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Interest on Reserves and Daylight Credit

Huberto M. Ennis and John A. Weinberg

Banks hold reserves in the form of account balances at the central bank and vault cash. The average aggregate reserves of depository institutions in the United States during 2005 was $46 billion. Banks use these reserves to settle payments to other banks (and other participants in financial markets) during the day. In 2005, the average daily value of Fedwire fund transfers—the primary means by which banks transfer funds to one another—was approximately $2 trillion; that is, nearly 50 times the quantity of reserves. When reserves do not pay interest overnight, banks face an opportunity cost from holding reserves overnight. However, if overnight overdrafts resulting from ending the day with insufficient reserves imply a penalty (in terms of higher interest rates or other types of penalties), then holding reserves may also be associated with the benefit of avoiding potential overdrafts. On average, during 2005 banks held a total of $1.7 billion in excess reserves; that is, reserves in excess of required reserves (see Table 1).

In September 2006, Congress passed legislation that authorized the Federal Reserve to pay interest on banks' reserve balances, beginning in 2011. The legislation also granted the Board of Governors additional flexibility in setting reserve requirements for depository institutions after October 1, 2011. According to this new legislation, the Federal Reserve can pay interest on all types of balances, including required reserves, supplemental reserves, and contractual clearing balances, held by or for depository institutions at a reserve bank. Such interest, if authorized by the Board, may be paid at least once each

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Table 1 Total Reserves at Depository Institutions (Million $)

Date	Total Reserves	Non-Borrowed Reserves	Required Reserves	Excess Reserves
June 2003	42,025	41,864	39,980	2,046
December 2003	42,949	42,903	41,906	1,043
June 2004	45,720	45,540	43,787	1,933
December 2004	46,848	46,785	44,938	1,909
June 2005	45,950	45,701	44,176	1,774
December 2005	45,406	45,237	43,497	1,909
June 2006	45,067	44,814	43,282	1,785
October 2006	41,756	41,528	40,058	1,698

Notes: Federal Reserve Statistical Release H.3, Table 2, Aggregate Reserves of Depository Institutions and the Monetary Base (not adjusted for changes in reserve requirements and not seasonally adjusted) (http://www.federalreserve.gov/releases/h3/).

calendar quarter at a rate or rates not to exceed the general level of short-term interest rates.

This new legislation represents a significant change in policy that could affect the choices that banks make about reserve holdings. And, since most central banks conduct monetary policy by intervening in the daily market for banks' reserves, this change could affect the implementation of monetary policy as well by altering the behavior of the demand for reserves. Since paying interest reduces the opportunity cost for a bank of being "stuck" with unused reserves overnight, banks may become willing to hold greater reserves. But the demand for reserves depends not only on this opportunity cost, but also on the benefit of avoiding the need to borrow to make up for a reserves shortfall. It is also likely that the demand for reserves depends on the nature of the payments for which the reserves will be used.

In the settlement of payments during a business day, banks' reserves are supplemented by access to intraday credit from the Fed. If one bank seeks to send funds in excess of its reserve balance through the Fedwire system to another bank, the sender incurs a daylight overdraft. So reserves and daylight credit act as a substitute means of funding transfers during the day. The treatment of reserves overnight, though, can influence the degree to which banks rely on daylight credit to cover their daylight payment activity.

The opportunity cost of holding reserves is most directly affected by the central bank's interest rate policy. A bank's willingness to substitute away from reserves for payment purposes is directly affected by the terms on which the central bank provides daylight credit. In this article, we are interested in the link between these terms and the terms on overnight reserves. We provide a simple model of the demand for reserves by banks (in line with the classic contribution by Poole 1968) and study the implications of paying interest on reserves on the conduct of monetary policy and the use of daylight credit by banks.

One important public policy dimension with regard to daylight credit is absent from our model. Specifically, the model abstracts from credit risk incurred by the central bank. When banks settle payments by drawing on central bank credit, the result is to shift credit risk exposure from private counterparties to the central bank. Central banks have a number of tools available for managing this exposure, from the pricing of daylight credit to the imposition of credit caps or collateral requirements. To address these and other public policy questions adequately would require a more complete, general equilibrium model. The model we examine is meant to isolate some key forces that we think would be at work in the joint determination of the demand for reserves and for daylight credit in a reasonable, more general, model. Understanding the forces driving daylight credit is important because of the potential for overuse of underpriced central bank credit and the associated misallocation of risk.

Before presenting the model, we discuss in Section 1 some basic observations about reserves, payments, and credit in the Fed's large-value payment system. Section 2 introduces the basic model of banks' demand for reserves and the determination of the equilibrium interest rate in the market for reserves. Banks' demand for reserves in our model is purely voluntary. No reserve requirements are assumed. The reason banks hold reserves in our model is because reserves are useful for making payments. The alternative assets, in our case bonds, have a positive overnight rate of return premium but cannot be used to make payments. If the bank does not have enough reserves to settle its payments, it has to resort to central bank credit. Overnight overdrafts, in particular, are subject to a penalty rate that banks want to avoid paying. In other words, banks hold reserves to limit their exposure to overdraft penalties.

In Section 3, we introduce the central bank's ability to pay interest on unused reserves. We show how interest on reserves allows the central bank to fix the market interest rate at a target level by "flooding" the market with reserves and fixing the interest on reserves at the chosen target. This policy was first proposed by Goodfriend (2002) and the model provides a formalization of his argument. The model also allows a precise description of an alternative approach to paying interest on reserves. In this approach, the central bank pays a rate at a fixed spread *below* the target market rate, which, together with an overnight lending rate at a fixed spread *above* the target, creates a "corridor" around the market rate.

Sections 2 and 3 consider the demand for reserves in the absence of a potential payments-related need for daylight credit. However, as noted by Lacker (2006) and as suggested by the interdependence discussed above, the ability of the central bank to pay interest on reserves may have relevant implications for the daylight credit policy that the central bank may find optimal.

Section 4, then, extends the model in Sections 2 and 3 to take into consideration the determinants of the daylight credit decisions of banks. We show how interest on reserves can motivate banks to economize the use of daylight credit without reducing their access to liquidity during the day.

Our simple model allows us to demonstrate a number of interesting features of the mechanics of the markets for reserves. For instance, in a corridor system, there are circumstances in which a central bank can implement a change in the target market rate without changing the supply of reserves, simply by moving its lending rate and its rate on reserves together. But this result requires that aggregate demand for reserves—which is driven in the model by aggregate payment requirements—be relatively stable. With greater variability in demand, the task of implementing the target rate is simplified by the approach proposed by Goodfriend of paying interest at the target rate. When intraday variation in the timing of payments is added, which creates a potential demand for daylight credit, eliminating the opportunity cost of holding reserves by paying interest at the target rate has the added effect of greatly reducing the demand for daylight credit.

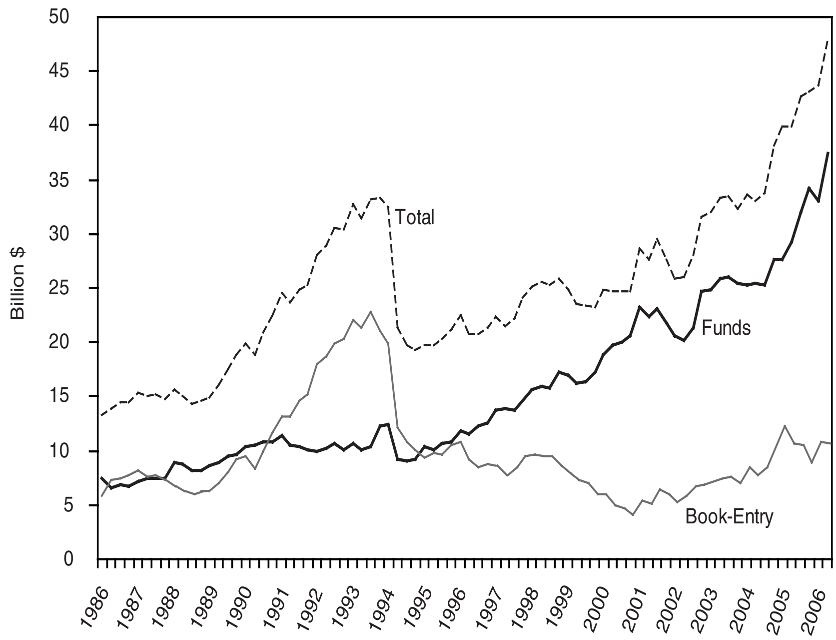
1. U. S. PAYMENTS AND RESERVES

Systems for clearing and settling large-value payments among banks are often categorized according to their approach to settlement. Systems in which payments are settled one-by-one through the transfer of central bank money throughout the day are typically referred to as real-time gross settlement systems (RTGS). The alternative is net settlement, in which payments are held until the end of a settlement period, typically a day, and only net obligations are actually transferred. Zhou (2000) provides a good introduction to these differences, and Kahn and Roberds (1999) discuss in detail the comparative advantages and disadvantages of the alternative systems.

A notable difference between these two alternative ways of organizing (large-value) payment systems is that a daily net settlement arrangement involves the creation of intraday credit exposures among its members. By contrast, in an RTGS system, bilateral obligations are extinguished throughout the day. Because of possible mismatches in the timing of receipts and payments during the day, participants in an RTGS system may demand credit to cover early payments when they are expecting later receipts. In some of these systems, intraday credit is provided by the central bank.

For the most part, large-value payments in the United States are executed using one of the two main systems: Fedwire and CHIPS (Clearing House Interbank Payments System). Fedwire has two subsystems: Fedwire Funds Transfer and Fedwire Book-Entry Securities. The Fedwire Funds Transfer system is a real-time gross settlement system of funds transfers across Federal Reserve accounts of participants. The Fedwire Book-Entry Securities system

Figure 1 Average Daylight Overdraft (Quarterly Data)



Notes: Federal Reserve Board Payment System Risk data. Average daylight overdrafts are calculated based on a 21.5-hour Fedwire operating day (http://www.federalreserve.gov/paymentsystems/psr/).

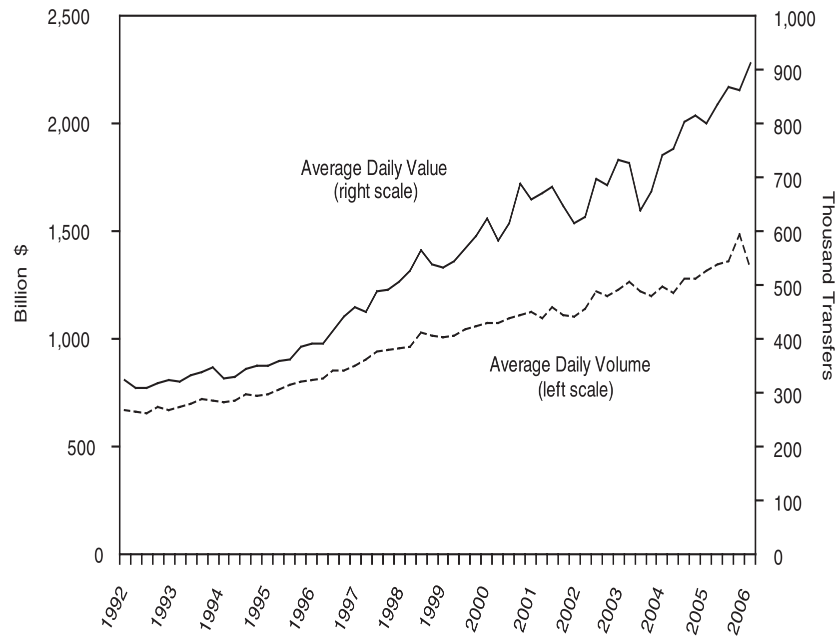
is a real-time, delivery-versus-payment, gross settlement system that allows for the immediate, simultaneous (electronic) transfer of government securities against payment.[1]

CHIPS is a bank-owned payment system operated by the New York Clearing House to clear and settle business-to-business transactions. On January 22, 2001, CHIPS converted from an end-of-day, multilateral net settlement system to one that provides final settlement for all payment orders as they are released. Payment instructions submitted to the queue that remain unsettled at the end of the day, known as the residual, are tallied on a multilateral net basis. Banks pre-fund their CHIPS payments with a Fedwire transfer from their reserve accounts at the Fed at the beginning of the day.

To facilitate the normal flow of payments in the system, the Federal Reserve provides daylight credit to depository institutions. In this context, the Federal Reserve has adopted an explicit program to control the use of intraday

[1] A delivery-versus-payment system is a mechanism that ensures that the final transfer of one asset occurs if and only if the final transfer of another asset occurs.

Figure 2 Fedwire Funds Transfers (Quarterly Data)



Notes: Federal Reserve Board Fedwire Funds Service quarterly data. The average daily volume and average daily value of transfers are based on the number of business days in each period (http://www.federalreserve.gov/paymentsystems/fedwire/).

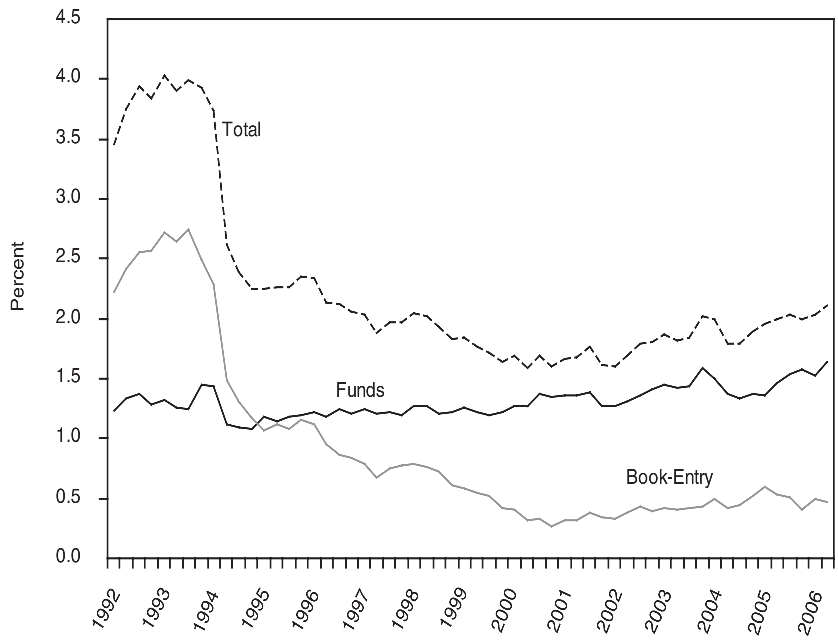
credit, the Payments System Risk (PSR) policy (Coleman [2002] provides a good introduction to the evolution of the PSR policy of the Fed). The two main instruments of the PSR policy are the imposition of net debit caps and interest rate fees on daylight overdrafts. The objective is to limit excessive use of daylight credit and, therefore, reduce the Fed's exposure to credit risk.

In 1985, the Fed introduced net debit caps for the first time. Net debit caps limit the maximum daylight overdraft position that a depository institution can incur in its Federal Reserve account. These debit caps did not have a great influence on the expansion of daylight credit that was taking place at the time, and, in 1994, the Federal Reserve started imposing a minute-by-minute interest charge on the average daylight overdraft that each institution incurred during the business day.

Figure 1 shows the large drop on average daylight overdraft after that change in policy.[2] (For a careful statistical analysis of the effect of caps and

[2] Most of the reduction in daylight overdrafts after the change in policy in 1994 was due to a reduction in securities-related overdrafts. Charging fees provided a strong incentive for securities

Figure 3 Average Daylight Overdraft as a Percentage of Average Daily Volume of Transfers (Quarterly Data)



Notes: Federal Reserve Board data (see Figures 1 and 2). Average daylight overdrafts are calculated based on a 21.5-hour Fedwire operating day (http://www.federalreserve.gov/paymentsystems/psr/).

fees on the level of daylight overdrafts in the United States, see Hancock and Wilcox 1996. See also Mills and Nesmith 2006.) It is important to note, however, that the number and value of Fedwire transactions has been trending upwards during this period (see Figure 2) and that, in fact, the value of the average daylight overdraft, as a percentage of the value of total transactions over Fedwire, has remained relatively stable at around 1.8 to 2.0 percent for the last ten years, displaying perhaps a slight upward trend (see Figure 3).

The need for credit in the payment system is determined by banks' holdings of reserves. The reserve positions of banks are, in turn, determined by an array of factors, including legal reserve requirements and the price of borrowing reserves, either on the federal funds market or from the Fed's discount window.

dealers to adopt practices that reduced the use of intraday credit; in particular, they substantially revised their repo settlement practices. Net debit caps were reduced by 25 percent in early 1988 and daylight credit initially fell (approximately 5.5 percent) but then continued growing at an accelerated pace until 1994, as seen in Figure 1. The data on peak daylight credit present a similar pattern to that for average daylight credit presented here.

Desired reserves could also depend on the cost of borrowing from the Fed within the day. For a given level of payment activity, daylight overdrafts will typically decrease as reserve holdings of banks increase. The model in the next section provides a first step in formalizing some of the relationships among payments, demand for reserves, and interest rates, which are essential for understanding how modern payment systems function.

2. A SIMPLE MODEL

We start our analysis with a very simple model that allows us to capture some of the tradeoff faced by banks in their management of reserves. In fact, in the next two sections we abstract from issues related to daylight credit and keep the model and analysis as simple as possible. These sections provide a good introduction to the main forces driving the determination of the interest rate in the market for reserves. Later, in Section 4, we extend the model in a natural way and discuss the connection among daylight credit, reserve management by banks, and the market interest rate.

We start our study of the simple model by first describing the decision problem faced by a typical bank. The solution to this problem delivers the demand for reserves for each individual bank (Poole 1968). After that, we consider the situation in which there are many banks and aggregate their demands to obtain the demand for reserves in the market. Finally, we study the determination of interest rate as a result of a standard equalization of (aggregate) demand and supply.

The Bank's Problem

Let us start with a simplified setup. Suppose the bank has a given amount of funds, F, that will be used to execute some payments for the same amount. While the the total amount of payments, F, is known with certainty, payments can happen at the end of the day or next morning. The bank can decide to hold these funds in either of two possible assets, reserves (R) or bonds (B). That is, we have that

$$F = R + B.$$

Let P be the payment that the bank has to make at the end of the day. The next-morning payment will then be equal to $F - P$. Suppose that bonds cannot be used to settle payments, and the bank must decide the allocation of funds between reserves and bonds before knowing the exact amount P. Also, for simplicity, assume that payments today are only credited in the recipient's account the next day; that is, the bank does not expect to receive new funds that would increase its end-of-day balances. Then, if the amount of reserves

R held by the bank is lower than the required end-of-day payment P, the bank incurs an overnight overdraft for the value $P - R$.[3]

For concreteness, assume that the size of the end-of-day payment P is uniformly distributed in the interval $\left[0, \overline{P}\right]$. The assumption about the distribution of P is just for the sake of simplicity; it implies that the size of the payment can take any value in the interval $\left[0, \overline{P}\right]$, with the probability of observing any particular one of these values being the same. More importantly, the probability that P is smaller than an arbitrary value $x \in \left[0, \overline{P}\right]$ is given by $p(x) = x/\overline{P}$, and the average value of P conditional on being greater than x is given by

$$E_{x+}P = \frac{\overline{P} + x}{2}.$$

Let r be the (overnight) rate of return on bonds and r_o the interest rate on overnight overdrafts. We consider the case where the overnight overdraft rate implies a penalty; that is, $r_o > r$. Reserves give no return but can be used to cover part (or all) of the payment P. Throughout this article we assume that $F > \overline{P}$.

The overnight expected return for the bank, denoted by Π, is then given by

$$\Pi = [1 - p(R)] \left[rB - r_o (E_{R+}P - R)\right] + p(R)rB.$$

The first term tells us that, with probability $1 - p(R)$, the bank needs to make an end-of-day payment P greater than R and, hence, the bank has to incur an overdraft. The expected overdraft is given by the amount $E_{R+}P - R$. With probability $p(R)$, the payment P is smaller than the total reserves held by the bank and the bank just gets the normal return on its bond holdings r, B.

Rearranging the expression for the bank's return we have that

$$\Pi = rB - [1 - p(R)]\, r_o (E_{R+}P - R).$$

Using the equation $F = B + R$ and substituting the expression for $E_{R+}P$ and $p(R)$, we can rewrite the expression for Π as

$$\Pi = r(F - R) - \left(1 - \frac{R}{\overline{P}}\right) r_o \left(\frac{\overline{P} - R}{2}\right),$$

which again can be rewritten as

$$\Pi = rF - \left[rR + \frac{r_o}{2\overline{P}} \left(\overline{P} - R\right)^2\right].$$

[3] Alternatively, we could interpret P as the value of the required payments net of any new balances arriving late in the day. Our simplistic assumption about turnover of reserve flows facilitates the analysis of equilibrium but is not essential for the results. However, within-the-day turnover brings about a number of other interesting issues that we do not discuss in this article (see Beyeler et al. 2006 for a careful study regarding this issue).

The bank will choose its level of reserves, R, to maximize its overnight expected return Π. Then, when $r_o > r > 0$, the demand for reserves by the typical bank is given by

$$R^* = \frac{(r_o - r)}{r_o}\overline{P}. \tag{1}$$

This expression tells us that, when the interest rate on bonds, r, increases, the bank will lower the amount of reserves held (the opportunity cost of holding reserves is higher). Also, as the size of the possible payments increases (that is, as $\overline{P}$ increases), *ceteris paribus*, the bank will choose to hold higher levels of reserves (reserves are more likely to be useful in avoiding overdrafts). It is a little less obvious to see, yet still true, that if the value of the overdraft interest rate, r_o, increases, the optimal level of reserves, $R,^*$ also increases. Finally, notice that for $r \geq r_o$ the bank will demand zero reserves and for $r = 0$ the bank will hold any amount of reserves between $\overline{P}$ and F.

We have assumed that the total amount of funds held by the bank is fixed, and equal to F.[4] Under this assumption, equation (1) has an alternative interpretation. The equation tells us that the penalty premium on overnight overdrafts, given by $r_o - r$, determines the *composition* of the bank's portfolio between bonds and reserves. Reserves are held to avoid paying the penalty premium. However, reserves do not gain interest overnight. Hence, holding reserves also has an opportunity cost. The bank balances these costs and benefits to determine the optimal composition of its portfolio. In this simple model, the only reason for banks to hold reserves is to avoid paying the overnight penalty rate. If $r_o = r$, then there are no benefits of holding reserves (while there is still an opportunity cost), and the proportion of funds held as reserves is zero.

