market-value corporate debt is the sum of the face value of short-term debt from the Flow of Funds and an estimate of the market value of long-term debt. The market value of long-term debt is estimated by capitalizing the difference between gross nonfinancial corporate interest expenses and interest expenses on short-term debt by the average corporate bond rate. The estimates of leverage represent end-of-quarter figures.

4. Bernanke and Campbell (1988) and Strong (1988), using different measures of aggregated corporate leverage, also find that market-value leverage among nonfinancial corporations did not increase much on balance in the 1980s.

5. In a two period model, distinguishing between dividends and capital gains is somewhat contrived. Also, unless the tax on capital gains realized in Period 1 is taken into account, the model will not work if the proceeds from selling capital assets are reinvested in the firm.

6. With \( \nu_E > I \), it is possible for the initial investor to issue debt such that \( a > 1 \). In that case, the initial investor presumably would have to pay taxes on the proceeds in excess of the book-value of equity in Period 1.

7. This differs from the assumption in Hochman and Palmon (1985) in which the interest rate on debt is fixed for a given expected interest rate.

8. This would not necessarily be the case if the initial investors financed the entire project and merely designated a portion of \( I \) as debt since it must be the case that \( (Y/I) \geq R \) for all equity financing to be feasible. If the original investor designated all of the initial investment as debt, the market-value of the debt (as well as that of the firm) in Period 1 would be

\[
D' = \frac{l + Y(1-t_p) - \rho l}{1 + r} \geq D.
\]

The nominal before-tax rate-of-return on \( D' \) also would be \( R \). The measured rate-of-return on \( I \), which would represent the book-value of debt, would be \( (Y/I) \geq R \).

9. The expression for the marginal tax effect when debt is used to replace existing equity is somewhat different. In that case, the initial investor can be assumed to invest \( I \) in the project before issuing debt. If \( \nu_E > I \), then, replacing the initial funds (the equity) with debt will involve capital gains realized in Period 1. The tax on the capital gains would reduce the marginal benefit from using debt when replacing existing equity relative to the effect in (5). Using \( E^g \) to represent the book-value of equity, which is equal to \( I \) with all equity financing, and \( E^m \) to represent the market-value of equity, which is equal to \( V_z \) with all equity financing, the marginal effect from replacing equity with debt is

\[
g' = R [(1-t_p) - (1-t_c) [w(1-t_p) + (1-w)(1-t_k)]]
- \frac{(E^m - E^g) t_k}{E^m} < g.
\]

Strictly speaking, (5') represents the marginal effect from leveraging on the value of the firm plus the wealth of the initial investor. The last term in (5') represents the effect on the wealth of the initial investor from the taxation of capital gains in Period 1.

10. In the two-period model, with inflation equal to zero, the marginal value of leveraging is:

\[
g = \frac{r}{1 + r} \left\{ 1 - \frac{(1-t_c)[w(1-t_p) + (1-w)(1-t_k)]}{1-t_p} \right\}.
\]
However, when the analysis is extended to an infinite period model with perpetual debt the interest rate terms no longer enter the expression for \( g \). In that case, the expression is:

\[
g = 1 - \left(1-t_c\right)[w(1-t_p)+(1-w)(1-t_k)]/1-t_p,
\]

which is the Miller (1977) expression for the gains from leverage per dollar of debt. With an inflation premium in the nominal interest rate, the interest rate terms remain in the expression for \( g \).

11. In a Miller (1977) type world, tax rates on interest and equity income for the marginal investor are equal and inflation does not affect leverage for an individual firm. On the other hand, Modigliani (1982), allows for benefits from diversification, and argues that the incentive for leveraging are positively related to inflation. Rangazas and Abdullah (1987) also show that tax incentives for leveraging are positively related to nominal interest rates under the assumption that firms minimize costs. That study, however, assumes that the before-tax nominal interest rate is constant for a given expected rate of inflation.

12. Hochman and Palmon (1985) also argue that the theoretical effects of inflation on leverage are ambiguous. However, they assume a Miller (1977) type world, so to get this result they have to introduce into their model other leverage-related costs. Without such costs in their model, the effects of inflation (without inflation indexation) are unambiguously negative because only the bracket creep effect comes into play.

13. See, for example, Auerbach (1989b) and Gertler and Hubbard (1989).

14. From (2) in the text,

\[
\frac{\partial R}{\partial t_p} = \frac{r + p}{(1-t_p)^2} = \frac{R}{1-t_p} > 0.
\]

15. Another complication in assessing the sign of (7) is that the proportions of profits distributed as dividends and capital gains likely are related to the tax rates on the two types of incomes. In practice, a decrease in \( t_p \), for example, should lead to a larger portion of profits distributed as dividends—that is, the weight on \( t_p \) should be negatively related to \( t_p \). In this case, as long as the marginal tax rate on capital gains, \( t_k \), is less than the marginal tax rate on ordinary income, \( t_p \), an increase in the weight on \( t_p \) increases the marginal gain from issuing debt. Thus, even if the proportion of profits paid out as dividends changes with \( t_p \), (7) remains negative for values of \( w \) less than one.

16. In the case where debt is used to retire existing equity, it can be seen from the expression in Note 9 that the effect of a change in \( t_k \) on the marginal benefit from leveraging will involve another term.

17. DeAngelo and Masulis (1980) are responding to Miller (1977), who argues that tax considerations can determine leverage at the aggregate level, without doing so at the firm level. DeAngelo and Masulis argue that, as leverage increases, the earnings that can be sheltered by non-debt shields decline. As leverage increases, then, the marginal tax advantage of issuing debt (net of the loss in value of the non-debt shields) should eventually decrease and can go to zero. This means that factors affecting the value of non-debt shields can affect the marginal tax benefit of debt financing.

DeAngelo and Masulis also point out that inflation can reduce the value of certain non-debt shields. They note that for depletion and depreciation allowances the deductions are fixed at the time of the relevant investment. Therefore a rise in inflation and the nominal income of a firm would diminish the effects of non-debt shields and enhance the effect of the debt shield. This effect would reinforce the positive effects that inflation has on the incentives for leveraging in (7).

18. Bernanke and Campbell (1988) argue that “near-bankruptcy” costs, such as curtailment of projects due to a lack of funding, also can serve to reduce the attractiveness of debt financing.

19. Information asymmetries exist because a firm insider, like an owner-manager, knows more about the ex ante investment opportunities, as in Meyer and Mujlid (1984), or about the ex post returns, as in Williamson (1986). These information asymmetries affect the cost of outside funds because the interests of the insider (agent) often do not coincide with those of the outsiders (principals).


21. Financing by insiders still would be preferred, all else equal. The amount of internal funds presumably would be related to the net worth of insiders.

22. The tax effects relate strictly to the firms choice between debt and equity and not necessarily to the choice between inside and outside financing.

23. The expression for the tax incentive for debt financing becomes:

\[
G = R\{(1-t_{pi})-(1-t_p)[w(1-t_{pe})+(1-w)(1-t_k)]\},
\]

where \( t_{pi} \) is the personal tax rate on interest income and \( t_{pe} \) is the personal tax rate on dividends.

24. This approach is used in Wright (1969) and Rangazas and Abdullah (1987), though the latter use the average rate based on dividend income for both interest and dividend income.

By using gross adjusted income categories, rather than income actually taxed, this approach should overstate the marginal tax rates. Also, using only data on personal income tax rates could overstate the average rate given that certain holders of debt and equity are argued to face very low or even zero marginal tax rates (see, for example, Summers (1989), Auerbach (1989b), King and Fullerton (1984)). Nevertheless, the estimates of tax rates on interest and dividend income should be useful for examining the movements in the income-tax incentive for leveraging over time.
26. See King and Fullerton (1984), page 222.
27. See, for example, Rangazas and Abdullah (1987).
28. Gandolfi (1982) and Rose (1986) show that, with taxes on capital gains (and the depreciation allowances based on historical costs), the tax-amended Fisher equation is more complicated than the Darby (1975) specification.
29. As a reminder, the average tax rates on interest and dividends are estimated separately. Following the notation in Note 23, (12) is

\[ G = (r + p) \left\{ 1 - \frac{(1 - t_c)[w(1 - t_{pe}) + (1 - w)(1 - t_x)]}{1 - t_{pri}} \right\} \]

30. This decline for the most part reflects the impact of bracket creep on income tax rates and some rise in the average marginal tax rate on capital gains.
31. Changes in the statutory tax rates are known ahead of time, though exact income distributions are not.
32. In this case, firms would make decisions regarding debt and equity based on the level of stock prices at the beginning of the period. The change in leverage can be rewritten as

\[ \log\left( \frac{(D/E)_t}{(D/E)_{t-1}} \right) = \log(\frac{D_t}{D_{t-1}}) - \log(\frac{N_tSP_{t-1}}{N_{t-1}SP_{t-1}}) - \log(\frac{SP_t}{SP_{t-1}}) \]

where \( N \) is the number of shares. In a given period, the first two right-hand-side terms are the ones that would reflect the decisions of firms.
33. The specification in (14) raises the issue of simultaneity bias, since changes in leverage can affect stock prices. However, it seems reasonable that the dominant channel of causation is from exogenous shocks to prices affecting the market value of equity, and, thus, market-value leverage.
34. The magnitude of the coefficient also could be due to the use of the S&P500 index to measure the change in stock prices for all nonfinancial corporations.
35. Lagged values of the change in leverage were not significant, so the regression for Column 4 was estimated without those variables.
REFERENCES


Managing Risk in Japanese Interbank Payments Systems

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The author, currently at the Information and Computer System department of the Bank of Japan, was a visiting scholar at the Federal Reserve Bank of San Francisco from January to June 1990. He appreciates many helpful and constructive comments from the editorial committee and seminar participants at the Federal Reserve Bank of San Francisco. Members of the editorial committee were Reuven Glick, Elizabeth Laderman, and Ramon Moreno. Opinions expressed in this article do not necessarily reflect the views of the Bank of Japan.

As a result of differences in the approach to managing payments systems, Japanese payments systems may differ in their risk and efficiency characteristics from U.S. payments systems. The existence of a facility for real-time transfers, prohibitions on daylight overdrafts, certain collateral requirements, loss-sharing among banks, and the pricing of credit in half-day markets are arrangements in Japanese payment systems that have historically not been present in U.S. payment systems. Three possible measures could further reduce risk in Japanese payments systems: (i) improve the balance between real-time transfers and designated-time transfers; (ii) expedite payment transactions; and (iii) introduce delivery-versus-payment.

Over the past fifteen years, the financial markets of the major industrial economies have become larger and more integrated as a result of deregulation and advances in telecommunications and electronics technology. The accompanying growth of national and international financial transactions has spurred a rapid expansion in payment volume and linked national payments systems more closely. In Japan, for example, the payments value of the major private payments systems increased five-fold over the past 15 years to a level of about 25 times nominal GNP. In the United States, the total payment value for CHIPS and Fedwire rose from about 20 times nominal GNP in the mid-1970s to over 50 times nominal GNP in 1986.1

The enormous growth and globalization of payments systems have generated increasing concern about the possibility of default within those systems. Risk of default arises when financial institutions extend credit to each other by making payment before receipt. “Systemic” risk of default arises when default by an individual payments system participant adversely affects the position of a large number of other participants and thereby produces a chain of additional defaults.

While some degree of systemic risk is inherent in all financial transactions, many policy makers are concerned that this risk has increased significantly in recent years, due to the large increases in payment volumes and closer international integration of payments systems. In particular, closer integration of payments systems may have increased systemic risk because default on the part of one participant can now spread more widely beyond national borders, and national authorities can neither monitor nor control the riskiness of activities of participants in foreign payments networks.

The perception that systemic risk in payments system may have increased has prompted policy makers and financial institutions to focus attention on the operation and risk characteristics of different payments systems. In
the United States, a privately formed committee, called the Large-Dollar Payments Systems Advisory Group, and a group formed by the Federal Reserve, called the Task Force on Controlling Payments Systems Risk, published reports on the daylight overdrafts of Fedwire in August 1988. In Japan, as well, efforts are being made to make payments system safer and more efficient.

This paper describes Japanese payments systems, discusses how risk is managed in these systems, and reviews possible measures for further reducing risk. This paper is meant to provide not only an understanding of Japanese payments system themselves, but also of the Japanese financial system as a whole, since a payments system is closely related to the market practices and historical background of the country concerned. Reference is made to the payments systems of other countries as well.

The paper is organized as follows: Section I briefly discusses general features of different types of payments systems. In Section II, the four Japanese payments systems are described in detail. Section III discusses how payment system risk in Japan may be reduced and Section IV presents several conclusions.

I. Payments Systems and Risk

Interbank payments arise from the transfer of funds between the account holders of different banks. Interbank payments systems can be categorized into two types according to whether funds are transferred among banks on a net or a gross basis.

A clearing system is a payments system that transfers funds among banks on a net basis. In such a system, all payment instructions (information which causes a receiver's account to be credited) are cleared through a single location where differences between the total amount due to be received and the total amount due to be paid by each bank, i.e. the net credit/debit positions, are calculated. Typically, these net positions are then settled at a predetermined time through the transfer of funds between reserve accounts at the central bank. The check clearing systems of most countries generally are clearing systems. The Zengin System and the Gaitame-Yen System in Japan, and CHIPS and ACH in the U.S. can be classified in this category.

A settlement system is a payments system that transfers funds among participants' reserve accounts with a central bank on a gross basis; i.e. on a payment instruction by payment instruction basis. Net positions from a clearing system usually are settled through a settlement system. BOJ-NET, managed by the Bank of Japan, Fedwire, managed by the Federal Reserve System in the U.S., and Swiss Interbank Clearing (SIC), managed by the Swiss National Bank in Switzerland, are examples of settlement systems.

Clearing systems and settlement systems differ from each other in terms of operational efficiency. A clearing system, in which only participants' net positions are settled, is operationally more efficient than a settlement system, in which every payment instruction generates an interbank settlement. Hence, the workload for a given number of payments is considerably less in a clearing system.

However, this difference in terms of efficiency is not as significant as in the past, when payment instructions were exchanged on a paper basis. Now that payment instructions are exchanged electronically, the cost associated with processing each payment instruction in a settlement system has decreased dramatically. As a result, the relative advantage of a clearing system in terms of operational efficiency is disappearing.

The two types of systems differ more importantly with respect to the degree of systemic risk. In general, settlement systems entail less systemic risk than do clearing systems. The reason is that in a settlement system payment instructions and interbank settlements through the central bank are processed at the same time. This is important because once reserve funds with a central bank are transferred, the transfer is said to be "final," meaning that the central bank guarantees that the receiving bank will never lose the amount received, even as a result of the sending bank's default or legal proceedings stemming from its insolvency. (A payments system in which settlement is guaranteed at the same time that the payment instruction is received is often said to have "finality." Therefore, in a settlement system, no chain of credit is generated among participants and systemic risk is minimized.

