

# Is “High” Capacity Utilization Inflationary?

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Capacity utilization in U.S. industry features prominently in discussions of inflation. This prominence derives from the widely held viewpoint that “high” rates of capacity utilization are tantamount to resource-shortage conditions or “bottlenecks” that inevitably erupt into price inflation. For instance, an article in Citicorp’s *Economic Week* (January 18, 1994) argues: “In the past, a utilization rate swinging up toward the 84%–85% range was a source of much anxiety. Usually when the rate got that high, production bottlenecks started to appear. . . . Shortages developed. And soon, key price indexes were shooting up.” In the *American Banker* (January 12, 1994), Stephen Davies points out: “Economists say that, historically, there has been a connection between a healthier industrial sector and rising prices. That’s because factories start to run into bottlenecks in which supplies and labor are short” (p. 1). And in an article in *Barron’s* (June 20, 1994), Gene Epstein states, “Capacity utilization should remain below 85%, the assumed inflationary danger zone” (p. 48). A concomitant viewpoint is that when capacity utilization is “low,” the economy is in the inflationary *safe* zone. The threshold defining “high” and “low” rates of capacity utilization is often 85 percent, as the above quotations exemplify.

The purpose of the present study is to outline the theory underlying these popular views and to evaluate it in terms of its ability to explain the facts about the U.S. economy. To accomplish this task, the article proceeds as follows. Section 1 isolates and discusses the core features of the theory. Section 2 presents the evidence on the relationship between capacity utilization and inflation and related evidence on the linkage between cyclical GDP and inflation. Section 3 assesses the theory in terms of its ability to explain the evidence. Finally, Section 4 concludes with a summary and suggestions for future research.

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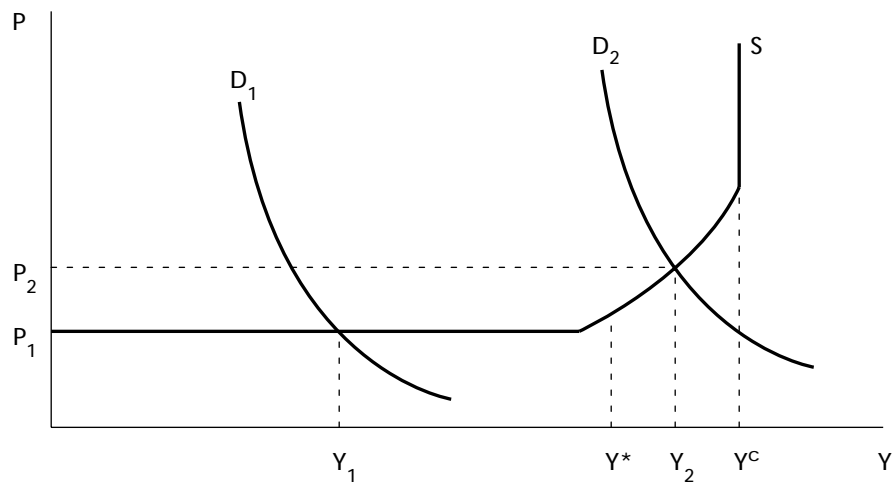
## 1. THE THEORY

The theory supporting the view that high rates of capacity utilization are inflationary has not been fully articulated. Yet, it seems to be a variant of traditional Keynesian theory. Figure 1 illustrates the key elements of this theory. In this figure,  $P$  is the general price level,  $Y$  is aggregate real output,  $Y^*$  is the full-employment level of output,  $Y^c$  is the capacity level of output,  $D_1$  and  $D_2$  denote alternative aggregate demand curves, and  $S$  is the aggregate supply curve.

Figure 1 shows that the intersection between aggregate demand and supply determines the price level and output. It is a snapshot of the economy over the time horizon relevant for the study of business cycles, the short run. For this horizon, the fixed capacity output level,  $Y^c$ , provides the effective “lid” on the economy. Cyclical fluctuations in output correspond to deviations of actual output,  $Y$ , from the constant full-employment output level,  $Y^*$ . Resources are less than fully employed when  $Y < Y^*$ , while they are more than fully employed when  $Y > Y^*$ —that is, people and capital work overtime. The cyclical fluctuations in output are driven by shifts in the aggregate demand curve, stemming from changes in consumption, investment and government expenditures or in the stock of money. The figure abstracts from economic growth, a long-run phenomenon, which can be imagined as increasing  $Y^*$  and  $Y^c$  gradually over time and also shifting the demand and supply curves outward slowly over time. Such growth is due to improvements in technology and increases in the stock of capital and the work force.

In this theory, the aggregate supply curve is nonlinear. This nonlinearity implies that the relationship between the price and output responses to demand

**Figure 1 A Keynesian Theory**



shifts depends on the *level* of real output or, alternatively, the *level* of overall resource use in the economy. At low levels of output, say  $Y = Y_1$ , resources such as labor and capital are underemployed. Firms can obtain as much of these resources as they wish at constant wages and rents. Therefore, firms are willing to supply whatever amount of final goods is demanded at the existing price level,  $P_1$ . When output is high, say  $Y = Y_2$ , the story is different. Now resources are more than fully employed—capital and labor are working overtime. If firms want to increase resource usage, they must offer higher wages and rents. Consequently, firms are willing to accommodate demand expansions only if they can pass along the resource price increases to consumers in the form of higher final goods prices. Moreover, these price increases occur at an increasing rate as  $Y^c$  is approached and resources become increasingly overworked. In short, at low output levels, a rise in demand causes a rise in output with little or no accompanying price inflation. When output is high, demand expansions cause both output increases and rising inflation.<sup>1</sup>

The Federal Reserve’s capacity utilization rate for total U.S. industry is the ratio of actual to capacity industrial production. The capacity industrial production index attempts “to capture the concept of sustainable practical capacity, which is defined as the greatest level of output that a plant can maintain within the framework of a realistic work schedule, . . . assuming sufficient availability of inputs to operate the machinery and equipment in place” (Federal Reserve Statistical Release G.17, March 15, 1994, p. 18). Thus, the capacity utilization rate intends to measure  $Y/Y^c$ . The above theory explains the attention devoted to tracking the capacity utilization rate in discussions of inflation.

The isolation of the central features of the theory immediately invites some criticism and questions. First, the concept of capacity output relies on the *impossibility* of quickly expanding the work force, capital stock and technological knowledge. But the work force *is* elastic *even* in the short run—for example, retired workers and young adults acquiring education *can* be induced to enter the work force. New investments and bringing back “on line” previously obsolete capital *can* rapidly increase the stock of capital. Improvements in technology, leading to more efficient production techniques and labor-saving capital equipment, *can* quickly occur as well. These points are emphasized in recent discussions in *The Wall Street Journal* (Harper and Myers, June 6, 1994) about “companies forced to buy equipment to keep up with technological improvements” (p. A6) and in *The New York Times* (Uchitelle, April 24, 1994): “Labor-saving machinery permits the extra production with fewer workers” (p. 24).

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<sup>1</sup> Notice that there is an ambiguity in the theory. It is not clear how the theory develops the relationship between the *level* of output and the *rate of change* of prices. A strict interpretation of the theory’s underlying arguments points to a relationship between the *levels* of output and prices.

Second, the short-run changes in the work force, capital stock and technology will impact on deviations of  $Y$  from  $Y^*$ , since they shift the aggregate supply curve. Therefore, cyclical output fluctuations may not always be demand driven. Indeed, quantitative real business cycle theory shows that between 54 and 70 percent of the postwar output fluctuations in the United States can be explained by short-run changes in technology (see Kydland and Prescott [1991] and Aiyagari [1994]).

Third, the possible variation in the relationship between price and output responses to demand shocks rests on the presumed existence of a critical level, or levels, of output at which underemployment of resources sets in and prices no longer adjust so as to clear the goods and factor markets. It is difficult to see why such critical levels should exist. The theory is silent on this issue.

Thus, there are some good reasons for wondering whether the theory connecting high capacity utilization with inflation provides a useful guide in explaining the real world. Therefore, the questions arise: What are the facts on the relationship between capacity utilization and inflation? Is the Keynesian theory consistent with these facts?

## 2. THE EVIDENCE

### The Empirical Data

The evidence examined here involves seasonally adjusted, quarterly data for the United States over the period 1953:1–1994:1. A complete description of the data is in the appendix. The individual series will be gradually introduced into the discussion.

Regarding the above theory, two *theoretical* variables measure the overall degree of resource utilization:  $Y/Y^c$  and  $Y/Y^*$ . Trend output growth does not affect these variables since it affects the numerators and denominators to the same extent. Consequently, movements in these variables are purely cyclical in nature. Furthermore, they bear a perfect positive relationship to one another—stemming from the common cyclical variation in  $Y$ . Also notice that the average values of  $Y/Y^c$  and  $Y/Y^*$ , when taken over the long run, are  $Y^*/Y^c$  and  $Y^*/Y^*$  respectively—since in the long run the temporary cyclical deviations of  $Y$  from  $Y^*$  do not, by definition, occur.

The closest *empirical* counterparts to  $Y/Y^c$  and  $Y/Y^*$  are the capacity utilization rate and cyclical per-capita GDP (henceforth referred to as cyclical GDP), respectively. The former measure was described earlier. Cyclical GDP is the percentage deviation of per-capita GDP from its smoothly evolving time trend.<sup>2</sup> Both empirical variables exhibit purely cyclical variations. The mean

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<sup>2</sup> This trend is derived by using the Hodrick-Prescott filtering method (with the “smoothing” parameter value set equal to 1600). See Kydland and Prescott (1990) for a description of the method.

value of utilization provides an estimate of  $Y^*/Y^c$ ; the mean value of cyclical GDP is zero (i.e., it is an estimate of the logarithm of  $Y^*/Y^*$ ). In contrast to the perfect synchronization between movements in the theoretical variables,  $Y/Y^c$  and  $Y/Y^*$ , movements in the utilization rate and cyclical GDP may diverge for at least two reasons.<sup>3,4</sup> First, the underlying output measures are different—the utilization rate uses industrial production, while cyclical GDP uses the more comprehensive production measure, GDP. Second, cyclical GDP captures output relative to its smoothly evolving trend, while capacity utilization is output relative to its capacity level. Even though the theory assumes that the trend growth in  $Y^*$  and  $Y^c$  is the same, there is no reason why this has to hold in the data.

Figure 2 shows the capacity utilization rate and cyclical GDP. The two series move together closely but not perfectly. The contemporaneous correlation between the series is 0.74.<sup>5</sup> Movements in the two variables are sufficiently different that both merit attention as alternative measures of resource utilization from the point of view of the Keynesian theory.

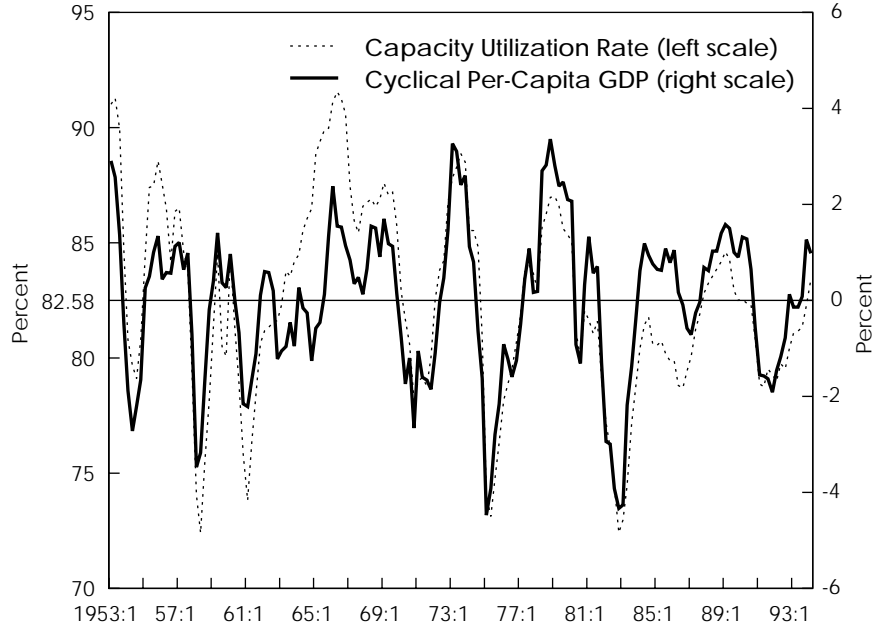
The capacity utilization rate is shown with the inflation rate in Figure 3. Inflation is measured by the quarter-to-quarter annualized percentage change in the CPI. Notice that there are time periods when the utilization rate is both high (in excess of 85 percent) and rising and inflation is also rising—for example, during 1964 and 1972. This is consistent with the theory. But, there are also periods of high utilization when utilization and inflation move in opposite directions. For example, early in 1955 and 1965, *rising* utilization simultaneously occurs with *falling* inflation, and during most of 1973–74 and 1979, *falling* utilization coincides with *rising* inflation. Furthermore, during much of the time when utilization is low and variable, inflation exhibits substantial variation. These episodes are inconsistent with the theory. A similar story emerges regarding cyclical GDP and inflation. It is depicted in Figure 4. The upshot is that the linkage between high utilization or high cyclical GDP and inflation is not immutable; neither is the linkage between low utilization or low cyclical GDP and inflation.

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<sup>3</sup> Notice that cyclical GDP measures  $\log(Y/Y^*)$ . The exponential of cyclical GDP gives a measure of  $Y/Y^*$ , which is in comparable units to the utilization rate. This exponential transformation makes no substantive difference to the quantitative analysis, a reflection of the transformation being close to a linear one (at least for the underlying range of variation in cyclical GDP). It is not undertaken for the quantitative analysis discussed in the article since it is common practice to measure cyclical GDP as a percentage *deviation*, not a *ratio* (again see Kydland and Prescott [1990]).

<sup>4</sup> See Shapiro (1989) for a detailed description and discussion of the capacity utilization rate.

<sup>5</sup> The correlation between capacity utilization and cyclical per-capita industrial production is higher. It is 0.82. This shows that the second reason explaining the difference between utilization and cyclical per-capita GDP is the more important of the two reasons.

**Figure 2 Capacity Utilization Rate and Cyclical Per-Capita GDP**

Notes: (1) Cyclical per-capita GDP is the percentage deviation of per-capita GDP from its Hodrick-Prescott trend component. (2) The horizontal line is drawn at the mean of both variables.

### Regression Analysis

Here regression analysis quantifies the *average* historical relationships between inflation and each of the two real economic activity variables. The analysis also tests the significance and possible asymmetries in those relationships.

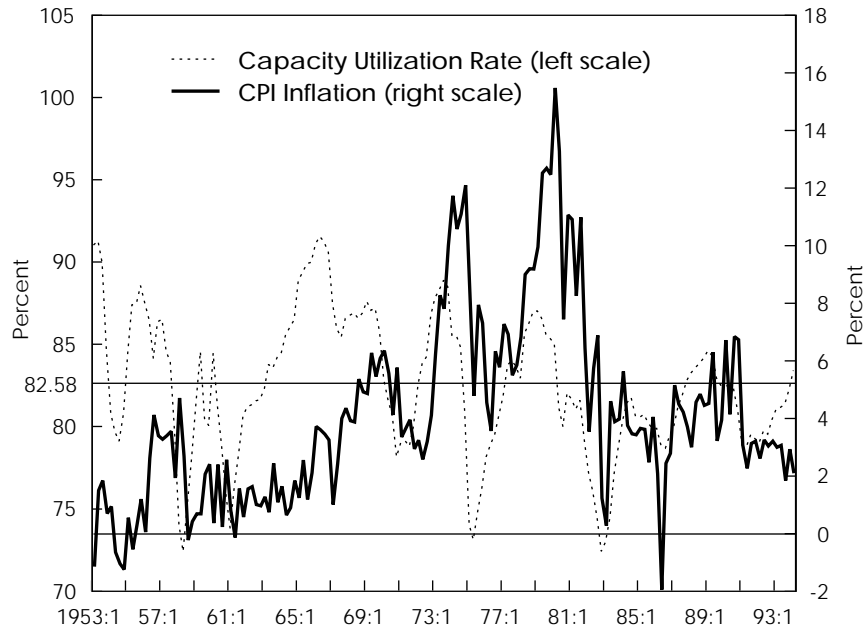
The regression equation with capacity utilization is

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta_h u_t^h + \beta_l u_t^l + \epsilon_t. \quad (1)$$

Including cyclical GDP, the regression equation is

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \gamma_h y_t^h + \gamma_l y_t^l + v_t. \quad (2)$$

$\pi_t$  is the time  $t$  inflation rate,  $u_t^h(u_t^l)$  is the time  $t$  utilization rate series pertaining to high (low) utilization rates and  $y_t^h(y_t^l)$  is the time  $t$  cyclical GDP series corresponding to high (low) cyclical GDP. These high and low series are explained more fully below.  $\epsilon_t$  and  $v_t$  are disturbance terms at time  $t$ . The  $\alpha_i$  ( $i = 0, 1, 2, 3$ ),  $\beta_i$  ( $i = h, l$ ) and  $\gamma_i$  ( $i = h, l$ ) are parameters.

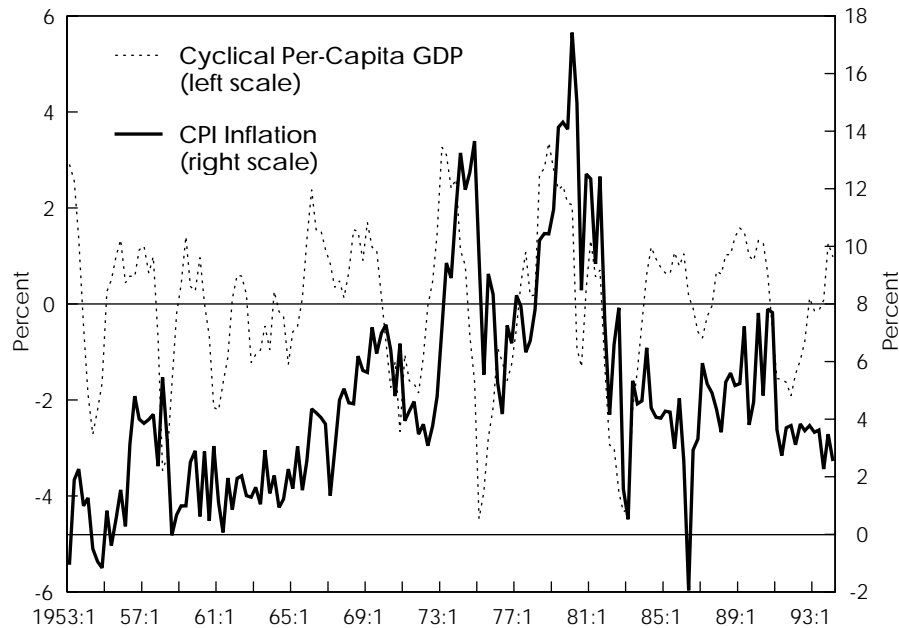
**Figure 3 Capacity Utilization Rate and CPI Inflation**

Notes: (1) CPI inflation is measured quarter to quarter at an annualized rate. (2) The horizontal lines are drawn at the mean of the utilization rate and zero CPI inflation.