The Market for Reserves

Normally, there are many banks interacting in the market and deciding their optimal level of reserves.[5] In principle, we can aggregate all their demands to obtain the market demand. Recall that the demand for reserves of bank i is given by

$$R_i^* = \begin{cases} 0 & \text{if} \quad r \geq r_o \\ \frac{(r_o - r)}{r_o}\overline{P}_i & \text{if} \quad r_o > r > 0 \\ \left[\overline{P}_i, F\right] & \text{if} \quad r = 0. \end{cases}$$

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[4] Then, in fact, we could normalize F to unity, in which case R and B could be interpreted as the proportion of funds held in reserves and bonds, respectively.

[5] See Bartolini et al. (2005) for a more detailed discussion of the overnight federal funds market in the United States where over 7,500 institutions with accounts at the Federal Reserve borrow and lend reserve balances on an uncollateralized basis.

Note that r_o and r are market prices common to all banks, but the distribution of likely end-of-day payments may differ across banks, and hence $\overline{P}_i$ may differ across banks.

For simplicity we will assume that there is a large number (a continuum) of banks (with mass equal to one). Then, when $r_o > r > 0$ the total demand for reserves in the market is given by

$$R^d = \int_0^1 R_i^* di = \frac{(r_o - r)}{r_o} E\overline{P},$$

where $E\overline{P}$ is the average value (across banks) of the maximum possible required payment. Note that in our simple model, the expected payment requirement for bank i is given by $\overline{P}_i/2$ and the average across banks is then equal to $E\overline{P}/2$. Hence, the variable $E\overline{P}$ characterizes the level of payment requirements in the economy.

Let us also assume that, each day, the aggregate volume of end-of-day payments can be in either of two possible states, high or low.[6] In other words, some days the required payments (on average) tend to be high, and some days they tend to be low. We capture this idea by allowing $E\overline{P}$ to take two possible values, $E\overline{P}_H$ and $E\overline{P}_L$, with $E\overline{P}_H > E\overline{P}_L$ and the probability of $E\overline{P}_H$ equal θ (hence, the probability of $E\overline{P}_L$ equals $1 - \theta$).

Here, for simplicity, we assume that banks know the level of aggregate required payments before choosing their demand for reserves. Note, then, that for a given value of r_o, the aggregate demand for reserves is a function of the interest rate r and the level of required payments indexed by $E\overline{P}_j$. Then, we can write $R_j^d(r) \equiv R^d(r; E\overline{P}_j)$ with j being equal to H and L.

By buying and selling bonds in exchange for reserves, the central bank controls the relative supply of reserves available in the system. Given a value of the supply of reserves, R^s, there exists an interest rate r^* that clears the market; that is, there exists an interest rate r_j^* such that

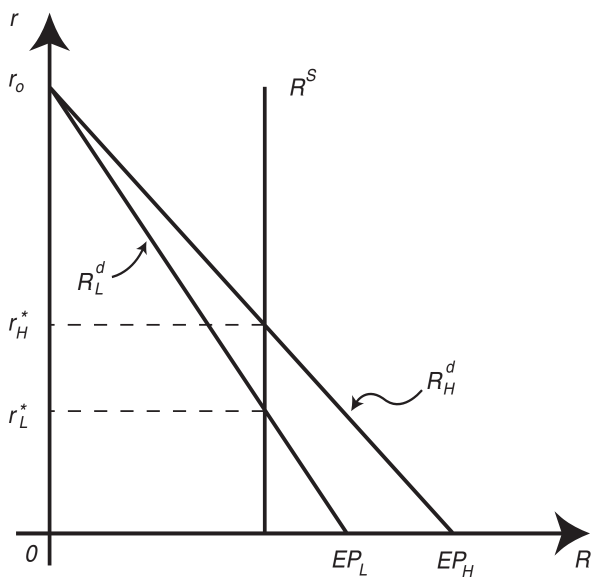
$$R_j^d(r_j^*) = R^s, \tag{2}$$

with $j = H, L$. Figure 4 provides a graphic representation of this market-clearing condition.

Equation (2) defines an implicit function $r_j^*(R^s)$ for the market-clearing interest rate given a supply of reserves, R^s. In particular, if $R^s \in (0, E\overline{P}_L)$

[6] One possible interpretation of the fluctuations in the aggregate (net) volume of payments by banks is that the fluctuations originate in flows between the government and the banking system. Dotsey (1991) and Guthrie and Wright (2000) take this interpretation in their study of New Zealand's system. See also Bartolini, Bertola, and Prati (2002) for a study of the U.S. system that uses a similar interpretation.

Figure 4 The Market for Reserves



we have that

$$r^*_j(R^s) = r_o\left(1 - \frac{R^s}{E\overline{P}_j}\right). \tag{3}$$

Then, a higher supply of reserves, R^s, implies a lower market interest rate. Also, it is easy to see that (for a given R^s) the market interest rates satisfy $r^*_H > r^*_L$. In other words, if the central bank were to fix the supply of reserves, R^s, the interest rate would be higher in periods of high payment requirements and lower in periods of low payment requirements.

We can also think of this relationship between the market interest rate and the supply of reserves in a slightly different way. Suppose that the central bank wants the interest rate to be equal to some target level r_T. Then, there is a level of the supply of reserves R^s_j such that

$$r^*_j(R^s_j) = r_T.$$

In particular, we have that if $r_T \in (0, r_0)$ then

$$R^s_H = \frac{r_o - r_T}{r_o}E\overline{P}_H > \frac{r_o - r_T}{r_o}E\overline{P}_L = R^s_L;$$

that is, to maintain a given target interest rate, the central bank has to provide a higher supply of reserves in periods of high payment requirements.

Now, suppose that the central bank has a target interest rate but, for some reason, has to decide the supply of reserves before knowing whether the level of payment requirements in the banking system will be high or low. In principle, the central bank will try to predict the level of demand for reserves. However, the predictions may not be perfect. In this case, a possible strategy the central bank could follow is to fix the supply of reserves such that the *average* interest rate equals the target rate. The market rate, then, will fluctuate around the target rate, being higher than the target rate in periods of high demand (that is, when $E\overline{P} = E\overline{P}_H$) and lower than the target rate during period of low demand (that is, when $E\overline{P} = E\overline{P}_L$).

To see this in the model, notice that the central bank would choose the supply of reserves R^{sT} such that

$$\theta r_H^*(R_T^s) + (1-\theta)\, r_L^*(R_T^s) = r_T. \qquad \text{(TC)}$$

To simplify the exposition, let us define the variable $\eta_j \equiv 1/E\overline{P}_j$ and concentrate our attention on the case where r_j^* is lower than r_o and positive for both $j = H$ and $j = L$.[7] Then, using expression (3) we have that the average interest rate can be rewritten as follows:

$$\theta r_H^*(R_T^s) + (1-\theta)\, r_L^*(R_T^s) = \theta r_o \left(1 - \eta_H R_T^s\right) + (1-\theta)\, r_o \left(1 - \eta_L R_T^s\right),$$

Reorganizing terms, the target condition (TC) becomes

$$r_o \left(1 - \overline{\eta} R_T^s\right) = r_T,$$

where $\overline{\eta} = \theta \eta_H + (1-\theta)\, \eta_L$. Equivalently, we can rewrite the above condition as

$$R_T^s = \frac{r_o - r_T}{r_o} \frac{1}{\overline{\eta}},$$

which tells us that to implement a higher average (target) rate the central bank will need to provide a lower supply of reserves. Note that $\eta_H \leq \overline{\eta} \leq \eta_L$ and that $\overline{\eta}$ is a decreasing function of θ. This property of $\overline{\eta}$, in turn, implies that when the probability of a high demand for reserves increases, the central bank, to target the same average rate of interest, will need to supply a higher amount of reserves. Finally, note that the market interest rate will be given by

$$r_j^*(R_T^s) = r_o \left(1 - R_T^s \eta_j\right).$$

Using the expression for R_T^s we have that

$$r_j^* = r_T + (r_o - r_T) \left(\frac{\overline{\eta} - \eta_j}{\overline{\eta}} \right),$$

which implies that $r_H^* \geq r_T$ (since $\overline{\eta} \geq \eta_H$), $r_L^* \leq r_T$ (since $\overline{\eta} \leq \eta_L$), and $r_L^* < r_H^*$. These inequalities confirm our previous claim stating that the market

[7] While the other possible cases are similar, they are less interesting.

rate will be higher than the target rate in periods of high demand and lower than the target rate during periods of low demand.

3. INTEREST ON RESERVES

In the previous section, we considered the case in which reserves held by banks and not used in payments yielded no interest overnight. In general, banks hold reserves as balances in an account at the central bank, and in principle the central bank could pay interest on those unused reserves. We consider this possibility in this section.

There are different ways the central bank can pay interest on reserves. Here we concentrate on one possible scheme that has been discussed in policy circles (see Lacker 2006). Under this scheme, the central bank automatically pays interest overnight on all unused reserves held by banks at the end of the day after all payments have been executed. We call this scheme a sweep facility.

Overnight Sweep of Reserves

Suppose unused reserves are "swept" overnight into bonds that pay an interest rate r_s. Then, banks obtain a return of r_s on the amount $R - P$, whenever this difference is positive. In this case, the overnight expected return for the bank is given by

$$\Pi(R) = [1 - p(R)] \left[rB - r_o \left(E_{R_+} P - R\right)\right] + p(R) \left[rB + r_s \left(R - E_{R_-} P\right)\right],$$

where $E_{R_+} P$ is the expected value of P conditional on being greater than R, and $E_{R_-} P$ is the expected value of P conditional on being smaller than R. Here, it is important to note that if $R \geq \overline{P}$ then $p(R) = 1$, $E_{R_+} P = 0$, and $E_{R_-} P = \overline{P}/2$. After some manipulations, the expression for $\Pi(R)$ can be rewritten as

$$\begin{aligned} \Pi(R) &= rF - r\frac{\overline{P}}{2} \\ &\quad - [1 - p(R)] (r_o - r) \left(E_{R_+} P - R\right) \\ &\quad + p(R)(r_s - r) \left(R - E_{R_-} P\right). \end{aligned}$$

The second term in this expression ($-r\overline{P}/2$) is the average forgone interest from making the required end-of-day payment. The third term is the cost of covering the high end of the distribution of payments with overnight overdrafts, and the fourth term is the (potential) net benefit of getting the sweep interest rate on unused reserves (on the low end of the distribution of required payments).

Recall that $F \geq \overline{P}$. Then, if $r_s > r$, it is clear that the bank would choose the level of R to equal F; that is, the bank would maintain all its funds in the form of reserves. To see this, note that, for all $R \geq \overline{P}$ the overnight expected return is given by

$$\Pi(R \mid R \geq \overline{P}) = rF - r\frac{\overline{P}}{2} + (r_s - r)\left(R - \frac{\overline{P}}{2}\right),$$

and when $r_s > r$ we find that $\Pi(F) > \Pi(R)$ for all $R \leq F$ and, hence, $\Pi(R)$ is maximized at $R = F$.[8] It is not hard to see that even if $r_s = r$ we still find that $\Pi(F) \geq \Pi(R)$ for all $R \leq F$. However, as long as R is greater than $\overline{P}$, the bank is indifferent over the composition of its portfolio; that is, the bank makes the same return independent of how much of its funds are held in reserves (as long as they are enough to cover all possible end-of-day payments).

When $r_s < r \leq r_o$, the bank's demand for reserves is given by

$$R^* = \frac{(r_o - r)}{(r_o - r_s)}\overline{P},$$

where R^* is the (interior) value of R that maximized Π.[9] Note that, since in this case $r_s < r$, we find that $R^* < \overline{P}$ and, for some high possible realizations of the size of the payment P, the bank will not have enough reserves and will take an overnight loan at the penalty rate r_o.

The demand for reserves of an individual bank is then given by

$$R^* = \begin{cases} 0 & \text{if} \quad r \geq r_o \\ \frac{(r_o - r)}{(r_o - r_s)}\overline{P} & \text{if} \quad r_o > r > r_s \\ \left[\overline{P}, F\right] & \text{if} \quad r = r_s \\ F & \text{if} \quad r < r_s. \end{cases}$$

The Market for Reserves Under Sweeps

Using the demand function for individual banks, we can aggregate across banks and obtain the market demand for reserves under a sweep system. Following the aggregation procedures used before, we find that

$$R^d = \begin{cases} 0 & \text{if} \quad r \geq r_o \\ \frac{(r_o - r)}{(r_o - r_s)}E\overline{P} & \text{if} \quad r_o > r > r_s \\ \left[E\overline{P}, F\right] & \text{if} \quad r = r_s \\ F & \text{if} \quad r < r_s. \end{cases}$$

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[8] Here we are not allowing the bank to hold a negative position on bonds. If banks could short-sell bonds then r_s would be the effective "floor" of the market interest rate. None of our results depend on this assumption.

[9] See Whitesell (2006) for a similar analysis that would correspond to the case when P has a normal distribution. Also, Dotsey (1991) and Guthrie and Wright (2000) use versions of this theory to explain monetary policy implementation by the Reserve Bank of New Zealand.

As long as the supply of bonds is positive, the equilibrium interest rate r^* cannot be lower than r_s. The reason for this result is that if $r^* < r_s$ then all banks will want to hold all their funds as reserves. In this case, the demand for bonds in the market is equal to zero and the market for bonds does not clear (since the supply was positive). Figure 5 illustrates the determination of the market-clearing interest rate under a sweeps system. Note that if the supply of reserves by the central bank R^s is greater than $E\overline{P}$ and less than F, then the market-clearing interest rate r^* equals r_s.

Two important insights, useful in understanding monetary policy implementation, result from studying the determination of interest rates in a market in which banks have available a sweep facility that allows them to earn interest on reserves. The first insight is related to the role of the net supply of reserves in a so-called "corridor" system (see Guthrie and Wright 2000 and Whitesell 2006 for recent, more thorough discussions of corridor systems).[10] The second insight, discussed extensively by Goodfriend (2002) (see also Woodford 2000), is related to the advantages of "flooding" the market with reserves as a means of targeting a specific market interest rate.

In fact, the corridor and Goodfriend systems have been regarded as two alternative schemes for the implementation of interest rate policy. Next, we provide a brief introduction to these systems in the context of our model. While abstracting from many important issues, we believe that the discussion that follows can be helpful in understanding the relative advantages of each of the systems.

The Corridor System

For simplicity, let us concentrate on the case where $E\overline{P}$ is constant. Suppose that the monetary authority wishes to target a given rate r_T. One alternative is to use a corridor system, in which the overnight overdraft and sweep rates are set as follows:

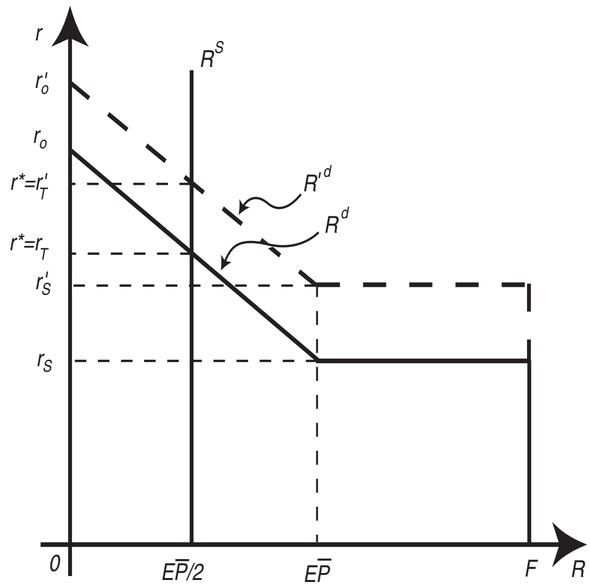
$$r_o = r_T + \frac{\delta}{2} \text{ and } r_s = r_T - \frac{\delta}{2}.$$

We will call δ the size of the corridor. Then, by setting the supply of reserves equal to $E\overline{P}/2$, the monetary authority can drive the market interest rate r^* to equal the target rate r_T.[11] What is most interesting about this system is that, to the extent that the value of $E\overline{P}$ is fairly stable, the monetary authority can drive the market rate to *any* target it wishes, just by changing proportionally the rates r_o and r_s (or, in other words, given the size of the corridor, by

[10] See also Berentsen and Monnet (2006) for a general equilibrium analysis of a corridor system.

[11] To see this, substitute the formula for the demand of reserves (the case when $r_o > r > r_s$) in the equation $R^d = R^s = E\overline{P}/2$.

Figure 5 The Corridor System



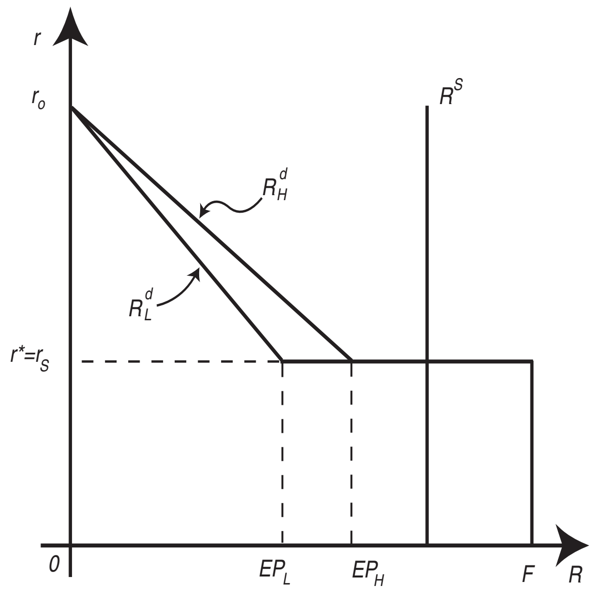
changing the target rate r_T), without changing (in any significant way) the supply of reserves. In fact, the market rate will jump to the new target just as a consequence of its announcement.

Figure 5 illustrates this case. If the monetary authority intends to increase the target rate from r_T to r_T', then it only needs to increase proportionally the rates r_o and r_s (to r_o' and r_s', respectively). The demand for reserves, as a result, will shift upward (in a parallel manner) and even if the supply of reserves remains unchanged (at the $E\overline{P}/2$ level) the market will clear at the higher, desired rate r_T'.

Alternatively, the corridor may be centered at the market interest rate (and not the target rate).[12] In such a case, Guthrie and Wright (2000) explain how the central bank can still use announcements to influence the overnight market interest rate without the need for explicit open market operations. Their explanation uses an arbitrage argument based on the expectations hypothesis of the term structure of interest rates. The key element in Guthrie and Wright's theory is the ability of the central bank to use open market operations, if necessary, to influence the overnight rates in the future. They call their

[12] Recall that, for example, when demand is stochastic, the market rate can be different from the target rate.

Figure 6 The Goodfriend System



strategy "threat-based monetary policy" (i.e., a threat to influence future rates, if necessary).

The Goodfriend System

Consider now the case where $E\overline{P}$ can take two values, $E\overline{P}_H > E\overline{P}_L$, as in the second part of Section 2. Then, the central bank can make the market interest rate always equal to a given target rate r_T by fixing the sweep rate $r_s = r_T$ and supplying $R^s > E\overline{P}_j$ for $j = H, L$ (see Figure 6). Clearly, the forecasting process required to assure that $R^s > E\overline{P}_j$ for $j = H, L$ is much simpler than the one that requires forecasting the exact values of $E\overline{P}_j$ for $j = H, L$.

The Goodfriend system requires that banks hold large amounts of reserves, which may result in large interest payments associated with the sweep facility.[13] The corridor system is less subject to this qualification, but when the payment requirements by banks fluctuate (as represented by fluctuations in $E\overline{P}$ in the model), the interest rate will be harder to target precisely. An

[13] See Goodfriend (2002, Section IV) for a careful discussion of these issues. Paying interest on reserves tends to encourage banks to substitute reserves for bond holdings in their balance sheet. Goodfriend discusses how the central bank could exploit the yield curve to finance interest on reserves by increasing its holdings of longer-term bonds (which, in principle, will result from the initial exchange of bonds for reserves when banks increase their demand for the latter).

exhaustive discussion of the pros and cons of corridor systems is beyond the scope of this article. Whitesell (2006) provides some interesting perspective on these issues. Here, it is sufficient to note that even when $E\overline{P}$ fluctuates, if the fluctuations are not very significant, a corridor system still allows the central bank to change the (average) target rate (by announcement) without major revisions to the supply of reserves.

4. DAYLIGHT CREDIT

The previous section dealt with the decision of banks, which, as the end of the day approaches, do not want to find themselves holding unused reserves that will earn zero interest overnight. To discuss the issue of daylight credit and how it relates to the end-of-day decisions, we need to extend the model to include some daytime decisions. However, the analysis in the previous section will constitute an integral part of the analysis in this more complicated case.

The Bank's Problem

We start the analysis, again, by studying the decisions of an individual bank. Relative to the bank's problem in the previous section, we add an extra decision that will allow us to capture some of the tradeoffs faced by the bank during the day. In particular, we will consider the situation in which the bank expects to make *two* payments before the night. We will denote by P^E the early payment and P^L the late payment. Both payments, as before, are uniformly distributed in the interval $\left[0, \overline{P}\right]$.

The bank starts the day with a given amount of funds F, with $F \geq 2\overline{P}$. These funds are allocated to holdings of bonds B_1 and reserves R_1. The bank observes the value of P^E and is required to settle this payment (real-time gross settlement). If $P^E > R_1$, then the bank needs to obtain daylight credit. Let r_e be the daylight credit interest rate. After that, with probability q, the bank finds a counterparty to trade bonds for reserves and adjust the composition of its portfolio. Later, with potentially an adjusted portfolio, the bank faces the arrival of a second payment P^L and is required to settle that payment. No new trading opportunities (or chances to adjust the portfolio) exist after the second payment. The left-over (positive or negative) balances are carried overnight.

First, we consider the case in which unused reserves earn no interest (and no sweep service is in place). To solve the problem of the bank, we start by studying the decision of the bank in the later part of the day when it finds a counterparty (that is, with probability q). Let us define the value Π_2 as follows:

$$\Pi_2^*\left(P^E\right) = r\left(F - P^E\right) + \max_{R_2}\left\{-rR_2 - \left[1 - p(R_2)\right] r_o \left(E_{R_2+}P^L - R_2\right)\right\},$$

where R_2 is the amount of reserves chosen by the bank in the rebalancing stage (after finding the counterparty). The maximization problem in the expression for Π_2^* is the same as the one studied in the previous section and describes the quantity of reserves the bank would like to carry to fulfill the late payment P^L (before knowing the exact value of that payment).

