Japanese Monetary Policy, 1991–2001

Bennett T. McCallum

During recent years, Japanese monetary policy has been the topic of a great deal of discussion, commentary, and debate. This is not only because of the great practical importance of the long-lasting slump of the world's second largest national economy, but also because the situation in Japan has raised interesting issues concerning some fundamental topics in monetary theory. Accordingly, this paper considers issues relating to recent and prospective policy measures of the Bank of Japan (BOJ).

It is hard to avoid the impression that Bank of Japan (BOJ) policy has been overly restrictive for approximately a decade. That statement does not imply that Japan's poor economic performance during the 1990s was entirely or even primarily attributable to monetary policy, for structural flaws have also been very important.[1] It does suggest, however, that Japanese economic performance would have been less undesirable if BOJ policy had been less restrictive. In the pages that follow, I will attempt to support the foregoing claim, discuss the difficulty faced by the BOJ because of the zero lower bound on nominal interest rates, and illustrate this difficulty with a small quantitative study. Then I will take up some of the nonstandard policy approaches that have been proposed and will argue that the most promising of these would entail rapid monetary base growth effected largely through purchases of foreign

---

H. J. Heinz Professor of Economics, Carnegie Mellon University, and Visiting Scholar, Federal Reserve Bank of Richmond. This paper has been prepared for the Federal Reserve Bank of Richmond. It is in part based upon position papers presented at meetings of the Shadow Open Market Committee held in Washington, D.C., in April and October of 2001. For useful comments, the author is indebted to Marvin Goodfriend, Bob Hetzel, Tom Humphrey, Allan Meltzer, Edward Nelson, Roy Webb, and John Weinberg. The views expressed in this article are not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

[1] Major banking-system difficulties are widely recognized, and in addition it is likely that the growth rate of "potential" or "natural-rate" output has fallen from the level of the 1970s and 1980s. But the severity of the bank-solvency problem has been increased by the deflation of the past several years, and it is almost certainly the case that actual output has fallen far below potential.

exchange. Such a strategy has faced two major objections, however, so much of the paper is devoted to counterarguments to these objections. The first objection is based on legal provisions of the Bank of Japan Law and the second on the concern that such actions would constitute a "beggar-thy-neighbor" policy that would reduce Japanese demand for imports. It is argued that neither of these objections is appropriate. With respect to the former, it is suggested that the BOJ Law, as written, includes conflicting provisions and that foreign exchange purchases for the purpose of monetary control could be conducted if the BOJ were to request permission of the government. In this regard, the intimate connection between monetary and exchange-rate policies is emphasized. With respect to the beggar-thy-neighbor issue, it is argued that in fact an expansionary monetary policy of the type recommended would increase net Japanese imports. In this regard, a major portion of the paper is devoted to a quantitative analysis of the trade-balance effects of a policy of the recommended type. The analysis is carried out in the context of a dynamic optimizing model of an open economy, which is exposited in some detail. Policy simulation exercises conducted with this model represent a major feature of the paper.

1. HAS BANK OF JAPAN POLICY BEEN TIGHT?

That BOJ policy has been quite tight—low interest rates notwithstanding—is suggested by the most prominent and widely-respected guideline for the conduct of monetary policy, i.e., the policy rule developed by John Taylor (1993a). The Taylor rule can be expressed as

$$R_t = 3 + \Delta p_t^a + 0.5(\Delta p_t^a - 2) + 0.5(y_t - \bar{y}_t), \tag{1}$$

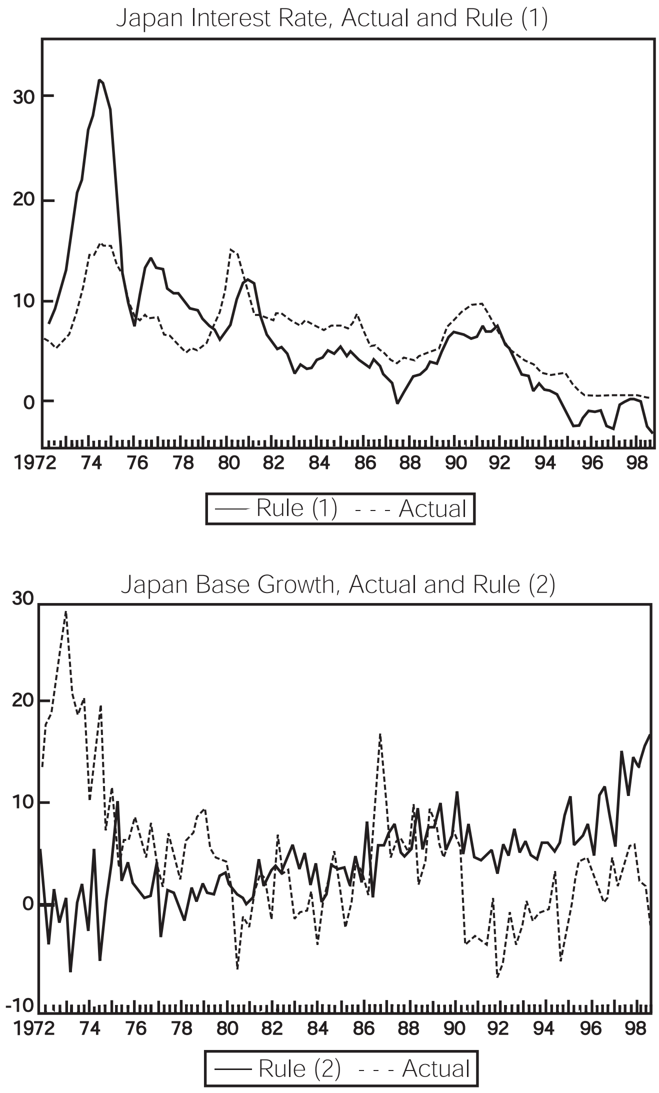
where R is the call rate, Δp_t^a is the average inflation rate (GDP deflator) over the previous four quarters, y is real GDP and $\bar{y}$ is its potential value.[2] A chart contrasting Taylor-rule prescriptions for the overnight call rate[3] with actual values of this rate over the years 1972–1998 appeared in a recent paper in this journal (McCallum, 2000b) to which the reader is referred for various details.[4] That comparison is reproduced in the top half of Figure 1. There it is clear that the actual value exceeded the setting prescribed by Taylor's rule during almost every quarter beginning with 1993Q1 and continuing through 1998Q4. Of course, the negative values called for by the rule are not feasible,

[2] Here the long-run average real rate of interest is taken to be 3 percent per annum (p.a.) and the inflation target rate to be 2 percent. Some versions of the rule use other values for these and for the coefficients attached to the target variables.

[3] The (uncollateralized) overnight call rate was the BOJ's operating target or instrument variable through the period of the 1990s. The procedure was changed in March 2001.

[4] The most important of these, of course, is the measurement of "potential" output—which has been especially problematic for Japan in recent years. Its reliance on this inherently difficult concept is one weakness of the Taylor rule.

Figure 1 Policy Rule Indications



but that does not alter the fact that Taylor's policy guideline has called for greater monetary ease through this period.

An alternative rule involving management of the monetary base has been promoted in several of my papers (e.g., McCallum 1988, 1993, 2000b). It can be written as

$$\Delta b_t = 5 - \Delta v_t^a + 0.5(5 - \Delta x_{t-1}), \tag{2}$$

where b and x are logs of the monetary base and nominal GDP, while Δv_t^a is the average rate of base velocity growth over the previous four years. Here 5 is the target value for nominal GDP growth, obtained from a 2 percent inflation target and a 3 percent assumed long-run average growth rate for real GDP. This rule is much less prominent than Taylor's, primarily because actual central banks focus upon interest rates, not monetary base growth rates, in designing their policy actions. Especially in an environment with near-zero call rates, however, its prescriptions may be of interest. In any event, the actual and rule (2) settings for base growth rates are shown in the lower panel of Figure 1.[5] There the indication is that actual BOJ policy has been too tight virtually all of the time since the middle of 1990![6]

Increased base money growth rates have been recommended for several years by Mr. Nobuyuki Nakahara, a member of the BOJ's Monetary Policy Board (MPB).[7] But until the change that was announced at the MPB meeting on March 19, 2001, the BOJ's position was that additional base growth would have no stimulative effect since short-term nominal interest rates were close to zero. With such low rates, base money and short-term government securities (bills) become almost perfect substitutes, so purchases of the latter by the BOJ have no effect on asset markets and consequently none on the economy, according to the BOJ view. That position will be discussed in the next two sections.

2. THE BANK OF JAPAN'S DIFFICULTY

Over the period 1999–2001, commentary in influential nonacademic publications including the *Economist*, the *Financial Times*, and the *Wall Street Journal* became increasingly critical of the BOJ for not providing more monetary stimulus to aggregate demand in Japan. The plots presented in the previous section also suggest that more stimulus is needed and has been needed for years, but nevertheless I believe that much of the press commentary has failed to recognize the difficulty of the problem that has faced the BOJ. It is not just stubbornness that has prevented the BOJ from providing such stimulus, for the nature of monetary policy actions is sharply different when short-term interest rates are effectively equal to zero. It is not true that there has been "nothing more that the BOJ can do," but what needs to be done is different than in normal conditions and the policy actions are more difficult to design.

[5] The plot is reproduced from the same source as before, which provides details.

[6] Some early indication that BOJ policy was too tight during 1990–92 appears in McCallum (1993, 35–36). Also see McCallum and Hargraves (1995).

[7] Mr. Nakahara's term as an MPB member ended in April 2002. The MPB currently includes Mr. Shin Nakahara, who is not related to Mr. Nobuyuki Nakahara.

For some years, the BOJ took the position that nothing more could be done, beyond lowering its overnight call rate well below one percent and finally almost to zero. These statements were of questionable validity, as we shall see, and perhaps reflected a fundamentally misguided tendency to think of levels of nominal interest rates as direct indicators of monetary conditions, with low rates representing expansionary policy. In fact, nominal rates will be low (for given real rates) when expected inflation is low; thus low rates are in large part an indication that monetary policy has been tight in the past, not that it is loose in the present. Recognizing this last point, several critics have argued that the BOJ should gauge its actions in terms of monetary base growth rates, rather than interest rates, and should provide stimulus by increasing the growth rate of the monetary base. As can be seen in the bottom half of Figure 1, my base-growth-oriented policy rule would have called for about 11 percent (per annum) growth rates over the period 1996–1998, rather than the values of about half that magnitude that were actually recorded.

It is crucial to recognize, however, that just expanding the base growth rate will not be effective, in the face of zero interest rates, unless nontraditional assets are purchased. Normally, open market operations are conducted by exchanging base money for short-term government bills. But when short-term interest rates are near zero, such purchases will have virtually no effect. One way to understand this point is to recall that both base money and bills are nominally-denominated paper assets that are virtually free from default risk. What then is the difference between them as assets; why do people and firms hold money when bills normally provide the holder with a higher rate of interest? The answer, from traditional monetary theory, is that money is a generally accepted medium of exchange that provides transaction-facilitating services to its holders—services not provided by bills.[8] Rational economic agents then adjust their holdings of these two assets so as to equalize their *net* benefits at the margin. The sum of pecuniary interest earnings plus transaction-facilitating services is equated at the margin, for the two assets, with interest earnings usually being lower and services higher for base money assets.

But when short-term interest rates fall to zero, then there is no difference in the interest component of the net yield for the two assets, so their marginal service yields will also be equal. That condition is brought about by holders choosing to keep on hand a quantity of money large enough that its service yield at the margin is driven down to zero. But then, at the margin, base money and bills become perfect substitutes—the distinguishing characteristic of base money is lost (at the margin, not overall)! Consequently, open-market operations that exchange bills for money in private portfolios have effects that are like those of replacing a billion dollars' worth of $5 Federal Reserve Notes with

[8] Or provided to a lesser extent by bills. For a review of traditional monetary analysis, see McCallum and Goodfriend (1988).

a billion dollars' worth of $10 Federal Reserve Notes. To an approximation, in other words, there is no effect. Accordingly, an expansionary monetary policy needs to be implemented in some nontraditional manner, e.g., by purchase of nontraditional assets. Such a purchase would alter the composition, in private portfolios, of these other assets relative to the sum of money plus government bills, thereby stimulating some response on the part of private asset holders.[9]

3. SOME QUANTITATIVE RESULTS

Is there any empirical evidence supportive of the theoretical view just described? A very simple but straightforward way to approach that question is to examine the relationship between base money growth and the growth rate of nominal GDP. To that end, let us consider an updated and modified version of the simplest macroeconomic model of aggregate demand utilized in McCallum (1993). It is a single-equation dynamic relationship of nominal income growth and its dependence on money base growth. Let x_t and b_t denote logarithms of nominal GDP and the adjusted monetary base, respectively, so that Δx_t and Δb_t are quarterly growth rates. The data series utilized extend from 1970Q1 through 2001Q3 and are seasonally adjusted.[10]

Least-squares estimation over the period 1970Q3–2001Q3 yields the following relationship:

$$\begin{aligned} \Delta x_t \quad = \quad & \underset{(.0019)}{-0.0002} + \underset{(.0873)}{0.261}\ \Delta x_{t-1} + \underset{(.0840)}{0.344}\ \Delta x_{t-2} + \underset{(.0887)}{0.248}\ \Delta b_{t-1} \end{aligned} \tag{3}$$

$$R^2 = 0.483 \quad SE = 0.0116 \quad DW = 2.15.$$

The numbers in parentheses are standard errors, so Δb_{t-1} evidently has a highly significant effect on Δx_t and its subsequent values. Thus if this relation were structural, it would indicate that a money base rule could be devised to keep nominal GDP growth reasonably close to a desired target path. A similar relationship was utilized in that manner in McCallum (1993), where it provided results quite comparable to those of small but somewhat more complex models intended to be structural.[11]

The issue to be examined here, by contrast, is whether the relationship between base growth and nominal income has "broken down" in recent years—as

---

[9] One reader has suggested that it would be stimulative for the central bank to simply print base money and give it to private individuals. In fact such a process would create an imbalance in private portfolios and thereby lead to some type of reaction. But such a scheme combines both monetary and fiscal policy. It is equivalent to a fiscal transfer of government bills to private agents plus an open-market purchase of bills.

[10] These series were obtained from the web pages of the BOJ (monetary base) and the Japanese government's Economic and Social Research Institute (GDP).

[11] For a discussion of the potential vulnerability of the relationship to the Lucas critique, see McCallum (1993, 37–38).

would arguably be the case with near-zero interest rates and traditional open-market purchases of government bills. In fact, such an impression is supported by visual inspection of a simple plot of those two variables against time. To consider the matter more formally, however, I have reestimated relationship (3) permitting crucial parameters to change in 1995Q1.[12] Inclusion of a 0-1 dummy variable, that changes from 0 to 1 in 1995Q1, indicates a downward shift in the equation's constant term, with a highly significant t-statistic of –3.05. If instead the slope coefficient on the base growth variable is permitted to change at that time, again a significant decrease is detected, with the t-statistic being –3.31. Inclusion of both effects seems most appropriate (since the two variables are highly collinear) and leads to the following estimates:

$$\begin{aligned}
\Delta x_t \;=\; & \underset{(.0022)}{0.0031} + \underset{(.091)}{0.137}\Delta x_{t-1} + \underset{(.090)}{0.210}\Delta x_{t-2} + \underset{(.103)}{0.399} \\
& \Delta b_{t-1} - \underset{(.192)}{0.318}\ D95 \cdot \Delta b_{t-1} - \underset{(.0041)}{0.0045}\ D95
\end{aligned} \tag{4}$$

$$R^2 = 0.531 \quad SE = 0.0111 \quad DW = 2.10.$$

Here we see that the estimate of the net effect of Δb_{t-1} on Δx_t for the post-1995 observations is $0.399 - 0.318 = 0.081$, a very small value. Furthermore, a Wald test of the hypothesis that the net effect equals zero gives a P-value of 0.617, indicating that a zero-effect hypothesis could not be rejected at any conventional significance level.[13] For all practical purposes, then, the recent effect on nominal GDP growth of additional money base growth has been zero, according to these last estimates. This finding is consistent with the notion that, in a situation with near-zero short-term interest rates, BOJ purchase of treasury bills will be ineffective as means of stimulating aggregate demand.

Of course, the simple investigation just conducted falls well short of what would be required for a convincing counterfactual policy exercise, which would require a well-specified structural model. But the results here are not being used in that manner, i.e., to assess the effects of an alternative policy rule. Instead they are being used only to indicate that a substantial breakdown in the money-GDP relationship has occurred. For that purpose, the foregoing exercise should be adequate.

---

[12] This break date, or one close to it, is suggested by the extensive recent empirical study by Miao (2000).

[13] Together the two shift terms are highly significant; a Wald test of the hypothesis that both coefficients equal zero results in a P-value of 0.0022.

4. POLICY PROPOSALS

Let us now turn to the crucial issue, namely, how monetary policy can be conducted in a situation with interest rates near zero. Several prominent monetary economists have taken up this issue, including Marvin Goodfriend (1997, 2000), Paul Krugman (1998, 2000), Allan Meltzer (1998, 1999, 2000), Athanasios Orphanides and Volker Wieland (2000), Willem Buiter and Nikolaos Panigirtzoglu (2001), Lars Svensson (2001), and myself (McCallum 2000a, 2002). Goodfriend (2000) and Buiter and Panigirtzoglu propose a tax on base money that would keep it from being a perfect substitute for short-term securities and thereby open the way for an effective monetary policy even when a zero-lower-bound situation is in effect. This scheme's logic is evidently impeccable, but the probable unpopularity of the explicit tax on money would seem to present a formidable practical barrier (even though it would make possible a reduced average level of the implicit tax on money). Accordingly we will focus on the other proposals, which involve the central bank purchase with base money of assets other than the traditional short-term yen securities. Meltzer (2000), for example, suggests that the purchase of long-term Japanese government bonds would be stimulative. McCallum and Svensson suggest instead the purchase of foreign exchange (i.e., short-term securities that are claims to dollars or other non-yen currencies). The general ideas behind these asset-purchase proposals are basically similar, but there are important practical differences.

As explained above, the basic idea is that for increased growth of base money to be stimulative, it is necessary that the assets bought from private portfolios be ones that are not perfect substitutes for government bills (or for money). Otherwise, the composition of private portfolios will not be affected in an economically relevant manner so no response will be induced. Obviously, longer term government bonds represent one leading possibility. But according to the expectations theory of the term structure, which says that long-term interest rates are appropriate averages of expected short-term rates, long-term and short-term government securities are perfect substitutes. Now, there is evidence strongly suggesting that this theory is not empirically accurate, but there is no widely accepted alternative to rely upon. What is needed is a theory of the term premium that relates *variations* in that premium to asset positions. In the absence of any widely accepted theory of that type, it is not obvious how to design an appropriate policy or even that purchases of long-term bonds would have an effect on aggregate demand in the appropriate direction.[14]

Consequently, I have suggested that the best course of action would be for the BOJ—or any central bank in a similar situation—to purchase foreign exchange (McCallum, 2000a, 2002). Lars Svensson (2001) has made a closely

[14] A different and more optimistic position has recently been expressed by Goodfriend (2001).

related proposal.[15] The difference is that in this case there is a more well understood transmission channel, working via the exchange rate. It is clear that the purchase of foreign exchange tends to depreciate the value of the yen. With prices in Japan initially rising less rapidly than the price of foreign exchange,[16] a real exchange rate depreciation would result, and this would tend to stimulate exports and to increase Japanese income and production. That is of course what is desired—to increase Japanese income and spending.

It is important to keep in mind, in this regard, that increases in income have strong and reliable positive effects on imports. Indeed, the strength of income effects on imports is probably strong enough that the overall effect of the stimulative policy would be to *increase* Japan's imports (in real terms) from its trading partners. Under that assumption it is not the case that the recommended policy would tend to depress aggregate demand in other nations. Fear of that outcome is therefore not a sensible reason for avoiding stimulative monetary policy.[17] Indeed, it is not even clear that such a policy would induce the real exchange rate to appreciate for more than a short period of time. These issues will be quantitatively explored below, in Sections 5 and 6.

A few critics of the foreign-exchange strategy have contended that a central bank cannot reliably influence its currency's exchange rate. In that regard it is widely believed by analysts that *raising* a currency's real foreign-exchange value by monetary policy is not possible, and that keeping its nominal value high would require extreme measures that would not be tolerated for long in a nation with democratic political processes. But to depreciate a fiat currency in nominal terms is not difficult; the basic requirement is simply the creation of an excess quantity of the currency.[18] And a reduction in value is what is needed in the case of Japan.[19]

Proceeding under the presumption that a central bank can exert adequate control over its currency's nominal exchange rate, McCallum (2000a, 2002) has considered a policy rule for use in a zero-lower-bound situation of the following form, with s_t representing the log of the home-country price of

---

[15] It should be noted that a few economists, including myself, Marvin Goodfriend (1997), Allan Meltzer (1998, 1999, 2000), and John Taylor (1997), have been urging a more expansionary policy for the BOJ at least since 1995. Our first proposals did not, however, emphasize purchases of foreign exchange per se.

[16] Even in the event that Japanese domestic prices increased along with the price of foreign exchange, there would be a benefit—this would raise nominal interest rates, leading to an escape from the "liquidity trap" situation described above.

[17] It is my impression that fear of this outcome did, in fact, keep U.S. and international agencies from supporting policy proposals of the type expressed here, until recently. See Section 5 below.

[18] For a more detailed argument, see McCallum (2000a, 2002).

[19] Even a depreciation could not be effected if the currency were literally a perfect substitute for foreign currencies, but such is not the case. Interesting new evidence of a market-microstructure type has recently been developed by Evans and Lyons (2000, 2002).

foreign exchange:

$$\Delta s_t = \mu_0 + \mu_1(2 - \Delta p_t) + \mu_2(\bar{y}_t - y_t), \quad \mu_1, \mu_2 > 0. \tag{5}$$

Here the rate of depreciation of the exchange rate Δs_t is increased when inflation and/or output are below their target values. Such a rule would be implemented in a manner similar to that typically used with an interest-rate instrument. Specifically, the central bank would observe the relevant asset price almost continuously and make open-market purchases (sales) when a depreciation (appreciation) is indicated.[20] It is important to note that rule (5) does not represent a fixed exchange rate. To the contrary, it represents a regime that subordinates the exchange rate entirely to macroeconomic conditions.

Quite recently, in 2001 and 2002, the BOJ has taken actions that indicate an intention to pursue a more stimulative policy than in the past. To date, however, it has not given any official recognition to the possibility of purchasing foreign exchange as a way of providing a more stimulative monetary policy.[21] We need to look into the reasons for this neglect, of which two are prominent. One of these involves the BOJ's legal charter and the other stems from beliefs concerning the effects on other nations' balance of payments magnitudes. Since the latter topic is the more analytical in nature, it will be considered first.

