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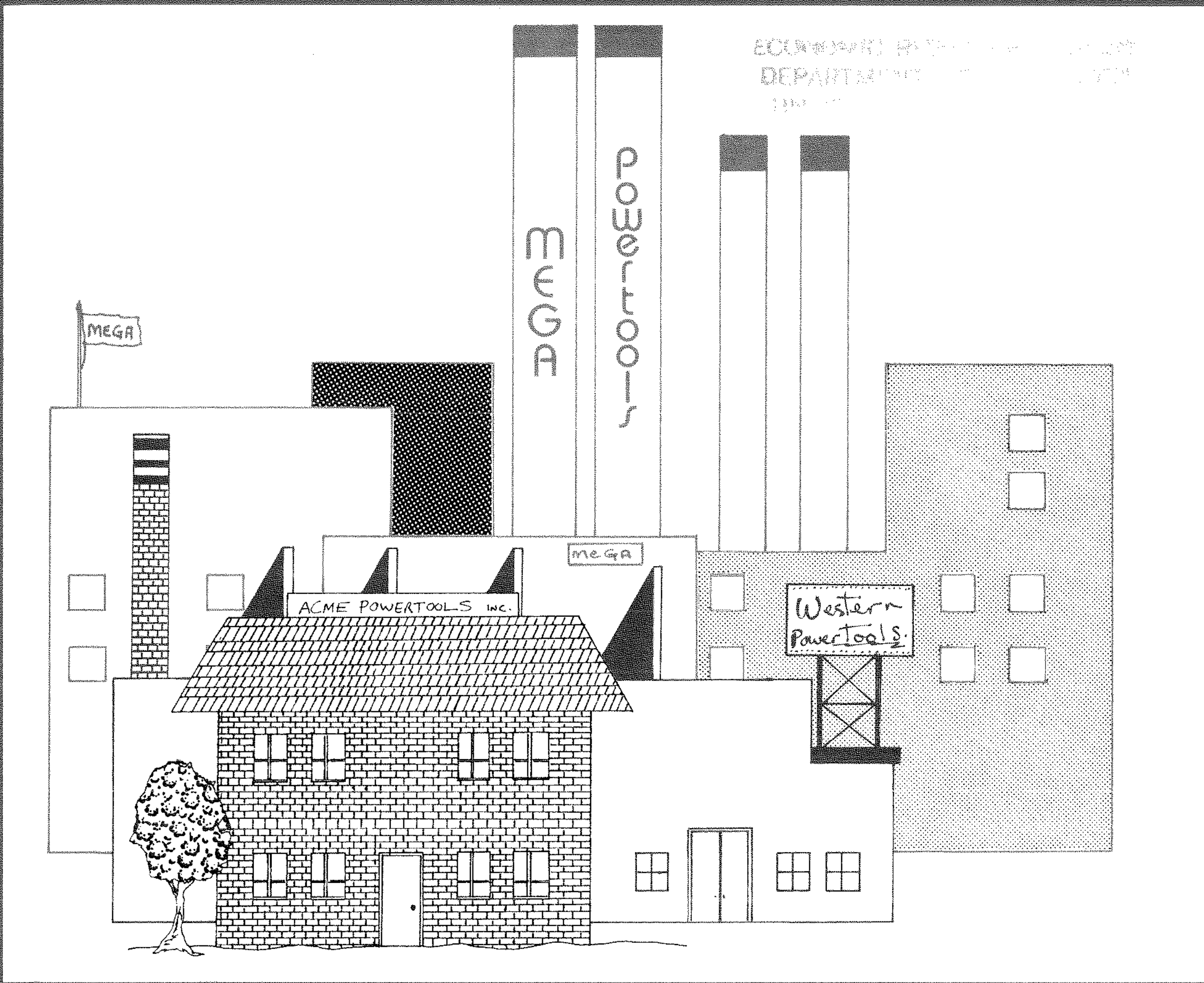

Firm Size, Exchange Rates, and Interest Rates

Winter 1984

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Firm Size, Exchange Rates, and Interest Rates

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The Economics of Firm Size: Implications from Labor-Market Studies

Michael C. Keeley*

One hypothesis about why large and small firms coexist in many industries is that entrepreneurial ability determines firm size. More able entrepreneurs manage firms whose sizes vary more than proportionately with their talents. However, because larger firms also face higher costs (per worker) in monitoring worker performance, they have more specialized methods of production, require more specialized training and hire more skilled workers to economize on monitoring. Empirical results tend to confirm this hypothesis. Moreover, the evidence implies that significant economic losses may be associated with public policies that prohibit firms from attaining their optimum size.

Many industries are characterized by the coexistence of firms of widely varying size, often with output concentrated in a few large firms. Understanding why this is so is extremely important for regulatory and antitrust policy.

On the one hand, it is widely believed that a high concentration of output (that is, a positively skewed distribution of firm size) leads to reduced competition.[1] One of the rationales for the regulation of many industries is to reduce concentration, prevent increased concentration, or to restrict how firms in concentrated industries may operate, especially those that are believed to be natural monopolies.[2] Antitrust law also grew out of a concern with concentration and may have had substantial effects on market structure through the prohibition of mergers, and until recently, suits aimed at breaking up successful large businesses.[3] Regulation in industries such as trucking, airlines, rail transportation and banking, may also have had significant effects on the firm-size distribution in these industries. For example, in banking, various restrictions on geographic competition, such as prohibitions against interstate banking, may have resulted in less concentrated market structures.

On the other hand, there is a concern about the potential costs of preventing firms from operating at their most efficient or optimal scales. For example, breaking up a large firm into several smaller ones, or prohibiting the merger of smaller firms, may have substantial costs if there are economies of scale. Thus, possible anti-competitive effects due to concentration should be balanced against possible increases in productive efficiency in formulating regulatory or antitrust policy.

For example, some economists have argued for a repeal of the antitrust laws so that the United States can compete more effectively with Japan.[4] Proposals for the elimination of restrictions on interstate banking are also often based on the notion that a more efficient provision of services would result and benefit consumers. Opponents to this type of deregulation are concerned that the elimination of restrictions would lead to a more concentrated banking industry with less competition.

Until recently, there was no theory of firm-size that was able simultaneously to explain actual firm-size distributions and several empirical regularities in production associated with firm-size. However, new developments in economic theory that focus on entrepreneurial ability as a fixed factor of produc-

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tion and on the organization of production are capable of explaining firm-size distributions and differences in production methods among different-sized firms. In this paper, these new theories are reviewed, some of their implications for differences in production are tested indirectly, and the public policy implications are explored. The empirical analysis focuses on differences in the employment practices among different-sized firms, in particular, differences in wage levels, wage growth and turnover. Hopefully, the results provide useful evidence about the determinants of the distribution of firm-size so that public policies aimed at influencing market structure can be better informed.

The organization of this paper is as follows. In Section I, I review the implications of neoclassical theory regarding firm-size distributions and present evidence that contradicts these implications. Next, in Section II, several new developments in economic theory explaining the determinants of firm size are reviewed and their empirical implications regarding differences between various characteristics of the labor forces of large and small firms are discussed. Then, in Section III, empirical evidence is presented that tends to support these hypotheses. Finally, the summary and conclusions are presented along with policy implications.

I. Neoclassical Theory and Facts

To analyze the determinants of the firm-size distribution, it is first necessary to define what a firm is and explain why firms exist. Our current understanding of what firms are and why they exist is due to Coase (1937), who hypothesized that economic activity takes place in firms instead of markets because of the transactions costs involved in organizing economic activities in markets. A firm substitutes a command-and-control system for the allocation of resources that could have been achieved through the market economy. Presumably, an economic activity (production) takes place within a firm if it is less costly than if it took place using the market. Alchian and Demsetz (1972) extended this basic concept by emphasizing the importance of group or team production. Group production implies the need for monitoring workers because the separate contribution of each individual cannot be assessed simply by observing output. Thus, group production might be very difficult to achieve in a market setting.

If entrepreneurs in a given competitive industry have available the same technology of production, face the same relative prices for transactions within the firm and outside, and face identical input prices, all firms in the industry (that is, firms producing the same products) would have identical cost functions. Assuming U-shaped average cost functions, all firms would produce at the minimum point on their average cost curves and the number of firms would equal total market demand (at a price equal to minimum average cost) divided by the output of a typical firm at its minimum average cost. Thus, if such conditions held, all firms in an industry would be the same size.[5]

However, even casual observation contradicts the notion that all firms in an industry are the same size. For example, in banking, the size distribution is positively skewed and the variance in firm-size is very large (with a standard deviation approximately four times the mean), with firms ranging in size from those with less than \$5 million in assets to those with over \$100 billion. Furthermore, the relative variance of firm-size in many other industries is even larger, and the skewness of the size distribution function in banking is considerably less than that in many other industries.

To analyze the extent to which industry type can explain firm-size, I have estimated the fraction of the overall variance in firm-size that is explained by industry type. The variance in firm-size is analyzed using three regressions of firm size on 1-digit, 2-digit, and 4-digit SIC (Standard Industrial Classification) codes. The regressions are of the following general form:

$$\mathrm{SIZE}_i = A + \Sigma_j B_j \mathrm{SIC}_{ij} + e_i \quad (1)$$

where: SIZE_i = the size of firm i, measured at the establishment level, in terms of number of workers,

SIC_{ij} = a set of dummy variables indicating whether firm i is in the Standard Industrial Classification code (SIC) category j,

e_i = a random error term.

The data for these regressions are from an employer survey of over 5000 establishments.[6] (Establishments are defined as places of business regardless of ownership, and thus differ from firms.) Establishment-level data are used (as opposed to firm-level data), and since the overall variance in establishment size is probably less than the variance in firm-size because some firms are comprised of many establishments, the variance in establishment size within an industry is probably less than the variance in firm-size.

The regressions indicate that the SIC code explains 6 percent at the 1-digit level, 14 percent at the 2-digit level, and 47 percent at the 4-digit level of the variance in establishment size. Thus, even at the 4-digit level, over 50 percent of the variance in establishment size is within industries, and this may be an understatement since in some cases the 4-digit SIC code level may be too detailed in disaggregating firms that are really in the same industry.[7]

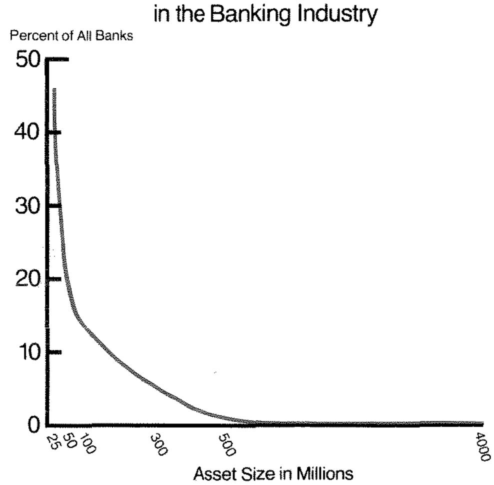
These regression results do support the hypothesis that production technology (and hence, industry) is related to firm-size because firm-size does depend on industry, but they also strongly suggest that other factors must be responsible for the large within-industry variation in firm size. One reason may be that average cost functions are not U-shaped but flat, exhibiting constant returns to scale over a wide range, perhaps with initial economies of scale (and declining average costs) over only a relatively small initial interval.[8] This type of cost function would imply a rectangular distribution of firms within the flat portion of the average cost function, since firms outside this range would be unlikely to survive (because of their higher average costs), and because within the range there is no reason for any particular size to be observed more frequently.

However, this implication of neoclassical theory is also contradicted by the data: the distribution of firm-size within industry is highly skewed, not rectangular. For example, in Chart 1, the distribution of firm-size in the banking industry shows a highly skewed distribution with a long right tail, and distributions in most industries are very similar.[9]

One attempt to reconcile this skewness with neoclassical theory is the application of stochastic models to the growth of individual firms. Much of this analysis is derived from the work of Gibrat (1931), who showed that if all firms have an equal chance of growing (or declining) by a given percentage amount, then a log normal size distribution of firms will result even if all firms are initially the same size. There does seem to be evidence that the growth of firms is independent of size, as well as evidence that, at least for some industries, the observed distribution is log normal. This theory of the firm-size distribution, taken literally, implies that there are no differences in production technologies of firms of different sizes (in the same industry), that average costs are independent of size, and that the existing size distribution at any moment in time is simply due to the cumulative effects of luck (random factors). That is, there should be no systematic differences between large and small firms since they have the same cost functions, and since differences in size are due only to random shocks.

Because this theory implies that the size distribution of firms under competition is due simply to randomness, one very important policy implication is that there would be no losses in economic efficiency with the dissolution of large firms, or opposition to the consolidation of smaller firms through mergers, as long as firms were operating on the flat portion of their average cost curves. For example, interstate banking could be prohibited without any concern for possible losses in output due to an

Chart 1
Distribution of Firm Size
in the Banking Industry



Source: 1981 *Statistics on Banking*, Washington: FDIC, 1982, Table 104.

inefficient scale of operation. Furthermore, since less concentrated industries *may be* more competitive, one need not worry about balancing gains from increased competition against losses due to non-optimal size when breaking up large firms or prohibiting mergers. Much of the empirical evidence presented in this paper, however, shows that there are many systematic differences between large and small firms that cannot be explained by purely statistical models of firm size.

II. New Explanations of the Firm-Size Distribution

To explain differences in firm size, recent theoretical research has emphasized differences in entrepreneurial ability, which is assumed to be in perfectly inelastic supply. For example, Lucas (1978) developed a model in which individuals are alike as workers but differ in entrepreneurial ability. In his model, the distribution of entrepreneurial ability leads to an equilibrium distribution of firm size, with more able entrepreneurs running larger firms.

Rosen (1981a, b, 1982) also developed a model in which entrepreneurial ability explains the size distribution of firms. His model focuses on the hierarchical structure of production in firms where decisions at each level of the hierarchy affect the efficiency of labor inputs at the next lower level. This production technology implies that there is a multiplicative effect in assigning persons of superior talent to the top ranks because they increase productivity by more than the increments of their talents. A more productive chief executive officer, for example, affects the productivity of everyone below him or her in the organization, and thus even a small increment in his or her talent may have a very large overall effect on productivity. Even if entrepreneurial ability were distributed normally, the firm-size distribution would be skewed because, in equilibrium, this production technology implies that output and labor input rise more than proportionately with talent.

These same characteristics of production imply that managerial compensation rises more than proportionately with ability. This, in turn, implies a skewed distribution of compensation as well, a finding confirmed by the work of Roberts (1956) and Fox (1978), who both find managerial compensation varying with the log of the number of employees. The reason this type of production technology does not lead to an equilibrium in which there is only one firm managed by the most talented individual (at least in each industry) is that the economies due to the hierarchical organization are limited by the increasing costs of monitoring workers. Monitoring workers requires direct worker contact and, therefore, is not subject to the same sorts of scale economies as the coordination and allocation of resources.

Rosen's model has some interesting implications for the structure of firms' cost functions. First, in equilibrium, any given firm will be subject to increasing average costs if it expands output beyond its equilibrium level, holding managerial talent constant. Second, firms of different sizes, in equilibrium, may well have identical *measured* average costs because the economies realized by the larger firm due to its superior management talent will be captured by the management whose talent is assumed to be in perfectly inelastic supply and because managers' compensation is often recorded as a cost, not a profit. Thus, evidence that measured average costs are independent of firm size does not necessarily imply that breaking up large firms would not result in significant economic losses. Rosen's model of firm size implies quite the opposite: there are economic benefits from allowing managers with superior talent to manage larger firms.

Oi (1982a, b) also developed a model that emphasizes entrepreneurial ability, but Oi's model more fully develops the implications of monitoring costs. His model, like Rosen's, assumes entrepreneurs perform two functions: they coordinate production and they monitor the performance of workers, but Oi does not assume a hierarchical production technology and thus Oi's model is unable to explain the skewed distribution of firm size in most industries. In Oi's model, more able entrepreneurs are assumed to be more productive at coordination but all entrepreneurs are assumed to be equally productive at monitoring. Although it might seem somewhat arbitrary to assume more able coordinators are not also more able monitors, Rosen's model of hierarchical production suggests why this might be so.

A more efficient monitor cannot reap the same sort of efficiencies in monitoring that he or she can in making coordinating and allocative decisions because monitoring cannot be delegated in a hierarchical structure.

Since an increase in entrepreneurial talent leads to an increase in the number of employees, and more employees implies more time spent monitoring and less time spent making coordinating and allocative decisions, the implicit costs of monitoring worker performance should be higher in larger firms. They should be higher because the alternative use of time is coordination (or allocation of resources), for which there are economies of scale implicit in hierarchical organizations and because of the lack of such scale economies in monitoring. These scale economies make coordination more valuable in large firms, and, as Rosen points out, this strengthens the implication that firm size will be skewed because superior managerial talent can be economized by "subordinating" monitoring through a more hierarchical structure.

The dependence of monitoring costs on firm size means that firms of different sizes will have different types of workers, different types of managers, different types of capital, and different sorts of production methods. Thus, this new theory stands in sharp contrast to the stochastic explanations of the firm-size distribution that predict no systematic structural differences among firms of different sizes. For example, the monitoring-cost hypothesis predicts that large firms should have more hierarchical structures, less flexible production methods (which are presumably associated with less monitoring), more reliable workers (who require less monitoring), and more reliable capital equipment (which requires fewer repairs and therefore less monitoring of workers who perform the repairs).

The monitoring-cost hypothesis has several empirical implications. First, it implies that larger firms invest in more specific human capital (that is, skills specific to the firm) than small firms because larger firms have higher costs per worker in monitoring worker performance. These higher monitoring costs lead larger firms to have more rigid and specialized methods of production, which in turn require more firm-specific training (that is, specific human capital). Since large firms find it optimal to invest in more specific human capital than small firms, turnover rates (both quit and fire/layoff rates) should be lower in large firms.

Second, this theory implies that large firms will hire more productive employees—that is, employees with higher levels of human capital (which depends on commonly measured variables such as education as well as traits such as intelligence or reliability that are more difficult to measure). This is the case because more productive workers allow firms to lower monitoring costs per unit of output and thus to economize on those costs, assuming that they depend on the number of workers and not total output.

Below, we present some indirect tests of these hypotheses by estimating differences in wage levels, wage growth, and turnover among firms of different sizes and types.

III. Empirical Evidence on the Monitoring-Cost Hypothesis

The monitoring-cost hypothesis implies that large firms invest more in specific human capital than small firms because large firms have higher costs (per worker) of monitoring worker performance. These higher monitoring costs lead large firms to institute rigid and specialized methods of production, which, in turn, require more specialized training. This hypothesis also implies that large firms hire workers with more general human capital in order to reduce the costs (per unit of output) of monitoring, assuming that monitoring costs depend on the number of workers and not total output. Although specific and general human capital are not directly observable, wage rates, wage growth and turnover are. Thus, one can test the monitoring-cost hypothesis indirectly by analyzing these observable variables. Below, I first discuss the implications for wage levels.

According to human capital theory, (see Becker (1964)), wage rates depend on general and specific human capital. General human capital is a set of skills and knowledge that can be transferred from one employer to another, while specific human capital is a set of skills and knowledge that are useful only in a specific firm. Since this theory predicts that wage levels differ among firms of different sizes because of differences in human capital, to analyze the implications of the monitoring-cost hypothesis for differences in wage rates one needs to assess its implications for human capital.

General human capital is, by definition, equally valuable in any firm. Workers are therefore expected to bear the full costs of acquiring general human capital since they take their general human capital with them when they leave the firm. If such general human capital is acquired on the job, then observed earnings would be net of investment costs (that is, observed earnings would equal the potential marginal product minus the costs to the employer of providing general training). Since such general human capital is equally valuable at all firms, workers must be paid the full return on any investments in such capital (or they will leave and find employment in a new firm).

The allocation of costs and returns between employer and employee of specific and general human capital is quite different. Since specific human capital is only valuable in the firm where it is acquired, this type of capital completely depreciates when the worker leaves the firm. Thus, if the firm had entirely borne the costs and received the benefits of such investments, the firm would lose its investment if a worker quit. Similarly, if the worker paid the entire cost of the investment and received the full benefit and then was fired, the worker would lose his capital. Becker shows that such considerations lead workers and firms to share in the costs and benefits of investments in specific human capital to ensure that both decisions to fire and to quit take into account the loss of specific human capital that would result.[10]

To summarize, human capital theory implies that observed wages depend on (opportunity) marginal products,[11] the amount of training currently being undertaken, and the share of the training costs being paid implicitly by the worker. Higher marginal products lead to higher wages as do lower workers' shares of training costs (because workers pay for their training implicitly with lower wages) and greater amounts of on-the-job general or specific training (holding constant the share of costs that are borne by the worker) lead to lower wages. Thus, all other things being equal, a given amount of specific training will lead to higher observed current wages than the same amount of general training because only a fraction of specific training costs are borne by the worker. Marginal products, of course, depend on the accumulation of human capital, and the share of the marginal product being paid to the worker depends on the share of the training costs that are borne by the worker. Wage levels thus may differ among firms of different sizes and types because of differences in the levels of their workers' general and specific human capital and differences in the provision of current general and specific training.[12]

Workers with more general human capital require less monitoring per unit of output if monitoring costs depend on the number of workers and not total output, or if there are other aspects of production in which managers' and workers' skill levels complement each other. Large firms thus should be more likely to hire highly skilled workers than small firms; that is, there should be a positive matching of more able entrepreneurs with more able workers. This consideration alone implies that larger firms would pay higher wages. However, if large firms provided sufficiently more training and this training were paid for by employees, such a training effect could conceivably offset the higher wages in large firms due to the greater skill levels of their employees.

Whether large firms would provide more on-the-job general training than small firms depends on whether training activities require more monitoring than production activities because large firms face higher monitoring costs. It seems likely that such training does require much individual attention (monitoring). If so, large firms would simply hire workers with more general training that had been acquired elsewhere. This lower rate of general human capital accumulation would lead to even greater wage differences between large and small firms. Although large firms are expected to offer more specific training (which would depress observed wages somewhat), it is unlikely this would dominate the higher wages large firms pay due to higher skill levels and their practice of providing less general training. Thus, on balance, large firms are expected to pay higher wages. Below, this hypothesis is tested.

Data and Samples

The data analyzed come from the Employment Opportunity Pilot Projects (EOPP) baseline household and employer surveys. These surveys were designed to obtain pre-program measures of a variety of variables, such as wage rates, earnings, employment, and unemployment as part of the evaluation of EOPP.[13] The household survey covered the period January 1, 1979 through the date of the interview (most interviews occurred between May and September 1980).[14]

The household survey was conducted in 10 pilot and 10 matching control sites throughout the U.S.; and the employer survey was conducted in 10 pilot sites and 19 control sites. The employer and household surveys have in common 7 pilot and 7 matching control sites where the two surveys are linked. The linked surveys are used when analyzing the household data since firm size comes from the employer survey.[15]

When analyzing differences among firms of different types and sizes in wage levels, wage growth, and turnover, all firms are classified into one of five types:

Large private business (an establishment with 500 or more employees)
Small private business (an establishment with less than 500 employees)
Government—federal, state or local
Self employed
Special government—(CETA, WIN, EOPP, Manpower or youth program)

Analysis of Firm Size on Wage Levels

To test the hypothesis that wage rates are higher in large firms, the following sorts of models were estimated:

$$\ln w = A_0 + A_1 C + B_1 D_1 + B_2 D_2 + B_3 D_3 + B_4 D_4 + e \qquad (2)$$

where: $\ln w$ is the natural log of an employee's wage rate, C is a vector of control variables, $D_i = 1$ if the job is of the ith type ($i = 1, \ldots, 4$), and the As and Bs are parameters to be estimated. In this analysis, wage rates are defined as earnings divided by hours of work, and include tips, bonuses and commissions but not fringe benefits. (There were no measures of fringe benefits in the EOPP household survey.) Three different specifications of this model, with different control sets, were estimated: one with no control variables, one with a set of 2-digit SIC industry dummies, and one with the same industry dummies and other control variables.