We are now ready to describe the decision problem of the bank choosing daylight reserves R_1. The final return of the bank will depend on the value of the early payment P^E and the late payment P^L, relative to the chosen value of reserves R_1. The bank does not know the value of the required payments at the time of choosing R_1 so the value of P^E may turn out to be lower or greater than R_1. In fact, under some values of the interest rates r and r_o, the optimal value of R_1 may be larger than $\overline{P}$, in which case the bank always has enough reserves to cover the early payments P^E (and no daylight credit is used).

For given values of P^E and R_1, we describe in the Appendix the expected payoff to a bank that does not get to rebalance its portfolio after its early payment. We denote this payoff by $\pi_1\left(P^E, R_1\right)$. The total expected payoff to the bank also involves the payoffs when it can rebalance its portfolio and the charges from using daylight credit, if there is any. Explicitly, if P^E is greater than R_1, then the bank's total return is given by

$$\Pi_1\left(P^E, R_1\right) = q\Pi_2^*\left(P^E\right) - r_e(P^E - R_1) + (1-q)\,\pi_1\left(P^E, R_1\right),$$

where the second term represents the interest paid on daylight credit.

If P^E is lower than R_1, then the bank's total return after making the early payment is given by

$$\Pi_1\left(P^E, R_1\right) = q\Pi_2^*\left(P^E\right) + (1-q)\,\pi_1\left(P^E, R_1\right).$$

Here, since $P^E < R_1$, the bank does not need daylight credit and the interest rate r_e does not appear in the expression.

The bank will choose daylight reserves R_1 to maximize the expected value (over possible realizations of P^E) of the total return $\Pi_1\left(P^E, R_1\right)$.[14] The solution to this problem is given by

$$R_1^* = \begin{cases} \frac{\left[r_e^2+2(1-q)r_o c\right]^{\frac{1}{2}}-r_e}{(1-q)r_o}\overline{P} & \text{if} \quad r_o > r \geq 0.5r_o, \\ \left[2-\left(\frac{2r}{r_o}\right)^{\frac{1}{2}}\right]\overline{P} & \text{if} \quad 0.5r_o > r > 0, \end{cases}$$

[14] It is worth mentioning that to perform these computations one needs to take into account that, for some interest rates, the optimal amount of reserves R_1 would be greater than $\overline{P}$. In such case, the bank will not require daylight credit regardless of its realization of P^E.

where $c = (1 - q)(r_o - r) + r_e$. Note that R_1^* is equal to $\overline{P}$ and continuous in r at $r = 0.5r_o$. If $r = 0$ then the bank is indifferent between holding any amount of reserves in the interval $\left[2\overline{P}, F\right]$.[15]

Note first that given all other values of the relevant variables, increases in the value of the market interest rate r decrease the value of R_1^*. In other words, the demand for reserves of an individual bank is a decreasing function of the interest rate, as in the previous sections. Similarly, the demand for reserves is an increasing function of the size of the highest possible payment $\overline{P}$.

It is also not hard to show that if $r \in (0.5r_o, r_o)$, then R_1^* is an increasing function of r_e. That is, when the interest rate on daylight overdrafts increases, the bank holds more reserves. When $r \in (0, 0.5r_o)$, the demand for reserves does not depend on the interest rate on daylight credit because the bank chooses a level of reserves $R_1^* > \overline{P}$ and never incurs a daylight overdraft.

The last comparative statics that we consider is with respect to the probability of being able to rebalance the portfolio after the early payment, that is, the probability q. In this model the optimal amount of reserves, as long as it is smaller than $\overline{P}$, is increasing in q. The reason for this result is that the probability of holding unused reserves overnight increases as q decreases (this is because rebalancing is not possible and the value of both payments P^E and P^L may happen to be low). For a high opportunity cost of holding unused reserves (that is, for high values of r), the bank will lower reserves if these are more likely to become excess overnight reserves.

When $r \in (0.5r_o, r_o)$, the average daylight credit incurred by the bank can be computed as

$$DC = \int_{R_1}^{\overline{P}} \frac{P - R_1}{\overline{P}} dP = \frac{\left(\overline{P} - R_1\right)^2}{2\overline{P}}.$$

Clearly, this quantity decreases when R_1 increases. In this model then, an increase in the interest rate on daylight credit tends to increase the level of reserves and, hence, decrease the average daylight credit incurred by banks.

The Market for Reserves

The overnight interest rate on bonds, r, will result from the interactions late in the day between banks that get to rebalance their portfolio and the central bank. The demand for reserves early in the day results from the anticipation by banks of the value that the interest rate will take in these late-in-the-day interactions. This is the case since what matters to banks is the opportunity cost of holding

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[15] Note that when $r = r_o$, we have that R_1^* is positive. Hence, in contrast with the analysis in the previous section, in this case r could potentially be greater than r_o and the demand for reserves still be positive. Recall that reserves now also allow the bank to economize in daylight credit. We do not study the (unusual) case of $r > r_o$ here.

reserves given by the overnight interest rate on bonds, r. In our model, banks can perfectly predict this interest rate, r. Clearly, this result is a simplification. In reality, the overnight interest rate tends to fluctuate during the day (although such fluctuations are not very significant in the United States). Bartolini et al. (2005) extensively document the behavior of the overnight interest rate in the U.S. federal funds market and provide interesting discussions of the reasons for the observed interest rate fluctuations.[16]

In our model, only a proportion q of the banks are active late in the day. Then, the aggregate demand for reserves (late in the day) is given by

$$R_2^d = q \int_0^1 R_{2i}^* di.$$

Given the supply of reserves, R_2^s, provided by the central bank (also, late in the day), the market-clearing interest rate, r^*, will be such that $R_2^d(r^*) = R_2^s$. Note that we have assumed here that those reserves that have been used to make early payments during the day do not become available as new reserves for the recipient until the next day. Also, in most cases, the central bank does not intervene in the bond market late in the day. Assuming that it does intervene, as we do in this article, simplifies the exposition but is not essential for the argument. In summary, these extreme assumptions are just simplifications that keep the market-clearing conditions easy to manipulate. There are, of course, alternative ways of setting up the market-clearing condition that would result in similar conclusions.

Note that the demand for reserves obtained in this way will behave similarly to the demand obtained in Sections 2 and 3. Then, it is easy to demonstrate that, for large enough values of R_2^s, the market interest rate r^* will be lower than $0.5r_o$ and, hence, there will be no demand for daylight credit. However, as in the previous section, if the demand for reserves is not perfectly predictable, some fluctuations in the market interest rate will persist. Also, the market interest rate that implies no demand for daylight credit may be too low, relative to some specific target that the central bank may have in mind. In the rest of this section, we demonstrate that a sweep facility that amounts to paying interest on reserves can resolve these two potential shortcomings.

Overnight Sweeps and Daylight Credit

Suppose the central bank automatically sweeps overnight all unused reserves held by banks into bonds that pay a return r_s, which is also fixed by the central

[16] Many of the key elements in Bartolini et al. (2005) discussions are captured in our model in a stylized and easy-to-study manner (for example, the inability by banks to perfectly predict their payment needs late in the day and the risk of overdraft faced by banks that might not be able to find a lender before the market closes). We abstract from studying within-the-day interest rate fluctuations but believe that our model, after some minor modifications, could be used as a first step in a formal study of these issues.

bank. Then, the demand for reserves by an individual bank is given by

$$R_1^* = \begin{cases} \frac{\left[r_e^2+2(1-q)(r_o-r_s)c\right]^{\frac{1}{2}}-r_e}{(1-q)(r_o-r_s)}\overline{P} & \text{if} \quad r \geq 0.5\,(r_o+r_s)\,, \\ \left[2-\left(\frac{2(r-r_s)}{r_o-r_s}\right)^{\frac{1}{2}}\right]\overline{P} & \text{if} \quad r < 0.5\,(r_o+r_s)\,, \end{cases}$$

where again $c = (1-q)(r_o - r) + r_e$. Here, if $r = r_s$ then the bank is indifferent among holding any amount of reserves in the interval $\left[2\overline{P}, F\right]$. Note that, not surprisingly, the demand for reserves under a system with no sweeps is the same as the one in a system where the interest on sweeps, r_s, is set to equal zero.

For $r \geq r_s$ the demand is decreasing in r and continuous (and, in particular, when $r = 0.5\,(r_o + r_s)$ the demand for reserves R_1^* is equal to $\overline{P}$). Hence, when the interest rate r is smaller than $0.5\,(r_o + r_s)$, the bank holds enough reserves to never require daylight credit (since $P^E < R_1$ for all possible values of P^E).

If the central bank wishes to set the market interest rate at a given target level r_T, and simultaneously drive the use of daylight credit to zero, then an effective mechanism is to set $r_s = r_T < r_o$ and supply enough reserves to make $r = r_s$. This is basically the same Goodfriend idea that we explained for the simpler model with only one payment per period (see Figure 6). Here, the Goodfriend system has the added (potential) benefit of significantly reducing the demand for daylight credit by banks.

5. DISCUSSION AND EXTENSIONS

In this section, we discuss some important aspects of the interbank payment system that were left out in our simple model. First, we discuss how the model would work in the presence of reserve requirements. After that, we discuss the important issue of credit risk that partly motivates many of the most significant policy questions in this general subject. Finally, we provide some discussion of a few other assumptions that are associated with important issues related to the workings of the market for reserves. In all the cases, we make an explicit effort to provide adequate references to the relevant literature that extend the analysis in this article.

Reserve Requirements

Our model of the demand for reserves by banks does not rely on the imposition of reserve requirements. However, reserve requirements are a common feature of many payment systems and, in particular, of the U.S. system. A thorough discussion of the functioning of the market for reserves when there are reserve requirements is beyond the scope of this article. Here, we only present a short introduction to the issue that exploits the simplicity of our model (see Whitesell 2000 and Bartolini, Bertola, and Prati 2002 for more comprehensive, related studies).

Let us go back to the simpler setup of Section 3 and suppose that the central bank imposes a minimum reserve requirement equal to $\underline{R}$. Also assume that banks that cannot satisfy the reserve requirement have to pay the overnight overdraft rate r_o per units of reserve deficiency (i.e., borrowed reserves).[17] In such a case, the demand for reserves would take the form in Figure 7. Clearly, if the supply of reserves is lower than $\underline{R}$ then banks would only agree to hold bonds if the rate of return for holding bonds, r, is greater than or equal to r_o. In fact, if r is greater than r_o then it is better to hold only bonds and pay the penalty rate r_o to obtain borrowed reserves that cover the reserve requirement. Some of the return from the interest payments accrued on bonds can later be used to cover the interest on borrowed reserves.

Hence, if both bonds and reserves are to be held in equilibrium the market interest rate, r, must equal r_o when the supply of reserves is lower than $\underline{R}$. If the supply of reserves is greater than $\underline{R}$ then the analysis is similar to the one in Section 2, where banks choose their balance to cover the reserve requirement and the expected late payment P.

In reality, banks could face some uncertainty about the value of $\underline{R}$ since it depends on their holding of deposits subject to reserve requirement, a variable that is not fully predictable at all times. In the United States, the system is set up so as to minimize this uncertainty. Banks have to satisfy an average level of reserves over a reserve maintenance period, where the required reserves are calculated based on the holding of deposits in a previous period. Also, failure to meet the requirement implies a penalty that is different than the overnight overdraft rate. While the analysis would be more complicated in this case, the basic logic described here would still apply.[18]

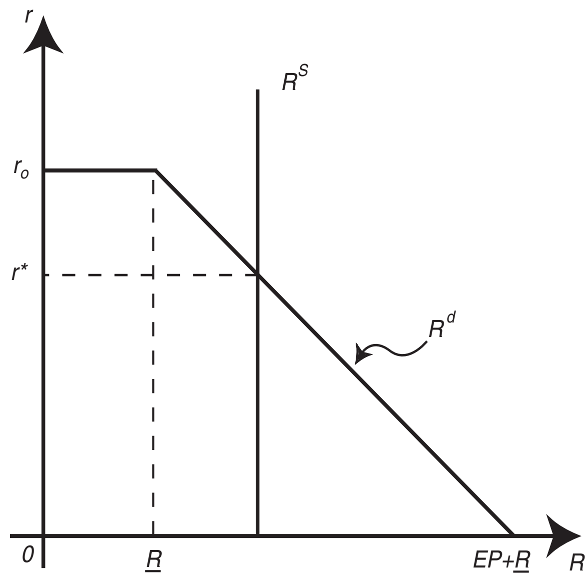
Credit Risk

The model does not deal with the role of credit risk in determining the behavior and outcomes in the payment system. Clearly, paying attention to credit risk considerations would be essential to reach any definite policy conclusion. For

[17] This is not exactly how deficiencies in reserve requirements are treated in the U.S. system, but this simpler case serves for the purpose of illustration.

[18] Whitesell (2000) analyzes a similar model with a two-day reserve requirement maintenance period and a corridor system. See also Bartolini, Bertola, and Prati (2002) for a model more closely motivated by the features of the U.S. system.

Figure 7 Reserve Requirements and the Market for Reserves



example, credit risk would play a role in explaining why the central bank may want to economize on bank usage of daylight credit (see Kahn and Roberds 1999 for a description of a model in which credit risk plays a crucial role). Zhou (2000), following the original contribution by Freeman (1996), shows that if credit risk is not relevant, then the intraday interest rate r_e should be set to equal zero in the optimal policy.[19] The model in this article artificially abstracts from credit risk and is not designed to provide direct insight into the optimal determination of the rates r_o, r_e, and r_T.[20] Rather, it shows how, given the values for the relevant interest rates, a system of sweeps, which amounts to paying interest on reserves, can facilitate the implementation of a target rate r_T.

An important feature of the U.S. intraday credit policy that was left out of the analysis is the imposition of quantitative limits (or "caps") on the amount of intraday credit. At any time during the day, each bank should hold intraday

[19] This conclusion also depends on the assumption that daylight credit will be exclusively devoted to payment purposes and cannot be diverted into short-term speculative investment.

[20] Optimal policy in payment arrangements is a difficult question (see Zhou 2000, Temzelides and Williamson 2001, Martin 2004, Mills 2006, and Berentsen and Monnet 2006 for recent contributions). Even the basic question of whether the central bank should play a role in the payment system does not have a commonly accepted answer among academic economists. Green (1999) provides a careful study of this important issue.

credit to an amount that does not exceed the bank's cap.[21] In the model, there is no role for caps. In part, this is a consequence of our explicit abstraction of any credit risk consideration. To a first approximation, caps are imposed to limit the ability of banks to take large negative positions in their accounts at the central bank when they are likely to fail. Temzelides and Williamson (2001) provide a related justification for caps in a dynamic model with explicit informational frictions (see also Koeppl, Monnet, and Temzelides 2004).

Two other instruments that can be used to limit the credit risk exposure associated with the provision of daylight credit are daylight interest rates and collateral. Interest rates on daylight credit have other implications (aside from accounting for credit risk) for the management of reserves by banks. Even though our model does not take into account credit risk, we have considered the case of positive daylight credit interest rates to study such alternative implications on banks' management of reserves. Lacker (1997) points out that interest on daylight credit could reduce the distortion created by not paying interest on reserves, and in this case finds that daylight overdraft should be charged an interest rate at least as high as the market rate.

Collateral, in the form of repos, could certainly be used in the environment of our model. Recall that total funds, F, are allocated between bonds and reserves. Since F is greater than P^E, the sum of bonds and reserves is also greater than P^E. Therefore, even if payment P^E is greater than reserves, the bank can always use bonds to collateralize the necessary daylight credit.[22] Since there is no explicit consideration of credit risk in our model, even though the use of collateral is possible, it is inconsequential (see Martin 2004 and Mills 2006 for environments in which collateral requirements play an important role).

There are other ways to influence the amount of daylight credit in the system. For example, McAndrews and Rajan (2000) propose the use of explicit policies to encourage synchronization of payments during the day and suggest that such policies would tend to limit banks' reliance on daylight credit to cover intraday payments (see also Martin and McAndrews 2006).

Other Important Assumptions

In the model, the connection between the payment of interest on reserves and daylight credit comes from the fact that banks can only adjust their portfolio

[21] There is some flexibility implicit in these caps. Basically, banks can exceed the caps in unusual situations. After repeated violations of the cap, banks can be placed under a strict system of monitoring and, if necessary, some of their requested payments may be rejected to avoid overdrafts. See Federal Reserve System (2005) for details.

[22] There is no opportunity cost of holding collateral in the model. For a model exploiting a mechanism similar to this one, but where collateral bears an opportunity cost, see Berentsen and Monnet (2006).

during the day with some probability. In other words, banks face a trade friction in the asset market that limits their ability to adjust their holdings of reserves. Interestingly, there is a growing amount of literature that aims at capturing these trading frictions in financial markets (see Duffie, Gârleanu, and Pedersen 2005, Weill 2005, and Lagos and Rocheteau 2006, for example). In principle, we can expect that some of the ideas in this article will extend to environments that more closely follow the new literature on financial markets with (search) frictions.

The model also assumes that the size of payments P^E and P^L and the probability of being able to adjust the portfolio after a payment, q, are exogenous and cannot be modified by the bank. However, in principle the bank could influence the size and timing of payments at some cost. This flexibility is not present in the model and, if introduced, would highlight the potentially distortionary effects of certain payment system policies, as for example, not paying interest on reserves (see Lacker 1997 for a model in which this type of distortion is possible). Similarly, the efforts to find counterparties to trade and adjust portfolios are also part of an explicit decision by banks facing costs and benefits that are implied by the system in place. If we change the system, for example to evaluate different policies, such decisions by banks may also change. The model abstracts from this type of so-called "Lucas critique" effect.

The sizes of P^E and P^L can be interpreted as proxies for the volume of payment requirements arriving early and late in the day. These values are random in the model. Since the size of the payments are random, the bank cannot perfectly predict them. However, there is no relevant decision in the model that influences the values of P^E and P^L that are likely to be observed. It is in this sense that we say that P^E and P^L are exogenous. In the real world, banks have some degree of discretion regarding the timing of payments during the day. McAndrews and Rajan (2000) discuss some evidence that suggests that U.S. banks actively synchronize payments to affect payment flows during the day. The discretion over the timing of payments opens the door to strategic behavior by banks. A number of articles have formally studied the possibility of delays and gridlocks in real-time gross settlement systems (see Bech and Garratt 2003, Martin and McAndrews 2006, Mills and Nesmith 2006, and Beyeler et al. 2006 for recent contributions).[23]

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[23] See Rochet and Tirole (1996) and Freixas and Parigi (1998) for discussions of contagion and systemic risk in payment systems. Furfine (2003) provides an interesting empirical investigation of interbank exposures in the United States.

6. CONCLUSION

The model in this article is not suitable to analyze the welfare economics of the topic, as the underlying real economic activity that drives payment needs (and hence the demands for reserves and daylight credit) is hidden from view and treated as exogenous. Still, even the partial equilibrium analysis of this article points to some tentative conclusions. By paying interest on reserves at less than the market rate of interest, the central bank essentially imposes a tax on reserves. This tax encourages banks to hold no more reserves than is necessary to just meet their payment needs. But uncertainty in the timing of payments means that "just meeting" only happens by accident. Hence, the desire to hold down reserves leads banks to demand central bank credit. The provision of such credit presents the central bank with a new set of challenges, from finding an appropriate price to managing the credit risk exposures that could result.

In short, like any tax, a tax on reserves creates distortions, including distortions in the use of daylight credit. Against these, a complete analysis would have to consider what costs might be associated with the greater reserve holdings implied by the Goodfriend system of paying interest at the target market rate. Our model does not address this issue, but to the extent that such costs are small, our analysis suggests the optimality of eliminating the tax on reserves.

In a more general context, Goodfriend (2006) discusses the role of interest on reserves as part of a comprehensive proposal for determining and implementing an optimal rate of inflation in the economy. He argues that the provision of currency card accounts, combined with the payment of interest on banks' reserves, would allow the monetary authority to achieve the optimum quantity of money (as in the Friedman rule) at any inflation rate. This would, in turn, free the monetary authority to set the inflation rate at the optimum level based only on considerations related to the existence of price rigidities (relative price distortions and mark-ups) and the zero-lower-bound on nominal interest rates.

This article has emphasized the interdependence of banks' demand for daylight credit in the payment system and for overnight reserves. The use of reserves as the medium of settlement for interbank payments means that changes in the central bank's treatment of overnight reserves could also affect the operation of the intraday, interbank settlement system. If one of the forces driving the demand for daylight credit has been the desire by banks to avoid the opportunity cost of holding sterile reserves, then reducing that opportunity cost by paying interest on reserves should reduce the demand for such credit. We have considered a simple model that deals with these interdependencies explicitly and allows us to better understand their origin and consequences.

APPENDIX

Let us denote by π_1 the expected payoff to the bank if it does not get a chance to rebalance its portfolio after the first payment, P^E. To specify this expected payoff, there are three relevant ranges of P^E that need to be considered. First, when $P^E < R_1 - \overline{P}$, the amount of extra reserves R_1 left after making the early payment P^E is enough to cover all possible late payments. This can only happen if $R_1 > \overline{P}$, of course. In this case, no overnight overdraft will be incurred and the expected payoff to the bank is given by

$$\pi_1\left(P^E, R_1\right) = rB_1.$$

Under a sweep system this payoff becomes

$$\pi_1\left(P^E, R_1\right) = rB_1 + r_s\left(R_1 - P^E - \frac{\overline{P}}{2}\right),$$

where the second term represented the interest earned on unused reserves overnight.

When $\max\left\{0, R_1 - \overline{P}\right\} \leq P^E < \min\left\{R_1, \overline{P}\right\}$, there are always some possible values of P^L such that the bank will have to incur an overnight overdraft. In this case, the expected payoff π_1 is given by

$$\pi_1\left(P^E, R_1\right) = rB_1 - \left[1 - p\left(R_1 - P^E\right)\right] r_o\left(P^E + E_{(R_1-P^E)+}P^L - R_1\right).$$

Under a sweep system we need to add to this payoff the expected interest earned on unused reserves, which is given by

$$p\left(R_1 - P^E\right) r_s\left(R_1 - P^E - E_{(R_1-P^E)-}P^L\right),$$

where $E_{(R_1-P^E)-}P^L$ represents the conditional expectation over values of P^L smaller than $R_1 - P^E$.

Finally, if $R_1 < \overline{P}$, then it can happen that P^E is greater than R_1 and, in this case, the bank will incur an overnight overdraft equal to the sum of the daylight credit balance ($P^E - R_1$) and the full amount of the second payment P^L. The expected payoff π_1 is then given by

$$\pi_1\left(P^E, R_1\right) = rB_1 - r_o(P^E + \frac{\overline{P}}{2} - R_1),$$

where $\overline{P}/2$ represents the average value of P_L. In this case, the bank does not hold any unused reserves and, hence, the payoff is the same under a sweep system.

