The postwar U.S. labor market has been characterized by steadily rising employment rates. An examination of the data suggests that two factors, namely demographic trends and increased female participation, have been largely responsible for this phenomenon; we focus on the latter factor. Within a dynamic stochastic general equilibrium model, we study the implications of a gradual reduction in both the implicit household cost of female employment and employer-based discrimination. For both experiments, the model predicts an endogenous decline in male employment rates and hours worked. We also find that the presence of employer-based discrimination gives rise to a countercyclical wage gap when the model is driven mainly by shocks to total factor productivity. These results are all consistent with U.S. labor market data.

The “cost-push” view of the inflation process implies that higher wage growth leads to higher future inflation, notwithstanding the nature of Federal Reserve policy and the inflation regime. I test this implication using wage-price data over the sample period 1952Q1–1999Q2, during which both inflation and Fed policy vary considerably. Although wage growth does help predict inflation over the full sample period, this result is due to the high inflation subperiod 1966–1983. Wage growth does not help predict inflation during low inflation subperiods 1952–1965 and 1984–1999, suggesting wage growth has not been an independent source of inflation in the United States.

Macroeconomists are increasingly using a New IS-LM model to discuss the economy’s response to shocks and the design of monetary policy rules. This new model has better microfoundations than earlier IS-LM models and explicitly incorporates expectations about future economic conditions. Price level, or inflation rate, targeting is desirable in the New IS-LM model, for it stabilizes output at capacity. Such neutral policies require adjustments to the nominal interest rate as changes occur in the real economy.
A Study of U.S. Employment Rates with Emphasis on Gender Considerations

Carl D. Lantz and Pierre-Daniel G. Sarte

Policymakers who are concerned with understanding the cyclical state of the economy will be interested in identifying the sustainable rate of employment growth. If productivity growth is taken to be exogenous, the rate at which jobs are created will determine growth in output. Assuming that employment is measured reasonably well, a policymaker armed with a measure of trend job growth will be able to glean information about the economy’s status from the monthly employment report.

When attempting to determine this trend rate, one might posit as a first approximation that employment growth should be equal to the growth rate of the population. At the very least, this assumes that the fraction of the population employed will remain constant over time. However, a cursory look at U.S. data reveals that this fraction has been rising persistently over the postwar period. A robust determination of the sustainable rate of job creation should therefore take account of this trend. Upon further inspection of the available data, it becomes apparent that the driving forces behind this process have been demographic trends, i.e., the “baby boom,” and, to a larger extent, the increased participation of women in the labor market. In this article we focus our attention on the latter determinant.

Female employment, expressed as a fraction of the working age population, has been increasing steadily over the past 50 years. While most work in this area has focused on the socioeconomic factors underlying this phe-
nomenon, we explore the phenomenon’s effects on the larger economy within the framework of a stochastic dynamic general equilibrium model. Specifically, we use a model that allows households to make labor supply decisions along both the extensive margin—the decision to work outside the home or not—and the intensive margin, along which workers adjust the number of hours supplied to the outside market.

In our first experiment, we model a reduction in participation costs for female workers, that is, the implicit utility cost to the representative household incurred when a female member devotes time to outside work. Several factors may have contributed to such a phenomenon. For instance, advances in household technology such as those embedded in modern appliances have automated many of the chores previously accomplished entirely through physical labor. A general decline in birthrates over much of the period in question may have reduced the cost of childcare for the average working family. Furthermore, part of the utility cost to females choosing to work outside the home can be interpreted as the unfavorable social stigma historically associated with working women. As the severity of this stigma has declined in recent decades, the cost of participating in the outside labor market has fallen for women. We find that this decline in participation costs leads not only to an increase in female participation, but also to a decline in the fraction of males who are market employed. The latter result follows naturally as women replace men in the workplace, but it is also reinforced by a wealth effect that leads men optimally to choose greater leisure as overall consumption increases. We also find that the real rate of interest in our model economy is temporarily affected by this process. What is left unexplained in this first experiment is the well-documented earnings gap that has existed historically between male and female workers.

In our second experiment, we attempt to capture the effects of demand-side, or employer-based, discrimination against female workers in order to partially explain the behavior of the gender earnings gap. We do not attempt to identify which of the potential causes of discrimination is most relevant, nor do we take a stand on its microeconomic underpinnings. Instead, we model discrimination as a reduced form constraint on the amount of female labor that firms are willing to employ. We then allow for this exogenous constraint to be relaxed over time. In this scenario, we find that reduced employment opportunities for women generate both higher male employment and a larger gender earnings gap than can be explained by human capital differences alone. As opportunities for women expand in the model, female employment naturally increases and the earnings gap narrows. As in the first experiment, male employment falls partly as the result of optimal household behavior with respect to leisure. Interestingly, our model predicts that the gender earnings gap will be countercyclical when changes in total factor productivity are the main sources
of shocks to the economy. These results are all consistent with postwar U.S. labor market experience.

The article is divided into three parts. In the first section, we analyze some labor market data and attempt to estimate some reasonable approximations of trend employment growth. The fact that employment rates have been far from constant over time complicates this estimation. Two of the main forces driving the secular change in employment rates have been changes in demographics and the gender composition of employment. We leave the analysis of demographics to further research, but understanding the role of gender motivates the work in sections two and three.

1. SOME DATA ANALYSIS OF U.S. LABOR MARKETS

To start, let us think of the working population (i.e., age 16 and over), \( P \), in terms of three distinct pools: employed workers, denoted \( E \); the unemployed, \( U \); and those outside the labor force who are not actively looking for work, \( Z \). We denote the labor force by \( N \). Thus, we have that

\[
P = E + U + Z. \tag{1}
\]

Given equation (1), perhaps the simplest way in which to think about the evolution of aggregate employment is to consider a long-run equilibrium where each pool on the right-hand side of the equation grows at a constant rate. Letting \( \gamma_X \) denote the constant growth rate of pool \( X \) over a given period, it must then be the case that \( \gamma_P = \gamma_E = \gamma_U = \gamma_Z \). In this scenario, we can think of the level of job creation, \( \Delta E \), over the period used to measure growth as

\[
\Delta E = \gamma_P E. \tag{2}
\]

With employment currently at about 130 million and population growth hovering around 1 percent annually, it turns out that \( \Delta E \) equals roughly 110,000 at a monthly rate.

Equation (2) is often used to approximate a sustainable level of job creation, that is, a level of job creation that does not generate undue strain in labor markets. Accordingly, any job creation in excess of 110,000 might be considered above trend. However, the simple calculation we have just carried out is subject to an important problem. If the growth rate of each pool is constant and equal to population growth in some long-run equilibrium, the number of employed workers, unemployed workers, and people outside the labor force as a fraction of the total population should also be constant. Looking at Figure 1, we can see that this has never in fact been the case.

While the unemployment rate, \( U/P \), is often the focus of attention in both academic research and the popular press, Figure 1 clearly shows that other considerations also deserve attention. Specifically, in Figure 1, panel b, we
Figure 1 U.S. Employment Trends

- **a. Total Population**
  - Population: 100, 120, 140, 160, 180, 200, 220

- **b. Fraction of Population Employed**
  - Fraction: 0.54, 0.58, 0.62, 0.66

- **c. Fraction of Population Unemployed**
  - Fraction: 0.01, 0.03, 0.05, 0.07

- **d. Fraction of Pop. Outside Labor Force**
  - Fraction: 0.32, 0.34, 0.36, 0.38, 0.40, 0.42
see that the employment rate, $E/P$, has shown a steady but important change over the years. It was as low as 0.55 in 1950 and has gradually increased to a high of 0.64 in 1999. Similarly, the fraction of population outside the labor force, $Z/P$, has steadily fallen from approximately 0.41 in the early 1950s to 0.33 today. In contrast, Figure 1, panel c, suggests that the unemployment rate has remained roughly constant throughout the years at around 0.045. Thus, it seems that to acquire a grasp of U.S. labor markets, it is as important to understand movements in and out of employment that possibly stem from changes in the population outside the labor force as it is to understand variations in unemployment. Moreover, because changes in $E/P$ and $Z/P$ appear to have been very gradual over time, the source of these changes is likely to have been structural in nature.

As we have just argued, it is not entirely clear in Figure 1, panel b, that the fraction of employed workers has ever been in a steady state. More importantly, at first glance it seems anyone’s guess where this fraction might settle, if ever. One way to approach this issue would be simply to make a statistical guess as to the behavior of $E/P$ that allows for non-constant growth over time. We would then be able to infer sustainable levels of job creation without assuming that the share of each labor market pool in population is constant. Thus, define $\rho_E$ as $E/P$ and let

$$\rho_E(t) = \frac{a_0}{1 - a_1 e^{-a_2(t-1948)^2}},$$  \hspace{1cm} (3)

where $t > 0$ denotes time. We choose the functional form depicted in (3) because it will generally give rise to S-shaped curves as suggested by Figure 1, panel b. Furthermore, this functional form is convenient since the function starts at $a_0/(1 - a_1)$ in 1948 and eventually asymptotes to $a_0$. The parameters $a_0$ and $a_1$, therefore, determine the bounds of the function while $a_2$ controls its degree of curvature. These features will allow us to keep $\rho_E(t)$ bounded between zero and one. Under different assumptions about what $\rho_E(t)$ might be in the long run (i.e., different values of $a_0$), we can then use the data available on $E/P$ to estimate $a_1$ and $a_2$ by Non-Linear Least Squares. The results from this estimation are presented in Figure 2.

Under the assumption that the fraction of employed workers should eventually settle at 0.65, close to its current level, it turns out that a sustainable level of job creation for the year 2000 would be around 110,000 jobs at a monthly rate. However, this number may be as high as 130,000 jobs for 2000 if we expect that 3/4 of the working population should eventually be employed in the long run. It is interesting to note that in all reasonable scenarios, current employment to population ratios are above trend. In fact, for current employment to population ratios to be considered on trend, we would have to expect all of the working population to be employed in the long run, which is perhaps a somewhat unrealistic scenario.
Once it has been established that the fraction of employed workers has been far from constant over the last 50 years, a natural question to ask is: What structural changes have driven the evolution of $E/P$ during this period? We explore the two most important sources of structural change in this section: the continually changing role of gender in labor markets and the impact of demographic considerations.

The Role of Gender

Figure 3 illustrates employment rates by gender and the fraction of females and males outside the labor force relative to their working population. The
most striking feature of Figure 3 is that the employed males to male population ratio and the males outside the labor force to male population ratio have steadily moved in a direction opposite to that suggested by their aggregate counterparts. The fraction of employed males began to increase in 1990 but currently falls well short of its historic high in the early 1950s. Similarly, the fraction of males not looking for employment (relative to the male population) is now twice what it was in the late 1940s. In sharp contrast, the employment rate of women has gradually risen from a low of 0.30 in the late 1940s to a high of 0.58 today; that is, only 30 percent of the female working population was employed at the end of World War II. The fraction of females outside the labor force has also steadily fallen over the postwar period. Hence, it appears that aggregate employment rates are largely driven by the increasing participation of women in the labor force. Furthermore, assuming that there are no fundamental differences between men and women in their preferences towards work, we might expect that $E/P$ and $Z/P$ should converge to the same values for these two groups.

Figure 4 shows employment rates by gender as well as cohort and further reinforces the ideas we have just presented. Consider, for instance, the cohort of women who are 20 years old in 1948–1949. The employment rate for women actually falls between ages 20 and 30, increases up to age 50, and drops off as they retire. The fall in the employment rate between ages 20 and 30 can presumably be attributed to childbirth or the rearing of young children. In contrast, consider the cohort of women 20 years of age in 1968–1969. Not only is their employment rate higher across the board, but also the dip that occurs at age 30 does not stand out. It is noteworthy that the male employment rate only shows a small decrease by cohort. Overall, therefore, increases in the employment rate at all ages are mostly driven by female labor behavior.

The continuously increasing female employment rate just documented is generally thought to be the result of both demand- and supply-side factors. On the demand side, Jacobsen (1994) argues that part of the increase can simply be attributed to a general rise in labor demand stemming from technological advances in production. In addition, the noticeable rise in women’s education over the past 50 years has led to increased demand for female workers in a world that is becoming more and more service oriented. Of course, a question immediately arises as to what factors prompted the growth in women’s education in the first place. According to Jacobsen (1994, p. 128), these factors include “a relaxation of social restrictions on appropriate levels and types of education for women, and greater resources on the part of families who might previously have had to ration education among their children.” Finally, a decline in labor market discrimination against women may also have contributed to a rise in female labor demand.

Explanations of demand-side discrimination usually fall into two broad categories. On one hand, neoclassical models propose that labor market out-

C. D. Lantz and P.-D. G. Sarte: U.S. Employment Rates
Figure 3  U.S. Employment Rates by Gender

- a. Fraction of Female Population Employed

- b. Fraction of Female Pop. Outside Labor Force

- c. Fraction of Male Pop. Employed

- d. Fraction of Male Pop. Outside Labor Force
Figure 4 Employment Rates by Cohort

a. Females

b. Males

c. Aggregate
comes are in part a function of agents’ personal prejudices or tastes against associating with particular demographic groups, as in Becker (1957). In the case of gender discrimination, these tastes may be a function of employers’ or customers’ perception of appropriate roles for women and men. On the other hand, models of statistical discrimination assume that employers must make hiring decisions in the face of incomplete information or uncertainty, as in Phelps (1972). Since it is impossible to assess the exact level of productivity associated with a particular job candidate, employers make inferences based on observed or perceived correlation between productivity and various employee characteristics. To the degree that women are perceived to be less productive or dependable than their male counterparts, they will face reduced employment opportunities and wages. One might expect such perceptions to be biased by long-standing attitudes toward gender differences, which may adjust only slowly over time, even in the face of accumulated evidence to the contrary.

Antidiscrimination legislative efforts may have played an important role in reducing the constraints on female employment opportunities during the postwar period. The Equal Pay Act of 1963 required employers to pay the same wage to women and men who do substantially equal work. The notion that this act was even necessary is suggestive that some degree of discrimination against women was taking place in the years that followed World War II. Title VII of the Civil Rights Act of 1964 prohibited employment discrimination on the basis of race, religion, national origin, or sex. The Equal Opportunity Act of 1972 strengthened the 1964 legislation by expanding its coverage to state and local governments as well as educational institutions. Furthermore, it granted the Equal Employment Opportunity Commission (EEOC) the power to sue private sector respondents. Beller (1982) finds that enforcement of Title VII, as measured by the number of completed EEOC investigations and successful settlements, was significant in reducing the gender gap both in wages and in the probability of being employed in male dominated occupations for the period 1967 to 1974. Furthermore, these studies indicate that Title VII was more effective following the passage of the 1972 amendment.

On the supply side, economic considerations that are likely to have induced increased female employment rates include rising wages for women, changes in family composition, and especially changes in non-market production technology. Jacobsen (1994, p. 129) writes that during the “twentieth century, technology has been widely adopted that has enabled families to produce non-market output at lower cost. In particular, we have seen the spread of market goods and services that serve as critical inputs into non-market production. In 1920, one-third of homes had electricity; ... by 1960, practically all homes were electrified. In 1940, 17 percent of farm homes had indoor running water, ... by 1970, 93 percent of rural homes had running water.” Of course, for advances in home production technology to cause a rise in female
employment rates in the postwar period, we must assume that the responsibility for housework (e.g., meal preparation and cleanup, clothing maintenance, housecleaning, etc.) has disproportionately fallen on women. A 1965 study found that women spent an average of 37.8 hours per week engaged in unpaid household chores and childcare. Men, on average, spent only 10.0 hours per week involved in these tasks.\(^1\) A later study, conducted in 1986, revealed that women were spending 31.9 hours per week on these duties, compared to 18.1 hours for men.\(^2\) These data clearly suggest that female household members have more likely been responsible for household work over the years.

It is interesting that as the female employment rate steadily increases in Figure 3, panel a, the male employment rate progressively falls in panel c. This observation is often interpreted to mean that as the male employment rate fell, women were hired to fill the newly created vacancies. However, this reasoning sidesteps the question of why men were gradually less willing to work at particular jobs. We shall argue in this article that, in fact, the direction of causality may well run the other way. If important changes in the economic environment made it less costly for women to work in the marketplace, the resulting increase in family income may have led to a wealth effect that reduced male labor supply. Note that in the latter scenario, males are not displaced by females in jobs but would optimally choose to work less.

2. GENDER AND LABOR MARKETS: AN APPLICATION OF THE CHO-COOLEY MODEL

Although both demographic considerations and the changing role of gender have been crucial determinants of the U.S. aggregate employment rate, for simplicity we shall confine our theoretical analysis to the role of gender within a general equilibrium dynamic framework. In exploring how to think about gender within a neoclassical model, we shall address the various implications of advances in home production technology, the different factors that might underlie differences in gender earnings, and the role of discrimination in general equilibrium.

A Basic Framework without Discrimination

As we saw in the previous section, both the female and male employment rates have displayed considerable variation over the postwar period. To address this fact within the context of an artificial economy, we shall need a model

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\(^2\) Source: Study of Americans’ Use of Time, 1986 (Survey Research Center, University of Maryland).
that allows workers to adjust their labor supply both along the intensive and extensive margins as in Cho and Cooley (1994).

Consider a closed economy populated by a large number of households that comprise a continuum of members uniformly distributed on \([0, 2]\). Households are composed of men and women in equal proportion, working members of the current generation. Each household cares about the welfare and resources of its present as well as future descendants. As in Barro and Sala-i-Martin (1995, chapter 2), this intergenerational consideration may be modeled by assuming that the current generation maximizes utility subject to a budget constraint over an infinite horizon. Thus, we shall in effect analyze the decisions of an immortal extended family.3 For the sake of transparency, we assume that the size of each extended family is constant over time.

Let \(0 \leq h_f \leq 1\) represent the normalized number of hours supplied by a female worker to the market on any given day. (Throughout the remainder of the article, the subscripts \(f\) and \(m\) will stand for females and males respectively.) Further, we define \(a_f h_f^{\gamma+1}/(\gamma + 1)\) as the disutility that a female household member experiences when she provides \(h_f\) hours of work. If \(0 \leq e_f \leq 1\) stands for the measure of female workers within the household, then \(a_f h_f^{\gamma+1}/(\gamma + 1) e_f\) designates total household disutility derived from female labor. Similarly, we let \(a_m h_m^{\gamma+1}/(\gamma + 1) e_m\) denote total family disutility derived from male labor. Observe that if \(a_f = a_m\), then men and women have identical preferences with respect to the number of hours spent working and, consequently, should have identical labor supply schedules along the intensive margin.

Turning our attention to the extensive margin, we follow Cho and Cooley (1994) and assume that there exists a cost associated with each time a household member chooses to work. The idea is that participating in the labor force requires that real resources be spent in the replacement of household production. For example, we can think of these resources as the cost of replacing services such as child care or domestic maintenance while agents are away at work.4 Cho and Cooley (1994) express the utility costs associated with the replacement of household services as an increasing function of the number of working household members, \(b_i e_i^\tau/(\tau + 1), i = m, f\). In the context of the model presented here, the assumption that the burden of housework fell more heavily upon women for a considerable fraction of the past 50 years may be interpreted as \(b_f > b_m\). To a degree, this inequality in the cost of market work can also be interpreted as the unfavorable social stigma that working women may have carried earlier in this century. Observe that from the vantage point

---

3 See Barro (1974) for a formal derivation of this modeling assumption with parental altruism.

4 Cho and Cooley (1994) show explicitly how to map household production into agents’ preferences.
of the household as a whole, the total costs associated with having female members work is \(b_f e_{t+1}^i / (\tau + 1)\) and is similar for male members.

In each period, female and male workers receive wages \(w_{f,t}\) and \(w_{m,t}\) respectively in exchange for their labor services. Households also own capital, \(K_t\), from which they earn interest income, \(r_t K_t\), and discount the future at the rate \(0 < \beta < 1\). Income is either saved in the form of capital accumulation or used to purchase consumption goods, \(C_t\). Given the features of the model we have just described, the representative household maximizes its expected utility into the infinite future,

\[
\max \ U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^\sigma - 1}{\sigma} - \sum_{i=f,m} a_i h_{i,t}^{\gamma+1} e_{i,t} \gamma + 1 - \sum_{i=f,m} b_i e_{i,t}^\tau \tau + 1 \right], \quad (4)
\]

subject to the following budget constraint,

\[
C_t + K_{t+1} - (1 - \delta) K_t = \sum_{i=f,m} w_{i,t} h_{i,t} + r_t K_t, \quad (5)
\]

where \(0 < \delta < 1\) is the depreciation rate on capital. Note that in this framework, \(e_{i,t}\) carries the interpretation of the employment rate of gender \(i\) in period \(t\). The solution to this problem yields the following first order conditions:

\[
-a_i h_{i,t}^{\gamma} + C_t^{\sigma-1} w_{i,t} = 0, \quad i = f, m, \quad (6)
\]

\[
-a_i h_{i,t}^{\gamma+1} / \gamma + 1 - b_i e_{i,t}^\tau + C_t^{\sigma-1} w_{i,t} h_{i,t} = 0, \quad i = f, m, \quad (7)
\]

\[
-C_t^{\sigma-1} + \beta E_t \left[ C_{t+1}^{\sigma-1} (r_{t+1} + 1 - \delta) \right] = 0. \quad (8)
\]

Equation (6) can be thought of as a labor supply schedule for each gender along the intensive margin. As noted earlier, when \(a_f = a_m\), these labor supply schedules are identical. In fact, the number of weekly hours spent at work has been slightly lower for women than for men over the last two decades. However, the ratio of female hours to male hours has been creeping up somewhat during that period. In addition, differences between genders in weekly hours at work are nowhere near as large as differences in employment rates. In our model economy, employment rates for each gender are determined by equation (7). The first two terms of this equation denote the marginal costs of having an additional household member work in the marketplace while the third term captures its marginal benefit in utility terms. Finally, equation (8) equates the marginal benefit and the marginal cost of saving an additional unit of the consumption good.
Figure 5  Trend and Cyclical Components of the Gender Earnings Ratio

a. Median Weekly Earnings

b. Gender Earnings Ratio

c. Cyclical Components in Gender Earnings Ratio and GDP
Firms in the economy produce goods, pay wages to female and male labor $L_f$ and $L_m$ respectively, and make rental payments on capital. Each firm has access to an identical constant-returns-to-scale production technology,

$$Y_t = A_t K_t^{1-\alpha} \left[ z_f L_{f,t}^{\rho} + z_m L_{m,t}^{\rho} \right]^\frac{\alpha}{\rho}, \quad 0 < \alpha < 1, \text{ and } \rho \leq 1, \quad (9)$$

where $A_t$ embodies shifts in total factor productivity. We interpret $z_f$ and $z_m$ as factors that might influence the productivity of genders differently in the market. For example, we noted earlier the substantial increase in women’s education over the postwar period. In principle, therefore, both $z_f$ and $z_m$ should be endogenously determined and time varying. However, in the case of education, the gradual narrowing of differences in education between genders was partly due to a relaxation of social restrictions on women. It will be simplest, therefore, to take $z_f$ and $z_m$ as reduced form parameters. The parameter $\rho$ captures the elasticity of substitution between male and female labor in production, which in this case is given by $1/(\rho - 1)$. Hence, male and female labor are perfect substitutes when $\rho = 1$, and one might think of this scenario as a suitable benchmark.

