Staggered Price Setting and the Zero Bound on Nominal Interest Rates

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The nominal interest rate cannot be less than zero: no one would choose to hold assets bearing a guaranteed negative nominal return when they could instead hold money, which bears a guaranteed zero nominal return. Does the zero bound have normative implications for monetary policy? The nominal interest rate tends to be low when expected inflation is low, so the lower expected inflation is, the more likely it is that zero nominal interest rates would be encountered. Some have argued that the zero bound's proximity at low inflation constitutes an argument against policy that results in low inflation or deflation.[1] Here we compare moderately deflationary and moderately inflationary regimes using a macroeconomic model to evaluate whether the zero bound introduces distortions that make low inflation undesirable.

The model and the method distinguish our analysis from other recent research on the same topic.[2] The model has optimizing behavior by individuals and firms, with the qualification that firms' price setting is staggered. Other analyses of the zero bound have also used sticky-price models; the zero bound is more likely to be important if nominal disturbances have real effects, as they do with sticky prices. Individuals in the model choose to hold money because it decreases the time they must spend shopping. Other analyses have not modeled money demand. The method we employ involves solving the entire model nonlinearly, which means directly imposing the zero bound on nominal

Alexander.Wolman@rich.frb.org. I thank Mike Dotsey, Marvin Goodfriend, Bob Hetzel, Andreas Hornstein, Tom Humphrey, Bob King, Jeff Lacker, Bennett McCallum, Yash Mehra, Athanasios Orphanides, David Small, and Alex Tabarrok for helpful discussions and comments. The views expressed herein do not represent the views of the aforementioned individuals, the Federal Reserve Bank of Richmond, or the Federal Reserve System.

[1] Notable examples are Vickrey (1954), Okun (1981), and Summers (1991).

[2] Notable examples include Fuhrer and Madigan (1997), Rotemberg and Woodford (1997), and Orphanides and Wieland (1998). Section 1 contains a discussion of these articles.

interest rates. We then compare the two inflation regimes in several ways, one of which involves using an explicit welfare metric, the representative agent's expected utility.

In the model, a deflationary regime where nominal interest rates are occasionally zero generates *higher* welfare than a moderate inflation regime where nominal interest rates are always positive. This striking result—which conflicts with the spirit if not the letter of previous work—can be attributed to two factors mentioned above. First, the fact that money demand is explicitly modeled means that there is a distortion associated with *positive* nominal interest rates: individuals waste resources economizing on real money balances. Second, while the two-period staggered price-setting requirement makes prices sticky, it does not make *inflation* sticky. When inflation is sticky, as in the models used by Fuhrer and Madigan (1997) and Orphanides and Wieland (1998), for example, the zero bound on nominal interest rates effectively means that real interest rates are constrained in low-inflation regimes. In contrast, in the sticky-*price* model used here, real interest rates are not constrained at low inflation. The monetary authority can create temporary expected inflation when nominal rates are zero, thereby pushing real rates down, as described by Mishkin (1996).

1. BACKGROUND AND RELATED WORK

Nominal interest rates are interest rates on marketable securities or loans denominated in an economy's unit of account. In contrast, real interest rates apply to assets denominated in a market basket of goods and services. Irving Fisher, who used the terms "money interest" and "real interest," is traditionally credited with being the first to distinguish between nominal and real interest rates. Fisher himself acknowledged, however, that he had many predecessors who understood the distinction between nominal and real interest rates to some degree.[3]

In Fisher's original analysis, the relationship between the nominal (money) interest rate and the real interest rate is but a special case of the relationships between interest rates denominated in any two standards of value. The celebrated Fisher equation first appears in "Appreciation and Interest," in an example where the two standards are gold and wheat. But, when Fisher introduces that analysis, he poses the general question, "If a debt is contracted in either of two standards and one of them is expected to change with reference to the other, will the rate of interest be the same in both? Most certainly not" (Fisher 1896, p. 6). The Fisher equation follows three pages later: $1 + j = (1 + a)(1 + i)$,

[3] Fisher's important works on this subject are "Appreciation and Interest," *The Rate of Interest*, and *The Theory of Interest*. See Humphrey ([1983] 1986), and Laidler (1991) for a discussion of Fisher's predecessors.

where j and i are the rates of interest in wheat and gold, respectively, and a is the (certain) expected rate of appreciation of gold in terms of wheat. This generality on Fisher's part is important, because it provides him with a principle for understanding why the money interest rate is bounded by zero. Fisher states that the interest rate cannot be negative in any standard that can be hoarded without loss. The argument is straightforward: individuals would choose to hoard the standard itself rather than hold securities or loans denominated in that standard and yielding negative interest. For perishable standards, however, the situation is different: "One can imagine a loan based on strawberries or peaches contracted in summer and payable in winter with negative interest" (Fisher 1896, p. 32). Since fiat money is storable at near zero cost, it follows that the nominal interest rate in a modern, fiat-money economy is approximately bounded by zero.

The zero bound is clearly a constraint on monetary policy, but is it an important constraint? In order to answer this question, one needs a macroeconomic model and a criterion for measuring importance. To understand the contribution made by this article, one should first know something about the models and criteria used in recent analyses by Fuhrer and Madigan (1997), Rotemberg and Woodford (1997), and Orphanides and Wieland (1998).[4]

Fuhrer and Madigan (1997) and Orphanides and Wieland (1998) use similar models, so we will consider them together. As with our analysis below, they assess the zero bound's importance by comparing their models' performance at a moderate inflation target to that at an inflation target low enough to make the nominal interest rate occasionally zero. Fuhrer and Madigan use a small model that contains (i) a backward-looking IS curve, (ii) an overlapping price-contracting specification, and (iii) a monetary policy reaction function.[5] Orphanides and Wieland's model shares the same contracting specification but disaggregates the IS curve into separate spending equations for consumption, fixed investment, inventory investment, net exports, and government spending. Neither model includes money. Monetary policy operates by changing the short-term nominal interest rate. Long-term real interest rates enter the spending equations, but because the contracting specification makes inflation sticky, persistent changes in the short-term nominal rate generate changes in the long-term real rate. Thus, monetary policy can affect real spending and hence output. In both models, the equations representing private sector behavior are posited rather than derived from explicit optimization problems.

Fuhrer and Madigan evaluate the zero bound's importance by comparing their model's responses to IS curve shocks at inflation targets of zero and 4

[4] The "liquidity trap" literature associated with Keynes ([1936] 1964) and Patinkin (1965) concerned the possibility of a positive lower bound on nominal interest rates. Relating that literature to recent work would be an article by itself.

[5] It is the same model used in Fuhrer and Moore (1995).

percent. In contrast, Orphanides and Wieland simulate their model using estimated shock processes and compare the variance of output at different inflation targets. The general conclusion of these papers is that at a zero inflation target, monetary policy is significantly constrained by the zero bound, in the sense that the zero bound is encountered regularly, and output is consequently more variable than at a moderate inflation target. The easiest explanation for this result comes from the first example in Fuhrer and Madigan, a permanent shock to the IS curve. The monetary authority responds to this shock by lowering short-term nominal interest rates. When the inflation target is zero, the monetary authority cannot lower the nominal rate by as much as it would choose if the inflation target were 4 percent. With sticky inflation, the decline in the real interest rate is also smaller, and therefore—because of the interest rate effect on spending—there is a larger fall in output at the zero inflation target. This fall in output is presumed to be bad, although that presumption is not implied by the model.

The principal virtue of the analysis conducted by Fuhrer and Madigan (1997) and Orphanides and Wieland (1998) is that it is performed using models that fit a particular sample of data quite well. However, their low inflation experiments are conducted in an economic environment quite different from the data sample. Therefore, the fact that the models' equations are not derived from explicit objective functions makes it doubtful that those equations would be stable in the face of the contemplated policy experiments. Although the model we use has not been shown to fit recent data well, it is valuable because it is set up with explicit objective functions for individuals and firms. This means that the model can legitimately be used for policy and welfare analysis.

Rotemberg and Woodford (1997) come to a slightly different conclusion about the importance of the zero bound as a constraint on monetary policy, using a different model and approach from those of Fuhrer and Madigan and Orphanides and Wieland. As we will also, Rotemberg and Woodford use a sticky-price model whose equations are derived from explicit optimization problems, and they use the utility function of agents in the model to measure the welfare associated with different monetary policy rules. However, Rotemberg and Woodford linearize their model to simplify the analysis, and this precludes them from directly imposing the zero bound. They account for the zero bound indirectly by assuming that the variability of the monetary authority's interest rate instrument is constrained by the average level of interest rates, that is, by the inflation target. Specifically, they assume that the ratio of the standard deviation of the nominal interest rate to the average level of the nominal interest rate can be no greater than the ratio that describes their 1980–1995 U.S. sample. Thus, policy rules that generate high variability of nominal rates are incompatible with low inflation targets. Since a generic implication of models such as theirs is that stable inflation requires volatile nominal interest rates, their assumption implies a sharp tradeoff between the level of inflation and its variability. While this assumption has the effect of increasing the optimal inflation target from zero in their model, the optimum does not move far from zero.

All three papers discussed above exclude money from the models. Rotemberg and Woodford correctly state that the behavior of their model would be unchanged if they used a money-in-the-utility function specification where money was additively separable in the period utility function. However, ignoring money demand also means ignoring the welfare costs of positive nominal interest rates. That is, while the behavior of real and nominal variables may be invariant to incorporating money in an additively separable way, the welfare implications of different monetary policies are not invariant to this modification. Since concern about the zero bound on nominal interest rates boils down to concern about the welfare level associated with very low inflation targets, leaving money out of the model may be an important omission.

2. A MODEL WITH STAGGERED PRICE SETTING

Our analysis of the zero bound's importance for monetary policy is based on an explicit optimizing sticky-price model similar to, but simpler than, the one in Rotemberg and Woodford (1997). As in Fuhrer and Madigan (1997) and Orphanides and Wieland (1998), we impose the zero bound directly, rather than measuring its importance indirectly.[6] However, we take our analysis two steps further. First, we explicitly model money demand (using a shopping-time technology), so there is a force working in *favor* of zero nominal interest rates. Second, no linear approximations are employed to solve the model, which is fundamentally nonlinear.

The model follows the tradition of Taylor (1980), in that price setting is staggered: each firm sets its price for two periods, with one half of the firms adjusting each period.[7] As in Taylor's model, monetary policy is nonneutral in the model because stickiness in individual prices gives rise to stickiness in the price level. There are a continuum of firms, and they produce differentiated consumption goods using labor provided by consumers at a competitive wage as the sole input. Consumers are infinitely lived and use income from their labor, which is supplied elastically, to purchase consumption goods. Consumers hold money in order to economize on transactions time, as in McCallum and Goodfriend ([1987] 1988).

[6] Fuhrer and Madigan use three different approaches, one of which involves directly imposing the zero bound.

[7] The remainder of this section is loosely based on Section 2 in King and Wolman (forthcoming 1999). The model analyzed here differs in that it explicitly motivates money demand with a shopping-time technology.

Consumers

Consumers have preferences over a consumption aggregate (c_t) and leisure (l_t) given by

$$E_t \sum_{t=0}^{\infty} \beta^t \cdot [\ln(c_t) + \chi_t \cdot l_t]\,. \tag{1}$$

The discount factor β is set to 0.985, and the variable χ_t is a random preference shock.[8] The consumer's budget constraint is

$$c_t + \frac{M_t}{P_t} + \frac{B_t/P_t}{1+R_t} = \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + w_t n_t + d_t + \frac{S_t}{P_t},$$

and the time constraint is

$$n_t + l_t + h[M_t/(P_t c_t)] = E, \tag{2}$$

where P_t is the price level, M_t is nominal money balances chosen in period t to carry over to $t+1$, B_t is holdings of one-period nominal zero-coupon bonds maturing at $t+1$, R_t is the interest rate on nominal bonds, w_t is the real wage, n_t is time spent working, d_t is real dividend payments from firms, S_t is a lump-sum transfer of money from the monetary authority, $h[M_t/(P_t c_t)]$ is time spent transacting, and E is the time endowment. Defining real balances to be $m_t \equiv M_t/P_t$, the function $h(\cdot)$ is parameterized as in Wolman (1997):

$$h(m_t/c_t) = \phi \cdot (m_t/c_t) - \frac{\nu}{1+\nu} A^{-1/\nu} (m_t/c_t)^{\frac{1+\nu}{\nu}} + \Omega, \text{ for } m_t/c_t < A \cdot \phi^{\nu},$$

$$h(m_t/c_t) = \Omega, \text{ for } m_t/c_t \geq A \cdot \phi^{\nu}, \tag{3}$$

with $\phi = 1.4 \times 10^{-3}$, $A = 1.7 \times 10^{-2}$, and $\nu = -0.7695$. Transactions time is thus decreasing in real balances and increasing in consumption, up to a satiation level of the ratio of real balances to consumption.

Goods Market Structure

As in Blanchard and Kiyotaki (1987), we assume that every producer faces a downward-sloping demand curve with constant elasticity ε. [9] The consumption aggregate is an integral of the differentiated products $c_t = [\int c(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega]^{\frac{\varepsilon}{\varepsilon-1}}$, as in Dixit and Stiglitz (1977).

[8] This value of β implies a steady-state real interest rate of 6.5 percent per annum and hence a steady-state nominal interest rate of about 11.5 percent when there is 5 percent annual inflation. While the number assigned to β has quantitative implications for the results reported below, it does not have qualitative implications.

[9] We assume $\varepsilon = 10$.

Since all producers that adjust their prices in a given period choose the same price, it is easier to write the consumption aggregate as

$$c_t = c(c_{0,t}, c_{1,t}) = \left(\frac{1}{2} \cdot c_{0,t}^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} \cdot c_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where $c_{j,t}$ is the quantity consumed in period t of a good whose price was set in period $t-j$. The constant elasticity demands for each of the goods take the form

$$c_{j,t} = \left(\frac{P^*_{t-j}}{P_t} \right)^{-\varepsilon} \cdot c_t, \tag{5}$$

where P^*_{t-j} is the nominal price at time t of any good whose price was set j periods ago, and P_t is the price index at time t, given by

$$P_t = \left[\frac{1}{2} \cdot \left(P^*_t\right)^{1-\varepsilon} + \frac{1}{2} \cdot \left(P^*_{t-1}\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{6}$$

Optimization

If we attach Lagrange multipliers λ_t and μ_t to the budget and time constraints, respectively, so that λ_t is the marginal value of real wealth and μ_t is the marginal value of time, the first-order conditions for the individual's maximization problem, with respect to c_t, l_t, n_t, B_t, and M_t, are

$$\frac{1}{c_t} = \lambda_t - \mu_t \cdot h'(\cdot)(\frac{m_t}{c_t^2}), \tag{7}$$

$$\chi_t = \mu_t, \tag{8}$$

$$\mu_t = w_t \cdot \lambda_t, \tag{9}$$

$$\frac{\lambda_t}{P_t} = \beta \cdot (1 + R_t) \cdot E_t \frac{\lambda_{t+1}}{P_{t+1}}, \tag{10}$$

and

$$\frac{\lambda_t}{P_t} + \frac{\mu_t}{P_t} \cdot h'(\cdot)(\frac{1}{c_t}) = \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}. \tag{11}$$

In choosing consumption optimally (as in [7]), the individual weighs the benefit of consuming a marginal unit, which is the left-hand side of (7), against the cost, which consists of both forfeited real wealth (the first term on the right-hand side) and time spent transacting (the second term on the right-hand side). In choosing leisure and labor supply optimally (as in [8] and [9]), the individual weighs the marginal value of time against both the marginal utility of leisure and the wage earnings that the time would yield. The choice of bond holdings (equation [10]) equates the marginal value of nominal wealth today to $(1 + R_t)$ times the marginal value of nominal wealth tomorrow. And finally, optimal money holdings (equation [11]) imply that the individual equates the transactions-facilitating benefit to the foregone interest cost of holding money.[10]

Firms

Each firm produces with an identical technology:

$$c_{j,t} = n_{j,t},\ j = 0, 1, \tag{12}$$

where $n_{j,t}$ is the labor input employed in period t by a firm whose price was set in period $t-j$. Given the price a firm charges, it hires enough labor to meet the demand for its product at that price. Firms that do not adjust their price in a given period can thus be thought of as passive, whereas firms that adjust their price do so optimally, that is, in order to maximize the present discounted value of their profits. Given that it has set a relative price $\frac{P^*_{t-j}}{P_t}$, real profits for a firm of type j are

$$\frac{P^*_{t-j}}{P_t} \cdot c_{jt} - w_t \cdot n_{jt}, \tag{13}$$

that is, revenue minus cost.

Optimal Price Setting

Maximization of present value implies that a firm chooses its current relative price, taking into account the effect on current and expected future profits. Substituting into (13) the demand curve (5) and the technology (12), the present discounted value of expected profits is given by

$$c_t \cdot \left[\left(\frac{P^*_t}{P_t} \right)^{1-\varepsilon} - w_t \cdot \left(\frac{P^*_t}{P_t} \right)^{-\varepsilon} \right] +$$

$$\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \cdot c_{t+1} \cdot \left[\left(\frac{P^*_t}{P_{t+1}} \right)^{1-\varepsilon} - w_{t+1} \cdot \left(\frac{P^*_t}{P_{t+1}} \right)^{-\varepsilon} \right] \tag{14}$$

for the two periods over which a price will be in effect. Differentiating (14) with respect to P^*_t and setting the resulting expression equal to zero, one sees that the optimal relative price satisfies

$$\frac{P^*_t}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{\sum_{j=0}^{1} \beta^j E_t[\lambda_{t+j} \cdot w_{t+j} \cdot (P_{t+j}/P_t)^{\varepsilon} \cdot c_{t+j}]}{\sum_{j=0}^{J-1} \beta^j E_t[\lambda_{t+j} \cdot (P_{t+j}/P_t)^{\varepsilon-1} \cdot c_{t+j}]}. \tag{15}$$

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[10] The transactions-facilitating benefit is given by $\frac{\mu_t}{P_t} \cdot h'(\cdot)(\frac{1}{c_t})$, and the foregone interest cost is $\frac{\lambda_t}{P_t} - \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}$ (see [10]). A conventional money demand equation can be derived by combining (9)–(11): $m_t/c_t = A \cdot \{[R_t/(1 + R_t)] \cdot (c_t/w_t) + \phi\}^{\nu}$.

Essentially, the optimal relative price equates discounted marginal revenue with discounted marginal cost; the numerator of (15) represents marginal cost and the denominator marginal revenue.[11] In a noninflationary steady state, the firm would choose a markup over marginal cost of $\frac{\varepsilon}{\varepsilon-1}$. In an inflationary or deflationary steady state, the markup would differ from $\frac{\varepsilon}{\varepsilon-1}$, as adjusting firms would take into account the future erosion (or inflation) of their relative price (see King and Wolman [forthcoming 1999] for details). With uncertainty, the markup becomes time varying: it depends on the current and expected future marginal utility of wealth, price level, aggregate demand, and real wage.