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How Accurate Are Real-Time Estimates of Output Trends and Gaps?

Mark W. Watson

Trends and gaps play an important role in macroeconomic discussions. For example, the output gap (the deviation of output from its trend, or potential value) and the unemployment gap (the deviation of the unemployment rate from its trend, or "NAIRU") are standard business cycle indicators and key ingredients for Phillips curve forecasts of inflation, and likewise the trend, or long-run level of inflation is a central concern of central banks.

Trends and gaps (deviations of series from trends) are inherently two-sided concepts. By this I mean that the value of the trend in real GDP in 1987, for example, depends on how the observed value of GDP in 1987 compares to its past values (in 1986, 1985, and so forth) and to future values (in 1988, 1989, etc.). For historical analysis, the need for past and future values of the series does not pose a problem. Looking at a plot of the postwar values of real GDP, it is fairly easy to estimate its trend value by drawing a smooth curve through the plot, and various statistical formulae have been developed to mimic this freehand trend estimation procedure. However, because trends and gaps require both past and future data, it is much more difficult to estimate their values at the beginning of the sample (where there is no past data) and at the end of the sample (where there is no future data). The end-of-sample uncertainty

I have benefited from discussions with Robert Hall, Robert King, Athanasios Orphanides, and James Stock, and comments from colleagues at the Federal Reserve Bank of Richmond. Data and replication files for this research can be found at http://www.princeton.edu/~mwatson.
Mark W. Watson is a professor in the Department of Economics and the Woodrow Wilson School at Princeton University, Research Associate at the National Bureau of Economic Research, and Visiting Scholar with the Federal Reserve Bank of Richmond. The views expressed in this article are those of the author and not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

in the trend is particularly problematic because these are the observations most relevant for real-time policy analysis.

The accuracy of real-time estimates of trends and gaps depends on the series under study. For example, if a series shows essentially random fluctuations around a linear trend, then the value of the trend can be accurately estimated from past observations. On the other hand, when a series shows serially correlated fluctuations around a slowly evolving trend, then future values of the series are critical to accurately separate the trend from the fluctuations. This article studies four economic indicators: industrial production, unemployment rate, employment, and real GDP to quantify the accuracy of real-time or one-sided estimates of output trends, gaps, and business cycle components.[1]

The analysis must begin with a definition of a trend and several reasonable definitions suggest themselves. Low-order polynomials in time are natural candidates, but these methods can yield unrealistic estimates of estimation errors at the ends of the sample. Martingales (or "random walks") and integrated martingales (processes for which first differences are random walks) can approximate smooth sample paths and are used to represent trends in unobserved component models (see Harvey 1989 for a detailed discussion). However, these models imply that the trend value cannot be estimated with certainty, even using an infinite amount of past and future data. This feature may or may not be reasonable, but it often leads to the conclusion that the estimated trend is inaccurate.

This article defines trends, gaps, and business cycle components using band-pass filters. These filters are moving averages of the data designed to isolate variation at specific frequencies; they are analogous to filters on an audio system that allow a user to eliminate specific frequency bands (for example, the sound from a low-frequency bass guitar or a high-frequency piccolo). In this article, the trend is defined as the cyclical movements in the time series with periods longer than the business cycle (that is, longer than 8 years); the gap includes components with periods shorter than 8 years; and the business cycle component includes components with periods between $1\frac{1}{2}$ and 8 years.

The advantage of this definition is twofold. First, it produces reasonable-looking and flexible estimates of trends, gaps, and business cycle components

[1] There are two distinct problems using real-time data to estimate trends and gaps. First, data published in real time are often subsequently revised, and these revisions can be large. Second, for the purpose of estimating trends and gaps, future values of the series are needed, so that estimates of a trend at time t will change as data becomes available for time $t+1, t+2$, etc., even if the data at time t is not revised. This article is concerned with the second problem. In particular, all analysis in this article is carried out using a 2006-vintage dataset, and to avoid confusion with actual real-time estimates, I will refer to estimates constructed using current and past values of a series as "one-sided" estimates. Orphanides (2003a) studies many of the same problems studied here and also includes analysis of data revisions.

(see the discussion and examples in Baxter and King 1999 and Stock and Watson 1999), and second, it means that historical values of these components can be estimated precisely allowing a sharp distinction between historical and one-sided analysis. Importantly, for interpreting the results shown in this article, uncertainty about the correct definition of trends, gaps, and business cycle components will only increase the real-time uncertainty of the estimates.

This is not the first article to look at this issue. For example, Staiger, Stock, and Watson (1997, 2002) quantify the uncertainty in estimates of the NAIRU; Orphanides and van Norden (2002) and Orphanides (2003a) discuss uncertainty in estimates of the output gap; Orphanides (2003b) and Orphanides and Williams (2002) discuss the effects of output gap uncertainty on monetary policy; and Hall (2005) contains a thoughtful critique of the usefulness of decomposing series in smooth trend and gap components.

The following section provides a brief review (or primer) on band-pass filtering and the Appendix contains some additional details. Section 2 presents benchmark results for one-sided estimates of the gaps based on the index of industrial production, the unemployment rate, payroll employment, and real GDP. As it turns out, the one-sided gap estimates are quite imprecise and capture only 50 percent of the variability in the gap as determined by two-sided estimates. Section 3 discusses improving the precision by using multivariate methods, but these produce only marginal improvements in the precision of the one-sided estimates. This section also shows that the reduction in volatility associated with the "Great Moderation" has greatly increased the (absolute) precision of one-sided estimates. The final section contains a brief summary and some concluding remarks.

1. A REVIEW OF BAND-PASS FILTERING

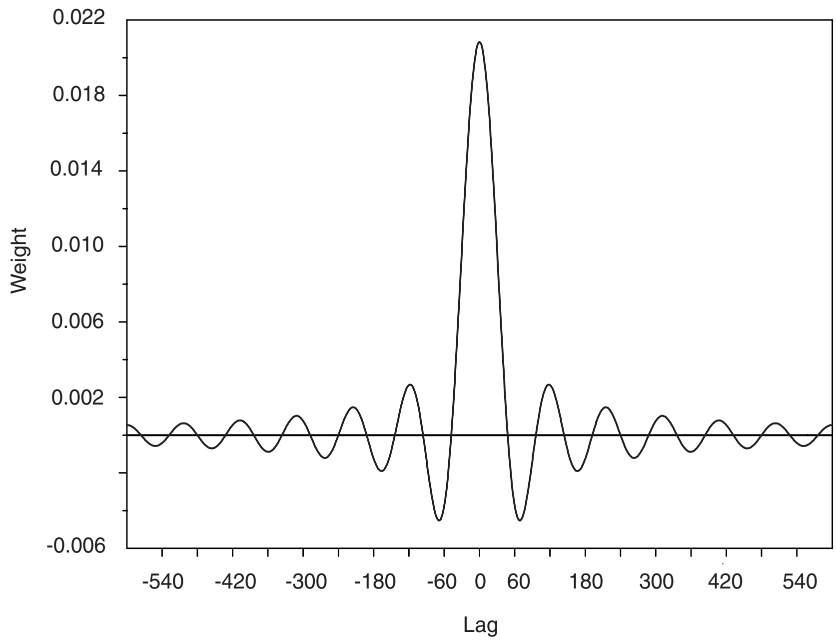
Let Y_t denote a stationary scalar stochastic process. The Spectral Representation (sometimes called the Cramér Representation) of Y is given by

$$Y_t = \int_0^{\pi} \cos(\omega t) d\alpha(\omega) + \int_0^{\pi} \sin(\omega t) d\delta(\omega), \tag{1.1}$$

where $d\alpha(\omega)$ and $d\delta(\omega)$ are zero-mean random variables that are mutually uncorrelated, are uncorrelated across frequency, and have variances that depend on frequency. The representation decomposes Y_t into a set of heteroskedastic, mutually uncorrelated, strictly periodic components. The business cycle component of Y can be defined as $Y_t^{BC} = \int_{\omega_1}^{\omega_2} \cos(\omega t) d\alpha(\omega) + \int_{\omega_1}^{\omega_2} \sin(\omega t) d\delta(\omega)$, where ω_1 and ω_2 demarcate business cycle frequencies, for example, frequencies with periods between $1\frac{1}{2}$ and 8 years. Similarly, the trend component of Y can be defined as the lower-than-business-cycle components of Y, $Y_t^{Trend} = \int_0^{\omega_1} \cos(\omega t) d\alpha(\omega) + \int_0^{\omega_1} \sin(\omega t) d\delta(\omega)$, and the gap is $Y_t^{Gap} = Y_t - Y_t^{Trend}$.

Band-pass filtering uses moving averages of the data to estimate frequency components of Y over specific frequency bands. To see how a band-pass filter

Figure 1 Band-Pass Filter Weights (Periods Less Than Eight Years—Monthly Data)



Notes: Let $c(L) = \sum_{j=-\infty}^{\infty} c_{|j|}L^{-j}$ denote the band-pass filter for monthly data frequencies with periods greater than 96 months. This figure shows the first 600 values of c_j, corresponding to weights that are used in a symmetric 100-year moving average of monthly data.

works, let X_t denote a moving average of Y_t with moving average weights c_j

$$X_t = \sum_{j=-r}^{s} c_j Y_{t-j} = c(L)Y_t, \tag{1.2}$$

where $c(L) = \sum_{j=-r}^{s} c_j L^j$ is a polynomial in the lag-operator L with coefficients c_j. As shown in the Appendix, the ω^{th} cyclical component of X_t is the ω^{th} component of Y_t transformed in two distinct ways: (1) it is shifted backward or forward in time, and (2) it is amplified or attenuated. Specifically, letting $X(t, \omega)$ and $Y(t, \omega)$ denote the ω^{th} components, $X(t, \omega) = g(\omega)Y(t - \rho(\omega), \omega)$, so that $\rho(\omega)$ denotes the time shift and $g(\omega)$ denotes the amplification factor. A calculation presented in the Appendix shows that $g(\omega) = |\, c(e^{-i\omega})|$ and $\rho(\omega) = \omega^{-1} \times \tan^{-1}\{Im[c(e^{-i\omega})/Re[c(e^{-i\omega})]\}$, where $i = \sqrt{-1}$ is a complex number and $c(e^{-i\omega}) = \sum_{j=-r}^{s} c_j e^{-ji\omega}$, with imaginary and real parts given by $Im[c(e^{-i\omega})]$ and $Re[c(e^{-i\omega})]$. Because the moving average operation modifies the cyclical components, $c(L)$ is called a *filter.*

A band-pass filter chooses the coefficients c_j to isolate (or "pass") a specific range (or "band") of cyclical components. To be specific, a band-pass filter that isolates frequencies between ω_{Lower} and ω_{Upper} chooses the moving average weights c_j so that that $g(\omega)$ and $\rho(\omega)$ satisfy two properties:

$$\rho(\omega) = 0 \text{ and} \tag{1.3}$$

$$g(\omega) = \left\{ \begin{array}{c} 1 \text{ for } \omega_{Lower} \leq \omega \leq \omega_{Upper} \\ 0 \text{ otherwise} \end{array} \right\}. \tag{1.4}$$

The restriction (1.3) means that the series is not shifted in time. This constraint can be satisfied by making the filter symmetric, that is, by choosing $c_j = c_{-j}$ for all j. (This makes $Im[c(e^{-i\omega})] = 0$, so that $\rho(\omega) = 0$.) The restriction (1.4) is more complicated. The Appendix shows that this constraint is satisfied by choosing

$$c_j = \left\{ \begin{array}{c} \frac{1}{j\pi}[\sin(j\omega_{Upper}) - \sin(j\omega_{Lower})] \text{ for } j \neq 0 \\ \frac{1}{\pi}[\omega_{Upper} - \omega_{Lower}] \text{ for } j = 0 \end{array} \right\}. \tag{1.5}$$

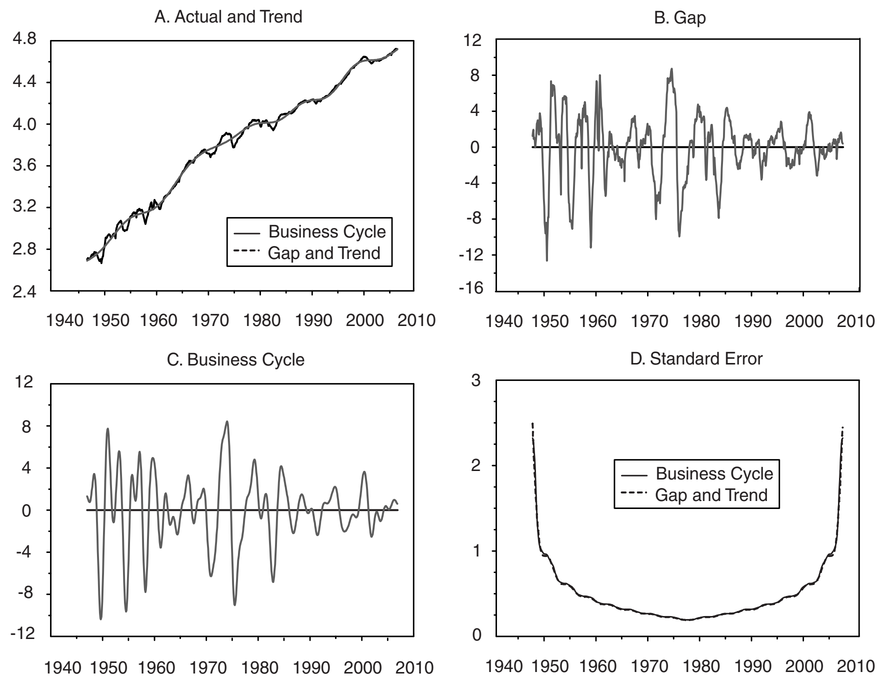
Figure 1 plots these weights for the monthly trend band-pass filter with $\omega_{Upper} = \frac{2\pi}{96}$ and $\omega_{Lower} = 0$, which passes components with periods greater than 8 years (= 96 months). The weights die out slowly. The figure plots the weights for the first 600 values of c_j, corresponding to a symmetric 100-year moving average of the data. Evidently, the weights are nonnegligible even outside this 100-year window.

The weights shown in Figure 1 produce the estimated trend in a series. The deviation of the series from the trend is the gap: $Y_t^{Gap} = Y_t - Y_t^{Trend} = [1 - c^{Trend}(L)]Y_t$, so that the band-pass filter for the gap is $1 - c^{Trend}(L)$. Thus, the weights used to construct the band-pass estimates of the gap will also decay very slowly.

Evidently, accurate estimates of the trend or gap in a series require a long two-sided moving average. This leads to problems for estimating the trend or gap for dates near the beginning of the sample period (when long lags of the series are not available) and near the end of the sample (when long leads are not available).

Two results suggest how these problems are best handled. First, Baxter and King (1999) consider the problem of constructing a finite order filter $\hat{c}(L) = \sum_{j=-s}^{s} c_{|j|}L^j$ that provides the best L^2 (or "least squares") approximation to the ideal $g(\omega)$ given in (1.4). They show that the best approximation is simply the truncated version of the infeasible infinite order filter. Second, Geweke (1978) makes the following general observation about constructing optimal estimates of filtered series: Let $X_t = \sum_{j=-r}^{s} c_j Y_{t-j}$, and suppose that data are available on a vector of random variables Z_τ from $1 \leq \tau \leq T$. Then the best

Figure 2 Two-Sided Band-Pass Estimates: Logarithm of the Index of Industrial Production



Notes: These panels show that estimated values of the band-pass estimates of the trend (periods > 96 months), the gap (periods < 96 months), and the business cycle (periods between 18 and 96 months). Panel D shows the standard errors of the estimates relative to values constructed using a symmetric 100-year moving average. Values shown in Panel A correspond to logarithms, while values shown in Panels B-D are percentage points.

(minimum mean square error) estimator of X_t is given by $E(X_t \mid \{Z_\tau\}_{\tau=1}^T) = \sum_{j=-r}^{s} c_j E(Y_{t-j} \mid \{Z_\tau\}_{\tau=1}^T)$.

Taken together, the Baxter and King (1999) and Geweke (1978) results suggest the following procedure for constructing band-pass estimates of the trend and gap. First, approximate the ideal filter using $\hat{c}(L) = \sum_{j=-s}^{s} c_{|j|} L^j$ with filter weights given by (1.4) and s chosen sufficiently large ($s = 600$). Second, letting $\{Z_\tau\}_{\tau=1}^T$ denote the sample observations on Y, construct $Y_{t|T}^{Trend} = \sum_{j=-s}^{s} c_{|j|} Y_{t/\tau}$, where $Y_{t/T} = E(Y_t \mid \{Y_\tau\}_{\tau=1}^T)$. That is, $Y_{t|T}^{Trend}$ is constructed using the ideal filter, truncated after a large number of terms and applied to the Y_t series padded into the future and past using forecasts and backcasts of the series.[2] Truncating the filter using a small value of s (an approach used by some applied researchers) is not necessary when the series is padded with forecast values of the series, and as Geweke's (1978) analysis implies, this produces a more accurate estimate of the ideal band-pass filtered series. Readers familiar with seasonal adjustment will recognize that this essentially is the procedure used in the Census X-12-ARIMA seasonal adjustment procedure (see Findley et al. 1998), and Christiano and Fitzgerald (2003) propose a one-sided band-pass filtered estimator using this procedure implemented with random-walk forecasts of Y_t.

The error in $Y_{t|T}^{Trend}$ is

$$Y_{t|T}^{Trend} - Y_t^{Trend} = \sum\nolimits_{j=-s}^{s} c_{|j|}(Y_{t-j|T} - Y_{t-j}) + \sum\nolimits_{|j|>s} c_{|j|} Y_{t-j}. \quad (1.6)$$

With s chosen sufficiently large, the second term is negligible and the variance of the first term can be computed from the autocovariances of the forecast/backcast errors of the Y process. (Details are provided in the Appendix.) Standard errors based on this variance formula will be used in the next section, which studies estimates of the trend, gap, and business cycle component of several economic time series.[3]

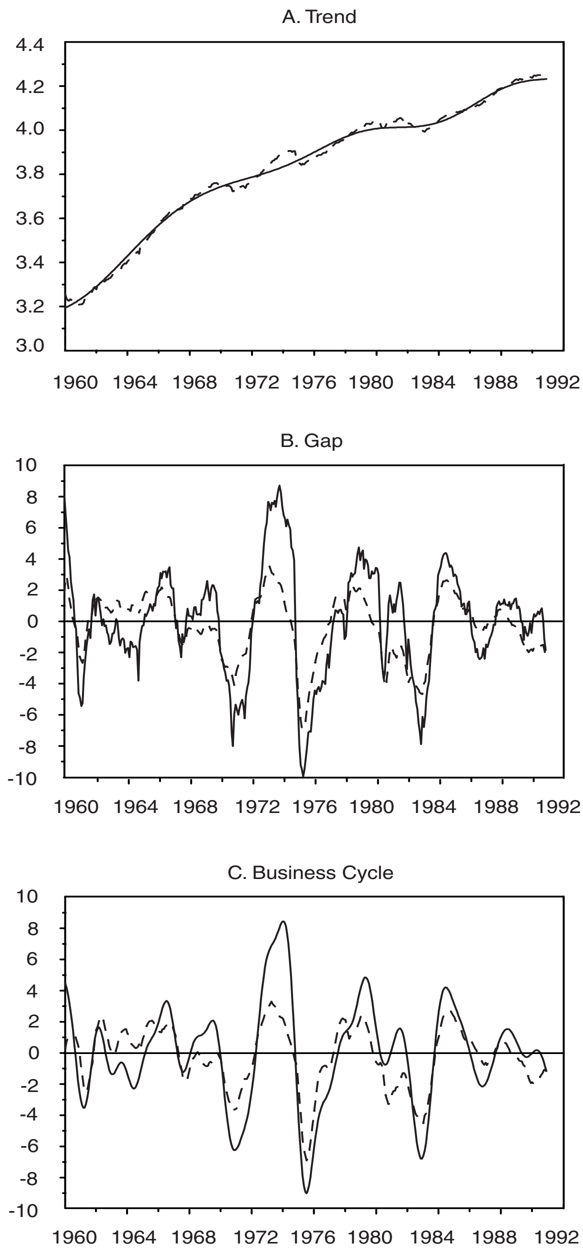
2. EMPIRICAL RESULTS

Figure 2 shows the results for computing the estimated trend (Panel A), gap (Panel B), and business cycle component (Panel C) of the logarithm of the index of industrial production (*IP*) using data from 1947:2–2006:11. These estimates are computed using a 600-term approximation to the band-pass filters and forecasts and backcasts constructed from an AR(6) model for $\Delta ln(IP_t)$. Panel D of the figure shows the standard error of the estimated components, where the standard error is computed by estimating the standard deviation of the first term on the right-hand side of (1.6) using the estimated parameters of the AR model. The estimated trend and gap components have the same standard error (because the gap and trend add to the observed series), while the estimated business cycle component is slightly smaller.

[2] As a practical consideration, it is useful to follow a suggestion by Baxter and King (1999) and modify the coefficients in the truncated trend filter so that they sum to unity. This produces an $I(0)$ estimate of the gap when the filter is applied to an $I(1)$ process and assures a bounded mean square error for the one-sided band-pass filtered estimates. The empirical analysis presented in the next section uses this modification.

[3] Harvey and Trimbur (2003) suggest an alternative procedure for approximate band-pass filtering based on an unobserved components model. An attractive feature of their proposal is that the end-of-the-sample problem is easily handled by the Kalman filter.

Figure 3 Two-Sided (Solid) and One-Sided (Dashed) Band-Pass Estimates: Index of Industrial Production



Notes: The solid lines are the two-sided estimates shown in Figure 2. The dashed lines are one-sided estimates that do not use data after the date shown on the horizontal axis.

Table 1 Joint Frequency Distribution of the Sign of $Y_{2-sided}^{BusinessCycle}$ and $Y_{1-sided}^{BusinessCycle}$—Industrial Production: 1960–1990

	$Y_{2-sided}^{BusinessCycle} > 0$	$Y_{2-sided}^{BusinessCycle} < 0$
$Y_{1-sided}^{BusinessCycle} > 0$	0.36	0.12
$Y_{1-sided}^{BusinessCycle} < 0$	0.15	0.37

Notes: This table shows the relative frequency of the events $Y_{2-sided}^{BusinessCycle} > 0$, $Y_{2-sided}^{BusinessCycle} < 0$, $Y_{1-sided}^{BusinessCycle} > 0$, and $Y_{1-sided}^{BusinessCycle} < 0$ for 1960–1990. $Y_{2-sided}^{BusinessCycle}$ is computed using the logarithm of the index of industrial production over 1947:2–2006:11, while $Y_{1-sided}^{BusinessCycle}$ uses a one-sided sample from 1947 through the date of the index.