In the model with no control variables, B_i measures the mean difference in the natural log of the wage rate between category i and small private businesses (the excluded category). The natural-log specification is used since it fits somewhat better than a linear model, and because the coefficients can be interpreted as percentage effects (if they are small).[16] Industry control variables are included to control for possible differences in wages that might be correlated with firm size across industries. It should be noted that the coefficient estimates of the government category, when industry dummies are included, cannot be easily interpreted since government employment is largely but not entirely captured in SIC codes 91 through 97.

Finally, the complete set of control variables are added to hold observable differences in the characteristics of the individuals and their employers constant. These control variables are described in Appendix A: they include not only the usual human capital variables (education, experience, demographic characteristics) but also dummies for occupation of the worker and whether the worker was a union member. Finally, there are a number of site characteristics (unemployment rate and SMSA size). This control set is far more comprehensive than those normally available.[17]

In Table 1, the estimated coefficients of these models are presented for four demographic groups. The results from the specification with no control variables indicate that for all groups, workers in large private businesses receive considerably higher wage rates than workers in small private businesses. The effects are large and statistically significant for all groups, indicating that employees of large firms earn between 17 and 40 percent (exp. .16 = 1.17 and exp. .34 = 1.40) more than workers in small private businesses. This table also shows some statistically significant differences for other types of employers, with government workers earning less than workers in small private businesses for married men and youth, but earning more in the case of married women and single female heads. (F Tests of the joint significance of the four firm-size-and-type variables presented in Table 1 indicate they are jointly significant at the 1% level or better.) Likewise, special government workers earn less—probably an indication of the low-paying nature of these special jobs. Self-employed married men and women earn significantly less than their counterparts in small private businesses, but there are no statistically significant differences for self-employed single female heads and youth.

These results strongly confirm the hypothesis that workers in large firms are paid more than workers in small firms.[18] These results affirm those of other researchers [see Schiller (1982), Mellow (1981), and Oi (1982)].

When industry-control variables are included, the results show similar, although generally somewhat smaller, effects. F tests indicate one can reject the hypothesis that industry has no effect on wages, holding firm size constant. This is not surprising since different industries probably have workers with different skill levels and, hence, different wages.

The results with the complete set of control variables are generally similar except that estimated wage differences are smaller, ranging from 10 to 15 percent depending on the demographic group. However, all estimated differences between large and small private businesses are statistically significant at the 1 percent level or better, except for youth for whom the effects are significant at the 5% level.

These results show that large firms do hire workers with higher levels of general human capital (since the inclusion of the human-capital control variables leads to smaller differences) than do small firms, but they also show that, even controlling for industry type and a large number of observable differences between the employees of large and small businesses, large businesses pay significantly higher wages. These results are consistent with the monitoring-cost hypothesis that large firms hire more productive workers, both in terms of higher levels of measurable characteristics and factors such as intelligence that are not observed. This would explain the smaller wage differences when

Table 1
Natural Log of Wage Rates as a Function of Employer Type
(Compared to Small Private Businesses)
(Standard Errors in Parentheses)

	Married Men			Married Women		
Sample Size		9,369			4,683	
Dependent Variable Mean (ln wage rate)		1.87			$ 1.37	
Wage Rate Mean		$ 7.33/hr			$ 4.41/hr	
	No Control Variables	**Industry Control Variables**	**All Control Variables**	**No Control Variables**	**Industry Control Variables**	**All Control Variables**
R^2	.04	.16	.38	.05	.16	.32
Large Private Business	.24*** (.018)	.19*** (.018)	.098*** (.016)	.30*** (.027)	.22*** (.027)	.14*** (.025)
Self Employed	−.16*** (.019)	−.10*** (.019)	−.21*** (.017)	−.11*** (.034)	−.07** (.034)	−.15*** (.032)
Government	−.072*** (.016)	−.005 (.019)	−.08*** (.017)	.15*** (.017)	.08*** (.022)	.02 (.020)
Special Government	−.56*** (.05)	−.44*** (.05)	−.33*** (.04)	−.15*** (.048)	−.12*** (.048)	−.05 (.044)
F Test for inclusion of additional variables	107.07***	15.66***	95.84***	55.45***	7.87***	29.21***

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

all control variables are included, as well as the persistence of significant wage differences even after controlling for many factors.

However, the results also would be consistent with the idea that lower levels of specific or general training are being provided by large firms (whose costs are partially or fully borne by workers), although our theory predicts the opposite: that large firms provide more specific and less general training than small firms. Since training leads to more rapid wage growth, we can test this alternative theory by analyzing wage growth.

Firm Size and Type and Wage Growth on the Job

Real wage growth on a particular job, according to human capital theory, is due to the accumulation of human capital. More rapid human capital accumulation that is paid for by the employee leads to more rapid wage growth. Although we expect large firms to provide more specific training, they would provide less general training if providing general training requires relatively more monitoring than other activities. Thus, there is no strong prior reason to expect differences in wage growth between small and large firms since the more rapid wage growth in large firms due to the higher rate of specific human capital accumulation may be fully or partially offset by less rapid general human capital accumulation.

Starting and ending wage rates for hourly workers are used to analyze wage growth on a particular job. (For salaried employees, however, no measure of wage growth on the job is available, so such workers are excluded.) The empirical models employed to analyze wage growth are very similar to those used to analyze wage levels. They are of the general form:

$$\ln (w_e/w_s)/\text{length} = A_0 + A_1 C + B_1 D_1 + B_2 D_2 + B_3 D_3 + B_4 D_4 + e \quad (3)$$

where: w_s = starting wage rate

w_e = ending wage rate

Table 1, continued
Natural Log of Wage Rates as a Function of Employer Type
(Compared to Small Private Businesses)
(Standard Errors in Parentheses)

	Single Female Heads			**Youth**	
	2,900			2,228	
	1.37 $ 4.41/hr			1.14 $ 3.43/hr	
No Control Variables	**Industry Control Variables**	**All Control Variables**	**No Control Variables**	**Industry Control Variables**	**All Control Variables**
.08	.19	.35	.01	.16	.24
.34*** (.032)	.23*** (.034)	.15*** (.031)	.16*** (.050)	.16*** (.048)	.12 ** (.047)
.0063 (.052)	.03 (.051)	−.01 (.047)	−.07 (.072)	−.06 (.070)	−.04 (.068)
.26*** (.021)	.21*** (.025)	.14*** (.024)	−.09** (.041)	−.04 (.042)	−.07*** (.041)
−.052 (.043)	−.065 (.044)	−.014 (.040)	−.11*** (.032)	−.06* (.035)	−.02 (.035)
61.70***	4.81***	18.32***	7.31***	5.14***	6.02***

length = length of the period (in years) over which the starting and ending wages are measured, and the D_is, A_is, B_is and C are defined as before.

The model therefore measures the relative wage growth of various sizes and types of employers compared to small private businesses. The same three specifications of the control set that were used in the wage-level regressions also are used here.

The estimates of this wage-growth model are presented in Table 2. When no control variables are included, there are no statistically significant differences in the rate of wage growth between small and large firms, with the exception of the youth demographic group. For youth, wage growth on the job is greater in large businesses. The only pattern that is consistent across demographic groups is a lower rate of growth for special government jobs.

Estimates using industry control variables only and estimates using industry, human capital and additional control variables for the reason the job spell ended, if it ended (quit, laid off, or fired) were not significantly different from those using no control variables. The one difference was that they showed no significant differences between large and small private businesses for youth.

Since wage growth does not appear to differ significantly between large and small firms, to the extent that large firms provide more specific training (which is paid for by employees) than small firms (which would lead to more rapid wage growth in large firms), it must be offset by less general training being provided by large firms. These results, along with the existence of higher wage levels in large firms, suggest that such firms are hiring more highly skilled workers (in terms of both measurable and unmeasurable characteristics), and that observed wage differences are due to differences in skill levels and not differences in on-the-job human capital accumulation. Also, if large firms do provide more specific human capital (as both theory and the empirical work on turnover presented below suggest), then these results imply that small firms provide more general training.[19] The monitoring-

Table 2
Annual Wage Growth as a Function of Employer Type
(Compared to Small Private Businesses)
(Standard Errors in Parentheses)

	Married Men			Married Women		
Sample Size		5,968			3,104	
Dependent Variable Mean (annual wage growth)		.085			.077	
	No Control Variables	**Industry Control Variables**	**All Control Variables**	**No Control Variables**	**Industry Control Variables**	**All Control Variables**
R^2	.003	.022	.06	.006	.048	.079
Large Private Business	−.0007 (.005)	−.0017 (.006)	−.0052 (.006)	.0127 (.008)	.010 (.009)	.005 (.009)
Self Employed	−.038*** (.012)	−.034*** (.012)	−.034*** (.012)	.0008 (.021)	.007 (.021)	.007 (.021)
Government	.0028 (.0061)	.0008 (.007)	−.0079 (.007)	−.02*** (.006)	−.016** (.008)	−.016** (.008)
Special Government	−.039** (.016)	−.033* (.017)	−.036** (.017)	−.0217 (.015)	−.0127 (.015)	−.015 (.015)
F Test for inclusion of additional variables	4.21***	1.42***	6.43***	4.29***	1.68***	2.71***

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

cost hypothesis predicts higher skill levels, more specific training and less general training in large firms. Thus, these results confirm the predictions of the monitoring-cost hypothesis.

Analysis of Turnover Rates

Since the monitoring-cost hypothesis predicts more specific human capital in large firms, turnover rates should be smaller. To determine if turnover rates depend on firm size and type, models determining the (instantaneous) transition rates between employment and leaving that employment (either finding new employment or not working) are estimated.

The instantaneous rate of an event occurring, r(t), is the limit, as Δt approaches zero, of the probability of the event occurring between t and $t+\Delta t$ $[P(t, t+\Delta t)]$ per unit of time:

$$r(t) = \lim_{\Delta t \to o} \frac{P(t,t+\Delta t)}{\Delta t} \cdot \qquad (4)$$

If r(t) were constant over time, then the expected duration until the event occurs would be l/r and the duration until the event occurs would be distributed exponentially.

This framework can be used to model various discrete events. For example, to model job turnover, let the rate of leaving employment for the *jth* person be given by r_{jk}, where k is the destination state, and k=1 indicates being fired or laid off and k=2 indicates quitting. We assume that the turnover rate depends on the type and size of the employer and various control variables:

$$\ln r_{jk} = A_{ok} + A_k C_j + B_{1k} D_{1j} + B_{2k} D_{2j} + B_{3k} D_{3j} + B_{4k} D_{4j} \qquad (5)$$

where,

D_{ij} = a dummy variable indicating a firm of size and type i.

C_j = a vector of control variables including those described in Appendix A and, in addition, 1-digit SIC industry dummies.[20]

The As and Bs are parameters to be estimated.

In this model, the rate of leaving employment depends on whether the person quits or was fired, the characteristics of the individual, and the size and type of the person's employer.[21]

The vector of parameters in equation (5) can be

Table 2, continued

Single Female Heads			Youth		
	1,923			2,163	
	.071			.048	
No Control Variables	**Industry Control Variables**	**All Control Variables**	**No Control Variables**	**Industry Control Variables**	**All Control Variables**
.003	.08	.13	.011	.064	.13
.011	.004	−.004	.025*	.015	.010
(.010)	(.010)	(.011)	(.013)	(.013)	(.013)
−.002	.016	.013	−.037	−.032	−.022
(.022)	(.022)	(.022)	(.025)	(.026)	(.025)
−.0006	.009	.003	−.0046	−.0035	.0019
(.007)	(.009)	(.009)	(.011)	(.012)	(.012)
−.03**	−.010	−.011	−.034***	−.021**	−.012
(.014)	(.014)	(.014)	(.008)	(.009)	(.010)
1.56	2.063***	2.51***	6.06***	1.64***	4.40***

Table 3
Turnover Rates as a Function of Employer Type
(Compared to Small Private Businesses)
(In Percentage Terms)
(Standard Errors of the Antilog of the Parameters in Parentheses)[a]

	Married Men	Married Women	Single Female Heads	Youth
Sample Size	11,197	5,637	3,319	2,867
Fraction Leaving Employment	.28	.35	.34	.54
Mean Rate (per year)	.39	.53	.53	1.51
X_2	2,556.93***	1,361.32***	855.09***	419.94***
Pseudo R_2	.05	.05	.05	.02
Large Private Business	−44.1***	−34.5***	−57.8***	−41.5***
	(4.5)	(7.7)	(7.4)	(10.9)
Self-Employed	−42.1***	−46.1***	−46.9***	−53.2***
	(4.1)	(5.7)	(9.8)	(10.2)
Government	−48.7***	−29.5***	−36.9***	−14.1
	(4.1)	(5.4)	(6.4)	(11.1)
Special Government	6.4	6.3	−32.0***	18.4
	(16.9)	(15.4)	(10.5)	(12.7)

[a] The coefficients in this table are percentage effects calculated as $((e^b - 1)100)$, where b is the maximum likelihood estimate of the coefficient of a log-linear rate model, similar to that described by equation (5).

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 4
Voluntary and Involuntary Turnover as a Function of Employer Type
(Compared to Small Private Businesses) — (In Percentage Terms)
(Standard Errors in Parentheses)

	Married Men		Married Women	
	Fired/Laid off	Quit	Fired/Laid off	Quit
Sample size	11,019		5,466	
Fraction Leaving Employment	.11	.16	.08	.25
Mean Rate (per year)	.15	.22	.12	.38
x^2	1555.06***	1338.44***	543.73***	1098.89***
Pseudo R^2	.07	.04	.07	.05
Large Private Business	−29.8***	−54.4***	−4.9	−40.4***
	(7.9)	(5.6)	(21.9)	(8.5)
Self Employed	−31.3***	−49.8***	−35.3	−52.2***
	(8.00)	(4.8)	(14.9)	(6.1)
Government	−64.2***	−43.8***	49.2**	−54.4***
	(5.7)	(5.7)	(21.0)	(4.8)
Special Government	−12.4	16.0	32.8	−5.4
	(23.4)	(24.3)	(39.1)	(17.3)

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

estimated by the method of maximum likelihood using individual data on observed lengths of employment spells.[22] The observed length of employment (in a particular job) equals the last time the person is employed (either when the person leaves his or her job or when the observation period ends) minus the time the person is first employed in that job (or July 1, 1979, depending on which is later).

In Table 3, we present the results of a simplified version of the model described by equation (5) in which there is only one destination state—leaving the current job. Mean annual turnover rates (from a rate model with only a constant term) show that turnover rates are relatively high in our sample, ranging from .39 per year for men to 1.51 per year for youth. Since the inverse of the turnover rate gives the expected duration at that particular job, these numbers imply lengths of employment at a particular job ranging from 2.56 years for married men to .66 years for youth.[23]

The results in this table strongly suggest that small businesses have much higher turnover rates than large businesses, government, or the self-employed. For example, for married men, turnover rates are 44 percent less in large businesses than small businesses, and for single female heads they are 58 percent less. The only type of employer that has higher turnover rates than small private firms is special government programs, which is not surprising given that the intention of these programs is to provide short-term employment. These results also suggest that turnover rates are not, in general, significantly different among the government, the self-employed, and large private businesses.

In Table 4, we present the coefficients of the turnover model described by equation (5), which is identical to the model in Table 3 except that quits are distinguished from lay-offs. Sample sizes are somewhat lower because some observations lacked information on why a job ended. The results suggest that for all demographic groups, quit rates are considerably higher than fire/lay-off rates, with the smallest differences for married men and the largest differences for youth. These results also suggest that large private businesses have lower quit rates and lower fire/lay-off rates than small businesses, with somewhat larger differences for quit rates. Both the self-employed and large private businesses generally have lower quit and fire/lay-off rates than small businesses. Not unexpectedly, the

Table 4, continued

Single Female Heads		Youth	
Fired/Laid off	Quit	Fired/Laid off	Quit
3,250		2,814	
.09	.24	.15	.38
.14	.37	.42	1.06
363.27***	604.33***	286.57***	308.47***
.07	.05	.04	.02
−47.8***	−61.9***	−21.5***	−50.1***
(16.8)	(8.33)	(26.6)	(11.7)
−50.3***	−43.1***	−78.4***	−60.00***
(19.5)	(12.2)	(7.3)	(10.6)
−18.2	−48.7***	34.2	−33.7***
(16.1)	(6.6)	(27.8)	(11.4)
8.6	−47.6***	98.0***	−22.2*
(27.6)	(10.7)	(32.4)	(12.0)

government has lower fire/lay-off rates than large private businesses.

The results in Tables 3 and 4 are consistent with the hypothesis that large private firms provide more specific training than small businesses. Since specific human capital is fully depreciated when a worker leaves, both the firm and the worker have an incentive to avoid this potential loss of wealth. Much smaller turnover rates for large firms are consistent with the notion that large firms provide more specific on-the-job training than small firms. Also, we find that small firms generally have the highest turnover rates of all five categories of firms analyzed. This suggests that very little specific human capital is accumulated by workers in small firms.

However, the fact that the self-employed have significantly lower turnover rates than employees of small businesses indicates that owners of small businesses are much less likely to quit and close their businesses than the employees of small businesses are likely to leave. This is consistent with the notion that many owners of small businesses have substantial specific capital, both human and physical, invested in their businesses.

IV. Summary and Conclusions

After controlling for industry type and a large number of individual characteristics, we find striking differences in wage levels and turnover among different-sized firms. Large private businesses pay significantly higher wages than small private businesses. However, no significant differences are found between the *rates* of wage growth of large and small businesses. This suggests that large private firms are hiring more highly skilled workers, both in terms of measurable and unmeasurable characteristics, than small private firms, and that the observed wage differences are due to differences in levels of human capital, not differences in the rate of accumulation. This is consistent with the hypothesis that large firms hire more highly skilled workers because such workers have lower monitoring costs (per unit of output). The finding that large firms also have significantly lower turnover than small firms supports the hypothesis that large firms have more rigid and specialized methods of production and therefore provide more on-the-job training. These results taken together provide strong support for the hypothesis that, even holding constant industry, occupation and various individual characteristics, employment practices depend strongly on firm size.

To summarize, large firms hire more highly skilled workers and consequently pay higher wages, provide more specific on-the-job training, provide less general on-the-job training, and retain their workers much longer than small firms. Stochastic models of firm size (taken at face value) are not consistent with these systematic differences among firms of different sizes. Thus, the results in this paper support the monitoring-cost hypothesis.

The evidence supports the notion that firm size does matter—that firm size is the result of a deterministic process depending on the distribution of managerial or entrepreneurial talents, the economics of a hierarchical organization of production within firms and the costs of monitoring workers' performance. This model of the firm also explains the skewed distributions of firm size within industries, why large and small firms coexist, and suggests that there may be economic losses associated with public policies that prohibit firms from attaining their optimum size.

Appendix A
Control Variables Employed in the Wage-Level Regression

Site Characteristics

Large SMSA dummy (population over 1 million)
Small SMSA dummy (population under 1 million)
Not SMSA dummy—excluded category
Site unemployment rate

Spell Characteristics

Dummy for spell being truncated by the end of the period
Duration of the spell
Dummy for spell being truncated at 7-1-79
Dummy for job coming from 2-job file
Dummy for job being continued from prior spell

Demographic Characteristics

Race dummy for Black
Race dummy for Hispanic
Low-income strata dummy (from EOPP survey)
Age in years
Number of persons in the family

Human Capital Variables

Disability dummy for disability that limits the amount of work
Number of years of school
Number of years worked since age 17
Number of years worked squared

Occupation Characteristics

Dummy for union member
Dummy for occupation executive/administration
Dummy for occupation engineer/scientist/doctor
Dummy for occupation teacher/librarian
Dummy for occupation health technician/nurse/pharmacist
Dummy for occupation marketing/sales
Dummy for occupation clerical
Dummy for occupation service
Dummy for occupation transportation
Dummy for occupation mechanical
Dummy for occupation production
Dummy for occupation not known because person was not working when occupation question was asked (excluded occupations include material handler, technologists, writer/artist, and any unknown occupations)

Income and Labor Force Characteristics

Dummy for not working first half of 1979
Dummy for receiving AFDC first half of 1979
Dummy for receiving UI first half of 1979
Dummy for receiving Food Stamps first half of 1979
Non-Labor income first half of 1979

FOOTNOTES

1. There is also concern about the concentration of political power as well as the distribution of income that would result from such concentration.

2. For example, it is widely believed that public utilities represent natural monopolies, which, if unregulated, would restrict output and charge higher prices to consumers.

3. For example, the government's IBM case was brought primarily because of IBM's large market share in mainframe computers.

4. See Lester Thurow, "Abolish the Antitrust Laws," **Dun's Review** (February 1981, p. 72.)

5. See Baumol (1982) for a recent discussion of industry structure that emphasizes the technology of production. Viner (1932) originally developed this theory of market structure.

6. Data are from the first wave of the employer survey that was performed as part of the evaluation of the Employment Opportunity Pilot Projects (EOPP). See Section IV for a description of the data.

7. In the data set analyzed, the 4-digit SIC code resulted in 548 different categories of firms (i.e., industries) out of a sample of 5271 observations.

8. See Stigler (1958), Simon and Bonini (1958), Hoit and Prais (1956), and Ijiri and Simon (1964).

9. Studies by Hart and Prais (1956), Simon and Bonini (1958), Quandt (1966) and Ijiri and Simon (1964) all show that the distribution of firm-size within specific industries is skewed. Analysis of the EOPP data indicates that the firm-size skewness in banking is less than most, but not all, other industries defined at the 2-digit SIC level.

10. This framework has been used by Pencavel (1972) to explain differences in turnover rates.

11. That is the marginal product that could be achieved if no time were devoted to training. Opportunity marginal products themselves depend, of course, on the accumulation of both general and specific human capital.

12. Another reason why observed wages may differ among firms is because of differences in the nonpecuniary

conditions of work. For example, Masters (1969) has argued that large firms have to pay higher wages because of their more rigid and inflexible working schedules. However, one can think of many cases where the working environment is superior in large firms.

13. EOPP was designed to test a structured job-search program combined with a work and training program that was a key part of President Carter's welfare reform proposal. The program began in some sites on a very limited basis in the summer of 1979 but was not into full operation until the summer of 1980. The program never reached the scale of operation originally intended and was soon phased out during 1981 under the Reagan Administration. However, the operations and purpose of this program are not pertinent to this study in which only preprogram data are analyzed.

14. An important characteristic of the sample is that the period covered by the interview (January 1, 1979 through June 1, 1980, on average) is artificially divided into a six-month "control" period from January 1, 1979 through June 30, 1979, and an analysis period from July 1, 1979 through the end of the interview. This is done because the statistical models employed in this paper are based on the assumption that variables measured during the first six-month period are exogenous with respect to the dependent variables that are analyzed during the second period. If such variables were calculated during the analysis period it might be difficult to infer the direction of causality.

15. In this study, firm size is from the employer survey. Since all firms with 500 or more employees were included in the sample frame of the employer survey, it is possible to determine firm size for the employers of all individuals in the household survey by inference. Thus, samples are many times larger than they would have been were we to restrict the analysis to only matching cases. Only 6,788 jobs in the household survey were matched to the employer survey out of approximately 35,000 jobs. For all matching cases, firm size is taken directly from the employer survey and for all non-matches firm size can be inferred to be less than 500, assuming that the matching was done accurately. For matching cases, firm size is taken from the employer survey sample records, which contain information on the entire sample frame of employers in the sites common to both the household and employer surveys regardless of whether the employer survey was actually completed.