Driving Process

The only exogenous variable in the model is the preference shock χ_t, and it is assumed to follow a two-state Markov process:

$$\Pr\left(\chi_t = \overline{\chi} \mid \chi_{t-1} = \overline{\chi}\right) = 0.8, \text{ and}$$

$$\Pr\left(\chi_t = \underline{\chi} \mid \chi_{t-1} = \underline{\chi}\right) = 0.8, \ \underline{\chi} < \overline{\chi}. \tag{16}$$

Thus, χ_t varies between high and low values, and on average each value persists for five periods before switching. This process is not meant to replicate actual features of the U.S. economy. Rather, it is chosen to make the economy alternate between periods of high and low output in a way that makes the real interest rate vary over time. It is by no means the only process that would yield such behavior. The equilibrium behavior of the real interest rate will be affected by monetary policy as well as by the shock process.

Monetary Policy

As described below, we assume that policy is characterized by a feedback rule for the nominal interest rate. One component of the feedback rule is a "target" inflation rate, an inflation rate that the rule would deliver in the absence of shocks. In general, the feedback rule makes the nominal rate a differentiable function of observable variables. In certain states of the world, however, that differentiable function would make the nominal rate negative. In those states of the world, we assume that the policy rule sets $R_t = 0$. Given the nominal interest rate implied by the policy rule, the monetary transfer (S_t) is determined by money demand. Note that money demand is an integral part of the model. It is sometimes asserted that when the monetary authority follows an interest rate rule, money demand can be left out of the model, as it only serves to determine the value of the money supply. Here that is not the case, because the quantity

[11] Note that in this sentence, marginal revenue and cost are with respect to price, not quantity.

of money enters other equations of the model in addition to the money demand equation (specifically [7] and [2]).

The nominal interest rate is the rate on one-period bonds, which are assumed to be in zero net supply. This is somewhat problematic from the standpoint of justifying the zero bound. That is, the zero bound is a necessary characteristic of nominal bonds that are willingly held, but nominal bonds are not actually held in the model (they are priced). This inconsistency can be rectified by assuming that there is a fixed real quantity of outstanding government bonds, and the government pays the interest on those bonds by levying lump-sum taxes as necessary.

Solving the Model

The standard method used for solving dynamic stochastic models such as this one is to calculate the steady state for a given inflation rate, and then linearize the model's equations around that steady state. Linearization would be inappropriate here, because it would rule out imposing the zero bound on nominal interest rates. Instead of linearizing, then, we solve the model using a crude version of the finite element method (see McGrattan [1996]). This method involves picking a grid of points for the model's state variable, P^*_{t-1}, and then finding values of the "control" variables numerically for each grid point and for each value of the preference shock such that the model's equations are approximately satisfied. The solution consists of mappings from the state variable to each of the other variables. Those mappings can be used in conjunction with the stochastic process for the preference shock to simulate the model. Because this solution method involves a finite number of grid points, it necessarily yields only an approximate solution. However, to the extent that the true mappings from the state variables to the other variables are smooth functions, the grid method can yield an extremely accurate solution. Furthermore, the extent that the mappings appear nonlinear gives an indication of the error that would be associated with linearization methods.

3. IMPLICATIONS OF THE ZERO BOUND IN THE MODEL

Using the model described above, one can determine whether the zero bound means that a very low inflation target (here it will be deflation) significantly modifies economic performance relative to a moderate inflation target. For a particular specification of monetary policy, we will simulate the model at moderate inflation and then at moderate deflation, and compare the results along three dimensions. The first involves simulating the model for 30 periods with the same shocks at high and low inflation, and informally comparing the results. The second involves the variances of inflation and output, which has been the conventional metric in the literature on monetary policy rules (see the papers in

Taylor [forthcoming 1999]). Given that the model yields an obvious choice for a welfare function (the representative agent's expected utility), we also compare the two regimes in terms of welfare.

Model Simulations

Recent research on monetary policy has emphasized "Taylor rules," that is, specifications of policy where the monetary authority sets a short-term interest rate as a linear combination of deviations of inflation from a target and deviations of output from some trend or potential level. These rules, popularized by John Taylor (1993), have been shown to be parsimonious approximations of the behavior of actual central banks and to have reasonable properties in certain theoretical models. The rule used below is similar to a Taylor rule, except that instead of inflation on the right-hand side it uses the price level. Concretely,[12]

$$R_t = \max \left\{ \begin{array}{c} R^* + 1.5 \cdot [\ln(P_t) - \ln(\overline{P}_t)] + 1.0 \cdot [\ln(c_t) - \ln(\overline{c})], \\ 0 \end{array} \right\}, \qquad (17)$$

where R^* is the steady-state nominal interest rate consistent with the chosen inflation target, $\overline{P}_t$ is a target price-level path that grows at the targeted inflation rate, and $\overline{c}$ is the steady-state level of consumption associated with the inflation target.[13] This rule implies that the price level will always be expected to return to the same trend path. In contrast, the standard Taylor rules imply that inflation will always be expected to eventually return to target, but the price level will be expected to drift away from any previous trend path.[14]

Introduction to the Functions Describing General Equilibrium

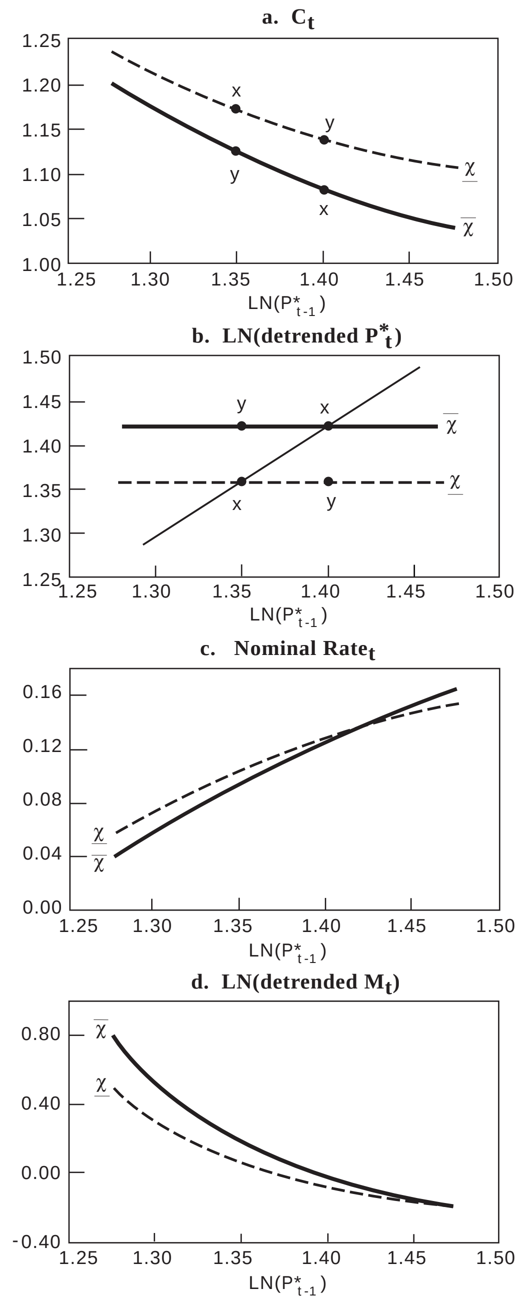
As background to the simulation results, Figure 1 displays the relationships between key endogenous variables and the state variable, which is the price set last period by adjusting firms. Figure 1 is generated with an inflation target of 5 percent. The solid lines show the relationship between P^*_{t-1} (detrended by the targeted inflation rate) and each endogenous variable when the preference shock takes on a high value, and the dashed lines show the relationships when the preference shock takes on a low value. Using panel b, and with knowledge of P^*_0, one can trace out a path for P^*_t by drawing values of χ_t from the stochastic

[12] The interest rate in (17) is a quarterly interest rate, whereas the rates plotted in Figures 1–4 are annual rates.

[13] The inflation target affects steady-state consumption for two reasons. First, the markup chosen by adjusting firms varies with the inflation target in a way that does not exactly offset the inflation erosion of nonadjusting firms' markups. Second, by lowering real balances, higher inflation effectively makes consumption more expensive.

[14] The original motivation for using a price-level target here instead of an inflation target was computational ease. It turns out, however, that analyzing an inflation-targeting policy is no more demanding than analyzing price-level targeting. We are studying inflation-targeting policies in ongoing research.

Figure 1 Functions Mapping State Variable (P^*_{t-1}) to Other Variables at 5 Percent Inflation Target



process governing it. Then, with the path for P_t^* in hand, the relationships in panels a, c, and d can be used to generate paths for the other variables for the given sequence of χ_t. What follows is a discussion of the model's principal mechanisms in light of the relationships shown in Figure 1.

There are essentially two determinants of current-period variables in the model. One is the value of the stochastic preference parameter (χ_t), and the other is the value of the price that adjusting firms set last period. When χ_t takes on a high value, the marginal utility of leisure is high. Agents react by supplying less labor to the market, and this reaction brings with it a decrease in consumption. Thus, in panel a, the level of consumption is low when $\chi_t = \bar{\chi}$. For low values of P_{t-1}^*, the lower level of consumption causes the monetary authority to set a lower value for the nominal interest rate, as in the left-hand part of panel c, and the lower nominal interest rate in turn drives up money demand (panel d). However, when P_{t-1}^* is especially high, the nominal rate is lower in the $\underline{\chi}$ (high-consumption) state. Why is this the case? The feedback rule for monetary policy sets the nominal rate as an increasing function of both consumption and the price level, so it must be that in the high-P_{t-1}^* region the price-level effect dominates in the feedback rule. The policy functions for the price level (not shown) indeed reflect this fact. The price level is higher in the $\bar{\chi}$ state than in the $\underline{\chi}$ state, and the gap between the price levels in the two states is increasing in P_{t-1}^*.

Another perspective on the nominal interest rate functions in panel c comes from thinking about two relationships emphasized by Irving Fisher. We have already seen the "Fisher Equation: I," which states that the nominal interest rate is approximately equal to the sum of the real interest rate and expected inflation.[15] But Fisher also provided the seminal discussion of the relationship between real interest rates and current and future marginal utilities of consumption. Since the real interest rate is the price at which agents can trade current consumption for future consumption, it follows that agents will choose an expected consumption path to equate the real interest rate to the ratio of marginal utilities of current and future consumption. When utility is logarithmic in consumption, as it is here, this "Fisher Equation: II" implies that the real interest rate is approximately equal to expected consumption growth.[16]

From panel a, we know that consumption and the preference parameter move in opposite directions. Further, the stochastic process for the preference parameter is mean reverting, so that when χ_t is low it is expected to increase, and, therefore, consumption is expected to fall. From Fisher's second equation,

[15] For an explanation of why the "Fisher Equation: I" is only approximately correct, see Sarte (1998).

[16] The relationship is only approximate here because the shopping time requirement means that the marginal utility of consumption is greater than the marginal value of a unit of real wealth. To derive this approximate relationship, combine (7) and (10) above.

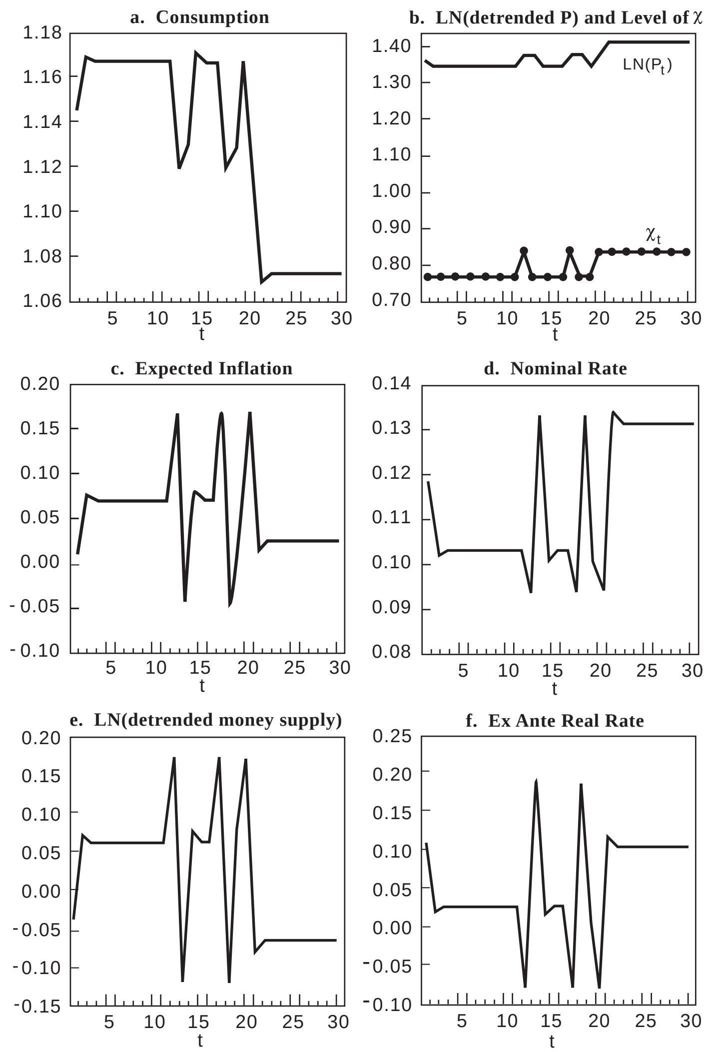
real interest rates are then low when the preference parameter is low. Note, however, that the policy rule typically makes nominal rates high in those cases when we have just argued that real rates are low. From Fisher's first equation, it must then be that high nominal rates correspond to high enough expected inflation to counteract the low real rates. From panel d we can see that monetary policy does in fact deliver high expected inflation when the preference parameter is low. The money supply is low when the preference parameter is low, and mean reversion implies that the money supply is expected to increase in those periods, generating high expected inflation.

Note that the behavior of real interest rates conflicts with the behavior displayed in the other articles discussed above. There the monetary authority lowers nominal interest rates when output is low, and real rates fall as well. Here, for the most part, the monetary authority also decreases nominal interest rates when output is low. However, real interest rates are to a great extent determined by the shock process in conjunction with Fisher's second equation. For a large class of such processes that includes the one used here, real interest rates are low when output is *high*. More generally, it has proven difficult to produce models where the cyclical behavior of real rates matches the data without resorting to the type of reduced form modeling employed by Fuhrer and Madigan (1997) and Orphanides and Wieland (1998).

Simulated Time Paths

Figure 2 displays the time paths of the variables from Figure 1 other than P_t^*, as well as the price level, the real interest rate, and expected inflation, for a sequence of 30 χ_t drawn from the stochastic process described above. This sequence will be a benchmark for comparison with the low inflation target case below. Focusing first on consumption (panel a), note that there are essentially three regions: low, high, and intermediate. The high-consumption region is attained with any sequence of at least two consecutive low values for χ_t (the realizations of χ_t are plotted in panel b). Likewise, the low-consumption region is attained with any sequence of at least two consecutive high values for χ_t. These regions correspond to the points marked **x** in Figure 1a and b. The intermediate-consumption region corresponds to the transition from one value of the preference shock to the other; these are the points marked **y** in Figure 1a and b. The fact that it takes two periods to transit between the high- and low-consumption regions is an implication of two-period price stickiness. To see this, suppose the economy had been in the low preference parameter/high-consumption state for several periods. If χ_t then took on a high value, in the initial period the state variable (P_{t-1}^*) would be at the level associated with $\underline{\chi}$, so that the economy could not immediately transit to low consumption. If χ_t remained high in the next period, consumption would settle at a lower level, because the state variable had changed; by the period after the shift in χ_t, all

Figure 2 Time Paths from 30-Period Simulation (5 Percent Inflation Target)



firms would have had a chance to adjust their price. If prices were flexible, the transition would be immediate, whereas with prices set for more than two periods the transition would be correspondingly longer.

Note that in some of the periods when consumption takes on an intermediate value, the real rate is negative (Figure 2f). Specifically, this occurs in periods when $\chi_t = \overline{\chi}$ and $\chi_{t-1} = \underline{\chi}$ (periods 12, 17, and 20). Referring back to Figure 1, one can see that in this situation consumption is expected to fall towards the low level associated with $\bar{\chi}$. With consumption expected to fall significantly, the real rate must be negative. Because the inflation target is 5 percent, the zero bound does not inhibit the real rate from going negative. However, one might expect that with a very low inflation target, the real rate would be inhibited from going negative, and thus the zero bound would interfere with the economy's "natural" behavior.

Figures 3 and 4 correspond to Figures 1 and 2, with an inflation target of -5 percent. From Figure 3a–c, we see that for a wide range of values of the state variable, including the region corresponding to high consumption, the nominal rate is zero. This drastically different behavior of the nominal rate, however, does not correspond to significantly different functions for consumption (Figure 3a). The simulation in Figure 4 confirms these results. Whereas we surmised that the nominal rate might hit the zero bound when $\chi_t = \bar{\chi}$ and $\chi_{t-1} = \underline{\chi}$, in fact it hits the bound whenever $\chi_t = \underline{\chi}$. However, consumption behavior is almost indistinguishable from Figure 2, the 5 percent inflation target. From Fisher's second equation, we know that similar consumption behavior must correspond to similar real rate behavior, as confirmed in Figure 4f. How is a zero nominal rate consistent with a negative real rate in periods 12, 17, and 20? From Fisher's first equation, the real rate is the difference between the nominal rate and expected inflation, so in those periods the monetary authority is making expected inflation positive (panel c). The targeted rate of deflation is consistent with periods of high expected inflation, because the policy rule unambiguously makes the expected inflation temporary, and there is no uncertainty about whether the monetary authority will adhere to the policy rule.

Simulations such as those in Figures 2 and 4 are an informal means of evaluating whether the zero bound is important. However, those simulations provide clear evidence—at least in the model used here—that monetary policy can offset the zero bound by generating temporary expected inflation. With real rates thus unconstrained, the existence of the zero bound does not appear to constitute an argument against a low inflation target. Figure 4 illustrates an additional feature of the model that *favors* a very low inflation target. In panels a and f, the series for consumption and real rates from Figure 2, corresponding to a 5 percent inflation target, are reproduced along with the new series corresponding to 5 percent deflation. In panel a, we see that consumption is actually higher in every period with the 5 percent deflation target than it is with the 5 percent inflation target. The lower inflation target corresponds to lower nominal interest rates on average, as is shown clearly in panel d of Figures 2 and 4. Lower nominal interest rates in turn correspond to a smaller money demand distortion, as in Bailey (1956) and Friedman (1969). Individuals hold higher real balances because the opportunity cost of real balances has fallen, and higher real balances effectively make consumption cheaper, because they decrease the time that an individual must spend transacting.