At each point in time, firms maximize profits and solve,

$$\max \Pi_t = Y_t - r_t K_t - w_{f,t} L_{f,t} - w_{m,t} L_{m,t}, \quad (10)$$

which gives the following first order conditions,

$$(1 - \alpha)A_t K_t^{-\alpha} \left[ z_f L_{f,t}^{\rho} + z_m L_{m,t}^{\rho} \right]^\frac{\alpha}{\rho} - r_t = 0, \quad (11)$$

$$\alpha A_t K_t^{1-\alpha} \left[ z_f L_{f,t}^{\rho} + z_m L_{m,t}^{\rho} \right]^\frac{\alpha-\rho}{\rho} z_f L_{f,t}^{\rho-1} - w_{f,t} = 0, \quad (12)$$

$$\alpha A_t K_t^{1-\alpha} \left[ z_f L_{f,t}^{\rho} + z_m L_{m,t}^{\rho} \right]^\frac{\alpha-\rho}{\rho} z_m L_{m,t}^{\rho-1} - w_{m,t} = 0. \quad (13)$$

Combining equations (12) and (13) immediately yields an expression for the gender earnings ratio,

$$\frac{w_{f,t}}{w_{m,t}} = \frac{z_f}{z_m} \left( \frac{L_{f,t}}{L_{m,t}} \right)^{\rho-1}. \quad (14)$$

Thus, when male and female workers are perfect substitutes into production, only factors that affect differences in gender productivity affect the gender earnings ratio. Of course, at this stage, equation (14) abstracts from discrimination. Figure 5, panel a, shows that over the last 20 years, male median weekly earnings have consistently exceeded those of females (i.e., $w_{f,t}/w_{m,t} < 1$). However, panel b shows an increasing trend in the gender earnings ratio during that period. Interestingly, this reduction in the male/female earnings gap appears to have slowed down as of 1994.

An equilibrium for this economy consists of households’ optimality conditions (6) through (8), firms’ optimality conditions (11) through (13), and
goods and labor market clearing conditions,

\[ C_t + K_{t+1} - (1 - \delta)K_t = Y_t \]  

(15)

and

\[ L_{i,t} = e_{i,t}h_{i,t} \quad i = f, m. \]  

(16)

3. NUMERICAL EXAMPLES

In this section, we study several numerical examples in order to gain insight into the dynamic general equilibrium effects of different changes in the economic environment. We investigate the effects of a reduction in the female cost of market work. We also examine how a loosening of discriminatory hiring practices against women affects the overall economy. The idea of women facing reduced employment opportunities in the workplace may seem somewhat outdated by today’s standards. Nevertheless, it remains that this notion may have substantially contributed to the gender wage gap over the last 50 years.

To explore these issues in greater detail, we must first assign values to the exogenous parameters of the model we have just presented. The parameters \( a_f \) and \( a_m \) are set to 19 and 16 respectively, implying that \( h_f = 0.313 \) while \( h_m = 0.374 \). Assuming that agents can work a maximum of 16 hours a day, 7 days a week, these values translate into 35 weekly hours spent working for females and 42 hours for males. We set \( \tau_f \) and \( \tau_m \) to 1.31 and 1.10 respectively to generate employment rates of 0.58 for women and 0.72 for men. This calibration assumes that the present employment rates depicted in Figure 3 are approximately at their steady state. We normalize the Total Factor Productivity parameter to 1 and assume that male and female workers are perfect substitutes in production, \( \rho = 1 \). We normalize \( z_f \) to 1 and set \( z_m \) to 1.20 so as to obtain a gender earnings ratio of 0.83. While Figure 5, panel b, suggests that the gender earnings ratio is currently 0.76, Blau and Kahn (1997) find that 43 percent of the gap cannot be explained by human capital differences with the implication that much of the unexplained portion results from discriminatory practices.5 On the other hand, Kim and Polacheck (1994) have conducted empirical research suggesting that the “unexplained” portion can be reduced 50 percent when estimates allow for unobservable individual-specific effects, which the authors think of as individual differences in motivation. We choose an intermediate level and model 30 percent of the gender earnings ratio as

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5 Blau and Kahn consider 1979 and 1988 PSID data. They find that the earnings ratio increased from 0.62 to 0.72 over that period. Adjusting for human capital variables, the ratios are 0.72 and 0.80, respectively. Adjusting for human capital and industry, occupation and collective bargaining status, the ratios are 0.78 and 0.88. The unexplained portion of the pay gap (in 1988) is therefore given by \( \frac{1-0.88}{1-0.72} = .43 \).
resulting from discrimination. In other words, the women’s to men’s wage ratio is 0.76 and 70 percent of that gap is attributable to factors other than discrimination. This means that absent discrimination, this ratio would be 0.83, and that is how the no-discrimination model is calibrated. We shall think of a period as a year in the numerical examples so that $\beta$ is set to 0.98. All other parameters are chosen symmetrically for men and women and set to the values in Cho and Cooley (1994). In particular, we have $b_m = b_f = 0.8$, $\sigma = 2$, $\gamma = 0.8$, and $\alpha = 0.64$. Table 1a summarizes key aspects of the model steady state that arise from the calibration presented here.

The first experiment we carry out considers the effects of a protracted fall in the female labor market participation cost. As advances in household production—as well as changing attitudes—have made it progressively easier for women to join the labor force, we wish to analyze their general equilibrium implications for other variables. In the second experiment, we introduce reduced employment opportunities for women as a way of modeling discrimination. We explain how such reduced opportunities can generate a gender wage gap and analyze the effects of a permanent change in total factor productivity in that environment. As in the data, changes in total factor productivity

### Table 1a Benchmark Steady Rate

<table>
<thead>
<tr>
<th>Aggregate Characteristics</th>
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<tbody>
<tr>
<td>Consumption</td>
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</tr>
<tr>
<td>Rate of Interest</td>
<td>0.070</td>
</tr>
<tr>
<td>Investment:Output Ratio</td>
<td>0.255</td>
</tr>
<tr>
<td>Gender Earnings Ratio</td>
<td>0.833</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Labor Force Gender Characteristics</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly Hours</td>
<td>42</td>
<td>35</td>
</tr>
<tr>
<td>Employment Rate</td>
<td>0.720</td>
<td>0.577</td>
</tr>
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</table>

### Table 1b Steady State with Reduced Female Employment Opportunities

<table>
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<th>Aggregate Characteristics</th>
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<tbody>
<tr>
<td>Consumption</td>
<td>0.452</td>
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<tr>
<td>Rate of Interest</td>
<td>0.070</td>
</tr>
<tr>
<td>Investment:Output Ratio</td>
<td>0.255</td>
</tr>
<tr>
<td>Gender Earnings Ratio</td>
<td>0.760</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Labor Force Gender Characteristics</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly Hours</td>
<td>42</td>
<td>33</td>
</tr>
<tr>
<td>Employment Rate</td>
<td>0.73</td>
<td>0.523</td>
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</table>
will generate a countercyclical gender earnings ratio. Finally, we shall analyze
the effects of a gradual reduction in discrimination against female labor.

The Dynamic Effects of a Reduction in Female Labor
Market Participation Costs

Figure 6 illustrates the effects of a gradual fall in the cost of market work for
women. In Figure 6, panel a, we show that this is modeled as a permanent
10 percent fall in the value of $b_f$ over 16 years. This gradual reduction in the
dynamics of $b_{f,t}$ can be modeled as

$$\ln b_{f,t} = \sum_{j=1}^{16} \omega_j u_{t-j},$$

where

$$u_t = u_{t-1} + \varepsilon_t$$

and

$$\sum_{j=1}^{16} \omega_j = 1.$$  

Since $b_{f,t}$ falls gradually, we should expect that both male and female em-
ployment rates should also respond incrementally over time as suggested in
Figures 1 and 3. Observe in Figure 6, panel d, that the reduction in women’s
labor market costs directly implies a 4 percent permanent rise in the female
employment rate. As we had conjectured, therefore, historical changes in the
female employment rate are consistent with technological advances in home
production. In addition, some interesting general equilibrium effects emerge.
Because the utility cost of market work for women is permanently lower, ag-
gregate consumption eventually rises to a higher steady state in Figure 6, panel
b. This rise in aggregate consumption translates into a wealth effect that actu-
ally causes a fall in the male employment rate (and male work hours) in Figure
6, panel c. Note that equations (6) and (7), which characterize labor supply for
each gender, both depend on aggregate consumption. Of course, wages also
adjust downward in this experiment, leading to a substitution effect, which
reinforces the reduction in male employment. In other words, some degree
of crowding out does take place as women enter the labor market. On the
whole, in a manner consistent with Figure 3, panels a and c, advances in home
production technology lead not only to a rise in the female employment rate
but also a fall in its male counterpart. However, in the final steady state,
the magnitude of the change in the male employment rate is relatively small
compared to that in the female employment rate.

It is also worth noting that a reduction in the labor participation cost of
women examined here implies a temporary rise in the rate of interest. As
is typical of neoclassical frameworks, changes in the rate of interest mimic
changes in the growth rate of consumption (see equation (8)). Finally, observe
Figure 6 The Effects of a Reduction in the Female Labor Participation Cost

a. Female Labor Market Participation Cost

b. Consumption

c. Male Employment

d. Female Employment
that this particular example is also useful in illustrating the forward-looking behavior of household members. Since advances in home production technology are gradual, reductions in women’s cost of market work today signal further reductions in the future. Anticipating these future reductions, male workers cut back their labor supply contemporaneously despite the fact that the initial decline in market costs is quite small. Given that the capital stock is predetermined when the shock occurs, production also falls contemporaneously. It follows that consumption falls on impact as shown in Figure 6, panel b.

The General Equilibrium Impact of Reduced Employment Opportunities for Women

In the numerical experiment we have just carried out, the gender earnings ratio remained unaffected and constant as suggested by equation (14). However, this equation only captures one notion of the earnings gap based on differing gender productivity. We argued earlier, for instance, that the human capital embodied in female workers had risen substantially over the past five decades because of increased education. We now show how demand-side discrimination in the form of reduced employment opportunities for women can contribute to lowering female earnings.

Consider the model presented in the previous section. With the household side unchanged, male and female labor supply continue to obey equations (6) and (7). However, suppose that firms are unwilling to manage more than \( \tilde{L}_f > 0 \) units of female labor. As in Phelps (1972), we can imagine that this labor demand constraint stems from the perception that female labor is less dependable or productive than male labor. According to Goldin (1990), firms have often viewed gender as a sign of shorter expected job tenure, leading to job segregation and limited opportunities for women. Firms then maximize profits in (10) subject to

\[
L_{f,t} \leq \tilde{L}_f. \tag{19}
\]

Letting \( \tilde{w}_{f,t} \) denote the new female wage that emerges from this constrained maximization problem, we obtain

\[
\frac{\tilde{w}_{f,t}}{w_{m,t}} = \frac{z_f}{z_m} - \frac{\phi_t}{w_{m,t}} \leq \frac{w_{f,t}}{w_{m,t}}, \tag{20}
\]

where \( \phi_t \geq 0 \) is the Lagrange multiplier associated with the constraint (19). Note that in the previous section, the market equilibrium value of female labor was increasing in female productivity, \( z_f \). Thus, the more productive female labor, the more likely equation (19) is to bind. When this is the case, \( \phi_t > 0 \) and a lower gender earnings ratio emerges.

Table 1b describes the steady state that obtains when \( \tilde{L}_f \) is calibrated to generate a gender earnings ratio of 0.76 as suggested by the most recent ratios
Table 2a  Gender Earnings Ratio and GDP

| Corr(Yt, wf, t+k/wm, t+k) for k = |  |
|---|---|---|---|---|---|---|---|
| -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| -0.050 | -0.107 | -0.169 | -0.234 | -0.310 | -0.330 | -0.332 |

Table 2b  Gender Earnings Ratio and GDP

| Corr(Yt, wf, t+k/wm, t+k) for k = |  |
|---|---|---|---|---|---|---|
| -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| -0.354 | -0.550 | -0.762 | -0.985 | -0.661 | -0.395 | -0.165 |
| [0.14] | [0.11] | [0.07] | [0.00] | [0.07] | [0.11] | [0.13] |

in Figure 5, panel b. Aside from the fall in both the female employment rate and female work hours that naturally follows, we wish to stress the following two points. First, the male employment rate rises somewhat to compensate for the drop in female labor input; however, this increase is less than one-for-one since $z_m > z_f$. Second, and more important, steady state household consumption falls by 3.62 percent. This decline in consumption reflects the efficiency loss that emerges when 30 percent of the gender wage gap is due to discriminatory hiring practices against women.

Figure 5, panel c, suggests a negative correlation between the cyclical components of the gender earnings ratio and real GDP. Table 2a confirms this negative correlation at different leads and lags. An argument often cited to explain this negative correlation is that in periods of boom, women are more likely to be new entrants or re-entrants in the job market since the male participation rate is already high. Relative to women who are already part of the labor force, however, the marginal female worker is likely to be less productive. Hence, one expects the median female wage to decrease in times of economic expansion. An implicit assumption here is that there exists some heterogeneity among female workers.

Aside from the compositional effect we have just described, there may be other factors that contribute to a countercyclical gender earnings ratio. In fact, in the framework with reduced female employment opportunities presented in this section, the gender earnings ratio will move in a direction opposite to that of output when the economy is mainly driven by shifts in total factor productivity. Figure 7 shows the impulse responses of output and the gender
Figure 7 The Effects of an Increase in Total Factor Productivity

a. Total Factor Productivity

b. Output

c. Interest Rate

d. Gender Earnings Ratio
earnings ratio to a 2 percent permanent increase in total factor productivity. As expected, output rises on impact and increases monotonically to a higher steady state level. In contrast, the gender wage ratio falls both contemporaneously and in the long run. Because firms are unwilling to hire more than a given level of female labor input, the female wage exhibits inertia despite the rise in productivity. The male wage, on the other hand, unambiguously increases as the demand for male labor shifts out in response to the productivity shock. Consequently, the gender earnings ratio falls in periods of boom.

In Table 2b, we present the cross-correlations of output and the gender earnings ratio obtained at different leads and lags when the productivity process is calibrated as

$$\ln A_t = \rho A \ln A_{t-1} + \epsilon_{A,t},$$

where $\rho_A = 0.95$ and $\epsilon_{A,t}$ is a random variable with mean zero and standard deviation of 0.01. The model statistics are the mean values calculated from 200 simulations of samples with 80 observations each, the size of the sample for the data in Figure 3. In square brackets are the standard deviations of the sample statistics. As suggested by the impulse responses in Figure 7, output and the gender earnings ratio are negatively correlated at all leads and lags, much more so in fact than in the data. While our model misses some important dimensions, such as heterogeneity among workers that would give rise to the type of compositional effect discussed above, our result also suggests that there may be other key sources of shocks aside from shifts in total factor productivity.

Finally, Figure 8 shows the dynamic effects of a gradual 2 percent improvement in female employment opportunities (i.e., $\Delta \tilde{L}_f > 0$). As the constraint on female employment becomes less binding, the economy becomes more efficient and aggregate consumption rises in Figure 8, panel a. At the same time, the female wage rises closer to its unconstrained equilibrium so that the gender earnings ratio increases in panel b. Note in Figure 8 that as the female employment rate rises in response to a looser employment constraint, the male employment rate correspondingly falls. This is not only because the rise in female employment is making work opportunities scarcer for male labor; in this case, the fall in male employment is partly a reflection of the wealth effect induced by the increase in aggregate consumption. Thus, as the economy becomes more efficient, men choose to reduce how much they work. Further, this exercise suggests that the rise in the female employment rate in Figure 3, panel a, and the simultaneous decline in the male employment rate in panel c, are consistent with a continuing relaxation of discriminatory hiring practices against female labor. A loosening of the constraint on female employment

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6 This particular process for total factor productivity is standard in the real business cycle literature.
Figure 8 The Effects of Relaxing the Female Labor Constraints

a. Consumption

b. Gender Earnings Ratio

c. Male Employment

d. Female Employment
opportunities has the same effects on gender employment rates as a decrease in women’s costs of market work. Both of these changes in the economic environment are consistent with the behavior of employment rates by gender since the end of World War II.

4. CONCLUDING REMARKS

We have documented that U.S. employment rates have changed considerably over the postwar period. The data suggest that both demographic and gender specific factors have been important forces underlying the evolution of these rates. We have focused specifically on the role of gender heterogeneity in determining aggregate employment rates, and we have also developed the implications of gender differences for the overall economy.

Within a stochastic general equilibrium framework, we have modeled the effects of a reduction in female employment participation costs, as well as a reduction in employer-based gender discrimination. Reduced employment costs, while giving rise to higher levels of female employment, also generate lower rates of male employment through both a wealth and a substitution effect at the household level. Reduced participation costs do not, however, explain the historical behavior of the gender earnings gap. Employer based discrimination is then introduced as a constraint on the measure of female labor that firms are willing to employ. In the presence of such discrimination, a gender earnings gap emerges in excess of what can be explained by relative differences in productivity. This model-generated pay gap is countercyclical in nature, rising in periods of economic contraction and diminishing when the economy booms, thus approximating what is seen in U.S. data. When the constraint on female labor is relaxed over time, the model predicts gradually rising levels of female employment, which endogenously lead to lower rates of male employment and a narrowing gender wage gap. The latter predictions have also been stylized features of U.S. labor market experience.
REFERENCES


Wage-Price Dynamics: Are They Consistent with Cost Push?

Yash Mehra

For gauging inflationary pressures, many policymakers and financial market analysts pay close attention to the behavior of wages. It is widely believed that if wage costs rise faster than productivity, the price level may rise as firms pass forward increased wage costs in the form of higher product prices. Hence changes in productivity-adjusted wages are believed to be a leading indicator of future inflation.

One problem with this popular “cost-push” view of the inflation process is that it does not recognize the influences of Federal Reserve policy and the resulting inflation environment on determining the causal influence of wage growth on inflation. If the Fed follows a non-accommodative monetary policy and keeps inflation low, then firms may not be able to pass along excessive wage gains in the form of higher product prices. In fact, an alternative view is that inflation is a “monetary” phenomenon and is caused by excess aggregate demand. According to this view, the causation runs from inflation to wage growth: firms are able to raise the price of their products because of excess aggregate demand caused by an expansionary monetary policy. The resulting increase in prices leads workers to demand higher wages.

In this article, I investigate whether wage-price dynamics are consistent with the cost-push view of the inflation process. The cost-push view implies that Fed policy and the resulting inflation environment do not matter in determining the ability of firms to pass forward higher wage costs in the form of higher product prices. Higher wage growth should lead to higher future

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1 The term “productivity-adjusted wage growth” refers to wage growth in excess of productivity gains, measured here by growth in unit labor costs. The empirical work here focuses on this measure of wage growth.

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The author wishes to thank Robert Hetzel, Huberto Ennis, and Roy Webb for helpful comments. The views expressed are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.
inflation irrespective of what Fed policy has been and whether inflation has been high or low. I test this implication in two ways. First, I investigate whether there exists a long-term equilibrium relation between the price level and the level of wages and, if it exists, whether that equilibrium relation can be interpreted as the long-term price equation, meaning the price level is causally related to wages. The cost-push view implies that the long-term equilibrium relation is in fact the long-term price equation in which wages can be considered exogenous. Then, the estimated coefficient that appears on the wage variable measures the long-term response of the price level to wages. Second, even if a long-term relation between the levels of price and wage series does not exist, short-term changes in them may still be correlated. If the cost-push view is correct, then wage growth should help predict inflation and such predictive content should be invariant to changes in Fed policy and the inflation regime.

I test these implications of the cost-push view using data on the U.S. sample period 1952Q1 to 1999Q2. During this sample period both the nature of monetary policy pursued by the Federal Reserve and the behavior of inflation have varied considerably. In particular, this period contains two subperiods, 1952 to 1965 and 1983 to 1999, during which inflation remained low to moderate and one subperiod, 1966 to 1982, during which inflation steadily accelerated. Furthermore, the descriptive analysis of monetary policy in Goodfriend (1993) and the monetary policy reaction functions estimated more recently in Clarida, Gali, and Gertler (2000) and Mehra (1999) indicate that since 1979 the Fed has concentrated on maintaining low inflation. It is widely believed that as a result of such policy, inflation declined sharply in the early 1980s and has remained low to moderate since then. The cost-push view implies that the predictive content of wage growth for future inflation should be stable over this sample period. As in some previous research, these wage-price dynamics are investigated using techniques of cointegration, Granger-causality, and weak exogeneity.

Other studies have previously investigated whether wage growth helps predict inflation. Mehra (1991), Hu and Trehan (1995), and Gordon (1998) report evidence that indicates wage growth has no predictive content for future inflation. Emery and Chang (1996) and Hess (1999) point out that the find-

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2 The breakpoints used here are suggested by a cursory look at inflation data, measured by the behavior of the GDP deflator. The hypothesis that holds that breaks in inflation are related to changes in monetary policy has been tested in Webb (1995). Based on his reading of FOMC minutes and fiscal and political commentary, Webb points out monetary policy changed first around the mid-1960s and then the early 1980s. He sees monetary policy making a discrete move toward more inflation at some point in the middle of the 1960s and another move toward lower inflation in the early 1980s. Thus three periods identified are: an early low inflation period from 1952Q2 to 1966Q4; a middle inflationary period from 1967Q1 to 1981Q2; and a disinflationary period from 1981Q3 to 1990Q4. The three sample periods are close to those studied here. However, one is likely to get qualitatively similar results if instead these breakpoints are used.
Y. Mehra: Wage-Price Dynamics

ing that wage growth helps predict inflation is sensitive to the sample period chosen. In contrast, Ghali (1999) reports Granger-causality test results that indicate wage growth does predict inflation, implying the cost-push view is correct. The main criticism of the previous empirical work, including the one in Ghali (1999), is the absence of discussion about the influences of Fed policy and the resulting inflation environment on determining the causal influence of wage growth on inflation. Hence the issue of the stability of wage-price dynamics has essentially been ignored. Furthermore, the predictive content in previous work is generally investigated using tests of Granger-causality summarized by the conventional F-statistic. However, it is important to assess the quantitative size of the predictive content. I therefore provide evidence on the size of the feedback between wages and prices, reporting the “magnitude” of the sum of coefficients that appear on lagged wage growth in predicting inflation.3

The empirical work presented here does not favor the cost-push view. The results indicate that the price level and the productivity-adjusted wage (measured here by the level of unit labor costs) are indeed cointegrated over the full sample period 1952Q1 to 1999Q2, meaning there exists a long-term, equilibrium relation between prices and wages. However, this equilibrium (cointegrating) relation cannot be interpreted as the long-term price equation because wages are not found to be exogenous. Thus, the estimated coefficient that appears on the wage variable does not measure the long-term response of the price level to wages, as implied by the cost-push view. The evidence rather indicates this equilibrium relation is the wage equation, in which prices can be considered exogenous.

The test results for Granger-causality indicate that over the full sample period 1952Q1 to 1999Q2, higher wage growth does lead to higher future inflation, as predicted by the cost-push view. However, the estimated, short-run feedback from wage growth to inflation is quantitatively modest and quite unstable during the sample period. In particular, the full sample result that wage growth helps predict future inflation is mainly due to the inclusion of observations from the high inflation subperiod 1966 to 1983. Wage growth does not help predict inflation during two low inflation subperiods, 1952Q1 to 1965Q4 and 1984Q1 to 1999Q2. In contrast, inflation always helps predict future wage growth, this result holding in all subperiods. Furthermore, the estimated, short-run feedback from inflation to wage growth is quantitatively large, with the estimated feedback coefficient close to unity, indicating wage growth has adjusted one-for-one with inflation. Consequently, the result that wage growth helps predict inflation only during the subperiod of high and rising inflation does not favor the cost-push view of the inflation process. It

3 An exception is the recent work in Gordon (1998). He does not, however, investigate the stability of the wage-price feedback.
is only during a highly inflationary environment that firms are able to pass forward higher wage costs in the form of higher product prices, suggesting causation that runs from excess aggregate demand to inflation and to wages.