16. Since $\ln w_{Di=1} - \ln w_{Di=0} = Bi$, the exponential of the coefficient is the ratio of the wage when Di = 1 to the wage when Di = 0. That is,

$$\ln\left[\frac{w_{Di=1}}{w_{Di=0}}\right] = B_i, \text{ or } \frac{w_{Di=1}}{w_{Di=0}} = \exp B_i \quad .$$

17. In addition to these sorts of control variables there are also variables that hold constant various ways in which the observations were created. There is control for left- and right-censoring of the spell whether the job was continued from a previous spell, whether the job was from the 2-job spell file (indicating that the person held 2 jobs at least part of the time during which the job in question is being analyzed), and whether this job was a second job (indicating that two jobs were held and that the job being analyzed is less important in terms of hours worked).

18. Workers in large private firms also earn more than government workers or self-employed workers.

19. Schiller (1982) finds that workers in small firms have more rapid rates of wage growth than workers in large firms for new entrants to the labor force. This evidence is also consistent with the notion that small firms provide more general training.

20. Two digit SIC industry dummies were not used in the analysis of turnover because of computational cost.

21. This equation is based on a number of assumptions. For example, it assumes that the explanatory variables and their coefficients do not vary over time, that the rate of leaving employment does not depend on the length of time of employment, that the rate of leaving one spell is independent of characteristics of previous spells and that unobserved variables do not affect the rate (heterogeneity). By including a large number of variables in C, we hope to account for some of these effects.

22. If we define e_j to equal one if individual j is observed leaving his or her job due to being fired or laid off and zero otherwise, o_j to equal one if individual j is observed leaving his or her job due to quitting, and zero otherwise and then the likelihood function, assuming independence among length of spells, may be written as:

$$L = \prod_{J=1}^{n} \left[r_{1j} H(t_j)\right]^{ej} \left[r_{2j} H(t_j)\right]^{oj} \left[H(t_j)\right]^{1-ej-oj}$$

where $$H(t_j) = \exp\left[-r_{1j}t_j - r_{2j}t_j\right]$$

is the probability the individual is still employed at the same job at time t_j is the length of the observed spell. Maximization of L with respect to the Bs from the above equation gives the maximum likelihood estimates of the Bs. For further details on the structure of this model, see Tuma (1976), Tuma and Robins (1980), or Tuma, Hannan, and Groeneveld (1979).

23. Very high turnover rates for youth are one reason why youth have such high observed unemployment rates.

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Dynamic Adjustment in Money Demand

Brian Motley*

Models of money demand generally assume that in the short-run, actual real balances may diverge from their desired level. This paper compares two alternative explanations. The first is that the central bank fixes the nominal money stock but prices change slowly so that the real *money stock adjusts to its desired level with a lag. The second is that transactions costs cause individuals to change their nominal money holdings slowly. Empirical evidence mildly supports the second hypothesis, even during the 1979-82 period when the Federal Reserve closely monitored the nominal M1 stock.*

Models of money demand generally take it for granted that, in the short-run, the actual stock of real money balances may diverge from the "desired stock" determined by the prevailing levels of income and interest rates. The standard explanation for this divergence is that there are "transaction costs" to adjusting money holdings, which render it non-optimal for individual transactors to alter their stocks of money continually to hold them at desired levels. These transaction costs include not only such explicit charges as brokerage fees for buying and selling financial assets but also the psychological and "shoe-leather" costs of deciding upon and then implementing a change in average money holdings.

An alternative reason for expecting divergences between actual and desired real balances is that although individual transactors can increase or decrease their nominal money holdings, the economy as a whole cannot, as long as the central bank is controlling the stock of money closely. In other words, if the Federal Reserve fixes the nominal stock, money becomes a "hot potato", so that the desired stock of money must adjust to match the actual stock rather than conversely. As a result, divergences between the desired and actual stocks may occur not because economic agents are individually slow to adjust their actual money holdings to their desired levels but because the factors determining those desired holdings—prices, income and interest rates—do not adjust instantaneously to close such gaps.

The so-called "buffer-stock" approach to the demand for money in a sense lies between these two views of the adjustment process. According to this approach, an essential function of money is to serve as a "buffer" between streams of receipts and expenditures, both of which are somewhat unpredictable. The phrase "quantity of money demanded... does not refer to an amount of money which an (individual economic) agent will want to hold at each and every moment, but rather, to an amount which he will want to hold on average over some time interval."[1] The agent anticipates that his actual money holdings will vary around this average, rising when outlays are unexpectedly low or receipts are unexpectedly high, and falling when the converse is the case.

Indeed, it is largely because agents anticipate such variations (that is, they expect the unexpected) that they hold stocks of money.[2] As a result, when money holdings *do* rise or fall, agents do not immediately seek to move their holdings back to their desired average level. This explains why—even at the level of the individual agent—"actual" money may diverge from "desired" money, and also why the factors determining the total desired

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stock—income, prices and interest rates—do not adjust rapidly to eliminate such divergences.

In a recent article in this *Review*, Judd and Scadding compared a variety of alternative dynamic models of money demand.[3] They concluded that models in which divergences between desired and actual real balances resulted from slow adjustment in *prices* provided the best explanation of U.S. data. Judd and Scadding used quarterly data and their estimation period ended in 1974. The purpose of this short paper is to compare the performance of this "price-adjustment" model with the conventional approach using monthly data over the period since mid-1976 and, in particular, to examine whether changes in the central bank's policy target have affected the results of this comparison.

The use of monthly data raises the issue of whether the "adjusting variable" may be different according to the length of time considered. Most theoretical models assume that, in the short run, income and prices are essentially fixed and that it is the interest rate that moves to equate money supply and money demand. Interest rate changes are then transmitted to income and prices over a longer time period. Judd and Scadding found that a model in which interest rates move in response to divergences between money supply and money demand was unable to explain U.S. quarterly data. A possible explanation for their result would be that the interest rate adjustment process is completed within a quarter and so cannot be captured in quarterly data.

However, in preliminary tests, I obtained similarly poor results using monthly data, casting some doubt on the standard view that interest rates are the adjusting variable in the short run. In view of these initial results, I decided to limit this study to a comparison between the conventional money-adjustment approach, in which the *nominal* money supply adjusts to the public's demand for *nominal* money balances, and the price-adjustment approach in which the nominal supply of money is fixed and prices adjust to equate the *real* money supply with the public's demand for *real* money balances.

Alternative Models of Dynamic Adjustment

The *price adjustment model* begins with the assumption that prices change in response to the difference between the actual money stock determined by the central bank and the desired stock:

$$\Delta P_t = \gamma[M_t - (m_t^d + P_{t-1})] \qquad (1)$$

where all variables are in logarithms and m_t^d represents the desired stock of real balances given the levels of real income, y_t, and interest rates, i_t,

$$m_t^d = a_0 + a_1 y_t + a_2 i_t \qquad (2)$$

Substituting Equation (2) into Equation (1) and adding an error term yields

$$P_t = (1\text{-}\gamma)\, P_{t-1} + \gamma M_t - \gamma a_0 - \gamma a_1 y_t - \gamma a_2 i_t + u_t^P \qquad (3)$$

This equation also may be written in an alternative form:

$$M_t - P_t = (1-\gamma)(M_t - P_{t-1}) + \gamma a_0 + \gamma a_1 y_t + \gamma a_2 i_t - u_t^P \quad (3a)$$

The Judd-Scadding version of the *money adjustment model* begins with the assumption that the nominal money stock responds to the difference between the currently desired money stock and the actual stock in the preceding period, after adjustment for the effect on the actual stock of the change in bank loans:

$$\Delta M_t = \delta \Delta BL_t + \lambda[m_t^d + P_t - (M_{t-1} + \delta \Delta BL_t)] \qquad (4)$$

Substituting Equation (2) into Equation (4) and adding an error term yields

$$M_t = (1-\lambda)M_{t-1} - \lambda P_t + \delta(1-\lambda)\Delta BL_t + \lambda a_0 + \lambda a_1 y_t + \lambda a_2 i_t + u^M_t \qquad (5)$$

As in the case of the price-adjustment model, this equation also may be written in an alternative form:

$$(M_t - P_t) = (1-\lambda)(M_{t-1} - P_t) + \delta(1-\lambda)\Delta BL_t + \lambda a_0 + \lambda a_1 y_t + \lambda a_2 i_t + u^M_t \qquad (5a)$$

It is important to note that in Equation (3) the price level is the dependent variable and the nominal money stock is exogenous, whereas in Equation (5) nominal money is endogenous and prices are exogenous. Although the models may be transformed algebraically to *appear* to have the same dependent variable, such transformations do not alter the estimated parameters as long as the appropriate coefficient restrictions are imposed. In comparing the models, it is important to bear this in mind, since the variance of prices is substantially less than the variance of the nominal money stock.[4]

Empirical Results

The results of estimating Equations (3a) and (5a) over the period 1976.08-1983.08 are shown in Table 1. The entries represent the underlying structural parameters of the models: the long-run elasticities with respect to real income and the interest rate [a_1 and a_2 in Equation (2)] and the adjustment coefficients (λ and γ). It is striking that the elasticity estimates are extremely close. Moreover, both the income elasticity (not significantly different from unity) and the interest rate elasticity are close to estimates made with the San Francisco Money Market Model.[5]

The standard error of the price-adjustment equation is noticeably lower than that of the money-adjustment model. This result is the same as that reached by Judd and Scadding using quarterly data over an earlier sample period. However, this finding does *not* necessarily imply that the price-adjustment model is superior because the variance of the dependent variable also is lower in the price-adjustment equation. In terms of the proportion of total variance explained, the models are quite similar and, in fact, the money-adjustment model provides a slightly better fit. Allowing for degrees of freedom, the price model explains 28.5 percent of total variance while the money model explains 34.4 percent.[6]

The underlying theory of these models suggests that their appropriateness should depend on the monetary policy rule being followed by the central bank. If, for example, the authorities pursue an interest rate target in the short run, the stock of nominal money is endogenous and hence the money-adjustment model is appropriate. With a money-stock target, on the other hand, money becomes a "hot-potato," making the price-adjustment model more appropriate.

To test the idea that the policy rule may influence the adjustment mechanism, the full sample period (1976.08-1983.08) was divided into two subsamples. In 1979.10-1982.07, the Federal Reserve was assumed to be fixing the nominal money stock in the short run; hence the price adjustment model should be the appropriate one. In 1976.08-1979.09 and 1982.08-1983.08, the Federal Reserve was assumed to be allowing the nominal money stock to be endogenous in the short run; hence the money-adjustment model should be appropriate. Separate equations were estimated for these two periods. The results are shown in Table 2. The coefficients shown in the first and second columns of the table are unconstrained estimates, while those in the third and fourth columns were made subject to the restriction that the underlying long-run income and interest rate elasticities remained constant over the full 1976-83 sample period. This restriction reflects the assumption that these long-run elasticities do not depend on the policy regime. Under this restriction, the estimated elasticities are not much different from those estimated in Table 1.

The results in Table 2 cast some doubt on the price-adjustment model. When this model is estimated over the period for which theory suggests it should be most appropriate (the period from October 1979 to July 1982, during which the Federal Reserve was targeting M1), the income and interest rate elasticities are implausibly low. These elasticity estimates change dramatically when the constraint that they are constant over the full 1976-83 period is imposed[7] [compare columns (1) and (3) of

Table 1
Money Demand Equations (1976.08—1983.08)

	Price Adjustment Model	Money Adjustment Model
Adjustment Factor*	0.038	0.109
	(2.760)	(3.768)
Real Personal Income	1.110	1.018
(Elasticity)	(2.635)	(3.958)
Commercial Paper Rate	-0.194	-0.182
(Elasticity)	(3.221)	(4.645)
Change in		0.276
Bank Loans**		(2.070)
Constant	-2.872	-2.021
(Long Run Value)	(0.927)	(1.087)
RHO 1	0.247	0.186
	(2.207)	(1.523)
RHO 2	-0.020	-0.177
	(0.183)	(1.453)
SEE	0.002095	0.004504
"Explained" Sum of Squares***	0.285	0.344

* γ in Equation (3); λ in Equation (5)
** δ in Equation (5)
*** Based on monthly growth rates of prices and nominal money, and adjusted for degrees of freedom. See footnote 6.

the table]. By contrast, the money equation yields plausible parameters that do not change much when the restriction is imposed.

I would interpret these results as giving some support to the money-adjustment model and casting some doubt on the price-adjustment model, but the evidence is not strong either way. One possible explanation for the results is that although the Federal Reserve *targeted* M1 over the 1979-82 period, it did not control it sufficiently closely in the short run for it to be genuinely exogenous. Even over this period, then, M1 was not a true "hot potato." Another explanation could be that the sample period for the price-adjustment model is short and includes highly unusual interest rate and income volatility as well as a period of credit controls.

Summary and Conclusions

The principal purpose of this paper was to reexamine the question raised by Judd and Scadding as to the appropriateness of the conventional assumption in money demand studies that nominal money demand responds to changes in its determinants (income, prices, interest rates) with a lag. The alternative view considered here is that the real money supply adjusts to real demand with a lag because prices adjust slowly. Judd and Scadding concluded that the price-adjustment model outperformed the money-adjustment specification.

I have argued that the appropriate specification, in principle, depends on the policy regime in effect. If the central bank is pursuing a nominal M1 target, money becomes a "hot potato" and the conventional specification in which nominal money is the adjusting variable will not be appropriate. Such a specification would be suitable if the Federal Reserve had an interest rate or nominal income target and, in the short run, allowed the nominal supply of money to adjust in response to changes in demand.

I also argue that it is not appropriate to compare these models in terms of the standard errors of the estimated equations under the rival specifications. Under the conventional specification, the dependent variable is the nominal money stock, whereas under the alternative, the dependent variable is the price level. Since the variance of prices is less than that of nominal money, one expects an equation with prices as the dependent variable to have a lower standard error. The models cannot be rearranged to have the same dependent variable, so

Table 2
Money Demand under Varying Policy Regimes

	Unconstrained Price Adjustment Model (79.10-82.07)	Unconstrained Money Adjustment Model (76.08-79.09, 82.08-83.08)	Constrained Price Adjustment Model (79.10-82.07)	Constrained Money Adjustment Model (76.08-79.09, 82.08-83.08)
Price Adjustment Coefficient	0.0953 (1.925)		0.0236 (1.197)	
Money Adjustment Coefficient		0.0707 (4.50)		0.0739 (4.65)
Real Personal Income (Elasticity)	0.353 (1.41)	1.155 (9.68)	1.096 (11.70)	1.096 (11.70)
Commercial Paper Rate (Elasticity)	- 0.006 (1.41)	- 0.182 (7.54)	- 0.171 (8.24)	- 0.171 (8.24)
Change in Bank Loans[1]		0.184 (1.35)		0.192 (1.38)
Constant	2.871 (1.62)	- 2.93 (3.39)	- 2.837 (3.85)	- 2.503 (3.68)
RHO1	0.230	- 0.338	0.256	- 0.331
RHO2	0.180	- 0.257	0.182	- 0.249
SEE[2]	0.003053		0.003087	

1. δ in Equation (5)
2. These standard errors refer to the full sample period and measure the ability of the combined models to explain the data.

that it is not possible to set up a "nested" equation that includes each model as a special case.

There is some weak evidence that the money-adjustment specification provides a superior explanation of the U.S. experience. The estimated parameters appear to be more stable under this specification. On the other hand, the empirical results suggest that the estimates of the underlying long-run parameters are not sensitive to the dynamic structure chosen. This is a very useful result since it implies that, at least in the long-run, predictions of the effects of changes in the stock of nominal money on income, prices and interest rates will not be affected much by the adjustment assumptions made in estimating the money-demand relation.

FOOTNOTES

1. David Laidler, "The Buffer Stock Notion in Monetary Economics," 1983 Harry Johnson Lecture, April 12, 1983.

2. The fact that agents hold this buffer in the form of money rather than of other assets which yield interest is explained by the existence of transactions costs of switching between money and these other assets.

3. John P. Judd and John L. Scadding, "Dynamic Adjustment in the Demand for Money: Tests of Alternative Hypotheses," *Economic Review*, Federal Reserve Bank of San Francisco, (Fall 1982).

4. The variance of the monthly change in the logarithm of prices (i.e. the monthly growth rate) over the period from August 1976 to August 1983 is 6.30×10^{-6} whereas that of the monthly change in the logarithm of nominal money is $3.17 \times 10.^{-5}$ Thus nominal money was approximately five times more variable than was the price level.

5. John P. Judd, "A Monthly Model of the Money and Bank Loan Markets" Working Papers in Applied Economic Theory and Econometrics, No. 83-01, May 1983.

6. For each equation this proportion is computed as

$$1-[(RSS/n-k)/(TSS/n-1)]$$

where n is the number of observations, k is the number of parameters estimated, RSS is the residual sum of squares of each equation and TSS is the total sum of squares of the monthly growth rate of the dependent variable in each equation (i.e., prices and nominal money respectively).

7. An F-test of this restriction cannot reject it at the 5% level of significance. The computed F-statistic is 1.72, compared to a critical value of 3.13.

Intervention, Deficit Finance and Real Exchange Rates: The Case of Japan

Michael M. Hutchison*

How official sterilized (non-monetary) foreign exchange market intervention may influence the exchange rate by changing the relative supply of government bonds denominated in domestic and foreign currencies is shown in this paper. Recent Japanese experience is investigated in the context of a simple asset market model of exchange rate determination. The empirical estimates suggest that Japanese official intervention has had only a small influence on the real value of the yen-dollar exchange rate largely because its impact was dwarfed by the role of large fiscal deficits in changing the relative supply of government bonds.

The Williamsburg economic summit took place last Spring amidst great concern over the general strength and dramatic volatility of the dollar in world currency markets. A major issue on the agenda was whether central banks should increase their intervention in the foreign exchange market and attempt both to hinder the dollar's climb and to reduce its fluctuations. As a basis for discussion, summit participants used the Report of the Working Group on Exchange Market Intervention (the Jurgensen report) commissioned at the 1982 Versailles summit. Although the report gave few policy recommendations, it did help to identify the important issues and to clarify the meaning of intervention. In particular, the Jurgensen report distinguished between "monetary" intervention and "sterilized" intervention, and emphasized that the effectiveness of central bank foreign exchange market operations will largely depend upon the distinction.

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Official foreign exchange market intervention may be viewed as a process through which the central bank shifts the composition of its portfolio between foreign and domestic assets. In the case of monetary intervention, the central bank changes its net foreign asset holdings through purchases and sales of foreign exchange and allows a corresponding change in its monetary liabilities, that is, the monetary base (total reserves plus currency held by the non-bank public). Sterilized intervention, on the other hand, means that the central bank allows the change in its net foreign asset holdings to be offset by a corresponding change in its net domestic assets. Monetary liabilities of the central bank remain unchanged in this case, and the monetary base is "sterilized" from foreign exchange market intervention operations. In both instances, foreign assets held by the public will change. Monetized intervention, however, will change the public's holdings of base money, while sterilized intervention will change the public's holdings of domestic bonds.

Table 1 illustrates the effect of intervention on asset supplies in more concrete terms. The table shows a stylized central bank balance sheet and the consolidated private sector claims on foreign and domestic governments. The domestic central bank holds two assets, domestic government bonds and foreign government bonds, and has one liability, domestic base money. (For the sake of simplicity, we assume that central bank domestic credit equals domestic bond holdings and that central bank net foreign asset holdings equal net foreign bond holdings). The consolidated private sector aggregates the foreign private sector and the domestic private sector and focuses on total claims on government, that is, "outside" assets. Consolidated private sector claims on foreign and domestic governments are distributed between domestic base money, foreign base money, domestic government bonds and foreign government bonds.

Monetary foreign currency support intervention occurs when the central bank purchases foreign currency from the private sector with domestic currency, and in turn purchases a foreign bond with its foreign exchange receipts.[1] The net effect on the central bank balance sheet and private asset holdings from this operation is indicated by (1) in Table 1. The central bank increases its holdings of foreign bonds and increases domestic base money. Reflecting the new composition of the central bank portfolio, the private sector holds more domestic base money and fewer domestic bonds.

Table 1

(A)

Domestic Central Bank Balance Sheet

Assets		Liabilities	
Foreign Bonds	1 + 2 +	Domestic Base Money	1 + 2 nc
Domestic Bonds	1 nc 2 −		

(B)

Consolidated Private Sector Claims on Governments (Foreign and Domestic)

Domestic Base Money	1 + 2 nc
Foreign Base Money	1 nc 2 nc
Domestic Bonds	1 nc 2 +
Foreign Bonds	1 − 2 −

Notes: +: increase
−: decrease
nc: no change

With a stylized case of sterilized foreign currency support intervention, the domestic central bank follows all of the steps outlined above for monetary intervention and then takes one additional step—it sells a domestic bond in order to hold domestic base money unchanged. The net effect, shown by (2) in Table 1, is that the central bank holds more foreign bonds and fewer domestic bonds, leaving base money unchanged, while the private sector holds fewer foreign bonds and more domestic bonds.

The Jurgensen report emphasized the distinction between monetary and sterilized intervention because the form intervention takes will largely determine its effectiveness. Few disagree that monetary intervention may have a significant influence on the exchange rate. By changing base money, monetary intervention will influence the broader money aggregates, interest rates, and prices in the economy. These variables are important determinants of exchange rates. Sterilized intervention, on the other hand, amounts to a switch of a domestic (foreign) bond for a foreign (domestic) bond held in private portfolios. This form of intervention will only be effective if investors view the bonds as less than perfect substitutes, and relative yields adjust as a consequence of the change in their relative supply. The change in relative yields is the channel through which sterilized intervention may exert an influence on the exchange rate. The debate surrounding the efficacy of intervention thus largely concerns sterilized intervention, and the degree to which changes in the distribution of foreign and domestic bonds in private portfolios will influence relative yields.

A major difficulty facing the central bank, however, is that sterilized intervention is but one source of change in the public's bond holdings. The sale of government bonds to finance government budget deficits will likely play a larger role than intervention in determining the bond mix in private portfolios. The public's domestic government bond holdings (B) is determined by the interaction of monetary policy, government budget deficits, and official exchange market intervention. A government budget deficit over any given period, for example, must be financed by the private sector. The government bond can be sold either to the public or to the central bank. In the former case, B will increase; in the latter case, base money will increase. On the other hand, B will also be influenced by the central bank when it purchases or sells domestic bonds in conducting monetary policy through open market operations, even in the absence of current government budget deficits. In short, the public's domestic bond holdings equal the cumulative domestic budget deficit ($\int$DEF) minus cumulative open market purchases of government bonds in exchange for domestic base money (MB^d)[2] plus cumulative sales of home-country domestic bonds by official foreign-exchange intervention authorities ($\int$INT), that is, sterilized intervention in both the home and foreign countries combined:[3]

$$B = \int DEF - MB^d + \int INT \qquad (1)$$

From equation (1), it is apparent that a domestic bond sale either to finance a home-country budget deficit, or to finance exchange market intervention in support of the foreign currency will increase the supply of domestic currency denominated bonds in private portfolios. The ability of the central bank to influence the relative supply of bonds through sterilized foreign exchange operations alone during any given period will therefore be complicated by these other sources of supply.

Purpose

This paper investigates the Japanese experience with sterilized intervention in the foreign exchange market over the 1973–82 floating rate period. The Japanese case is particularly interesting both because the Bank of Japan pursues an active foreign exchange market intervention policy—it is among the most active of central bank participants in the foreign exchange market—and because large central government budget deficits have increased the stock of yen-denominated government bonds in private portfolios at a rate far surpassing that of most other major industrial nations since the mid-seventies. An empirical study of the Japanese case therefore provides some interesting insights into the complications central banks face in their attempts to manage exchange rates.