Panel D shows that there is substantial uncertainty associated with the estimated value of the trend, gap, or business cycle components near the beginning and ends of the sample. For example, the business cycle component has a standard deviation of 2.3 percentage points at the end of the sample, which corresponds to an R^2 of only slightly greater than 50 percent. The uncertainty falls as data accumulates: when there are 15 years of data after the endpoint, the standard error falls to less than 0.4 percentage points, which corresponds to an R^2 of 99 percent.

Figure 3 shows the full-sample estimates of the components over the period 1960–1990, together with the one-sided estimates of the components. The one-sided estimates are computed using "pseudo-real-time" methods; that is, the results shown for date t are constructed using data from the beginning of the sample through time period t. Thus, for example, to compute the one-sided estimate for 1969:12, data from 1947:2–1969:12 are used to estimate an AR(6) model. This model, in turn, is used to forecast and backcast 300 observations, and the 600-term band-pass filter is applied to the resulting series.

Figure 3 shows that the one-sided estimates are considerably different than the historical estimates, consistent with the standard error results shown in Figure 2. The one-sided estimates of the gap and business cycle components are less variable than their two-sided counterparts. This "attenuation" is a property of optimal estimates: the difference between the two-sided and one-sided estimates reflects unforecastable shocks that are uncorrelated with the one-sided estimates. The figure shows the underestimation of the output gap in the late 1960s and early 1970s as highlighted in Orphanides's (2003a) discussion of the "Great Inflation."

While there is substantial error in the level of the business cycle gap, the sign of the one-sided estimate of the output gap is a useful indicator of the sign of the two-sided gap. Table 1 summarizes the joint distribution of the signs of the one-sided and two-sided estimates of the business cycle component of industrial production. During the 1960–1990 sample, $\hat{P}(Y_{2-sided}^{BusinessCycle} > 0) = 0.51$, while $\hat{P}(Y_{2-sided}^{BusinessCycle} > 0) \mid Y_{1-sided}^{BusinessCycle} > 0) = 0.71$, where $\hat{P}$ denotes the relative frequency in the sample. Similarly $\hat{P}(Y_{2-sided}^{BusinessCycle} < 0) = 0.49$, while $\hat{P}(Y_{2-sided}^{BusinessCycle} < 0 \mid Y_{1-sided}^{BusinessCycle} < 0) = 0.76$. Thus, at least over this sample period, positive and negative realizations of $Y_{1-sided}^{BusinessCycle}$ served as reasonably reliable indicators of the sign of $Y_{2-sided}^{BusinessCycle}$.

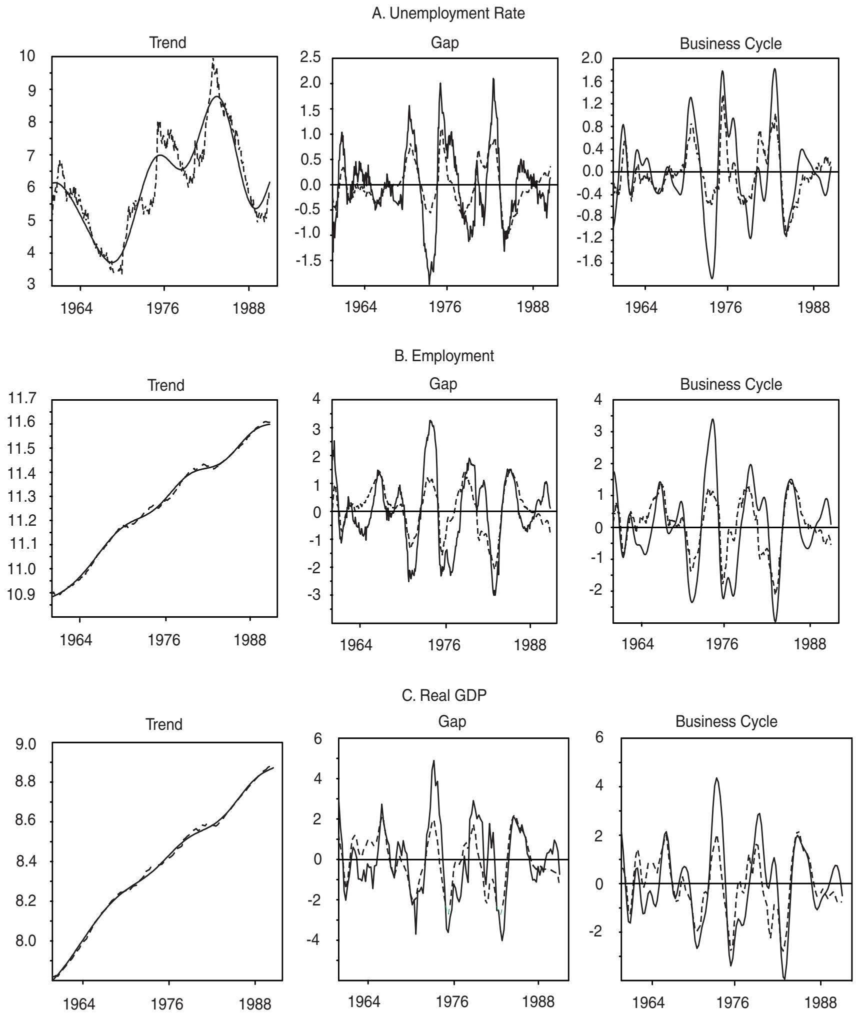
The index of industrial production is one of several cyclical indicators. Figure 4 summarizes results for three other indicators: the civilian unemployment rate and the logarithm of employment, both available monthly, and the logarithm of real GDP, a quarterly time series. The figure compares the two-sided and one-sided estimates of the trend, gap, and business cycle component for each of these series over 1960–1990. Table 2 summarizes uncertainty in the one-sided estimates by showing the estimated standard error associated with the one-sided band-pass filter estimate, the corresponding R^2, and the values of $\hat{P}(Y_{2-sided}^{BusinessCycle} > 0 | Y_{1-sided}^{BusinessCycle} > 0)$ and $\hat{P}(Y_{2-sided}^{BusinessCycle} < 0 | Y_{1-sided}^{BusinessCycle} < 0)$. The results for these series are similar to those obtained from the index of industrial production. There is significant error in the end-of-sample estimates with R^2 values of approximately 50 percent. That said, the sign of the filtered estimates predicts the sign of the two-sided estimates with a probability of approximately 70 percent.

3. IMPROVING THE ACCURACY OF ONE-SIDED BAND-PASS ESTIMATES

The error in one-sided band-pass estimates arises from the use of forecasts of future values of Y_t in place of true values. The resulting forecast errors lead to errors in the one-sided band-pass estimates. More accurate forecasts have smaller forecast errors, and, therefore, result in more accurate one-sided band-pass estimates. Forecasts may become more accurate through the use of improved forecasting methods or because of good luck associated with smaller shocks. This section quantifies the effect of both of these sources of increased accuracy for one-sided band-pass estimates of the output gaps.

The forecasts constructed in the last section were based on univariate information sets; that is, future values of Y_t were forecast using current and lagged values of Y_t. Several authors have noted that multiple indicators can, in principle, be used to increase the accuracy of output gaps. For example, Basistha and Startz (2005), Kuttner (1994), and Orphanides and van Norden

Figure 4 Two-Sided (Solid) and One-Sided (Dashed) Band-Pass Estimates: Unemployment Rate, Employment, and Real GDP



Notes: See the notes in Figure 3 for a description of the series plotted. Panel A uses monthly data on the civilian unemployment rate beginning in 1948; Panel B uses monthly data on the logarithm of nonfarm payroll employment beginning in 1947; and Panel C uses data on the logarithm of real GDP (chained $2,000) beginning in 1947.

(2002) discuss the issue in the context of Kalman filter estimates in unobserved components models, and Altissimo et al. (2006) and Valle e Azevedo (2006) discuss the issue in the context of one-sided band-pass filtered estimates.

Table 2 Summary of Results for Four Cyclical Indicators

	$Y^{Gap}_{1-sided}$		$Y^{BusinesCycle}_{1-sided}$		$\hat{P}[Y^{BusinessCycle}_{2-sided} > 0$	$\hat{P}[Y^{BusinessCycle}_{2-sided} < 0$
Series	**SE**	**R^2**	**SE**	**R^2**	$\mid Y^{BusinessCycle}_{1-sided} > 0]$	$\mid Y^{BusinessCycle}_{1-sided} < 0]$
Industrial Production	2.49	0.50	2.32	0.53	0.76	0.71
Unemployment Rate	0.56	0.48	0.53	0.52	0.63	0.66
Employment	0.99	0.50	0.94	0.53	0.69	0.68
Real GDP	1.12	0.55	1.05	0.58	0.71	0.71

Notes: This table summarizes results for the four series shown in the first column. SE indicates the standard error of the end-of-sample band-pass estimate of Y^{Gap} (column 2) or $Y^{BusinessCycle}$ (column 4), and R^2 is the corresponding R^2 of the one-sided estimate. $\hat{P}[Y^{BusinessCycle}_{2-sided} > 0 \mid Y^{BusinessCycle}_{1-sided} > 0]$ shows the relative frequency of $Y^{BusinessCycle}_{2-sided} > 0$ conditional on $\mid Y^{BusinessCycle}_{1-sided} > 0$ over the 1960–1990 sample period, and similarly for $\hat{P}[Y^{BusinessCycle}_{2-sided} < 0 \mid Y^{BusinessCycle}_{1-sided} < 0]$. Estimates were constructed using data beginning in 1947:2, except those involving the unemployment rate began in 1948:2.

Table 3 Standard Errors of One-Sided Band-Pass Estimates: AR (Univariate) and VAR (Multivariate) Forecasts

	$Y_{1-sided}^{Gap}$		$Y_{1-sided}^{BusinessCycle}$	
Series	**AR**	**VAR**	**AR**	**VAR**
Industrial Production	2.01	1.88	1.88	1.80
Unemployment Rate	0.46	0.41	0.43	0.40
Employment	0.78	0.77	0.75	0.75
Real GDP	1.03	0.86	0.95	0.83

Notes: This table summarizes results for the four series shown in the first column. The entries under "AR" are the standard errors of one-sided band-pass estimates constructed using forecasts constructed by univariate AR models with six lags. The entries under "VAR" are the standard errors of one-sided band-pass estimates constructed using forecasts constructed by VAR models with six lags for monthly models and four lags for quarterly models. The VAR models included the series of interest and first difference of inflation, the term spread, and building permits. Monthly models were estimated over 1960:9–2006:11, and quarterly models were estimated over 1961:III–2006:III.

Table 3 shows results from constructing one-sided estimates using forecasts from univariate time series models with forecasts constructed from Vector Autoregressive (VAR) models. The VAR models include the first difference of inflation (for personal consumption expenditures [all items] deflator), the term spread (the difference between ten-year Treasury bond yields and three-month Treasury bill rates), and housing starts (new permits). Inflation is included because it is often used as an indicator for the output gap, and the other variables are standard leading indicators of economic activity. VARs for the monthly series (industrial production, unemployment rate, and employment) use six lags of each of the variables and the quarterly VAR for real GDP uses four lags. Results are shown for the VAR estimated from 1960:9–2006:11 for the monthly series and 1961:III–2006:III for real GDP. The autocovariances of the forecast errors, which together with the band-pass filter weights, determine the standard error of the one-sided band-pass estimates, and were computed from the estimated parameters of the VAR.

The univariate standard errors are in the columns labeled "AR" in Table 3, and the multivariate standard errors are in the columns labeled "VAR".[4] There is a small but nonnegligible increase in precision associated with the VAR forecasts. For example, the standard error of $Y^{BusinessCycle}$ falls by approximately 5 percent (from 1.88 to 1.80) for industrial production and by

[4] The univariate standard errors shown in Table 3 are slightly smaller than the values shown in Table 2 because the standard errors in Table 2 included observations from the late 1940s and 1950s, which were somewhat more volatile than those in the 1960–2006 sample period used in Table 3.

Table 4 Standard Errors of One-Sided Band-Pass Estimates

	$Y_{1-sided}^{Gap}$		$Y_{1-sided}^{BusinessCycle}$	
Series	**1960–1983**	**1984–2006**	**1960–1983**	**1984–2006**
Industrial Production	2.27	1.47	2.12	1.37
Unemployment Rate	0.54	0.39	0.51	0.37
Employment	0.84	0.37	0.80	0.35
Real GDP	1.28	0.58	1.20	0.54

Notes: This table summarizes results for the four series shown in the first column. The standard errors for the one-sided band-pass estimates are computed using the same AR models as Table 2, but the standard deviation of the AR residual is computed over the sample period shown in the column headings.

over 10 percent (from 0.95 to 0.83) for real GDP. That said, the standard errors of the one-sided estimates remain large.

The standard errors for the one-sided band-pass estimates shown in Table 2 were based on autoregressive models estimated using data from the late 1940s through 2006, and those in Table 3 used estimates from 1960 through 2006. But, as is now widely appreciated, the volatility of real economic activity over the past 20 or so years has been much lower than the volatility in the preceding 30 years.[5] This Great Moderation is evident in Figures 2–4. For real variables, such as those considered here, the reduction in volatility is well characterized as a reduction in the volatility in the "shocks" to the AR model, rather than a change in the AR coefficients. (See Ahmed, Levin, and Wilson 2004, Blanchard and Simon 2001, and Stock and Watson 2002.) This implies that AR forecasting formulae have been relatively constant over the postwar period, but that the variance of forecast errors has fallen. This, in turn, implies that the standard error of one-sided band-pass estimates has fallen.

Table 4 presents estimates of the standard errors for one-sided band-pass estimates of Y^{Gap} and $Y^{BusinessCycle}$ over the 1960–1983 and 1984–2006 sample periods. These estimates are based on the same full-sample estimated AR models used in Table 2, but with error standard deviations that are allowed to be different in the two sample periods. The standard errors shown in Table 4 for 1960–1983 are computed using the AR error standard deviation estimated over 1960–1983, and the results for 1984–2006 use standard deviations estimated over 1984–2006. The reduction in volatility has been large: the standard deviation of the AR errors has fallen by approximately 50 percent, and this reduction is reflected in an increase in the precision of the one-sided band-pass estimates. For example, these results suggest that the one-sided

[5] For example, see Blanchard and Simon (2001), Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Stock and Watson (2002).

estimate of the GDP output gap was 1.3 percentage points during 1960–1983, but fell to 0.6 percentage points in the post-1984 period.

4. SUMMARY AND CONCLUSIONS

This article has discussed the problem of estimating output trends, gaps, and business cycle components using the "one-sided" data samples that are available in real time. The results indicate that one-sided estimates necessary for real-time policy analysis are substantially less accurate than the two-sided estimates used for historical analysis. The quantitative results suggest that one-sided estimates of gaps and business cycle components have an R^2 of approximately 0.50; that is, they forecast only 50 percent of the variability in historically measured gaps and business cycle components. Thus, the answer to the question posed in the title of this article, "How Accurate are Real-Time Estimates of Trends and Gaps?" is "not very." Small improvements can be achieved using leading indicators to help forecast future values of the output series used in the construction of the one-sided estimates. The Great Moderation has led to an increase in the accuracy of forecasts of real economic variables and this accuracy, in turn, has led to an increase in the precision of one-sided output trend, gap, and business cycle estimates.

The analysis in this article was based on one-sided estimates constructed using band-pass filters, but the conclusion coincides with the conclusion reached by other authors using different methods (see, for example, Staiger, Stock, and Watson 1997 for an analysis of the unemployment rate gap using spline methods and unobserved component models and Orphanides and van Norden 2002 for an analysis of output gaps using a wide variety of methods).

APPENDIX: LINEAR FILTERS

This Appendix reviews some key results on linear filters. Let $X_t = c(L)Y_t$, where $c(L) = c_{-r}L^{-r} + \ldots + c_{-1}L^{-1} + c_0L^0 + c_1L + \ldots + c_sL^s$ is a time-invariant linear filter. From (1.1), the ω^{th} component of Y_t, $Y(t, \omega)$ is a weighted average of $cos(\omega t)$ and $sin(\omega t)$. For notational simplicity, suppose that $Y(t, \omega) = 2cos(\omega t) = e^{i\omega t} + e^{-i\omega t}$. In this case, $X(t, \omega)$ has a simple representation:

$$\begin{aligned} X(t,\omega) &= \sum_{j=-r}^{s} c_j Y(t-j,\omega) \\ &= \sum_{j=-r}^{s} c_j[e^{i\omega(t-j)} + e^{-i\omega(t-j)}] \\ &= e^{i\omega t}\sum_{j=-r}^{s} c_j e^{-i\omega j} + e^{-i\omega t}\sum_{j=-r}^{s} c_j e^{i\omega j} \\ &= e^{i\omega t}c(e^{-i\omega}) + e^{-i\omega t}c(e^{i\omega}). \end{aligned}$$

To simplify this expression further, write the complex number $c(e^{i\omega})$ in polar form, as $c(e^{i\omega}) = a + ib$, where $a = Re[c(e^{i\omega})]$ and $b = Im[c(e^{i\omega})]$. Then $c(e^{i\omega}) = (a^2 + b^2)^{\frac{1}{2}}[\cos(\theta) + i\sin(\theta)] = ge^{i\theta}$ where $g = (a^2 + b^2)^{\frac{1}{2}} = [c(e^{i\omega})c(e^{-i\omega})]^{\frac{1}{2}}$ and $\theta = \tan^{-1}[\frac{b}{a}] = \tan^{-1}[\frac{Im[c(e^{i\omega})]}{Re[c(e^{i\omega})]}]$. This means that $X(t,\omega)$ can be written as

$$\begin{aligned} X(t,\omega) &= e^{i\omega t}ge^{-i\theta} + e^{-i\omega t}ge^{i\theta} \\ &= g[e^{i\omega[t-\frac{\theta}{\omega}]} + e^{-i\omega[t-\frac{\theta}{\omega}]}] \\ &= 2g\cos(\omega(t-\omega^{-1}\theta)) \\ &= gY(t-\omega^{-1}\theta,\omega). \end{aligned}$$

This expression shows that the filter $c(L)$ "amplifies" $Y(t,\omega)$ by the factor g and shifts $Y(t,\omega)$ back in time by $\omega^{-1}\theta$ time units.

Note that g and θ depend on ω, and so it makes sense to write them as $g(\omega)$ and $\theta(\omega)$. $g(\omega)$ is called the filter *gain* (or sometimes the *amplitude gain*), $g(\omega)^2 = [c(e^{i\omega})c(e^{-i\omega})]$ is called the *power transfer function* of the filter, and $\theta(\omega)$ is called the filter *phase*. In the expression below equation (1.2), $\rho(\omega) = \omega^{-1}\theta(\omega)$.

To derive the band-pass filter, first consider the problem of constructing the low-pass filter with frequency cutoff $\underline{\omega}$. Then, the gain of the band-pass filter is given by

$$gain(c(L)) = \mid c(e^{i\omega}) \mid = c(e^{i\omega}) = \left\{ \begin{array}{c} 1 \text{ for} -\underline{\omega} \le \omega \le \underline{\omega} \\ 0 \text{ elsewhere} \end{array} \right\},$$

where the second equality follows because $c(e^{i\omega})$ is real, ($c(L)$ is symmetric). Since $c(e^{-i\omega}) = \sum_{j=-\infty}^{\infty} c_j e^{-i\omega j}$, then $c_j = (2\pi)^{-1}\int_{-\pi}^{\pi} e^{i\omega j}c(e^{-i\omega})d\omega$ follows generally from $\int_{-\pi}^{\pi} e^{i\omega k}d\omega = \left\{ \begin{array}{c} 2\pi \text{ for } k=0 \\ 0 \text{ for } k \ne 0 \end{array} \right\}$. Setting the gain equal to unity over the desired frequencies and carrying out the integration yields

$$c_j = (2\pi)^{-1}\frac{1}{ij}e^{i\omega j} \mid_{-\underline{\omega}}^{\underline{\omega}} = \left\{ \begin{array}{c} \frac{1}{j\pi}\sin(\underline{\omega}j) \text{ for } j \ne 0 \\ \frac{\underline{\omega}}{\pi} \text{ for } j = 0 \end{array} \right\}.$$

The difference between low-pass filters with cutoffs ω_{Lower} and ω_{Upper} is a band-pass filter that passes frequencies between ω_{Lower} and ω_{Upper}, and this difference yields the filter weights given in (1.5).

To compute the standard error of the one-sided band-pass filtered estimate, suppose initially that Y_t is $I(0)$ with moving average representation $Y_t = \theta(L)\varepsilon_t$. For any date t, Y_t^{BP} is a function of values of Y_j with $j \leq T$ and values of Y_j for $j > T$, where T represents the final date in the sample. Write these two components as $Y_t^{BP} = w(L)Y_T + v(L^{-1})Y_T$, where $w(L)$ is a polynomial in nonnegative powers of L, and $v(L^{-1})$ is a polynomial in negative powers of L. The term $w(L)Y_T$ represents the part of Y_t^{BP} determined by values of Y_j with $j \leq T$, and the term $v(L^{-1})Y_T$ represents the part of Y_t^{BP} determined by Y_j with $j > T$. The variance of the one-sided estimate of Y_t^{BP} is then the variance of $\{v(L^{-1})Y_T - E[v(L^{-1})Y_T \mid Y_j, j \leq T]\}$. Write $v(L^{-1})Y_T = v(L^{-1})\theta(L)\varepsilon_T$, so that $v(L^{-1})Y_T - E[v(L^{-1})Y_T \mid Y_j, j \leq T] = d(L^{-1})\varepsilon_T$, where $d(L^{-1}) = [v(L^{-1})\theta(L)]_{-}$, and the polynomial operator $[.]_{-}$ retains terms involving negative powers of L. The variance of $v(L^{-1})Y_T - E[v(L^{-1})Y_T \mid Y_j, j \leq T]$ is then $\sigma_\varepsilon^2 \sum_j d_j^2$. Because the autocovariance generating function is symmetric, the variance associated with pre-sample values of Y_j can be computed using the same formula after time reversing the stochastic process. Finally, the same type of calculations can be used for $I(1)$ processes by computing the variance of $(v(L^{-1})-1)Y_T - E[(v(L^{-1})-1)Y_T \mid Y_j, j \leq T]$.