The plan of the rest of the paper is as follows. Section 1 reviews the economic rationale of why wages and prices may move together over time. Section 2 presents empirical results. Section 3 contains concluding observations.

1. SHORT-RUN WAGE-PRICE DYNAMICS: ECONOMIC RATIONAL AND TESTING

The view that systematic movements in wages and prices are related can be rationalized in a number of ways. One such rationalization can be derived from the expectations-augmented Phillips curve view of the inflation process. Consider the price and wage equations that typically underlie such Phillips curve models described in Gordon (1985, 1988) and Stockton and Glassman (1987).

\[
\Delta p_t = h_0 + h_1 \Delta (w - q)_t + h_2 x_t + h_3 s p_t
\]

\[
\Delta (w - q)_t = k_0 + k_1 \Delta p^e_t + k_2 x_t + k_3 s w_t
\]

\[
\Delta p^e_t = \sum_{j=1}^{n} \lambda_j \Delta p_{t-j}
\]

where all variables are in their natural logarithms and where \( p \) is the price level, \( w \) is the nominal wage rate, \( q \) is labor productivity, \( x \) is a demand pressure variable, \( p^e \) is the expected price level, \( sp \) represents supply shocks affecting the price equation, \( sw \) represents supply shocks affecting the wage equation, and \( \Delta \) is the first difference operator. Equation (1) describes the price markup behavior. Prices are marked over productivity-adjusted wage costs and are influenced by cyclical demand and the exogenous, relative supply shocks. This equation implies that productivity-adjusted wages determine the price level, given demand pressures. Equation (2) is the wage equation. Wages are assumed to be a function of cyclical demand and expected price level, the latter modeled as a lag on past prices as in (3). The wage equation, together with equation (3), implies that wages depend upon past prices, ceteris paribus.

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4 As shown in Nordhaus (1972), this pricing equation can be derived from the optimizing behavior of firms. Under assumptions of Cobb-Douglas production function, constant returns, the constant relative price of capital, and profit maximizing behavior, the optimal price equation looks like one as in (1), with \( h_1 = 1 \). The last result implies that prices adjust one-for-one with wages in the long run.
The price and wage behavior described above suggests that long-run movements in wages and prices must be related. Furthermore, if one allows for short-run dynamics in such behavior, the analysis above would also suggest that past movements in wages and prices should help predict future changes in those same variables, ceteris paribus. In previous research these implications have been tested using tests for cointegration and Granger-causality between wage and price series.\textsuperscript{5} In this article, I also use tests for weak exogeneity proposed by Hendry, Engle, and Richard (1983). Furthermore, following Gordon (1998) I also present evidence on the magnitude of the feedback between wage growth and inflation.

To illustrate, assume that wage and price series are cointegrated, indicating that wages and the price level comove in the long run and that the cointegrating relation is given in (4).

\begin{align*}
  p_t &= a_0 + a_1 (w - q)_t + U_t \quad (4.1) \\
  (w - q)_t &= -(a_0/a_1) + (1/a_1) p_t - (1/a_1) U_t, \quad (4.2)
\end{align*}

where $U_t$ is the disturbance term. In (4.1) the cointegrating relation is normalized on the price variable, whereas in (4.2) it is normalized on the productivity-adjusted wage. That the finding wages and prices are cointegrated simply implies that these two variables are correlated in the long run, but correlation does not necessarily indicate causation. In order to investigate whether this finding about the presence of cointegration between prices and wages can be given a causal interpretation, Hendry, Engle, and Richard (1983) have proposed tests for weak exogeneity. In particular, the wage variable in the cointegrating regression (4.1) can be considered given in determining the response of the price level to wages if wages are weakly exogenous with respect to the long-term parameter $a_1$. In that case, one can causally interpret (4.1) as the long-term price equation, the parameter $a_1$ measuring the long-term response of the price level to wages. Conversely, if wages are not weakly exogenous but the price level is, then one can reformulate (4.1) as (4.2) and interpret it as the long-term wage equation with the parameter $(1/a_1)$ measuring the long-term response of wages to prices. If the “price markup” hypothesis holds in the long run, and hence the popular cost-push view of the inflation process is correct, then wages should be weakly exogenous in (4.1).

The test for weak exogeneity amounts to examining whether the residual $U$ enters significantly in inflation and wage regressions of the forms (5) and (6).

\footnote{See, for example, Mehra (1991), Hu and Trehan (1995), Gordon (1998), and Ghali (1999).}
\[
\Delta p_t = b_0 + \lambda_1 U_{t-1} + \sum_{j=1}^{s} b_{1j} \Delta p_{t-j} + \varepsilon_{1t} \quad (5)
\]

\[
\Delta (w - q)_t = c_0 + \lambda_2 U_{t-1} + \sum_{j=1}^{s} c_{1j} \Delta (w - q)_{t-j} + \varepsilon_{2t} \quad (6)
\]

Wages are weakly exogenous in (4.1) if \( \lambda_2 \) is zero, whereas prices are weakly exogenous in (4.2) if \( \lambda_1 \) is zero. Intuitively, the test for weak exogeneity amounts to determining whether the long-run comovement of prices and wages is the result of wages adjusting to prices, prices adjusting to wages, or both. The cost-push view of the inflation process implies that the long-run comovement is the result of prices adjusting to wages, not wages adjusting to prices. Hence cointegrating regression (4.1) is the price equation in which the price level is causally determined by wages in the long run.

The tests for cointegration and weak exogeneity discussed above focus on the presence and the nature of the long-run correlation between wage and price series. However, even if a long-term cointegrating relation does not exist, we may still find that wage growth and inflation are correlated in the short run. If the price markup hypothesis holds in the short run, then wage growth may help predict future inflation. In previous research, this issue has been investigated using tests for Granger-causality. In particular, consider the following inflation and wage growth equations:

\[
\Delta p_t = b_0 + \lambda_1 U_{t-1} + \sum_{j=1}^{s} b_{1j} \Delta p_{t-j} + \sum_{j=1}^{s} b_{2j} \Delta (w - q)_{t-j} + \sum_{j=1}^{s} b_{3j} CD_{t-j} + \varepsilon_{1t} \quad (7)
\]

\[
\Delta (w - q)_t = c_0 + \lambda_2 U_{t-1} + \sum_{j=1}^{s} c_{1j} \Delta (w - q)_{t-j} + \sum_{j=1}^{s} c_{2j} \Delta p_{t-j} + \sum_{j=1}^{s} c_{3j} CD_{t-j} + \varepsilon_{2t} \quad (8)
\]

where \( CD \) stands for cyclical demand and where other variables are defined as before. These equations include the error-correction variable \( U_{t-1} \), in case wage and price series are cointegrated. The test of the hypothesis that wages help predict inflation in the Granger-causal sense is that all \( b_{2j} \neq 0 \) and/or \( \lambda_1 \neq 0 \). In previous work this test has usually been carried out using the conventional
F-statistic, without paying much attention to the issue of whether the estimated effect of wage growth on inflation is quantitatively large or modest. Hence, in order to estimate the relative quantitative effects of lagged wage growth and inflation on each other, I consider below the following transformation of inflation and wage-growth equations (Gordon 1998).6

\[
\Delta p_t = b_0 + \lambda_1 U_{t-1} + \sum_{j=1}^{s} (b_{1j} + b_{2j}) \Delta p_{t-j} + \sum_{j=1}^{s} b_{2j} \Delta (w - q - p)_{t-j} + \sum_{j=1}^{s} b_{3j} C D_{t-j} + \varepsilon_{1t} \tag{9}
\]

\[
\Delta (w - q)_t = c_0 + \lambda_2 U_{t-1} + \sum_{j=1}^{s} (c_{1j} + c_{2j}) \Delta (w - q)_{t-j} + \sum_{j=1}^{s} c_{2j} \Delta (p - (w - q))_{t-j} + \sum_{j=1}^{s} c_{3j} C D_{t-j} + \varepsilon_{2t} \tag{10}
\]

Equation (9) comes about if we add and subtract \(b_{2j} \Delta p_{t-j}\) terms in (7), whereas equation (10) comes about if we add and subtract \(c_{2j} \Delta (w - q)_{t-j}\) terms in (8). In inflation equation (9), the “freely estimated”7 sum of coefficients \(\sum_{j=1}^{s} b_{2j}\) indicate the weight on lagged wage growth in the determination of inflation, once we control for the influence of lagged inflation. Hence, wage growth is an independent source of cost-push inflation if the estimated sum of coefficients \(\sum_{j=1}^{s} b_{1j}\) is positive and statistically different from zero.

6 The reformulated inflation equation (9) incorporates the view that inflation and wage growth comove in the short run. Consider an increase in past wage growth. If this increase in past wage growth is simply due to past inflation, then such increase in past wage growth should have no additional effect on current inflation if the influence of past inflation is already accounted for in the inflation equation. Thus, in order to capture the additional influence of past wage growth on inflation, past wage growth in excess of past inflation is included \((w - q - p)\) in the inflation equation, which also includes past inflation itself. Past wage growth, then, is an independent source of inflation if past wage growth measured as deviations from past inflation is significant. There are several advantages of this reformulation. First, the sum of coefficients appearing on past wage growth directly measures the magnitude of the effect of past wage growth on inflation. Second, one can impose and test restrictions on the sum of coefficients appearing on lagged inflation. For example, the “natural-rate hypothesis” implies coefficients on past inflation sum to unity. One can test whether past wage growth is still relevant when the natural-rate hypothesis holds, as in Gordon (1998). Third, including wage growth measured as deviations from inflation reduces the degree of multicollinearity among right-hand-side variables, enabling more precise estimation by ordinary least squares. Fourth, if one estimates without imposing any restrictions on the coefficients that appear on past inflation, tests for Granger-causality conducted using the reformulated inflation equation should yield results similar to those conducted using the conventional inflation equation (7). Similar considerations hold for the reformulated wage-growth equation (10). The empirical work reported here does not impose any restrictions on coefficients in the inflation equation, meaning wage-growth coefficients are “freely estimated.”

7 See footnote 6.
its estimated magnitude measuring the size of the feedback from wage growth to inflation. Similarly, in wage growth equation (10) the freely estimated sum of coefficients $\sum_{j=1}^s c_{2j}$ measures the feedback from inflation to wage growth, once we control for the influence of lagged wage growth. Therefore, inflation is an independent source of wage growth if the estimated sum $\sum_{j=1}^s c_{2j}$ is positive and statistically different from zero. I test these hypotheses using the conventional F- and t-statistics, besides reporting the estimated sum of coefficients to gauge the quantitative significance of the feedback.

2. EMPIRICAL RESULTS

The price level is measured by the log of the chain-weighted GDP deflator ($p$); the productivity-adjusted wage by the log of the index of unit labor costs of the non-farm business sector ($w - q$); and the cyclical demand ($CD$) by the log of real-over-potential GDP or by first differences of the civilian unemployment rate. Since supply shocks could have short-term effects on wages and prices, tests of predictive content are conducted including some of these in the system. The supply shocks considered here include the relative price of imports. Dummy variables for the period of President Nixon’s wage and price controls and for the period immediately following the wage and price controls are also included. Potential GDP is the series generated using the Hodrick-Prescott (1997) filter. The data used are quarterly and cover the sample period 1952Q1 to 1999Q2.

A Preliminary Look at the Data

Figure 1 takes a preliminary look at prices and unit labor costs. It charts year-over-year growth rates of productivity-adjusted wages measured by unit labor costs and the general price level over 1952Q1 to 1999Q2. This figure clearly indicates that in the short run, productivity-adjusted wage growth and inflation appear to comove closely only over a subperiod that begins in the mid 1960s and ends in the early 1980s. This subperiod is the one during which inflation steadily accelerated. In the remaining subperiods, there does not appear to be much strong comovement between wage growth and inflation, at least in the short run. During these two subperiods, inflation remained low to moderate. Figure 1 clearly suggests that the relationship between inflation and wage growth may not be stable during the sample period considered here.

Figure 2 is similar to Figure 1 except that it charts wage growth measured by compensation per hour. It also reveals that wage growth and inflation do not comove strongly over the full sample period. Sometime since the early 1980s, however, the short-run relationship between inflation and wage growth appears to have weakened significantly.
Figure 3 provides a perspective on the long-run relation between the price level and the level of unit labor costs. As in previous research, the evidence discussed below indicates that the price level and unit labor costs series \((p, w - q)\) are cointegrated over the full sample period 1952Q1 to 1999Q2 (Engle and Granger 1987). Table 1 presents the estimated cointegrating regression. Figure 3 charts the actual price level and the price level predicted by the cointegrating regression, which includes only the level of unit labor costs. As can be seen, these two series move together over most of the sample period, even during the period since the early 1980s when short-run correlation between the growth rates of these two series weakened significantly. However, one still cannot tell from Figure 3 whether this long-run comovement is the result of the price level adjusting to the level of unit labor costs, unit labor costs adjusting to the price level, or both. In order to determine the source of the long-run comovement, I now turn to results from tests performed for Granger-causality and weak exogeneity.
Tests for Weak Exogeneity

Before I perform tests for Granger-causality, I need to investigate whether there exists a cointegrating relation between the price level and unit labor costs during the sample period studied here. Though in many previous studies it has been shown that these two series are indeed cointegrated,8 I repeat the test because the sample period covered in previous research differs from the one used here.

Panel A in Table 1 presents the Engle and Granger (1987) test for cointegration.9 The test results indicate that the price level and unit labor costs are cointegrated over 1952Q1 to 1999Q2, implying the presence of a long-run equilibrium relation between the level of unit labor costs and the price level as in (4).

In order to determine if this cointegrating regression can be interpreted as the long-term price or wage equation, I present test results for weak exogeneity in panel B of Table 1. Those results indicate that in the system \((p_t, (w - q_t))\), unit labor costs are not weakly exogenous but the price level is, meaning

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8 See, for example, Mehra (1993), Hu and Trehan (1995), Hess (1999), and Ghali (1999).

9 The unit root tests (not reported) indicate that wage and price series are integrated of order one, meaning these series are stationary after differencing once.
wages, not prices, adjust in response to deviations in the long-run cointegrating relation. Hence the cointegrating regression here cannot be interpreted as the price equation and thus contradicts the cost-push view.

If we rewrite the estimated cointegration regressions reported in panel A of Table 1 as wage regressions, we get (11).

\[
\begin{align*}
\text{Without Trend} & : (w - q)_t = .25 + .94p_t; \\
\text{With Trend} & : (w - q)_t = -.05 + .001TR_t + 1.0p_t
\end{align*}
\]

The test results for weak exogeneity imply that one can interpret (11) as the long-run wage equation. The estimated coefficient that appears on the price level in (11) is not different from unity, indicating that unit labor costs have adjusted one-for-one with the price level in the long run. Together these results suggest that the long-run comovement of the price level and unit labor costs charted in Figure 3 has arisen mainly as a result of unit labor costs adjusting to the price level rather than the other way around.
Table 1 Engle-Granger Test for Cointegration

<table>
<thead>
<tr>
<th>Panel A: Cointegrating Regressions: 1952Q1–1999Q2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without Trend</td>
</tr>
<tr>
<td>$p_t = -.26 + 1.06(w-q)t$;</td>
<td>$p_t = .05 + .001TR_t + .94(w-q)t$;</td>
</tr>
<tr>
<td>ADF = $-3.52^*$</td>
<td>ADF = $-5.02^*$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Error-Correction Coefficients: Test for Weak Exogeneity</th>
<th>Inflation Regression (t-statistic)</th>
<th>Unit Labor Costs Regression (t-statistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Trend</td>
<td>.017(1.9)</td>
<td>.16(5.2)</td>
</tr>
<tr>
<td>With Trend</td>
<td>.007(.58)</td>
<td>.27(5.9)</td>
</tr>
</tbody>
</table>

*Significant at the 5 percent level.

Notes: $p$ is the chain-weighted GDP deflator; $w$ is compensation per man hour; $q$ is output per man hour; and $TR$ is a linear trend. All variables except $TR$ are in their natural logarithms, so that $w-q$ is unit labor costs. ADF is the augmented Dickey-Fuller statistic; it tests $\rho = 0$ in regressions of the form

$$
\Delta U_t = \rho U_{t-1} + \sum_{s=1}^{k} b_s \Delta U_{t-s};
$$

where $U$ is the residual from the cointegrating regression reported in panel A. The coefficients reported in panel B are from error-correction regressions of the form

$$
\Delta p_t = b_0 + \lambda_p U_{t-1} + \sum_{j=1}^{k} b_{1j} \Delta p_{t-j}; \Delta(w-q) = c + \lambda w U_{t-1} + \sum_{j=1}^{k} c_{1j} \Delta(w-q)_{t-j},
$$

where all variables are defined as above. The optimal lag length $k$ used is 4, as indicated by the Akaike information criterion plus 2 (Pantula, Gonzalez-Farias, and Fuller 1994).

Granger-Causality Tests

The discussion above has focused on sources of the presence of the long-run correlation between the price level and unit labor costs as reflected in the cointegrating regression. Now I present results on the nature of the short-run feedback between the growth rates of these variables, using tests for Granger-causality. The short-run inflation and wage equations that underlie these tests are given in (9) and (10), which include one-period lagged value of the residual from the cointegrating regression reported in Table 1 (the cointegrating regression used is the one without trend). I present results for the full sample 1953Q1 to 1999Q2 as well as for subsamples 1953Q1 to 1965Q4, 1966Q1 to 1983Q4, and 1984Q1 to 1999Q2. Table 2 presents results with the cyclical demand measured by the output gap, and Table 3 contains results with the cyclical demand measured by changes in the civilian unemployment rate.

---

10 I get qualitatively similar results if the cointegrating regression used is the one with trend.

11 The estimation period for inflation and wage growth regressions begins in 1953Q1, earlier observations being used to capture lags in these equations.
Table 2  Testing for Short-Run Feedback: GDP Deflator and Unit Labor Costs

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Output Gap as a Measure of Cyclical Demand</th>
<th>Inflation Equation</th>
<th>Unit Labor Costs</th>
<th>Growth Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \lambda_p )</td>
<td>SUM(_w)</td>
<td>( \lambda_w )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(t-value)</td>
<td>(t-value)</td>
<td>(t-value)</td>
</tr>
<tr>
<td>1953Q1–1999Q2</td>
<td>0.01</td>
<td>3.5*</td>
<td>0.08</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.5)</td>
<td>(1.2)</td>
<td>(1.0)</td>
<td>(4.9)</td>
</tr>
<tr>
<td>1953Q1–1965Q4</td>
<td>0.01</td>
<td>1.1</td>
<td>0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(1.0)</td>
<td>(0.4)</td>
<td>(0.2)</td>
<td>(4.0)</td>
</tr>
<tr>
<td>1966Q1–1983Q4</td>
<td>0.01</td>
<td>2.4**</td>
<td>0.17</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.9)</td>
<td>(1.3)</td>
<td>(0.6)</td>
<td>(2.1)</td>
</tr>
<tr>
<td>1984Q1–1992Q2</td>
<td>-0.01</td>
<td>1.1</td>
<td>0.00</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td>(0.09)</td>
<td>(1.6)</td>
<td>(2.3)</td>
</tr>
</tbody>
</table>

*Significant at the .05 percent level.
**Significant at the .10 percent level.

Notes: The coefficients reported above are from regressions of the form

\[
\Delta p_t = b_0 + \lambda_p U_{t-1} + \sum_{j=1}^{4} b_1 j \Delta p_{t-j} + \sum_{j=1}^{4} b_2 j \Delta (w-q-p)_{t-j} + \sum_{j=1}^{4} b_3 j CD_{t-j} + \varepsilon_{1t}
\]

\[
\Delta (w-q)_t = c_0 + \lambda_w U_{t-1} + \sum_{j=1}^{4} c_1 j \Delta (w-q)_{t-j} + \sum_{j=1}^{4} c_2 j \Delta (p-(w-q))_{t-j} + \sum_{j=1}^{4} c_3 j CD_{t-j} + \varepsilon_{2t},
\]

where CD is cyclical demand measured by the output gap; and U is the residual from the cointegrating regression (without the trend) reported in Table 1. F1 tests \( \lambda_p, b_{2j} = 0 \); F2 tests \( \lambda_w, c_{2j} = 0 \); SUM\(_w\) is the sum of \( b_{2j} \) coefficients and SUM\(_p\) is the sum of \( c_{2j} \) coefficients. The lag length used is indicated by the Akaike information criterion plus 2 (Pantula, Gonzalez-Farias, and Fuller 1994).

As indicated before, wage growth is an independent source of cost-push inflation if the sum of estimated coefficients \( \sum_{j=1}^{4} b_{2j} \) that appear on lagged wage growth in (9) is positive and statistically different from zero, its estimated magnitude measuring the size of the feedback from wage growth to inflation. Table 2 presents the pertinent F- and t-statistics, as well as estimates of the sum of coefficients \( \sum_{j=1}^{4} b_{2j} \). If we focus on full sample estimates, the estimated coefficients, \( b_{2j}, j = 1, 4 \), are significant by the conventional F-statistic, indicating that wage growth Granger-causes inflation (see the F1-statistic in Table 2).\(^{12}\) However, the full sample estimates also indicate that the estimated sum \( \sum_{j=1}^{4} b_{2j} \) is small in magnitude and not different from

\(^{12}\) The lag length used here is selected by the Akaike information criterion plus 2 (Pantula, Gonzalez-Farias, and Fuller 1994). With this selection of lag length, tests discussed in Pantula, Gonzalez-Farias, and Fuller (1994) are shown to have more power. Nevertheless, results here are robust to changes in lag length.
Table 3 Testing for Short-Run Feedback: GDP Deflator and Unit Labor Costs

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Inflation Equation</th>
<th>Unit Labor Costs</th>
<th>Growth Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_p$ (t-value)</td>
<td>$F1$ SUM$w$ (t-value)</td>
<td>$\lambda_w$ (t-value)</td>
</tr>
<tr>
<td>1953Q1–1999Q2</td>
<td>0.00 (0.3)</td>
<td>2.5* (0.4)</td>
<td>0.07 (2.1)</td>
</tr>
<tr>
<td>1953Q1–1965Q4</td>
<td>0.02 (0.8)</td>
<td>0.8 (0.4)</td>
<td>−0.02 (0.1)</td>
</tr>
<tr>
<td>1966Q1–1983Q4</td>
<td>0.01 (0.9)</td>
<td>1.7 (0.4)</td>
<td>0.05 (0.8)</td>
</tr>
<tr>
<td>1984Q1–1992Q2</td>
<td>−0.02 (1.0)</td>
<td>0.8 (0.1)</td>
<td>−0.01 (1.6)</td>
</tr>
</tbody>
</table>

Notes: See notes in Table 2. Table 3 is similar to Table 2, except that cyclical demand (CD) is measured instead by changes in the civilian unemployment rate.

zero (see the t-statistic in Table 2). These results indicate that the short-run feedback from wage growth to inflation, though statistically significant, is quantitatively small and transient in nature, disappearing within a year. In contrast, the estimated sum of coefficients ($\sum_{j=1}^{s} c_{2j}$) that appear on lagged inflation in wage growth equation (10) is statistically significant and large in magnitude (see F2- and t-statistics in Table 2). In fact, the estimated sum of these coefficients $\sum_{j=1}^{s} c_{2j}$ is not different from unity, indicating that wage growth adjusts one-for-one with inflation. Together the full sample results are consistent with the presence of a bi-directional feedback between inflation and wage growth, though the size of the feedback from wage growth to inflation is quantitatively modest and transitory in nature.