Figure 3 Functions Mapping State Variable (P^*_{t-1}) to Other Variables at 5 Percent Deflation Target

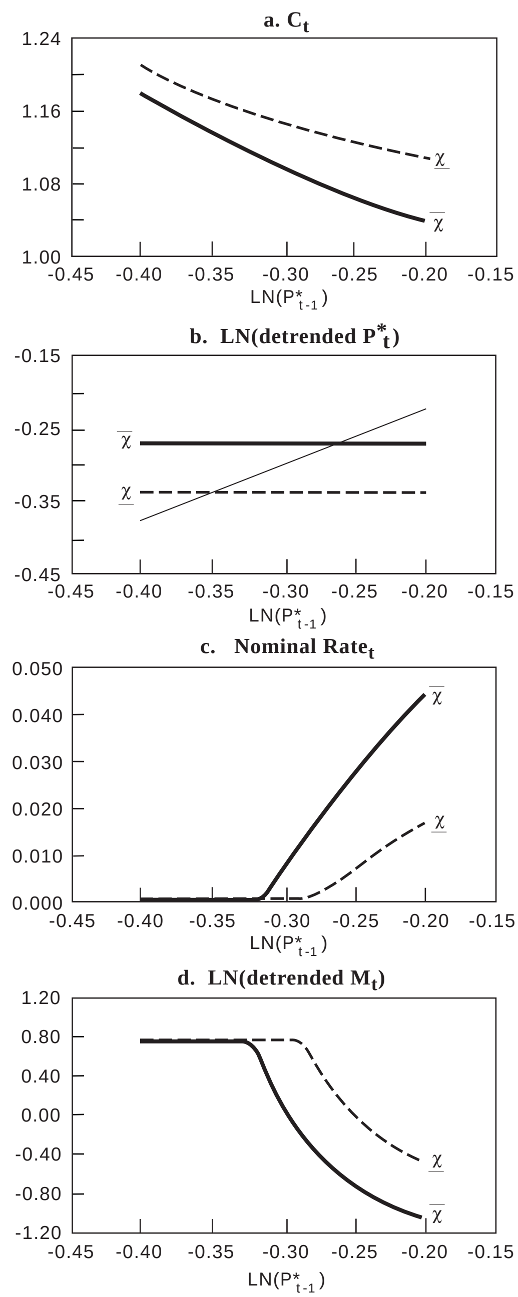
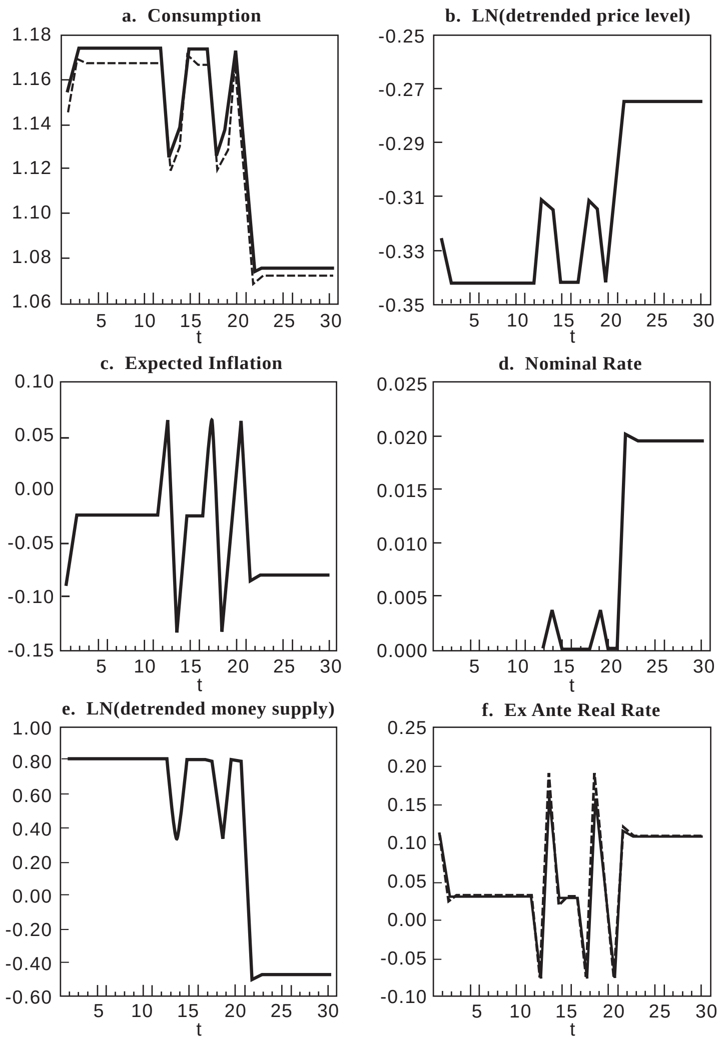


Figure 4 Time Paths from 30-Period Simulation (5 Percent Deflation Target)



Note: Dashed series are from Figure 2 (5 percent inflation target).

Variances

The simulations in Figures 2 and 4 provide strong evidence on the importance of the zero bound, and the welfare results below give the bottom line. To enhance comparability with the articles by Rotemberg and Woodford (1997) and Orphanides and Wieland (1998), we also provide information on variability at high and low inflation targets. Table 1 shows the standard deviations of some of the main variables in the model for both regimes, based on simulations of 5,000 periods. As suggested by Figures 1–4, the variability of consumption is barely affected by the inflation target. On the other hand, the nominal interest rate is much less variable when the inflation target makes zero occasionally binding. There is a tradeoff in the model between the average level of inflation and the minimum feasible variability of inflation, just as described in Rotemberg and Woodford (1997). Also as in that paper, the large difference in nominal interest rate variability in the two regimes translates into only a small difference in inflation variability. A striking feature of Table 1 is the tremendous increase in money supply variability in the deflation regime. This can be traced to the fact that the money demand function exhibits increasing sensitivity to nominal interest rates as the nominal interest rate falls.

Table 1 Standard Deviations in the Two Policy Regimes

	Consumption	**Inflation**	**Nominal rates**	**Money**
5 percent inflation	0.0427	0.0706	0.0145	0.0910
5 percent deflation	0.0435	0.0786	0.0093	0.7562

Welfare

The motivation for this article came from the idea that low inflation targets might be bad because of distortions introduced by the zero bound on nominal interest rates. It is clear from the simulations presented thus far that in fact the real (as opposed to nominal) distortions associated with the zero bound are small. Nevertheless, it is interesting to know whether the inflation or deflation regime is preferred on welfare grounds. When the zero bound is not a factor, a welfare comparison will hinge on the other distortions present in the model. Those other distortions involve the inflation tax and the interaction between sticky prices and monopolistic competition. The inflation tax distortion makes deflation preferable to inflation. Sticky prices and monopolistic competition make the optimal inflation target near zero, so neither 5 percent inflation nor deflation targets would obviously be preferred to the other on that basis. It therefore seems likely that the unambiguous effect of the inflation tax will dictate that the lower inflation regime is preferred. However, to resolve the issue definitively, we must compare the representative individual's expected utility in the inflation and deflation regimes.

We calculate expected utility by performing 1,000 simulations of 1,000 periods each, with each simulation beginning from a random value for the state variable. The initial condition is chosen by simulating the model for 50 periods, starting from the steady state, and then setting $P_0 = P_{50}$. Each simulation ($k = 1$ to $1,000$) yields a value for $U_k \equiv \sum_{t=0}^{1,000} \beta^t \cdot [\ln(c_t) + \chi_t \cdot l_t]$, and then expected utility is given by $E(U) = 1,000^{-1} \cdot \sum_{k=1}^{1,000} U_k$. With values for expected utility in both regimes, we compare the regimes by pretending that they were generated in a steady state. We calculate the average per-period utility in the two regimes and then the percentage increase in consumption that would make an agent living in the lower utility regime just as well-off as an agent in the higher utility regime. The results of this exercise are that an agent living in the inflationary regime would be indifferent between receiving a 2.6 percent increase in per-period consumption and switching to the deflationary regime.

To illustrate the importance of the inflation tax in these results, we can repeat the comparison of the two inflation regimes with a slight modification. That modification is to eliminate the money demand distortion; we modify (7) to $\lambda_t = 1/c_t$ and replace (11) with $M_t = P_t \cdot c_t$. With the inflation tax eliminated, the 5 percent inflation target regime is marginally preferred to the 5 percent deflation target regime, although the difference in welfare is minuscule compared to the difference found (with opposite sign) when the inflation tax played a role. The results from eliminating the money demand distortion mean that money demand is crucial in making the deflationary regime welfare-superior to the inflationary regime. However, even without the money demand distortion, the fact that the nominal interest rate is occasionally zero in the deflationary regime does not significantly affect the behavior of real variables. In particular, the policy rule is still able to generate temporarily high expected inflation when real rates need to be negative.

Open Questions

With respect to the specific model used here, at least three modifications would be interesting to analyze. The first modification deals with the specification of price stickiness. Structural models of sticky inflation are ad hoc, but they have been shown to fit recent data well. It should be possible to modify the price block of the current model to make inflation sticky. The resulting specification would not simply repeat the work of Orphanides and Wieland (1998) and Fuhrer and Madigan (1997), because it would incorporate money demand. Solving such a model would be more computationally intensive than solving the model in this article, because it would include additional state variables associated with the pricing specification.

The second modification is related to the first; it involves changing the policy rule from the price-level form to the more common inflation form. Possibly with such a rule and a low inflation target the monetary authority would be less able to generate the temporary expected inflation necessary to drive real rates negative. More generally, it would be interesting to study the properties of a wide range of rules and to find out what the optimal rule is. Experiments with a rule that specifies the money supply instead of the nominal interest rate as the policy instrument yield similar results to those above, in that the deflationary regime is preferred to the inflationary regime. The interest rate rule generates higher welfare than the money rule, but that comparison is limited, focusing on two specific rules as opposed to classes of rules. In terms of optimal rules, King and Wolman (forthcoming 1999) find that it is optimal to stabilize the price level if the money demand distortion is nonexistent. With that distortion present, optimal policy will undoubtedly involve some deflation, but it is not clear exactly what the optimal policy rule is.[17]

The third modification is one that takes more seriously the fiscal aspect of monetary policy. Work by Woodford (1996) and Sims (1994) emphasizes the joint behavior of fiscal and monetary policy. This joint behavior might be especially relevant when interest rates are near zero, because at zero nominal interest rates, fiscal and monetary policy effectively become unified; money and government bonds are perfect substitutes.

Apart from the specifics of the model, the assumption that agents in the model have perfect information about the policy rule is crucial. We found that zero nominal interest rates did not prevent the real rate from falling, because the monetary authority could generate expected inflation when the nominal rate was zero. Agents know that any inflation that ensues will be temporary, and that the monetary authority remains committed to its stated inflation target, so these occasional periods of high expected inflation do not trigger inflation scares. In practice, central banks might have concerns about being able to generate occasional episodes of high expected inflation without endangering the credibility of their low inflation target. In principle it would be possible to analyze this sort of issue in an extension of the current framework.

A fundamental assumption underlying all recent work on the zero bound is that negative ex ante real interest rates are occasionally a natural characteristic of the U.S. economy. It is a trivial matter to look at data on ex post real rates and see that at the short end of the yield curve they have been negative on many occasions. It is less clear that ex ante real rates have been negative. From Irving Fisher, we know that real rates defined by the CPI can be negative only to the extent that the market basket that makes up the CPI is not storable at zero cost. Undoubtedly the inclusion of various services and perishable goods means that in principle the ex ante real rate can be negative. Nonetheless, lack of consensus about how to estimate inflation expectations means that widely accepted series for ex ante real rates do not exist.

[17] The approach taken in this article would suggest defining the optimal policy rule as the rule that generates the highest level of unconditional expected utility. King and Wolman (forthcoming 1999) use a different criterion; they ask what policy rule is implied by assuming that the monetary authority maximizes agents' expected utility given some arbitrary initial conditions.

4. CONCLUSIONS

Two general conclusions are supported by the theoretical analysis in this article. First, the way money demand is modeled is important for how one evaluates the zero bound on nominal interest rates. Existing work presumes that the zero bound makes low inflation bad, because it prevents monetary policy from optimally responding to shocks. But monetary theory supports a strong benefit to zero nominal interest rates, namely, eliminating inefficiencies associated with holding "too little" money. The existence of those inefficiencies contributed to the result in this article that, taking into account the zero bound, a regime with moderate deflation yields higher welfare than a regime with moderate inflation. The second conclusion is that stickiness of inflation is crucial in generating costs of low inflation associated with the zero bound. If prices are sticky but inflation is not, then real rates can fall even if nominal interest rates are very low: the monetary authority simply creates some expected inflation if it wants to drive real rates down.

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Government Loan, Guarantee, and Grant Programs: An Evaluation

Wenli Li

Recently, there has been a trend toward loan guarantee programs over other programs that support the credit market. From 1970 to 1998, the real value of outstanding federal loan guarantees rose at an accelerated pace, while the real value of direct loans, the other major government loan program, has remained about the same (see Figure 1). In particular, the Small Business Administration (SBA), which has provided government loan guarantees to small businesses since 1953, has experienced an unprecedented increase in its loan volume over the past three years. In December 1997, with the growing popularity of SBA loans, Congress passed an SBA funding bill that set aside \$39.5 billion and \$11 billion, respectively, for the SBA's 7(a) and 504 business loan programs over the next three years. This more than tripled the current 7(a) level which was \$10.3 billion in fiscal year 1997.[1]

The surge in loan guarantee programs prompts the question: Are loan guarantees the best way to provide benefits to targeted borrowers or to channel additional resources to targeted sectors? As the following paragraphs show, not in all cases. This article explains that conclusion by examining the economic consequences of three distinct methods of channeling resources to targeted borrowers: direct government lending, loan guarantees, and outright grants. While the logic applies to any credit market segment, the article particularly focuses on the small business sector. The analysis studies the changes in firm investment, bankruptcy cost, and business entry under each loan program in a

I would like to thank Tom Humphrey, Jeff Lacker, Pierre Sarte, and John Weinberg for many important suggestions. Jeff Walker provided excellent research assistance. The views expressed are the author's and not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

[1] Bureau of National Affairs, Inc. (1997).

theoretical model economy designed to capture the essential features of small business borrowing.

One thing is sure. These credit policies cannot make the private economy any more efficient, the reason being that the government does not have information or technology advantage over private agents. Therefore, there will not be any efficiency gain associated with credit policies. (In other words, the absence of efficiency gains means that policies cannot make any agent better off without hurting other agents.) In this article, we take as given a political desire to assist a particular group of borrowers and look at how the different alternate credit programs redistribute resources.

Perhaps it is most appropriate to explore the effects of government credit programs within a model of financial frictions. It is natural to do so because many economists contend that such frictions have a greater effect on certain kinds of borrowers, such as small businesses and students, than on others. Accordingly, the environment studied here is one in which financial frictions are caused by private information: in particular, moral hazard.[2] Moral hazard occurs when the very act of insuring a borrower against risk induces him to take on additional risk. Such frictions drive a wedge between the cost of internal funds and that of external funds as in Townsend (1979) and Gale and Hellwig (1985). The central notion is that wealth affects people's decisions, creating liquidity constraints.

The relevance of such a model is supported by empirical evidence. Holtz-Eakin, Joulfaian, and Rosen (1994), Evans and Leighton (1989), Blanchflower and Oswald (1998), and Evans and Jovanovic (1989) among many others, find that a lack of wealth affects people's ability to become self-employed, even after accounting for the possible correlation between entrepreneurial ability and wealth. In a more recent study, Bond and Townsend (1996) reported on the results of a survey of financial activity in a low-income, primarily Mexican neighborhood in Chicago and found that borrowing is not an important source of finance for business start-ups. In their sample, only 11.5 percent of business owners financed their start-up with a bank loan, while 50 percent of the respondents financed their start-up entirely out of their own funds.

1. AN OVERVIEW OF GOVERNMENT CREDIT PROGRAMS

In the United States, the federal government regularly proposes and endorses programs that are designed to direct and encourage the flow of funds to

[2] Adverse selection—namely, situations in which borrowers have unverifiable hidden knowledge about their likelihood of repayment—is another form of private information that gives rise to financial frictions. See de Meza and Webb (1987), Gale (1991), Innes (1991), and Lacker (1994) for discussion.

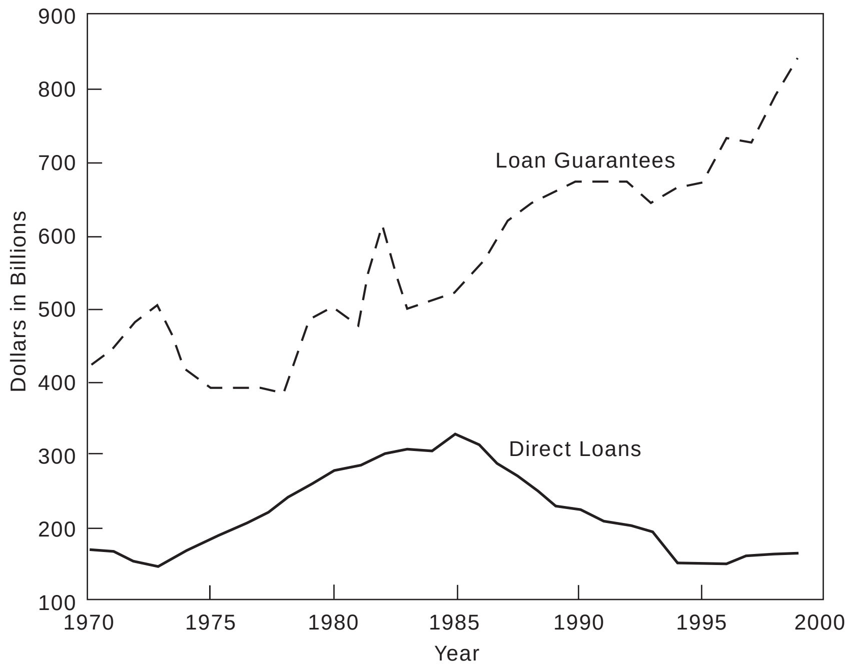
selected consumers and businesses. For instance, the Community Reinvestment Act (CRA) attempts to increase the flow of funds to disadvantaged communities or persons by requiring depository institutions to make a minimum effort to fund these groups. Similarly, the SBA's section 7(a) loan program and its Small Business Investment Company program encourage the flow of funds to small businesses through government guarantees of debt issued by the financial intermediaries providing the funds to the small business. Numerous other government-sponsored enterprises (GSEs) such as Fannie Mae, Sally Mae, Freddie Mac, etc., operate on secondary markets and provide credits for targeted groups in exchange for preferential treatment from the government.

