The Decline in Currency Use at a National Retail Chain

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The composition of US retail payments is changing rapidly. According to the Federal Reserve's triennial Payments Study (2013, 2016), from 2012 to 2015 the value of debit and credit card payments increased at annual rates of 7.1 percent and 7.4 percent, respectively. Over this same period, nominal GDP rose at less than a 4 percent annual rate, which suggests that the increase in card payments came at the expense of some other form(s) of payments, the obvious candidates being checks and cash. The value of check payments did fall over this period, but it is possible that the fall in check payments was offset by an increase in ACH rather than card payments; ACH tends to be used in business and financial transactions while cards are used in consumer payments. The Payments Study covers only noncash payments, but Wang and Wolman (2016a) provide direct evidence about cash use at a large discount retailer, finding that the cash share of the number of payments fell by 2.46 percentage points per year from 2010 to 2013. In their study, an increase in card use was almost the mirror image of a decrease in cash use.

At least four sets of factors could be contributing to the apparent shift from cash to card in retail payments. First, Wang and Wolman (2016a) documented a negative relationship between transaction size and the share of cash transactions; thus, some of the decline in observed cash shares could be due to an increase in average transaction size. Second, Wang and Wolman also documented systematic relationships between the cash share of payments in a location and the

demographic and economic characteristics of the location; over time, changes in those characteristics may explain changes in the cash share. Third, changes in technology may be reducing the cost and increasing the availability and security of debit and credit cards. And fourth, consumers' perceptions of cards may be improving slowly, generating a gradual expansion in card use. This paper brings new evidence to bear on the contributions of the first two factors to the decline in cash payments. Using an updated version of the data from Wang and Wolman (2016a,b), we study the association between changes in payment shares and changes in the size of transactions as well as changes in location-specific economic and demographic variables over the period from February 2011 to February 2015. While we cannot distinguish the third and fourth factors listed above, the portion of the decline in cash shares that is unexplained by our analysis represents the sum of these two sets of factors.

There are important public policy questions for which it matters what explains the decrease in cash use. Cash remains an important means of payment in the United States, and in the wake of the long recent experience with interest rates at their effective lower bound, some economists have advocated policies that would reduce or even eliminate the availability of paper currency (Rogoff 2016). Without paper currency, the argument goes, monetary policy would no longer be constrained by a lower bound on nominal interest rates.[1] Against this, the benefits of cash must be considered, and the accounting we provide for the decline in cash use can contribute to the debate over the benefits of cash. To the extent that the decline in cash use is accounted for by changing demographics or changing transaction size, there may be greater scope for concern about the effects of a (hypothetical) elimination of currency on particular segments of society.

In Wang and Wolman (2016a), and in this paper, we analyze transactions data from a discount retailer with thousands of stores across the US. In the earlier paper, we combined the transactions data with fixed demographic data and other data across locations.[2] With almost two million transactions every day, we were able to precisely characterize the daily and weekly patterns of payment use. And, with thousands of zip-code locations, we were also able to precisely estimate the relationships between cash shares and location-specific variables. However, the fact that our data covered only three years meant that we could not

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[1] Rogoff (2016) also sees benefits from eliminating cash related to the fact that cash is heavily used in the underground economy.

[2] In Wang and Wolman (2016b), we conducted a similar analysis that concentrated on retail outlets in the Fifth Federal Reserve District.

incorporate time variation in the location-specific data: the Census Bureau's American Community Survey (ACS) data were not available at the zip-code level for more than one year in our dataset. In the current paper, we do not attempt to capture the daily variation in payment shares but instead focus on the "medium-term" shift in the cash share of transactions from February 2011 to February 2015, using only data from those two months. While we sacrifice on one dimension, we are able to incorporate time variation in the location-specific data using the five-year ACS estimates at the zip-code level for 2011 and 2015.

On average, across the stores in our study, the share of cash transactions fell by 8.6 percentage points from February 2011 to February 2015. Our statistical model attributes approximately 1.3 percentage points of that decline to increasing transaction sizes. Changes in demographic and other location-specific variables contribute between 0.5 and 1.3 percentage points, so our analysis attributes approximately three-quarters of the decline in cash use to a pure time effect, which stands in for the third and fourth factors listed above, and any other factors omitted from our analysis.

1. TRANSACTIONS DATA: THE DECLINE IN CURRENCY USE

Our payments data come from a US retail chain selling a wide variety of goods, with a majority of its revenue accounted for by household consumables such as food and health-and-beauty aides. The chain has thousands of stores and is located in most states. Although there is not a specific geographic focus, the stores tend to be located in relatively low-income zip codes.[3] While the raw data are at the level of individual transactions (time and location, size, means of payment), our analysis uses aggregated data: for each zip code, we compare the shares of transactions in each of the four main payment types (cash, debit card, credit card, and check) in February 2011 to the corresponding shares in February 2015. One month is a long enough time period to get a relatively large number of transactions: most zip codes had more than 7,000 transactions in each of the two months. The total number of zip-code locations is more than 5,000. We chose February 2011 and February 2015 to balance two considerations. A longer time span provides a better sense of the trend decrease in cash use, but we needed

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[3] See Wang and Wolman (2016a) for some additional information. Our use of the data is governed by a confidentiality agreement that limits the degree of detail we may disclose.

to choose years for which zip-code-level data are available from the ACS.

Figure 1 is a scatterplot of the share of cash transactions in each zip code in 2015 and 2011, on the y- and x-axes respectively. The solid grey line is the locus of points for which the cash share is equal in the two years, and points below (above) the line indicate a decrease (increase) in the cash share. This figure provides a nice overview of the data and the properties we want to study. First, there is significant variation in the share of cash transactions in both years. Second, the share of cash transactions declined from 2011 to 2015 in almost every zip code, as indicated by the small number of observations that lie above the y=x line. And third, while the decrease in the cash share does not seem closely related to the level of the cash share, the decrease is also not constant across zip codes. The first and third properties—cross-zip-code variation in both the level and change in the cash share—provide motivation for using demographic and other zip-code-level variables in our statistical analysis. The second property—a significant common component in the change in the cash share across zip codes—could partly reflect changes in demographics that are common across locations. However, the common component also reflects changes in payments technology and consumer perceptions that are not captured by our analysis.

Table 1 displays summary statistics for the data in Figure 1, as well as the corresponding data for shares of debit, credit, and check transactions. From February 2011 to February 2015, the average cash share of transactions across zip codes declined from 78.2 percent to 69.5 percent, or 2.18 percentage points per year. Our focus is primarily on the decline in cash and the combined increase in credit and debit use; the total card share of transactions increased by an average of 2.3 percentage points per year, with the difference, 0.12 percentage points per year, accounted for by a decrease in the share of transactions conducted with checks. Our data are not well-suited to distinguishing credit and debit transactions because the category we call "debit" includes only PIN debit transactions—signature debit and most prepaid cards are included in "credit."[4] PIN debit transactions increased by an average of 1.63 percentage points per year, approximately 70 percent of the overall increase in card use.

Table 1 also shows that from 2011 to 2015 both the standard deviation of cash transaction shares and the interquartile range (difference between the 75th and 25th percentiles) increased. This corresponds to

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[4] PIN debit is a debit card transaction that requires the consumer to enter a PIN number, whereas signature debit is a debit card transaction that requires the consumer to sign their name (like a credit card transaction).

Figure 1 Zip-Code-Level Cash Shares for 2015 and 2011
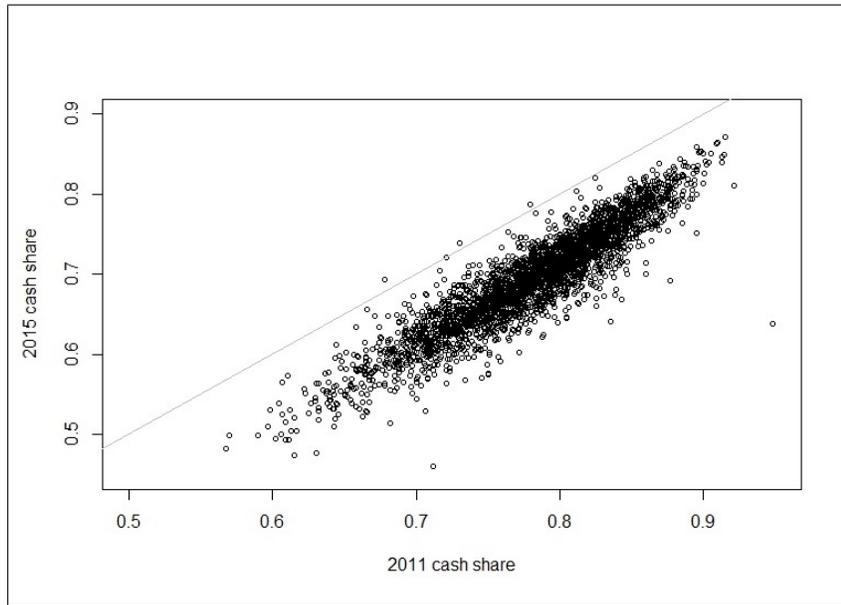
the third property noted in reference to Figure 1: the distribution of cash shares across zip codes did not shift down in a uniform manner. Figure 2 illustrates this explicitly, showing that the histogram of cash shares across zip codes was more spread out in 2015 than in 2011, in addition to shifting to the left.

Dispersion across locations in the *change* in cash shares is illustrated in the third row of Table 1 and in Figure 3. Cash shares declined by an average of 8.6 percentage points, but there is significant dispersion: in 25 percent of zip codes, the cash share decreased by at least 9.9 percentage points, and in 25 percent of zip codes the cash share decreased by less than 7.0 percentage points.

As mentioned in the introduction, one factor that could help account for the changes in cash shares depicted in Figures 1 through 3 is a change in the distribution of transaction sizes. Our econometric analysis of the change in cash shares below will explicitly take into account transaction size, but for now we simply report on the distributions of median transaction size and change in median transaction size by location. Table 2 provides various statistics for the distributions: for example, the mean value of median transaction size rose from $7.26 to $7.96, and the mean change in median transaction size is $0.70.

**Table 1 Payment Shares Across Zip Codes, February 2011
vs. February 2015**

|	Mean	Std. dev.	1%	25%	50%	75%	99%
Cash: 2011	0.782	0.056	0.636	0.747	0.787	0.822	0.891
2015	0.695	0.063	0.532	0.653	0.699	0.740	0.824
change	-0.086	0.025	-0.150	-0.099	-0.085	-0.070	-0.031
Debit: 2011	0.161	0.050	0.062	0.127	0.156	0.192	0.292
2015	0.226	0.058	0.095	0.187	0.222	0.261	0.380
change	0.064	0.028	-0.016	0.049	0.065	0.081	0.128
Credit: 2011	0.047	0.034	0.008	0.024	0.036	0.059	0.171
2015	0.074	0.049	0.015	0.039	0.060	0.096	0.246
change	0.027	0.029	-0.017	0.009	0.019	0.039	0.121
Check: 2011	0.010	0.011	0.000	0.002	0.006	0.014	0.051
2015	0.005	0.006	0.000	0.001	0.003	0.007	0.026
change	-0.006	0.006	-0.027	-0.008	-0.004	-0.001	0.001

Note: Rows titled "change" show distributions of changes in payment shares from
2011 to 2015. These may show different means than the change in the mean share
for a particular payment type because the set of stores is not identical in the two
years (e.g., for cash, change in mean is 0.087 and mean change is 0.086).

**Table 2 Median Size of Transactions Across Zip Codes,
February 2011 vs. February 2015**

	Mean	Std. dev.	1%	25%	50%	75%	99%
2011	7.26	1.02	5.35	6.56	7.15	7.81	10.12
2015	7.96	1.10	5.87	7.20	7.88	8.66	10.90
Change	0.70	0.78	-1.28	0.27	0.67	1.12	2.52

Note: The third row is the distribution of change in median transaction size from
2011 to 2015.

Figures 4 and 5 display histograms of the two distributions of median
transaction size (Figure 4) and the distribution of changes in median
transaction size (Figure 5). The distribution of transaction sizes shifted
to the right from 2011 to 2015 and became slightly more spread out.
The dispersion in changes in median transaction size (Figure 5) is in-
deed consistent with the behavior of transaction size accounting for
some of the shift in the cash share distribution from 2011 to 2015.