5. THE BALANCE OF PAYMENTS ISSUE

In this section we take up a major analytical issue concerning the policy position presented above. During the late 1990s, some leading officials of the International Monetary Fund were opposed to monetary stimulus as a means for combating Japan's ongoing economic weakness.[22] Their reason was a belief that monetary stimulus would lead to exchange rate depreciation, which would be harmful to other nations seeking to expand (or, during the Asian crisis, maintain) exports to Japan. This source of objection to a more stimulative monetary policy is, however, inappropriate. First, it is highly unlikely that such a policy would lead to lessened imports by Japan, for an increase

[20] As with current practice, market participants may to some extent move rates as desired by the central bank, even without actual open-market operations, if the central bank's intentions are made clear.

[21] In an interview with Bloomberg reported on July 19, 2001, Dr. Kunio Okina, Director of the BOJ's Institute for Monetary and Economic Studies, suggested that the BOJ should consider purchase of foreign exchange as a tool of monetary policy, while leaving exchange rates to the currency market. But on July 25, Mr. Sakuya Fujiwara, Deputy Governor of the BOJ, indicated (in a question-and-answer session at the Tokyo Foreign Correspondents' Club) that Okina's suggestion does not reflect BOJ policy.

[22] This claim is based in part on personal conversations. For some evidence, see Fischer (1998), which proposes fiscal expansion and banking reforms but does not mention monetary policy. In his very recent comment in the *Brookings Papers*, Fischer accepts the need for Japanese monetary stimulus, but still labels this a "beggar-thy-neighbor" policy (2001, 165).

in Japanese real income would tend to increase imports and most likely to an extent greater than any decrease brought about by Japanese exchange rate depreciation. Second, according to recent views of most academics and policymakers alike, monetary policy should be directed primarily toward keeping inflation low (but non-negative!), with the avoidance of real cyclical fluctuations a possible secondary objective.[23] From this perspective, fiscal and structural policies are more appropriate tools to use in managing balance-of-payments problems. Thus, if Japan is not going to share a common currency with, e.g., the United States, then their bilateral exchange rate should be free to float with each country managing its monetary affairs so as to keep a low inflation rate.[24] From this perspective, one could argue that the United States should not try to use its political influence to prevent a depreciation of the yen. More generally, it would seem undesirable for any country to attempt to induce other nations to manage their monetary policy in ways that are domestically harmful but temporarily helpful for the country in question.[25] From a long-term perspective, the United States will benefit from having other important nations conduct their monetary policies in a manner that yields low inflation with domestic macroeconomic stability.

Not all analysts would agree, however, with the contention that monetary stimulus of the type here suggested would not have a depressing effect on other nations' exports to Japan. Accordingly, this section and the next will be devoted to a substantial consideration of that position. For such an issue it is necessary to conduct analysis in a quantitatively specified structural model, and the convincingness of the exercise will depend upon the qualifications of the model utilized. The one that will be used was developed by McCallum and Nelson (1999) and utilized subsequently by them (2000) in an exploration of relationships between CPI inflation and exchange-rate depreciation. The model is not econometrically estimated using Japanese time series data, but is a "new open-economy macroeconomic model"—i.e., is based on dynamic optimizing analysis assuming sticky-price adjustments and solved assuming rational expectations—that has been calibrated to match certain characteristics of the Japanese economy. It differs from other contributions in the area, however, in the manner in which imported goods are treated. In particular, the M-N model treats imports not as finished goods, as is common, but instead as raw-material inputs to the home economy's producers. This alternative

[23] Real cyclical conditions should provide only a secondary objective for monetary policy because monetary effects on these conditions are temporary and poorly understood, whereas monetary effects on prices (and thus on inflation rates) are long-lasting and well understood.

[24] Moreover, decisions to share a common currency should be made on grounds of microeconomic efficiency, not in an attempt to solve macroeconomic stabilization difficulties.

[25] Indeed, it may well have been U.S. pressure that led the BOJ to be somewhat too loose (even on traditional standards that ignore asset price movements—see Figure 1, lower panel) during 1986–88, a stance that permitted Japan's asset price boom of the late 1980s and set the stage for a clampdown that began the past decade's slump.

modelling strategy leads to a cleaner and simpler theoretical structure, relative to the standard treatment, and is empirically attractive. Since the optimizing, general equilibrium analysis has previously been presented in McCallum and Nelson (1999), here I will take an informal expository approach designed to facilitate understanding of the model's basic structure.

It is well known that optimizing analysis leads, in a wide variety of infinite-horizon models that involve imperfect competition, to a consumption Euler equation that can be expressed or approximated in the form

$$c_t = E_t c_{t+1} + b_0 + b_1 r_t + v_t, \tag{6}$$

where $c_t$is the log of a Dixit-Stiglitz consumption-bundle aggregate of the many distinct goods that a typical household consumes in period t.[26] In (6), r_t is the real interest rate on home-country one period bonds (private or government) and v_t is a stochastic shock term that pertains to household preferences regarding present versus future consumption. In closed-economy analysis, relation (6) is often combined with a log-linearized, per-household, overall resource constraint to yield an "expectational IS function," to use the term of Kerr and King (1996). That step presumes that investment and capital are treated as exogenous. The simplest version of that assumption is that the capital stock is fixed; since that assumption is rather common in the literature, it is adopted here.

For an open-economy extension, one might be tempted to write the resource constraint as

$$y_t = \omega_1 c_t + \omega_2 g_t + \omega_3 x_t - \omega_4 im_t, \tag{7}$$

where y_t, g_t, x_t, and im_t are logarithms of real output, government consumption, exports, and imports while ω_1, ω_2, ω_3, and ω_4 are steady state shares of output for consumption, government purchases, exports, and imports. But if imports are exclusively material inputs to the production of home-country goods, and $Y_t = \ln^{-1} y_t$ is interpreted as units of output, then the relevant resource constraint is

$$y_t = \omega_1 c_t + \omega_2 g_t + \omega_3 x_t. \tag{7$'$}$$

It is desirable that import demand be modelled in an optimizing fashion. Toward that end, assume that output of all consumer goods is effected by producers that are constrained by production functions all of the same CES form, with labor and material imports being the two variable inputs. Then the cost-minimizing demand for imports is

$$im_t = y_t - \sigma q_t + const., \tag{8}$$

---

[26] Thus $c_t = \ln C_t$, with $C_t = [\int C_t(z)^{(\theta-1)/\theta}\, dz]^{\theta/(\theta-1)}$, where $\theta > 1$, z indexes distinct goods, and the integral is over $(0, 1)$, while the corresponding price index is $P_t = [\int P_t(z)^{1-\theta} dz]^{1/(1-\theta)}$.

where σ is the elasticity of substitution between materials and labor in production, and where "*const*." denotes *some* constant.[27] Also, q_t is the log price of imports in terms of produced consumption goods. We will refer to $Q_t = \ln^{-1} q_t$ as the real exchange rate. Let P_t and S_t be the home-country money price of goods and foreign exchange, with P_t^* the foreign money price of home-country imports. Then if p_t, s_t, and p_t^* are logs of these variables, we have

$$q_t = s_t - p_t + p_t^*. \tag{9}$$

Symmetrically, we assume that export demand is given as

$$x_t = y_t^* + \sigma^* q_t + const., \tag{10}$$

where y_t^* denotes production abroad and σ^* is the price elasticity of demand from abroad for home-country goods.

Now consider output determination in a flexible-price version of the model. Taking a log-linear approximation to the home-country production function, we have

$$y_t = (1-\alpha)a_t + (1-\alpha)n_t + \alpha\, im_t + const.,$$

where n_t and a_t are logs of labor input and a labor augmenting technology shock term, respectively. Suppose for simplicity that labor supply is inelastic, with 1.0 units supplied per period by each household. Thus with full price flexibility we would have $n_t = 0$ and the flexible-price, natural rate (or "potential") value of y_t will be $\bar{y}_t = (1-\alpha)a_t + \alpha\ im_t + const.$ so that $\bar{y}_t = (1-\alpha)a_t + \alpha[\bar{y}_t - \sigma q_t] + const.$, or

$$\bar{y}_t = a_t - [\sigma\alpha/(1-\alpha)]q_t + const. \tag{11}$$

But while $\bar{y}_t$ would be the economy's output in period t if prices could adjust promptly in response to any shock, we assume that prices adjust only sluggishly. And if the economy's demand quantity as determined by the rest of the system (y_t) differs from $\bar{y}_t$ then the former quantity prevails—and workers depart from their (inelastic) supply schedules so as to provide whatever quantity is needed to produce the demanded output, with im_t given by (8).

In such a setting, the precise way in which prices adjust has a direct impact on demand and consequently on production. There are various models of gradual price adjustment utilized in the recent literature that are intended to represent optimizing behavior. In the analysis that follows, I will use

$$\Delta p_t = 0.5(E_t\Delta p_{t+1} + \Delta p_{t-1}) + \phi_2(y_t - \bar{y}_t) + u_t, \tag{12}$$

where u_t is a behavioral disturbance. This form of equation has been fairly prominent, primarily because it tends to impart a more realistic degree of

[27] That is, the expression "*const*." in different equations appearing through the remainder of the article will typically refer to different constant magnitudes.

inflation persistence than does the Calvo-Rotemberg model (which is theoretically more attractive).[28]

A standard feature of most current open-economy models is a relation implying uncovered interest parity (UIP). Despite its prominent empirical weaknesses, accordingly, the basic M-N model includes one:

$$R_t - R_t^* = E_t \Delta s_{t+1} + \xi_t. \tag{13}$$

We include a time-varying "risk premium" term ξ_t, however, that may have a sizeable variance and may be autocorrelated.

It remains to describe how monetary policy is conducted. In most recent research in monetary economics, it is presumed that the monetary authority conducts policy by adjusting a one-period nominal interest rate in response to prevailing (or forecasted future) values of inflation and the output gap, $\tilde{y}_t = y_t - \bar{y}_t$:

$$R_t = (1 - \mu_3)[\mu_0 + \Delta p_t + \mu_1(\Delta p_t - \pi^*) + \mu_2 \tilde{y}_t] + \mu_3 R_{t-1} + e_t. \tag{14}$$

Here $\mu_3 > 0$ reflects interest rate smoothing. Quantitative results reported by M-N (1999, 2000) are based on estimated or calibrated versions of this rule, in most cases with E_{t-1} applied to $\tilde{y}_t$ and Δp_t.

To complete the model, we need only to include the Fisher identity, $(1 + r_t) = (1 + R_t)/(1 + E_t \Delta p_{t+1})$, which we approximate in the familiar fashion:

$$r_t = R_t - E_t \Delta p_{t+1}. \tag{15}$$

Thus we have a simple log-linear system in which the ten structural relations (6)–(15) determine values for the endogenous variables y_t, $\bar{y}_t$, Δp_t, r_t, R_t, q_t, s_t, c_t, x_t, and im_t. Government spending g_t and the foreign variables p_t^*, y_t^*, R_t^* are taken to be exogenous—as are the shock processes for v_t, a_t, e_t, and ξ_t.

Of course, it would be possible to append a money demand function such as

$$m_t - p_t = \gamma_0 + \gamma_1 y_t + \gamma_2 R_t + \eta_t, \tag{16}$$

and one of this general form—perhaps with c_t replacing y_t—would be consistent with optimizing behavior.[29] But, as many writers have noted, that equation would serve only to determine the values of m_t that are needed to implement the R_t policy rule.

With the structure given above, a useful measure of the balance on goods and services account—i.e., net exports—is

$$net_t = x_t - (im_t + q_t), \tag{17}$$

[28] See Fuhrer and Moore (1995) and Clarida, Gali, and Gertler (1999).

[29] See McCallum and Nelson (1999) or many other papers.

where it is assumed that $\omega_3 = \omega_4$. This measure is used in what follows. Also, incidentally, it is possible to calculate the log of the GDP deflator as

$$p_t^{DEF} = [p_t - \omega_3(s_t + p_t^*)]/(1 - \omega_3). \tag{18}$$

Before moving on, it should be noted that an advantage of our strategy of modelling imports as material inputs to the production process is that the relevant price index for produced goods is the same as the consumer price index, which implies that the same gradual price adjustment behavior is relevant for all domestic consumption. In addition, it avoids the unattractive assumption, implied by the tradeable versus nontradeable goods dichotomization, that export and import goods are perfectly substitutable in production.

Theoretical advantages would not constitute a satisfactory justification, of course, if in fact most imports were consumption goods. Such is not the case, however, at least for the United States. Instead, an examination of the data suggests that (under conservative assumptions) intermediate productive inputs actually comprise a larger fraction of U.S. imports than do consumer goods (including services).[30]

There is one way in which the model developed in McCallum and Nelson (1999) differs significantly from the 10-equation formulation just presented. Specifically, the M-N model includes a somewhat more complex form of consumption versus saving behavior, one that features habit formation. Thus in place of the time-separable utility function that leads to equation (6), we assume that each period-t utility term includes $c_t/(c_{t-1})^h$, with $0 \leq h < 1$, rather than c_t alone. That specification gives rise to the following replacement for (6):

$$c_t = h_0 + h_1 c_{t-1} + h_2 E_t c_{t+1} + h_3 E_t c_{t+2} + h_4(\log \lambda_t) + v_t. \tag{6'}$$

Here λ_t is the Lagrange multiplier on the household's budget constraint, which obeys

$$\log \lambda_t = const. + E_t \lambda_{t+1} + r_t, \tag{19}$$

and there are constraints relating the h_j parameters to others in the system. For details and additional discussion, see M-N (1999) and the recent study of Fuhrer (2000).

Calibration of the model draws on M-N (1999) but differs in a few ways that, in retrospect, seem appropriate. For the parameters governing spending

[30] For the year 1998, imported consumer goods amounted to \$453 billion while imports of business inputs came to \$624 billion, approximately. These figures are based on an examination of categories reported in the August 1999 issue of the *Survey of Current Business*. For several categories it is clear whether they are composed predominantly of consumer or business goods. For others, judgmental assignments were required. Those assignments are as follows, with the reported figure being the fraction of the category classified as "business inputs": automotive vehicles, engines, and parts, 25 percent; travel, 25 percent; passenger fares, 25 percent; foods, feed, and beverages, 50 percent; and other private services, 75 percent.

behavior, I retain here the $h = 0.8$ value taken from an early version of Fuhrer (2000), but for the counterpart of b_1 I now use -0.4, rather than $-1/6$, in order to reflect the greater responsiveness of investment spending, which is not included explicitly in the model.[31] For σ, the elasticity of substitution in production (and therefore the elasticity of import demand with respect to Q_t), I again begin with 1/3—and for the elasticity of export demand with respect to Q_t the same value is used—but also consider larger values. In (11), the labor-share parameter $1 - \alpha$ equals 0.64. The steady state ratio of imports (and exports) to domestic production is taken to be 0.10, a slightly lower value than in M-N (1999) so as to reflect the Japanese degree of openness. For the present application government consumption is included, with $\omega_2 = 0.2$.

In the price adjustment relation, the specification is that $\phi_2 = 0.03$. The latter value is based on my reading of a wide variety of studies, plus conversion into nonannualized fractional terms for a quarterly model. Policy rule parameters should be thought of in relation to realistic values close to $\mu_1 = 0.5$, $\mu_2 = 0.4$, and $\mu_3 = 0.8$, the latter reflecting considerable interest rate smoothing. In the experiments reported in this paper, however, rule (14) is replaced by the rule (5) that is designed for the zero-lower-bound situation. In most cases, expectations based on $t - 1$ data are used for the Δp_t and $\tilde{y}_t$ variables appearing in the policy rule, in order to make the latter operational.

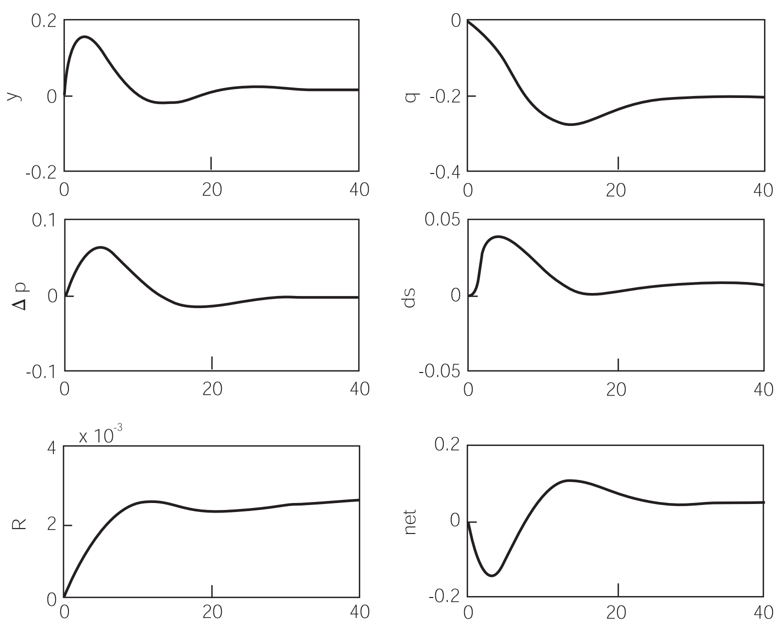
The stochastic processes driving the model's shocks must also be calibrated, of course. For both foreign output and the technology shock, I have specified AR(1) processes with AR parameters of 0.95, rather than the 1.0 values used in M-N (1999). The innovation standard deviations (SD) are 0.03 and (as before) 0.0035. The latter value might appear smaller than is usual, but is appropriate to generate a realistic degree of variability in $\bar{y}_t$ when the latter is not exogenous but instead is dependent on q_t. The UIP risk premium term ξ_t is generated by an AR(1) process with AR parameter 0.5 and innovation 0.04; these values are based on work reported in Taylor (1993b). Government consumption (in logs) follows an AR(1) process, with AR parameter 0.99 and innovation SD of 0.02. Finally, the v_t, u_t, and e_t shock processes are taken to be white noise with SD values of 0.011, 0.002, and 0.0017, respectively.

6. SIMULATION RESULTS

In McCallum (2000a, 2002) I have conducted exercises with this model under the assumption that the nominal interest rate is immobilized at zero, in order to show that monetary policy conducted by means of a rule such as (5) would provide stabilizing influence despite the "liquidity trap" situation. Those policy experiments were not designed, however, to reflect the transitional effects of

[31] The parameter in question is the intertemporal elasticity of substitution in consumption when $h = 0$.

Figure 2 Responses to Policy Change, Initial Case

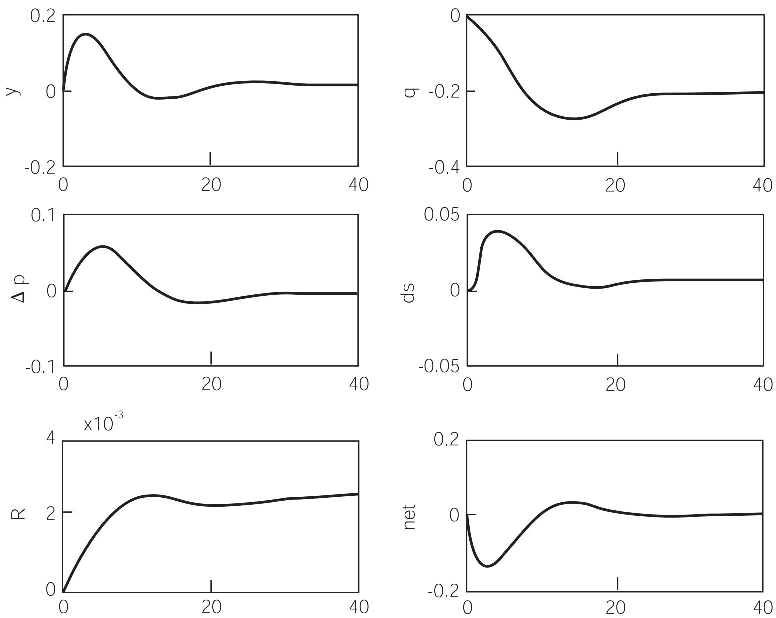


the *adoption* of such a rule; they were conducted as if the rule had been in effect for a long period of time. In what follows, I will use a different strategy. It will again be assumed that an exchange-rate rule has been in effect, but the initial equilibrium is one that leaves the zero-interest situation intact. The objective is to break out of that situation, so the "shock" to which the system is subjected is an increase in the target rate of inflation. This is represented as a permanent upward shock to π^*, the inflation target in the policy rule. Arbitrarily, the experiment assumes a 2.0 percent per-annum shock, e.g., from –1.0 percent inflation to +1.0 percent. In quarterly fractional units, that amounts to an increase of 0.005 in π^*.[32] The precise rule utilized is as follows, with $\mu_1 = 0.5$ and $\mu_2 = 0$:

$$\Delta s_t = E_{t-1}\Delta p_t + \mu_1(\pi^* - E_{t-1}\Delta p_t) + \mu_2(\bar{y}_t - y_t). \tag{20}$$

[32] It should be said that I am not entirely comfortable with analysis of this type of "shock," which seems more like a regime change than the type of shock that RE policy analysis is best designed to handle. Consequently, I would not take details of the dynamics too seriously, but would limit attention to the general nature of the responses. (Many economists do, of course, use rational expectations to analyze the effects of policy regime changes—i.e., to study transition periods—but I have generally been skeptical of such studies.)

Figure 3 Responses to Policy Change, Preferred Case

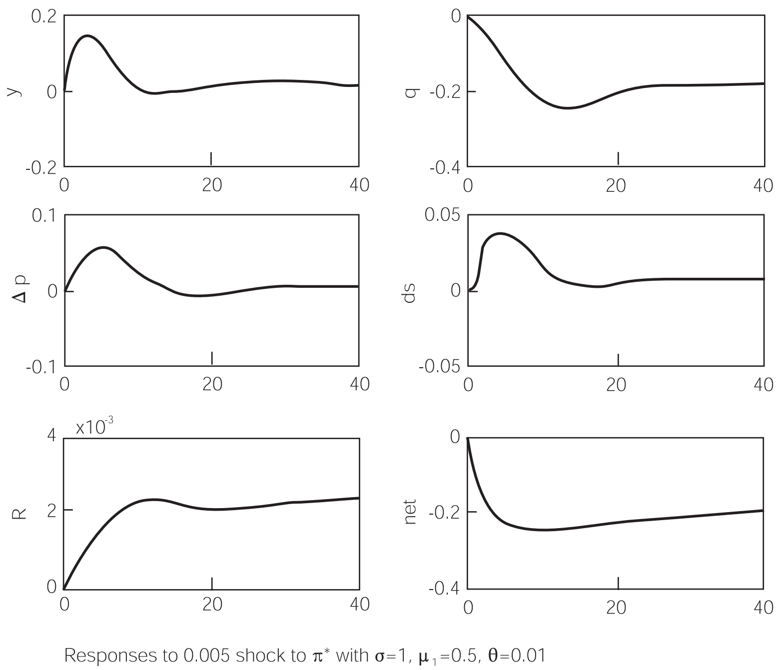


Responses to 0.005 shock to π* with σ=1/2, μ_1=0.5, θ=0.01

The variable on whose response we shall focus is the home country's—i.e., Japan's—net export balance in real terms. Since the model is formulated to be linear in logarithms of most variables, the measure actually calculated is the log of real exports minus the log of real imports. These have to be expressed in common price units, so induced changes in the real exchange rate have to be taken into account. The negative of that measure is taken to reflect the increase in net real exports by Japan's trading partners.