Section I presents a model of real (price-adjusted) exchange rate determination that is later employed to measure the effectiveness of Japanese sterilized intervention operations. The model is a variant of the "asset market approach" which views the exchange rate as an asset price whose value is largely determined by the relative demand and supply of asset stocks denominated in various national currencies and which is prone to large fluctuations as asset risk-return characteristics are perceived to change. The foreign exchange market is viewed as an asset market in this model, with exchange rates determined not by the balancing of flow demands and supplies of currencies, but rather by the values at which the market as a whole is willing to hold the outstanding stocks of assets.[4] Sterilized intervention can be conveniently analyzed within this framework because of its influence on relative bond supplies.

Section II discusses Japanese foreign exchange market intervention policy and describes the growth of fiscal deficits in Japan. This provides the background for an account of the growth of Japanese government bonds in private portfolios in comparison to the United States. An empirical test of the model using Japanese-U.S. data from 1973 through 1982 is the subject of the third section. The paper concludes with a short summary and discussion of several policy implications.

I. Real Exchange Rate Model

The basic formulation of the real exchange rate model follows Isard (1980a, 1980b) and may be viewed as a simplification of Hooper and Morton's (1982) extension of the sticky-price monetary model of exchange rate determination developed by Dornbusch (1976).[5] The focus dependent variable is the real (price-adjusted) value of the exchange rate. Because the real exchange rate is a measure of domestic and foreign relative prices, it is an important factor in determining a country's position in international competition, and its determinants are of special policy significance.

The exchange rate equation is derived initially from the covered interest parity condition. This condition states that the return on domestic securities must equal the return available on (equally risky) foreign securities once investors have covered their open positions (and hedged against exchange risk) by forward market purchases or sales:[6]

$$(1 + R) = (F/S)(1 + R^*) \tag{2}$$

where: R = domestic nominal interest rate over maturity t

R* = foreign nominal interest rate over maturity t

S = spot exchange rate (domestic price of foreign currency)

F = forward exchange rate

In log form, (2) may be expressed as (3)

$$r = f - s + r^* \quad \text{or} \tag{3}$$

$$f - s = r - r^* \tag{3'}$$

where $r = \log(1 + R)$, $r^* = \log(1 + R^*)$, $s = \log S$ and $f = \log F$

The expected yield differential between domestic and foreign bonds is the difference between the forward rate and the future spot rate,[7] $\phi^e = f - s^e$. To see this, we may substitute $f = \phi^e + s^e$ into (3) and solve for the expected yield differential: $\phi^e = r - [r^* + (s^e - s)]$. The return on domestic bonds is given by the domestic interest rate (r) and the return on foreign bonds is given by the foreign interest rate (r^*) plus the expected percent appreciation of the foreign currency, $s^e - s$. The forward rate will generally not equal the expected future spot rate $\phi^e \neq 0$ (and expected yields on government bonds denominated in different national currencies will generally not be equal), if investors view the bonds as imperfect substitutes. A simple model relating the yield differential to government bond supply and demand is developed below.

Rearranging and substituting the yield differential and expected future spot exchange rate for the forward rate gives:

$$s^e - s = r - r^* - \phi^e \tag{4}$$

or

$$s = (r^* - r) + \phi^e + s^e \tag{4'}$$

Equation (4) is a condition which will hold in internationally integrated financial markets with rational behavior on the part of investors. It simply states that the market expectation of domestic currency depreciation over a given period will be equal to the interest differential between domestic and foreign securities over a similar holding period, less any expected yield differential.

It is convenient to think of the future expected exchange rate as linked to the current spot exchange rate through the interest differential. Equation (4′) illustrates this relationship. A given interest differential (including ϕ^e) is consistent with any given spot exchange rate level, and only indicates the expected change in the (log) level of the exchange rate over the maturity of the bonds in question. Once expectations about the expected future spot rate are identified, however, the spot rate is determined at a given level. The link between the current (spot) price of a currency and its expected future price is hence quite strong, as it is in the case of any asset price.

To show that (4′) holds in real (price-adjusted) terms as well as nominal, we define the real exchange rate (q) and expected future real exchange rate (q^e) as the difference between the nominal exchange rate and current and expected future relative prices expressed in logs, respectively:

$$q = s - (p - p^*) \tag{5}$$

$$q^e = s^e - (p^e - p^{*e}). \tag{5'}$$

where: q = real exchange rate (log)

q^e = expected future real exchange rate (log)

p = domestic price level (log)

p* = foreign price level (log)

p^e = expected future domestic price level (log)

p^{*e} = expected future foreign price level (log)

Equations (5) and (5′) are both identities and differ only in that (5′) is an *ex ante* relationship based on market expectations of future spot exchange rates and of future domestic and foreign price levels. The real exchange rate measures deviations from purchasing power parity, that is, the extent to which the nominal exchange rate moves beyond simple adjustment to relative price shifts.

Expected future price levels may now be disaggregated into their current price and expected inflation components,

$$p^e = p + \pi \tag{6}$$

$$p^{e*} = p^* + \pi^* \quad (6')$$

where $\pi = \ln(1 + \dot{p})$, $\pi^* = \ln(1 + \dot{p}^*)$, and $\dot{p}$, $\dot{p}^*$ equal the expected percent change in the domestic and foreign price levels, respectively. Substituting (5) − (6′) into (4′) then gives the basic real exchange rate identity:

$$q = (r^* - \pi^*) - (r - \pi) + \phi^e + q^e \quad (7)$$

This equation expresses the real exchange rate as a function of domestic and foreign real interest rates, the yield differential, and the expected future real exchange rate. Up to this point, we have made no behavioral assumptions about the way expectations are formed, and have derived (7) by manipulating the covered interest arbitrage condition, introducing the definition of the real exchange rate and disaggregating the expected future price level into its current price and expected inflation components. The formation of expectations regarding the future real exchange rate and the yield differential need to be identified, however, before the model can be implemented.

Expected Equilibrium Real Rate

Among the more important influences of expectations about future real exchange rates (q^e), is the market's perception of a sustainable balance of payments. The current account of the balance of payments is perhaps the most logical choice as a measure of external equilibrium from an asset market view of exchange rate determination. This is because the current account equals the difference between the sale of domestic goods and services (and net transfers) to foreigners and the purchase of foreign goods and services by domestic residents. Whenever the current account does not equal zero, a nation exports more or less than it imports. In the absence of official reserve flows, the current account imbalance reflects the net accumulation (current account surplus) or decumulation (current account deficit) of private domestic resident claims on foreigners. It is in this sense that a sustainable long-run current account must be consistent with the rate of foreign asset accumulation or decumulation desired by private investors (both foreign and domestic) in the long-run. If an unexpected current account surplus or deficit arises, some adjustment may be necessary to restore the current account to a path moving towards its long-run sustainable value. The real exchange rate is a major component in the current account adjustment process because it reflects the relative price between domestic and foreign goods and services. An unexpected current account surplus or deficit, to the extent that it is considered permanent, is therefore assumed to indicate to market participants that a real exchange rate shift may be necessary to return the current account back to its sustainable level or path.[8]

This process may be formalized by stating the change in the expected long-run future real exchange rate (dq^e) as a function of unexpected current account movements or "surprises" (CA^s) over period t:

$$dq^e = g[CA^s(t)]\, dt \quad (8)$$

This relationship assumes that all changes over period t in the expected future long-run real exchange rate are unexpected, and that no time trend, and hence predictable element, is discernable during the period under investigation. Market participants thus employ all available current account information in their formation of expectations about the future real exchange rate, and only additional information imbedded in the unexpected component of the contemporaneous current account will cause them to revise their expectations. We are, in effect, assuming that market participants form their expectations "rationally," but we limit the set of relevant information upon which they base their expectations to the current account.

Integrating (8) from an initial period to the present gives the level of the expected long-run future real exchange rate at a point in time t as a function of unexpected cumulative current account developments:

$$q^e = \int g[CA^s(t)]\, dt \quad (9)$$

It is assumed that the (log) level of the expected future real exchange rate, q^e, may be expressed as a simple linear function of the cumulative sum of unexpected current account developments:[9]

$$q^e = \propto_0 + \propto_1 \Sigma CA^s \quad (10)$$

Yield Differential Determinants

The expected yield differential is of particular importance because it is through this channel that foreign exchange market intervention influences the exchange rate. The yield differential is determined by the interaction of demand and supply for assets in both home and foreign countries. Under certain assumptions, it may be expressed as a function of the relative supply of government bonds, relative wealth, the perceived exchange rate risk associated with holding foreign bonds, the degree of investor risk aversion and the currency "habitat" preferences of investors (Dornbusch, 1980).

For the purpose of empirical estimation, we assume a simple bond demand function similar to Frankel (1982) to derive the yield differential determinants.[10] Total private demand for domestic government bonds (B) is the sum of domestic private demand (B_d) and foreign private demand (B_f). The domestic and foreign demand functions, in turn, may be expressed as a proportion (β) of domestic and foreign wealth invested in domestic government bonds:

$$B_d = \beta_d W_d \tag{11}$$

$$B_f = \beta_f W_f \tag{12}$$

where $W_d(W_f)$ is domestic (foreign) wealth and $\beta_d(\beta_f)$ is the proportion of domestic (foreign) wealth devoted to domestic government bonds. By assuming that β_d and β_f are simple linear functions of the expected yield differential, that is,

$\beta_d = a_d + b\,\phi^e$ and $\beta_f = a_f + b\,\phi^e$, we may write (11) and (12) as:

$$B_d = (a_d + b\,\phi^e)\,W_d \tag{13}$$

$$B_f = (a_f + b\,\phi^e)\,W_f \tag{14}$$

where a_d, a_f are the desired percentages of total domestic and foreign wealth, respectively, held in domestic government bonds independently of expected relative yields, and $b\phi^e$ measures the proportion of wealth devoted to domestic bonds in response to differential yields. This formulation assumes that domestic and foreign demand for domestic government bonds, measured as a percentage of total wealth, differs only by a constant term which is presumably higher in the domestic country because domestic investors prefer the home currency "habitat."

To close the model we need to introduce the foreign bond market and develop demand functions analogous to (13) and (14). We do so by defining wealth as total holdings of bonds, both foreign and domestic. Domestic wealth (foreign wealth) relevant for portfolio choice in this model thus consists of domestic resident (foreign resident) holdings of both domestic and foreign bonds. This implicitly assumes that investors first allocate a portion of their total wealth to total bond holdings, and then decide between foreign and domestic bonds. This assumption, though restrictive, allows us to drop the foreign bond demand functions explicitly, and to infer the percentage of domestic and foreign residents' wealth devoted to foreign bonds directly from β_d and β_f. For example, the percentage of total domestic wealth, so defined, invested in foreign bonds equals one minus the percentage of domestic wealth invested in domestic bonds, $(1-\beta_d)$.

W can now define total "world" wealth as $W = W_d + W_f$; sum (13) and (14) to get total domestic bond demand ($B = B_d + B_f$); and solve for the yield differential:

$$\phi^e = \frac{1}{b}\left\{\frac{(B)}{W}\right\} - \frac{(a_d - a_f)}{b}\left\{\frac{(W_d)}{W}\right\} - \frac{a_f}{b} \tag{15}$$

From (1) and (15), it is apparent that a domestic bond sale either to finance a home-country budget deficit, or to finance foreign-currency support exchange market intervention, will increase the supply of domestic currency denominated bonds in private portfolios and, consequently, increase their expected relative return (that is, increase the yield differential), if investors view the bonds as less than perfect substitutes ($b \neq \infty$). This will immediately depreciate the exchange rate through (7) in our model. The larger the degree of substitutability between foreign and domestic bonds, that is, the larger the value of b, the smaller will be the increase in the yield differential, and hence, the effect on the exchange rate, of any given change in relative bond supply. An increase in domestic wealth relative to total wealth, on the other hand, will increase the relative demand for domestic bonds (assuming that domestic investors display a greater preference for home securities than do foreign investors, that is, $a_d > a_f$) and lower the yield differential. This will appreciate the value of the domestic currency.

Real Exchange Rate Equation

Substituting the expected equilibrium real exchange rate determinants (10) and yield differential determinants (15) into the basic real exchange rate equation (7), and rearranging, gives the model to be estimated in Section III of this article:

$$q = b_o + b_1 (r^* - \pi^*) + b_2 (r - \pi) + b_3 (B/W) + b_4 (W^d/W) + b_5 (\Sigma CA^s) \qquad (16)$$

where: $b_o = \alpha_o - (a_f/b)$;
$b_1 > 0$
$b_2 < 0$
$b_3 = 1/b > 0$
$b_4 = [-(a_d - a_f)/b] < 0$
$b_5 = \alpha_1 < 0$

Equation (16) expresses the real exchange rate as a function of foreign and domestic real interest rates, the proportion of domestic government bonds in total wealth, the proportion of domestic wealth in total wealth and the cumulative sum of current account surprises. The real exchange rate will depreciate (that is, increase) with a rise in the foreign real interest rate, and appreciate with a rise in the home real interest rate. The size of the real interest rate coefficients (b_1 and b_2) equal, in theory, the term to maturity (in years) of the foreign and domestic bonds whose interest rates are included in the equation. For example, a one percentage point increase in the five-year domestic bond rate (expressed at annual rates), *ceterius paribus*, should immediately appreciate the real exchange rate by five percent. The exchange rate appreciation keeps expected relative yields unchanged by creating an expected home currency depreciation of approximately one percent annually over the five-year maturity of the domestic bond. Unexpected current account surpluses in the home country (b_5), on the other hand, will appreciate the domestic currency, as market participants perceive a shift in the terms of trade necessary to return the current account to a sustainable long-run equilibrium position.

An increase in the share of domestic government bonds in total wealth, other things being equal, will depreciate the home currency as investors demand a higher yield to absorb the bonds into their portfolios. Holding interest rates and the future expected real exchange rate constant, the increased yield may come about through an exchange rate depreciation. An increase in the share of domestic wealth in total wealth, on the other hand, will increase the relative demand for domestic bonds and appreciate the exchange rate, *ceteris paribus*.

This process can be explained in intuitive terms as the macro-economic adjustment process set in motion from domestic disturbances that work partially through the foreign exchange market. In this case, the disturbance is an increase in domestic government bonds which necessitates an increase in their expected relative return. Part of this adjustment may be reflected in higher relative interest rates in the home country, and part may come about through an increase in the expected appreciation of the domestic currency. The increase in the expected appreciation of the currency may come about by an immediate spot depreciation beyond any drop in expectation regarding the future equilibrium rate (which is determined by the long-run current account equilibrium condition). This then sets up a greater appreciation of the home currency, or less of a depreciation, than was expected before the sale of a domestic bond, either to finance a domestic budget deficit or to finance foreign-currency support intervention (purchases of foreign bonds with domestic bonds).[12]

II. Exchange Market Intervention and Budget Deficit Finance in Japan

The real exchange rate model outlined above, with its focus on the influence of privately held government bonds and financial wealth on the exchange rate, is particularly appropriate for an analysis of the Japanese experience since the mid-1970s. On the one hand, Japan's large fiscal deficits and active exchange market intervention policy have combined to increase greatly the supply of Japanese government bonds in private portfolios[13] relative to other major industrial nations. On the other hand, the surplus of Japanese private savings over investment during the last decade has meant that financial wealth in Japan available for portfolio investment has also grown at a more rapid rate than most other industrial nations.

Table 2 gives two measures of the Bank of Japan's foreign exchange market intervention in relation to changes in the yen-dollar exchange rate over the course of the floating rate period. The intervention proxies are gross changes in official Japanese reserves (less gold) and changes in the Japanese foreign exchange funds account.[14] By either measure, Japanese intervention in foreign exchange markets is significant and the Bank of Japan is generally recognized as among the most active of the central bank participants in the foreign exchange markets. During 1982, for instance, Japan lost almost $5 billion in foreign exchange reserves in an attempt to slow the slide of the yen against the dollar. The foreign exchange funds measure suggests that the Bank of Japan sold $8.9 billion (net) foreign exchange during 1982. Furthermore, Japan in the past has gained or lost similar amounts of international reserves in a single quarter in its foreign exchange operations. The sharp drop of the yen following the oil price shock in the fourth quarter of 1979, for instance, was met with strong yen-support operations by the Bank of Japan and a $5 billion decline in official reserves. The foreign exchange funds account indicates that the Japanese sold $6.2 billion in foreign exchange during the quarter.

The Bank of Japan has generally followed a "leaning against the wind" intervention policy, that is, it usually sells yen when it appreciates and buys yen when it depreciates. This is indicated by the figures in the table. Japanese reserves generally fall when the yen is depreciating and rise when it is appreciating, and the Bank of Japan both buys and sells large amounts of foreign exchange over a period of several quarters. Empirical evidence derived from estimating the Bank of Japan's foreign exchange market intervention functions also supports this conclusion. Quirk (1977) estimates that the

Table 2
Official Japanese Foreign Exchange Market Intervention: Reserve Change, Foreign Exchange Funds and Exchange Rate Change

		Gross Change in[1] Official Reserves (less gold) (Billion Dollars)	Foreign Exchange[2] Funds Account (Billion Dollars)	Percent Change in[3] Yen-Dollar Exchange Rate (Yen Depreciation (+))
1974		U.S. $ 1.3	U.S. $−1.3	7.5%
1975		−0.6	−2.1	1.6%
1976		3.8	2.6	0.0%
1977		6.6	6.2	− 9.4%
1978	I	6.3	7.7	− 3.8%
	II	−1.9	1.1	− 7.0%
	III	1.9	2.9	−12.7%
	IV	3.8	3.1	− 1.1%
1979	I	−4.2	−3.9	5.7%
	II	−4.1	−4.1	8.0%
	III	0.4	0.8	0.5%
	IV	−5.0	−6.2	9.0%
1980	I	−1.8	−3.7	2.0%
	II	4.1	2.4	− 4.5%
	III	1.1	0.9	− 5.4%
	IV	1.6	1.2	− 4.2%
1981	I	1.9	1.5	− 2.4%
	II	1.0	0.3	7.0%
	III	0.2	−1.1	5.4%
	IV	0.6	−0.6	− 3.1%
1982	I	−1.1	−1.9	3.9%
	II	−1.7	−2.6	4.6%
	III	−1.4	−2.5	6.0%
	IV	−0.7	−1.9	0.3%

Notes: (1) Source: IMF, *International Financial Statistics*
(2) Bank of Japan, *Economic Statistics Monthly*; converted to dollars with period average yen-dollar exchange rate.
(3) Period average yen-dollar exchange rate.

Bank of Japan bought (sold) U.S. $160 million in foreign currency in the current month in response to a one-percent appreciation (depreciation) of the yen against the dollar during the March 1973–October 1976 period. Argy (1982) calculates a somewhat stronger response (U.S. $210 million) for a given percent change in the effective (trade-weighted) value of the yen over the March 1973–December 1979 period. Policy statements by Japanese central bank officials do not refute these findings.[15]

One implication of the Japanese leaning-against-the-wind intervention policy, however, is that, while it may influence relative government bond supplies in the short-term, it will have little effect over extended periods as intervention on either side of the market is eventually reversed. This suggests that month-to-month and perhaps, quarter-to-quarter, changes in the yen exchange rate may be influenced by Japanese intervention policy through this channel, but that longer-term effects are probably small. This is, of course, the intent of a leaning-against-the-wind policy—to slow, but not to reverse, exchange rate movements in the hopes of reducing exchange volatility, while not hindering longer-term trends.

Table 3 illustrates the development of Japanese general government budget deficits, as a percent of GNP, in comparison to the U.S. and Germany. The rapid growth of these deficits and their large size, both by historical standards in Japan and relative to the U.S., Germany and other major industrial countries, is a fairly recent phenomenon.

Japanese government budget deficits surged in the mid-1970s, as Table 3 demonstrates. The rising fiscal deficits are attributable to both cyclical and structural developments in the pattern of revenues and expenditures. On the revenue side, the 1974–1975 recession in Japan brought on a cyclical shortfall in government revenues, more so than in most industrial countries, because Japan is highly dependent on corporate taxes, and consequently highly variable corporate profits, for its tax revenue. The structural problem on the revenue side is that the Japanese income growth rate, and hence, the growth of tax receipts, has slowed substantially since 1973.

Table 3
International Comparison of General Government Fiscal Balance, 1972–81
(In percent of GNP)

	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
Japan[1]										
Receipts	22.1	24.0	25.1	23.1	24.0	24.7	26.5	27.2	28.7	30.0
Disbursements	21.8	21.9	25.0	26.7	27.5	28.8	30.7	31.7	32.7	33.7
Balance	0.3	2.1	0.1	− 3.6	− 3.5	− 4.1	− 4.1	− 4.4	− 4.0	− 3.7
United States[2]										
Receipts	31.4	31.5	32.2	30.6	31.3	31.6	31.5	31.6	31.8	32.6
Disbursements	31.7	31.0	32.5	34.9	33.5	32.5	31.5	31.1	33.1	33.5
Balance	− 0.3	0.5	− 0.3	− 4.2	− 2.1	− 0.9	–	0.6	− 1.3	− 1.0
Germany[2]										
Receipts	38.7	41.2	41.5	40.8	42.3	43.5	43.3	42.9	42.8	–
Disbursements	39.2	40.0	42.9	46.6	45.9	45.9	46.0	45.8	46.3	–
Balance	− 0.5	1.2	− 1.4	− 5.8	− 3.6	− 2.4	− 2.7	− 2.9	− 3.5	–

[1] Fiscal year.
[2] Calendar year.

Chart 1
Relative Japanese/U.S. Government Debt

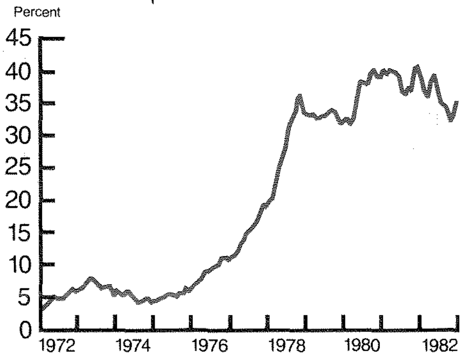


Table 4
International Comparison of Gross Private Savings Less General Government Fiscal Deficit, 1972–80 (percent of GNP)

	1972	1975	1978	1980
Japan	31.7	25.7	24.9	24.9
United States	15.1	12.8	17.3	15.3
Germany	21.0	16.0	18.1	17.3

Notes: Calculated as gross private savings less general budget deficit, as percent of GNP. Japan data is for fiscal year; U.S. and German data cover calendar years.

On the expenditure side, the government sector in Japan has grown rapidly in recent years, led by increases in social security benefits, social assistance grants, and interest on the public debt. These developments have increased the size of general government expenditures from its historically small percentage of GNP to a ratio approximately equal to that of the U.S. in 1981—33 percent.

The net result of these developments is that the government began issuing "deficit finance" bonds in 1975 to cover the shortfall in revenues over *current* expenditures. The amount of deficit-financing bonds issued by the government increased rapidly from 1975 through the remainder of the decade. Total government bond issues as a percent of both general account expenditure and GNP peaked in fiscal 1979-80. These developments have correspondingly spurred the growth of Japanese government bonds held privately. The ratio of outstanding government bonds to GNP increased in Japan from 6% to 36% from the end of fiscal 1974/75 to the end of 1982/83. In less than a decade, Japan moved from having a ratio of (central government) outstanding bonds to GNP that was among the lowest in industrial countries to one that was among the highest. Chart 1 illustrates the growth of Japanese national government debt held in private portfolios in comparison to the U.S. over the last decade.