Figure 2 Histograms of Zip-Code-Level Cash Share
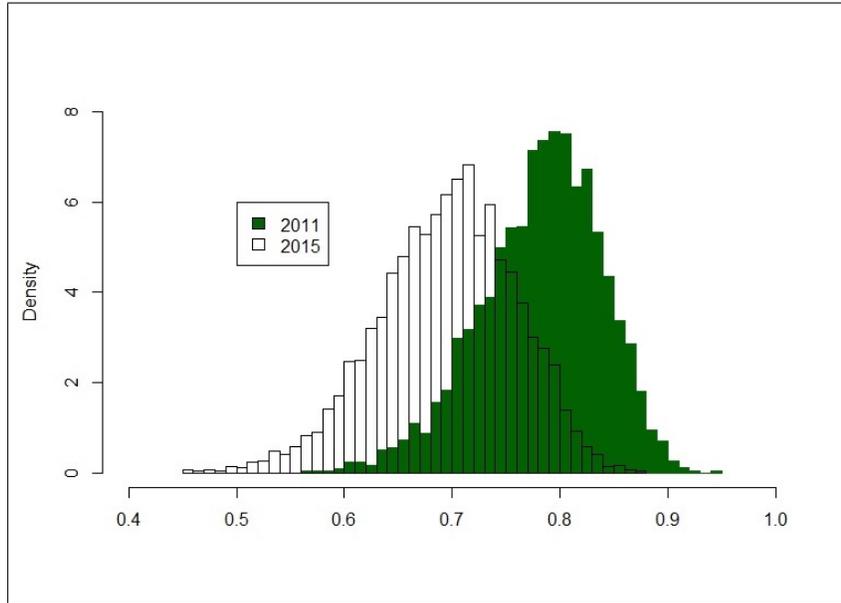
2.   LOCATION-SPECIFIC DATA

Table 3 provides summary statistics for the location-specific data used
in our analysis, comparing the 2011 and 2015 values. Wang and Wol-
man (2016a) provide a discussion of why one would expect these vari-
ables to be relevant for explaining payment choice, arguing that each
consumer has a threshold transaction size below which they will use
cash and above which they will use a noncash form of payment. The
threshold may vary over the week, month, and year, and it will likely be
related to the consumer's financial situation, their demographic char-
acteristics, and their surrounding environment (including banking op-
tions, population density, and crime rates). The overall cash share in
a particular location at a particular time will thus depend on the char-
acteristics of the consumers in that location, the characteristics of the
location, and the size distribution of transactions.

In Wang and Wolman (2016a), we used the same demographic vari-
ables to account for variation in cash shares across locations, but our
data did not allow for the possibility of using changes in those variables
to account for the change over time in cash shares; the location-specific
variables were necessarily treated as fixed over the three-year sample
of data due to limitations of the Census Bureau data. Here, the longer

Figure 3 Histogram of Change in Zip-Code-Level Cash Share
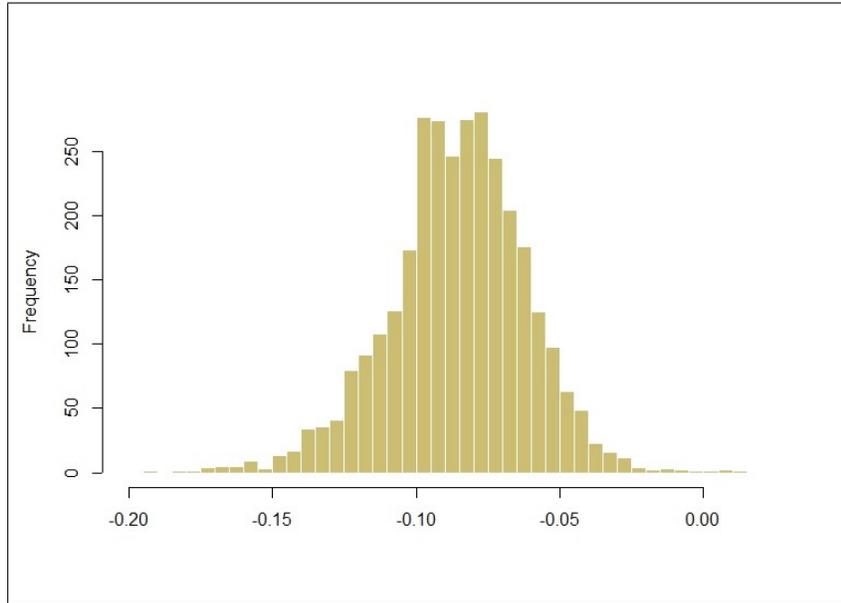
span of the transactions data means we can incorporate distinct de-
mographic data for 2011 and 2015 for each zip code to decompose the
changes in cash shares. Our earlier paper used forecasted nationwide
changes in the location-specific variables to project *future* changes in
cash shares and attributed up to 15 percent of the overall projected
decline in cash shares to forecasted changes in location-specific vari-
ables. Below, we will compare that number to our decomposition of
actual changes in cash shares.

The demographic variables (sex, age, race, and education) and the
housing variables in Table 3 are all from the ACS. We use ACS five-year
estimates at the zip-code level for 2011 and 2015. Note that for age we
report only the 2011 data. We fix the age data at 2011 levels because
we think that cohort is more important than age for payment behav-
ior.[5] The banking variables—market concentration, as measured by the

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[5] In principle, we would like to use data on the distribution of cohorts in each
year. However, because the age data in our regression are in relatively large bins (e.g.,
fifteen years), it will not provide an acccurate picture of how the cohort distribution
changes across the four-year span of our data. In Section 4, we will use the estimated
coefficients together with more detailed age data to construct a rough measure of the
cohort contribution to the change in cash shares.

Figure 4 Histogram of Zip-Code-Level Median Transaction Size
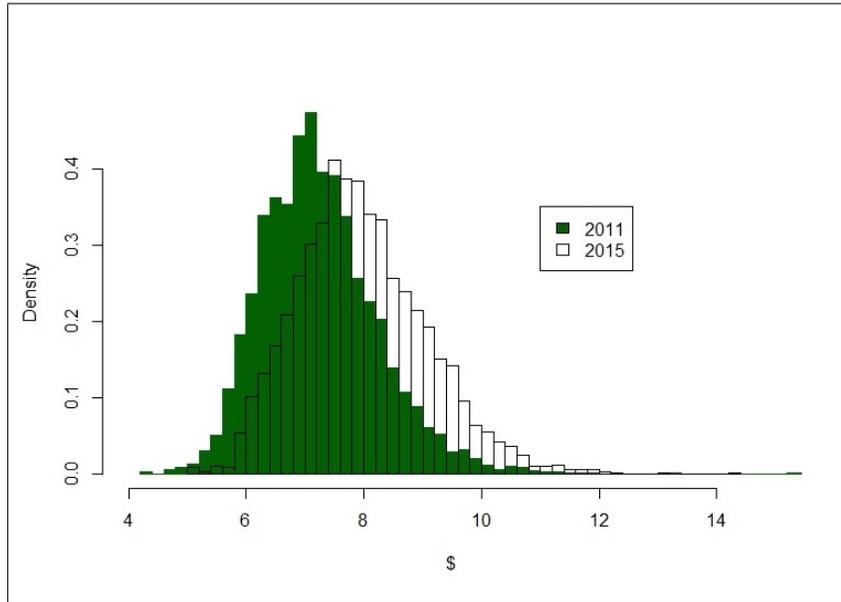
Herfindahl-Hirschman index (HHI), and the number of bank branches per capita—are from the FDIC's Summary of Deposits. Banking HHI is calculated by squaring each bank's share of deposits in a zip code and then summing these squared shares. We allow the HHI effect to differ between rural and urban areas because of the possibility that high concentration in an urban area may reflect the presence of a small number of high-productivity banks. The robbery rate is from the FBI's uniform crime report (note that the robbery rate is at the county level). In most cases, the changes from 2011 to 2015 appear to be small.[6] However, the examples of median household income and education show that changes in location-specific variables have the potential to account for some of the decline in cash use. Across locations, Wang and Wolman (2016a) found that higher educational attainment and higher income were associated with lower cash use; Table 3 shows that both educa-

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[6] One exception is the HHI index. Note that in our earlier work the HHI was measured at the level of metropolitan statistical area (MSA) or rural county. Here it is measured at the zip-code level. In Wang and Wolman (2016b), we found that variation in HHI explained little of the variation in payment shares across zip codes.

**Figure 5 Histogram of Change in Zip-Code-Level Median
                Transaction Size**
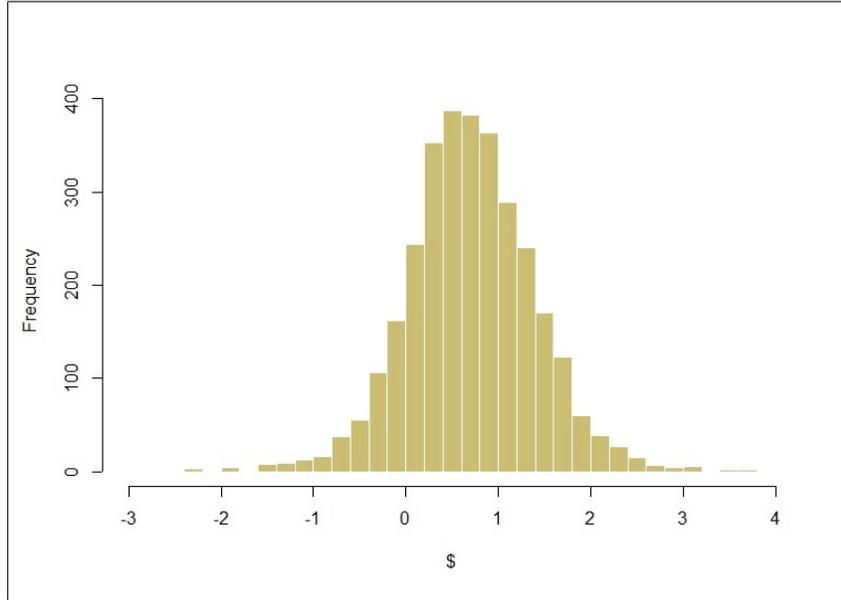
tional attainment and income increased on average from 2011 to 2015, which would be consistent with a decrease in cash use assuming the relationship found by Wang and Wolman also holds across time. In the next section, we will report estimates of a statistical model similar to that in our 2016a paper using the variables in Table 3. Then in Section 4, we will quantify the contributions of changes in transaction size and in the demographic variables to the decline in cash use.

3.   EMPIRICAL FRAMEWORK AND ESTIMATES

In this section, we describe the statistical model used to analyze payment shares and provide a summary of the estimates. The statistical model is tailored to the properties of the variable we are seeking to explain: in a particular time period in a particular location, the shares of cash and other payment types are each between zero and one, and they must sum to one. These properties mean that linear regression is not appropriate.

Table 3 Summary Statistics of Zip-Code Variables

Variable (unit)	Mean		Std. dev.	
	2011	**2015**	**2011**	**2015**
Banking HHI	0.43	0.46	0.26	0.26
Banking HHI $\times$ Metro	0.28	0.29	0.29	0.30
Branches per capita $(1/10^3)$	0.38	0.36	0.36	0.32
Robbery rate $(1/10^5)$	13.17	12.34	28.477	26.02
Median household income ($)	43,221	43,818	12,289	12,621
Population density (per mile^2)	1479	1484	2614	2493
Family households (%)	66.50	65.52	8.65	8.85
Housing: Renter-occupied (%)	28.18	30.14	11.21	11.79
Owner-occupied	57.33	55.28	12.86	12.77
Vacant	14.49	14.58	8.59	8.63
Female (%)	50.87	50.74	2.87	2.92
Age: < 15 (%)	20.03	-	4.08	-
15-34	26.65	-	5.88	-
35-54	27.36	-	3.28	-
55-69	16.16	-	3.77	-
$\geq$ 70	9.81	-	3.81	-
Race: white (%)	74.88	75.62	22.80	22.18
black	16.61	15.85	21.65	20.94
Hispanic	13.55	15.26	19.39	20.83
Native	1.07	1.06	4.20	4.08
Asian	1.42	1.58	2.34	2.61
Pac-Islr	0.06	0.06	0.28	0.30
other	3.81	3.31	6.31	5.36
multiple	2.15	2.51	1.76	1.92
Educ below high school (%)	18.36	16.89	8.70	8.61
high school	34.22	33.62	7.33	7.41
some college	21.28	21.76	4.34	4.21
college	26.14	27.72	10.18	10.47

Note: The sum for race percentage is greater than 100 because Hispanic includes other categories.

Description of model

The purpose of the statistical model is to provide estimates of the relationship between the levels of payment shares and a set of explanatory variables comprising transaction size, the time- and location-specific variables, state-level fixed effects, and year fixed effects. We pool the data for the two years, restricting the relationship between payment and the explanatory variables to be the same across the two years. Changes in payment shares can be captured by changes in the explanatory variables and by the year fixed effects.