Results of the first experiment are shown in Figure 2. Responses over 40 quarter-years are shown for six variables: the log of real output (y), the inflation rate (Δp), the nominal interest rate (R), the log of the real exchange rate (q), the rate of depreciation of the nominal exchange rate (ds), and the net export variable in (17) (net). In Figure 2 we see that the upward jump in the target inflation rate (π), which occurs in period 1, does indeed induce an exchange-rate depreciation rate that remains positive for over two years. Inflation, not surprisingly, rises and stays above its initial value for over two years, then oscillates and settles down at a new steady state rate of 0.005 (in relation to its starting value). Quite surprisingly, p responds more strongly

Figure 4 Responses to Policy Change, Alternative Case



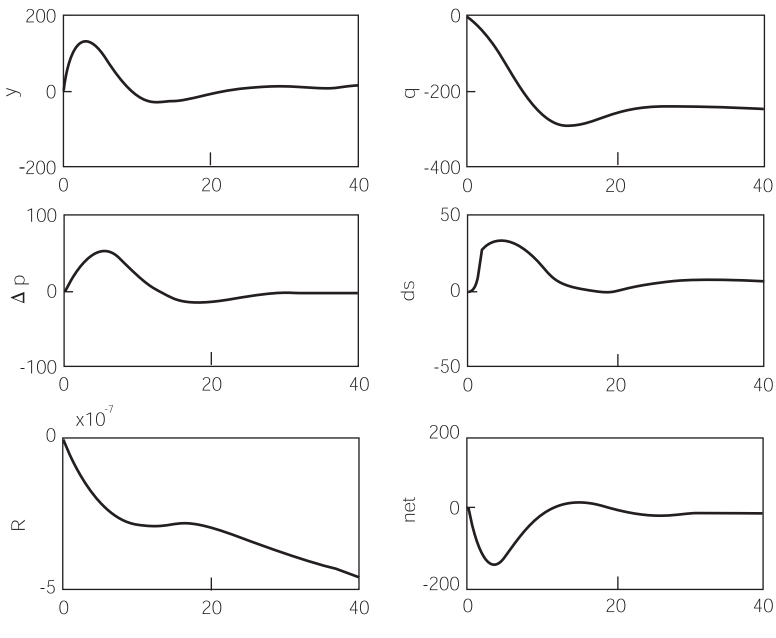
than s so the real exchange rate appreciates.[33] As expected, however, real output rises strongly for two years. Most importantly, the real (Japanese) export balance is so affected by the two-year increase in real output that it turns negative and stays negative for almost two years, although it levels off at a positive value. This pattern is only partly supportive of the argument advanced above, but a single plausible change in the calibration alters it so as to be almost entirely supportive.

The relevant point is that the parameter values used in Figure 2 include figures of 1/3 for the import price elasticities (σ) both at home and abroad.[34] That figure, originally selected by McCallum and Nelson (1999) for reasons that do not pertain in the present exercise, are quite small. Most specialists contend that such magnitudes are substantially larger, at least large enough to satisfy the venerable Marshall-Lerner condition (i.e., that their sum equals 1.0 or more). Accordingly, in Figure 3 the same experiment is repeated but with

[33] It has been verified that steady state response value is zero, reflecting long-run monetary neutrality. But it takes many years for q to return to its original vicinity.

[34] In what follows, I will describe these elasticities as if they were both positive numbers.

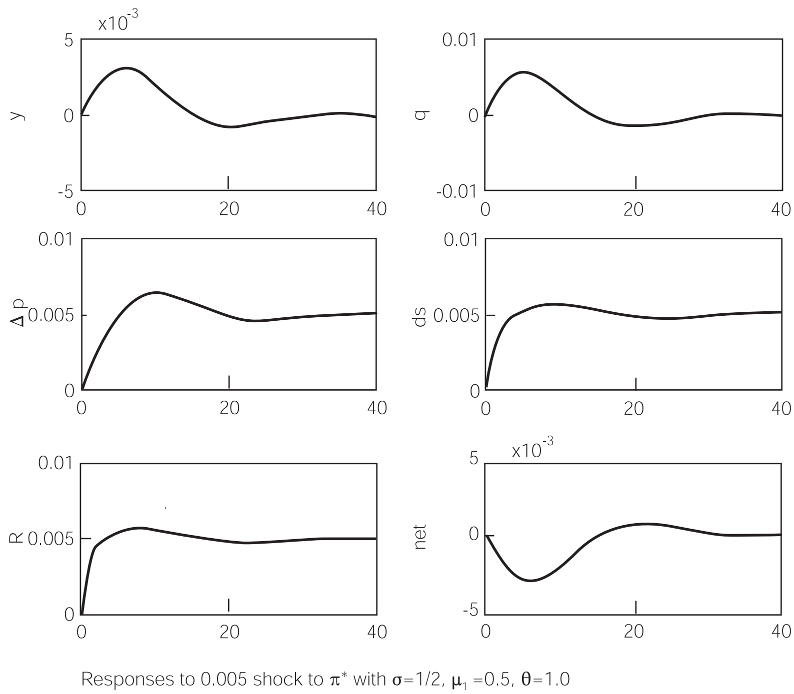
Figure 5 Responses with R_t Kept at Zero



values of 1/2 for each of these import price elasticities. There the effect of the real exchange rate appreciation is eliminated, and the net export balance reflects only the movement of output. Thus Japan's net exports remain negative for about two years, briefly turn positive, and then finally stay slightly negative for a long time (despite long-run neutrality). Setting each country's import price elasticity instead at 1.0, close to the conventional wisdom, gives an entirely different picture, with net exports staying strongly negative for a very long time—see Figure 4.

There are many parameter changes that could be considered, but the more important order of business is to discuss the upward movement of the interest rate R_t that occurs in Figures 2–4. It is clear that the long-run response is a rise of 0.005, which must of course be the case if there is monetary superneutrality and an upward jump in target inflation of that magnitude. But how are the dynamics in R_t being modeled? One extreme possibility is that uncovered interest parity is maintained throughout. But that would be inconsistent with my basic position (and with huge quantities of empirical research). Accordingly, my first attempts at this simulation exercise assumed that the interest rate remains immobilized at its initial zero-lower-bound level.

Figure 6 Responses with UIP Maintained Throughout

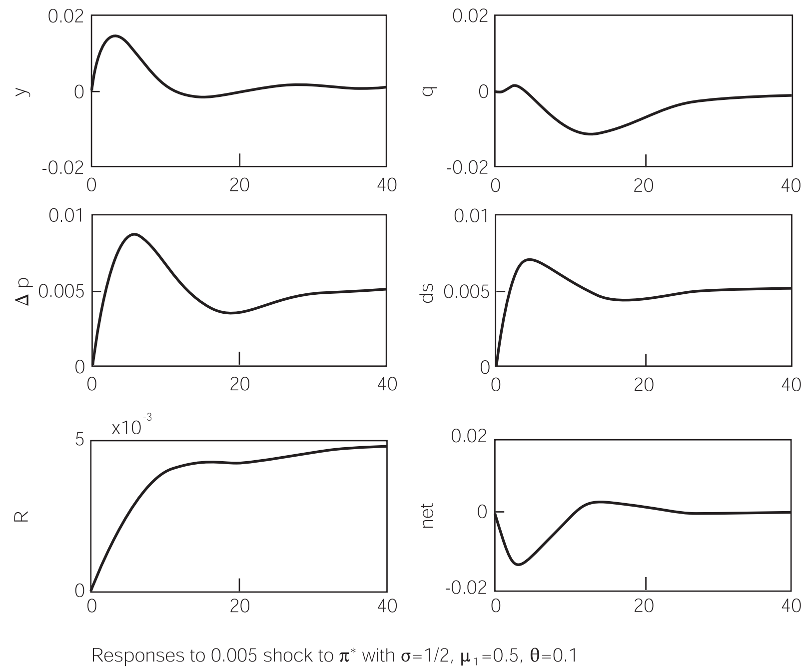


That specification leads, however, to the results shown in Figure 5. There the responses are implausibly large, with inflation rising to almost 20,000 percent per year. (Recall that the numbers shown are in fractional quarterly units.) This might seem to reflect some kind of calculation error, but actually the point is that if R_t were held unchanged, the increased inflation rate would imply a reduction in the real interest rate of 2 percent per year, maintained forever. In a forward-looking rational expectations model, such a change has enormous effects. Furthermore, this way of treating the nominal interest rate is inconsistent with superneutrality and inconsistent with what one believes would happen in the face of a permanently increased inflation rate.

The other extreme treatment of R_t dynamics is to impose uncovered interest parity in all periods. In that case, which I have already described as unrealistic, we have the results shown in Figure 6, where the responses are all very small. Consequently, for the experiments reported in Figures 2–4, I have imposed the following compromise formula:

$$R_t = \theta R_t^{uip} + (1 - \theta) R_{t-1}, (0 \leq \theta \leq 1), \qquad (21)$$

Figure 7 Responses with Fast R_t Adjustment



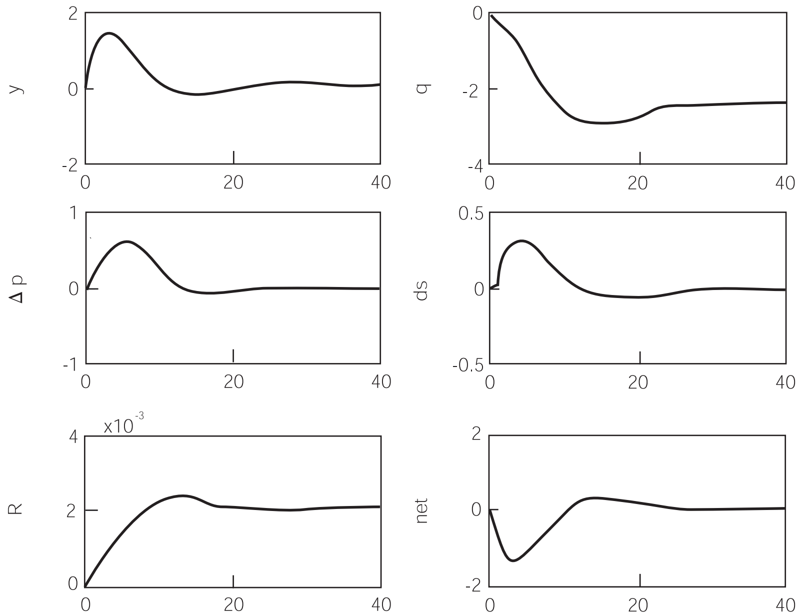
where R_t^{uip} is the value that would prevail if uncovered interest parity held in each period. The value used for θ in Figures 2–4 is 0.01, but the results for x_t are not much different qualitatively if one adopts a value of 0.1 or 0.001—see Figures 7 and 8. That is, the net export variable follows a pattern of the same shape; quantitatively the effects are larger the smaller is θ. In all of Figures 5–8, the import price elasticity was kept at 1/2.

In sum, the simulation results suggest strongly that the move to a more expansionary monetary policy by the BOJ, implemented by policy rule (5), would not have beggar-thy-neighbor effects on Japan's trading partners but instead would induce an increase in their net exports to Japan.

7. THE BANK OF JAPAN LAW, MONETARY POLICY, AND EXCHANGE-RATE POLICY

Let us now turn to the second major objection that has been voiced to the adoption of a policy rule such as (5), namely, that foreign exchange purchases and sales cannot legally be conducted by the BOJ according to its charter. Only a few years ago, in 1998, did the BOJ gain monetary policy independence,

Figure 8 Responses with Slow R_t Adjustment



Responses to 0.005 shock to π^* with $\sigma=1/2$, $\mu_1=0.5$, $\theta=0.001$

i.e., the right and duty to conduct monetary policy as judged appropriate by itself (rather than by the Ministry of Finance).[35] The provisions of this independence are codified in a legal document that, in English, is termed "The Bank of Japan Law." The provisions of this law are of direct relevance because the BOJ evidently has seen the Law as an obstacle to a policy of the type recommended above. Purchases of foreign exchange, it is contended, are the province of the Ministry of Finance, not the BOJ. An unofficial English translation of the Law, made by the BOJ, can be found on the BOJ's web site (http://www.boj.or.jp). The following comments and interpretation are based on that version, as amended January 6, 2001.

The BOJ Law mentions foreign exchange purchases only in Articles 15, 40, 41, and 42. These references all simply *presume* that such purchases will be made either for the purpose of "cooperating. . . with foreign central banks and international institutions. . . " or else "to stabilize the exchange rate of the national currency." Those activities, furthermore, are to be conducted

[35] The law was promulgated on June 11, 1997, and put into effect on April 1, 1998. It has been amended several times.

in a manner specified by the Ministry of Finance. So viewed alone these passages do apparently suggest that the BOJ has no mandate to purchase foreign exchange in the manner suggested above, i.e., for macroeconomic demand management.

However, Articles 1 and 2 of the Law stipulate that a primary duty of the BOJ is to "carry out currency and monetary control. . . " in a manner "aimed at, through the pursuit of price stability, contributing to the sound development of the national economy." Also, Article 3 states that "the BOJ's autonomy regarding currency and monetary control shall be respected." Thus the Law also gives support to the idea that foreign exchange purchases *for the purpose of monetary control* would be consistent with the duties that are explicitly assigned to the BOJ. Evidently, then, there is some internal inconsistency in the Law.

Furthermore, Article 15 states that the Policy Board will decide on matters inclusive of "determining or altering the guidelines for currency and monetary control in other forms," i.e., forms other than money-market operations. This suggests that the Law could be interpreted as stating that the Policy Board has the authority to adopt policies *for exerting monetary control* by the purchase or sale of foreign exchange. In that regard it is important to emphasize again that the purpose of the foreign exchange transactions in question is definitely not to stabilize the exchange rate. Instead, the recommended policy makes the level of the exchange rate subservient to monetary policy, with the latter directed at maintaining price stability so as to promote the sound development of the Japanese national economy. So Article 15 adds to the evident inconsistency in the Law.

Finally, however, we need to consider Article 43, which states that the BOJ ". . . may not conduct any business other than those prescribed by this Law unless such business is necessary to achieve the Bank's objectives prescribed by this Law and the Bank obtains authorization from the Minister of Finance and the Prime Minister." It would appear that this article does not rule out the suggested activities *per se*, because they are integral to the BOJ's achievement of its assigned objectives. Under recent conditions, moreover, they might well be deemed "necessary." Nevertheless, it would seem to be appropriate for the BOJ to seek approval from the Minister of Finance and the Prime Minister, since such a step would keep the proposed actions from conflicting with Article 43. If the government were to favor more monetary stimulus, a well-formulated proposal would presumably meet with approval (although it is possible that political infighting could interfere).

That the BOJ Law does not recognize foreign exchange transactions as a means for conducting monetary policy is illogical but not actually surprising, partly because transactions involving government bills are satisfactory and desirable under normal conditions—i.e., with interest rates substantially above zero. Furthermore, it must be noted that the Japanese arrangements are not out of line with those pertaining to central banks in other economies. In the United States, for example, it is generally understood (despite unclear legislation) that foreign exchange policy is primarily the province of the Treasury.[36] That assignment has not been troublesome for U.S. monetary policy in recent years, but arguably that is so because the Treasury has seen fit to let the foreign exchange value of the dollar be determined by market forces without substantial intervention. Even in the euro area, where monetary legislation for the European Central Bank is expressly designed to protect central bank independence and direct it toward price level stability, there is an anomalous provision regarding exchange rates of the euro vis-a-vis the dollar, the yen, and other currencies. This anomaly appears in Article 109 of the Maastricht Treaty, which gives the E.U. Council of Ministers (i.e., the member nations' finance ministers[37]) the power to make agreements on an exchange-rate system for the euro (relative to non-EU currencies) or to adopt "general orientations" for exchange-rate policy. These actions are supposed not to conflict with the goal of price stability, but the provision could nevertheless create difficulties.

Despite the existence of these actual arrangements, I suggest that it is a mistake to view monetary policy and exchange-rate policy as independent entities, as they implicitly suggest. Indeed, although it would be a slight exaggeration to claim that monetary and exchange-rate policies are merely different aspects of one macroeconomic policy tool, that claim comes closer to the truth than the view suggesting that there are two distinct tools. (In making this statement, I am assuming that the nation under discussion does not attempt to manage exchange rates by means of direct controls, which would of course introduce serious microeconomic inefficiencies and inducements for corruption.) Let us develop that argument in the remainder of this section.

One way to proceed is to recall the nature of monetary arrangements under a gold standard (or any other metallic standard). Any such arrangement on an international basis clearly dictates exchange rates among all nations that adopt gold-standard regimes. But such regimes are simultaneously specifications of domestic monetary standards, ones that require monetary policy to be governed by the overriding obligation of maintaining the domestic-money price of gold—and consequently the value of money in terms of gold.

For fiat money systems the relevant analytical point is that, from a long-run perspective, money stock and exchange-rate paths cannot be independently controlled or managed, basically as a consequence of the long-run neutrality of money. Short-run non-neutralities are a fact of life, of course, so there is some scope for temporary departures of exchange rates from the paths implied by monetary policy. These departures can be effected by fiscal actions or

[36] On this topic see Broaddus and Goodfriend (1996), which takes a position similar to that of the present section, and Hetzel (1996). The quotes on page 21 of the latter are useful.

[37] The Council members are finance or economics ministers when the business is finance or economics, in which case the Council is known as Ecofin. For other issues, other ministers will represent the member countries. When the Council is attended by the countries' prime ministers, the meeting becomes a "summit."

possibly by sterilized—hence nonmonetary—exchange market interventions. But since such departures will only be temporary, it is inappropriate (and dangerous) to think of them as reflecting distinct macroeconomic policies.

A counterargument that some might raise would point out that real exchange rates can be affected permanently by fiscal stances. A higher steady state ratio of government spending to income tends, for example, to generate a higher real foreign-exchange value of a nation's currency. But an increased ratio of government consumption to income has a one-time effect on the real exchange rate, not a continuing or ongoing effect. Thus a monetary policy that generates an average inflation rate that is inconsistent with a fixed nominal exchange rate—or more generally a specified nominal exchange-rate path featuring a nonzero rate of depreciation or appreciation—will eventually lead to a breakdown. Fiscal policy cannot, that is, be used to overcome long-run inconsistencies between money stock, price level, and exchange-rate paths. Useful papers elaborating on this point have been written by Bordo and Schwartz (1996), Garber and Svensson (1995), and Obstfeld and Rogoff (1995).

Furthermore, it is important to keep in mind that a large fraction of fiscal policy actions involves switches between bond finance and tax finance for given streams of government purchases. This reminder is relevant because many standard and widely-used macroeconomic models incorporate the property of Ricardian equivalence, i.e., the property that switches between bond and tax finance have no effect on macroeconomic variables of primary importance, including real and nominal exchange rates (and net exports).[38] Admittedly, it is quite unlikely that actual economies possess this Ricardian property in full, but evidence suggests that deviations are fairly minor. Thus for most fiscal policy actions, there will be at most minor or short-lived effects on exchange rates.

The other possible way of exerting a policy effect on exchange rates is via sterilized interventions, i.e., foreign exchange transactions that are offset so as to result in no net change in the economy's outstanding stock of base money. It is widely agreed by students of the issue, however, that effects of sterilized interventions are at most small and temporary.[39] Thus they too cannot provide a means for escaping the long-run links between money stock and exchange rate magnitudes.

Yet another way to put the argument is as follows. Most economists agree that central banks possess only one significant monetary policy tool. Some would describe it as control over the monetary base whereas others would emphasize the setting of short-term interest rates. But that distinction is unimportant with regard to the issue at hand; what matters is that there is only one significant tool. Consequently, if the central bank is required (externally or

[38] An early statement of this result is provided by Stockman (1983, 151–2).
[39] For a survey of the literature, see Edison (1993).

by its own choice) to devote that policy tool to the achievement of some target path for an exchange rate, then the tool is not available for achievement of a domestic macroeconomic objective—be it expressed in terms of inflation alone or, e.g., some combination of inflation and output deviations from their target values. In short, legislation or arrangements that give exchange-rate control to the finance ministry, or some other branch of government, are basically inconsistent with central bank independence.

8. CONCLUSION

On the basis of the arguments developed above, plus those presented in previous papers, it would appear that an appropriate policy would be for the Bank of Japan to temporarily maintain a growth rate of base money of 10–15 percent per year, with most of the newly created base used to purchase foreign exchange (the remainder being used to purchase long-term government bonds).[40] After a growth rate of nominal GDP of 4–5 percent has been achieved, policy could then revert to more normal arrangements, with a target of about 2 percent measured inflation or 4–5 percent nominal GDP growth.[41]

There have been two prominent objections to this type of proposal. One is that the proposed policy would have undesirable "beggar-thy-neighbor" effects on Japan's trading partners. Simulations with a calibrated model of the "new open-economy macroeconomics" type suggest, however, that the policy's expansionary effects on output would lead to an increase, not a decrease, in Japanese imports. Presentation of the model and the simulation study constitutes a major undertaking of the paper.

The second main objection has been that purchase of foreign exchange is inconsistent with the Bank of Japan Law. But the arguments developed above indicate that purchase of foreign exchange for the purpose of monetary control is basically consistent with those provisions of the Law that call for the BOJ to exert monetary control so as to contribute to the sound development of the Japanese economy. Therefore, since the Law does not mention this reason for conducting foreign exchange transactions, the BOJ could overcome the Law's internal inconsistencies by requesting approval from the Minister of Finance and the Prime Minister. It could also seek amendment of the Law so as to recognize the close relationship between monetary policy and exchange-rate policy, thereby strengthening Japan's statutory basis for central bank independence.

---

[40] The range 10–15 percent is suggested, admittedly loosely, by the plots in the bottom panel of Figure 1.

[41] Studies including Shiratsuka (1999) suggest that measured overstates actual inflation in Japan by about 1 percent per year.

REFERENCES

Bordo, Michael D., and Anna J. Schwartz. 1996. "Why Clashes Between Internal and External Goals End in Currency Crises." *Open Economies Review* 7 (Sup.1): 437–68.

Broaddus, J. Alfred, Jr., and Marvin Goodfriend. 1996. "Foreign Exchange Operations and the Federal Reserve." Federal Reserve Bank of Richmond *Economic Quarterly* 82 (Winter): 1–19.

Buiter, Willem, and Nikolaos Panigirtzoglu. 2001. "Liquidity Traps: How to Avoid Them and How to Escape Them." In *Refections on Economics and Econometrics: Essays in Honor of Martin Fase*, ed. by W.F.V. Vantoor and J. Mooij. Boston: Kluwer Academic Publishers.

Clarida, Richard, Jordi Gali, and Mark Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (December): 1661–1707.

Edison, Hali J. 1993. "The Effectiveness of Central-Bank Intervention: A Survey of the Literature after 1982." Special Papers in International Economics, No. 18, Princeton University, Department of Economics, International Finance Section.