The equations include three lagged values of the inflation series. Their inclusion is essential to ensure an adequate specification—omitting lagged inflation results in equations with almost no explanatory power. The choice of lag length is predicated on a sequence of F-tests, which establish that additional lagged values of the inflation series are statistically insignificant. Allowing inflation to depend on its lagged values constitutes a generalization of the simple, static Keynesian theory to admit persistence in the inflation process.

In line with the theory, it is the *contemporaneous* values of the real economic activity variables that enter into the regression equations. Their coefficients indicate the magnitude of the average, contemporaneous relationship obtaining between them and inflation, given the past path of inflation. A sequence of F-tests establishes that lagged values of the real economic activity series are insignificant once the contemporaneous values are accounted for in the regression equations.

To test for possible asymmetry in the relationship between utilization and inflation, one must first specify a threshold value defining high and low

**Figure 4 Cyclical Per-Capita GDP and CPI Inflation**

Notes: (1) Cyclical per-capita GDP is the percentage deviation of per-capita GDP from its Hodrick-Prescott trend component. CPI inflation is measured quarter to quarter at an annualized rate. (2) The horizontal lines are drawn at the mean of cyclical per-capita GDP and zero CPI inflation.

utilization rates and then create corresponding high and low utilization rate series. This exercise is accomplished as follows. Initially the utilization rate is expressed in terms of deviations from its mean. If  $r_t$  denotes the time  $t$  utilization rate and  $\bar{r}$  its mean (equaling 82.58 percent), then  $u_t = r_t - \bar{r}$  is the time  $t$  deviation of utilization from its mean. The threshold value is set at 85 percent since that value is the one most often used in media discussions. Two new variables,  $u_t^h$  and  $u_t^l$ , are then derived as follows:

$$\begin{aligned} u_t^h &= u_t \text{ if } r_t \geq 85 \text{ percent and zero otherwise;} \\ u_t^l &= u_t \text{ if } r_t < 85 \text{ percent and zero otherwise.} \end{aligned}$$

Should a difference exist between the coefficients of  $u_t^h$  and  $u_t^l$ , it signifies a difference in the relationship between inflation and each of high and low rates of utilization.<sup>6</sup>

<sup>6</sup> It is important to express the utilization rate,  $r_t$ , as a deviation from its mean before deriving the  $u_t^h$  and  $u_t^l$  series. Not doing so results in series, say  $\bar{u}_t^h$  and  $\bar{u}_t^l$ , that have a correlation equaling  $-0.99$ , causing extreme multicollinearity problems in the regression equation. By contrast,  $u_t^h$  and  $u_t^l$  are mildly correlated (with correlation 0.32), so multicollinearity problems do not arise.



A similar procedure is followed regarding cyclical GDP, the time  $t$  value of which is denoted by  $y_t$ . By construction,  $y_t$  has a zero mean. Using a threshold value equal to one standard deviation above the mean of  $y_t$  (equaling 0.0175), one can derive the two variables,  $y_t^h$  and  $y_t^l$ :

$$\begin{aligned} y_t^h &= y_t \text{ if } y_t \geq 0.0175 \text{ and zero otherwise;} \\ y_t^l &= y_t \text{ if } y_t < 0.0175 \text{ and zero otherwise.} \end{aligned}$$

A difference between the coefficients of  $y_t^h$  and  $y_t^l$  would mean a difference in the relationship between inflation and each of high and low cyclical GDP.<sup>7</sup>

The regression results for equation (1) are presented in the top panel of Table 1. The coefficients of  $u_t^h$  and  $u_t^l$  are  $\beta_h = 0.10$  and  $\beta_l = 0.19$ . They are individually significant (that is, significant at the 5 percent level). An F-test of the hypothesis  $\beta_h = \beta_l$  strongly indicates nonrejection at significance level 0.28. In other words, the evidence suggests that the relationship between inflation and utilization is the *same* regardless of whether or not the utilization rate is high.

Setting  $\beta_h = \beta_l = \beta$  improves the precision of the estimation and therefore leads to the preferred regression equation:

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta u_t + \epsilon_t. \quad (3)$$

The findings for this equation are displayed in the lower panel of Table 1. The coefficient of  $u_t$  is  $\beta = 0.15$ , which is significant. The relationship between capacity utilization and inflation is a significantly positive one—with a one percentage point increase in utilization being associated, on average, with a 0.15 percentage point increase in inflation.

The top panel of Table 2 gives the regression results for equation (2). The coefficients of  $y_t^h$  and  $y_t^l$  are found to be  $\gamma_h = 0.58$  and  $\gamma_l = 0.36$ , respectively. Both are individually significant. The hypothesis  $\gamma_h = \gamma_l$  cannot be rejected, using an F-test, at a fairly high significance level of 0.38. Once again, the evidence reveals no important asymmetry.

Setting  $\gamma_h = \gamma_l = \gamma$  yields a more precise estimation, leading to the preferred specification:

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \gamma y_t + v_t. \quad (4)$$

The lower panel of Table 2 shows the findings for this equation. The coefficient of  $y_t$  is  $\gamma = 0.45$ . It is significant. A one percentage point increase in the deviation of per-capita GDP from its trend path has been linked with a 0.45 percentage point increase in inflation, on average.

<sup>7</sup> The findings, to be discussed below, are robust to alternative choices of thresholds. More exactly, for utilization threshold values equaling 82.58 percent and 80 percent, and for cyclical GDP threshold values equal to zero and one standard deviation *below* the mean of  $y_t$  ( $-0.0175$ ), similar findings obtain.

**Table 1 Regression Results for Inflation-Utilization Relationship, 1953:1–1994:1**


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Regression Equation (with utilization threshold value = 85%)

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta_h u_t^h + \beta_l u_t^l + \epsilon_t$$

$\alpha_0 = 0.38$	$\alpha_1 = 0.58$	$\alpha_2 = -0.05$	$\alpha_3 = 0.41$	$\beta_h = 0.10$	$\beta_l = 0.19$
(1.70)	(8.20)	(-0.61)	(5.77)	(2.07)	(4.04)

$\bar{R}^2 = 0.80$     DW = 1.94

For test of hypothesis  $\beta_h = \beta_l$ ,  $F(1, 159) = 1.19$  (0.28)

Regression Equation (imposing  $\beta_h = \beta_l$ )

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta u_t + \epsilon_t$$

$\alpha_0 = 0.25$	$\alpha_1 = 0.58$	$\alpha_2 = -0.05$	$\alpha_3 = 0.41$	$\beta = 0.15$
(1.32)	(8.25)	(-0.62)	(5.73)	(5.12)

$\bar{R}^2 = 0.80$     DW = 1.94

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Notes: (1) The number of observations is 165. (2) t-statistics are in parentheses below the corresponding coefficient values. (3)  $\bar{R}^2$  is the regression goodness-of-fit statistic. DW is the Durbin-Watson statistic. (4) The F-statistic with the relevant degrees of freedom is denoted by  $F(\cdot, \cdot)$ ; its significance level follows in parentheses.

**Table 2 Regression Results for Inflation-Cyclical GDP Relationship, 1953:1–1994:1**


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Regression Equation

(with cyclical GDP threshold value = one standard deviation above its mean)

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \gamma_h y_t^h + \gamma_l y_t^l + v_t$$

$\alpha_0 = 0.26$	$\alpha_1 = 0.55$	$\alpha_2 = -0.05$	$\alpha_3 = 0.40$	$\gamma_h = 0.58$	$\gamma_l = 0.36$
(1.21)	(7.94)	(-0.58)	(5.88)	(3.59)	(2.78)

$\bar{R}^2 = 0.81$     DW = 1.99

For test of hypothesis  $\gamma_h = \gamma_l$ ,  $F(1, 159) = 0.78$  (0.38)

Regression Equation (imposing  $\gamma_h = \gamma_l$ )

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \gamma y_t + v_t$$

$\alpha_0 = 0.36$	$\alpha_1 = 0.55$	$\alpha_2 = -0.04$	$\alpha_3 = 0.41$	$\gamma = 0.45$
(1.94)	(8.02)	(-0.54)	(5.94)	(6.23)

$\bar{R}^2 = 0.81$     DW = 1.99

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Notes: See notes for Table 1.

### Forecasting Analysis

Another way of gauging the linkages between inflation and each of the two real economic activity variables is to assess the marginal predictive content of the latter variables for inflation. This assessment is done as follows. First,  $u_{t-1}$  and  $y_{t-1}$  replace  $u_t$  and  $y_t$ , respectively, in equations (3) and (4) to give the forecasting equations:

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta u_{t-1} + \epsilon_t \quad (5)$$

and

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \gamma y_{t-1} + v_t. \quad (6)$$

Omitting the real variables gives the univariate forecasting equation:

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + d_t, \quad (7)$$

where  $d_t$  is the time  $t$  disturbance term. All right-hand-side variables in these equations are in the time  $t - 1$  information set. Consequently, the disturbance terms  $\epsilon_t$ ,  $v_t$  and  $d_t$  are the one-period-ahead forecasting errors.

The estimation results for equations (5)–(7), over the period 1953:1–1994:1, are presented in Table 3. The findings for equations (5) and (6) are similar to those for equations (3) and (4), respectively—a reflection of  $u_t$  and  $y_t$  being highly autocorrelated.

By sequentially estimating (5), (6) and (7) over the 17 sample periods that start in 1953:1–1990:1, increasing by one quarter at a time and ending with 1953:1–1994:1, one can generate a sequence of 17 one-period-ahead forecasting errors for each equation.<sup>8</sup> Computing and then comparing the mean squared forecast errors (MSE) of the equations allows one to assess the marginal predictive content of utilization and cyclical GDP for inflation. Ireland (1995) introduces this measure of predictive contribution.

Table 4 lists the MSE for each forecasting equation. The ratio of the MSE for the utilization-inflation equation to the MSE for the univariate equation is 0.87. In the case of the cyclical GDP-inflation equation, the ratio of MSE is 0.82. These findings highlight that both utilization and cyclical GDP have substantial predictive power for inflation, over and above that stemming from the accounting for past inflation behavior. Also cyclical GDP has the edge on the utilization rate in this respect.

The forecasting results naturally lead to the question: Does utilization have predictive content for inflation once account is taken of past inflation *and* cyclical GDP? To answer this, Table 3 gives the estimation findings (1953:1–1994:1) for the forecasting equation that includes *both* utilization and cyclical GDP:

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta u_{t-1} + \gamma y_{t-1} + b_t, \quad (8)$$

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<sup>8</sup> Each forecasting equation is stable over various forecasting periods.

**Table 3 Regression Results for the Forecasting Equations, 1953:1–1994:1**

Forecasting Equation with Utilization						
$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta u_{t-1} + \epsilon_t$						
$\alpha_0 = 0.34$ (1.73)	$\alpha_1 = 0.58$ (8.02)	$\alpha_2 = -0.06$ (-0.66)	$\alpha_3 = 0.40$ (5.45)	$\beta = 0.13$ (4.47)		
$\bar{R}^2 = 0.79$	DW = 1.88					
Forecasting Equation with Cyclical GDP						
$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \gamma y_{t-1} + v_t$						
$\alpha_0 = 0.45$ (2.37)	$\alpha_1 = 0.54$ (7.36)	$\alpha_2 = -0.05$ (-0.62)	$\alpha_3 = 0.41$ (5.78)	$\gamma = 0.41$ (5.34)		
$\bar{R}^2 = 0.80$	DW = 1.90					
Univariate Forecasting Equation						
$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + d_t$						
$\alpha_0 = 0.36$ (1.73)	$\alpha_1 = 0.66$ (8.89)	$\alpha_2 = -0.06$ (-0.64)	$\alpha_3 = 0.31$ (4.21)			
$\bar{R}^2 = 0.77$	DW = 1.88					
Forecasting Equation with Utilization and Cyclical GDP						
$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta u_{t-1} + \gamma y_{t-1} + b_t$						
$\alpha_0 = 0.43$ (2.23)	$\alpha_1 = 0.54$ (7.36)	$\alpha_2 = -0.05$ (-0.63)	$\alpha_3 = 0.42$ (5.81)	$\beta = 0.03$ (0.69)	$\gamma = 0.34$ (2.83)	
$\bar{R}^2 = 0.80$	DW = 1.90					

Notes: See notes for Table 1.

where  $b_t$  is the one-period-ahead forecast error. The coefficient of  $u_{t-1}$ ,  $\beta = 0.03$ , is insignificant, while that of  $y_{t-1}$ ,  $\gamma = 0.34$ , is significant.

This finding suggests that utilization does not have independent predictive content for inflation. But this conclusion, based on the relative significance of  $\beta$  and  $\gamma$ , must be tempered by the recognition that the high collinearity between  $u_{t-1}$  and  $y_{t-1}$ , noted earlier, makes it difficult to disentangle the relative predictive contributions of  $u_{t-1}$  and  $y_{t-1}$ . What *is* revealing is that the predictive relationship between inflation and cyclical output, specified by equation (6), is sufficiently stronger than that between inflation and utilization, given by equation (5), so that  $y_{t-1}$  is still significant even when  $u_{t-1}$  is included in the forecasting equation for inflation.

**Table 4 Forecasting Results for Inflation, 1990:1–1994:1**


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Forecasting Equation with Utilization, Equation (5)	
$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta u_{t-1}$	
MSE <sup>5</sup> = 1.4599	
Forecasting Equation with Cyclical GDP, Equation (6)	
$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \gamma y_{t-1}$	
MSE <sup>6</sup> = 1.3723	
Univariate Forecasting Equation, Equation (7)	
$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3}$	
MSE <sup>7</sup> = 1.6818	
Mean Squared Error Ratios	
MSE <sup>5</sup> /MSE <sup>7</sup> = 0.8681	MSE <sup>6</sup> /MSE <sup>7</sup> = 0.8159

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Notes: (1) Forecasts are one-period-ahead forecasts. The number of forecasts is 17. (2) MSE<sup>*i*</sup> is the mean squared forecasting error for forecasting equation *i* (*i* = 5, 6, 7).

### 3. ASSESSING THE THEORY

The empirical evidence, presented above, on the relationship between inflation and either capacity utilization or cyclical GDP provides a basis for evaluating the Keynesian theory that connects high resource usage with inflationary conditions.

Analysis of the time profiles of the data establishes that utilization or cyclical GDP does not always move together with the inflation rate. There are several episodes during which inflation and utilization or cyclical GDP move in the opposite direction. For example, during 1973–74 and 1979, *falling* utilization rates and cyclical GDP coincided with *rising* inflation rates. The theory cannot account for episodes such as these. Thus, the theory’s assumption that the aggregate supply curve and associated capacity output level are constant over the business cycle is called into question. For if the supply curve were fixed, inflation and utilization or cyclical GDP would always move in the same direction.

The empirical regression analysis confirms, on average, the theory’s prediction of a positive relationship between utilization or cyclical GDP and inflation. A one percentage point increase in the utilization rate (cyclical GDP) is associated, on average, with a 0.15 (0.45) percentage point increase in the rate of inflation. But, the regressions also indicate that the asymmetries predicted by the

theory are not present in the data. That is, the utilization-inflation relationship is the same regardless of whether the utilization rate is high or low, and the stage of the cycle is immaterial to the cyclical GDP-inflation relationship. This casts doubt on the theory's joint assumption of a fixed and nonlinear supply curve. Alternatively expressed, there is no evidence of the existence of immutable threshold levels of utilization or cyclical GDP, levels at which underemployment of resources sets in and prices become relatively unresponsive to market conditions, as asserted by the theory.

The forecasting exercise lends support to the Keynesian theory by showing that both capacity utilization and cyclical GDP have substantial marginal predictive content for inflation. The alternative inclusion of utilization and cyclical GDP in an otherwise univariate forecasting equation for inflation reduces the mean squared one-period-ahead forecast error by 13 and 18 percentage points, respectively. Of the two measures of real economic activity, cyclical GDP exhibits the stronger predictive relationship with inflation, presumably because it is the broader measure of real economic activity. This finding, together with the high degree of correlation between utilization and cyclical GDP, suggests that there is nothing special about the capacity utilization rate relative to cyclical GDP in predicting price inflation. In other words, the empirical linkage between utilization and inflation evidently obtains only in so far as utilization is highly correlated with cyclical GDP.

#### **4. CONCLUSION**

Media discussions of inflation inextricably link high capacity utilization with inflationary conditions. The present study outlines the Keynesian theory underlying this viewpoint and then evaluates it in terms of its ability to explain the facts about the U.S. economy over the period 1953:1–1994:1. The outcome is summarized as follows.

First, capacity utilization has often moved in the opposite direction to that of inflation. Of particular note are the oil-price shock periods, 1973–1974 and 1979, when capacity utilization plummeted while inflation soared. The theory cannot explain such occurrences. Second, the relationship between utilization and inflation is, on average, a positive one as asserted by the theory; however, it is the same regardless of whether utilization is high or low, which conflicts with the theory. Third, the theory derives support from the facts that utilization helps predict future price inflation and that the broader measure of economic activity, cyclical GDP, works better than utilization as an inflation predictor.

The upshot of this evaluation is that problems for the Keynesian theory include not only misspecification of the channels through which shocks impact on the economy but also its ignoring of shocks to the supply side of the economy. More exactly, the evidence is not supportive of the theory's assumption of a nonlinear aggregate supply curve that remains constant over the business

cycle. An alternative theory that drops the assumption of a nonlinear relationship between inflation and real economic activity and that incorporates both demand and supply shocks is called for. In particular, by including supply shocks the alternative theory has the potential to explain why inflation and real economic activity sometimes move in opposite directions.