We assume that the relationship between payment shares and explanatory variables is captured by a *fractional multinomial logit* (FM-Logit) model, which states the expected share of each payment type, conditional on the explanatory variables, is a multinomial logit function of the explanatory variables:

$$E[s_{k,j,t} \mid x_{j,t}] = \frac{\exp(x'_{j,t}\beta_k)}{\sum_{m=1}^{4} \exp(x'_{j,t}\beta_m)}, \tag{1}$$
$$k = 1, 2, 3, 4.$$

Before explaining each of the terms in this expression, it will be helpful to understand the subscripts: k and m denote the payment types, cash, debit, credit, and check; j denotes zip code; and t denotes year. The left-hand-side variable, $E[s_{k,j,t} \mid x_{j,t}]$, is the expected value of the share of type k payments in zip code j in year t, conditional on the time- and location-specific variables $x_{j,t}$ (a vector), which can be thought of as including the state and the year as well as the median transaction size and the demographic and other variables summarized in Table 3. The right-hand side is a function of the explanatory variables as well as coefficients; β_k is a vector of coefficients that multiply the explanatory variables.[7]

By construction, the right-hand side is a number between zero and one as long as the data and coefficients are real numbers. And, by construction, the expected shares always sum to one: $\sum_{k=1}^{4} E[s_{k,j,t} \mid x_{j,t}] = 1$. Note, however, that from (1), for any β_k, $k = 1, 2, 3, 4$, the expected shares are invariant to the transformation $\tilde{\beta}_k = \beta_k + c$, where c is a vector the same length as β_k. In order to achieve identification of β_k, a normalization is needed. We use the standard normalization of setting $\beta_4 = 0$, where $k = 4$ denotes cash. This implies

$$E[s_{4,j,t} \mid x_{j,t}] = \frac{1}{1 + \sum_{m=1}^{3} \exp(x'_{j,t}\beta_m)}. \tag{2}$$

In the Appendix, we present this model in somewhat more detail and explain how the coefficients can be estimated.

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[7] As an alternative to the FMLogit model of payment shares, we could estimate a multinomial logit model at the individual transaction level. By aggregating transactions and modeling shares, we are able to use a larger number of transactions and smooth out the "noise" in individual transactions.

Basic results

We follow the approach described in the Appendix to estimate the model in (1) and (2). In a linear regression model, the usual way to report results is in the form of the estimated coefficients and P-values (or standard errors). With the nonlinear model used here, it is more informative to report *marginal effects* and their P-values; they are presented in Table 4.[8] For continuous variables, the marginal effect we report (on cash) is the derivative of the predicted share with respect to the variable. For the state and time fixed effects (the former are not reported in the table), the marginal effects we report are the difference between the predicted cash share when the indicator variable is one and when it is zero.

Many of the marginal effects reported in Table 4 are highly significant and have similar magnitudes to those reported in Wang and Wolman (2016a). For example, the median transaction effect is -0.019, compared to -0.018 in the earlier paper. Some of the estimates do differ, however, and not all the marginal effects reported in Table 4 are estimated precisely, in contrast to Wang and Wolman (2016a). The number of different zip codes is roughly comparable in the two papers, but here we use fewer days of data for each zip-code-level observation of the demographic variables. In our earlier paper there were more than 1,000 days of data for each observation of a demographic variable; here there is just one month of data—either February 2011 or February 2015, and this leads to the marginal effects being estimated less precisely.

With respect to age, as discussed above, we interpret the age distribution as the cohort distribution and therefore fix it at its 2011 value. Of course, this means we treat the cohort distribution as fixed so that it cannot explain any of the change in cash shares. In Section 4, we delve into the cohort effect in more detail and present some calculations that represent a rough estimate of the contribution of changes in the cohort distribution to changes in the cash share.

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[8] The dependent variables are the fractions of each of the four general payment instruments used in transactions at stores in a zip code in February 2011 and February 2015. The independent variables take their values in 2011 and 2015. Metro is a dummy variable taking the value of one when the zip code is in an MSA, otherwise it is equal to zero. We rescale some of the variables relative to the levels reported in Table 3 in order to make the marginal effects of common magnitude. Branches per capita is measured as the number of bank branches per 100 residents in a zip code. Robbery rate is defined as the number of robberies per 100 residents in a county. Median household income is measured in units of $100,000 per household in a zip code. Population density is measured in units of 100,000 residents per square mile in a zip code. All the demographic variables are expressed as fractions.

Table 4 Marginal Effects on Cash

Variable	Estimate at mean	P-value
Med. transaction size	-0.019	0.000
(Year=2015) - (Year=2011)	-0.068	0.000
Banking HHI	-0.002	0.469
Banking HHI $\times$ Metro	-0.022	0.000
Branches per capita	-0.040	0.127
Robbery rate	-0.062	0.005
Median household income ($)	-0.017	0.153
Population density (per mile^2)	0.016	0.535
Family households	-0.089	0.000
Housing: Owner-occupied	-0.364e-04	0.969
Vacant	.013	0.178
Female	-0.027	0.186
Age: 15-34	-0.147	0.000
35-54	-0.114	0.000
55-69	0.016	0.531
≥ 70	6.80e-04	0.981
Race: black	0.063	0.000
Hispanic	0.011	0.050
Native	0.141	0.000
Asian	-0.062	0.007
Pac-Islr	-0.073	0.627
other	0.009	0.434
multiple	-0.001	0.964
Educ: high school	-0.279	0.000
some college	-0.463	0.000
college	-0.309	0.000

Turning to the model's overall fit, Figures 6 and 7 show that it does a reasonable job of explaining the variation in cash shares across time and locations: Figure 6 compares the actual distribution of 2011 cash shares to the model's predicted distribution, and Figure 7 does the same thing for 2015. The pseudo-R^2 values are 0.55 for 2011 and 0.59 for 2015.

4.   ANALYSIS OF DECLINE IN CASH SHARES

Table 1 shows that the mean cash share of transactions declined by 8.7 percentage points from 2011 to 2015. Our model does a good job of capturing this decline: the predicted cash share evaluated at the means of the 2015 data is 8.8 percentage points lower than the predicted cash share evaluated at the means of the 2011 data. Alternatively, we can calculate the predicted cash share for every observation and compare the mean predicted shares for 2011 and 2015: the difference is

Figure 6 2011 Actual (Green) and Predicted Cash Shares
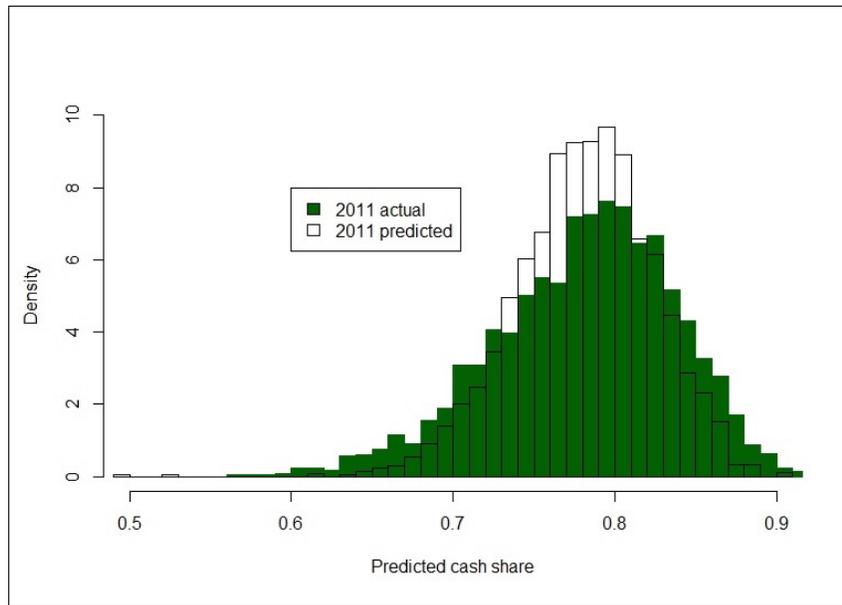
8.7 percentage points. In a linear regression, these two objects would be identical, but because the FMLogit model is nonlinear, the mean predicted value may differ from the predicted value evaluated at the mean of the explanatory variables. We will report both numbers at various points below; they never differ by much.

The empirical framework suggests three types of factors to account for the decline in cash shares from 2011 to 2015. First, given a relationship between transaction size and cash shares, an upward shift in the distribution of median transaction sizes (Figure 4) can account for some of the decline in cash shares. Second, given a relationship between demographic variables and cash shares (Table 4), changes in the demographic variables might account for some of the decline in cash shares. And finally, a portion of the decline in cash shares is accounted for by the year dummy; this portion is effectively unexplained and likely attributable to changes in the attributes of noncash payments (e.g., cost, availability, and security) and changing preferences on the part of consumers.

Figure 7 2015 Actual (Green) and Predicted Cash Shares
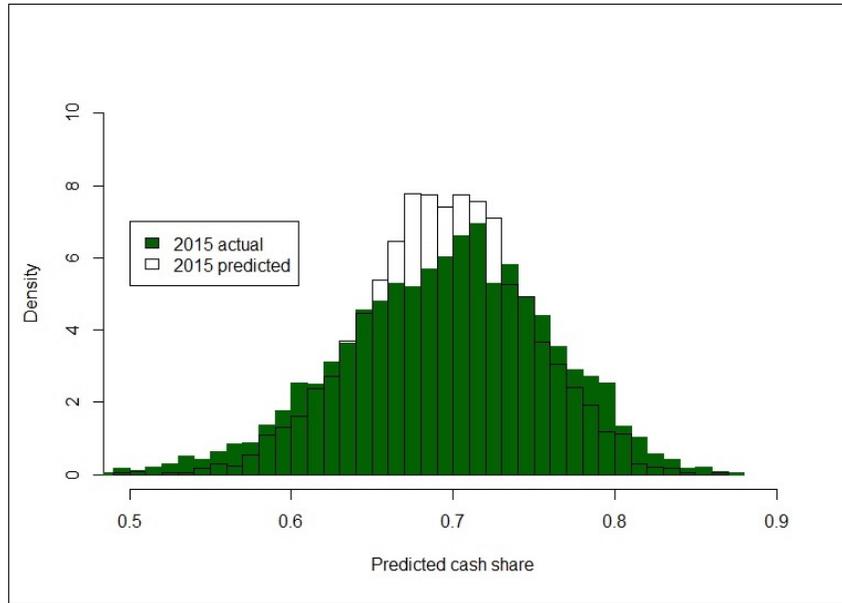
Increasing average transaction size

The average value of median transaction size increased by \$0.70 from 2011 to 2015. A simple measure of the contribution of changing transaction size to the decline in cash shares is the product of the \$0.70 increase with the marginal effect for transaction size, -0.019. According to this measure, increasing transaction size can account for a decrease of 1.35 percentage points in the cash share, roughly 15 percent of the total decline. This simple measure ignores nonlinearity in the empirical model. We can take into account the nonlinearity by comparing 2011 predicted cash shares to the shares the model would predict if transaction size changed to its 2015 level but all other variables were fixed at their 2011 values. This approach yields a decrease of 1.33 percentage points in the predicted cash share evaluated at the mean of the explanatory variables and a decrease of 1.44 percentage points in the mean predicted cash share across zip codes. Thus, the linear approximation (1.35 percentage points) turns out to be quite accurate.

The smoothed density functions in Figure 8 are based on the same approach: the black line represents the density function of predicted cash shares for 2011, whereas the red line represents the density func-

Figure 8 The Transaction Size Effect
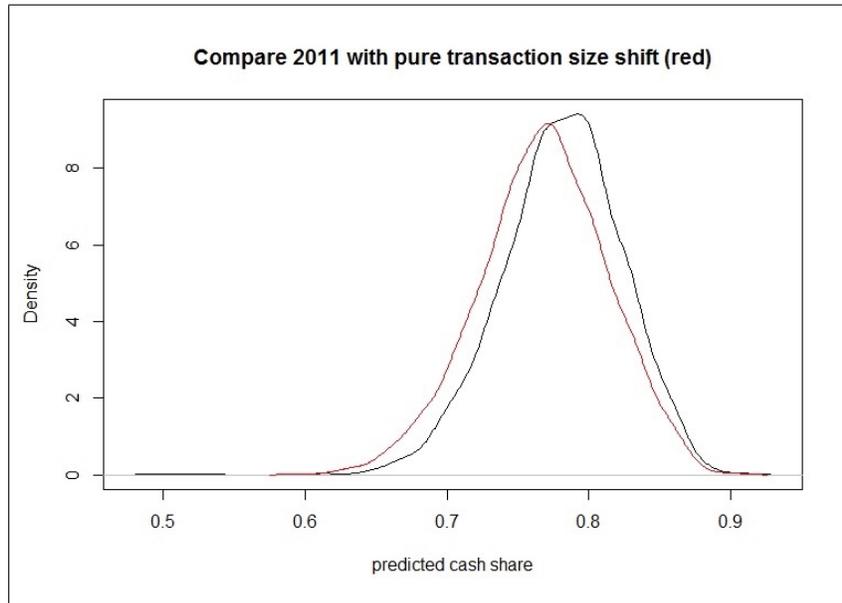
tion of counterfactual predicted values, calculated with 2015 transaction size but 2011 values of all other variables. There is a notable leftward shift in the distribution explained by the increase in transaction size, but the shift is small relative to the overall change shown in Figure 2. Note finally that our estimates of the contribution of increasing transaction size to the decrease in cash shares may be affected by correlations between transaction size and some of the zip-code-level variables. This means that a portion of the effect attributed to transaction size could instead be attributed to changes in the zip-code-level variables. In Wang and Wolman (2016b), we explore this idea in more detail by regressing transaction size on the zip-code-level variables and then including the residual portion of transaction size in the FMLogit regression in place of actual transaction size. We find that indeed the marginal effects of other variables change when they include indirect effects of transaction size.