Evans, Martin D. D., and Richard K. Lyons. 2000. "The Price Impact of Currency Trades: Implications for Intervention." Working Paper (June).

__________. 2002. "Order Flow and Exchange Rate Dynamics." *Journal of Political Economy* 110 (February): 170–80.

Fischer, Stanley. 1998."The Asian Crisis, the IMF, and the Japanese Economy." Speech prepared for Asahi Shimbun symposium, Tokyo, 8 April.

__________. "Comments." 2001. *Brooking Papers on Economic Activity* (No. 2): 161–66.

Fuhrer, Jeffrey C. 2000. "Habit Formation in Consumption and its Implications for Monetary-Policy Analysis." *American Economic Review* 90 (June): 367–90.

__________, and George R. Moore. 1995. "Inflation Persistence." *Quarterly Journal of Economics* 109 (February): 127–59.

Garber, Peter M., and Lars E. O. Svensson. 1995. "The Operation and Collapse of Fixed Exchange Rate Regimes." *Handbook of International Economics*, vol. 3, ed. by G. Grossman and K. Rogoff. Amsterdam: North-Holland Publishing Co.

Goodfriend, Marvin. 1997. "Comments." In *Towards More Effective Monetary Policy*, ed. by I. Kuroda. New York: St. Martin's Press.

______________. 2000. "Overcoming the Zero Bound on Interest Rate Policy." *Journal of Money, Credit, and Banking* 32 (November, Part 2): 1007–35.

______________. 2001. "Financial Stability, Deflation, and Monetary Policy." Federal Reserve Bank of Richmond, Working Paper 01-1, (January).

Hetzel, Robert L. 1996. "Sterilized Foreign Exchange Intervention: The Fed Debate in the1960s." Federal Reserve Bank of Richmond *Economic Quarterly* 82 (Spring): 21–46.

Kerr, William, and Robert G. King. 1996. "Limits on Interest Rate Rules in the IS Model." Federal Reserve Bank of Richmond *Economic Quarterly* 82 (Spring): 47–75.

Krugman, Paul. 1998. "It's Baaack: Japan's Slump and the Return of the Liquidity Trap." *Brookings Papers on Economic Activity* (No. 2): 137–87.

______________. 2000. "Thinking About the Liquidity Trap." *Journal of the Japanese and International Economies* 14 (December): 221–37.

McCallum, Bennett T. 1988. "Robustness Properties of a Rule for Monetary Policy." *Carnegie-Rochester Conference Series on Public Policy* 29 (Autumn): 173–203.

______________. 1993. "Specification and Analysis of a Monetary Policy Rule for Japan." Bank of Japan *Monetary and Economic Studies* 11 (November): 1–45.

______________. 2000a. "Theoretical Analysis Regarding a Zero Lower Bound on Nominal Interest Rates." *Journal of Money, Credit, and Banking* 32 (November, Part 2): 870-904.

______________. 2000b. "Alternative Monetary Policy Rules: A Comparison with Historical Settings for the United States, the United Kingdom, and Japan." Federal Reserve Bank of Richmond *Economic Quarterly* 86 (Winter): 49–79.

______________. 2002. "Inflation Targeting and the Liquidity Trap." *Inflation Targeting: Design, Performance, Prospects*, ed. by N. Loyaza and R. Soto. Santiago: Central Bank of Chile.

______________, and Monica Hargraves. 1995. "A Monetary Impulse Measure for Medium-Term Policy Analysis." *Staff Studies for the World Economic Outlook* International Monetary Fund, (September): 52–70.

______________, and Marvin S. Goodfriend. 1988. "Theoretical Analysis of the Demand for Money." Federal Reserve Bank of Richmond *Economic Review* 74 (January/February): 16–24.

______________, and Edward Nelson. 1999. "Nominal Income Targeting in an Open-Economy Optimizing Model." *Journal of Monetary Economics* 43 (June): 553–78.

______________, and ______________. 2000. "Monetary Policy for an Open Economy: An Alternative Framework with Optimizing Agents and Sticky Prices." *Oxford Review of Economic Policy* 16 (Winter): 74–91.

Meltzer, Allan H. 1998. "Time to Print Money." *Financial Times*, 17 July.

______________. 1999. "Comments: What More Can the Bank of Japan Do?" Bank of Japan *Monetary and Economic Studies* 17 (December): 189–91.

______________. 2000. "The Transmission Process." In *The Monetary Transmission Process*, ed. by the Deutsche Bundesbank. New York: Palgrave.

Miao, Ryuzo. 2000. "The Role of Monetary Policy in Japan: A Break in the 1990s?" *Journal of the Japanese and International Economies* 14 (December): 366–84.

Obstfeld, Maurice, and Kenneth Rogoff. 1995. "The Mirage of Fixed Exchange Rates." *Journal of Economic Perspectives* 9 (Fall): 73–96.

Orphanides, Anastasios, and Volker Wieland. 2000. "Efficient Monetary Policy Design Near Price Stability." *Journal of the Japanese and International Economies* 14 (December): 327–65.

Shiratsuka, Shigenori. 1999. "Measurement Errors in the Japanese Consumer Price Index." Bank of Japan *Monetary and Economic Studies* 17 (December): 69–102.

Stockman, Alan C. 1983. "Real Exchange Rates under Alternative Nominal Exchange Rate Systems." *Journal of International Money and Finance* 2 (August): 147–66.

Svensson, Lars E. O. 2001. "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap." Bank of Japan *Monetary and Economic Studies* 19 (February, special edition S-1): 277–312.

Taylor, John B. 1993a. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39 (November): 195–214.

______________. 1993b. *Macroeconomic Policy in a World Economy*. New York: W.W. Norton and Co.

______________. 1997. "Policy Rules as a Means to More Effective Monetary Policy." In *Towards More Effective Monetary Policy*, ed. by I. Kuroda. New York: St. Martin's Press.

Efficient Public Investment in a Model with Transition Dynamics

Pierre-Daniel G. Sarte and Jorge Soares

A long history of debate exists among macroeconomists regarding the degree to which public capital contributes to overall economic activity. Estimates of the output elasticity with respect to public capital range anywhere from 0.06 in early work by Ratner (1983) to as high as 0.39 in a widely cited study by Aschauer (1989). This elasticity refers to the percentage change in GDP induced by a given percentage change in government capital.[1] Glomm and Ravikumar (1997, 197) remark that economists have generally been skeptical of Aschauer's estimate, mainly because "the productivity of public capital is simply not believed to be larger than the productivity of the private capital stock (which is roughly 0.36)." That being said, few would argue that public infrastructure plays no productive role, and estimates of the public capital elasticity of output lying between 0.05 and 0.15 are often put forward.[2]

In an economy where government investment in equipment and structures complements private investment, it is only natural to ask what factors determine the optimal share of public investment in output. The trade-off involves public capital that is productive but that must also be financed through distortional taxation.

[redacted] Jorge Soares is assistant professor in the Department of Economics at George Washington University, 2201 G Street NW, Washington, DC 20052.

[redacted] We wish to thank Margarida Duarte, Marvin Goodfriend, and Alex Wolman for all of their helpful suggestions. Any errors, however, are our own. The views expressed in this article are the authors' and do not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

[1] For any variables x and y, the elasticity of y with respect to x is defined as $\varepsilon \equiv \left(\frac{dy}{dx}\right)\left(\frac{x}{y}\right)$. Thus, if $y = x^{\theta}$, $\varepsilon = \theta$.

[2] See Glomm and Ravikumar (1997) for a survey of recent estimates.

The literature has shown that the optimal share of gross public investment should equal the government capital elasticity of output. However, most influential papers in this literature utilize a very special class of growth models: endogenous growth models without transition dynamics.[3] In these models, the elasticities of output with respect to private and public capital must add up to one. This restriction is empirically implausible because the private capital elasticity of output is approximately 0.36 in the U.S. economy. Thus, if this restriction were true, then the optimal share of public investment in GDP would be 0.64.

In this article, we show that this implausible restriction is an implication of the lack of transition dynamics. We revisit the optimal choice of public investment in a more general and plausible model that allows for gradual transitions between steady states. Since endogenous growth is not essential to our argument, we revert to the more conventional growth model with exogenous technical progress.

Contrary to previous work, we show that (i) the optimal share of gross public investment in output should be less than its elasticity along a balanced growth path; (ii) this share depends in important ways on assumed preferences and technology, including the underlying rate of productivity growth; (iii) the optimal sequence of public investments is not time invariant, and furthermore, a policy aimed at implementing it is not time consistent; and (iv) the government capital elasticity of output is likely to be relatively low at less than 0.1 if the observed U.S. ratios of gross public investment to output and of public capital to private capital are approximately optimal.

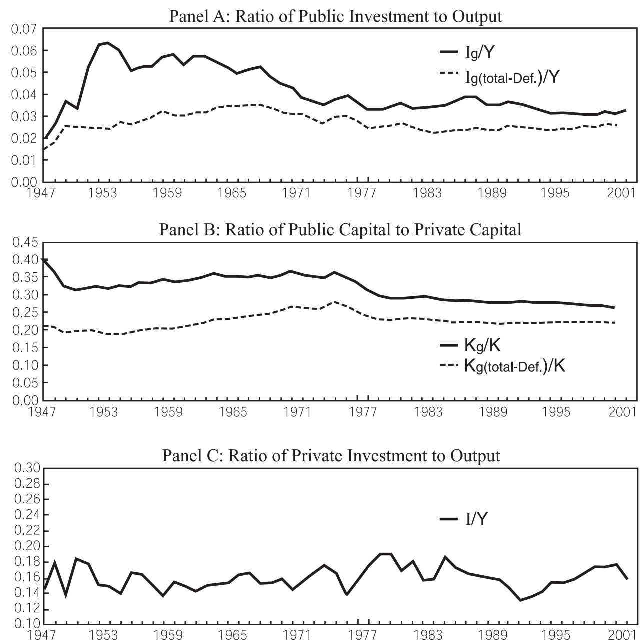
This article is organized as follows: Section 1 gives an overview of public investment in U.S. data. Section 2 sets the basic theoretical framework and derives the optimal steady state share of public investment in output. In contrast, Section 3 describes the solution to the full optimal policy problem with commitment. Section 4 offers concluding remarks.

1. PUBLIC INVESTMENT IN U.S. DATA

Panel A in Figure 1 depicts the behavior of U.S. gross public investment relative to GDP during the postwar period. Observe first that the degree of public investment in the United States is nonnegligible and roughly comparable as a percentage of GDP to that of its net exports. Public investment has amounted to as much as 6 percent of GDP at its peak in the 1950s, but it has also been relatively constant. The run-up in public investment apparent during the 1950s and early 1960s captures a discrete increase in military spending related to the Korean War, as well as increases in spending on schools and

[3] See Barro (1990), Glomm and Ravikumar (1997), and Aschauer (2000), among many others.

Figure 1 Public Investment and Public Capital over the Postwar Period



highways. Outside of changes in military spending over time, the share of public investment in output has stayed mostly flat at approximately 3 percent.

Panel B in Figure 1 shows the ratio of public to private capital from 1947 to today. Abstracting from public capital tied to national defense, which includes aircrafts, ships, vehicles, electronic equipment, and missiles, this ratio has never shown much variation, moving only between 0.20 and 0.25 during the entire period. We see a slight run-up in public capital during the mid-1960s, which corresponds to the construction of the interstate highway system. Interestingly, Fernald (1999) suggests that this construction provided a significant onetime increase in productivity. Note in Panel C in Figure 1 that private investment relative to output has always been much larger than public investment, averaging around 15 percent since World War II.

The fact that some degree of public investment has consistently taken place over the years suggests that the provision of infrastructure such as highways, airports, and even public sector research and development indeed contributes to economic activity. Thus, we will now study a simple economic environment where public capital plays a productive role and examine the factors that determine efficient public investment.

2. THEORETICAL FRAMEWORK

Consider a closed economy in which a large number of firms produce a single final good according to the technology:

$$Y_t = K_t^{\alpha}(z_t l_t)^{1-\alpha} K_{gt}^{\theta}, \tag{1}$$

where $0 < \alpha < 1$ and $0 < \theta < 1 - \alpha$. The condition $\theta < 1 - \alpha$ prohibits the possibility of endogenous growth (see Barro and Sala-i-Martin [1995], 153). In equation (1), K_t and K_{gt} denote private and public capital at time t, respectively, and $z_t l_t$ represents the quantity of skill-weighted labor input. We allow for exogenous technical progress in the use of labor input, l_t, so that z_t grows at a constant rate over time:

$$z_t = \gamma_z z_{t-1}, \gamma_z > 1, \text{ and } z_0 = 1. \tag{2}$$

This feature of the technology will make it possible for the economy to experience balanced growth in the long run at a rate related to γ_z. We treat K_{gt} as a pure public good that enhances each firm's production.[4]

We assume that public investments are financed by a flat tax on income, $0 < \tau_t < 1$, that can vary with time. Hence, we can express new outlays of public capital, K_{gt+1}, as

$$K_{gt+1} = \underbrace{\tau_t Y_t}_{\text{Public Investment}} + (1 - \delta)K_{gt}, \tag{3}$$

$$K_{g0} > 0 \text{ given,}$$

where $0 < \delta < 1$. Observe that τ_t also represents the share of gross public investment in GDP. Our main objective is to characterize the efficient ratio of public investment to output.

Firms

Let r_t and W_t denote, respectively, the rental price of private capital and the wage at date t. Then, taking as given the sequence $\{r_t, W_t\}_{t=0}^{\infty}$, each firm maximizes profits by solving

$$\max_{K_t, l_t} K_t^{\alpha}(z_t l_t)^{1-\alpha} K_{gt}^{\theta} - r_t K_t - W_t l_t. \tag{4}$$

The first order conditions associated with this problem imply that

$$r_t = \alpha K_t^{\alpha-1}(z_t l_t)^{1-\alpha} K_{gt}^{\theta} \tag{5}$$

[4] Thus, we abstract from congestion considerations and think of the aggregate stock of public capital as being available to all firms. This assumption typically subjects the economic environment to scale effects with respect to steady state allocations since public infrastructure is non-rival (see Barro and Sala-i-Martin [1992]).

and

$$W_t = (1-\alpha)K_t^{\alpha} z_t^{1-\alpha} l_t^{-\alpha} K_{gt}^{\theta}. \tag{6}$$

Households

The economy is inhabited by a large number of identical households. Their preferences are given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}, \text{ with } \sigma > 0, \tag{7}$$

where $0 < \beta < 1$ is a subjective discount rate. At each date, households decide how much to consume or save and how much capital to rent to firms. Each household is assumed to be endowed with one unit of time that they supply inelastically. Their budget constraint is given by

$$C_t + I_t = (1-\tau_t)\left[W_t l_t + r_t K_t\right], \tag{8}$$

where

$$\begin{aligned} K_{t+1} &= I_t + (1-\delta)K_t, \\ K_0 &> 0 \text{ given.} \end{aligned} \tag{9}$$

In equation (8), C_t and I_t denote consumption and gross investment, respectively. Before proceeding with the household's problem, we find it useful to first derive the economy's constant balanced growth rate in the steady state. This will allow us to define a normalization of the economy's variables that will show explicitly why the optimal rate of public investment may depend on the exogenous rate of technical progress, $\gamma_Z > 1$. We denote the long-run growth rate of a given variable x by γ_x.

Equation (3) implies that if public investment ultimately grows at the constant rate γ_{I_g}, then $\gamma_{I_g} = \gamma_{K_g}$. In addition, the household's budget constraint (8) implies that $\gamma_C = \gamma_I = \gamma_{I_g} = \gamma_Y$ so that $\gamma_{K_g} = \gamma_Y$.[5] Since equation (9) also means that $\gamma_I = \gamma_K$, we further have that $\gamma_K = \gamma_Y$. Therefore, in the end, all variables in the economy grow at the common growth rate γ_Y. To determine this growth rate, observe from the production technology described in (1) that

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha} \gamma_z^{1-\alpha} \left(\frac{K_{gt+1}}{K_{gt}}\right)^{\theta}$$

or

$$\gamma_Y = \gamma_K^{\alpha} \gamma_z^{1-\alpha} \gamma_{K_g}^{\theta}.$$

[5] Note that $W_t l_t + r_t K_t = Y_t$.

Since $\gamma_{K_g} = \gamma_Y = \gamma_K$ above, we can immediately obtain the economy's balanced growth rate, which we simply denote by γ:

$$\gamma = \gamma_z^{\frac{1-\alpha}{1-\alpha-\theta}}. \tag{10}$$

Given this growth rate, a sensible normalization for our economy is one that expresses our model's variables in detrended form. Specifically, for any variable X_t that grows at rate γ along the balanced growth path, we express its detrended counterpart in lowercase form:

$$x_t = \frac{X_t}{\gamma^t}.$$

In the steady state, detrended variables will then be constant, while non-detrended variables will grow at the constant rate, γ.[6]

Taking the sequence of prices $\{r_t, w_t\}_{t=0}^{\infty}$, as well as the sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$, as given, the household's problem may now be expressed as[7]

$$\max_{\{c_t,\, k_{t+1}\}_{t=0}^{\infty}} \mathcal{U} = \sum_{t=0}^{\infty} \underbrace{\left(\beta\gamma^{(1-\sigma)}\right)}_{\beta^*}{}^{t} \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{11}$$

subject to

$$\begin{aligned} c_t + \gamma k_{t+1} - (1-\delta)k_t &= (1-\tau_t)\left[w_t l_t + r_t k_t\right], \\ k_0 &> 0 \text{ given.} \end{aligned} \tag{12}$$

The solution to the household's dynamic optimization problem yields the familiar Euler equation,

$$\gamma c_t^{-\sigma} = \beta^* c_{t+1}^{-\sigma}\left[(1-\tau_{t+1})r_{t+1} + 1 - \delta\right]. \tag{13}$$

In the above expression, taxes distort private incentives to consume and save but also finance public infrastructure that raises future returns to private investment; recall that $r_{t+1} = \alpha k_{t+1}^{\alpha-1} k_{gt+1}^{\theta}$. Given this trade-off, how should society choose the share of public investment in output? An intuitive answer to this question might be to select the ratio of public investment to output that maximizes steady state welfare. This captures perhaps the simplest notion of optimal policy. We show that such intuition indeed replicates conventional wisdom obtained from endogenous growth frameworks. However, in both our model and endogenous growth settings, ignoring transition dynamics turns out to have crucial implications for policy.

[6] Observe that if X_t grows at the rate γ, then $X_t = \gamma^t X_0$. Therefore $x_t = \frac{X_t}{\gamma^t} = X_0$.

[7] It should be noted that the modified discount rate, β^*, implicitly imposes a restriction on the extent of technical progress, since $\gamma^{(1-\sigma)}$ must be less than $1/\beta$ in order that the maximization problem be well defined.

The Decentralized Steady State Equilibrium and a Policy Golden Rule

We can describe the decentralized steady state equilibrium of the environment laid out in this section most easily as a vector $\{c, i, k, k_g\}$ such that, given a constant share of public investment in output, τ,[8]

$$\text{A)} \quad \gamma = \beta^* \left[(1-\tau)\alpha k^{\alpha-1} k_g^{\theta} + 1 - \delta\right],$$

$$\text{B)} \quad c + i = (1-\tau) k^{\alpha} k_g^{\theta},$$

$$\text{C)} \quad \left[\gamma - (1-\delta)\right] k = i,$$

$$\text{D)} \quad \left[\gamma - (1-\delta)\right] k_g = \tau k^{\alpha} k_g^{\theta}.$$

Conditions A) and B) are the households' long-run consumption/savings decision and budget constraint, respectively. Conditions C) and D) are the normalized accumulation equations for private and public capital.

By combining conditions A) and D), and given the definition of β^*, it is straightforward to show that the decentralized equilibrium ratio of public to private capital satisfies

$$\frac{k_g}{k} = \frac{\left[\frac{\gamma^{\sigma}}{\beta} - (1-\delta)\right]\tau}{\alpha\left[\gamma - (1-\delta)\right](1-\tau)}. \tag{14}$$

In particular, the higher the tax rate, the more public investment takes place and the higher the steady state ratio of public to private capital. Furthermore, one can show that the detrended level of private capital in the steady state is given by

$$k^{1-\alpha-\theta} = \left[\frac{(1-\tau)\alpha}{\frac{\gamma^{\sigma}}{\beta} - (1-\delta)}\right]^{1-\theta} \left[\frac{\tau}{\gamma - (1-\delta)}\right]^{\theta}. \tag{15}$$

Observe in the equation above that k is a strictly concave function of τ. In particular, setting $\tau = \theta$ maximizes the steady state quantity of private capital. Furthermore, we can make use of conditions A), B), and C) to solve for consumption. Given the preferences in (7), it follows that steady state welfare is

$$\mathcal{U} = \frac{\left\{\frac{1}{\alpha}\left[\frac{\gamma^{\sigma}}{\beta} - (1-\delta)\right] - \left[\gamma - (1-\delta)\right]\right\} k^{(1-\sigma)}}{\left(1 - \beta\gamma^{(1-\sigma)}\right)(1-\sigma)}, \tag{16}$$

[8] We use the expression "decentralized" to emphasize that steady state allocations obtain from household and firm optimization conditional on policy.

so that $\mathcal{U}$ depends on τ only through private capital. Because $\mathcal{U}$ is a strictly increasing function of k, while k is concave in τ, setting $\tau = \theta$ also maximizes steady state welfare.

To recapitulate, we have just shown in an exogenous growth context that along the balanced growth path, the optimal share of gross public investment in output, τ, must equal the public capital elasticity of output, θ. As it happens, this is equivalent to the *full* optimizing solution in an endogenous growth model without transition dynamics. Since endogenous growth requires $\alpha + \theta = 1$, and since α is approximately 0.36 in the U.S. economy, endogenous growth models imply that the optimal share of public investment in output should be 0.64, an implausibly large number.[9] Nothing in our exogenous growth framework requires that $\alpha + \theta = 1$, and consequently, τ does not have to be implausibly large at $1 - \alpha$.

There remains, however, another empirical puzzle to solve. Most estimates would place θ between 0.05 and 0.15. In contrast, we have seen that the share of gross public investment in output (τ in our model) has consistently remained only around 0.03 over the postwar period. To address this puzzle, we now study the full welfare-maximizing policy problem that takes into account both steady state considerations and transition dynamics.

3. EFFICIENT PUBLIC INVESTMENT WITH COMMITMENT: THE RAMSEY PROBLEM

Unfortunately, there is no simple way to characterize the Ramsey problem for our economy. Setting up and solving the full welfare maximization problem, however, allows us to address explicitly two important notions associated with efficient policy.

First, we show why the efficient sequence of public investment is not time consistent. Second, we explain why Woodford's (1999) "timeless perspective" provides potentially one answer to this problem. In essence, the timeless perspective advocates the implementation of the steady state solution to the Ramsey problem at all dates. It thus considers optimality from the vantage point of a date far in the past. Because the implied policy is one to which any welfare-maximizing government would have wished to commit itself on that date in the past, this solution concept avoids the problem of time consistency. We will now address these issues in detail.