One such alternative theory is that of Greenwood and Huffman (1987) and Coleman (1994). This theory emphasizes technology shocks as the main source of cyclical output fluctuations. It also stresses the endogenous responsiveness of the money supply to those shocks. Other alternative theories are advanced by Lucas (1975), Cho and Cooley (1992) and Ireland (1994). These theories feature significant sources of monetary nonneutralities, thereby allowing not only technology shocks but also money supply shocks to play an important role in driving cyclical output fluctuations. All of these theories offer explanations of the empirical relationship between inflation and cyclical output. They all show a relationship that is positive, but only on average, since opposite movements sometimes occur. Further research will reveal which of these theories or alternative theories provides the best explanation.

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## DATA APPENDIX

The data are quarterly, seasonally adjusted (unless otherwise indicated) and for the United States over the period 1952:1–1994:1. The 1952 data are used only in forming lagged variables for the regression analysis. The source for all data is the FAME database. A description of the individual series follows.

Inflation Rate:	CPI inflation measured quarter to quarter at an annualized rate. The average value of the underlying CPI index is 100 over the period 1982–1984.
Utilization Rate:	Total industry (consisting of manufacturing, mining and utilities) utilization rate for the period 1967:1–1994:1. Manufacturing industry utilization rate for the period 1952:1–1966:4.
Industrial Production:	Total industry output index, 1987 = 100.
Gross Domestic Product:	Gross domestic product measured at constant 1987 prices and in billions of dollars.
Population:	Civilian, noninstitutionalized, aged 16 and above, measured in thousands. This series is not seasonally adjusted.

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# An Empirical Measure of the Real Rate of Interest

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Robert Darin and Robert L. Hetzel

The interest rate adjusted for expected inflation, the real rate of interest, is a key variable in macroeconomics. It is the price one pays for currently available resources expressed in terms of future resources. How does the real rate of interest behave? Despite the importance of this question, there is no generally available measure of the real rate of interest one can use to answer it. Economists who have studied the real rate have had to create their own series. The purpose of this article is to construct and make available a number of alternative empirical measures of the real rate of interest.

As noted above, the real rate of interest is the difference between the observed market rate of interest and the inflation rate expected by the public. Expected inflation, however, is not directly observable.<sup>1</sup> In order to construct a real rate series, one must select a proxy for expected inflation. We examine two possibilities—inflation forecasts made by Data Resources Incorporated (DRI) and by the staff of the Board of Governors of the Federal Reserve System. The DRI forecasts are made monthly. Through 1978, the Board staff produced monthly forecasts. Thereafter, it produced them eight times per year. These forecast series, therefore, allow construction of real rate series that are observed frequently enough to study cyclical timing relationships.

As an illustration of the usefulness of having a real rate series, we first review recent public debate over the typical level of the real rate. The main part of the article provides a defense of the plausibility of the real rate series constructed here and listed in the appendix. We compare forecasts of inflation from four different sources: the staff of the Board of Governors, DRI, the Michigan Survey of Consumers, and the Livingston Survey. We argue that the

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■ The views expressed are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

<sup>1</sup> Hetzel (1992) makes a proposal for indexed bonds that would render expected inflation directly observable. Expected inflation would be the difference in yields between nonindexed and indexed Treasury securities of the same maturity.

broad agreement exhibited among all these series is evidence that the series used here (Board of Governors staff and DRI) have been representative of the expectations of inflation affecting financial markets.

We then discuss other approaches to estimating the real rate. In this context, we examine the predictive ability of the four forecast series. We point out the persistent underprediction of inflation by survey forecasts in the 1970s. We argue that this underprediction does not reflect a basic defect in the survey data, but rather the special difficulty in predicting inflation during the final transition from a commodity to a fiat money standard.

## 1. CONTROVERSY OVER THE BEHAVIOR OF THE REAL RATE

Recently, the behavior of the real rate of interest has become an issue in debates over monetary policy. For example, during Humphrey-Hawkins testimony in July 1993, the chairman of the Federal Reserve System, Alan Greenspan (1993), drew attention to the unsustainably low value of the then current short-term real rate:

Currently, short-term real rates, most directly affected by the Federal Reserve, are not far from zero; long-term rates, set primarily by the market, are appreciably higher, judging from the steep slope of the yield curve and reasonable suppositions about inflation expectations. This configuration indicates that market participants anticipate that short-term real rates will have to rise as the head winds diminish, if substantial inflationary imbalances are to be avoided. (P. 853)

In spring 1994, after the Federal Reserve began to raise the funds rate, controversy arose over what constitutes typical behavior of the real rate of interest. This controversy is illustrated by the following excerpts from *The Wall Street Journal*.

[W]ith the economy now growing at a robust pace . . . the Fed has concluded that it is time to take the foot off the accelerator and put monetary policy into a “neutral” stance. . . . Robert Reischauer, director of the Congressional Budget Office, said neutral probably means inflation-adjusted rates of somewhere between  $\frac{3}{4}\%$  and  $1\frac{1}{2}\%$ . But chief White House economist Laura Tyson has said that—excluding the anomalous 1980s—inflation-adjusted interest rates “have always been below 1%.” (Wessel, 4/19/94, p. A2)

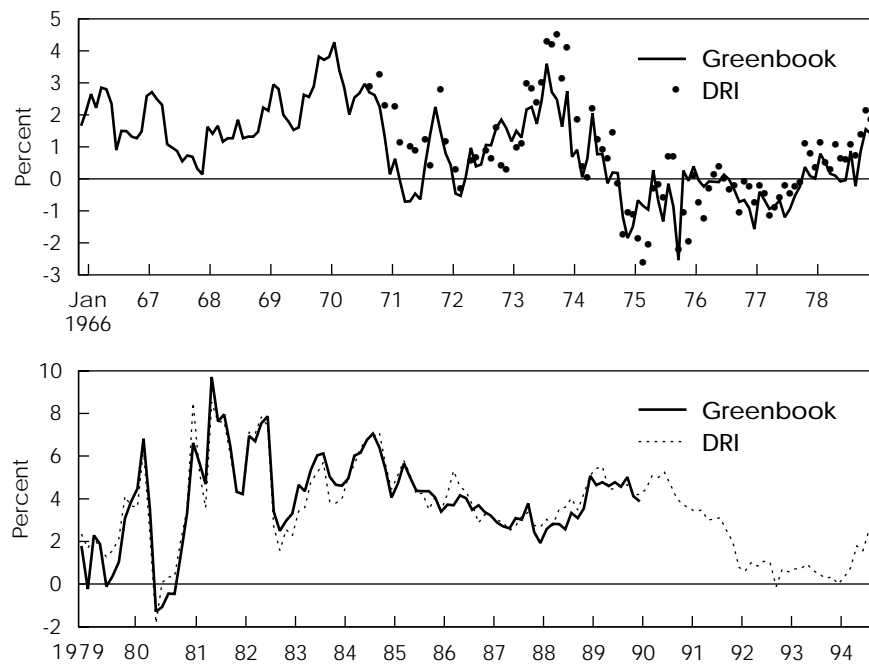
The federal funds rate . . . now stands at 3.5%. And inflation is running at roughly 3%. That means the “real” interest rate . . . is only .5%. That is well below historical experience, says Barry Bosworth of the Brookings Institution, who adds that the norm is “1.5% to 2% real short-term rates in the middle of an economic expansion.” (Murray, 4/11/94, p. A1)

In 1992 and 1993 real interest rates had been stuck around zero because of a weak world economy. Rates have since increased with global economic prospects, but the recent level of real rates, 1.9% to 2%, is not high by historical standards; it is just about the average since 1961. Real rates remain well below the average of 3% that prevailed during the period of high growth and robust investment from 1984 to 1989. (Barro, 8/19/94, p. A10)

## 2. REAL RATE SERIES

Figure 1 shows two real rate series for Treasury bills, one using inflation forecasts from the staff of the Board of Governors and one using forecasts from DRI. (The data appendix provides a detailed discussion of data sources and the

**Figure 1 Greenbook's and DRI's One- to Two-Quarter Real Treasury Bill Rates**



Notes: The Greenbook real rate is calculated for dates on which Greenbooks were published. It is the difference between the yield on those dates on a Treasury bill maturing at the end of the subsequent quarter and a weighted average of Greenbook inflation forecasts for the contemporaneous and subsequent quarter. Inflation is measured by the GNP (GDP from 1992 on) implicit price deflator. The DRI real rate is calculated using the same Treasury bill yield and DRI predictions of inflation from the DRI publication immediately preceding publication of the Greenbook. Observations in the top panel are monthly. Observations in the bottom panel correspond to FOMC meetings, which have been held eight times a year since 1981, and tick marks indicate the first FOMC meeting of the year.

construction of the real rate series.) The Board staff forecasts are contained in a document referred to as the Greenbook, which is prepared prior to Federal Open Market Committee (FOMC) meetings. Because Greenbooks remain confidential for five full calendar years after the year in which they are published, the Greenbook real rate series ends in 1989. The DRI forecasts are from the table “Quarterly Summary for the U.S. Economy—Control” in the DRI/McGraw-Hill monthly publication *Review of the U.S. Economy*. Observations in the top part of Figure 1 are monthly. In the bottom part of the graph, they correspond to FOMC meetings, which have occurred eight times a year since 1981.

We calculate the Greenbook real rate series for dates on which the Board staff issued Greenbooks. The real rate is the difference between the yield (recorded on the Greenbook issue date) on a Treasury bill that matures on the last working day of the subsequent quarter and a weighted average of the Greenbook inflation forecasts for the contemporaneous and subsequent quarter. Inflation is measured by the GNP (GDP from 1992 on) implicit price deflator. We calculate the DRI real rate series using the same Treasury bill yield and DRI predictions of inflation from the most recent monthly DRI *Review of the U.S. Economy* available as of the issue of the Greenbook. The Greenbook and DRI real rate series generally move together. Some discrepancies in the two series arise because the dates on which the Greenbook and DRI inflation forecasts are made can differ by as much as a month.

### 3. HOW SIMILAR ARE THE INFLATION FORECASTS?

We now examine the correspondence among four inflation forecasts: the Greenbook, the DRI *Review*, the Livingston Survey, and the Michigan Survey. Different groups make the four forecasts. The staff of the Board of Governors makes the Greenbook forecasts. The 19 members of the FOMC critically examine the Greenbook forecasts at their meetings. Professional forecasters trained as economists make the DRI forecasts and sell them to a variety of corporations and state governments. Economists working for banks, corporations, and in financial markets make the forecasts in the Livingston Survey. The Survey Research Center of the University of Michigan randomly selects respondents from the public for the inflation forecasts in its Survey of Consumers. A straightforward explanation for the similar behavior among these different measures of expected inflation is that they do in fact capture movements in the public’s expectation of inflation. This similarity suggests that the real rate series proposed here capture, at least broadly, the real rate as perceived by the public.

The Livingston Survey is available starting in June 1946. Joseph Livingston was a financial columnist from Philadelphia who surveyed business economists twice yearly on their expectations of CPI inflation. Among others, Carlson (1977), Caskey (1985), Hafer and Resler (1980), Jacobs and Jones (1980), and

Mullineaux (1978) examine the properties of this series. A series for the real rate of interest constructed from Livingston Survey data on expected inflation consists of only two observations per year.

The Survey Research Center of the University of Michigan has collected data on expected inflation quarterly since 1966 and monthly since 1978. (Before 1966, it asked respondents only whether they expected prices to go up or down.) Starting in 1978, the median, as well as the mean, of the individual respondents' forecasts from the Michigan Survey becomes available. The survey median has been lower than the survey mean in 95 percent of the observations. For all observations, the median prediction is lower than the mean prediction by an average of 1.0 percentage points. (This fact indicates that a small number of respondents regularly expected inflation to be unusually high relative to the group forecast.)

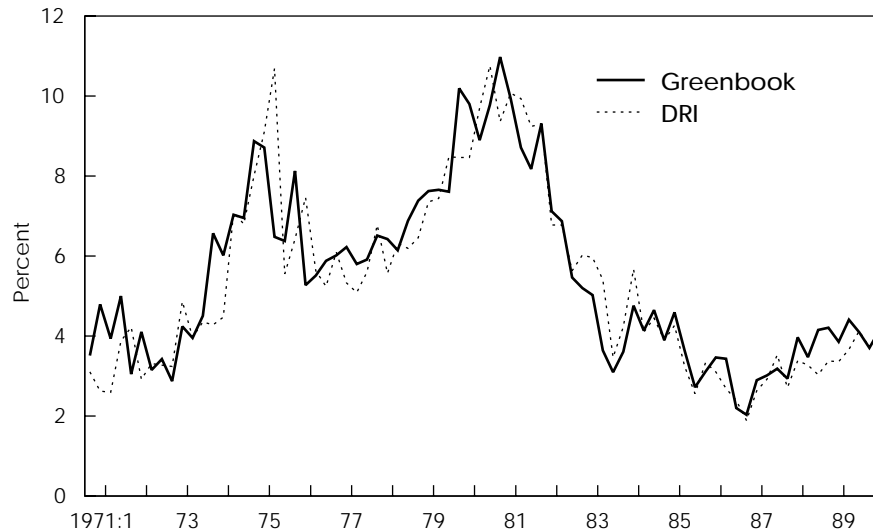
Figures 2 and 3, which compare Greenbook forecasts with DRI and Livingston forecasts, respectively, reveal a great deal of similarity between the paired forecasts. The standard deviation of the difference between the Greenbook and DRI forecasts from 1970Q3 to 1989Q4 is about 1 percent. The standard deviation of the difference between the Livingston and matching (May and November) Board staff forecasts from 1968 to 1989 is 0.60 percent.

Figure 4 plots Livingston Survey forecasts of four-quarter CPI inflation. It also plots the Michigan four-quarter mean forecasts of CPI inflation made in the same month and, beginning in 1978, the median forecasts as well. Although the Livingston and Michigan series move together, the Livingston series regularly lies below the Michigan series until 1980. From 1982 through the middle of 1988, the Livingston and Michigan mean forecasts are close. Thereafter, the mean of the Michigan forecast lies above the Livingston forecast. Over the period starting with the November survey of 1967 and ending with the May survey of 1994, the standard deviation of the difference in the Michigan (mean value) predictions and the Livingston predictions is 1.1 percent.

We maintain that the broad underlying similarity among the series examined above indicates that they capture movements in the public's expectations of inflation. Of course, as indicated by the discrepancies in inflation forecasts among the series, individual observations from a particular series are only rough estimates of the consensus view of expected inflation that shapes the behavior of market rates. Nevertheless, we believe the real rate series contained in Table 1, which are constructed from Greenbook and DRI inflation forecasts, do capture the general behavior of the short-term real rate of interest.

#### **4. COMPARING PREDICTED AND ACTUAL INFLATION**

Two characteristics of the various inflation forecasts examined above warrant close scrutiny. First, through the 1970s, the inflation forecasts generally fall short of subsequently realized inflation. These persistent forecast errors could

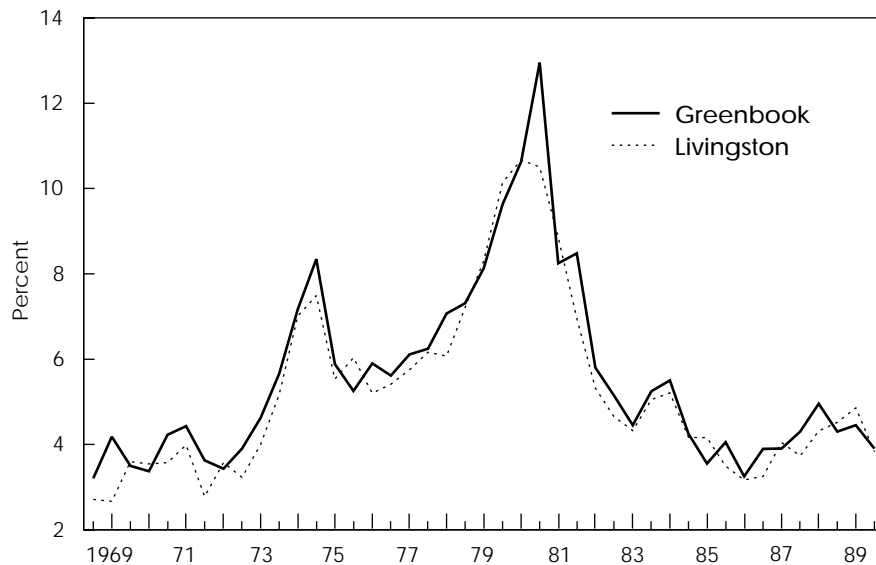
**Figure 2 Greenbook's and DRI's Inflation Predictions**

Notes: Observations of predicted inflation are from Greenbooks for February, May, August, and November FOMC meetings and are of the annualized quarterly percentage change in the GNP price deflator for the subsequent quarter. DRI predictions are from DRI publications with the same monthly date as the Greenbook.

indicate a defect in the survey data on expected inflation. Second, the forecasts do have some predictive power. That is, they perform more accurately than naive forecasts that simply employ past observations of inflation to predict future inflation. This latter characteristic, however, is not a necessary property of forecasts. The process generating inflation could be such that predicting inflation is simply very hard.

Figure 5 compares quarterly predictions of CPI inflation over future four-quarter periods from the Michigan Survey with the subsequently realized CPI inflation. It illustrates the persistent underprediction of inflation over much of the period shown. The Michigan Survey respondents underpredict inflation except during the early 1970s, the mid-1970s, the mid-1980s, and the early 1990s. They underpredict inflation whenever inflation rises. From 1973 through 1981, the average underprediction is 1.6 percentage points. (The standard deviation of the prediction errors is 2.3 percent.) This pattern of errors in predicting inflation is similar for the other sources—Greenbook, DRI, and Livingston. From 1966 to 1981, the Livingston Survey underestimates inflation by 1.8 percentage points on average. (The standard error of the forecast errors is 2.1 percent.)