In contrast, in a clearing system the processing of payment instructions (including calculation of each participant's net position and transmission of payment instructions among banks) and interbank settlement are conducted separately. Typically, settlement does not take place, and therefore payments are not final until the end of the day or on the next day. Therefore, before the interbank settlement takes place, the receiving bank gets an instruction that it should credit funds to a receiving customer's account. By crediting the funds, the receiving bank in effect grants credit to the sending bank until the interbank settlement occurs. Whether or not the receiving bank credits funds to a customer before settlement is left to its discretion, but it often does so for the customer's convenience.
It is the buildup of such a chain of credit within a clearing system that creates systemic risk. Since default by one member can lead to default by others, even those which have no direct transaction with it, the risk of a series of defaults within a clearing system exists. This arises especially in a clearing system with "unwinding," a procedure under which participants' net positions are recalculated with payment instructions of the defaulting participant put aside.²

Both types of payments systems involve some form of credit risk. In clearing systems, credit flows between banks whenever receiving customers are credited before settlement of net debit and credit positions occurs. The time lag between the initiation of a payment instruction and the settlement of reserve accounts influences the amount of credit that is generated, and hence the level of risk. A payments system in which settlement occurs within the same day that a sender instructs payment is called a "same-day settlement" system. A payments system in which settlement occurs on the day after a sender instructs payment is called a "next-day settlement" system. Because unsettled time in a next-day system is longer than in a same-day system, more credit is accumulated in the former and risk is correspondingly greater.

In settlement systems credit risk arises from the intraday credit markets which become sometimes necessary for providing banks with needed reserve funds. In the U.S., for example, the Federal Reserve Banks grant free intraday credit to sending banks on Fedwire. They do this by crediting receiving banks reserve accounts as soon as they receive the payment message, but debiting sending banks' reserve accounts only at the end of the day. The resulting "daylight overdrafts" on Fedwire represent credit risk to the Federal Reserve.

Daylight overdrafts are not permitted on Japan's BOJ-NET, nor on Switzerland's SIC. In BOJ-NET, however, there is a designated-time transfer facility by which participants can send payment instructions in advance for settlement at a designated time of a business day. With this, they can input instructions for any amount even if the payment exceeds the current reserve account at the input time, therefore, it can be said that implicit free credits flow among private banks. Furthermore, there are private half-day interbank credit markets called Asa-han (morning session) and Go-han (afternoon session), where banks lend reserve funds to one another.³ It is these private bank lenders of reserve funds in Japan that bear the credit risk.

The presence of credit risk raises potential problems for both settlement systems and clearing systems concerning the appropriate amount of credit. In an economically efficient payments system, the equilibrium amount of interbank credit⁴ accurately reflects both the private and social marginal costs, and benefits, of credit creation. In either a settlement or a clearing system, if interbank credit is mispriced, economic inefficiency is generated. For instance, it can be argued that unpriced daylight overdrafts, as on Fedwire, create too much credit risk exposure for the Federal Reserve.⁵ Free credit associated with interbank payments is not restricted to the United States, and it is likely that its prevalence is one of the reasons for the recent expansion of worldwide payment volume.

One solution to the problem of too much credit risk is to set a positive price for interbank credit. However, choosing the proper price may be difficult. In some situations, the private market can be relied upon to arrive at an economically efficient price. In Japan, in the half-day interbank private credit markets, credit is priced at about 3.65 basis points for lending and about 14.6 basis points for borrowing. Because of the presence of the negative externality related to systemic risk, however, private market pricing might underprice credit to the extent it does not take into account the costs associated with systemic risk. As a result, some countries have made attempts to administer the pricing of interbank credit, with the goal of balancing the social benefits of credit against the social costs, including systemic risk. On the basis of an assessment of this type, the Federal Reserve is scheduled to charge 25 basis points annually for daylight overdrafts on Fedwire beginning in mid-1991.

Another option is to attempt to control the quantity of interbank credit directly. Quantity controls typically are not administered by a government financial system authority, but are exercised at the option of payments system participants.⁶ The idea here is that the inability of payments system participants to control their own positions in clearing systems is one factor that may contribute to credit quantities exceeding their efficient levels. Bilateral credit limits and sender net debit caps are typical quantity controls.

Bilateral credit limits constitute upper limits that participants set on net positive positions (net amount received) vis-a-vis other individual participants. A participant can refuse to accept payment instructions from another participant if doing so would cause its credit limit vis-a-vis that participant to be exceeded. A sender net debit cap is a participant's upper limit on its aggregated negative position (total amount sent).

Although such direct restrictions may be very effective in reducing the credit associated with interbank payments, and the corresponding credit risk, they likely have a
negative effect on operational efficiency by raising the processing costs in clearing systems. In addition, they may introduce the risk that customers may claim damages against their bank if the bank misses making time-critical payments due to the existence of these restrictions. There may be other means whereby systemic risk can be reduced. One is to set entrance requirements for payments system participants that assure that only relatively financially strong institutions have access. Another is to design loss-sharing rules and/or collateral requirements that reduce the probability of a chain reaction when one participant defaults in a clearing system.

II. Japanese Payments System

There are four major interbank yen payments systems in Japan—BOJ-NET, the Zengin System, the Gaitame-Yen System, and the check clearing system. Except for the check clearing system, these systems are all generated electronically. BOJ-NET is a settlement system, and is similar to Fedwire and SIC. The other three systems are clearing systems. As in other countries, the settlement system, BOJ-NET, is managed by the central bank, the Bank of Japan. The three clearing systems in Japan all are managed privately.

BOJ-NET, the Zengin System, and the check clearing system handle mostly domestic payments. BOJ-NET handles high value institutional transactions for the money and security markets; the number of transactions handled is

| Table 1 | Features of Large-Value Interbank Yen Payments Systems in Japan |
|--------------------------|--------------------------|--------------------------|--------------------------|
| Features               | BOJ-NET               | Zengin System           | Gaitame-Yen System        | Check Clearing System   |
| Settlement system      | • Settlement system    | • Clearing system       | • Clearing system         | • Clearing system       |
| for domestic wholesale | for domestic retail    | for cross border         | for domestic commercial   |
| transactions           | transactions          | transactions            | trades.                   |
| Funds transfer         | • Funds transfer and   | • Electronic basis      | • Electronic basis       |
| and Japanese Government | Japanese Government    |                          |                          |
| Bond transfer system   | Bond transfer system  |                          |                          |
| • Electronic basis     | • Electronic basis    |                          |                          |
| Established            | 1988                   | 1973                     | 1980                     | 1879                    |
| Managed by             | BOJ                    | TBA¹                     | TBA¹                     | Check clearing houses   |
| No. of Participants²   | 351                    | 4,870                    | 151                      | 599³                    |
| (of which foreign      | (82)                   | (3)                      | (61)                     | (87)                    |
| banks)                 |                        |                          |                          |
| Risk Structure         | • Same-day transfer    | • Same-day transfer      | • Same-day transfer       | • Next-day transfer     |
|                        | for both customers and| for customers, next-day | for both customers and    | for both customers and   |
|                        | banks                  | transfer for banks       | banks                    | banks                   |
|                        | • Overnight credit risk|                          |                          | • Overnight credit risk  |
| Risk Management        | • No daylight overdraft| • BOJ’s guarantee        | • Bilateral net credit    | • Suspension of         |
|                        | • Real-time and        |                          | limit                    | transaction system      |
|                        | designated-time transfer| • Collateral             | • Liquidity sharing among|                          |
|                        | • Third party transfer | • Loss sharing among all| banks with positive       |                          |
|                        |                        | the participants         | balance against the       |                          |
|                        |                        | • Sender net debit cap   | defaulting bank           |                          |
|                        |                        |                          |                          |                          |

¹TBA: Tokyo Bankers Association
²As of the end of December 1989
³Tokyo Clearing House only
relatively small. The Zengin System handles lower value individual transactions in large volume. The check clearing system handles comparatively small value bills or checks originating from commercial trade in local areas. The Gaitame-Yen System, which is similar to CHIPS in the U.S., handles international payments, such as yen payments arising from foreign exchange, Euro-Yen trading and other cross-border transactions. Table 1 provides a summary of the important features of the various payments systems, while Tables 2 through 4 provide some indicators of size and use.

The operation of these payments systems is supplemented by an intraday interbank credit market, which provides funds for clearing at designated times. Funds for the morning session are borrowed at opening (9:00 a.m.) and repaid at check-clearing time (1:00 p.m.). Funds for the afternoon session are borrowed at check-clearing time and repaid at the day’s closing (3:00 p.m.). As mentioned earlier, credit in this intraday market is priced (about 3.65 basis points for lending, about 14.6 basis points for borrowing).

**BOJ-NET**

BOJ-NET is composed of two systems—the funds transfer system, which began operations in October 1988 and the security (Japanese Government Bond) transfer system, initiated in May 1990. The funds transfer system is a same-day gross settlement system, similar to Fedwire in the United States. Most of the transfers are executed for the settlement of wholesale transactions, such as in the money market or the securities market. Payment orders on BOJ-NET are processed through an on-line computer network among account holders and the BOJ. A payment order instructs the user’s reserve account at the BOJ to be debited and another account holder’s reserve account to be credited. Though non-on-line users continue to rely on paper-
based orders, even in this case all data are input at terminals in the BOJ's head office and branches by the BOJ’s operators. Out of 651 account holders with the Bank of Japan, 351 currently are participants of the funds transfer network (as of the end of August 1990). Participants mainly are banking institutions, but securities houses and money market dealers (tanshi) also are included. All 82 foreign banks that have offices in Japan hold accounts at the Bank of Japan, and 80 of them are participants in BOJ-NET.

BOJ-NET allows both real-time instantaneous transfers and transfers at designated times of the day, specifically at 9:00 a.m., 1:00 p.m. and 3:00 p.m. Funds transfer orders for designated times can be revoked until the designated times, unlike real-time transfers.11 Users also can send post-dated instructions, for settlement at one of the designated times on the following business day.

The Bank of Japan offers finality on BOJ-NET transfers. However, unlike many other central banks, the Bank of Japan does not permit daylight overdrafts in reserve accounts.12 This limits the amount of transfer to the institution’s reserve balance at the time of the transfer, whether it be a real-time transfer or a transfer taking place at a predetermined designated time. If a participant has insufficient funds in its reserve account at the time of the transfer, the payment instruction automatically is rejected. This implies that the Bank of Japan bears relatively little risk.

### Table 3

**Settlement value and volume**

(per day in 1989)

<table>
<thead>
<tr>
<th></th>
<th>BOJ-Net funds transfers</th>
<th>Zengin System</th>
<th>Gaitame-Yen System</th>
<th>Check clearing System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Value</strong> (¥ billion)</td>
<td>118,613²</td>
<td>6,396</td>
<td>21,257</td>
<td>17,876 (15,020)</td>
</tr>
<tr>
<td><strong>Volume</strong> (in thousands)</td>
<td>11</td>
<td>2,288</td>
<td>21</td>
<td>1,526 (564)</td>
</tr>
<tr>
<td><strong>Value per one transaction</strong> (¥ million)</td>
<td>15,300³</td>
<td>3</td>
<td>1,012</td>
<td>11.7 (26.6)</td>
</tr>
</tbody>
</table>

1Figures for March to December 1989.

2Transactions among BOJ accounts (including those of non-financial institutions).

3Sample survey conducted on 9/28/89. The sample consists of 18 accounts at the head office of the Bank of Japan, whose activity in June 1989 accounted for 28.8% of the total transactions.

4Figures in parentheses indicate transactions through the Tokyo Clearing House.

In the case of designated-time transfers, BOJ-NET participants extend payment instructions to another bank in expectation of an incoming transfer at a future designated time. It can be argued that this facility is very similar to a clearing system in terms of the interdependent credit that it generates, and thus in terms of its systemic risk.

In addition to funds transfers among account holders with the Bank of Japan, BOJ-NET provides the facility for large value funds transfers for the customers of account holders (so-called “third-parties”). This facility provides the convenience of same day funds transfer with payment finality for large payments. The services provided here are similar to those offered by other central banks’ systems, such as Fedwire or SIC. The minimum transfer amount on BOJ-NET is set at a relatively high 300 million yen for third-party transfers.

There are other restrictions in connection with third-party transfers that are initiated by security firms and other non-bank participants in BOJ-NET. Specifically, if the

### Table 4

**Share of Transaction Volume at Posting Time**

(Sample survey conducted on 9/28/89)

<table>
<thead>
<tr>
<th>Posting Time</th>
<th>Tanshi Companies</th>
<th>Non-Tanshi Companies</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume</td>
<td>Share %</td>
<td>Volume</td>
</tr>
<tr>
<td>9 AM</td>
<td>32</td>
<td>(4.4)</td>
<td>33</td>
</tr>
<tr>
<td>1 PM</td>
<td>2,484</td>
<td>(12.9)</td>
<td>651</td>
</tr>
<tr>
<td>3 PM</td>
<td>684</td>
<td>(18.9)</td>
<td>538</td>
</tr>
<tr>
<td>Real-Time</td>
<td>31</td>
<td>(1.4)</td>
<td>76</td>
</tr>
<tr>
<td>Total</td>
<td>3,231</td>
<td>100.0</td>
<td>1,298</td>
</tr>
</tbody>
</table>

**notes**

1. The sample consists of 18 accounts of major brokers and banks at the head office of the Bank of Japan, whose activity in June 1989 accounted for 28.8% of the total transactions.
2. The figures represent actual transfer orders originating from the accounts of sample institutions to other accounts at BOJ. The figures differ from those in other charts in this respect.
3. The figures given in parentheses in the volume column are the average amount of transfer orders in billions of yen.
transferring entity is a non-bank institution, it cannot specify its paying customer's name on the payment instruction. Likewise, if the transferee is a non-bank institution, the transferring institution cannot specify the payee's name on the payment instruction. The rationale for these restrictions is that a full third-party transfers are considered to be part of the funds transfer business, which may not legally be conducted by non-bank institutions due to the separation between the banking and securities businesses in Japan. Due to such restrictions, the use of third-party transfers is not as large as that of funds transfers among account holders with the Bank of Japan.

The input hours of funds transfer are from 9:00 a.m. to 4:30 p.m. However, transfer instructions for the same day are accepted only until 3:00 p.m. (in the case of third-party transfer instructions, until 2:00 p.m.). Post-dated instructions are accepted until 4:30 p.m. The operational cycle of funds transfers on BOJ-NET is shown in Table 5.

The number of transfers of reserve accounts, almost all of which stem from the use of BOJ-NET, is approximately eleven thousand a day. This is very small compared with the private payments system in Japan and also is smaller than that of major central banks abroad. The average value per transfer is 15.3 billion yen.

To sum up, BOJ-NET is a large-value settlement system. BOJ-NET does offer finality, but because of the prohibition on daylight overdrafts, this entails relatively little risk for the Bank of Japan. The real-time transfer portion of BOJ-NET has relatively little systemic risk, but its operational efficiency is likely to be relatively low. The designated-time transfer system may help to improve operational efficiency, but may introduce significant systemic risk (See Section IV).

The Zengin System

The Zengin System, which is managed by the Tokyo Bankers Association, processes nationwide domestic funds transfers by translating each bank's position against that of other banks into a bilateral position against the Bank of Japan. These positions are then settled through adjustment of BOJ reserve accounts. The Zengin System thus is a clearing system.

The Zengin System is used mainly for relatively small value transfers such as private funds transfers, direct deposits, pension payments, stock dividend payments, etc. It started its operation in 1973 as a nationwide electronic clearing system, and now has grown to a large network system of nearly five thousand banks and other deposit-taking institutions as participants. Its original members were nationwide banks (city banks, long-term credit banks, trust banks, regional banks) and the Shoko-chukin Bank (a financial institution for small businesses), but other institutions joined later. In addition, three foreign banks are members. At the end of December 1989, the system included 4,870 institutions with 43,684 places of business.