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The Economics of Sovereign Defaults

Juan Carlos Hatchondo, Leonardo Martinez, and Horacio Sapriza

Sovereign defaults are widespread throughout history. In particular, after Russia defaulted on its sovereign debt in 1998, numerous episodes of sovereign default followed. These recent episodes invigorated the study of sovereign defaults, giving rise to much interesting work that added to the large body of literature on this topic. This article discusses the economics of sovereign defaults, summarizing lessons from existing work on this issue. First, we describe the costs associated with a sovereign default episode. We discuss costs imposed by creditors and those implied by the information revealed by the default decision. Second, we identify circumstances that are likely to lead to a default episode. We explain that sovereign defaults are likely to be observed when resources available to the sovereign are low, borrowing costs are high, or there is a change in political circumstances. Finally, we discuss how understanding sovereign defaults may help to account for distinctive economic features of emerging economies. We conclude that even though there is a large body of literature studying default episodes, there is still a great deal that is not known.

1. SOVEREIGN BORROWING AND DEFAULTS

Sovereign debt refers to debt incurred by governments. Sovereign borrowing can be a key policy tool to finance investment or to respond to a cyclical downturn.

There are different definitions of a sovereign default. First, from a legal point of view, a default event is an episode in which a scheduled debt service

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E-mails: JuanCarlos.Hatchondo@rich.frb.org; Leonardo.Martinez@rich.frb.org; and Hsapriza@andromeda.rutgers.edu.

is not paid beyond a grace period specified in the debt contract. Second, credit-rating agencies consider a "technical" default an episode in which the sovereign makes a restructuring offer that contains terms less favorable than the original debt.[1]

Sovereign defaults do not necessarily imply a total repudiation of outstanding debt. Most default episodes are followed by a settlement between creditors and the debtor government. The settlement may take the form of a debt exchange or debt restructuring. The new stream of payments promised by the government typically involves a combination of lower principal, lower interest payments, and longer maturities. Credit-rating agencies define the duration of a default episode as the amount of time that passes between the default event and when the debt is restructured (even though there may be holdout creditors).

Sturzenegger and Zettelmeyer (2005) propose a methodology to compute estimates of debt recovery rates. They also describe recent debt restructurings and estimate the recovery rates for these episodes. Table 1 presents these estimates, which are equal to the market value of the new instruments obtained by the creditors in the debt exchange plus any cash payment they received divided by the net present value of the remaining contractual payments on the old instruments (inclusive of any principal or interest that remained unpaid after the date of maturity). The present values are discounted using the yield of the new instruments immediately after the results of the exchange offer became public information.[2]

Sovereign Borrowing Versus Private Borrowing

There are some similarities between sovereign borrowing and private sector borrowing. For example, like private agents (households and corporations), governments can borrow to finance long-lived investments. Furthermore, in the same way households borrow to preserve living standards through periods of temporary hardship, governments borrow if they do not want to decrease expenditures when tax revenues are low. In addition to seeking to smooth private consumption, households want to smooth their consumption of services provided by the government (law enforcement, justice, defense, public health, public education, parks, etc.). Consequently, benevolent governments would rather provide a smooth flow of services than have this flow fluctuating with tax revenues. Moreover, some government expenditures are not contingent on tax revenues, and therefore governments may want to borrow when tax revenues are low in order to afford these expenditures. For instance, gov-

[1] See Peter (2002) for further discussion on rating agencies' definitions of default.

[2] Sturzenegger and Zettelmeyer (2005) also discuss alternative ways of estimating recovery rates.

Table 1 Average Recovery Rates for Recent Debt Restructuring (1998–2005)

Country	Debt Restructuring Episodes	Rate (percent)
Russia	GKO/OFZs-residents	55.0
	GKO/OFZs-nonresidents	38.9
	MinFin3	36.8
	PRINs/IANs	47.4
Ukraine	OVDPs-residents	93.1
	OVDPs-nonresidents	43.7
	Chase Loan	69.3
	ING Loan	62.0
	International Bonds	62.2
Pakistan	Eurobonds	69.1
Ecuador	International Bonds	62.6
Argentina	Phase 1 (residents)	58.3
	Pesification	54.4
	2005 International	27.1
Uruguay	External	87.1
	Domestic	76.7

Source: Sturzenegger and Zettelmeyer (2005).

ernment employee wages are typically not contingent on tax revenues. Thus, governments may want to borrow when tax revenues are low in order to pay wages to their employees.

There are also significant differences between the borrowing problem faced by governments and the borrowing problem faced by private agents. The distinctive features of governments' borrowing problems imply that the economics of sovereign defaults may differ from the economics of personal or corporate bankruptcy.

First, the most important difference is that it is easier for households and firms to post appropriable collateral in order to improve borrowing conditions.[3] If a private agent defaults, the government forces him to hand over the assets posted as collateral. On the other hand, a sovereign cannot commit to hand

[3] This is the case, at least, in developed countries. Djankov, McLiesh, and Shleifer (forthcoming) argue that in developing countries, creditor protection is poor (as are information-sharing institutions), and therefore it is more difficult for lenders to force repayment or grab collateral. Berger and Udell (1990) explain that "Collateral plays an important role in U.S. domestic bank lending, as evidenced by the fact that nearly 70% of all commercial and industrial loans are currently made on a secured basis." In contrast, Fleisig (1996) states that only 10 percent of all loans are secured by collateral in Argentina.

Table 2 Selected Government Defaults and Rescheduling of Privately Held Bonds and Loans (1824–2003)

	1824–1834	1867–1882	1890–1900	1911–1921	1931–1940	1976–1989	1998–2003
Europe							
Austria		1868		1914	1932		
Bulgaria				1915	1932		
Germany					1932		
Greece	1824		1893				
Hungary					1931		
Italy					1940		
Moldova							2002
Poland					1936	1981	
Portugal	1834		1892				
Romania				1915	1933	1981	
Russia				1917			1998
Serbia-Yugoslavia			1895		1933	1983	
Spain	1831	1867, ’82					
Turkey		1876		1915	1940	1978	
Ukraine							1998
Latin America							
Argentina	1830		1890	1915	1930s	1982	2001
Bolivia		1874			1931	1980	
Brazil	1826		1898	1914	1931	1983	
Chile	1826	1880			1931	1983	
Columbia	1826	1879	1900		1932		
Costa Rica	1827	1874	1895		1937	1983	
Cuba					1933	1982	
Dominica							2003
Dom. Republic		1869	1899		1931	1982	
Ecuador	1832	1868		1911, ’14	1931	1982	1999
El Salvador	1827			1921	1931		
Guatemala	1828	1876	1894		1933		
Honduras	1827	1873		1914		1981	
Mexico	1827	1867		1914		1982	
Nicaragua	1828		1894	1911	1932	1980	

over its assets if it defaults, and in general there is no authority that can force it to do so. Few government assets are located outside of national borders, and even if there was a significant amount of government assets abroad, there are legal obstacles that would prevent them from being confiscated. Wright (2002) presents a case study that shows how attempts to attach sovereign assets have had limited success. Thus, sovereign debt is typically unsecured.

Second, the costs of bankruptcy are different from those of sovereign default. While for households and firms an important part of the costs of debt repudiation is determined by bankruptcy law, there is no international legal

Table 2 (Continued) Selected Government Defaults and Rescheduling of Privately Held Bonds and Loans (1824–2003)

	1824–1834	1867–1882	1890–1900	1911–1921	1931–1940	1976–1989	1998–2003
Latin America (continued)							
Panama					1932	1982	
Paraguay	1827	1874	1892	1920	1932	1986	
Peru	1826	1876			1931	1978, '83	
Uruguay		1876	1891	1915	1933	1983	2003
Venezuela	1832	1878	1892, '98			1982	
Africa							
Angola						1988	
Cameroon						1989	
Congo						1986	
Cote d'Ivoire						1984	
Egypt		1876				1984	
Gabon						1986	
Gambia						1986	
Liberia		1874		1912		1980	
Madagascar						1981	
Malawi						1982	
Morocco						1983	
Mozambique						1984	
Niger						1983	
Nigeria						1983	
Senegal						1981	
Sierra Leone						1977	
South Africa						1985	
Sudan						1979	
Tanzania						1984	
Togo						1979	
Uganda						1981	
Zaire						1976	
Zambia						1983	
Other							
Jordan						1989	
Pakistan						1981	1999
Philippines						1983	
Vietnam						1985	

Notes: Defaults are excluded unless they coincide with a cluster. Russia also defaulted in 1839; Venezuela in 1847 and 1864; and Spain, in 1820 and 1851. U.S. southern states defaulted in the 1840s. Defaults are federal except for Argentina's defaults in 1915 and during the 1930s, which were at the provincial level. The year listed refers to the initial rescheduling or default.

Source: Sturzenegger and Zettelmeyer (2006a) Table 1.1.

framework that imposes costs on a defaulting sovereign. The next section discusses costs of sovereign defaults.

Third, politico-economic factors affect the issuance of government debt (see Alesina and Tabellini 2005 and Persson and Svensson 1989). For example, a politician who cares mostly about the period during which he will be in office may not fully internalize the costs of issuing debt. Moreover, governments can borrow strategically to bind the hands of future governments with different preferences. Such strategic behavior would be more important in economies where policymakers' interests are more polarized.

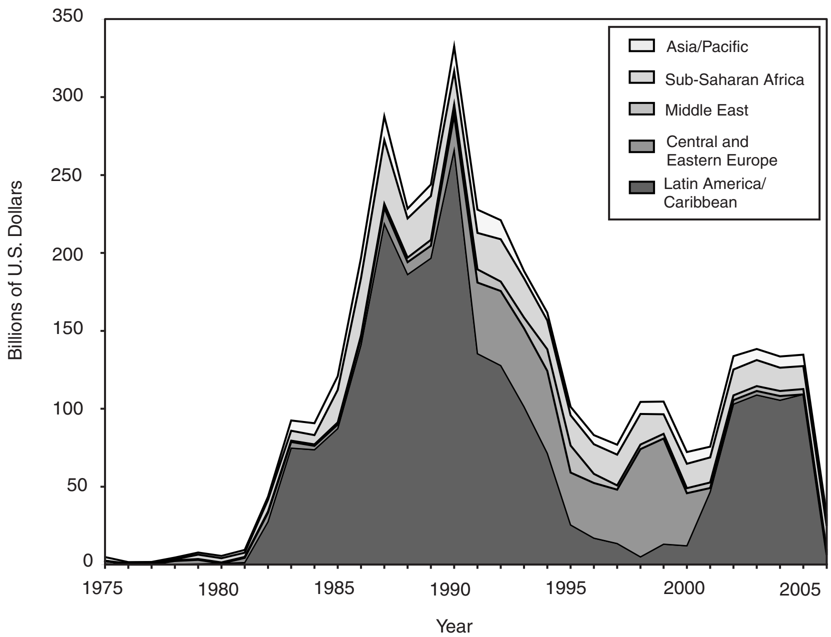
Historical Evidence

Sovereign defaults are not a novel feature of financial markets; and their incidence is widespread throughout history. For example, Spain defaulted six times between 1550 and 1650, and France defaulted eight times between 1550 and 1800 (see Reinhart, Rogoff, and Savastano 2003). Tomz and Wright (forthcoming) document 250 sovereign defaults by 106 countries between 1820 and 2004. Moreover, there are no reasons that rule out the occurrence of default episodes in the future (see Beers and Chambers 2006). Sturzenegger and Zettelmeyer (2006a) explain that sovereign defaults have occurred in temporal and sometimes regional clusters that correspond to the end of a boom-bust cycle in international capital flows. Table 2 presents a list of default events since 1824 grouped into seven temporal clusters. Figures 1 and 2 show the amount of sovereign debt in default and the number of countries in default since 1975, respectively. The amount of sovereign debt in default peaked at more than $335 billion in 1990. This debt was issued by 55 countries (Beers and Chambers 2006). One of the largest defaults in history occurred in late December 2001, when Argentina defaulted on $82 billion.

2. COSTS OF SOVEREIGN DEFAULTS

Identifying the costs of a sovereign default is essential in understanding why we observe sovereign debt in the first place. If there were no costs of defaulting, the sovereign would default under all circumstances. Anticipating this behavior, investors would never lend to sovereigns and there would be no sovereign debt. That is, for sovereign debt to exist, it is necessary that at least in some circumstances it would be more costly for a sovereign to default than to pay back its debt. Similarly, for sovereign defaults to exist, it is necessary that at least in some circumstances it would be more costly for a sovereign to pay back its debt than to default. There is an ongoing debate about the importance of different costs of a sovereign default. The remainder of this section describes two costs that are often mentioned in the literature: sanctions imposed by creditors and signaling costs.

Figure 1 Sovereign Debt in Default (1975–2006)



Notes: Sovereign loans from multilateral lending institutions (such as the World Bank) are not considered.

Source: Beers and Chambers (2006).

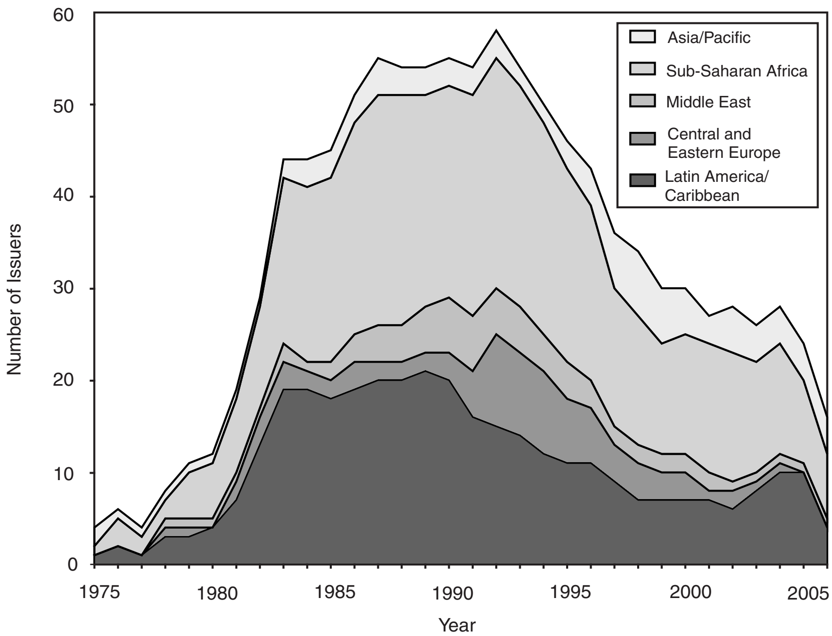
Costs Imposed by Creditors

It has been argued that creditors of defaulted debt can impose sanctions on defaulting sovereigns. In this subsection, we present arguments on the plausibility of punishments imposed by creditors that have been discussed in the sovereign default literature. First, we concentrate on the ability of creditors to increase the borrowing cost of defaulting sovereigns. Later, we focus on other sanctions.

Higher Borrowing Cost

The ability of creditors to impose higher borrowing costs on defaulting sovereigns has received a great deal of attention in the literature. In general, increasing a defaulting sovereign's borrowing costs would require coordination among holders of defaulted debt and all other potential lenders. It would require that potential creditors who find it beneficial to lend to a sovereign

Figure 2 Number of Countries in Default (1975–2006)



Notes: Sovereign loans from multilateral lending institutions (such as the World Bank) are not considered.

Source: Beers and Chambers (2006).

that has defaulted in the past would choose not to give credit to this sovereign, because these creditors want to punish the defaulter for its past behavior. In models of sovereign default, coordination among lenders can be sustained in infinitely repeated games in which a creditor wants to maintain his good reputation by not deviating from his agreement with other creditors in order to keep his share of the profits obtained through coordination (see Wright 2002).

Such a degree of coordination seems unlikely to occur in competitive credit markets with a large number of potential lenders. Wright (2005) discusses how in the past three decades, the sovereign debt market has become more competitive and explains how an increase in competition (number of creditors) may diminish the creditors' ability to coordinate (see also Wright 2002).[4] With more creditors, the share of the benefits from coordination for each creditor

[4] A similar point is raised by Cole, Dow, and English (1995) and Athreya and Janicki (2006), among others.

is smaller, and therefore deviations from a coordination agreement become relatively more attractive. This indicates that coordination was more likely to occur during periods in which the number of potential lenders was small, as in the 19th century when a large fraction of international capital flows was channeled through a few creditors. But coordination is less likely nowadays, when almost anyone can buy sovereign bonds.

Lenders can also try to impose financial sanctions that do not require such coordination. In their analysis of the legal consequences of sovereign default episodes, Sturzenegger and Zettelmeyer (2006b) discuss how holders of defaulted bonds succeeded in interfering with cross-border payments to other creditors who had previously agreed to a debt restructuring. If all cross-border payments could be blocked, a defaulting sovereign would not be able to borrow abroad—no creditor would lend if it were unable to collect the payments. From this, Sturzenegger and Zettelmeyer (2006b) infer that holders of defaulted bonds may have been able to "exclude" defaulting economies from international capital markets. Yet, at the same time they conclude that "legal tactics are updated all the time, and new ways are discovered both to extract payment from a defaulting sovereign as well as to avoid attachments." In particular, they expect that "the threat of exclusion may be less relevant for some countries or to all countries in the future." In any case, there are other financial alternatives available to defaulting economies. They could issue bonds in local markets, obtain aid, or ask for official credit (from other governments or multilateral financial institutions). It is not obvious whether a sovereign forced to use these alternatives would face a higher borrowing cost.

At the extreme level, instead of imposing a higher borrowing cost on defaulting sovereigns, creditors may exclude these sovereigns from capital markets. Punishment by exclusion is often discussed in sovereign default literature. For instance, it is one of the costs assumed in recent quantitative models of sovereign default.[5] There is also extensive empirical literature that attempts to identify whether creditors punish defaulting sovereigns by excluding them from capital markets. A common finding is that a default leads to a drainage in capital flows (this may be in part because sovereign defaults often occur together with devaluations; see IMF 2002 and Gelos, Sahay, and Sandleris 2004). However, the observed difficulties in market access after a default may be the result of the same factors that triggered the default decision itself. For example, both default and the difficulties in market access after default may be triggered by political turnover (see end of Section 3). The empirical literature finds no clear evidence of defaulters being

[5] See Aguiar and Gopinath (2006); Arellano (2005); Arellano and Ramanarayanan (2006); Bai and Zhang (2005); Cuadra and Sapriza (2006a, 2006b); Lizarazo (2005a, 2005b); and Yue (2006). Hatchondo, Martinez, and Sapriza (forthcoming) discuss the role of the exclusion assumption in quantitative models of sovereign default.

punished by creditors through exclusion or higher interest rates on new loans when sufficient control variables are used (see Eichengreen and Portes 2000; Gelos, Sahay, and Sandleris 2004; and Meyersson 2006).

Other Sanctions

On a number of occasions, a government has intervened actively in support of its constituents who are holders of defaulted debt issued by other sovereigns (see Sturzenegger and Zettelmeyer 2006a). These interventions have taken the form of diplomatic dissuasion, withholding of official credit, threat of trade sanctions, and in exceptional cases, armed interventions.

For instance, Mitchener and Weidenmier (2005) provide a case study of gunboat diplomacy.[6] They study the economic effects of the announcement of Theodore Roosevelt's 1904 Corollary to the Monroe Doctrine, which proclaimed that the United States would intervene in the affairs of unstable Central American and Caribbean countries that did not pay back their debts. Mitchener and Weidenmier (2005) find a drastic increase in Latin American sovereign bond prices following this announcement. This is an example of how an increase in the costs of defaulting that, in turn, implies a decrease in the default probability (other things being equal) can decrease a sovereign's borrowing cost.[7]

On the other hand, in their analysis of the importance of government interventions in favor of its constituents who are harmed by the default of other sovereigns, Sturzenegger and Zettelmeyer (2006a) conclude that "Creditor country government intervention in debt disputes has been the exception rather than the rule." This may be the case because the interests of holders of defaulted debt are not necessarily aligned with those of their governments. Furthermore, even though these interventions may have been important before World War II, no explicit sanctions or armed interventions were triggered by default episodes occurring after World War II.

It has also been argued that the IMF's role as crisis creditor has been used to increase the bargaining power of lenders in debt restructuring negotiations. However, recent changes in the IMF's policy indicate that the strength of IMF pressure declined over time. For instance, the IMF moved from its policy of not lending to countries that were in arrears (that is countries with debt that

[6] In international politics, gunboat diplomacy refers to the pursuit of foreign policy objectives with the aid of conspicuous displays of military power, which constitutes a direct threat of warfare should terms not be agreeable to the superior force.

[7] The price of a sovereign bond that promises to pay one unit the next period and satisfies the lenders' zero profit condition is given by $q = \frac{1-default\ probability}{1+r}$, where r is the interest rate at which lenders can borrow. The interest rate the sovereign would pay (if it does not default) is given by $\frac{1}{q}-1$ and is increasing with respect to the default probability (because the bond price, q, is decreasing with respect to the default probability).

remains unpaid following the date of maturity) in the 1980s to a policy of not lending to defaulting countries that were not in "good faith" negotiations with creditors in the 1990s (the exact meaning of good faith is unclear but seems to imply a weaker IMF stand toward defaulters).

Signaling Costs

Numerous theoretical studies of sovereign defaults present models in which a default is costly because of the information it signals (see Sandleris 2006). For example, a default decision may signal that the policymakers in office are less prone to respect property rights. Furthermore, government officials' assessments of the fundamentals of the economy may be different from the ones of private agents. As long as the default decision depends on these assessments, a default discloses some of the government's private information to market participants. For instance, if the government finds it optimal to default in bad circumstances (see Section 3), a default could signal poor economic conditions.

If there is persistence in the variables that are signaled by the default decision (the government type or economic conditions), defaulting increases the perceived probability of a future default (other things being equal). Thus, there is a signaling cost of defaulting because information revelation results in an increase in the borrowing cost.[8] In contrast with the costs discussed in the beginning of this section, signaling costs reflect the increased perceived probability of a future default and not a punishment imposed by creditors.

Furthermore, the signal transmitted by a default decision may have other consequences besides increasing the cost of future borrowing. Cole and Kehoe (1998) argue that a sovereign default may imply that the government is considered untrustworthy in other areas besides the credit relationship with lenders. Sandleris (2006) explains how by revealing negative information about itself or the economy, the government may affect firms' net worth and their ability to borrow, which may lower the desired level of investment. This can generate a contraction in foreign lending to domestic firms, and a credit crunch—a sudden reduction in the availability of loans or other forms of credit in the economy—in domestic credit markets. Using micro-level data, Arteta and Hale (2006) find that sovereign debt crises are systematically accompanied by a large decline in foreign credit to domestic private firms. IMF (2002), Kumhof (2004), and Kumhof and Tanner (2005) explain that domestic financial crises are observed after sovereign defaults—similarly, Kaminsky and

[8] Similarly, Chatterjee, Corbae, and Ríos-Rull (2005) study household bankruptcy in a model in which borrowers have different discount factors and there is asymmetric information about the borrower's type. In this model, there is a signaling cost of filing for bankruptcy because someone who files for bankruptcy is believed to be more likely to file again in the future.