In evaluating the Greenbook forecasts, we use forecasts of one-quarter-ahead (nominal output deflator) inflation made for FOMC meetings held in

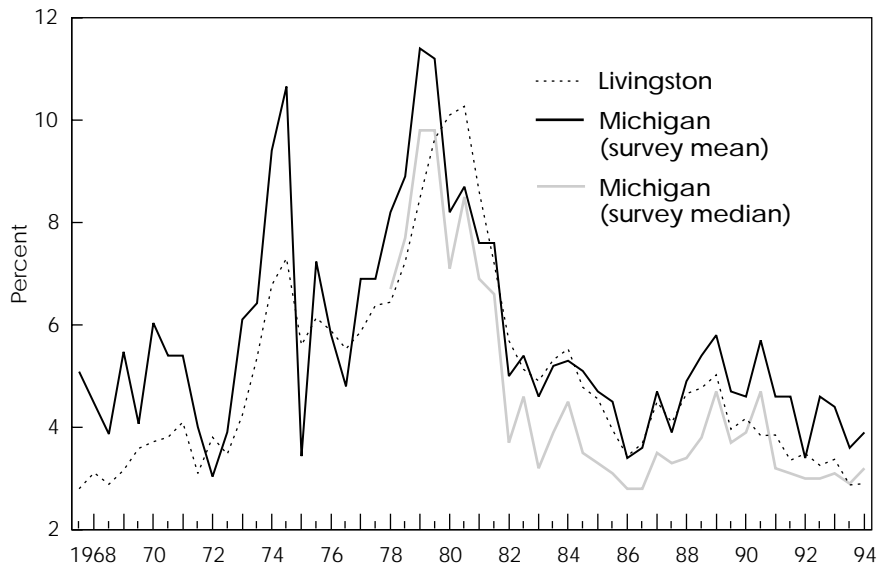
**Figure 3 Greenbook's and Livingston's Two-Quarter Inflation Predictions**

Notes: Greenbook predictions of inflation are for the GNP implicit price deflator before 1980 and for the CPI thereafter. Livingston predictions are for the CPI. The tall tick marks correspond to May Greenbook predictions of the annualized inflation rate for the last two quarters of the year. Tall tick marks also correspond to Livingston predictions of the annualized inflation rate for the eight-month period ending December and are from the June release. The short tick marks correspond to the December Greenbook predictions of the annualized inflation rate for the first two quarters of the following year. Short tick marks also correspond to Livingston predictions of the annualized inflation rate for the eight-month period ending in June and are from the December release.

February, May, August, and November (Figure 2). In general, from 1966 through 1981, subsequently realized inflation exceeds predicted inflation. During this period, the Greenbook underpredicts inflation by 1.1 percentage points on average. (The standard deviation of the one-quarter-ahead prediction errors is 1.6 percent.) In 1982, actual inflation falls below predicted inflation. The predictions are then fairly accurate from 1983 through 1989. For the corresponding DRI forecasts, from 1970Q3 through 1981Q4, the average underprediction is 1.2 percentage points and the standard deviation of the one-quarter-ahead prediction errors is 2.4 percent.<sup>2</sup>

One way to assess whether the forecasts shown in Figure 2 have predictive value is to compare them with naive forecasts made by simply extrapolating

<sup>2</sup> The actual inflation series changes over time as nominal and real output are rebenchmarked and as seasonal factors change. The original forecasts, therefore, were for a somewhat different inflation series than the one to which they are compared here.

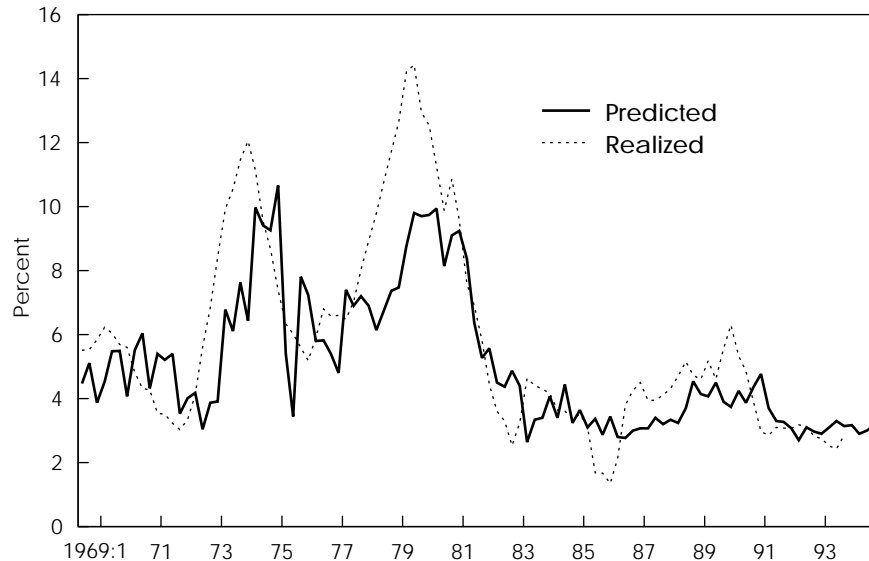
**Figure 4 Michigan's and Livingston's Inflation Predictions**

Notes: Observations of predicted CPI inflation are for the subsequent four-quarter period. The Livingston Survey was conducted in May and November. Its forecasts are matched with Michigan Survey of Consumers forecasts also made in the months of May and November. Michigan forecasts are the mean (black line) and the median (grey line) of respondents' forecasts. The median is available only starting in 1978. Tall tick marks indicate first observation of the year.

past inflation. Accordingly, we use the inflation rate from the quarter prior to the quarter in which the inflation forecast was made as a simple benchmark forecast. For the period 1966 through 1981, if the Greenbook's *forecast* of (GNP deflator) inflation for the subsequent quarter is replaced with the past quarter's *actual* inflation rate, the correlation with subsequently realized inflation is 0.63. For this period, the correlation between the Greenbook predictions of inflation and subsequently realized inflation is 0.79. This latter correlation represents an improvement of 25 percent over the naive prediction made using the prior quarter's actual inflation figure. For the period 1982Q1 to 1994Q2, the correlation between the prior quarter's inflation rate and the subsequently realized inflation rate is 0.42, while the correlation between the predicted and subsequently realized inflation rate is 0.62, an improvement of 48 percent.

In evaluating the DRI predictions, as with the Greenbook, we use forecasts of one-quarter-ahead (nominal output deflator) inflation dated as of February, May, August, and November. (The forecasts were made at the end of the preceding month.) For the period 1973Q1 through 1981Q4, the correlation between the naive prediction (using the actual inflation rate two quarters in the past) and



**Figure 5 Michigan's Inflation Predictions and Realized Inflation**

Notes: Observations of predicted inflation are from the Survey of Consumers conducted by the Survey Research Center of the University of Michigan. Before 1978, predicted inflation consists of quarterly observations of the mean inflation rate predicted by respondents. From 1978 on, observations are quarterly averages of monthly observations of the median inflation rate predicted by respondents. Observations of actual inflation are the subsequently realized annual percentage changes in the CPI (all urban consumers).

subsequently realized inflation is 0.49, while the correlation between predicted and subsequently realized inflation is 0.60, an improvement of 22 percent.<sup>3</sup>

## 5. IS THERE A BETTER WAY TO ESTIMATE THE REAL RATE?

Empirical work on the real rate of interest divides two groups. In one group, researchers use survey data to measure expected inflation. They regress the observed market rate of interest on a proxy for expected inflation derived from survey data (almost invariably the Livingston Survey) and on a collection of variables believed to be determinants of the real rate (government deficit, price of oil, etc.). Makin (1983) and Mehra (1985) represent examples of this methodology. Researchers in the other group assume that expected inflation

<sup>3</sup> For the same period, the Greenbook's average underprediction is 1.0 percentage points and the standard deviation of the one-quarter-ahead prediction errors is 1.9 percent. The correlation between predicted and subsequently realized inflation is 0.72.

equals subsequently realized inflation plus a white-noise error term. They use subsequently realized inflation over the relevant forecast period as a proxy for expected inflation (see Fama [1975]).

Researchers in the latter group use the ex-post real rate of interest (the market rate minus subsequently realized inflation) as a noisy measure of the ex-ante real rate. Using either a time-series or a structural model of the real rate, they then often fit a regression explaining this ex-post real rate. Then they use the fitted parameters of the model to generate a less noisy, smoother series for the real rate. For example, Antoncic (1986) generates estimates of the real rate by assuming the real rate is a random walk. (See also Garbade and Wachtel [1978] and Fama and Gibbons [1982].) Huizinga and Mishkin (1986) generate estimates of the real rate by assuming it can be represented as a linear combination of variables, that is, as a distributed lag of ex-post real rates, inflation rates, and the price of energy. (See also Bonser-Neal [1990].)

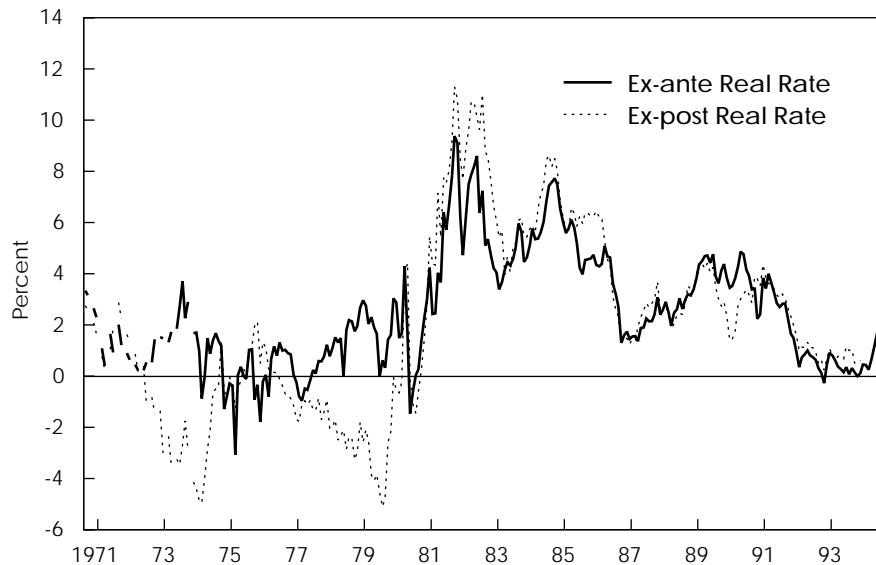
The approaches used by each group yield quite different measures of real rates over the earlier and latter parts of the 1970s.<sup>4</sup> Figure 6 plots a one-year real rate calculated as the difference between the one-year Treasury bill rate and predictions of four-quarter CPI inflation from DRI. It also plots the realized real rate for the corresponding four-quarter period, that is, the one-year bill rate minus the subsequently realized inflation rate. The increases in the rate of inflation that began in 1973, 1977, and, to a lesser extent, 1989 are associated with a realized real rate significantly less than the real rate calculated using inflation forecasts. Conversely, when inflation falls starting in 1981, the realized real rate lies well above the predicted real rate.

Researchers in the second group discussed above justify their use of realized inflation as an unbiased measure of expected inflation through the assumption of rational expectations. Specifically, they assume that participants in financial markets understand the nature of the monetary regime that generates inflation. The assumption of rational expectations, together with the assumption that individuals make efficient use of information, implies that forecast errors, apart from special cases, will not exhibit persistent bias. Because measures of expected inflation derived from survey data persistently underpredict inflation through the end of the 1970s, they fail to meet the requirements set by this second group.

A variant of the rational expectations approach is to assume that the public understands the time-series behavior of inflation. One can then use past observations of inflation to recreate the public's predictions of inflation. (For an interesting application, see Choi [1994].) Under the assumption that inflation is an autoregressive process, we regress inflation on its lagged values to generate

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<sup>4</sup> The issue of which approach generates better measures of the real rate will be settled only when a consensus develops over the validity of a structural model of the real rate. The predictions of that model can then be compared with the alternative empirical measures of the real rate.

**Figure 6 DRI's Ex-ante Real Rate and the Ex-post Real Rate**

Notes: The one-year real ex-ante rate is the Salomon Brothers one-year government bond yield read from a yield curve minus four-quarter predicted CPI inflation from DRI publications. The bond yield is for the last working day of the month. Observations are dated as of the subsequent month. If that month is the first or second month of a quarter, the quarter in which that month falls is the first quarter used in the four-quarter inflation forecast. If that month is the third month of a quarter, the subsequent quarter is the first quarter used in the four-quarter inflation forecast. No observation is plotted in cases in which the DRI forecast was unavailable. The ex-post one-year real rate is the bond yield minus the subsequent four-quarter CPI inflation rate. Tick marks indicate first observation of the year.

predictions of inflation. Equation (1) is a regression of contemporaneous (implicit GNP deflator) inflation on its three lagged values for the period 1966Q1 to 1979Q4.

$$\pi_t = 0.45\pi_{t-1} + 0.31\pi_{t-2} + 0.24\pi_{t-3} + \hat{u} \quad (1)$$

$$\bar{R}^2 = 0.32 \quad \text{SEE} = 1.96 \quad \text{DW} = 2.0 \quad \text{Degrees of Freedom} = 50$$

We employ regressions like (1) to generate inflation forecasts whose predictive accuracy can be compared to the Greenbook and DRI predictions displayed in Figure 2.

The forecasts of Figure 2 were made close to the middle of a quarter (February, May, August, and November) and were for the succeeding quarter. For example, the prediction of 1970Q4 inflation shown in Figure 2 was made by the Board staff in the August 12, 1970, Greenbook and by DRI at the end of July. At the time these predictions were made, the forecasters would have just received GNP data for the preceding quarter, 1970Q2. We therefore conduct the comparison as follows.

To begin, we regress inflation on its three lagged values over the period 1966Q1 to 1970Q2.<sup>5</sup> We then use this regression to forecast inflation for 1970Q3. Next, we substitute the resulting prediction for 1970Q3 and the realized inflation rates for 1970Q2 and 1970Q1 into the regression equation to obtain a prediction of inflation for 1970Q4. This predicted value is comparable to the Greenbook and DRI predictions of one-quarter-ahead inflation made in 1970Q3 for 1970Q4 and shown in Figure 2: all three predictions use data for the period predating 1970Q3. We repeat this procedure for each quarter through 1980Q4. That is, we run a series of rolling regressions, each of which starts in 1966Q1, with each successive regression containing one additional quarter.

The resulting comparison of forecast errors highlights the Board staff's and DRI's persistent underprediction of inflation through the 1970s (see Cullison [1988]). From 1970Q3 through 1980Q4, Greenbook and DRI forecasts underestimate inflation by 1.3 percent and 1.6 percent on average, respectively, while the time-series forecasts slightly overestimate inflation by  $-0.2$  percent. The time-series predictions, however, are not superior on all dimensions. The sum of the squared errors of the predictions from 1970Q3 to 1980Q4 is lower for the Greenbook than for the autoregressive predictions, 171 compared to 204 (267 for DRI). Also, the autocorrelation in the Greenbook and DRI forecast errors is negligible, while the autocorrelation in the autoregressive predictions is 0.4.

We conduct one final test in the spirit of the rational expectations literature to see whether Greenbook forecasts made efficient use of information. We calculate the correlation between the forecast errors of one-quarter-ahead inflation (derived again from the series shown in Figure 2) and the figure for the most recently available rate of growth of GNP as of the date of the forecast. (The latter figure is taken from the Greenbook.) It seems likely that when the rate of growth of GNP was high, the Board staff would underestimate inflation, and vice versa. In this event, the correlation between forecast errors and GNP growth would be positive. However, the correlation is in fact negligible ( $-0.03$ ). In this case, the Board staff was making efficient use of available information.

## 6. WHY THE PERSISTENT FORECAST ERRORS?

We maintain that the underprediction of inflation exhibited by survey data reflected the long period of time required for the public to realize that the process generating inflation under the prior commodity and Bretton Woods monetary standards had disappeared irrevocably. (See Caskey [1985] for a thorough

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<sup>5</sup> We choose 1966 as a starting date under the assumption that as of this date the FOMC no longer conducted monetary policy subject to constraints imposed by the Bretton Woods system. We choose the end date on the basis of when DRI predictions become available.

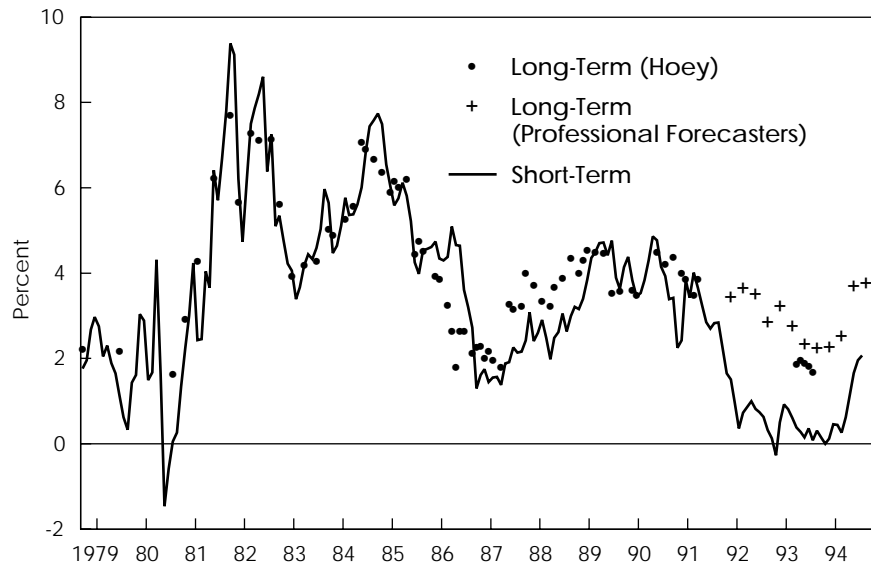
exposition of this view.) Before World War II, the quantity of money had been determined through fixing its value in terms of gold or silver. After World War II, the United States was part of the Bretton Woods system, which mimicked the international gold standard. Under the Bretton Woods system, the Federal Reserve maintained a dollar price of gold. In order to maintain the reserves necessary to peg the price of gold, the Fed had to respond to reserve outflows by raising rates, just as central banks had responded to gold outflows under the gold standard.

After the mid-1960s, the monetary regime changed to a pure fiat money regime. In 1968, Congress eliminated the gold cover on Federal Reserve notes. With the closing of the gold window in August 1971, the last vestigial, institutional relationship tying the value of the dollar to the value of a commodity disappeared. Under the new fiat money regime, there were no institutional arrangements to tie down the inflation rate. Moreover, monetary policy from the mid-1960s until the end of the 1970s was unique in the history of the United States through the emphasis placed on controlling growth of real output and unemployment. As a consequence, the character of the process generating inflation changed. The level of the inflation rate began to move randomly instead of reverting to a low average value.

Given that prior to World War II the United States had been on a commodity standard for all of its history apart from wars and that the Bretton Woods system replaced the gold standard after the War, it is no surprise that the public required some time in order to understand that “high and rising” inflation would not necessarily entail subsequent reductions in inflation. Furthermore, because of the particular historical circumstances surrounding the appearance of inflation, the public was slow to develop an understanding of the new, nonstationary character of inflation. Inflation surged first in conjunction with the Vietnam War. Inflation in wartime had been the historical norm, however. Inflation then surged after two oil-price shocks, one in 1973 and one in 1978. Given the association of inflation with these real shocks, the public required considerable time to realize that changes in the inflation rate were likely to be persistent rather than transitory.

