MONETARY POLICY
AND
INTEREST RATES
The Federal Reserve Bank of San Francisco's Economic Review is published quarterly by the Bank's Research and Public Information Department under the supervision of Michael W. Keran, Senior Vice President. The publication is edited by William Burke, with the assistance of Karen Rusk (editorial) and William Rosenthal (graphics). Opinions expressed in the Economic Review do not necessarily reflect the views of the management of the Federal Reserve Bank of San Francisco, or of the Board of Governors of the Federal Reserve System.

For free copies of this and other Federal Reserve publications, write or phone the Public Information Section, Federal Reserve Bank of San Francisco, P.O. Box 7702, San Francisco, California 94120. Phone (415) 544-2184.
I. Introduction and Summary

II. Why Have Interest Rates Been So Volatile?

Paul Evans

... The 1979 change in monetary policy produced only about 30 percent of the increased volatility in long-term interest rates, and the rest came from sources not directly under Federal Reserve control.

III. Liability Management, Bank Loans and Deposit "Market" Disequilibrium

John P. Judd and John L. Scadding

... Large swings in bank loans, induced primarily by the Special Credit Restraint Program, were the major source of money's variability in 1980.

IV. The Response of Real Output and Inflation to Monetary Policy

Rose McElhattan

... Both inflation and real GNP respond quickly to a change in monetary policy, with the major effects occurring within two years of the initial change.

Editorial committee for this issue:
Hang-Sheng Cheng, Brian Motley, and Kenneth Bernauer
Monetary Policy and Interest Rates

October 6, 1979 may represent one of the key dates in the nation's monetary history, along with December 23, 1913 (the passage of the Federal Reserve Act) and March 4, 1951 (the signing of the "Accord" which removed Treasury domination of central-bank policy). On that October day, the Federal Reserve began to improve its monetary control by changing its operating techniques—that is, by controlling the quantity of bank reserves rather than by tightly pegging the cost of those reserves (the Federal-funds rate). The Fed subsequently has been broadly successful in meeting its monetary-control objectives, but its operational shift also has been accompanied by increased volatility in both interest rates and the monetary aggregates. This issue of the Economic Review examines these several aspects of post-1979 monetary policy, and also analyzes the response of major economic variables to policy changes.

Paul Evans investigates how much of the recent increase in interest-rate volatility stemmed from the October 1979 change in monetary policy. He finds that this policy change produced only about 30 percent of the increased volatility in long-term interest rates, and that the rest came from sources not directly under Federal Reserve control. "Almost all of this 30 percent resulted from the Fed's adherence to its monetary targets; by itself, the freeing of the funds rate had little to do with the increased rate volatility."

Evans' findings thus indicate that the Federal Reserve has been responsible for only a small part of the increase in interest-rate volatility. Citing a number of national and international political events of the period, he argues, "Clearly, none of these events was a direct consequence of the monetary-policy change. Furthermore, future years may see a return to normalcy, with a sharp reduction in interest-rate volatility."

Continuing, he argues that the Federal Reserve's decision to move the Fed-funds rate more in response to monetary surprises entails more volatility of both long- and short-term interest rates. "It probably also helps the Federal Reserve to hit its targets for money growth and hence for inflation. For this reason, the increased volatility—and the resultant reduction in capital formation and redirection of capital towards shorter-lived assets—may be the price that must be paid to hit these targets."

Turning to the surprisingly high variability of the monetary aggregates in the post-1979 environment, John Judd and John Scadding have developed a monthly money-market model which explains this phenomenon. The model shows how certain types of financial-market disturbances, such as sharp changes in bank loans, can affect the money supply and thus cause problems of monetary control. The evidence indicates that large swings in bank loans, induced primarily by the Special Credit Restraint Program, were the major source of money's variability in 1980.

Conventional money-market models reflect the view that the monetary aggregates are determined primarily by the public's demand for money. The Judd-Scadding model reflects an alternative view—that the monetary aggregates are determined in the short-run primarily by the supply of money, which rises out of the behavior of banks and the public in credit markets (in the markets for bank loans and banks' liabilities like large certificates of deposit). As evidence, they note that banks' demand for reserves responds to its financial-market determinants with very short lags, consistent with the typical speed of adjustment in
credit markets, but not with the typical sluggishness of money demand. Also, bank loans had a potent influence on the monetary aggregates in 1980, as their model predicts. Finally, they note that the market for money is often characterized by disequilibrium in the short-run: money-supply shocks temporarily push the public off its short-run money-demand curve, which allows the money supply to exert a large short-run influence on the stock of money observed in the economy.

The Judd-Scadding results imply that policymakers should pay close attention to financial-market developments, especially when signs appear of a shift in the conventional money-demand function. They cite in particular the second quarter of 1980, when conventional models severely overpredicted the money stock. “Evidence of a downward shift in the money-demand relationship would imply that the money supply should be allowed to fall commensurately to avoid an overly expansionary monetary policy. On the other hand, our model explains the decline in money as supply shock, induced by the decline in bank loans that followed from the Special Credit Control Program of 1980. Such a conclusion implies that monetary-control efforts should be directed toward more rapid money-supply growth to avoid an overly contractionary policy.”

Next, Rose McElhattan presents a small model of the U.S. economy for estimating the response of inflation and real output to a change in monetary policy. Measures obtained from the model’s reduced-form equations provide estimates of the complete adjustment paths of inflation and real output to a monetary disturbance. In her model, prices continue to change until both the inflation rate and the level of real money balances reach their respective long-run values, while real output continues to adjust until it equals the level of potential output and is growing at the rate of potential output. Most other reduced-form models focus only upon the adjustment of rates of change in prices and output to a monetary disturbance. In contrast with these, McElhattan’s model provides results which are consistent with the neutrality of money, which is one of the most generally accepted properties regarding economic behavior. It holds that changes in the money supply ultimately affect only nominal variables, such as prices and wages, leaving all real quantities, such as goods and services, unchanged.

In McElhattan’s model, both inflation and real GNP respond quickly to a change in monetary policy, with the major stimulative or deflationary phase occurring within two years of the initial change. Her findings thus conflict with most of the published literature, which suggests that output and prices require about five years to respond to a change in money growth.

McElhattan thus provides an alternative to the viewpoint that it will take a long time to bring down the inflation rate, and that we risk an economic recession in the process. “Changes in monetary growth, at least since the mid-1960’s, apparently have acted fairly rapidly upon inflation — and hence upon aggregate demand as well. Thus, since a monetary contraction is likely to bring inflation down faster than previously anticipated, less of the brunt of that contraction need be borne by real GNP, so that a major decline or loss of real income need not result when we adopt a policy which gradually reduces monetary growth.”
On October 6, 1979, the Federal Reserve announced that henceforth it would tightly control the money supply while letting the Federal-funds rate respond freely to market forces. The Federal Reserve hoped that this policy change would help it to stabilize employment and real income while bringing inflation down to a tolerable level.

The Federal Reserve did indeed free the Federal-funds rate — the rate clearly was much more volatile after October 6, 1979 than it was before that date, as can be seen from Panel a of Figure 1. That action, however, has produced no clear victory against inflation, and 1980 could hardly be considered a year of great stability in the real economy. Furthermore, both short-term and long-term interest rates have become much more volatile, as panels b and c of Figure 1 demonstrate.

Volatile interest rates — especially volatile long-term rates — may impose burdens on the real economy. Savers may find their portfolios riskier, and may therefore save less and shift from long-term to short-term securities. Purchasers of houses, plant and equipment, and other long-lived physical assets may then be forced to finance their purchases with short-term debt, thus making these purchases riskier. If the additional risk reduces long-term investment as well as saving, the economy will experience less capital formation, and a smaller portion of that capital formation will go into long-lived assets. As a result, the economy’s growth rate will slacken.

This paper investigates how much of the recent increase in interest-rate volatility stemmed from the change in monetary policy of October 6, 1979. It finds that this policy change produced only about 30 percent of the increased volatility in long-term interest rates, and that the rest came from sources not directly under Federal Reserve control. Almost all of this 30 percent resulted from the Fed’s adherence to its monetary targets; by itself, the freeing of the funds rate had little to do with the increased rate volatility. Therefore, panel c of Figure 1 gives a misleading picture of the new monetary policy’s impact on rate volatility and hence on investment and growth. The actual effect was substantially smaller.

The next section of this paper formulates a model of interest rates, and Sections II and III discuss the estimates resulting from the fitting of this model to weekly U.S. data. Next, Section IV decomposes the recent increase in interest-rate volatility into several components, and discusses each of those components. Finally, Section V summarizes the paper and draws some policy conclusions.
To begin, we formulate a model based on the hypothesis that the securities markets are efficient. Simply speaking, the efficient-markets hypothesis claims that readily available information is so efficiently processed that no market participant can do systematically better than any other participant.

Samuelson (1965) and Sargent (1976) have shown that, to a close approximation, interest rates in an efficient market respond immediately and completely to any new information reaching the bond markets. The reason is that interest-rate changes generate capital gains or losses, which dominate the short-period returns to all but the shortest-term bonds. If interest rates in fact changed slowly in response to new information, a savvy investor could use that information accurately to predict future capital gains or losses. Being able to make accurate predictions would be a veritable license to print money, because the investor could hold bonds when they were going to rise in price and sell them short when they were going to fall in price. For example, if the investor knew that government deficits raise interest rates gradually, he or she would react to an unusually low deficit by buying bonds and to an unusually high deficit by selling them short. By the time the interest rate actually changed, he or she would probably have made a fortune.

The efficient-markets hypothesis essentially
assumes that a large number of savvy investors participate in the securities markets. Therefore, when these investors think, on the basis of new information, that interest rates will rise and hence that bond prices will drop, their efforts to sell their bonds immediately will drive up interest rates. In principle, the rise will be so rapid that no one can manage to sell a single bond before interest rates rise as far as they are going to rise.

The efficient-markets hypothesis implies that DR, the interest-rate change, is given approximately by

$$\text{DR} = K + BE + V,$$  

(1)

where $K$ is a parameter, $B$ is a vector of parameters, $E$ is a vector of new information used in the empirical analysis, and $V$ is an error term that captures the effects of all other information and that moves independently of $E$. The longer the term to maturity of the bond, the better is the approximation.

By definition, new information is that part of current information that was not known in the past. In order to give content to this definition, we must add an hypothesis about knowledge — namely, that one part of currently available information can be predicted from past information with the use of standard econometric techniques, and the other part (to be called "new information") cannot be so predicted. Specifically, let $Z$ be the vector of variables used in the empirical analysis. Then, if bond-market participants knew $Z(-1)$, the value of $Z$ in the previous period, they would predict $Z$ as $AZ(-1)$, where $A$ is a matrix of coefficients obtained by regression $Z$ on $Z(-1)$. Therefore, $E$, the vector of new information about $Z$, is simply the error vector in the equation

$$Z = AZ(-1) + E.$$  

(2)

Note that $E$ is serially uncorrelated, because past values of $E$ are known by assumption and serial correlation would imply (contrary to assumption) that past values of $E$ are useful in predicting current values of $E$. Similarly, $V$ in equation (1) should be serially uncorrelated because it, too, is new information that should not depend on such information as its own past history.

Equations (1) and (2) suggest the following strategy. First, collect some series that are readily available (say, from government publications). Then estimate prediction equations, like (2), for these series and obtain the residuals, which are consistent estimates of $E$. Finally, regress changes in various interest rates on these residuals to obtain consistent estimates of $K$ and $B$ in equation (1). If the efficient-markets hypothesis is correct, past information should not change interest rates. Therefore, this hypothesis is refuted if any lagged $E$'s have statistical significance in equation (1) or if $V$ is serially correlated.

II. Prediction Equations

Bond-market participants surely pay attention to a great many series of data — such as real GNP, the inflation rate, the government deficit, corporate credit demands, and monetary-policy variables. However, many of these series cannot be used here, because they are not available on a weekly basis. Moreover, since this paper is mainly concerned with the impact of monetary policy on interest rates, we limit the analysis to appropriate monetary series.

We assume that GM, the growth rate of the (unadjusted) money supply ($M-1B$), and DFFR, the change in the Federal-funds rate, adequately characterize monetary policy. To extract the new information from these series, one must estimate prediction equations that relate them to past information known by bond-market participants. Essentially, one must estimate an equation system of the form (2) — first for the sample period extending from the first full week of 1977 to the last full week before October 6, 1979, and again for the sample period extending from the first full week after October 6, 1979, to the week ending on October 22, 1980. Using the
methodology advocated by Box and Jenkins (1976), we obtained the following results for the sample period before October 6, 1979 (the figures in parentheses are standard errors):

\[
GM = GM(-52) + .00126 + EM (.000203) - .673 EM(-1) + .232 EM(-13) (.063) (.085) - .156 EM(-14), (.057)
\]

S.E. = .006013, R\(^2\) = .357; (3)

\[
DFFR = .0515 + 4.72 EM(-1) + EFFR (.0114) (1.89)
\]

S.E. = .1361, R\(^2\) = .042; (4)

where EM is a residual from the regression for the money growth rate, EFFR is a residual from the regression for the Federal-funds rate, and \((-i)\) attached to a symbol indicates that it is lagged \(i\) weeks.

For the sample period after October 6, 1979, we obtained

\[
GM = (-52) + .000413 + EM (.001201) - .290 EM(-1) + .587 EM(-13) (.137) (.175) - .170 EM(-14) + .493 EM(-26) (.110) (.233) - .143 EM(-27), (.122)
\]

S.E. = .007767, R\(^2\) = .443; (5)

\[
DFFR = .074 + 37.2 EM(-2) (.100) (13.8) - 0.93 DR12MO(-2) (0.35) + 3.05 DR12MO(-2) (0.68)
\]

\[
- 2.07 DR10YR(-2) + EFFR, (0.67)
\]

S.E., = .7158, R\(^2\) = .478;

where DR3MO is the weekly difference in the three-month Treasury-bill rate, DR12MO is the weekly difference in the twelve-month Treasury-bill rate, and DR10YR is the weekly difference in the ten-year Treasury bond rate. Even though equations (3) and (4) or equations (5) and (6) may not look like the equation system (2), it is easy to show that they take that form. This is because EM and EFFR in equations (3) - (6) are unknown only until GM and DFFR become known.

Equations (3) and (5) imply that only three kinds of effects are relevant in determining the money-supply growth rate in any given week. First, if all other effects are zero, GM equals GM(-52), the value that it assumed in the same week of the previous year. Thus, GM tends to keep any weekly seasonal pattern it has assumed. Second, the term EM combines all of the influences on the money growth rate that could not have been predicted in the previous week. Third, the terms in lagged values of EM determine how GM will tend to move the year after EM has assumed a non-zero value.

To illustrate, suppose that EM rises by one percentage point in the first week of January some year, but is left unchanged in all other weeks. Equation (3) implies that the money growth rate is one percentage point higher in the first week of January of that year, .673 percentage points lower in the second week of January, .232 percentage points higher in the first week of April, .156 percentage points lower in the second week of April, and is unchanged in all other weeks of the year. At year’s end the money supply is .403 \((=1 -.673 + .232 - .156)\) percentage points higher than at the end of the previous year. Consequently, the average annual money growth rate rises by .403 percentage points. Moreover,
since money growth will follow this same scenario in future years — note that GM (−52), the growth rate one year earlier, appears in the right-hand member of equation (3) — average money growth also rises by .403 percentage points in every future year.

A similar calculation using equation (5) yields a money growth rate that is one percentage point higher in the first week of January, .290 percentage points lower in the second week of January, .587 percentage points higher in the first week of April, .170 percentage points lower in the second week of April, .493 percentage points higher in the first week of July, and .143 percentage points lower in the second week of July. The money growth rate also follows this same pattern in every future year; the average rate rises by 1.477 (=}1 -.290 + .587 -.170 + .493 -.143) percentage points.

It is important to note that a monetary surprise (EM≠0) permanently changes the growth rate of the money supply and not just its level. A positive surprise raises the growth rate; a negative surprise lowers the growth rate.

The results suggest that the Federal Reserve, before October 6, 1979, would have responded to a monetary surprise of one percentage point by raising the Federal-funds rate only about 4.72 basis points. (See equation (4).) No other variable was helpful in predicting changes in the funds rate. Surprises in the funds rate were usually small: about two-thirds of them were between −13.61 and +13.61 basis points. This fact demonstrates that the Federal Reserve more or less pegged the Federal-funds rate and responded sluggishly to monetary surprises.

The results also suggest a quite different Federal Reserve response since October 6, 1979 (See equation (6).) In this period, the Federal-funds rate responded to monetary surprises with a two-week lag rather than a one-week lag, and the response was much larger. Furthermore, lagged interest rates began to affect the funds rate.

The two-week lags in equation (6) have special significance. For a number of years, reserve requirements have been lagged two weeks, rather than imposed contemporaneously. This institutional feature implies that an increase in the money supply, and hence deposits, generates an increase in demand for reserves two weeks later. For this reason, the Federal-funds rate will tend to rise two weeks later unless the Federal Reserve completely accommodates this increase in demand. Therefore, because EM(−2) had no statistically significant effect on the funds rate before October 6, 1979, the Federal Reserve in that period must have largely accommodated changes in the demand for reserves. Since then, however, the Federal Reserve has apparently let the banks largely fend for themselves, for EM(−2) has a larger and statistically significant coefficient in equation (6).

The two-week lag on the interest-rate terms suggests that these rates affect the Federal-funds rate by operating first on the demand for reserves. For example, a change in bond rates might drive businesses to borrow more from the banks, and this in turn would push up the demand for reserves two weeks later. This explanation, however, would lead to the conclusion that business-loan demand is roughly independent of the level of interest rates (−0.93 + 3.05 − 2.07 is roughly zero), but rises when the term structure becomes more humped (the twelve-month Treasury-bill rate rises relative to the three-month Treasury-bill rate and the ten-year Treasury-bond rate.)

Equations (4) and (6) provide one more insight. Since October 6, 1979, the Federal Reserve has evinced a much greater willingness to tolerate large movements in the Federal-funds rate: the standard error rises from .1361 percent a year in equation (4) to .7158 percent a year in equation (6). The greater movement in the funds rate, as well as the Federal Reserve’s apparent willingness to let the banks bear some of the brunt of adjustment in the market for reserves, suggests only one conclusion: the Federal Reserve tried much harder to control the money supply after October 6, 1979 than it ever did before. Nevertheless, the short-term variability of the money supply has also risen (the standard
error of equation (5) is larger than that of equation (3). Moreover, movements in the money supply have become more persistent, as we have seen in our analysis of equations (3) and (5).

III. Interest-Rate Equations

The residuals EM and EFFR represent "new information" about monetary policy. In this section, we estimate how bond markets have used this new information in setting interest rates, and then test whether these markets are efficient.

In Equation (1), DR referred to changes in end-of-period (say, end-of-week) interest rates. Our interest-rate data, however, are not end-of-week data, but rather averages of daily data for weeks beginning on Sundays and ending on Saturdays. This complication, and two others discussed below, imply that the appropriate equation is:

\[
DR = k + bEM(-1) + cEM(-2) + dEFFR(+1) + eEFFR + V, \quad (7)
\]

where DR is the difference in the bond rate averaged over weeks beginning on Sundays and ending on Saturdays; k, b, c, d and e are parameters; and V is an error term. In this

Figure B

Three-Month Treasury Bill Rate
equation, $EM(-1)$, $EM(-2)$, $EFFR(+1)$ and $EFFR$ take the place of $E$, and the error term $V$ is serially correlated, unlike its counterpart in equation (1). To be specific, $V$ should be a first-order moving average.\(^{10}\) The Federal Reserve generally releases M-1B data with an eight-or nine-day lag. Moreover, these data are averages of daily data for weeks ending on Wednesdays rather than Saturdays. For this reason, the bond markets know only $EM(-2)$ during the first few days of any week, and then learn $EM(-1)$.\(^{11}\) Therefore, both $EM(-1)$ and $EM(-2)$ belong in equation (7).

The Federal-funds rate data used to fit equations (4) and (6) are averaged over weeks ending on Wednesdays. Since bond-market participants probably keep track of the funds rate on an hour-to-hour (or even minute-to-minute) basis, they already know part of $EFFR$ before the week begins on Sunday, and then learn part of $EFFR(+1)$ before the week ends on Saturday. Therefore, including $EFFR(+1)$ and $EFFR$ in equation (7) is appropriate.\(^{12}\)

We used Treasury-bill and Treasury-bond rate data, as well as the residuals from equations (3) - (6), to fit equation (7) for the sample periods before and after October 6, 1979. According to the efficient-markets hypothesis, including $EM(-3)$, $EM(-4)$, $EFFR(-1)$, $EFFR(-2)$, etc., in equation (7) — or specifying its error term to be a second (or higher) order moving average — should add no statistically significant explanatory power. We have found this hypothesis to be true for the long-term interest rates. The regressions for

---

**Figure C**

**Twenty-Year Treasury Bond Rate**

![Graph showing the twenty-year treasury bond rate from 1977 to 1980. The graph indicates a trend with a significant increase in 1979.]
some of the short-term interest rates, however, improved somewhat when we lengthened their lags or error-term structures.\textsuperscript{13} Tables 3 and 4 report the best regressions.\textsuperscript{14} Table 1 summarizes those results, which have the following implications.

1. Unexpected monetary increases tend to raise interest rates, and unexpected monetary decreases tend to lower them — contrary to a common belief among economists. In that popular view, if the Federal Reserve increases the money supply and nothing else happens, the public will hold more money than it wants. In the short run, incomes and goods prices will not change very much; therefore, interest rates must fall to make the public content to hold the increased money supply. That analysis is faulty, however, perhaps because it assumes that the stock of money rises because the Federal Reserve consciously chooses to increase it. Suppose instead that the money stock rises because of a rise in the quantity of money demanded at prevailing interest rates.

If the Federal Reserve does not entirely accommodate the increased demand, interest rates must rise to make the public content to hold less money than desired at the initial interest rates. Incidentally, if this analysis is valid, monetary surprises are primarily due to changes in money demand rather than in money supply.

2. Since October 6, 1979, the bond markets have responded about ten times as much as before to weekly money-supply data. Specifically, a monetary surprise of one percentage point tended to raise interest rates only 3-7 basis points before October 6, 1979, but tended to raise them 31-86 basis points after that date. Since interest rates respond to monetary surprises as useful economic indicators, bond-market participants must believe that monetary surprises tell them more now about the future state of the economy than they used to do. Apparently, the monetary-policy change has increased the information content in weekly money-supply data (or at

### Table 1

**Cumulative Interest-Rate Effects of Various Surprises**

<table>
<thead>
<tr>
<th>Security</th>
<th>Effect of One-Percentage-Point Increase in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-month Treasury Bill</td>
<td>5.62</td>
</tr>
<tr>
<td></td>
<td>(2.94)</td>
</tr>
<tr>
<td>Six-month Treasury Bill</td>
<td>6.60+</td>
</tr>
<tr>
<td></td>
<td>(2.14)</td>
</tr>
<tr>
<td>Twelve-month Treasury Bill</td>
<td>6.98+</td>
</tr>
<tr>
<td></td>
<td>(1.94)</td>
</tr>
<tr>
<td>Three-year Treasury Bond</td>
<td>6.77+</td>
</tr>
<tr>
<td></td>
<td>(1.76)</td>
</tr>
<tr>
<td>Five-year Treasury Bond</td>
<td>5.99+</td>
</tr>
<tr>
<td></td>
<td>(1.46)</td>
</tr>
<tr>
<td>Seven-year Treasury Bond</td>
<td>4.76+</td>
</tr>
<tr>
<td></td>
<td>(1.32)</td>
</tr>
<tr>
<td>Ten-year Treasury Bond</td>
<td>4.02+</td>
</tr>
<tr>
<td></td>
<td>(1.24)</td>
</tr>
<tr>
<td>Twenty-year Treasury Bond</td>
<td>3.11+</td>
</tr>
<tr>
<td></td>
<td>(1.02)</td>
</tr>
</tbody>
</table>

*The effects are measured in basis points. The figures in parentheses are standard errors.
+ Statistically significant at the .05 level.
least has made the market believe so).