If we focus on subsample estimates, we find that they indicate the result above: Wage growth Granger-caused inflation is not robust to changes in the sample period. In particular, wage growth does not Granger-cause inflation over two low inflation subperiods considered here, 1953Q1 to 1965Q4 and 1984Q1 to 1999Q2, and the sum of estimated coefficients $\sum_{j=1}^{s} b_{2j}$ that appear on lagged wage growth is also zero (see F1- and t-statistics in Table 2). However, if we consider the high inflation subperiod 1966Q1 to 1983Q4, then results indicate wage growth does Granger-cause inflation. Moreover, the estimated sum $\sum_{j=1}^{s} b_{2j}$ is large, indicating the presence of a significant feedback from wage growth to inflation. Together these results indicate that wage growth is not an “independent” source of inflation because it helps predict inflation only during the sample period of high and rising inflation. In contrast, the subperiod estimates of the coefficients, $c_{2j} j = 1, 4$, that measure feedback from inflation to wage growth are not sensitive to changes in the sample.
period. In all three subperiods considered here, inflation does Granger-cause wage growth. Furthermore, the estimated sum $\sum_{j=1}^{s} c_{2j}$ is always statistically significant and mostly close to unity, implying that wage growth adjusts one-for-one with inflation (see F2- and t-statistics in Table 2).

The Granger-causality test results presented in Table 2 are based on inflation and wage growth regressions estimated with the output gap as a measure of the cyclical demand. In order to assess whether Granger-causality results are robust to changes in the measure of the cyclical demand used, Table 3 contains results when the cyclical demand is measured instead by changes in the civilian unemployment rate. As can be seen, these alternative Granger-causality test results yield inferences about the nature of feedback between inflation and wage growth that are qualitatively similar to those produced by results in Table 2. In particular, wage growth does Granger-cause inflation in the full sample period, but then this result is not robust across all subperiods.

3. CONCLUDING OBSERVATIONS

The cost-push view of the inflation process that is implicit in the expectations-augmented Phillips curve model assigns a key role to wage growth in determining inflation. In this article, I evaluate this role by investigating empirically both the presence and stability of the feedback between wage growth and inflation during the U.S. postwar period, 1952Q1 to 1999Q2. The results indicate that wage growth helps predict future inflation over the full sample period considered here. However, this finding is very fragile, and it appears in the full sample because the estimation period includes the subperiod 1966Q1 to 1983Q4 during which inflation steadily accelerated. Wage growth does not help predict inflation in two other subperiods, 1953Q1 to 1965Q4 and 1984Q1 to 1999Q2, during which inflation remained low to moderate. In contrast, inflation always helps predict wage growth, a finding that is both quantitatively significant and stable across subperiods. These results thus do not support the view that wage growth has been an independent source of inflation in the U.S. economy.

The results here confirm what few others have found in recent work on this issue: Wage growth no longer helps predict inflation if we consider subperiods that begin in the early 1980s (Emery and Chang 1996; Hess 1999). The period since the early 1980s is the period during which the Fed has concentrated on keeping inflation low. What is new here is the finding that even in the pre-1980 period there is another subperiod, 1953Q1 to 1965Q4, during which wage growth does not help predict inflation. This is also the subperiod during which inflation remained mostly low, mainly due to monetary policy pursued by the Fed (Webb 1995).
REFERENCES


The New IS-LM Model: Language, Logic, and Limits

Robert G. King

Recent years have witnessed the development of a New IS-LM model that is increasingly being used to discuss the determination of macroeconomic activity and the design of monetary policy rules. It is sometimes called an “optimizing IS-LM model” because it can be built up from microfoundations. It is alternatively called an “expectational IS-LM model” because the traditional model’s behavioral equations are modified to include expectational terms suggested by these microfoundations and because the new framework is analyzed using rational expectations. The purpose of this article is to provide a simple exposition of the New IS-LM model and to discuss how it leads to strong conclusions about monetary policy in four important areas.

- **Desirability of price level or inflation targeting:** The new model suggests that a monetary policy that targets inflation at a low level will keep economic activity near capacity. If there are no exogenous “inflation shocks,” then full stabilization of the price level will also maintain output at its capacity level. More generally, the new model indicates that time-varying inflation targets should not respond to many economic disturbances, including shocks to productivity, aggregate demand, and the demand for money.

- **Interest rate behavior under inflation targeting:** The new model incorporates the twin principles of interest rate determination, originally developed by Irving Fisher, which are an essential component of modern macroeconomics. The real interest rate is a key intertemporal relative...
price, which increases when there is greater expected growth in real activity and falls when the economy slows. The nominal interest rate is the sum of the real interest rate and expected inflation. Accordingly, a central bank pursuing an inflation-targeting policy designed to keep output near capacity must raise the nominal rate when the economy’s expected growth rate of capacity output increases and lower it when the expected growth rate declines.

• **Limits on monetary policy:** There are two limits on monetary policy emphasized by this model. First, the monetary authority cannot engineer a permanent departure of output from its capacity level. Second, monetary policy rules must be restricted if there is to be a unique rational expectations equilibrium. In particular, as is apparently the case in many countries, suppose that the central bank uses an interest rate instrument and that it raises the rate when inflation rises relative to target. Then the New IS-LM model implies that it must do so aggressively (raising the rate by more than one-for-one) if there is to be a unique, stable equilibrium. But if the central bank responds to both current and prospective inflation, then it is also important that it not respond too aggressively.

• **Effects of monetary policy:** Within the new model, monetary policy can induce temporary departures of output from its capacity level. However, in contrast to some earlier models, these departures generally will not be serially uncorrelated. If the central bank engineers a permanent increase in nominal income, for example, then there will be an increase in output that will persist for a number of periods before fully dissipating in price adjustment. Further, the model implies that the form of the monetary policy rule is important for how the economy responds to various real and monetary disturbances.

In summary, the New IS-LM model instructs the central bank to target inflation. It indicates that there are substantial limits on the long-run influence that the monetary authority can have on real economic activity and that there are also constraints on its choice of policy rule. But the New IS-LM also indicates that the monetary authority can affect macroeconomic fluctuations through its choice of the monetary policy rule, as well as via monetary policy shocks.

The plan of the article is as follows. Section 1 provides some historical background on the evolution of the IS-LM model since its origin in Hicks (1937). Section 2 then quickly lays out the equations of the closed economy version of the New IS-LM model. Section 3 uses the framework to show how a neutral monetary policy—a policy which keeps output close to its capacity level—implies a specific inflation targeting regime and, if certain exogenous shocks are small, rationalizes a full stabilization of the price level. Following
Goodfriend and King (1997), such a policy is called a “neutral monetary policy” and the new model is used to determine some rules for the setting of alternative monetary instruments that would yield the neutral level of output.

The article next turns to understanding the mechanics of the New IS-LM model. Proponents of IS-LM modeling typically stress that sticky prices are central to understanding macroeconomic activity (e.g., Mankiw [1990]) so that the discussion begins in Section 4 with this topic. Firms are assumed to set prices and adjust quantity in response to changes in demand. But in the New IS-LM model, firms are assumed to be forward-looking in their price-setting, in line with research that begins with Taylor (1980). Forward-looking price-setting has major effects on the linkage between nominal disturbances and economic activity, endowing the model with a mix of Keynesian and Classical implications. Section 5 considers the long-run limits on monetary policy given this “supply side” specification and several related topics.

Turning to the aggregate demand side, the new model’s IS schedule is also forward looking. Section 6 starts by discussing why this is the inevitable attribute of optimizing consumption-investment decisions and then considers some macroeconomic implications of the new model’s IS schedule.

The macroeconomic equilibrium of the New IS-LM model is employed to analyze three key issues that are relevant to monetary policy. Section 7 considers limits on interest rate rules. Section 8 highlights how monetary policy can produce short-run departures of output from its capacity level, either as a result of monetary shocks or as a result of a policy rule which differs from the neutral rules developed in Section 3. It also considers the origin and nature of the tradeoff between inflation and output variability that is present in this model. The article is completed by a brief concluding section.

1. THE EVOLUTION OF THE IS-LM MODEL

Before detailing the model, it is useful to briefly review the historical process that has led to its development and influences its current uses. Since the 1930s, variants of the IS-LM model have been the standard framework for macroeconomic analysis. Initially, Hicks’s (1937) version was used to explain how output and interest rates would be affected by various shocks and alternative policy responses. Subsequent developments broadened the range of issues that could be studied with the model, notably the introduction of an aggregate production function and a labor market by Modigliani (1944). With the rise of quantitative frameworks for monetary policy analysis—such as the PennFRB-MIT model, which was employed by the Federal Reserve System—the role of the IS-LM model changed in a subtle manner. After detailed explanations were worked out in these policy laboratories, the IS-LM model was used to give a simple account of the findings.
While the initial IS-LM model did not determine how the price level evolved through time, the addition of a price equation—or a wage/price block that featured a Phillips (1958) curve—made it possible to explore the implications for inflation.\(^1\) The simultaneous occurrence of high inflation and high unemployment in the 1970s led macroeconomists to question this aspect of theoretical and quantitative macromodels. Further, during the rational expectations revolution spurred by Lucas (1976), fundamental questions were raised about the value of the IS-LM model and the related quantitative macroeconomic policy models. The IS-LM model was portrayed as being fatally inconsistent with optimizing behavior on the part of households and firms (Lucas 1980). The quantitative macropolicy models were criticized for not using microfoundations as a guide to the specification of estimable equations and also for avoiding central issues of identification (Sims 1980, Sargent 1981). The rational expectations revolution suggested that new macroeconomic frameworks were necessary—both small analytical frameworks like the IS-LM model and larger quantitative macropolicy models—and that these would lead to a substantial revision in thinking about the limits on monetary policy and the role of monetary policy.

One initial attempt at updating the IS-LM model was initiated in Sargent and Wallace (1975), who incorporated a version of the aggregate supply theory developed by Lucas (1972, 1973) in place of the Phillips curve or wage/price block. According to this rational expectations IS-LM model, systematic monetary policy could not influence real economic activity, although monetary shocks could cause temporary departures of output from its capacity level. This finding that systematic monetary policy was irrelevant led the related literature to be described, by some, as the New Classical macroeconomics. Sargent and Wallace also used their framework to argue against use of the nominal interest rate as the instrument of monetary policy—suggesting that this practice was inconsistent with a unique macroeconomic equilibrium. While this rational expectations IS-LM model was subsequently used to clarify issues of importance for monetary policy—for example, Parkin (1978) and McCallum (1981) showed that an appropriate nominal anchor could allow the interest rate to be used as the instrument of monetary policy—it did not gain widespread acceptance for three reasons. First, some economists—particularly macroeconomic theorists—saw the model as flawed, because its lack of microfoundations led it to lack the behavioral consistency conditions which are the inevitable result of optimization and the expectational considerations which are at the heart of dynamic economic theory. Second, other economists—particularly applied macroeconomists—were suspicious of the

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\(^1\) With this addition, the Hicksian setup was sometimes and more accurately called an IS-LM-PC model, but it has been more commonly referred to by its shorter title, as will be the practice in this article.
model because it suggested that departures of output from capacity should be serially uncorrelated. Third, many economists—including central bankers—remained convinced that the systematic choices of the monetary authority were important for the character of economic fluctuations and thus rejected the model due to the “policy irrelevance” implication.

In recent years, there has been the development of small, optimizing macro models that combine Classical and Keynesian features in a “New Neoclassical Synthesis.” The New IS-LM model is an outgrowth of this more general research program and is thus designed to incorporate the major accomplishments of the rational expectations revolution, including a more careful derivation from microfoundations, while retaining the stark simplicity that made the earlier IS-LM frameworks much employed tools. One important use of the New IS-LM model is to communicate results from other, more complicated macroeconomic models that are relevant to monetary policy. For example, Kerr and King (1996) first used the core equations of the New IS-LM model to exposit issues involving interest rate rules for monetary policy that had arisen in my research on small, fully articulated macroeconomic models with sticky prices and intertemporal optimization (King and Watson 1996; King and Wolman 1999). The current article shows how the New IS-LM model is also useful in expositing many issues that arise in these sorts of small, fully articulated models and also in larger quantitative macroeconomic models that are currently employed for monetary policy analysis, including the new rational expectations framework of the Federal Reserve (the FRB-US model) and the various U.S. and international models developed by Taylor (1993). In fact, in using the model to discuss the implications of sticky prices, restrictions on interest rate policy rules, and the trade-off between the variability of inflation and output, the article will touch repeatedly on themes which have been central parts of Taylor’s research program.

2. THE NEW IS-LM MODEL

Like its predecessors, the New IS-LM model is a small macroeconomic model designed to describe the behavior of economy-wide variables that enter in most discussions of monetary policy. There are five endogenous variables: the log level of real output/spending $y$, the log price level $P$, the real interest rate $r$, the inflation rate $\pi$, and the nominal interest rate $R$.

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2 See Goodfriend and King (1997) for a detailed discussion of these developments.

3 Bernanke and Woodford (1997) and Clarida, Gali, and Gertler (1999) have since made similar use of essentially the same framework to study various monetary policy issues. Related analyses using variations on the New IS-LM approach include McCallum and Nelson (1999) and Koenig (1993a,b); these authors use an alternative approach to aggregate supply.

4 The New IS-LM model is most frequently presented in discrete time so as to keep the mathematical analysis as simple as possible (see Kimball [1995] for a continuous time analysis of
The Core Equations

Three speciﬁcations are present in all of the recent papers that employ the New IS-LM model. These are an IS equation, a Fisher equation, and a Phillips curve equation.

The forward-looking IS equation makes current real spending \( y_t \) depend on the expected future level of real spending \( E_t y_{t+1} \) and the real interest rate \( r_t \). There is also an aggregate demand shock \( x_{dt} \): a positive \( x_{dt} \) raises aggregate spending at given levels of the endogenous determinants \( E_t y_{t+1} \) and \( r_t \).

\[
IS : y_t = E_t y_{t+1} - s[r_t - r] + x_{dt} \tag{1}
\]

The parameter \( s > 0 \) determines the effect of the real interest rate on aggregate demand: If \( s \) is larger then a given rise in the real interest rate causes a larger decline in real demand. The parameter \( r > 0 \) represents the rate of interest which would prevail in the absence of output growth and aggregate demand shocks. The new IS equation is described as forward-looking because \( E_t y_{t+1} \) enters on the right-hand side.

The Fisher equation makes the nominal interest rate \( R_t \) equal to the sum of the real interest rate \( r_t \) and the rate of inﬂation that is expected to prevail between \( t \) and \( t+1, E_t \pi_{t+1} \).

\[
F : R_t = r_t + E_t \pi_{t+1} \tag{2}
\]

This conventional speciﬁcation of the Fisher equation omits any inﬂation risk premium in the nominal interest rate.

The expectational Phillips curve relates the current inﬂation rate \( \pi_t \) to expected future inﬂation \( E_t \pi_{t+1} \), the gap between current output \( y_t \) and capacity output \( \bar{y}_t \), and an inﬂation shock \( x_{\pi t} \).

\[
PC : \pi_t = \beta E_t \pi_{t+1} + \psi(y_t - \bar{y}_t) + x_{\pi t} \tag{3}
\]

The parameter \( \beta \) satisﬁes \( 0 \leq \beta \leq 1 \). The parameter \( \psi > 0 \) governs how inﬂation responds to deviations of output from the capacity level. If there is a larger value of \( \psi \) then there is a greater effect of output on inﬂation; in this sense, prices may be described as adjusting faster—being more ﬂexible—if \( \psi \) is greater.

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5 The notation used in this case will carry over to the rest of the article: shocks are called \( x \) and their nature is identiﬁed with a subcript, such as \( d \) for demand in this case. The exact statistical properties of \( x_{dt} \) are not speciﬁed at present, but they are taken to be stationary random variables with a zero mean.

6 See McCallum and Nelson (1999a) for additional discussion of this issue.
Using the definition of the inflation rate $\pi_t = P_t - P_{t-1}$, this specification might alternatively have been written as $P_t = P_{t-1} + \beta E_t \pi_{t+1} + \phi(y_t - \bar{y}_t) + x_{\pi_t}$. This alternative form highlights why (3) is sometimes called a “price equation” or an “aggregate supply schedule.” It is a price equation in the sense that it is based on a theory of how firms adjust their prices, as discussed further in Section 4 below. It is an aggregate supply schedule because it indicates how the quantity supplied depends on the price level and other factors. But this article uses the Phillips curve terminology because this is the dominant practice in the new and old IS-LM literature.

The relationship between the output gap and the steady-state rate of inflation gap is given by $y - \bar{y} = 1 - \beta \phi \pi$ according to this specification. In fact, experiments with fully articulated models that contain the structural features which lead to (3)—including those of King and Wolman (1999)—suggest a negligible “long-run effect” at moderate inflation rates. Prominent studies of the monetary policy implications of the New IS-LM model—including that of Clarida, Gali, and Gertler (1999)—accordingly impose the $\beta = 1$ condition in specifying (3). In this article, $\beta$ will be taken to be less than but arbitrarily close to one.

**Money Demand and Monetary Policy**

To close the model and determine the behavior of output, the price level and other variables, it is necessary to specify the monetary equilibrium condition. Researchers presently adopt two very different strategies within the literature on the New IS-LM model.

**Specifying money demand and money supply.** Under this conventional strategy, the money demand function is typically assumed to take the form

$$MD : M_t - P_t = \delta y_t - \gamma R_t - x_{vt}$$

(4)

with $M_t - P_t$ being the demand for real balances. This demand for money has an income elasticity of $\delta > 0$ and an interest semielasticity of $-\gamma < 0$. There is a shock which lowers the demand for money, $x_{vt}$: this is a shock to velocity when $\delta = 1$ and $\gamma = 0$.

The money supply function is assumed to contain a systematic monetary policy component, $f_{Mt}$, and a shock component $x_{Mt}$:

$$MS : M_t = f_{Mt} + x_{Mt}.$$ 

(5)

The monetary authority’s systematic component may contain responses to the current state, lagged or expected future level of economic activity. Taken together, these equations determine the quantity of money and also provide

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7 Sargent (1973) showed that this semilogarithmic form is very convenient for small rational expectations models.
Figure 1

a. The New IS Schedule

b. The New Phillips Curve
one additional restriction on the comovement of output, the price level and interest rates.

*Specifying an interest rate rule for monetary policy.* An alternative—and increasingly popular—strategy is to simply specify an interest rate rule for monetary policy,

$$IR : R_t = f_{Rt} + x_{Rt}, \quad (6)$$

which contains a systematic component, $f_{Rt}$, and a shock component $x_{Rt}$.

Under this rule, the quantity of money is demand-determined at the $R_t$, which is set by the monetary authority. Thus, the behavior of the money stock can be deduced, from (4) and (6), as $M_t - P_t = \delta y_t - \gamma [f_{Rt} + x_{Rt}] - x_{vt}$. But since the stock of money is not otherwise relevant for the determination of macroeconomic activity, some analysts proceed without introducing money at all.8

**What Is New about This Model?**

The answer to this question depends on the chosen starting point in the history of macroeconomic thought.

Relative to the original model of Hicks, the New IS-LM model is different in that it makes the price level an endogenous variable, which is influenced by exogenous shocks and the monetary policy rule. In the language of Friedman (1970) and other monetarists, the New IS-LM model views the price level as a monetary phenomenon rather than as an unexplained institutional phenomenon. In terms of formal modeling, the idea that the price level is a monetary phenomenon is represented in two ways. First, the model cannot be solved for all of the endogenous variables without the specification of a monetary policy rule. Second, under a money stock rule, even though some individual prices are sticky in the short run, the price level responds to exogenous, permanent changes in the level of the money stock in both the short run and the long run. But, since the 1970s, textbook presentations of the IS-LM model have added a pricing block or aggregate supply schedule, which makes the price level endogenous.

The New IS-LM model also incorporates expectations in ways that the traditional IS-LM model did not. But the rational expectations IS-LM model of Sargent and Wallace (1975) also incorporated the influence of expectations of inflation into both the Fisher equation and the aggregate supply schedule. Modern textbook treatments discuss these expectations mechanisms in detail.

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8 For example, Kerr and King (1996) discuss how one can manipulate an “IS model” to study limits on interest rate rules and Clarida, Gali, and Gertler (1999) conduct their discussion of the “science of monetary policy” within this model without specifying the supply and demand for money.
Figure 1 shows two of the New IS-LM model’s key equations. As in modern textbooks, there is an IS curve which makes output depend negatively on the (real) interest rate and a Phillips curve or aggregate supply schedule which makes output depend positively on the inflation rate. Relative to these presentations, the New IS-LM model differs (i) in the stress that it places on expectations in both aggregate demand and aggregate supply and (ii) in the particular ways in which expectations are assumed to enter into the model. In particular, the new IS schedule (1) identifies expected future income/output as a key determinant of current output, while this is missing in the Sargent-Wallace model. The new aggregate supply schedule or Phillips curve (3) identifies expected future inflation as a key determinant of current inflation, while in the Sargent-Wallace model it is yesterday’s expectation of the current inflation rate that is relevant for supply.

These channels of influence are highlighted in Figure 1. In panel a of the figure, an increase in expected future output shifts the IS curve to the right, requiring a higher real interest rate at any given level of output. In panel b of the figure, an increase in expected future inflation shifts the Phillips curve to the left, requiring a higher current inflation rate at any given level of output.

However, while it is possible to express these behavioral equations in familiar graphical ways, the reader should not be misled into thinking that macroeconomic analysis can be conducted by simple curve-shifting when expectations are rational in the sense of Muth (1961). Instead, it is necessary to solve simultaneously for current and expected future variables, essentially by determining the complete path that the economy is expected to follow. Once this path is known, it is possible to return to the individual graphs of the IS curve or the Phillips curve to describe the effects of shocks or policy rules. But this is not the same as deriving the result by shifting the curves.

3. NEUTRAL MONETARY POLICY

If the monetary authority’s objective is to stabilize real economic activity at the capacity level, the New IS-LM model provides a direct case for an inflation-targeting monetary policy.

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9 Expectations are assumed to be rational in Muth’s sense in this article and related literature. It is also worth noting that this article and much of the related literature also assumes that there is full current information and that monetary policy rules are credible.

10 This point is related to the discussion in King (1993), where I argued that the traditional IS-LM model is flawed due to its treatment of expectations and could not be resurrected by the New Keynesian research program. In particular, while I noted that “every macroeconomic model contains some set of equations that can be labelled as its IS and LM components, since these are just conditions of equilibrium in the goods and money markets,” I also stressed that “while some of us may choose to use the IS-LM framework to express results that have been discovered in richer models, it is not a vehicle for deriving those results. To simplify economic reality sufficiently to use the IS-LM model as an analytical tool, economists must essentially ignore expectations....”
Inflation Implications

In the New IS-LM model, there is a direct link between the objective of keeping output at a capacity level—which Goodfriend and King (1997) call a neutral monetary policy objective—and the dynamics of inflation. Setting $y_t = \bar{y}_t$ in (3) and solving this expression forward implies that

$$\pi_t = \beta E_t \pi_{t+1} + x_{\pi_t} = \sum_{j=0}^{\infty} \beta^j E_t x_{\pi,t+j}. \tag{7}$$

This solution has three direct implications.