## **7. WHAT IS THE NORMAL LEVEL OF THE REAL RATE?**

What is the average level of the real rate of interest? Before examining this question, we would like to know to what extent generalizations about the short-term rate of interest carry over to the long-term rate of interest. Figure 7 displays a ten-year real rate constructed using forecasts from two surveys of ten-year expected inflation. The initial forecasts are from a survey conducted by Richard Hoey. The first observation in this series is for September 1978. Hoey conducted

**Figure 7 Long-Term and Short-Term Real Rates**

Notes: The long-term real rate is the ten-year bond yield minus the predicted ten-year inflation rate from the "Decision Makers Poll" conducted in the 1980s by Richard B. Hoey (for Warburg, Paribus, Becker; Drexel, Burnham, Lambert; and Barclay's de Zoete Wedd). The Hoey Survey was discontinued in March 1991, but was reinstated by Cowens Investment Strategy for five months beginning in March 1993. Starting in October 1991, the Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia (formerly conducted by the American Statistical Association and the National Bureau of Economic Research) began to collect data in its quarterly survey on expected CPI inflation for a ten-year horizon. The long-term real rate is calculated using both series whenever possible. Observations of the long-term real rate are matched with monthly observations on the short-term real rate calculated as the difference between Salomon Brothers one-year government bond yields and DRI predictions of four-quarter CPI inflation. Tick marks indicate first observation of the year.

his survey only intermittently before 1981. He conducted it more frequently starting in 1983 and discontinued it in March 1991. Cowens Investment Strategy conducted the survey again for five months in 1993. Toward the end of 1991, in its quarterly Survey of Professional Forecasters, the Federal Reserve Bank of Philadelphia began to collect predictions of CPI inflation over future ten-year intervals. This latter series fills in most of the missing observations from the Hoey Survey, although the observations are less frequent.

Figure 7 also plots the DRI one-year real rate from Figure 6. For the period 1978 to 1991, short- and long-term rates are quite close.<sup>6</sup> From July 1980 through March 1991, the long-term real rate averages 4.25 percent, while

<sup>6</sup> For the period shown in Figure 7, the standard deviation of the one-year real rate (2.2) is slightly higher than that for the Hoey long-term real rate series (1.5).

the short-term real rate averages 4.3 percent. With the sharp fall in short-term market rates in 1991, however, long-term and short-term real rates diverge. From November 1991 through August 1994, the long-term real rate averages 3.0 percent, while the short-term real rate averages only 0.63 percent. This divergence suggests that statements about the behavior of the short-term real rate of interest do not necessarily carry over to the behavior of the long-term rate of interest.

From November 1965 through the end of 1993, the mean of the Treasury bill real rate calculated using Greenbook inflation forecasts (the series shown in Figure 1) is 2.3 percent.<sup>7</sup> As a check on this figure, we calculate a semiannual, one-year Treasury bill real rate using inflation forecasts from the Livingston Survey for the period June 1951 through June 1965. The mean of this series is 2.1 percent, which lies close to the first estimate.<sup>8</sup> Trehan (1995) calculates the realized real rate on one-year Treasury bonds as 1.8 percent over the period 1954 to 1993.

The real rate, however, is variable over time. From November 1965 to June 1974, the Greenbook Treasury bill real rate is 1.6 percent. It falls to  $-0.38$  from July 1974 to September 1978. It then begins to rise and is 1.1 percent from October 1978 to October 1979. From November 1979 to October 1990, the real rate averages 4.3 percent. It reaches its maximum value of 9.7 percent in May 1981. The DRI one-year Treasury bill real rate, whose monthly observations are shown in Figure 6, falls in the 1990s, to 2.1 percent over the interval November 1990 to June 1992 and then to 0.5 percent over the interval July 1992 to the end of 1993.

## 8. CONCLUDING COMMENTS

We have examined four sources of short-term inflation forecasts: the Greenbook issued by the staff of the Board of Governors before FOMC meetings, the DRI monthly publication *Review of the U.S. Economy*, the semiannual Livingston Survey, and the Survey of Consumers conducted by the Survey Research Center of the University of Michigan. The inflation forecasts in these series can diverge significantly for individual observations and moderately over extended periods.

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<sup>7</sup> The standard deviation of the real rate is somewhat lower than for the nominal rate. From November 1965 to July 1979, the standard deviation of the one- to two-quarter Greenbook real Treasury bill rate shown in Figure 1 is 1.3, while the standard deviation of the nominal bill rate is 1.6. The corresponding figures for the period August 1979 through July 1994 are 2.1 and 3.4.

<sup>8</sup> We use the one-year Treasury bill rate from Salomon Brothers' "Analytical Record of Yields and Yield Spreads." From 1951 through 1958, the bill rate is for mid-month. For this period, in matching the Livingston inflation forecasts, we use the Treasury bill rate from May and November. From 1959 through June 1965, the bill rate is for the first of the month. For this period, we use an average for May and June and also for November and December.

Nevertheless, they display the same broad patterns. We conclude that these series can be used to construct measures of the real rate of interest. The average short-term real rate on Treasury bills is about 2 percent. The real rate exhibits considerable variation, however, and at times has remained considerably above or below the 2 percent norm.

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## DATA APPENDIX

### Sources of Inflation Forecasts

1. The Greenbook, formally titled “Current Economic and Financial Conditions,” is prepared by the staff of the Board of Governors of the Federal Reserve System and is circulated prior to FOMC meetings. Part 1 of the Greenbook, “Summary and Outlook,” has made forecasts for nominal and real output and the implicit output deflator since November 1965. Since 1980, the Greenbook has also made predictions for CPI inflation. Greenbooks remain confidential for five full calendar years after the year in which they were published.

Initially, Greenbook forecasts were entirely judgmental. The Board staff first made a forecast using a large-scale econometric model in May 1969, although model forecasts did not influence the Greenbook forecasts until the early 1970s. Since the early 1970s, Greenbook forecasts have made use of a judgmental forecast and a model forecast. Senior staff decide how to weight these two kinds of forecasts in the combined forecast that appears in the Greenbook. Once a combined forecast for nominal and real GNP is arrived at, the equations in the staff’s econometric model are adjusted to produce the combined forecast. This adjusted model is then estimated to provide consistent forecasts of the various components of the National Income and Product Accounts.

2. The DRI/McGraw-Hill monthly publication *Review of the U.S. Economy* publishes quarterly forecasts of CPI and implicit GNP deflator inflation. Forecasts are taken from the table “Quarterly Summary for the U.S. Economy—Control.” We have issues of the DRI *Review* starting in March 1973. (We are indebted to John Caskey for these issues. We are indebted to Steve McNees and Delia Sawhney of the Boston Fed for the earlier observations.)

3. Begun in 1947 by Joseph Livingston, the Livingston Survey is currently conducted by the Federal Reserve Bank of Philadelphia. Twice annually (in June and December) the Philadelphia Fed asks about 50 business economists for their forecasts of the level of the CPI at six- and twelve-month horizons. The forecasts of inflation in the article follow Carlson (1977). Carlson notes that the December survey is mailed early in November when respondents have available the October CPI. The respondents forecast the level of the CPI for the following June. The forecast of inflation, therefore, is assumed to be the



annualized rate of growth of the CPI over the eight-month period from October to June. Similarly, the inflation forecast based on the forecasted December level of the CPI for the following year is assumed to be the annualized rate of growth of the CPI over the 14-month period ending in December of the following year.

4. The Survey of Consumers conducted by the Survey Research Center of the University of Michigan includes questions on expected price changes in the following 12 months. The survey consists of a random telephone sample of 500 or more individuals and asks the questions “During the next twelve months, do you think prices in general will go up, or go down, or stay where they are now?” and “By about what percent do you expect prices to go up, on the average, during the next twelve months?” The survey begins in 1946, but quantitative estimates of the predicted inflation rate are continuously available only since May 1968. Before 1978 the survey is quarterly; thereafter, it is monthly. The mean of the individual survey responses is available from 1966 to the present. The mean and median are available from 1978 to the present.

5. Richard B. Hoey in “Decision Makers Poll” conducted irregularly timed surveys of inflation expectations when he worked for Bache, Halsey, Stuart & Shields; Warburg, Paribus, & Becker; Drexel, Burnham, Lambert; and Barclays de Zoete Wedd Research, respectively. The first ten-year inflation forecast is from September 1978. The survey begins collecting shorter-term (approximately one-year) forecasts in October 1980. The number of respondents varies between 175 and 500 and includes chief investment officers, corporate financial officers, bond and stock portfolio managers, industry analysts, and economists. The survey dates are the dates on which the polls were mailed to Hoey. The survey was discontinued in March 1991, resumed in March 1993, and ended again definitively in August 1993.

6. The Survey of Professional Forecasters was first conducted by the American Statistical Association and National Bureau of Economic Research in 1968Q4. It is currently conducted quarterly by the Federal Reserve Bank of Philadelphia. In 1981Q3, the survey begins collecting forecasts of four-quarter rates of CPI inflation. In 1991Q4, it begins to collect forecasts of CPI inflation over the next ten years.

### **Constructing the Real Rate Series**

#### ***Greenbook Real Rate Series***

- a) Real rate series of one- to two-quarter maturity calculated as the difference between the Treasury bill rate and expected inflation measured by the implicit output deflator—Table 1, column (4)

This series is shown in Figure 1. It is calculated as the difference between the Treasury bill yield and predicted inflation from the Greenbook. Inflation is

for changes in the implicit GNP (GDP from 1992 on) deflator. A weighted-average inflation rate for the period from the Greenbook date to the end of the succeeding quarter is calculated from the Greenbook's inflation forecasts for the current and succeeding quarters. The weight given to the current quarter's inflation rate is the ratio of the number of days left in the current quarter to the number of days from the Greenbook date until the end of the succeeding quarter. The weight given to the succeeding quarter's inflation rate is the ratio of the number of days in that quarter to the number of days from the Greenbook date until the end of the succeeding quarter. This weighted-average expected inflation rate is subtracted from the Treasury bill yield. The Treasury bill yield is for the date the Greenbook appeared and is for the bill maturing on the last working day of the succeeding quarter. It is copied from the Federal Reserve Bank of New York's daily release "Composite Quotations for U.S. Government Securities." (For August 1972, the Treasury bill yield is for January 4, 1973, instead of December 31, 1972.)

In the 1960s, the FOMC usually met more than 12 times per year. For example, it met 15 times in 1965. In order to make the real rate series monthly through 1978, we record an observation for only the first FOMC meeting of the month for those months in which there was more than one meeting. The FOMC met only nine times in 1979. (Because the October 6, 1979, meeting was unscheduled, there was no Greenbook and no real rate is calculated for this date.) It met 11 times in 1980. Starting in 1981, it has met eight times a year. For this reason, starting in 1979, the observations of the Greenbook real rate series are less frequent than monthly.

The real rate series begins in November 1965 because the Greenbook first began to report predictions of inflation for the November 1965 meeting. Until November 1968, for FOMC meetings in the first two months of a quarter, the Greenbook often reported a forecast of inflation for only the contemporaneous quarter. For this reason, for the following FOMC meeting dates, the real rate calculated is for the period only to the end of the contemporaneous quarter, not to the end of the succeeding quarter: 11/23/65, 1/11/66, 2/8/66, 4/12/66, 5/10/66, 6/7/66, 7/26/66, 11/1/66, 12/13/66, 1/10/67, 7/18/67, 10/24/67, 11/14/67, 1/9/68, 2/6/68, 4/30/68, 5/28/68, 7/16/68, 10/8/68, 10/17/72, and 11/21/72. For these dates, the maturity of the Treasury bill used to calculate the real rate varies between one and three months. For other dates, the maturity varies between three and six months. For this reason, some of the variation in real rates reflects term-structure considerations. This variation is a consequence of the fact that the FOMC meets at different times within a quarter and the Greenbook inflation forecasts are for the quarters of the year.

- b) Real rate series of one- to two-quarter maturity calculated as the difference between the commercial paper rate and expected inflation measured by the implicit output deflator—Table 1, column (5)

This series is calculated like the one above except that the interest rate is the commercial paper rate for prime paper placed through dealers. Observations are matched with the publication dates of the Greenbook. From 1965 through 1969, rate data are from the New York Fed release “Commercial Paper.” Subsequently, they are from the Board’s FAME database. From 1965 through April 1971, the paper rate is for four- to six-month paper. Thereafter, if there are fewer than 135 days from the Greenbook date to the end of the subsequent quarter, the three-month paper rate is used; otherwise, the six-month paper rate is used.

### ***DRI Real Rate Series***

- a) Real rate series of one- to two-quarter maturity calculated as the difference between the Treasury bill rate and expected inflation measured by the implicit output deflator

This series is shown in Figure 1. It is calculated like the Greenbook series discussed above except for the substitution of predictions of (implicit GDP deflator, GNP before 1992) inflation from the most recent DRI *Review of the U.S. Economy* available as of the publication of the Greenbook. In order to keep the Greenbook and DRI real rate forecasts as closely comparable as possible, we keep the interest rate the same. Consequently, unlike the Greenbook forecasts, the matching between the date on which the interest rate is recorded and the date of the inflation forecast is not exact.

- b) Real rate series of one-year maturity calculated as the difference between the Treasury bill rate and expected inflation measured by the consumer price index—Table 1, column (3)

This series is the difference between the one-year Treasury bill rate and the four-quarter inflation rate predicted by DRI. The one-year Treasury bill rate is from Salomon Brothers “Analytical Record of Yields and Yield Spreads” and is read from a yield curve. The yield for each month is for the last business day for the preceding month. Because the DRI forecasts for a particular “control” month are made at the end of the preceding month, the date of the interest rate and forecast are fairly closely matched.

Four-quarter predicted inflation is a geometric average of the quarterly DRI predictions of CPI inflation. When the control date on the DRI forecasts is the first or second month of the quarter, the initial quarterly inflation forecast is the one reported for the contemporaneous quarter. For example, if the control date is January or February, then the initial quarter used in constructing the inflation forecast is the first quarter of the year. If the control date is the third month of the quarter, the initial quarter used in constructing the four-quarter forecast is the inflation forecast for the subsequent quarter. For example, if the control date is March, then the initial quarter of the four-quarter forecast is the second quarter.

- c) Real rate series of two-quarter maturity calculated as the difference between the Treasury bill rate and expected inflation measured by the consumer price index—Table 1, column (1)

The calculations for this series are like those for the preceding series with two changes. First, the interest rate is the six-month Treasury bill yield from Salomon Brothers. Second, the geometric average of the quarterly predictions of inflation is for two quarters.

- d) Real rate series of two-quarter maturity calculated as the difference between the commercial paper rate and expected inflation measured by the consumer price index—Table 1, column (2)

The calculations for this series are like those for the preceding series with two changes. First, the interest rate is the 180-day commercial paper rate for the last working day of the month preceding the control date on the DRI forecast of inflation. Second, the geometric average of the quarterly DRI predictions of CPI inflation is for two quarters.

#### *Hoey and Survey of Professional Forecasters Ten-Year Real Rate Series*

These series are shown in Figure 7. The ten-year market rate is the ten-year Treasury constant maturity yield taken from the Federal Reserve's Statistical Release G.13, "Selected Interest Rates."

#### **Real Rate Series**

Table 1 presents five series for the real rate of interest. The first three are constructed using CPI inflation predictions from DRI. The first two are for interest rates of two-quarter maturity and the third is for one-year maturity. The first and third use the Treasury bill real rate, while the second uses the six-month commercial paper rate.<sup>9</sup> The last two real rate series use inflation forecasts from the Greenbook. Depending upon when the Greenbook was published within the quarter, the maturity of the real rate varies from slightly more than three months to almost six months. One series uses the Treasury bill rate and the other the commercial paper rate.

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<sup>9</sup> In periods such as 1969–1970 and 1973–1974, when market rates were high relative to Regulation Q ceilings, disintermediation out of bank deposits apparently drove down the bill rate relative to the paper rate. Consequently, in these periods the two real rate series differ.