Changing demographic and other variables

Table 4 shows that many location-specific variables have a systematic relationship with the cash share of transactions. Since these variables

Figure 9 The Zip-Code-Level Effect
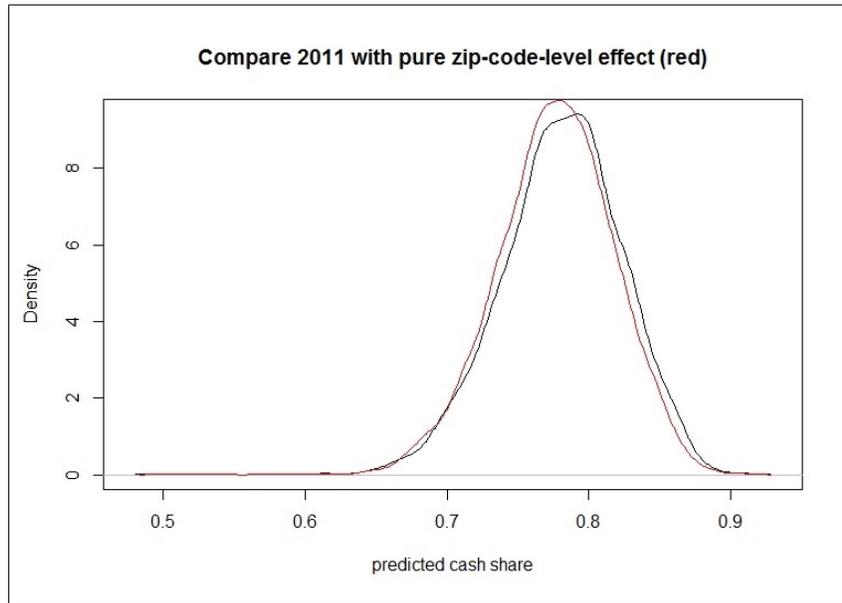
take on different values in 2011 and 2015, they may be able to account
for some of the decline in cash shares over that period. In contrast,
Wang and Wolman (2016a) used only a three-year span of data with
fixed values of the location-specific variables. As mentioned above,
that paper included a rough forecasting exercise that took into account
projected changes in the location-specific variables, but the projected
changes were identical across locations. In order to quantify the effect of
the zip-code-level variables, here we use an analogous approach to that
used for transaction size: we compare the predicted cash shares for 2011
with the predicted cash shares implied by holding fixed transaction size
and the year dummy at their 2011 values but allowing all the location-
specific variables to take on their 2015 values. Comparing the predicted
value of cash share conditional on 2011 means to that conditional on
2015 zip-code-level variable means, the 2011 year dummy, and 2011
mean transaction size yields a decline of 0.5 percentage points. This
estimate does not change if we instead compare means of predicted
values across zip codes.

Figure 9 plots the smoothed density function for 2011 predicted
cash shares and compares it to the density of predicted cash shares un-
der the assumption that the zip-code-level variables take on their 2015

Figure 10 Transaction Size and Zip-Code-Level Effects

Compare 2011 (black), size+zip-code-level effects (red), 2015 (blue)
(vertical lines represent means)
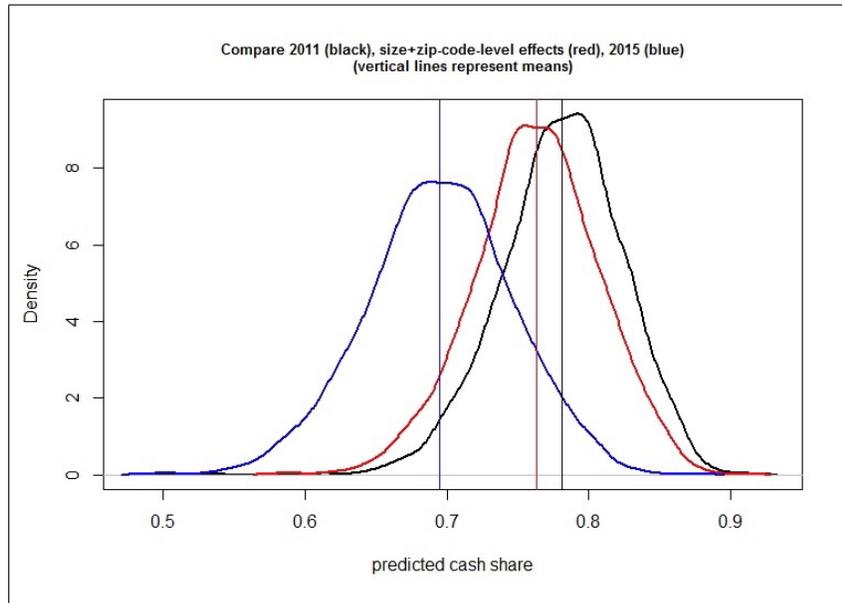
values but the year dummy and transaction size are fixed at their 2011 values. There is a small but discernible leftward shift in the distribution of predicted cash shares, consistent with the mean estimate. As discussed above, Figures 8 and 9 attribute any effects of transaction size that work through zip-code-level variables to transaction size. In Figure 10, we combine both effects, so that the precise decomposition is irrelevant: the black line is the density of 2011 predicted cash shares; the red line is the density of predicted cash shares holding fixed the year dummy at 2011 but allowing all other variables to change; and the blue line is the density of 2015 predicted cash shares. In Figure 10, the vertical lines represent the respective means. Consistent with our previous calculations, the combination of changes in transaction size and changes in zip-code-level variables accounts for a 1.8 percentage point decline in the mean predicted cash share across zip codes or 1.7 percentage points if we instead use the predicted change in the cash share at the means of the data.

In Wang and Wolman (2016a), the forecasting exercise attributed a relatively large fraction of the projected decrease in the cash share to a cohort effect: a shift in the population toward later-born cohorts who were accustomed to using cards would drive down the cash share of

transactions. Thus far, the calculations here do not take into account that effect because they hold fixed both the age and cohort distribution of the population and the coefficients on age or cohort. Ideally, we would like to treat the cohort distribution just like the other zip-code-level variables in our study: this would involve allowing the cohort distribution to change from 2011 to 2015, estimating a common cohort effect, and then calculating the contribution of the changing cohort distribution to the change in the cash share. The difficulty with this approach is that our data are on age distribution, and in fifteen- and twenty-year bins. Age and cohort are interchangeable at a point in time; for example, the fraction of the population in 2011 that was between 15 and 34 years old (=age) is identical to the fraction of the population in 2011 that was born between 1977 and 1996 (=cohort). However, across time, cohort distributions and age distributions need to be tracked separately unless they are in one-year bins. For example, if we know the fraction of the population that was between 15 and 34 in 2011 and the fraction of the population that was between 15 and 34 in 2015, we have information about two different cohorts in the two years, not the same cohort. For 2011 we have the 1977 to 1996 cohort, and for 2015 we have the 1981 to 2000 cohort. If we knew the age distribution in one-year increments for 2011 and 2015, then it would be trivial to calculate the corresponding cohort distribution in one-year increments.

Without precise data on how the cohort distribution evolved from 2011 to 2015, we nonetheless computed a rough estimate of the contribution of shifts in the cohort distribution to the decrease in cash shares from 2011 to 2015. The idea behind this estimate is to use aggregate census data on a finer gradation of the age distribution to come up with an educated guess about how the cohort distribution changed from 2011 to 2015 across the large bins in our study. Then, we will combine that educated guess with our estimated marginal effects for the different cohorts. Note first that, from Table 4, the cash marginal effect for population aged 35-54 in 2011 is -0.114, compared to 0.016 for age 55-69. The 35-54 age group is the cohort born between 1957 and 1976, and the 55-69 age group is the cohort born between 1942 and 1956. For ages less than 34, the marginal effect is even more negative, and for ages above 69, it is close to zero. According to nationwide census data, the 2011 population share of ages 50-54 was 7 percent. We thus pose the following question: How would the predicted cash share change if there were a 7 percentage point increase in the fraction of the population for whom the cash marginal effect is -0.114, and a 7 percentage-point decrease in the fraction of the population for whom the cash marginal effect is 0.016? The answer is that the predicted cash share would fall by 0.8 percentage points. Adding this to the 1.7

percentage points accounted for by transaction size and other location-specific variables would allow us to account for nearly 30 percent of the overall 8.7 percentage-point predicted decline in the cash share.

The remainder of the predicted decrease in cash shares at the mean of the data—either 7 percentage points or 6.2 percentage points if we include the imputed age effect—is attributed to the year dummy, although this decomposition is not exact: the marginal effect for the year dummy is 6.8 percentage points, and if we compare predicted means for 2011 variables with the year dummy changing, the difference is 6.6 percentage points. Regardless of how we measure it, between 70 and 80 percent of the decline in cash shares cannot be explained by either an increase in transaction size or changes in location-specific variables. We attribute that unexplained decline to a pure "time effect," which is standing in for all other factors that play a role in payment choice but are not included in the model. The leading candidates for these factors are wider availability, better security, and lower cost of cards, as well as evolving consumer perceptions of each of those factors.

5.   CONCLUSION

The cash share of transactions at a large national discount retailer declined by approximately 8.6 percentage points from February 2011 to February 2015. Following up on Wang and Wolman (2016a,b), we use a FMLogit model to study the cash share of transactions across time and locations. The geographic coverage is similar to our earlier paper: thousands of store locations, at the zip-code level. The time coverage is more sparse here: two months, four years apart, as opposed to three years of daily transaction shares in our earlier paper. By restricting the time dimension to low-frequency changes, in this paper we are able to introduce time variation in the zip-code-level variables. Previously, we *measured* the trend decrease in cash shares but were able to attribute it only to a pure time trend or an increase in transaction sizes. We used forecasts of demographic variables to produce a crude measure of the projected contribution of changes in those variables to changes in the cash share. The main contribution of this paper is to explicitly decompose the trend decrease in cash use into a component due to changes in demographic and location-specific variables, as well as a transaction-size component and a pure time effect. We find that location-specific changes in demographic and other variables account for between 0.5 and 1.3 percentage points of the 8.6 percentage-point overall decline. Increasing transaction sizes account for 1.3 percentage points, which leaves between 70 and 80 percent of the decline in cash use unexplained. The unexplained portion is likely being driven

by improved actual characteristics of payment cards as well as slowly evolving consumer perceptions of those characteristics.

Referring back to the introduction, although we attribute a relatively small portion of the decline in cash use to location-specific factors, it would be premature to dismiss distributional arguments about the benefits of currency. First, evaluating those arguments requires quantifying the benefits of currency and payment cards to different groups; that is not part of our analysis and would require an economic model. Second, for the stores and time period in our study, the share of cash transactions declined from 78 percent to 70 percent. Whether our results would carry over to a much larger decline in cash use is an open question, to which time may help provide the answer. Finally, our focus has been on demographic and other location-specific factors across the store locations in our study. As discussed in Wang and Wolman (2016a), those stores are generally located in relatively low-income zip codes. It is possible that analysis of additional retailers in other locations would reveal that demographics account for a greater proportion of the change in cash shares; that is, part of the change in cash shares that we label unexplained may be accounted for by characteristics that are common to the stores and customers studied here but that are distinctive in the context of the entire US economy.

APPENDIX: THE FRACTIONAL MULTINOMIAL LOGIT MODEL

The regression analysis in the paper uses the FMLogit model. The FMLogit model conforms to the multiple fractional nature of the dependent variables, namely that the fraction of payments for each instrument should remain between 0 and 1, and the fractions add up to 1. The FMLogit model is a multivariate generalization of the method proposed by Papke and Wooldridge (1996) for handling univariate fractional response data using quasi-maximum likelihood estimation. Mullahy (2010) provides more econometric details.