3. Before October 6, 1979, monetary surprises affected short-term interest rates more than long-term interest rates, and affected intermediate-term rates even more than short-term rates. To explain this finding, suppose that a security’s term to maturity indicates the type of new information to which its interest rate is most sensitive. For instance, the three-month Treasury-bill rate is most sensitive to new information about what the economy will do in the next three months, whereas the ten-year Treasury-bond rate is sensitive to new information about what the economy will do for the next ten years. It then follows that, before October 6, 1979, monetary surprises conveyed relatively more information about what the economy would do six months to a year in the future (intermediate-term) than about what it would do for the next six months (short-term) or after a year (long-term). Apparently, during the period when the Federal-funds rate was pegged, changes in money demand (which produce monetary surprises) took six months to a year to exert their greatest effects on the economy.

4. After October 6, 1979, monetary surprises have affected short-term interest rates much more than long-term rates. Apparently, when nonborrowed reserves are used as the operating instrument, as they are today, changes in money demand exert their greatest effects on the economy almost immediately.

5. Surprises in the Federal-funds rate have affected short-term interest rates much more than they have long-term rates. For example, before October 6, 1979, a surprise of one percentage point would have raised the three-month Treasury-bill rate 61 basis points, while raising the twenty-year bond rate by only 14 basis points. Presumably, surprises in the funds rate tell the bond markets less about the far future than about the near future.

6. Surprises in the Federal-funds rate have affected interest rates less since October 6, 1979, than before. In particular, a surprise of one percentage point raised interest rates by 14-61 basis points before October 6, 1979, but only by 11-35 basis points since then. Apparently, letting the Federal-funds rate respond freely to market forces has reduced the information content of rate surprises for predicting the future state of the economy. 

<table>
<thead>
<tr>
<th>Security</th>
<th>Increased Coefficients on Surprises in</th>
<th>Increased Variances of Surprises in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Money Supply</td>
<td>Federal Funds Rate</td>
</tr>
<tr>
<td>Three-month Treasury Bill</td>
<td>28.2</td>
<td>-1.9</td>
</tr>
<tr>
<td>Six-month Treasury Bill</td>
<td>35.0</td>
<td>-9.7</td>
</tr>
<tr>
<td>Twelve-month Treasury Bill</td>
<td>27.1</td>
<td>-18.2</td>
</tr>
<tr>
<td>Three-year Treasury Bill</td>
<td>28.1</td>
<td>-5.0</td>
</tr>
<tr>
<td>Five-year Treasury Bond</td>
<td>25.7</td>
<td>-3.2</td>
</tr>
<tr>
<td>Seven-year Treasury Bond</td>
<td>24.5</td>
<td>-3.4</td>
</tr>
<tr>
<td>Ten-year Treasury Bond</td>
<td>27.8</td>
<td>-2.2</td>
</tr>
<tr>
<td>Twenty-year Treasury Bond</td>
<td>27.4</td>
<td>-2.4</td>
</tr>
</tbody>
</table>
### Table 3
Interest-Rate Regressions for Period Before October 6, 1979

<table>
<thead>
<tr>
<th>Security</th>
<th>EM(-1)</th>
<th>EM(-2)</th>
<th>EFFR(+1)</th>
<th>EFFR</th>
<th>Lag 1</th>
<th>Lag 2</th>
<th>Constant</th>
<th>R²</th>
<th>S.E.</th>
<th>Q(12)**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-month Treasury Bill</td>
<td>0.97</td>
<td>4.65</td>
<td>.389</td>
<td>.223</td>
<td>-.080</td>
<td>-.349</td>
<td>.0402</td>
<td>.211</td>
<td>.1551</td>
<td>7.3</td>
</tr>
<tr>
<td></td>
<td>(1.22)</td>
<td>(2.03)</td>
<td>(0.093)</td>
<td>(0.095)</td>
<td>(0.082)</td>
<td>(0.082)</td>
<td>(0.0075)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Six-month Treasury Bill</td>
<td>2.39</td>
<td>4.21</td>
<td>.425</td>
<td>.151</td>
<td>-.009</td>
<td>-.246</td>
<td>.0381</td>
<td>.254</td>
<td>.1103</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>(1.54)</td>
<td>(1.48)</td>
<td>(0.070)</td>
<td>(0.070)</td>
<td>(0.086)</td>
<td>(0.086)</td>
<td>(0.0070)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Twelve-month Treasury Bill</td>
<td>2.39</td>
<td>4.59</td>
<td>.392</td>
<td>.105</td>
<td>.131</td>
<td>.0343</td>
<td>.310</td>
<td>.0971</td>
<td>10.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.39)</td>
<td>(1.35)</td>
<td>(0.060)</td>
<td>(0.061)</td>
<td>(0.086)</td>
<td>(0.092)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three-year Treasury Bond</td>
<td>2.63</td>
<td>4.14</td>
<td>.286</td>
<td>.058</td>
<td>.200</td>
<td>.0242</td>
<td>.196</td>
<td>.255</td>
<td>.0733</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>(1.26)</td>
<td>(1.23)</td>
<td>(0.055)</td>
<td>(0.055)</td>
<td>(0.085)</td>
<td>(0.089)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Five-year Treasury Bond</td>
<td>2.57</td>
<td>3.42</td>
<td>.203</td>
<td>.024</td>
<td>.230</td>
<td>.196</td>
<td>.073</td>
<td>.255</td>
<td>.0733</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>(1.04)</td>
<td>(1.02)</td>
<td>(0.046)</td>
<td>(0.046)</td>
<td>(0.085)</td>
<td>(0.0076)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seven-year Treasury Bond</td>
<td>2.25</td>
<td>2.51</td>
<td>.170</td>
<td>.021</td>
<td>.176</td>
<td>.168</td>
<td>.217</td>
<td>.0661</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.94)</td>
<td>(0.92)</td>
<td>(0.041)</td>
<td>(0.041)</td>
<td>(0.086)</td>
<td>(0.0065)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ten-year Treasury Bond</td>
<td>1.57</td>
<td>2.45</td>
<td>.136</td>
<td>.030</td>
<td>.185</td>
<td>.148</td>
<td>.185</td>
<td>.0624</td>
<td>8.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.89)</td>
<td>(0.87)</td>
<td>(0.039)</td>
<td>(0.039)</td>
<td>(0.086)</td>
<td>(0.0062)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Twenty-year Treasury Bond</td>
<td>1.34</td>
<td>1.77</td>
<td>.116</td>
<td>.019</td>
<td>.254</td>
<td>.123</td>
<td>.203</td>
<td>.0514</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.73)</td>
<td>(0.71)</td>
<td>(0.032)</td>
<td>(0.032)</td>
<td>(0.084)</td>
<td>(0.0054)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The standard error of each coefficient appears below it in parentheses.
**Q(12) is the Box-Pierce statistic, a measure of serial correlation. None of the entries in this column indicates significant serial correlation at any conventional significance level.
+Statistically significant at the .05 level.

### Table 4
Interest-Rate Regressions for Period After October 6, 1979

<table>
<thead>
<tr>
<th>Security</th>
<th>EM(-1)</th>
<th>EM(-2)</th>
<th>EM(-3)</th>
<th>EFFR(+1)</th>
<th>EFFR(+2)</th>
<th>EFFR(+3)</th>
<th>Lag 1</th>
<th>Lag 2</th>
<th>Constant</th>
<th>R²</th>
<th>S.E.</th>
<th>Q(12)**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(7.8)</td>
<td>(7.5)</td>
<td>(7.8)</td>
<td>(0.099)</td>
<td>(0.110)</td>
<td>(0.104)</td>
<td>(0.094)</td>
<td>(0.089)</td>
<td>(0.131)</td>
<td>(1.31)</td>
<td>(1.32)</td>
<td>(1.106)</td>
</tr>
<tr>
<td>Six-month Treasury Bill</td>
<td>25.1</td>
<td>35.7</td>
<td>22.2</td>
<td>.079</td>
<td>.137</td>
<td>.007</td>
<td>.116</td>
<td>.199</td>
<td>.106</td>
<td>.420</td>
<td>.564</td>
<td>.009</td>
</tr>
<tr>
<td></td>
<td>(6.5)</td>
<td>(6.8)</td>
<td>(6.5)</td>
<td>(0.085)</td>
<td>(0.095)</td>
<td>(0.089)</td>
<td>(0.082)</td>
<td>(0.075)</td>
<td>(0.139)</td>
<td>(0.140)</td>
<td>(0.091)</td>
<td></td>
</tr>
<tr>
<td>Twelve-month Treasury Bill</td>
<td>21.5</td>
<td>26.9</td>
<td>12.8</td>
<td>.143</td>
<td>.149</td>
<td>.046</td>
<td>.170</td>
<td>.314</td>
<td>.034</td>
<td>.560</td>
<td>.3603</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>(7.0)</td>
<td>(7.4)</td>
<td>(7.2)</td>
<td>(0.077)</td>
<td>(0.075)</td>
<td>(0.080)</td>
<td>(1.48)</td>
<td>(1.55)</td>
<td>(0.073)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three-year Treasury Bond</td>
<td>26.3</td>
<td>27.7</td>
<td>13.2</td>
<td>.148</td>
<td>.046</td>
<td>.050</td>
<td>.046</td>
<td>.441</td>
<td>.3854</td>
<td>7.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.4)</td>
<td>(7.4)</td>
<td>(7.4)</td>
<td>(0.080)</td>
<td>(0.080)</td>
<td>(0.080)</td>
<td>(1.55)</td>
<td>(1.55)</td>
<td>(0.055)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Five-year Treasury Bond</td>
<td>22.0</td>
<td>22.6</td>
<td>12.6</td>
<td>.060</td>
<td>.046</td>
<td>.049</td>
<td>.174</td>
<td>.393</td>
<td>.3090</td>
<td>4.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.5)</td>
<td>(6.5)</td>
<td>(6.5)</td>
<td>(0.071)</td>
<td>(0.071)</td>
<td>(0.071)</td>
<td>(1.52)</td>
<td>(1.52)</td>
<td>(0.050)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seven-year Treasury Bond</td>
<td>20.2</td>
<td>18.2</td>
<td>9.04</td>
<td>.046</td>
<td>.174</td>
<td>.049</td>
<td>.098</td>
<td>.419</td>
<td>.2646</td>
<td>3.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.0)</td>
<td>(5.9)</td>
<td>(5.9)</td>
<td>(0.064)</td>
<td>(0.064)</td>
<td>(0.064)</td>
<td>(1.54)</td>
<td>(1.54)</td>
<td>(0.040)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ten-year Treasury Bond</td>
<td>19.9</td>
<td>15.8</td>
<td>.087</td>
<td>.053</td>
<td>.046</td>
<td>.048</td>
<td>.092</td>
<td>.395</td>
<td>.2364</td>
<td>3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.1)</td>
<td>(5.1)</td>
<td>(5.1)</td>
<td>(0.055)</td>
<td>(0.055)</td>
<td>(0.055)</td>
<td>(1.53)</td>
<td>(1.53)</td>
<td>(0.035)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Twenty-year Treasury Bond</td>
<td>17.5</td>
<td>13.7</td>
<td>.042</td>
<td>.065</td>
<td>.049</td>
<td>.049</td>
<td>.092</td>
<td>.50</td>
<td>.395</td>
<td>3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.5)</td>
<td>(4.5)</td>
<td>(4.5)</td>
<td>(0.049)</td>
<td>(0.049)</td>
<td>(0.049)</td>
<td>(1.53)</td>
<td>(1.53)</td>
<td>(0.035)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The standard error of each coefficient appears below it in parentheses.
**Q(12) is the Box-Pierce statistic, a measure of serial correlation. None of the entries in this column indicates significant serial correlation at any conventional significance level.
+Statistically significant at the .05 level.
IV. Decomposition of the Increase in Rate Volatility

In this section, we attempt to explain why interest rates have become much more volatile since October 6, 1979, than they were previously.

Equation (7) implies that \( VR \), the volatility of the bond rate \( R \), is

\[
VR = (b^2 + c^2)VEM + (d^2 + e^2)VEFFR + VV,
\]

where \( VEM \), \( VEFFR \) and \( VV \) are the variances of surprises in money, the Federal-funds rate, and the error term; and \( b, c, d \) and \( e \) are the coefficients of \( EM(-1), EM(-2), EFFR(+1) \) and \( EFFR \). To a first approximation, differencing this equation then yields

\[
DVR = (D(b^2 + c^2))VEM + (D(d^2 + e^2))VEFFR + DVV
\]

where \( DVR \), \( D(b^2 + c^2) \), \( DVEM \), \( D(d^2 + e^2) \), \( DVEFFR \) and \( DVV \) are the differences in \( VR \), \( b^2 + c^2, VEM, d^2 + e^2, VEFFR \) and \( VV \) between the two sample periods; and \( VEM, (b^2 + c^2), VEFFR \) and \( (d^2 + e^2) \) are the average values of \( VEM, b^2 + c^2, VEFFR \) and \( d^2 + e^2 \) in the two sample periods.

One can decompose the increase in volatility of each interest rate into five components due to 1) larger coefficients on monetary surprises; 2) larger variance of monetary surprises; 3) larger coefficients on the surprises in the Federal-funds rate; 4) larger variance of Funds-rate surprises; and 5) a more variable error term. Since the first four terms are supposed to capture the effects of monetary changes, the last term may be called the nonmonetary component.

We have used equation (8) and the empirical results reported in Tables 3 and 4 to calculate the fraction of the interest-rate volatility attributable to each component. The results appear in Table 2. Since equation (8) approximates an identity, the decomposition in Table 2 has no economic content by itself. To give it content, we have made four specific assumptions, as follows.

1. The nonmonetary component is independent of monetary policy. If the money supply and the Federal-funds rate provide a sufficiently complete characterization of monetary policy, and if equations (3)-(6) adequately describe that policy, this assumption follows immediately.

2. Changes in monetary policy have little effect on the variance of monetary surprises. This variance presumably reflects the weekly variance of money demand or of bank-loan demand, as Judd and Scadding argue elsewhere in this issue of the Economic Review. Monetary policy may well be able to control the money supply closely over periods as long as a quarter or two, but has little control on a weekly basis. In other words, a shift in monetary policy can change the coefficients in an equation like (3) or (5), but can have little effect on the standard error.

3. The coefficients \( b \) and \( c \) rose because the equation generating the money supply changed from (3) to (5), and because the Federal Reserve raised the coefficient on \( EM \) substantially (see equations (4) and (6)). These changes were part of the Fed's effort to target the money supply. Since stricter targeting makes monetary surprises more informative, interest rates responded much more to monetary surprises after October 6, 1979 than before.

4. The coefficients \( d \) and \( e \) fell because of a rise in the variance of surprises in the Funds rate. This increased variance reduced the information contained in monetary surprises, thereby causing interest rates to respond less to any given surprise.

Given these assumptions, the decomposition procedure (Table 2) suggests several important implications. First, factors beyond the Federal Reserve's direct control accounted for most of the increased volatility of interest rates. Nonfinancial factors accounted for about 45 percent of the increased volatility of short-
term interest rates, and for up to 65 percent of the volatility of intermediate-and long-term rates. Factors causing monetary surprises contributed about 7 percent more, so that all sources together accounted for 52-72 percent of the increased volatility. Second, making the Federal-funds rate more sensitive to monetary surprises generally resulted in 25-30 percent of the increased interest-rate volatility. Third, any Federal Reserve attempt to reduce the variance of surprises in the Federal-funds rate after October 6, 1979 would have reduced interest-rate volatility, but significantly so only for short-term rates. For example, preventing the variance of surprises in the funds rate from rising would have reduced the volatility of the three-month Treasury-bill rate by 19.4 percent \((=21.3 - 1.9)\), but would have reduced the volatility of the twenty-year Treasury-bond rate by only 3.3 percent \((= 5.7 - 2.4)\).

V. Summary and Conclusions

The efficient-markets hypothesis implies that interest rates adjust immediately to new information. Our empirical results support this hypothesis for long-term interest rates, since they suggest that bond markets quickly use new information about the money supply and the Federal-funds rate.

The Federal Reserve's October 6, 1979 change in monetary policy altered the way that bond markets set interest rates. Previously, a monetary surprise of one percentage point raised interest rates by 3-7 basis points, and a surprise of one percentage point in the Federal-funds rate raised rates by 14-61 basis points. After October 1979, such surprises would have raised interest rates by 31-86 and 11-35 basis points, respectively. Clearly, monetary surprises have become rather important, while surprises in the Federal-funds rate have become substantially less important.

Analysis of the decomposition of rate volatility suggests that 52-72 percent of the increased volatility resulted from factors not under the Federal Reserve's direct control. About 25-30 percent of the increased volatility resulted from making the Federal-funds rate respond to monetary surprises. The rest came from freeing the Federal-funds rate to respond to nonmonetary market forces; this source was responsible for as much as 20 percent of the increased volatility of short-term rates, but for as little as 3 percent of the increased volatility of long-term rates.

These findings suggest several public-policy implications — primarily, that the Federal Reserve has not been responsible for most of the increase in interest-rate volatility. The post-October 1979 period has seen many unexpected events that could have changed interest rates or shifted the demand for money. For example, militant students seized hostages in Iran, the Russians invaded Afghanistan, decontrol of oil prices began, President Carter authorized credit controls, the silver market collapsed, and the U.S. underwent a radical change in political direction. Clearly, none of these events was a direct consequence of the monetary-policy change. Furthermore, future years may see a return to normalcy, with a sharp reduction in interest-rate volatility.

Second, the Federal Reserve's decision to move the Federal-funds rate more in response to monetary surprises entails more volatility of both long-and short-term interest rates. It probably also helps the Federal Reserve to hit its targets for money growth and hence for inflation. For this reason, the increased volatility — and the resultant reduction in capital formation and redirection of capital towards short-lived assets — may be the price that must be paid to hit these targets. The price has certainly proven to be higher than many believed before October 6, 1979. Whether this price has been too high depends on how important it is to hit monetary targets, and how much the increased volatility reduces savings and changes the composition of investment.

Finally, Federal Reserve intervention in the market for reserves to eliminate surprises in the Federal-funds rate would mean only a slight reduction in the volatility of long-term interest rates. If the Federal Reserve inter-
vened, however, it would simply replace private agents as the speculator in that market. This paper has established no presumption that the Fed is a better speculator than private agents — and even if it were, it would not need to intervene directly itself. A timely and credible public announcement of the Fed’s superior information would make the market as efficient as it could ever be — simply because efficient securities markets make optimal use of all the information available to them.

FOOTNOTES

1. For example, see Herman (1981).
2. See Fama (1970) for a discussion of the efficient-markets hypothesis and for a review of some empirical work supporting it.
3. Mishkin (1980) has shown that one-quarter holding-period yields of long-term bonds are indeed dominated by capital gains and losses.
4. Clearly, I am assuming here that a linear predictor is best.
5. I have used unadjusted data, because I believe that the method by which the Federal Reserve obtains its seasonally adjusted data does more harm than good.
6. Henceforth, I shall refer to these sample periods as “before October 6, 1979” and “after October 6, 1979.”
7. This methodology entails examining the sample autocorrelations and partial autocorrelations of these series, identifying univariate processes for each series, fitting these processes, subjecting each fitted process to tests of model adequacy, crosscorrelating the residuals of these processes, identifying the bivariate process generating the two series, fitting this bivariate process, and testing whether the fitted process is adequate. See Box and Jenkins (1976) and Granger and Newbold (1977) for a complete description of this methodology.
8. For example, equations (3) and (4) take the form (ignoring the constant term)

\[ z = H(L)e, \]

where \( z \) is a vector composed of GM, -GM(-52) and DFFR; \( e \) is a vector composed of EM and EFFR; and \( H(L) \) is a 2x2 matrix in polynomials in the lag operator \( L \), which is defined such that \( Lz = Z(-i) \). Since \( H(L) \) is invertible,

\[ H^{-1}(L)z = e. \]

Suppose that

\[ H^{-1}(L) = I - J_1 L - J_2 L^2 - \ldots \]

Then \( z = J_1 z(-1) + J_2 z(-2) + \ldots + e. \)

Let \( Z \) be the vector obtained by stacking \( z, z(-1), \ldots, \) and let \( E \) be the vector with EM in the first entry, EFFR in the second entry, and zeros in the remaining entries.

Then

\[ Z = AZ(-1) + E, \]

where

\[
A = \begin{bmatrix}
J_1 & J_2 & J_3 & \ldots \\
0 & 1 & 0 & \ldots \\
0 & 0 & 1 & \ldots \\
\vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\]

9. This statement ignores the constant term and assumes that a year has exactly 52 weeks.
10. To keep the analysis as simple as possible, I assume that

\[ dr_{it} = v_{it}, \]

where \( dr_{it} \) is the change in the bond rate from the end of day \( i-1 \) of week \( t \) to the end of day \( i \) of week \( t \) (day 0 of week \( t \) is day 7 of week \( t - 1 \)), and \( v_{it} \) is a serially uncorrelated error term. This equation implies that \( DR_{it} \), the change in the bond rate from the end of day \( i \) of week \( t-1 \) to the end of day \( i \) of week \( t \), is

\[ DR_{it} = \sum_{i=1}^{7} v_{i-1} + \sum_{i=1}^{7} v_{i}/7 \]

Averaging this equation then yields

\[ (*) \] DR = \[ \frac{7}{1} \left( \sum_{i=1}^{7} v_{i-1} + \sum_{i=1}^{7} v_{i} \right)/7 \]

Therefore, DR and DR (-1) have the nonzero covariance

\[ (**) \] \[ \frac{7}{1} \left( \sum_{i=1}^{7} \text{Var} \left( v_{i-1} \right) \right)/49 \]

and DR and DR(-1), i > 1, have a zero covariance, it follows that DR is the first-order moving average. Note that DR has the representation

\[ DR = U + gU(-1), \]

where \( U \) is a serially uncorrelated error term and \( g \) is a parameter. The parameter \( g \) and the variance of \( U \)
are chosen so that DR has the same variance as (*) implies and DR and DR(-1) have the covariance (**).

11. It is hard to be more specific about when EM(-1) affects the bond markets, because M-1B data may leak out before its official release date. I assume, however, that leakage occurs after the beginning of the week.