Central financial institutions such as the Zenshinren Bank, the National Federation of Credit Co-operatives, and the Norinchukin Bank act on behalf of small financial

<table>
<thead>
<tr>
<th>Table 5</th>
<th>BOJ-NET Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Day</strong></td>
<td><strong>Funds transfers</strong></td>
</tr>
<tr>
<td><strong>T-3</strong></td>
<td></td>
</tr>
<tr>
<td><strong>T-1</strong> (settlement day)</td>
<td>Advance input for next day's transfers possible</td>
</tr>
<tr>
<td>9:00—Opening</td>
<td>Funds for the day's opening available</td>
</tr>
<tr>
<td>10:00—</td>
<td>Receive data on check clearings and Zengin System</td>
</tr>
<tr>
<td>11:00—</td>
<td>Input for the day's and next day's transfers possible at any time during the business hour</td>
</tr>
<tr>
<td>12:00—</td>
<td></td>
</tr>
<tr>
<td>13:00—Check-clearing time</td>
<td>Settlement at check-clearing time</td>
</tr>
<tr>
<td>14:00—</td>
<td>End of input for the day's transfers for third parties</td>
</tr>
<tr>
<td>15:00—Closing</td>
<td>Settlement at the day's closing</td>
</tr>
<tr>
<td>16:00—</td>
<td>End of input for post-dated transfers</td>
</tr>
<tr>
<td>16:30—</td>
<td>Processing of the next day's opening</td>
</tr>
</tbody>
</table>

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institutions in their respective sectors (respectively the small business shinkin banks (454), credit co-operatives (414), and agricultural cooperatives (3,685)). The exchange balances of these central institutions with the Bank of Japan therefore include their own exchange balances plus those of the related financial institutions. This system is called a proxy settlement system and was initiated in February 1979.

The Zengin system processes as many as two million transactions per day. However, it is mainly used for bulk payments, and as a result, the average value per transaction is comparatively small, about 3 million yen.

The processing of domestic funds transfers in the Zengin System is depicted in Chart 1. Assume at time T a sender a asks a participant bank (Bank A) to send his money to a beneficiary b who has an account at another participant bank (Bank B). The sender bank debits a’s account and instructs the beneficiary bank to credit b’s account through the Zengin System Center.

This process is virtually real-time, in the sense that the transferred amount becomes almost immediately available to the beneficiary b. Thus, the Zengin System is a same-day settlement system from the customer’s viewpoint. However, this system is a next-day settlement system for the participant banks. The balance between the sending and receiving banks, which is calculated at the Zengin System Center, is settled on the following day (T + 1 day) at the check clearing time of 1:00 p.m. by BOJ-NET. Under the Zengin System, the beneficiary bank bears an overnight credit risk since funds become available to the beneficiary on the transaction day while interbank funds settlement is conducted on the following day.

To cope with this risk and secure the clearing process, the Bank of Japan stands ready to provide provisional liquidity if a transferor becomes insolvent at the time of final settlement on the following day. Under this agreement, every participating bank is required to deposit collateral with the Bank of Japan, and all participants assume joint responsibility when unsettled liabilities exceed the collateral value of the failed bank. To further control the risk exposure, a sender net debit cap was introduced in July 1990.

The BOJ’s provisional liquidity guarantee and the participants’ joint loss sharing agreement contribute to the avoidance of unwinding. Moreover, the loss sharing agreement and the required collateral reduce the credit exposure from receiving banks to sending banks. Although the system is thus very well organized to provide liquidity and limit credit risk, the Bank of Japan’s deep involvement might encourage the misconception that it is willing to bear unlimited losses.

The Zengin System, being a clearing system, does entail some systemic risk, but the BOJ’s provisional liquidity guarantee, the collateral requirements and the joint loss sharing agreement reduce this risk somewhat. The netting of debits and credits in the Zengin System is operationally efficient, but the involvement of the BOJ in the System

---

**Chart 1**

**Zengin System Flow of Funds**

1. **Sender a** requests for transfer withdraw from account a
2. **Sender bank A** sends transfer instruction
3. **Beneficiary b** sends notice of credit account b
4. Zengin Center conducts calculation for the accumulated debit and credit balance for each bank
5. Results of the calculation settled among banks’ accounts
6. **Bank B’s account**
7. **Bank A’s account**
8. **BOJ**
may lead participants to underprice risk and therefore overextend credit beyond economically efficient levels.

The Gaitame-Yen System

The Gaitame-Yen System, similar to CHIPS in the United States, is a same-day, net settlement facility that settles yen payments arising from foreign exchange and other cross-border transactions.24 It began operations in October 1980. Like the Zengin System, the Gaitame-Yen System is managed by the Tokyo Bankers Association. Since March 1989, the Bank of Japan has operated the Gaitame-Yen System as a part of the BOJ-NET. All procedures are currently conducted electronically.

The number of transactions conducted through the Gaitame-Yen System is about 21 thousand per day and the average value of transactions is approximately one billion yen. Foreign banks are the big players in the Gaitame-Yen System; 61 out of 151 participants are foreign banks as of the end of December 1989, and they handle one fourth of the total volume of transactions.

Banks sending or receiving a large volume of settlement orders connect their computers directly with the BOJ’s host computer. When a bank receives transfer orders via SWIFT,25 the data are automatically input into its computer and finally processed by BOJ’s host computer.

Payment instructions on the Gaitame-Yen System are netted out among participants, and the net position of each is settled through reserve accounts with the Bank of Japan. Transfer orders for the same day are input at the sending bank’s terminal by 1:45 p.m. (for post-dated transfer by 4:00 p.m.) and output at the receiving bank’s terminal immediately.26 How soon the payee has funds available for use depends on when the receiving bank credits his account after it receives a payment instruction. Usually, it credits the payee’s account immediately after receiving payment instructions.

While the Gaitame-Yen system is similar to the Zengin System (see Chart 1), it differs in that net debit and credit positions of Gaitame-Yen System participants are settled at the same-day’s closing (3:00 p.m.) while those of Zengin System participants are settled at check clearing time (1:00 p.m.) on the next-day. Therefore, a receiving bank can obtain final funds on the same business day as they were sent in the Gaitame-Yen System. Even in this case, though, the receiving bank bears a credit risk to the sending bank if the receiving bank credits its beneficiary’s account before settlement.27

To cope with this credit risk, a bilateral net credit limit facility is offered at the receiver’s option. With this option, each bank can set an upper limit on the net credit position it will accept from any other bank. Such a limit can be changed at the bank’s discretion through its terminals. When the BOJ receives an on-line funds transfer order, it checks to see that it does not exceed the limit placed by the receiving bank. If the amount exceeds the given limit, the BOJ notifies the remitting bank of the error. Another rule, introduced in March 1989, states that when a bank defaults, all banks with bilateral net credit balance against the former will jointly bear the shortage of liquidity so that there will be no need for unwinding.

Unlike in the Zengin System, settlement of net positions of participating banks in the Gaitame-Yen System is not insured by the BOJ. However, it is not necessary to unwind in the event of a participant’s default because of the existence of the liquidity sharing rule. Nevertheless, since there are no clear agreements on how to share the loss among participants,28 this rule does not give clear incentives to participants to reduce risk.

The Gaitame-Yen System has some systemic risk, but it is reduced by the bilateral net credit limits and the liquidity sharing rules. To the degree that unsettled amount accumulates with the time lag between transfer instructions and settlement, risk in the Gaitame-Yen System is reduced by its being a same-day system. The netting of transactions makes the Gaitame-Yen System relatively efficient, in an operational sense.

The Check Clearing System

The check clearing system is a paper-based payment system under which financial institutions of a specified area come together at a clearing house at a specified time every day in order to exchange checks payable at other institutions as well as bills, receipts, bond coupons, and other such instruments. Checks and bills are the most popular means of payment between corporations and individual participants in financial markets.29

Checks and bills usually are passed through clearing houses by banks, and are not processed by the Bank of Japan. As of March 1989, there were 183 clearing houses legally designated by the Minister of Justice as well as 595 other private clearing houses managed either as associate institutions of the Bankers’ Associations of the various regions or as independent corporations.30 The exchanges and settlements at the clearing houses include not only city banks, regional banks, trust banks, and long-term credit banks, but also the second tier regional banks, shinkin...
banks, credit co-operatives, and other institutions either as direct participants or as participants through correspondents with which they have dealings. The clearing balances of individual banks normally are settled through transfers among reserve accounts at the Bank of Japan. For locations where the Bank of Japan does not have branches, settlements are carried out through interbank deposits at specified banks. The number of transactions through clearing houses is about 1.5 million per day, and the average value of transactions is about 12 million yen.

The rules and procedures in Japanese check clearing systems basically are similar to those in other countries. A typical transaction in the Tokyo Clearing House, for example, is processed as follows: the beneficiary presents a check to his bank (the transferee bank), and this is passed on to the Clearing House in the evening, where net balances are calculated and the transferor bank receives the check (the T day). Settlement between reserve accounts at the BOJ is then carried out at 1:00 p.m. ("check clearing time") on the following business day (T + 1 day).

In the case of a failure to pay, the transferor bank returns the dishonored check to the transferee bank and requests that its reserve account be credited the previously debited amount on the next day (the T + 2 day). This means that a series of transactions are not final but provisional on confirmation on the T + 2 day that the check has not been dishonored. Thus, this check clearing system can be described as a next-day settlement system, since beneficiaries have to wait at least another day (until the T + 1 day) for the fund to be credited to them, and since interbank settlement cannot be finalized until the day after the clearing (the T + 2 day).

The transferee bank bears short-term insolvency risk if it credits funds to the beneficiary’s account before confirmation on the T + 2 day, since the transferor bank can claim refund of reserve funds in case of a default. In this system, therefore, transferee banks have to be particularly careful in determining when to credit funds to the beneficiary’s accounts.

Collateral usually is used to control risk in the check clearing system. In the case of the Tokyo Clearing House, participants must deposit bonds (namely public and corporate bonds, with total face value of 3 million yen) as collateral with the Clearing House. However, this amount of collateral is too small to cover participating banks’ default, should it occur. Therefore, the possibility of unwinding, and hence systemic risk, still remains in the check clearing system.31

III. Reducing Risk in Japanese Payments Systems

It is apparent from the discussion in Section II that the approach to managing payments systems in Japan differs in some respects from that in the U.S. This implies some differences in risk and efficiency characteristics. In particular, the existence of a facility for real-time transfers and the prohibition on daylight overdrafts (BOJ-NET); collateral requirements, loss sharing among banks (Zengin system); and the pricing of credit by participants in half-day credit markets are all arrangements that tend to reduce risk, and that have historically not been present in U.S. payments systems.32 Although the risk-management measures adopted in Japanese payments systems have proved to be adequate, continuing developments in financial markets and technological innovations suggest that efforts to reduce risk even further may be desirable.

There are three ways in which the risk in Japanese payments systems may further be reduced: (i) improving the balance between real-time and designated-time transfers; (ii) expediting payment transactions; (iii) introducing delivery versus payment. In all three cases, a major concern in attempting to manage payments system risk entails a loss of efficiency in payments. While it can be argued that less risk in payments systems is generally desirable, risk reduction measures may lead to less credit than is socially optimal.

Improving the balance between real-time and designated-time transfers

Though there are two facilities in BOJ-NET, namely the real-time instantaneous transfer and the designated-time transfer, the use of the real-time transfer is very small. Only 2.4 percent of all transactions are in real-time,33 while 97.6 percent are delayed to designated times, with 69.2 percent at the check clearing time (1:00 p.m.) and 27.0 percent at the business closing time (3:00 p.m.).34 (see Table 4.)

Real-time transfers are more desirable than designated-time transfers from the point of view of risk. First, with real-time transfers, payment instructions are independently executed, transaction by transaction, on a final basis. This limits any chain reaction to a single participant’s default. On the other hand, with designated-time transfers, payment instructions are concentrated at a specific time, and therefore are interdependent. This generates greater systemic risk of a chain response to individual defaults. Second, with real-time transfers, users cannot
input instructions for payments which exceed their current account balance at the input time. In contrast, with designated-time transfers, they can input instructions for any amount of payment in advance, even if the payment exceeds the current account balance at the input time. This potentially allows implicit interest-free credit and hence greater default risk to be generated among private financial institutions. Third, since in Japan most interbank settlements are processed at two designated times, increases in daylight overdrafts in customers' accounts with banks are likely to arise with designated-time transfers because customers are not fully aware of the settlement times.

However, designated-time transfers have certain advantages. First, users of BOJ-NET are less constrained by the prohibition on daylight overdrafts when they use the designated-time facility rather than the real-time facility. One of the reasons is that they can resort to credit in the half-day markets to obtain funds to settle at the designated times. This is much more difficult when using the real-time facility. Second, participants tend to prefer the designated-time transfer to the real-time transfer because the fixed settlement times of the former enable banks to enjoy the advantages of concentrating their transactions. In particular, banks can synchronize funds transfers arising from the inflow and outflow of funds at the designated times.

Given this trade-off between risk and efficiency, it is not entirely clear whether measures to encourage greater use of the real-time facility in order to reduce risk in the Japanese payments system are called for. Further research on this issue may shed light on appropriate ways to weight the risk and efficiency in payments systems, and the relative merits of designated-time and real-time transfers from a social point of view.

**Expediting Payment Transactions**

The credit risk in payment systems can be viewed as proportional to the amount of unsettled payments balances. This amount depends in turn on the time lag between contract and settlement. The longer the lag, the larger the accumulated unsettled balances. This suggests that measures that shorten the lag may reduce the risk in payments systems. In the Japanese stock market the time lag between contract and settlement is shorter than the international standard. For government bonds, however, the lag runs up to ten business days (see also note 34), making it one of the longest among the major industrial countries. Hence, expediting settlement in the government bond market in Japan is very important.

With regard to the unsettled balances, it is important to recognize a difference between same-day and next-day settlement systems. With a next-day settlement system, unsettled balances remain until settlement is finally completed on the next day, while with a same-day settlement system, they disappear within the same day that a sender instructs payment.

Both same-day and next-day settlement systems are employed in Japanese financial markets. For example, transactions in the call and bill markets are mostly settled through BOJ-NET and Euro-yen transactions are settled through the Gaibame-Yen System (both are same-day settlement systems). Other short-term money markets transactions, such as those involving CDs and commercial paper, large value financial transactions involving government bonds, and foreign exchange, utilize BOJ-NET, a same-day settlement system, as well as the Zengin System and the check clearing system, both of which are next-day settlement systems.

In contrast, most trading in overseas financial markets now is settled through same-day settlement systems. In the United States, transactions in the short-term money markets (federal funds, CDs, commercial paper and bankers acceptances), government bond markets (T-Bills, T-Notes, T-Bonds), financial futures, and options markets all are settled through Fedwire, which is a same-day settlement system. A large portion of Euro-dollar and foreign exchange transactions are settled through CHIPS, a same-day settlement system.

From the standpoint of minimizing settlement risk as well as enhancing global interdependence, it may be desirable for Japan to move to a system where the settlement of large value transactions in major financial markets are processed through same-day settlement systems. Given developments in technology, such a change would seem to impose no significant operational or economic costs.

**Delivery-Versus-Payment**

Delivery-versus-payment is a mechanism under which a fund transfer and a security transfer are conducted simultaneously. When delivery-versus-payment is not available, the possibility arises that a party to a transaction might fail to receive the funds expected in return for the completed delivery of a security, or, conversely might fail to receive the security expected in return for a completed payment, owing to the counterparty’s default.

The development of electronic funds transfer technology has increased the feasibility of delivery-versus-payment system. Electronic delivery-versus-payment systems already are operating in government bond markets in the United Kingdom (CGO System), the United States (Fedwire), and in the Euro-market (Euroclear, CEDEL).
Japan, however, although individual participants have some devices for ensuring delivery-versus-payment, an electronic delivery-versus-payment system does not exist.