Formally, consider a random sample of $i = 1, ..., N$ zip-code-day observations, each with M outcomes of payment shares. In our context, $M = 4$, which corresponds to cash, debit, credit, and check. Letting s_{ik} represent the kth outcome for observation i, and x_i, $i = 1, ..., N$, be a vector of exogenous covariates, the nature of our data requires that

$$s_{ik} \in [0, 1] \qquad k = 1, ..., M;$$

$$\Pr(s_{ik} = 0 \mid x_i) \geq 0 \quad \text{and} \quad \Pr(s_{ik} = 1 \mid x_i) \geq 0;$$

$$\text{and} \quad \sum_{m=1}^{M} s_{im} = 1 \quad \text{for all } i.$$

Given the properties of the data, the FMLogit model provides consistent estimates by enforcing conditions (3) and (4),

$$E[s_k|x] = G_k(x; \beta) \in (0, 1), \quad k = 1, ..., M; \qquad (3)$$

$$\sum_{m=1}^{M} E[s_m \mid x] = 1; \qquad (4)$$

and also accommodating conditions (5) and (6),

$$\Pr(s_k = 0 \mid x) \geq 0 \qquad k = 1, ..., M; \qquad (5)$$

$$\Pr(s_k = 1 \mid x) \geq 0 \qquad k = 1, ..., M; \qquad (6)$$

where $\beta = [\beta_1, ..., \beta_M]$.[9] Specifically, the FMLogit model assumes that the M conditional means have a multinomial logit functional form in

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[9] To simplify the notation, we suppress the "i" subscript in Eqs (3)-(9).

linear indexes as

$$E[s_k \mid x] = G_k(x; \beta) = \frac{\exp(x\beta_k)}{\sum\limits_{m=1}^{M} \exp(x\beta_m)}, \qquad k = 1, ..., M. \qquad (7)$$

As with the multinomial logit estimator, one needs to normalize for identification purposes, and we choose the normalization $\beta_M = 0$. Therefore, Eq (7) can be rewritten as

$$G_k(x; \beta) = \frac{\exp(x\beta_k)}{1 + \sum\limits_{m=1}^{M-1} \exp(x\beta_m)}, \qquad k = 1, ..., M - 1; \qquad (8)$$

and

$$G_M(x; \beta) = \frac{1}{1 + \sum\limits_{m=1}^{M-1} \exp(x\beta_m)}. \qquad (9)$$

Finally, one can define a multinomial logit quasilikelihood function $L(\beta)$ that takes the functional forms (8) and (9) and uses the observed shares $s_{ik} \in [0, 1]$ in place of the binary indicator that would otherwise be used by a multinomial logit likelihood function, such that

$$L(\beta) = \prod_{i=1}^{N} \prod_{m=1}^{M} G_m(x_i; \beta)^{s_{im}}. \qquad (10)$$

The consistency of the resulting parameter estimates $\hat{\beta}$ then follows from the proof in Gourieroux et al. (1984), which ensures a unique maximizer. In our regression analysis, we use Stata code developed by Buis (2008) for estimating the FMLogit model.

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Idiosyncratic Sectoral Growth, Balanced Growth, and Sectoral Linkages

Andrew Foerster, Eric LaRose, and Pierre-Daniel Sarte

I n general, there is substantial heterogeneity in value added, gross output, and production patterns across sectors within the US economy. There is also considerable asymmetry in intermediate goods linkages; that is, some sectors are much larger suppliers of intermediate goods to different sectors, on average, than others. Such heterogeneity suggests that there may be significant differences in the extent to which shocks to individual sectors not only affect aggregate output, but also transmit to other sectors.[1]

In this paper, in contrast to previous literature focusing on shorter-run variations in economic activity, we explore how longer-run growth in different sectors affects other sectors and overall aggregate growth. We consider a neoclassical multisector growth model with sector-specific capital and linkages between sectors in intermediate goods. In particular, we investigate the properties of a balanced growth path where total factor productivity (TFP) growth is sector-specific. We derive a relatively simple formula that simultaneously captures all relationships between value-added growth and TFP growth across sectors. We then study the effect of changes in TFP growth in one sector on value-added growth in every other sector. In addition, we can use the Divisia index for aggregate value-added growth to calculate the effect of a change in TFP growth in a given sector on aggregate GDP growth. Finally, using

data on value-added growth for each sector over the period 1948-2014, we recover each sector's model-implied mean TFP growth over this period and examine how sectoral changes in TFP growth in practice carry over to other sectors.

In all three of the above exercises, we also consider a special case of our model without capital. This case collapses to the model considered by Hulten (1978), or Acemoglu et al. (2012). In that model, absent capital, the impact of a level change in sectoral TFP on GDP is entirely captured by that sector's share in GDP.[2] We show that a version of this result also holds in growth rates along the balanced growth path. In that special case, other microeconomic details of the environment become irrelevant as long as we can observe the distribution of value-added shares across sectors.

More generally, in the benchmark model, value-added growth and the effects of changes in TFP growth in a given sector on GDP growth depend on that sector's capital intensity, its share of value added in gross output, and the degree to which its goods are used as intermediates by other sectors. In this regard, in a multisector model with capital, it becomes important to have information pertaining to the underlying microeconomic structure of the economy beyond what is captured in shares. Fortunately, the model delivers a simple expression of relevant parameters that can easily be constructed from sectoral-level data provided by government agencies.

Using such data, we can quantify the effects of changes in sectoral TFP growth and compare these results to the special case of our model where a version of Hulten (1978) holds in growth rates. In the seven sectors we consider in this paper, sectors vary widely in their shares of capital in value added and value added in total output, and some sectors are considerably more important suppliers of intermediate goods than others. Overall, we find that adding capital to the model creates substantial spillovers across sectors resulting from TFP growth changes that, for every sector, substantially increase the responsiveness of GDP growth to such changes. These spillover effects are larger for sectors more integral to sectoral linkages in intermediates, a finding consistent with the literature we discuss below.

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[2] Pasten, Schoenle, and Weber (2018) and Baqaee and Farhi (2018) show that, even in a model without capital, this result may not hold due to factors such as heterogeneous price rigidity and nonlinearities in production.

1.   RELATED LITERATURE

The modern literature on multisector growth models started with the real business cycle model presented in Long and Plosser (1983). In their model, a representative agent chooses labor inputs and commodity inputs to n sectors, with linkages between sectors in inputs and uncorrelated exogenous shocks to each sector. Taking the model to the data with six sectors, they found substantial comovement in output across sectors; furthermore, shocks to individual sectors generally led to large aggregate fluctuations, particularly for sectors that heavily served as inputs in production.

For many years, there existed a sense that at more disaggregated levels than that of Long and Plosser (1983), idiosyncratic sectoral shocks should fail to affect aggregate volatility. Lucas (1981), in particular, argued that in an economy with disaggregated sectors, many sector-specific shocks would occur within a given period and roughly cancel each other out in a way consistent with the Law of Large Numbers. Dupor (1999) helped formalize the conditions under which the intuition in Lucas (1981) would apply. He considered an n-sector economy with linkages between firms in intermediates as well as full depreciation of capital. Assuming all sectors sold nonzero amounts to all other sectors, and that every row total in the matrix of linkages was the same (i.e., every sector is equally important as an input supplier to all other sectors), Dupor found that aggregate volatility converged toward zero at a rate of $\sqrt{n}$; the underlying structure of the input-output matrix was irrelevant as long as it satisfied those conditions.

Horvath (1998) countered that Dupor's irrelevance theorem failed to hold because, in practice, sectors are not uniformly important as input suppliers to other sectors. He observed that at high levels of disaggregation in US data, the matrix of input-output linkages became quite sparse, with only a few sectors selling widely to others; consequently, sectoral shocks could explain a significant share of aggregate volatility, which would decline at a rate much slower than $\sqrt{n}$. (Horvath [2000] showed that his earlier result still held in more general models including, among other things, linkages between sectors in investments.) Acemoglu et al. (2012) expand on Horvath's idea by analyzing the network structure of linkages and conclude that it is the asymmetry, rather than the sparseness, of input-output linkages that determines the decay rate of aggregate volatility. In a multisector model with linkages between sectors in investment as well as intermediates, Foerster, Sarte, and Watson (2011) find evidence of a high level of asymmetry in the data, consistent with Acemoglu et al. (2012). They also show that, starting with the Great Moderation around 1983, roughly half the variation in aggregate output stems from sectoral shocks.

As an additional perspective on the failure of sectoral shocks to average out, Gabaix (2011) also points out that the "averaging out" argument will not hold when the distribution of firms (or sectors) is fat-tailed, meaning a few large firms (or sectors) dominate the economy. In such a case, aggregate volatility decays at rate $\frac{1}{\ln n}$, and idiosyncratic movements can cause large variations in output growth.

While it should be clear from this section that the literature on multisector growth models has mostly focused on the relationship between aggregate and sectoral volatility, this paper focuses instead on the relationship between aggregate and sectoral growth. The arguments of Horvath (1998), Acemoglu et al. (2012), and others regarding the nature of input-output linkages still hold relevance for sectoral growth. In that vein, the analysis herein builds more directly on the work of Ngai and Pissarides (2007). In that paper, the authors focus on the effects of different TFP growth rates across sectors on sectoral employment shares. The model we present extends their work by explicitly capturing all pairwise linkages in intermediate goods in the economy while additionally allowing every sector to produce capital.

2.   ECONOMIC ENVIRONMENT

We consider an economy with n sectors. For simplicity, we assume that utility is linear in the final consumption good. Preferences are given by

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t C_t$$

$$C_t = \prod_{j=1}^{n} \left(\frac{c_{j,t}}{\theta_j} \right)^{\theta_j}, \ \sum_{j=1}^{n} \theta_j = 1,$$

where C_t represents an aggregate consumption bundle taken to be the numeraire good.

Gross output in a sector j results from combining value added and materials output according to

$$y_{j,t} = \left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1 - \gamma_j} \right)^{1-\gamma_j},$$

where $y_{j,t}$, $v_{j,t}$, and $m_{j,t}$ denote gross output, value added, and materials output, respectively, used by sector j at time t. Materials output in a given sector j results from combining different intermediate materials

from all other sectors, as described by the production function,

$$m_{j,t} = \prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \quad \sum_{i=1}^{n} \phi_{ij} = 1,$$

where $m_{ij,t}$ denotes the use of materials produced in sector i by sector j at time t.

Value added in sector j is produced using capital and labor,

$$v_{j,t} = z_{j,t} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j} \left(\frac{\ell_{j,t}}{1 - \alpha_j} \right)^{1 - \alpha_j},$$

where $z_{j,t}$ denotes a technical shift parameter that scales production of value added, which we refer to as value-added TFP.

Capital is sector-specific, so that output from only sector j can be used to produce capital for sector j, and it accumulates according to the law of motion,

$$k_{j,t+1} = x_{j,t} + (1 - \delta) k_{j,t},$$

where $x_{j,t}$ represents investment in sector j at time t and δ denotes the depreciation rate of capital.

Goods market clearing requires that

$$c_{j,t} + \sum_{i=1}^{n} m_{ji,t} + x_{j,t} = y_{j,t},$$

while labor market clearing requires that

$$\sum_{j=1}^{n} \ell_{j,t} = 1.$$

Here, we assume that aggregate labor supply is inelastic and set to one. We also assume that labor can move freely across sectors so that workers earn the same wage, w_t, in all sectors.

Finally, we assume that TFP growth in sector j, $\Delta \ln z_{j,t}$, follows an AR(1) process,

$$\Delta \ln z_{j,t} = (1 - \rho) g_j + \rho \Delta \ln z_{j,t-1} + \eta_{j,t},$$

where $\rho < 1$ and $\eta_{j,t} \sim \mathcal{D}$ with mean zero for each j.

3.   PLANNER'S PROBLEM

The economy we have just described presents no frictions, so that decentralized allocations in the competitive equilibrium are optimal. Thus, we derive these allocations by solving the following planner's

problem:

$$\max \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \prod_{j=1}^{n} \left(\frac{c_{j,t}}{\theta_j} \right)^{\theta_j} \tag{1}$$

such that $\forall\, j$ and t,

$$c_{j,t} + \sum_{i=1}^{n} m_{ji,t} + x_{j,t} = \left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1 - \gamma_j} \right)^{1-\gamma_j}, \tag{2}$$

$$m_{j,t} = \prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \tag{3}$$

$$v_{j,t} = z_{j,t} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j} \left(\frac{\ell_{j,t}}{1 - \alpha_j} \right)^{1-\alpha_j}, \tag{4}$$

$$k_{j,t+1} = x_{j,t} + (1 - \delta)\, k_{j,t}, \tag{5}$$

and $\forall\, t$,

$$\sum_{j=1}^{n} \ell_{j,t} = 1. \tag{6}$$

Let $p_{j,t}^y$, $p_{j,t}^v$, $p_{j,t}^m$, and $p_{j,t}^x$ denote the Lagrange multipliers associated with, respectively, the resource constraint (2), the production of value added (4), the production of materials (3), and the capital accumulation equation (5) in sector j at date t.

The first-order conditions for optimality yield

$$\frac{\theta_j C_t}{c_{j,t}} = p_{j,t}^y.$$

This expression also defines an ideal price index,

$$1 = \prod_{j=1}^{n} \left(p_{j,t}^y \right)^{\theta_j}. \tag{7}$$

We additionally have that

$$p_{j,t}^v v_{j,t} = \gamma_j p_{j,t}^y y_{j,t}.$$

Likewise,

$$p_{j,t}^m m_{j,t} = \left(1 - \gamma_j \right) p_{j,t}^y y_{j,t}.$$

The above two expressions define a price index for gross output,

$$p_{j,t}^y = \left(p_{j,t}^v \right)^{\gamma_j} \left(p_{j,t}^m \right)^{1-\gamma_j}.$$

In addition, we have that

$$p^y_{i,t} m_{ij,t} = \phi_{ij} p^m_{j,t} m_{j,t},$$

which gives material prices in terms of gross output prices,

$$p^m_{j,t} = \prod_{i=1}^{n} \left(p^y_{i,t} \right)^{\phi_{ij}},$$

and

$$w_t \ell_{j,t} = (1 - \alpha_j) p^v_{j,t} v_{j,t},$$

where w_t is the Lagrange multiplier associated with the labor market clearing condition (6).