12. The sum of the coefficients on EFFR(+1) and EFFR consistently estimates the coefficient that one would obtain using Federal-funds rate data averaged over weeks ending on Saturdays. First, let $X_1$ be EFFR(+1), $X_2$ be EFFR, $Y$ be DR, and $X$ be the EFFR that would be used if the right data were available. Next, let $Z_1$ and $Z_2$ be the parts of $X$ that $X_1$ and $X_2$ give to the bond markets, and let $E_1$ and $E_2$ be defined by

$$X_1 = Z_1 + E_1$$
$$X_2 = Z_2 + E_2$$
$$X = Z_1 + Z_2$$

By construction, $Z_1$, $Z_2$, $E_1$, and $E_2$ are mutually orthogonal, and $X_1$, $X_2$, and $X$ have the same variance. Then, let $\alpha$ be the fraction of the variance of $X$ contributed by $Z_1$, and $1 - \alpha$ be the fraction contributed by $Z_2$. Finally, let

$$Y = \beta X + V,$$

where $V$ is orthogonal to $Z_1$, $Z_2$, $E_1$, and $E_2$. Then

$$Y = \beta (X_1 + X_2) + V - \beta (E_1 + E_2).$$

Since $X_1$ and $X_2$ are orthogonal, the least-squares estimator $b_1$ of the coefficient on $X_1$ is

$$\Sigma X_1 Y / \Sigma X_1^2.$$ 

Its probability limit is therefore

$$\text{plim} \left( \Sigma X_1 (\beta (X_1 + X_2) + V - \beta (E_1 + E_2)) / \Sigma X_1^2 \right) = \beta + \beta \text{plim} \left( \Sigma X_1 X_2 / \Sigma X_1^2 \right) = - \beta \text{plim} \left( \Sigma X_1 V / \Sigma X_1^2 \right) - \beta \text{plim} \left( \Sigma X_1 (E_1 + E_2) / \Sigma X_1^2 \right) = (1 - \text{plim} \left( \Sigma E_1^2 / \Sigma X_1^2 \right)) \beta = 1 - \text{plim} \left( \Sigma E_1^2 / \Sigma X_1^2 \right) \beta = \alpha \beta.$$

Similarly, the probability limit of the least-squares estimator $b_2$ of the coefficient on $X_2$ is

$$\text{plim} \left( b_2 = (1 - \alpha) \beta \right)$$

Hence

$$\text{plim} \left( b_1 + b_2 \right) = \beta.$$

13. Strictly speaking, the efficient-markets hypothesis only rules out long lag structures in the equations for long-term interest rates. I therefore conclude that the data support the efficient-markets hypothesis.

14. If the error term $V$ is a first-order moving average, it takes the form

$$V = U + g U(-1),$$

where $U$ is a serially uncorrelated error term and $g$ is a parameter. If $V$ is a second-order moving average,

$$V = U + g U(-1) + h U(-2),$$

where $h$ is a parameter. The columns labeled Lag 1 and Lag 2 provide the estimates of $g$ and $h$.

15. Since October 6, 1979, the Federal-funds rate has conveyed more information about supply and demand in the market for reserves, even though it has conveyed less information about the aggregate economy.

16. I define the volatility of an interest rate to be the variance of its weekly differences.

17. Some of the equations reported in Tables 1 and 2 have longer lag structures than equation (8) recognizes. For these equations, I have modified equation (8) appropriately.

REFERENCES


Liability Management, Bank Loans, And Deposit “Market” Disequilibrium

John P. Judd and John L. Scadding*

High rates of inflation during the past decade have increasingly focused the attention of policy makers and the general public on the importance of bringing the monetary aggregates under control. The Federal Reserve System now has an official goal of slowly reducing growth rates in the monetary aggregates over the next few years in order to lower rates of inflation gradually. Since October 1979, the Fed has attempted to improve monetary control by focusing its short-run operations on achieving targets for bank reserves, and by letting the Federal-funds rate vary more widely than previously had been the case.

Despite these procedural changes, the monetary aggregates gyrated widely in 1980, and were significantly above or below the Fed’s longer-run targets at various times during the year. This paper discusses a monthly money-market model which provides an explanation for the surprisingly high variability of money in 1980. The model shows how certain types of financial-market disturbances, such as sharp changes in bank loans, can affect the money supply and thus cause problems of monetary control. The evidence indicates that large swings in bank loans, induced primarily by the Special Credit Control Program, were the major source of money’s variability in 1980.

This explanation has no role in conventional models, which view the supply of deposits as being determined by the public’s demand, given short-term rates of interest, income and prices. With a conventional model, unexpected movements in the monetary aggregates often reflect changes in the past relationship between the public’s demand for money and its determinants — that is, reflect a “shift” in the demand function for money. There is little doubt, in retrospect, that such a downward shift occurred in 1975-76, when historically high interest rates induced the public to economize on money balances. In far greater doubt, however, are assumed subsequent “shifts” of shorter duration, such as the one in the second quarter of 1980. The present paper argues that the rapid monetary deceleration in the second quarter of 1980 (as well as the rapid growth in the first and third quarters) was caused, not by a money-demand shift, but by a money-supply “shock” induced by changes in bank loans. This is a crucial distinction for policymakers. A downward shift in the demand for money makes a given money supply more expansionary. Thus the appropriate policy is to lower the money supply. On the other hand, a downward money-supply “shock” for a given demand for money makes policy more contractionary. Thus the appropriate policy response is to offset the money-supply “shock” through faster growth in bank reserves.

Whereas conventional models emphasize the demand for money, the model in this paper emphasizes the supply of money, with banks playing an important role in determining that supply. In particular, it explicitly incorporates bank loans and banks’ management of non-deposit liabilities into the determination of transaction deposits. In this approach, banks maximize profits by satisfying the public’s

---

*The authors are Senior Economists, Federal Reserve Bank of San Francisco. Lloyd Dixon and Steven Kamin provided research assistance for this article.
demand for loans with funds raised with the least costly mix of managed liabilities (such as large certificates of deposit and repurchase agreements). The outcome of this process is an aggregate "supply" of transaction deposits, which varies inversely with the Federal-funds rate and directly with the commercial-paper rate and with bank loans.

The model treats money as a buffer stock in the public's portfolio. Loan-induced increases in the money supply thus exert an especially powerful impact on the monetary aggregates in the model. When the public demands additional bank loans, it temporarily absorbs the deposits that are created in the process without significant interest-rate changes in the short-run: i.e., money-supply shocks induced by bank-loan movements can put the market for money into temporary disequilibrium. This means that changes in bank loans have a large short-run effect on the public's money holdings and a relatively small effect on interest rates.

The model therefore provides a theoretical rationale to explain why changes in the supply of money can dominate short-run movements in the monetary aggregates. The empirical section provides three pieces of evidence consistent with this hypothesis. They involve the speed with which banks adjust reserves when interest rates change, with the contribution that bank-loan changes make to explaining movements in money, and with the extent to which money-supply shocks temporarily shift the public's demand curve for money.

Section I of the paper describes the theoretical model. Here we show how the model determines the stock of transaction deposits, total reserves, and the Federal-funds and commercial-paper rates. Section II reports the results of estimating the model on lunar-monthly data (four-week blocks) for the sample period July 1976 to September 1979. This section also considers the results of simulating the model both over the sample period and out of sample over the post-October 1979 period — the period marked by the Federal Reserve's adoption of a new reserve-operating procedure. Section III uses the simulation results to assess the cause of the volatility in the monetary aggregates in 1980. Section IV summarizes the conclusions and the policy implications of the model.

I. Theoretical Model

The model is designed to analyze the behavior of the commercial banks, nonbank public and Federal Reserve in the markets for transaction deposits and bank reserves. Thus the primary variables determined by the model include the stock of transaction deposits and the commercial-paper rate in the deposit market, and the stock of reserves and the Federal-funds rate in the reserves market. The underlying characteristics of the model are described in three distinct stages, which are summarized in Table 1. Each stage includes the preceding stage(s), so that by stage 3, the model is complete. A formal specification of the model is presented in Appendix A.

Stage 1 analyses the markets in which commercial banks sell nondeposit liabilities (such as large certificates of deposit and repurchase agreements) to the nonbank public. As shown below, demands for and supplies of these instruments — expressed as functions of own and substitute yields as well as the sizes of the banks' and public's portfolios — are sufficient to determine the banking system's mix of liabilities between deposits and nondeposits. The level of deposits implied by this mix constitutes the banking system's "supply" of transaction deposits. Note that in Stage 1, the "supply" of deposits is defined as a function of the Federal-funds rate, and therefore abstracts of conditions in the market for reserves.

Stage 2 introduces the Federal Reserve by adding to the analysis the market for bank reserves. The banking system's desired mix between nondeposit liabilities and deposits determined in Stage 1, together with the reserve-requirement ratios on these categories of bank liabilities, define the banking system's
demand for total reserves. The supply of reserves comes from 1) the amount of borrowing from the Federal Reserve, and 2) the amount of nonborrowed reserves supplied by the Fed. The addition of the supply of reserves allows the reserves market to clear at equilibrium values of the funds rate and total reserves. In Stage 2, both the reserves and nondeposit-liabilities markets clear. Hence the supply of deposits at this stage is consistent not only with the banks’ preference among liabilities, but also with the banks’ and the Fed’s desired level and composition of reserves.

Stage 3 introduces the public’s demand for transaction deposits. The interaction of this demand with the supply of deposits determined in Stage 2 completes the solution of the model. Here it is not strictly accurate to speak of market equilibrium because the market for deposits allows for short-run disequilibrium. Nevertheless, since the model defines the source and size of that disequilibrium, the deposit market can determine the stock of deposits and the commercial-paper rate. The remainder of this section describes each stage in more detail.

### Stage 1: Nondeposit Liabilities
The analysis begins with the description of the portfolio behavior of an individual bank (Figure 1). A minimum of seven categories of bank assets and liabilities is necessary to preserve the model’s usefulness as a foundation for empirical research. These categories are total reserves, \( R \); bank loans, \( BL \); private transaction deposits (including demand, ATS and NOW accounts), \( DB \); other deposits (primarily small time and savings), \( TB \); managed liabilities less security holdings, \( IMB \); member-bank borrowing, \( RB \); and net Federal funds purchased and repurchase agreements, \( FF/IRP \). The last three items together constitute what we call nondeposit liabilities.

The short-run problem of a representative bank involves financing a given stock of loans. Banks consider loans to be exogenous on a monthly basis, because the short-run demand is relatively interest inelastic — and because banks often respond sluggishly in altering their loan rates when their marginal costs of funds change, waiting for signs that such cost changes are not transitory.

Part of the bank’s loan portfolio is financed by transaction and other deposits, which it

### Table 1

#### Stages of the Model

<table>
<thead>
<tr>
<th>Markets</th>
<th>Behavioral Relations</th>
<th>Variables or Relations Solved For</th>
<th>Variables Affecting Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1: Banks' nondeposit liabilities</td>
<td>1. Banks' supplies of nondeposit liabilities</td>
<td>i) Funds rate</td>
<td>i) Nonborrowed reserves target</td>
</tr>
<tr>
<td></td>
<td>2. Public's demand for nondeposit liabilities</td>
<td>ii) Commercial paper rate</td>
<td>ii) Discount rate</td>
</tr>
<tr>
<td>Stage 2: Bank reserves</td>
<td>1. Banks' demand for total reserves</td>
<td>iii) Discount rate</td>
<td>iii) Bank loans</td>
</tr>
<tr>
<td></td>
<td>2. Federal Reserve's supply of reserves</td>
<td>iv) Bank loans</td>
<td>iv) Personal income</td>
</tr>
<tr>
<td></td>
<td>2. Public's demand for deposits</td>
<td>b. Commercial paper rate</td>
<td>ii) Discount rate</td>
</tr>
</tbody>
</table>

23
regards as exogenous in the short run. The bank adjusts implicit deposit rates sluggishly — as it does the loan rate — viewing the quantity of deposits in the short run as being essentially determined by the public’s demand. Banks must finance the excess of loans over deposits by selling nondeposit liabilities to the public. The individual bank’s short-run portfolio choice involves choosing the structure of nondeposit liabilities — among IMB, FF/RP and RB.

The bank’s portfolio choices are the outcomes of maximizing expected profits subject to the balance-sheet constraint. In the very short run, only IMB, FF/RP and RB can be adjusted. The factors influencing expected profits include, among other things, the explicit interest costs of each of three liability items — the rate on longer-term managed liabilities (such as CDs), denoted by \( i_o \); the discount rate, \( i_B \); and the Fed-funds rate, \( i_F \). As well, expected profits depend on the risk and liquidity characteristics of assets and liabilities, so that the marginal returns or costs of each portfolio item include a marginal non-interest element in addition to the explicit interest cost. For example, banks’ borrowings from the Federal Reserve depend not only on the discount rate, but also on banks’ traditional “reluctance to borrow” from the Fed. Given these variables — as well as the (exogenous) size of the portfolio to be financed, \((BL + R - DB - TB)\) — individual banks sell optimal quantities of IMB, FF/RP and RB to the non-commercial-bank sectors.

The quantities of these instruments actually observed depend on the interaction of the banks’ supplies of various types of nondeposit liabilities with the nonbank public’s demands for them. The latter depends upon relative yields and other characteristics (e.g., risk) of the bank and nonbank assets in the public’s portfolio, together with the overall size of that portfolio.

The interaction between the banks and the nonbank public in the markets for banks’ nondeposit liabilities is critical to the model, because equilibrium in these markets determines the “supply” of deposits created by the banking system. Equilibrium is depicted in Figure 2 by the curve EQ. This curve represents all combinations of the funds rate and bank nondeposit liabilities \((IMB + FF/RP)\) which are consistent with equilibrium between the banks’ supplies of and the public’s demands for IMB and FF/RP (for expositional purposes we assume that \(RB = 0\)).

Movements along EQ are determined in the following manner. Assume that the funds rate rises. Since banks consider FF/RP a substitute for IMB as a source of funds, they will respond by raising their offer rates on IMB. Since rates on both IMB and FF/RP have risen, the public’s demand for the total of those instruments would also have risen, with the net inflow of funds having been drawn from nonbank instruments (such as commercial paper), whose rates had not increased. Thus an increase in the funds rate induces an increase in \(i_o\), which results on balance in a rise in banks’ total nondeposit funds.

The increased purchases of \(IMB + FF/RP\) extinguish demand deposits as the public draws down its checking accounts to pay for them. This process ensures that the banks’ balance sheets will be in equilibrium. If banks attract more nondeposit funds, their need for deposits decreases, given the size of the loan portfolio to be financed. The destruction of deposits that accompanies the inflow of non-
Figure 2
Equilibrium in the Markets for Banks' Nondeposit Liabilities

\[ i_F \]

\[ EQ \]

IMB+FF/RP

deposits ensures that the new mix of liabilities is consistent with the banking system’s portfolio needs. Thus the combination of EQ — which describes the nondeposit funds supplied by the public for each level of the funds rate — and the bank’s portfolio constraint implicitly defines the stock of deposits which is consistent with both the banks’ and public’s preferences for nondeposits.

The combinations of \( i_F \) and \( DB \) that satisfy both EQ and the bank’s portfolio constraint constitute the Stage 1 supply of deposits (\( DB_{s1} \)). A higher funds rate leads to fewer deposits being supplied. This occurs because the inflow of nondeposit funds to banks resulting from the funds-rate increase causes banks to extinguish deposits as their need for them declines. \( DB_{s1} \) is also a function of the nonfinancial commercial-paper rate (representing the rate on the public’s nonbank instruments) and the banking system’s portfolio scale variable, \( BL + R - TB \) (referred to as SCALE below).

\( DB_{s1} \) is positively related to the nonfinancial commercial-paper rate, which means that its curve shifts to the right when \( i_{cp} \) rises. The public regards commercial paper as a substitute for bank liabilities like RPs and large CDs. Hence a rise in the paper rate will reduce the public’s demand for bank nondeposit liabilities as they shift funds into commercial paper. Banks will respond by raising offer rates on nondeposit liabilities, but this will be insufficient to stem completely the exodus of funds. As a result, banks will end up supplying more transaction deposits, which they create as they buy back managed liabilities from the nonbank public.

A rise in SCALE also shifts \( DB_{s1} \) to the right. A rise in bank loans, for example, increases the size of the portfolio banks must finance, with a consequent increase in SCALE. For given \( i_F \) and \( i_D \), the amount of nondeposit liabilities is fixed by the public’s demand for these liabilities. Consequently, the supply of bank deposits must increase by the increase in loans if rates are not to change. These deposits constitute the proceeds of loans, which are spent by the initial borrower and flow into the accounts of his suppliers, employees and the like.

Stage 2: Reserves

The deposit-supply function of Stage 1 was defined as a function of the funds rate. In Stage 2, we add the reserves market; this determines the funds rate along with a more comprehensive definition of the supply of deposits, denoted \( DB_{s2} \), which includes the influence of the Federal Reserve’s conduct of monetary policy.

The right-hand diagram of Figure 3 shows \( DB_{s1} \) from Stage 1. In the left-hand diagram, \( R^d \) plots the amounts of required reserves the banking system would need to hold for each point on \( DB_{s1} \). This will depend upon the required-reserve ratio for transaction deposits. (For expositional purposes, only transaction deposits are considered reservable.) The higher the level of transaction deposits supplied, the larger are required reserves. Hence lower funds rates, which are consistent with a larger quantity of deposits supplied, are in turn associated with a greater need or “demand” for reserves. The graph of all such combinations of funds rates and required reserves therefore can be thought of as defining the banking system’s demand function for reserves, depicted in Figure 3 as \( R^d \).
The description of the factors determining the total amount of reserves available — the supply of reserves — is conditional on the Federal Reserve's choice of operating procedure. We assume the current procedure, whereby the Federal Reserve determines a target for nonborrowed reserves; in Figure 3 one such target is illustrated by the vertical line RU*.

Total reserves available can be larger than RU*. Banks may borrow reserves from the Federal Reserve on a temporary basis, instead of borrowing in the Federal-funds market. Consequently, a higher funds rate leads banks to switch from the Fed-funds market to the Federal Reserve's discount window, adding to the aggregate stock of reserves in the system. The amount borrowed will also depend on the discount rate \( i_d \), the rate charged by the Fed for such borrowing. At funds rates below the discount rate, banks have little incentive to borrow, so that total reserves are roughly equal to nonborrowed reserves (this accounts for the vertical portion of \( R^s \) below the "kink" at \( i_f = i_d \)). But as the funds rate rises above the discount rate, banks respond to a profit incentive and expand their borrowing from the Fed. However, the amount of this borrowing is limited by the banks' reluctance to borrow, which effectively determines the slope of \( R^s \) at funds rates above the kink. Since the reluctance to borrow tends to rise as the level of

Figure 3
Derivation of Stage 2 Deposit Supply
borrowing rises, \( R_s \) becomes more steeply sloped at higher funds rates. In Figure 3, discount-window borrowing as a function of the funds rate is added to the nonborrowed-reserves target to obtain total reserves available, or what we call reserves supply, \( R_s \).

The interaction of reserves supply, reserves demand and \( DB_{-1} \) determine market-clearing levels for the funds rate, total reserves and the Stage 2 supply of deposits (\( DB_{-2} \)). As noted earlier, \( DB_{-1} \) is defined for different funds rates, whereas \( DB_{-2} \) is co-determined with the funds rate for any given level of the Federal Reserve's monetary-control instrument. Point A in the upper two graphs of Figure 3 illustrates the determination of \( DB \) at stage 2 for the case in which the Fed uses nonborrowed reserves as its instrument.

The movement from A to B shows the effect on Stage 2 \( DB \) of an increase in the commercial-paper rate. As seen from the discussion of Stage 1, a rise in the commercial-paper rate shifts \( DB_{-1} \) to the right. This shift, shown in the NE diagram of Figure 3, is associated with an increase in the demand for reserves in the NW diagram. The increased demand for reserves puts upward pressure on the funds rate. Hence the increase in \( i_{cp} \) causes both \( i_F \) and \( DB \) to rise from A to B. Levels of Stage 2 deposits are plotted against the commercial-paper rate in the SE diagram, and denoted by \( DB_{-2} \).

An increase in the Federal Reserve's nonborrowed-reserves operating instrument causes \( DB_{-2} \) to rise. For example, a larger stock of nonborrowed reserves puts downward pressure on the funds rate. As a result, borrowed reserves fall, offsetting part of the increase in RU. In addition, the lower funds rate induces banks to cut the rates they pay on other nondeposit liabilities, so that the public reduces its holdings of these instruments, causing banks to create more deposits. The net effect in the reserves market is a movement down along the \( R^d \) curve, with a lower funds rate and a higher level of total reserves. In the deposit market, the Stage 2 supply curve shifts to the right. For any given commercial-paper rate, a lower funds rate induces a lower equilibrium quantity of IMB + FF/RP, and thus a larger supply of deposits.

An increase in bank loans also has a positive effect on \( DB_{-2} \). When bank loans rise, banks' managed liabilities and deposits rise at unchanged interest rates: i.e., both \( R_s \) and \( DB_{-2} \) shift to the right. The increased demand for reserves causes the funds rate to rise, as banks are "forced" to the discount window for a larger quantity of reserves when nonborrowed reserves are held constant. The higher funds rate eliminates part of the increase in banks' reserves demand and deposit supply, but on balance both quantities rise.

Note that the influence of bank loans on deposit supply depends heavily on the behavior of the Federal Reserve. If, for example, the Fed held the funds rate constant in the face of an increase in bank loans, the partial offset of the increase in \( DB_{-2} \) could not occur. As a consequence, the impact of a bank-loan increase would be larger than in the case where the Fed held nonborrowed reserves constant and allowed the funds rate to rise. By an analogous argument, the Fed could reduce nonborrowed reserves to such an extent that a change in bank loans would have no influence on the quantity of deposits supplied.

**Stage 3: Transaction Deposits**

Only in the last stage is the public's demand for transaction deposits introduced. This demand is used in conjunction with the Stage 2 deposit supply to solve for the commercial-paper rate and the stock of transaction deposits. The model allows for the possibility of market disequilibrium by distinguishing two concepts of deposit demand. The first — short-run equilibrium demand — is the conventional relationship in which deposit demand is a function of short-term interest rates, income and lagged deposits. We include lagged deposits in this function to allow for incomplete adjustment of the public's demand in the short-run to changes in interest rates and income.

Conventional practice treats this short-run equilibrium demand as equal to the actual stock of deposits: i.e., it views the public as always being on its demand function. The pre-
sent model, however, allows for temporary disequilibrium in the deposit market, in which the commercial paper rate does not adjust to make the actual stock equal to the short-run equilibrium demand at each moment of time.\textsuperscript{11}

Actual deposits are therefore identified with the second concept of short-run demand — the disequilibrium demand for deposits. This differs from its equilibrium counterpart to the extent that market disturbances originating in certain types of shifts in the Stage 2 money supply temporarily force the public off the equilibrium demand curve. This approach makes an important distinction between the demand for money and the demand for credit. Changes in the quantity of bank loans, for example, are assumed to be in accordance with the equilibrium in the bank-loan market. However, these loan changes have an important by-product: the creation or destruction of deposits. Since changes in credit demand are not necessarily associated with equal changes in deposit demand, the public ends up temporarily holding deposits it does not want: i.e., it only accepts the deposits because this is a necessary part of accepting the credit it does want.

The important question is whether the public remains in disequilibrium for a long enough time to permit this effect to show up in monthly observations. The persistence of disequilibrium will depend, in part, upon the size of transaction costs involved in adjusting money balances to desired levels, and will vary among classes of depositors. Transaction costs may be relatively small for large businesses, who have at their disposal a number of highly liquid financial instruments (e.g., repurchase agreements) with which to adjust deposit holdings. In contrast, households and others could face relatively large transaction costs. Inflows of “unwanted” deposit balances do not lead them to make immediate portfolio adjustments by the full amount necessary to restore equilibrium.

Disequilibrium in the deposit market could persist longer than it takes an individual depositor to adjust to desired balances. One depositor’s equilibrium may be another depositor’s disequilibrium. To the extent that depositors reduce their unwanted balances by purchasing goods and services and securities from other members of the public, the latter’s deposit balances may exceed desired levels. This process of spending and respending persists until the unwanted deposits are “sold” back to banks for nondeposit liabilities (reducing deposit supply) and/or income and prices rise enough (raising deposit demand) to restore equilibrium to the deposit market.