The introduction of electronic delivery-versus-payment in Japan would reduce risk in payment systems without any cost in efficiency, and thus appears to be desirable. Since the establishment of the Japanese government securities system in BOJ-NET in May 1990, it has become feasible to introduce delivery-versus-payment by integrating the cash and securities delivery systems of BOJ-NET.

IV. Concluding Remarks

The discussion in this paper has highlighted two important features of Japanese payments systems. First, the risks associated with payments systems in Japan are common to those of payments systems in other countries. Delivery lags and the interdependence of transactions produce credit and systemic risk in Japanese payments systems, as they do elsewhere. Second, the approach to managing payments systems in Japan in some respects differs from the U.S. approach. As a result, Japanese payments systems may differ in their risk and efficiency characteristics from U.S. payments systems. In particular, the existence of a facility for real-time transfers and the prohibition on daylight overdrafts (BOJ-NET), collateral requirements, loss sharing among banks (Zengin system), and the pricing of credit in half-day credit markets are all arrangements that tend to reduce risk, and that have historically not been present in U.S. payments systems.

Three possible measures for further reducing risk in Japanese payments systems also have been discussed: (i) improving the balance between real-time transfers and designated-time transfers, (ii) expediting payment transactions (iii) introducing delivery-versus-payment. Further research is required to determine the advisability of the first measure, but it is apparent that the second and third measures could reduce risks without introducing significant operational or economic costs.

Further research also is required to identify other measures that may reduce payments systems risk in the face of the growing integration of world financial markets. As a result of such integration, payments systems in one country are now easily influenced by incidents in other countries. Various kinds of international cooperation as well as a better understanding of foreign systems will become increasingly important components of efforts to manage payments systems risks.
1. CHIPS stands for Clearinghouse Interbank Payments System, and is a clearing system for dollar payments arising from international transactions. It is owned and operated by the New York Clearinghouse banks. Fedwire is managed by the Federal Reserve System, and is used for transferring reserve account balances of depository institutions, as well as government securities.

2. Unwinding, which is characteristic of clearing systems, is a procedure under which participants' net positions are recalculated with payment instructions of the defaulting participant put aside and the recalculated new positions are settled. The risk in unwinding arises because settling the new positions may be difficult because the net positions of all participants will have changed. In order to settle, participants may need to make arrangements for raising additional funds or cancel certain prior commitments. Such rearrangements or contract cancellations may cause other participants to default.

3. There are also longer-term private interbank credit markets in Japan.

4. "Interbank credit" here includes both credit among private banks and credit from a central bank to private banks.

5. Since the Federal Reserve has unlimited financial strength compared with private banks, the problem is not that the Federal Reserve bears too much risk by itself, but that private banks have an incentive to use daylight overdrafts excessively.

6. In some countries, it may be politically infeasible for a central bank or other authority to limit the credit exposure of individual payments system participants due to possible charges of discrimination.


9. Funds for the morning session are utilized either for the large withdrawal of cash from BOJ windows or large funds transfer from Tokyo to cities nationwide in the early morning. Funds for the afternoon session are mostly in demand by institutions who need to cover a shortage of funds in BOJ accounts arising from bill/check clearings, Zengin System transfers, etc., at check-clearing time (1:00 p.m.).

10. Before the introduction of BOJ-NET, BOJ-checks had been mainly used for transfers among account holders with the BOJ. A payee who gets a BOJ-check from a payer (drawer) presents it at a BOJ window for funds transfer within the Bank. Although BOJ-checks were largely replaced by computer transactions, they are still used for several purposes, for example, settlements by relatively small, non-BOJ-NET-participant account holders with the Bank of Japan. The Bank of Japan does not accept payment orders by telephone, so BOJ-checks remain one of the important large-scale means of payment in Japan despite the Bank’s effort to enhance the convenience and applicability of BOJ-NET.

11. A real-time transfer cannot be revoked because it is final. However, if both parties agree to it, a reverse transaction can be used to undo a transfer.

12. The Federal Reserve Banks and the Bundesbank allow daylight overdrafts up to a limit and the Bank of England and the Bank of France allow them without any limit. Like the Bank of Japan, the Swiss National Bank does not allow daylight overdrafts.

13. Major securities firms hold accounts with BOJ, so that they can also be users of BOJ-NET. However, they are not allowed to engage in traditional commercial banking operations, such as taking deposits or providing funds transfer services to customers. They can, therefore, only effect funds transfer orders with customer information through BOJ-NET if they themselves are either the ultimate beneficiary or the originator. This ensures that securities firms themselves do not engage in the business of funds transfer for customers. In the U.S., such an issue would not arise because securities firms are not participants in Fedwire.

14. Transaction volumes using Fedwire and SIC are approximately 200,000 (1986) and 170,000 (November 1988) per day, respectively.

15. Founded in 1945, the Tokyo Bankers Association comprises 131 banks (city banks, long-term credit banks, trust banks, regional banks and second-tier regional banks). Its main functions are to study the financial system, rationalize banking activities, and manage the Zengin System, the Gaitame-Yen System, and the Tokyo Clearinghouse.

16. Actually, this scheme is called the "domestic exchange settlement system." The Zengin System is used only for the electronic exchange of payment instructions. Payment instructions are exchanged on a paper basis, too; however, the use of such paper exchange is very small. For simplicity, the paper transactions are ignored in the text.

17. For one participant, aggregated receiving and sending amounts are debited from and credited to his reserve account, respectively. In this sense, this procedure is a little different from that of an ordinary clearing system which transfers one net position for one participant through a shadow account (or nominal account) with a central bank.

18. The Zengin System is a retail funds transfer system with no exact equivalent in the U.S. Although the Automated Clearing House is used by many private firms in the United States to make recurring payments, such as payroll payments, electronically, it is not used nearly as extensively by the private sector as the Zengin System is used in Japan.

20. Foreign bank participation in the Zengin small-value-transaction system is limited because foreign banks in Japan generally do not do much retail business.
21. Figures in parentheses indicate the number of institutions as of the end of March 1990.
22. The collateral required of a participant is its average daily net debit position over the previous year.
23. The sender net debit cap was introduced as an informal guideline in 1987 at 15 times the required collateral level. It was strengthened to a formal restriction at 10 times as much as the required collateral level ("warning line" at 5 times) in July 1990.
24. For example, the Gaitame-Yen System handles yen transfers based on correspondent agreements, yen remittances, payments resulting from export/import trade, yen settlement of foreign exchange, etc. As far as foreign exchange transactions are concerned, though almost all the transactions outside the Tokyo market and transactions between domestic banks and offshore banks use the Gaitame-Yen System, transactions between domestic units are settled through the check clearing system.
25. SWIFT (Society for Worldwide Interbank Financial Telecommunications) is a nonprofit, cooperative organization that facilitates the exchange of payment instructions between financial institutions around the world. SWIFT is considered to be a mere message transfer system, not a payments system per se. Used worldwide, the SWIFT format is the most popular world standard. As of August 1989, 1,487 banks from 65 countries were participating in SWIFT.
26. A transfer order can be input up to three business days prior to the settlement date. This helps to reduce the peak workload of each participant bank.
27. In this case, if the beneficiary withdraws from his account, the receiving bank bears a liquidity risk in addition to the credit risk.
28. Participants are required to discuss how to share the loss later, but there is no clear agreement on how this is to be done.
29. However, very few personal checks are used in Japan.
30. The clearing houses designated by the Minister of Justice are exempt from antitrust laws. Moreover, the presentation of bills or checks to the designated clearing houses as opposed to private clearing houses is more effective from a legal viewpoint.
31. The issuer of a bill or check that is dishonored because of insufficient funds or for other reasons is subject to the posting of a notice of failure to collect. Those who issue dishonored bills again within six months are subject to a two-year prohibition from current account transactions and lending transactions with member financial institutions of the clearing house. This suspension from the transaction system is designed to deter default by customers, not by banks.
32. Note that pricing of credit does exist in the Federal funds market, but since it is not an intraday market, it cannot meet liquidity needs for intraday transactions. Instead, participants in U.S. payment systems incur unpriced daylight overdrafts in the course of their daily transactions subject to limits that have been introduced in recent years. Also note that in 1986, the Federal Reserve adopted a policy under which banks are encouraged to voluntarily establish limits on the net amount they can owe at any one time across all large-dollar networks. Although the program is voluntary, only those institutions that set a cap are permitted to incur daylight overdrafts on Fedwire.
33. Almost all users of real-time transfers are regional banks and second-tier regional banks (former sogo banks), which have relatively large intraday reserve balances. City banks usually use the designated-time transfer facility.
34. There also are business customs in Japan that concentrate funds transfers on specific days of the month, particularly on the 10th, 20th and the end of the month. These customs also apply in settling maturity dates of corporate financial investment in large-scale-time deposits, CDs and other instruments as well as those Japanese government bonds. The settlement dates for Japanese government bonds, however, have been changed to 5, 10, 15, 20, 25 and the end of the month since August 1987 (so-called 5/10 days settlement).
35. There is a custom of designated-time transfer in the BOJ-checks system too. The drawer of a BOJ check stamps either "check clearing" or "the day's closing" on it to show when the settlement should be executed. When the payee presents the check at a BOJ window, BOJ debits/credits the account of the payer/payee at the time duly designated on the check (the designated time should come after the check presentation, and on the same business day). Checks without stamps of time designation are processed immediately after their presentation (immediate processing).
36. The shortening of the time lag between contract and settlement will lessen settlement risk. At the same time, however, the probability of settlement failures due to operational mistakes may rise.
37. With the bankruptcy of the Herstatt Bank in 1974, CHIPS, initially a next-day settlement system, shifted to a same-day settlement system. In the United Kingdom, funds transfers in the short-term money, foreign exchange and futures markets are handled by CHAPS or Town Clearing, which is a same-day settlement system, and gilt trades are settled through CGO (Central Gilt's Office), which is a same-day settlement system operated by the Bank of England. In Switzerland, most funds transfers in financial markets are settled through SIC (Swiss Interbank Clearing), which is a same-day settlement system.
38. A BOJ-check is one of the devices for ensuring delivery-versus-payment. For example, an overseas institutional investor sells Japanese stocks through a securities house in Japan. The securities house presents a BOJ-
check, which may be received from a bank where it has an account if the securities house does not have an account with the Bank of Japan, to the proxy bank where the overseas investor holds an account. The proxy bank presents stocks to the securities house. In this way, the BOJ-check and the stock are physically exchanged at the same time. This is a delivery-versus-payment system because a BOJ-check is regarded as cash by financial institutions in Japan. This is one of the reasons why the BOJ-checks system remains in use despite the introduction of the electronic BOJ-NET system.

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This paper studies a situation in which two previously isolated countries decide to unite their currencies and their fiscal policies. We assume that initially there is a “soft currency” country and a “hard currency” country. Given fiscal policy, we study the range of exchange rates of “soft” for “hard” currency that are feasible set. The inflation rate under the new consolidated government depends on the fiscal policy it follows, but does not depend on the exchange rate selected.

On July 2, 1990, East and West Germany became united through a common currency. The West German Deutsche Mark (DM) became the only legal tender on both sides of the border, and debts and payments denominated in the East German Ostmark (OM) were converted to DM at rates stipulated in an agreement signed by both governments on May 2.

The monetary union of East and West Germany raises a variety of issues, including the consequences of choosing one conversion rate over other possible rates, the price level implications of the conversion, and the welfare implications of the conversion for citizens of the two countries. To shed light on some the issues involved, this paper provides a theoretical analysis of German monetary unification.

Our analysis relies on a standard model of money, specifically, the overlapping generations model of Samuelson (1958). Although other models, such as the cash-in-advance model, are available, our key conclusions depend on aspects of the model that would appear in virtually any model of money, namely, the budget constraints of the two governments and the demand for fiat currency in each of the two countries being a function of the rate of return on currency. Thus, very similar results would emerge from these other models.

We analyze two countries which initially manage to isolate themselves, so that neither country trades with or borrows from the other, nor do the residents of one country hold the currency of the other. One country balances its budget and thereby supports a zero-inflation monetary system. There is also a country that runs a persistent government deficit and finances the deficit by a combination of inflation tax and repressed inflation. We model repressed inflation as a legal restriction or rationing scheme that forces citizens to hold more currency than they voluntarily would. This produces a “currency overhang” and repressed inflation. These legal restrictions are to be interpreted in the manner of Bryant and Wallace (1984) as devices to increase the base of the inflation tax.

We refer to the first country as the “hard currency country” because the value of its currency is stable over
time (there is zero or low inflation), and people hold and exchange its currency voluntarily. We refer to the other country as the “soft currency country” because its currency lacks one or both of those attributes: the value of its currency is deteriorating over time, and/or particular classes of people (typically, citizens of the soft currency country) are required to hold some of its currency involuntarily, either through explicit savings requirements or as a consequence of a commodity rationing scheme.

We compare the initial situation with a second one which we call monetary union: in the former soft currency country, the controls that forced residents to hold the soft currency are dismantled. The currency and credit markets are united with those of the hard currency country. In the process, the new, consolidated government chooses a rate at which the old, soft currency will be exchanged for the new, single currency. We study how the inflation rate in the unified monetary system depends on the fiscal policy of the new government. We show that there is a range of rates that can be sustained as equilibrium exchange rates, and we study the welfare consequences of a choice in this range.

I. Overview

In this section, we provide a brief overview of our arguments and results. Our reasoning exploits properties of two basic relationships: a demand function for government-issued currency, and the government’s budget constraint.

In the model we use, money is held voluntarily by agents to an extent determined by the return on currency. Since currency does not pay explicit interest, the real rate of return on currency is the change in its purchasing power. Since we prefer to work with gross rates of return (one plus the net change), we denote the rate of return on currency from \( t \) to \( t+1 \) as \( R_t = \frac{p(t)}{p(t+1)} \), where \( p(t) \) is the price level at \( t \). We assume that the real demand for currency in a country is an increasing function of \( R_t \), which we denote by \( f(R_t) \); the nominal supply, or stock of currency at \( t \) is \( H(t) \), and \( f(R_t) = H(t)/p(t) \).

A government can raise real revenues by generating inflation, thereby imposing an inflation tax on people who hold currency from \( t \) to \( t+1 \). The base for the tax is \( f(R_t) \), the real amount of currency held, while the rate of the tax is \( 1 - R_t \). The government’s budget constraint at \( t \) can be written as

\[
\frac{H(t) - H(t-1)}{p(t)} = D, \tag{1}
\]

where \( D \) is the real value, assumed constant over time, of that portion of the deficit financed by currency creation. This budget constraint can be written as

\[
\frac{H(t)}{p(t)} - \frac{H(t-1)}{p(t-1)} - \frac{p(t-1)}{p(t)} = D
\]

or

\[
f(R_t) - f(R_t-1)R_{t-1} = D.
\]

In a steady state situation, \( R_{t-1} = R_t = R \), so the above equation becomes

\[
f(R) \times (1-R) = D
\]

which decomposes the amount of inflation tax collected into the product of the base for the tax and the tax rate.

When the demand for currency is an increasing function of \( R \), the inflation tax revenue function \( f(R)(1-R) \) is as depicted in Figure 1. As \( R \) rises from some low value, \( f(R)(1-R) \) initially rises because the base of the tax \( f(R) \) rises faster than the rate \( 1-R \) falls. Eventually, however, as \( R \) rises toward 1, that is, as inflation falls to 0, \( f(R)(1-R) \) begins to fall toward 0. Notice that, as a result of the curve’s shape, if there exists one tax rate that finances a
steady state deficit $D$, then there are in general two such rates. For reasons indicated below, we will assume that we are always in the “good” equilibrium (with a higher $R$ or, equivalently, a lower inflation rate).