Another reason the model developed above is particularly relevant for the Japanese case is that it explicitly accounts for the influence of relative financial wealth on the exchange rate. Japanese financial wealth, similar to government debt, has grown at a rapid pace over the last decade, and at a rate much faster than that of other major industrial nations.[16] This phenomenon is due both to the high private savings rate in Japan, which has continued unabated over the decade, and the sharp drop in the growth of Japanese private domestic investment since the 1973 oil-price shock. The combination of the two factors has resulted in a large increase in Japanese financial wealth that has been available to finance domestic government budget deficits (see Table 4) as well as to export capital (through current account surpluses) to the rest of the world over the greater part of the period. One advantage of the present model is that it allows us to identify the separate influences of relative wealth from relative bond supply on the real exchange rate.

While it is true that the rapid growth of financial wealth in Japan has largely "filled in" the financing requirements of government budget deficits, it is nevertheless useful to look at the partial influence of each variable on the exchange rate.[17]

III. Empirical Results

This section presents the empirical results from testing the two-country real exchange rate model given by equation (16) with monthly data for Japan and the U.S., over the 1973-82 period of floating exchange rates. Long-term real interest rates are used as the interest rate variable in the model because expectations of the future real exchange rate, q^e, are based on an equilibrium current account condition that will hold only in the long run.[18]

The real value of the yen/dollar in log form (q) is calculated as the log of the spot yen/dollar exchange rate less the log of the Japanese wholesale price index plus the log of the U.S. wholesale price index. Long-term real interest rates (r_{lt}) are constructed from the nominal interest rate on long-term governmental securities less a two-year centered moving average of the monthly percent change in the consumer price index.[19]. Japanese government bonds held privately (B^j) is measured by total Japanese national government debt, net of Bank of Japan and government agency holdings.

"World" financial wealth (W) is calculated as the sum of U.S. base money, U.S. national government debt privately held, Japanese base money and Japanese national government debt privately held.[20] Japanese financial wealth (W_j) is calculated as the sum of Japanese national government debt privately held, Japanese base money, and the cumulative sum of Japanese current account surplus from March 1973, the beginning of the floating rate period in Japan. B^j/W and W^j/W state total Japanese privately held bonds and financial wealth, respectively, as a percent of total financial wealth.[21]

The current account surprise variable is the cumulative sum of the residuals from the equation regressing the difference in the Japanese and U.S. current account (JCA-USCA) on six of its lagged values and three lagged values of the real exchange rate.

Estimation results are given in Table 5. Column (4) of the table shows the fully specified model given by equation (16), and columns (1)-(3) show the sensitivity of the coefficient estimates to dropping particular variables from the regressions. Turning to column (4), all of the signs of the variable coefficients conform to theoretical predictions, and are significant at the 90% level of confidence or higher with the exception of the current account surprise variable. A one percentage point increase in the Japanese long-term real interest rate is estimated to appreciate the real value of the yen by 0.77 percent, *ceterius paribus*. The U.S. real interest rate estimate is significantly higher than its Japanese counterpart, however, and suggests that a one percentage point increase will depreciate the yen by 1.35 percent.

The estimates for the yield differential determinants, relative Japanese debt (B^j/W) and relative

Table 5
Regression Estimates of the Real Yen/Dollar Exchange Rate
(Monthly, March 1973–December 1982)

Explanatory Variables	(1)	(2)	(3)	(4)
Intercept	5.46	5.66	5.44	5.62
	(145.80)	(130.90)	(164.27)	(110.90)
r^j_{lt}	− 0.08	− 0.79	− 0.09	− 0.77
	(− 0.13)	(− 1.91)	(− 0.15)	(− 1.81)
r^{us}_{lt}	0.99	1.42	0.97	1.35
	− (1.96)	(3.20)	(2.02)	(2.96)
B^j/W		1.14		1.17
		(1.83)		(1.86)
W^j/W		− 1.47		− 1.39
		(− 3.34)		(− 3.19)
ΣCA^s			− 0.004	− 0.003
			(− 1.83)	(− 1.18)
R^2	.86	.86	.87	.86
Rho	.91	.81	.89	.82
OBS	118	118	118	118

Notes: (1) r^j_{lt} (r^{us}_{lt}) is constructed from nominal long-term rates less a twelve month centered moving average of monthly percentage change in the Consumer Price Index. B^j/W is the ratio of Japanese national government securities to "world" private financial wealth, proxied by the sum of Japanese and U.S. private financial wealth. W^j/W is the ratio of Japanese private financial wealth to world private financial wealth. CA^s is the cumulative current account surprise variable. The real yen/dollar rate in log form is constructed as the log of the nominal yen/dollar rate less the log of the Japanese Wholesale Price Index (WPI) plus the log of the U.S. WPI index. See statistical appendix for data sources and complete variable definitions.

(2) All equations are estimated with the Fair technique, with correction for first-order autocorrelation ($\hat{\rho}$) and instruments for r^j and B^j/W. The t-statistics are in parentheses below each coefficient.

wealth (W^j/W), suggest that these factors also exert an important influence on the yen rate. A one percentage point increase in the ratio of Japanese national government debt to total financial wealth depreciates the real value of the yen 1.17 percent. A one percentage point increase in the ratio of Japanese financial wealth to total financial wealth, on the other hand, appreciates the yen 1.39 percent. A one billion dollar unexpected increase in the net Japanese-U.S. current account is estimated to appreciate the yen by 0.3 percent. This coefficient is not statistically significant, however.

The first three columns in the table show the results from regressing the real exchange rate on Japanese and U.S. real interest rates alone (Column 1), real interest rates together with relative debt and relative wealth (Column 2), and real interest rates with the cumulative current account surprise variable (Column 3). All of the coefficient signs in the regressions again conform to theory and, in addition, long-term U.S. interest rates, relative Japanese debt, and relative Japanese wealth are significant at the 90% level of confidence or higher. The long-term Japanese real interest rate, however, only becomes significant when relative debt and relative wealth are included in the regression. The cumulative current account surprise variable, in contrast, is only significant when these variables are excluded.

Overall, the regressions lend some support to the real exchange rate model. Considering the length of the period under investigation, and the great deal of volatility the real yen-dollar rate has experienced, the results may be viewed in a favorable light. Of particular interest for our purposes is that relative debt and relative wealth both enter significantly in the regressions, and that the Japanese real interest rate only becomes significant with the yield differential determinants included. This indicates that the Japanese real interest rate is an important determinant of the real exchange rate, but that its influence can only be measured when the model is fully specified and other factors (relative debt and relative wealth) are held constant.

To get a rough idea of the policy significance of the B^j/W coefficient estimate, consider the size of the coefficient in relation to the total stocks of Japanese government debt and total financial wealth. For December 1982 values, a one percentage point increase in the ratio of Japanese debt to total wealth would entail a U.S. $13.28 billion increase in Japanese government bonds outstanding in private portfolios (assuming financial wealth remains unchanged). This increase causes an estimated 1.1 percent depreciation in the real value of the yen.

A comparison of this figure to the recent Japanese budget deficit was 10,853 billion yen in 1982, or approximately U.S. $43 billion calculated at the period average yen/dollar exchange rate. At given financial wealth and Japanese real interest rates, the floatation of these additional government bonds to finance this revenue shortfall would have caused the real value of the yen to depreciate by an estimated 3½ percent over the course of the year.

Turning to Japanese intervention, the yen was generally weak in comparison to the dollar through most of 1982 and, in response, the Bank of Japan bought yen and sold dollars in the Tokyo foreign exchange market. Assuming that the Bank of Japan sold between $5-$9 billion in foreign exchange over the year, the estimates suggest that this would have appreciated the yen during the year by less than one percent in real terms holding other factors unchanged.

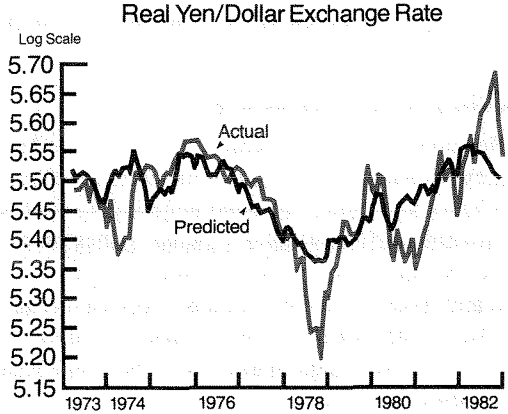
These examples serve to illustrate the large sums involved if Japan wishes to influence the real value of the yen through sterilized intervention operations. The sums are large because existing stocks of government bonds in private portfolios are large, and very large purchases or sales of these assets are involved in bringing about a significant change in their relative supply. The examples also point out that the large financial requirements of the central government have caused new bond issues to dominate the limited domestic bond purchases by the Bank of Japan needed to finance yen-support intervention operations in 1982. More generally, in the Japanese case, it appears that the influence of sterilized intervention on the movements in the yen-dollar real exchange rate since the mid-seventies has probably been small. In addition, sterilized intervention may become even less effective as an independent policy instrument. In the face of large, and rapidly growing, privately held stocks of government bonds, it may take an enormous amount of sterilized intervention to influence the exchange rate.

Some words of caution regarding the interpretation of these results are in order. First, while the model appears to perform well overall by standard goodness-of-fit (R^2) measures, it has only moderate success in tracking some of the more dramatic swings in the real value of the yen. This is illustrated in Chart 2 which shows actual and predicted values of the real yen value (log) from the fourth regression (Column 4) in Table 5.[22]

A second point concerns the coefficient estimates on the real interest rate variables. Both coefficients are substantially less (in absolute value) than theory suggests should be the case for long-term interest rates. In addition, the Japanese real interest rate coefficient is significantly less than that of the U.S., whereas theory predicts coefficient values roughly equal in absolute value. These shortcomings in the empirical estimates are important because interest rates play a central role in our model of exchange rate determination. Both the generally low coefficient values and the asymmetry between Japanese and U.S. variables shed some doubt on the plausibility of the underlying model. On the other hand, the inherent difficulties in measuring expected long-term inflation in both Japan and the U.S. may explain these apparent inconsistencies.

Third, and perhaps most important, the model's estimates only measure the direct or partial influence of relative debt on the exchange rate. To the extent that additional bond issues increase domestic real interest rates, the exchange rate will tend to appreciate in value, and, thereby, offset some of the initial downward pressure on the currency. That is, budget deficits may cause real interest rates to rise and appreciate the domestic currency. This constitutes an indirect channel through which government bond issues may influence the exchange rate, and it will work against the direct effect channeled through the risk premium.

Chart 2
Real Yen/Dollar Exchange Rate



But, while the link between real interest rates and real exchange rates is well-documented, there is only scant evidence of a predictable link between government bond supplies and real interest rates. In the Japanese case, a recent empirical study by Fukao and Okubo (1982) appears to refute the hypothesized positive link indirectly when it found no evidence of such a link between government bonds outstanding and *nominal* long-term interest rates. One explanation for their finding may be, as the theory presented here suggests, that the foreign exchange market absorbs some of the initial impact resulting from government bond issues in Japan. In the short run, the yen exchange rate is clearly more flexible than Japanese long-term interest rates, a reason being that Japanese long-term interest rates have been subject to fairly rigid government controls until recent years. Because of this rigidity, the exchange rate may be the channel through which relative yields adjust to reach a new equilibrium following a shift in relative government bond supplies. The empirical results do not refute this hypothesis.

Parameter Shift

The Japanese financial system has undergone a process of rapid liberalization in recent years[23] that has included the deregulation of interest rates and relaxation of capital controls. These developments suggest that Japanese and U.S. government securities have probably been viewed by investors as better substitutes during the latter part of the sample period than previously, making the yield differential variable less pronounced and its determinants less important in our exchange rate equation. To investigate this proposition, we include a test to see if the slope parameters on relative debt and relative wealth have significantly changed from the 1973:03 -1978:12 period to the 1979:01-1982:12 period. The *a priori* expectation is that both coefficients are significantly less in absolute value in the latter period than in the earlier period.

Test results are given in Table 6. Column (2) presents estimates of the full model with the shift dummy variables and, for purposes of comparison, column (1) restates previous model coefficient estimates. Neither parameter shift is significant at the 90% level of confidence. However, the estimated coefficients for the earlier period do increase substantially in absolute value—from 1.17 to 1.78 for relative debt and from -1.39 to -1.69 for relative wealth—and the estimated coefficients in the latter period approach zero. Both results are consistent with our expectations. The estimated coefficients in the latter period equal -0.48 [1.78 + (-2.26)] for relative debt and -0.32 (-1.69 + 1.37) for relative wealth.

These results suggest that the yield differential variable may have become less important because of the ongoing liberalization of the Japanese financial system. This implies that the relative supply of government securities and relative financial wealth will probably have less of an impact on the yen exchange rate in the future than it had in the past. In practical terms, this means that the *direct* influence of Japanese government budget deficits on the real exchange rate may be less currently than what the model estimates suggest was the case during the greater part of the seventies. It also means, however, that sterilized intervention as a separate policy instrument may be even less effective now than it was during the earlier period.

Table 6
Regression Estimates of the Real Yen/Dollar Exchange Rate with Parameter Shift Dummy between 1973:03–1978:12 and 1979:01–1982:12

Explanatory Variables	(1)	(2)
Intercept	5.62	5.62
	(109.57)	(108.59)
r^{j}_{lt}	− 0.77	− 0.95
	(− 1.81)	(− 2.01)
r^{us}_{lt}	1.35	1.61
	(2.96)	(3.39)
B^j/W	1.17	1.78
	(1.86)	(1.77)
W^j/W	− 1.39	− 1.69
	(− 3.19)	(− 2.94)
ΣCA^s	− .003	− .003
	(− 1.18)	(− 1.35)
$D^*(B^j/W)$		− 2.26
		(− 1.54)
$D^*(W^j/W)$		1.37
		(1.58)
R^2	.86	.86
Rho	.82	.81
OBS	118	118

Note: See Notes for Table 4. Dummy variable equals zero for 1973:03–1978:12 period and unity for 1979:01–1982:12.

From this viewpoint, a loss in effectiveness of the sterilized intervention policy instrument may have been a necessary cost associated with the increased internationalization of the Japanese financial system. The results are inconclusive, but they do suggest that even the limited effectiveness of this policy instrument may overstate its importance.

West Germany also has generally unregulated capital flows. Consistent with the results presented above, a recent study of the German case by Obstfeld (1982) also found that sterilized intervention has only a small effect on the exchange rate. Through a series of experiments simulating a structural model of German asset markets and prices, he concluded that the German central bank's "... ability to influence the exchange rate without altering monetary conditions is very limited. It is accurate to assert that the sterilized intervention considered above has essentially no effect on the exchange rate." (p. 24). This conclusion for Germany is consistent with the empirical results obtained in the case of Japanese sterilized intervention in this study.

VI. Conclusions and Policy Implications

This paper has analyzed how sterilized foreign exchange market intervention works within a simple model of real exchange rate determination. We demonstrated that this form of intervention changes the relative supply of bonds denominated in domestic and foreign currency. It will only influence the exchange rate when investors view the bonds as imperfect substitutes and relative yields adjust as a consequence.

Because the central bank does not allow base money to change in response to its foreign exchange purchases and sales, sterilized intervention may be of limited usefulness as a separate policy instrument. Nevertheless, the distinction between sterilized and monetary intervention is important because it isolates the direct effects of intervention operations on the exchange market from those of monetary policy operations.

Sterilized intervention has two important limitations in particular. First, while sterilized intervention may influence the relative supply of government bonds at the margin, bond issues floated to finance government budget deficits have played a predominant role in changing relative bond supplies. In theory, the shift in the relative supply of bonds is what influences the exchange rate, not whether the source of the change is due to foreign exchange operations or to finance budget deficits. Sterilized intervention will shift relative bond supplies to a greater extent than bond issues to finance deficits for a given purchase or sale of domestic bonds (because foreign bonds privately held will correspondingly increase or decrease with central bank domestic bond purchases or sales in the intervention case). For a given shift, however, the simple asset model predicts an equal influence on the exchange rate.

Second, is the related point that the central bank's ability to use sterilized intervention as an effective policy instrument will be smaller the larger the outstanding stocks of government liabilities. Quite simply, the rapid growth of government bonds since the mid-seventies has correspondingly increased the amount of sterilized intervention necessary to bring about a given exchange rate change (assuming the elasticity of substitution between foreign and domestic bonds remains unchanged) in Japan. In this sense, the central bank must be willing to commit ever growing resources in pursuit of their exchange rate objectives.

A simple model of real exchange rate determination, applied to Japanese-U.S. data over the 1973-82 floating rate period, provided some interesting results. The empirical results generally support the theoretical model and suggest that the relative supply of government bonds has influenced the yen-dollar real exchange rate. However, the coefficient estimates also suggest that a very substantial amount of sterilized intervention may be necessary to bring about a noticeable exchange rate movement. This result is primarily due to the large existing stocks of Japanese and U.S. government bonds outstanding. Another complication is that the Bank of Japan's intervention operations since the mid-seventies, though sizable by international standards, have been small relative to Japanese government bond issues financing budget deficits. In addition, some tentative evidence indicates that the effectiveness of sterilized intervention in Japan is further limited by the financial market liberalization measures implemented in recent years. These measures have tended to increase the substitutability between Japanese and U.S. bonds, and reduces the effectiveness of sterilized intervention.

It is difficult to draw strong policy conclusions about the overall usefulness of sterilized intervention from these results, however. We have used a very simple model in the analysis, and have identified only one channel through which sterilized intervention may influence the exchange rate. Sterilized intervention may influence the exchange rate through other channels as well. For example, sterilized intervention may have a "demonstration" effect, that is, it may influence expectations about underlying economic conditions or policies. This in turn might directly shift the expected future real exchange rate, q^e, and the current rate. In addition, the results presented here are based on monthly data while central bank intervention objectives may be of a shorter duration. Japanese intervention, even when sterilized, may have a considerable influence on short-term exchange rates (for example, daily) that is not picked up by monthly data.

Nevertheless, these results do support the simple asset market model of exchange rate determination presented here and suggest that the Bank of Japan's sterilized intervention operations have had only a small influence on the yen-dollar real exchange rate. However, it appears likely that sterilized intervention will become an even less potent policy instrument as the Japanese financial system becomes more closely integrated with its western counterparts.

Data Appendix

Data Sources: International Monetary Fund, *International Financial Statistics (IFS)*; Bank of Japan, *Economic Statistics Monthly (EMS)*; U.S. Treasury Department, *Treasury Bulletin*.

q = real yen-dollar exchange rate (log) = $\log(s) - \log(P) + \log(P^*)$;

P = Japanese Wholesale Price Index, (*IFS*, line 63);

P^* = U.S. Wholesale Price Index, (*IFS*, line 63);

r^j_{lt} = Japanese long-term real interest rate; weighted average of 5-10 year bond rates in Japan (Source: FRB Macrodata library); less expected inflation (see text for construction of expected inflation)

r^{us}_{lt} = U.S. long-term real interest rate; 10-year U.S. treasury bonds (Source: FRB macrolibrary); less expected inflation (see text for definition)

B^j = Japanese national government debt held privately; equal to total national government debt less government and Bank of Japan holdings. Converted to billion U.S. dollars with period average exchange rate. Source: *ESM* Table: "National Government Debt."

W = U.S. and Japanese financial wealth proxy; $W = B^j + B^{US} + M^j + M^{US}$

B^{US} = U.S. central government debt held privately; calculated as (1) "Estimated Ownership of Public Debt Securities by private investors—total privately held" (Table OFS-2) less (2) "selected U.S. liabilities to foreigners—total, official institutions" (Table IFS-2) less (3) "nonmarketable U.S. Treasury bonds and notes issued to official institutions and other residents of foreign countries" (Table IFS-3). *Treasury Bulletin*

M^j = Bank of Japan Reserve Money, (*IFS* line 14); (converted to billion U.S. dollars with period average exchange rate).

M^{US} = U.S. Federal Reserve, Reserve Money, (*IFS* line 14);

W^j = Japanese financial wealth; $W^j = M^j + B^j + JCASUM$

JCASUM = cumulative sum of Japanese current account from 1973:03 (billion U.S. dollars) (*ESM*) plus $11.5 billion (U.S. liabilities to Japan, 1973:03, *IFS* line 9a.d.)

ΣCA^s = cumulative current account surprise; cumulative sum of residuals from regressing Japanese current account less U.S. current account on six own lag values and three q lag values (from 1973:03).

FOOTNOTES

1. Central banks generally hold their foreign reserve assets which are used for intervention operations in the form of government interest-bearing securities, primarily U.S. Treasury securities, as the dollar is the predominant intervention currency.

2. Note that MB^d created through open market purchases of government securities does not necessarily equal the total monetary base. Base money may be supplied through central bank loans to commercial banks or through open market operations using commercial bills, for instance.

3. See Dooley and Isard (1979), p. 5, for a discussion of this issue.

4. See Mussa (1979) and J. A. Frankel (1981) for convincing arguments in support of an asset market approach as opposed to the more traditional "flow" models of exchange rate determination.

5. Isard (1980a, 1980b, 1982) derives this model in a similar fashion (in both nominal and real values) and terms his an "accounting identity" approach because it is based on an arbitrage condition and several identities. It is also derived and tested for a weighted average of the dollar's real value by Hooper (1983). Hooper ignores the risk premium in his empirical tests, however.

6. The covered interest rate parity condition, equation (2), generally holds in Japan once transactions costs are taken into account. See Seo (1981).

7. The yield differential is usually called the risk premium in the exchange rate literature. This is because risk adverse speculators must expect some profit if they are to hold an uncovered futures contract (and bear the risk associated with unexpected exchange rate fluctuations) which the interest rate arbitragers have purchased or sold to hedge their portfolios against exchange rate risk.

8. This condition also implicitly sets the condition for goods market equilibrium and has been introduced in this context by Kouri (1976) and Hooper (1983), amongst others. Note also that the sustainable long-run current account is assumed constant in this model.

9. A more complicated function is derived in Hooper and Morton (1982). They attempt to distinguish between transitory and permanent elements in the current account.

10. The asset demand function employed here is a simplification of Frankel's (1982) general approach. Frankel

extends the single foreign demand for domestic securities function developed here into two components: a "focus" foreign country demand function and a third-country investor demand function.

11. The few studies that have embedded exchange risk determinants into empirical exchange rate models have generally employed "abbreviated specifications" (e.g. Hooper and Morton (1982), p. 45) because of data considerations. These attempts have met with limited success in modeling the risk premium, however. Misspecification of the model may be one reason for the generally poor results heretofore. This model derives the determinants of the risk premium along the lines of Dornbusch (1980), Dooley and Isard (1979) and Frankel (1982), and employs this more complete specification in the exchange rate equation estimates. This formulation presents the risk premium as a function of both the relative supply of government bonds demonimated in domestic anf foreign currency and the international distribution of wealth among investors.

12. Another explanation of the adjustment process given changes in relative asset supplies or relative wealth considers the role of private capital flows. An increase in domestic assets at unchanged interest rates, for instance, will increase the proportion of home securities in private portfolios beyond the ratio desired by investors. As investors begin to sell home securities in favor of foreign securities in an attempt to bring portfolios back into balance, the resulting capital outflows from the domestic country puts downward pressure on the exchange rate. Increases in relative domestic wealth, on the other hand, will shift upward the demand for home securities, cause an incipient capital inflow and appreciate the domestic currency. If assets are considered perfect substitutes, i.e., investors do not differentiate between securities donominated in the home and foreign securities, then neither relative asset supplies nor relative wealth should influence the exchange rate through this channel (i.e., through the risk premium or "differential return" channel).

13. This measures Japanese government securities not held by the Bank of Japan or other Japanese government agencies (e.g., Trust Fund Bureau, Industrial Investment Special Account, Postal Life Insurance and Annuity). Of the total government debt held privately in December 1982, 99% was held in long-term instruments (internal bonds—consisting of construction bonds and deficit-finance bonds).