Government intervention in the financial market has occurred mainly via direct loans, grants, and indirect loan guarantees. In the case of direct loans, a government agency acts as an intermediary in place of banks; it issues loans directly to the targeted group, obtaining funds from the capital markets by issuing Treasury securities and/or imposing taxes. Direct loans typically offer large subsidies, usually to the agricultural and rural sectors. Unlike direct loans, grants and loan guarantees do not involve any repayment from the recipients. Grants, provided by the government directly to the targeted recipients, are often received at the end of the period when they are added to business profits to help defray costs. Loan guarantees provide investors with assurance that the government will make up any difference between a given guaranteed loan payment and an agent's actual loan payment. A loan guarantee requires the participation of three parties: the government agency, the borrower, and the private lender. The government agency deals indirectly with the borrower through a private lender. Typically, the acquisition of an SBA loan proceeds as follows. The borrower first presents the appropriate financial data for the lender to review. Based on the lender's evaluation, three courses of action are possible: the lender (1) may decide to finance the loan without an SBA loan guarantee; (2) may provide financing conditional upon obtaining an SBA loan guarantee; or (3) may reject the loan. If the lender approves the loan based on the SBA's willingness to provide a guarantee, then the lender must help the borrower prepare the SBA loan application. Upon completion of the application, the SBA reviews the loan. Over 90 percent of all loan guarantee applications are approved by the SBA (Haynes 1996). Of course loan guarantee programs assist a wide range of borrowers besides small businesses, including homeowners, students, and exporters.[3]

Figure 1 depicts the recent trend in government direct loan and loan guarantee programs (GSEs included). As shown here, federal credit outstanding in

[3] In addition to direct loans and loan guarantees, GSEs aid borrowers in housing, agricultural, and student loan markets, primarily through the operation of secondary markets. The tax-exempt status of state and local governments allows them to borrow at reduced cost and to direct the interest savings to preferred borrowers.

Figure 1 Real Value of Federal Credit and Guarantees Outstanding (1992 dollars)



the form of loan guarantees has experienced an explosive growth relative to that of direct loans.[4]

Tables 1 and 2 present the various direct loan and guaranteed loan programs that existed in the 1996 fiscal year. As the tables show, virtually every sector of the economy is covered by some type of program, and assistance to some sectors takes the form of both direct loans and guaranteed loans. In this article, we focus on the kinds of programs associated with investment behavior. Examples of such programs include those targeted to the entrepreneurial community and students.

2. THE THEORETICAL MODEL

A sensible model for our purpose must have two key features. First, the model should display asymmetric information that gives rise to financial frictions so

[4] Grants are not used as much as direct loans and loan guarantees. We do not have time-series data on the spending of government grants in the United States.

Table 1 Direct Loan Transactions of the Federal Government: 1996 Fiscal Year (Millions of Dollars)

	Net Outlays	Outstandings
National defense		1,384
Internal affairs	1,674	38,983
Energy	1,036	34,125
Natural resources and environment	34	294
Agriculture	6,183	15,580
Commerce and housing credit	1,570	40,897
Transportation	47	314
Community and regional development	1,963	17,739
Education, training, employment and social services	9,120	12,431
Health	25	834
Income security	93	2,303
Veteran benefits and services	1,442	1,188
General government direct loans	379	462
Total	23,566	166,534

Source: The Budget of the United States Government, 1996.

that agents' wealth affects their investment demand. Second, the model should also demonstrate that the amount of desired investment (not simply whether to invest) varies with the cost of borrowing.

Here we describe an economic environment that contains the above features. It is a simple environment with borrowing and lending occurring under the condition of moral hazard. The main characteristic of this environment is that some information regarding the return to investment projects is concealed and is observable to project owners but not to lenders. Because lenders do not have full information, they cannot determine the state of the projects so they have to spend real resources to verify borrowers' reports. The economy studied here also includes another important characteristic: agents decide whether to start a new business or remain an employee. Since imperfect information limits risk-sharing, this self-selection turns out to be correlated with the amount of assets that agents hold, as well as the quality of their business projects. Therefore, both margins of business activity are captured in the model, namely, the intensive margin of business investment and the extensive margin of entry.

To introduce some notation, we refer to a two-period economy with a continuum of agents of measure one. Consumption takes place in both periods, and we denote them by c_i, $i = 1, 2$. The utility function is assumed to take the form $U(c_1) + c_2$. In the first period, each agent receives some wealth w and a project that can be operated in the second period. Wealth w has a cumulative distribution function $G(w)$ on the interval $[\underline{w}, \overline{w}]$, where $0 < \underline{w} < \overline{w}$. The

Table 2 Guaranteed Loan Transactions of the Federal Government: 1996 Fiscal Year (Millions of Dollars)

	Net Outlays	**Outstandings**
National defense	276	441
Internal affairs	8,418	34,341
Energy		691
Natural resources and environment		
Agriculture	5,082	12,309
Commerce and housing credit	181,277	987,420
Transportation	826	2,154
Community and regional development	839	2,565
Education, training, employment and social services	19,816	101,874
Health	210	3,113
Income security	5	3,867
Veteran benefits and services	28,676	154,762
General government direct loans	379	462
Subtotal	245,425	1,303,537
less secondary guaranteed loans	−101,540	−497,433
Total	143,885	806,104

Source: The Budget of the United States Government, 1996.

project is indexed by its probability of success p: if a project succeeds, it produces output $f(k)$, where k is total investment; if it fails, no output will be produced. Function $f(k)$ is assumed to be increasing in k and concave, i.e., $f'(.) > 0$ and $f''(.) < 0$. The project success probability p is characterized by a cumulative distribution function denoted by $\Gamma(p)$ with support $[\underline{p}, \overline{p}]$. The probability of success p is a measure of business quality.

In the first period, after receiving his endowment of assets and a project, an agent determines his consumption for this period and his saving for the second period. He also decides whether he wants to carry out his project. In period 2, the agent, if he is an entrepreneur, decides how much to invest. If the total amount of investment exceeds his saving, then he needs to borrow. If the agent is a worker, he draws his income from lending and a fixed income q from working an outside option in period 2.[5] The following timeline describes the sequence of actions.

[5] We could assume that production takes both capital and labor as inputs and that q corresponds to the wage that is endogenously determined. This assumption would further complicate the analysis here without much gain.

period 1: all agents receive w and learn p
⇓
formulate period 1 consumption
make occupational decision that will affect period 2
⇓
period 2:

entrepreneurs:	workers:
borrow or lend	lend
⇓	⇓
manage the project	work the outside option
⇓	⇓
repay loan	receive interest payment from deposit
⇓	⇓
consume	consume

The information structure of the economy is as follows. Everything in the first period is public information: the level of assets, the quality of the project, and the decision about whether or not to be an entrepreneur. In period 2, however, when production takes place, only those carrying out the project observe the outcome of the project. An outsider can learn the outcome only after bearing a verification (auditing) cost. Given that financing a project may require loans from more than one lender, the optimal financial structure is one where all lending is transacted by a large financial intermediary who lends to a large number of borrowers and borrows from a large number of depositors. Because it has a comparative advantage in doing so, the financial intermediary monitors the borrowers to economize on verification costs; if there were direct lending, each of the lenders who lent to an entrepreneur would have to verify the investment project's return in the event of default.

Those wishing to borrow attempt to do so by announcing loan contract terms: the amount of loans borrowed, repayment after production conditional on borrowers' report, and when monitoring occurs. If the financial intermediary accepts the terms, it then takes deposits, makes loans, and monitors project returns as required by the contracts it accepts. We assume perfect competition in the financial sector. Then, in equilibrium, the financial intermediary will be perfectly diversified, will earn zero profits, and will have a nonstochastic return on its portfolio. Therefore, the intermediary need not be monitored by the depositors.

The two-outcome distribution of returns is a special case of the more general distributions discussed in Townsend (1979) and Gale and Hellwig (1985). We rule out randomized verification strategies, that is, the financial intermediary cannot verify the return of an agent's project with some probability. The optimal contract in this setting is a debt contract where entrepreneurs pay a fixed amount if the project succeeds and default if the project fails, in which case verification takes place. We can interpret the act of verification as implying bankruptcy for two reasons. First, in the more general setup, the optimal contract turns out to be the standard debt contract under which the return is observed if and only if the firm is insolvent. Second, real-world bankruptcy does appear to involve a transfer of information. The cost of bankruptcy can be substantial and is likely to be a function of the level of the firms' debt. For simplicity, we assume that bankruptcy cost takes the form of $\beta + \gamma b$, where β corresponds to the fixed cost, and γ is the per-unit variable cost. The amount of borrowing is denoted by b. Firms' total investment k is then the sum of its own internal fund or savings from first period s and loan borrowing b.

Let x denote the payment by the entrepreneur to the financial intermediary, and let r be the interest rate the financial intermediary pays to investors. It follows that the financial intermediary is willing to accept loan contract offers yielding an expected rate of return of at least r. Borrowers differ in the amount s of their initial wealth that they save, and their project's probability of success p. A loan contract with a borrower (s, p) must satisfy the following constraint,

$$px = rb + (1 - p)(\beta + \gamma b), \tag{1}$$

if the intermediary is willing to accept it. Investment k is the total of saving s and loan borrowed b. The loan contract also has to be feasible for the borrower

$$x \leq f(k). \tag{2}$$

This expression says that the borrower has enough to repay the loan in the good state.

Borrowers will then maximize their own expected utility by setting investment level k, subject to the constraints just described. Therefore, announced loan contracts will be selected so that they solve

$$\pi(s, p) = \max_b \{pf(b + s) - px\} = \max_b \{pf(b + s) - rb - (1 - p)(\beta + \gamma b)\}, \tag{3}$$

where $b = k - s$, subject to conditions (1) and (2). The function $\pi(s, p)$ is the expected second-period consumption of a borrower with saving s and business project p.

The return v to a representative worker (s, p) is equal to

$$v(s) = q + rs, \tag{4}$$

consisting of the income q plus the gross return rs on savings. In period 1, an agent chooses his period 1 consumption c_1, saving s, period 2 consumption c_2, and occupational decision δ to solve the following problem:[6]

$$\max U(c_1) + Ec_2, \tag{5}$$

subject to

$$Ec_2 = \delta\pi(s,p) + (1-\delta)v(s), \tag{6}$$

$$s = w - c_1, \tag{7}$$

$$\delta \in \{0,1\}. \tag{8}$$

Condition (6) says the second-period consumption depends on the agent's occupation, $\pi(s,p)$ for an entrepreneur and $v(s)$ for a worker. Condition (7) indicates that saving is the difference between an agent's asset endowment and his first-period consumption. Condition (8) restricts δ to be a binary variable that takes a value 1 when the agent chooses to be an entrepreneur in the second period and 0 when he chooses to be a worker.

Saving in period 1 is a solution to the following first-order condition:

$$U'(w-s) = \begin{cases} \pi_1(s,p) & \text{if } \pi(s,p) > v(s), \\ r & \text{otherwise.} \end{cases} \tag{9}$$

Figures 2 and 3 describe the determination of occupational choice for a given project and a given endowment of asset. The asset level is measured on the horizontal axis in Figure 2, the project success probability is measured

[6] Another way of writing an agent's problem is as follows:

$$\max_{\delta}\{U^w, U^e\},$$

where U^w is the utility of being a worker in the second period, and U^e is the utility of being an entrepreneur in the second period. The occupational decision is denoted by δ; it takes a value of 1 when $U^w < U^e$ and 0 otherwise. Moreover,

$$U^w = \max_{c_1,s,c_2} U(c_1) + E(c_2),$$

subject to

$$Ec_2 = v(s),$$

$$s = w - c_1.$$

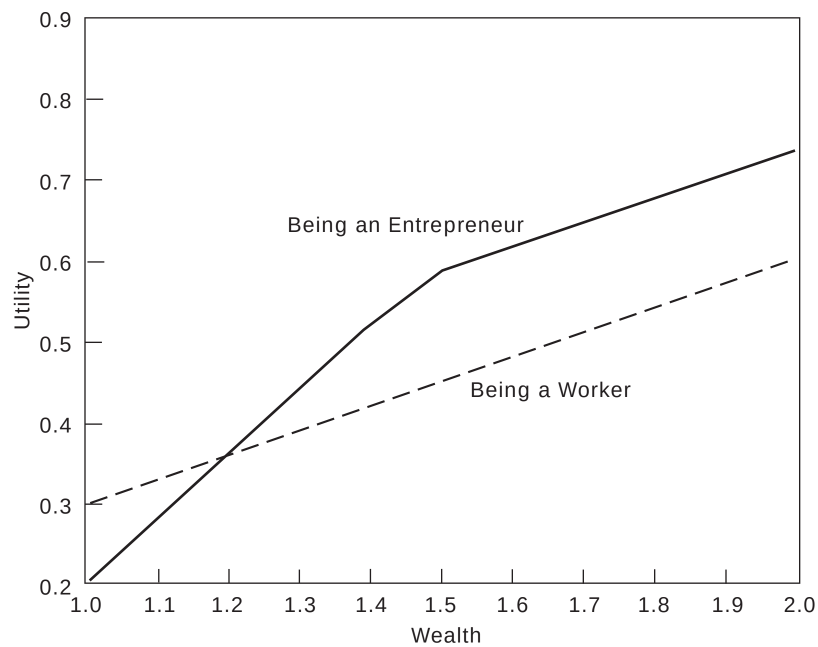
$$U^e = \max_{c_1,s,c_2} U(c_1) + E(c_2),$$

subject to

$$Ec_2 = \pi(s,p),$$

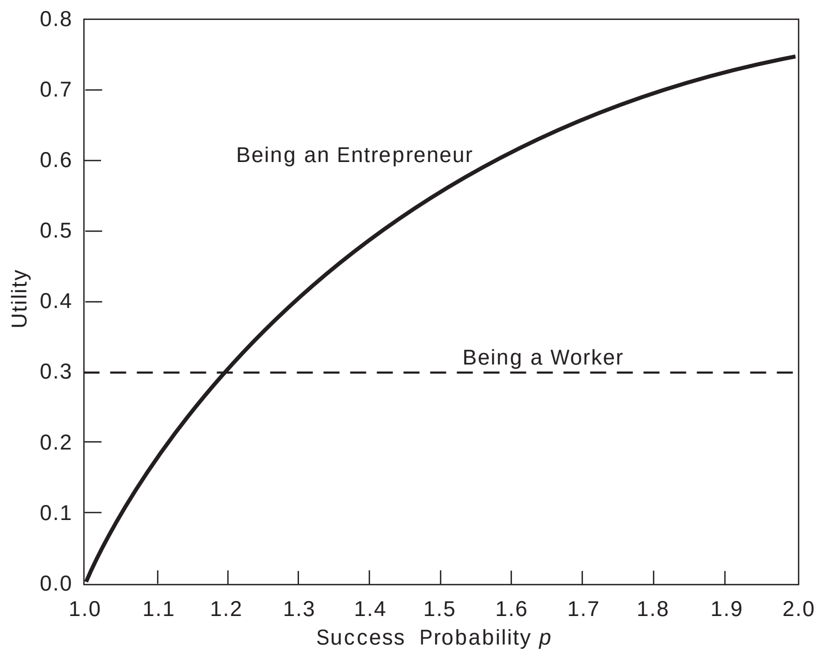
$$s = w - c_1.$$

Figure 2 Determination of Occupational Choice - I



on the horizontal axis in Figure 3. The utility of being either an entrepreneur or a worker is measured in the vertical axis of both figures. Note first that all entrepreneurs equate the marginal product of investment to the marginal cost of funds, which includes the monitoring cost associated with lending, i.e., $pf'(k) = r + (1 - p)\gamma$. Workers save additional wealth, so utility rises with wealth at rate r for workers. Entrepreneurs also save any additional wealth, and additional saving for this group reduces future borrowing needs, saving $r + (1 - p)\gamma$. This holds as long as saving is less than desired capital stock. If saving is greater than that, investment is self-financing, and extra wealth will first increase utility at rate $pf'(s)$ (which is less than $r + (1 - p)\gamma$), then r. Thus, there is a cutoff level of wealth, as shown in Figure 2, such that agents with wealth higher than the cutoff level will become entrepreneurs. It is clear from the profit function that entrepreneurs' utility increases with the quality of their business, while the utility of workers does not vary with their endowed project. Hence, as shown in Figure 3, there exists a cutoff level of business quality for each wealth level so that agents with projects above the cutoff level will become entrepreneurs. Results 1 and 2 summarize the analysis.

Figure 3 Determination of Occupational Choice - II



Result 1. Given a project, there is a threshold asset level such that agents with assets higher than the threshold will choose to undertake their projects.

Result 2. Given the asset endowment, there is a threshold probability of success such that agents whose projects have a higher probability of success become entrepreneurs.

The competitive equilibrium of this economy is defined as a resource allocation for workers and entrepreneurs together with an interest rate for which two conditions hold. First, agents maximize expected utility by choosing several decision variables, including their consumption in both periods, their saving in period 1, their occupational decisions and, in the case of entrepreneurs, their investment and loan size in period 2. Second, the market for capital clears, i.e.,

$$\int_p \int_w b(w,p)\delta(w,p)dG(w)d\Gamma(p) = \int_p \int_w \max\{s(w,p)-k,0\}\delta(w,p)dG(w)d\Gamma(p) + \int_p \int_w s(w,p)[1-\delta(w,p)]dG(w)d\Gamma(p), \quad (10)$$

where δ denotes the occupational choice. The left-hand side of (10) is demand for loans by entrepreneurs; the first term on the right-hand side is saving by entrepreneurs, and the second term is saving by workers. Agents' saving s, investment k, and occupational decision δ are all functions of their assets w and their project quality p.

The Case of the First Best without Information Asymmetry

We now briefly analyze the economy without information asymmetry in order to draw comparisons. Starting with period 2, in the absence of information asymmetry, the interest rate charged by intermediaries is equal to their cost of funds. Hence direct lending performs equally as well as financial intermediation, and there will also be no need for financial intermediaries. Agents face the same interest rate regardless of their asset holdings. The entrepreneurial decision will be determined solely by the quality of the business project. To see this, note that the profit function for an entrepreneur with saving s and success probability p is

$$\begin{aligned}\pi(s, p) &= \max_b \{pf(s+b) - rb\} \\ &= pf(k^*) - r(k^* - s),\end{aligned} \tag{11}$$

where k^* is the solution to the following first-order condition

$$pf'(k^*) = r.$$

The income for a worker with saving s and project p is

$$v(s) = rs + q.$$

It is clear that the difference between $\pi(s, p)$ and $v(s)$ is independent of s. Additional saving has the benefit of reducing required borrowing for the entrepreneur, which is worth r per unit in period 2. Rate r is the same as the rate of return that workers obtain on their savings. Therefore, greater initial wealth does not make entrepreneurship any more attractive than working.

The key difference between the economy with imperfect information and the economy examined here is that wealth enters into the decision rules of agents in the information-constrained economy. Private information reduces aggregate output in two ways. First, as Result 2 demonstrates, it is not always true that the most efficient projects are chosen. Some inefficient projects are carried out simply because the owners have higher internal funds, and some efficient projects are not activated because the owners have insufficient funds. Second, there is a social cost associated with monitoring. This cost does not accrue to any member of the economy and hence is viewed as a deadweight loss. The discussion of government policies in the credit market in the next section will be centered around these two dimensions. The first relates to the extent of business activity in the economy, while the second is a measure of the transaction costs associated with financial intermediation.

3. GOVERNMENT CREDIT PROGRAMS

The government finances loans by borrowing from lenders at a competitive, risk-free interest rate. Correspondingly, it finances subsidies through imposition of an income tax, which we assume is a lump-sum levy.[7] The government has access to the same information and verification technology as the private financial intermediary, therefore, as shown earlier, government subsidies cannot be Pareto-improving. However, government subsidies and taxation do have distributive effects. We will focus on the use of government credit programs for redistributive purposes and will ask which programs are most efficient in channeling resources to the desired groups.