For a single closed economy, Figure 1 can be used to determine the steady state equilibrium value of $R$, and an initial price level $p(1)$ at some time $t=1$. First, the equilibrium $R$ is determined by the intersection of $f(R)(1-R)$ with the deficit $D$. Then, given that value of $R$, equation (1) written at $t=1$ can be manipulated to yield an equation that determines $p(1)$ as a function of some initial inherited currency stock $H(0)$:

$$\frac{H(1) - H(0)}{p(1)} = D$$

or

$$f(R) = \frac{H(1)}{p(1)} - \frac{H(0)}{p(1)} + D.$$  

This equation can be solved for $p(1)$ as a function of $D$ and $H(0)$. We can use Figure 1 to pick off the value of $f(R)$ associated with the equilibrium $R$.

Our model of East and West Germany before unification describes the two separate economies using two versions of Figure 1, one with a very low $D$, the other with a high $D$. The country that runs a low deficit $D$ attains a high return on money $R$ and a low inflation rate. The country with a higher $D$ attains a lower $R$, assuming it is willing to allow the price level to be determined freely by the supply of and demand for its currency. Later in this paper, we describe some measures that a government can take to enhance artificially the demand for its currency. Using a version of Figure 1, we shall show how such measures can be used to raise the base of the inflation tax and reduce the tax rate needed to finance a given deficit. We represent East Germany as having resorted to such measures.

Our approach to studying currency unification can be summarized by constructing a figure as the vertical summation of the two versions of Figure 1. At some time $t=1$, we suppose that the two countries open their borders and consolidate both their currencies and their government budgets. The stock of the new currency is the sum of the old western currency and the old eastern currency multiplied by an exchange rate $e$: the old eastern currency is, in effect, exchanged for the new currency at a rate of $e$ DM per OM. This means that the currency stock inherited at time $t=1$ from the old regime is $H_W(0) + eH_E(0)$, where the subscripts $W$ and $E$ refer to West and East, respectively. We want to study the consequences of alternative values of $e$. The unified monetary-fiscal authority assumes the old deficits, so that the deficit of the unified government is simply $D = D_E + D_W$. The demand for the new currency is $f(R) = f_E(R) + f_W(R)$, so that the inflation tax revenue is $(1-R)[f_E(R) + f_W(R)]$.

Figure 2 depicts the equilibrium values for $R$ and $p(1)$ in the new regime. Inspection of that figure shows that whether an equilibrium exists in the new regime does not depend on the value of the exchange rate $e$. Indeed, if an equilibrium exists, there are many values of $e$ compatible with that equilibrium. A stationary equilibrium depends only on the size of $D_E + D_W$ relative to the maximum height attained by the inflation tax revenue function $(1-R)[f_E(R) + f_W(R)]$. When a stationary equilibrium exists, the value of $e$ influences the value of the price level $p(1)$: the higher is $e$, the higher $p(1)$ will be. Thus, our apparatus distinguishes sharply between the “level” and “rate of change” effects. The setting of $e$ is irrelevant for the steady state inflation rate under the new regime, but $e$ does influence the “one-time” inflation at the start of the new regime.

In the remainder of this paper we use this model to elaborate on the consequences of the move to monetary unification. We study what difference the choice of $e$ makes, and to whom. We find that the choice of $e$ matters to easterners and westerners who enter unification with either assets or debts denominated in either former currency, but that it doesn’t affect the welfare of others. Although the exact detail of who wins and loses in the process of unification may depend on our particular model (which is the overlapping generations model of Samuelson, as noted above), the general macroeconomic features

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**Figure 2**

![Figure 2](image-url)
of our results, are much more robust, because they depend only on features of the demand for money and the government budget constraint that are embodied in Figures 1 and 2.

The remainder of this paper is organized as follows. Section II presents the basic economic model we use to describe a closed monetary economy, and some of the policy options open to the monetary-fiscal authorities. In Section III we indicate which options are assumed to have been chosen by the authorities of the two countries. Section IV describes the consequences of a monetary unification hitherto unforeseen and suddenly implemented. Section V examines the effects of an anticipated monetary unification. Section VI examines anticipated unification when there is uncertainty about the exact terms of unification. Finally, Section VII discusses some qualifications.

II. The Model

We will be using an overlapping generations model of a simple kind. Models of the type used in this paper were used by Wallace (1980), Sargent and Wallace (1981,1982), Bryant and Wallace (1984) and Sargent (1987). The presentation in this paper most closely follows Sargent (1987).

Time is discrete and starts at \( t = 1 \). Each period, a generation is born, which is destined to live two periods, and is indexed by the subscript \( t \); also, in period 1, there are agents called the initial old, who live only one period. There is a single consumption good in each period. The agents' identical preferences are defined over consumption in each period of their lives; these preferences are represented by \( u(c_t(t), c_{t+1}) \); the initial old have preferences \( u_0(c_{t+1}) \). The vector of endowments in both periods is represented by the pair \((\omega^h_t(t), \omega^h_{t+1})\), where \( h \) indexes the agent. We allow the possibility that some agents have different endowments from others. There is no production in this model, nor is there any uncertainty.

There are two countries, called East and West. Variables that are specific to either country carry an \( E \) or \( W \) subscript. Each country has a constant population of size \( 2N_i \) for \( i \in \{E, W\} \). Before the monetary reform, each country has a government which can collect lump-sum taxes on agent \( h \) of generation \( t \). After-tax endowments will be called \((\omega^h_{t+1}(t), \omega^h_{t+1}(t+1))\), where \( h \) indexes the agent. We allow the possibility that some agents have different endowments from others. There is no production in this model, nor is there any uncertainty.

A government can also issue intrinsically useless pieces of paper called East or West Marks (and denoted EM or WM). The total amount of currency outstanding at the end of period \( t \) is written \( H_t(t) \). The initial old in both countries are endowed with an aggregate amount \( H_0(0) \) of their currency. Governments purchase the consumption good in the amounts \( G_t(t) \), and dispose of it in ways that procure utility for no one.

Each period, there is a market for the consumption good in each country, and the price of the good in Marks is written \( p_t(t) \). There is also a market for loans among young agents. We will assume that these loans are denominated in Marks, and carry a nominal interest rate denoted \( r_t \). The real interest rate on these loans, by definition, is \( R_t = r_t p_t(t) / p_{t+1} \).

We assume that an impermeable separation stands between the two countries (a Wall), so that no interaction takes place between East and West. This Wall was erected before period 1, and is initially expected to stand indefinitely.

We begin the analysis with a study of some of the policies that the two governments can conduct. For simplicity, we represent a government's task as the financing of a constant deficit of taxes with respect to expenditures, denoted \( D \geq 0 \). A government can require the young in each generation to hold a minimum amount \( \lambda \geq 0 \) of the currency in real terms. The parameter \( \lambda \) is a policy instrument that is designed to influence the base of the inflation tax.

We will study two possible regimes; in the first one, \( \lambda \) is set equal to 0, so that constraint (2), below, is only the traditional one which forbids agents to issue currency. In the second regime, \( \lambda \) is positive, and the corresponding constraint is binding. These options are available in either country, and this section sets forth the analytics in the context of a single, closed economy with general endowment patterns. We will later specify which regime will prevail in each country.

All young agents solve the following problem:

\[
\max_{c_t(t), c_{t+1}} u(c_t(t), c_{t+1}) \quad (P)
\]

\[
c_t(t), c_{t+1}, I(t)
\]
subject to the constraints
\[ c_i(t) + \frac{m(t) + l(t)}{p(t)} \leq \omega_i(t) \]
\[ c_{i(t+1)} \leq \omega_i(t+1) + \frac{m(t) + r_l(t)}{p(t+1)} \]
\[ \lambda \leq \frac{m(t)}{p(t)} \]  \hspace{1cm} (2)

where \( l(t) \) denotes the amount lent (or borrowed, if negative) by the young agent, and \( m(t) \) the agent’s choice of money holdings.

The equilibrium is the solution to the agents' maximization problem, the government’s budget constraint, as well as an equilibrium condition in the credit market.

**Regime 1:** Either \( \lambda = 0 \) or the currency constraint is never binding.

A classic arbitrage argument shows that equilibrium requires
\[ r_t = 1 \text{ or } R_t \leq \frac{p(t)}{p(t+1)}. \]

Agents’ decisions can be represented by a saving function, which is the solution to the maximization problem above. Letting \( f_i(R_t) \) be the saving of agent \( h \) of generation \( t \), we have
\[ f_i(R_t) = \omega_i(t) - c_i(t), \]
where \( R_t = p(t)/p(t+1) \) is the rate of return on money holdings. The function \( f_i \) will be strictly increasing in \( R_t \), under the assumption of gross substitutability of consumption in the two periods. It should be kept in mind that this function depends on the after-tax endowment of each agent.

The government’s budget constraint is

\[ D = \frac{H(t) - H(t-1)}{p(t)}, t \geq 1 \]
\[ = \frac{H(t)}{p(t)} - R_{t-1} \frac{H(t-1)}{p(t-1)}, t \geq 2 \]  \hspace{1cm} (3)

and the equilibrium condition in the credit market is

\[ \sum_h f_h(R_t) = N f(R_t) = \frac{H(t)}{p(t)}. \]  \hspace{1cm} (4)

This equation states that the net saving of generation \( t \) equals the net dissaving of generation \( t-1 \) and of the government.

Equation (3) defines a one-to-one mapping between \( R_t \) and \( h(t) = H(t)/Np(t) \). We use it to replace \( H(t)/p(t) \) in

(3). Writing \( d = D/N \), we express condition (3) as
\[ f(R_t) = R_{t-1} f(R_{t-1}) + d \]
\[ R_t = f^{-1}(R_{t-1} f(R_{t-1}) + d) \]
\[ = \phi(R_{t-1}). \]

An equilibrium sequence \( \{R_t\}_{t=1}^\infty \) will solve this first-order non-linear difference equation.

The function \( \phi \) can take many forms, depending on the utility function \( u \). In the case where \( u \) takes the form
\[ u(c_t, c_{t+1}) = \ln(c_t) + \ln(c_{t+1}) \]
\[ f \] is found to be
\[ f(R) = \frac{\Omega_1}{2} - \frac{\Omega_2}{2R} \]  \hspace{1cm} (6)

where \( \Omega_i = \sum_h \omega_i h \) for \( i \in \{1, 2\} \), and (5) becomes
\[ \frac{\Omega_2}{R_t} + 2d - \Omega_1 - \Omega_2 + \Omega_1 R_{t-1} = 0 \]

which is shown in Figure 3. If \( \Omega_1 > \Omega_2 \) holds, then for values \( 0 \leq d \leq d^* = (\sqrt{\Omega_1} - \sqrt{\Omega_2})^2 \) there are two stationary solutions for \( R \) (and for \( h \)), found by intersecting the graph of \( d + R f(R) \) with that of \( f(R) \). Figure 4 shows the function \( (1-R)f(R) \), and the two stationary solutions can be found for any deficit \( d \leq d^* \). In the case \( d = 0 \), the two solutions are \( \beta \) and 1, where we define \( \beta = \frac{\Omega_1}{\Omega_2} < 1 \).

Under rational expectations dynamics, the lower gross rate of return on currency, \( R_t \), is stable, while the higher \( \bar{R} \), is unstable. Any path starting at \( h(1) \in [0, \bar{h}] \) (respectively \( R_1 \in [\frac{\Omega_1}{\Omega_2}, \bar{R}] \)) will converge to \( h \) (respectively \( \bar{R} \)). Paths starting at \( h(1) > \bar{h} \) (respectively \( R_1 > \bar{R} \)) are not feasible because they eventually drive \( h(t) \) to \(+\infty\), which would
eventually mean negative consumption. Hence \( R \) is necessarily in \([\frac{\omega_2}{\omega_1}, \bar{R}]\).

Notice that the comparative dynamics associated with the "stable" stationary values \( \bar{R} \) are in some sense perverse: an increase in the deficit raises \( R \), and lowers inflation. Thus, we can not rely on the rational expectations dynamics of this model to focus attention on government deficits as a cause of inflation. However, it has been shown in several contexts, both theoretical and experimental, that learning reverses the stability of the stationary points \((R, \bar{R})\) relative to the rational expectations dynamics. \(^4\) Such learning schemes suggest that we select the higher stationary point \( \bar{R} \) as our equilibrium. Point \( \bar{R} \) is associated with "classical" comparative dynamics: a higher deficit lowers \( \bar{R} \), and thus raises the inflation rate. We appeal to these learning dynamics as our justification for focusing on the \( \bar{R} \) stationary equilibrium.

A young agent's budget set is depicted in Figure 5: point C is attained when an interest rate of 1 prevails (in other words when the price level is constant) whereas point B is attained for \( R < 1 \). The seigniorage function \( f(R)(1-R) \) can be read as the distance \( A \omega \), when the line \( AB \) has a slope of \(-1\).

**Regime 2:** \( \lambda > 0 \), and the currency constraint is always binding

We now consider a regime in which \( \lambda \) is positive and binding.

Evidently, if the currency constraint is binding, \( h(t) = \lambda \) for all \( t \geq 1 \), and

\[
d = \lambda (1 - R_t) \quad \text{or} \quad R_t = 1 - \frac{d}{\lambda}.
\]

Thus, the inflation rate is unique, constant, and positive. Note that increasing \( \lambda \) raises \( R \), thereby lowering the inflation rate. Note also that the nominal amount of forced savings per capita grows with time, since it is \( \lambda p(t) \).

Chart 1 shows the actual data for East Germany. \(^5\) For the constraint to be binding, we must verify that

\[
\lambda \geq f^h(R_t) \quad \text{for all} \quad h \quad \text{and} \quad t \geq 1,
\]

which translates into the condition

\[
d \geq (1-R)f(R).
\]

Another condition must also hold, namely, that consumption remain positive. This imposes on \( \lambda \) the condition that

\[
\lambda \leq \min_{\omega} (\omega^h) = \omega_1,
\]

which translates into the following condition on \( R \):

\[
R < 1 - \frac{d}{\omega_1} = \bar{R}\star
\]

**Figure 4**

**Figure 5**

**Chart 1**

Nominal Savings Per Capita in East Germany (1949 - 1989)
Thus $R$ is bounded above, away from 1; furthermore, $R$ must lie in the regions of $(0, \bar{R}^*)$ where condition (7) is satisfied.

In the case of the logarithm utility function, (7) is satisfied if: a) $d > d^*$, and then it is true for all $R \in (0, \bar{R}^*)$; or b) $0 \leq d \leq d^*$, and then it is true for $R \in (0, \bar{R}) \cup (\bar{R}, \bar{R}^*)$. Note that a) corresponds to values of the deficit that cannot be financed in regime 1. Moreover, in b) the return on money $R$ can be chosen to be higher than in regime 1.

Figure 6 illustrates this: the seigniorage function $(1 - R)f(R)$ is represented and the region below that curve is shaded. When the deficit is $d_2$, it cannot be financed by voluntary holdings of money. A solution with forced savings can be found as the intersection of the $d_2$ line with the graph of $\lambda(1 - R)$, with the resulting rate $R_2$. If the deficit is $d_1$, it can be financed with or without the currency constraint; with the constraint, a rate such as $R_1$ can be achieved, which is higher than $\bar{R}$. With a lower value of $\lambda$, lower rates of return are achieved, such as $R_3$.