14. Quirk (1977) argues that the foreign exchange funds is the most appropriate proxy for Japanese official intervention because it includes "hidden intervention," i.e., Bank of Japan foreign exchange deposits with its member commercial banks, and excludes certain transactions with the U.S. military in Japan which are included in official reserve figures.

15. See, for instance, "A Japanese View of Exchange Rate Policy" written in 1982 by Shijuro Ogata, Bank of Japan Executive Director, for **Aussenwirtsehalf** of the University of St. Gallen.

16. By the national income accounting identity, it can be shown that the excess of private savings (S) over private domestic investment (I) equals the current account surplus (X-M) plus the government budget deficit (G-T): $S-I=(X-M)+(G-T)$. Beyond running large government budget deficits (measured either by the central or consolidated general government accounts), Japan has run a current account surplus over the greater part of the last decade and has generally been a net exporter of capital to the rest of the world.

17. See Hang-Sheng Cheng, "Crowding-Out: Japanese Experience," Federal Reserve Bank of San Francisco **Weekly Letter**, March 19, 1982, for an engaging discussion of the "crowding out" versus "filling-in" issue in the Japanese context. The common opinion expressed by government publications in Japan is that there is no evidence of government credit demands crowding out private investment thus far, but it is feared that it may become a problem in the future if budget deficits are not reduced. See **Economic Survey of Japan 1979/1980**.

18. The Fair estimation procedure is employed in all regressions. This is a statistical technique designed to provide consistent coefficient estimates of an equation with both autocorrelated error terms and endogenous explanatory variables. Which variables are assumed endogenous is particularly important because the empirical results are sensitive to this choice. Japanese relative bond supplies and Japanese real interest rates are treated as endogenous variables in the short-run. To the extent that Bank of Japan reacts to real exchange rate movements in its intervention operations, Japanese bond supply will be correlated with the error term in the exchange rate equation. Interest rates in Japan, both nominal and real, may also be systematically influenced by real exchange movements and are treated as an endogenous variable in the model estimation. The instruments for both endogenous explanatory variables are formed from the predicted values of a reduced form equation which includes contemporaneous and lagged exogenous variables in the system (r^{us}, W^{j}/W, CA^{s}), the lagged endogenous variables (q, B/W, r^{j}) and a time trend. See Fair (1970).

19. This measure is similar to Hooper (1983) and Shafer and Loopesko (1983).

20. The financial wealth measure has been broadened from that defined in the theoretical section. To private bond holdings have been added (U.S. and Japanese) base money. This adjustment has been made to help distinguish better empirically between domestic bond supply and domestic financial wealth.

21. All Japanese data used in computing B^{j}, W and W^{j} are converted from yen to U.S. dollars at the average month-to-month yen/dollar exchange rate.

22. A related point concerns the structural stability of the exchange rate equation and estimated coefficients to different sample periods, changes in variable definitions, and the choice of estimation technique. Exchange rate models

have generally had poor out of sample forecasting performance in recent years, and have demonstrated significant structural instability. The results here should therefore be interpreted with this important qualification in mind.

23. See Charles Pigott's "Financial Reform in Japan," in Federal Reserve Bank of San Francisco **Economic Review**, Winter 1983, for a comprehensive review of the process of liberalization in Japanese financial markets.

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Indicators of Long-Term Real Interest Rates

Charles Pigott*

Longer term real interest rates cannot be measured directly, but their movements can be estimated from economic indicators they affect, particularly foreign exchange rates and nominal interest rates. An increase in U.S. nominal interest rates that is accompanied by a rising dollar indicates that U.S. longer term real interest rates probably also have risen. On this basis, the unprecedentedly high level of the dollar in recent years strongly suggests that U.S. long-term real interest rates remain very high by historical standards.

Since October 1979, when the Federal Reserve announced a major change in its operating procedures, interest rates here and abroad have fluctuated to a degree unprecedented in post-war experience. These fluctuations have generated great controversy, both about their origins and their consequences. Most perplexing of all have been the gyrations in longer-term interest rates, particularly their apparent tendency to vary with seemingly short-term disturbances in the markets.[1] This turmoil and confusion has come at a particularly unwelcome time, as financial innovation and deregulation sometimes have made it more difficult to predict the impact of the monetary aggregates targeted by the Federal Reserve, and hence increased the need for other indicators of the effect of policy on the economy.

These circumstances have underscored the need for measures of medium and long-term real interest rates and expected inflation. In theory, medium and long-term real interest rates are important determinants of investment and other real spending decisions. Knowledge of their level could be helpful in gauging the future course of economic activity, as well as the effect of current monetary and fiscal policies on the economy. Inflation anticipated over the next several years would provide an indication of public perceptions about the future course of monetary policy, and thus about the credibility of the authorities' public commitments to maintain price stability. Unfortunately, it is very difficult to measure longer-term real interest rates or expected inflation, mainly because inflation expected over the next several years need not depend in any predictable way on past trends.

The basic objective of this paper is to demonstrate a practical method for measuring medium and long-term real interest and expected inflation rates for the U.S. This method uses several economic variables affected by real interest rates and/or expected inflation as "indicators" of their movements. Included among these variables are the spot and forward exchange values of the dollar, which are shown to be closely related to long-term real interest rates and expected inflation. Estimates of longer-term real interest rates and expected inflation can then be calculated from weighted averages of the indicators. As explained in the next section, underlying this approach is the observation that real interest rates and expected inflation have very different impacts on certain other financial variables, such as exchange rates. Hence, the way in which these variables move when nominal interest rates vary provides a clue about the extent to which real interest rates and expected inflation have changed.

The next section describes the relations among

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real interest rates and expected inflation and the financial variables used as their indicators. An intuitive description of how the indicators can be used to measure real interest rates (and expected inflation as well) is also given. A more precise and technical description of the approach is given in the Appendix. Our estimates of the actual variations in monthly real interest rates over 1976—mid-1983 are given in Section II. One important finding is that the variability of long-term real interest rates has apparently increased dramatically since the change in Federal Reserve operating procedures in 1979. Another is that long-term real interest rates have remained relatively high in 1982 and 1983, despite a substantial fall in nominal interest rates.

I. Indicators of Real Interest Rates

Any interest rate can be conceptually divided into two parts: an *inflation premium* and a (before tax) *real interest rate*. The inflation premium is equal to the amount of inflation expected *over the life* of the investment and serves to compensate for the erosion of the purchasing power of the funds lent. For example, an individual who lends $100 for one year at a 10% rate is not really better off at the end of the year if inflation during the year is also 10%; then the $110 repaid at the end of the year buys the same amount of goods and services as the amount lent could have purchased a year earlier, and so no real return on the investment is gained.[2]

The real interest rate, which equals the nominal rate less the inflation premium, thus measures the amount of additional purchasing power an investment yields. So if the nominal interest rate were 12% while inflation was expected to be 10%, the real interest rate would be 2%. This relation can be written for reference as:

$$iu_t(n) = ru_t(n) + \Pi u_t(n) \qquad (1)$$

where iu(n) is the U.S. nominal interest rate on a n-year security, ru(n) is the corresponding U.S. real interest rate, and Πu(n) is the expected inflation rate over the next n years, that is, the inflation premium.

The real interest rate and the inflation premium are likely to have very different impacts on economic behavior. Economic theory implies that individuals' and businesses' real spending decisions are influenced by the real interest rate, but little, if at all, by the inflation premium. Inflation expectations, as reflected in the inflation premium, are likely to be important determinants of wages and prices set in contracts, besides serving as a gauge of the credibility of authorities' policies to ensure price stability. The effects of fluctuations in nominal interest rates on the economy thus will depend upon the extent to which they reflect changes in real rates or in expected inflation. For this reason, real interest rates and inflation premia are generally more useful to know than nominal rates alone.

Only nominal interest rates are actually quoted in financial markets, however. Their real and expected inflation components are not directly observable. Plainly, there is no way to determine from changes in nominal interest rates *alone* how much these components have varied.

Thus to measure real interest and expected inflation rates requires some information in addition to that provided by nominal interest rates. A common approach to this problem is to use an independent measure of expected inflation. For example, inflation over the near-term future is generally most closely related to that experienced in the recent past. The inflation premium on a short-term security can often be approximated by the current trend in actual inflation, providing a rough measure of the short-term real interest rate. On this basis it seems fairly clear that U.S. short-term real interest rates have fluctuated considerably over the last several years, much more so than during the 1970s.[3]

Unfortunately, this approach is not appropriate to the measurement of medium and longer-term real interest rates. Inflation expected over the coming *years* depends critically upon the macroeconomic policies authorities follow in the future. Public perceptions about these future policies—indeed any reasonable guess about them—need not be related in any obvious or dependable way to past trends, and therefore are apt to be extrememly difficult to gauge correctly. Who, after all, can pretend to know with any confidence what the stance of monetary and fiscal policies will be several years from now?

Indicators

An alternative approach is to look for economic variables whose movements provide clues about the variations in real interest rates and/or expected inflation, and hence serve as *indicators* of their values. In principle, any variable that is affected by real interest rates or expected inflation rates could serve as such an indicator. In this sense, nominal interest rates are indicators of their real and expected inflation components.

Particularly helpful as indicators are financial variables which react differently to real interest rates than to expected inflation. Suppose that a certain financial variable tended to increase when real interest rates rose, but generally was unaffected by fluctuations in expected inflation. Then a rise in nominal interst rates that was accompanied by an increase in this variable would suggest that real interest rates had increased. A variable that was affected by expected inflation but not by real interest rates could be used in an analogous fashion.

Admittedly, no particular indicator is likely to provide a completely accurate measure of either real interest rates or expected inflation. Still, it ought to be possible to use several such indicators to estimate, or approximate, these components of nominal interest rates. This approach, which attempts to 'read' the signals provided by financial markets, is the one taken here.

But What Indicators?

Changes in interest rates expected to prevail in the distant future ("long-run") are apt to be especially good indicators of *expected long-run* inflation. Consider the nominal (say one-year) interest rate now (at t) expected to prevail 'many,' or N, years in the future. This will be referred to as the *forward* interest rate and denoted fiu_t (it being understood that it is the forward rate corresponding to many years in the future).

$$fiu_t = E_t iu_{t+N}$$

($E_t iu_{t+N}$ stands for the "expected value" of iu_{t+N} based on information available at t). This can be *approximated* from the term structure of nominal interest rates, since long-term rates are approximately averages of expected future shorter-term rates.[4] As with any nominal interest rate, the forward rate is composed of the real interest and the inflation rates expected to prevail in the "long-run," that is, N years from now.

In the long-run though, the real interest rate is mainly determined by the productivity of capital, which in turn reflects the savings decisions of households, businesses, and government, the growth of the labor force, and the rate of progress of technology. Generally these conditions change slowly, so that the expected *long-run* real interest rate (as reflected in the forward interest rate) can be regarded as essentially constant when considering a period of several years. Variations in current real interest rates can be viewed as resulting from *non-persistent* imbalances in supply and demand in money and credit markets. For example, economic theory suggests that because of the lag between money and inflation, increases in money growth temporarily lower real interest rates by raising real balances; real interest rates return to their original values once inflation "catches up," however.[5]

This reasoning implies that changes in the long-term forward interest rate mainly reflect shifts in long-run expected inflation. So a rise in the forward rate corresponding to ten years in the future would measure the change in inflation expected to prevail ten years from now, or

$$\Delta fiu_t = \Delta E_t \Pi u_{t+N} \qquad (2)$$

where $E_t \Pi u_{t+N}$ is inflation expected to prevail *beginning* N years in the future. The forward rate also provides a more indirect indication of inflation expected to prevail over the *next* ten years, that is of the inflation premium in medium and long-term nominal interest rates. The reason is that an increase in inflation expected ten years from now suggests that inflation over the next several years has also increased. Thus it is more likely that an increase in, say, the 5 year nominal interest rate reflects an increase in expected inflation over the next five years if the forward interest rate has also increased than if it has not.

Conversely, a rise in the nominal n-year interest rate relative to the forward interest rate is more likely to signal an increase in the real interest rate than would an increase in the nominal interest rate taken by itself. Hence, the change in the *difference* between the rate on a n-period asset and the forward rate,

$$I1: \Delta diu_t(n) = \Delta iu_t(n) - \Delta fiu_t$$

is apt to be a better indicator of variations in the n-year real interest rate than changes in the nominal interest rate itself. This indicator is composed of changes in the real interest rate plus changes in the difference between inflation anticipated over the next n years and that expected in the long run.

$$\Delta diu_t(n) = \Delta ru_t(n) + \Delta d\Pi u_t(n), \quad (3)$$
$$\text{where } d\Pi u_t(n) = \Pi u_t(n) - E_t \Pi u_{t+N} .$$

Thus this new indicator effectively removes from nominal interest variations that portion of shifts in expected n-year inflation that simply reflect movements in anticipated long-run inflation. Since inflation expected over the next several years is apt to be closely related to long-run inflation expectations, this implies that real interest rates are likely to account for a larger proportion of the variations in the indicator I1 than those of the nominal interest rate. This suggests that I1 will be the better indicator of real interest rate movements (although by no means an exact one).[6]

Exchange Rate Indicators

The foreign exchange value of the dollar is closely related to U.S.—and foreign—interest rates simply because in comparing the yields on investments in different currencies an individual must take account of the expected change in the exchange rate between them. For example, if the interest rate on a one-year German-mark denominated-security were 5 percent while the mark were expected to appreciate by 3 percent over the year vis a vis the dollar, its expected yield *in dollars* would be 8 percent.

Because of the risk that exchange rates will not change by exactly the amount originally anticipated, the expected dollar yields on securities identical in all respects except the currencies they are denominated in may differ.[7] However available evidence suggests that in the absence of capital controls, such currency 'risk' premia are not very large, at least among the U.S. and other major industrial nations.[8] Thus the difference between U.S. and foreign interest rates for a given maturity can be viewed as a reasonable approximation of the expected change in foreign currency value of the dollar, expressed at an annual rate:

$$iu_t(n) - if_t(n) \cong (1/n)[e_t - E_t e_{t+n}] \quad (4)$$

where e_t is the current foreign currency price of the dollar (expressed in logarithms) and $E_t e_{t+n}$ is its expected value n years from now.

This relation between nominal interest rates and the nominal exchange rate is easily converted to one between real interest rates and the real exchange rate. The *real* exchange rate, x_t, is simply the nominal exchange rate 'deflated' by the ratio of the foreign to the U.S. price level:

$$x_t = e_t + (pf_t - pu_t)$$

where pf and pu are the logarithms of the foreign and U.S. price levels.

The real exchange rate measures the value of foreign goods and services in terms of our own, or the rate at which U.S. and foreign products can be exchanged for one another. Suppose a 'basket' of U.S. goods sells for one dollar (our price level is one) while a 'basket' of German goods sells for one mark. Then if the nominal exchange rate is 2 marks/dollar, 2 baskets of German goods are needed to obtain one basket of U.S. products. Hence, the real exchange rate for the dollar is two.

As this suggests, the real exchange rate is a reflection of the relative value of U.S. versus foreign products. Ultimately, this rate will be determined by supply and demand conditions in product and factor markets. Furthermore, in the long-run, the level of the real exchange rate should be largely unaffected by inflation (since inflation's effect on *relative* prices is neutral, at least approximately) or by real interest variations (since these result from temporary disturbances in financial markets).

Subtracting the U.S. minus the foreign expected inflation rate from (2) and rearranging gives,

$$ru_t(n) - rf_t(n) = (1/n)[x_t - E_t x_{t+n}]$$

or

$$x_t = n[ru_t(n) - rf_t(n)] + E_t x_{t+n} \quad (5)$$

where $E_t x_{t+n}$ is the *future* real exchange rate expected to prevail after n years. The relation shows that the n-year real interest differential effectively measures the divergence between the current real exchange rate and that expected to prevail *at maturity*.

This relation also implies that increases in the long-term U.S.–foreign real interest differential raise the *current* real exchange rate, x_t. For example, an increase of one percentage point in the (annualized) 5 year U.S. real interest relative to abroad, all other factors the same (that is, no change in the expected 'long-run' real exchange rate) will raise the real value of the dollar by five percent. In this sense, variations in long-term real interest rates can have very substantial impacts on actual real exchange rates.[9]. It follows that variations in the current real exchange rate are an indicator of the U.S. (and foreign) long-term real interest rates; a rise in x_t suggests that our real interest rate may have gone up.[10]

$$I2: \Delta x_t = n[\Delta ru_t(n) - \Delta rf_t(n)] + \Delta E_t x_{t+n}$$

Finally, the two indicators defined above can be combined with the foreign interest rate to yield an indicator of the change in our expected inflation—again expressed *relative* to that anticipated for the distant future. Define

$$y_t = x_t + n[dif_t(n) - diu_t(n)]$$

where the foreign interest rate indicator, $dif_t(n)$, is defined analogously to that for the U.S. Now using the expressions for the real exchange and interest rate indicators (see I1 and I2) gives:

$$I3: \Delta y_t = n[\Delta d\Pi f_t(n) - \Delta d\Pi u_t(n)] + \Delta E_t x_{t+n}$$

where again $d\Pi f_t(n)$ is analogous to the corresponding U.S. variable. The variable defined in I3 can be regarded as a third indicator of the U.S. long-term real interest rate. The reason is that its variations provide information about the expected inflation component of the U.S. nominal interest rate indicator, I1, and hence *indirectly* about its real interest component. In particular, a rise in this indicator suggests a *fall* in U.S. expected inflation, and therefore an increase in the U.S. real interest rate for any given value of the nominal interest rate indicator. This third indicator will be referred to as the deflated 'forward exchange rate,' since it is effectively the n-year forward exchange value of the dollar (the currently quoted value of the dollar for delivery n years from now) deflated by the current U.S.-foreign price level ratio, and expressed relative to the U.S.-foreign forward interest differential.[11]

How Do We Use Them?

The analysis has identified three potential indicators of the real and expected inflation portions of the long-term nominal interest rate, namely changes in: the n-year nominal interest rate relative to the forward interest rate; the current real exchange rate; and the (n-year) deflated forward exchange rate. The relations between these indicators and real interest rates are summarized in Table 1. The likely increase in the real interest rate accompanying a given rise in the nominal interest rate (iu) is greater:

(i) the larger the accompanying *rise* in the interest rate indicator I1;

(ii) the larger the accompanying *increase* in the real exchange value of the dollar, I2;

(iii) the smaller the *decline* in the forward exchange rate indicator, I3 (since a decline in y suggests a rise in expected U.S. inflation).

These observations suggest that movements in real interest rates and expected inflation can be estimated from variations in the indicators. An obvious course is to use weighted averages of the indicators as these estimates, say:

$$\Delta ru_t(n) = W1\Delta diu_t(n) + W2\Delta x_t + W3\Delta y_t. \quad (6)$$

Ideally the weights used should reflect the *average* degree to which the real interest rate changes with the indicators. For example, W1 should reflect the average change in the real interest rate corresponding to a given change in the interest rate indicator, all other indicators being constant.

Table 1
The Indicators and Their Relations

Indicators

I1: $\Delta diu_t(n)$ — change in the n-year U.S. nominal interest rate (expressed in logarithms) less the (log of) the forward interest rate, fiu_t

I2: Δx_t — change in the logarithm of the U.S. real exchange rate, calculated as the spot foreign currency/\$ rate times the ratio of U.S. to foreign price level (using consumer prices)

I3: Δy_t — change in the deflated forward exchange value of the dollar (relative to the U.S.-foreign forward interest rate differential), again in logarithms. $y_t = x_t + n\ (dif_t(n) - diu_t(n))$

Relations

(i) $\Delta diu_t(n) = \Delta ru_t(n) + \Delta d\Pi u_t(n)$

(ii) $\Delta x_t = n\ [\Delta ru_t(n) - \Delta rf_t(n)] + \Delta E_t x_{t+n}$

(iii) $\Delta y_t = n[\Delta d\Pi f_t(n) - \Delta d\Pi u_t(n)] + \Delta E_t x_{t+n}$

Of course such estimates of changes in the U.S. real interest rate cannot be expected to be exact, mainly because, as can be seen from Table 1, the indicators are affected by other variables as well. For example, the real exchange rate indicator is affected by the foreign real interest rate and the expected future real exchange rate, as well as the U.S. real interest rate. In fact, there are five "underlying" variables making up the set of indicators (the U.S. and foreign real interest rates, the U.S. and foreign expected inflation rates, and the expected future real exchange rate) none of which are directly observable. Given that there are only three indicators, none of these underlying variables can be determined exactly.

If direct observations of real interest rates were available, the weights, W, could be estimated simply by performing a *regression* of the form in (6), that is of changes in the real interest rate on the indicators. The problem then is, how can this regression be performed *without* any direct measurements of the dependent variable, namely the changes in the U.S. long-term real interest rate?

As explained in more detail below (and more completely in the Appendix), this regression, that is the estimation of the weights, can actually be carried out indirectly given certain additional assumptions discussed below. In effect, the weights can be inferred from clues provided by the relations among the indicators. Recall, for example, that an increase in the U.S. nominal interest rate indicator due to a rise in the real interest component, will, all other factors held constant, be associated with an increase in the real exchange rate indicator. This suggests that the greater the extent to which U.S. nominal interest rate and exchange rate indicators actually tended to move together, the greater the weight, W2, is apt to be.

Obtaining the Weights

To see more precisely what is involved in estimating these weights, let I_t stand for the vector of the indicators at 't.' (The following discusion is a bit technical; readers interested mainly in the results can skip to Section II: Empirical Results.)

$$I_t = [\Delta diu_t(n), \Delta x_t, \Delta y_t]$$

Then the weights given in (5) are defined by:

$$W = Cov[\Delta dru_t(n), I_t] Var(I_t)^{-1}, \qquad (7)$$

where W=(W1,W2,W3), Cov( ) stands for the covariance of changes in the U.S. real interest rate with the three indicators, and Var( ) is the variance matrix of those indicators.

The weights, W, are those that would be estimated if the regression (6) could actually be performed directly. Estimates of the change in the real interest rate using these weights are 'optimal' in the sense that they minimize the average (squared) divergence between the estimated and actual values of $\Delta ru_t(n)$ (in comparison with any other weighted average of the indicators).[12]

With no direct observations of the real interest rate, the technical problem becomes that of estimating the covariance of real interest rate changes with the indicators. (Clearly, the variance matrix of the indicators can be estimated directly). However it can be seen from Table I that these are determined by the relations—that is the variances and covariances—among the underlying variables that make up the indicators, the U.S. and foreign real interest and expected inflation changes, and changes in the expected future real exchange rate. For example, the covariance of the U.S. real interest rate change with the nominal interest rate indicator is determined by the variance of fluctuations in the real interest rate *and* its covariance with changes in U.S. expected inflation.

As suggested earlier, the relations among the indicators provide the primary source of information about the relations among the variables underlying them. A simplified example illustrates this. Suppose that changes in the U.S. real interest rate were independent of (uncorrelated with) the other underlying variables. Then the observed covariance between the interest rate and real exchange rate indicators is:

$$Cov[\Delta diu_t(n), \Delta x_t] = nVar[\Delta ru_t(n)]$$

In short, *under these assumptions*, the variance of the real interest rate, and hence its covariance with the nominal interest rate indicator, could be calculated from the observed relation between the U.S. nominal interest rate and the real exchange rate.

Proceeding in this way, it would appear possible to estimate the relations among all the underlying variables (their variances and covariances) from the relations among the indicators. This in turn would define the relation of real interest rate changes to the indicators, allowing the weights, W, to be estimated. This is the sense in which the approach taken here amounts to an 'indirect' regression.