---

[9] An implicit assumption here is that the share of private capital is measured adequately. An alternative model that allowed for investment in human capital might allow θ to be significantly less than 0.64. Indeed, with a broader concept of capital that allowed for not only physical but also human capital, the share of capital in output might be closer to 0.75 (see Barro and Sala-i-Martin [1992, 38]). Observe also that in an endogenous growth model that allowed for transition dynamics (i.e., where the balanced growth path was reached asymptotically), the full welfare-optimizing solution would not necessarily prescribe $\tau = \theta$ at all dates.

Consider a benevolent government that, at date zero, is concerned with choosing a sequence of tax rates consistent with the development of public infrastructure that maximizes household welfare. In choosing policy, this government takes as given the decentralized behavior of firms and households in the spirit of Chamley (1986). We further assume that at date zero, it can credibly commit to any sequence of policy actions. The problem faced by this benevolent government would then be to maximize (11) subject to equations (12), (13), and (3) (in normalized form), and the corresponding Lagrangean can be written as

$$\max_{\{c_t,\tau_t,k_{t+1},k_{gt+1}\}_{t=0}^{\infty}} \mathcal{L} = \sum_{t=0}^{\infty} \beta^{*t} \frac{c_t^{1-\sigma}}{1-\sigma} \tag{17}$$

$$+\sum_{t=0}^{\infty} \beta^{*t} \mu_{1t} \left\{ \beta^* c_{t+1}^{-\sigma} [(1-\tau_{t+1}) r_{t+1} + 1 - \delta] - \gamma c_t^{-\sigma} \right\}$$

$$+\sum_{t=0}^{\infty} \beta^{*t} \mu_{2t} [\tau_t y_t + (1-\delta) k_{gt} - \gamma k_{gt+1}]$$

$$+\sum_{t=0}^{\infty} \beta^{*t} \mu_{3t} [(1-\tau_t) y_t + (1-\delta) k_t - \gamma k_{t+1} - c_t],$$

where $y_t = k_t^{\alpha} l_t^{1-\alpha} k_{gt}^{\theta}$ and $r_t = \alpha(y_t/k_t)$.

The first constraint in (17) makes clear that our benevolent planner takes households' consumption/savings behavior as given. It can, however, influence the intertemporal allocations they choose by altering tax policy over time. The optimal selection of τ_t is governed by the following two equations:

$$\frac{\partial \mathcal{L}}{\partial \tau_t} : \mu_{20} - \mu_{30} = 0 \text{ for } t = 0 \tag{18}$$

and

$$\frac{\partial \mathcal{L}}{\partial \tau_t} : \mu_{2t} - \mu_{3t} - \mu_{1t-1} \frac{\alpha c_t^{-\sigma}}{k_t} = 0 \;\forall t > 0. \tag{19}$$

The fact that these first order conditions differ for $t = 0$ and $t > 0$ suggests an incentive to take advantage of initial conditions in the first period with the promise never to do so in the future. It is exactly in this sense that the optimal policy is not time consistent, since once date zero has passed, a new planner wishing to solve for the optimal policy at some date $t > 0$ would always choose a tax rate on that date different from what had been prescribed at time zero. For the purpose of this section, therefore, we imagine that the optimization takes place only once, in period zero. Once our benevolent planner has decided on a course of action, his hands are tied and he is precommitted to that course of action.

It should be noted that the choice of τ_t introduces a lagged predetermined variable, $\mu_{1,t-1} \geq 0$. At a purely mechanical level, the corresponding initial condition, $\mu_{1,-1}$, serves as an artificial device that helps make stationary the final system of difference equations that characterizes the optimal solution. However, unlike with fundamental state variables such as the private or public capital stock, this initial condition is not arbitrary. Instead, since the optimal choice of τ_0 should satisfy equation (18), it must be the case that $\mu_{1,t-1} = 0$ in (19) at $t = 0$. Alternatively, Dennis (2001, 6) points out that the lagged Lagrange multiplier, $\mu_{1,t-1}$, may be interpreted as the "current value of promises not to exploit the initial state" and, in particular, to abide by past commitments. However, since no history exists prior to period zero, there are no past commitments on which to assign any value at that date. It is optimal, therefore, to set $\mu_{1,-1} = 0$.

The fact that the optimal policy is chosen once and for all in period zero does not necessarily imply that it is not flexible. On the contrary, the solution to the Ramsey problem provides a description of where to set τ_t in every state of the world. We shall see that it is also explicit about how the share of public investment in output depends on the economic environment. Therefore, as noted in Dennis (2001, 6), "if a change to one or more parameters takes place, the policy rule automatically reflects this change; [and] there is no need for re-optimization to take place." Accordingly, the remaining first order conditions associated with problem (17) are

$$\frac{\partial \mathcal{L}}{\partial c_t} : c_0^{-\sigma} + \sigma\gamma\mu_{10}c_0^{-\sigma-1} - \mu_{30} = 0 \text{ for } t = 0,$$

and, $\forall t > 0$,

$$\frac{\partial \mathcal{L}}{\partial c_t} : c_t^{-\sigma} - \sigma\mu_{1t-1}c_t^{-\sigma-1}\left[r_t(1-\tau_t) + 1 - \delta\right] + \sigma\gamma\mu_{1t}c_t^{-\sigma-1} - \mu_{3t} = 0, \tag{20}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial k_{t+1}} \quad : \quad & \mu_{1t}\beta^* c_{t+1}^{-\sigma}\alpha(\alpha-1)\left(\frac{y_{t+1}}{k_{t+1}^2}\right)(1-\tau_{t+1}) + \beta^*\mu_{2t+1}\tau_{t+1}r_{t+1} \\ & -\mu_{3t}\gamma + \beta^*\mu_{3t+1}\left[(1-\tau_{t+1})r_{t+1} + 1 - \delta\right] \\ & = 0, \end{aligned} \tag{21}$$

and

$$\begin{aligned} &\frac{\partial \mathcal{L}}{\partial k_{gt+1}} : \mu_{1t}\beta^* c_{t+1}^{-\sigma}\alpha\theta\left(\frac{y_{t+1}}{k_{t+1}k_{gt+1}}\right)(1-\tau_{t+1}) - \mu_{2t}\gamma \\ &+\beta^*\mu_{2t+1}\left[\tau_{t+1}\theta\frac{y_{t+1}}{k_{gt+1}} + 1 - \delta\right] + \beta^*\mu_{3t+1}(1-\tau_{t+1})\theta\left(\frac{y_{t+1}}{k_{gt+1}}\right) \\ &= 0. \end{aligned} \tag{22}$$

With these first order conditions in hand, we now turn to the long-run properties of the efficient solution for public investment.

The Timeless Perspective and a Modified Policy Golden Rule

As we have already seen, one of the features associated with the optimal policy described in this section is that it requires the policymaker to commit once and for all to his chosen rule at time zero. That is, optimal commitments are generally not time consistent. However, in the context of monetary policy, Woodford (1999) points out that "the optimal commitment fails to be time consistent only if the central bank considers 'optimality' at each point in time in a way that allows it to consider the advantages, from the vantage point of that particular moment, of a policy change at that time that was not previously anticipated." One way around this problem, therefore, would be for the policymaker to adopt a pattern of behavior "to which it would have wished to commit itself to at a date far in the past, contingent upon the random events that have occurred in the meantime." Woodford refers to this pattern of behavior as the "timeless perspective."

The attraction of Woodford's timeless perspective is that it retains the solution to the Ramsey problem as the concept of optimal policy while getting rid of the unique nature of date zero. Two potential problems with the notion of timeless perspective are that it may be consistent with multiple policy outcomes (see Dennis 2001) and that welfare may be lower than that obtained under alternative time consistent solutions (see Jensen and McCallum 2002).

Under the timeless perspective, the policy rule must ensure that the optimal stationary equilibrium is eventually reached or, if already reached, that it continues in that state. In our context, the optimal stationary equilibrium is given by a vector $\{c, y, \tau, k, k_g, \mu_1, \mu_2, \mu_3\}$ that solves equations (19) through (22), along with the Euler equation (13), the resource constraint (12), the equation describing the evolution of public capital (3) (in normalized form), and the definition of output, $y_t = k_t^{\alpha} l_t^{1-\alpha} k_{gt}^{\theta}$, all without time subscripts. The optimal steady state equilibrium, therefore, can be characterized as a system of eight equations in eight unknowns, which is shown in detail in the Appendix.

In the steady state, households' optimal consumption/savings decisions satisfy the familiar Euler equation,

$$\gamma - \beta^* \left[(1-\tau)\alpha \left(\frac{y}{k} \right) + 1 - \delta \right] = 0. \tag{23}$$

Furthermore, the Appendix shows that the efficient share of public investment in output must be such that

$$\gamma - \beta^* \left[\theta \left(\frac{y}{k_g} \right) + 1 - \delta \right] = 0. \tag{24}$$

It follows from equations (23) and (24) that the Ramsey solution equates the after-tax return to private investment, $(1-\tau)\alpha(y/k)$, with the marginal return to public investment, $\theta(y/k_g)$. Consequently, we can immediately pin down the ratio of public to private capital, given τ, as

$$\frac{k_g}{k} = \frac{\theta}{\alpha(1-\tau)}. \tag{25}$$

As expected, the higher the tax rate, the more public capital can be generated relative to private capital.

The idea that the after-tax return to private investment must equal the marginal return to public investment at the optimum also helps determine the efficient share of public investment in output. Note that in our framework the opportunity cost of one unit of resources invested in the public sector is the after-tax return this unit would have otherwise earned in the private sector, or, by equation (23),

$$\underbrace{(1-\tau)\alpha\left(\frac{y}{k}\right)}_{\text{return to private investment}} = \frac{\gamma}{\beta^*} - (1-\delta). \tag{26}$$

The marginal benefit of investing one unit of resources in the public sector is $\theta(y/k_g)$, and since public capital accumulates according to the law of motion $k_g[\gamma - (1-\delta)] = \tau y$ in the steady state, we have

$$\theta\left(\frac{y}{k_g}\right) = \theta\left(\frac{\gamma-(1-\delta)}{\tau}\right). \tag{27}$$

Hence, equating marginal cost and marginal benefit (i.e., the RHS of equations 26 and 27) directly yields the efficient solution for public investment as a fraction of output:

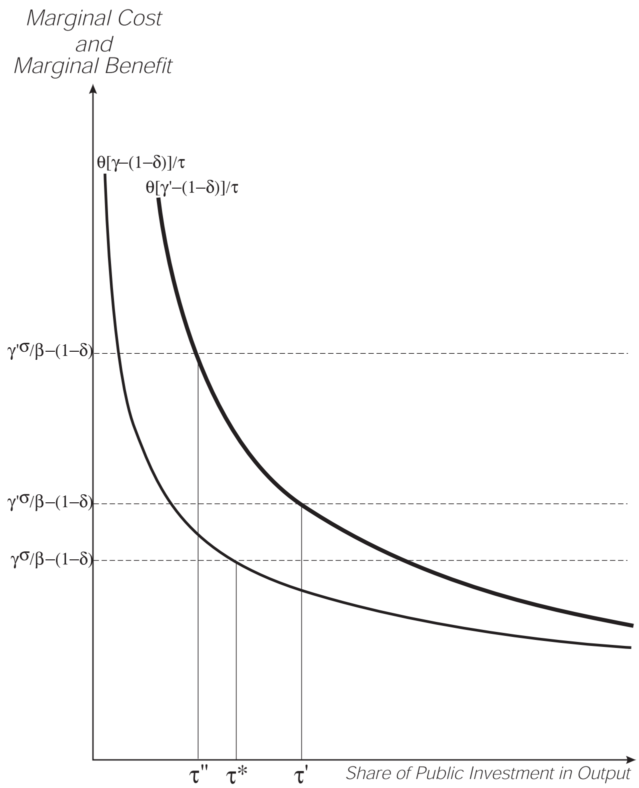
$$\tau = \theta\left\{\frac{\gamma-(1-\delta)}{\frac{\gamma^\sigma}{\beta}-(1-\delta)}\right\} < \theta. \tag{28}$$

We can think of the equation above as a modified golden rule for policy. Analogous to the modified golden rule for private capital in the one sector growth model, the share of public investment in output given by (28) falls short of the policy golden rule outlined in the previous section by an amount that depends importantly on discounting. To see this, observe that when $\delta = 1$ in equation (28), $\tau = \left(\beta\gamma^{1-\sigma}\right)\theta = \beta^*\theta < \theta$. Moreover, from (14) and (25), it would then be the case that

$$\underbrace{\frac{k_g}{k} = \frac{\theta}{\alpha(1-\beta^*\theta)}}_{\text{Modified Policy Golden Rule}} < \underbrace{\frac{\theta}{\alpha\beta^*(1-\theta)} = \frac{k_g}{k}}_{\text{Policy Golden Rule}}. \tag{29}$$

In other words, although the policy golden rule eventually leads to more public infrastructure, and although public capital matters in production, the impatience reflected in the rate of time preference means that it is not optimal to

Figure 2 Efficient Public Investment



reduce current consumption through higher taxes to reach this higher ratio of public to private capital.

Relative to most earlier work, the modified policy golden rule condition is an important one in at least two respects. First, it implies that a high elasticity of output with respect to public capital, θ, does not necessarily have to translate into a large share of public investment in output, τ. Therefore, observed empirical estimates of θ lying between 0.05 and 0.15 do not have to be inconsistent with the 0.03 share of public investment in output. Second, the efficient share of public investment in GDP now depends on a variety of preference and technology considerations, including exogenous productivity growth, the rate of depreciation, and the coefficient of intertemporal substitution.

Comparative Statics

Figure 2 depicts the effects of a rise in exogenous labor productivity growth on the efficient ratio of public investment to output. We can see that an increase in the rate of labor augmenting technical progress from γ to γ' raises both the marginal cost, $\frac{\gamma}{\beta^*} - (1 - \delta)$, and the marginal benefit, $\theta\left(\frac{\gamma-(1-\delta)}{\tau}\right)$, of public investment. Intuitively, the returns to both types of capital increase, and it is not clear whether investment in public capital should increase or fall. Depending on the degree to which the marginal cost of public investment rises in Figure 2, we can see that the optimal rate of public investment, τ^*, may rise to τ' or instead decrease to τ''. In the case with 100 percent depreciation, equation (28) reduces to $\tau^* = \beta\gamma^{(1-\sigma)}\theta$. Hence, it follows that

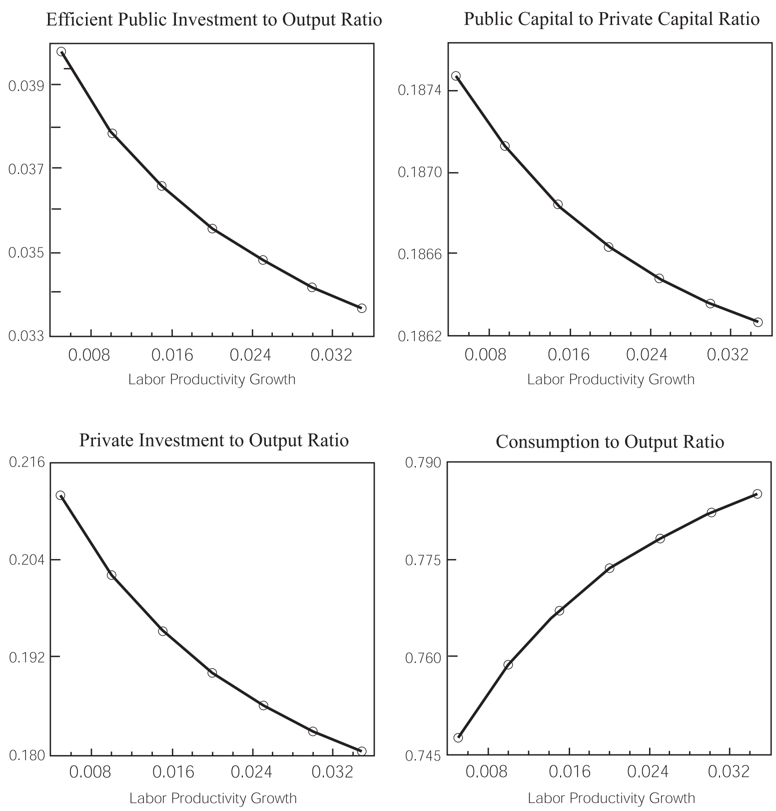
$$\frac{\partial\tau^*}{\partial\gamma} = (1 - \sigma)\beta^{-\sigma}\theta \gtreqless 0 \Leftrightarrow \frac{1}{\sigma} \gtreqless 1. \qquad (30)$$

Put another way, when households are not particularly willing to substitute consumption across time (i.e., $1/\sigma < 1$), an increase in exogenous labor productivity growth leads them to want to increase present consumption relative to output. Therefore, with fewer resources available for investment, the optimal steady state share of public investment in output must fall.

To conclude this section, we calibrate the above example in order to determine what value of θ is implied by the theory. We shall think of our benchmark as that of an economy resembling the United States. Other than for the public capital elasticity of output, θ, the parameter values of the economy are selected along the lines of conventional general equilibrium quantitative studies. Thus, a time period represents one quarter, and we set the subjective discount rate, β, to 0.99. The share of private capital in output, α, is set to 1/3, and σ is chosen so as to make the coefficient of intertemporal substitution 1/2. Capital is assumed to depreciate at a 10 percent annual rate. We let the rate of technical progress vary so that the rate of growth in per capita output lies between zero and 3.5 percent. In the United States, the rate of growth of per capita GDP has averaged 2.5 percent since World War II.

Given these parameter values, equations (28) and (29) indicate that the choice of θ simultaneously pins down two measures, namely, the share of public investment in GDP and the ratio of public to private capital. There is an obvious sense, then, in which our model can fail, since choosing θ to match one measure for the United States would leave it free to miss its mark on the other. We find that setting the public capital elasticity of output to 0.06 helps generate a share of public investment in output between 3.3 and 4 percent as in U.S. data. This is shown in the upper left-hand panel of Figure 3. Interestingly, we find that this same public capital elasticity of output leads to a ratio of public to private capital that matches well U.S. postwar experience at approximately 20 percent (see the upper right-hand panel of Figure 3). We conclude that if the share of public investment and the ratio of public to

Figure 3 Steady State Responses to Changes in Labor Productivity Growth



private capital are approximately efficient in the United States, then the public capital elasticity of output implied by the theory lies in the lower range of most empirical estimates at around 0.06.

Figure 3 also shows that the efficient share of public investment in GDP falls with increases in exogenous labor productivity growth. At the same time, the consumption to output ratio rises in the lower right-hand panel of Figure 3. In this calibrated example, as exogenous labor productivity growth increases, households can afford to consume more relative to output and both private and public investment as a fraction of GDP decreases.

4. SUMMARY

In a setting where public infrastructure plays a productive role, we have characterized the main features of efficient public investment under commitment. In contrast to most previous studies, we have shown that the optimal share of public investment in output may be less than its elasticity in the long run and, moreover, that this share may depend in important ways on assumed preferences and technology. We have also stressed the crucial nature of the commitment assumption, as the optimal sequence of public investments is not time consistent. Finally, if the observed ratios of public investment to GDP and of public to private capital are approximately optimal in the United States, then our model suggests that the government capital elasticity of output is likely to be relatively low at 0.06.

APPENDIX

As described in the text, the optimal stationary equilibrium in our framework is a vector $\{c, y, \tau, k, k_g, \mu_1, \mu_2, \mu_3\}$ that solves eight equations. The first four equations are

$$y - k^{\alpha} l^{1-\alpha} k_g^{\theta} = 0 \tag{31}$$

$$\gamma - \beta^* \left[(1-\tau)\alpha\left(\frac{y}{k}\right) + 1 - \delta\right] = 0 \tag{32}$$

$$\tau y + (1-\delta)k_g - \gamma k_g = 0 \tag{33}$$

$$(1-\tau)y + (1-\delta)k - \gamma k - c = 0. \tag{34}$$

The next four equations are worked out in more detail.

Equation (21) reads as:

$$\mu_1 \beta^* c^{-\sigma} \alpha(\alpha-1)\left(\frac{y}{k^2}\right)(1-\tau) + \beta^* \mu_2 \tau \alpha \left(\frac{y}{k}\right)$$

$$-\mu_3 \left[\underbrace{\gamma - \beta^* \left[(1-\tau)\alpha\left(\frac{y}{k}\right) + 1 - \delta\right]}_{= 0 \text{ by equation (32)}}\right]$$

$$= 0.$$

Thus, we have

$$\mu_1 \beta^* c^{-\sigma} \alpha(\alpha-1)\left(\frac{y}{k^2}\right)(1-\tau) + \beta^* \mu_2 \tau \alpha \left(\frac{y}{k}\right) = 0,$$

or more simply

$$\left(\frac{\mu_1 c^{-\sigma}}{k}\right)(\alpha - 1)(1-\tau) + \mu_2\tau = 0. \tag{35}$$

In the steady state, equation (19) reads as

$$-\mu_1 c^{-\sigma}\alpha\left(\frac{y}{k}\right) + \mu_2 y - \mu_3 y = 0,$$

so that

$$\left(\frac{\mu_1 c^{-\sigma}}{k}\right)\alpha = \mu_2 - \mu_3. \tag{36}$$

Equation (20) is

$$c^{-\sigma} - \sigma\mu_1 c^{-\sigma-1}\left[(1-\tau)\alpha\left(\frac{y}{k}\right) + 1 - \delta\right] + \sigma\gamma\mu_1 c^{-\sigma-1} - \mu_3 = 0,$$

or

$$c^{-\sigma} - \sigma\mu_1 c^{-\sigma-1}\left\{\left[(1-\tau)\alpha\left(\frac{y}{k}\right) + 1 - \delta\right] - \gamma\right\} - \mu_3 = 0,$$

so that by equation (32),

$$c^{-\sigma} - \sigma\mu_1 c^{-\sigma-1}\gamma\left[\frac{1}{\beta^*} - 1\right] - \mu_3 = 0. \tag{37}$$

Finally, equation (22) reads as

$$\mu_1\beta^* c^{-\sigma}\left[\frac{\alpha\theta y}{kk_g}\right](1-\tau) - \mu_2\gamma + \beta^*\mu_2\left[\tau\theta\left(\frac{y}{k_g}\right) + 1 - \delta\right]$$
$$+\beta^*\mu_3(1-\tau)\theta\left(\frac{y}{k_g}\right) = 0.$$

This simplifies to

$$\beta^*\theta\left(\frac{y}{k_g}\right)(1-\tau)(\mu_2 - \mu_3) - \mu_2\gamma + \beta^*\mu_2\left[\tau\theta\left(\frac{y}{k_g}\right) + 1 - \delta\right]$$
$$+\beta^*\mu_3(1-\tau)\theta\left(\frac{y}{k_g}\right) = 0$$

by equation (36), which implies

$$\beta^*\theta\left(\frac{y}{k_g}\right)(1-\tau) - \gamma + \beta^*\left[\tau\theta\left(\frac{y}{k_g}\right) + 1 - \delta\right] = 0,$$

or

$$\gamma - \beta^*\left[\theta\left(\frac{y}{k_g}\right) + 1 - \delta\right] = 0. \tag{38}$$

This last equation and equation (32) imply that the Ramsey solution equates the marginal return to government capital with the after-tax return to private capital.