It is possible, depending on the utility function and endowments, that every agent would prefer regime 2 to regime 1. This situation is illustrated in Figure 7: point A is that attained in regime 1, point B in regime 2: the utility level is higher under the forced savings regime. Thus regime 2 could be justified on two grounds, depending on the level of deficit the government has chosen to finance via inflation: that this deficit is too high to be financed with voluntary holding of money by agents, or that the government can improve agents' welfare by moving from regime 1 to regime 2.

Repressed Inflation

There are two senses in which we can speak of repressed inflation in regime 2: one is that the rate of return on money can be made higher (and the inflation rate lower) in regime 2, as we saw. The other is that the initial price level $p(1)$ is higher in regime 1.

To see this, we solve for $p(1)$. The government budget constraint at $t = 1$ is

$$d = \frac{H(1) - H(0)}{Np(1)}.$$

In regime 1, the equilibrium condition yields

$$p(1)\frac{\Omega_1}{2} - p(2)\frac{\Omega_2}{2} = \frac{H(1)}{N}$$

$$p(1)(\frac{\Omega_1}{2} - \frac{\Omega_2}{2R_1}) = p(1)d + \frac{H(0)}{N}$$

$$p(1) = \frac{H(0)}{N(\Omega_1/2 - \Omega_2/2R_1 - d)} = \frac{H(0)}{N(f(R_1) - d)}.$$

In regime 2, it yields

$$Np(1)\lambda = H(1)$$

$$p(1) = \frac{H(0)}{N(\lambda - d)}.$$

Thus, as long as the legal constraint on money holdings is binding, the initial price level is higher in regime 1.

This result can be reformulated in the following terms: suppose that regime 2 has been in force from $t = 1$ on, and that, at time $t = t_0$, the legal restriction on money holdings is removed unexpectedly, all other parameters of the problem remaining unchanged. Then, either the deficit is too high to be financed and money becomes worthless immediately, or else it can be financed, in which circumstance the actual price level $p(t_0)$ is higher than was previously expected, and the inflation rate is higher from $t_0$ on than at any time before. This is the content we give here to the phrase "repressed inflation."
III. East and West before Unification

In country East, appropriate social arrangements ensure that all agents receive identical after-tax endowments \((\gamma_1, \gamma_2), \gamma_1 > \gamma_2\), in all generations \(t \geq 1\). Agents within a generation are identical in preferences and endowments, which implies that there will be no intra-generational lending: each agent chooses \(l^E_E(t) = 0\).

The government of East faces a constant positive deficit of tax revenues with respect to its expenditures, so that for all \(t \geq 1\)
\[
G_E(t) = \sum \tau^E_i(t) - \sum \tau^E_{i-1}(t) = D_E
\]
with \(D_E > 0\). It has chosen to resort to a currency constraint, so that regime 2 as described above prevails in East. This means that the equilibrium price level path is of the form:
\[
p^E(t) = p^E(1) \left( \frac{1}{R_E} \right)^{t-1}
\]
with \(R_E = 1 - \frac{d_E}{\lambda} = 1 - \frac{D_E}{N_E \lambda}\),
\[
p(1) = \frac{H_E(0)}{N_E(\lambda - d_E)}.
\]

In country West, \(N_1\) agents have the endowment \((\alpha_1, \alpha_2)\) while \(N_2 = N_W - N_1\) agents have the endowment \((\beta_1, \beta_2)\). We assume that
\[
\alpha_1 > \beta_2 > \beta_1,
\]
which makes the first type of agents (indexed by \(W\alpha\)) "lenders" and the second type (indexed by \(W\beta\)) "borrowers". A consequence of this assumption will be to introduce some distributional effects of the events which will happen in Sections V and VI. It is assumed that
\[
\frac{N_1 \alpha_2 + N_2 \beta_2}{N_1 \alpha_1 + N_2 \beta_1} = \frac{\Omega^W W}{\Omega^W} < 1,
\]
which insures existence of equilibria with valued fiat currency.

Agents solve the maximization problem \((P)\) referred to above and choose to hold private debt as well as money: since we still assume that private debt is not indexed, the budget constraint of a young agent in the West endowed with \((\omega_1, \omega_2, \omega_3)\) is
\[
c^W(t) + \frac{m^W_W(t) + l^W_W(t)}{p^W_W(t)} \leq \omega_1
\]
\[
c^W(t+1) \leq \omega_2 + \frac{m^W_W(t) + l^W_W(t)}{p^W_W(t+1)}
\]
\[
m^W_W(t) \geq 0.
\]
Lenders are indifferent between holding money or private debt, while borrowers will set \(m^W_W(t) = 0\) and \(l^W_W(t) \leq 0\).

The government of country West is assumed to be running a "tight" policy: the deficit is set to \(D = 0\) in all periods, and the money stock is constant, \(H(t) = H(0)\) for all \(t\). Taxes are set so as to achieve this goal.

This is merely a particular case of regime 1, with \(D = 0\); with the logarithmic utility functions, we know that there may be two stationary solutions \(\beta\) and 1. Indeed, the equilibrium condition is
\[
\sum \frac{m^W_W(t) + l^W_W(t)}{p^W_W(t)} = \sum f^W_W(t) = H_W(0) \frac{p^W_W(t)}{p^W_W(t)}, t \geq 1
\]
and with logarithmic utility functions (8) becomes
\[
\frac{\Omega^W W}{2} - \frac{p^W_W(t+1)}{p^W_W(t)} \frac{\Omega^W W}{2} = H_W(0) \frac{p^W_W(t)}{p^W_W(t)}.
\]
The general solution to this first-order difference equation in \(p(t)\) is found to be
\[
p^W_W(t) = \bar{p}_W + (p^W_W(0) - \bar{p}_W) \left( \frac{1}{R_s} \right)^t
\]
where we defined
\[
\bar{p}_W = \frac{2H_W(0)}{\Omega^W W - \Omega^W W} > 0 \text{ and } R_s = \frac{\Omega^W W}{\Omega^W} < 1.
\]
The constant \(\bar{p}_W\) is the unique non-inflationary solution, in which \(R_s = 1\). For all other solutions, \(R_s < 1\) is a constant, and \(\lim_{t \to \infty} p(t) = \infty\). The same argument about stability under learning, as described above, will serve to select the non-inflationary equilibrium, in other words the one with the highest return on money. We will consider this equilibrium to be prevailing in West.
IV. Monetary Unification

We consider the following situation. At some date, which we renormalize to be \( t=0 \), the Wall separating East and West unexpectedly disappears. The two countries unite, and become provinces of a single country. The two governments merge to form a single government. This new government inherits the stream of expenditures and pre-unification taxes, and has the power to impose new taxes on the citizens of both (former) countries. We will assume that the new government enacts the following rule: residents of each half of the new country may move to the other half, in which case they will receive an endowment of (0,0).\(^7\) This ensures that the distribution of population remains the same after unification: agents will not move between the two provinces, and they can be taxed at different rates, depending on prior citizenship (that is, on their current place of residence). The single government also has the ability to issue a currency called the Mark (denoted \( M \)). These arrangements prevail for \( t \geq 1 \).

At the beginning of period 1, all West Marks are exchanged for Marks one for one, and all East Marks are exchanged at the rate of one EM for \( e \) M. The government chooses \( e \), and sets \( \lambda = 0 \), which means that in the East the compulsion to hold currency has been eliminated.

Our purpose in this section is to describe the class of exchange rates, interest rates, price levels, and inflation rates that are consistent with these new arrangements. We establish the following:

1. If the consolidated government adopts the fiscal policies of the two preexisting governments, so that the deficit of the consolidated government is simply the sum of the deficits of the two prior governments, it may or may not be feasible to effect monetary unification without fiscal changes, depending on how big the consolidated deficit is.

2. If it is feasible for the new government to effect monetary unification under a fixed policy that simply consolidates and continues the deficits of the old governments, the move to monetary unification raises the inflation rate in the West and may or may not reduce it in the East, depending on the real value of the constraint previously imposed. All western lenders born at \( t \geq 1 \) are made better off by this change. Western borrowers born at \( t \geq 1 \) are made worse off by this change.

4. The increased inflation rate imposed on westerners by the monetary unification can be avoided by reducing the deficit of the consolidated government. The consequences for different citizens' welfare of this deficit reduction depends on precisely which people's taxes are raised.

The new government has the possibility to depart from prior taxing practices; any new taxes it decides upon will be denoted \( \tau^h(i) \) (tax on agent \( h \) of generation \( i \) in period \( i \in \{1,2\} \) of his life). The resulting after-tax endowment will be denoted \( \bar{\omega}^h(i) \). The aggregate tax burden on the young (respectively old) in period \( t \) is denoted \( T_1(t) \) (respectively \( T_2(t) \)). Our assumptions imply that the government may forever tax young and old in each (former) country separately; therefore both \( T_1(t) \) and \( T_2(t) \) may carry \( E \) and \( W \) superscripts.

The old generation at time \( t=1 \), who are indexed 0, have the budget constraints:

- **eastern borrowers:** \( c^h_k(1) \leq e \frac{m^h_k(0)}{p(1)} + \gamma_2 \)
- **western lenders:** \( c^h_w(1) \leq m^h_w(0) + l^h_w(0) \frac{p(1)}{p(1)} + \alpha_2 \)
- **western borrowers:** \( c^h_w(1) \leq m^h_w(0) + l^h_w(0) \frac{p(1)}{p(1)} + \beta_2 \)

The young in all generations will henceforth face the following problem:

\[
\max u(c_t(t), c_t(t+1))
\]

subject to the constraints:

\[
c_t(t) + \frac{m(t)+l(t)}{p(t)} \leq \bar{\omega}_x(t)
\]

\[
c_t(t+1) \leq \bar{\omega}_x(t+1) + \frac{m(t)+l(t)}{p(t+1)},
\]

the solution to which is represented by the saving function

\[
f^h(R_t) = \frac{(m^h(t)+l^h(t))}{p(t)}.
\]

The government faces the budget constraint:

\[
D(t) = \frac{H(t)}{p(t)} - R_{t-1} \frac{H(t-1)}{p(t-1)}, \quad t > 1 \quad (11a)
\]

\[
D(1) = \frac{H(1)}{p(1)} - \frac{H_w(0) + eH_E(0)}{p(1)} \quad (11b)
\]
\[ D(t) = D_W(t) + D_E(t) = (-T_f^W(t) - T_f^W(t-1)) + (D_E - T_f^E(t) - T_f^E(t-1)). \]

The equilibrium condition is, for all \( t \geq 1 \):
\[ F_t(R_t) = N_1 f^W_t(R_t) + N_2 f^W_t(R_t) + N_E f^E_t(R_t) = \frac{H(t)}{p(t)}. \]  

(12)

The following proposition is a straightforward application of the Kareken and Wallace (1981) result on the indeterminacy of exchange rates.

**Proposition 1.** Given an equilibrium \( \{\bar{R}, \bar{p}, \bar{H}, (\bar{\tau}_{t-1}(t), \bar{\tau}_t(t), \bar{c}_{t-1}(t), \bar{c}_t(t))_{t=1}^\infty \} \), for any \( \bar{e} \in (0, \infty) \) there exists another equilibrium satisfying \( R_t = \bar{R}, \tau_t(t) = \bar{\tau}_t(t), c_t(t) = \bar{c}_t(t), c_t(t+1) = \bar{c}_t(t+1) \) for all \( t \); and \( \bar{p}(t) \neq \hat{p}(t), \bar{H}(t) \neq \hat{H}(t), \) for all \( t, \) \( \hat{c}_t(t) \neq \hat{c}_t(t)(1). \)

**Proof:**

We take as given that a monetary equilibrium exists; the macron-bearing equilibrium, \( \{\bar{R}, \bar{p}, \bar{H}, (\bar{\tau}_{t-1}(t), \bar{\tau}_t(t), \bar{c}_{t-1}(t), \bar{c}_t(t))_{t=1}^\infty \} \), solves (11) and (12). For any choice of \( \bar{e} \in (0, \infty) \), we can construct a caret-bearing equilibrium as follows. Given a choice of \( \bar{e} \), combine (11b) and (12) into
\[ \bar{D}(1) = F_t(\bar{R}) - \frac{H_W(0) + \bar{e}H_E(0)}{\bar{p}(1)} \cdot \hat{p}(1) \]

Solve this equation for \( \hat{p}(1) \) to get
\[ \hat{p}(1) = \frac{H_W(0) + \bar{e}H_E(0)}{F_t(\bar{R}) - \bar{D}(1)} \cdot \hat{H}(t) \]  

(13)

Since the macron-bearing equilibrium solves (11) and (12) with positive money stocks, the denominator on the right hand side of (13) is positive, and (13) can be solved for \( \hat{p}(1) \). Then \( \hat{p}(t+1) = \hat{p}(t) / \bar{R} \), and (12) gives \( \hat{H}(t) = F_t(R) \hat{p}(t) \). Since \( \hat{H}(t) / \hat{p}(t) = \bar{H}(t) / \bar{p}(t) \), (11a) will be satisfied.\(^8\)

One can interpret this proposition in the following sense: for a given fiscal policy \( \{(\tau_{t-1}(t), \tau_t(t), c_{t-1}(t), c_t(t))_{t=1}^\infty \} \), such that money has value in equilibrium, there are corresponding sequences of “real” variables \( \{\bar{D}, \bar{R}, (\bar{\tau}_t(t), \bar{c}_t(t), \bar{c}_t(t+1))_{t=1}^\infty \} \), such that money has value in equilibrium, there are corresponding sequences of “real” variables \( \{\bar{D}, \bar{R}, (\bar{\tau}_t(t), \bar{c}_t(t), \bar{c}_t(t+1))_{t=1}^\infty \} \). There is a continuum of price paths \( \{\hat{p}(t)\}_{t=1}^\infty \) (and corresponding paths \( \{\hat{H}(t)\}_{t=1}^\infty \) ) consistent with these sequences, indexed by \( p(1) \); the choice of \( e \in (0, \infty) \) is sufficient to select the price path via equation (13) (which gives \( p(1) \) as an affine function of \( e \), without altering any other aspect of the equilibrium. The existence itself of the equilibrium is a disjoint issue from the choice of the exchange rate, and is amenable to the same analysis as was conducted in Section II. Moreover, since the welfare of generations \( t \geq 1 \) depends only on \( R_t \) and not on the specific price level path, the choice of \( e \) affects only the consumption of the old at \( t = 1 \). For the latter, each choice of \( e \) corresponds to a particular distribution of consumption good.

When does a monetary equilibrium exist? Figure 8 will be helpful in this context. The seigniorage functions of both provinces \( f^E(R)(1 - R) \) and \( f^W(R)(1 - R) \) have been represented, as well as the sum \( F(R)(1 - R) \). Since the unified country will not resort to the \( \lambda \) constraint, a monetary solution is found as the intersection of the \( y = d \) line with the graph of \( F(R)(1 - R) \). If the new government simply consolidates East’s deficit without raising taxes, that is, \( D(t) = D_E \), then a monetary equilibrium may or may not exist. In Figure 8, the deficit \( d_2 \) cannot be financed, although it was financed by East under regime 2. On the other hand, \( d_1 \) can be financed. The value \( d^* \) is the largest deficit that can be financed.