Direct Loans

Suppose the government institutes a direct loan program that is available to a subset of the population, identified by race or location. The targeted group otherwise has the same characteristics as the population as a whole and is a fraction μ of the general population. We assume that direct government loans will bear a below-market interest rate, and we denote the difference between this interest rate and that of the market rate by ε.[8] A lump-sum income tax τ is levied on all agents in order to finance the subsidy.

We examine the subsidized entrepreneurs first. It is convenient to consider the situation where the private financial intermediary administers all the loans and is compensated by the government for the amount of the loan subsidy. Using the same notation as before, in period 2 the break-even condition for the financial intermediary becomes

$$px = (r - \varepsilon)b + (1 - p)(\beta + \gamma b), \tag{12}$$

[7] There is another potential avenue for the government to finance its loans that is not captured by the model: the government can issue securities and require private financial intermediaries and households to hold a certain proportion of these securities. An increase in the number of government subsidies will then increase the amount of government securities that must be held by banks or by households. This increase in private agents' holding of government securities can in turn affect the behavior of households and private intermediaries. For example, the U.S. Farm Credit System has at least the implicit support of the U.S. government, permitting it to issue bonds at an interest rate only very slightly above Treasury security yields. Effectively, this support lowers the opportunity cost of funds to the lender. Interested readers can find related discussion in Fried (1983).

[8] In our setup, it does not matter whether entrepreneurs receive all their loans from the government at a below-market interest rate or only receive a fraction of their loans at a below-market interest rate. That is, the two cases are the same as long as the net subsidy is the same in both cases.

where εb is the direct loan subsidy. The profit function for a subsidized entrepreneur (s, p) is

$$\pi^s(s,p) = \max_b \{pf(b+s) - px\} \tag{13}$$

$$= \max_b \{pf(b+s) - (r-\varepsilon)b - (1-p)(\beta + \gamma b)\}. \tag{14}$$

An entrepreneur decides loan borrowing b according to

$$pf'(b+s) = r - \varepsilon + (1-p)\gamma. \tag{15}$$

Consider first the partial equilibrium effects of the direct loan program where the effect of the change of interest rate is not taken into account. Agents now borrow more and have a lower marginal productivity of capital.[9] Given our monitoring technology, this increases social cost in the sense that additional resources will be allocated to monitoring. The decrease in marginal productivity of capital is independent of the success probability of the project p.

Since profits are strictly increasing in the loan subsidy rate $\varepsilon \left(\frac{\partial \pi^s}{\partial \varepsilon} = b > 0\right)$, subsidized entrepreneurs will benefit. Moreover, it is the cash-poor entrepreneurs with good projects who benefit the most. The intuition is clear. The direct loan subsidy studied here is proportional to the amount of loans borrowed, and it is precisely those who are either poor or have a good business who need to borrow the most.

An unsubsidized entrepreneur's profit function remains the same as equation (3). We denote it by $\pi^u(s, p)$, where the superscript u stands for unsubsidized. A worker's income also remains the same as equation (4).

The agent's problem is now

$$\max U(c_1) + Ec_2, \tag{16}$$

subject to

$$Ec_2 = \delta[\xi\pi^s(s,p) + (1-\xi)\pi^u(s,p)] + (1-\delta)v(s), \tag{17}$$

$$s = w - c_1 - \tau, \tag{18}$$

$$\delta \in \{0, 1\}, \tag{19}$$

where ξ is 1 if the agent belongs to the targeted group and 0 if not.

[9] We limit our attention to cases where $(1-p)\gamma \geq \varepsilon$. If the inequality is not satisfied, external funds will be more attractive than internal funds, and entrepreneurs will choose to deposit all their savings with the financial intermediary—an unrealistic situation.

The corresponding first-order condition that solves for saving is as follows:

$$U'(w-s-\tau)=\begin{cases}\frac{\partial\pi^{s}(s,p)}{\partial s}, & \text{subsidized entrepreneurs;}\\ \frac{\partial\pi^{u}(s,p)}{\partial s}, & \text{unsubsidized entrepreneurs;}\\ r, & \text{worker.}\end{cases} \tag{20}$$

The imposition of a lump-sum tax reduces the incentive to save for all agents in the economy, while the reduction in the marginal productivity of saving (and therefore of capital) further discourages subsidized entrepreneurs from saving. Taxation and public provision of the subsidy thus crowd out private saving. This reduction in private saving would further increase the demand for external funding and hence increase the monitoring cost associated with external finance in the event of failure. Moreover, loan subsidies give the targeted group an advantage over the nontargeted group: holding everything else the same, an agent belonging to the targeted group is more likely to become an entrepreneur. Therefore, some agents in the nontargeted group will be crowded out of entrepreneurship.

To summarize, the partial equilibrium analysis above indicates that on one hand a direct loan encourages cash-poor agents with good projects to carry out their projects. On the other hand, it creates an incentive for subsidized entrepreneurs to overinvest beyond the desired investment level; a disincentive for all agents, particularly entrepreneurs, to save; and a disincentive for unsubsidized agents to become entrepreneurs.

The competitive general equilibrium of this economy with government subsidy rate ε is easily defined. It is a resource allocation of workers, entrepreneurs, an interest rate, and a lump-sum tax rate τ that satisfies three conditions. First, agents choose their consumption in both period 1 and period 2; their savings in period 1; their occupational decisions and, in the case of entrepreneurs, their borrowing in period 2 to maximize the expected discounted utility from consumption. Second, the market for capital clears. Third, government balances its budget, i.e.,

$$\int_p\int_w \mu\varepsilon b(s,p)\delta(s,p)dG(w)d\Gamma(p)=\int_p\int_w \tau dG(w)d\Gamma(p), \tag{21}$$

the left-hand side represents government expenditure on direct loan subsidies, and the right-hand side represents government revenue from lump-sum tax.

The general equilibrium effect of direct loans from the government is more involved. The increase in loan demand and the decrease in private saving will drive the interest rate up, the increase in interest rate will have offsetting effects on savings and the demand for loans. Therefore, in equilibrium, the above partial equilibrium results will be lessened. Moreover, fewer unsubsidized entrepreneurs will choose to become entrepreneurs, and those that do will invest less in response to the increased interest rate, i.e., the government subsidy will crowd out unsubsidized entrepreneurs and their investment. We summarize these findings in Result 3 and plot them in Figure 4. This figure shows how the population is divided into workers and entrepreneurs for the benchmark case and for the case of direct subsidies. In the benchmark model, the cutoff line for being an entrepreneur is downward sloping. Any agents above the cutoff line will become entrepreneurs, any below will be workers. Under direct loans, the cutoff line for the targeted group shifts downward and becomes steeper, reflecting that cash-poor entrepreneurs with good business prospects benefit the most from direct loans. For the nontargeted group, the cutoff line shifts upward, reflecting the crowding effect caused by the advantage that subsidized entrepreneurs have over the unsubsidized, along with the effect of taxation.

Result 3. Under direct loans from the government, subsidized entrepreneurs will for a given interest rate invest more in their projects, reducing their marginal return on capital. Entrepreneurs in the targeted group with few assets and good projects (low w and high p) benefit most from a direct loan subsidy. Savings for all agents decline, but savings for subsidized entrepreneurs decline even more. Unsubsidized entrepreneurs have less incentive to carry out their projects, hence some of them will be crowded out of entrepreneurship. These results are likely to be weakened in general equilibrium because the interest rate is higher.

Loan Guarantees

Now consider a government loan guarantee program. Motivated by SBA practices, we assume that the government guarantees a proportion η of each private loan made by targeted entrepreneurs. In other words, the private lender, in case of default, is guaranteed η percent of the loan payment. Again to facilitate comparison, we assume that only a fraction μ of the population are members of the targeted group.

We consider first the entrepreneurs who receive loan guarantees. Let x denote loan payment in the event of success. Then the break-even condition for the financial intermediary is

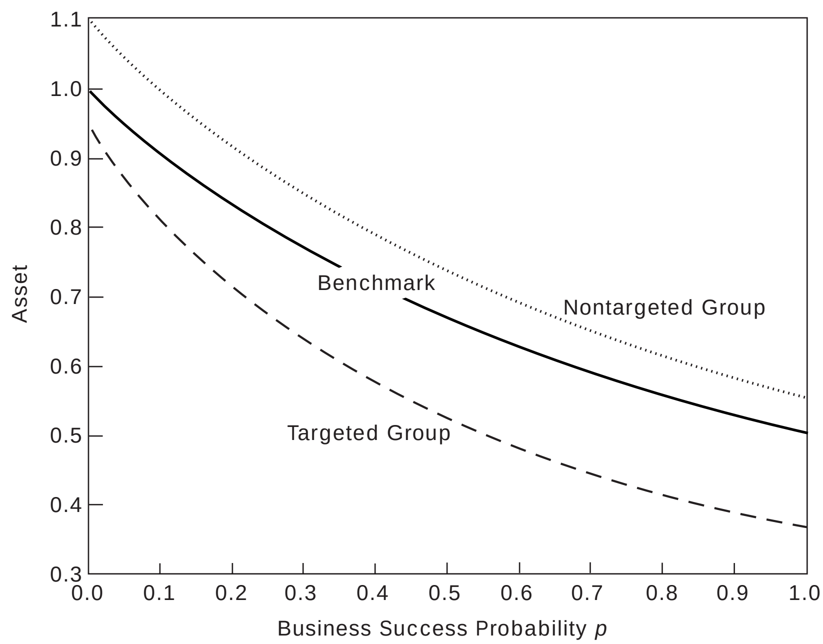
$$px + (1-p)\eta x = rb + (1-p)(\beta + \gamma b). \tag{22}$$

The corresponding profit function for a subsidized entrepreneur becomes

$$\begin{aligned}\pi^s(s,p) &= \max_b \{pf(b+s) - px\} \\ &= \max_b \{pf(b+s) - p[rb + (1-p)(\beta+\gamma b)]/[p + (1-p)\eta]\}, \end{aligned} \tag{23}$$

where $x = \frac{rb+(1-p)(\beta+\gamma b)}{p+(1-p)\eta}$ by equation (22).

Figure 4 Determination of Occupational Choices under Direct Loans



Loan borrowing b is determined by the following equation, which requires the marginal productivity of capital to be equal to the marginal cost

$$pf'(b+s) = r + (1-p)\gamma - (1-p)\eta\frac{r+(1-p)\gamma}{p+(1-p)\eta}$$
$$= \frac{p[r+(1-p)\gamma]}{p+(1-p)\eta}. \tag{24}$$

Again, we will only study the case where agents weakly prefer internal funds to external funds even under loan guarantees. As in the case of direct loan programs, the marginal productivity of capital is smaller than the benchmark case. However, unlike direct loan programs, the difference $(1-p)\eta\frac{r+(1-p)\gamma}{p+(1-p)\eta}$ is a function of both the loan guarantee percentage and the success probability of the project. In fact, the difference decreases with p, implying that the investment behavior of agents with riskier projects is more distorted; that is, there is more overinvestment, compared with the benchmark economy.

To find out how loan guarantees affect entrepreneurs, we can examine the profit function of a typical subsidized entrepreneur $\pi^s(s,p)$,

$$\frac{\partial \pi^s}{\partial \eta} = (1-p)\frac{rb + (1-p)(\beta + \gamma b)}{p + (1-p)\eta} - (1-p)^2 \eta \frac{rb + (1-p)(\beta + \gamma b)}{[p + (1-p)\eta]^2}$$

$$= (1-p)\frac{rb + (1-p)(\beta + \gamma b)}{[p + (1-p)\eta]^2} p > 0. \tag{25}$$

The derivative of expected utility with respect to the subsidy rate η is positive, indicating that all subsidized entrepreneurs benefit from the loan guarantee. To see which subsidized entrepreneurs benefit most, we can examine how the effect of the subsidy rate varies with saving and project quality.

$$\frac{\partial \left(\frac{\partial \pi^s}{\partial \eta}\right)}{\partial s} = \frac{\partial \left(\frac{\partial \pi^s}{\partial \eta}\right)}{\partial b}\frac{\partial b}{\partial s}$$

$$= -(1-p)\frac{r + (1-p)\gamma}{[p + (1-p)\eta]^2}\, p < 0, \tag{26}$$

$$\frac{\partial \left(\frac{\partial \pi^s}{\partial \eta}\right)}{\partial p} = -\{b[r + (1-p)\gamma] + (1-p)\beta\}\frac{p}{[p + (1-p)\eta]^2}$$

$$-(1-p)(b\gamma + \beta)\frac{p}{[p + (1-p)\eta]^2}$$

$$+(1-p)\{b[r + (1-p)\gamma] + (1-p)\beta\}\frac{\eta - p(1-\eta)}{[p + (1-p)\eta]^3}. \tag{27}$$

Intuitively, given that a fixed proportion of a loan is guaranteed in the event of failure, those who borrow more and/or have a higher probability of failure will benefit more from loan guarantees. This explains why those with low savings enjoy relatively more benefits. The effect of loan guarantees on an agent with a good project is determined by two forces. On the one hand, having a good project means borrowing more and hence being able to enjoy the benefits of large loan guarantees in the event of failure; on the other hand, a good project means a lower probability of failure and therefore less need for a loan guarantee. The first two terms on the right-hand side of equation (27) are negative, while the sign of the third one is ambiguous. Since $\frac{\partial \left(\frac{\partial \pi^s}{\partial \eta}\right)}{\partial p}|_{p=0} > 0$ in the neighborhood of $p = 0$, agents will benefit more if they have a higher probability of success. Conversely, $\frac{\partial \left(\frac{\partial \pi^s}{\partial \eta}\right)}{\partial p}|_{p=1} < 0$ indicates that, in the neighborhood of $p = 1$, agents with a lower probability of success will benefit more. These results suggest that a middle range of entrepreneurs benefits the most from the loan guarantees.

An unsubsidized entrepreneur has the same profit function and investment behavior as in the benchmark economy. We denote an unsubsidized entrepreneur's profit function by $\pi^u(s, p)$. The income of workers remains the same.

An agent's problem in period 1 is defined as follows:

$$\max U(c_1) + Ec_2, \tag{28}$$

subject to

$$Ec_2 = \delta[\xi\pi^s(s, p) + (1 - \xi)\pi^u(s, p)] + (1 - \delta)v(s), \tag{29}$$

$$s = w - c_1 - \tau, \tag{30}$$

$$\delta \in \{0, 1\}, \tag{31}$$

where ξ takes a value of 1 if the agent belongs to the targeted group and 0 otherwise.

Agents in period 1 will determine their saving for period 2 so that the marginal gains from saving in the latter period equal the marginal cost of reduced consumption in the former one. Under loan guarantees, the marginal gains from saving are lower than in the benchmark economy, thus inducing subsidized entrepreneurs to reduce their savings. For unsubsidized entrepreneurs and workers, the lump-sum income tax acts to increase their marginal benefits of consumption in period 1; accordingly, under loan guarantees, unsubsidized entrepreneurs and workers will increase their consumption and reduce their savings. Moreover, unsubsidized entrepreneurs will receive less from their projects than their subsidized counterparts and as a result are likely to be crowded out of entrepreneurship.

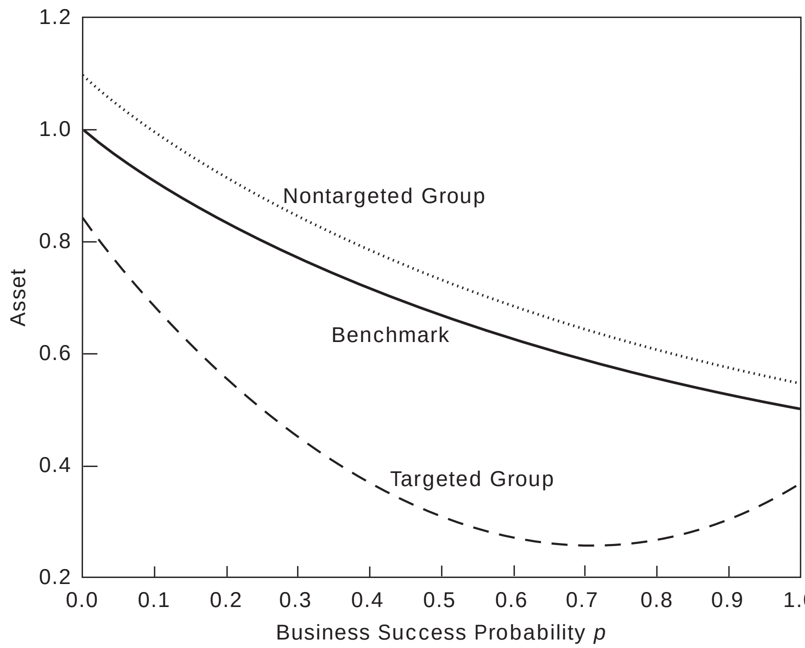
The competitive equilibrium can be defined similarly to that of an economy with direct loans with the government budget constraint being

$$\int_p \int_w \mu(1 - p)\eta x(s, p)\delta(s, p)dG(w)d\Gamma(p) = \int_p \int_w \tau dG(w)d\Gamma(p), \tag{32}$$

where $\delta(s, p)$ is the indicator for occupational decision; it has a value of 1 for entrepreneurs and 0 for workers. The left-hand side is the government's expense to guarantee a fraction μ of entrepreneurs a portion η of their loans in the event of default, and the right-hand side is government revenue from the lump-sum tax.

In general equilibrium, the increased loan demand and decreased loan supply raise the equilibrium interest rate, in which case borrowing is more expensive for entrepreneurs and saving is more attractive for all agents. Consequently, the partial equilibrium results will be lessened. Moreover, unsubsidized entrepreneurs will reduce their investment in response to the higher interest rate. We summarize these findings in Result 4. Figure 5 describes the determination of occupational choices under loan guarantees. Agents above the cutoff lines

Figure 5 Determination of Occupational Choices under Loan Guarantees



become entrepreneurs, and those below become workers. Under loan guarantees, the cutoff line for targeted entrepreneurs shifts downward and becomes more convex, indicating that entrepreneurs with businesses of mediocre quality benefit the most from the loan guarantees; the cutoff line for nontargeted entrepreneurs shifts upward, reflecting the crowding out of unsubsidized entrepreneurs.

Result 4. With government loan guarantees, investment by subsidized entrepreneurs for a given interest rate is higher, and marginal returns to capital are lower, than in the benchmark economy by an amount that decreases with p. Poor entrepreneurs with mediocre projects (low w and medium p) benefit more than others from the loan guarantees. Private savings are lower, especially for entrepreneurs. The increase in the equilibrium interest rate in general equilibrium will lessen these results.

Grants

Instead of lending directly to entrepreneurs or providing investors with a guarantee on entrepreneurial loans, the government can offer targeted entrepreneurs a grant of ϕ, payable at the end of the period and financed by lump-sum income tax τ. Added to firm profits, the grant would be available for investors. Again, we assume that the targeted group is a fraction μ of the population and that they share the same wealth and business quality characteristics as the general population.