**Table 1 Real Rate of Interest**

Year	Month	(1)	(2)	(3)	(4)	(5)
		DRI			Greenbook	
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper
1965	11				1.66	2.27
	12				2.1	2.34
1966	1				2.65	2.82
	2				2.22	2.32
	3				2.85	3.38
	4				2.8	3.59
	5				2.35	3.07
	6				0.9	2.16
	7				1.5	2.36
	8				1.49	2.38
	9				1.32	2.1
	10				1.27	2.67
	11				1.48	3.1
	12				2.59	3.23
1967	1	(DRI data are not available until August 1970.)			2.71	3.84
	2				2.49	3.24
	3				2.31	3.25
	4				1.08	1.95
	5				0.98	1.9
	6				0.87	1.89
	7				0.55	1.48
	8				0.73	1.5
	9				0.69	1.45
	10				0.32	1.19
	11				0.14	1.18
	12				1.63	2.24
1968	1				1.41	2.08
	2				1.66	2.04
	3				1.16	1.73
	4				1.27	1.81
	5				1.27	2.1
	6				1.85	2.29
	7				1.27	2.28
	8				1.32	2.14
	9				1.32	1.98
	10				1.47	2.09
	11				2.23	2.49
	12				2.13	2.25
1969	1				2.95	3.1
	2				2.81	3.27
	3				2	2.96

**Table 1 Real Rate of Interest (Continued)**

Year	Month	(1)	(2)	(3)	(4)	(5)	
		DRI			Greenbook		
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper	
1969	4				1.79	2.71	
	5				1.53	2.85	
	6				1.61	3.43	
	7				2.63	4.41	
	8				2.56	4.12	
	9				2.89	4.19	
	10				3.82	5.23	
	11				3.72	4.96	
	12				3.81	4.79	
	1970	1				4.27	5.34
		2				3.39	4.47
		3				2.86	4.45
4					2.01	3.66	
5					2.54	3.83	
6					2.68	4.19	
7					2.97	4.53	
8		3.75	5	3.34	2.7	4.37	
9		3.7	4.5	3.17	2.62	3.7	
10					2.25	2.83	
11		3.24	3.625	2.66	1.3	2.18	
12		2.3	2.715	2.3	0.14	0.96	
1971	1				0.63	1.41	
	2	1.15	1.525	1.09	-0.04	0.52	
	3	-1.5	-0.84	0.41	-0.71	0.26	
	4				-0.7	0.22	
	5	1.1	1.8	1.65	-0.46	0.08	
	6	-0.39	0.21	0.93	-0.64	0.26	
	7				0.5	0.81	
	8	1.25	1.11	2.02	1.4	2.62	
	9	1.58	2.325	1.22	2.25	3.32	
	10				1.49	2.59	
	11	0.58	1.3	0.91	0.8	1.42	
	12	-0.42	-0.065	0.64	0.43	1.05	
1972	1				-0.46	0.03	
	2	0	0.125	0.45	-0.53	0.14	
	3	-1.14	-1.065	0.23	0.06	0.49	
	4				0.97	1.31	
	5	-0.45	-0.05	0.23	0.39	0.92	
	6	-0.69	-0.39	0.54	0.45	1.05	
	7				1.06	1.66	
	8	-0.17	0.2	0.54	1.05	1.63	

**Table 1 Real Rate of Interest (Continued)**

Year	Month	(1)	(2)	(3)	(4)	(5)
		DRI			Greenbook	
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper
1972	9	0.04	0.16	1.44	1.58	2.09
	10				1.85	2.57
	11	1.04	1.05	1.53	1.58	2.1
	12	0.5	0.46	1.48	1.17	1.44
1973	1				1.5	1.75
	2	0.61	0.825	1.33	1.29	1.84
	3	1	1.2	1.73	2.17	2.73
	4				2.25	2.79
	5	1.33	1.68	1.86	1.72	2.46
	6	2.42	2.7	2.73	2.44	3.16
	7	3.34	3.9	3.71	3.6	4.46
	8	1.1	2.18	2.27	2.71	4.01
	9	2.53	3.93	2.9	2.48	3.94
	10				1.63	3.33
	11	1.31	1.91	1.69	2.74	3.37
	12	0.91	1.93	1.71	0.69	2.43
1974	1	0.46	1.75	0.99	0.91	1.51
	2	-1.7	-0.95	-0.87	0.05	0.55
	3	-0.82	-0.56	-0.04	0.64	1.08
	4	0.57	1.21	1.49	2.05	2.87
	5	-0.08	1.46	0.88	0.77	3.32
	6	-0.06	1.63	1.4	0.78	3.36
	7	0.08	3.53	1.68	-0.14	4.05
	8	0.9	3.15	1.39	0.2	2.78
	9	0.17	2.13	1.17	0.18	2.92
	10	-2.82	-0.02	-1.28	-1.19	0.65
	11	-1.13	-0.37	-0.76	-1.85	-0.71
	12	0.24	1.5	-0.28	-1.5	0.13
1975	1	-1.16	0.7	-0.38	-0.67	0.2
	2	-4.17	-3.66	-3.08	-0.84	-0.2
	3	-1.4	-0.9	0.04	-0.96	-0.33
	4	-0.3	-0.07	0.36	0.27	0.39
	5	0.41	0.475	1.02	-0.66	-0.17
	6	-0.85	-0.95	-0.11	-1.33	-0.87
	7	0.67	0.73	1.03	-0.15	-0.18
	8	0.72	0.38	1.05	-0.88	-0.91
	9	-2.21	-2.65	-0.92	-2.54	-2.32
	10	-1.45	-1.82	-0.33	0.26	0.41
	11	-2.51	-2.35	-1.78	-0.06	0.24
	12	-0.63	-0.97	-0.2	0.39	0.65
1976	1	-0.5	-0.4	0.06	-0.05	0.05

**Table 1 Real Rate of Interest (Continued)**

Year	Month	(1)	(2)	(3)	(4)	(5)
		DRI			Greenbook	
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper
1976	2	-1.31	-1.27	-0.79	-0.24	-0.06
	3	0.45	0.05	0.62	-0.08	0.01
	4	0.95	0.65	1.16	-0.09	-0.09
	5	0.39	0.15	0.81	-0.01	-0.2
	6	0.58	0.33	1.32	0.13	0.42
	7	0.7	0.5	1.01	-0.02	0.06
	8	0.55	0.35	1.04	-0.34	-0.21
	9	0.64	0.48	0.9	-0.72	-0.55
	10	0.91	0.83	0.85	-0.66	-0.67
	11	-0.17	-0.27	0.06	-0.92	-0.91
	12	-0.19	-0.02	-0.24	-1.57	-1.26
	1977	1	-0.85	-0.87	-0.78	-0.4
2		-2.39	-2.71	-0.96	-0.67	-0.73
3		-1.15	-1.5	-0.47	-0.94	-0.96
4		-1.22	-1.27	-0.52	-0.85	-0.85
5		-0.85	-1.07	-0.16	-0.67	-0.74
6		0.17	0.38	0.24	-1.2	-0.97
7		-0.04	0.05	0.1	-0.94	-1.03
8		0.57	0.15	0.6	-0.52	-0.55
9		0.65	0.55	0.58	-0.25	-0.13
10		0.96	0.8	0.77	0.37	0.09
11		1.27	1.05	1.23	0.09	0.21
12		0.25	0.18	0.78	0.01	0.36
1978	1	0.5	0.49	1.08	0.79	0.47
	2	1.1	0.92	1.52	0.5	0.53
	3	0.87	0.64	1.32	0.16	0.41
	4	0.99	0.75	1.5	0.1	0.03
	5	0.56	0.34	1.35	-0.08	0.07
	6	1.5	1.28	1.8	-0.04	0.67
	7	1.33	1.36	2.2	0.87	1.11
	8	1.47	1.63	2.14	-0.23	0.62
	9	1.63	1.61	1.76	0.86	1.37
	10	2.02	1.94	1.96	1.55	1.73
	11	2.39	2.05	2.68	1.41	3
	12	2.42	2.92	2.97	1.38	2.45
1979	1	1.73	2.26	2.75	1.79	2.17
	2	1.38	1.55	2.05		
	3	1.69	1.64	2.3	-0.23	0.05
	4	1.35	1.06	1.89	2.28	2.2
	5	0.59	0.48	1.65	1.88	2.01
	6	0.33	0.25	1.15		



**Table 1 Real Rate of Interest (Continued)**

Year	Month	(1)	(2)	(3)	(4)	(5)
		DRI			Greenbook	
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper
1979	7	-0.23	-0.08	0.62	-0.12	0.12
	8	-0.38	-0.24	0.33	0.36	0.7
	9	0.92	1.46	1.43	1.04	2.11
	10	0.5	1.34	1.61		
	11	1.76	2.98	3.04	3.08	3.98
	12	2.42	2.54	2.89		
1980	1	1.58	1.5	1.49	3.86	3.67
	2	1.39	1.36	1.67	4.48	4.53
	3	2.77	2.44	4.31	6.81	7.21
	4	-1.21	-0.24	1.63	3.51	5.05
	5	-4.7	-3.91	-1.46	-1.29	-0.5
	6	-0.67	-0.51	-0.59		
	7	0.3	0.2	0.05	-1.08	-0.96
	8	0.11	-0.52	0.26	-0.45	0.69
	9	1.06	1.02	1.34	-0.46	0
	10	1.76	2.23	2.24	1.33	1.78
	11	2.13	1.89	3	3.3	3.6
	12	4.51	4.36	4.24	6.62	9.04
1981	1	2.37	2.47	2.43		
	2	2.35	2.74	2.45	5.67	5.36
	3	4.14	3.68	4.04	4.68	4.67
	4	3.78	3.73	3.66		
	5	6.6	6.06	6.41	9.71	9.39
	6	6.34	6.86	5.71		
	7	6.96	7.13	6.69	7.62	7.48
	8	7.47	7.55	7.8	7.95	7.97
	9	9.75	8.74	9.38		
	10	8.85	8.5	9.12	6.49	7.13
	11	5.58	5.84	6.21	4.33	4.51
	12	4.79	4.07	4.73	4.22	5.2
1982	1	6.28	6.13	6.16		
	2	7.53	7.29	7.52	6.95	6.97
	3	8.52	7.82	7.87	6.7	7.23
	4	9.07	8.85	8.2		
	5	9.8	9.67	8.6	7.54	7.49
	6	6.57	6.87	6.38		
	7	7.17	7.33	7.25	7.86	8.82
	8	4.02	4.75	5.1	3.4	3.95
	9	4.6	5.36	5.35		
	10	3.92	5.06	4.77	2.5	4.67
	11	4.7	4.72	4.22	2.97	3.45

**Table 1 Real Rate of Interest (Continued)**

Year	Month	(1)	(2)	(3)	(4)	(5)
		DRI			Greenbook	
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper
1982	12	3.65	3.34	4.06	3.31	3.9
1983	1	3.25	3.33	3.39		
	2	3.58	3.3	3.67	4.66	4.62
	3	4.53	4.15	4.16	4.36	4.33
	4	4.6	4.32	4.44		
	5	4.52	4.28	4.33	5.36	5.25
	6	4.43	3.94	4.59		
	7	4.68	4.39	5.03	6.04	6.07
	8	5.62	5.06	5.97	6.12	5.98
	9	5.15	4.7	5.65		
	10	3.91	3.63	4.47	5.03	4.84
	11	4.08	3.77	4.64	4.65	4.5
	12	4.51	4.02	5.11	4.61	4.91
1984	1	5.62	5.34	5.77		
	2	5.25	4.87	5.36	4.95	4.66
	3	4.94	4.5	5.38	6.02	5.98
	4	5.32	5.04	5.61		
	5	5.8	5.54	6.01	6.15	6.61
	6	5.9	5.88	6.84		
	7	6.3	6.5	7.44	6.75	7.17
	8	7.32	7.16	7.58	7.05	7.5
	9	7.39	7.14	7.74		
	10	7.14	6.76	7.49	6.41	6.54
	11	5.98	5.8	6.55	5.39	5.39
	12	5.59	5.24	6.03	4.06	4.3
1985	1	5.04	4.72	5.59		
	2	5.28	4.83	5.75	4.71	4.6
	3	5.86	5.71	6.12	5.66	5.89
	4	5.43	5.26	5.81		
	5	4.95	4.78	5.22	5	5.17
	6	4.15	4.12	4.25		
	7	3.81	4.04	3.98	4.37	4.48
	8	4.53	4.63	4.54	4.36	4.77
	9	4.28	4.24	4.57		
	10	4.37	4.6	4.61	4.36	5.02
	11	4.61	4.65	4.74	4.07	4.28
	12	3.98	4.09	4.34	3.4	3.92
1986	1	4.21	4.41	4.3		
	2	4.53	4.71	4.38	3.72	3.97
	3	5.8	5.88	5.09		
	4	5.24	5.69	4.66	3.7	4.26

**Table 1 Real Rate of Interest (Continued)**

Year	Month	(1)	(2)	(3)	(4)	(5)	
		DRI			Greenbook		
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper	
1986	5	5.66	5.72	4.64	4.16	4.4	
	6	2.75	2.73	3.61			
	7	2.29	2.55	3.2	4.02	4.31	
	8	2.8	2.95	2.72	3.49	3.71	
	9	0.55	0.61	1.3	3.7	4.17	
	10	1.04	1.18	1.61			
	11	1.24	1.31	1.75	3.38	3.63	
	12	1.42	1.51	1.45	3.21	3.49	
	1987	1	1.34	1.51	1.55		
		2	1.24	1.2	1.57	2.89	3
		3	0.96	1.32	1.38	2.71	3.24
		4	1.74	1.83	1.88		
5		1.38	2.17	1.91	2.62	3.6	
6		1.86	2.54	2.26			
7		1.52	2.34	2.14	3.09	3.95	
8		1.72	2.12	2.16	3.02	3.55	
9		2	2.41	2.42	3.78	4.81	
10		2.39	3.15	3.08			
11		1.86	3.04	2.41	2.43	3.94	
12		2.05	3.11	2.6	1.93	4.05	
1988	1	2.87	3.16	2.9			
	2	2.52	2.87	2.48	2.59	3.14	
	3	1.55	2.13	1.98	2.83	3.52	
	4	2.26	2.66	2.48			
	5	2.34	2.78	2.62	2.81	3.51	
	6	2.86	3.28	3.05	2.57	3.35	
	7	2.37	2.96	2.63			
	8	2.83	3.47	2.99	3.33	4.22	
	9	2.79	3.35	3.21	3.09	3.82	
	10	2.9	3.32	3.16			
	11	3.13	3.52	3.4	3.55	4.02	
	12	3.55	4.08	3.84	5.08	5.82	
1989	1	4.08	4.24	4.36			
	2	4.41	4.53	4.5	4.64	4.98	
	3	4.55	5.14	4.7	4.77	5.57	
	4	4.57	5.08	4.72			
	5	4	4.52	4.42	4.6	5.21	
	6	5.06	5.27	4.76			
	7	3.96	4.63	3.89	4.77	5.56	
	8	3.94	4.23	3.63	4.56	4.81	
	9	4.17	4.45	4.14			

**Table 1 Real Rate of Interest (Continued)**

Year	Month	(1)	(2)	(3)	(4)	(5)
		DRI			Greenbook	
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper
1989	10	4.31	4.65	4.38	5.02	5.71
	11	4.11	4.21	3.83	4.11	4.3
	12	3.59	3.72	3.45	3.87	4.48
1990	1	3.63	3.66	3.53		
	2	3.63	3.55	3.83		
	3	4.8	4.71	4.3		
	4	5.49	5.53	4.86		
	5	5.31	5.35	4.77		
	6	4.42	4.41	4.15		
	7	4.27	4.32	3.93		
	8	3.36	3.33	3.39		
	9	2.49	2.54	3.42		
	10	0.33	0.82	2.25		
	11	1.04	1.27	2.42		
	12	3.36	3.89	3.84		
1991	1	3.48	4.07	3.43		
	2	4.33	4.73	4.01		
	3	4.21	4.37	3.65		
	4	3.75	3.96	3.25		
	5	3.37	3.48	2.85		
	6	2.92	2.98	2.7		
	7	2.88	3.16	2.83		
	8	2.96	3.11	2.84		
	9	2.25	2.41	2.25		
	10	1.54	1.71	1.65		
	11	1.43	1.54	1.51		
	12	0.74	1.04	0.97		
1992	1	0.29	0.5	0.36		
	2	0.79	0.86	0.73		
	3	0.72	0.84	0.86		
	4	0.86	0.92	1		
	5	0.51	0.59	0.82		
	6	0.51	0.57	0.74		
	7	0.32	0.5	0.63		
	8	0.08	0.19	0.32		
	9	-0.01	0.11	0.13		
	10	-0.57	-0.26	-0.27		
	11	0.15	0.3	0.5		
	12	0.88	1.17	0.92		
1993	1	0.89	1.01	0.81		

**Table 1 Real Rate of Interest (Continued)**

Year	Month	(1)	(2)	(3)	(4)	(5)
		DRI			Greenbook	
		Two-Quarter T-bill	Two-Quarter Commercial Paper	One-Year T-bill	One- to Two- Quarter T-bill	One- to Two- Quarter Commercial Paper
1993	2	0.51	0.63	0.61		
	3	0.4	0.49	0.38		
	4	0.34	0.49	0.28		
	5	0.27	0.35	0.15		
	6	0.17	0.18	0.36		
	7	-0.24	-0.05	0.08		
	8	0.13	0.23	0.31		
	9	-0.35	-0.26	0.15		
	10	-0.68	-0.52	0		
	11	-0.36	-0.33	0.12		
	12	0.14	0.21	0.46		
	1994	1	0.14	0.18	0.44	
2		-0.06	-0.03	0.26		
3		0.2	0.39	0.62		
4		0.61	0.84	1.12		
5		0.99	1.15	1.66		
6		1.35	1.45	1.96		
7		1.12	1.32	2.07		
8			1.84			
9			1.96			
10			2.48			
11			2.63			
12			3.07			

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# Using the Permanent Income Hypothesis for Forecasting

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Peter N. Ireland

Personal consumption expenditures grew by almost 2 percent during 1993 in real, per-capita terms. Real disposable income per capita, meanwhile, actually fell slightly. By definition, households draw down their savings when consumption grows faster than income. In fact, the figures for consumption and income just mentioned underlie a decline in the personal savings rate from over 6 percent in the fourth quarter of 1992 to only about 4 percent in the fourth quarter of 1993.<sup>1</sup>

One popular interpretation of these data starts with the idea that reductions in the savings rate cannot be permanently sustained. Eventually, households must rebuild their savings by cutting back on consumption; to the extent that lower consumption leads to lower income, income must fall as well. Thus, in *U.S. News & World Report*, David Hage (1993–94) used the behavior of consumption, income, and savings to forecast that the economy would slow in 1994: “[A] slowdown in consumer spending is likely, and it could trim an additional 0.6 percentage point off growth” (p. 43). Around the same time, Gene Epstein (1993) of *Barron’s* quoted economist Philip Braverman as saying that “consumers don’t have the wherewithal to keep up the current spending pace. The prevailing euphoria will get knocked for a loop” (p. 37). Similarly, in DRI/McGraw-Hill’s *Review of the U.S. Economy*, professional forecaster Jill Thompson (1993) wrote: “All is not rosy, of course. Consumers went out on a limb to give the economy a needed jump-start. . . . They have pushed the saving rate very low and incurred more debt. . . . Consumption must slow” (pp. 16 and 18).

In light of this conventional wisdom, which suggests that a decline in savings presages a slowdown in economic growth, the continued strength of

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<sup>1</sup> Appendix B describes all data and their sources.

the U.S. economy in 1994 came as a surprise, raising the question of whether an alternative framework can better reconcile the recent behavior of consumption, income, and savings. This article considers one such alternative: Milton Friedman's (1957) permanent income hypothesis. This hypothesis implies that households save less when they expect future income to rise. Thus, according to the permanent income hypothesis, a decline in savings like that experienced during 1993 signals that faster, not slower, income growth lies ahead.

Although developed in detail by Friedman in his 1957 monograph, the permanent income hypothesis has its origins in Irving Fisher's (1907) theory of interest. Thus, the article begins by reviewing Fisher's graphical analysis and by indicating how this analysis extends to a full statement of the permanent income hypothesis. The article goes on to show how Robert Hall (1978) derives the permanent income hypothesis from a mathematical theory that has very specific implications for the joint behavior of consumption, income, and savings. Following John Campbell (1987), it draws on Hall's version of the hypothesis to formulate a simple econometric model that exploits data on savings to forecast future income growth. Estimates from this model show that the U.S. data conform not to conventional wisdom, but to the intuition provided by the permanent income hypothesis: historically, declines in savings have preceded periods of faster, not slower, income growth. Finally, the article uses the model to generate forecasts for the U.S. economy.