Finally, the actions of the Federal Reserve can significantly influence disequilibrium in the deposit market. If the Fed moves RU so as to peg the funds rate, for example, it would in effect allow the full impact of bank-loan changes on deposit supply to be felt in the deposit market. If, on the other hand, the Fed hits its nonborrowed reserves targets and lets the funds rate vary, the impact of bank loans on deposit-market disequilibrium will be muted. Furthermore, under such a reserves-control procedure, the Fed could be an important source of disequilibrium itself. Assume, for example, that the Fed exogenously increased total reserves in excess of required reserves. Banks might lend out these excess reserves by purchasing Treasury securities

---

**Figure 4**

**Effect of Deposit Supply “Shock” on Observed Deposits**

- DB<sub>d</sub>
- DB<sub>r</sub>
- DB<sub>s</sub>
- DB<sub>r</sub>-2
- DB<sub>s</sub>-2
- DB<sub>b</sub>
- DB<sub>b</sub>-2

Note: \( \Delta DB^b = \lambda DB^b \)
from the public. The Treasury-bill rate would fall enough to induce the public to sell bills, but the associated increase in deposits (i.e., the proceeds of the bill sales) would not necessarily be demanded in the equilibrium sense in the short-run. The deposit market would be in (temporary) disequilibrium to the extent that this occurs.

The process by which bank loans influence the deposit market is illustrated in Figure 4. The curve $DB^d$ denotes the short-run equilibrium demand for deposits as a function of the commercial-paper rate, $i_{cp}$, with nominal income, $Y$, held constant. A decrease in bank loans is illustrated by a leftward shift in the deposit-supply function by the horizontal distance, $\Delta DB^s$. This disturbance causes the public to end up holding fewer deposits than the equilibrium-demand curve would indicate. In the short-run, $i_{cp}$ and $DB^d$ move from point A to point B rather than to the point C indicated by the equilibrium-demand function. At point B, $DB^d$ differs from $DB^*$ by some fraction $\lambda$ of the initial $DB^*$ "shock". This disequilibrium reduces interest-rate variability in response to deposit-supply disturbances such as changes in bank loans. (The same may also be true for changes in nonborrowed reserves when they are a source of deposit-market disturbances.) Graphically, the process of the move back to equilibrium can be thought of as made up of 1) movements along $DB^a$ as interest rates adjust, and 2) leftward shifts in $DB^d$ (shown by $DB^d$) as income and prices change until equilibrium is reached at $D$.

The theoretical model is completed with the addition of descriptions of the public's demands for currency (C) and other deposits (TB) as functions of income, the commercial-paper rate and other variables. These equations will be described in more detail in the next section.

**II. Empirical Model**

We summarize the empirical version of the theoretical model in Table 2, and report the corresponding estimation results in Table 3. (Appendix B contains a glossary of variable names.) The empirical version of the model recapitulates, with modifications, the theoretical model, but it also includes additions to explain other components of MIB besides demand deposits, and to account for the other important uses of reserves besides those held against demand deposits and nondeposit funds. (A fuller accounting for the uses of reserves, along with a more complete description of some of the modifications discussed below, can be found in Appendix C.) But more importantly, the empirical model includes modifications to the core equations dictated by the fairly complex structure of reserve requirements in the real world.

Two of the equations from the theoretical model carry over with minor changes. They are the banks' aggregate demand for reserves against demand deposits and nondeposit funds, denoted $RA$ and described in equation (2.1), and the banks' demand for borrowed reserves described in equation (2.3). Reserves demand now includes the discount rate, $i_B$, which had previously been assumed to be constant, and the reserve ratio against nondeposit liabilities, $r_n$, which had been assumed to be zero. Equations (2.1), (2.3), the specification of the Federal Reserve's supply of nonborrowed reserves (equation (2.11)) and the supply of deposits (2.12) together constitute the empirical version of Stage 2 of the model, which is used to solve for the funds rate, the quantity of reserves, and $DB^s-2$.

The empirical counterpart of Stage 3 is used to solve for the commercial-paper rate and the quantity of demand deposits. This version consists of the public's demand for demand deposits, equation (2.5), and the corresponding supply of demand deposits ($DB^s-2$). The latter relationship is where the empirical version departs most significantly from the theoretical model.

In the theoretical discussion, the derivation of deposit supply in Stage 2 was trivial: the
### Table 2
Empirical Model

#### Behavioral Relations

<table>
<thead>
<tr>
<th>Description</th>
<th>Banks, Thrifts and Public</th>
<th>Estimated Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2.1) Banks' demand for reserves</td>
<td>RA = RA(i_F, i_C, i_B, SCALE, r_D, r_L)</td>
<td>(3.1)</td>
</tr>
<tr>
<td>(2.2) Banks' demand for reserves (two week lag)</td>
<td>RA_{t+\frac{1}{2}} = RA'(i_F, i_C, i_B, SCALE, r_D, r_L)</td>
<td>(3.2)</td>
</tr>
<tr>
<td>(2.3) Borrowing from Federal Reserve</td>
<td>RB = RB(i_F, i_B, RU, RB_{t-1})</td>
<td>(3.3)</td>
</tr>
<tr>
<td>(2.4) Multiplier</td>
<td>DB/RA_{t+\frac{1}{2}} = MULT(r_D, r_L, SCALE, (LTB/DB)_{t-1})</td>
<td>(3.4)</td>
</tr>
<tr>
<td>(2.5) Public's demand for demand deposits</td>
<td>DB^d = DB^d(\Sigma_iC, \Sigma Y, \Delta BL)</td>
<td>(3.5)</td>
</tr>
<tr>
<td>(2.6) Public's demand for savings deposits</td>
<td>SB = SB(i_C, Y, DUM(\cdot), SB_{t-1})</td>
<td>(3.6)</td>
</tr>
<tr>
<td>(2.7) Public's demand for small time deposits</td>
<td>STB = STB(i_C, Y, DUM(\cdot), STB_{t-1})</td>
<td>(3.7)</td>
</tr>
<tr>
<td>(2.8) Public's demand for currency</td>
<td>C^d = C^d(\Sigma Y)</td>
<td>(3.8)</td>
</tr>
<tr>
<td>(2.9) Public's demand for checkable deposits at banks</td>
<td>OCDB = OCDB(OCDB_{t-1}, OCDB_{t-2})</td>
<td>(3.9)</td>
</tr>
<tr>
<td>(2.10) Public's demand for other checkable deposits at banks and thrifts</td>
<td>OCD = OCD(OCD_{t-1}, OCD_{t-2}, OCD_{t-3})</td>
<td>(3.10)</td>
</tr>
<tr>
<td><strong>Federal Reserve</strong></td>
<td><strong>Supply of Nonborrowed Reserves</strong></td>
<td></td>
</tr>
<tr>
<td>(2.11a) Funds rate operating procedure</td>
<td>RU = RU(i_F^*, R^d)</td>
<td></td>
</tr>
<tr>
<td>(2.11b) Reserves operating procedures</td>
<td>RU = RU^*</td>
<td></td>
</tr>
<tr>
<td>(2.12) Supply of demand deposits</td>
<td>DB = MULT * RA_{t+\frac{1}{2}} = \frac{1}{1/r_D + r_L(LTB/DB))RA_{t+\frac{1}{2}}</td>
<td></td>
</tr>
<tr>
<td>(2.13) Total reserves</td>
<td>R = RA + r_D DBG_{t-1/2} + r_L(SB_{t-1/2} + STB_{t-1/2} + OCDB_{t-1/2}) + RTH + RE</td>
<td></td>
</tr>
<tr>
<td>(2.14) M-1A</td>
<td>M1A = DB + C</td>
<td></td>
</tr>
<tr>
<td>(2.15) M-1B</td>
<td>M1B = M1A + OCD</td>
<td></td>
</tr>
<tr>
<td>(2.16) Excess reserves</td>
<td>RE = RE</td>
<td></td>
</tr>
<tr>
<td>(2.17) Reserves against thrift deposits</td>
<td>RTH = RTH</td>
<td></td>
</tr>
<tr>
<td>(2.18) Treasury deposits at commercial banks</td>
<td>DBG = DBG</td>
<td></td>
</tr>
</tbody>
</table>

30
reserves-demand function was simply multiplied by the inverse of the required-reserve ratio against demand deposits. This approach implicitly assumed that changes in deposits were fully reflected contemporaneously in reserves, and that only demand deposits were reservable. In the real world, neither is true.

With lagged reserve accounting, changes in deposits do not show up in reserves demand until two weeks later. Even with monthly data, reserves of the current month only partly reflect contemporaneous deposit changes. The full effect of deposits shows up in reserves centered two weeks later: i.e., in the average of the last two weeks of this month and the first two weeks of next month. Clearly, if we want to predict deposits from reserves, we must use this measure of reserves, i.e., reserves shifted forward half a month.

Hence, two estimates of reserves demand are needed for the empirical model. The first, RA or contemporaneous reserves, is used to explain the funds rate, and is described by equation (2.1). The second, RA_{t+1/2}, or reserves shifted forward half a month, is used to predict the supply of deposits for Stage 2, and is described by equation (2.2). To make a uniform two-week lag from deposits to reserves, we must respecify all of the data in the model in lunar months of four weeks each.

Predicting the multiplier is also no longer trivial. The complication arises not because nondeposit funds are reservable, but because the requirement is not uniform across all types of such funds. Consequently, the average reserve-requirement ratio is a function not only of the split between demand deposits and nondeposit funds, but also of the allocation of the latter among reservable and nonreservable categories. As a result, the arguments of RA, which explain the split only, are not necessarily suited to predicting the average reserve-requirement ratio and hence its inverse, the multiplier. Preliminary estimation indicated that the SCALE variable of RA (a measure of the aggregate size of banks' portfolios) helped to explain the multiplier, but that the interest-rate arguments of RA (the funds rate, the commercial-paper rate and the discount rate) did not. Since large CDs (LTB) accounted for almost all of the reserve requirements against nondeposit funds, we used the lagged ratio of large CDs to demand deposits to help predict the multiplier, as shown in equation (2.4). We then multiplied this prediction by RA_{t+1/2} to obtain the deposit-supply function for Stage 2.

**Estimation Results**

All equations were estimated in seasonally-adjusted lunar-monthly observations (four-week periods) from 1976:Lunar 8 (begins July 21, 1976) through 1979:Lunar 10 (ends October 3, 1979). The ending date coincided with the Federal Reserve's adoption of a monetary-control procedure which focuses primarily on reserves in day-to-day operations. We chose the beginning date to avoid entangling the estimation of the model with the bias inherent in the (mid-1974 to mid-1976) shift in money demand. (Now that the model has been estimated over a fairly "clean" sample period, we are working to extend the sample back to 1973.)

We aggregated seasonally-adjusted weekly figures (where available) to give lunar-month observations, or interpolated where only calendar-month data were available. Both the funds rate and commercial-paper rate are endogenous in the model, so that we used two-stage least squares wherever these rates appeared as explanatory variables in a regression equation. Even though the funds rate was a policy variable under the Fed's pre-October 1979 operating regime, it was not strictly exogenous. The Fed adjusted the rate when money deviated from target, and because money is one of the endogenous variables in the model, this practice effectively made the funds rate endogenous as well. We corrected for first-order serial correlation where the autocorrelation coefficient was significant at the 10-percent level.

The results of estimating the reserves-market equations and the demand-deposit multiplier are reported in Table 3 as equations (3.1) to (3.4). Recall that reserves demand is viewed as reflecting primarily the behavior of deposit supply. The latter in turn is regarded as
<table>
<thead>
<tr>
<th>Equation</th>
<th>Estimated Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.1) ( \ln RA_i = 3.8 - .36(\ln F_i - \ln CP_i) - .07\ln B_{i-1} + .23\ln SCALE_i )</td>
<td>( R^2 = .98 ) ( \text{SEE} = .0069 ) ( \text{DW} = 1.89 )</td>
</tr>
<tr>
<td>(3.2) ( \ln RA_{i+1/2} = 3.1 - .51(\ln F_i - \ln CP_i) )</td>
<td>( R^2 = .96 ) ( \text{SEE} = .0087 ) ( \text{DW} = 1.85 )</td>
</tr>
<tr>
<td>(3.3) ( RB_i = .008 + .64(i_F_i - i_B_i)^{1/2}Z_1 - .54\Delta R_iZ_1 + .59 RB_{i-1} )</td>
<td>( R^2 = .92 ) ( \text{SEE} = .14 ) ( \text{DW} = 1.84 )</td>
</tr>
<tr>
<td>(3.4) ( \ln MULT_i = .01 - .075\ln(LTB_{i+1}/DB_{i+1}) - .04\ln SCALE_i )</td>
<td>( R^2 = .97 ) ( \text{SEE} = .0026 ) ( \text{DW} = 1.99 )</td>
</tr>
<tr>
<td>(3.5) ( \ln DB_i = .8\ln DB_{i-1} = .17 + .66\Delta \ln B_{i+1} + \sum_{a_i \ln CP_{i+1}}^6 )</td>
<td>( R^2 = .88 ) ( \text{SEE} = .0038 ) ( \text{DW} = 1.74 )</td>
</tr>
</tbody>
</table>

where

\[
\begin{align*}
a_0 &= - .016 (1.24) & b_0 &= .33 (1.67) \\
a_1 &= - .015 (2.02) & b_1 &= .19 (4.36) \\
a_2 &= - .014 (2.75) & b_2 &= .10 (1.57) \\
a_3 &= - .012 (2.39) & b_3 &= .02 (0.31) \\
a_4 &= - .010 (1.81) \\
a_5 &= - .008 (1.46) \\
a_6 &= - .006 (1.22) \\
\Sigma &= - .081 (2.75) & \Sigma &= .64 (7.47) \\
R^2 &= .88 \\
\text{SEE} &= .0038 \\
\text{DW} &= 1.74
\end{align*}
\]
Table 3 (continued)

(3.6) \[ \ln SB_t = .44 + .11 (1/i_{CP,t}) + .13 \ln Y_t + .65 \ln SB_{t-1} \]
\[ (3.07) (2.44) (4.07) (11.03) \]
\[ - .02 \text{MMCDUM}_t - .13 \text{BUSDUM}_t - .02 \text{ATSDUM}_t + .56 U_{t-1} \]
\[ (2.63) (2.78) (5.21) (4.36) \]
\[ R^2 = .998 \]
\[ \text{SEE} = .0024 \]
\[ \text{SW} = 1.76 \]

(3.7) \[ \ln SB_t = -0.05 + .16 (1/i_{CP,t}) - .15 (1/i_{CP,t}) \text{MMCDUM}_t + .16 \ln Y_t \]
\[ (1.90) (1.54) (2.63) (2.52) \]
\[ + .77 \ln SB_{t-1} + .03 \text{MMCDUM}_t + .008 \text{ATSDUM}_t + .007 \text{SPRDUM}_t + .19 U_{t-1} \]
\[ (13.93) (2.92) (4.69) (3.02) (1.29) \]
\[ R^2 = .999 \]
\[ \text{SEE} = .0029 \]
\[ \text{DW} = 2.19 \]

(3.8) \[ \ln C_t = -1.64 + \sum_{i=0}^{8} a_i \ln Y_{t+i} + .87 U_{t-1} \]
\[ (12.2) (11.5) \]

where

\[ \begin{align*}
    a_0 &= .12 \quad (1.62) \\
    a_1 &= .12 \quad (2.95) \\
    a_2 &= .12 \quad (8.07) \\
    a_3 &= .11 \quad (17.92) \\
    a_4 &= .11 \quad (5.14) \\
    \Sigma &= .83 \quad (46.34)
\end{align*} \]

\[ R^2 = .999 \]
\[ \text{SEE} = .0016 \]
\[ \text{DW} = 1.57 \]

(3.9) \[ \text{OCDB}_t = .03 + 1.00 \text{OCDB}_{t-1} + .55 \Delta \text{OCDB}_{t-1} \]
\[ (1.43) (60.83) (3.96) \]

\[ R^2 = .989 \]
\[ \text{SEE} = .254 \]
\[ \text{DW} = 1.99 \]

(3.10) \[ \text{OC}_{D_t} + .80 + 1.57 \text{OC}_{D_{t-1}} - .73 \text{OC}_{D_{t-2}} + .12 \text{OC}_{D_{t-3}} \]
\[ (2.02) (5.32) (1.39) (0.40) \]

\[ R^2 = .986 \]
\[ \text{SEE} = .473 \]
\[ \text{DW} = 2.11 \]

NOTE:

* t-statistics are in parentheses.
* Estimation method is two-stage least squares with Cochrane-Orcutt adjustment where indicated by the variable $U_{t-1}$.
* Instrumental variables used for $i_F$ and $i_{CP}$. Sample period was 1976: Lunar 8 - 1979: Lunar 10.
* Distributed lags in (3.5) and (3.8) are second-degree Almon with the tail tied to zero.
being determined by the aggregate size of banks' portfolios, measured by SCALE, and by the fraction financed by nondeposits, which is a function of \( i_{cp}, i_F, \) and \( i_B \). Hence \( RA^d \) depends on the same variables and is influenced in the same direction by them. In particular, higher \( i_F \) and \( i_B \) would be expected to lower \( RA^d \), while increases in SCALE and \( i_{cp} \) would raise it. Also, increases in the required-reserve ratios, \( r_D \) and \( r_I \), should raise \( RA^d \).

The first two lines report the results for the two estimates of reserves demand. Both equations fit the data quite well. All the estimated coefficients have the right signs, and all pass a test of significance at the 95-percent confidence level, except for the coefficients on \( r_I \). Both measures of \( RA^d \) are relatively elastic with respect to the funds rate, especially \( RA_{t+1/2} \), which determines the elasticity of demand-deposit supply. The \( RA_t \) measure of reserves demand should be less responsive to its arguments than is \( RA_{t+1/2} \), which in fact is true. The reason is that \( RA_t \) reflects only a partial response of demand deposits to changes in the funds rate and the other arguments, because it excludes the requirements against deposits created in the last half of the month. \( RA_{t+1/2} \) on the other hand includes reserves against all deposits of the current month, and therefore more accurately measures their response to interest rates and SCALE.

The two versions of reserve demand adjust rapidly to their explanatory variables, with full adjustment occurring in one month. Although we tried a number of distributed-lag specifications, lagged effects of the explanatory variables were consistently insignificant. These findings — rapid speeds of adjustment and relatively large interest elasticities — are consistent with one of our central hypotheses: the supply of deposits results from the interaction of banks and the public in various credit markets, where participants actively maximize profits on a day-by-day and hour-by-hour basis. As noted earlier, this part of the model differs from conventional models, which view deposit supply as accommodating the public's demand for deposits. Since many deposit holders inactively manage their balances, conventional models produce the result that deposits (and thus reserves) respond to interest rates with long lags and low elasticities.

Next, we present the model's representation of the supply of total reserves. Under the funds-rate regime of the estimation period, total reserves supply is simply equal to banks' demand for total reserves, \( R^d \). The only remaining issue concerns what part of this demand is supplied through borrowed and what part through nonborrowed reserves. The estimated member-bank borrowing function is reported in Table 3 equation (3.3). Its arguments are the square root of the differential of the funds rate over the discount rate (defined to be zero when the funds rate is below the discount rate), changes in nonborrowed reserves, and lagged borrowing. It was observed that, when the funds rate fell below the discount rate, member-bank borrowing shrank to a small frictional amount. Thus, we hypothesized that banks borrow from the Federal Reserve primarily when there is sufficient incentive in the form of a positive funds rate/discount rate differential. Tests of this hypothesis were strongly confirmed. As a consequence, we imposed the constraint on the estimated equation that borrowing responds only to positive differentials.

We used the square root of the differential to reflect the increasing administrative pressure and/or reluctance to borrow accompanying a rise in the spread (and therefore in RB). With the square root, the RB equation has the property that RB's responsiveness to a given change in the spread declines as the level of the spread rises.

We also hypothesized that because of lagged reserve accounting, changes in nonborrowed reserves would have a transitory effect on borrowing. Under lagged accounting, required reserves this week are fixed, being determined by deposits of two weeks ago. A reduction in nonborrowed reserves therefore forces banks in the short-run to replace them with borrowed reserves, because the total demand for reserves is unchanged. Thus we should observe a negative relationship between changes in nonborrowed and borrowed reserves.
In the borrowing equation, first, all explanatory variables have the expected signs and are highly significant. Second, the speed of adjustment is again relatively fast — the mean lag is 1.4 lunar months. However, even this relatively quick adjustment seems surprisingly slow when compared to the even faster adjustment in the reserves-demand equations noted earlier. Third, the implied contemporaneous response of borrowing to the funds rate is very large, especially when the spread is very low. Thus a 10-basis-point rise in the funds rate increases borrowing by $64 million when the spread is 25 basis points. When the spread rises to 50, 100 or 200 basis points, a 10-basis-point increase in the funds rate produces $45, $32 and $22 million of additional borrowing, respectively. The long-run responses are about 2½ times larger.

To complete the banking side of the model, we need a prediction of the supply of deposits. This we obtain by multiplying the equation for RA\textsuperscript{1+1/2} (equation (3.2)) by the estimate of the multiplier in equation (3.4). The multiplier is simply a weighted average of the reserve-requirement ratios on demand deposits and nondeposit funds. For reasons explained in Appendix C, large certificates of deposit (LTB) are the only significant reservable "nondeposit" liability. Hence, the multiplier can be written as 1/(r\textsubscript{D} + r\textsubscript{L} (LTB/DB)). For reasons discussed above, the ratio LTB/DB is approximated as a function of its lagged value and SCALE. Hence we estimated the multiplier as a function of these two variables and the required-reserve ratios. The coefficients on the latter had the correct negative signs. The coefficient on SCALE was also negative, indicating perhaps that as banks’ portfolios increased, they raised nondeposit rates to attract more funds, causing the ratio of CDs to demand deposits to rise.

The demand for demand deposits can be viewed as a disequilibrium process in which deposit-supply shocks move the public away from its equilibrium demand. Over the sample period of this study, bank loans were found to be the major source of money-supply shocks. Disequilibrium caused by past shocks is worked off at a rate of (1 - \rho) per month, so that a fraction \rho of last month’s disequilibrium persists into the current period. At the same time, the fraction \lambda of this month’s shock is held temporarily, and thus adds to the measure of current disequilibrium. Observed deposits therefore can be written,

\[
\ln DB_t = \ln DB\textsubscript{t-1} + \rho (\ln DB\textsubscript{t-1} - \ln DB\textsubscript{t-1}) + \lambda \Delta \ln BL_t
\]

The short-run equilibrium demand function for deposits, DB\textsuperscript{d}, is a function of ic\textsubscript{CP} and nominal income (Y) — which determine the long-run equilibrium demand for deposits — and lagged values of DB\textsuperscript{d} represent partial adjustment of money demand in the short-run to the long-run equilibrium level. Since we cannot directly observe DB\textsuperscript{d} — it does not equal DB when there is disequilibrium — we solve for it in terms of interest rates and income by successive substitution, i.e.,

\[
\ln DB\textsubscript{t} = \Sigma a_i \ln i_{CP,t-1} + \Sigma b_i \ln Y_{t-1}
\]

Substituting this result into (1) and rearranging we have\textsuperscript{15}

\[
\ln DB\textsubscript{t} = \Sigma a_i \ln i_{CP,t-1} - \rho \Sigma a_i \ln i_{CP,t-1-i} + \Sigma b_i \ln Y_{t-1} - \rho \Sigma b_i \ln Y_{t-1,i} + \rho \ln DB_{t-1} + \lambda \Delta \ln BL_t
\]

Estimates of the demand-deposit demand equation are shown in (3.5). The long-run elasticities on income and the commercial-paper rate are highly significant, and their values are in the “normal” range for traditional money-demand equations. Second, the change in the bank-loan variable is significant, with the expected positive sign. Third, the coefficient on \Delta \ln BL is relatively large. For example, the decline in BL in May 1980 is estimated to have held observed demand deposits to a 1/2-percent growth rate, compared to the 13-percent growth which would have otherwise occurred. Fourth, the estimate of \rho at .8
indicates that deposit-market disequilibrium induced by bank loans persists with a mean lag of four months.