For subsidized entrepreneurs in period 2, using the same notation as before, the loan payment x for an entrepreneur with saving s, project success probability p, and borrowing b satisfies the break-even condition

$$px = rb + (1 - p)(\beta + \gamma b). \tag{33}$$

A subsidized entrepreneur (s, p) chooses b to maximize his profit function in the second period,

$$\begin{aligned} \pi^s(s, p) &= \max_b \{pf(s + b) + \phi - px\} \\ &= \max_b \{pf(s + b) + \phi - rb - (1 - p)(\beta + \gamma b)\}. \end{aligned} \tag{34}$$

It is easy to see that the first-order condition that determines firms' investment is unchanged so that a grant does not alter an entrepreneur's investment choices. Additionally, from the first-order condition (9), a grant does not change an entrepreneur's saving decision in period 1 either.[10] However, it does increase an agent's incentive to become an entrepreneur since carrying out a risky activity is associated with a higher payoff now. Obviously, since $\frac{\partial \pi(s,p)}{\partial \phi} = 1$, the benefit is fixed for all entrepreneurs regardless of their assets and business projects.

The problem of an unsubsidized entrepreneur remains the same as in the benchmark economy. An agent's problem at period 1 is now

$$\max U(c_1) + Ec_2, \tag{35}$$

subject to

$$Ec_2 = \delta(\xi\pi^s(s, p) + (1 - \xi)\pi^u(s, p)) + (1 - \delta)v(s), \tag{36}$$

$$s = w - c_1 - \tau, \tag{37}$$

$$\delta \in \{0, 1\}, \tag{38}$$

where δ is 1 if the agent chooses to be an entrepreneur in period 2 and 0 otherwise; ξ takes a value of 1 if the agent belongs to the targeted group and 0 otherwise.

The marginal gain from saving is unaffected by the grant. However, the marginal cost of saving at period 1 is increased by the imposition of a lump-sum

[10] As with direct loans and loan guarantees, the associated lump-sum income tax has a distortionary effect on agents' saving in period 1.

tax. Therefore, all agents will reduce their saving. The incentive to consume more in period 1 is smaller for grants than for those of direct loans and loan guarantees.

The definition of general equilibrium under grants is similar to the cases of direct loans and loan guarantees except for the government's budget constraint

$$\int_p \int_w \mu\phi\delta(w, p) dG(w) d\Gamma(p) = \int_p \int_w \tau dG(w) d\Gamma(p). \tag{39}$$

Here the left-hand side is the government's expense from giving out a fixed grant ϕ to targeted entrepreneurs, and the right-hand side is lump-sum tax revenue.

As with direct loans and loan guarantees, in general equilibrium increased loan demand and decreased loan supply drive up the interest rate, loan borrowing becomes more expensive and saving more attractive. The partial equilibrium results discussed will be lessened. These findings are summarized in Result 5. Figure 6, which depicts how grants affect agents' occupational choices, shows the asset-project success probability cutoff line shifting downward for the targeted group and upward for the nontargeted group.

Result 5. With grants, the investment behavior of subsidized entrepreneurs for a given interest rate is unaffected. All entrepreneurs benefit equally from the subsidies regardless of their asset holdings and project quality. Agents in period 1 will reduce their saving in response to the imposition of the lump-sum tax. These effects are reduced in general equilibrium due to the increase in the equilibrium interest rate.

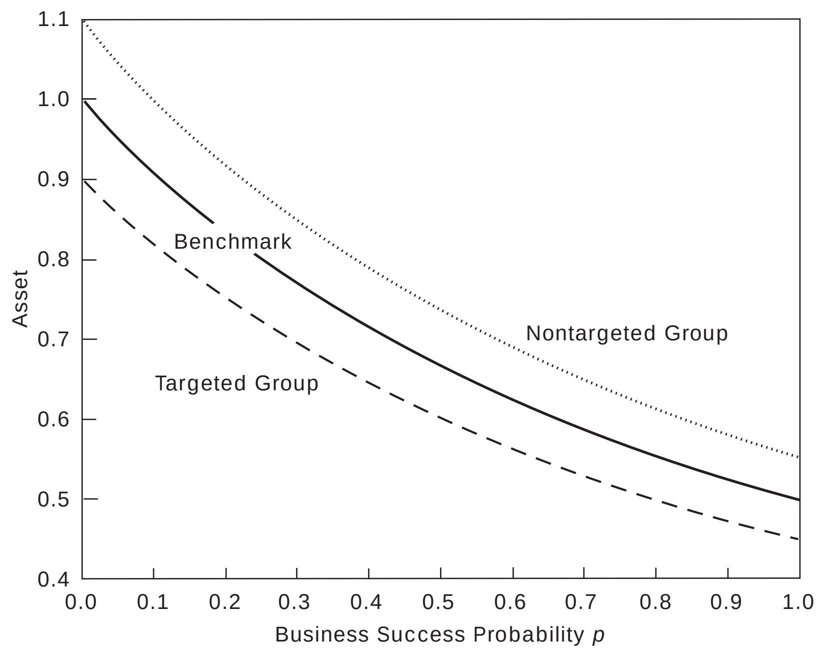
Our analysis, despite its partial equilibrium nature, provides some evidence on the direction and magnitude of the many channels through which agents are affected under different loan programs. First, along the investment margin, both direct loans and loan guarantees create incentives for entrepreneurs to overinvest (compared with the benchmark economy). The incentive is stronger for owners of poor projects under loan guarantees. Grants, on the contrary, do not alter investment behavior.

Second, with respect to risk-shifting, owners of good projects who are less wealthy benefit the most from direct loans. While poor agents do benefit more from loan guarantees, those with medium-quality projects benefit the most. Grants are nondiscriminatory; a fixed amount is assigned to all entrepreneurs.

Third, government subsidies in the form of direct loans and loan guarantees crowd out the saving of all agents in the economy, especially those of the entrepreneurs. Lump-sum income taxation reduces consumption in period 1 and hence increases the marginal utility of consumption in period 1. Moreover, it reduces savings for all agents under all loan programs.

To summarize, grants have the least distortionary effect, direct loans are capable of targeting efficient projects, and loan guarantees are more likely to

Figure 6 Determination of Occupational Choices under Grants



attract relatively riskier entrepreneurs. Since direct comparison of the general equilibrium impact of different government credit programs is not as transparent as that of partial equilibrium analysis, we now turn to numerical analysis for some insights.

4. A NUMERICAL EXAMPLE

This section reiterates the lessons of the previous analysis in general equilibrium by incorporating the effect of loan programs on the interest rate. These lessons are conducted by applying a hypothetical numerical example.

Before we launch our numerical analysis, note that all these forms of government subsidies shift loan demand outward, while lump-sum taxation shifts private loan supply inward so that in the new equilibrium, the interest rate will go up. This rise in the equilibrium interest rate offsets some of the benefits created by government subsidies for entrepreneurs, since loans are more expensive now. In contrast, the rise in the interest rate benefits workers who are disadvantaged by taxation.

Table 3 A Numerical Example

	Benchmark	Direct Loan	Guarantee	Grant
Entrepreneurs				
total	0.2204	0.2205	0.2207	0.2227[1]
targeted	0.0441	0.0463	0.0467	0.047[2]
nontargeted	0.1763	0.1742	0.1740	0.1753
Monitoring cost	0.02066	0.02073[3]	0.02081	0.02073[3]
Total output	0.64543	0.64611[4]	0.64542	0.64548
Cutoff w level				
targeted	1.2574	1.2433	1.2344	1.2336
nontargeted	1.2574	1.2610	1.2613	1.2592
Cutoff p level				
targeted	0.6379	0.6341	0.6316	0.6307
nontargeted	0.6379	0.6392	0.6392	0.6385
Average w for entrepreneurs	0.9715	0.9663[5]	0.9664	0.9668
Average p for entrepreneurs	0.7895	0.7898[6]	0.7896	0.7876

[1] most overall entrepreneurial activity
[2] most entrepreneurial activity within targeted group
[3] least monitoring cost
[4] most total output
[5] least average wealth for entrepreneurs
[6] highest average business quality for entrepreneurs

In our numerical example, the utility function is chosen to be of *log* form in the first period, and linear in the second period, i.e., $U(w, p) = \frac{c_1^{1-2.5}}{1-2.5} + 3c_2$. The wealth variable w is a random draw from a uniform distribution over the interval $[0.2, 1.6]$, in which the richest person with wealth 1.6 is 8 times richer than the poorest person having wealth 0.2. The success probability p of an agent's endowed project follows a uniform distribution over the domain $[0.3, 0.85]$. The production function takes the form $1.7k^{0.67}$. The fixed monitoring cost β is set to be 0.1, and the unit cost γ is 0.4. The wage that workers get from the outside option q is 0.4.

We fix the lump-sum tax to be 0.001 per person; the fraction of agents who are eligible for subsidies μ is 0.2. Then we study the different loan programs whose rates—ε for direct loans, η for loan guarantees, and ϕ for grants—are chosen so that the government balances its budget in equilibrium. Table 3 reports the results.

The results are consistent with our analysis in the previous section. One thing common with all three loan programs is that agents in the targeted group are helped at the cost of the agents in the nontargeted group. Though entrepreneurial activity increases under all loan programs in the targeted group, it declines in the nontargeted group. The threshold levels of both wealth and project quality increase for the nontargeted agents.

When comparing direct loans and loan guarantee programs, we find that loan guarantees are better at promoting entrepreneurship at the cost of lower average business quality and higher bankruptcy cost. The reason is straightforward. As shown in Section 3, direct loan programs benefit poor agents with good projects the most. So these agents tend to borrow more and therefore require most of the subsidies. Under loan guarantees, however, entrepreneurs with few assets and mediocre projects benefit the most. The resulting benefits are somewhat more evenly distributed. For the same reason, under direct loans the average wealth of entrepreneurs is lower and the average quality of their projects is higher than under loan guarantees. Since entrepreneurs with low quality projects are more likely to bankrupt, the bankruptcy cost is higher under loan guarantees. These results survive different parameter specifications in our experiments.

Another interesting result is that grants seem to outperform loan guarantees in promoting entrepreneurship at lower monitoring cost. However, grants induce the lowest average business quality among all the programs and do not seem to help the poor. This has to do with the nondiscriminatory nature of grants.

5. CONCLUSION

Are loan guarantees the best way to channel assistance to targeted classes of borrowers? Our analysis of a credit market with asymmetric information indicates that grants are most effective at promoting entrepreneurship. Loan guarantees attract relatively riskier businesses with few assets. Direct loans do best at targeting cash-poor borrowers with good projects. Subsidized entrepreneurs overinvest under direct loans and loan guarantees.

All of the programs, especially direct loan and loan guarantee programs, discourage private saving. So why are loan guarantees so popular? Although there is no clear answer, it may be that differences in government budgetary accounting allow guarantees to be passed easily since loan guarantees often do not appear in the budget until a payment is made. Webb (1991) provides an excellent review and an estimate of the unfunded liabilities of the U.S. government budget. Another possibility, as suggested by the model, is that the benefits of guarantees spread more evenly over a broad set of agents than do the benefits of direct loan subsidization. This more equitable distribution of benefits perhaps appeals to the public’s conception of fairness and therefore can help generate more political support for guarantees.

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Fisher's Equation and the Inflation Risk Premium in a Simple Endowment Economy

Pierre-Daniel G. Sarte

One of the more important challenges facing policymakers is that of assessing inflation expectations. Goodfriend (1997) points out that one can interpret the meaning of a given interest rate policy action primarily in terms of its impact on the real rate of interest. However, evaluating this impact requires not only that one understands the various links between the nominal rate and expected inflation but also that one can quantify these relationships.

To find an approximate measure of expected inflation, one often turns to the behavior of long bond rates. Two key ideas explain why this approach might be appropriate. First, Fisher's theory holds that the real rate of interest is just the difference between the nominal rate of interest and the public's expected rate of inflation. Second, the long-term real rate is generally thought to exhibit very little variation.[1] Alternatively, and still based on Fisher's theory, one might use the yield spread between the ten-year Treasury note and its inflation-indexed counterpart as an estimate of expected inflation. In January 1997, the U.S. Treasury indeed began issuing ten-year inflation-indexed bonds.

While economic analysts typically attempt to capture inflation expectations using Fisher's equation, this method has its flaws. When inflation is stochastic, Fisher's relation may not actually hold. Barro (1976), Benninga and Protopapadakis (1983), as well as Cox, Ingersoll, and Ross (1985), show that the decomposition of the nominal rate into a real rate and expected inflation should

The opinions expressed herein are the author's and do not represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System. For helpful suggestions and comments, I would like to thank Mike Dotsey, Tom Humphrey, Yash Mehra, and Alex Wolman. Any remaining errors are, of course, my own.

[1] See Simon (1990).

include an additional component excluded from Fisher's equation: the inflation risk premium. This premium reflects the outcome of random movements in inflation that effectively cause nominal bonds to be risky assets relative to inflation-indexed bonds. As we shall see in this article, the sign of the premium may be positive or negative, depending on how unexpected movements in inflation co-vary with surprises in consumption growth.

Another reason the Fisher equation may not hold is that when one links the nominal rate to the real rate and expected inflation, one must consider the nonlinearity inherent in inflation when calculating expectations. Specifically, inflation is a ratio of prices. We shall see that this nonlinearity works through the variance of inflation surprises.

Since it is evident that Fisher's equation does not work in all situations, why should one consider the equation useful? (Note that if both the inflation risk premium and the variance of inflation surprises are negligible, then Fisher's equation holds precisely.) This article answers the question by building on earlier work by Labadie (1989, 1994). In particular, the analysis below relies upon three key building blocks. First, to study the effect of inflation risk on nominal rates, we formally incorporate uncertainty as part of the environment surrounding households' optimal bond purchasing decisions. Second, we assume that a bivariate vector autoregression (VAR) in the logs of consumption growth and inflation drives the model. This assumption makes it possible to work out exact analytical solutions for bond yields and expected inflation. Finally, we estimate the driving process empirically by using U.S. consumption-growth data to calibrate the model's analytical solutions. In contrast to Labadie (1989), we are able to derive solutions consistent with a general-order VAR process instead of a VAR(1). This allows us to better capture the joint time-series properties of consumption growth and inflation. Moreover, whereas Labadie's work focuses on the equity premium, we will concentrate mainly on the model's quantitative implications for the inflation risk premium.

Two important conclusions emerge from the analysis. One is that the model's quantitative estimates of the inflation risk premium are insignificant. This result occurs primarily because little covariation exists between shocks to consumption growth and unexpected movements in inflation in U.S. data. In other words, since inflation surprises are as likely to occur whether consumption growth is high or low, there is no reason why the inflation risk premium should be substantially positive or negative. This notion is unrelated to the fact that the equity premium tends to be very small in consumption-based asset pricing models. We will show that adopting a pricing kernel that helps explain the equity premium does not necessarily change the size of the inflation risk premium in any meaningful way. The implication is that, in practice, Fisher's equation may be a reasonable approximation even when inflation is stochastic.

The other important conclusion (for the sample period covering 1955 to 1996) is that the model's historical estimates of the yield on a one-year nominal bond match the actual yield on one-year Treasury notes relatively well. However, the model's estimates of the one-year nominal rate perform very poorly during the late 1970s. The model's inability to track the nominal rate during that period may reflect the unusual tightening by the Federal Reserve (the Fed) in an effort to bring down inflation at that time. Our benchmark model suggests a consumption-based real rate whose standard deviation is around 1 percent. Surprisingly, this is more than half the standard deviation of the ex post real rate despite the fact that consumption growth is relatively smooth. Using a different methodology, we find additional supporting evidence in favor of Fama (1990), who suggests that expected inflation and the real rate move in opposite directions. Finally, our model indicates that it is difficult to determine whether expected inflation is more or less volatile than the real rate at short horizons. Although conventional wisdom suggests that the real rate varies more than expected inflation in the short run, we find that the choice of preference specification is crucial for this result.

This article is organized as follows. Section 1 presents the basic framework used to price nominal and inflation-indexed bonds. Section 2 describes the joint driving process linking consumption growth and inflation. Sections 3 and 4 present the results which obtain under different preference specifications. Finally, Section 5 offers some concluding remarks.

1. PRICING NOMINAL AND INFLATION-INDEXED BONDS

The economy is populated by a continuum of infinitely lived households. These households are identical in terms of their preferences and endowments. The per capita endowment is nonstorable, exogenous, and stochastic. The typical household's wealth consists of currency, one-period inflation-indexed and one-period nominal discount bonds. Thus, an indexed bond purchased at time t pays one unit of the endowment good with certainty at time $t + 1$. As in Labadie (1989, 1994), this instrument provides a benchmark that helps isolate real from inflationary effects. Contrary to the indexed bond, the nominal bond is subject to inflation risk. That is, a nominal bond purchased at date t pays one unit of currency, say dollars, at date $t + 1$.

Each household maximizes its lifetime utility over an infinite horizon. The timing of trade follows that of the cash-in-advance economy described in Lucas (1982). Specifically, at the beginning of each period and before any trading takes place, a stochastic monetary transfer, $\nu_t M_{t-1}$, and a real endowment shock, y_t, are realized and observed publicly. After receiving the money transfer, as well as any payoffs on maturing bonds, the representative household decides on how to allocate its nominal wealth between money balances, M_t^d, indexed bonds, z_t, and nominal discount bonds, z_t^N. Once the asset market has closed, the household uses its money balances acquired at the beginning of the period M_t^d to finance its consumption purchases $p_t c_t$, where p_t is the price level at date t. The household then receives its nominal endowment income $p_t y_t$, which it cannot spend until the subsequent period. To summarize, the representative household solves

$$\max\ \mathcal{U} = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s), \quad 0 < \beta < 1, \tag{1}$$

subject to the constraints

$$\frac{p_{t-1}}{p_t} c_{t-1} + q_t z_t + \frac{x_t}{p_t} z_t^N + \frac{M_t^d}{p_t} = \frac{p_{t-1}}{p_t} y_{t-1} + \frac{M_{t-1} + \nu_t M_{t-1}}{p_t} + z_{t-1} + \frac{z_{t-1}}{p_t}, \tag{2}$$

and

$$c_t \leq \frac{M_t^d}{p_t}. \tag{3}$$

We denote by q_t and x_t the real price of a one-period indexed bond and the price of a one-period nominal bond, respectively. E_t is the conditional expectations operator where the time t information set includes all variables dated t and earlier.

Appendix A contains the first-order conditions associated with the above problem. These optimality conditions yield the following Euler equation,

$$u'(c_t) = \beta E_t (1 + r_t) u'(c_{t+1}), \tag{4}$$

where we have defined $1/q_t$ as $(1 + r_t)$. Equation (4) states that in choosing how much to consume versus how much to save in the form of an indexed bond, the representative household explicitly compares marginal benefit and marginal cost. The marginal benefit, in utility terms, of consuming one additional unit of the endowment good today is given by $u'(c_t)$. Alternatively, the household could save that additional unit and use it to purchase an indexed bond that would yield $(1 + r_t)$ with certainty in the following period. Therefore the right-hand side of equation (4) captures the marginal cost of consuming one additional unit of the endowment good today in utility terms. As equation (4) indicates, the optimal consumption/savings allocation naturally equates marginal benefit and marginal cost.