REFERENCES

Aschauer, David A. 1989. "Is Public Expenditure Productive?" *Journal of Monetary Economics* 23 (March): 177–200.

____________. 2000. "Do States Optimize? Public Capital and Economic Growth." *The Annals of Regional Science* 34 (August): 343–63.

Barro, Robert J. 1990. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98 (October): S103–25.

____________, and Xavier Sala-i-Martin. 1992. "Public Finance in Models of Economic Growth." *Review of Economic Studies* 59 (October): 649–61.

Chamley, C. 1986. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica* 54 (May): 607–22.

Dennis, R. 2001. "Pre-Commitment, the Timeless Perspective, and Policymaking from behind the Veil of Uncertainty." Working Paper 2001-19, Federal Reserve Bank of San Francisco (August).

Fernald, John G. 1999. "Roads to Prosperity? Assessing the Link between Public Capital and Productivity." *American Economic Review* 89 (June): 619–38.

Glomm, G., and B. Ravikumar. 1997. "Productive Government Expenditures and Long-Run Growth." *Journal of Economic Dynamics and Control* 21 (January): 183–204.

Jensen, C., and Bennett T. McCallum. 2002. "The Non-Optimality of Proposed Monetary Policy Rules under Timeless-Perspective Commitment." Working Paper 8882, National Bureau of Economic Research (April).

Ratner, J. B. 1983. "Government Capital and the Production Function of U.S. Private Output." *Economic Letters* 13 (2/3): 213–17.

Woodford, M. 1999. "Commentary on: How Should Monetary Policy Be Conducted in an Era of Price Stability?" *New Challenges for Monetary Policy*, a symposium sponsored by the Federal Reserve Bank of Kansas City, 26–28 August.

Potential Consequences of Linear Approximation in Economics

Alexander L. Wolman and Elise A. Couper

The equilibrium of a dynamic macroeconomic model can usually be represented by a system of nonlinear difference equations, and in contemporary models these systems can be large and difficult to solve. Even for small models, it is common research practice to use approximations that allow for analytical statements about the model's behavior. The most popular form of approximation is linearization around a steady state.[1]

To study linear approximations, economists have access to the methods for solving dynamic linear models described in Sargent (1979) and Blanchard and Kahn (1980). Blanchard and Kahn provide conditions under which there exists a unique nonexplosive solution to a system of linear difference equations. In his analysis of first- and second-order linear difference equations, Sargent explains how, in some cases, models based on optimizing behavior justify the exclusion of explosive solutions as equilibria. Subsequently, attention has shifted toward explicitly nonlinear optimization-based models, but the methods described by Sargent and Blanchard and Kahn have been widely used to study the linear approximations to nonlinear models.

For as long as linear approximations have been used, economists have been aware of certain limitations. In particular, linear approximations may be quantitatively inaccurate unless one restricts attention to the model's behavior near the point around which the linearization was taken. Recent work by Benhabib, Schmitt-Grohé, and Uribe (2001) has highlighted an additional limitation of linearization that is potentially more severe: linearization may lead

■ This paper does not necessarily represent the views of the Federal Reserve System or the Federal Reserve Bank of Richmond. The authors thank Marvin Goodfriend, Andreas Hornstein, Tom Humphrey, and Ned Prescott for helpful comments.

[1] A steady state is a point $\bar{x}$ such that if $\bar{x}$ is an equilibrium in any period t, then $\bar{x}$ is also an equilibrium in period $t+1$.

one to incorrect conclusions about the existence or uniqueness of equilibrium. These scholars have argued, based on this reasoning, that a monetary policy rule widely advocated for its stabilization properties may actually subject the economy to multiple equilibria. Our purpose in this article is to provide a simple exposition of the type of problems highlighted by Benhabib, Schmitt-Grohé, and Uribe. While we do not advocate that linearization be abandoned entirely, it is important for users to be aware of the risks.

We use two simple models to illustrate the risks of linearization. In both models, the dynamics boil down to one equation in one variable. This simplicity means that it is straightforward to compare the results based on linearization to the model's global properties.

In the first model, the single equation concerns the evolution of the stock of government debt. We will show that analysis of this model based on linearization can lead one to an erroneous conclusion about whether an equilibrium exists. A researcher might conclude, for example, that a particular tax policy rule leads to the nonexistence of equilibrium when, in fact, an equilibrium does exist for all but extreme initial values of debt. Or, linearization could suggest that an equilibrium always exists when actually there is none outside a narrow range of initial levels of debt.

In the second model, the single equation concerns the evolution of the inflation rate. There, an equilibrium always exists, but naive analysis based on linearization can lead one to erroneously conclude that there is only one equilibrium when in fact there are many. Thus, a researcher using linearization might advocate a particular policy rule based on its promise of delivering a unique equilibrium when in fact that rule is susceptible to multiple equilibria. This precise critique has been made by Benhabib, Schmitt-Grohé, and Uribe against recent work advocating "active Taylor rules" for monetary policy.

In more complicated models, it may not be possible to determine whether a linear approximation results in misleading conclusions about the uniqueness and existence of equilibrium. However, in closing we will offer some suggestions for minimizing the risk of being misled.

1. MACROECONOMIC EQUILIBRIUM

A typical macroeconomic model consists of a set of maximization problems and a set of market clearing conditions pertaining to a vector of variables. Solving a model involves two steps: The first step is to derive the optimality conditions that describe solutions to the maximization problems in isolation. The second step is to collect these conditions with the market clearing conditions and manipulate them so that the variables whose values are not known at the beginning of a period (for example, the price of a unit of capital) are expressed as functions of the variables whose values are known at the beginning of a period (for example, the capital stock). We refer to the former set of variables as nonpredetermined, and the latter set as predetermined. If at least one such vector-valued function exists, then an equilibrium of the model exists. If there is exactly one such function, equilibrium is unique, whereas multiple functions correspond to multiple equilibria. The second step can be difficult, especially for models with many variables, and it often requires some numerical approximations. The most popular approximation method is linearization around a steady state.

We will describe two simple models, which will then be used in our analysis of linearization. In both models, there is assumed to be an infinitely lived representative consumer who receives a constant endowment of consumption goods each period. The consumer discounts future utility at rate β per period. In the first model, there is a government that purchases a constant amount of the consumption goods each period. The government issues debt and levies lump- sum taxes in order to pay for its consumption. In the second model, there is no government spending; however, the consumer derives utility from real money balances as well as consumption. The government issues nominal money by making lump-sum transfers to consumers.

A Model with One Dynamic Variable, Predetermined

The representative consumer has preferences for current and future consumption (c_t) given by the maximization problem

$$\max \sum \beta^t u(c_t), \tag{1}$$

subject to the budget constraint

$$c_t + b_{t+1} + \tau_t \leq y + r_{t-1} b_t, \tag{2}$$

where u () is an increasing and concave function; b_t is the quantity of one-period, real government debt maturing in period t, paying a gross interest rate of r_{t-1}; τ_t is the lump-sum tax levied in period t; and y is the constant endowment received each period. Denoting by λ_t the Lagrange multiplier on the budget constraint at time t, the first order condition for consumption is

$$u'(c_t) = \lambda_t; \tag{3}$$

the marginal value to the consumer of an additional unit of income is equal to the marginal utility associated with using that income for consumption. The first order condition for bond holding is

$$\lambda_t = \beta r_t \lambda_{t+1}; \tag{4}$$

the marginal utility of income in the current period is equated to the present discounted utility of converting current income into future income at the given interest rate r_t. The transversality condition is

$$\lim_{t \longrightarrow \infty} \beta^t \lambda_t b_{t+1} = 0; \tag{5}$$

this condition can be viewed as the first order condition for bond purchases in the "final period." It would be suboptimal for a consumer to accumulate bonds so that the present utility value of consumption that could be realized by selling those bonds in the distant future did not go to zero.

The government budget constraint is

$$b_{t+1} + \tau_t \geq g + r_{t-1} b_t, \tag{6}$$

where g is the constant level of per-period government purchases of goods. The left-hand side is the government's sources of revenue, and the right-hand side is the government's uses of revenue. In equilibrium the goods market clears, implying that government consumption plus private consumption equals the total endowment of goods:

$$c_t + g = y. \tag{7}$$

Because the endowment and government consumption are constant, private consumption must also be constant:

$$c_t = c \equiv y - g \qquad \forall t. \tag{8}$$

Since equilibrium consumption is constant, the marginal utility of consumption is constant, and thus, from (3) and (4), the real interest rate is constant in equilibrium:

$$r_t = \beta^{-1}. \tag{9}$$

It remains to solve for the equilibrium quantity of government debt (b_{t+1}) and the tax rate (τ_t). The two equations left to determine these variables are the government budget constraint (6) and the transversality condition (5). One might think that the consumer's budget constraint (2) is an additional equation. However, if we substitute the market clearing condition (7) into the consumer's budget constraint, the consumer's budget constraint and the government budget constraint become identical:

$$b_{t+1} = (g - \tau_t) + \beta^{-1} b_t. \tag{10}$$

This result is an implication of *Walras's law* (see Varian [1992, 317]).

It is clear that the government budget constraint and the transversality condition are not sufficient to determine a unique equilibrium path, or even a finite number of equilibrium paths for b_{t+1} and τ_t. In order to narrow the set of equilibria, the standard research practice is to specify a policy rule for the quantity of debt issued or, more commonly, for the tax rate. Given a rule that sets the tax rate as a function of other variables, one can determine whether equilibrium exists and is unique.

Note that if the rule makes the tax rate a function of no variables other than b_{t+1} or b_t, substituting the tax rule into (10) yields one equation that implicitly determines the current period debt as a function of the predetermined debt from the previous period. Henceforth, we will assume that the tax rule sets the lump-sum tax as a function of only the predetermined level of debt, $\tau_t = h(b_t)$. In this case, (10) explicitly describes the evolution of government debt:

$$b_{t+1} = g - h(b_t) + \beta^{-1} b_t. \quad (11)$$

Below we will assume a particular form for $h()$, and thus we will be able to determine whether equilibrium exists and is unique.

A Model with One Dynamic Variable, Nonpredetermined

The second model we will consider is one in which we again end up with a single dynamic equation, although in this case the equation will not contain a predetermined variable. Here the representative consumer has preferences for current and future real money balances (m_t) as well as consumption (c_t), given by the maximization problem

$$\max \sum \beta^t [u(c_t) + v(m_t)], \quad (12)$$

where $u()$ and $v()$ are increasing, concave functions.[2] In this model, the government issues non-interest-bearing money by making lump-sum transfers to consumers.[3] The consumer maximizes utility subject to the budget constraint

$$c_t + \frac{M_t}{P_t} + \frac{B_{t+1}}{P_t} = y + \frac{M_{t-1}}{P_t} + \frac{R_{t-1}B_t}{P_t} + \frac{TR_t}{P_t}. \quad (13)$$

In (13), c_t and y are as defined above. The new variables in (13) are the nominal money supply ($M_t = m_t P_t$), the price level (P_t), the quantity of one-period nominal bonds maturing in periods $t+1$ and t (B_{t+1}, B_t), the nominal interest rate on bonds maturing in the current period (R_{t-1}), and the quantity of nominal transfers from the government to the household (TR_t).[4]

Just as in the first model, the first order condition for consumption is given by (3), with λ_t now the Lagrange multiplier on the budget constraint (13). The first order condition for real money balances is

$$v'(m_t) - \lambda_t + \beta \frac{P_t}{P_{t+1}} \lambda_{t+1} = 0. \quad (14)$$

If the consumer increases real balances marginally in period t, he or she gains current utility directly ($v'(m_t) > 0$) but sacrifices current period consumption

[2] We also assume $\lim_{m \to 0} v'(m) < u'(y)$ and $v'(\breve{m}) = 0$, where $\breve{m} < \infty$. The former condition implies that, at a sufficiently high (finite) nominal interest rate, the economy will demonetize. The latter condition implies that, at a nominal interest rate of zero, individuals become satiated with a finite level of real balances.

[3] See Brock (1975) and Obstfeld and Rogoff (1983) for further discussion of related models.

[4] Although we have included nominal bonds in the consumer's budget constraint, their quantity will be zero in equilibrium (we assume that the government does not issue or purchase bonds, and since households are identical, the quantity of bonds must be zero).

valued at λ_t. The same *nominal* balances are available in the next period as a source of income to be used for consumption. However, the real value in the next period of those nominal balances is deflated by the inflation rate P_{t+1}/P_t, and the marginal utility is discounted back to the current period by the factor β. Condition (14) states that these effects are mutually offsetting: optimal behavior implies that a marginal change in real balances leaves utility unchanged.

The first order condition for holdings of nominal bonds is

$$\lambda_t = \beta \frac{P_t}{P_{t+1}} R_t \lambda_{t+1}. \tag{15}$$

The interpretation of (15) is similar to that of (4). However, because here the bonds pay off in dollars instead of goods, current real income is converted into future real income at rate $\frac{P_t}{P_{t+1}} R_t$. Finally, the transversality condition for money is[5]

$$\lim_{t \longrightarrow \infty} \beta^t \lambda_t m_t = 0. \tag{16}$$

This has a similar interpretation to the bond transversality condition in the first model.

The government budget constraint is

$$M_t/P_t + TR_t = M_{t-1}/P_t; \tag{17}$$

the left-hand side is the government's sources of revenue, and the right-hand side is the government's uses of revenue. Because we assume that any changes in the money supply are automatically accomplished by lump-sum transfers, the government budget constraint does not play any role in the determination of equilibrium.

In equilibrium the goods market clears, implying that private consumption equals the total endowment of goods:

$$c_t = y. \tag{18}$$

As before, constant consumption implies that the marginal utility of consumption is constant, and in this case, from (15), the equilibrium nominal interest rate is equal to expected inflation divided by the discount factor

$$R_t = \beta^{-1} \frac{P_{t+1}}{P_t} \tag{19}$$

(this is a version of the Fisher equation relating nominal and real interest rates; see Fisher [(1930) 1954]). Combining (15), (3), and (19), we see that there is a simple relationship between the nominal interest rate and the marginal

[5] There is also a transversality condition for bonds. However, since the quantity of bonds is zero, this condition is automatically satisfied.

utilities of consumption and real balances:

$$v'(m_t) = u'(c)\left(1 - \frac{1}{R_t}\right). \tag{20}$$

The marginal utility of consumption is known, so equation (20) then can be used to express m_t as a function of R_t; it is a money demand function. In turn, equation (19) determines R_t as a function of expected inflation. Without a specification of monetary policy, however, we cannot determine the price level or expected inflation. The standard research practice is to specify a policy rule for the quantity of money or the nominal interest rate. Given a rule that sets one of these variables as a function of other variables, one can determine whether equilibrium exists and is unique.

Note that if the rule makes the nominal interest rate depend only on P_t or P_{t+1}, substituting the policy rule into (19) yields one forward-looking difference equation in the price level. Henceforth, we will assume that the monetary policy rule sets the nominal interest rate as a function of the current inflation rate, $R_t = \mathfrak{R}(P_t/P_{t-1})$. In this case, the difference equation describes inflation, which we will denote by π (that is, $\pi_t = P_t/P_{t-1}$):

$$\beta^{-1}\pi_{t+1} = \mathfrak{R}(\pi_t). \tag{21}$$

Below we will assume a particular form for $\mathfrak{R}()$, and thus we will be able to determine whether there is a unique nonexplosive equilibrium.

The reader may be struck by the fact that the difference equation in (21) is independent of the preference specification in (12). It is a common feature of simple monetary models that one can derive a difference equation in either real balances or the price level (inflation is a transformation of the price level). However, in general, one must bring in information from the "other" part of the model in order to determine whether candidate paths are equilibria. Anticipating the discussion below, here the linearization ignores information from preferences, whereas the global analysis does not.

2. LINEARIZATION

The models we are working with contain just one dynamic variable and can be written in the form

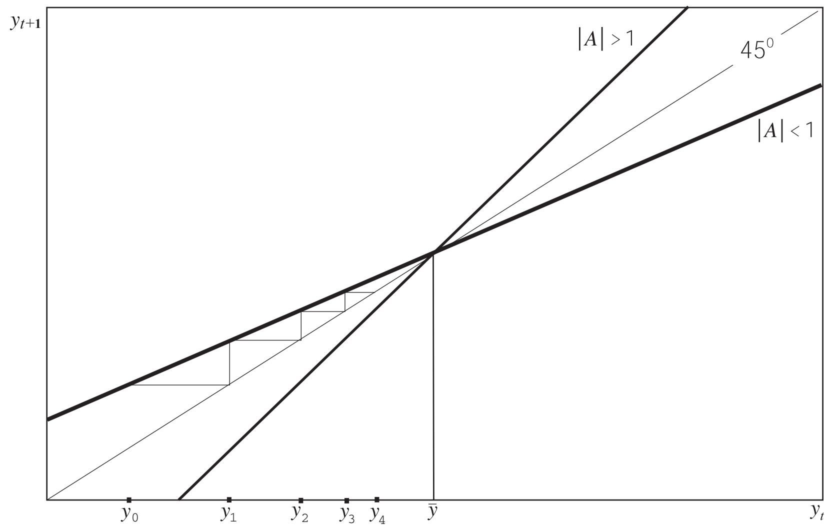
$$E_t y_{t+1} = f(y_t), \tag{22}$$

where y_t is the endogenous dynamic variable.

Linearization involves first computing a steady state of this equation and then taking a first order Taylor-series approximation around that steady state. A steady state of the difference equation system is a point $\bar{y}$ such that $\bar{y} = f(\bar{y})$, and the linear approximation around this steady state is

$$(E_t y_{t+1} - \bar{y}) = f'(\bar{y})(y_t - \bar{y}). \tag{23}$$

Figure 1 Dynamics of a Univariate Linear Difference Equation



This approximation is guaranteed to be valid only for small deviations from the steady state. The univariate linear difference equation system (23) can be written

$$E_t \tilde{y}_{t+1} = A\tilde{y}_t, \tag{24}$$

where $\tilde{y}_t \equiv y_t - \bar{y}$.

Once we have the linearized equation, we can ask how many nonexplosive solutions there are in the neighborhood of the steady state. In general, when y_t is predetermined, $|A|$ must be less than one for a unique nonexplosive solution to exist; when y_t is nonpredetermined, $|A|$ must be greater than one.

The logic behind these conditions is easy to see when A is positive.[6] Figure 1 plots two possible cases for this univariate linear difference equation: $A > 1$ and $0 < A < 1$. The interpretation of these two cases depends on whether the variable y_t is predetermined.[7]

[6] The case where $A < 0$ is similar. There, $-1 < A < 0$ produces dampened oscillations rather than monotone convergence; $A < -1$ produces explosive oscillations. The number of equilibria can then be determined in the same manner as in the case where $A > 0$.

[7] Using a plot of y_{t+1} versus y_t (such as Figure 1), it is simple to trace the time path for y_t starting from an initial point y_0. Start with y_0 on the horizontal axis, and draw a vertical line up to the function y_{t+1}. Then draw a horizontal line to the 45-degree line and a vertical line back to the horizontal axis. This is y_1. Repeat to get y_2, etc. If y_t is a predetermined variable, then the initial condition is known, and the process just described reveals the path of y_t (if there

First, suppose y_t is not predetermined, so that the initial condition y_0 is not known but instead needs to be determined in equilibrium. Then if $A > 1$, any initial value y_0 other than the steady state leads to y_t exploding either upward or downward: the steady state $\bar{y}$ is the unique nonexplosive solution. If $A < 1$, then y_t will converge back to the steady state regardless of the initial condition y_0: at any point in time there is a continuum of nonexplosive solutions, one of which is the steady state.

Now consider the case where y_t is predetermined, so that at any point in time y_t is known and y_{t+1} can be read off the graph. Then if $A > 1$, unless y_0 happens to be equal to the steady state value, y_t will explode over time: for most initial conditions, a nonexplosive solution does not exist. If $A < 1$, then y_t will converge back to the steady state regardless of the initial condition y_0: at any point in time there does exist a unique nonexplosive solution.

For models containing more than one variable, there are related conditions involving the eigenvalues of a matrix A (see Blanchard and Kahn [1980]). Sargent (1979, 177) describes the general principal as that of "solving stable roots backward and unstable roots forward."

Note that these conditions are concerned with the existence of nonexplosive solutions. It is common practice for researchers to restrict attention to nonexplosive solutions. Sometimes equilibrium must be nonexplosive because of a transversality condition. In other cases, nonexplosiveness is not a requirement of equilibrium, but researchers find other equilibria unappealing on a priori grounds. Our tax model falls into the former category: explosive behavior (at a rate greater than β^{-1}) cannot occur in equilibrium because of the transversality condition. In our monetary model, explosive behavior of the price level cannot be ruled out as an equilibrium per se, but we will nonetheless restrict attention to nonexplosive equilibria.[8]

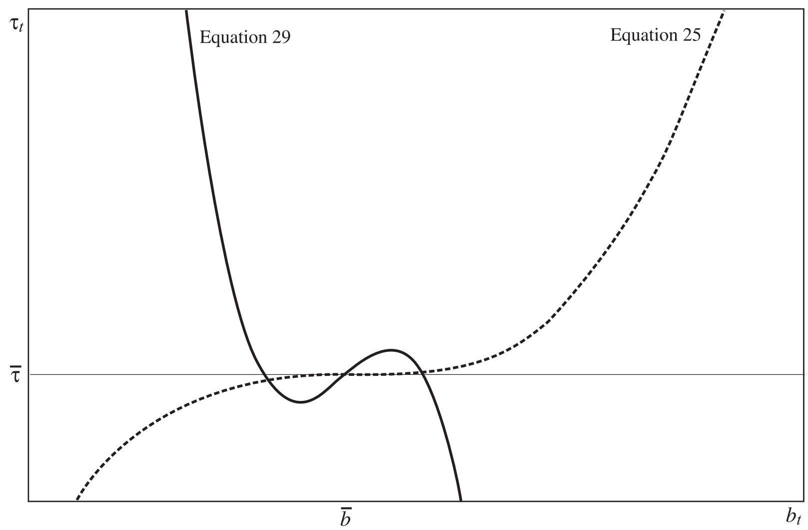
3. THREE PITFALLS OF EXCESSIVE RELIANCE ON LINEARIZATION

We have mentioned that linear approximations are less reliable far from the steady state. This fact typically motivates researchers who use linearization to study only examples in which there are small deviations around the steady state. However, linearization may even give incorrect answers near the steady state by suggesting an incorrect number of nonexplosive equilibria. This possibility exists because a linear approximation treats the local properties of the dynamic system as though they govern the model's global behavior, and

is an equilibrium path). If y_t is not predetermined, then the process reveals whether there is a unique initial condition for which y_t does not exhibit explosive behavior.

[8] As is clear from Obstfeld and Rogoff (1983), in monetary models, ruling out candidate equilibria based on simple explosiveness conditions is inappropriate. We use these conditions to make our points more clearly.