From the law of motion for capital accumulation, we have that

$$p^x_{j,t} = p^y_{j,t}.$$

Finally, the Euler equation associated with optimal investment dictates

$$p^x_{j,t} = \beta \mathbf{E}_t \left[\alpha_j \frac{p^v_{j,t+1} v_{j,t+1}}{k_{j,t+1}} + p^x_{j,t+1} (1 - \delta) \right].$$

The first-order conditions give rise to natural expressions of the model parameters as shares that are readily available in the data. In particular, θ_j represents the share of sector j in nominal consumption, and γ_j represents the share of value added in total output in sector j, while ϕ_{ij} represents materials purchased from sector i by sector j as a share of total materials purchased in sector j. Furthermore, $1 - \alpha_j$ equals the share of total wages in nominal value added in sector j, and consequently, α_j represents capital's share in nominal value added. Nominal value added in sector j in this economy is then given by $p^v_{j,t} v_{j,t} = \gamma_i p^y_{j,t} y_{j,t}$, and it follows that $GDP_t = \sum_j p^v_{j,t} v_{j,t}$.

In the remainder of this paper, we adopt the following notation: $\Gamma_d = diag\{\gamma_j\}$, $\alpha_d = diag\{\alpha_j\}$, $\Theta = (\theta_1, ..., \theta_n)$, and $\Phi = \{\phi_{ij}\}$.

Some Benchmark Results in Levels

A special case of the economic environment presented above is one where $\alpha_j = 0 \ \forall j$, which, absent any growth in sectoral TFP or shocks, reduces to the static economies of Hulten (1978) or Acemoglu et al. (2012). In this case, aggregate value added, or GDP, is given by the consumption bundle C_t and

$$\frac{\partial \ln GDP_t}{\partial \ln z_{j,t}} = s^v_j \ \forall t,$$

where s^v_j is sector j's value-added share in GDP, and we summarize these shares in a vector, $\mathbf{s}^v = (s^v_1, ..., s^v_n)$, given by

$$\mathbf{s}^v = \Theta(I - (I - \Gamma_d)\Phi')^{-1} \Gamma_d. \tag{8}$$

As shown in Hulten (1978), in this special case, a sector's value-added share entirely captures the effect of a *level* change in TFP on GDP. Accordingly, Acemoglu et al. (2012) refer to the object $\Theta(I - (I - \Gamma_d)\Phi')^{-1}\Gamma_d$ as the *influence vector*.

A model with capital is dynamic but, in the long run, converges to a steady state in levels absent any sectoral TFP growth. With a discount factor β close to 1, the effect of a level change in sectoral log TFP on log GDP continues to be given primarily by sectoral shares, as in equation (8). In other words, Hulten's (1978) result continues to hold in an economy with capital in that the variation in the effects of sectoral TFP changes on GDP is determined by the variation in sectoral shares. In this case, however, sectoral shares need to be adjusted by a factor that is constant across sectors and approximately equal to the inverse of the mean employment share.

With exogenous sectoral TFP growth, the economy no longer achieves a steady state in levels. Instead, with constant sectoral TFP growth, the steady state of the economy may be defined in terms of sectoral growth rates along a balanced growth path. Along this path, the effects of TFP growth changes on GDP growth involve additional considerations. In particular, sectoral linkages in intermediates mean that changes in sectoral TFP growth in one sector potentially affect value-added growth rates in every other sector and, therefore, can impact overall GDP growth beyond changes in shares. These sectoral linkages consequently create a multiplier effect that, as we show below, can lead to a total impact of a TFP growth change in a given sector that is several times larger than that sector's share in GDP.

4. SOLVING FOR BALANCED GROWTH

We now allow for each sector to grow at a different rate along a balanced growth path. In particular, we derive and explore the relationships that link different sectoral growth rates to each other and study how TFP growth rates in one sector affect all other sectors and the aggregate balanced growth path.

Consider the case where $z_{j,t}$ is growing at a constant rate along a nonstochastic steady-state path, that is $\eta_{j,t} = 0$ and $\Delta \ln z_{j,t} = g_j \ \forall j$, t. Moreover, the resource constraint (2) in each sector requires that all variables in that equation grow at the same constant rate along a balanced growth path. Therefore, we normalize the model's variables in each sector by a sector-specific factor $\mu_{j,t}$. In particular, we define $\tilde{y}_{j,t} = y_{j,t}/\mu_{j,t}, \tilde{c}_{j,t} = c_{j,t}/\mu_{j,t}, \tilde{m}_{ji,t} = m_{ji,t}/\mu_{j,t}$, and $\tilde{x}_{j,t} = x_{j,t}/\mu_{j,t}$. We show that detrending the economy yields a system of equations that is stationary in the normalized variables along the balanced growth path

and where the vector $\mu_t = (\mu_{1,t}, ..., \mu_{n,t})'$ can be expressed as a function of the underlying parameters of the model only.

Detrending the Economy

The capital accumulation equation in sector j can be written under this normalization as

$$k_{j,t+1} = \tilde{x}_{j,t}\mu_{j,t} + (1-\delta)\, k_{j,t},$$

so that

$$\tilde{k}_{j,t+1} = \tilde{x}_{j,t} + (1-\delta)\, \tilde{k}_{j,t}\left(\frac{\mu_{j,t-1}}{\mu_{j,t}}\right),$$

where $\tilde{k}_{j,t} = k_{j,t}/\mu_{j,t-1}$.

Using this last equation, we can write value added in sector j as

$$v_{j,t} = z_{j,t}\left(\frac{\tilde{k}_{j,t}\mu_{j,t-1}}{\alpha_j}\right)^{\alpha_j}\left(\frac{\ell_{j,t}}{1-\alpha_j}\right)^{1-\alpha_j}.$$

The aggregate labor constraint in each period, $\sum_j \ell_{j,t} = 1$, implies that the labor shares, $\ell_{j,t}$, are already normalized: $\tilde{\ell}_{j,t} = \ell_{j,t}$. Then defining $\tilde{v}_{j,t} = v_{j,t}/\left(z_{j,t}\left(\mu_{j,t-1}\right)^{\alpha_j}\right)$, the expression for value added becomes

$$\tilde{v}_{j,t} = \left(\frac{\tilde{k}_{j,t}}{\alpha_j}\right)^{\alpha_j}\left(\frac{\ell_{j,t}}{1-\alpha_j}\right)^{1-\alpha_j}.$$

The equation for materials used in sector j can be written in normalized terms as

$$\tilde{m}_{j,t} = \prod_{i=1}^{n}\left(\frac{\tilde{m}_{ij,t}}{\phi_{ij}}\right)^{\phi_{ij}},$$

where $\tilde{m}_{j,t} = m_{j,t}/\prod_{i=1}^{n}\mu_{i,t}^{\phi_{ij}}$. It follows that gross output in sector j becomes, in normalized terms,

$$\tilde{y}_{j,t}\mu_{j,t} = \left(\frac{\tilde{v}_{j,t}z_{j,t}\mu_{j,t}^{\alpha_j}}{\gamma_j}\right)^{\gamma_j}\left(\frac{\tilde{m}_{j,t}\prod_{i=1}^{n}\mu_{i,t}^{\phi_{ij}}}{1-\gamma_j}\right)^{1-\gamma_j},$$

which may be rewritten as

$$\tilde{y}_{j,t} = \left(\frac{\tilde{v}_{j,t}}{\gamma_j}\right)^{\gamma_j}\left(\frac{\tilde{m}_{j,t}}{1-\gamma_j}\right)^{1-\gamma_j}\left[\frac{z_{j,t}^{\gamma_j}\mu_{j,t-1}^{\gamma_j\alpha_j}}{\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\right]. \qquad (9)$$

Observe that for the detrended variables to be constant along a balanced growth path, it must be the case that the expression in square

brackets is also constant along that path. Thus, we can use equation (9) to solve for $\mu_{j,t}$ as a function of the model parameters. In particular, we can rewrite the term in square brackets as

$$\frac{z_{j,t}^{\gamma_j}\mu_{j,t-1}^{\gamma_j\alpha_j}\mu_{j,t}^{\gamma_j\alpha_j-1}}{\mu_{j,t}^{\gamma_j\alpha_j}} \prod_{i=1}^{n}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}},$$

where we aim for the growth rate of $\mu_{j,t}$ to be constant. Thus, without loss of generality, we choose $\mu_{j,t}$ such that

$$z_{j,t}^{\gamma_j}\mu_{j,t}^{\gamma_j\alpha_j-1}\prod_{i=1}^{n}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}} = 1,$$

which in logs gives

$$\gamma_j \ln z_{j,t} + \left(\gamma_j\alpha_j - 1\right)\ln u_{j,t} + \sum_{i=1}^{n}\left(1-\gamma_j\right)\phi_{ij}\ln\mu_{i,t} = 0. \qquad (10)$$

In matrix form, with $z_t = (z_{1,t}, ..., z_{n,t})'$, equation (10) becomes

$$\Gamma_d \ln z_t + (\Gamma_d\alpha_d - I)\ln\mu_t + (I - \Gamma_d)\,\Phi'\ln\mu_t = 0.$$

It follows that along a balanced growth path,

$$\Delta\ln\mu_t = \left(I - \Gamma_d\alpha_d - (I - \Gamma_d)\,\Phi'\right)^{-1}\Gamma_d g_z, \qquad (11)$$

where $g_z = (g_1, ..., g_n)'$.

Sectoral Value Added and GDP along a Balanced Growth Path

Having derived expressions in terms of the normalizing factors for $\mu_{j,t}$, we now derive the normalizing factors for value added in each sector. By construction, these factors in turn will grow at the same rate as value added in each sector. As given above, the normalizing factor for value added in sector j, denoted as $\mu_{j,t}^v$, is $z_{j,t}\mu_{j,t-1}^{\alpha_j}$. In vector form, this becomes

$$\Delta\ln\mu_t^v = \Delta\ln z_t + \alpha_d\left(I - \Gamma_d\alpha_d - (I - \Gamma_d)\,\Phi'\right)^{-1}\Gamma_d\Delta\ln z_{t-1},$$

so that along a balanced growth path,

$$\Delta\ln\mu_t^v = \left[I + \alpha_d\left(I - \Gamma_d\alpha_d - (I - \Gamma_d)\,\Phi'\right)^{-1}\Gamma_d\right]g_z. \qquad (12)$$

In other words, in this economy, TFP growth in each sector potentially affects value-added growth in every other sector through a matrix that summarizes all linkages in the economy, $\left[I + \alpha_d\left(I - \Gamma_d\alpha_d - (I - \Gamma_d)\,\Phi'\right)^{-1}\Gamma_d\right]$. Moreover, these effects may be

summarized analytically by

$$\frac{\partial \Delta \ln \mu_t^v}{\partial g_z} = \left[I + \alpha_d \left(I - \Gamma_d \alpha_d - (I - \Gamma_d)\, \Phi'\right)^{-1} \Gamma_d\right], \qquad (13)$$

where the element in row i and column j of this matrix represents the effect of an increase in TFP growth in sector j on value-added growth rates in sector i:

$$\frac{\partial \Delta \ln \mu_{i,t}^v}{\partial g_j} = 1 + \alpha_i \gamma_j \xi_{ij} \text{ if } i = j,$$

where $(I - \Gamma_d \alpha_d - (I - \Gamma_d)\, \Phi')^{-1} = \{\xi_{ij}\}$, or

$$\frac{\partial \Delta \ln \mu_{i,t}^v}{\partial g_j} = \alpha_i \gamma_j \xi_{ij} \text{ if } i \neq j.$$

As mentioned above, growth rates in every sector depend on TFP growth rates in every sector because of the linkages between sectors in intermediate goods. The matrix $(I - \Gamma_d \alpha_d - (I - \Gamma_d)\, \Phi')^{-1} \Gamma_d$ suggests that, all else equal, TFP growth changes in sectors that are more capital intensive (i.e., where α_j is higher) and have higher shares of value added in gross output (i.e., where γ_j is higher) will tend to have larger effects on other sectors. Additionally, more capital-intensive sectors will tend to have larger responses to TFP growth changes in other sectors.