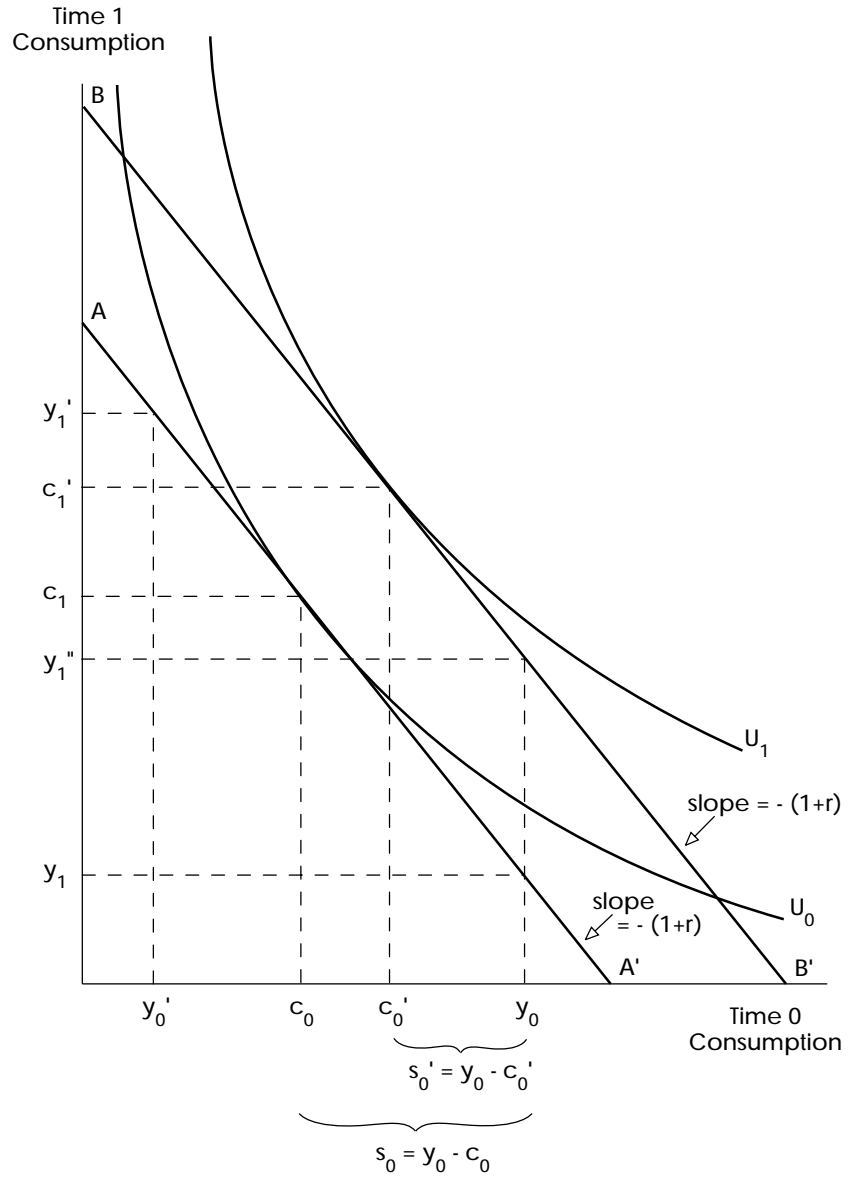
## 1. FISHER'S THEORY OF INTEREST AND THE PERMANENT INCOME HYPOTHESIS

In presenting his theory of interest, Irving Fisher (1907) uses a graph like that shown in Figure 1 to illustrate how a household makes its consumption and savings decisions. To simplify his graphical analysis, Fisher considers only two periods. His horizontal axis measures goods at time 0, and his vertical axis measures goods at time 1. Fisher's representative household receives income  $y_0$  during time 0 and  $y_1$  during time 1.

The representative household faces the fixed interest rate  $r$ , which serves as an intertemporal price. It measures the rate at which the market allows the household to exchange goods at time 1 for goods at time 0. In particular, if the household lends one unit of the good at time 0, it gets repaid  $(1 + r)$  units of the good at time 1. Similarly, if the household borrows one unit of the good at time 0, it must repay  $(1 + r)$  units of the good at time 1. Thus, in Figure 1, the household's budget constraint  $AA'$ , which passes through the income point  $(y_0, y_1)$ , has slope  $-(1 + r)$ .

The household's preferences over consumption at the two dates are represented by the indifference curves  $U_0$  and  $U_1$ , each of which traces out a set of consumption pairs that yield a given level of utility. Utility increases with

**Figure 1 Fisher's Diagram**



consumption in both periods; hence,  $U_1 > U_0$ . The slope of each indifference curve is determined by the household's marginal rate of intertemporal substitution, the ratio of its marginal utilities of consumption at dates 0 and 1, or the rate at which it is willing to exchange goods at time 1 for goods at time 0.

To maximize its utility, the household chooses the consumption pair  $(c_0, c_1)$ , where the indifference curve  $U_0$  is tangent to the budget constraint  $AA'$ . At  $(c_0, c_1)$ , the household's marginal rate of intertemporal substitution equals the gross rate of interest  $(1 + r)$ . The household saves amount  $s_0 = y_0 - c_0$ .

Now suppose that the household's income pair changes to  $(y'_0, y'_1)$ . Since this new income point lies on the same budget constraint as  $(y_0, y_1)$ , the household continues to select  $(c_0, c_1)$  as its optimal consumption combination. In fact, the household chooses  $(c_0, c_1)$  starting from any income point along  $AA'$ . Since all income points along  $AA'$  have the same present value, equal to

$$PV = y_0 + \frac{y_1}{(1 + r)}, \quad (1)$$

this example illustrates the first implication of Fisher's theory: the household's consumption choice depends only on the present value of its income pair  $(y_0, y_1)$ , not on  $y_0$  and  $y_1$  separately.

Next, hold  $y_0$  constant and suppose that the household's income at time 1 increases to  $y'_1$ . This change increases the present value of the household's income pair. It shifts the budget constraint out from  $AA'$  to  $BB'$  and leads the household to choose the preferred consumption pair  $(c'_0, c'_1)$ . Since  $c'_0 > c_0$ , the increase in time 1 income allows the household to reduce its time 0 savings from  $s_0 = y_0 - c_0$  to  $s'_0 = y_0 - c'_0$ . This example illustrates the second implication of Fisher's theory: the household saves less when it expects future income to be high. Conversely, the household saves more when it expects future income to be low.

This second implication makes Fisher's model useful for forecasting. It suggests, in particular, that data on household savings help forecast future income. A low level of savings today indicates that households expect higher income in the future. A high level of savings today signals that households expect lower income in the future. Note that both of these predictions contradict the conventional wisdom, which indicates that low savings predate lower income.

Milton Friedman's (1957) permanent income hypothesis generalizes Fisher's analysis to a model in which there are more than two periods and the representative household may be uncertain about its future income prospects. Thus, Friedman also derives the result that a representative household's consumption depends not on its current income but on the present value of its future income. With an infinite number of periods and uncertainty, this present value can be written

$$PV = \sum_{t=0}^{\infty} \frac{Ey_t}{(1 + r)^t}, \quad (2)$$

where  $Ey_t$  denotes the household's expected income at time  $t$ .

Friedman defines the household's permanent income  $y^p$  as the constant income level that, if received with certainty in each period  $t$ , has the same present value as the household's actual income path. That is,  $y^p$  satisfies

$$\sum_{t=0}^{\infty} \frac{y^p}{(1+r)^t} = PV = \sum_{t=0}^{\infty} \frac{Ey_t}{(1+r)^t}. \quad (3)$$

In light of the formula

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r}, \quad (4)$$

equation (3) simplifies to

$$y^p = \frac{r}{1+r} PV. \quad (5)$$

Thus, the first implication of the permanent income hypothesis is that the household's consumption at date 0 can be written as a function of its permanent income:

$$c_0 = f(y^p). \quad (6)$$

Similarly, Friedman generalizes the second implication of Fisher's analysis. Friedman's representative household borrows to increase consumption today when it anticipates higher income in the future. In other words, it saves less when it expects future income to be high. Conversely, the household uses additional savings to buffer its consumption against expected declines in income; it saves more when it expects future income to be low. Thus, like Fisher's theory of interest, Friedman's permanent income hypothesis suggests that data on savings help forecast future income.

## 2. HALL'S VERSION OF THE PERMANENT INCOME HYPOTHESIS

Robert Hall (1978) develops a mathematical version of the permanent income hypothesis that makes the relationship between savings and expected future income identified by Fisher and Friedman more precise. In fact, the details of Hall's model indicate exactly how data on savings can be used to forecast future changes in income.

Hall, like Friedman, assumes that there are many time periods  $t = 0, 1, 2, \dots$  and that the representative household is uncertain about its future income prospects.<sup>2</sup> Hall's infinitely lived representative household has expected utility

$$E \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (7)$$

where  $E$  once again denotes the household's expectation,  $u(c_t)$  measures its utility from consuming amount  $c_t$  at time  $t$ , and the discount factor  $\beta$  lies between zero and one.

The household begins period  $t$  with assets of value  $A_t$ . It earns interest on these assets at the constant rate  $r$ ; its capital income during period  $t$  is therefore  $y_{kt} = rA_t$ . The household also receives labor income  $y_{lt}$  during period  $t$ .

At the end of period  $t$ , the household divides its total income  $y_t = y_{kt} + y_{lt}$  between consumption  $c_t$  and savings  $s_t = y_t - c_t$ . It then carries assets of value

$$A_{t+1} = A_t + s_t = (1 + r)A_t + y_{lt} - c_t \quad (8)$$

into period  $t + 1$ .

The household is allowed to borrow against its future labor income at the interest rate  $r$ ; because of borrowing and the associated accumulation of debt, its assets  $A_t$  may become negative. Its borrowing is constrained in the long run, however, by the requirement that

$$\lim_{t \rightarrow \infty} \frac{A_t}{(1 + r)^t} = 0. \quad (9)$$

To see how equation (9) limits the household's borrowing, note that equation (8) is a difference equation in the variable  $A_t$ . Using equation (9) as a terminal condition, one can solve equation (8) forward to obtain

$$A_t = \sum_{j=0}^{\infty} \frac{c_{t+j} - y_{lt+j}}{(1 + r)^{j+1}}. \quad (10)$$

Equation (10) shows that the household must repay any debt owed today ( $-A_t$ ) by setting future consumption  $c_{t+j}$  below future labor income  $y_{lt+j}$ . Equation (10) also implies that

$$A_t = \sum_{j=0}^{\infty} \frac{E_t c_{t+j}}{(1 + r)^{j+1}} - \sum_{j=0}^{\infty} \frac{E_t y_{lt+j}}{(1 + r)^{j+1}}, \quad (11)$$

where  $E_t$  denotes the representative household's expectation at time  $t$ . This condition states that the household's current level of assets  $A_t$  must be sufficient to cover any discrepancy between the present value of expected future consumption and the present value of expected future labor income.

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<sup>2</sup> This presentation of Hall's model follows Sargent (1987, Ch. 12) quite closely.

The representative household chooses consumption  $c_t$  and asset holdings  $A_{t+1}$  for all  $t = 0, 1, 2, \dots$  to maximize the utility function (7) subject to the constraints (8) and (9). The solution to this problem dictates that

$$u'(c_t) = \beta(1+r)E_t u'(c_{t+1}). \quad (12)$$

Equation (12) simply generalizes the optimality condition shown in Figure 1 as the tangency between the household's indifference curve and its budget constraint. It indicates that the household sets its expected marginal rate of intertemporal substitution, the ratio of its marginal utility of consumption at time  $t$  to its expected marginal utility of consumption at time  $t+1$ ,  $u'(c_t)/\beta E_t u'(c_{t+1})$ , equal to the gross rate of interest  $(1+r)$ .

Assume that the interest rate  $r$  is related to the household's discount factor  $\beta$  via  $\beta = 1/(1+r)$ . Assume also that the household's utility is quadratic, with  $u(c) = u_0 + u_1 c - (u_2/2)c^2$  for some positive constants  $u_0$ ,  $u_1$ , and  $u_2$ .<sup>3</sup> Under these additional assumptions, equation (12) reduces to

$$c_t = E_t c_{t+1}. \quad (13)$$

Equation (13) states Hall's famous result that under the permanent income hypothesis, consumption follows a random walk.

Equation (13) implies that  $E_t c_{t+j} = c_t$  for all  $j = 0, 1, 2, \dots$ . Substituting this result into equation (11) yields

$$c_t = rA_t + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j}. \quad (14)$$

The right-hand side of equation (14), equal to current capital income plus  $r/(1+r)$  times the present value of expected future labor income, defines the representative household's permanent income in Hall's model. Equation (14), like equation (6), states the first main implication of the permanent income hypothesis: consumption is determined by permanent income.

Using  $y_{kt} = rA_t$  and  $s_t = y_{kt} + y_{lt} - c_t$  and denoting the change in labor income by  $\Delta y_{lt} = y_{lt} - y_{l,t-1}$ , one can rearrange equation (14) to reveal the second main implication of the permanent income hypothesis:

$$s_t = - \sum_{j=1}^{\infty} \frac{E_t \Delta y_{l,t+j}}{(1+r)^j}. \quad (15)$$

According to equation (15), the household's current savings  $s_t$  equals the present value of expected future declines in its labor income. Thus, equation (15) states that the household saves less when it expects future gains in income, that is,

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<sup>3</sup> These assumptions, while restrictive, greatly simplify the analysis. Hansen and Singleton (1982) derive and test the implications of Hall's (1978) model under more general assumptions about  $r$  and  $u(c)$ .

positive values for  $\Delta y_{t+j}$ . Conversely, the household saves more when it anticipates future declines in income, that is, negative values for  $\Delta y_{t+j}$ . Once again, this second implication of the permanent income hypothesis suggests that data on savings help forecast future changes in income.

### 3. THE PERMANENT INCOME FORECASTING MODEL

John Campbell (1987) shows exactly how Hall's version of the permanent income hypothesis can be used to formulate an econometric forecasting model for the U.S. economy. Since the permanent income hypothesis implies that data on savings will help forecast future changes in labor income, Campbell starts with a bivariate vector autoregression (VAR) for  $\Delta y_{it}$  and  $s_t$  of the form

$$\begin{bmatrix} \Delta y_{it} \\ s_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta y_{it-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}, \quad (16)$$

where, for example,  $a(L) = a_1 + a_2L + a_3L^2 + \dots + a_pL^{p-1}$ ,  $L$  is the lag operator, and  $u_{1t}$  and  $u_{2t}$  are serially uncorrelated errors.<sup>4</sup> Campbell then shows how the relationship (15) between savings and future labor income identified by Hall's model translates into a set of parameter restrictions on the VAR (16).

Campbell works through the series of linear algebraic manipulations outlined in Appendix A. First, he uses the VAR (16) to compute the expected future declines in labor income  $-E_t\Delta y_{t+j}$  that appear on the right-hand side of equation (15). Next, he demonstrates that these expected future declines depend on the coefficients of the lag polynomials  $a(L)$ ,  $b(L)$ ,  $c(L)$ , and  $d(L)$ . In particular, if the present value of the expected future declines in income are to equal the current value of savings  $s_t$ , as required by equation (15), the following parameter restrictions must hold:

$$a_1 = c_1, \dots, a_p = c_p, d_1 - b_1 = (1+r), b_2 = d_2, \dots, b_p = d_p. \quad (17)$$

Equation (17) gives the restrictions imposed by Hall's version of the permanent income hypothesis on the VAR (16).

### 4. PERFORMANCE OF THE PERMANENT INCOME FORECASTING MODEL

Quarterly data, 1959:1–1994:3, are used to estimate the VAR in equation (16) both with and without the permanent income restrictions (17). The specification (16) assumes that  $\Delta y_{it}$  and  $s_t$  have zero mean; in practice, adding constant terms to the VAR removes each variable's sample mean. The estimated models include six lags of each variable on the right-hand side.

<sup>4</sup> See Sargent (1987, Ch. 9) for details about the lag operator.



Panel (a) of Table 1 shows the unconstrained equation for labor income growth. The negative sum of the coefficients on lagged savings indicates that a decrease in savings translates into a forecast of faster income growth, exactly as implied by the permanent income hypothesis. Moreover, an F-test easily rejects the hypothesis that the savings data do not help to forecast future income growth; again as predicted by the permanent income hypothesis, the coefficients on the lags of  $s_t$  are jointly significant at the 0.00037 level.

Panel (b) of Table 1 displays the equation for labor income when the permanent income constraints (17) are imposed on the VAR. The estimates assume that  $r = 0.01$ , which corresponds to an annual real interest rate of 4 percent. The coefficients of the constrained equation closely resemble those of the unconstrained equation, indicating once again that the data are consistent with the permanent income hypothesis.

**Table 1 Estimated Labor Income Equation from the Permanent Income Forecasting Model**

(a) Unconstrained Model				
$\Delta y_{lt} = 88.9$	$+ 0.107\Delta y_{lt-1}$	$- 0.0286\Delta y_{lt-2}$	$+ 0.236\Delta y_{lt-3}$	$- 0.111\Delta y_{lt-4}$
	(0.118)	(0.121)	(0.118)	(0.121)
	$+ 0.0261\Delta y_{lt-5}$	$- 0.0636\Delta y_{lt-6}$	$- 0.379s_{t-1}$	$+ 0.218s_{t-2}$
	(0.120)	(0.0876)	(0.102)	(0.143)
	$- 0.140s_{t-3}$	$+ 0.172s_{t-4}$	$- 0.109s_{t-5}$	$+ 0.170s_{t-6}$
	(0.144)	(0.142)	(0.145)	(0.110)
(b) Constrained Model				
$\Delta y_{lt} = 106$	$+ 0.0448\Delta y_{lt-1}$	$- 0.0734\Delta y_{lt-2}$	$+ 0.168\Delta y_{lt-3}$	$- 0.0417\Delta y_{lt-4}$
	(0.109)	(0.111)	(0.109)	(0.111)
	$+ 0.0115\Delta y_{lt-5}$	$- 0.0742\Delta y_{lt-6}$	$- 0.375s_{t-1}$	$+ 0.235s_{t-2}$
	(0.111)	(0.0806)	(0.0942)	(0.131)
	$- 0.109s_{t-3}$	$+ 0.0899s_{t-4}$	$- 0.0840s_{t-5}$	$+ 0.159s_{t-6}$
	(0.132)	(0.131)	(0.133)	(0.101)

Note: Standard errors are in parentheses.