Equation (3.8) presents the public’s demand for currency as a function of a distributed lag on nominal GNP. The commercial-paper rate could theoretically enter this equation, but did not do so significantly during the sample period. The combination of DB and C indicates the model with the stock of MIA.

In order to determine MIB, we must explain MIA plus total other checkable deposits (OCD). The latter includes deposits both at banks and thrifts, although thrift deposits were relatively small, being confined to NOW accounts at institutions in Northeastern states. The major component of OCD during the sample period was commercial-bank ATS (automatic transfer from savings) accounts. These deposits were introduced in November 1978; hence the growth in OCD represents almost entirely the public’s accumulation of desired stocks of ATS accounts. This stock adjustment in the public’s demand was modelled most effectively as a function of past OCD. (3.10)

The model includes three more demands by the public for bank liabilities: banks’ other checkable deposits, equation (3.9); small time deposits, equation (3.7); and passbook savings deposits, equation (3.6). These variables enter the model because banks are required to hold reserves against them. Other checkable deposits at banks (OCDB), like OCD, is modelled as a time series. For savings (SB) and small time deposits (STB), the public’s demands determine their quantities. The arguments of these functions include personal income, the commercial-paper rate, and a number of (dummy) variables capturing the effects of various regulatory changes during the sample period (see Appendix B for definitions).

**Simulation Results**

While Table 3 shows how the estimated equations perform individually, it does not indicate how well all of the model’s equations and identities simultaneously predict the endogenous variables of the system. Consequently, we made a full-model static simulation of the sample period, using actual values for lagged dependent variables and applying autocorrelation corrections to preceding month’s errors. Table 4 presents the results of this simulation for the four major variables of the model (M1A, M1B, R, iex).

The model fits the in-sample data for the monetary and reserve aggregates quite well, producing root-mean-squared errors (RMSE) ranging from 0.21 to 0.30 percent of the average levels of M1A, M1B, and R. As is typi-

<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Simulations</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>In-Sample†</td>
</tr>
<tr>
<td>Static</td>
</tr>
<tr>
<td>M1A $883 million</td>
</tr>
<tr>
<td>(0.21 percent)</td>
</tr>
<tr>
<td>M1B $1,016 million</td>
</tr>
<tr>
<td>(0.30 percent)</td>
</tr>
<tr>
<td>Total reserves $108 million</td>
</tr>
<tr>
<td>(0.24 percent)</td>
</tr>
<tr>
<td>Commercial paper rate 14 basis points</td>
</tr>
<tr>
<td>(1.7 percent)</td>
</tr>
</tbody>
</table>

†Federal-funds rate exogenous. All exogenous variables set at actual values.
cal of money-market models, the interest-rate forecasts are less accurate than the monetary and reserve-aggregate forecasts. The RMSE for the commercial-paper rate is 1.7 percent, which amounts to 14 basis points.

The right-hand column of Table 4 shows RMSEs for the same four variables from a dynamic out-of-sample simulation over 1979/L11 - 1980/L11. In this simulation, lagged dependent variables took on values predicted by the model in previous periods, and serial correlation adjustments were applied only to the error in the final in-sample month. Not surprisingly, the RMSEs from this experiment are larger than the in-sample results — for the aggregates, they range from 0.45 to 0.60 percent, while for the commercial-paper rate the RMSE is 15.9 percent. In view of the extreme volatility of the post-October 1979 period compared to the earlier estimation period, we may

Figure 5
Out-of-Sample Predictions of M-1B and the Commercial Paper Rate: Post-October 6, 1979
take the out-of-sample results as a measure of the model’s success.

Even more encouraging is the success of the model at predicting the turning points during the period. As shown in Figure 5, the model was able to simulate the rather wild gyrations of M1B, whereas a wide variety of more traditional models missed these turning points. The model did not do quite as well on \( i_{CP} \), specifically missing the large drop in 1980/L5. In other months, however, the simulation tracked reasonably well.

III. Why Were the Aggregates So Volatile in 1980?

Analysis of the model’s exogenous variables indicates that changes in bank loans were by far the most important contributor to M1B’s rapid growth in the first and third quarters of 1980 — and also to its rapid second-quarter decline. Evidence for this conclusion is presented in Figure 6, which compares two dynamic simulations of M1B. The solid line is a full dynamic simulation — i.e., the same one shown in Figure 5. The dashed line is a simulation with bank loans constant, but identical in every other respect to the full simulation. This experiment indicates that without the post-1979 volatility in bank loans, M1B would not have gyrated as it did.

What accounts for the erratic pattern of bank-loan movements in 1980? The most plausible explanation is the Special Credit Control Program of March 1980, which put binding constraints on bank-credit growth. In the first quarter of 1980, the financial press had reported that businesses were anticipating credit controls. This probably contributed to the rapid growth of loans in that quarter, as
firms attempted to obtain bank credit while it was still available. In the second quarter, loans declined absolutely in response to the binding constraints of the credit controls. Finally, loans spurted in the summer period as firms attempted to make up for the lack of loans in the preceding quarter.

IV. Conclusions

Conventional money-market models reflect the view that the monetary aggregates are determined primarily by the public’s demand for money. The money-market model presented in this paper reflects an alternative view — that the monetary aggregates are determined in the short run primarily by the supply of money, which arises out of the behavior of banks and the public in established financial markets. Several pieces of evidence support this hypothesis. First, the money supply responds to its financial-market determinants with very short lags, consistent with the typical speed of adjustment in financial markets, but not with the typical sluggishness of money demand. Second, bank loans can have — and in 1980, did have — a potent influence on the monetary aggregates. Third, the market for money is often characterized by disequilibrium in the short-run. Money-supply “shocks” temporarily push the public off its short-run money-demand curve, which allows the money supply to exert a large short-run influence on the stock of money observed in the economy.

These results have important implications for Federal Reserve monetary policy. First, policy makers should pay close attention to financial-market developments, which can influence the growth of money in a quick and potent fashion. Second, policy makers should be especially careful to evaluate financial-market developments when signs appear of a shift in the conventional money-demand function. A good case in point is the second quarter of 1980, when conventional models severely overpredicted the money stock. Evidence of a downward shift in the money-demand relationship would imply that money supply should be allowed to fall commensurately to avoid an overly expansionary monetary policy. On the other hand, the model in this paper explains the decline in money as a supply shock, induced by the decline in bank loans that followed from the Special Credit Control Program of 1980. Such a conclusion implies that monetary-control efforts should be directed toward more rapid money-supply growth to avoid an overly contractionary policy.

Appendix A

Formal Representation of the Model

The model describes the portfolio behavior of the Federal Reserve, commercial banks and the nonbank public over monthly observations. The balance sheets of commercial banks and the nonbank public are shown below. See Appendix B for definitions of variables.
The Federal Reserve is assumed to control $RU = R - RB$ and $i_B$, making them exogenous. In addition, the model takes as exogenous $BL$ and $Y$. $BL$ is exogenous because the public's demand for loans is unresponsive to the contemporaneous (monthly) loan rate, while $Y$ is exogenous because the lag between monetary policy and $Y$ is greater than one month. In addition, individual banks take deposits ($DB$ and $TB$) as determined entirely by the public's demand for deposits. Since the yields banks pay on these assets are legally held below market-clearing levels, individual banks will supply any quantity demanded by the public. Finally, it is assumed that any quantity of currency demanded by the public will be supplied. Given these assumptions and the profit-maximizing behavior described in Section I of the text, the following structural model may be specified.

\begin{align*}
IMB &= IMB_0 + i_F + i_B + BL - DB - TB + R \quad (1) \\
IMB^d &= IMB^d_0 + i_F + i_C + Y \quad (2) \\
FF/RP &= FF^s_0 + i_F + i_B + BL - DB - TB + R \quad (3) \\
FF/RP^d &= FF^d_0 + i_F + i_C + Y \quad (4) \\
RB^d &= RB (i_F, i_B) \quad (5) \\
OA^d &= OA^d_0 + i_F + i_C + Y = 0 \quad (6) \\
DB^d &= DB^d_0 + i_C + Y \quad (7) \\
DB &= DB^d + DB^P (ADB^s) \quad (8) \\
TB^d &= TB^d_0 + i_F + i_C + Y \quad (9) \\
C^s &= C^s_0 + i_F + i_C + Y \quad (10) \\
R^d - RB^d &= RU^s \quad (11) \\
R^d &= r_0 DB \quad (12) \\
NW &= DB + TB - DBG - BL + C + IMB + FF/RP + OA - OL \quad (13)
\end{align*}

Only twelve of these thirteen equations are independent, and thus any one of them can be dropped from the solution of the model's reduced form. We chose to drop equation 6. The remaining twelve equations can be solved for the following twelve unknowns: $IMB$, $FF/RP$, $DB$, $DB^d$, $TB$, $C$, $R$, $RB$, $NW$, $i_o$, $i_F$, $i_C$.

In Section I of the text, the model is solved in three stages as follows. In Stage 1, equations 1, 2, 3, 4, 5 and 12 are solved for $IMB$, $FF/RP$, $RB$, $i_o$, $DB$ and $R$ as functions of $i_F$, $i_C$, $i_B$, $(BL - TB)$, $Y$ and other variables. The sum of the equations for $IMB$, $FF/RP$, and $RB$ provides the EQ equation of the text. The equations for $DB$ and $R$ are the Stage 1 deposit-supply and reserves-demand equations.

In Stage 2, equation 11 is added to the Stage 1 equations, to provide solutions for $IMB$, $FF/RP$, $RB$, $i_o$, $DB$, $R$ and $i_F$, as functions of $i_C$, and $i_B$, $(BL - TB)$, $Y$ and other variables. The equation for $DB$ is the Stage 2 deposit-supply equation.

In Stage 3, equations 7 and 8 are added to the Stage 2 equations, providing solutions for $IMB$, $FF/RP$, $RB$, $i_o$, $DB$, $R$, $i_F$, and $DB^d$ and $i_C$, as functions of $i_B$, $(BL - TB)$, $Y$ and other variables. Finally, the model can be completed by using equations 9, 10, and 13 to provide solutions for $C$, $TB$, and $NW$.

### Appendix B

**Glossary of Symbols**

- **ATSDUM**: Dummy variable for the introduction of ATS accounts at commercial banks: $\text{In1, In2, In3, \ldots, In13}$, during 1978/L11-1979/L10.
- **BUSDUM**: Dummy variable for the introduction of business and state-and-local government saving deposits at banks: $\text{In20, In21, In22, \ldots, In26}$, during 1976/L7-1976/L12, and $\text{In26}$ during 1976/L13-1979/L10.
- **C**: Currency in the hands of the public.
- **DB**: Private demand deposits at commercial banks.
- **DBG**: U.S. Treasury demand deposits at commercial banks.
- **DUM(\cdot)**: Institutional changes affecting the public's demand for SB and STB; includes ATSDUM, BUSDUM, MMCDUM and SPRDUM.
FF/RP  Net federal funds purchased plus security repurchase agreements at commercial banks.
IMB  Total nondeposit funds plus time deposits in denominations of $100,000 or more, less total holdings of securities at commercial banks, less FF/RP.
LTB  Time deposits in denominations of $100,000 or more.
i_B  Federal Reserve discount rate.
i_CP  Three-month nonfinancial commercial-paper rate.
i_F  Federal-funds rate.
i_O  Ninety-day large negotiable certificate-of-deposit rate.
i_SB  Passbook-savings rate at commercial banks.
BL  Total loans at commercial banks.
MMCDUM  Dummy variable for the introduction of six-month money market certificates at commercial banks: 1 during 1978/L7-1979/L10; 0 elsewhere.
M1B  C + DB + OCD.
MULT  DB/RA_t+1/2.
NW  Net worth of the nonbank public = DB + TB - DBG - BL + C + IMB + FF/RP + OA - OL.
OCD  Other checkable deposits at commercial banks and thrift institutions.
OCDB  Other checkable deposits at commercial banks.
OA  Other assets of the nonbank public.
OL  Other liabilities of the nonbank public.
R  Total member-bank reserves, adjusted for Regulations D and M.
RA  Reserve requirements against demand deposits and managed liabilities, adjusted for Regulations D and M.
RB  Borrowed reserves from the Federal Reserve.
RE  Member bank excess reserves.
RR  Member bank required reserves, adjusted for Regulations D and M.
RU  Member bank nonborrowed reserves, adjusted for Regulations D and M.
r_D  Reserve-requirement ratio against demand deposits.
r_I  Reserve-requirement ratio against time deposits in denominations of $100,000 or more.
r_O  Reserve-requirement ratio against IMB.
r_T  Reserve-requirement ratio against SB, STB and OCDB.
SB  Passbook-savings deposits at commercial banks.
SCALE  IMB + FF/RP + RB + DB - RA = BL - TB + (R - RA).
SPRDUM  Dummy variable for the elimination of the 25-basis-point spread between yields on money-market certificates at thrift institutions over commercial banks: 1n1, 1n2, ..., 1n7 during 1979/L4-1979/L10.
STB  Time deposits in denominations of less than $100,000 at commercial banks.
TB  Other deposits = DBG + OCDB + SB + STB.
Y  Personal income in current dollars.
Z1  Zero when funds rate below or at discount rate. Unity when funds rate above discount rate.
Appendix C
Other Reserve Requirements

The theoretical model focuses on the way that portfolio decisions of banks and the public affect the stock of demand deposits, and through them, the demand for reserves. In reality, other items besides demand deposits are reservable. Small time and savings deposits (SB and STB), government deposits (DBG), other checkable deposits (OCDB), and certain nondeposits also have reserve requirements, and therefore affect the amount of required reserves. In addition, required reserves contain the reserves that thrift institutions must hold (RTH) with the phasing in of the universal reserve requirements mandated by the Monetary Control Act. And finally, measured reserves also include the small amount of excess reserves (RE) that banks hold.

The behavioral relationship underlying reserves demand is framed in terms of demand deposits and nondeposit liabilities only. Hence the other components of reserve requirements must first be stripped away before reserves demand can be estimated. This refined version is called adjusted reserves, RA; its relation to total reserves, R, is shown in the reserves identity (equation 2.13 of Table 2 in the text.)

The other components of total reserves must still be accounted for. This is done in two ways. Excess reserves and requirements against thrift deposits and Treasury deposits are treated as constants over the sample period (equations 2.16, 2.17, and 2.18), since they are small and exhibit only slight variation. The others are treated by estimating the quantities of corresponding deposits and multiplying them by the appropriate reserve ratio. For small time and savings deposits, the public’s demands are viewed as determining their quantities, because the banks’ scope for altering rates is constrained by interest-rate ceilings. Thus the public’s demands for SB and STB are estimated as functions of interest rates, income, and a number of variables representing institutional changes (DUM(.)). The resulting estimates are multiplied by the corresponding reserve ratio to predict the amount of reserves held against them. Estimates of the public’s demand for other checkable deposits at banks (equation 2.9) are used in the same way to estimate the reserves held against them.

Recognizing that both demand deposits and some non-deposit liabilities are reservable makes the analysis of the multiplier somewhat more complicated than our theoretical discussion would indicate. In that discussion, we could think of demand deposits alone as having reserve requirements, which meant that the multiplier — the ratio of demand deposits to reserves — was simply the reciprocal of the demand-deposit required-reserve ratio, $r_D$. With managed funds also reservable, we must also take account of the fact that part of RA will not be available to support demand deposits. The larger the amount of reserves absorbed in requirements against nondeposit liabilities, the smaller will be the amount of demand deposits outstanding per dollar of RA, i.e. the smaller will be the multiplier.

Not all nondeposit liabilities are reservable. For all intents and purposes, large time deposits (LTB) are the only significant ones that are. This is because the model uses a reserve series that abstracts from changes in Regulations D and M, which define reserve requirements. That is, the measure removes discontinuities in the reserves numbers caused by changes in required-reserve ratios. If a liability item has incurred reserve requirements only part of the time, its reserves will not show up in the smoothed series because its benchmark ratio is zero. Most reserve requirements on nondeposits have been on-again, off-again (on Eurodollar borrowing, for example) and therefore are not included in our reserve series. The important exception is reserves against LTB, which are included because these large CDs have always been covered by reserve requirements.

42
Hence our adjusted reserves series, RA, is essentially composed of required reserves against demand deposits and large time deposits. The multiplier therefore depends not only on $r_D$ but as well on $LTB$ (relative to DB) and its reserve ratio, $r_L$. In the empirical model, the multiplier is estimated as a function of these variables.

FOOTNOTES


4. The money-supply part of the present model is in the spirit of the model in Franco Modigliani, Robert Rasche, and J. Philip Cooper, "Central Bank Policy, the Money Supply, and the Short Term Rate of Interest," *Journal of Money, Credit and Banking*, May, 1970, pp. 166-217.


6. Ernst Baltensperger, "Alternative Approaches to the Theory of the Banking Firm," *Journal of Monetary Economics*, January 1980, pp. 1-38. He distinguishes the following non-interest costs: for liabilities, their associated costs of liquidity management (e.g., differences in withdrawal risk) and costs of producing and maintaining deposit contracts; on the asset side, risks of default, and information and transaction costs associated with extending different types of credit. Baltensperger also includes differences in the cost of acquiring or disposing of an asset or liability.

7. The nonbank public's balance-sheet constraint is derived from the following balance sheet.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency</td>
<td>Loans from Banks</td>
</tr>
<tr>
<td>Demand Deposits DB</td>
<td>Other Liabilities OL</td>
</tr>
<tr>
<td>Other Deposits TB-DBG</td>
<td>Net Worth NW</td>
</tr>
<tr>
<td>Managed liabilities of bank (net of bank securities holdings) IMB</td>
<td>Net Federal Funds lent plus repurchase agreements FF/RP</td>
</tr>
<tr>
<td>Other assets OA</td>
<td></td>
</tr>
</tbody>
</table>


10. Until October 6, 1979, the Fed used the funds rate as its operating instrument. To achieve its targets, the Fed set RU so that the funds rate which cleared the reserves market equaled the funds-rate target. This procedure makes DB$^1$ and DB$^2$ empirically indistinguishable. However, the theoretical distinction noted above still applies: DB$^2$ includes Fed behavior, whereas DB$^1$ remains the same no matter what the Fed's operating procedures are.


14. There is no theoretical reason why this must be so. Rather, the importance of bank loans presumably reflects in part the fact that the Federal Reserve used a Federal-funds instrument to try to control money during the sample period. As discussed in the theoretical section, this procedure allowed bank...
loans the fullest scope to affect the supply of money. But during the simulation period (1980) — despite the Fed's reserves-control procedure — bank loans remained an important source of disequilibrium because of the very large impact of the Special Credit Control Program.

15. Equation 3 resembles an autoregressive transformation of a conventional short-run deposit-demand function, in which actual deposits are identified with short-run equilibrium demand. This transformation is frequently used in conventional estimates of short-run money-demand functions because of evidence of significant serial correlation in the residuals. The disequilibrium specification (1) suggests that there is a structural explanation for this serial correlation: namely, the process by which equilibrium is restored in the money market. As well, however, the disequilibrium specification differs from the conventional by a term which measures the effect on money demand of current money-supply disturbances.


17. Reserve-requirement ratios include the effects of the proportion of each deposit type held at member banks: i.e., \( r_D = r_m(DM/DB) \), where \( r_m \) is the average ratio imposed on member banks, DM is member-bank demand deposits subject to reserve requirements, and DB is all bank demand deposits. Weekly data are available for DM/DB. Member-to-total bank ratios of .70 and .63 were used for SB and STB throughout the sample period. The member-to-total bank series for LTB was calculated for each month as the residual from the above data and assumptions, and from currently available reserve-requirement data.


19. These consist of the following: the introduction of business and state and local government passbook-savings deposits; the introduction of six-month MMCs at banks; the introduction of ATS accounts; and the removal of the 25-basis-point thrift differential on MMCs.
This paper presents a small model of the U.S. economy for estimating the response of inflation and real output to a change in monetary policy. Measures obtained from the model’s reduced-form equations provide estimates of the complete adjustment paths of inflation and real output to a monetary disturbance. By complete adjustment we mean that the response of each variable to a change in monetary policy continues until the variable’s level and rate of change have both reached their respective long-run values. In other words, prices will continue to change until both the inflation rate and the level of real money balances reach their respective long-run values, while real output will continue to adjust until it equals the level of potential output and is growing at the rate of potential output. Most other reduced-form models focus only upon the adjustment of rates of change in prices and output to a monetary disturbance. In contrast with these, our model provides results which are consistent with the neutrality of money, which is perhaps one of the most generally accepted properties regarding economic behavior. It holds that changes in the money supply ultimately affect only nominal variables, such as prices and wages, leaving all real quantities, such as goods and services, unchanged.

Our empirical estimates provide the adjustment patterns of both real GNP and inflation to their respective long-run values implied by the neutrality property. Notably these time patterns show a relatively quick, short-run response of both inflation and real GNP to a change in monetary growth, with those responses completed within about two years’ time. This contrasts with conventional model estimates which range from three to five years. Our results suggest that a monetary contraction is likely to bring inflation down faster with less adverse affect upon real economic activity than previously anticipated.

In the next section we describe the model and detail its long- and short-run properties, emphasizing the expected lag pattern between changes in monetary policy and changes in the level and rates of change of prices and real output. In the following section, we estimate the reduced-form equations of the model, utilizing an estimation technique suggested by John Scadding (see Appendix 1). Using this method, we are able to place restrictions on both the steady-state level of a variable and its rate of change. The final section provides policy implications and conclusions.

I. Model of Real Output and Inflation

The structure of the model is concerned directly with behavior in the markets for goods, money and labor. Each market is characterized by fairly standard economic relationships which are detailed in Appendix 2, and which may be combined to provide the aggregate-demand and aggregate-supply equation shown in Table 1. Each of the variables is measured in terms of its natural logarithm.

Aggregate demand, which is stated in terms of the level of output relative to its potential, \((Y/YP)^{3}\), is inversely related to the rate of inflation, \(dP\). This occurs because — given the rate of growth in the nominal money supply, \(dM\) — a reduction in the rate of inflation raises...
real money balances at each level of output. Higher real balances lead to a fall in real interest rates, which in turn increases interest-sensitive spending. By the same line of reasoning, aggregate real demand declines when the inflation rate increases.

However, demand is positively related to expected changes in the rate of inflation, as indicated by the positive sign associated with the coefficient a12. For example, if people anticipate an increase in the inflation rate, their demand for goods and services will increase in the current period as they take advantage of this period’s relatively lower prices. The negative value associated with the coefficient a13 indicates that demand is inversely associated with changes in the government-budget surplus. For example, an increase in government revenues relative to spending acts to depress aggregate demand. Also, past increases in the real-GNP gap tend to reduce current real growth. This may occur, for instance, when past increases in income tend to increase current saving more than investment. Finally, wi represents the random disturbance or error term in estimating aggregate demand. It captures the sometimes sizable but unsystematic effect upon aggregate demand of many factors whose total impact over time averages zero. For instance, the impact of both labor troubles and unusual weather would generally be included in this term.