The case for price stability: If there are no inflation shocks ($x_{\pi_t} = 0$ for all $t$) then the solution is that the inflation rate should always be zero. This is a striking, basic implication of the New IS-LM model. Reversing the direction of causation, it means that a central bank which keeps the price level constant also makes output always equal to the capacity level. Finally, it means that shocks to aggregate demand such as $x_{dt}$ and to the determinants of capacity output $y_t$ do not affect the price level under a neutral monetary policy regime.

The case for simple inflation targets: If there are inflation shocks, there continues to be an average inflation rate of zero under a neutral monetary policy. However, as Clarida, Gali, and Gertler (1999) stress, the New IS-LM model suggests that there may be sustained departures from the zero long-run inflation target as a result of inflation shocks. For example, if the shock term is a first-order autoregression, $x_{\pi_t} = \rho x_{\pi,t-1} + e_{\pi_t}$, then the solution for the neutral inflation rate is

$$\bar{\pi}_t = \frac{1}{1 - \beta \rho} x_{\pi_t} = \rho \bar{\pi}_{t-1} + \frac{1}{1 - \beta \rho} e_{\pi_t},$$

so that the inflation target inherits the persistence properties of the inflation shock. If the persistence parameter $\rho$ is positive, then a higher-than-average current inflation target implies that there will be, on average, a higher-than-average inflation target in the future.

In this setting, a central bank must more actively manage inflation in order to keep output at its capacity level. The New IS-LM model, however, implies that many shocks do not affect the inflation rate if it is managed to keep output at capacity, including aggregate demand shocks $x_{dt}$, shifts in determinants of capacity output $\bar{y}_t$, and shocks to the demand for money $x_{mt}$.

Appraising This Policy Implication

This strong policy conclusion raises a number of questions, which are considered in turn. In trying to answer these questions, we encounter a natural

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11 Recall that the inflation shocks are assumed to have a zero mean.
limitation of IS-LM models, new and old. Since these models are not built up from microfoundations, the answers frequently will require stepping outside the confines of the model to discuss other, related research.

Is this result a special one or does it hold in other related models? In fact, King and Wolman (1996) found that a constant inflation target causes real activity to remain at essentially the capacity level when there are changes in productivity or money demand within a fully articulated, quantitative model (a setting where sticky prices, imperfect competition and an explicit role for monetary services were added to a standard real business cycle model). The generality of this conclusion is suggested by the fact that Rotemberg (1996) was led to call it a “mom and apple pie” result in his discussion of King and Wolman (1996).12

What is capacity output? When explicit microfoundations are laid out, it is potentially possible to define a measure of capacity output more precisely. Goodfriend and King (1997) followed this approach—within a class of models with sticky prices, imperfect competition, and flexible factor reallocation—to identify capacity output as the level of output which would obtain if all nominal prices were perfectly flexible, but distortions from imperfect competition remained present in the economy.

Is stabilization at capacity output desirable? If output is inefficiently low due to monopoly or other distortions, then it may not be optimal to always keep output at its capacity level: optimal monetary policy may seek to produce deviations of output from capacity in response to underlying shocks. To study this issue carefully, though, it is again necessary to develop microeconomic foundations and to consider the design of monetary policies which maximize the welfare of agents in response to various shocks (as with the productivity shocks analyzed in Ireland [1996]). Studying a fully articulated economy with multiperiod price stickiness, King and Wolman (1999) show it is efficient—in the sense of maximizing welfare—to fully stabilize the price level and to keep output at its capacity level in response productivity shocks.13

**Economic Activity under Neutral Policy**

In the analysis above, the Phillips curve (3) was used to determine the behavior of inflation which is consistent with output being at its capacity level \( y_t = \bar{y}_t \). The other equations of the model economy then restrict the behavior of the remaining variables.

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12 He also verified that it held in other, related fully articulated models (Rotemberg and Woodford 1997, 1999).
13 See also Goodfriend and King (1997) and Rotemberg and Woodford (1997).
Given that output is at its capacity level, the IS curve then implies that the real rate of interest is
\[ r_t = \frac{1}{s} [E_t \bar{y}_{t+1} - \bar{y}_t + x_{dt}] \]  
(8)
This is a neutral or “natural” real rate of interest, the idea of which is developed in more detail in Section 5.2 below. The real rate of interest is positively affected by growth in capacity output \( E_t \bar{y}_{t+1} - \bar{y}_t \) and by aggregate demand shocks \( x_{dt} \).

Taking this natural rate of interest \( r_t \) together with expected inflation, the Fisher equation (2) then implies that the nominal interest rate is
\[ R_t = r_t + E_t \pi_t + \frac{1}{s} \]  
(9)
That is, a neutral interest rate policy must make the nominal interest rate vary with the natural rate of interest and the inflation target (7). For example, if the real economy is expected to display strong real growth in capacity output, then the nominal interest rate must be raised.\(^{14}\)

Finally, the money demand function (4) implies that the stock of money evolves according to
\[ M_t = (\pi_t + P_{t-1}) + \delta \bar{y}_t - \gamma \bar{R}_t - x_{ut} \]  
That is, money growth obeys
\[ \bar{M}_t - \bar{M}_{t-1} = \pi_t + [\delta (\bar{y}_t - \bar{y}_{t-1}) - \gamma (\bar{R}_t - \bar{R}_{t-1}) - (x_{ut} - x_{u,t-1})] \]  
(10)
which is the sum of the chosen inflation target and the change in the real private demand for money.

**Implementation via a Money Stock Rule**

One way to implement a neutral monetary policy is via a money stock rule. The solution (10) indicates that in order for the economy to stay at capacity output, the money stock must respond to the state of the economy. In particular, the growth of the neutral money stock is a complicated function of the exogenous variables of the model. Money growth must move one-for-one with the target rate of inflation \( \pi_t \), which in turn depends on the inflation shock \( x_{\pi t} \). Money growth must also accommodate the changes in real demand for money brought about by growth in the capacity level of output \( \bar{y}_t \), as stressed by Ireland (1996). It must also accommodate shocks to the demand for money and changes in the neutral nominal interest rate (which in turn depend on changes in the expected growth in capacity output and changes in the inflation target from (7)). This policy rule involves choices in the general money supply function (5), namely that there are no money supply shocks \( x_{Mt} = 0 \) and that the systematic

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\(^{14}\) Unless there is simultaneously a negative price shock for some reason.
The component of policy is given by

\[ f_{Mt} = M_{t-1} + \pi_t + [\delta(\bar{y}_t - \bar{y}_{t-1}) - \gamma(\bar{R}_t - \bar{R}_{t-1})] - (x_{vt} - x_{t-1}). \]

Under this rule, the central bank is not responding directly to output, inflation and so forth. Instead, it is responding to the fundamental determinants of economic activity.\(^{15}\) Further, implicit in treating the solution (10) as a policy rule is the statement by the monetary authority, “if inflation deviates from the neutral level then no adjustment in the path of the money stock will occur.” In the rational expectations equilibrium of the New IS-LM model, this statement turns out to be sufficient to assure that no departures of inflation from the neutral inflation rate ever occur.

### Implementation via an Interest Rate Rule

There has been a great deal of research on interest rate rules in recent years for at least three reasons. First, as argued by Goodfriend (1991), this research focus matches well with the fact that the Federal Reserve actually implements monetary policy by choosing the setting of the federal funds rate, a very short-term nominal interest rate. Second, as shown by Taylor (1993), some simple interest rate rules appear to yield a quantitative match with the behavior of the FRS over various time periods. Third, there are interesting conceptual issues that arise regarding the determination of macroeconomic activity under an interest rate rule.

In looking for an interest rate rule that would yield the neutral level of output, a reasonable first idea would be to select the interest rate solution (9). In the New IS-LM model, as in other many frameworks considered by monetary economics dating back at least to Wicksell, this choice would not be enough to assure that the neutral level of real activity would occur. It might, but other levels of economic activity could also arise. One way of thinking about why multiple equilibria may occur is that money is demand-determined under an interest rate rule, so that the monetary authority is implicitly saying to the private sector, “any quantity of money which you desire at the specified nominal interest rate \( \bar{R} \) will be supplied.”

To eliminate the possibility of multiple equilibria, it is necessary for the monetary authority to specify how it would behave if the economy were to depart from the neutral level. For example, a specific interest rate rule—which responds to deviations of inflation from neutral inflation—is

\[ R_t = \bar{R}_t + \tau(\pi_t - \bar{\pi}_t) = [\bar{r}_t + E_t\bar{\pi}_{t+1}] + \tau(\pi_t - \bar{\pi}_t). \]

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\(^{15}\) From this standpoint, it is clear that the assumption above—that the central bank and other actors have complete information about the state of the economy—is a strong one.
By specifying $\tau > 0$ then, the monetary authority would be saying, “if inflation deviates from the neutral level, then the nominal interest rate will be increased relative to the level which it would be at under a neutral monetary policy.” If this statement is believed, then it may be enough to convince the private sector that the inflation and output will actually take on its neutral level.

Thus, a substantial amount of work on the New IS-LM model has concerned finding the conditions which assure a unique equilibrium. Section 7 below exemplifies this research. For the interest rate rule above, it shows that one way of assuring a unique equilibrium is to have a strong positive response, $\tau > 1$, as Kerr and King (1996) previously stressed. But, it also stresses that (i) a rule which specifies a strong negative responses to current inflation may also lead to a unique equilibrium, and (ii) that strong positive responses may lead to multiple equilibria if policy is forward looking.

4. PRICE STICKINESS AND ECONOMIC ACTIVITY

Milton Friedman (1970, p. 49) focused attention on the importance of determining how a change in nominal income is divided between responses of real output and the price level at various horizons. In the New IS-LM model, changes in monetary policy can affect real output because there is price stickiness of a sort long stressed in Keynesian macroeconomics. But since stickiness of prices is modeled in a New Keynesian manner—with pricing rules based on firms’ optimizing behavior—there are some novel implications for the dynamics of real output and the price level.

The Structure of the New Phillips Curve

The New Keynesian research on aggregate supply was designed to produce an “an old wine in a new and more secure bottle” by providing a better link between inflation and real activity, with microfoundations that earlier Keynesian theories lacked.16 Four key ideas are stressed in the twin volumes edited by Mankiw and Romer (1991) on this topic: costly price adjustment, asynchronous price adjustment, forward-looking price setting, and monopolistic competition.

These ideas have been implemented in a variety of applied macroeconomic models beginning with Taylor’s (1980). All of these sticky price models contain two central ingredients. First, since price adjustment does not take place simultaneously for all firms, the price level is a weighted average of current and past prices. Second, since firms have market power and recognize that their nominal prices may be fixed for some time, the models display a

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richer, forward-looking pattern of price-setting than that which arises in the standard, static monopoly pricing model.

These general ideas have been implemented in a variety of different approaches to pricing. Models in the style of Taylor (1980) assume that firms adjust their prices every \( J \) periods, where \( J \) is assumed to be fixed. Calvo (1983) proposed an alternative stochastic adjustment model, in which each firm has a constant probability of being able to adjust its price every period. The Calvo model has been incorporated into the New IS-LM model for four reasons. First, it seems to capture a key aspect of price dynamics at the level of individual firms, which is that these involve discrete adjustments which occur at irregularly spaced intervals of time. Second, it leads to price level and price-setting expressions which can be readily manipulated analytically. Third, this approach has provided a tractable base for recent studies which have provided empirical support for the New Keynesian approach to pricing. Fourth, it also turns out to be observationally equivalent at the aggregate level to a popular alternative model of price adjustment—the quadratic cost of adjustment model for prices—as shown by Rotemberg (1987). At the same time, the Calvo and Taylor models are similar in the broad predictions developed in this section, so that the increased tractability comes at a small apparent cost.

In the Calvo model, the microeconomic extent of price stickiness is determined by a single parameter, the probability that a firm will be unable to adjust its price in a given period, which will be called \( \eta \). Since a firm’s adjustment probabilities do not depend on the duration of its interval of price fixity, there is a probability \( \eta^j \) of being stuck in period \( t + j \) with the price that is set at \( t \) and the probability of first adjusting in \( j \) periods is \( (1 - \eta)(1 - \eta^{j-1}) \). Accordingly, the expected duration of price stickiness is 
\[
0 + 1(1 - \eta) + 2(1 - \eta)\eta + \ldots + (j - 1)(1 - \eta)\eta^{j-2} + \ldots + \frac{1}{\eta},
\]
which depends on \( \eta \) in a convenient manner.

This degree of microeconomic stickiness plays a role in both the nature of the price level and the nature of the pricing decision. In the model economy, there are many, essentially identical firms which face stochastic individual opportunities to adjust prices. With a large number of firms in the economy,

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17 Recent interesting empirical studies of this approach include Roberts (1995), Gali and Gertler (1999), and Sbordonne (1998).

18 Rotemberg (1982) used the quadratic cost of adjustment model to study U.S. price dynamics. Generalizations of this approach, developed in Tinsley (1993) are employed in the Federal Reserve System’s new rational macroeconometric model.

19 However, Wolman (2000) stresses that they can be quite different in some detailed implications for price dynamics.

20 This model is sometimes criticized on a number of grounds. First, the probability of being able to adjust price is independent of the time since the last price adjustment, so that firms face some chance of being trapped with a fixed price for a very long time. Second, the probability of price adjustment is exogenous. Dotsey, King, and Wolman (1999) study time-dependent and state-dependent pricing that overcomes each of these objections.
the fraction of firms adjusting price in a period is equal to the probability of price adjustment \((1 - \eta)\) and the fraction of firms stuck with a price that is \(j\) periods old is \((1 - \eta)\eta^j\).

A backward-looking price level: In general, the price level is an average of prices. In any model with staggered price-setting, some of these prices will be newly set by firms which are adjusting prices and some will have been set in prior periods. Taking \(P^*_t\) to be the price chosen by all adjusting firms in period \(t\) and \(P_t\) to be the price level as above, the following simple loglinear specification captures the idea that the price level is an average of prices:

\[
P_t = (1 - \eta) \sum_{j=0}^{\infty} \eta^j P^*_{t-j} = \eta P_{t-1} + (1 - \eta) P^*_t.
\] (11)

The second equality derives from the definition of the lagged price level: it is a convenient expression for many analytical purposes. Notably, (11) can be rewritten as a partial adjustment mechanism, \(P_t - P_{t-1} = (1-\eta)(P^*_t - P_{t-1})\), so that the price level responds only gradually when \(P^*_t\) is raised above \(P_{t-1}\) with the extent of price level adjustment just being the microeconomic probability of price adjustment.

Forward-looking price-setting: A key aspect of New Keynesian models is that firms know that their prices may be sticky in future periods. For this reason, they rationally consider future market conditions when they set prices. The idea of forward-looking price-setting by firms may be captured with the specification

\[
P^*_t = (1 - \beta \eta) \sum_{j=0}^{\infty} (\beta \eta)^j E_t[\psi_{t+j} + P_t] + x_{Pt},
\] (12)

which can be developed from the Calvo model as in Rotemberg’s survey of New Keynesian macroeconomics (1987). The price chosen by firms adjusting at date \(t\), \(P^*_t\), is a distributed lead of nominal marginal cost (real marginal cost is \(\psi_t\) so that nominal marginal cost is \(\psi_t + P_t\) in this loglinear world). There are two parts to the discounting: \(\beta\), which represents a conventional market discount factor (so that \(\beta\) is very close to, but less than one) and \(\eta\), which reflects the fact that firms know that there is a lower probability of being stuck with today’s price as they look further ahead. The shock \(x_{Pt}\) is a structural shock to the level of prices set by firms in period \(t\) and its relationship to the inflation shock introduced earlier in (3) will be determined later. The second line of (12) involves using the definition of \(P^*_{t+1}\) to eliminate the distributed lead of future nominal marginal cost.

The forward-looking pricing rule (12) implies that a current change in nominal marginal cost affects \(P^*_t\) very differently if it is expected to be permanent than if it is expected to be temporary. If nominal marginal cost is
expected to be the same in all future periods, then there is a one-for-one effect of its level on $P_t^\ast$, since $(1 - \beta \eta) \sum_{j=0}^{\infty} (\beta \eta)^j = 1$: a firm will raise its price proportionately if changes in marginal cost are expected to be permanent. By contrast, $P_t^\ast$ will respond by a smaller amount, $(1 - \beta \eta)$, if the change in marginal cost is expected to be temporary, affecting only date $t$ marginal cost.

**Output and demand:** New Keynesian macroeconomists stress that an optimizing, monopolistically competitive firm will rationally supply additional output in response to an expansion of demand, rather than rationing customers, when its price is sticky (see, for example, Romer [1993]). This output response is profitable so long as the firm’s sticky nominal price is greater than its nominal marginal costs. The specification (3) assumes that this is true over the range of disturbances considered in the New IS-LM model.

**A heroic assumption:** To generate (3), a final—heroic—assumption is needed. In particular, assume that real marginal cost is positively related to the output gap, with the parameter $h$ being the elasticity of this response. That is,

$$\psi_t = h(y_t - \bar{y}_t).$$

The parameter $h$ is positive under conventional assumptions about the aggregate production function and factor supply elasticities. Real marginal cost would necessarily rise with the level of economic activity if the economy had some fixed factors (such as a predetermined capital stock) or if higher real wage rates were necessary to induce workers to supply additional hours.

The specification involves a shortcut that avoids modeling of the labor market, which is complicated, difficult, and controversial. Some fully articulated models suggest that (14) is a useful approximation and also suggest particular values of $h$. Others may suggest that this assumption is a weakness of the New IS-LM model.

**Putting the elements together:** Combining (11), (12), and (14), as is done in Appendix A, leads to

$$P_t - P_{t-1} = \beta(E_t P_{t+1} - P_t)$$

$$\quad + [h \frac{(1 - \eta)(1 - \beta \eta)}{\eta}] (y_t - \bar{y}_t)$$

$$\quad + \frac{(1 - \eta)}{\eta} [x_{Pt} - \beta \eta E_t x_{P,t+1}].$$

This is identical to (3), but there is an explicit linking of the parameter $\psi = h \frac{(1 - \eta)(1 - \beta \eta)}{\eta}$ to deeper parameters of the price adjustment process and the elasticity of marginal cost with respect to the output gap.  

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21 There is also a linking of the inflation shock $x_{\pi t}$ to underlying shocks to the price setting equation $x_{Pt}$ above, which is $x_{\pi t} = (\frac{1 - \eta}{\eta})[x_{Pt} - \beta \eta E_t x_{P,t+1}]$. This latter linkage is important
Long-run neutrality: The form of the equation (15) highlights the fact that a purely nominal disturbance, which permanently affects the level of prices at all dates by the same amount, will have no effect on the level of real economic activity within the New IS-LM model. Specifically, if the price level is constant at all dates \((E_t P_{t+1} = P_t = P_{t-1} = P)\) and there are no inflation shocks \((\pi_t = 0)\), then output is equal to capacity \((y_t = \bar{y}_t)\).

The Nonneutrality of Nominal Shocks

Many New Keynesian authors, including Taylor (1980) and Mankiw (1990), have stressed that the new Phillips curve implies that nominal disturbances can have effects on real economic activity because prices are sticky and output is demand-determined. In this subsection, the implications of price stickiness for the division of nominal income changes into prices and output are explored.

Implications from analytical solutions for output and prices: Suppose that nominal income is exogenous and governed by the simple rule \(Y_t - Y_{t-1} = \rho(Y_{t-1} - Y_{t-2}) + x_Y t\) with \(x_Y\) being a series of “white noise” shocks.\(^{22}\) For simplicity, assume that capacity is expected to be constant through time at \(\bar{y}\) and that there are no price shocks.

Since (15) is a much-studied second order expectational difference equation, whose solution is reported in Appendix B of this article, it is easy to compute the solution for the price level. The solution takes the form

\[
P_t = \theta P_{t-1} + (1 - \theta)(1 - \beta \theta) \sum_{j=0}^{\infty} (\beta \theta)^j E_t(Y_{t+j} - \bar{y})
\]

\[= \theta P_{t-1} + (1 - \theta)(Y_{t-1} - \bar{y}) + \frac{1 - \theta}{1 - \theta \beta \rho}(Y_t - Y_{t-1})\]

where \(\theta\) is the smaller root of the equation \(\beta z^2 - [1 + \beta + \varphi]z + 1 = 0\), which may be shown to be between zero and one (see Appendix B). Further, since \(y_t = Y_t - P_t\), the model’s implications for output are readily calculated

\[y_t - \bar{y} = (\theta - \frac{1 - \beta \rho}{1 - \theta \beta \rho})(Y_t - Y_{t-1}) + \theta [y_{t-1} - \bar{y}]\]  (17)

There are several aspects of these solutions that warrant discussion. First, the coefficient \(\theta\) provides one measure of the degree of gradual price level

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\(^{22}\) There are two alternative ways to rationalize this. One is that there is a strong form of the quantity equation, with the money demand function (4) satisfying \(\delta = 1\) and \(\gamma = 0\) and the money supply equation (5) taking the form \(M_t = x_{Mt}\) with \(x_{Mt}\) being a random walk. Another is that the monetary authority follows a monetary policy rule which makes nominal income equal to an exogenous random walk.
adjustment at the macroeconomic level, since it indicates the extent to which
the past price level influences the current price level. This is different from the
extent of price stickiness $\eta$ at the microeconomic level, although increases in
$\eta$ lead to larger values of $\theta$. In this example, $\theta$ is influenced by the elasticity
of marginal cost $h$ as well as $\eta$. If inflation is more responsive to departures of
output from the capacity level, then the current price level becomes less sticky,
in the sense that it is less dependent on the past price level. (More specifically,
lower values of $\eta$ or higher values of $h$ lead to higher values of $\varphi$, which
in turn make for smaller solutions for $\theta$.) More generally, the importance of
predetermined prices to the current price level depends on the structure of the
entire macroeconomic model, i.e., it is a system property rather than a property
of just the equations of the “price block”, such as (11) and (12).23

Second, the degree of gradual price level adjustment is important for the
persistence of output fluctuations: $\theta$ enters (16) as the coefficient on the lagged
price level and enters (17) as the coefficient on the lagged output level. The
simplicity of this linkage reflects the fact that nominal income is evolving
exogenously in this model, but the general relationship between the extent of
gradual price level adjustment and the degree of output persistence also carries
over to richer setups.

Third, when the growth rate of nominal income is white noise (so that the
level of nominal income is a random walk), then $\theta$ also controls the split of
a change in nominal income between output and the price level. If prices are
more sticky, then nominal income changes have a greater effect on real output.

Fourth, when the growth rate of nominal income becomes more persistent,
then there is a larger effect of a surprise nominal income change on the price
level and a correspondingly smaller one on output. In fact, if the changes
in nominal income growth are permanent ($\rho = 1$) and market discounting is
small ($\beta = 1$) then the coefficient on $Y_t - Y_{t-1}$ in the price level equation
(16) becomes one and the coefficient in the output equation (17) becomes
zero. In this limiting situation, there is neutrality independent of the degree
of underlying price stickiness or the value of $\theta$ which is the indicator of the
gradual adjustment of the price level.