The complication is that it cannot plausibly be assumed that real interest rates are independent of all other underlying variables. More generally, the relations among these variables could be fairly complex; for example, there might be complex interactions between U.S. and foreign real interest rates and expected inflation, and, if so, these would affect the way in which the relations between the interest rate and exchange rate indicators are interpreted. Once these possibilities are allowed for, the information provided by the relations among the indicators is no longer sufficient to determine those among the variables underlying them. The reason is that there are five underlying variables, and hence more relations among them than for the three indicators. Thus while the relations among the indicators will continue to be the primary basis for the estimation of weights, some additional assumptions, suggested by economic theory or other data, must be made.[13]

Assumptions

One assumption is suggested by the earlier discussion, where it was argued that the real exchange rate is unaffected in the *long-run* by inflation or real interest fluctuations. This implies (*assuming* that n-years is sufficient for this long-run condition to hold):[14]

(A1) Changes in the expected future (n-years from now) real exchange rate, $\Delta E_t x_{t+n}$, are uncorrelated with changes in the n-year U.S. and foreign real interest and expected inflation components.

It is also reasonable (and necessary) to restrict the cross-country relations among real interest rates and expected inflation by assuming that foreign expected inflation changes have no direct impact on U.S. real interest rates, and similarly for U.S. expected inflation and the foreign real interest rate. This can be stated as:

(A2) Foreign expected inflation affects the U.S. real interest rate only to the extent to which it affects U.S. expected inflation. Similarly, U.S. expected inflation affects the foreign real interest rate only via its impact on foreign expected inflation.[15]

Finally, the estimation also requires some assumption, that is, prior estimates, concerning the *average* response of U.S. and foreign real interest rates to their respective changes in expected inflation. These responses are measured by the 'coefficients' bu and bf defined as:

bu = average change in $ru_t(n)$ given a one percent change in $d\Pi u_t(n)$

bf = average change in $rf_t(n)$ given a one percent change in $d\Pi f_t(n)$

Similarly, some prior assumption must also be made about the average response of variations in foreign real interest rates to changes in the U.S. real interest rate, measured by the coefficient g defined as:

g = average change in $rf_t(n)$ given a one percent change in $ru_t(n)$.

Given these assumptions, the relations (covariances) among the five underlying variables can be expressed in terms of (see Appendix): their (5) variances; and the relation (covariance) between U.S. and foreign expected inflation. These parameters can then be calculated from the six independent variables provided by the covariance matrix of the indicators—once, that is, the values of bu, bf and g are specified. The way in which these coefficients are estimated is described briefly in the next section and in more detail in the Appendix.

II. Empirical Results

The analysis of the previous section will now be applied to estimate actual changes in U.S. real interest rates and expected inflation for the period 1976–mid-1983. These estimates will be based on the five year U.S. and *German* government bond rates (n=5), which are taken to be the 'long-term' nominal interest rates. The forward interest rates correspond to 7 years in the future for the U.S., and five years for Germany, while the exchange rate indicators are based on the foreign exchange value of the dollar vis a vis the German mark.[16] Separate estimates are calculated for the sub-periods before and after June 1979. The reason is that the variability of nominal interest rates changed dramatically in 1979 (especially after the change in Federal Reserve operating procedures in October of that year), as did their relation to exchange rates. This suggests that the behavior of real interest rates, and their relation to the indicators, also changed, and that the appropriate weights prior to mid-1979 are not the same as those applying after that date.[17]

A First Look

Its useful to begin by examining relations among the indicators to see what they tentatively suggest about the extent of fluctuations in real interest rates and expected inflation. For this purpose, Table 2 lists measures of the actual extent to which a given indicator tended to vary with a given change in each of the others (these are based on the variances of the indicators and correlations among them).

Several tentative conclusions are suggested by the figures in the table. First, the very weak relation between changes in the U.S. interest rate indicator and the real exchange rate for the earlier period suggests that the real interest rate's variability was low in comparison with that of the nominal interest rate itself. The analysis of the last section implies that a one-percent increase in the 5-year real interest rate will, all other factors held constant, raise the real exchange rate by 5 percentage points. Thus if real interest rates were the main source of changes in the U.S. nominal interest rate indicator, that indicator could be expected to be associated with more than proportionate changes in the real exchange rate in the same direction.

In fact, during the earlier period, a one percentage point rise in the nominal U.S. interest rate indicator was, on average, associated with only a 0.9 percent increase in the real exchange rate. This suggests that changes in expected inflation, rather than in real interest rates, were the main sources of variations in U.S. nominal interest rates during this period—a conclusion supported by a number of previous studies of short-term interest rates.[18] Similar reasoning suggests that the variability of real interest rates rose substantially from the first to the second period: on average the real exchange rate increased by about 3 percent for each 1 percentage point rise in the U.S. interest rate indicator after mid-1979.

Second, the data suggest that the variability of (changes in) expected inflation may also have risen substantially from the first to the second period. This is suggested by the fact that the variability of the forward exchange rate indicator (which helps measure foreign relative to U.S. expected inflation) rose dramatically. (In addition, the U.S. forward interest rate variability also increased sharply after mid-1979).

Third, the data also suggest that there may be considerable variability in the expected future real

Table 2
Relations Among The Indicators[1]

	Standard Deviation (Basis Points)	Average response to 1 percentage point change in:[2] $\Delta diu_t(5)$	Δx_t	Δy_t
I. First Period (1976.01–1979.06)				
a. $\Delta diu_t(5)$	17.3	—	.01	−.07
b. Δx_t	177.9	.92	—	.42
c. Δy_t	306.3	−2.50	1.24	—
Memo: $\Delta dif_t(5)$	37.3	−.37	.06	.10
II. Second Period (1979.07–1982.12)				
a. $\Delta diu_t(5)$	40.3	—	.05	−.05
b. Δx_t	300.7	3.06	—	.65
c. Δy_t	342.9	−2.48	.03	.04
Memo: $\Delta dif_t(5)$	26.0	.11	.03	.04

[1] For variable definitions, see Table I.

[2] This is the coefficient in a *bivariate* regression of the column variable on the row variable; for example, the response of $\Delta diu_t(5)$ to Δx_t is: $\mathrm{Cov}(\Delta diu_t(5), \Delta x_t) / \mathrm{Var}(\Delta x_t) = .01$ for the first period.

exchange rate, $E_t x_{t+5}$. The analysis in the last section (see Table 1) showed that movements in the actual real exchange rate reflect changes in the U.S.-foreign real interest differential, or shifts in the expected future real exchange rate, or both. The actual real exchange rate indicator was in fact highly variable in both periods. Yet, as argued above, the Table 2 figures do not point to much variability of the U.S. real interest rate, or indeed (similar reasoning would show) to much variability in the foreign real interest rate, over the first period. This suggests that much of the variability of the actual real exchange rate was due to changes in its expected future value. The same conclusion is suggested by the fact that the actual real exchange rate and deflated forward exchange rate indicators are very positively correlated. The expected future real exchange rate is the factor common to variations in these two indicators, and so if fluctuations in $E_t x_{t+5}$ were substantial, the real and deflated forward exchange rates could be expected to move closely together—as in fact they did.[19]

Variability of Real Interest Rate Changes

Table III lists estimates of the variability of the U.S. five year real interest and expected inflation changes obtained using the procedures outlined in the previous section. (The 'memo' lines in the Table are intended to provide an indication of how the estimates are affected by alternative choices of the prior-estimated parameters, bu, bf, and g.)

For the first period, the estimates are based on measures of the average response of U.S. and foreign real interest rates to their respective expected inflation rates estimated from observed *short-term* interest rates and expected inflation (see Appendix for details). This amounts to assuming that the average response of longer-term real interest rates to expected inflation (that is, bu and bf as defined earlier) is essentially the same as that for short-term rates and is plainly only an approximation.[20] It was also assumed for the period prior to mid-1979 that U.S. real interest rates had no *direct* impact on foreign real interest rates, which is consistent with previous studies suggesting that authorities abroad did *not* systematically vary their domestic interest rates in response to variations in U.S. (real) interest rates.[21]

The second period results are based on the assumption that the variability of changes in the expected future real exchange rate is the same as that estimated for the first period. (This leads to estimates that seem more plausible than those based on bu and bf estimates from short-term interest rates). This amounts in effect to assuming that all of the increased variability in the actual real exchange rate from the first to the second period is due to

Table 3
Estimates of the Variability of Real Interest Rates and Expected Inflation

				— Standard Deviation (Basis Points) —						
	bu	**bf**	**g[1]**	**$\Delta ru(5)$**	**$\Delta d\Pi u(5)$**	**$\Delta \Pi u(5)$**	**$\Delta rf(5)$**	**$\Delta d\Pi f(5)$**	**$\Delta \Pi f(5)$**	**$\Delta E_t x_{t+5}$**
I. 1976.01–1979.06										
Estimates:	0	−.33	0	10.0	14.1	23.6	22.3	51.1	26.0	128.1
Memo: estimates with alternative bu, bf, g[3]	−.30	−.33	−.10	13.4	16.8	17.2	22.9	50.5	25.3	113.1
II. 1979.07–1982.12										
Estimates:	−.63	−.63	.20	47.0	47.4	95.4	41.1	62.2	99.0	123.0
Memo: estimates with alternative bu, bf, g[3]	−.44	−.20	.20	39.6	20.8	76.6	10.5	30.7	65.4	253.3

Notes:

[1] 'g' is the average change in the foreign real interest rate for a given change in the U.S. real interest rate; $g = \mathrm{Cov}\,[\Delta ru_t(5), \Delta rf_t(5)]/\mathrm{Var}[\Delta ru_t(5)]$

[2] See Appendix for details on how the estimated variance of the 5-year expected inflation change, $\Delta \Pi u_t(5)$ and $\Delta \Pi f_t(5)$, is obtained.

[3] The 'memo' estimate for bu for the first period is taken from Mishkin's (1981) estimates; the 'memo' bu and bf for the second period are taken from Appendix Table A1, using short-term nominal interest rates and inflation.

increased variations in real *interest* rates. In addition, the average response of foreign real interest rates to those in the U.S. is estimated from short-term interest rates for this period. The reason is that there is at least casual evidence to suggest that foreign authorities may at times have 'reacted' to interest rate changes in the U.S. after 1979.[22]

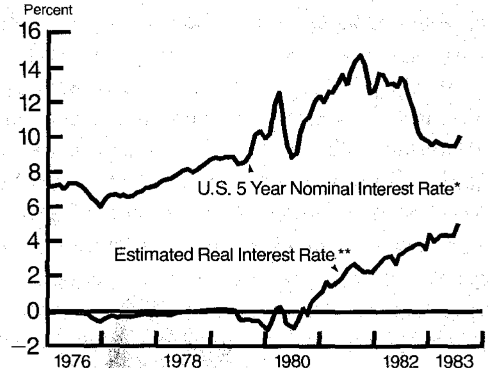
The results in Table 3 have three major implications. First, as suggested earlier, the variability of the U.S. real interest rate in the first period was relatively low compared to that of expected inflation. Apparently, movements in expected inflation dominated fluctuations in nominal long-term interest rates, as previous studies have suggested is the case for short-term rates. This conclusion seems to be reasonably robust, in the sense that it remains even if U.S. real interest rates and expected inflation are assumed to be substantially negatively correlated.

Second, the variability of U.S. real interest rates rose dramatically after the Federal Reserve stopped 'smoothing' nominal interest rates in 1979. This conclusion too is very robust, since it holds even if the prior-estimated parameters (bu, bf, g) are assumed the same as for the first period. More surprising, perhaps, is that the variability of U.S. expected inflation has also increased and apparently continues to be greater than that of the real interest rate.

Finally, the results imply that variations in real interest rates have *not* accounted for *all* the variations in the 'long-run' real exchange rate. For the first period, variations in the long-run real exchange rate accounted for about half of the variations in the current real exchange rate. This result is of interest, since it suggests that *purchasing power parity*, that is U.S. versus foreign inflation, is not the *sole* determinant of nominal exchange rates in the long-run, as is often asserted to be the case.[23]

Chart 1

Nominal and Estimated Real Interest Rates



* U.S. 5-year government bond rate.

** Real interest rate measured as the cumulative change since December 1975.

Estimates of Real Interest Rates

It is now straightforward to estimate the actual variations in U.S. long-term real interest rate. The weights on the indicators corresponding to the estimates in Table 3 are given in Table 4. In some cases, these weights are more easily interpreted by rewriting the estimating relations in terms of the U.S. interest rate, the real exchange rate, and the *foreign* interest rate, as is done in the last three columns of Table 4 (see the relations in Table 1). Note that in this rewritten form the coefficients of the U.S. nominal interest rate indicator are all positive, as are those on the real exchange rate.[24]

Chart 1 plots the estimates of the cumulative change in the U.S. real interest rate for January 1976 through July 1983 obtained from these weights

Table 4
Relations For Estimating Real Interest Variations

	Coefficient of $\Delta\hat{ru}_t$ on:				Implied Coefficients of:[2]		
	Δdiu_t	Δx_t	Δy_t	'R^2'[1]	Δdiu_t	Δx_t	Δdif_t
Period I (1976.01–1979.06)	.335	.012	.001	.39	.330	.013	.005
Period II (1979.07–1983.07)	−.583	.275	−.164	.60	.237	.111	−.816

Notes:

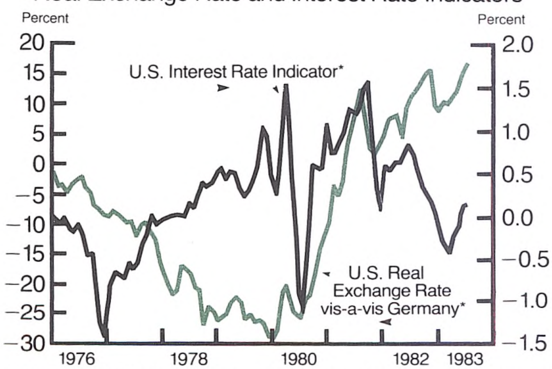
[1] *Estimate* of the fraction of variance of changes in the real interest rate accounted for by the indicators: $V(\Delta\hat{ru}_t)/V(\Delta ru_t)$ where $V(\Delta\hat{ru}_t)$ is the variance of the *estimates* of Δru_t calculated from the above relations.

[2] Obtained using the expression for Δy_t in Table 1.

and the indicators.[25] As expected, the estimates imply that real interest rates fluctuated little before 1979, but considerably more after then. Apparently, real interest rates rose from mid-1979 through April 1980, fell back through the following June, and then generally rose over the next several years. A particularly interesting implication of these results is that U.S. long-term real interest rates remained quite high over August 1981 through December 1982 even as our nominal interest rates declined sharply. The nominal 5-year interest rate fell by nearly 5 percentage points over this period, yet the estimates suggest that the real interest rate actually increased, by nearly one percent. This suggests that the decline in U.S. nominal interest rates during this period reflected a very sharp drop in expected inflation, rather than any substantial decline in real interest rates. Note also that the estimates of the long-term real interest rate generally

Chart 2A

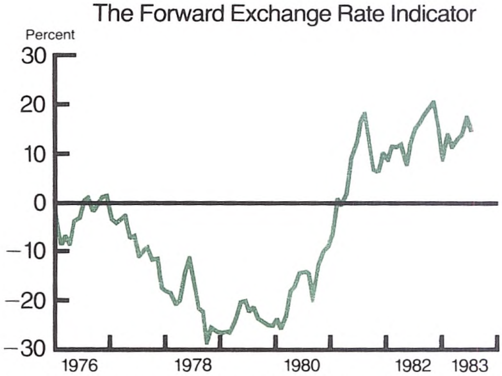
Real Exchange Rate and Interest Rate Indicators



* Both variables measured as the cumulative percentage change since December 1975.

Chart 2B

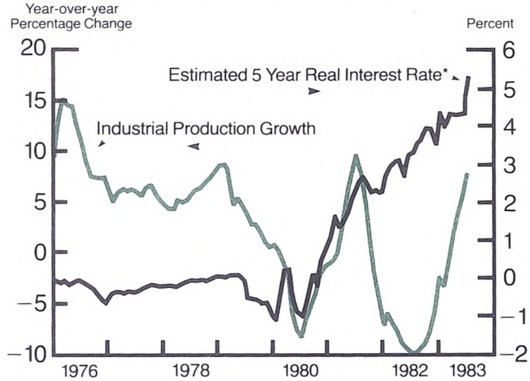
The Forward Exchange Rate Indicator



* Cumulative percentage change in the indicator since December 1975.

Chart 3

Industrial Production Growth and Estimated Real Interest Rate



* Real interest rate measured as the cumulative change (percentage points) since December 1975.

moved with short-term real interest rates until mid-1982, when the latter fell sharply.

That the U.S. real interest rate remained very high during 1982 is strongly suggested by the fact that both the real exchange rate and forward exchange rate indicators rose substantially during the period (the U.S. *forward* interest rate also fell by nearly as much as the 5 year nominal interest rate—see Chart 2.) Recall that increases in the real exchange rate suggest a rise in our real interest rate, while increases in the forward exchange rate signal a drop in our expected inflation. It is also interesting to note that the rise in the U.S. real interest rate from August 1981 through mid-1982—despite a nearly 200 basis point fall in the nominal rate—nearly coincided with a sharp drop in U.S. growth (Chart 3).

Less plausible, perhaps, is the results' implication that the long-term U.S. real interest rate increased by nearly two percentage points from mid-1982 to mid-1983. This result is a reflection of the sharp increase in the real value of the dollar during this period, as the other two indicators were essentially unchanged. Some increase in the U.S. real interest rate during 1983 is not implausible as nominal interest rates (and proxies for short-term real interest rates) did rise. However in view of the robust real growth during this period, it seems less reasonable to suppose that the real interest rate increased as much as the results here imply.[26]

Consider now the implications of these estimates for the behavior of expected inflation over the last several years. The estimates suggest that inflation anticipated for the next five years actually increased during 1980 and 1981, even though actual inflation began to decline in mid-1979. Is this pattern plausible? While inflation began to fall in 1980, actual inflation over 1980-1981 was actually higher than during the two previous years. Hence, the drop in actual inflation beginning in mid-1980 may not have affected longer-term inflation expectations much by the end of 1981.

Furthermore, actual and prospective U.S. government budget deficits rose substantially during 1980-1981, as the Administration's "supply-side" fiscal package was put in place. Many market commentators (although certainly not all) have argued that these developments substantially increased the risk that the Federal Reserve would have to raise money growth to accommodate huge deficits, and as a result raise inflation in the future. If so, inflation expected several years in the future could have been rising even as actual inflation was coming down. That expected future inflation did rise substantially over this period is also suggested by the fact that the forward U.S. interest rate increased by nearly 3 percentage points during this period.

However these results do conflict with survey evidence gathered by Richard Hoey that suggests a fall in the public's expected 5 year future inflation rate of about 1.5 percentage points during 1980-1981.[27] If the Hoey data is correct, the results here underestimate both the fall in expected inflation *and* the rise in real interest rates over this period.

The results also suggest a very dramatic decline in the expected inflation rate over the last eighteen months, indeed by nearly as much as the fall in the nominal U.S. interest rate. According to the estimates, expected inflation in mid-1983 was about 2 percentage points below its level in mid-1979. The downward trend in expected inflation (although not, perhaps, the implied magnitude of the decline) is very plausible in view of the dramatic drop in actual inflation during 1982. In addition, the substantial slowing of money growth from mid-1981 to mid-1982 may well have raised the credibility of the Federal Reserve's anti-inflation resolve, and so contributed to a further easing of market expectations of inflation. (Again, however, the Hoey survey suggests a milder—although still substantial—fall in expected inflation since the end of 1981, and a substantial fall in real interest rates as well).

Assessment

Overall, the results point to two conclusions about the 'indicator' approach to measuring real long-term interest rates taken here. First, the general pattern traced by the estimates for U.S. real interest rates and expected inflation seems generally plausible. The results suggest a substantial decline in expected five-year inflation over the last several years, as seems reasonable in view of the sharp drop in actual inflation and the course of Federal Reserve policy. The estimates also suggest that our longer-term real interest rates have remained very high over the last eighteen months in comparison to their level prior to the initiation of the Fed's anti-inflation drive in 1979. This too is very plausible since nominal longer-term interest rates are now much higher than in 1979, while, again, expected inflation almost certainly has fallen greatly.

Second, it is evident that the use of exchange rate indicators can greatly alter the impression of movements in longer-term real interest rates that would be conveyed by variations in nominal interest rates alone. The fall in long-term nominal interest rates here between August 1981 and March 1982 would, of itself, have suggested a substantial decline in longer-term real interest rates. The behavior of exchange rates, though, suggests a very different pattern, one which seems more plausible given the behavior of other economic variables. Thus exchange rate indicators do appear to provide useful information about long-term real interest variations in addition to that conveyed by nominal interest rates.

Needless to say, these results are highly experimental, and subject to substantial error in measurement. More accurate estimates may well be obtainable by using several exchange rates (rather than one), and by adding proxies for short-term real interest rates (or other variables related to long-term real interest rates) or expected inflation to the set of indicators. Nonetheless, the results do suggest that an indicator approach to measuring long-term real interest rate movements is practical and of potential use for policy-guidance.

III. Conclusion

The last several years have provided ample reminders that there are many factors that critically affect economic behavior that cannot be measured or observed directly. Economists and business analysts have long known that 'business confidence' is an important influence on investment, but they still have not found a way to measure this confidence with any precision. More recently, we have come to appreciate the impact of real interest rates and expected inflation on our economy, and thus to regret even more our inability to observe them.

In measuring longer-term real interest rates and expected inflation, this paper has attempted to apply systematically an approach long used implicitly by economists and others. That is, movements in variables that cannot be observed directly—in this case real interest rates and expected inflation—have been inferred from variations in other variables to which they are related, and which are directly measurable. The main basis for this analysis is economic theory, which specifies the relations that are likely to hold between the unobservable variables of interest and the indicators which are used to measure them. This process amounts to a bit of economic 'detective work,' with the observable indicators providing the 'clues' and economic analysis providing the rules by which they are used. The resulting estimates of real interest rates and expected inflation are, in effect the *most likely explanations* for the observed movements in the indicators, *given* the assumptions supplied by economic theory.

Here it has been argued that exchange rates, spot and forward, are likely to be especially good sources of 'clues' about movements in longer-term real interest rates and expected inflation. The main reason is that real spot exchange rates are *directly* affected only by the real interest component of nominal interest rates, while (long-term) forward exchange rates are directly affected by expected inflation, but not by real interest rates. For this reason, movements in exchange rates provide information about how to separate changes in real interest and expected inflation rates that underlie observed movements in nominal interest rates. Similarly, the term structure of interest rates has been used to provide information about the source of changes in nominal interest rates, in the sense that when current nominal interest rates vary with long-term forward interest rates, the most likely cause is a change in expected inflation.

The analysis has also illustrated some of the practical difficulties of implementing what is, in theory, a fairly simple and straightforward idea. Of necessity, the estimates are based on certain assumptions which are not easily tested, and on parameters about which neither economic theory nor available evidence supply much definite information. This is one reason why the results must be regarded as provisional. Another is that more information, supplied by exchange rates vis a vis other countries, measures of short-term real interest rates, or other variables might well be added to the set of indicators to obtain more accurate and reliable estimates.

Nonetheless, the estimates are both plausible and surprising in ways that suggest that useful information can indeed be extracted from foreign exchange and other financial markets. Viewed *by itself*, the sharp fall in nominal interest rates over late 1981 and early 1982 would have suggested a significant drop in long-term real interest rates. Yet a very different impression, that real interest rates remained high and did not drop much, if at all, was suggested by the continued strength of the dollar in the foreign exchange markets, a conclusion supported by this paper's formal analysis based on both sets of indicators. And again, the actual behavior of the real sector of the U.S. economy during this period (although not that later in 1982 and during 1983) supports this latter impression more than that conveyed by nominal interest rates alone. This experience suggests that while using economic knowledge to 'read' the signals from financial markets is not an exact science, it is still of considerable potential use for policy guidance and worth further study.