Now, in this setup, the representative household also has the option of saving through a nominal pure discount bond. Optimality implies that

$$\frac{u'(c_t)}{p_t} = \beta E_t (1 + r_t^N) \frac{u'(c_{t+1})}{p_{t+1}}, \tag{5}$$

where $(1 + r_t^N)$ is defined as $1/x_t$. Analogous to the situation we have just described, the marginal benefit of consuming one additional dollar's worth of the endowment good today, where one dollar is worth $1/p_t$ units of the endowment good, is $u'(c_t)/p_t$. By instead saving this additional dollar in a nominal bond, the representative household would reap $(1+r_t^N)/p_{t+1}$ units of the endowment good next period. The right-hand side of equation (5), therefore, represents the marginal cost of consuming an additional dollar's worth of the endowment good in the current period. As in equation (4), the optimal consumption/savings allocation still dictates equating marginal benefit to marginal cost.

Since equations (4) and (5) simply show different methods of how to best allocate income towards consumption and savings, one might naturally expect a precise link to emerge between the real rate and the nominal rate. Using equation (5) yields

$$\frac{1}{1+r_t^N} = \beta E_t \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}},$$

which may be rewritten[2] as

$$\frac{1}{1+r_t^N} = \left(\frac{1}{1+r_t}\right) E_t \left(\frac{p_t}{p_{t+1}}\right) + \beta \mathrm{cov}_t \left(\frac{u'(c_{t+1})}{u'(c_t)}, \frac{p_t}{p_{t+1}}\right). \tag{6}$$

Note that if inflation is deterministic, then the covariance term on the right-hand side of equation (6) disappears and the above equation reduces to Fisher's relation,

$$(1+r_t^N) = (1+r_t)\left(\frac{p_{t+1}}{p_t}\right). \tag{7}$$

To understand the nature of the differences between the modified Fisher equation and equation (7), let us first examine the covariance term in (6). This term is known as the inflation risk premium and already emerges in Benninga and Protopapadakis (1983) or Cox, Ingersoll, and Ross (1985). Recall that saving one additional dollar in period t yields $(1+r_t^N)/p_{t+1}$ units of the endowment good in period $t+1$. However, the price level next period, p_{t+1}, is unknown at date t. Inflation, therefore, makes the nominal discount bond a risky asset; the premium in effect alters the nominal rate to account for this additional risk.

To make matters more concrete, let us temporarily suppose that momentary utility is given by the Constant Relative Risk Aversion (CRRA) function $u(c) = c^{1-\gamma} - 1/1-\gamma$, $\gamma > 0$. Consequently, the ratio of marginal utilities in equation (6) is decreasing in consumption growth and given by $(c_{t+1}/c_t)^{-\gamma}$. Therefore, when the conditional covariance term is negative, inflation is likely to be high when consumption growth is low. In other words, the return on the nominal bond is adversely affected by inflation precisely when the household suffers from low consumption growth. Now observe that relative to a world without inflation uncertainty, a negative conditional covariance raises the nominal rate. We may, therefore, interpret this higher nominal yield as compensating

[2] Here we use the fact that for any two random variables x and y, $\mathrm{cov}(x,y) = E(xy) - E(x)E(y)$.

the household for the additional inflation risk associated with the nominal bond. The reverse is true when the conditional covariance term is positive.

Another reason equation (6) does not correspond to Fisher's relation when inflation is stochastic, even if the conditional covariance term were zero, has to do with Jensen's Inequality. In particular, $E_t(p_{t+1}/p_t)$ is generally not equal to $1/E_t(p_t/p_{t+1})$. As one might expect, we shall see below that the difference between $E_t(p_{t+1}/p_t)$ and $1/E_t(p_t/p_{t+1})$ rises with the volatility of inflation surprises. In a world without such surprises, the conditional expectations operator is irrelevant, so this difference would vanish.

To close the model, we simply note that in equilibrium, $c_t = y_t$, while $M_t^d = M_t$. In addition, since households are identical, indexed and nominal bonds are in zero net supply so that $z_t = z_t^N = 0$. In what follows, we assume for simplicity's sake that $\nu_t \geq \beta$ so that the cash-in-advance constraint always binds.

2. THE ENDOWMENT AND INFLATION PROCESSES

We now define a driving process for this economy. Let endowment growth and the inflation rate be denoted by $y_{t+1}/y_t = \zeta_{t+1}$ and $p_{t+1}/p_t = \phi_{t+1,}$ respectively. We assume that the joint time-series behavior of $\ln \zeta_{t+1}$ and $\ln \phi_{t+1}$ can be described by a covariance stationary bivariate VAR (p).[3] The law of motion for the endowment process is

$$\ln \zeta_{t+1} = \delta_{\zeta 0} + \sum_{j=0}^{p} \delta_{\zeta\zeta,j} \ln \zeta_{t-j} + \sum_{j=0}^{p} \delta_{\zeta\phi,j} \ln \phi_{t-j} + \varepsilon_{\zeta,t+1}. \tag{8}$$

Similarly, the inflation rate follows a process that can be described by

$$\ln \phi_{t+1} = \delta_{\phi 0} + \sum_{j=0}^{p} \delta_{\phi\zeta,j} \ln \zeta_{t-j} + \sum_{j=0}^{p} \delta_{\phi\phi,j} \ln \phi_{t-j} + \varepsilon_{\phi,t+1}. \tag{9}$$

Shocks to endowment growth and inflation, $(\varepsilon_{\zeta,t}, \varepsilon_{\phi,t})$, are assumed to be jointly distributed normal random variables such that $E(\varepsilon_{\zeta,t}) = E(\varepsilon_{\phi,t}) = 0$, $\text{var}(\varepsilon_{\zeta,t}) = \sigma_\zeta^2$, $\text{var}(\varepsilon_{\phi,t}) = \sigma_\phi^2$, and $\text{cov}(\varepsilon_{\zeta,t}, \varepsilon_{\phi,t}) = \sigma_{\zeta\phi}$. Moreover, as in Labadie (1989), the shocks satisfy $E(\varepsilon_{\zeta,t}, \varepsilon_{\phi,s}) = E(\varepsilon_{\zeta,s}, \varepsilon_{\phi,t}) = 0$, for $s \neq t$.

3. RESULTS WITH CRRA UTILITY

Analytical Solutions

In this section, we assume that momentary utility is of the CRRA form. Our main focus will be to derive and interpret solutions for bond prices or,

[3] Since the cash-in-advance constraint is assumed to bind, this bivariate system implicitly dictates the behavior of money growth.

alternatively, rates of return on the indexed and nominal discount bonds. The goal is to assess to what degree Fisher's equation approximates its generalized version in (6) in a calibrated consumption-based asset pricing model. With CRRA utility, equation (4) becomes

$$q_t = \beta E_t \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}, \tag{10}$$

which may also be written as

$$\ln q_t = \ln \beta + \ln E_t \zeta_{t+1}{}^{-\gamma}.$$

Using the properties of log-normal random variables described in Appendix B, as well as those of the driving process in Section 2, it immediately follows that

$$\ln q_t = \ln \beta - \gamma\delta_{\zeta 0} + \frac{\gamma^2\sigma_\zeta^2}{2} - \gamma \sum_{j=0}^{p} \delta_{\zeta\zeta,j} \ln \zeta_{t-j} - \gamma \sum_{j=0}^{p} \delta_{\zeta\phi,j} \ln \phi_{t-j}.$$

The real price of the one-period inflation-indexed bond can therefore be expressed as

$$q_t = \beta \left[\exp(-\gamma\delta_{\zeta 0} + \frac{\gamma^2\sigma_\zeta^2}{2})\right] Q_t, \tag{11}$$

where $Q_t = \Pi_{j=0}^{p} \zeta_{t-j}^{-\gamma\delta_{\zeta\zeta,j}} \Pi_{j=0}^{p} \phi_{t-j}^{-\gamma\delta_{\zeta\phi,j}}$. Equation (11) suggests that the real rate, $1/q_t$, is not only a function of past endowment growth but also of past inflation rates. This result arises since, by equation (8), past inflation rates help forecast endowment growth next period, ζ_{t+1}. In addition, observe that greater volatility in unexpected endowment growth movements, as captured by σ_ζ^2, raises q_t and, therefore, lowers the real rate. Put another way, a more risky endowment growth process serves to lower the real rate of return. This latter effect, however, is only present to the degree that households care about risk so that $\gamma > 0$. When households are risk-neutral and $\gamma = 0$, q_t is independent of σ_ζ^2.

Turning our attention to the behavior of the nominal rate, equation (5) can be rewritten as

$$x_t = \beta E_t \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \frac{p_t}{p_{t+1}} \tag{12}$$

so that $\ln x_t = \ln \beta + \ln E_t (c_{t+1}/c_t)^{-\gamma} (p_t/p_{t+1})$. Again, using the properties of log-normal random variables yields

$$\ln E_t \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \frac{p_t}{p_{t+1}} = E_t \ln \zeta_{t+1}^{-\gamma} + E_t \ln \phi_{t+1}^{-1} + \tag{13}$$
$$\frac{\gamma^2}{2} \mathrm{var}_t \ln \zeta_{t+1} + \frac{1}{2} \mathrm{var}_t \ln \phi_{t+1} + \gamma \mathrm{cov}_t \left(\ln \zeta_{t+1}, \ln \phi_{t+1}\right).$$

As before, we can use the properties of the driving process to obtain

$$x_t = \beta \left[\exp(-\gamma\delta_{\zeta 0} - \delta_{\phi 0} + \frac{\gamma^2\sigma_\zeta^2}{2} + \frac{\sigma_\phi^2}{2} + \gamma\sigma_{\zeta\phi}) \right] X_t, \tag{14}$$

where $X_t = \Pi_{j=0}^{p} \zeta_{t-j}^{-(\gamma\delta_{\zeta\zeta,j}+\delta_{\phi\zeta,j})} \Pi_{j=0}^{p} \phi_{t-j}^{-(\gamma\delta_{\zeta\phi,j}+\delta_{\phi\phi,j})}$. As expected, the behavior of the nominal rate depends on the time-series characteristics of both endowment growth and the inflation rate. In particular, observe that the greater the unconditional variance of inflation surprises, the lower the nominal rate, since $1 + r_t^N = 1/x_t$. Furthermore, a larger negative covariance between unexpected movements in endowment growth and inflation surprises raises the nominal rate (so long as $\gamma > 0$). As mentioned earlier, this result reflects that when $\sigma_{\zeta\phi} < 0$, high inflation shocks tend to occur when endowment growth is unexpectedly low. In this case, the household, therefore, requires a higher yield on nominal bonds to account for the inflation risk. Alternatively, we can see this notion by tracing the effect of the covariance between endowment growth shocks and inflation shocks on the inflation risk premium directly. By using equation (6) and solving for $\ln 1/(1+r_t)E_t(p_t/p_{t+1})$ as we have done above, one sees that

$$\begin{aligned} &\text{cov}_t \left(\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}, \frac{p_t}{p_{t+1}} \right) = \\ &\qquad \left[\exp \left(-\gamma\delta_{\zeta 0} - \delta_{\phi 0} + \frac{\gamma^2\sigma_\zeta^2}{2} + \frac{\sigma_\phi^2}{2} \right) \right] \left[\exp(\gamma\sigma_{\zeta\phi}) - 1 \right] X_t. \end{aligned} \tag{15}$$

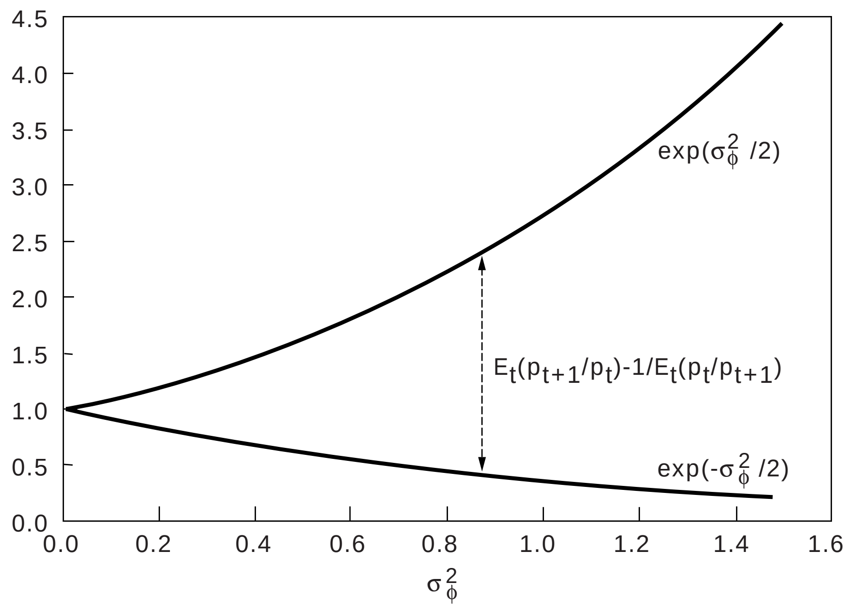
Hence, it is now clear that $\text{cov}_t \left((c_{t+1}/c_t)^{-\gamma}, p_t/p_{t+1} \right) \leq 0$ whenever $\sigma_{\zeta\phi} \leq 0$, regardless of the other terms in equation (15). As suggested by equation (6), this effect raises the nominal rate over and above that implied by movements in the real rate and expected inflation alone. Finally, it should be intuitive that when households are risk-neutral and $\gamma = 0$, the inflation risk premium is identically zero irrespective of $\sigma_{\zeta\phi}$.

Earlier in the analysis, we hinted that even if the inflation risk premium were zero at all dates, equation (6) would not necessarily reduce to the Fisher equation when inflation is stochastic. We argued that, generally, $E_t(p_{t+1}/p_t) \neq 1/E_t(p_t/p_{t+1})$ and that this difference would rise with the volatility of inflation surprises. This result is shown formally in Appendix C and, in particular,

$$E_t \frac{p_{t+1}}{p_t} - \frac{1}{E_t(p_t/p_{t+1})} = \exp(\delta_{\phi 0}) \left[\exp \left(\frac{-\sigma_\phi^2}{2} \right) - \exp \left(\frac{\sigma_\phi^2}{2} \right) \right] P_t^{-1}, \tag{16}$$

where $P_t = \Pi_{j=0}^{p} \zeta_{t-j}^{\delta_{\phi\zeta,j}} \Pi_{j=0}^{p} \phi_{t-j}^{\delta_{\phi\phi,j}}$. Figure 1 illustrates how the right-hand side of this last equation varies as a function of ϕ_ϕ^2. Since the result in equation (16) is essentially driven by Jensen's Inequality, the greater the variance of inflation

Figure 1 Effect of Jensen's Inequality on the Simple Fisher Equation



shocks, the more the convexity inherent in the price ratio matters. In a world without inflation surprises, $\sigma_\phi^2 = 0$, and the right-hand side of (16) vanishes. Note that in the latter case, inflation is not necessarily constant but is deterministic and described by equation (9), without the $\varepsilon_{\phi,t+1}$ shock. Therefore the conditional expectations operator in (16) becomes, in some sense, irrelevant.

Thus far, we have been able to show that the discrepancy between the modified Fisher equation in (6) and the Fisher equation in (7) ultimately boils down to two crucial aspects of the environment; namely, the covariance between unexpected movements in endowment growth and inflation surprises, as well as the unconditional volatility of inflation surprises. However, whether this difference is quantitatively significant remains to be seen.

Quantitative Implications

To address the quantitative features of the model just presented, we must first tackle the issue of calibration. As a benchmark case we first fix the discount rate, β, to 0.996 and set the risk-aversion parameter, γ, to 0.75. The value of the discount rate is chosen so that, in the benchmark scenario, the mean of the model-implied ex post real rate matches its counterpart in the data at 2.32 percent. Note that since U.S. real consumption has generally been growing at about 2 percent over the sample, our discount factor is scaled up by a factor of

$(1.02)^{\gamma}$ relative to one that would be appropriate for stationary data. We then examine how the results vary with changes in the risk-aversion parameter. The only other necessary parameters of the model relate to the exogenous driving process. To this end, we estimate the bivariate VAR described by equations (8) and (9) using the following data:

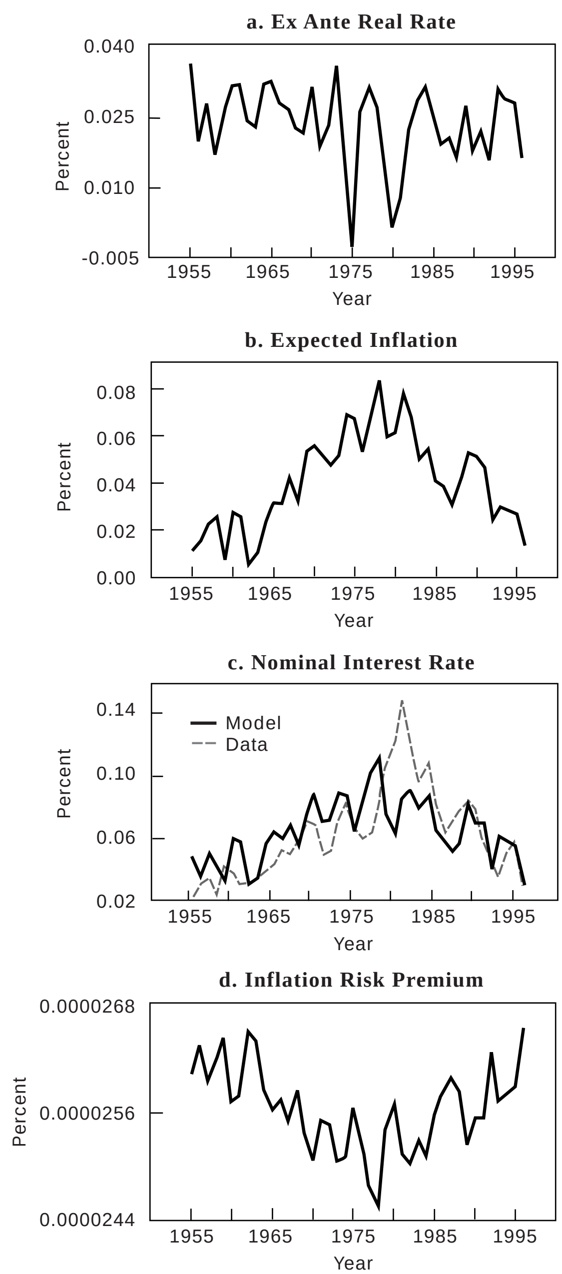
- Consumption refers to per capita annual U.S. consumption of nondurable goods, durable goods, and services, spanning 1955 to 1996 and expressed in 1992 dollars.
- Price level refers to the ratio of nominal consumption to real consumption.