Reinhart (1999) show that debt devaluations in developing countries are followed by banking problems. IMF (2002), Kumhof (2004), and Kobayashi (2006) argue that financial crises may lead to severe recessions.[9]

The signals implied by a government's default decision may also have political consequences. The default may reveal important characteristics of the incumbent policymakers, such as their competence. For instance, the poor economic conditions that trigger a default decision can be interpreted as the result of bad policies. Moreover, because the holdings of sovereign debt are not uniformly distributed across the population, a government's default may signal, to some extent, its redistributional goals.

Although the existence of signaling costs of defaulting seems plausible, it is not clear how important these costs are. More specifically, it is not clear how important the government's private information is, the extent to which this information is transmitted through the default decision, and the importance of the effects of communicating this information.

3. DETERMINANTS OF SOVEREIGN DEFAULTS

This section discusses which circumstances are likely to lead to a sovereign default. Investors try to measure the probability of the realization of such circumstances in order to estimate the probability of a default and then compute the appropriate price of sovereign bonds.[10] Of course, identifying the set of states that are likely to trigger a sovereign default is closely related with identifying how the costs of defaulting discussed in the previous section depend on these states.

Resources

A sovereign may find it optimal to repudiate outstanding debt contracts when current resources are sufficiently low. In order to avoid a default in these situations, large adjustments to expenditures or revenues would be required and these adjustments can be costly. Empirical evidence indicates that a sovereign tends to default in periods of low available resources. Using a historical data set with 169 sovereign defaults, Tomz and Wright (forthcoming) report that 62 percent of these default episodes occurred in years when the output level in the defaulting country was below its trend. Cantor and Packer

[9] Default episodes are often observed in periods of recessions. This means that a fraction of the low economic activity that is observed after a default episode can be explained by weak fundamentals existing prior to the default decision, and thus cannot be interpreted as a cost of defaulting (it is not triggered by the default decision).

[10] Cantor and Packer (1996) find that higher sovereign credit ratings—which reflect lower believed probabilities of a borrower not paying back his debts—are associated with lower interest rates.

(1996) find that sovereign credit ratings strongly respond to macroeconomic factors, such as the GDP growth rate and per capita income.

Government resources are low, for example, during a cyclical downturn. The countercyclicality of the interest rate paid by governments in developing countries (see Section 4) is consistent with sovereigns being more likely to default when economic conditions are worse. Higher interest rates may reflect a higher compensation to lenders who estimate a higher default probability.

Fluctuations of terms of trade (ratio of the price of exports to the price of imports) are an important driving force behind the business cycles in some emerging economies (see Mendoza 1995, Kose 2002, Broda 2004, and Broda and Tille 2003).[11] At the same time, several emerging economies strongly rely on commodity taxation as a source of public revenues and depend largely on imported intermediate goods that have no close substitutes. Some authors find that terms of trade fluctuations are a significant predictor of sovereign default and interest rate spreads in emerging economies (see Catao and Sutton 2002; Catao and Kapur 2004; Min et al. 2003; Min 1998; Caballero 2003; Caballero and Panageas 2003; Hilscher and Nosbusch 2004; and Calvo, Izquierdo, and Mejia 2004). A recent example of the relevance of commodity prices is found in Ecuador, where falling commodity prices led to a deterioration of the macroeconomic conditions and a sovereign default in 1999.[12] The sharp declines in oil prices during the second half of the 1990s have also been linked to the worsening of the macroeconomic and fiscal situation that led to the Russian default of 1998 (see Sturzenegger and Zettelmeyer 2006a).

Furthermore, episodes of sovereign default may be triggered by wars or civil conflicts that adversely affect a country's productivity (Sturzenegger and Zettelmeyer [2006a] describe such episodes). Defaults may also be triggered by a devaluation of the local currency when a relatively large fraction of the sovereign's debt is denominated in foreign currency and its revenues rely heavily on the taxation of nontradable goods. The magnitude of crises triggered by a devaluation of the local currency is likely to be amplified by currency mismatches in the banking sector, the corporate sector, and households.

Borrowing Costs

External factors that increase the cost of borrowing may also trigger a default episode. Both international interest rates and the total net lending to emerging

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[11] For many countries, the term of trades of a few goods significantly affect their income. For example, according to the United Nations Conference on Trade and Development, 57 developing countries depended on three commodities for more than half of their exports in 1995 (see World Bank 1999).

[12] Oil and bananas together accounted for 59 percent of Ecuadorian exports in 2001. Ecuador was the first country to default on Brady bonds (Brady bonds arose from an effort in the late 1980s to reduce the debt held by less-developed countries that were frequently defaulting on loans).

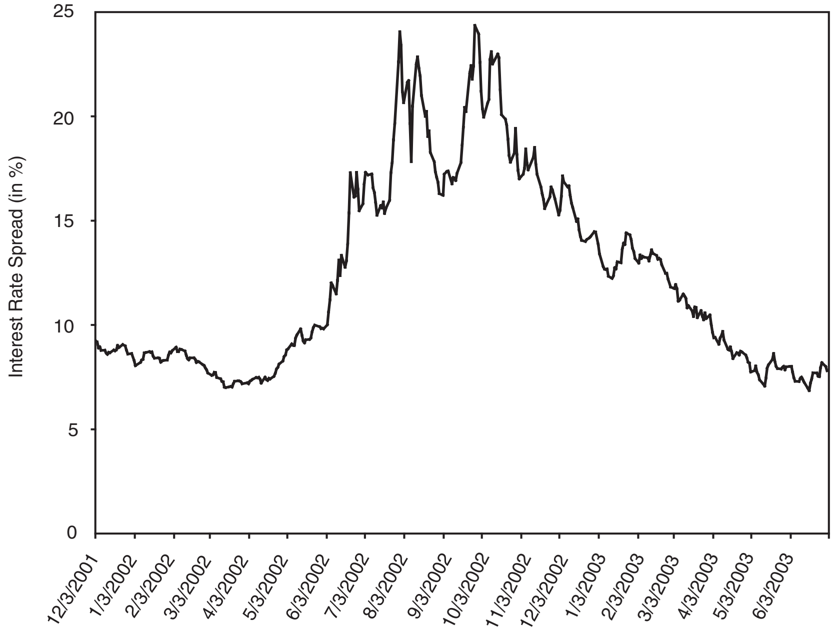
economies may influence lending to a particular developing country. Borrowing costs are particularly important in periods in which a country is trying to roll over its debt. The importance of external factors for the borrowing cost of developing countries is suggested by empirical studies that find that the interest rates paid by these countries have tended to move in the same direction as U.S. interest rates (see Lambertini 2001, Arora and Cerisola 2001, and Uribe and Yue 2006).

Political Factors

In addition to pure economic variables, political factors may also play a nontrivial role as determinants of defaults. There is a large literature discussing the links between political risk and sovereign defaults. Bilson, Brailsford, and Hooper (2002) define political risk as "the risk that arises from the potential actions of governments and other influential domestic forces, which threaten expected returns on investment." Sturzenegger and Zettelmeyer (2006a) conclude that "a solvency crisis could be triggered by a shift in the parameters that govern the country's willingness to make sacrifices in order to repay, due to changes in the domestic political economy (a revolution, a coup, an election etc.)." Similarly, Van Rijckeghem and Weder (2004) explain that it is reasonable to infer that a country's willingness to pay is influenced by politics, i.e., by the distribution of interests and by the institutions and power structures. Santiso (2003) writes, "One basic rule of the confidence game [in international financial markets] is then to be very careful when nominating the official government voicer. For investors it is mainly the ministry of economics or finance or the governor of the central bank."

Figure 3 illustrates the behavior of the sovereign spread in Brazil before and after the run-up to the presidential elections in October 2002. This behavior is often mentioned as an example of the importance of political factors as determinants of default decisions. The concerns raised by the left-wing presidential candidate Luiz Inácio "Lula" da Silva because of his past declarations in favor of debt repudiations is the most accepted explanation for the sharp increase in the country spread preceding the Brazilian election (see Goretti 2005). Spreads may have increased because of a decrease in the expected willingness to pay by the future government. More recently, the newly elected president of Ecuador, Rafael Correa, declared his intentions to restructure the country's debt. On January 17, 2007, two days after taking office, Ecuador's Minister of the Economy told a group of investors that the government may repay only 40 percent of its foreign debt as part of an effort to free up funds for health care and education. The day after, Ecuador's benchmark government foreign securities tumbled, driving the yield up 1.1 percentage points to 14.32 percent (see Pimentel 2007).

Figure 3 Sovereign Bonds Spread in Brazil



Source: JPMorgan (EMBI Global).

Empirical studies suggest that political factors are important in understanding sovereign default. Citron and Nickelsburg (1987) find that political instability is statistically significant as a determinant of a country's default probability. Along the same line, Balkan (1992) considers two dimensions of the borrower's political environment, a democracy index and a political instability index, and finds them statistically significant in explaining default probabilities. Rivoli and Brewer (1997) find that long- and short-term armed conflict in a country and changes in the long-term political legitimacy of the government are the most significant political predictors of debt reschedulings during the 1980s. Kohlscheen (2003) finds that parliamentary democracies experience a lower probability of default than presidential systems. He argues that this is explained by the higher number of veto players (i.e., political players with power to prevent a default) in parliamentary systems. Moser (2006) finds a significant effect of changes of the finance minister and/or the minister of the economy on a country's interest rate spreads. He argues that such events may reveal important signals about the government's future policy course. These signals may contain information that affect expectations both about how the government will influence future growth and the policymakers' willingness to service debt.

The International Country Risk Guide's index of political risk for investors is used in recent empirical studies to account for the effect of political circumstances in environments where government priorities shift frequently (see Arteta and Hale 2006). The index is an attempt to evaluate the political risk faced by businesses in different countries. It is computed based on experts' subjective analysis. The index has also been interpreted as a proxy of the quality of institutions (see Reinhart, Rogoff and Savastano 2003, and Meyersson 2006). Similarly, the World Bank's Country Policy and Institutional Assessment database (CPIA) summarizes assessments on 20 scores in the general areas of economic management, structural policies, policies for social inclusion, and public sector management and institutions (see Gelos, Sahay, and Sandleris 2004).

Alfaro and Kanczuk (2005), Cole, Dow, and English (1995), and Hatchondo, Martinez, and Sapriza (2007) present models in which both default and difficulties in market access after a default may be triggered by political turnover. In their models, policymakers with different willingness to pay, alternate in power. When policymakers with a weaker willingness to pay take power, they may default on the debt issued by investor-friendly governments (those with a stronger willingness to pay). Following this, as long as policymakers with a weaker willingness to pay stay in power, governments experience difficulties in market access. It is more costly for these governments to borrow (because lenders understand that, other things being equal, these governments are more willing to default), and therefore they borrow less. Market access improves after the defaulting policymakers lose power. A clear example of this is discussed in Cole, Dow, and English (1995). They explain that "the ability of Reconstruction governments in Florida and Mississippi to borrow after the Civil War suggests that the old creditors could not block new loans once the states' reputations had been restored by an observable change in regime."

In Hatchondo, Martinez, and Sapriza (2007), we argue that the stability of investor-friendly regimes is key for these political defaults to occur. In our model, political defaults occur only if governments with a stronger willingness to pay are expected to stay in power long enough. Recall that the price received by the government for the bonds it issues incorporates a discount that mirrors the default probability. If an investor-friendly government chooses borrowing levels that would lead a less-friendly government to default, it has to compensate lenders for this contingency, i.e., for the contingency that less-friendly policymakers become the decisionmakers in the future. If the probability of this contingency is high enough (investor-friendly regimes are not stable), it is too expensive for a friendly government to choose borrowing levels that would lead less-friendly governments to default. In this scenario, friendly governments choose borrowing levels that even less-friendly governments will most likely choose to pay, and therefore it is unlikely that under these circumstances political turnover triggers a default.

In order to gauge the importance of political factors as determinants of some recent default episodes, in Hatchondo, Martinez, and Sapriza (2007) we study the behavior of International Country Risk Guide's index of political risk for investors in these episodes (we study Argentina, Ecuador, Pakistan, Russia, and Uruguay). We conclude that the Argentinean default in 2001 is the most likely to have been triggered by a change in political circumstances. Only Argentina and Uruguay exhibit a relatively high degree of political stability that is necessary in our model for a default to be triggered by political turnover. But while Argentina exhibits the widest difference in the average levels of political risk before and after its default, these two levels are almost identical in Uruguay.

4. BUSINESS CYCLES IN EMERGING ECONOMIES AND SOVEREIGN DEFAULT

This section discusses how understanding the economics of sovereign defaults helps to account for distinctive features of business cycles in emerging economies. The link between interest rates and business cycles has recently been the subject of intense research in international economics. For example, Neumeyer and Perri (2005) find that the dynamics of interest rates are important for understanding business cycle fluctuations in emerging economies. A similar finding is presented by Uribe and Yue (2006). To the extent that the interest rate paid by sovereigns is influenced by the probability of default, understanding the rationale of sovereign defaults may help one to understand business cycles in emerging economies.

Several studies have documented that business cycles in small emerging economies differ from those in small developed economies (see Aguiar and Gopinath 2007, Neumeyer and Perri 2005, and Uribe and Yue 2006). Table 3 presents average business cycle statistics for emerging economies (Argentina, Brazil, Korea, Mexico, and the Philippines) and developed economies (Australia, Canada, the Netherlands, New Zealand, and Sweden) computed by Neumeyer and Perri (2005); in the table, σ denotes a standard deviation and ρ denotes a correlation.

Table 3 shows that some moments are similar across the two groups of countries but other moments are noticeably different. For example, compared with developed economies, emerging economies feature:

1. More volatility—the volatilities of output, real interest rates, and net exports are higher.

Table 3 Average Business Cycle Statistics for Emerging and Developed Economies

	Emerging Economies	Developed Economies
σ(GDP)	2.79	1.37
σ(R)	2.32	1.66
σ(NX)	2.40	0.92
σ(PC)/σ(GDP)	1.30	0.92
σ(TC)/σ(GDP)	1.71	1.08
σ(INV)/σ(GDP)	3.29	3.44
ρ(R,GDP)	-0.55	0.20
ρ(NX,GDP)	-0.61	-0.23
ρ(PC,GDP)	0.80	0.67
ρ(TC,GDP)	0.79	0.68
ρ(INV,GDP)	0.88	0.73
ρ(NX,R)	0.51	-0.22
ρ(PC,R)	-0.55	0.24
ρ(TC,R)	-0.56	0.25
ρ(INV,R)	0.48	0.21

Notes: Net exports (NX) are exports minus imports over GDP. Real interest rates (R) are in percentage points. Total consumption (TC) includes private (PC) and government consumption, changes in inventories, and statistical discrepancy. Investment (INV) is gross fixed-capital formation. All series except net exports and real interest rates are in logs. All series have been Hodrick-Prescott filtered. Statistics are based on quarterly data.

Source: Neumeyer and Perri (2005).

2. Higher volatility of consumption relative to income—the ratio of volatilities is typically higher than one in emerging economies, while it is roughly equal to one in developed economies.

3. Countercyclical real interest rates in contrast with the procyclical real interest rates in developed economies.

4. More countercyclical net exports.

Other distinctive features of emerging economies are that most of these economies exhibit a procyclical government expenditure (government expenditure is acyclical or slightly countercyclical in developed countries) and a countercyclical inflation tax (the inflation tax is procyclical in developed countries). These features are documented by Gavin and Perotti (1997), Talvi and Vegh (2005), and Kaminsky, Reinhart, and Vegh (2004).

Default risk may help explain some of the distinctive features of emerging economies. In recent years, several authors have used the sovereign default framework proposed by Eaton and Gersovitz (1981) to account for the business cycle regularities of emerging economies.[13] In this framework, the high interest rates paid by developing countries reflect a compensation for the default probability. Furthermore, the countercyclicality of spreads paid by developing countries is consistent with the fact that sovereigns are more likely to default when economic conditions are relatively bad (see beginning of Section 3). The tendency of sovereigns to default in bad times implies that in such times, borrowing is more expensive, and thus borrowing levels may be lower. This is consistent with the more countercyclical net exports in developing countries.[14] Lower borrowing levels in bad times may explain the higher volatility of consumption relative to income observed in emerging economies. Similarly, if borrowing is more expensive in bad times, then it may be optimal to tax more and decrease government expenditures in such times, which would help to explain the procyclicality of public expenditures and the countercyclicality of tax rates in emerging countries. Of course, a complete understanding of the differences between developed and developing countries would require a theory of why default risk is higher in developing countries.

5. CONCLUSIONS

Sovereign default episodes are widespread throughout history and are likely to continue to occur in the future. The discussion in this article suggests that even though there is a large literature studying default episodes, there is still a great deal to be learned. More research is necessary to assess the magnitude of the different costs of defaulting and to understand the precise role played by the determinants of a sovereign default. There are also open questions in other dimensions. For instance, it is not clear what explains differences in recovery rates on defaulted debt or differences in the duration of a default episode. Answering these questions, and thus advancing our understanding of the economics of sovereign default, seems a necessary step in order to completely comprehend the distinctive economic features of emerging economies.

[13] See Aguiar and Gopinath (2006); Arellano (2005); Arellano and Ramanarayanan (2006); Bai and Zhang (2005); Cuadra and Sapriza (2006a,b); Eyigungor (2006); Hatchondo, Martinez, and Sapriza (2006, 2007, forthcoming); Lizarazo (2005a, 2005b); and Yue (2006).

[14] Similarly, in an environment with moral hazard and risk of repudiation, Atkeson (1991) shows that the optimal contract specifies that the borrowing country experience a capital outflow when the worst realizations of national output occur.

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Banks and Liquidity Creation: A Simple Exposition of the Diamond-Dybvig Model

Douglas W. Diamond

Banks make loans that cannot be sold quickly at a high price. Banks issue demand deposits that allow depositors to withdraw at any time. This mismatch of liquidity, in which a bank's liabilities are more liquid than its assets, has caused problems for banks when too many depositors attempt to withdraw at once (a situation referred to as a bank run). Banks have followed policies to stop runs, and governments have instituted deposit insurance to prevent runs. Diamond and Dybvig (1983) develop a model to explain why banks choose to issue deposits that are more liquid than their assets and to understand why banks are subject to runs. The model has been widely used to understand bank runs and other types of financial crises, as well as ways to prevent such crises. This article uses narrative and numerical examples to provide a straightforward explanation of the ideas in Diamond and Dybvig (1983).

Diamond and Dybvig (1983) argue that an important function of banks is to create liquidity, that is, to offer deposits that are more liquid than the assets that they hold. Investors who have a demand for liquidity will prefer to invest via a bank, rather than hold assets directly. Before discussing the

I am grateful to Elizabeth Cammack for helpful comments on this article. The article gives an alternative exposition of Diamond-Dybvig (1983), and my understanding of these topics relies not only on my joint work with Phil Dybvig, but also on many subsequent conversations with him. This article is a revised and extended version of materials that I have presented to students at The University of Chicago. Douglas W. Diamond is Professor of Finance in the Graduate School of Business at The University of Chicago, Research Associate at the National Bureau of Economic Research, and Visiting Scholar with the Federal Reserve Bank of Richmond. The views expressed in this article are those of the author and not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

methods by which banks might create liquidity, it is important to understand why there is a demand for liquidity by consumers or producers. I begin with the consumer demand for liquidity. Investors demand liquidity because they are uncertain about when they need to consume and, thus, how long they wish to hold assets. As a result, they care about the value of liquidating their assets on several possible dates, rather than on a single date.

Creating deposits that are more liquid than the assets held by banks can be viewed as an insurance arrangement in which depositors share the risk of liquidating an asset early at a loss. This model explains an important function of banks. It also shows that offering these demand deposits subjects the banks to bank runs if too many depositors withdraw.

Creating liquid deposits is one important function of financial intermediaries like banks. Another function is monitoring borrowers and enforcing loan covenants. The latter function is modeled in Diamond (1984), and is described in a simple framework in Diamond (1996).

1. THE DEMAND FOR LIQUIDITY

This section first analyzes some important reasons for the demand for more liquid assets by investors who are consumers. It then provides an alternative motivation for a demand for liquid assets by entrepreneurs. When the assets that investors can hold directly are illiquid, there is a demand for creating more liquid assets.

An illiquid asset is one in which the proceeds available from physical liquidation or a sale on some date are less than the present value of its payoff on some future date. In the extreme, a totally illiquid asset is worthless (cannot be sold or physically liquidated for a positive amount) on some date but has a positive value on a later date. The lower the fraction of the present value of the future cash flow that can be obtained today, the less liquid is the asset.

Consider the following asset on three dates, $T = 0$, $T = 1$, and $T = 2$. If one invests one unit at date 0, it will be worth r_2 at date 2, but only $r_1 < r_2$ at date 1. The lower $\frac{r_1}{r_2}$ is (holding constant market discount rates), the less liquid is the asset.

The Uncertain Horizon of Investors

Investors face an uncertain horizon to hold the asset. Each will need to consume either at date $T = 1$ or $T = 2$. However, as of date 0, an investor does not know at which date he will need to consume. Each begins with 1 with which to invest on date $T = 0$. An investor cannot buy direct insurance against his need for liquidity, because the need is private information and, thus, is not due to some observable event such as a hurricane. However, contracts can be designed that indirectly provide this insurance. Liquid assets that offer a smaller loss when liquidated early can provide indirect insurance. As a result, for some risks, investors can save and liquidate the liquid assets as needed. I will call an investor a "type 1" if he needs to liquidate at $T = 1$ and a "type 2" if he can wait until $T = 2$. For this example, this means he will consume only at $T = 1$ if of type 1, and only at $T = 2$ if of type 2 (or he can store date 1 consumption goods and consume them at date 2).

As of date 0, an investor does not know which type he will be, but each investor has a probability t of being of type 1 and $1 - t$ of being of type 2. There is no aggregate uncertainty, and there will be a fraction t of investors of type 1. To be concrete, suppose that $t = \frac{1}{4}$ and that there are 100 investors. As a result, 25 will be of type 1 and 75 will be of type 2, but it is not known at date 0 *which* investors will be of each type.