Figure 2 Two Policy Rules for Lump-Sum Taxes



the global behavior can be crucial in determining the number of nonexplosive equilibria. Locally the dynamics may imply that a variable explodes away from the steady state, whereas the global dynamics exhibit sufficient nonlinearity so that the explosiveness is shut off at some point. The opposite situation can also occur. Using the models described earlier, we illustrate three ways in which linear approximation can lead to an incorrect conclusion about the number of nonexplosive equilibria in a model.

Spurious Nonexistence

It is possible that linearization suggests that there is no nonexplosive solution when global analysis reveals that one in fact exists. In the tax model above, the following tax policy gives such a result:

$$\tau_t = h(b_t) = \bar{\tau} + \tau_1 (b_t - \bar{b})^3, \tag{25}$$

where

$$\bar{b} = \frac{1}{1 - \beta^{-1}} (g - \bar{\tau}). \tag{26}$$

This policy rule is plotted in Figure 2 as the dashed line; it raises the lump-sum tax when the level of debt is above $\bar{b}$ and lowers the lump-sum-tax when the level of debt is below $\bar{b}$. The responsiveness of taxes to debt is nonlinear, rising in magnitude the further the stock of debt is from $\bar{b}$. This behavior appears reasonable, in that it might be expected to bring the level of debt back toward a steady state from any initial condition.

Combined with the government budget constraint (10), the tax rule yields an equation describing the evolution of the stock of government debt:

$$b_{t+1} = g - \bar{\tau} - \tau_1 (b_t - \bar{b})^3 + \beta^{-1} b_t. \quad (27)$$

Linearizing (27) around $\bar{b}$, which is a steady state, we get

$$b_{t+1} - \bar{b} = \beta^{-1} \left(b_t - \bar{b} \right). \quad (28)$$

Notice that in the linearized form of the tax policy, taxes do not respond to debt: $\tau_t = \bar{\tau}$. Given this nonresponsiveness, it is not surprising that an application of the conditions discussed in Section 2 indicates that an equilibrium does not exist unless the initial debt stock is equal to $\bar{b}$. According to the linearized model, for any initial debt level other than $\bar{b}$, the quantity of debt will grow at rate $1/\beta$, violating the transversality condition.

The global analysis of (27) tells a very different story. A plot of b_{t+1} versus b_t is given in Figure 3a.[9] It turns out that there are three steady states: $\bar{b}_1$, $\bar{b}$, and $\bar{b}_2$. If the initial debt happens to be equal to one of those steady state values, there is a unique equilibrium with constant debt. If the initial debt is not equal to one of the steady state values, but it lies in one of the intervals $(\bar{b}_1, \bar{b})$ or $(\bar{b}, \bar{b}_2)$, then there is a unique equilibrium in which the debt converges to $\bar{b}_1$ or $\bar{b}_2$, respectively. The debt levels b_l and b_u correspond to a unique equilibrium in which the debt cycles between those two levels. If the initial debt is between b_l and the steady state $\bar{b}_1$ (or between $\bar{b}_2$ and b_u), then there is a unique equilibrium in which the debt converges to one of the steady states. Finally, if the initial debt is either below b_l or above b_u, then there is no equilibrium, because the debt path implied by (27) violates the transversality condition.

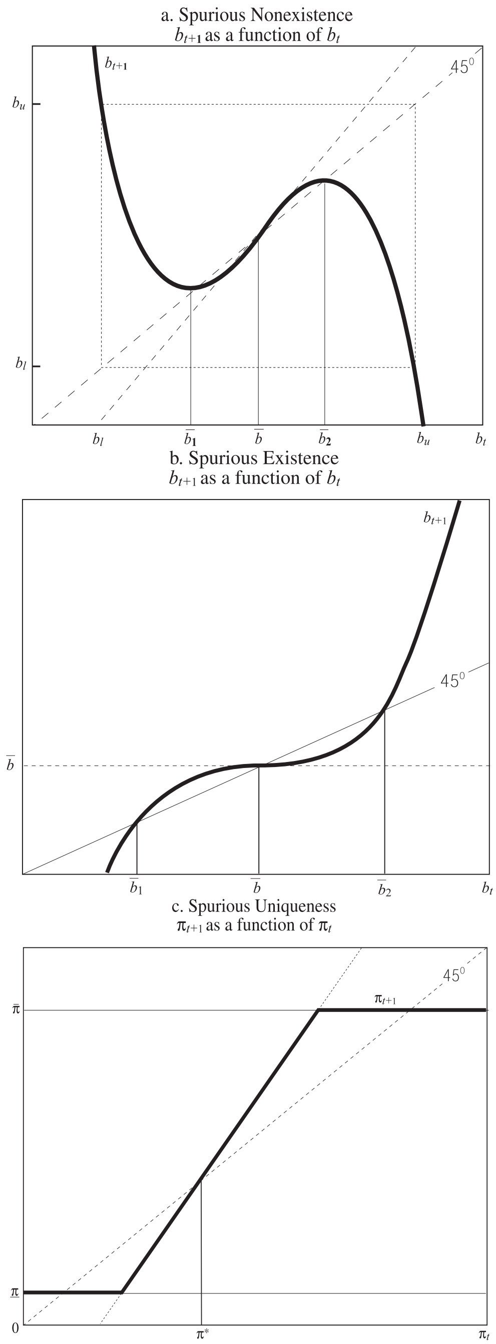
In this example, linearization leads us to conclude that an equilibrium does not exist when in fact our analysis of the global dynamics shows that there is an equilibrium for a wide range of initial conditions on the debt. Since the nonexistence of equilibrium suggests that there is a fundamental problem with a model, this possibility should lead to caution in interpreting linearization when it results in a finding of nonexistence.

Spurious Existence

A second possibility is that there is a unique equilibrium to the linearized model, but global analysis shows that there are no equilibria for a wide range of initial conditions. Returning to the tax model, consider a tax policy given

[9] In each panel of Figure 3, the dynamics of the linear approximation are indicated by a dashed line.

Figure 3 Three Nonlinear Difference Equations



by

$$\tau_t = h(b_t) = -(b_t - a)(b_t - \bar{b})(b_t - c) + b_t. \tag{29}$$

This function is plotted as the solid line in Figure 2, for carefully chosen values of the parameters a, $\bar{b}$, and c. For this rule, the tax rate rises with the debt stock near the level $\bar{b}$ but decreases with the debt stock far away from $\bar{b}$. Substituting (29) into the government budget constraint (10), the equation describing the evolution of government debt is

$$b_{t+1} = g + (b_t - a)(b_t - \bar{b})(b_t - c) - b_t + \beta^{-1} b_t. \tag{30}$$

Linearizing (30), we find

$$b_{t+1} - \bar{b} = \overbrace{((\bar{b} - a)(\bar{b} - c) - 1 + \beta^{-1})}^{\gamma}(b_t - \bar{b}), \tag{31}$$

and we choose the parameters a, $\bar{b}$, and c so that $(\bar{b} - a)(\bar{b} - c) = 1 - \beta^{-1}$, that is, $\gamma = 0$. For any starting value of b_t, the linearized version implies that government debt would converge immediately to the steady state $\bar{b}$. This is an example of the case where $|A| < 1$ and the one variable is predetermined. Therefore, there appears to be a unique nonexplosive solution to the linearized equations and thus a unique equilibrium.

The nonlinear difference equation (30) is graphed in Figure 3b. If the initial debt is between $\bar{b}_1$ and $\bar{b}_2$, then there is a unique equilibrium in which the debt converges to $\bar{b}$. However, if the initial debt is outside this range, no equilibrium exists; the debt path implied by (30) violates the transversality condition.

Here linearization leads us to conclude that there is a unique equilibrium, whereas global analysis reveals that existence depends on the initial debt stock. One could argue that if the initial debt stock is within a reasonable range, then there is a unique equilibrium and the linear dynamics give a good approximation to that equilibrium. However, one could choose the parameters of this example so that there is an arbitrarily small region in which the existence results from the linear analysis hold.

Spurious Uniqueness

Finally, we can imagine a situation in which linearization suggests that there is a unique nonexplosive equilibrium when in fact there are multiple nonexplosive equilibria. This is the case that has been highlighted in the recent work by Benhabib, Schmitt-Grohe, and Uribe.[10] Turning to the monetary model,

[10] The research of Christiano and Rostagno (2001) is a related work that also uses global analysis.

consider the following interest rate rule:

$$R_t = \frac{1}{\beta} + \gamma \left(\pi_t - 1\right), \tag{32}$$

with $\gamma > 1/\beta$. This rule represents a well-defined, feasible policy for setting the nominal interest rate, as long as the gross inflation rate is close to its *targeted* steady state value of 1. Furthermore, this type of rule has been studied in both empirical and theoretical contexts by authors such as Clarida, Gali, and Gertler (2000). It is known as an active Taylor rule, because it is a Taylor-style rule that raises the nominal interest rate more than one-for-one with the current inflation rate.

Combining the policy rule with the Fisher equation relating inflation to the nominal interest rate, we arrive at the following equation describing the evolution of inflation:

$$\pi_{t+1} = 1 + \beta\gamma \left(\pi_t - 1\right). \tag{33}$$

This difference equation has a unique steady state $\bar{\pi} = 1$ (the targeted steady state). Furthermore, the equation is already linear, so we need merely note that the coefficient on π_t is greater than one to see that any path for inflation other than the steady state will lead to inflation exploding upward (if $\pi_0 > 0$) or downward (if $\pi_0 < 0$). Thus, there appears to be a unique nonexplosive equilibrium.

The problem with the above reasoning is that along the explosive paths on either side of the steady state, the policy rule (33) eventually implies an infeasible choice of the nominal interest rate. First, consider an inflation path in which the initial inflation rate is positive ($\pi_0 > 1$). Along such a path, the inflation rate becomes arbitrarily high, and the path is hence ruled out as explosive. But if the inflation rate becomes arbitrarily high, at some point the gross nominal interest rate exceeds $R^* \equiv u'(y) / \left(u'(y) - \lim_{m\to 0} v'(m)\right)$. At a gross nominal interest rate of R^*, the model economy demonetizes; consumers will hold no money at interest rates equal to or greater than R^*, because the marginal benefit of real balances is bounded above by a number less than the marginal interest cost of holding real balances. The economy thus does not have a well-defined point-in-time equilibrium at a nominal interest rate above R^*. Similarly, if the initial inflation rate is negative ($\pi_0 < 1$), the dynamics in (33) indicate that the inflation rate will eventually become arbitrarily large with a negative sign. But then at some point the policy rule (32) requires that the gross nominal interest rate be less than unity. At a gross nominal interest rate equal to unity, consumers are satiated with real balances. The nominal interest rate cannot fall below unity because all agents would choose to hold negative quantities of nominal bonds (money would have a negative opportunity cost), and bonds are in zero net supply.

Because the interest rate rule given by (32) implies infeasible policy actions in certain situations, that rule is not a complete description of policy. A slightly modified rule that implies feasible policy actions in any situation is

$$R_t = \begin{cases} 1, \quad \text{if } \pi_t < 1 + \frac{1}{\gamma}\left(1 - \frac{1}{\beta}\right) \\ \frac{1}{\beta} + \gamma\left(\pi_t - 1\right), \quad \text{if } \frac{1}{\gamma}\left(1 - \frac{1}{\beta}\right) < \pi_t - 1 < \frac{1}{\gamma}\left(R^* - \frac{1}{\beta}\right) \\ R^*, \quad \text{if } \frac{1}{\gamma}\left(R^* - \frac{1}{\beta}\right) < \pi_t - 1. \end{cases}$$

The difference equation describing equilibrium then becomes

$$\pi_{t+1} = \begin{cases} \beta, \quad \text{if } \pi_t < 1 + \frac{1}{\gamma}\left(1 - \frac{1}{\beta}\right) \\ 1 + \beta\gamma\left(\pi_t - 1\right), \quad \text{if } \frac{1}{\gamma}\left(1 - \frac{1}{\beta}\right) < \pi_t - 1 < \frac{1}{\gamma}\left(R^* - \frac{1}{\beta}\right) \\ \beta R^*, \quad \text{if } \frac{1}{\gamma}\left(R^* - \frac{1}{\beta}\right) < \pi_t - 1. \end{cases}$$

This nonlinear difference equation is illustrated in Figure 3c. The consequences of modifying the policy rule so that it always delivers feasible policy actions are dramatic. In the linear difference equation (33), paths beginning from an initial inflation rate away from the steady state generated explosive behavior of inflation (see the dashed line in Figure 3c). By contrast, the modified policy rule implies that there are two steady states ($\underline{\pi} = \beta$ and $\bar{\pi} = \beta R^*$) in addition to the targeted steady state, and paths that begin away from the targeted steady state converge to one of these new steady states. Thus, instead of there being a unique nonexplosive equilibrium, there is a continuum, indexed by the initial inflation rate.

4. DISCUSSION

We have shown how approximating economic models by linearization around a steady state may lead to incorrect conclusions about the existence or uniqueness of equilibrium. In each of our examples, the misleading results implied by linearization were associated with a particular government policy rule. However, there is no reason to believe that it is only government policies that can lead to these problems with linear approximations. One should not assume that because a particular model has no role for government policy, a linear approximation will necessarily give the right answers about the existence and uniqueness of equilibrium.

In the simple models studied here, it was easy to see—and hence avoid—the problems associated with linearization. Unfortunately, with larger models it is harder to see the red flags signaling that linearization may be giving incorrect answers. Furthermore, global analysis (i.e., analysis of the model without any approximations) is infeasible with larger models.[11] There are,

[11] We should note that Benhabib, Schmitt-Grohé, and Uribe (2001) conduct global analysis of a two-variable system in continuous time. Kuznetsov (1998) discusses global analysis of a two-variable system in discrete time.

however, steps one can take to minimize the risk of falling victim to the problems described above. Before linearizing it is important to determine the number of steady states. If there is more than one steady state, it may not be advisable to work with a linear approximation unless one has a strong reason for believing that only one of the steady states is relevant. If there is a unique steady state, then in some models a check on the results of linear approximation can be provided by analyzing a simplified version of the model in which it is feasible to compare the local linear and global dynamics. In the case of a unique steady state, a promising approach currently receiving much attention involves taking a local *higher order* approximation to the model's system of difference equations.[12]

REFERENCES

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martin Uribe. 2001. "The Perils of Taylor Rules." *Journal of Economic Theory* 96 (January/ February): 40–69.

Blanchard, Olivier J., and Charles M. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica* 48 (July): 1305–12.

Brock, William A. 1975. "A Simple Perfect Foresight Monetary Model." *Journal of Monetary Economics* 1 (April): 133–50.

Christiano, Lawrence J., and Massimo Rostagno. 2001. "Money Growth Monitoring and the Taylor Rule," Working Paper 8539. Cambridge, Mass.: National Bureau of Economic Research. (October).

Clarida, Richard, Jordi Gali, and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics* 115 (February): 147–80.

Fisher, Irving. 1954 [1930]. *The Theory of Interest.* New York: Kelley and Millman, Inc.

Kuznetsov, Yuri A. 1998. *Elements of Applied Bifurcation Theory*. New York: Springer Verlag.

Obstfeld, Maurice, and Kenneth Rogoff. 1983. "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy* 91 (August): 675–87.

[12] See Sims (2000) and Schmitt-Grohé and Uribe (2002).

Sargent, Thomas J. 1979. *Macroeconomic Theory.* New York: Academic Press.

______________, and Neil Wallace. 1975. " 'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule." *The Journal of Political Economy* 83 (April): 241–54.

Schmitt-Grohé, Stephanie, and Martin Uribe. 2002. "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function." http://papers.ssrn.com/sol3/papers.cfm?abstract_id=277709 (January).

Sims, Christopher. 2000. "Second Order Accurate Solution of Discrete Time Dynamic Equilibrium Models." http://eco-072399b.princeton.edu/yftp/gensys2/Algorithm.pdf (December).

Varian, Hal R. 1992. *Microeconomic Analysis.* New York: W.W. Norton and Co.

The Cyclical Behavior of Prices and Employee Compensation

Roy H. Webb

Are prices procyclical? For many economists, they clearly are. As Lucas (1976, 104) put it, "The fact that nominal prices and wages tend to rise more rapidly at the peak of the business cycle than they do in the trough has been well recognized from the time when the cycle was first perceived as a distinct phenomenon." More recently, however, other researchers have challenged the prevailing view. According to Kydland and Prescott (1990, 17), "[T]he U.S. price level has clearly been countercyclical in the post–Korean War period."

The issue is of particular importance to macroeconomists who must choose a model to work with. A monetary sector was an integral part of equilibrium dynamic macro models that gained popularity in the 1970s, such as Lucas (1972). Monetary misperceptions could then give rise to procyclical movements in prices. In contrast, the real business cycle models that later gained popularity, such as Prescott (1986), did not have that property. If the behavior of prices over the business cycle were a clearly established empirical regularity, that information would help choose the type of model to use for economic analysis.

This paper attempts to better understand how respected economists can hold such seemingly divergent views of the same data. By closely examining the data on aggregate price measures, I will try to clarify why each view could be correct under specific definitions of important terms. In doing so, I will propose a way of viewing the data that may be useful in other circumstances.

The author gratefully acknowledges helpful comments from Marvin Goodfriend, Robert Hetzel, and Raymond Owens. Valuable research assistance was provided by Elliot Martin. The views and opinions expressed in this article are solely those of the author and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

In particular, the methodology that is employed to assess price cyclicality can be easily used to study other variables of interest. The cyclical behavior of wages has been a subject of controversy for over a half century and is examined in the final section of the paper.

1. PRICES AND THE BUSINESS CYCLE

Much of our understanding of the complex phenomena that are unified under the idea of the business cycle was developed by researchers associated with the National Bureau of Economic Research (NBER) in the first half of the twentieth century. Their initial approach was to describe the cycle, either verbally or with voluminous statistics. Their conception of a typical business cycle is now part of our common language, and many statistical regularities that are usually thought to characterize cycles were first noted in their early publications. An important early example of this line of research is Mitchell (1913). Although his observations were based on an American economy much different from our own, much of his account of the behavior of economic aggregates anticipated later developments in economic activity. Prices played a key role in his view of the cycle, as the following passages attest:

> A revival of activity, then, starts with this legacy from depression: a level of prices low in comparison with the prices of prosperity, [and]. . . drastic reductions in the cost of doing business (150). While the price level is often sagging slowly when a revival begins, the cumulative expansion in the physical volume of trade presently stops the fall and starts a rise (151). Like the increase in the physical volume of business, the rise in prices spreads rapidly; for every advance of quotations puts pressure upon someone to recoup himself by making a compensatory advance in the prices of what he has to sell. . . . Retail prices lag behind wholesale. . . and the prices of finished products [lag] behind the prices of their raw materials (152). [O]ptimism and rising prices both support each other and stimulate the growth of trade (153). Among the threatening stresses that gradually accumulate within the system of business during seasons of high prosperity is the slow but sure increase in the costs of doing business (29). The price of labor rises. . . . The prices of raw materials continue to rise faster on the average than the selling prices of products (154). [T]he advance of selling prices cannot be continued indefinitely. . . [because] the advance in the price level would ultimately be checked by the inadequacy of the quantity of money (54). [Once a downturn begins] with the contraction in trade goes a fall in prices (160). [T]he trend of fluctuations [in prices] continues downward for a considerable period. . . . [T]he lowest level of commodity prices is reached, not during the crisis, but toward the close of the subsequent depression, or even early in the final revival of business activity. The chief cause of this fall is the shrinkage in the demand for consumers' goods, raw materials, producers' supplies, and construction

> work (134). [E]very reduction in price facilitates, if it does not force, reductions in other prices (160). Once these various forces have set trade to expanding again, the increase proves cumulative, though for a time the pace of growth is kept slow by the continued sagging of prices (162).

Note that this account was based on economic activity under the gold standard at a time when no trend would be expected in the price level. Evidence during that time generally supported the behavior Mitchell described. Zarnowitz (1992, ch. 4), for example, found strong evidence of procyclical prices in the first 150 years of U.S. history. In contrast, under our current fiat money system, the CPI has risen in each of the past 47 years, with an average annual increase of 4.1 percent. This change in monetary regime leads to an immediate modification of Mitchell's analysis that preserves its spirit while conforming to recent evidence. Inflation can be substituted for the level of prices in the writing above, and the logic is preserved; a recession[1] is thus associated with falling inflation and consequently the inflation rate is relatively low at the beginning of a cyclical expansion. Then, as the expansion progresses, the rate of inflation rises, led by relatively large increases in commodity prices. The evidence presented below is consistent with that analysis.

The controversy, though, concerns the cyclical behavior of the price *level* in the last half century. In order to understand the challenge to the conventional wisdom that the price level is procyclical, we need to investigate the exact meaning of cyclical price movements when prices are continually rising. The following section thus examines filtering, that is, removing some measure of a long-run trend from a series in order to study shorter-run movements.

2. FILTERING ECONOMIC TIME SERIES

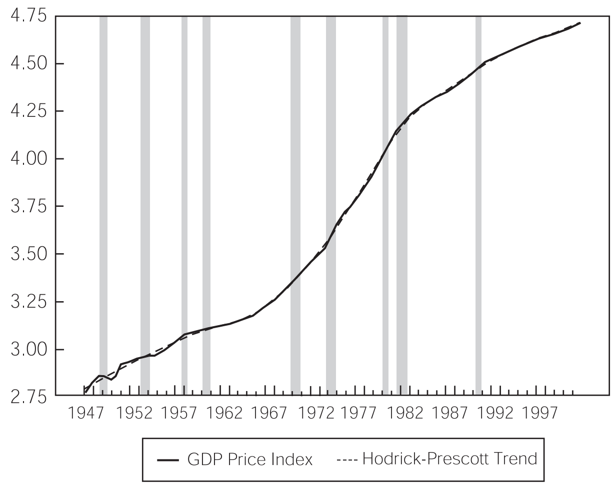
Consider a series of data generated as

$$X_t = (1 + g)X_{t-1}(1 + \varepsilon_t), \tag{1}$$

where X is a data series, the subscript t indexes time, g is a fixed positive number, and ε is a random variable with zero mean. The series would grow, on average, at rate g, and a graph of X versus time would eventually appear nearly vertical. A common first step in studying the series would be to take logarithms, which would change the time-series plot to a series fluctuating around a straight line with slope $1 + g$. In this case, an obvious filter for removing the long-run trend would be to divide each observation X_t by $(1+g)^t$. In the typical case where g is not known, one can estimate the coefficients in

[1] Mitchell used the word *depression* where we would use *recession* today.