Implications from simulated responses to an increase in nominal income:
Figure 2 highlights some implications of (3) and a similar figure will be used
later to highlight some implications of the full New IS-LM model. In constructing
these figures, the time unit is taken to be one quarter of a year, which
is a conventional macroeconomic modeling interval. The response of the price

23 For this reason, it is affected by other parameters of the New IS-LM model when the full
model is solved, as in Section 8 below.
Figure 2

(a) Price Level

(b) Output

(c) Inflation
level and output will be measured in percentage points and the responses of inflation rates and interest rates will be measured in percent per annum.24

The solid line in panel b of Figure 2 shows the (impulse) response of output to an unexpected and permanent one percent increase in nominal income which takes place at date 0. Given that nominal income is exogenously one percent higher and since $y = Y - P$, the path for output is a mirror image of the path for prices: output is high when prices are low relative to the level of nominal income. On impact, output rises by $0 < \theta < 1$ percent, with the figure constructed under the assumption that $\theta = .20$.25 The price level rises by $0 < (1 - \theta) < 1$ percent, with the figure constructed under the assumption that $1 - \theta = .8$.

In subsequent periods, the price level gradually adjusts up to its new higher long-run level, while output falls back toward the capacity level. The speed of adjustment is again given by the value of $\theta$. There is an output effect of $\theta$ percent in the first period, $\theta^2$ in the second period, and so forth.

The inflation rate is shown by the solid line in panel c of Figure 2 and is given mathematically by differencing (16) under the assumption that $\rho = 0$, which results in $\pi_t = \theta \pi_{t-1} + (1 - \theta)(\Delta Y_t - \Delta \bar{Y})$.26 This is exactly the same solution as for the level of the output gap, so that a crude Phillips curve relationship of the form $\pi_t = (\frac{1 - \theta}{\theta})(y_t - \bar{Y})$ would work perfectly in this economy, given the assumed driving process. More generally, under a variety of driving processes, the model predicts that a rising price level (inflation) is positively associated with high output (relative to capacity).27 In this sense, the model can generate a traditional empirical Phillips correlation between inflation and real activity.

**Persistent Output Effects**

Many empirical studies suggest that business cycles arising from nominal disturbances display considerable persistence, lasting for many quarters. Taylor (1980) and other New Keynesian macroeconomists have suggested that price stickiness can lead to persistent effects of various disturbances on output.

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24 These conventional measurement choices will require some care when comparisons are made across the panels of the figures, as discussed further below.

25 This value of $\theta$ obtains when $\beta = .99$ and $\varphi = .05$, which are the parameter values used in sections below.

26 Since the inflation rate is stated at an annualized percentage rate of change, the .2 percentage point increase in the price level (shown in panel a of Figure 2) that occurs at the initial date corresponds to a $4 \times .2 = .8$ rise in the annualized inflation rate at the initial date (shown in panel c of Figure 2). By contrast, all of the mathematical relationships described in the text and appendices involve the quarterly inflation rate, i.e., the percentage change in the price level between $t - 1$ and $t$.

27 In particular, (17) implies that $\pi_t = \frac{1 - \theta}{\theta} \frac{1 - \beta \rho}{1 - \beta \rho \varphi} (y_t - \bar{Y})$. Thus, the slope of the Phillips curve depends negatively on the persistence of nominal income growth.
If persistent business cycles arise from changes in nominal income then (3) implies that nominal income must itself be persistent.\footnote{But, as discussed above, its growth rate cannot be too persistent or there will be no effect of a surprise change.}

To illustrate this point, (16) can be used to recompute the solution for the price level in the case of purely temporary variations in nominal income:

\[
P_t = \theta P_{t-1} + (1 - \theta)(1 - \beta \theta)(Y_t - \bar{y})
\]

and the comparable solution for output is

\[
y_t - \bar{y} = [(1 - (1 - \theta)(1 - \theta \beta))[Y_t - \bar{y}]
+ \theta [y_{t-1} - \bar{y}] - \theta [Y_{t-1} - \bar{y}].
\]

Thus, the effect of a purely temporary change in aggregate demand is to raise the price level somewhat and to raise output considerably on impact. But in subsequent periods, the economy will be stuck with a price level above its long-run level and have a smaller than capacity level of output. These dynamics are the dashed lines in panels a–c of Figure 2. In the impact period, the rise in nominal income produces a large increase in real output and a small increase in the price level, because price-setters correctly understand that the increase in nominal income is temporary. These analytical and simulation results highlight the fact that the pricing dynamics underlying the New Keynesian Phillips curve (3) do not themselves make business cycles persistent.\footnote{Chari, Kehoe, and McGrattan (2000) question whether even permanent movements in the money stock can cause persistent movements in output. In terms of the present model, they do so by imposing restrictions on \( \eta \) and \( h \).}

5. **LONG-RUN LIMITS ON MONETARY POLICY**

While allowing for short-run effects of nominal income on output, the New IS-LM model embodies the idea—put forward by Friedman (1968) and Phelps (1967)—that the monetary authority cannot engineer permanent departures of output from its capacity level. This idea was formalized in an earlier generation of rational expectations macromodels and is sometimes described as involving a vertical long-run Phillips curve. For this reason, this section considers the long-run limits on monetary policy under some alternative specifications of aggregate supply, ending with a discussion of the relationship in the New IS-LM model.

**The Price Surprise Supply Curve**

A previous generation of IS-LM macromodels incorporated an alternative “expectations augmented” Phillips curve (notably, see Sargent and Wallace...
In particular, these models used an aggregate supply curve of the “price surprise” form

\[ y_t - \bar{y}_t = l(P_t - E_{t-1} P_t), \quad (18) \]

where \( l \) is a positive parameter that governs the influence of increases in prices on output. This aggregate supply curve was rationalized by Lucas (1972a, 1973) as arising from incomplete information on the part of suppliers and by Phelps and Taylor (1977) as arising from sticky prices.

By subtracting the past price level from both \( P_t \) and \( E_{t-1} P_t \), an expectation Phillips curve quite similar to (3) can be derived:

\[ \pi_t = E_{t-1} \pi_t + \frac{1}{l} (y_t - \bar{y}_t). \quad (19) \]

Modern presentations of aggregate supply theory—such as those in the textbooks referenced above—stress two implications of (18) or (19) that were developed in the 1970s. First, if there is surprise expansion of demand—taken as in Section 4.2 to be an increase in nominal output \( y + P \)—then there is an increase in both output and the price level, with the split between these depending on the size of the supply elasticity \( l \). Thus, there is a positive relationship between inflation and output when there are shocks to nominal demand, i.e., a short-run correlation of the form discovered by Phillips. Second, any expected expansion of demand would raise expected and actual inflation by the same amount, thus neutralizing the real consequences.\(^{30}\) Thus, there is no long-run Phillips curve and the position of the short-run Phillips curve (in \( \pi, y_t \) space as in Figure 1) shifts with the expected rate of inflation.

**The Long-run Effect of Inflation**

The analysis of Section 4.2 demonstrated a similar link between temporary movements in inflation and output for the New IS-LM model’s Phillips curve (3), \( \pi_t = \beta E_{t-1} \pi_{t+1} + \varphi (y_t - \bar{y}_t) \). To explore the long-run implications in the new model, suppose that the economy is in an inflationary steady-state with \( \pi_t = E_{t-1} \pi_{t+1} = \pi \). Then, output will be

\[ y_t = \bar{y}_t + \frac{1 - \beta}{\varphi} \pi \]

so that we can say that the “long-run slope” of the Phillips curve is \( \frac{1 - \beta}{\varphi} \). This slope measures the response of output to changes in the long-run rate of inflation.

\(^{30}\) More specifically, the response of output can be calculated as follows. First, it is direct from (18) that \( E_{t-1} y_t = E_{t-1} \bar{y}_t \), i.e., that the economy is expected to be at capacity each period. Second, the response of real output can be calculated by using \( Y_t - E_{t-1} Y_t = (P_t - E_{t-1} P_t) + (y_t - E_{t-1} y_t) \) together with (18) to determine that \( P_t - E_{t-1} P_t = \frac{1}{l} (Y_t - E_{t-1} Y_t) \) and \( y_t - E_{t-1} y_t = \frac{1}{l} (Y_t - E_{t-1} Y_t) \).
inflation, after the economy has made a transition from one inflationary steady state to another. With \( \beta \) close to unity, then, (3) implies there is a negligible long-run slope to the Phillips curve.

Experiments with fully articulated models—such as that constructed by King and Wolman (1996)—suggest that the effect of inflation on output relative to capacity is very small. Accordingly, the condition \( \beta = 1 \) is imposed in the remainder of this section. The fully articulated models provide this quantitative result because (i) firms do not allow sustained inflation to have much effect on their monopoly profits and (ii) households do not allow sustained inflation to have much effect on their factor supply.

**Estimating the Long-run Effect**

Lucas (1972b) and Sargent (1971) showed that it was a subtle matter to estimate the long-run effect if the economy possessed an economy with an aggregate supply equation of the form (18) or a price equation of the form (19). Earlier, Gordon (1970) and Solow (1969) had proposed to estimate the long-run slope by specifying a hybrid model that nested expectational and nonexpectational forms of the Phillips curve. A simple form of this hybrid empirical model is

\[
\pi_t = g E_{t-1} \pi_t + \varphi (y_t - \bar{y}_t).
\]

With \( g < 1 \), this specification would imply a long effect of inflation on output, with a slope of \( \frac{1 - g}{\varphi} > 0 \). Solow and Gordon estimated this specification using adaptive expectations proxies for \( E_{t-1} \pi_t \), with the simplest variant of their procedure assigning \( E_{t-1} \pi_t = \pi_{t-1} \). In general, these studies found \( g \) to be significantly less than one through the 1970s.

Lucas and Sargent argued that this procedure was flawed in a setting with rational expectations. To illustrate their point, suppose that \( \pi_t = \rho \pi_{t-1} + e_t \) with \( \rho < 1 \). Then the rational expectations solution for inflation is \( \pi_t = \rho \pi_{t-1} + \varphi (y_t - \bar{y}_t) \). Application of the Solow-Gordon method would thus estimate that \( g = \rho < 1 \). Therefore, as stressed by Lucas and Sargent, the reduced form relationship would indicate an exploitable long-run trade-off, with a 1 percent higher inflation rate yielding \( \frac{1 - \rho}{\varphi} \) percentage points higher output, even though no tradeoff was actually present.

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31 The closely related model of Yun (1996) eliminates effects of sustained inflation by essentially allowing firms to index their nominal prices by the trend inflation rate.

32 There is a subtlety here, in that sticky price models built up from micro foundations can imply that there is a small effect of inflation on the volume of physical output—a quantity aggregate—while there is a larger effect of inflation on the value that households place on this output, due to relative price distortions that emerge when prices are sticky.

33 Lucas (1972b) worked with a supply schedule, while Sargent (1971) worked with a wage equation.
The Phillips curve (3) in the New IS-LM model also implies that there is this set of problems. Supposing as above that \( \pi_t = \rho \pi_{t-1} + e_t \) with \( \rho < 1 \) to illustrate this point, it follows that (3) implies that \( \pi_t = \frac{\psi}{1-\rho} (y_t - \bar{y}_t) \). An econometrician conducting Solow and Gordon’s test would estimate \( g = 0 \) and calculate that a 1 percent higher inflation rate would yield \( \frac{1-\rho}{\psi} \) percentage points of output.\(^{34}\)

Overall, the New IS-LM model thus embodies the consensus among macroeconomists that there is little long-run trade-off between inflation and real activity. It also suggests, as did earlier rational expectations for IS-LM models, that the existence of a short-run Phillips curve could mislead applied econometricians and central bankers into believing that there is a long-run trade-off.

**Disinflation Dynamics**

In terms of permanent changes in the inflation rate, such as that engineered by the Federal Reserve System during the “Volcker deflation” of 1979–1983 and more recently by other central banks around the world, there are some very classical implications of the Phillips curve, stressed by Buiter and Miller (1985), that is incorporated in the New IS-LM model. While these implications are not strictly the limits on monetary policy which are the focus of this section, they are related to the shifts in trend inflation considered here.

Within the “surprise” form of the Phillips curve, which developed from Lucas’s (1973) analysis, there is only a one-time real effect of an unanticipated, permanent, and credible change in the inflation rate since (19) implies that \( \pi_t = E_{t-1} \pi_t + \frac{1}{\psi} (y_t - \bar{y}_t) \). To illustrate this point, suppose that the inflation rate is governed by the random walk specification, \( \pi_t = \pi_{t-1} + e_t \), which implies that all inflation changes are unexpected and permanent. Then, \( E_{t-1} \pi_t = \pi_{t-1} \) and a decline in the rate of inflation causes an output decline of \( (y_t - \bar{y}_t) = \psi e_t \) with no expected consequences for future output.

The new Phillips curve (3) has a related, but stronger implication: There is no effect of an unanticipated, permanent and credible shift in the inflation rate since \( \pi_t = E_t \pi_{t+1} \) in this case and the above analysis (with \( \beta = 1 \)) that changes in the trend rate of inflation have no effect on real activity.\(^{35}\) Ball (1995) emphasizes the importance of policy credibility to this implication of snap disinflation.

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\(^{34}\) The assumption of exogenous inflation is simply for analytical convenience: a similar spurious long-run tradeoff appears, as in Section 4.2, when the model is solved with exogenous nominal income.

\(^{35}\) A slight modification of the structure of the current model—requiring that firms post prices prior to receiving information about date \( t \)—is employed in Bernanke and Woodford (1997). This has the implication that \( \pi_t = E_{t-1} \pi_{t+1} + \psi (y_t - \bar{y}_t) \) so its has the same implication for an unanticipated, permanent disinflation as does (19).
6. THE NEW IS CURVE

In this section, three aspects of the new IS curve are discussed. Section 6.1 explains the role of expected future output in the new IS curve. Section 6.2 considers the implications of omitting expectational terms for traditional IS specifications. Section 6.3 discusses two key implications of the new IS curve, the natural rate of interest and the cyclical behavior of the real interest rate, which can be obtained without full solution of the New IS-LM model.

To begin, let’s return to panel a of Figure 1, which may be viewed as the familiar, traditional IS curve. In this graph, a higher real interest rate leads to a lower level of aggregate demand. Given that output is demand-determined and the economy under study is closed, a higher rate thus leads to a lower level of output/income. The negative slope of this specification reflects the idea that an increase in income is partly saved by households, with a lower real interest rate required to stimulate additional investment. The traditional IS curve is viewed as fairly steep by many economists, who believe that large changes in interest rates are necessary to produce macroeconomically important changes in aggregate demand. This steep slope corresponds to a small value of $s$ in (1).36

The new IS curve also implies a negative relationship between interest rates and output, holding fixed expected inflation and expected future output. In this sense, the New IS-LM model is very traditional. As stressed by McCallum and Nelson (1999b), it is also very traditional in that no asset stocks—neither the capital stock nor the quantity of real balances—enter anywhere in these specifications.

But it also predicts that shifts in expectations about future output can be a very important determinant of the level of aggregate demand. For example, if output is expected to be 1 percent higher in the future, then the new IS specification implies that aggregate demand will be 1 percent higher today.

Importance of Expected Future Output

The potential importance of this expectations effect raises two related questions. First, why is the new IS curve written as in (1), rather than as $y_t = \chi E_t y_{t+1} - s r_t + x_{dt}$ with $\chi$ being a parameter governing the size of these expectations effects? Second, is the actual behavior of income likely to mean that there is an important difference between the two specifications?

Rationalizing the unit coefficient on $E_t y_{t+1}$: Total demand in a closed economy involves consumption, investment, and government components. In

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36 The traditional view that the IS schedule is relatively interest-inelastic also means that many economists have downplayed the importance of shifts in expected inflation for aggregate demand, since the effect of these is captured by $s E_t \pi_{t+1}$ in (1). Without taking a stand on the interest-elasticity of aggregate demand, the present discussion therefore plays this channel.
the United States and most other economies, consumption is by far the largest part of this demand. The modern theory of consumption, developed by Hall (1978, 1988) and others along the lines first sketched by Irving Fisher, implies that an intertemporally efficient consumption plan equates the cost of foregone consumption today and the benefits of increased future consumption. More specifically, Hall (1978) shows that efficient consumption growth should be positively related to the real interest rate. If we let \( c_t \) be the logarithm of consumption, Hall’s finding suggests that the dominant component demand should obey

\[
E_t c_{t+1} - c_t = s[r_t - r],
\]

which alternatively implies that

\[
c_t = E_t c_{t+1} - s[r_t - r].
\]

To simply apply the consumption equation to total demand, it is necessary to make one of two assumptions: either consumption is assumed to be all of aggregate demand, or the residual components of demand move exactly with total demand or consumption. Neither of these is likely to be true exactly, with investment being proportionately more volatile than total demand and government purchases being proportionately less volatile. While government demand may not be forward looking, neoclassical investment theory suggests that expectations about future output will be a very important determinant of current investment, with potentially much larger effects than are present in consumption. Overall, though, the consumption theory makes (1), with a unit coefficient, the natural first approximation to the forward-looking theory of aggregate demand.

**Implications for the Traditional IS Curve**

Suppose that there was really a new IS curve of the form (1), but that a macroeconomic analyst worked with a traditional IS curve.

*Instability and lags in the traditional IS Curve:* Written in terms of the nominal interest rate and organized so as to facilitate comparison with the traditional IS curve, the new IS curve is

\[
y_t = -sR_t + \{E_t y_{t+1} + sE_t \pi_{t+1} + x_{dt}\}.
\]

The term \( E_t y_{t+1} + sE_t \pi_{t+1} + x_{dt} \) combines the actual aggregate demand shock \( x_{dt} \) with the expectational elements that are omitted in the traditional approach.

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37 Woodford (1996) and Dotsey, King, and Wolman (1999) are examples of economies in which (a) there is no capital or investment and (b) there are separability restrictions on preferences; these conditions guarantee that there is exactly an IS curve of the form (1). McCallum and Nelson (1999) detail the necessary separability conditions. They also argue that (1) is a good approximation to an economy with investment because there is a small cyclical variation in the capital stock.
There are thus two key implications. First, if output and inflation expectations are substantially variable, there will be large shifts in the position of the traditional IS curve. Second, variables that are useful for forecasting $E_t y_{t+1}$ and $E_t \pi_{t+1}$ will improve the empirical fit of a traditional IS curve: since both output and inflation display important persistence empirically, lagged values of these variables can enter.\footnote{The new IS curve also can explain why empirical researchers have found it hard to isolate effects of interest rates on aggregate demand. Shifts in expected income and interest rates should be correlated with nominal interest rates, leading to biased estimates of the interest sensitivity $s$.}

The long-term interest rate and the traditional IS Curve: Many economists believe that the long-term interest rate is more important for aggregate demand than the short-term interest rate (see, for example, Goodfriend (1998)). The new IS curve also helps explain why long-term interest rates can appear more important in practice even if it is the short-term interest rate that is behaviorally relevant for certain parts of aggregate demand. For this purpose, let’s assume that the expectations theory of the term structure holds exactly, without a term premium, so that the $n$ period real interest rate is $r^n_t = \frac{1}{n} [r_t + E_t r_{t+1} + \ldots E_t r_{t+n-1}]$. Let’s also assume that output is expected to be equal to its capacity level after $n$ periods. Then, iterating the new IS curve, output can be shown to be

$$y_t = -sr_t + E_t y_{t+1} + x_{dt}$$
$$= -s[r_t + E_t r_{t+1} + \ldots E_t r_{t+n-1}] + E_t \bar{y}_{t+n} + x_{dt}$$
$$= -\sigma r^n_t + E_t \bar{y}_{t+n} + x_{dt}$$

with $\sigma = sn$. Thus, the implied coefficient on the long rate is much larger than $s$ and the fit of this expression should be much better because there is no longer the omitted variable $E_t y_{t+1}$. Each of these implications occurs because the long-term real interest rate “stands in” for the influence of expected future output $E_t y_{t+1}$.

Persistence of output and the importance of expectations effects: Macroeconomists agree that fluctuations in output are highly persistent, even though there is disagreement about the precise extent of this persistence. Persistence in output makes it possible to forecast output, which in turn means that there are important variations in the $E_t y_{t+1}$ term on the right hand side of (1). Yet it is only if output variations are close to temporary that there is little practical difference between the new and old IS schedules.

Interest Rate Implications

The new IS curve also embodies two modern ideas about the link between the real interest rate and real economic activity.
The natural rate of interest: If the economy is operating at its capacity level of output, then there is a particular level of the real interest rate which one may call the natural rate of interest. The new IS curve indicates that this natural rate of interest is given by

\[
\bar{r}_t = \frac{1}{s}[E_t \bar{y}_{t+1} - \bar{y}_t + x_{dt}].
\]

Thus, the natural rate of interest rises when the capacity level of output is expected to grow more rapidly. It also rises if there are shocks to demand at a given real interest rate. If there is a steep IS curve (small \(s\)) then the required increase in the real interest rate for a given growth rate of capacity output or demand shock is larger.

The real interest rate and the business cycle: The new IS schedule implies that the real interest rate also rises, more generally, when output growth is expected to be higher:

\[
r_t = \frac{1}{s}[E_t y_{t+1} - y_t + x_{dt}].
\]

Thus, the new IS curve implies that an economy recovering from a temporarily low level of output—one which has a high expected growth rate—would have a high real interest rate. A low real interest rate would be associated with an economy experiencing a temporarily high level of output. This implication will be very useful in interpreting the comovement of the real interest rate with cyclical fluctuations in output in Section 8.

7. LIMITS ON INTEREST RATE RULES

There has been substantial recent research on interest rate rules, since these strategies appear to describe some aspects of the actual instrument choice and policy actions of the Federal Reserve System (Goodfriend 1991, Taylor 1993). Specifically, Taylor (1993) studied the properties of an interest rate rule of the form

\[
T : R_t = [r + \pi] + \tau_\pi (\pi_t - \pi) + \tau_y (y_t - \bar{y}_t),
\]  

(20)

where \(r\) is the steady state real interest rate, \(\pi\) is the long-run inflation, and \(y_t - \bar{y}_t\) is the deviation of output from capacity.\(^{39}\)

Taylor proposed that a relatively aggressive response to inflation was important; in particular, he suggested that the FRS should raise the nominal interest rate more than one-for-one in response to inflation \(\tau_\pi > 1\). He also suggested that the central bank should lower the nominal interest rate when output was less than capacity, thus implying a positive value for \(\tau_y\).

\(^{39}\)In Taylor’s (1993) setting, the inflation measure was a four quarter average, but the current discussion will follow the recent literature in representing this as the current quarterly inflation rate.
In work on the consequences of alternative interest rate rules within the New IS-LM model and related fully articulated models, it is common for the space of policy rule parameters to be divided into two parts. In the first part of the parameter space, which is extensively studied, there is a unique stable rational expectations equilibrium. In the second part, which is avoided, there are multiple stable equilibria. This section describes how multiple equilibria can arise under an interest rate rule. It derives some standard restrictions on the parameters of an interest rate policy rule—some of which turn out to be related to \( \tau_\pi > 1 \)—that lead to a unique stable equilibrium.\(^{40}\)

The main focus of this section, however, is on the more specific question raised in Section 2 above: What restrictions on an interest rate rule must be imposed if the central bank seeks to obtain a neutral path of economic activity—of real output, inflation, and interest rates—as a unique outcome? To aid us in answering this question, the monetary policy rule is specified as

\[
R_t = \bar{R}_t + \tau_1 (E_t \pi_{t+1} - E_t \bar{\pi}_{t+1}) + \tau_0 (\pi_t - \bar{\pi}_t) + x_{R_t}. \tag{21}
\]

The first term in this expression is the neutral interest rate, i.e., the level of the nominal interest rate under a neutral policy. As discussed above, the neutral nominal rate involves the sum of the natural real rate of interest and the expected future inflation target, \( \bar{R}_t = \bar{r}_t + E_t \bar{\pi}_{t+1} \). The rule (21) also specifies that the monetary authority adjusts the nominal interest rate relative to its neutral level \( \bar{R}_t = \bar{r}_t + E_t \bar{\pi}_{t+1} \) if there are current or expected future departures of inflation from the targeted levels. This is in keeping with the spirit of Taylor’s rule, involving deviations from normal values, but is appropriate for a setup with a stochastically varying neutral path of inflation and real activity.