Appendix

The following explains in more detail how the estimates of the real interest and expected inflation rates can be calculated from the variance-covariance relations among the indicators. In addition, Section C below explains how the estimates of bu, bf, and g are obtained from short-term interest rates and inflation.

A. As in the text define:

$$diu_t(n) = iu_t(n) - fiu_t = ru_t(n) + d\Pi u_t(n)$$

$$dif_t(n) = if_t(n) - fif_t = rf_t(n) + d\Pi f_t(n)$$

where $d\Pi u$ and $d\Pi f$ refer to the difference between the inflation premium on an n year asset and the forward interest rate. The results in the text are based on $n = 5$.

The basic assumptions discussed in Section III can be stated formally as:

(A1) Let $\Delta E_t x_{t+n}$ be denoted 's_t'. Then s_t, which refers to the change in the long-run expected real exchange rate ('long-run' being n years), is uncorrelated with changes in expected inflation or the real interest rates (including the zu and zf components of the latter).

(A2) $\Delta ru_t(n) = bu\Delta d\Pi u_t(n) + zu_t$;
$\Delta rf_t(n) = bf\Delta d\Pi f_t(n) + zf_t$
where zu_t and zf_t are both uncorrelated with $\Delta d\Pi u_t$ and $\Delta d\Pi f_t$.

The relation (A2) expresses the text assumption that any correlation between the U.S. real interest rate and foreign expected inflation be indirect, and similarly for the foreign real interest rate and U.S. expected inflation. In particular it implies:

$$\mathrm{Cov}(\Delta ru_t(n), \Delta d\Pi f_t(n)) = bu\mathrm{Cov}(\Delta d\Pi u_t(n), \Delta d\Pi f_t(n))$$

$$\mathrm{Cov}(\Delta rf_t(n), \Delta d\Pi u_t(n)) = bf\mathrm{Cov}(\Delta d\Pi u_t(n), \Delta d\Pi f_t(n))$$

The definitions of bu and bf in the text also imply that:

$$\mathrm{Cov}(\Delta ru_t(n), \Delta d\Pi u_t(n)) = bu\mathrm{Var}(\Delta d\Pi u_t(n));$$

$$\mathrm{Cov}(\Delta rf_t(n), \Delta d\Pi f_t(n)) = bf\mathrm{Var}(\Delta d\Pi f_t(n)).$$

B. The following relations are easily shown to hold among the variance-covariances of the three indicators, $\Delta diu_t(n)$, Δx_t, and $\Delta dif_t(n)$ (or, alternatively, Δyt, as defined in the text) and those of the underlying variables:

i) $\mathrm{Var}(\Delta diu_t) = \mathrm{Var}(\Delta ru_t) + (1+2bu)\mathrm{Var}(\Delta d\Pi u_t)$

ii) $\mathrm{Var}(\Delta dif_t) = \mathrm{Var}(\Delta rf_t) + (1+2bf)\mathrm{Var}(\Delta d\Pi f_t)$

iii) $\mathrm{Cov}(\Delta diu_t, \Delta x_t)/n = \mathrm{Var}(\Delta ru_t) - \mathrm{Cov}(\Delta ru_t, \Delta rf_t) - bf\mathrm{Cov}(\Delta d\Pi u_t, \Delta d\Pi f_t) + bu\mathrm{Var}(\Delta d\Pi u_t)$

iv) $\mathrm{Cov}(\Delta dif_t, \Delta x_t)/n = -\mathrm{Var}(\Delta rf_t) + \mathrm{Cov}(\Delta ru_t, \Delta rf_t) + bu\mathrm{Cov}(\Delta d\Pi u_t, \Delta d\Pi f_t) - bf\mathrm{Var}(\Delta d\Pi f_t)$

v) $\mathrm{Cov}(\Delta diu_t, \Delta dif_t) = \mathrm{Cov}(\Delta ru_t, \Delta rf_t) + (1 + bu + bf)\mathrm{Cov}(\Delta d\Pi u_t, \Delta d\Pi f_t)$

vi) $\mathrm{Var}(\Delta x_t)/n^2 = \mathrm{Var}(\Delta ru_t) + \mathrm{Var}(\Delta rf_t) - 2\mathrm{Cov}(\Delta ru_t, \Delta rf_t) + \mathrm{Var}(s_t)/n^2$

where the '(n)' have been dropped to simplify notation. These six relations—whose left hand sides are observable and come from the variance-covariance relations among the three indicators—are in terms of 7 variables, given estimates of bu and bf: the variances of the real interest and expected inflation rates (4); the variance of the change in the expected long-run real exchange rate (1); the covariance of U.S. and foreign real interest rates, and the covariance of U.S. and foreign expected inflation (2).

To close the model for their first period, it is assumed that zu and zf are uncorrelated, which implies:

vii) $\mathrm{Cov}(\Delta ru_t, \Delta rf_t) = bubf\mathrm{Cov}(\Delta d\Pi u_t, \Delta d\Pi f_t)$

that is, the correlation between real interest rates entirely reflects the correlation between expected inflation rates across countries.

For the second period estimates, it is assumed that:

viii) $\mathrm{Cov}(\Delta ru_t, \Delta rf_t) = g\mathrm{Var}(\Delta d\Pi u_t)$

where g is independently estimated. To obtain the bu and bf estimates for the second period, let rd and $d\Pi d$ refer to the U.S.-German real interest and expected inflation differentials, and did for the nominal interest differential. Then the relations (i)-(vi) can be combined to obtain:

ix) $\mathrm{Var}(\Delta did_t) = \mathrm{Var}(\Delta rd_t) + \mathrm{Var}(\Delta d\Pi d_t) + 2\mathrm{Cov}(\Delta rd_t, \Delta d\Pi d_t) = a1$

x) $\mathrm{Var}(\Delta x_t)/n^2 - \mathrm{Var}(s_t)/n^2 = \mathrm{Var}(\Delta rd_t) = a2$

xi) $\text{Cov}(\Delta did_t, \Delta x_t)/5 = \text{Var}(\Delta rd_t) + \text{Cov}(\Delta rd_t, \Delta d\Pi d_t) = a3$

For the second period estimates, $\text{Var}(s_t)$ is taken equal to the estimated value for the first period. This allows a2 to be calculated, so that:

$\text{Var}(\Delta rd_t) = a2$; $\text{Cov}(\Delta rd_t, \Delta d\Pi d_t) = a3 - a2$; $\text{Var}(\Delta d\Pi d_t) = a1 + a2 - 2a3$

The ratio of $\text{Cov}(\Delta rd_t, \Delta d\Pi d_t)/\text{Var}(\Delta d\Pi d_t)$ provides therefore an estimate of the 'average' value of bu and bf, and this average value is used for both countries for the estimates for the second period.

Finally, the variance of the actual expected five year inflation rate, $\Delta\Pi u_t(n)$ and $\Delta\Pi f_t(n)$, can be obtained as follows. Assume that changes in the forward interest rates (which measure changes in inflation anticipated many years from now, or changes in 'long-run' inflation) are related to changes in the n-year real interest rates only to the extent that they affect the expected inflation components, $\Delta d\Pi u_t(n)$ and $\Delta d\Pi f_t(n)$. This means, in effect, that changes in 'long-run' inflation are independent of the zu_t and zf_t components of the real interest rate defined earlier. This implies:

$$\text{Cov}[\Delta ru_t(n), \Delta fiu_t] = bu\text{Cov}(\Delta d\Pi u_t(n), \Delta fiu_t)$$

or,

$$\text{Cov}(\Delta d\Pi u_t(n), \Delta fiu_t) = (1+bu)^{-1}\text{Cov}(\Delta diu_t(n), \Delta fiu_t)$$

where the covariance on the right-hand-side of the latter expression is directly measurable. This allows the covariance of $\Delta d\Pi u_t(n)$ with changes in the forward interest rate, and hence its covariance with shifts in expected 'long-run' inflation, to be estimated. This, given that the observable variance of changes in fiu_t measures the variance of expected 'long-run' inflation, allows the variance of $\Delta\Pi u_t(n)$ given in Table 3 to be computed. The corresponding values for Germany are calculated similarly.

Note that these latter estimates do *not* affect the estimates of the weights, and hence the estimates of actual real interest variations. The reason is that the weights depend only upon the estimated variances and covariances of the variables underlying the indicators, that is $\Delta d\Pi u_t(n)$ and $\Delta d\Pi f_t(n)$, as well as of the real interest and expected future real exchange rate changes.

C. As indicated in the text, one way to derive estimates of bu and bf, and g, is to examine the relations between short-term interest rates and proxies for short-term expected inflation—the latter being much easier to obtain than proxies for longer-term expected inflation.

Let iu_t stand for the one-month U.S. interest rate, Πu_t for the actual one-month U.S. inflation rate for the month ending at t (at an annual rate), and $E_t\Pi u_{t+1}$ for the inflation rate anticipated to prevail over the *next* month. Then bu could be estimated from the regression,

$$\hat{ru}_t = iu_t - \Pi u_{t+1} = C + bu[E_t\Pi u_{t+1} - fiu_t]$$

(recall that fiu_t equals the expected 'long-run' nominal interest rate, which consists of a *constant* 'long-run' real interest rate and the expected 'long-run' inflation rate). The $\hat{ru}_t$ is the ex-post, or realized, real interest rate and is an unbiased measure of the

Table A.1.
Estimates of Real Interest-Expected Inflation Relation

	Using as Expected Inflation Proxy:	
	1-Month Inflation	**6-Month Average Inflation**
Earlier Period:		
bu	.04	.32
bf	−.33	.33
g^2	$(.13)^3$	$(.04)^3$
Later Period:		
bu	−.44	−.78
bf	−.20	−.80
g^2	.17	.20

[1]bu (bf) estimated from regression:
$iu_t - \Pi u_{t+1} = C + bu\,[E_t\Pi u_{t+1} - fiu_t]$ (similarly for bf)
where iu_t is the one-month eurodollar deposit rate, Πu_{t+1} is the actual (consumer price) inflation rate from month t to t+1 (at an annual rate), and $E_t\Pi u_{t+1}$ is a proxy for the expected value of that inflation rate as of t. This expectation is approximated as either the one month actual inflation for the period ending in month t, or the average of the six months inflation for the period ending t. Thus, the dependent variable is the 'ex-post' real interest rate.

[2]g estimated from regression:
$if_t - E_t\Pi f_{t+1} = C + g[iu_t - E_t\Pi u_{t+1}]$
Variable definitions are as in note 1.

[3]Recall that this value is not used for the first period estimates in the text. Neither value is significantly different from zero, however.

actual real interest rate, assuming that market expectations are rational. This form is used because the bu in the text refers to the relation between the long-term real interest rate and the *difference* between the n-year expected inflation rate and that in the 'long-run'. Estimates of the value of bu, using one and three month past inflation rates (consumer price) as proxies for the next period's rate, are given in Table A.1.*

The fact that the German interest rate indicator (that is, $\Delta dif_t(n)$) and real exchange rate changes are positively correlated (see Table 2) strongly suggests that variations in real interest and expected inflation rates are *negatively* correlated for that country, that is that bf must be negative for both periods.** The estimates in Table A.1. also suggest a negative correlation between expected inflation and real interest rates for the U.S. for the *second* period. On this basis, the estimates in the first column, using last month's inflation rate as the proxy for that expected the next month, are most plausible, since these lead to negative estimates of bf, and since the second column estimates for the second period, while negative, give a *negative* estimate of the variance of the expected future real exchange rate change (this means that they are too large in absolute value). The actual (bu, bf) values used in the text estimates for period two lie between the first and second column estimates.

Finally, the value of 'g' for the second period is estimated from a similar regression of the form:

$$if_t - E_t\Pi f_{t+1} = C + g(iu_t - E_t\Pi u_{t+1})$$

where the dependent and independent variables are proxies for the foreign and U.S. real interest rates. As before, one and six months past inflation are used as proxies for that expected over the next month. Note again that the value of g estimated in this way is used in the text estimates for the second period only (where I have taken the column two value). However, the estimates for the first period are reasonably close to the zero value assumed in the text.

*Admittedly, the relation should, ideally, be estimated in first-differenced form to be stricly consistent with the text analysis. However, the estimates of bu and bf (and g) obtained with a first differenced form of the above yield implausible estimates for the variances of the underlying variables (that is, one or more are negative).

**Recall that, ceteris paribus, a rise in the German real interest rate *lowers* the real exchange rate indicator. The positive correlation between the German nominal interest rate and real exchange rate changes thus implies a negative correlation between German nominal and real interest rate variations. This is most likely to occur if German real interest and expected inflation rate changes are strongly negatively correlated.

FOOTNOTES

1. See, for example, Cornell (1982) and the article by Joseph Bisignano in the Fall 1983 issue of this Review.

2. As defined here, the real interest rate simply refers to the expected real return of the investment. It thus includes any allowance for risk—from inflation, future interest rate changes, or default—investors demand, since these help determine that real rate.

3. For example, the standard deviation of the one-month U.S. interest rate less the average of the past six months inflation increased more than three-fold from the period 1976-mid 1979 to the period mid-1979 through 1982.

4. That is, we can suppose that approximately,

$$iu_t(N+1) = (1/N+1)\sum_{j=0}^{N} E_t iu_{t+j}$$

so, again approximately,

$$E_t iu_{t+N} = (N+1)iu_t(N+1) - Niu_t(n).$$

This must be regarded as an approximation for two reasons. First, longer-term interest rates may well deviate from an average of expected future shorter-term rates by a "risk" premium that compensates for uncertainties about the future. There is much evidence for the existence of such a premium, although there remains considerable dispute as to its size. Second, even in the absence of this premium, the formula holds exactly only for pure discount (zero coupon) bonds and can be regarded as only an approximation—sometimes a rough one—when there are coupons.

5. For an illustration, and the effect on exchange rates, see Dornbusch (1976).

6. Frankel (1979a) used the difference between the short-term and long-term nominal interest rates as a proxy for the real interest rate. This is, however, equal to the difference

between the short and long-term real interest rates **plus** the short-term-long-term inflation **differential**, and is unlikely to be a very good measure of the long-term real interest rate.

It is very important to note that in no sense can changes in $diu_t(n)$ be regarded as **exact** measures of variations in the n-year real interest rate. This would only be the case if changes in expected inflation were the same for all horizons, that is if shifts in the term-structure of expected inflation were "flat." This will not always be the case, as numerous past instances of **temporary** increases in inflation due to supply-demand imbalances in primary commodity markets suggest. However, if inflation increases tend to be fairly persistent, much of the variance in the expected inflation component of the nominal interest, $iu_t(n)$, will be "removed" by the transformation to $diu_t(n)$. The expected inflation component of this latter indicator **will** generally be correlated with shifts in expected "long-run" inflation, however.

7. The circumstances under which such premia will exist (and what they depend on) are described in Frankel (1979b).

8. See, for example, Frankel (1982).

9. In effect, the expected real depreciation of the dollar, which is the percentage difference between its current level and that expected in the long-run, compensates for the **real** difference in the total interest earned on the U.S. versus a foreign asset **over the entire life of that investment**. For example, an increase relative to abroad in the U.S. real ten year rate of one percentage point **annualized** implies that the U.S. instrument now earns ten percent more in real terms that its foreign counterpart over its ten-year life; hence the real value of the dollar must rise above its value expected ten years from now by 10 percent. If in addition the real value of the dollar expected ten years from now is unaffected—which means at the least that real interest fluctuations are expected to have ceased by then—then the current real dollar itself rises by ten percent. Note however that the above (and text) relations between exchange rates and the maturity of an asset's yield are strictly true only for pure discount instruments; for coupon instruments, the 'scale' factor is proportional to the *duration*.

As this suggests, only changes in long-term real interest rates are likely to have unambiguous impacts on the current real exchange rate. For any given term, the corresponding real interest differential measures the deviation of the current real exchange rate from the value expected to prevail at maturity. However it is only for longer-term maturities, that is for periods far enough into the future that real interest fluctuations and other temporary influences on exchange rates have ceased, that the value expected at maturity can be expected to be unaffected.

The impact of shorter-term real interest fluctuations on the current real exchange rate thus depends upon how real interest rates expected in the future are affected. As an example, suppose that the current 1-month U.S. real interest rate increases by one percentage point (annualized) above its long-run level, but is expected to fall one percentage point below that level next month, and then return to the long-run level in all subsequent months. The current long-run real interest rate is thus unaffected, and hence the current real exchange rate will not change. However, it is easy to see that the real exchange rate expected to prevail a month from now must **fall**, and that the actual real exchange rate a month from now will fall from its current level.

10. Of course, increases in x_t may also reflect declines in foreign real interst rates, or an increase in the expected future real exchange rate.

11. Alternatively, the third indicator can be thought of as the foreign interest rate indicator, $dif_t(n)$. Movements in foreign interest rates provide information about variations in our real interest rate, in part by helping to "interpret" changes in the actual real exchange rate: the U.S. real interest rate is more likely to have increased, given a rise in the real value of the dollar, if the foreign interest rate has remained unchanged than if it has fallen.

12. More precisely, relation (6) with the weights given in (7) gives the conditional expectation of changes in the real interest rate, given the observed changes in the indicators. The statement that the estimates are "optimal" is strictly true only when we have an exact, or correct, measure of $\mathrm{Cov}(\Delta ru_t, I_t)$. In practice, we have to estimate it.

13. The procedure for estimating the variances and covariances of the indicators from those of the underlying variables is described in detail in the Appendix.

14. Note that this assumption is strictly valid only for n long enough for real interest rates to have returned to their (constant) long-run values. Real interest rates on shorter-term assets generally will be correlated with the real exchange rate expected at maturity. Moreover, **permanent** shifts in the real interest rate, caused by changes in capital productivity or other factors, generally will lead to changes in long-run relative commodity prices, and hence will be correlated with the long-run real exchange rate. Therefore, the assumption that real interest rate fluctuations are the result of temporary financial market disturbances is closely related to (A1).

15. This does **not necessarily** imply that, say, changes in U.S. expected inflation are uncorrelated with changes in foreign real interest rates. An "indirect" correlation could arise if U.S. and foreign changes in expected inflation were correlated, **and** changes in foreign expected inflation and real interest rates were also related.

16. All exchange rate and interest rate data refer to monthly **averages**. The forward interest rate for the U.S. is effectively the 3-year bond rate expected to prevail seven years from now, while for Germany it is the 2-year rate expected to prevail five years from now. For example, for the U.S.

$$fiu_t = 1/3[\mathrm{Log}\,[(1+iu_t(10))^{10}] - \mathrm{Log}\,[(1+iu_t(7))^{7}]]$$

where the iu_t are expressed in decimals.

Ideally, in calculating the 5-year forward exchange rate from the current exchange rate, interest rates on assets that are identical except for their currency of denomination should be used. Since the government bonds used here are not strictly identical in this sense (their tax treatment, for example, will differ), the forward exchange rate as calculated from the bond rates will differ from the 'true' value somewhat. The analysis in the text implicitly assumes that the difference is **constant** over time, so that it does not affect the calculated **changes** in the forward exchange rate.

17. The break is made in mid-1979, rather than in October of that year, since the Fed began slowing money growth somewhat before its official change in monetary targeting procedures in October 1979.

18. See, for example, Mishkin (1981), Fama and Gibbons (1982), and Cornell (1982).

19. This positive association **could** also reflect a strong **negative** relation between the U.S.-German real interest differentiatial and their expected inflation differential (see Table 1). This illustrates that a negative real interest/expected inflation relation has many of the same implications for the behavior of the indicators as does a high degree of variability in the expected future real exchange rate. This is the reason that the more negative the estimated relation of real interest rates and expected inflation, generally the **lower** the estimated variability of the expected future real exchange rate, and the higher the estimated variability of real interest rates.

20. See Appendix for more details on how bu and bf are estimated.

The effect of estimating bu and bf from shorter term interest rates can be seen from the "memo" item for the second period in Table 3. Since the alternative (bu, bf) are lower in absolute value, the estimates of the variation in U.S. and foreign real interest rates are also lower. The resulting estimate also implies a very large increase in the variance of the expected future real exchange rate over the first period. Indeed, the "memo" estimates imply that the variance of changes in $E_t x_{t+n}$ is about twice as great as that of the actual real exchange rate during the first period—which does not seem very plausible.

The "memo" for the first period uses a value for bu suggested by a study by Mishkin (1981) of U.S. shorter-term real interest rates during the 1960's and 1970's. This suggests that the short-term real rate increased by about 30 basis points for a 100 basis point increase in short-term inflation.

21. See Throop (1980) and Bisignano (1983). The Appendix again explains how the relation between U.S. and foreign real interest rates is estimated for the two periods.

22. Again, see Bisignano (1983).

23. Strictly speaking, of course, the results suggest only that the real exchange rate expected to prevail five years from **now** varies substantially; it does **not** rule out the possibility that purchasing power parity might hold over some longer period. However, examination of correlations among longer-term interest rates and exchange rates suggest that the basic conclusion would not be substantially altered by considering, say, 10 year interest rates. Meyer and Startz (1982) use an inference approach analogous to that here and find that most of the error in short-term predictions of the nominal exchange rate reflects error in predicting the **real** exchange rate. Besides being applied to short-term interest-exchange rate relations, their analysis is somewhat more restrictive than that here. In particular, it assumes that real interest and expected inflation variations are uncorrelated.

24. In interpreting the individual coefficients, it is very important to note that a change in one indicator will typically imply a change in some other variable related to the U.S. real interest rate. For example, a rise in the U.S. nominal interest rate with **no** change in the current real or forward exchange rate indicators—whose implied impact on the estimate of the U.S. real interest rate change is given by W1—can **only** occur if the foreign interest rate has also risen. Thus W1 can, in effect, be interpreted as the increase in the U.S. real interest rate given an equal change in the U.S. and foreign nominal interest rates. The U.S. real and foreign nominal interest rates apparently are **negatively** associated for the second period, which is why W1 appears to have the "wrong" sign. (This negative association reflects the apparent negative correlation between the foreign nominal and real interest rates, combined with the positive association of U.S. and foreign real interest rates). The fact that the coefficient of the foreign interest rate indicator is generally negative in the "rewritten" equation given in the right portion of the table has an analogous interpretation.

25. Data on the 5-year and 7-year German bond rates were not available after 1982. The change in the 5-year German bond rate was then taken to equal the change in the 5-year euro-DM deposit rate during 1983. To 'extrapolate' the 7-year German bond rate (to estimate fif_f), this rate was regressed on the German long-term government bond rate (which corresponds to an average of several long-term maturities) over the period 1979.07- 1982. The relation was then used to estimate the seven year rate for 1983.

26. Interestingly, if the forward exchange rate indicator is dropped, the resulting estimates of the real interest rate behave very similarly to those shown in Chart 1 for 1982, but increase by considerably less during 1983 (about 60 basis points).

The "memo" estimates for the second period (using bu and bf estimated from short-term interest rates) imply a somewhat different pattern for the U.S. real interest rate during 1982 and 1983. These suggest that real interest rates fell nearly 1 percentage point during 1982, ending the year at about the level of mid-1979; the same estimates suggest that the real interest rate fell further during 1983. However the estimates also imply that expected future inflation in mid-1983 was actually several percentage points higher than it was in mid-1979, a period over which the actual inflation rate declined by nearly half. This is another reason why these estimates do not seem so plausible.

27. For a compilation of Hoey's estimates, see Peter Isard, "What's Wrong with Empirical Exchange Rate Models...," Discussion Paper #226 (August 1983) of The International Finance Division of The Board of Governors of the Federal Reserve.

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