Figure 1 GDP Price Index and Trend



the following regression

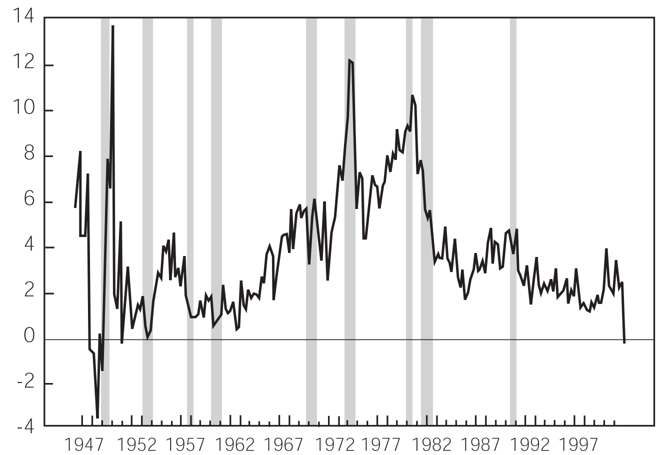
$$\ln X_t = \alpha + \beta T_t + \upsilon_t, \tag{2}$$

where T is a trend variable, taking a value of 1 in the first period, 2 in the second, and so forth; $\hat{\beta}$ is the estimated growth rate of the series; and υ is assumed to be white noise. In this case, the antilog of the estimated residual, e^{υ_t}, would be the detrended value of the observation X_t. This method is widely referred to as linear detrending. In some cases, a linear trend can fit the data well over a lengthy interval; for example, in Webb (1993) it is shown that real per capita GDP in the United States has fluctuated around a stable linear trend for over 100 years.

This method of detrending is not always appropriate. Suppose that g varied substantially over time in equation (2). Then imposing a linear trend could lead to long swings above or below trend, and the detrended data would be difficult to analyze. Price data, in particular, are not always and everywhere consistent with a fixed, linear trend; monetary regimes have varied, and within regimes the monetary authority may not have had a constant inflation target. Thus several methods of estimating a flexible, or time-varying, trend have been proposed that could be applied to prices. A conceptually simple method is to estimate the trend by a centered moving average. Thus letting the trend value of X be denoted X^*, then

$$X_t^* = \frac{1}{2k+1} \sum_{t=-k}^{k} X_{t-k}, \tag{3}$$

Figure 2 Annualized Percentage Change in GDP Price Index



and the detrended value can be either the difference between actual and trend, or the ratio of actual to trend.

Many macroeconomists use a flexible trend that is produced by a method known as the Hodrick-Prescott (HP) filter (1980). They calculate the trend terms x_t^* to minimize

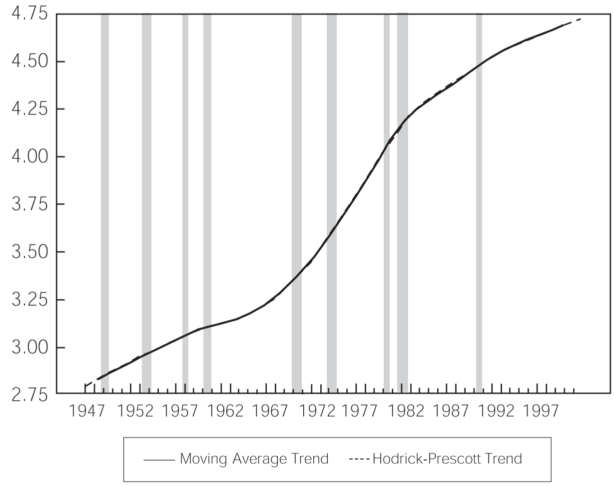
$$\sum_{t=1}^{N}(x_t - x_t^*)^2 + \lambda \sum_{t=2}^{N-1}[(x_{t+1}^* - x_t^*) - (x_t^* - x_{t-1}^*)]^2, \quad (4)$$

where the small x and x^* terms are logarithms of their counterparts using capital letters, N is the number of observations, and λ is a fixed number. For analyzing quarterly macroeconomic data, Hodrick and Prescott recommend a value of 1600 for λ, which will be used below. Intuitively, minimizing the expression (4) trades off deviations from trend, given by the first term, against changes in the trend value, given by the second term.

A final method of removing the trend is to simply take a difference in logs or, similarly, look at percentage changes in a variable. A disadvantage of this method is that the changes over a short period can be dominated by erratic factors.

These methods of removing the trend can be seen in Figures 1 through 4. In Figure 1 the logarithm of the GDP price index is first graphed, with shaded areas denoting cyclical recessions as defined by the NBER. Also included is the trend, estimated with the HP filter. In Figure 2, the quarterly percentage change is graphed, which effectively removes the trend. In Figure 3, the trend of the price index is estimated by the HP filter and a nineteen-quarter

Figure 3 GDP Price Index Trends



moving average filter. Both trends appear similar, and indeed, the correlation coefficient between the two is 0.999. Finally, the detrended values are plotted in Figure 4, and again both methods give somewhat similar estimates; in this case, the correlation coefficient is 0.95. Thus, when thinking about the meaning of filtered data, the intuitive moving average filter can be substituted for the less intuitive HP filter, if desired. All three methods indicate that inflation has been highly variable in the post–World War II period, and thus some form of a flexible trend is necessary in order to study price data.

3. THE ASSERTION OF COUNTERCYCLICAL PRICES

Kydland and Prescott (1990) studied the cyclicality of prices by examining the correlation of real GNP with the CPI and of real GNP with the GNP implicit price deflator. They found a sizable negative correlation between GNP and each price index and interpreted that negative correlation as demonstrating that the price level is countercyclical. In their words,

> This myth [that the price level is procyclical] originated from the fact that, during the period between the world wars, the price level was procyclical. But. . . no one bothered to ascertain the cyclical behavior of the price level since World War II. Instead, economists just carried on, trying to develop business cycle theories in which the price level plays a central role and behaves procyclically. The fact is, however, that whether measured by

Table 1 Series with the Segmented Cyclical Trend Removed

Series	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5
Real GDP	−1.0	0.6	1.1	−1.2	−3.3
GDP Price Index	0.4	−0.4	0	0.4	0.9
Personal Consumption Expenditure Price Index	0.5	−0.4	−0.1	0.6	1.0
Consumer Price Index	0.7	−0.6	−0.1	0.9	1.3
Producer Price Index	1.1	−0.9	0	1.1	1.4
Journal of Commerce Index	−1.6	0.8	1.3	0.1	−5.0
Average Hourly Compensation	0.9	−0.8	0.2	0.8	0.8
Real Average Hourly Compensation	0.3	−0.3	0.3	0.2	−0.2

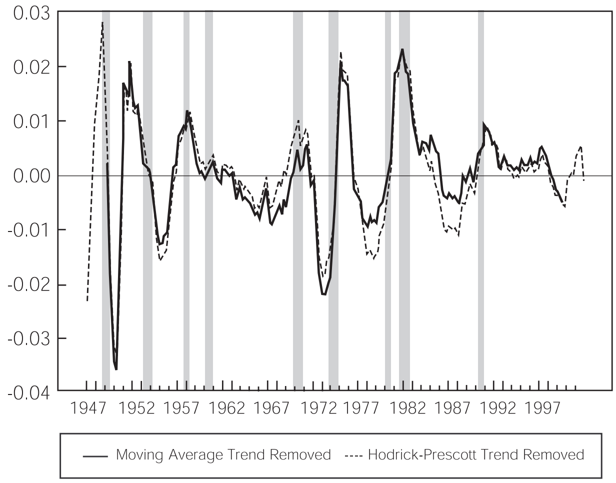
> the implicit GNP deflator or by the consumer price index, the U.S. price level clearly has been countercyclical in the post–Korean War period (17).

Cooley and Ohanian (1991) provided even more evidence of a negative correlation. They examined data over a longer time span and used a variety of methods to remove the trend in prices. An important part of their analysis was to apply the same filter to both prices and output data and then to examine the correlations. For 1948 Q2 to 1987 Q2, using a simple linear trend resulted in a correlation of –0.67; using log-differenced data resulted in a correlation of –0.06; and using HP-filtered data resulted in a correlation of –0.57. They interpreted these results as contradicting the view that prices are procyclical.

A common feature of these articles is that they discussed the cyclicality of prices by either redefining or ignoring the traditional business cycle. The traditional definition of business cycles was given by NBER researchers Burns and Mitchell (1946, 3):

> Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are

Figure 4 Detrended GDP Price Index



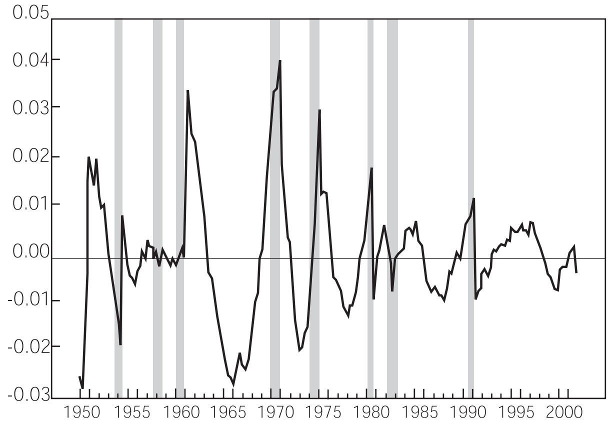
not divisible into shorter cycles of similar character with amplitudes approximating their own.

There are many valid reasons to study detrended macroeconomic variables, but they do not necessarily reveal much about business cycles as defined by the NBER. For example, by definition a detrended series will be symmetric, with positive observations balanced by negative observations. However, the business cycle has been notably asymmetric in the post–World War II United States. Most obviously, expansions last much longer than recessions. The length of the average recession has averaged 10.5 months, whereas expansions have averaged over five times as long, 56.9 months. In fact, the one expansion from 1991 to 2001 lasted 120 months, while all ten recessions from 1948 to date have totaled 105 months.[2]

Another property of focusing on detrended data is that results may be crucially dependent on the particular method used to detrend the data. As Canova (1998, 475) puts it, based on a study of data on real economic activity, "[Stylized facts] of U.S. business cycles vary widely across detrending methods, and . . . alternative detrending filters extract different types of information from the data." This effect can be seen in Figures 2 and 4 for the GDP price

[2] For this calculation it is assumed that the recession that began in March 2001 ended in December 2001.

Figure 5 GDP Price Index, Segmented Trend Removed



index. In Figure 2, differencing the data produces a series that tends to rise in cyclical expansions and fall in recessions. Conversely, applying the HP filter or a moving average filter to the same data series, as shown in Figure 4, yields a series that tends to fall in cyclical expansions and rise in recessions. Thus, whenever an assertion is based on detrended data, one should ask if the assertion is sensitive to the detrending method. Notice that the detrended prices in Figure 4 tend to be negative in the middle of cyclical expansions. That could be due to falling prices, but it could also be due to the rising trend, as both methods illustrated tend to have increasing trends in cyclical expansions. The next section thus takes a different approach to the question of price cyclicality.

4. NEW EVIDENCE ON PRICE CYCLICALITY

Assertions of procyclical prices have relied on purely statistical methods that ignored the traditional business cycle. Does that make a difference? This section looks at evidence based on a statistical method that is based on traditional business cycle dates. The method will be to take a simple trend, as shown in equation (2), that is defined only for a specific business cycle. In this paper, business cycles will be defined from trough to trough, where the date of the trough has been determined by the NBER. For the recession that began in March 2001, the NBER has not yet determined the date of the trough; in this paper, December 2001 will be used in place of an official date of the recession's trough. This method will be referred to below as the segmented cyclical trend, or SCT, method. It is illustrated in Figure 5. Most data series extend back to 1947, which allows nine complete busines cycles to be examined.

In order to assess the cyclicality of price movements, it is useful to divide each cycle into five separate phases to allow distinctive behavior to be observed. All calendar quarters will be classified as being in an expansion or a recession. An expansion begins in the quarter after the one that contains a trough, ends in the quarter containing the peak, and is divided into three phases.[3] The first phase, referred to here as early expansion, contains the first fourth of the number of quarters in the cyclical expansion. The second phase, or middle expansion, covers the next half of the number of expansion quarters. The final phase comprises the remaining one-fourth of the number of expansion periods. Recessions begin in the quarter following the peak and end in the quarter containing the trough. Since recessions are on average much shorter than expansions, they can be divided into a first half and a second half.[4] In the author's experience, this has been a useful classification for post–World War II business cycles, but many others can be imagined. In particular, Burns and Mitchell (1946) divided business cycles into nine phases for their analysis.

This cyclical classification is applied in Table 1. Several measures of prices are examined, including the price index for GDP and the price index for personal consumption expenditure from the Bureau of Economic Analysis; the consumer price index and the producer price index for finished goods from the Bureau of Labor Statistics; and the Journal of Commerce Index of commodity prices. The final two lines are discussed in the section below. All data series are seasonally adjusted. Each entry in the table is an average over a cyclical phase for the nine business cycles of items with the segmented linear trend removed.

The first series in the table is real GDP, which is often taken as the prototypical cyclical variable. Its high point is reached in Phase 3, which contains the cyclical peak. Similarly, its low point is reached in Phase 5, which contains the cyclical trough. Thus real GDP behaves as would be expected and is a useful benchmark for the series of prices.

The next four series are broad measures of prices of finished goods. Their behavior is quite different from real GDP. The GDP price index is typical, with its low point in Phase 2 and its high point in Phase 5. This different behavior of output and prices would seem to be consistent with the finding of countercyclical prices. This behavior can also be examined with other methods of detrending. Table 2 presents series of percentage changes, and Table 3 presents data detrended with the HP filter.

---

[3] This classification was motivated by the casual observation that growth was often very rapid near the beginning of expansions and was often subpar near the end of expansions.

[4] What if the length of expansion is not evenly divisible by four? For purposes of this section, if there is a nonzero remainder after dividing the number of quarters in an expansion by four, then the number of quarters in the remainder is added to the middle expansion phase. Similarly, if the number of quarters in a recession is odd, that first phase will be one quarter longer than the second.

Table 2 Series Expressed as Percentage Changes

Series	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5
Real GDP	2.6	0.8	−0.4	−6.2	−3.9
GDP Price Index	−0.5	−0.2	0.6	0.7	0.2
Personal Consumption Expenditure Price Index	−0.7	−0.3	0.9	1.0	0
Consumer Price Index	−1.0	−0.4	1.5	1.2	−0.4
Producer Price Index	−1.4	−0.4	1.9	1.5	−0.5
Journal of Commerce Index	4.1	−0.2	2.8	−9.0	−8.6
Average Hourly Compensation	−0.4	−0.3	1.1	0.2	−0.6
Real Average Hourly Compensation	0.3	0	0.2	−0.8	−0.6

The entries in Table 2 illustrate the importance of the detrending method. For real GDP, the highest value now occurs in Phase 1, rather than in Phase 3. This means that the real *growth* rate tends to be highest in the early phase of an expansion, even though from Table 1 we know that the *level* of GDP tends to be highest above trend in the late expansion phase. With prices, it is harder to discuss the detrended level of each series intuitively. Note in Figure 1 how the price level has risen consistently over the past half century. Any cyclical tendencies are small relative to the dramatic increase over time. Moreover, the rate of increase is significantly more rapid from the mid-1960s to the early 1980s than at other periods. For many purposes these broad trends may be more important than the cyclical movements. That said, in Table 2, the movements in prices over the business cycle are somewhat different than real GDP, which is again consistent with the assertion of countercyclical prices. Note that in this table inflation is highest when real growth is lowest, in Phase 4. Similarly, real growth is highest when inflation is lowest, in Phase 1. But also note that the entries for finished goods prices tend to increase during expansions, hit their highs in the early recession phase, and decline in the late recession phase, hitting their low point in the early expansion. Thus this general conformity with the business cycle could be viewed as a procyclical movement, but with a one-phase lag.

Finally, HP-filtered data are presented in Table 3. These data resemble those in Table 1. Real GDP is highest in the late expansion phase and lowest at the beginnning of expansions. Prices of final goods are below trend when GDP is above trend, and vice versa.

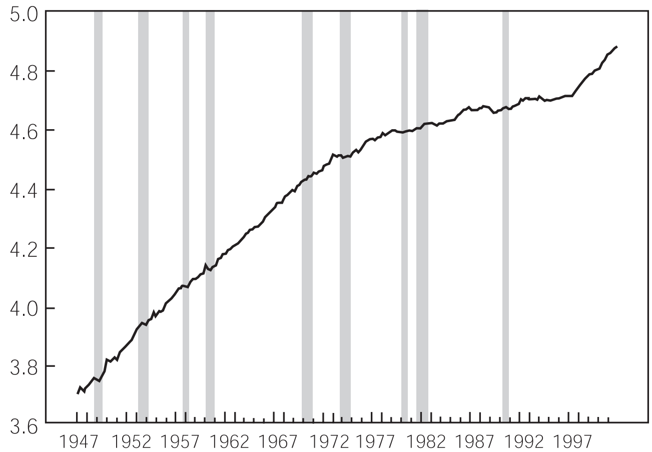
Table 3 Series with the HP Trend Removed

Series	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5
Real GDP	−1.3	0.5	1.5	−0.1	−0.3
GDP Price Index	0.2	−0.3	−0.1	0.6	0.8
Personal Consumption Expenditure Price Index	0.2	−0.4	−0.1	0.9	1.1
Consumer Price Index	0.2	−0.5	−0.1	1.3	1.5
Producer Price Index	0.3	−0.7	0	1.6	1.7
Journal of Commerce Index	−2.5	0.1	3.2	1.4	−4.9
Average Hourly Compensation	0.3	−0.4	0	0.8	0.7
Real Average Hourly Compensation	0.1	0	0.1	−0.1	−0.3

So far, then, the evidence seems, on balance, to support the assertion of countercyclical prices. Another interpretation is also possible. Until now the language of leading or lagging indicators has not been used, although it has a long tradition in discussions of cyclical behavior. Looking at the price indexes for finished goods in Table 1, one can see that these series reach their peak two phases after real GDP reaches its peak. This could be due to price stickiness, which is an integral feature of many macroeconomic models, for example, Goodfriend and King (1997). Thus, if changes in aggregate demand affect output before affecting prices of finished goods, that relationship could make a price index a lagging indicator as in Table 1.

Further evidence can be found by looking at commodity prices. Since commodity prices are often determined in spot markets, they can be immediately affected by supply or demand shifts. In contrast, finished goods prices are often set by explicit or implicit contracts and thus do not immediately display the total impact of supply or demand changes. That consideration suggests that commodity prices should be more of a coincident indicator. And in Tables 1 and 3, notice that the Journal of Commerce Index of commodity prices, like real GDP, hits its peak in Phase 3 and hits its low point in Phase 5. This behavior supports the view that commodity prices are a coincident indicator while finished goods prices are a lagging indicator. Thus, these data are consistent with many models that incorporate fluctuations of aggregate demand.

Figure 6 Real Average Hourly Compensation



5. EVIDENCE ON EMPLOYEE COMPENSATION

The cyclical behavior of wages has a long history of controversy, which began when Keynes (1936) asserted that real wages were countercyclical. Many articles have been written on the subject, and it is possible to find respected authors arguing for a procyclical pattern of real wages, a countercyclical pattern, or no meaningful pattern. For example, see Abraham and Haltiwanger (1995) for selected quotes and a discussion of recent evidence.

Unfortunately, consistent series on wages are not as plentiful as series on prices. This paper examines one particular series, employee compensation, which is available in quarterly form beginning in 1947. It includes wages, salaries, and fringe benefits. The nominal series is deflated with the PCE price index to obtain the real series and is graphed in Figure 6. Here the fluctuations around a trend are quite small, especially before 1973. For many analysts, the main issue is the significant growth in real wages over a half century, with a noticeable slowing between the early 1970s and the mid-1990s.

Both detrended nominal and real compensation are included in the tables. In Tables 1 and 3, the nominal series behaves somewhat like detrended prices of final goods. Both prices and compensation have low points in Phase 2 of the business cycle. Compensation is notably above trend in Phases 4, 5, and 1. In Table 2, the average growth rate of nominal wages is procyclical, hitting its high point in Phase 3 and its low point in Phase 5. Since nominal wage stickiness is often taken as a stylized fact, it may not be surprising that the nominal wage level behaves as a lagging indicator, too, as indicated in Tables 1 and 3.

The controversy has dealt with real wages, however, and the evidence is mixed. The growth rate of real wages seems procyclical in Table 2. That growth rate rises in expansions and declines in recessions. There is also evidence of procyclical real wage behavior in Table 3. But in Table 1 the real wage is above trend in Phases 1, 3, and 4, but below trend in Phases 2 and 5. Here again the choice of filter is important. This illustrates the limits of letting the data speak for themselves; in this case, some theory is needed just to choose a filter to remove the long-run growth trend of real compensation. And it is not surprising that authors have differed on the cyclicality of real wages. None of the evidence, though, supports Keynes's assertion of countercyclical real wages.

6. CONCLUSION

Data averaged over phases of post–World War II business cycles were examined for evidence of price cyclicality. The behavior of the level of final goods prices is consistent with the view that prices are countercyclical. Another interpretation, however, is that final goods prices are a lagging indicator, possibly due to price stickiness. Evidence of a procyclical level of commodity prices supports the latter interpretation. Phase-averaged data are also examined for employee compensation. Nominal compensation behaves much like finished goods prices, which would not surprise an analyst who believed that both wages and final goods prices are sticky. Real wage behavior is more difficult to characterize; however, it is difficult to reconcile the evidence presented with Keynes's original assertion of countercyclical wages.

REFERENCES

Abraham, Katharine G., and John C. Haltiwanger. 1995. "Real Wages and the Business Cycle." *Journal of Economic Literature* 33: 1215–64.

Burns, Arthur F., and Wesley C. Mitchell. 1946. *Measuring Business Cycles*. New York: National Bureau of Economic Research.

Canova, Fabio. 1998. "Detrending and Business Cycle Facts." *Journal of Monetary Economics* 41: 475–512.

Cooley, Thomas F., and Lee E. Ohanian. 1991. "The Cyclical Behavior of Prices." *Journal of Monetary Economics* 28: 25–60.

Goodfriend, Marvin, and Robert King. 1997. "The New Neoclassical Synthesis and the Role of Monetary Policy." *NBER Macroeconomics*

Annual: 231–95.

Keynes, John Maynard. [1936] 1964. *The General Theory of Employment, Interest and Money*. New York: Harcourt, Brace & Co.

Kydland, Finn E., and Edward C. Prescott. 1990. "Business Cycles: Real Facts and a Monetary Myth." Federal Reserve Bank of Minneapolis *Quarterly Review* (Spring): 3–18.

Lucas, Robert E., Jr. 1972. "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4 (April): 103–24.

______________. 1976. "Econometric Policy Evaluation: A Critique." *Journal of Monetary Economics* 2, Supplement, *Carnegie-Rochester Conference Series* 1.

Mitchell, Wesley Clair. [1913] 1941. *Business Cycles and Their Causes*. Berkley: University of California Press.

Prescott, Edward C. 1986. "Theory Ahead of Business Cycle Measurement." Federal Reserve Bank of Minneapolis *Quarterly Review* (Fall): 9–22.

Webb, Roy H. 1993. "Personal Saving Behavior and Real Economic Activity." Federal Reserve Bank of Richmond *Econoimic Quarterly* (Spring): 68–94.

Zarnowitz, Victor. 1992. *Business Cycles: Theory, History, Indicators, and Forecasting*. Chicago: University of Chicago Press.