The expression for GDP gives us

$$GDP_t = \sum_{j=1}^{n} p_{j,t}^v v_{j,t}.$$

Using a standard Divisia index, we can express aggregate GDP growth as a weighted average of sectoral growth rates in real value added,

$$\Delta \ln GDP_t = \sum_{j=1}^{n} s_{j,t}^v \Delta \ln v_{j,t}, \qquad (14)$$

where $s_{j,t}^v$ is the share of sector j in nominal value added,[3]

$$s_{j,t}^v = \frac{p_{j,t}^v v_{j,t}}{\sum_{j=1}^{n} p_{j,t}^v v_{j,t}}.$$

Define $\boldsymbol{\Delta} \ln \mathbf{v}_t = \Delta \ln \mu_t^v$ along the balanced growth path. We may then substitute our expression for $\ln \mu_t^v$ in terms of TFP to obtain the

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[3] These shares also hold in normalized form, so that $s_{j,t}^v = \frac{\tilde{p}_{j,t}^v \tilde{v}_{j,t}}{\sum_{j=1}^{n} \tilde{p}_{j,t}^v \tilde{v}_{j,t}}$, and are constant along the balanced growth path. Here we take the shares as exogenous parameters given in the data, but they can alternatively be solved as part of the steady state in normalized variables.

balanced growth rate of real aggregate GDP in terms of TFP growth:

$$\Delta \ln GDP_t = \mathbf{s}^v \left[I + \alpha_d \left(I - \Gamma_d \alpha_d - (I - \Gamma_d) \, \Phi' \right)^{-1} \Gamma_d \right] g_z.$$

This last expression implies that, with constant shares,

$$\frac{\partial \Delta \ln GDP_t}{\partial g_z} = \mathbf{s}^v \left[I + \alpha_d \left(I - \Gamma_d \alpha_d - (I - \Gamma_d) \, \Phi' \right)^{-1} \Gamma_d \right], \qquad (15)$$

with the effect of a change in TFP growth in sector j on GDP growth then given by the jth element,

$$\frac{\partial \Delta \ln GDP_t}{\partial g_j} = \left(s_j^v + \sum_{i=1}^{n} s_i^v \alpha_i \gamma_j \xi_{ij} \right).$$

The above equation shows that TFP changes in sectors with higher shares of value added in gross output, and whose intermediates are more heavily used by other sectors, will have larger effects on changes in GDP growth.

Balanced Growth with No Capital

Consider the special case of our model with no capital accumulation, $\alpha_j = 0 \; \forall j$. Then the formula for value added in sector j becomes

$$v_{j,t} = z_{j,t} \ell_{j,t}.$$

Since labor supply, $\ell_{j,t}$, is already normalized as implied by the labor supply constraint, the normalizing factor for value added in sector j at time t, $\mu_{j,t}^v$, is simply $\mu_{j,t}^v = z_{j,t}$, so that along a balanced growth path $\Delta \ln \mu_t^v = g_z$. Then we have

$$\frac{\partial \Delta \ln \mu_t^v}{\partial g_z} = I, \qquad (16)$$

so a change in TFP growth in sector j changes value-added growth in sector j by the same amount and has no impact on value-added growth in other sectors, even though sector j is linked to other sectors through intermediate goods. From equation (16), in the model without capital, we then have along a balanced growth path

$$\frac{\partial \Delta \ln GDP_t}{\partial g_z} = \mathbf{s}^v, \qquad (17)$$

which has jth element s_j^v. Put another way, a change in TFP growth in sector j increases the growth rate of real aggregate GDP by that sector's share of value added in GDP. To a first order, the intermediate goods matrix Φ and other details are irrelevant as long as we know the value-added distribution of sectors.

In the rest of this paper, we match this model to the data with $n = 7$ sectors in order to quantify equations (13) and (15), and we also invert $\left[I + \alpha_d \left(I - \Gamma_d \alpha_d - (I - \Gamma_d) \Phi'\right)^{-1} \Gamma_d\right]$ in equation (11) to obtain the implied TFP growth rates in each sector. We also use equations (16) and (17) to compare our quantitative benchmark results to those in the case without capital.

5. DATA

As described above, the natural expressions of several model parameters as shares make it easy to match this model to available data. All of the model parameters, consisting of the Φ matrix, the γ_j's, and the α_j's, can be obtained through the Bureau of Economic Analysis (BEA), which provides data at various levels of industry aggregation going back to 1947.

The highest level of aggregation reported by the BEA is the fifteen-industry level. We drop one industry corresponding to Government, and then we consolidate the fourteen remaining industries into seven broader sectors: Agriculture, Forestry, Fishing, and Hunting; Mining and Utilities; Construction; Manufacturing; Wholesale and Retail Trade; Transportation and Warehousing; and Services. The seven-sector level is a high enough level of aggregation to give us a broad overview of the economy, and these constructed sectors closely match the six sectors examined by Long and Plosser (1983).

To assemble the Φ matrix for our benchmark year, 2014, we rely on data from the BEA's Make-Use Tables, which at the fifteen-industry level provide a fifteen-by-fifteen matrix showing all pairwise combinations of intermediate goods purchases by one industry from another. From here, we sum intermediate goods purchases across all industries in a sector and then calculate shares of nominal intermediates from sector i in sector j's total nominal intermediates accordingly (dropping intermediate purchases from the Government sector from the total). In addition to calculating the Φ matrix for 2014, we also calculate it for 1948, the earliest year for which data on value-added growth are available. Later on, we will be interested in comparing our results when using the Φ matrix for 1948 to those using the Φ matrix for 2014 to see how changes in intermediate purchases patterns across sectors have affected growth and TFP throughout the economy. The BEA provides the pairwise intermediates purchases at a higher level of disaggregation in 1948, with forty-six industries. Since every industry at the fifteen-industry level is a grouping of industries at the forty-six-industry level, we can sum intermediate goods purchases across industries in a sector as before.

We also use the BEA's Make-Use Tables to calculate each sector's share of nominal value added in nominal gross output, γ_j, for 2014 by summing total value added and total gross output across industries in a sector and dividing accordingly. To calculate shares of capital in nominal value added, α_j, we use the BEA's data on GDP by industry, which breaks down value added within an industry into the sum of wages paid to employees, a gross operating surplus, and taxes minus subsidies. We sum the first two components across industries in a sector, ignoring taxes and subsidies, and calculate α_j as sector j's gross operating surplus divided by the sum of its gross operating surplus and wages.

Finally, the BEA's GDP data include the total nominal value added for each industry at the fifteen-industry level for each year going back to 1947. We use the BEA's chain-type price indexes for value added in each industry to calculate these numbers in real terms, then sum across industries in a sector to obtain real value added for each sector. From here, we can easily calculate the real value-added growth rates for each sector for each year from 1948 through 2014 and take an average for each sector over this period to get mean value-added growth rates. Additionally, we can calculate a sector's share in nominal value added for each year (excluding value added from the Government sector in total value added) and average across years to obtain each sector's mean share in nominal value added.

Table 1 displays the share of nominal value added in nominal gross output, γ_j, and the share of capital in nominal value added, α_j, for each sector. Some of these results are fairly intuitive; for instance, Construction and Wholesale and Retail Trade have the lowest (highest) shares of capital (labor) in value added, while Agriculture, Forestry, Fishing, and Hunting, and Mining and Utilities are the most capital-intensive. There is somewhat less variation in the shares of nominal value added in nominal gross output, with Manufacturing having the lowest share and Mining and Utilities having the highest.

Table 2 displays the matrix summarizing intermediate goods linkages, Φ, calculated for 2014, where the element in row i and column j represents the percentage of all intermediate goods purchased by sector j that come from sector i. First, it is not surprising that most sectors purchase a large share of intermediate goods from within their own sector: five of seven sectors have ϕ_{jj} values above 20 percent, with the Services sector purchasing over 75 percent of its intermediates from itself. It is also important to note that, in general, the Φ matrix displays substantial asymmetry. The average sector buys approximately 35 percent and 29 percent of its intermediates from Services and Manufacturing, respectively. If we exclude the diagonal entries of Φ, these

Table 1 Parameter Values for Each Sector

Sector	Sector Number	γ_j	α_j
Agriculture, Forestry, Fishing, and Hunting	(1)	0.4139	0.7493
Mining and Utilities	(2)	0.6845	0.7337
Construction	(3)	0.5419	0.3659
Manufacturing	(4)	0.3462	0.5205
Wholesale and Retail Trade	(5)	0.6558	0.3680
Transportation and Warehousing	(6)	0.4795	0.3865
Services	(7)	0.6123	0.4556

Table 2 Φ in 2014, with All Numbers Expressed as Percentages

Sector Number	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	39.72	0.04	0.27	7.20	0.31	0.02	0.19
(2)	2.88	32.76	2.47	15.70	1.66	1.84	2.65
(3)	0.96	3.86	0.03	0.36	0.41	1.01	2.64
(4)	29.16	21.40	52.72	50.37	9.12	31.90	12.98
(5)	10.30	4.10	24.00	8.03	7.26	9.23	3.31
(6)	5.58	9.27	3.85	4.11	12.53	23.85	2.73
(7)	11.39	28.57	16.65	14.24	68.70	32.15	75.51

numbers are still 29 percent and 26 percent. On the other hand, Agriculture, Forestry, Fishing, and Hunting, and Construction stand out as relatively unimportant suppliers of intermediate goods to other sectors.

6. QUANTIFYING BALANCED GROWTH RELATIONSHIPS

As derived in equation (13), $\frac{\partial \Delta \ln \mu_t^v}{\partial g_z} = \left[I + \alpha_d \left(I - \Gamma_d \alpha_d - \left(I - \Gamma_d \right) \Phi' \right)^{-1} \Gamma_d \right]$ in the benchmark model. Table 3 shows this matrix for our seven sectors. The element in row i and column j shows the percentage-point increase in value-added growth in sector i resulting from a 1 percentage point increase in TFP growth in sector j. Unsurprisingly, increases in TFP growth in sector j have by far the largest impact on value-added growth rates in that same sector; all the entries on the diagonal have magnitude greater than 1, with Mining and Utilities having the largest diagonal value and Construction having the smallest. However, the off-diagonal entries still indicate substantial effects of TFP growth changes in one sector on value-added growth in another. For instance, a 1 percentage point increase in TFP

Table 3 Effect of 1 Percentage Point Change in TFP Growth on Value-Added Growth in Percentage Points

Sector Number	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	1.7131	0.2099	0.0160	0.2751	0.1512	0.0726	0.4271
(2)	0.0187	2.3645	0.0221	0.1456	0.0615	0.0572	0.3818
(3)	0.0135	0.0692	1.2507	0.1032	0.0669	0.0189	0.1502
(4)	0.0536	0.2371	0.0090	1.4316	0.0801	0.0405	0.2862
(5)	0.0048	0.0295	0.0035	0.0332	1.3409	0.0211	0.2065
(6)	0.0118	0.0653	0.0059	0.0925	0.0454	1.2808	0.2153
(7)	0.0075	0.0500	0.0090	0.0538	0.0252	0.0146	1.7053

growth in the Services sector increases value-added growth in Agriculture, Forestry, Fishing, and Hunting by about 0.43 percentage points. Overall, increases in TFP growth rates in the Services sector have particularly strong effects on value-added growth rates in other sectors, reflecting the generally high usage of intermediate goods from Services by other sectors. On the other hand, changes in TFP growth in other sectors have small effects on value-added growth in Services, in part because Services purchases a small fraction of its intermediates from other sectors. (These observations apply, to a somewhat lesser extent, to the Manufacturing sector as well.) Increases in TFP growth rates in sectors such as Construction and Agriculture, Forestry, Fishing, and Hunting, whose intermediates are not heavily used by other sectors, have tiny effects on value-added growth in other sectors. Finally, it is worth noting that Mining and Utilities and Agriculture, Forestry, Fishing, and Hunting, whose α_j values are substantially higher than those of other sectors, are, on average, the most responsive to sectoral TFP growth changes.

In the case with no capital, a TFP growth change in sector j changes value-added growth in sector j by the same amount and has no impact on value-added growth in other sectors. Since all the diagonal entries of the matrix $\left[I + \alpha_d \left(I - \Gamma_d \alpha_d - (I - \Gamma_d) \Phi' \right)^{-1} \Gamma_d \right]$ have values above 1, linkages increase the own-sector effect of TFP growth rate increases on value-added growth rates in every sector.

Given data on shares of each sector in nominal value added, we can then calculate the effect of changes in TFP growth in each sector on changes in aggregate GDP in the benchmark model according to equation (15). As described above, we compile data on sectoral shares in nominal value added for each year in the period 1948–2014, and then we take the mean shares in nominal value added for each sector over this period. Table 4 shows $\frac{\partial \Delta \ln GDP_t}{\partial g_z}$ calculated from these mean shares

**Table 4 Effect of 1 Percentage Point Change in TFP Growth on GDP Growth in Percentage Points**

Sector	No Capital	Benchmark	Difference
Agriculture, Forestry, Fishing, Hunting	0.0297	0.0695	0.0398
Mining and Utilities	0.0457	0.2026	0.1569
Construction	0.0502	0.0712	0.0210
Manufacturing	0.2332	0.3868	0.1536
Wholesale and Retail Trade	0.1552	0.2505	0.0953
Transportation and Warehousing	0.0425	0.0794	0.0369
Services	0.4435	0.9020	0.4585

for both cases. The first column shows the case with no capital, where each entry just equals that sector's mean share in total nominal value added. Two of the seven sectors, Services and Manufacturing, account for over two-thirds of total nominal GDP, on average. The second column shows the benchmark case, and the difference between the two cases in the third column can be interpreted as the total multiplier effect of a change in TFP growth in one sector on other sectors (including itself).