Note that we are using annual data in order to avoid estimating equations (8) and (9) with variables averaged over extended periods. Using data averaged over a ten-year period, for instance, would result in a substantial loss of information. A VAR of order 4 is estimated with resulting R^2s of 0.75 and 0.90 for equations (8) and (9), respectively. The point estimates for $\sigma_{\zeta\phi}$ and σ_{ϕ}^2 are 3.20×10^{-5} and 2.43×10^{-4}. It directly follows that the inflation risk premium generated by this model is all but negligible. Observe that this result has little to do with the notion that the equity premium is typically small in this type of framework. Instead, it is driven almost exclusively by the fact that inflation surprises move in a way unrelated to unexpected changes in consumption growth. (We return to this point more fully in the next section.) Moreover, consistent with the high R^2 associated with the estimation of equation (9), the volatility of shocks to inflation also appears to be very small. Therefore, by equation (16), we may think of $E_t(p_{t+1}/p_t)$ as essentially equal to $1/E_t(p_t/p_{t+1})$.

Figure 2 presents the historical estimates generated from the model for the period 1955 to 1996 using the benchmark parameters. We chose this time span so that we could directly compare the model-implied nominal rate with the actual yield on one-year Treasury notes.

As we can see from Figure 2, panel c, except for the late 1970s and early 1980s, the model performs relatively well in matching the actual nominal rate. The model's inability to capture the sharp rise in nominal rates in the late 1970s can perhaps best be explained by the unusually aggressive disinflationary policy adopted by the Fed at that time. In response to strong inflationary pressures in the fall of 1980, Goodfriend (1993, pp. 11–12) notes that "the Fed began an unprecedented aggressive tightening. . . . Thus, the run-up of the funds rate to its 19 percent peak in January 1981 marked a deliberate return to the high interest rate policy." It may be, therefore, that the assumptions concerning the driving process described by equations (8) and (9) are not entirely justified. In particular, a specification for the driving process that included the possibility of a regime switch around 1980 might have been more appropriate.

Figure 2 Simulated Results with CRRA Utility



As shown in panel d, the inflation risk premium is insignificant over the entire period and since the variance of inflation surprises is small, the modified Fisher equation collapses almost exactly to the Fisher equation. To be specific, the gap that separates equation (6) from equation (7) is never more than 3 basis points over the entire period. Thus, while the Fisher equation does not hold in theory when inflation is stochastic, it may very well serve as a reasonable approximation in practice.

Panel a of Figure 2 also shows that the ex ante real rate can be quite volatile. Observe in particular the severe real rate drops that occur in 1975 and 1980. In the context of this model, recall that the real rate in equation (11) is in part a function of recent consumption growth. According to the driving process described in Section 2, past consumption growth helps predict future consumption growth in equation (10). Consequently, the sharp fall in real rates in 1975 and 1980 correspond respectively to the two recessions typically associated with the severe rise in oil prices and the credit controls imposed by the Carter Administration. Over the period under consideration, the one-year real rate fluctuates between 0.25 percent and 3.7 percent. This range is substantially greater than the 75-basis-point range found by Ireland (1996) for the ten-year real rate. Our findings therefore lend support to the stylized view that as maturity increases, variations in the nominal rate are more likely due to variations in expected inflation than variations in the real rate. Table 1 presents some key sample statistics concerning the time-series properties of the historical estimates generated by the model as we vary the risk-aversion parameter.

As suggested by the estimates in Table 1, the standard deviation of the real rate is about 1 percent in the benchmark case. This rate is more than half the standard deviation of the ex post real rate of 1.80 percent over the same period. Therefore, in spite of relatively smooth consumption growth, this framework generates a real rate with considerable volatility.

In Table 1, we also note that both the mean and the standard deviation of the real rate increase sharply with the risk-aversion parameter. This result emerges because a rise in the degree of risk aversion implies a fall in the elasticity of intertemporal substitution in consumption. Since the representative household is less willing to smooth consumption across periods, it generally requires a higher return on bonds in order to save. More importantly, this feature of the model is precisely that which makes it difficult to match the equity premium. As observed in Abel (1990), although the return on stocks typically rises with γ, the fact that the return on Treasury notes also rises with γ essentially leaves the difference between the stock return and the bond return unchanged, even for large increases in risk aversion. Ideally, to have a better chance of matching the equity premium without requiring extreme values of γ, one would like a framework in which increases in the degree of risk aversion do not necessarily yield increases in the real rate.

Table 1

	Ex Ante Real Rate: r_t	**Expected Inflation:** $E_t(p_{t+1}/p_t)$	**Nominal Rate:** r_t^N
$\gamma = 0.75$	mean: 2.14	mean: 4.03	mean: 6.23
	std: 1.00	std: 2.43	std: 2.39
	corr$(r_t, E_t(p_{t+1}/p_t))$: -0.29	var$(\frac{E_t(p_{t+1}/p_t)}{r_t})$: 5.99	
$\gamma = 1.75$	mean: 4.51	mean: 4.03	mean: 9.90
	std: 2.37	std: 2.43	std: 3.01
	corr$(r_t, E_t(p_{t+1}/p_t))$: -0.29	var$(\frac{E_t(p_{t+1}/p_t)}{r_t})$: 1.03	
$\gamma = 6$	mean: 15.10	mean: 4.03	mean: 19.64
	std: 9.02	std: 2.43	std: 8.98
	corr$(r_t, E_t(p_{t+1}/p_t))$: -0.28	var$(\frac{E_t(p_{t+1}/p_t)}{r_t})$: 0.07	

Finally, because the volatility of the real rate depends so crucially on γ in the above experiment, it is difficult to say whether the volatility of the real rate relative to that of expected inflation is greater or less than one. In addition, the model consistently generates a negative correlation between the real rate and expected inflation across all values of the risk-aversion parameter. The latter result supports earlier evidence to that effect by Fama (1990).

4. RESULTS WITH "KEEPING-UP-WITH-THE-JONESES" UTILITY

Analytical Solutions

Thus far, estimates of the inflation risk premium based on the above framework as well as U.S. consumption data appear to be quantitatively small. We have also suggested that this result is unrelated to the fact that the equity premium tends to be small in consumption-based asset pricing models. To see why this is true, we now adopt an alternative preference specification that we refer to as the "keeping-up-with-the-Joneses" (KUPJ) specification. Under this alternative way of modeling preferences, which defines utility as a function of *relative* consumption, Abel (1990) shows that while the return on stocks typically increases with the risk-aversion parameter, the real return on bonds generally remains constant. Therefore, when the degree of relative risk aversion is sufficiently high ($\gamma = 6$ in Abel [1990]), the author is able to generate an equity premium that is within the range of that observed in the data. We now formally show that even when utility is of the KUPJ form, the inflation risk premium remains small irrespective of the degree of risk aversion.

Following Abel (1990) and Gali (1994), momentary KUPJ utility is given by

$$u(c_t) = \frac{(c_t/C_{t-1})^{1-\gamma} - 1}{1-\gamma}, \tag{17}$$

where C_{t-1} denotes average consumption in the previous period. Thus, the specification in (17) captures the idea that it is not consumption per se but relative consumption that matters to households. Using $\widetilde{q}_t$ as the price of a one-period inflation-indexed bond under this alternative functional form for utility, equation (4) now reads as

$$\begin{aligned} \widetilde{q}_t &= \beta \left(\frac{C_t}{C_{t-1}} \right)^{\gamma-1} E_t \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \\ &= \left(\frac{C_t}{C_{t-1}} \right)^{\gamma-1} q_t, \end{aligned} \tag{18}$$

where q_t, given by equation (10), is the price of an inflation-indexed bond when utility is CRRA. Similarly, the inverse of the nominal rate in equation (12) is now given by

$$\begin{aligned} \widetilde{x}_t &= \beta \left(\frac{C_t}{C_{t-1}} \right)^{\gamma-1} E_t \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \frac{p_t}{p_{t+1}} \\ &= \left(\frac{C_t}{C_{t-1}} \right)^{\gamma-1} x_t. \end{aligned} \tag{19}$$

In equilibrium, $C_t = c_t$ when households are identical. Therefore, the solutions for $\widetilde{q}_t$ and $\widetilde{x}_t$ can simply be obtained by scaling up equations (11) and (14), respectively, by a power function of current consumption growth, $\zeta_t^{\gamma-1}$. More importantly, these results also indicate that the new inflation risk premium is now given by equation (15) multiplied by $\zeta_t^{\gamma-1}$. Since the inflation risk premium under KUPJ utility is simply the premium that emerges under CRRA utility scaled up by current consumption growth (to the power $\gamma - 1$), a value of $\sigma_{\zeta\phi} = 0$ still implies that the inflation risk premium is identically zero irrespective of γ. In other words, it is still true in this case that when unexpected movements in consumption growth and shocks to inflation are uncorrelated, the inflation risk premium is zero regardless of the degree of risk aversion. Given our estimate in the previous section of $\sigma_{\zeta\phi} = 3.20 \times 10^{-5}$, it follows that even when preferences follow the KUPJ specification, the simple Fisher equation in (7) remains a good approximation to the generalized Fisher equation in (6).

Quantitative Implications

Figure 3 presents the historical estimates from the benchmark case where utility is of the KUPJ form. The parameter values for the bivariate driving process are the same as those used in the previous section. A direct comparison with Figure 2 reveals little difference between the two sets of figures. In particular, observe that, as expected, the inflation risk premium continues to be negligible over the entire period under consideration. As in the earlier experiment, the model still fails to capture the behavior of the nominal rate at the end of the 1970s and beginning of the 1980s. However, it is interesting that both the ex ante real rate and the model-implied nominal rate seem to exhibit more variation relative to Figure 2. This result is consistent with the earlier work of Abel (1990) who finds that, while the mean return on bonds remains relatively constant as the degree of risk aversion rises with KUPJ preferences, the volatility of bond returns tends to exceed that which emerges with CRRA utility. The following table makes the last point more concretely.

When one compares Table 2 with Table 1, it is clear that under the alternative preference specification, the real rate is largely invariant with respect to the degree of risk aversion. This invariance property is precisely the mechanism that, for a high enough value of γ, allows Abel (1990) to generate an equity risk premium close to the one found in the data. Table 2 also clearly suggests that in all cases, the volatility of both the real rate and nominal rate is greater than its corresponding value in Table 1. As in the previous section, it remains that the volatility of the real rate increases sharply with the degree of risk aversion. Accordingly, whether the real rate varies more or less than expected inflation at short horizons still depends heavily on the particular preference specification adopted. In addition, as in Fama (1990), the model continues to suggest a consistent negative correlation between the real rate and expected inflation across different values of γ. Therefore, although we find that Fisher's equation holds relatively well in this framework, the nominal yield moves generally less than one-for-one with expected inflation at the one-year horizon.

5. CONCLUDING REMARKS

This article investigates the extent to which the simple Fisher equation can be interpreted as a reasonable approximation to its more complete counterpart in a dynamic endowment economy. The expanded Fisher equation, in addition to capturing movements in real rates and expected inflation, differs from its simpler version along two dimensions. First, it accounts for random movements in inflation through an inflation risk premium. Second, it acknowledges the inherent nonlinearity of inflation in drawing a link between the nominal rate and expected inflation.

Given U.S. consumption data, we find that the quantitative historical estimates of the inflation risk premium for the period 1955 to 1996 are small.

Figure 3 Simulated Results with KUPJ Utility

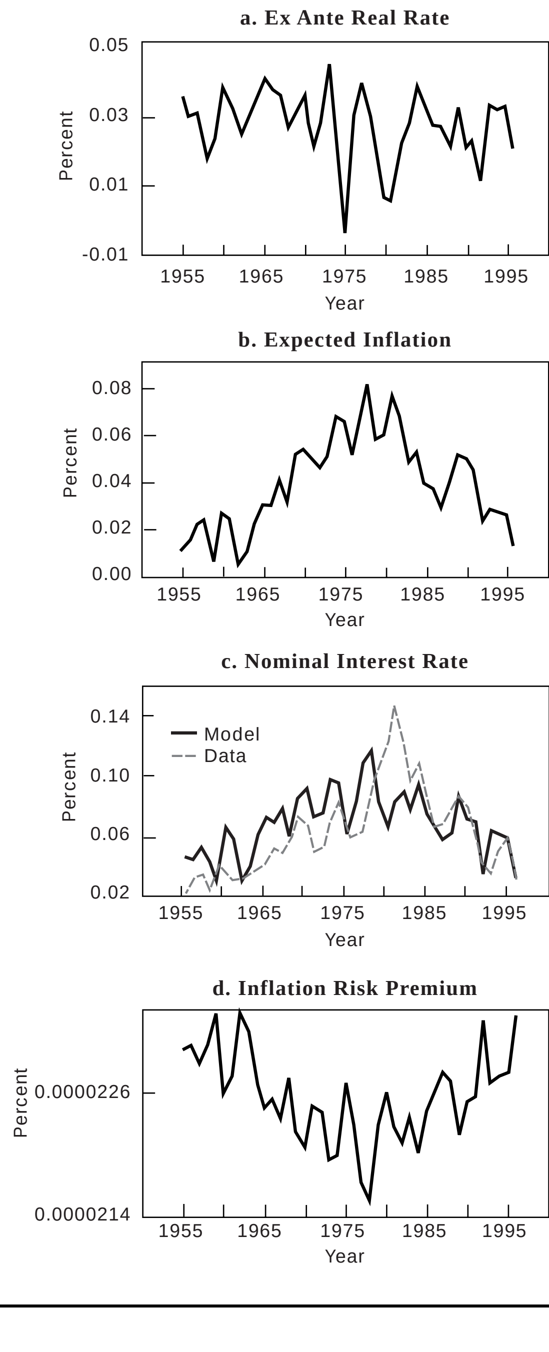


Table 2

	Ex Ante Real Rate: r_t	**Expected Inflation:** $E_t(p_{t+1}/p_t)$	**Nominal Rate:** r_t^N
$\gamma = 0.75$	mean: 2.75	mean: 4.03	mean: 6.86
	std: 1.24	std: 2.43	std: 2.62
	corr$(r_t, E_t(p_{t+1}/p_t))$: -0.16	var$(\frac{E_t(p_{t+1}/p_t)}{r_t})$: 3.80	
$\gamma = 1.75$	mean: 2.69	mean: 4.03	mean: 6.78
	std: 2.43	std: 2.43	std: 3.75
	corr$(r_t, E_t(p_{t+1}/p_t))$: -0.39	var$(\frac{E_t(p_{t+1}/p_t)}{r_t})$: 1.00	
$\gamma = 6$	mean: 2.65	mean: 4.03	mean: 6.65
	std: 11.06	std: 2.43	std: 10.75
	corr$(r_t, E_t(p_{t+1}/p_t))$: -0.37	var$(\frac{E_t(p_{t+1}/p_t)}{r_t})$: 0.04	

This result emerges primarily because unexpected movements in consumption and inflation surprises appear to have little covariation in U.S. data. In other words, since inflation surprises are largely unrelated to consumption growth, there is no reason why the inflation risk premium should be either positive or negative. Moreover, the latter notion was shown to have little to do with the equity premium being typically small in consumption-based asset pricing models. Therefore, although the Fisher equation does not theoretically apply in an environment with stochastic inflation, it may serve as an adequate approximation in practice.

Using two different preference structures, we also find that the model-implied nominal yield on one-year bonds matches the actual one-year yield on Treasury notes relatively well for most of the sample period. However, the model fails to track the nominal rate adequately in the late 1970s. We suspect that this latter result is partly driven by the singularly aggressive stance adopted by the Federal Reserve at that time in order to bring down very high inflation rates. In interpreting our results concerning the inflation risk premium, one needs to be cognizant of the model's failure along this dimension. Our benchmark cases also suggest a real rate whose volatility is more than half that of its U.S. ex post counterpart. Further, our framework in all cases provides additional evidence to support Fama's (1990) view that expected inflation and the real rate tend to move in opposite directions. Finally, we find that under both preference specifications, whether the real rate is more or less volatile than expected inflation depends heavily on households' degree of risk aversion. Taken together, these last two points suggest one should proceed with caution when interpreting movements in short-term nominal yields in terms of movements in expected inflation.

APPENDIX A: HOUSEHOLD OPTIMALITY CONDITIONS

Let λ_t and μ_t represent the Lagrange multipliers associated with constraints (2) and (3), respectively. Then, the first-order conditions associated with the household's problem are given by

$$u'(c_t) = \mu_t + \beta E_t \lambda_{t+1} \frac{p_t}{p_{t+1}}, \tag{20}$$

$$q_t \lambda_t = \beta E_t \lambda_{t+1}, \tag{21}$$

$$x_t \frac{\lambda_t}{p_t} = \beta E_t \frac{\lambda_{t+1}}{p_{t+1}}, \tag{22}$$

and

$$\lambda_t = \mu_t + \beta E_t \lambda_{t+1} \frac{p_t}{p_{t+1}}. \tag{23}$$

APPENDIX B: USEFUL PROPERTIES OF LOG-NORMAL RANDOM VARIABLES

This appendix describes properties of log-normal random variables that are useful in deriving the solution for bond prices described in Section 3. Let x be a log-normal random variable, then

- $\ln E(x) = E(\ln x) + (1/2)\text{var}(\ln x)$ and
- $\ln E(x^a) = aE(\ln x) + (a^2/2)\text{var}(\ln x)$ for $a \in \mathcal{R}$.

Furthermore, if y is a log-normal random variable, then so is $z = xy$. To see this, note that $\ln z = \ln x + \ln y$, which is the sum of two normal random variables and thus itself normally distributed. It directly follows from the first of the above properties that

- $\ln E(xy) = E(\ln x) + E(\ln y) + (1/2)\text{var}(\ln x) + (1/2)\text{var}(\ln y) + \text{cov}(\ln x, \ln y)$.

APPENDIX C: JENSEN'S INEQUALITY AND THE VARIANCE OF INFLATION SHOCKS

Since

$$\begin{aligned}\ln E_t \frac{p_{t+1}}{p_t} &= E_t \ln \frac{p_{t+1}}{p_t} + \frac{1}{2}\mathrm{var}_t \ln \frac{p_{t+1}}{p_t} \\ &= E_t \ln \phi_{t+1} + \frac{1}{2}\mathrm{var}_t \ln \phi_{t+1},\end{aligned}$$

equation (9) directly implies that

$$E_t \frac{p_{t+1}}{p_t} = \exp(\delta_{\phi 0} + \frac{\sigma_\phi^2}{2}) P_t, \tag{24}$$

where $P_t = \Pi_{j=0}^{p} \zeta_{t-j}^{\delta_{\phi\zeta,j}} \Pi_{j=0}^{p} \phi_{t-j}^{\delta_{\phi\phi,j}}$. Furthermore, since $\ln E_t p_t/p_{t+1}$ can simply be expressed as $\ln E_t(p_{t+1}/p_t)^{-1}$, we also have that

$$E_t \frac{p_t}{p_{t+1}} = \exp(-\delta_{\phi 0} + \frac{\sigma_\phi^2}{2}) P_t^{-1}. \tag{25}$$

It then follows that

$$E_t \frac{p_{t+1}}{p_t} - \frac{1}{E_t(p_t/p_{t+1})} = \exp(\delta_{\phi 0}) \left[\exp(\frac{-\sigma_\phi^2}{2}) - \exp(\frac{\sigma_\phi^2}{2}) \right] P_t^{-1}. \tag{26}$$

Hence, the difference on the left-hand side of equation (26) rises with σ_ϕ^2 as conjectured. This is shown in Figure 1.

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