If an equilibrium exists in the unified country, the inflation rate will rise in West, simply because it was 0 previously \( (R_W = 1) \), and because \( R = 1 \) is incompatible with a positive deficit. As for East, the inflation rate may be higher or lower, depending on the choice of \( \lambda \) that was made initially. For \( \lambda_1 \), the new rate of return \( R \) will be higher than \( R_1 \), and conversely for \( \lambda_2 \). It is also apparent that, should the deficit be lowered, the inflation rate may be made lower. How this affects agents’ welfare, however, will depend on who is taxed to finance this deficit reduction.

Thus, if we compare the welfare of generations \( t < 0 \) with that of generations \( t \geq 1 \) (and assume that taxes are unchanged), we need only consider real rates of return, and

![Figure 8](https://via.placeholder.com/150)

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we see that while western lenders will necessarily suffer (and western borrowers benefit) from the unification and the ensuing increase in inflation, easterners can be better or worse off. Which way easterners’ welfare goes does not depend on the exchange rate chosen, but rather on the extent to which they were constrained initially. We refer again to Figure 7 on this question.

Welfare implications for the \( t = 0 \) generation

We now consider the welfare implications of monetary unification for the old at time \( t = 1 \). For all save the first generation, welfare is identical under all the equilibria of Proposition 1 above. For the old at time \( t = 1 \), on the other hand, the reallocation effects of varying the exchange rate are important, simply because they are exchanging their old money for the new one, in both provinces. To see this, rewrite the eastern old’s consumption in period 1 as

\[
c^h_E(1) = \gamma_2 + e p_E(1) p_w(1) R_E f_E(R_E)
\]

where \( p_E(1) \) denotes the price level which would have prevailed had the Wall not fallen. For the western old, consumption is

\[
c^h_W(1) = \omega_2 + p_w(1) R_w f_w(R_w) = \omega_2 + \frac{p_w(1)}{f_w(1)}
\]

Remembering that \( f_w(1) > 0 \) for lenders, it is clear that the welfare of lenders worsens, the higher the actual price level is in period 1, and conversely for borrowers (inflation benefits debtors). Whether they are better off than if the Wall hadn’t fallen depends on whether \( \bar{p}_w = p_w(1) > p(1) \). The eastern old’s welfare falls when \( e/p(1) \) falls; whether they are better off without the Wall depends on whether \( e p_E(1)/p(1) > 1 \). Note that the eastern old’s interests do not necessarily conflict with that of either class of western old.\(^{10} \)

Thus, to evaluate the welfare consequences of the move to monetary union, we need to specify what fiscal policy the new government adopts. This fiscal policy will determine the new equilibrium return on currency \( R \), as well as the the price level \( p(1) \) as a function of \( e \). To compute solutions for various fiscal policies, we return to the assumption that preferences are identical in both countries and of the logarithmic form studied above.

Let us consider the case where the new government decides to tax the young of all generations and of both provinces by an amount \( T_1 = \sum_h \tau \), in the aggregate, and the old by an aggregate amount \( T_2 = \sum_t \tau b_t \), for \( t \geq 1 \) so as to set a constant deficit \( D = D_E - T_1 - T_2 \geq 0 \) for all \( t \geq 1 \) (recall that the previous deficit paths were \( D_E \) for East and 0 for West).

With logarithmic preferences, the saving function for each consumer is

\[
f^h(R_i) = \frac{\omega h - \tau h}{2} - \frac{\omega_2 - \tau b}{2 R_i}.
\]

The equilibrium condition becomes

\[
\sum f^h(R_i) = \frac{\Omega_1 - T_1}{2} - \frac{\Omega_2 - T_2}{2 R_i} = \frac{H(t)}{p(t)},
\]

and the government’s budget constraint

\[
D = \frac{H(t) - H(t-1)}{p(t)}.
\]

Equations (14) and (15) imply a second-order difference equation in \( p(t) \)

\[
(\Omega_1 - T_1)p(t + 1) - (\Omega_1 - T_1 + \Omega_2 - T_2 - 2D)p(t) + (\Omega_2 - T_2)p(t - 1) = 0
\]

which, under a boundedness condition on \( D \), has solutions of the form

\[
p(t) = a(\frac{1}{R_1})^{t-1} + b(\frac{1}{R_2})^{t-1} \text{ with } R_1 > R_2,
\]

where \( a \) and \( b \) are subject to the condition that \( p(t) \) remains positive, as well as to the initial condition

\[
Dp(1) = \frac{\Omega_1 - T_1 p(1) - \Omega_2 - T_2}{2} p(2) - H(0) - e H(0).
\]

A stationary or constant-inflation equilibrium corresponds to \( a = p(1), b = 0 \) or to \( a = 0, b = p(1) \). In both cases, the path \( \{p(t)\}_{t=1}^\infty \) is of the form

\[
p(t) = p(1)(\frac{1}{R_1})^{t-1}, \text{ } i \in \{1,2\}
\]

and imposing (17) determines \( p(1) \) as

\[
p(1) = \frac{2(H_w(0) + e H(0))}{\Omega_1 - T_1 + (\Omega_2 - T_2)/R_i - 2D}.
\]

Thus, \( p(1) \) is an affine function of the exchange rate chosen. From Section IV, we know that

\[
p_w(1) = \frac{2H_w(0)}{\Omega_1 - \Omega_2 W}
\]

Hence

\[
p(1) > p_w(1) \text{ if and only if } \frac{2H_w(0) e}{N_E(\gamma_1 - \gamma_2 / R_i) + \Omega_2 (1 - 1/R_i) - 2D + T_1 + (2 + 1/R_i)T_2} > p_w(1)
\]
It appears that there exists a critical value

\[ e^* = \frac{H_w(0)}{H_E(0)} \times \]

\[ \frac{N_E(y_1\gamma_2/R_1) + \Omega_2(l-1/R_1) - 2D_E + T_1 + (2+1/R_1)T_2}{\Omega_1 - \Omega_2} \]

such that \( p(1) > p_w(1) \) if and only if \( e > e^* \).

Note that \( e^* \) may possibly be negative. But if it is positive, and if the government chooses \( e < e^* \), a relative deflation in the West\(^{11} \) will take place in period 1, western lenders will be made better off and western borrowers worse off than with the Wall. Conversely, for \( e > e^* \), a relative inflation will occur in period 1. This critical value of the exchange rate does not depend on the price level in country East (which is determined by \( \lambda \)) but rather on the ratio of money stocks, on endowment and population parameters, and on the fiscal policy chosen. In particular, the value \( e^* = \frac{p_w(1)}{p_E(1)} \) is irrelevant to the occurrence of inflation in the West in period 1, and to the welfare of the western old. However, \( e^* \) matters for the eastern old’s welfare, which will be higher than with the Wall if and only if \( e/e^* > p(1)/p_w(1) \). The value \( e^* \) can be thought of as representing a “black market exchange rate” at the time of unification.

We can consider a few examples: one possibility open to the government is simply to leave after-tax endowments identical to what they were before unification. In other words, the East’s deficit is left intact and financed by inflation, and \( T_1 = T_2 = 0 \). We then rewrite (18) as

\[ p(1) = 2 \frac{H_w(0) + eH_E(0)}{\Omega_1 - \Omega_2/R_1 - 2D_E} \]

The critical value is

\[ e^*(T_1) = \frac{H_w(0)}{H_E(0)} \times \]

\[ \frac{N_Ey_1 - N_Ey_2/R_1(T_1) + \Omega_2(l-1/R_1(T_1) - 2D_E + T_1)}{\Omega_1 - \Omega_2} \]

Another possibility is for the government to tax only the young of each generation so that \( T_2 = 0 \), in which case

\[ e^*(T_1) = \frac{H_w(0)}{H_E(0)} \times \]

\[ \frac{N_Ey_1 - N_Ey_2/R_1(T_1)}{\Omega_1 - \Omega_2} \]

We must keep in mind that \( R_1 \) will change with \( T_1 \). If \( T_1 = D \), which corresponds to a balanced budget policy, then \( R = 1 \) or \( R = \Omega_2/\Omega_1 \).

These examples illustrate the way in which the government has the ability to choose an initial inflation or deflation (i.e., \( p(1) > p_w(1) \) or \( p(1) < p_w(1) \)), once it has chosen a fiscal policy.

V. The Effects of an Anticipated Unification

We examine the consequences of a delay between the announcement of monetary unification and the time at which it is implemented. We make the following assumptions.

All arrangements described in the first paragraph of Section IV are announced at time 1 to be prevailing for \( t \geq T \). In periods \( t = 1, \ldots, T - 1 \), the same arrangements as before are maintained, that is, both countries remain separate, government spending and taxes are unchanged, East still imposes savings restrictions, and so on.

We assume that at \( t = 1 \) a fiscal policy is specified for periods \( t \geq T \), by which we mean that \( \{ \{ \tau^T_{t-1}(t), \tau^T_{t}(t) \}_{t=T}^\infty \} \) are announced; a rate \( e \), at which East Marks will be received at \( t = T \) in exchange for new Marks, is also announced at \( t = 1 \). Agents can therefore compute the equilibrium allocations and price paths.

At time \( T \), everything will proceed exactly as in Section IV; \( E \) and \( W \) subscripts will disappear, the old of generation \( T - 1 \) will exchange their monies for mint-fresh Marks, markets will open, a price level \( p(T) \) (which can be computed given the fiscal parameters) will prevail.

The young of generation \( T - 1 \) in the West will thus face problem \( (P) \):

\[ \max u(c_{T-1}(T-1), c_{T-1}(T)) \]

subject to the constraints

\[ c_{T-1}(T-1) + \frac{m(T-1) + l(T-1)}{p_w(T-1)} \leq \bar{w}_1 \]

\[ c_{T-1}(T) \leq \bar{w}_2 + \frac{m(T-1) + l(T-1)}{p(T)} \]

the solution to which is again represented by the saving function \( f_{T-1}^w(p_w(T-1)/p(T)) \). The equilibrium condition can then be written

\[ \sum_h f_{T-1}^w(p_w(T-1)/p(T)) = \frac{H_w(0)}{p_w(T-1)} \quad (19) \]

which is then solved for \( p_w(T-1) \) as a function of \( p(T) \).

Young agents of previous generations \( 1 \leq t \leq T - 1 \) will be solving the same problem, and the path \( \{p_w(1), \ldots, \} \)
$p_w(T-1)$ can be computed through a backward recursion.

In the case of logarithmic utility functions, (19) takes the form

$$\frac{W_1}{2} - \frac{W_2}{2} p_w(T) = \frac{H_w(0)}{p_w(T-1)} \text{ or }$$

$$p_w(T-1) = R_s^{-1} p(T) + \frac{2H_w(0)}{W_1}.$$  \(20\)

This is solved backward to give

$$p_w(t) = p_w + (p(T) - p_w)(R_s)^{T-t} \text{ for } 1 \leq t \leq T-1 \quad (21)$$

which is just another version of (10), with a specific starting condition. Therefore, if $p(T) > p_w$ (as in the examples at the end of Section IV), there will be a progressive increase in the price level until it reaches $p(T)$; and $p(t)$ will increase at an accelerating rate as unification approaches. During that period, the inflation rate increases but remains bounded above by $1/R_s$. The time path of $p(t)$ is shown in Figure 9. The initial bout of inflation at the time of unification is

$$p(T) = p_w + (p(T) - p_w)(R_s)^{T-1},$$

which is increasing in $p(T)$, and, given $p(T)$, is decreasing in T. It can be shown that $R_s > .5$ is a sufficient condition for inflation to be higher in period 1 than in period 2, as illustrated by Figure 9.

VI. Anticipated Unification with Uncertainty

We now add a new wrinkle to the previous set-up, by introducing some uncertainty over the exchange rate to be chosen at time $T$.

At time 1, the same announcements are made as in Section V: the two countries will unite at time $T$, a consolidated government will take charge of both streams of government expenditures, and tax residents of both provinces. A fiscal policy is announced, which supports a monetary equilibrium. All parameters of the policy are made known, except for the exchange rate $e$. It is announced that the government will randomly choose among $n$ possible exchange rates ($e_1, \ldots, e_n$), with probabilities ($\pi_1, \ldots, \pi_n$) where $\sum \pi_i = 1$. The choice will be made at the beginning of period $T$. These induce $n$ states of the world in period $T$. There is no other uncertainty.

As Proposition 1 makes clear, the information available to agents allows them to compute the equilibrium sequences of consumptions and interest rates, for $t \geq T$, which will be identical in all states of the world. The price and money stock sequences, however, will depend on the (random) exchange rate: in particular, $n$ possible price levels may prevail in period $T$, namely ($p_1(T), \ldots, p_n(T)$), computed from $e_1$ and $e_2$ by using (13):

$$p_i(T) = \frac{H_w(T) + e_iH_e(T)}{F_T(R_T) - D(T)} \text{ for } i = 1, \ldots, n.$$

The probabilities attached to the price levels are ($\pi_1, \ldots, \pi_n$). It is more helpful to think of this distribution in terms of the value money may have in each state, that is, the reciprocals of the price levels ($1/p_1(T), \ldots, 1/p_n(T)$).

We will assume that agents maximize expected utility, and that utility is additively separable, of the form

$$u(c(t), c(t+1)) = u(c(t)) + u(c(t+1)).$$

We assume that financial markets available to agents of
generation $T-1$ can be represented by $n$ markets for claims on one unit of consumption in state $i$. We denote $q_i$ as the prices of these claims, and $s^h_i$ as the quantity of such claims bought (or sold) by the agent. The price of a real loan and the price of a nominal loan can be derived from these $n$ securities prices as

$$\sum_{i=1}^{n} q_i = \frac{1}{R_{T-1}}$$  \hspace{1cm} (22)

$$\sum_{i=1}^{n} \frac{q_i}{p_i(T)} = \frac{1}{p(T-1)}$$  \hspace{1cm} (23)

Money is therefore one of the assets available to the agent for purposes of transferring wealth across time and states of the world.

We will again proceed by backward recursion, starting from the generation born right before unification, at time $T-1$. The problem solved by an agent of generation $T-1$ will be

$$\max E \{u(c^h(T-1)) + u(c^h(T))\} = u(c^h(T-1))$$

subject to the constraints

$$c^h(T-1) + \sum_{i=1}^{n} q_i s^h_i \leq \omega^h$$  \hspace{1cm} (24)

$$c^h(T) \leq \omega^h + s^h$$  \hspace{1cm} (25)

Note that the agent now has $n+1$ budget constraints, which can be consolidated into a single budget constraint

$$c^h(T-1) + \sum_{i=1}^{n} q_i c^h_i(T) \leq \omega^h + \frac{\omega^h}{2R_{T-1}}$$  \hspace{1cm} (26)

The first order conditions are (26) and

for $i = 1, \ldots, n$ \hspace{1cm} $\frac{\pi_i}{q_i} u'(c^h_i(T)) = u'(c^h(T-1))$.  \hspace{1cm} (27)

Equations (26–27) describe each agent’s behavior. The market-clearing conditions on all financial markets

$$\sum_{h=1}^{n} s^h_i = \frac{H(0)}{p_i(T)}$$

can be written in the form

$$\sum_{h=1}^{n} (c^h_i(T) - \omega^h) = \frac{H(0)}{p_i(T)}$$  \hspace{1cm} (28)

$$\sum_{h=1}^{n} (\omega^h - c^h(T-1)) = \frac{H(0)}{p(T-1)}$$  \hspace{1cm} (29)

Equation (29) is redundant but convenient. Equilibrium is characterized by conditions (26–28).