It is a convenient choice for this article because (i) it contains a number of special cases which been studied previously in the literature, and (ii) it makes it easy to determine the restrictions on an interest rate policy rule that lead to a unique equilibrium under the neutral interest rate policy, which was the key question raised in Section 2.\(^{41}\)

\(^{40}\) There are two concerns which are frequently expressed about interest rate rules. First, there is a long branch of literature in monetary economics which suggests that interest rate rules can mean that there is not a unique equilibrium in macroeconomic models. Second, there is the concern of Friedman (1982) that an interest rate rule can lead the central bank to exacerbate macroeconomic fluctuations which arise from shocks to productive opportunities, changes in money demand, and so forth. The discussion in this section will be restricted to the former concern: If the central bank is responding to inflation and output as suggested by Taylor, when do interest rate rules lead to a unique outcome? But the second question is an open and important topic.

\(^{41}\) The specification of this rule leads to a subtle shift in the interpretation of the policy parameters \( \tau \); these involve specifying how the monetary authority will respond to deviations of inflation from target. But if these parameters are chosen so that there is a unique equilibrium, then no deviations of inflation will ever occur.

At the same time, the parameter restrictions developed here would also apply to a rule of the general form originally studied by Taylor, i.e.,

\[
R_t = r + \pi + \tau_1 (E_t \pi_{t+1} - \pi) + \tau_0 (\pi_t - \pi) + x_{R_t}.
\]
Potential Multiple Equilibria

It is useful to start by considering a simple, flexible price setup in which the monetary authority can affect the behavior of inflation but not the behavior of the real rate of interest. Suppose that the authority adopts the rule

\[ R_t = r_t + \pi + \tau (\pi_t - \pi) + x_{Rt}, \]  

(22)

where \( \pi \) is a constant trend rate of inflation and \( \tau \) governs the response of inflation to deviations from this level, which is a simplification of the two rules discussed above. Since the Fisher equation specifies that \( R_t = r_t + E_t \pi_{t+1} \), it follows that inflation is constrained by

\[ \tau [\pi_t - \pi] + x_{Rt} = [E_t \pi_{t+1} - \pi]. \]  

(23)

If \( \tau > 1 \), which is the case normally considered, then the unique stable rational expectations solution to this difference equation can be obtained by recursively solving the difference equation forward

\[ \pi_t - \pi = \frac{1}{\tau} \left\{ [E_t \pi_{t+1} - \pi] - x_{Rt} \right\} \]

and so forth until one concludes that

\[ \pi_t - \pi = -\left\{ \sum_{j=0}^{\infty} \left( \frac{1}{\tau} \right)^{j+1} E_t x_{t+j} \right\}. \]  

(24)

This unique stable solution makes inflation into a present value of expected monetary policy shocks.43

This is because the difference between these two rules is

\[ \bar{R}_t - (r + \pi) + \tau_1 (E_t \pi_{t+1} - \pi) + \tau_0 (\pi_t - \pi) \]

which is just a complicated “shock” term that depends on exogenous variables.

42 At the end of this process, one uses \( \lim_{j \to \infty} (\frac{1}{\tau})^j E_t x_{R,t+j} = 0 \), which surely obtains because \( \tau > 1 \) and \( x_{R,t} \) is stationary.

43 There are some puzzling aspects of this flexible price solution, which implies that the behavior of the nominal interest rate is

\[ R_t = r_t + \pi - \left\{ \sum_{j=0}^{\infty} \left( \frac{1}{\tau} \right)^j E_t x_{t+j} \right\} + x_{Rt}. \]

That is, when an \( x_t \) shock occurs so that the central bank’s chosen path is autonomously increased, then inflation must move to offset this response. For example, if \( x_t \) is serially uncorrelated, then inflation moves just enough so that the nominal rate is unresponsive to the shock (in this case \(-\left\{ \sum_{j=0}^{\infty} \left( \frac{1}{\tau} \right)^j E_t x_{t+j} \right\} = -x_t \) so that the interest rate is just \( R_t = r_t + \pi \)). For another example, if \( x_t \) is autoregressive with persistence parameter \( \rho \), then the nominal interest rate must actually fall in response to a positive policy shock.
By contrast, if $0 < \tau < 1$, there are multiple stable rational expectations solutions, which take the form

$$\pi_{t+1} - \pi = \tau (\pi_t - \pi) + x_{Rt} + \xi_{t+1}$$

(25)

with $\xi_{t+1}$ being an arbitrary random variable with $E_t \xi_{t+1} = 0$. These non-fundamental stochastic elements are sometimes referred to as “sunspots” or “animal spirits.”

Mathematically, they can enter in (25) because the perfect foresight solution displays an indeterminacy: any initial value of $\pi_0$ can be an equilibrium with the remainder of the stable perfect foresight equilibrium path being $\pi_{t+1} - \pi = \tau^t (\pi_0 - \pi)$. From this perspective, the $\xi_{t+1}$ can be interpreted as a randomly shifting set of initial conditions for the stochastic difference equation.

Economically, the equilibria described by (25) can be too volatile relative to the fundamental forces in the model economy. For example, even if the $x_{Rt}$ shocks are absent, inflation under such a policy rule can be arbitrarily volatile since the variance of $\xi$ is arbitrary. These multiple equilibria arise for a basic economic reason introduced in Section 2, which is that the central bank’s policy rule does not provide a sufficient nominal anchor.

Therefore, a simple flexible price model indicates that there could be a good reason for interest rate rules to be restricted to aggressive values of parameters, in line with Taylor’s (1993) suggestion that $\tau > 1$. The simple model also indicates, however, that there are other parameter choices which will lead to uniqueness. In particular, if the monetary authority aggressively lowers the rate in response to inflation (makes $\tau < -1$), then there will also be a unique equilibrium since the same logic employed in the derivation of (24) may be employed. Thus, in the simple flexible price model there is a “zone of indeterminacy” which includes all policy rules with $-1 < \tau < 1$.

**Limits in the New IS-LM Model**

In models with sticky prices, it is sometimes argued that there is a greater latitude for interest rate policies than in flexible price models. The New IS-LM model is simple enough that one can characterize analytically the parts of the parameter space in which there are unique equilibria and the parts in which

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44 Farmer (1999) has recently argued that understanding the effects of nonfundamental uncertainties of this form is very important for macroeconomics, echoing earlier assertions of Jevons and Keynes.

45 Another, less stressed, implication is that a shock which increases the nominal interest rate will raise the inflation rate under this solution.

46 With recent interest in the analysis of alternative interest rate rules under rational expectations, within the New IS-LM model and related fully articulated models, economists are beginning to explore new territory in terms of coefficients in interest rate rules within quantitative models (as in the recent volume of studies edited by Taylor [1999]).
Figure 3

a. Current Inflation Rule

b. Expected Inflation Rule

c. Composite Rule
there are multiple equilibria.\textsuperscript{47} However, modern literature on the design of monetary policy rules, as exemplified by the recent volume edited by Taylor (1999), typically proceeds by using graphical presentations of these rules, with some regions blocked out as “zones of indeterminacy.” Figure 3 is an example of this approach for the New IS-LM model, with some various versions of the general policy rule.

Response to the current inflation rate: Kerr and King (1996) used the New IS-LM model to study the case in which the central bank responds only to the current inflation rate.\textsuperscript{48} In panel a, the shaded region is the set of inadmissible settings for the response to current inflation ($\tau_0$) given that there is no response to expected inflation ($\tau_1 = 0$). As suggested by Taylor (1993) and the analysis of the flexible price model above, one boundary of the zone of indeterminacy is given by $\tau_0 = 1$, which was the restriction also focused on by Kerr and King. The figure implies that any rule of the form (21) with $\tau_1 = 0$ and $\tau_1 > 1$ is consistent with neutral behavior of output and inflation. Thus, in terms of the answer to the question raised in Section 2, the analysis indicates that there will be a unique equilibrium if the monetary says, “If inflation deviates from the neutral level, then the nominal interest rate will be increased by more than one-for-one relative to the level which it would be at under a neutral monetary policy.”

In the New IS-LM model, in contrast to conventional wisdom, the stickiness of prices implies that there is a larger zone of indeterminacy than in the flexible price model. This feature of the model was not stressed by Kerr and King because they did not focus on the lower boundary of the zone, which can be determined to be $\tau_0 = -\frac{2(1+\beta)}{\phi s} - 1$. Hence, as prices become more flexible or the IS curve becomes flatter—there is a larger value of $\phi s$—then the result approaches the boundary in the flexible price model of $\tau_0 = -1$, but the zone of indeterminacy is always larger with sticky prices. The monetary authority, however, may also insure a unique equilibrium by saying that it will very aggressively lower the inflation rate in response to deviations of inflation from its target.

Response to the expected inflation rate: Bernanke and Woodford (1997) studied a purely forward-looking rule in which $\tau_0 = 0$, which is the case illustrated in panel b of Figure 2. With a response to expected inflation (but no response to current inflation), there are two zones of indeterminacy. All policy responses with $\tau_1 < 1$ are precluded, so it is necessary for policy to be

\textsuperscript{47} Appendix C contains a detailed discussion of these regions. The approach is to (i) find the boundaries of the regions by learning when there are roots which are $\pm 1$ and (ii) determine which regions are zones of indeterminacy.

\textsuperscript{48} Comparison with Kerr and King (1996) highlights a feature of the current analysis. The earlier paper was concerned with rules of the form $R_t = r + \pi + \tau (\pi_t - \pi)$ so that the focus was on how the central bank should respond to deviations of inflation from a constant target. The current analysis focuses on deviations from a neutral inflation target.
aggressive in Taylor’s sense if it is forward looking. It is important, though, that it not be too aggressive, since the figure shows that some larger values are also ruled out because these lead to indeterminacies (the precise boundary is \( \tau_1 > 1 + \frac{2(1+\beta)}{\psi_s} \)).\(^{49}\) Forward-looking rules, then, suggest a very different pattern of restrictions are necessary to assure that there is a neutral level of output.

Response to both current and expected inflation: When the policy rule combines a mixture of current and expected inflation responses, there is a more complicated set of possibilities. In general, the results are closer to those in panel a when the forward-looking part of policy is not aggressive (\( \tau_1 < 1 \)) and closer to panel b when it is aggressive (\( \tau_1 > 1 \)).

For example, suppose that policy is mildly forward-looking, which is illustrated in panel c under the assumption that \( \tau_1 \) is set equal to .25. The key implication of the figure is that policy can then respond less aggressively to current inflation. There is now a larger range of admissible positive \( \tau_0 \) values, in the sense that values of \( \tau_0 < 1 \) lead to unique equilibria when they did not in panel a.

If monetary policy is to respond positively to both current and expected inflation, however, then it is necessary that the overall policy be aggressive. The upper boundary of the zone of indeterminacy is given by \( \tau_0 + \tau_1 = 1 \), so that \( \tau_0 > .75 \) leads to a unique equilibrium in the graph.\(^{50}\) Still, by responding partially to expected future inflation, monetary policy makes it less necessary to respond aggressively to current inflation.

If one takes all of these results together, one can see that the New IS-LM model suggests that there are important limits on interest rate rules if there is to be a unique equilibrium. There are important differences in the zones of indeterminacy for rules that respond to current inflation and prospective inflation.

An Alternative Nominal Anchor

While there is a substantial limit on the coefficients in “inflation” rules such as those put forward by Taylor (1993), it is important to note that interest rate rules with an alternative nominal anchor—a relationship to the price level—also

\(^{49}\) Michael Dotsey has stressed to me that there are no unique equilibria with forward-looking rules in a flexible price model, since the Fisher equation and policy rule are each equations linking the nominal rate to expected inflation. The restriction on inflation, analogous to (23), is \( \tau_1 [E_t \pi_{t+1} - \pi] + x_R t = [E_t \pi_{t+1} - \pi] \) and there is no possibility of a unique equilibrium. Hence, as described in the text discussion of the current inflation rule, an increase in \( \psi_s \) leads to a shrinking zone of admissable rules. But in this case the range of admissable rules is asymptotically negligible.

\(^{50}\) The appendix analysis also indicates that the lower boundary is given by \( \tau_0 = \tau_1 - \frac{2(1+\beta)}{\psi_s} - 1 \). Hence, a positive value of \( \tau_1 \) requires that even more negative values of \( \tau_0 \) are necessary to assure uniqueness relative to those shown in panel a.
can be used to insure neutral output under an interest rate rule. In particular, suppose that the nominal interest rate rule takes the form

\[ R_t = \bar{r}_t + E_t \bar{\pi}_{t+1} + f(P_t - \tilde{P}_t) + x_{R_t}, \tag{26} \]

which involves three components. First, as above, the nominal interest rate moves with the underlying neutral interest rate \( R_t = \bar{r}_t + E_t \bar{\pi}_t \) as above. Second, there are interest rate shocks \( x_{R_t} \) as above. Third, the nominal rate is adjusted whenever the price level deviates from a target path \( \tilde{P}_t \). Then, it is possible to show that there is a unique stable rational expectations equilibrium so long as \( f > 0 \), i.e., the nominal rate is raised whenever the price level exceeds the target path.\(^{51}\) This theoretical conclusion corresponds to an idea sometimes presented in discussions of monetary policy—for example, Goodfriend and King (1997)—that a central bank can have a greater degree of freedom in the short-run dimensions of its policy rule if it adopts a specification which recognizes the importance of the price level.

8. POLICY: SHOCKS, RULES, AND TRADE-OFFS

The New IS-LM model suggests that monetary policy may influence real economic activity in two distinct ways. First, the central bank may itself be a source of shocks, with the effects of monetary policy disturbances also depending on the form of the monetary policy rule in place. Second, by the choice of its monetary policy rule, the central bank can affect how macroeconomic activity responds to shocks originating elsewhere in the economy. The various influences of monetary policy may be summarized by a graph, as employed by Taylor (1979) and many subsequent studies, of the relationship between the variability of inflation and the variability of real activity. This section considers each of these ideas in turn.

Dynamic Response to an Interest Rate Shock

Increases in the target range for a short-term interest rate, such as the federal funds rate in the United States, are a monetary policy shock of sorts. These changes are typically suggested to lower the rate of inflation and to temporarily decrease real output as well.

In order to study the effects of such a shock within the New IS-LM model, it is necessary to choose parameters of the model—including those of the private economy (\( \beta, s, \varphi \)) and of the policy rule (\( \tau_0, \tau_1 \) and the process governing

\(^{51}\) The derivation in Appendix C assumes that the target path is the neutral price level path \( \bar{P}_t \) for ease of mathematical analysis. However, nearly any target path can be accommodated since the rule can be rewritten as \( R_t = \bar{R}_t + f(P_t - P_t) + x_{R_t} = \bar{R}_t + f(P_t - \bar{P}_t) + [x_{R_t} + f(\bar{P}_t - P_t)] \) with the deviation \( f(\bar{P}_t - P_t) \) being an additional shock of sorts.
and solve for the dynamic responses to the shock. As an example, Figure 4 displays the paths that arise when there is a simple rule that mandates a response to current, but not expected, inflation. The specific rule is

\[ R_t = r + \pi + \tau (\pi_t - \pi) + x_{Rt} \]

with \( \tau \) set equal to 1.05 so as to assure uniqueness. It is also assumed that there is an interest rate shock process that is first order autoregressive, \( x_t = \rho_R x_{t-1} + e_t \), and that \( \rho_R = .75 \). The policy shock is a rise in the nominal rate, \( e_0 = 1 \) with \( e_t = 0 \) for \( t > 0 \).

As discussed above, the time unit is taken to be one quarter of a year, which is a conventional macroeconomic modeling interval. The shock shown in the figure is a 100-basis-point rise in the annualized interest rate \( (e_0 = 1) \) as shown in panel a of the figure. Readers may find these graphs are most easily interpreted as representing the deviation from an initial zero inflation steady state in which the economy is operating at capacity output, although since the model is linear they also describe the effects of shocks on the economy more generally. This increase in interest rate is assumed to be followed by a 50-basis-point increase in the subsequent year, a 25-basis-point increase in the year after that, and so forth.

**Response of output:** The interest rate shock causes an immediate decline in output, with output reduced about \( 1/2 \) percent below capacity in the initial period (date 0) in panel b of Figure 4. The vertical axis can be interpreted as measuring the percentage deviation from the capacity level of output, so that it is about .45 in period 0, about .34 in period one, and so forth.

**Response of inflation:** The period of reduced output shown in panel b is accompanied by a similar interval of reduced inflation in panel c. As in Figure 2, the inflation rate is stated at an annualized percentage rate, so that it is four times the percentage change in the price level between \( t - 1 \) and \( t \). There is a relatively small reduction in inflation in the near term.

**Response of the nominal interest rate:** The behavior of inflation also is important for the path of the nominal interest rate in Figure 4; there is an important difference between the policy shock component of the interest rate (the ‘o’ path in panel a) and the actual behavior of the nominal interest rates. While there is a 100-basis-point increase in the policy shock component of the interest rate \( (x_{R0}) \), the decline in inflation means that this is not fully reflected

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52 In terms of the private sector parameters, \( \beta = .99 \), \( s = .5 \), and \( \phi = .05 \). The value of \( s \) is in line with estimates of the intertemporal elasticity of substitution, which typically exceed unity. The value of \( \beta \) is a conventional quarterly discount rate. The value of \( \phi = .05 \) is one of those employed by Taylor (1980).

53 Since output is depressed below capacity in period 0, it is expected to grow back toward its capacity level, with the one period growth rate being about .11 = \((- .34) - (- .45)\). At an annualized rate of growth, this is .45 percent.

54 There is a decrease in the annualized inflation rate of .35 percent in the initial period and a decrease of about .27 percent in the subsequent period.
in the nominal rate.\textsuperscript{55} The New IS-LM model therefore suggests that there may be a quantitatively large difference between monetary policy shocks and the innovations in the path of the interest rate.

Response of the real rate: There are two complementary ways of looking at the path for the real interest rate. One highlights the fact that the real interest rate rises by more than the nominal interest rate since there is a temporary period of expected deflation.\textsuperscript{56} The other derives from the link between the

\textsuperscript{55} The response of the nominal interest rate is given by $R_0 = \tau_0 \pi_0 + x_{R0}$ $= 1.05 \times (-0.35) + 1 = 0.63$.

\textsuperscript{56} In fact, at date 0, the nominal interest rate rises by 63 basis points and the real interest rate rises by 90 basis points (since inflation is expected to be $-0.27$ percent next period).
real interest rate and the growth rate of output, based on the specification of $r_t = \frac{1}{s}[E_t(\gamma_{t+1} - \gamma_t + x_{dt})]$. Each of these complementary descriptions of the real interest rate is a partial explanation of the workings of this simple dynamic general equilibrium model, but each also helps understand its operation.

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57 Recall from a previous footnote that $s = .5$. The real interest rate at date 0 is .90 percent higher because the economy is expected to grow about .45 percent between period 1 and period 0, so that the response of the real interest rate is .90% = $\frac{1}{4}[E_t(\gamma_{t+1} - \gamma_t)] = 2 * .45%$. 

Policy Rules and Macroeconomic Activity

To illustrate that alternative monetary policy rules can have a potentially important effect on how the macroeconomy responds to various shocks, it is easiest to modify the example studied in Section 4.2 above, which was used to trace out the dynamic response of prices and output to a change in nominal income. This is an interesting example from the standpoint of the design of monetary policy rules because some economists have suggested that the central bank should conduct monetary policy so that there is a target path of nominal output (see McCallum and Nelson [1999a] for one recent discussion of such nominal GDP rules).

One case for nominal GDP rules: It is sometimes argued that nominal GDP rules are desirable because they insulate output from various shocks. In the New IS-LM model, if monetary policy is structured so that nominal income is exogenous, then the analysis of Section 4 can be used to discuss the determination of output in the absence of price shocks or changes in capacity. In this case, with constant nominal income, the level of output would remain at capacity even if there were changes in the position of the IS curve, the LM curve, and so forth.

The case against nominal GDP rules if capacity changes: There is an important cost of such rules, which is that when there is an expansion of capacity output, the economy cannot immediately expand up to the new capacity level since the price level must gradually fall through time. By contrast, under the neutral monetary policy discussed earlier, a monetary expansion would have permitted an immediate output expansion while leaving the price level unaffected by the expansion of capacity.

The case against nominal GDP rules if there are price shocks: There is a similar case against nominal GDP rules if there are price shocks. In Section 3, it was shown that a neutral monetary policy would accommodate those disturbances, so that nominal income would change according to $\Delta Y_t = \Delta \pi_t + \Delta \bar{y}_t$ under a neutral policy. Price shocks would therefore also cause departures from capacity output if a nominal GDP rule were in place. For example, a positive price shock would raise the price level and lower output relative to capacity.

The relevance of alternative monetary rules for macroeconomic activity was originally stressed by Phelps and Taylor (1977), working in a loglinear macromodel with nominal stickiness. Dotsey (1999) has recently highlighted

58 Calculations similar to those in Section 4 can illustrate this point. Suppose that the path of nominal output ($Y_t = y_t + P_t$) is constant through time at $\bar{Y}$ and that capacity output is a random walk. $\bar{y}_t = \bar{y}_{t-1} + \varepsilon_t$ with $\varepsilon_t$ being white noise. The solution for the price level is $P_t = \theta P_{t-1} + (1 - \theta)(Y_t - \bar{Y}_t)$. Output is then $y_t - \bar{y}_t = \theta(y_{t-1} - \bar{y}_{t-1}) - (1 - \theta)(\bar{y}_t - \bar{y}_{t-1})$. Mechanically, this solution says that an increase in capacity output of $\Delta \bar{y}_t$ only affects current output by $\theta \Delta \bar{y}_t$; as discussed above, the stickiness of prices—as captured by $\theta$—implies that the economy cannot immediately expand up to the new level of capacity output.
this relevance, working in fully articulated models with a slightly different specification of price stickiness from that considered here. The fact that the form of the monetary policy rule matters for the response of the economy to real and nominal shocks is motivating many economists to study the performance of alternative monetary policy rules in forward-looking macromodels.

**The Variability Trade-off**

Taylor (1979) introduced the idea of summarizing the effects of alternative monetary policy rules in terms of their implications for the variability of inflation and real activity. He also suggested that there would typically be trade-offs between these two variability measures. Within the New IS-LM framework as developed here, both the internal logic of the model and a close reading of Taylor indicates that the natural trade-off to explore is that between inflation $\pi_t$ and the output deviation $z_t = y_t - \bar{y}_t$.

As an example, suppose that there are only inflation shocks $x_{\pi t}$ and that these are serially uncorrelated random variables. Suppose additionally that the monetary authority can respond directly to inflation shocks and does so to
make inflation equal to $\pi_t = fx_{\pi t}$, where $f$ is a parameter that governs the extent of the inflation response. Making use of the Phillips curve (3) and the fact that expected future inflation is zero, it follows that

$$\text{var}(\pi_t) = f^2 \text{var}(x_{\pi t})$$
$$\text{var}(z_t) = (f - 1/\phi)^2 \text{var}(x_{\pi t}),$$

where $\text{var}(\pi_t)$ is the variance of inflation, $\text{var}(z_t)$ is the variance of the deviation of output from capacity and $\text{var}(x_{\pi t})$ is the variance of $x_{\pi t}$.