The aggregate-supply equation states that deviations of output from potential are determined by unexpected changes in the current inflation rate and by past deviations of output from potential. Since these past deviations can in turn be related to past unexpected price changes, the supply equation indicates that the impact of unexpected inflation will persist for some time. The equilibrium condition states that aggregate demand equals supply in any given period of time.

The specification of inflation expectations for the current and subsequent periods, dP; and dP;+1, completes the model. The hypothesis used here, following John Muth, states that expectations are informed predictions of future events, based on the available information and the relevant economic theory at the time the expectations are formed. This is the well-known “rational expectations hypothesis.” We will leave the detailed specification of inflation expectations for later. For now, these general comments are sufficient to begin analysis of the model under steady-state and long-run conditions.

Long-run Behavior
Certain economic conditions characterize an economy in its steady-state or long-run equilibrium. In summary, they indicate that money is neutral in the long-run with respect to the level and rate of change of output.

First, there are no surprises regarding people’s expectations. Therefore, the actual money-supply growth rate and actual inflation rate are both equal to their respective anticipated rates.

Second, actual GNP is equal to potential GNP, as can be seen from the aggregate-supply equation in Table 1. In the long-run, unanticipated inflation is zero (dP − dP* = 0), and therefore the logarithm of the real GNP gap, Y/YP, is zero.

Third, in the long-run, the inflation rate is determined by the excess money supply by the growth rate of the money supply less the amount of money the public demands to finance the changing quantity of output. This result may be derived by considering the market conditions underlying aggregate demand which are stated in equations (1) through (5) in Appendix 2. In the long-run, several terms in those equations are zero, and we are left with the following specification for the inflation rate:

\[ dP_t = dM_t - a_6dYP_t \]

where a6 represents the income elasticity of money demand.

Fourth, in the long-run, changes in the money-supply growth rate are fully reflected in changes in the inflation rate: one percentage point more (less) monetary growth means one percentage point higher (lower) inflation. This result is represented in the above equation by the one-to-one relationship between those two rates — dP and dM, and by the earlier
assumption that potential GNP is exogenous with respect to the variables in our model.

Some critics argue that the long-run is an inappropriate state in which to analyze economic behavior; in other words, the economy is so frequently shocked away from its steady state that that state may never in fact be attained. However, long-run conditions may also provide useful “rules-of-thumb” regarding average economic behavior over spans of time. For instance, over long enough periods of time, the average rate of GNP will approach the economy’s potential growth rate as determined by its labor-force productivity growth. Similarly, the average rate of inflation will approach the rate of excess money supply.

Table 1
Summary of the Model

Aggregate Demand

\[(Y/YP)_t^d = k_0 - a_{11}(dP_t - dM_t) + a_{12}(dP_{t+1} - dP*) - a_{13} dG_t - c_1 B(L) d(Y/YP)_{t-1}^d + (Y/YP)_{t-1}^d + w_{lt} \]

Aggregate Supply

\[(Y/YP)_t^s = a_{10}(dP_t - dP*) + J(L) (Y/YP)_{t-1}^s + w_{lt} \]

Equilibrium Condition

\[(Y/YP)_t^d = (Y/YP)_t^s \]

where the coefficients and weights of the polynomials are combinations of the coefficients and weights in the structural equations (1) - (7) in Appendix 2*.

The unknown variables are \(Y_t^d, Y_t^s, dP_t^*, dP_{t+1}^*, \) and \(dP_t\).

List of Variables

\[G = \text{Federal government real high-employment surplus, measured as the ratio of revenues to expenditures,} \]
\[Y = \text{Real GNP,} \]
\[YP = \text{Real potential GNP,} \]
\[Y/YP = \text{Real GNP gap,} \]
\[M = \text{Money supply,} \]
\[P = \text{GNP implicit price deflator,} \]
\[P^* = \text{Expected GNP implicit price deflator.} \]

Each of the variables is measured in terms of its natural logarithm. Therefore, the change in real GNP, \(dY\), is a measure of the rate of change in real GNP, and \(Y/YP\) is a measure of real GNP as a percent of potential. The variables \(G, YP, UN\) and \(M\) are exogenous variables.

\[c_1 = \frac{a_7}{a_7 + a_4 a_6} \]
\[a_{10} = a_9 a_5 \]
\[a_{11} = \frac{a_4}{a_7 + a_4 a_6} \]
\[a_{12} = \frac{a_4 a_7}{a_7 + a_4 a_6} \]
\[a_{13} = \frac{a_3 a_7}{a_7 + a_4 a_6} \]
\[k_0 = (-dYP_t - c_1 B(L) dYP_{t-1}) \]
Short-run Adjustments
In the short-run, we focus on the transition period after a monetary change as the economy tends to move from one steady state to another. We emphasize the short-run adjustment paths of real GNP, inflation and real money balances which result from an increase in the money-supply growth rate. Similar reasoning may be applied to a decrease in monetary growth.

Any such monetary stimulus would lead initially to the creation of excess real money balances in the hands of the public. (This can be shown in the aggregate-demand equation by increasing the money-growth rate, $dM_t$, while leaving all other variables unchanged.) Individuals now will attempt to reduce those balances by increasing expenditures. Whether this increased demand will be met with greater production, higher prices, or some combination of the two depends upon aggregate supply behavior. If both employers and labor expect an increase in prices equal to the increased monetary growth, and if no contractual impediments exist, prices and wages will adjust quickly. Accordingly, there will be no change in the rate of output growth, and the increased monetary stimulus will result only in price and nominal-wage increases.

These results may be shown graphically with the use of the aggregate demand and supply equations. To simplify, we assume no change in fiscal policy and in the level of potential output, but a continued change in expected inflation. In addition, we assume zero weights for both polynomial functions, $B(L)$ and $J(L)$, in the aggregate demand and supply equations. With these assumptions, we can specify aggregate demand and supply with the following two simplified equations, each written with the inflation rate on the left-hand side.

\[
\begin{align*}
dP_t &= - \frac{1}{a_{11}} (Y/YP)^4 + k_0/a_{11} + dM_t \\
& \quad + \frac{1}{a_{11}} (Y/YP)_{t-1} \\
\text{Aggregate Demand (10.1)}
\end{align*}
\]

\[
\begin{align*}
dP_t &= dP^*_t + \frac{1}{a_{10}} (Y/YP)^5 \\
\text{Aggregate Supply (11.1)}
\end{align*}
\]

We begin with the economy in equilibrium for some time, illustrated in Chart 1 by the intersection of aggregate demand, $AD_0$, and aggregate supply, $AS_0$, at point $E_0$. At that point, the level of real output is equal to potential and the rate of inflation is equal to the money-supply growth rate, "$m_0$", on the vertical axis. Now, let the money growth rate increase to $m_1$, which shifts aggregate demand upward by the full amount of that increase to $AD_1$, according to equation (10). If price expectations increase by the same amount as money growth, aggregate supply will shift upward by the same amount as aggregate demand, to $AS_1$. The intersection of $AS_1$ and $AD_1$ at point $E_1$ provides the new solution: the change in money growth leads only to an equal increase in the inflation rate — without any change in the quantity of output — as inflation expectations change at the same time and by the same amount as the permanent change in money growth.

However, empirical evidence suggests that prices and inflation expectations do not adjust quickly to their new long-run values. Working in the face of uncertainty, individuals appear to rely heavily on observations of past behavior.
and other relevant information in forming expectations of the future. These expectations, although rational in the sense of being well-informed and based upon the relevant information, nevertheless provide imperfect predictions of the future at any given time. As a consequence, each increase in the money-growth rate may be followed by an increase in both real GNP and inflation.

Graphically, in Chart 2, the initial equilibrium position is again marked as point $E_0$. Let the rate of money growth increase to $M_1$. The aggregate-demand function will again shift upward to $AD_1$, and aggregate supply probably will also shift. But since we allow for inflation expectations which do not adjust immediately and completely to their new long-run value ($m_1$ on the vertical axis), the supply shift is smaller than the demand shift. For purposes of illustration, assume that aggregate supply shifts upward to $AS_1$. The new, short-run equilibrium is then at point B, which indicates an initial increase of both real output and inflation in response to the money-growth increase.

At the beginning of the next period, aggregate demand will shift upward again, according to equation 10, as past real income increases from the level associated with point $E_0$ to that of point B. Also, aggregate supply will shift upward again as market participants reevaluate price expectations in light of information not previously available. The intersection of aggregate demand and supply at point C illustrates the new short-run equilibrium position.

It is important to note that inflation increases from the level $m_0$ to a rate close to its new equilibrium rate, $m_1$, and then overshoots that value as continued adjustments in aggregate demand and supply lead to a new, short-run equilibrium at point C. Changes in price expectations (in light of revised forecasts) influence the shifts in supply, while past income and real money balances produce adjustments in aggregate demand. This characteristic overshooting property — “the key element in monetary theories of cyclical fluctuations,” according to Milton Friedman — is a widely noted economic phenomenon.

The inflation pattern is associated with a particular adjustment in real GNP, as shown in Chart 2. After the initial increase in monetary growth, real GNP increases above its potential level, and continues to increase as long as the inflation rate is below its new equilibrium rate, $m_1$. This occurs because as long as inflation increases more slowly than money growth ($m_1$), real money balances will increase and, accordingly, stimulate aggregate demand. Once the inflation rate starts to overshoot its new equilibrium level, real money balances will begin to decline. As a result of this contractionary force, real GNP begins to decline from its previous value.

The adjustments of real output, inflation and real money balances continue until each reaches a new, long-run value associated with the permanent increase in money-supply growth (Chart 3). First consider the response of inflation to an increase in the money rate, from $m$ to $m_1$ (Chart 3A). Initially, the inflation rate increases by less than the change in

---

**Chart 2**

Inflation and Output Adjustment to Increase in Money-Growth Rate: Lagged Adjustment in Price Expectations

---

49
money growth each period, but completes its initial adjustment phase at time $T_1$. At that time, the inflation rate will equal the long-run rate of change in money growth, although price expectations and real income will still not have made a complete adjustment. Subsequent to $T_1$, inflation overshoots and then returns to its long-run rate. In our illustration, we depict inflation as gradually returning to its new equilibrium position, although alternatively, it may exhibit a damped cyclical adjustment to its long-run value.

Next consider the response of real money balances to an increase in the money growth rate (Chart 3B). In the initial equilibrium, individuals hold a certain desired proportion of income in that form. For some time after money-supply growth increases, prices increase less rapidly than the nominal money supply in each period. As a result, real money balances increase and reach a maximum when the inflation rate initially reaches its long-run rate at time $T_1$. Subsequently, these balances steadily decline toward their new, long-run level, which we have shown to be equal to their initial level. With any sensitivity to interest rates, however, the level of real balances in the new equilibrium will be lower than initially. In the U.S., money demand generally moves inversely with interest rates. Therefore, we would expect that, after a permanent increase in monetary growth, real money balances will ultimately be slightly less than before the change.

Finally consider the response of real GNP to a change in the money growth rate (Chart 3C). In this adjustment process, the level of real output returns to its long-run path after rising above potential for some time. Meanwhile, the rate of change in real GNP increases and then declines in response to an increase in monetary growth, and finally increases again before approaching its long-run rate of change (Chart 3D). This pattern of growth is consistent with the adjustment in the level of GNP shown in Chart 3C.

In summary, a change in money-supply growth may have no significant effect on real output, at least as long as prices and price expectations adjust immediately. In other words, the entire change in money may be absorbed by a change in prices. In contrast, if there is no immediate price adjustment, monetary changes may be accompanied by changes in both inflation and real output. The key feature in the adjustment process is the cyclical response of both output and inflation. After a permanent increase in money growth, inflation will increase, overshoot, and then decline towards its long-run value. The real-output growth rate will at first increase, then decline,
and may increase again as the level of output returns to potential.

**Formation of Inflation Expectations**

Before deriving the final model equations, we must first express the functional forms for expected inflation. These will then be substituted into the aggregate demand and supply equations, which in turn can be solved for the final equations for real GNP and inflation.

To obtain the specifications for expected inflation, we follow a procedure suggested by Sargent and Wallace (1975). The mathematics are shown in Appendix 3. Given the rational-expectations feature of the model, expected inflation depends upon the public's forecasts of future monetary and fiscal policies, as well as the past history of deviations of output from its potential level. (These deviations directly affect current inflation, due to the lagged response of prices to past money-supply changes.) Particularly important is the process or rule by which the public forecasts future rates of change in the money supply and in Federal-budget surpluses. We assume that people forecast these values by considering their past history, and that they update their forecasts each period as new information becomes available. Consequently, expected inflation is determined by the past history of the money supply, Federal-budget surpluses and the GNP gap.

The derived specifications for expected inflation (shown in the appendix) can next be used to obtain the final equations for inflation and real GNP. However, changes in the processes by which the public predicts future monetary and fiscal policies will lead to changes in the parameters of the estimated equations. Consequently, those equations will provide appropriate means of forecasting real GNP and inflation as long as the public does not change the rule by which it predicts future government policies.

### II. Solution of the Model: Reduced-form Equations

In this section, we provide the reduced-form equations for the inflation rate and real GNP which are solutions of the system (equations 10, 11, 12, plus equations 16.1 and 17.1 from Appendix 3), and which serve as the basic equations estimated in the next section.

First, we obtain the equation for real GNP, which will be stated in terms of the GNP gap, \( Y/YP \), by substituting the specifications for inflation and expected inflation — provided by equations (13) and (14) in Appendix 3 — into the aggregate-supply equation.

\[
(Y/YP)_t = \sum_{i=0}^{N} m_{1i}(dM - E_{t-i-1}(dM))_{t-i} \\
- \sum_{i=0}^{N} g_{1i}(dG - E_{t-i-1}(dG))_{t-i}
\]  

(18)

where the coefficients \( m_{1i} \) and \( g_{1i} \) are combinations of the coefficients in equations (1) through (9) in Appendix 2. Also, we have assumed a finite length of the lags, \( N \). The equation thus states that the cyclical movement in real GNP is determined by distributed lags on unanticipated changes in money-supply growth and in the Federal-budget surplus. (An unanticipated change is defined as the actual less the expected value of a variable.)

\[ *m_{10} = a_{11}Z \]
\[ M_{1i} = a_{11}ZJ_i, \ i = 0, 1, 2, \ldots \]

(Recall that \( J_i \) are the weights of the polynomial \( J(L) \) in the aggregate supply equation.)

\[ g_{10} = a_{13}Z \]
\[ G_{1i} = a_{13}ZJ_i, \ i = 0, 1, 2, \ldots \]
We estimate the level of GNP relative to its potential and then derive the rate of growth of real GNP from that equation. We do this because the level specification possesses the desirable property that the long-run values of both the level and rate of change will be independent of the initial conditions of the forecast.

After a monetary shock, our real GNP estimates eventually return to their long-run values. This long-run property does not hold for conventional reduced-form equations, which estimate rates of change in real GNP directly. In such cases, the long-run value of income equals the initial level of income times the subsequent rate of change in potential GNP. As a result, those equations predict a persistent gap in real output equal to the initial gap at the time of the monetary shock. By doing so, the rates-of-change specifications ignore the economic impact resulting from deviations of unemployment from its natural rate. Consequently, these specifications may produce biased estimates of the impact of monetary growth upon real GNP.

To obtain the reduced form for the inflation rate, we substitute equations (16.1) and (17.1) into equation (13) in Appendix 3, and collect terms:

\[ dp_t = \sum_{i=0}^{K} m_{2i} dM_{t-i} - \sum_{i=0}^{J} g_{2i} dG_{t-i} + c' + \sum_{i=1}^{N} y_1 (Y/YP)_{t-i} - \sum_{i=1}^{N} y_2 (Y/YP)_{t-i} \]

where the coefficients are combinations of the coefficients in the equations (1) to (9) and of the money-supply and fiscal-policy processes.

The equation states that the rate of inflation is determined by 1) a distributed lag on current and past rates of growth in the money supply, 2) a distributed lag on current and past changes in the Federal surplus, and 3) past values of the cyclical component of GNP, Y/YP.

**Empirical Results**

Following the above specifications, we estimated equations for real GNP and the inflation rate over the sample period 1966.2-1978.1. The starting date was dictated by the availability of M-1B data and the length of the estimated lag distribution in the equations. The estimation period ends in 1978.1, to permit a relatively sizable time span outside the sample period for assessing the model's performance. The estimation results are shown in Box 1.

In estimating these equations, we applied a method suggested by John Scadding and detailed in Appendix 1. The method enables us to place restrictions on both the steady-state level of a variable and its range of change. For example, we can impose the restriction of the long-run neutrality of money, with respect to both the level and growth rate of real GNP. Similarly, in the inflation equation the restrictions may be imposed that the elasticity of the inflation rate with respect to the growth rate of money is unity, and that the level of prices is consistent with the level of money, given the demand for money.

In the real-GNP equation, we have added a lagged GNP-gap term to the reduced-form specification, equation 18. The equation then may be interpreted as estimating a rational distributed lag between the dependent variable and the $\bar{M}$ and $\bar{E}$ variables. In addition, in the inflation equation we have added two variables, $D_1$ and $D_2$, to capture the possible impact of Nixon-era wage and price controls. These same variables, although significant in the inflation equation, had no significant impact upon real GNP and are therefore not included in that equation.

\[ \sum_{i=0}^{K} m_{2i} dM_{t-i} = [w_1 H_1(L) + w_2 H_3(L) + w_4] dM_t \]

\[ \sum_{i=0}^{J} g_{2i} dG_{t-i} = [w_1 H_2(L) + w_2 H_4(H) + w_3] dG_t \]

\[ \sum_{i=0}^{N} y_{2i} = q_1 (1 + (w_1 + w_2) k_3) / (1 - w_i) \]

\[ c' = k_1 k_3 (w_1 + w_2) + c_0 \]

\[ k_3 = \sum_{j=0}^{\infty} v_j \]

\[ y_1 = k_3 v_3 (w_1 + w_2) + w_3 \]
Both the real GNP and inflation regressions account for over 80 percent of the variation in the dependent variables. The adjusted coefficient of determination, \( R^2 \), is .89 in the case of real GNP and .83 for inflation. The Durbin-Watson statistic (D.W.) indicates support for the hypothesis of no serial correlation in the error terms. Thus, on the basis of the relevant T-statistics, the explanatory variables in each equation have a statistically significant impact upon the determination of the dependent variables.

According to the estimated real-GNP equation, money is neutral in the long run with respect to the level of real GNP and its rate of change. These findings follow from several characteristics of the equation. First, in the long run, both unanticipated money and unanticipated federal expenditures equal zero. Second, the constant term does not significantly differ from zero, and the coefficient on the lagged gap term is less than unity, assuring the stable long-run result of a zero value for the log of \( \frac{Y}{YP} \).

According to the Scadding method, the coefficient on the current change in unanticipated money, \( \overline{M}_t \), in the real GNP equation provides an estimate of the total short-run effect of such a change. The estimate of .187 indicates that a one-percentage-point increase (decrease) in unanticipated money growth leads to a small transitory gain (loss) in real GNP. Since economic theory does not indicate an expected value of the transitory impact, we have left unconstrained the coefficient on the current value of unanticipated monetary growth.

The estimates also indicate that unanticipated increases in Federal expenditures, \( \overline{E} \), will at first lead to increases in real GNP, but after 14 quarters will have no significant impact upon either the level or rate of growth of real GNP. This result follows from the estimated sum of \(-0.068\), which is not significantly different from zero.

With regard to the inflation equation, we constrained the coefficient of \( dM_t \) to be unity, after testing for the appropriateness of that constraint. Consequently, in the long run, changes in money growth are fully reflected in changes in the inflation rate. The coefficient on the second difference of money, according to our estimation procedure, is equivalent to the long-run elasticity of the price level with respect to a change in the money-supply growth rate. In other words, the coefficient is equivalent to the long-run elasticity of real money balances with respect to money (with the signs reversed).

Our estimate indicates that real money balances, in the long-run, will decline by .21 percentage point when money growth increases by one percentage point each quarter. This value appears consistent with estimates for the long-run elasticity of money demand for our sample period. Accordingly, we did not constrain the estimate to take on any particular value. Together the two coefficients indicate that the inflation equation is consistent with the long-run neutrality of money. In the first instance, changes in the money growth rate are fully reflected in prices; and in the second, the price level is consistent with the level of money, given the demand for money in the steady state.

Only unanticipated changes in Federal spending significantly affected the inflation rate. There was no effect in the long run, because unanticipated changes are zero in that case. The results, however, indicate a short-run or transitory impact from spending changes which are initially unanticipated.

Finally, the sum of coefficients on the lagged gap measure was not significantly different from zero, so that we constrained their sum to that value. The estimated coefficients on the first difference of the gap indicate that a positive widening of the gap in period t-1 will lead to a small negative effect upon the current quarter’s inflation rate. Thereafter, the positive impact of the lagged-gap values offset the initial negative effect upon inflation.

Next, consider the response patterns of real GNP and inflation to changes in the money-supply growth rate. These patterns may be obtained from the empirical estimates by “unscrambling” the estimated coefficients, according to the method outlined in Appendix.
Box 1

REAL GNP EQUATION

\[
(Y/YP)_t = -0.0006 + 0.187M_t + \sum_{i=0}^{14} b_i M_{t-i} + \sum_{i=0}^{14} \theta_i E_{t-i} + 0.869(Y/YP)_{t-1}
\]

<table>
<thead>
<tr>
<th>Lag</th>
<th>(b_i)</th>
<th>t-statistic</th>
<th>(e_{ii})</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.207</td>
<td>3.7</td>
<td>0.0042</td>
<td>1.2</td>
</tr>
<tr>
<td>1</td>
<td>0.387</td>
<td>3.7</td>
<td>0.0074</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>0.542</td>
<td>3.7</td>
<td>0.0096</td>
<td>1.1</td>
</tr>
<tr>
<td>3</td>
<td>0.670</td>
<td>3.8</td>
<td>0.0107</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>0.773</td>
<td>3.8</td>
<td>0.0109</td>
<td>0.9</td>
</tr>
<tr>
<td>5</td>
<td>0.850</td>
<td>3.8</td>
<td>0.0100</td>
<td>0.8</td>
</tr>
<tr>
<td>6</td>
<td>0.900</td>
<td>3.9</td>
<td>0.0080</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>0.925</td>
<td>3.9</td>
<td>0.0050</td>
<td>0.4</td>
</tr>
<tr>
<td>8</td>
<td>0.924</td>
<td>3.9</td>
<td>0.0011</td>
<td>0.08</td>
</tr>
<tr>
<td>9</td>
<td>0.896</td>
<td>4.0</td>
<td>-0.0039</td>
<td>-0.32</td>
</tr>
<tr>
<td>10</td>
<td>0.843</td>
<td>4.0</td>
<td>-0.0100</td>
<td>-0.82</td>
</tr>
<tr>
<td>11</td>
<td>0.764</td>
<td>4.0</td>
<td>-0.0171</td>
<td>-1.34</td>
</tr>
<tr>
<td>12</td>
<td>0.659</td>
<td>3.9</td>
<td>-0.0252</td>
<td>-1.77</td>
</tr>
<tr>
<td>13</td>
<td>0.527</td>
<td>3.1</td>
<td>-0.0343</td>
<td>-2.03</td>
</tr>
<tr>
<td>14</td>
<td>0.370</td>
<td>2.1</td>
<td>-0.0444</td>
<td>-2.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sum -0.68</td>
<td>(.479)</td>
</tr>
</tbody>
</table>

The distributed lags are 2nd-degree polynomials, with a near-end constraint.