A type 1 investor with utility function $U(c)$ who consumes c_1 at $T = 1$ has utility $U(c_1)$. A type 2 investor who consumes c_2 at $T = 2$ (this c_2 includes any stored date 1 consumption goods) has utility $U(c_2)$. The utility function is the same for both types, but the date on which an investor wishes to consume depends on his type. An investor who holds the asset (r_1, r_2), which gives a choice of r_1 at date 1 or $r_2 > r_1$ at date 2, consumes $c_1 = r_1$ if of type 1 (with probability t) or $c_2 = r_2$ if of type 2 (with probability $1 - t$). The investor's expected utility is given by

$$tU(r_1) + (1 - t)U(r_2).$$

I assume that the investor has the risk-averse utility function of $U(c) = \frac{-1}{c}$. To simplify exposition, I add a constant of 1 to the utility (with no effect on any decisions) and use the utility function $U(c) = 1 - (\frac{1}{c})$. This allows the expected utility calculations to yield positive numbers.

Comparing More and Less Liquid Assets

Consider the following two assets, both of which cost 1 at date 0. The illiquid asset has $(r_1 = 1, r_2 = R)$ and a more liquid asset has $(r_1 > 1, r_2 < R)$. Investors have access only to the illiquid asset. Later, I will show how banks can create the more liquid asset, although there is no physical asset with its payoffs, but for now I simply illustrate the demand for liquidity with the following numerical example for the case where the probability of being of type 1 is $t = \frac{1}{4}$ and the illiquid asset has $(r_1 = 1, r_2 = R = 2)$. As a comparison, consider a hypothetical more liquid asset that has $(r_1 = 1.28, r_2 = 1.813)$. Section 2 explains why these particular numerical values are used. The expected utility from holding the illiquid asset is

$$\frac{1}{4}U(1) + \frac{3}{4}U(2) = 0.375.$$

The expected utility from holding the more liquid asset is

$$\frac{1}{4}U(1.28) + \frac{3}{4}U(1.813) = 0.391 > 0.375.$$

Each investor prefers the more liquid asset. A risk-averse investor prefers this smoother pattern of returns; holding the illiquid asset is risky because it delivers a low amount when liquidated early, on date 1.

Note that if investors were not risk averse and had constant marginal utility of consumption, they would not prefer this particular liquid asset. That is, if $U(c) = c$, then the expected utility of holding any asset is equal to its expected payoff given the policy of liquidating when of type 1. For the illiquid asset, the expected payoff is

$$\frac{1}{4}(1) + \frac{3}{4}(2) = 1.75.$$

The more liquid asset gives an expected payoff of:

$$\frac{1}{4}(1.28) + \frac{3}{4}(1.813) = 1.68 < 1.75.$$

The more liquid asset has a lower expected rate of return. Sufficiently risk-averse investors, but not risk-neutral investors, are willing to give up some expected return to get a more liquid asset.

Investors choose to liquidate assets when consumption is highly valuable to them. In particular, a type 1 investor liquidates the asset at a time when his marginal utility of consumption is high. An investor's demand for liquidity is greater the higher his (relative) risk aversion is, because liquidating early implies low consumption and, thus, high marginal utility of consumption.

Entrepreneurial Liquidity Demand

An alternative motivation for a large demand for liquid assets comes from an entrepreneur who may have a sudden need to fund a very high return project at date 1 (which cannot be funded elsewhere). The entrepreneur wishes only to consume on date 2 but may choose to liquidate assets on date 1 to fund this high return project. As a result, the entrepreneur places an especially high value on date 1 liquidation proceeds in the circumstances where he wants to liquidate early. Suppose that with probability t, the entrepreneur will be able to fund the high return investment project and that it returns $\Psi > R$ per unit invested. With probability $1 - t$, he does not get this opportunity and has access only to storage (storing one unit of goods at date 1 returns one unit at date 2). The availability of the high return is private information. Consider an asset that costs 1 at date 0 and offers either r_1 at date 1 or $r_2 > r_1$, at date 2. When the entrepreneur has access only to storage, he will not liquidate the asset. However, when he needs to fund the high return project he will liquidate it if the project's return Ψ exceeds $\frac{r_2}{r_1}$, the rate of return from continuing to hold the asset. As of date 0, the entrepreneur values an asset that can be liquidated for r_1 at date 1 or r_2 at date 2, as follows: $tr_1\Psi + (1-t)r_2$, if $\Psi > \frac{r_2}{r_1}$, and as $tr_1 + (1-t)r_2$, if $\Psi \leq \frac{r_2}{r_1}$. This is qualitatively similar to the risk-averse consumer, because the entrepreneur liquidates when the value of the proceeds is very high. Suppose that if $\Psi = 2.5$, $R = 2$, and $t = \frac{1}{4}$, the entrepreneur then values the illiquid asset ($r_1 = 1, r_2 = 2$) as $\frac{1}{4}\Psi(1) + \frac{3}{4}(2) = 2.125$, and the liquid asset ($r_1 = 1.28, r_2 = 1.813$) as $\frac{1}{4}\Psi(1.28) + \frac{3}{4}(1.813) = 2.160$. The entrepreneur prefers the more liquid asset.

The entrepreneurial demand for liquidity will be even more similar to the investor/consumer demand for liquidity if the high return project has decreasing returns to scale.

I do not continue to analyze the entrepreneurial demand for liquidity here, but refer the reader to Diamond and Rajan (2001) and Holmström and Tirole (1998). I now return to the consumer demand for liquidity.

2. BANK LIQUIDITY CREATION

I now show that a bank can provide the more liquid asset by offering demand deposits, even though the bank invests in the illiquid asset ($r_1 = 1, r_2 = 2$). I assume a mutual bank without equity (purely for expositional simplicity). Suppose that in return for a deposit of 1 at date $T = 0$, the bank offers to pay $r_1 = 1.28$ to those who withdraw at $T = 1$ or to pay $r_2 = 1.813$ to those who withdraw at $T = 2$.

If the bank receives \$1 from each of the 100 investors, it receives \$100 in deposits on date $T = 0$. If the bank invests in the illiquid asset, it will need to liquidate some of the illiquid asset at $T = 1$ to pay 1.28 to those who withdraw.

At $T = 1$, the bank's entire portfolio is worth \$100. Suppose 25 depositors withdraw 1.28 each, then $25(1.28) = 32$ assets must be liquidated: 32 percent of the portfolio must be liquidated. If 32 assets are liquidated, then 68 will remain until $T = 2$, when they will be worth $R = 2$ each. On date 2, there remain 75 depositors, each will receive

$$\frac{[100-32]2}{75} = \frac{[68]2}{75} = 1.813.$$

Depositors prefer the more liquid asset to the illiquid asset. A bank *can* provide the more liquid deposit which has a smaller loss from early liquidation than is available from holding the illiquid assets directly. This liquidity transformation service is one of the most important functions of banks. If the bank offers the more liquid deposits and invests in the illiquid assets, it can create liquidity. It is an equilibrium (a Nash equilibrium) for 25 depositors to withdraw at $T = 1$, because if *all* depositors expect 25 to withdraw at $T = 1$, only type 1 depositors will withdraw because the 75 type 2 depositors prefer the 1.813 available at $T = 2$ to the 1.28 available at $T = 1$.

When assets are illiquid and risk-averse investors do not know when they will need to liquidate, the bank can create a more liquid asset that allows investors to share the risk of liquidation losses. The bank can give a fraction t of investors r_1 at date 1 and a fraction $1-t$ of investors $r_2 = \frac{[1-tr_1]R}{1-t}$ at date 2, because if a fraction t of the depositors get r_1 in period $T = 1$, this will leave a fraction $[1-tr_1]$ of the assets unliquidated and in place until date 2. Each of the remaining fraction $(1-t)$ of depositors can receive $r_2 = \frac{[1-tc_1]R}{1-t}$ in period 2. Note that for the illiquid asset, $r_1 = 1$ and $r_2 = R$.

The Optimal Amount of Liquidity

This section derives the optimal amount of liquidity for the bank to create. The optimal levels of r_1 and r_2 will maximize the ex ante expected utility of each investor at date 0. The optimum is to provide a choice between $r_1 = 1.28$ at date 1 or $r_2 = 1.813$ at date 2. This derivation is not essential to understanding the balance of the article. The optimal amount of liquidity to create is the amount that maximizes each investor's ex ante expected utility, choosing $c_1 = r_1, c_2 = r_2$ to maximize $tU(r_1) + (1-t)U(r_2)$, subject to $r_2 \leq \frac{[1-tr_1]R}{1-t}$, $r_1 \geq 0$, $r_2 \geq 0$. For an interior optimum, the optimal values satisfy $U'(r_1) = RU'(r_2)$, so the marginal utility is in line with the marginal cost of liquidity, and $r_2 = \frac{[1-tr_1]R}{1-t}$, because no liquidity is wasted. For the case used in the example where $U(c) = 1 - \frac{1}{c}$, marginal utility is $U'(c) = \frac{1}{c^2}$ and the condition is $\frac{r_2^2}{r_1^2} = R$, or $\frac{r_2}{r_1} = \sqrt{R}$ because both r_1 and r_2 are positive.[1] From $r_2 = \frac{[1-tr_1]R}{1-t}$, this becomes $r_1 = \frac{\sqrt{R}}{1-t+t\sqrt{R}}$. For the example of $R = 2$, $t = \frac{1}{4}$, this optimum is $r_1 = 1.28$.

An Extension

When long-term assets are even more illiquid, there is an additional way that banks can help investors share the risk of liquidation losses. Suppose that the illiquid asset is as before, except that it returns $1-\tau$ (instead of 1) if liquidated at date 1, and $\tau > 0$. In this case a short-term liquid asset (equivalent to storage) that returns one unit per unit invested in the previous period offers a higher one-period return than investing in a long-term asset and liquidating it at date 1. However, because a bank knows that a fraction t of depositors needs to withdraw at $T = 1$, it can obtain the same set of payoffs as before:

[1] For general constant relative risk aversion utility functions $U(c) = \frac{c^{1-\rho}}{(1-\rho)}$, marginal utility is $U'(c) = c^{-\rho}$. The optimal r_1 is greater than 1 whenever the rate of relative risk aversion, ρ, is greater than 1 (as seems to be true in practice). At an interior optimum, $\frac{r_1^{-\rho}}{r_2^{-\rho}} = R$ or $\frac{r_2}{r_1} = R^{\frac{1}{\rho}}$. With $\rho > 1$, this implies $r_2 < R$, $r_1 > 1$. The example assumes $\rho = 2$.

$r_2 = \frac{[1-tr_1]R}{1-t}$, by investing in enough short-term assets to finance all of the date 1 withdrawals. If the bank pays r_1 at date 1 or r_2 at date 2, it puts a fraction (tr_1) of assets into short-term assets and $1 - (tr_1)$ into long-term illiquid assets and achieves the same payoffs as in the previous case. This holding of an inventory of liquid assets is referred to as the asset management of liquidity.

Investors holding the assets directly cannot perform as well as the bank. One possibility is for the bank to offer the choice between $r_1 = 1$ or $r_2 = R$. If an individual were to directly hold assets that allowed 1 to be obtained at date 1 (to consume 1 if of type 1), he would need to put 100 percent into short-term liquid assets and would consume only 1 if of type 2, by reinvesting in the short-term asset at date 1. The individual cannot achieve $r_1 > 1$ at all. To obtain $r_2 = R$, he must hold only illiquid long-term assets, implying that the largest r_1 available is $r_1 = 1 - \tau$. When long-term assets are more illiquid (τ is positive), then banks not only allow the risk of liquidating an illiquid asset to be shared, but also reduce the opportunity cost of creating a liquid date 1 payoff, r_1. This advantage of banks is present in the models of Bryant (1980), Jacklin (1987), Haubrich and King (1990), and Cooper and Ross (1998). An investor's opportunity set without the bank is worse than the bank's, because an investor needs all or none of his liquidity, while the bank knows that a fraction t of its depositors will need liquidity at date 1. To be precise, the individual investor without using the bank can put a fraction α in short-term assets and the remainder in long-term assets to obtain a choice between $r_1 = \alpha + (1-\alpha)(1-\tau)$ and $r_2 = \alpha + (1-\alpha)R$. Substituting out α, the individual's tradeoff between r_1 and r_2 is given by $r_2 = 1 + (1-r_1)\frac{(R-1)}{\tau}$. When the probability of being of type 1, t is not equal to 1 or zero, the individual's opportunity set is dominated by the bank's opportunity set of $r_2 = \frac{[1-tr_1]R}{1-t}$. For example when $t = \frac{1}{4}$, $\tau = \frac{1}{2}$, and $R = 2$, then without the bank the investor can get $r_2 = 1 - 2(r_1 - 1)$, so $r_1 = 0.9$ implies $r_2 = 1.2$. However, if the bank sets $r_1 = 0.9$, it can offer $r_2 = \frac{[1-0.25(0.9)]2}{0.75} = 2.07$. This provides an extra reason for banks to create liquidity when assets are illiquid. The ability of banks to offer a given amount of liquidity and the problems that this can possibly cause are identical to the original model with $\tau = 0$. As a result, for the rest of this article, I return to the original Diamond and Dybvig (1983) model with $\tau = 0$.

Bank Runs

Banks can create liquidity by offering deposits that are more liquid than their assets. If only the proper depositors withdraw, it works very well. However, creating this liquidity subjects the bank to bank runs. The *bank* may have liquidity problems. If a depositor's need for liquidity (the depositor's type) were a verifiable characteristic that could be written into contracts, the contract could specify that a type 1 be given r_1 at date 1, and a type 2 be given r_2 at date 2. However, on date 1 when each depositor learns his type, this is unverifiable private information. If a bank offers liquid deposits that offer each depositor the opportunity to withdraw r_1 on date 1 or r_2 on date 2, the depositors may select the appropriate withdrawal date for their type. That is, the type 1s take r_1 and the type 2s take r_2, and if all are expected to do this, each will choose the option that is best for him. It turns out, however, that there are multiple equilibria. That is, there is more than one self-fulfilling prophecy about who withdraws at date 1. There is a good equilibrium in which only the type 1 depositors withdraw and a bad equilibrium (a bank run) in which all withdraw at date 1 because they all expect each other to do the same.

To see why there are multiple equilibria, consider how much is left to pay depositors who wait until date 2 to withdraw if a fraction f of initial depositors withdraw at date 1. Because each asset is worth 1 at date 1, a fraction fr_1 of the total assets must be liquidated at date 1. This leaves $r_2(f) = \frac{\{1-[f \times r_1]\}R}{1-f}$ for each of the fraction $1 - f$ who wait until date 2. In any equilibrium, at least a fraction t of deposits will be withdrawn, or $f \geq t$, because type 1s always withdraw at date 1. The type 2 depositors will choose to withdraw at date 1 as well if $r_2(f) < r_1$. In the example with 100 depositors, $t = \frac{1}{4}$, or 25 are of type 1. If just the type 1 depositors withdraw, or $f = t = \frac{1}{4}$, and $r_1 = 1.28$, then $r_2 = 1.813 > r_1$, and the type 2 depositors will choose to wait until date 2 to withdraw. Depositors must choose simultaneously, before they know the actions of others. Each needs a forecast of f, denoted by $\hat{f}$. Given a borrower's forecast, he chooses whether to withdraw at date 1. A Nash equilibrium is a self-fulfilling prophecy of $\hat{f} = f$, and in the good equilibrium, $f = \hat{f} = t = \frac{1}{4}$.

However, suppose all depositors forecast that everybody else will withdraw (i.e., 99 depositors, so $\hat{f} \geq 0.99$). Then the bank will fail before $T = 2$. If 79 depositors or more are expected to withdraw, then the bank will be worthless at date 2: the bank can be liquidated for at most 100 at $T = 1$, and if 79 depositors were to each receive 1.28, at $T = 1$, the bank would not have sufficient assets, because $79 \times 1.28 = 101.12 > 100$. Note that a prophecy of $\hat{f} = 0.99$ is not self-fulfilling, because if it is believed by all, then every depositor will withdraw. The self-fulfilling prophecy of a bank run is $f = \hat{f} = 1$, where all rush to withdraw. Providing liquidity subjects the bank to runs. If a run is feared, it becomes a self-fulfilling prophecy.

The first paragraph of Diamond and Dybvig (1983) follows: "Bank runs are a common feature of the extreme crises that have played a prominent role in monetary history. During a bank run, depositors rush to withdraw their deposits because they expect the bank to fail. In fact, the sudden withdrawals can force the bank to liquidate many of its assets at a loss and to fail. In a panic with many bank failures, there is a disruption of the monetary system and a reduction in production."

Bank runs disrupt production because they force banks to call in loans early. This forces the borrowers to disrupt their production. The model does not have an explicit model of loans from the banks; it simply models the bank loans as illiquid. See Diamond and Rajan (2001) for a description of why bank loans are illiquid.

These two possible equilibrium beliefs (self-fulfilling forecasts of f) are locally stable. That is, if $t = \frac{1}{4}$, a type 2 depositor will not run given that a forecast $\hat{f}$ is just above $\frac{1}{4}$, for example $\hat{f} = 0.27$. Similarly, a type 2 depositor would run given a forecast $\hat{f}$ just below 1, for example, $\hat{f} = 0.97$. The tipping point for a run is a forecast implying that $r_1 \geq r_2$ or $r_1 > r_2(\hat{f}) = \frac{\{1-[\hat{f}\times r_1]\}R}{1-\hat{f}}$, which in the example is $\hat{f} > \frac{(R-r_1)}{r_1(R-1)} = \frac{2-1.28}{1.28(2-1)} = 0.5625$.

Because moving away from a good equilibrium requires a large change in beliefs, the initiation of a run when none was expected requires something that all (or nearly all) depositors see (and believe that others see). For example, a newspaper story that the bank is performing poorly could cause a run even if many knew that it was inaccurate, because those who know it is inaccurate can believe that the others will decide to withdraw based on the story. Even sunspots could cause runs if everyone believed that they did.

Using diversified funding sources can help insulate a bank from runs if diversified means that there is no commonly observed information source that is seen by a large number of the diverse depositors. An older example is also useful. It would make sense for a bank to have a large lobby (or fast bank tellers), because if a line to withdraw extended out to the street, passersby may conclude that a run is in progress. Conversely, once a run is in progress, it will be important to be able to convince all depositors that it will stop and to ensure all the depositors know that all others have been so convinced.

When all depositors do not observe the same news or other information sources, then the depositors will not have a way to tell if others are choosing to panic and run (they will have "incomplete common knowledge"). There are some very interesting analyses of runs in this context (see Morris and Shin 2003 and Goldstein and Pauzner 2005). In addition, there are some important, but somewhat difficult, analyses of bank policies when there is an unavoidable positive probability of a run (see Peck and Shell 2003, Ennis 2003, and Ennis and Keister 2006).

Suspension of Convertibility

In this simple model, a bank can suspend convertibility of deposits to cash in order to stop a run. That is, suppose the bank does not allow more than a fraction t of deposits to be withdrawn (does not allow $f > t$, or in the example, allows only 25 to withdraw). Then no matter how many depositors attempt to withdraw at date 1, a type 2 will get $r_2(t) = \frac{\{1-[t\times r_1]\}R}{1-t} > r_1$ at date 2. In the example, the type 2 would get 1.813 at date 2. As a result, the depositors would never panic and a run would never start. In this case, the suspension is a threat that need not actually be carried out. The problem lies in convincing potential participants in a run that convertibility will be suspended at the proper time. In the days before deposit insurance, banks regularly suspended convertibility to stop runs (see Friedman and Schwartz 1963). In a more general model, in which the fraction of type 1 depositors fluctuates sufficiently (and the realized fraction cannot be written into contracts), suspension cannot be used only as a threat. Some suspension would actually occur and would be unpopular. If suspension occurred regularly, depositors would desire another way of stopping runs caused by panics. In practice, government-provided deposit insurance has been instituted following many financial crises. Its effects are described in the next section.

Deposit Insurance

An alternative way to stop and prevent runs is deposit insurance, a promise to pay the amount promised by the bank no matter how many depositors withdraw, without suspension of convertibility. In the example, this is a promise of 1.28 to those who withdraw at $T = 1$ and 1.813 to those who withdraw at $T = 2$. How can this be accomplished if everyone withdraws? Unless there are outside resources that we did not account for in the model, the only way is to take some resources away from those who run and withdraw. Governments have taxation authority, which is the ability to take resources without prior contracts. This gives government deposit insurance an advantage over private deposit insurers who might themselves fail in a run, or who would need to hold sufficient liquid assets to prevent the financial system from creating liquidity.

In our example with $t = \frac{1}{4}$, where *exactly* 25 people ought to withdraw, suspension of convertibility works as well. However, if there is aggregate uncertainty about t, then suspension may prevent some type 1 depositors from withdrawing. In this case, suspension is costly. Deposit insurance can stop runs and avoid suspension of convertibility.

A bank with deposit insurance can credibly promise not to have runs. Government deposit insurance works because the government has taxation authority and, unlike most insurance companies, can provide a guarantee against large losses that are usually off the equilibrium path without holding reserves to back up their promise. In addition, a deposit insurance law commits the government to insure banks, which is an advantage over discretionary policies if self-fulfilling prophecies of runs need to be eliminated. Suspension of convertibility is usually a discretionary policy (see Gorton 1985). Another discretionary policy to prevent banks from liquidating illiquid assets and avoiding self-fulfilling runs is central-bank lending, financed by implicit taxation or money creation authority. The extent of the Great Depression in the United

States in the 1930s has been blamed on the lack of Federal Reserve discount window lending by Friedman and Schwartz (1963). Deposit insurance will solve this problem of discretionary lending, but its guaranteed bailout of depositors may cause incentive problems if bank regulation is poorly structured (see Barth, Caprio, and Levine 2006).

3. CONCLUSION

Banks create demand deposits to provide investors with liquid assets. When there is a demand for more liquid assets from investors or entrepreneurs, demand deposit contracts serve as a means for quick access to liquidity. Demand deposits work very well when investors forecast that banks will survive, but can cause severe damage if investors lose faith in banks. There is scope for banks to write more refined contracts, such as deposits with suspension of convertibility of deposits to cash. In addition, there may be a role for government policies to eliminate self-fulfilling runs on banks. The government plays a role because its taxation authority is not available to private firms.

The reasons why bank assets are illiquid and other reasons that banks help to create liquidity are explored in Diamond (1997) and Diamond and Rajan (2001, 2005). Diamond (2005) integrates these approaches with the role of banks in monitoring borrowers which is explored in Diamond (1984, 1996).

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