Once $p(T-1)$ and $R_{T-1}$ are solved for using these equations, the next steps are identical to those taken in Section V. An agent of generation $T-2$ faces a pair of prices $(p(T-2), p(T-1))$ and an interest rate $R_{T-2}$ (which must equal $p(T-2)/p(T-1)$ to preclude arbitrage). His saving function can be derived the same way as before, equilibrium will impose

$$\sum_{h=1}^{n} f^h_{T-2}(\frac{p(T-2)}{p(T-1)}) = \frac{H(0)}{p(T-2)}$$

which allows us to compute $p(T-2)$ given $p(T-1)$, and so forth to $p(1)$. The only generation to face uncertainty is generation $T-1$.

In the case of the logarithmic utility function $u(c) = \ln(c)$, (27) becomes

$$c^h_i(T) = \frac{\pi_i}{q_i} c^h(T-1).$$  \hspace{1cm} (30, i)

When these values are substituted into (26) we find

$$c^h(T-1) = \frac{\omega^h}{2} + \frac{\omega^h}{2R_{T-1}}.$$  \hspace{1cm} (31)

Equation (29) becomes

$$\sum_{h=1}^{n} c^h(T-1) = \frac{1}{2} (\Omega_1 + \frac{\Omega_2}{R_{T-1}}) = \Omega_1 - \frac{H(0)}{p(T-1)}$$

$$2\frac{H(0)}{p(T-1)} = \Omega_1 - \frac{\Omega_2}{R_{T-1}}$$  \hspace{1cm} (32)

This equation relates $p(T-1)$ and $R_{T-1}$.

We can use (28) and (30) to obtain

$$\sum_{h=1}^{n} c^h_i(T) = \frac{q_i}{\pi_i} \sum_{h=1}^{n} c_i^h(T)$$

$$\frac{q_i}{\pi_i} (\frac{H(0)}{p_i(T)} + \Omega_2) = \frac{q_j}{\pi_j} (\frac{H(0)}{p_j(T)} + \Omega_2) \quad \forall \ i, j$$

or

$$\frac{q_i}{p_i(T) k_i} = \frac{q_j}{p_j(T) k_j}$$

where we denote

$$k_i = \frac{\pi_i}{H(0) + \Omega_i p_i(T)}$$

and use (22) to solve for $q_i$ as functions of $R_{T-1}$:

$$q_i = 1 - \frac{R_{T-1}}{k_i p_i(T)} \quad \sum_{j=1}^{n} k_j p_j(T))$$  \hspace{1cm} (33)

We then invoke (23) to obtain another relation between $p(T-1)$ and $R_{T-1}$:

$$p(T-1) = \rho R_{T-1}$$  \hspace{1cm} (34)
with
\[ \rho = \sum_{i=1}^{n} \left( \frac{k_i}{\sum_{i=1}^{n} k_i} \right) \rho_i(T). \]

Equations (32) and (34) at last allow us to solve for \( p(T-1) \):
\[ p(T-1) = \frac{2H(0)}{\Omega_1} + \frac{\Omega_2}{\Omega_1} \rho \]  \hspace{1cm} (35)

Note the formal analogy between (20) and (35). This will allow an easy comparison with the case under certainty.

Since \( p(T-1) \) is solved as a function of the distribution of \( (p_1(T), \ldots, p_n(T)) \), the price sequence \( \{p(1), \ldots, p(T-2)\} \) can be solved for recursively, using equation (20):

for \( 1 \leq t \leq T-1 \),
\[ p(t) = \bar{p} + (p(T-1) - \bar{p})(\frac{\Omega_1}{\Omega_2})^{t-T+1} \]  \hspace{1cm} (36)

with \( \bar{p} \) being the zero-inflation price level prevailing before \( t = 0 \).

We establish the following result:

**Lemma.** In the logarithmic utility case, for any distribution \( (p_1(T), \pi_1, \ldots, p_n(T), \pi_n) \), the following holds:
\[ \rho > \left( \frac{E}{p(T)} \right)^{-1}. \]

**Proof:**

We wish to prove that
\[ \sum_{i=1}^{n} \frac{\pi_i}{p_i(T)} > \left( \sum_{i=1}^{n} \lambda_i p_i(T) \right)^{-1} \]
\[ \left( \sum_{i=1}^{n} \frac{\pi_i}{p_i(T)} \right) \left( \sum_{i=1}^{n} k_i p_i(T) \right) > \sum_{i=1}^{n} k_i \]
\[ \sum_{i=1}^{n} \pi_i \frac{1}{p_i(T)} \left( \sum_{i=1}^{n} \pi_i H(0) + \Omega_2 p_i(T) \right) \]
\[ > \sum_{i=1}^{n} \pi_i \frac{1}{p_i(T)} \frac{p_i(T)}{H(0) + \Omega_2 p_i(T)}, \]

if we denote \( \alpha_i = 1/p_i(T), \bar{\alpha} = \sum_{i=1}^{n} \alpha_i \) and \( f(x) = 1/(H(0) + \Omega_2 x) \), we want to prove
\[ \left( \sum_{i=1}^{n} \pi_i \alpha_i \right) \left( \sum_{i=1}^{n} \pi_i f(\alpha_i) \right) > \left( \sum_{i=1}^{n} \pi_i \alpha_i f(\alpha_i) \right); \]

Note that \( f \) is strictly decreasing in \( x \); therefore\( \alpha_j \geq \bar{\alpha} \) iff \( f(\alpha_j) \leq f(\bar{\alpha}) \)
\[ (\alpha_j - \bar{\alpha})(f(\alpha_j) - f(\bar{\alpha})) < 0 \text{ for all } j \]

We are now in a position to compare two possible policies. First, the government may announce a non-degenerate distribution of possible exchange rates \( (\varepsilon_1, \varepsilon_1, \ldots, \varepsilon_m, \pi_n) \). This distribution induces a distribution of price levels \( (p_1(T), \pi_1, \ldots, p_n(T), \pi_n) \), and a distribution of values of money \( (1/p_1(T), \pi_1, \ldots, 1/p_n(T), \pi_n) \). We call the mean value of money \( E(1/p(T)) = \sum_{i=1}^{n} \pi_i/p_i(T) \). This results in the equilibrium sequence \( \{p(1), R_1, \ldots, p(T-1), R_{T-1}\} \) which we just computed, and which we call the equilibrium under certainty.

Alternatively, the government, exactly as in Section V, may announce that an exchange rate \( \varepsilon \) will be chosen with certainty at time \( T \); we denote \( \{p(1), R_1, \ldots, p(T-1), R_{T-1}\} \) the corresponding equilibrium sequence, which we call the equilibrium under certainty for short.

We consider the case where \( \varepsilon \) is such that \( 1/p(T) = E(1/p(T)) \).

The lemma implies:

**Proposition 2.** Assume logarithmic utility functions. In the equilibrium under uncertainty, the price levels for \( t = 1, \ldots, T-1 \) are higher, and the rates of return lower, than in the equilibrium under certainty with \( 1/p(T) = E(1/p(T)) \).

**Proof:**

The lemma establishes that \( \rho > \bar{\rho}(T) \). From (35), it is apparent that \( p(T-1) > \bar{p}(T-1) \), and from (34) that \( R_{T-1} > R_{T-1} \). Since equation (36) describes both paths of price levels in both equilibria, it must be that \( p(t) > \bar{p}(t) \) for \( 1 \leq t \leq T-2 \) as well. As for the rates of return,
\[ R_t = \frac{p_{t-1}}{p_t} = \frac{\bar{p} + (p(T-1) - \bar{p})(\Omega_1/\Omega_2)^{-T}}{\bar{p} + (p(T-1) - \bar{p})(\Omega_1/\Omega_2)^{-T+1}} \]

and \( \Omega_1/\Omega_2 > 1 \) ensures the result.

The proposition confirms what intuition might suggest: we compare a world where money will have a certain value at time \( T \), to one where the future value of money is uncertain, but on average the same. In other words, in the second situation we have introduced some randomness in the value of money, around a given mean. The same way a risk-averse agent will prefer to receive with certainty the mean value of a lottery, rather than the lottery itself, we find that in our model the demand for money (which is
The proposition is set forth in terms of distributions of price levels at time $T$, and is not linked to the particular way in which randomness is introduced in the price level at time $T$. Other forms of randomness may be considered. Suppose, for example, that the exchange rate is determined with certainty at time $1$ ($\epsilon = 1$, say), but fiscal policy remains indeterminate. Assuming that the aggregate deficit can be financed by inflation, and that the government will choose to finance some constant fraction $\delta \in [0,1]$ of that deficit, the price level at time $T$ is given by equation (13), where the denominator $F(R_T) - \delta D = R_T F(R_T)$ is positive by assumption, and decreasing in $\delta$, as Figure 3 makes clear. Thus the uncertainty over $\delta$, if the government does not commit to a specific value before time $T$, will induce a distribution of possible values of money, the lowest one associated with a $\delta = 0$ and the highest one with a balanced budget.

The same result then applies: the added uncertainty has the effect of increasing the price levels and the inflation rates in all periods prior to unification, when compared to a choice of fiscal policy which would set the value of money $1/p(T)$ at the mean of the possible values of money.

### VII. Final Comments

The model we used in this paper has, as any model must have, a number of limitations. Some are the inevitable drawbacks which characterize any overlapping generations model; they are well known, and this is not the place to discuss them. We might mention that they often plague other workable models of money. We rather wish to point out drawbacks that are specific to the model we used, which should be borne in mind when trying to find similarities between this model and actual persons or events.

In our model, the country once unified remains closed, in the same sense the two countries were originally taken to be closed: there is no rest of the world, and consequently no foreign trade. As a result, we lose the ability to discuss consequences of monetary union on trade, and we miss an important consideration in the determination of the initial inflationary shock at unification. As some have pointed out, the DM is convertible, whereas the OM is not. East Germans endowed with hard Marks would presumably buy goods from abroad as well as from West Germany, and this may have a mitigating effect on inflation.

In our model, there are only two periods in agents' lives; therefore, at the time of unification only old people come in from the East to exchange their soft Marks for hard Marks, and these old people, by construction, only wish to spend their balances. Although the demographic structure of East Germany isn't extremely different from that of West Germany, in actuality some East Germans may not want to spend all their freshly minted DM on bananas. Again, this reduces the strength of inflationary forces.

Our model simply assumes that the new government converts all OM instantaneously into freely expendable Marks, and at a single exchange rate. The plan which will be implemented in Germany will not have this feature, although any legal restriction on the expendability of East German savings will have to be easily enforceable. A possible feature would have East Germans buy the State's capital stock with their savings; another would freeze part of their holdings for a period of time left to the Bundesbank's discretion. It is also possible that a fraction of East Germans' money holdings will be convertible at a rate, and the remainder at another, less favorable rate.

We have assumed that the good with which Easterners are endowed is of the same nature as the good available for purchase in the West. One might object to such a ruthless subsumption of BMWs and Trabants as identical commodities, and want to allow for less than perfect substitution. To illustrate the argument, the results of Section IV can be re-examined with $\gamma_1 = \gamma_2 = 0$, in other words with the assumption that goods produced in country East are considered worthless for consumption purposes, once agents are given a choice. Taking this consideration into account would reinforce the inflationary factors. We have also assumed that the Easterners' endowments would not change after unification. Incorporating such a feature would change conclusions about inflationary forces, but would also leave Proposition 1 unchanged.

On a theoretical level, one might object that we have assumed perfect foresight on the part of our agents, before as well as after, unification. But we have shown our agents expecting the Wall to remain in place indefinitely in Section III, and we have then betrayed their expectations in
Section IV (the element of surprise was of course crucial for the trick played on the old people at time 1). We would answer that we in fact assumed a particular probability distribution, namely that the status quo would remain with probability $1 - \epsilon$, and that the Wall would come down with probability $\epsilon$ (the latter is understood to be as small as usual). We would further argue that this representation is but a stylized version of most observers' probability distributions until the early days of October 1989.

NOTES

1. As we remark later, this result is simply an application and interpretation of the reasoning on which the exchange rate indeterminacy result of Kareken and Wallace (1981) is based.

2. Models of this type usually specify that loans are denominated in the consumption good (e.g. Sargent (1987)). A departure from this usage does not matter in a model with perfect foresight, such as ours, until such time as an unanticipated change in policy occurs.

3. It is possible to interpret the restriction on real balances as the outcome of a commodity rationing scheme which forces the young to hold more money than they would want by limiting the goods available for purchase. Notice that the scheme we use leaves old agents free to spend their accumulated cash balances.

4. See Marcet and Sargent (1989) and Arifovic and Sargent (1990) for some theoretical work on learning schemes in the context of this model. See Marimon and Sundar (1989) for some experimental evidence.

5. "The growth of the total balance of savings is the expression of the people's trust in the socialist development of the German Democratic Republic, and in the stability of its money" (DDR Handbuch (1979)).

6. The two regimes described here obviously do not cover all possibilities. For a given value of the deficit $d_1 \leq d^*$, and when $\lambda$ is set as low as $\lambda_2$ in Figure 5, then there are three stationary equilibria, one in which the constraint is binding with $R = R_5$, and two in which it is not binding, with $R = R_1$ or $R = R_2$. Thus, when the deficit can be financed by inflation alone, imposing the constraint does not necessarily imply that it will be binding, because multiple equilibria are possible.

7. This assumption is not excessive, in view of the severe restrictions recently placed on eligibility of East German citizens for social benefits in West Germany.

8. The allocations of the old at time 0 will be affected by $\rho(1)$: at an extreme, for low values of $\rho(1)$ the deflation could be so severe that Western borrowers would be unable to honor their commitments. In a sense, this is irrelevant because the only economic forces determining the equilibrium values of variables are the decisions of the young of generations $t \geq 1$. However, a government wishing to spare the original old Western borrowers this predicament would choose $\epsilon$ within a range $(\epsilon, +\infty)$, where $\epsilon$ verifies

$$\frac{H_W(0) + \epsilon}{F_i(R_i) - D_i} = \frac{H_W(0) + \epsilon}{F_i(R_i) - D_i} = \frac{1}{\beta_2}$$

so that old Western borrowers’ consumption after repayment of loans remains positive.

9. A variable $y$ is said to be an affine function of variables $x_1, x_2, \ldots, x_n$ if there exist constants $b_0, b_1, \ldots, b_n$ such that $y = b_0 + b_1 x_1 + \cdots + b_n x_n$.

10. Had we followed the usual practice of denominating private debt in real terms rather than nominal terms, western borrowers would have been unaffected by the unification, and western lenders would have been affected through their holdings of money only.

11. By relative deflation in the West we mean that $\rho(1) < \rho_W(1)$, that is, the price level actually prevailing at time 1 is lower than it would have been, had the Wall remained in place.

12. One East German out of four is over the age of 50, compared to one West German out of three.

13. This paper was written before the details of the currency unification were worked out.
Data Appendix

The following summarizes some of the available data on the German economies. All amounts (except population figures) are in billions of local currency. Sources are Statistisches Jahrbuch für die BRD 1989, Deutsche Bundesbank monthly report Apr. 1990, Encyclopaedia Britannica Yearbook 1989.¹

Federal Republic of Germany

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German Democratic Republic

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<td>—(end 89)</td>
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<td>—(as of late March 1990)</td>
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</tr>
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</table>

¹. TSP is Total Social Product (the socialist version of GNP, which excludes services, etc.). The 1990 figures for savings in East Germany and the black market exchange rate are the ones commonly cited (e.g. New York Times March 14, 1990; International Herald Tribune Feb.10-11, 1990; die Welt, March 6, 1990; Frankfurter Rundschau, April 2, 1990).

REFERENCES