Adjusted \(r^2 = .89\)   D.W. = 2.01   Standard Error = .00749

INFLATION EQUATION

\[
dP_t = -0.0004 + dM_t + 0.21d^2M_t + \sum_{i=0}^{20} b_i d^2 M_{t-i} + \sum_{i=0}^{12} \theta_i E_{t-i} - \sum_{i=0}^{16} \gamma d(Y/YP)_{t-i}
\]

\[-0.002D_1 + 0.009D_2\]

<table>
<thead>
<tr>
<th>Lag</th>
<th>(h_i)</th>
<th>t-statistic</th>
<th>(e_{2i})</th>
<th>t-statistic</th>
<th>(y_i)</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.23</td>
<td>(-10.0)</td>
<td>-0.001</td>
<td>(-7.7)</td>
<td>-0.083</td>
<td>(-2.70)</td>
</tr>
<tr>
<td>1</td>
<td>-2.23</td>
<td>(-10.1)</td>
<td>0.000</td>
<td>(0.01)</td>
<td>-0.081</td>
<td>(-3.03)</td>
</tr>
<tr>
<td>2</td>
<td>-3.00</td>
<td>(-10.1)</td>
<td>0.006</td>
<td>(0.9)</td>
<td>-0.078</td>
<td>(-3.35)</td>
</tr>
<tr>
<td>3</td>
<td>-3.57</td>
<td>(-10.0)</td>
<td>0.011</td>
<td>(1.7)</td>
<td>-0.075</td>
<td>(-3.61)</td>
</tr>
<tr>
<td>4</td>
<td>-3.97</td>
<td>(-9.9)</td>
<td>0.015</td>
<td>(2.2)</td>
<td>-0.072</td>
<td>(-3.75)</td>
</tr>
<tr>
<td>5</td>
<td>-4.20</td>
<td>(-9.8)</td>
<td>0.018</td>
<td>(2.5)</td>
<td>-0.069</td>
<td>(-3.75)</td>
</tr>
<tr>
<td>6</td>
<td>-4.29</td>
<td>(-9.6)</td>
<td>0.020</td>
<td>(2.6)</td>
<td>-0.065</td>
<td>(-3.65)</td>
</tr>
<tr>
<td>7</td>
<td>-4.25</td>
<td>(-9.2)</td>
<td>0.020</td>
<td>(2.7)</td>
<td>-0.061</td>
<td>(-3.47)</td>
</tr>
<tr>
<td>8</td>
<td>-4.09</td>
<td>(-8.8)</td>
<td>0.020</td>
<td>(2.7)</td>
<td>-0.056</td>
<td>(-3.26)</td>
</tr>
<tr>
<td>9</td>
<td>-3.85</td>
<td>(-8.3)</td>
<td>0.018</td>
<td>(2.7)</td>
<td>-0.046</td>
<td>(-2.84)</td>
</tr>
<tr>
<td>10</td>
<td>-3.53</td>
<td>(-7.6)</td>
<td>0.018</td>
<td>(2.7)</td>
<td>-0.051</td>
<td>(-3.04)</td>
</tr>
<tr>
<td>11</td>
<td>-3.15</td>
<td>(-6.9)</td>
<td>0.011</td>
<td>(2.6)</td>
<td>-0.040</td>
<td>(-2.65)</td>
</tr>
<tr>
<td>12</td>
<td>-2.74</td>
<td>(-6.1)</td>
<td>0.006</td>
<td>(2.6)</td>
<td>-0.034</td>
<td>(-2.48)</td>
</tr>
<tr>
<td>13</td>
<td>-2.29</td>
<td>(-5.3)</td>
<td></td>
<td></td>
<td>-0.028</td>
<td>(-2.34)</td>
</tr>
<tr>
<td>14</td>
<td>-1.87</td>
<td>(-4.5)</td>
<td></td>
<td></td>
<td>-0.021</td>
<td>(-2.21)</td>
</tr>
<tr>
<td>15</td>
<td>-1.43</td>
<td>(-3.7)</td>
<td></td>
<td></td>
<td>-0.015</td>
<td>(-2.09)</td>
</tr>
<tr>
<td>16</td>
<td>-1.04</td>
<td>(-2.9)</td>
<td></td>
<td></td>
<td>-0.007</td>
<td>(-1.99)</td>
</tr>
<tr>
<td>17</td>
<td>- .70</td>
<td>(-2.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>- .41</td>
<td>(-1.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>- .22</td>
<td>(-1.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>- .13</td>
<td>(-1.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sum .170   (2.14)

Adjusted \(r^2\) with unitary constraint on \(dM_t = .83\)   D.W. = 1.97   Standard Error = .0033

The distributed lags on expenditures and income are polynomial distributed lags, 2nd degree, with far-end constraints. On money a third-degree polynomial was used with a near-end constraint.
1. This "unscrambling" may be done either directly, through algebraic manipulations of the estimated coefficients — or by obtaining estimated lag patterns from dynamic multiplier simulations of the equations.

These computer simulations essentially solve the estimated equations for a given rate of money growth, and then repeat the solution with another rate of money growth exactly one percentage point higher in each and every period. The difference between the results in each period provides the estimate of the response of the dependent variable (such as real GNP or the deflator) to the specified increase in money growth. In these dynamic simulations, the lagged dependent variables are solutions of the model after the initial period.

Real GNP Estimates

Our model has been able to capture the complete adjustment paths of both the level and rate of change in real GNP to the change in monetary policy. This can be seen from the similarity between the empirical multiplier estimates (Charts 4A and 4B), and the expected estimates from the theoretical model (Charts 3C and 3D).

If we begin, say, with an initial equilibrium situation in which the level of real GNP is equal to potential (Chart 4A), an increase of one percentage point in the money growth rate will lead to a small initial response in the level of real GNP (Table 2).

After a rise of seven quarters, output reaches a peak .66 percentage point higher than potential, but it then returns to its potential level.
around the 14th quarter after the money change. The stimulus to real GNP thus appears to end within about two years, although the final adjustment does not end until about ten years after the initial change.

The growth rate of real GNP follows a cyclical pattern over time in response to a permanent increase in money growth. For instance, from an initial position of equilibrium with real GNP rising at a steady 3.2 percent rate each quarter, a one-percentage point rise in money growth would raise GNP growth to 3.7 percent within a half-year of the initial monetary change. The growth rate then begins to decline, and reaches its initial rate after the end of two years. Following a further decline, the growth rate increases again in the final phase of adjustment, settling at its long-run rate of 3.2 percent in about 10 years’ time.

With this adjustment pattern, the initial stimulus to real GNP is almost matched by an equivalent contraction of real GNP in the final phases of economic adjustment. As a result, the monetary change leads to a small but transitory gain in real output. However, in the long run, these gains disappear as real GNP returns to its potential path regardless of the rate of money growth. Moreover, most of the

---

**Chart 4**

Response of Selected Variables to Permanent Increase in Money-Growth Rate

- **Real-GNP Gap Multiplier***
- **Real-GNP Growth-Rate Multiplier**

*Percentage-point change in level of GNP relative to potential after a permanent one-percentage-point increase in the money growth rate, measured in natural logs.

**Percentage-point change in growth rate of real GNP after a permanent one-percentage-point increase in money growth rate, measured in natural logs.
stimulative effect of an increase in monetary growth appears to end within two years of the initial stimulus.

Our estimates capture the systematic changes in real GNP, although not the sharp saw-toothed variations characteristic of this series (Chart 5). The standard error of the quarterly estimates of GNP, at annual rates, is 2.92 percent within the estimation period (1962.2-1978.1) and 3.30 percent outside that period (1978.2-1980.4).\(^8\)

**Inflation Estimates**

Our model, again, permits us to estimate the complete adjustment of the level of prices and the rate of inflation to a change in money growth. This can be seen from the conformity of the estimated lag patterns in Charts 4C and 4D to those anticipated in Charts 3A and 3B.

In the long-run, the change in the rate of inflation equals the change in the rate of money growth. By the end of the first year, the inflation rate exhibits about 50 percent of its

---

*Percentage-point change in inflation rate after a permanent one-percentage-point increase in money growth rate, measured in natural logs.*

**Percentage-point change in level of real money balances after a permanent one-percentage-point increase in money supply, measured in natural logs.*
### Table 2

**Response of Real GNP to Permanent Percentage-Point Increase in Money-Supply Annual Growth Rate**

<table>
<thead>
<tr>
<th>Lag</th>
<th>Estimate</th>
<th>Lag</th>
<th>Estimate of $m_n$</th>
<th>Lag</th>
<th>Estimate</th>
<th>Lag</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.09</td>
<td>25</td>
<td>-.22</td>
<td>0</td>
<td>.38</td>
<td>25</td>
<td>.13</td>
</tr>
<tr>
<td>1</td>
<td>.20</td>
<td>26</td>
<td>-.19</td>
<td>1</td>
<td>.46</td>
<td>26</td>
<td>.12</td>
</tr>
<tr>
<td>2</td>
<td>.32</td>
<td>27</td>
<td>-.16</td>
<td>2</td>
<td>.48</td>
<td>27</td>
<td>.10</td>
</tr>
<tr>
<td>3</td>
<td>.43</td>
<td>28</td>
<td>-.14</td>
<td>3</td>
<td>.46</td>
<td>28</td>
<td>.09</td>
</tr>
<tr>
<td>4</td>
<td>.53</td>
<td>29</td>
<td>-.12</td>
<td>4</td>
<td>.39</td>
<td>29</td>
<td>.08</td>
</tr>
<tr>
<td>5</td>
<td>.60</td>
<td>30</td>
<td>-.11</td>
<td>5</td>
<td>.30</td>
<td>30</td>
<td>.07</td>
</tr>
<tr>
<td>6</td>
<td>.64</td>
<td>31</td>
<td>-.09</td>
<td>6</td>
<td>.18</td>
<td>31</td>
<td>.06</td>
</tr>
<tr>
<td>7</td>
<td>.66</td>
<td>32</td>
<td>-.08</td>
<td>7</td>
<td>.05</td>
<td>32</td>
<td>.05</td>
</tr>
<tr>
<td>8</td>
<td>.63</td>
<td>33</td>
<td>-.07</td>
<td>8</td>
<td>-.09</td>
<td>33</td>
<td>.04</td>
</tr>
<tr>
<td>9</td>
<td>.59</td>
<td>34</td>
<td>-.06</td>
<td>9</td>
<td>-.20</td>
<td>34</td>
<td>.04</td>
</tr>
<tr>
<td>10</td>
<td>.52</td>
<td>35</td>
<td>-.05</td>
<td>10</td>
<td>-.29</td>
<td>35</td>
<td>.03</td>
</tr>
<tr>
<td>11</td>
<td>.43</td>
<td>36</td>
<td>-.03</td>
<td>11</td>
<td>-.37</td>
<td>36</td>
<td>.02</td>
</tr>
<tr>
<td>12</td>
<td>.32</td>
<td>37</td>
<td>.00</td>
<td>12</td>
<td>-.45</td>
<td>37</td>
<td>.01</td>
</tr>
<tr>
<td>13</td>
<td>.21</td>
<td>38</td>
<td>.00</td>
<td>13</td>
<td>-.48</td>
<td>38</td>
<td>.00</td>
</tr>
<tr>
<td>14</td>
<td>.08</td>
<td>39</td>
<td>.00</td>
<td>14</td>
<td>-.54</td>
<td>39</td>
<td>.00</td>
</tr>
<tr>
<td>15</td>
<td>-.11</td>
<td>40</td>
<td>.00</td>
<td>15</td>
<td>-.78</td>
<td>40</td>
<td>.00</td>
</tr>
<tr>
<td>16</td>
<td>-.24</td>
<td>41</td>
<td>.00</td>
<td>16</td>
<td>-.56</td>
<td>41</td>
<td>.00</td>
</tr>
<tr>
<td>17</td>
<td>-.33</td>
<td>42</td>
<td>.00</td>
<td>17</td>
<td>-.37</td>
<td>42</td>
<td>.00</td>
</tr>
<tr>
<td>18</td>
<td>-.38</td>
<td>43</td>
<td>.00</td>
<td>18</td>
<td>-.21</td>
<td>43</td>
<td>.00</td>
</tr>
<tr>
<td>19</td>
<td>-.40</td>
<td>44</td>
<td>.00</td>
<td>19</td>
<td>-.08</td>
<td>44</td>
<td>.00</td>
</tr>
<tr>
<td>20</td>
<td>-.39</td>
<td>45</td>
<td>.00</td>
<td>20</td>
<td>.03</td>
<td>45</td>
<td>.00</td>
</tr>
<tr>
<td>21</td>
<td>-.37</td>
<td>46</td>
<td>.00</td>
<td>21</td>
<td>.10</td>
<td>46</td>
<td>.00</td>
</tr>
<tr>
<td>22</td>
<td>-.33</td>
<td>47</td>
<td>.00</td>
<td>22</td>
<td>.15</td>
<td>47</td>
<td>.00</td>
</tr>
<tr>
<td>23</td>
<td>-.29</td>
<td>48</td>
<td>.00</td>
<td>23</td>
<td>.18</td>
<td>48</td>
<td>.00</td>
</tr>
<tr>
<td>24</td>
<td>-.25</td>
<td>49</td>
<td>.00</td>
<td>24</td>
<td>.15</td>
<td>49</td>
<td>.00</td>
</tr>
</tbody>
</table>

### Table 3

**Response of Inflation Rate and Real Money Balances to Permanent Percentage-Point Increase in Money Supply Annual Growth Rate**

<table>
<thead>
<tr>
<th>Lag</th>
<th>Estimate</th>
<th>Lag</th>
<th>Estimate</th>
<th>Lag</th>
<th>Estimate</th>
<th>Lag</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-.02</td>
<td>16</td>
<td>1.52</td>
<td>0</td>
<td>.24</td>
<td>16</td>
<td>.53</td>
</tr>
<tr>
<td>1</td>
<td>-.02</td>
<td>17</td>
<td>1.54</td>
<td>1</td>
<td>.48</td>
<td>17</td>
<td>.39</td>
</tr>
<tr>
<td>2</td>
<td>.16</td>
<td>18</td>
<td>1.49</td>
<td>2</td>
<td>.69</td>
<td>18</td>
<td>.27</td>
</tr>
<tr>
<td>3</td>
<td>.32</td>
<td>19</td>
<td>1.46</td>
<td>3</td>
<td>.85</td>
<td>19</td>
<td>.16</td>
</tr>
<tr>
<td>4</td>
<td>.47</td>
<td>20</td>
<td>1.34</td>
<td>4</td>
<td>.98</td>
<td>20</td>
<td>.07</td>
</tr>
<tr>
<td>5</td>
<td>.60</td>
<td>21</td>
<td>1.37</td>
<td>5</td>
<td>1.07</td>
<td>21</td>
<td>-.02</td>
</tr>
<tr>
<td>6</td>
<td>.72</td>
<td>22</td>
<td>1.23</td>
<td>6</td>
<td>1.14</td>
<td>22</td>
<td>-.08</td>
</tr>
<tr>
<td>7</td>
<td>.84</td>
<td>23</td>
<td>1.20</td>
<td>7</td>
<td>1.18</td>
<td>23</td>
<td>-.13</td>
</tr>
<tr>
<td>8</td>
<td>.95</td>
<td>24</td>
<td>1.17</td>
<td>8</td>
<td>1.19</td>
<td>24</td>
<td>-.17</td>
</tr>
<tr>
<td>9</td>
<td>1.06</td>
<td>25</td>
<td>1.14</td>
<td>9</td>
<td>1.17</td>
<td>25</td>
<td>-.20</td>
</tr>
<tr>
<td>10</td>
<td>1.16</td>
<td>26</td>
<td>1.10</td>
<td>10</td>
<td>1.13</td>
<td>26</td>
<td>-.23</td>
</tr>
<tr>
<td>11</td>
<td>1.24</td>
<td>27</td>
<td>1.07</td>
<td>11</td>
<td>1.07</td>
<td>27</td>
<td>-.25</td>
</tr>
<tr>
<td>12</td>
<td>1.33</td>
<td>28</td>
<td>1.04</td>
<td>12</td>
<td>.99</td>
<td>28</td>
<td>-.26</td>
</tr>
<tr>
<td>13</td>
<td>1.39</td>
<td>29</td>
<td>1.01</td>
<td>13</td>
<td>.89</td>
<td>29</td>
<td>-.26</td>
</tr>
<tr>
<td>14</td>
<td>1.46</td>
<td>30</td>
<td>.99</td>
<td>14</td>
<td>.77</td>
<td>30</td>
<td>-.26</td>
</tr>
<tr>
<td>15</td>
<td>1.49</td>
<td>35</td>
<td>.93</td>
<td>15</td>
<td>.66</td>
<td>35</td>
<td>-.19</td>
</tr>
<tr>
<td>40</td>
<td>.95</td>
<td>40</td>
<td>.95</td>
<td>40</td>
<td>.40</td>
<td>40</td>
<td>-.12</td>
</tr>
<tr>
<td>50</td>
<td>.99</td>
<td>50</td>
<td>.99</td>
<td>50</td>
<td>.50</td>
<td>50</td>
<td>-.07</td>
</tr>
<tr>
<td>60</td>
<td>1.00</td>
<td>60</td>
<td>1.00</td>
<td>60</td>
<td>.60</td>
<td>60</td>
<td>-.05</td>
</tr>
</tbody>
</table>
ultimate change, and within two years exhibits a 100-percent change (Table 3). But over the next two years, it overshoots its ultimate change by about 50 percent. For instance, with a one-percentage-point increase in money growth associated with an inflation-rate increase from 7.0 percent to ultimately 8.0 percent, we expect to see inflation rise from 7.0 to 8.0 percent within two years, rise further to 8.5 percent within four years, and then slowly descend to 8.0 percent by the end of 10 years.9

Real money balances at first increase steadily in response to the monetary change, reaching a maximum at the end of two years (Chart 4D). Thereafter, they generally decline towards their new long-run value — about .05 percentage-point lower after a one-percentage-point annual rate of increase in money growth. Within the estimation period, 1966.2-1978.1, our equation follows the general movements in the inflation rate fairly closely (Chart 6). The model makes no adjustment for supply shocks, other than for the episodes of price control and decontrol, which sharply affected short-run inflation estimates. Within the sample period, the standard error is 1.24 percentage points, at an annual rate, or 22 percent of the mean inflation rate of 5.6 percent. Outside that period, from 1978.2-1980.4, the standard error is 1.8 percentage points, at an

Chart 5
Rate of Change in Real GNP

<table>
<thead>
<tr>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
</tr>
<tr>
<td>10.0</td>
</tr>
<tr>
<td>7.5</td>
</tr>
<tr>
<td>5.0</td>
</tr>
<tr>
<td>2.5</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>-2.5</td>
</tr>
<tr>
<td>-5.0</td>
</tr>
<tr>
<td>-7.5</td>
</tr>
<tr>
<td>-10.0</td>
</tr>
</tbody>
</table>


Outside estimation period

Estimated

Actual
annual rate, or 20 percent of the mean inflation rate of 9.0 percent.

**Consistency of the Estimates**

Our estimates of real GNP and inflation appear generally consistent. As long as real money balances increase, the level of real GNP also increases. Once the inflation rate overshoots its long-run value and real money balances begin to decline — that is, around the eighth quarter after the initial monetary change — the level of output begins to decline, falling below its potential around the fifteenth quarter. The rate of inflation then declines around the seventeenth quarter. Finally, both real GNP and inflation virtually complete their adjustments around 10 years after the initial monetary shock.
III. Policy Implications

Monetary policy makers often have tried to counteract the business cycle, with either stimulative or deflationary policies. In this paper, we have concentrated upon the effects of a stimulative policy. For a deflationary policy, the response patterns for real GNP and inflation will be the same, but with the signs reversed. Let us consider such a policy — especially one where the decrease in money growth is at first unanticipated.

Within the first year, inflation declines by about half of the decrease in money growth, and by the end of two years, by 100 percent of that change. Some overshooting then occurs, followed by a gradual and slow adjustment, so that inflation returns to its long-run value within 7 to 10 years. Thus, in the long-run, the permanent decrease in inflation matches the permanent decrease in the money growth rate.

Our results show a more substantial and rapid change in inflation than we are accustomed to expect from standard inflation models. According to conventional models, the initial inflation adjustment to a change in monetary growth takes about five instead of two years to complete. However, these standard models do not estimate the complete adjustment of prices. As a result, they fail to capture a period of overshooting, which is essential if the model is to capture the long-run effect of money on both the rate of inflation and the level of prices.

Recognizing this shortcoming, several authors have recently devised models which incorporate the long-run neutrality of money (as we have done), but with methods which are far more complicated than the Scadding method. Nonetheless, in each case, the initial adjustment period for inflation is much shorter than estimated from the standard model — from six to eight quarters for the U.S. (Giddings), and about five quarters on average for eleven developing countries (Khan). These results suggest that the inflation response has been relatively faster than what has been captured in standard-type estimates.

Our results also indicate that the major contraction in real GNP will be completed within two years following an unanticipated decrease in monetary growth. Within another year or two it will have returned to its initial level. Thereafter, the level of output will overshoot and, tracing a cyclical pattern, will return to its permanent level within about 10 years of the initial monetary change.

Point A in Chart 7 represents the initial position of the economy, where output equals potential and where the inflation rate is stable. In response to a one-percentage point decrease in monetary growth, which is initially not anticipated, in one year’s time inflation decreases by about .5 percentage points and real GNP falls .5 percent below what it otherwise would have been (point 1). By the end of the second year, inflation declines 1.0 percentage points.
and real GNP declines a maximum of about .7 percentage points below potential. Thereafter inflation overshoots its long-run value, while real GNP begins to approach potential after temporarily overshooting that level. Between the third and fourth years, real output has returned to its initial level. Between the seventh and tenth years, both the inflation rate and real GNP virtually attain their permanent values.

The cost of a deflationary policy in terms of lost output occurs within the first four years of the policy change. After that, real GNP overshoots potential and some gain occurs. On balance, we estimate that the short-run loss of real GNP exceeds the cumulative gain — specifically by a net $16 billion when real GNP equals $1540 billion (potential in 1981.1) at point A, in 1972 prices.

George Perry (Brookings Institution) argues that real GNP would have to decline by $33 billion annually for three years to bring inflation down by one percentage point, again in 1972 dollars. Similarly, our results indicate a $27-billion annual loss during the first 15 quarters after the initial shock, but the loss is partially offset and is reduced to $16 billion (as indicated) by the end of the adjustment period.

Separately, we can compare the results of two different policy assumptions — holding money growth constant at 7.5 percent a year — the average of the past three years — or reducing money growth gradually from 6.0 to 3.5 percent over the next half-decade — essentially what the Reagan Administration assumes. The results, shown in Chart 8, portray the consequences of the monetary assumptions but of no other outside shocks.

The policy of constant 7.5-percent money growth should raise real GNP from 1.5 percentage points below potential in 1980 until it equals potential GNP by the end of 1983. Thereafter some cyclical response occurs, producing some over-and undershooting, but not enough to lead to any recession. The inflation rate meanwhile should drop to around 7.0 percent by the end of 1982 and thereafter stay close to its long-run value of 7.3 percent.