Figure 1 plots the mean value-added shares against $\frac{\partial \Delta \ln GDP_t}{\partial g_z}$ computed in the benchmark. The size of the deviation from the forty-five-degree line indicates the size of the multiplier effects on other sectors. In absolute terms, this multiplier effect is by far the largest for the Services sector, in part reflecting the fact that the off-diagonal entries of the matrix $(I - \Gamma_d \alpha_d - (I - \Gamma_d) \Phi')^{-1} \Gamma_d$ are, on average, the highest for the column corresponding to Services. There are also large increases for Manufacturing, another sector important in the production of intermediate goods, and Mining and Utilities, which has a multiplier effect over three times as large as its share in GDP. This can be largely explained by the sector's high share of capital in value added and its importance as an intermediate goods supplier to itself and to the second-largest sector, Manufacturing.

To see the extent to which changes in the usage of intermediate goods across sectors, summarized in Φ, have impacted the effect of TFP growth changes in a sector on changes in the growth rate of GDP, we also recompute $\frac{\partial \Delta \ln GDP_t}{\partial g_z}$ using the Φ matrix in 1948. Figure 2 plots $\frac{\partial \Delta \ln GDP_t}{\partial g_z}$ calculated in the benchmark using Φ from 2014 against the values calculated from 1948. Because we hold the other parameters constant for each sector, any changes should result from changes in the relative importance of sectors as intermediate goods suppliers to other sectors. As noted by Choi and Foerster (2017), there have been

Figure 1 Derivative of GDP Growth with Respect to Sector TFP Growth
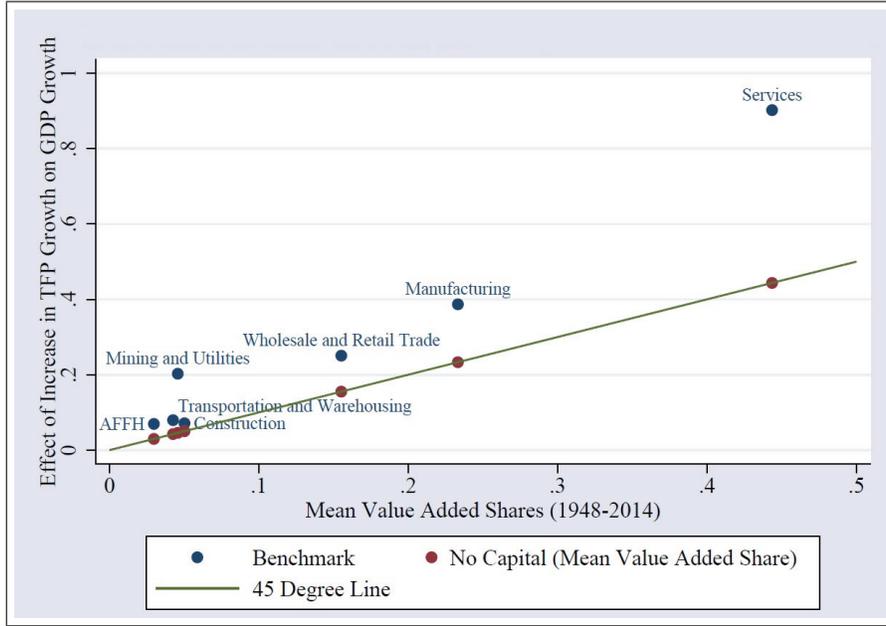
significant changes in the US economy's input-output network structure over this period. In particular, the Services sector is a markedly more important supplier of intermediate goods in 2014 than it was in 1948, driven by the increasing centrality of financial services, real estate, and other industries within this sector. On the other hand, sectors such as Manufacturing; Agriculture, Forestry, Fishing, and Hunting; and Mining and Utilities declined in importance over this period.

Consistent with these observations, Services saw the largest absolute increase in $\frac{\partial \Delta \ln GDP_t}{\partial g_z}$ over this period, while Manufacturing saw the largest absolute decrease, and Agriculture, Forestry, Fishing, and Hunting saw the largest percentage decrease. On the other hand, because $\frac{\partial \Delta \ln GDP_t}{\partial g_z}$ also depends on the shares of each sector in total nominal value added, a sector may decline in overall importance, as measured by its row total in Φ, over this period while still having an increasing value of $\frac{\partial \Delta \ln GDP_t}{\partial g_z}$. For example, Mining and Utilities declines in overall importance between 1948 and 2014 but it is a much more important supplier of intermediates for the Manufacturing sector

Figure 2 Effect of TFP Growth on GDP Growth, 1948 Φ vs. 2014 Φ
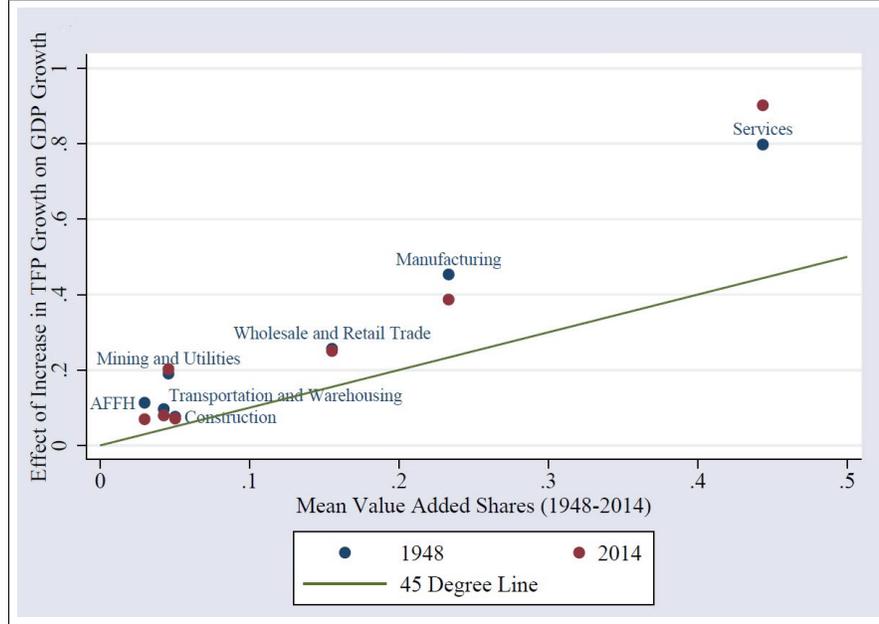
in 2014 than in 1948, largely explaining why Mining and Utilities sees a slight overall increase in $\frac{\partial \Delta \ln GDP_t}{\partial g_z}$.

As a final exercise, given data on value-added growth, we can invert the matrix $I + \alpha_d \left(I - \Gamma_d \alpha_d - (I - \Gamma_d) \Phi'\right)^{-1} \Gamma_d$ to obtain the implied TFP growth rates in the benchmark:

$$g_z = \left[I + \alpha_d \left(I - \Gamma_d \alpha_d - (I - \Gamma_d) \Phi'\right)^{-1} \Gamma_d\right]^{-1} \Delta \ln \mu_t^v. \qquad (18)$$

With no capital, this expression simply becomes

$$g_z = \Delta \ln \mu_t^v. \qquad (19)$$

For each of our seven sectors, we take an average of their real value-added growth rates over the period 1948-2014 and then calculate the implied mean TFP growth rates over this period. Figure 3 plots observed mean value-added growth against the model-implied mean TFP growth in the benchmark case and the case with no capital, where in the latter case all points lie on the forty-five-degree line. In the benchmark, all points lie well to the left this line. The decrease is largest in absolute terms for Agriculture, Forestry, Fishing, and Hunting and, consistent with intuition, is generally larger for sectors with larger val-

Figure 3 Implied Mean TFP Growth, 1948-2014
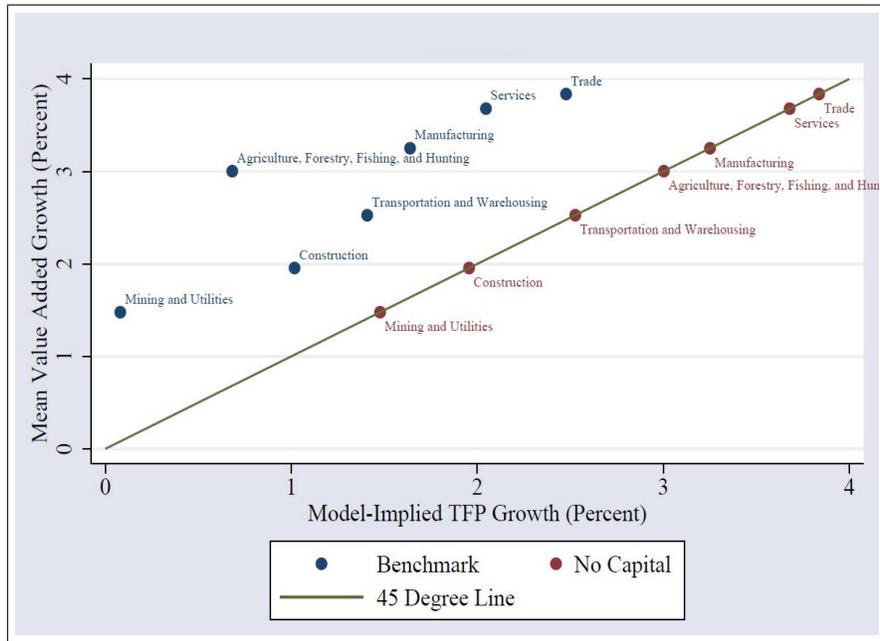
ues of α_j. The implied mean TFP growth for Mining and Utilities is just 0.08 percent.

Additionally, for the benchmark case, we calculate implied mean TFP growth rates using the Φ matrix for 1948 and compare the results to those using the Φ matrix for 2014. As shown in Figure 4, changes in patterns of intermediate goods usage between 1948 and 2014 have very little impact on implied mean TFP growth rates.

7.   CONCLUSION

Our analysis suggests that linkages between sectors in intermediate goods, and capital intensities of different sectors, lead to substantial effects of sector-specific TFP growth changes on value-added growth. TFP growth changes in sectors such as Manufacturing and Services, which account for a large share of the intermediate goods shares of other sectors, have especially large impacts on value-added growth in other sectors. On the other hand, changes in the input-output structure of the US economy from 1948 to 2014 have had a modest impact on

Figure 4 Implied Mean TFP Growth, 1948 Φ vs. 2014 Φ
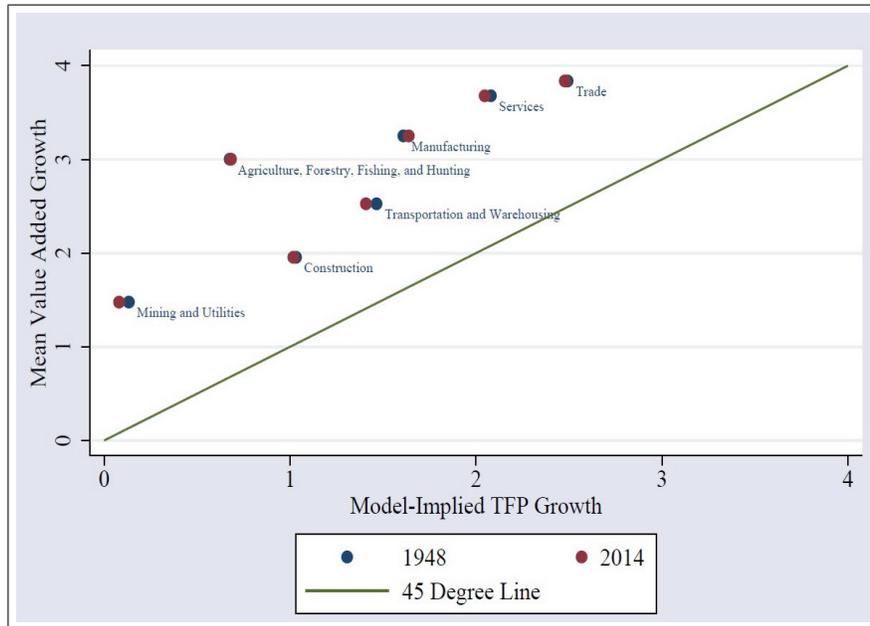
TFP growth in each sector and on the effect of TFP growth changes on GDP growth.

It is worth noting that our analysis here relies on a very high level of aggregation, with only seven sectors, and every sector uses some positive amount of intermediate goods from every other sector. Horvath (1998), Foerster, Sarte, and Watson (2011), and others have found that, at more disaggregated measures of sectors, there is more variability across sectors and the asymmetry of the matrix summarizing intermediate goods linkages substantially increases; many rows consist of mostly zeros, and a few sectors provide most of the economy's intermediate goods. Thus, our results most likely underestimate the degree of heterogeneity in the impact of sectoral changes at lower levels of aggregation.

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