To look at policies that minimize output variance given inflation variance, it is sufficient to restrict attention to values of $f$ between zero and one. Over the range between zero and one, there is indeed a trade-off. If there is a larger value of $f$, then there is more inflation variability but less output variability. This trade-off is illustrated with the downward sloping solid line in Figure 5. The neutral monetary policy discussed in Section 3 above corresponds to minimizing the variance of output deviations by setting $f$ equal to 1.

If inflation responds to another shock that is serially uncorrelated and uncorrelated with the inflation shock—for example, to productivity or money demand disturbances—according to a rule $\pi_t = f x_{\pi t} + g e_t$, then the frontier would shift upward, as illustrated by the dashed line in Figure 5. Proceeding as above, this alternative frontier is

$$\text{var}(\pi_t) = f^2 \text{var}(x_{\pi t}) + g^2 \text{var}(e_t)$$
$$\text{var}(z_t) = (f - 1/\phi)^2 \text{var}(x_{\pi t}) + (g/\phi)^2 \text{var}(e_t)$$

so that monetary policies allowing these influences would produce more inflation variability for a given amount of output variability.

9. SUMMARY AND CONCLUSIONS

The distinguishing characteristic of the New IS-LM model is that its key behavioral relations can be derived from underlying choice problems of households and firms and that these relations consequently involve expectations about the future in a central manner. The IS curve relates expected output growth to the real interest rate, which is a central implication of the modern theory of consumption. The aggregate supply/Phillips curve component of the model relates inflation today to expected future inflation and an output gap. This relationship can be derived from a monopoly pricing decision that is constrained by stochastic opportunities for price adjustment together with a consistent definition of the price level.

The New IS-LM model is increasingly being utilized to illustrate macroeconomic concepts that are robust across a variety of more detailed models and to exposit the implications of alternative monetary policy rules. This article has provided a description of this framework, highlighting its language and
logic. The article has also derived certain key implications of the framework for the conduct of monetary policy, which are summarized in the introduction: the case for inflation targets, the importance of adjustment of real and nominal interest rates to underlying real disturbances, the relevance of alternative monetary rules for the determination of output, and the potential consequences of monetary policy shocks.

Three aspects of this article may strike some readers as curious choices since my recent research has been aimed at developing small-scale fully articulated models of nominal frictions\textsuperscript{59} and exploring the implications of optimal and alternative policy rules within these models.\textsuperscript{60} First, the New IS-LM model is laid out with many free parameters and no attempt is made to compare its key predictions to the experience of the United States or other countries. Second, the New IS-LM model can be derived from first principles as a fully articulated model that arises from specifying preferences, technologies, and market institutions; poses and solves household and firm optimization problems; and finally imposes market equilibrium and other aggregate consistency conditions. This article, however, does not derive the behavioral relations from first principles. Instead, it follows the traditional IS-LM approach of postulating behavioral relations, with some background rationalization in terms of optimizing, and then manipulates these to study various monetary policy issues. Third, the New IS-LM model abstracts from investment and capital, while most of my research has placed these features at center stage.

At one level, this approach reflects the limited goal of the article—to provide a simple exposition of the New IS-LM model and to exemplify how it is currently being used to discuss monetary policy topics. This goal was itself chosen, however, because it is my belief that many macroeconomists will use the New IS-LM model without all of its background detail to discuss monetary policy and, in particular, to communicate results from other, more complicated macroeconomic models.

Yet the microeconomic foundations are not to be dismissed. In the course of this article, there were many critical junctures at which the New IS-LM model was silent on central questions because microfoundations were absent. For example, in the Section 3 discussion of why a neutral monetary policy—defined as one that stabilized output at a capacity level—was desirable, it was necessary to step outside the New IS-LM model to draw on alternative studies in which the concept of capacity output was carefully defined and in which the monetary policy conclusion was derived as one that maximized the welfare of the citizens of the economy. Otherwise, the neutral monetary policy, which is marked as the point ‘o’ in Figure 5, would simply be one of a menu of choices that the monetary authority might consider desirable, given some

\textsuperscript{59} King and Watson (1996), King and Wolman (1996), and Dotsey, King, and Wolman (1999).

\textsuperscript{60} King and Wolman (1996), Goodfriend and King (1997), and King and Wolman (1999).
posited preferences of its own. Further, the analysis of neutral and alternative monetary policies suggests that the case for inflation targets, as opposed to a policy of full price level stabilization, depends entirely on the existence of inflation shocks. If these shocks were absent, the Taylor frontier in Figure 5 would collapse to the origin, with no trade-off between the variability of inflation and the variability of output relative to capacity (this possibility is marked as * in the figure). Yet there is an increasing use of the New Keynesian Phillips curve and the New IS-LM model for monetary policy analysis without detailed consideration of a question which seems central: What are inflation shocks?61 Popular discussions sometimes point to changes in capacity output (supply shocks) or energy price variations as price shocks. Nevertheless, to study whether these are price shocks of the form incorporated in this model, it is necessary to develop additional microeconomic underpinnings of the New IS-LM model, working in detail with the pricing decisions of firms, the consumption decisions of households, and so forth. Changes in capacity output induced by fluctuations in productivity or the prices of inputs such as energy are not price shocks according to such a detailed analysis because these affect prices by shifting marginal cost, which is a key economic determinant included in the pricing equation. Within the basic framework of sticky price models, it is difficult to find price shocks that are not interpretable as behavioral errors on the part of price-setters, although perhaps the addition of a sector with flexible price firms would lead to changes in relative prices that might be interpreted in this manner.62 This issue illustrates well, I believe, an inevitable limitation of IS-LM style analysis, which is that it may be useful for illustrating new results but it will certainly not be useful for deriving them. Finally, my suspicion is that the omission of investment and capital from the New IS-LM model may be an important, if not fatal, flaw. But determining whether this suspicion is warranted will again require a more detailed analysis that builds up from the microfoundations.

Ultimately, the case for (or against) the New IS-LM model and its fully articulated relatives must involve a systematic exploration of their empirical implications. There is much recent progress on this important front that involves the evaluation of components of the models—notably the pricing and aggregate demand specifications—and full system implications. But a great deal of work remains to be done before we understand whether this new small model captures the reality of the choices facing monetary policy decisionmakers of major economies.

61 See, for example, Clarida, Gali, and Gertler (1999).
62 I have benefited from discussion of this topic with John Taylor.
APPENDIX A: DERIVING THE NEW PHILLIPS CURVE

Start with the equations describing the price level (11) and the optimal price (12), which are repeated from the main text as

\[ A - 1: \quad P_t = \eta P_{t-1} + (1 - \eta) P_t^* \]

\[ A - 2: \quad P_t^* = \eta \beta E_t P_{t+1}^* + (1 - \beta \eta) [\psi_t + P_t] + z_{pt}, \]

where \( z_{pt} = x_{Pt} - \beta \eta E_t x_{P,t+1} \). Update the first equation, take expectations, multiply by \( \eta \beta \) and subtract the result from (A-1), then rearrange the result to

\[ P_t - \eta \beta E_t P_{t+1} = \eta (P_{t-1} - \eta \beta P_t) + (1 - \eta)(P_t^* - \eta \beta E_t P_{t+1}^*). \]

Substitute in A-2:

\[ P_t - \eta \beta E_t P_{t+1} = \eta (P_{t-1} - \eta \beta P_t) + (1 - \eta - \eta \beta)[\psi_t + P_t] + (1 - \eta)z_{pt}. \]

Rearrange the result and substitute in the marginal cost specification.

\[
\begin{align*}
P_t - P_{t-1} & = \beta (E_t P_{t+1} - P_t) + \frac{(1 - \eta)(1 - \eta \beta)}{\eta} \psi_t + \frac{1 - \eta}{\eta} z_{pt} \\
& = \beta (E_t P_{t+1} - P_t) + \left[h \frac{(1 - \eta)(1 - \eta \beta)}{\eta} \right] (y_t - \bar{y}_t) + \frac{1 - \eta}{\eta} z_{pt}
\end{align*}
\]

so \( x_{Pt} = \frac{1 - \eta}{\eta} z_{pt} = \frac{1 - \eta}{\eta} [x_{Pt} - \beta \eta E_t x_{P,t+1}] \).

APPENDIX B: EXOGENOUS NOMINAL INCOME

The analysis begins by combining (3) with the definition link between nominal and real income, ignoring inflation shocks for mathematical simplicity.

\[ P_t - P_{t-1} = \beta (E_t P_{t+1} - P_t) + \varphi (Y_t - P_t - \bar{y}_t) \]

This can be written as the expectational difference equation

\[
-E_t P_{t+1} + (1 + \frac{1}{\beta} + \frac{\varphi}{\beta}) P_t - \frac{1}{\beta} P_{t-1} = \frac{\varphi}{\beta} (Y_t - \bar{y}_t),
\]

which has a polynomial \( \Phi(z) = [-\beta z^2 + (1 + \beta + \varphi) z - 1] \). The product of the roots of this polynomial is \( \frac{1}{\beta^2} \) and the sum of roots is \( (1 + \frac{1}{\beta} + \frac{\varphi}{\beta}) \). If \( \theta \) is the smaller of the roots, then the larger of the roots is \( \frac{1}{\beta \theta} > 1 \).
A graphical analysis of the roots of this familiar difference equation will provide some useful background for the analysis of more complicated models below. The graph is based on decomposing \( \Phi(z) = 0 \) into \( \Phi(z) = q(z) - l(z) \) with \( l(z) = -\varphi z \) and \( q(z) = [-\beta z^2 + (1 + \beta)z - 1] = -(1 - z)(1 - \beta z) \). Figure B-1 displays the quadratic equation \( q(z) \), which has roots of 1 and \( 1/\beta \), and the line \( l(z) \), which is negatively sloped if \( \varphi > 0 \) and passes through the origin. The intersection of these two curves implies that \( l(z) = q(z) \) and thus the values of \( z \) at the intersection points are the solutions to \( \Phi(z) = 0 \).

With \( \varphi = 0 \), \( l(z) = 0 \) and the solutions are thus 1 and \( 1/\beta \). For any \( \varphi > 0 \) the solution must be as displayed in Figure B-1, which is that there is one root less than 1 and one root that is greater than \( 1/\beta \). Finally, increases in \( \varphi \) will lower the smaller root \( \theta \).

With this information about the magnitude of the roots, the next task is to determine the solution, following Sargent (1978). Using the operator \( F \) which shifts the dating of the variable, but not the conditional expectation so that \( F^j E_t x_{t+k} = E_t x_{t+k+j} \), we can deduce that
\[ \frac{\varphi}{\beta} E_t (Y_t - \overline{Y}_t) = -E_t P_{t+1} + \left( 1 + \frac{1}{\beta} + \frac{\varphi}{\beta} \right) P_t - \frac{1}{\beta} P_{t-1} \]
\[ = -(F - \theta)(F - \frac{1}{\theta \beta}) E_t P_{t-1} \]
\[ = \left( \frac{1}{\theta \beta} \right) (F - \theta)(1 - \theta \beta F) E_t P_{t-1}. \]

The general solution to the difference equation can be produced by unwinding the unstable root forward, so that

\[ P_t - \theta P_{t-1} = \theta \varphi \frac{1}{(1 - \theta \beta F)} E_t (Y_t - \overline{Y}_t) \]
\[ = (1 - \theta)(1 - \beta \theta) \sum_{j=0}^{\infty} (\theta \beta)^j E_t (Y_{t+j} - \overline{Y}_{t+j}) \]

with one step in this derivation using the fact that \((1 + \frac{1}{\beta} + \frac{\varphi}{\beta}) = \theta + \frac{1}{\beta \theta}\) means that \(\theta \varphi = (1 - \theta)(1 - \beta \theta)\).

Under the assumed driving process, it follows that

\[ \sum_{j=0}^{\infty} (\theta \beta)^j E_t (Y_{t+j}) \]
\[ = \sum_{j=0}^{\infty} (\theta \beta)^j E_t [Y_{t-1} + (Y_t - Y_{t-1}) + \ldots (Y_{t+j} - Y_t)] \]
\[ = \sum_{j=0}^{\infty} (\theta \beta)^j [Y_{t-1} + (1 + \rho + \rho^2 \ldots + \rho^j)(Y_t - Y_{t-1})] \]
\[ = \frac{1}{1 - \theta \beta} Y_{t-1} + \left[ \frac{1}{1 - \theta \beta} \right] (1 - \theta \beta \rho)(Y_t - Y_{t-1}) \]

so that the specific solution for the price level is

\[ P_t = \theta P_{t-1} + (1 - \theta)(Y_{t-1} - \overline{Y}) + \left[ \frac{1 - \theta}{1 - \theta \beta \rho} \right] (Y_t - Y_{t-1}). \]

To find the behavior of output, we use the relationship between nominal and real income followed by some algebra:
\[
\begin{align*}
  y_t - \overline{y} & = Y_t - \overline{y} - P_t \\
  & = [Y_t - \overline{y}] - \\
  & \quad \left[ \theta P_{t-1} + (1 - \theta)(Y_{t-1} - \overline{y}) + \left( \frac{1 - \theta}{1 - \theta \beta \rho} \right)(Y_t - Y_{t-1}) \right] \\
  & = \theta [Y_{t-1} - P_{t-1} - \overline{y}] + \left( \frac{1 - \theta}{1 - \theta \beta \rho} - 1 \right)(Y_t - Y_{t-1}) \\
  & = \theta (y_{t-1} - \overline{y}) + \left( \frac{1 - \beta \rho}{1 - \theta \beta \rho} \right)(Y_t - Y_{t-1}).
\end{align*}
\]

**APPENDIX C: UNIQUENESS UNDER INTEREST RATE RULES**

To analyze the system dynamics under interest rate rules, it is convenient to subtract its neutral counterpart from each of the equations of the model. For example, the IS equation is \( y_t = E_t y_{t+1} + s r_t + x_{dt} \) and its neutral counterpart is \( \overline{y}_t = E_t \overline{y}_{t+1} - s \overline{r}_t + x_{dt} \), so that the result is

\[
IS : y_t - \overline{y}_t = E_t (y_{t+1} - \overline{y}_{t+1}) - s (r_t - \overline{r}_t).
\]

Similarly, the Fisher equation is

\[
F : r_t - \overline{r}_t = (R_t - \overline{R}_t) - E_t (\pi_{t+1} - \overline{\pi}_{t+1})
\]

and the Phillips curve is

\[
PC : (\pi_t - \overline{\pi}_t) = \beta E_t (\pi_{t+1} - \overline{\pi}_{t+1}) + \varphi (y_t - \overline{y}_t).
\]

The monetary policy rules can similarly be transformed, by simply subtracting \( R_t = \overline{r}_t + E_t \overline{\pi}_{t+1} \) from both sides of the equation.

For example, with the general specification (text ref) we have that

\[
R_t - \overline{R}_t = \tau_1 (E_t \pi_{t+1} - E_t \overline{\pi}_{t+1}) + \tau_0 (\pi_t - \overline{\pi}_t) + x_{Rt}.
\]

Thus, the analysis of system dynamics can be performed as if all shocks had been dropped—except for the policy shock—and the capacity output level had been treated as constant.

Similarly, with the price level specification (text ref) we have that

\[
R_t - \overline{R}_t = f (P_t - \overline{P}_t) + \{ f (\overline{P}_t - \overline{P}_t) + x_{Rt} \}
\]

so that the term in braces can be treated as a complicated interest rate shock.

Hence, in the remainder of this appendix, attention is restricted to analysis of a deterministic system—without any shocks or time variation in capacity—for the purpose of studying uniqueness issues.
The text discussion of interest rate rules involved the idea that there was a unique equilibrium so long as the central bank was willing to raise the real rate in specified circumstances, which suggests focusing on the real interest rate. To derive one restriction on the real rate, multiply IS by $\varphi$ and then eliminate output using the Phillips curve:

$$\varphi s \times r_t = [-\pi_t + (1 + \beta)E_{t}\pi_{t+1} - \beta E_{t}\pi_{t+2}] = -[(1 - \mathbf{F})(1 - \beta \mathbf{F})]E_{t}\pi_{t},$$

where $\mathbf{F}$ is the forward operator as in the main text. This is a private sector restriction on the behavior of the real interest rate, which links it to the inflation rate.

**Uniqueness with the Interest Rate Rule (21)**

Combining the Fisher equation (2) and the monetary policy rule (21), it is possible to determine an additional restriction on the real interest rate:

$$r_t = \tau_0 \pi_t + (\tau_1 - 1)E_{t}\pi_{t+1} = [\tau_0 + (\tau_1 - 1)\mathbf{F}]E_{t}\pi_{t}.$$

Combining this expression with the private sector restriction on the real rate leads to

$$l(\mathbf{F})E_{t}\pi_{t} = \varphi s[\tau_0 + (\tau_1 - 1)\mathbf{F}]E_{t}\pi_{t},$$

$$= -[(1 - \mathbf{F})(1 - \beta \mathbf{F})]E_{t}\pi_{t} = q(\mathbf{F})E_{t}\pi_{t}.$$

The left-hand side of this expression is a linear function $l$, and the right-hand side of this expression is a quadratic function $q$.

The nature of the system dynamics will depend on the roots of the quadratic polynomial $q(z) - l(z)$, which may be written as

$$-\beta z^2 + [\beta + 1 - \varphi s(\tau_1 - 1)]z - [1 + \varphi s \tau_0] = -\beta (z - \mu_1)(z - \mu_2).$$

This expression makes clear that the sum of the roots is $[1 + \frac{1}{\beta} + \frac{\varphi s(\tau_1 - 1)}{\beta}]$ and that the product of the roots is $[1 + \frac{\varphi s \tau_0}{\beta}]$. Since there are no predetermined variables in this system, there is a unique equilibrium only if there are two unstable roots, i.e., values of $\mu_i$ that are both larger than unity in absolute value. To study the magnitude of these, it is convenient to use a mixture of graphical and analytical techniques.

**Determining the boundaries:** The boundaries of the policy parameter regions can be determined by requiring that there is a root of exactly positive or negative one. Taking the positive unit root first,

$$l(1) = q(1)$$

$$\Rightarrow \varphi s[\tau_0 + (\tau_1 - 1)\mathbf{I}] = -[(1 - 1)(1 - \beta \mathbf{I})] = 0 \Rightarrow \tau_0 + \tau_1 = 1$$
so that there is a restriction that the sum of the policy rule coefficients must equal one from this source. Taking the negative unit root next,

\[ l(-1) = q(-1) \Rightarrow \varphi_s[\tau_0 + (\tau_1 - 1)(-1)] = \]

\[ -[(1 - (-1))(1 - \beta(-1))] = -2(1 + \beta) \Rightarrow \tau_0 - \tau_1 = -1 - \frac{2(1 + \beta)}{\varphi_s} \]

so that there is a restriction on the difference between the coefficients from this source.

**Graphing the functions \( l(z) \) and \( q(z) \) to determine the nature of the regions:**
A graph of the functions, similar to that used in Appendix B above, provides the easiest way of determining the nature of the roots in the regions defined by the above boundaries. Figure C-1 shows the nature of this pair of functions. The form of the quadratic equation \( q(z) \) is invariant to the nature of the policy rule; as is clear from the fact that \( q(z) = -[(1 - z)(1 - \beta z)] = 0 \) the two zeros are 1 and \( 1/\beta \). The figure is drawn for the case of a simple rule which involves only response to current, not expected inflation (\( \tau_1 = 0 \)) so that it corresponds to panel a of Figure 3 in the text. The function \( l(z) \) is downward sloping in this case since \( l(z) = s\varphi(\tau_0 - z) \) and \( s\varphi > 0 \). If \( \tau_0 = 1 \) then \( l(z) \) intersects with the quadratic at \( z = 1 \); this possibility is shown by the dashed line in B-1. If \( \tau_0 > 1 \), then this intersection is shifted to the right, i.e., all roots are greater than 1. In this case, there are two unstable roots and there is thus a unique stable rational expectations equilibrium. Hence, as \( \tau_0 \) is increased from the boundary region in panel a of Figure 3 in the main text, the region of unique equilibria is entered.

This graphical analysis can also be used to (i) confirm that a reduction in \( \tau_0 \) from the other boundary also produces an entry into the region of stability in panel a of Figure 3 of the text, and (ii) to determine that the other aspects of panels b and c are as described in the text.

**Uniqueness with the Interest Rate Rule (26)**

By combining the Fisher equation (2) and the monetary policy rule (26), it is possible to determine an additional restriction on the real interest rate:

\[ r_t = f P_t - E_t \pi_{t+1} = [F - F(F - 1)] E_t P_{t-1}. \]

Combining this expression with the private sector restriction on the real rate leads to

\[ a(F) E_t P_{t-1} = \varphi_s\varphi[F - F(F - 1)] E_t P_{t-1} \]

\[ = -[(1 - F)(1 - \beta F)][F - 1] E_t P_{t-1} = b(F) E_t P_{t-1}. \]

The left-hand side of this expression is a quadratic function, \( a(F) \), and the right-hand side of this expression is a cubic function \( b(F) \).
The nature of the system dynamics will depend on the roots of the polynomial \( c(z) = b(z) - a(z) \). To study the magnitude of these, it is again convenient to use a mixture of graphical and analytical techniques.

**Determining the roots of \( a(z) \) and \( b(z) \):** It turns out to be a simple matter to determine the roots of these expressions. The quadratic function \( q(z) \) has two roots, one of which is zero and the other of which is \( f + 1 \). The cubic equation \( b(z) \) has a root of \( \frac{1}{\beta} \) and two roots of 1.

**Graphing the functions \( a(z) \) and \( b(z) \) to determine the stability condition:**
A graph of the functions provides the easiest way of determining the nature of the roots of the cubic polynomial \( c(z) = b(z) - a(z) = 0 \).

Figure C-2 contains three functions. One of the solid lines is the cubic \( b(z) \), which highlights the fact that it has two repeated roots at \( z = 1 \) and a single root at \( z = 1/\beta \).

The dashed line is the quadratic \( a(z) \) with the parameter \( f = 0 \). There are two roots of this equation, one which is zero and the other which is unity. Hence, with \( f = 0 \), the graph highlights the fact—which can easily be determined using the definitions of \( a(z) \) and \( b(z) \)—that there is an exact root of unity in \( c(z) \). It also shows only one other intersection of the two lines, so that there is one unstable root and two unit roots of \( c(z) = b(z) - a(z) \).

The solid line which lies below the dashed line in the range \( 0 < z < 1 \) is an example of the quadratic \( a(z) \) with the parameter \( f > 0 \). Note that there is a zero root to this quadratic and a root greater than one (which was earlier determined to be \( 1 + f \)). Hence, with \( f > 0 \) there are three distinct roots, one of which is positive and less than unity and the other two are unstable. This is the configuration that insures uniqueness given that there is a single predetermined variable \( P_{t-1} \).
Figure C-1

\[ l(z) \quad \text{with} \quad \tau_0 > 1, \quad \tau_1 = 0 \]

\[ q(z) = -(1-z)(1-\beta z) \]

Figure C-2

\[ a(z) \quad \text{with} \quad f = 0 \]

\[ b(z) \quad \text{with} \quad f > 0 \]
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