In contrast, the policy of gradual reduction of money growth should hold real GNP below potential until mid-1988, and after some overshooting, should help the economy reach (and maintain) potential by 1991. With this approach, inflation should decline from around 6.5 percent by the end of 1982 to around 3.5 percent in 1985, and after some overshooting, should reach its permanent level of 3.5 percent in 1991.

The gradual policy could lead to $115 billion less output than the stable money-growth policy, in 1972 prices — or approximately 6 percent of the average level of potential over the 1980-91 period. Nonetheless, we may be able to obtain a gradual reduction in inflation without having to incur a recession in the process. But this gradual reduction in inflation generally would be associated with only moderate rates of real growth — averaging 3.9 percent in 1984 but thereafter remaining close to its potential rate of 3.2 percent.
In this paper, we consider the output and price effects of a permanent increase in the money-supply growth rate which is at first unanticipated. Initially, inflation steadily increases but by less than the permanent increase in money. During this period, real money balances expand, providing the stimulus for increases in real demand and real GNP.

Yet in the long-run, an increase in money growth apparently does not affect the level or the rate of growth of real GNP, at least to a first approximation. Consequently, the level of real money balances also should remain generally unchanged, since these are held in desired proportion to income. Interest rates could also be a determining factor, however, at least in this country. The desired level of real balances could be somewhat less in the new situation than before the monetary increases occurred, because the rate of interest can be expected to increase with the higher rate of inflation.

Our model of economic behavior is consistent with the behavior of real GNP and inflation just discussed. We have estimated reduced-form equations of the model for both the rate of inflation and the real GNP gap, defined as the level of output as a percent of its potential. For both variables, and also for the rates of change in real GNP and real money balances, our results indicate a cyclical response to a permanent change in money growth. Notably, both inflation and real GNP respond quickly to a change in monetary policy, with the major stimulative or deflationary phase occurring within two years of the initial change. Our findings thus conflict with most of the published literature, which suggests that output and prices require about five years to respond to a change in money growth.

Some analysts suggest that it will take a long time to bring down the inflation rate, and that we risk an economic recession in the process. Our results offer an alternative viewpoint. Changes in monetary growth, at least since the mid-1960’s, apparently have acted fairly rapidly upon inflation — and hence upon aggregate demand as well. Thus, since a monetary contraction is likely to bring inflation down faster than previously anticipated, less of the brunt of that contraction need be borne by real GNP, so that a major decline or loss of real income need not result when we adopt a policy which gradually reduces monetary growth.
Appendix I

John Scadding*

Simple Technique for Imposing Restrictions on Sums of
PDL coefficients

In estimating polynomial distributed lags, researchers typically are not so much interested in the individual coefficients as they are in certain sums of the coefficients. In many problems, for example, considerable importance attaches to whether the total sum of coefficients is unity or not. This appendix is designed to illustrate a simple method for estimating directly the sum of coefficients, or alternately for imposing on it any point restriction. The method illustrated can also be used to impose or estimate more complicated restrictions, and an illustration is given in the example below.

Suppose we take the familiar PDL relationship between money growth (measured as first differences in the log of money) and inflation (measured by first differences in the log of prices):

\[ \Delta \log P_t = \sum_{j=0}^{N} a_j \Delta \log M_{t-j}, \quad a_j = 0, \text{for } j \geq N. \]  

(1)

Interest usually focuses on how inflation adjusts to a permanent change in the rate of monetary growth. The answer to that question is given by the sequence of coefficients

\begin{align*}
w_0 &= a_0 \\
w_1 &= a_0 + a_1 \\
\vdots \\
w_i &= a_0 + a_1 + \ldots + a_i \\
w_N &= \sum_{j=0}^{N} a_j
\end{align*}

Thus \( w_0 \) gives the contemporaneous response of inflation to a one-percentage-point increase in money growth, \( w_1 \) measures how much higher inflation will be in the next period (compared to the rate before monetary expansion increased), and so on. The last coefficient, \( w_N \), measures the steady-state response of inflation to an increase in the rate of monetary growth. It is usual to inquire whether this long-run response is unity — i.e., whether ultimately changes in the rate of monetary expansion are fully reflected in the rate of inflation. The usual way to answer this question is to estimate (1) and sum the estimated \( a \)'s. An alternative is to rearrange (1) in such a way that \( w_N \) can be estimated directly. To do that, we integrate (1) by parts to obtain

\[ \Delta \log P_t = \sum_{j=0}^{N-1} w_j \Delta^2 \log M_{t-j} + w_N \Delta \log M_{t-N}, \]  

(2)

where \( \Delta^2 \) denotes second differences. Adding and subtracting \( w_N \) from each of the terms in the first summation yields

\begin{align*}
\log P_t &= \sum_{j=0}^{N-1} (w_j - w_N) \Delta^2 \log M_{t-j} \\
&\quad + w_N \Delta \log M_{t-N} + w_N \sum_{j=0}^{N-1} \Delta^2 \log M_{t-j} \\
\Delta \log P_t &= \sum_{j=0}^{N-1} (w_j - w_N) \Delta^2 \log M_{t-j} + w_N \Delta \log M_i \\
&\quad + \sum_{j=0}^{N-1} \Delta^2 \log M_{t-j} + w_N \Delta \log M_i \\
&= \sum_{j=0}^{N-1} \Delta^2 \log M_{t-j} + w_N \Delta \log M_i \\
w &\equiv w_j - w_N
\end{align*}

*Senior Economist, Federal Reserve Bank of San Francisco.
Thus it is possible to rewrite the distributed lag as another distributed lag in second differences of log $M$, plus a term in the contemporaneous growth rate of money whose coefficient is the sum of distributed-lag coefficients. Hence $w_N$ can be directly estimated; alternatively any restriction on $w_N$ can be imposed simply by taking the last term in (3) over to the left-hand side of the question.

To illustrate how the method can be used to impose more complicated restrictions, consider the question of whether a change in the rate of monetary expansion permanently affects the level of real money balances. Sufficient conditions that the level of real balances be unchanged in the new steady state (i.e. that the long-run elasticity of the level of prices with respect to money is unity) are:

$$w_N = 1$$

$$\sum_{j=0}^{N-1} \bar{w}_j = 0$$

Define the following coefficients:

$$v_0 = \bar{w}_0$$

$$v_1 = \bar{w}_0 + \bar{w}_1$$

$$\ldots$$

$$v_i = \bar{w}_0 + \bar{w}_1 + \ldots + \bar{w}_i$$

$$v_N = \sum_{j=0}^{N-1} \bar{w}_j$$

Next, integrate (3) by parts again to obtain

$$\Delta \log P_t = \sum_{j=0}^{N-2} (v_j - v_N) \Delta^3 \log M_{t-j} + w_N \Delta \log M_t$$

$$= \sum_{j=0}^{N-2} v_j \Delta^3 \log M_{t-j} + w_N \Delta \log M_t$$

$$+ v_N \Delta^2 \log M_t; \bar{v}_j \equiv v_j - v_N$$

Hence it is possible to rewrite the distributed lag as yet another distributed lag, this time in third differences of log $M$ and two other terms — a term in contemporaneous money growth, and a term in first differences of money growth. The coefficients on these two variables provide estimates respectively of the long-run elasticity of the inflation rate ($w_N$) and of the price level ($v_N$) with respect to money. Alternatively, equation (5) allows assumptions about either or both of these elasticities to be easily imposed.
Appendix 2
Model of Real Output and Inflation

The structure of the model is concerned directly with behavior in the goods, money and labor markets. Each market is characterized by simplified and highly aggregative relationships, and labor is the only factor market directly considered. The terms long-run, steady-state and potential are used interchangeably.

List of Variables
S = Total real savings in the national-income accounts;
I = Total real investment in the national-income accounts;
G = Federal government real high-employment surplus, measured as the ratio of revenues to expenditures;
Y = Real GNP,
YP = Real potential GNP;
Y/YP = Real GNP gap;
R = Nominal rate of interest;
M = Money supply;
P = GNP implicit price deflator;
U = Unemployment rate;
N = Natural rate of unemployment;
U/UN = Unemployment gap;
P* = Expected GNP implicit price deflator.

With the exception of R, the nominal rate of interest, each of the variables is measured in terms of its natural logarithm. Therefore, the change in real GNP, dY, is a measure of the rate of change in real GNP, and dP is a measure of the inflation rate. The variables G, YP, UN and M are exogenous variables.

Goods Market
\[
\text{d}S_t = F(L) \text{d}Y_t + a_2 \text{d}G_t \tag{1}
\]
\[
\text{d}I_t = a_3 (\text{d}R_t - (\text{d}P_{t+1}^* - \text{d}P_t^*)) \tag{2}
\]
\[
\text{d}S_t = \text{d}I_t \tag{3}
\]

Equation (1) indicates that the change in total savings depends upon a distributed lag in current and past changes in real income, F(L) dY, and the change in the high-employment Federal-government budget surplus, dG. The expression F(L) is a polynomial so that F(L) dY represents a polynomial distributed lag of the variable dY, and F(L) dY = f_0 dY_t + f_1 dY_{t-1} + f_2 dY_{t-2} + \ldots +. Equation (2) relates changes in real investment expenditures to changes in the real rate of interest, which in turn is represented by the change in the nominal rate of interest, R, minus the change in the expected rate of inflation. Equation (3) expresses equilibrium in the goods market.

From equations (1) - (3) we derive the IS function which expresses the equilibrium conditions in the goods market.

\[
\text{d}Y_t = - B(L) \text{d}Y_{t-1} - a_4 (\text{d}R_t - (\text{d}P_{t+1}^* - \text{d}P_t^*)) - a_5 \text{d}G_t \tag{3a}
\]

where the weights of the polynomial, B(L), and the coefficients, a_4 and a_5, are combinations of the coefficients in the structural equations (1) - (3).

Money Market
\[
\text{d}M_t^d = \text{d}P_t + a_6 \text{d}Y_t - a_7 \text{d}R_t \tag{4}
\]
\[
\text{d}M_t^d = \text{d}M_t \tag{5}
\]

Equation (4) states that the demand for real money balances (dM_t^d - dP_t) is positively related to current income and declines with increases in nominal interest rates. Equation (5) states the equilibrium conditions in the money market, that nominal money demanded equals nominal money supplied in any given period. From equations (4) and (5), we derive the LM function, which expresses the equilibrium conditions in this market.

\[
\text{d}M_t = \text{d}P_t + a_6 \text{d}Y_t - a_7 \text{d}R_t \tag{5a}
\]
where the E operator signifies expectations conditional on information available as of the end of period \((t-1)\). This hypothesis regarding the formation of expectations has become popularly known as “rational expectations.”

Unemployment is linked to levels of output via the production function, which is represented in the model as a type of Okun’s Law equation, equation (7).

The specification of inflation expectations completes the model. Our hypothesis, following John Muth’s proposal, states that expectations are informed predictions of future events, based on the available information and the relevant economic theory at the time the expectations are formed. This implies that expected inflation can be represented by the conditional mathematical expectation of inflation, \(dP_t\), based on the economic model and all the information assumed to be available as of the end of period \((t-1)\). Inflation expectations may then be represented with the following equations:

\[
\begin{align*}
    dP^*_t &= E_{t-1}dP_t \\
    dP^*_{t+1} &= E_{t-1}dP_{t+1}
\end{align*}
\]

where the E operator signifies expectations conditional on information available as of the end of period \((t-1)\). This hypothesis regarding the formation of expectations has become popularly known as “rational expectations.”

### Appendix 3

#### Formation of Inflation Expectations

Before the final equations for the inflation rate and real GNP can be derived from the model, we must express the functional forms for expected inflation, \(E_{t-1}dP_t\) and \(E_{t-1}dP_{t+1}\), equations (8) and (9) of the model. These will then be substituted into the aggregate demand and supply equations, which in turn can be solved for the final equations for the inflation rate and real GNP.

To obtain the specifications for expected inflation, \(E_{t-1}dP_t\) and \(E_{t-1}dP_{t+1}\), we follow a procedure suggested by Sargent and Wallace (1975). The first step is to obtain the solution of the system of equations (8)-(12) for the inflation rate, \(dP_t\). The expectations operator, \(E_{t-1}\), is then applied recursively to that equation to yield the equations for inflation expectations.

This procedure results in the following equations. First, the solution for the inflation rate is,

\[
dP_t = w_1E_{t-1}dP_t + w_2E_{t-1}dP_{t+1} + w_3(Y/YP)_{t-1} + \sum_{i=2}^{N} g_i(Y/YP)_{t,i} + w_4dM_t - w_5dG_t + c_0
\]
where the coefficients are functions of the parameters of the equations (1)-(9).

Applying the E operator to equation 13, we obtain,

\[ E_{t-1}P_t = w_1 E_{t-1}dP_t + w_2 E_{t-1}dP_{t+1} \]
\[ + w_3 (Y/YP)_{t-1} - \sum_{i=2}^{N} q_i (Y/YP)_{t-i} \]
\[ + w_4 E_{t-1}dM_t - w_5 E_{t-1}dG_t + c_0 \]

\[ E_{t-1}P_{t+1} = w_1 E_{t-1}dP_{t+1} \]
\[ + w_2 E_{t-1}dP_{t+2} + w_3 (Y/YP)_{t-1} \]
\[ - \sum_{i=1}^{N} q_i (Y/YP)_{t-i} + w_4 E_{t-1}dM_{t+1} \]
\[ w_5 E_{t-1}dG_{t+1} + c_0 \]

Applying equations (14) and (15) recursively and then gathering terms yields the solution for expected inflation,

\[ E_{t-1}dP_t = \sum_{i=0}^{\infty} v_i E_{t-1}dM_{i+1} + \sum_{i=0}^{\infty} v_i (Y/YP)_{t-i} \]
\[ - \sum_{i=0}^{\infty} v_i \sum_{j=0}^{i-1} \frac{q_i}{1-w_j} (Y/YP)_{t-i} \]
\[ - v_5 \sum_{i=0}^{\infty} v_i dG_{i+1} + \sum_{i=0}^{\infty} v_i k_i \]

where the coefficients are combinations of the coefficients in equation (13).

In equations (16) and (17), expected inflation for time t and time t+1 formed at time t-1 will depend upon the predictions formed at time t-1 for the exogenous variables, dM and dG, for all future periods. Particularly relevant, then, is the process or rule by which the public forms expectations of these variables. There are a number of alternatives for specifying such rules or processes. For instance, we may postulate a model which relates expected future monetary growth and fiscal policy to a set of predetermined variables relative to the model, or we may choose to postulate an ARIMA process for each variable. In the latter case, only past values of a variable are used to predict future values of the same variable. For now, we assume that the public forecasts future values of changes in M and G by considering the past history of these variables.

In addition, we assume that expectations regarding future policy variables are updated each period, as new information becomes available, in accordance with the theorem of optimal least-squares learning and expectations formation stated by Benjamin Friedman. This updating process is fully optimal in the sense of meeting the information-exploitation assumptions of Muth's rational-expectations hypothesis. We may write the expectation of future variables as a polynomial distributed lag

\[ *w_i = Z - (a_{12}/a_{10}) \]
\[ w_2 = (a_{12}Z)/a_{10} \]
\[ w_3 = (1 - (Z/a_{10})(j_0 + c_1b_0)) \]
\[ w_4 = (a_{11}Z)/a_{10} \]
\[ w_5 = (a_{13}Z)/a_{10} \]
\[ Z = (a_2 + a_a + a_6) / ((a_7 + a_a a_6) + a_4) \]
\[ q_i = (Z/a_{10})(j_i + c_1(b_0 - b_1)), i=1,2,\ldots \]
\[ c_0 = (r_0Z)/a_{10} \]

\[ **v = w_2/(1 - w_i) \]
\[ v_2 = w_4/(1 - w_i) \]
\[ v_3 = w_5/(1 - w_i) \]
\[ v_5 = w_7/(1 - w_i) \]
\[ k_1 = c_0/(1 - w_i) \]
of past values of that variable. Expected future money growth is then specified as follows:

\[ E_{t-1} dM_t = F_0(L) dM_t \]
\[ E_{t-1} dM_{t+1} = F_1(L) dM_{t+1} \]

or, in general

\[ E_{t-1} dM_{t+j} = F_j(L) dM_{t+j} \]

where \( F_j(L) \) indicates a polynomial in the lag operator, and \( j = 0, 1, 2, \ldots \). Similar specifications can be written for the fiscal variable, \( dG \).

These specifications for expected monetary growth and fiscal policy may next be substituted into the expected inflation equations (6) and (7). Expected inflation thus depends upon linear combinations of past monetary growth and past fiscal-budget changes. We may gather terms and write the expected inflation equations as polynomials in past exogenous variables and the lagged endogenous variables, as is done in the following equations.

Equations (16.1) and (17.1) represent the final equations for expected inflation. The model now may be represented by the equations shown in Table 1 of the text and quotations 16.1 and 17.1.

FOOTNOTES

1. For similar models, see Thomas J. Sargent and Neil Wallace and Roque B. Fernandez.
2. For a discussion of rational expectations, see Benjamin Friedman.
5. See Sargent and Wallace for an early discussion of this point.
6. This is shown in Appendix 1.
7. From 1960.4 - 1980.1, the money-demand equation estimated in first-difference form by Adrian Throop, (Federal Reserve Bank of San Francisco) is:

\[
\ln \frac{M1B_t}{P_t} = .00133 + .585 \ln \frac{M1B_{t-1}}{P_{t-1}} \\
(1.60) 
(6.29)
- .00965 \ln RTB_t \\
(1.73)
- .00654 \ln RCBPASS_t \\
(1.03)
+ .264 \ln (Y/P_t) \\
(3.72)
\]

\( R^2 = .576 \)
\( D.W. = 2.00 \)
\( S.E. = .00490 \)

The variables are defined as:

\( RTB \) = rate on three-month Treasury bills, annual effective yield;

\( RCBPASS \) = rate on commercial-bank passbook deposits, annual effective yield;

\( Y \) = nominal GNP;

\( P \) = GNP price deflator

The long-run elasticity of money demand with respect to the treasury-bill rate is \(-.0232\). According to that estimate, and assuming that nominal interest rates in the long-run fully reflect a change in the money growth rate, the expected change in real money balances with respect to a change in the level of interest rates would be .34. Our estimate is .21, which is not significantly different from .34 at the 10-percent level of significance. On the other hand, Throop’s estimate appears lower than Goldfeld’s estimate of \(-.048\) for the long-run elasticity of money demand with respect to the commercial-paper rate. This implies a .8 elasticity of real money balances with respect to a change in the level of short-term rates, and indicates that our estimate may be on the low side, given Goldfeld’s estimate.

8. These estimates were obtained from dynamic simulations in each sample period, in which only the initial lagged-dependent variable is the actual value, and actual money and Federal expenditures appear as the right-hand side of the equation.

9. Our estimates of the inflation multipliers shown in Table 3 and Chart 4C reflect what I shall call the “full effect” of a permanent change of 1.0 percentage point in the money growth rate upon the inflation rate. By “full effect” I mean that two effects are considered in deriving those multipliers: (1) the “direct” effect of
changes in money on the inflation rate, and (2), the
indirect effects of money as they work their way
through changes in the GNP gap (since lagged gap
values also appear in the inflation equation). We
derived these multipliers from dynamic simulation of
both the GNP and price equations. If we do not take
into consideration the indirect effects, we may easily
obtain estimates for the inflation multiplier and the
standard errors associated with those multipliers, as
shown below. The reader will notice that the multi­
pliers are slightly different from those reported in Table
3 in the text. For example, the initial adjustment period
is moved up by one-to-two quarters to the seventh
quarter following the initial change in monetary
growth when the indirect effect is not considered.
Also, in the latter instance, the total length of the lag is
22 quarters rather than about 10 years. When the
indirect effects are considered, then, they add a con­
siderable period of time in which slow adjustment
continues. Importantly, the small size of the standard
errors indicates that the estimated multipliers are
statistically significant, and that the overshooting pro­
erty of inflation is highly significant.

<table>
<thead>
<tr>
<th>Period</th>
<th>Inflation Multiplier</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-.019</td>
<td>.098</td>
</tr>
<tr>
<td>1</td>
<td>.008</td>
<td>.098</td>
</tr>
<tr>
<td>2</td>
<td>.224</td>
<td>.078</td>
</tr>
<tr>
<td>3</td>
<td>.424</td>
<td>.062</td>
</tr>
<tr>
<td>4</td>
<td>.605</td>
<td>.050</td>
</tr>
<tr>
<td>5</td>
<td>.768</td>
<td>.043</td>
</tr>
<tr>
<td>6</td>
<td>.914</td>
<td>.041</td>
</tr>
<tr>
<td>7</td>
<td>1.042</td>
<td>.042</td>
</tr>
<tr>
<td>8</td>
<td>1.152</td>
<td>.044</td>
</tr>
<tr>
<td>9</td>
<td>1.244</td>
<td>.046</td>
</tr>
<tr>
<td>10</td>
<td>1.319</td>
<td>.048</td>
</tr>
<tr>
<td>11</td>
<td>1.376</td>
<td>.048</td>
</tr>
<tr>
<td>12</td>
<td>1.415</td>
<td>.048</td>
</tr>
<tr>
<td>13</td>
<td>1.437</td>
<td>.048</td>
</tr>
<tr>
<td>14</td>
<td>1.440</td>
<td>.045</td>
</tr>
<tr>
<td>15</td>
<td>1.426</td>
<td>.045</td>
</tr>
<tr>
<td>16</td>
<td>1.394</td>
<td>.048</td>
</tr>
<tr>
<td>17</td>
<td>1.345</td>
<td>.055</td>
</tr>
<tr>
<td>18</td>
<td>1.277</td>
<td>.066</td>
</tr>
<tr>
<td>19</td>
<td>1.192</td>
<td>.082</td>
</tr>
<tr>
<td>20</td>
<td>1.089</td>
<td>.102</td>
</tr>
<tr>
<td>21</td>
<td>1.127</td>
<td>.105</td>
</tr>
</tbody>
</table>

10. See Gittings and Khan.

REFERENCES

Carlson, Keith M. “Money, Inflation, and Economic
Growth: Some Updated Reduced Form Results
and Their Implications,” Federal Reserve Bank

Fernandez, Roque B. “An Empirical Inquiry on the
Short-run Dynamics of Output and Prices,”
American Economic Review, (September 1977),
pp. 595-609.

Friedman, Benjamin M. “Optional Expectations and
the Extreme Information Assumptions of
‘Rational Expectations, Macromodels,” Journal
of Monetary Economics, 5(1979), 23-41.

Friedman, Milton. The Optimum Quantity of Money,
and Other Essays. Chicago, 1969.

Gittings, Thomas A., “A Linear Model of the Long-Run
Neutrality of Money,” Staff Memorandum,

Khan, Mohsin S., “Monetary Shocks and the
Dynamics of Inflation,” International Monetary
Fund Staff Papers (June 1980), pp. 250-284.

Laidler, David. “Money and Money Income: An Essay
on the ‘Transmission Mechanism,’” Journal of

Lucas, R.E., “Some International Evidence on Output,
Inflation Tradeoffs,” American Economic

Perry, George L. “Inflation in Theory and Practice,”
Brookings Papers on Economic Activity, 1,
(1980).

Sargent, Thomas J. and Neil Wallace. “Rational
Expectations, the Optimal Monetary Instrument,
and Optimal Money Supply Rule,” Journal of Po­
litical Economy, (April 1975), 241-254.