A statistical test rejects the constraints (17) at the 99 percent confidence level. As noted by King (1995), however, formal hypothesis tests seldom fail to reject the implications of detailed mathematical models such as Hall's.<sup>5</sup>

<sup>5</sup> Moreover, as indicated in footnote 3, Hall's model makes very restrictive assumptions about the interest rate and the household's utility function. The statistical rejection of the constraints (17) may therefore reflect the failure of one of these additional assumptions to hold in the data,

Ultimately, the permanent income hypothesis must be judged on its ability to forecast the data better than alternative models.

Thus, Table 2 reports on the forecasting performance of the permanent income model. First, the constrained VAR is estimated with data from 1959:1 through 1970:4 and is used to generate out-of-sample forecasts for the total change in labor income one, two, four, and eight quarters ahead. Next, the sample period is extended by one quarter, and additional out-of-sample forecasts are obtained. Continuing in this manner yields out-of-sample forecasts for 1971:1 through 1994:3.

The table computes the permanent income model's mean squared error at each forecast horizon. It expresses each mean squared error as a fraction of the mean squared error from a univariate model for labor income growth with six lags. Thus, figures less than unity in Table 2 indicate that the VAR's mean squared forecast error is smaller than the univariate model's.

The table shows that the permanent income forecasts improve on the univariate forecasts at all horizons. The gain in forecast accuracy exceeds 10 percent at horizons longer than one quarter. The permanent income model is especially valuable for forecasting at the annual horizon, where it reduces the univariate forecast errors by 25 percent.

**Table 2 Performance of the Permanent Income Forecasting Model**

<b>Horizon (Quarters Ahead)</b>	<b>Improvement Over Univariate Model</b>	<b>Improvement Over Unconstrained VAR</b>
1	0.95	1.00
2	0.87	0.97
4	0.75	0.92
8	0.89	0.78

Note: Performance is measured by the mean squared forecast error from the permanent income model expressed as a fraction of the mean squared forecast error from two alternative models: a univariate model for labor income and an unconstrained vector autoregression for savings and labor income.

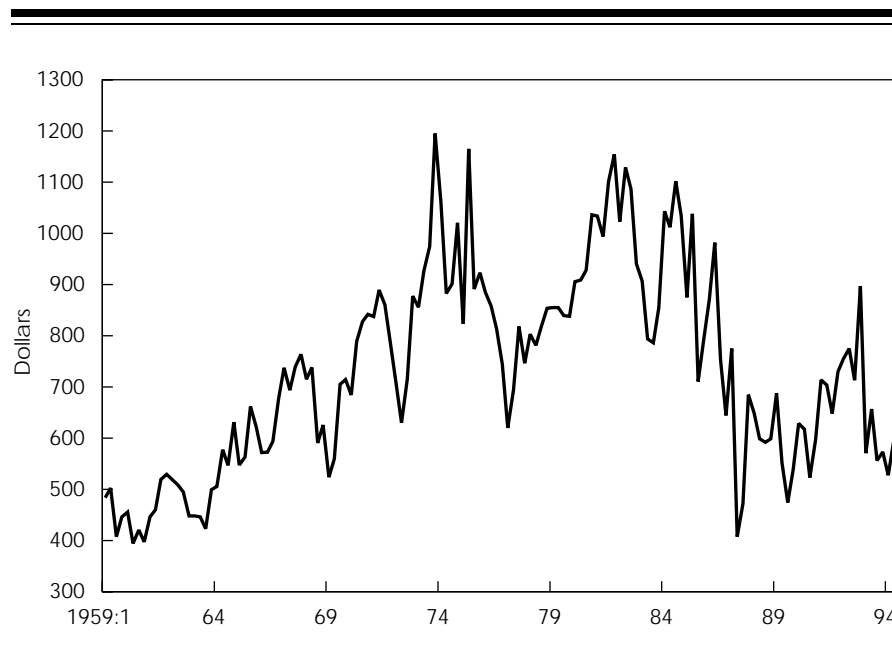
Table 2 also compares the forecasting performance of the constrained VAR to the performance of the VAR when the permanent income constraints (17) are not imposed. Once again, the figures less than unity indicate that the permanent income forecasts have lower mean squared error than the unconstrained forecasts. The improvement is most dramatic at longer horizons. Thus, Table 2 shows that the permanent income constraints help improve the model's out-of-sample forecasting ability relative to both a univariate model for labor income growth and an unconstrained VAR.

rather than a more general failure of the permanent income hypothesis itself.

Figure 2 plots the data for real personal savings per capita. It shows that savings increased from 1987 through the end of 1992, but have fallen since then. According to the permanent income hypothesis, this recent decline in savings indicates that households expect future gains in income.

Indeed, forecasts from the permanent income model reflect these expectations. When estimated with data through 1994:3, the constrained VAR predicts growth in real disposable labor income per capita of \$181 for 1995. Since real disposable labor income now stands at about \$12,300 and the population is growing at an annual rate of about 1 percent, this figure translates into a gain in aggregate real labor income of 2.5 percent. Thus, the permanent income model predicts that the U.S. economy will continue to expand in 1995.

**Figure 2 Real Personal Savings**  
1987 Dollars per Capita



## 5. CONCLUSION

Conventional wisdom suggests that the recent decline in personal savings cannot be sustained. Eventually, households will have to reduce their consumption, causing economic growth to slow. The permanent income hypothesis, however, contradicts this conventional wisdom. According to this hypothesis, households reduce their savings when they expect future income to be high; a low level of savings indicates that faster, not slower, income growth lies ahead.

This article uses a mathematical version of the permanent income hypothesis to formulate a simple econometric forecasting model for the U.S. economy. Estimates from the model reveal that the data are broadly consistent with the hypothesis' implications. Most important, the data indicate that declines in savings typically precede periods of faster, rather than slower, growth in income.

The results show that the permanent income model improves on univariate forecasts for annual labor income growth by 25 percent. The model also improves on the forecasting ability of an unconstrained vector autoregression for savings and labor income. In light of the recent decline in savings, the permanent income model forecasts continuing growth in personal income for 1995.

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## APPENDIX A: DERIVATION OF THE PERMANENT INCOME RESTRICTIONS

Campbell (1987) rewrites the vector autoregression (16) as

$$z_t = Az_{t-1} + v_t, \quad (18)$$

where

$$z_t = [\Delta y_{1t} \dots \Delta y_{1t-p+1} s_t \dots s_{t-p+1}]', \quad (19)$$

$$A = \begin{bmatrix} a_1 & \cdot & \cdot & \cdot & a_p & b_1 & \cdot & \cdot & \cdot & b_p \\ 1 & & & & & & & & & \\ & \cdot & & & & & & & & \\ & & \cdot & & & & & & & \\ & & & 1 & & & & & & \\ c_1 & \cdot & \cdot & \cdot & c_p & d_1 & \cdot & \cdot & \cdot & d_p \\ & & & & & 1 & & & & \\ & & & & & & \cdot & & & \\ & & & & & & & \cdot & & \\ & & & & & & & & 1 & \end{bmatrix}, \quad (20)$$

and

$$v_t = [u_{1t} \ 0 \ \dots \ 0 \ u_{2t} \ 0 \ \dots \ 0]'. \quad (21)$$

Equation (18), along with the conditions  $E_t v_{t+j} = 0$  for all  $j = 1, 2, 3, \dots$ , makes it easy to write the time  $t$  expectation of the vector  $z_{t+j}$  as

$$E_t z_{t+j} = A^j z_t. \quad (22)$$

Equation (22) implies, in particular, that

$$E_t \Delta y_{t+j} = e_1 A^j z_t, \quad (23)$$

where  $e_1$  is a row vector consisting of a one followed by  $2p - 1$  zeros.

Equation (23) shows that expected changes in future labor income depend on the coefficients of the lag polynomials  $a(L)$ ,  $b(L)$ ,  $c(L)$ , and  $d(L)$ , which are contained in the matrix  $A$ . The current value of savings, meanwhile, is just

$$s_t = e_{p+1} z_t, \quad (24)$$

where  $e_{p+1}$  is a row vector consisting of  $p$  zeros followed by a one and  $p - 1$  zeros. Equation (23) can be substituted into the right-hand side of equation (15), and equation (24) can be substituted into the left-hand side of equation (15), so that Hall's (1978) relationship between savings and income becomes

$$e_{p+1} z_t = - \sum_{j=1}^{\infty} (1+r)^{-j} e_1 A^j z_t. \quad (25)$$

Equation (25) must hold if, as required by equation (15), the current value of savings is to equal the discounted value of expected future declines in labor income.

Campbell uses the matrix analog to equation (4), which implies

$$\sum_{j=1}^{\infty} (1+r)^{-j} A^j = (1+r)^{-1} A [I - (1+r)^{-1} A]^{-1}, \quad (26)$$

to rewrite (25) as

$$e_{p+1} z_t = -e_1 (1+r)^{-1} A [I - (1+r)^{-1} A]^{-1} z_t, \quad (27)$$

or, more simply,

$$e_{p+1} [I - (1+r)^{-1} A] = -e_1 (1+r)^{-1} A, \quad (28)$$

where  $I$  is an identity matrix of the same size as  $A$ . The definition of  $A$  given by equation (20) implies that this last condition is equivalent to equation (17). Thus, equation (17) restates Hall's permanent income condition (15) as a set of parameter constraints that can be imposed on the VAR (16).

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## APPENDIX B: DATA SOURCES

All data used in this article come from the National Income and Product Accounts, as reported in the DRI/McGraw-Hill Data Base. The underlying quarterly series, 1959:1–1994:3, are the following:

WSD = Wage and Salary Disbursements  
 YOL = Other Labor Income  
 YENTADJ = Proprietors' Income  
 YRENTADJ = Rental Income  
 DIV@PER = Personal Dividend Income  
 YINTPER = Personal Interest Income  
 V = Transfer Payments  
 TWPER = Personal Contributions for Social Insurance  
 TP = Personal Tax and Nontax Payments  
 YD = Disposable Personal Income  
 C = Personal Consumption Expenditures  
 INTPER = Interest Paid by Consumers to Business  
 VFORPER = Personal Transfer Payments to Foreigners  
 YD87 = Disposable Personal Income, 1987 Dollars  
 NNIA = Population

From these series, disposable personal labor income (YDL) and disposable personal capital income (YKL) are constructed as

$$YDL = WSD + YOL + (2/3) * YENTADJ + V - TWPER \\ - (2/3) * TP - VFORPER$$

and

$$YDK = (1/3) * YENTADJ + YRENTADJ + DIV@PER + YINTPER \\ - (1/3) * TP - INTPER.$$

When converted into real, per-capita terms using the deflator for disposable personal income (YD/YD87) and the population NNIA, the series YDL, YKL, and C correspond to  $y_{lt}$ ,  $y_{kt}$ , and  $c_t$  in the text. Real savings per capita is defined by  $s_t = y_{lt} + y_{kt} - c_t$ .

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# The Rational Expectations Hypothesis of the Term Structure, Monetary Policy, and Time-Varying Term Premia

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Michael Dotsey and Christopher Otrok

Most empirical studies of the rational expectations hypothesis of the term structure (REHTS) generally find that the data offer little support for the theory.<sup>1</sup> In many cases this large body of empirical work indicates that the theory does not even provide a close approximation of market behavior. This feature has led some investigators to search for alternative “irrational” theories of behavior in order to explain the data. We, on the other hand, believe that the rejections are so striking that the large amount of irrationality implied by the data is too implausible for this avenue to be treated seriously. Since the rejection of rational expectations in these studies generally involves the rejection of more complicated joint hypotheses, we choose to focus our energies on exploring a broader class of models that are consistent with REHTS.

In particular, we examine a model that incorporates Federal Reserve behavior along with a reasonable parameterization of term premia to revise the theory. The consideration of Fed behavior was first suggested by Mankiw and Miron (1986), who found that REHTS was more consistent with the data prior to the founding of the Fed. Even stronger evidence is presented in Choi and

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<sup>1</sup> For an extensive set of results, see Campbell and Shiller (1991). Cook and Hahn (1990) and Rudebusch (1993) also give excellent surveys.

Wohar (1991), who cannot reject REHTS over the sample period of 1910–14. Cook and Hahn (1990) and Goodfriend (1991) argue persuasively that the Federal Reserve’s use of a funds rate instrument, and, in particular, the way in which that instrument is employed, is partly responsible for the apparent failure of REHTS.

Recently Rudebusch (1994), in a study very much in the spirit of ours, provides some empirical support for the Cook and Hahn (1990) and Goodfriend (1991) hypothesis. Further, McCallum (1994) shows the theoretical linkage between the Fed’s policy rule and the regression estimates in various tests of REHTS when the Fed responds to the behavior of longer-term interest rates.

While Fed behavior represents a potentially important component for explaining the empirical results of tests of REHTS, any explanation of these results that also maintains rational expectations must include time-varying term premia. Without time-varying term premia, tests of REHTS will not be rejected. This fact is pointed out in Mankiw and Miron (1986) and Campbell and Shiller (1991). Further, Campbell and Shiller indicate that white-noise term premia are insufficient to reconcile theory with data. We find this to be the case as well. Thus, we examine a more elaborate model of term premia coupled with Fed behavior in an attempt to explain some of the empirical results on REHTS.

Before developing a theory of Fed behavior and linking it to empirical work on REHTS, we present, in Section 1, a brief overview of the rational expectations hypothesis of the term structure. Then, in Section 2, we construct a model of Fed behavior that embodies the key elements described in Goodfriend (1991). We use the resulting model along with REHTS to generate returns on bonds of maturities ranging from one to six months. In Section 3 we focus, in essence, on the empirical regularities documented by Roberds, Runkle, and Whiteman (1993). We show that Fed behavior is not enough to reproduce their findings. Then, in Section 4, we turn our attention to incorporating a more realistic behavior of term premia. Combining these term premia with rational investor and Fed behavior generates data that is roughly consistent with the Roberds, Runkle, and Whiteman results. Section 5 concludes.

## **1. THE RATIONAL EXPECTATIONS THEORY OF THE TERM STRUCTURE**

Tests and descriptions of the rational expectations theory of the term structure constitute a voluminous literature. An excellent survey can be found in Cook and Hahn (1990), and an exhaustive treatment is contained in Campbell and Shiller (1991). The basic idea is that with the exception of a term premium, there should be no expected difference in the returns from holding a long-term bond or rolling over a sequence of short-term bonds. As a result, the long-term interest rate should be an average of future expected short-term interest

rates plus a term premium. Specifically, the interest rate on a long-term bond of maturity  $n$ ,  $r_t(n)$ , will obey

$$r_t(n) = \frac{1}{k} \sum_{i=0}^{k-1} E_t r_{t+mi}(m) + \phi_t(n, m), \quad (1)$$

where  $r_{t+mi}(m)$  is the  $m$  period bond rate at date  $t + mi$ ,  $E_t$  is the conditional expectations operator over time  $t$  information, and  $\phi_t(n, m)$  is the term premia between the  $n$  and  $m$  period bonds.<sup>2</sup> In equation (1),  $k = n/m$  and is restricted to be an integer.

The rational expectations hypothesis implies that  $r_{t+mi}(m) = E_t r_{t+mi}(m) + e_{t+mi}(m)$ , where  $e_{t+mi}(m)$  has mean zero and is uncorrelated with time  $t$  information. Using this implication, one can rearrange equation (1) to yield the following relationship:

$$\frac{1}{k} \sum_{i=1}^{k-1} [r_{t+mi}(m) - r_t(m)] = \alpha + r_t(n) - r_t(m) + v_t(n, m), \quad (2)$$

where  $v_t(n, m) = \frac{1}{k} \sum_{i=1}^{k-1} e_{t+mi}(m) - [\phi_t(n, m) - \alpha]$  and  $\alpha$  is the non-time-varying part of the term premium. Thus, future interest rate differentials on the shorter-term bond are related to the current interest rate spread between the long- and short-term bond.

Equation (2) forms the basis of the tests of the term structure that we focus on in this article. This involves running the regression

$$\frac{1}{k} \sum_{i=1}^{k-1} [r_{t+mi}(m) - r_t(m)] = \alpha + \beta[r_t(n) - r_t(m)] + v_t(n, m) \quad (3)$$

and testing if  $\beta = 1$ . We shall focus our attention on  $n = 2, 3, 4, 5$ , and 6 months and  $m = 1$  and 3 months. For  $m = 3$  and  $n = 6$  (implying  $k = 2$ ), the appropriate regression would be

$$^{1/2} [r_{t+3}(3) - r_t(3)] = \alpha + \beta[r_t(6) - r_t(3)] + v_t(6, 3). \quad (3')$$

That is, the change in the three-month interest rate three months from now should be reflected in the difference between the current six-month and three-month rates because the pricing of the six-month bill should reflect any expected future changes in the rate paid on the three-month bill.

In the absence of time-varying term premia, the coefficient  $\beta$  should equal one. In practice, however, that has not been the case. For example, Table 1

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<sup>2</sup> Term premia arise naturally in consumption-based asset pricing models and involve the covariance of terms containing the ratio of future price-deflated expected marginal utilities of consumption to the current price-deflated marginal utility of consumption, the price of the long-term bond, and future prices of the short-term bond. See Labadie (1994).

reports some estimates obtained by Roberds, Runkle, and Whiteman (1993) and Campbell and Shiller (1991). Not only is  $\beta < 1$ , but the degree to which  $\beta$  deviates from one increases as  $k$  increases. Also, the coefficient in the regression when  $n = 6$  and  $m = 3$  is of the wrong sign and insignificantly different from zero.

This latter result is in stark contrast to estimates obtained by Mankiw and Miron (1986) and Choi and Wohar (1991), who find that prior to the advent of the Fed, the theory fared much better. These two sets of results, which primarily involve  $r(6) - r(3)$ , imply a number of possibilities among which are the following: (1) REHTS once held but no longer does (perhaps because investors have become irrational), (2) the nature of term premia has changed, or (3) Federal Reserve policy has in some way affected the nature of the empirical tests.

In analyzing these possibilities, we first note that the term premia must be time-varying for  $\text{plim } \hat{\beta} \neq 1$  (i.e., the predicted value of  $\beta$  to be something other than one). To show this, we report the probability limit of  $\hat{\beta}$  in (3'), which is adopted from the derivation in Mankiw and Miron (1986):

$$\text{plim } \hat{\beta} = \frac{\sigma^2[E_t \Delta r_{t+1}(3)] + 2\rho\sigma[E_t \Delta r_{t+1}(3)]\sigma[\phi_t(6, 3)]}{\sigma^2[E_t \Delta r_{t+1}(3)] + 4\sigma^2[\phi_t(6, 3)] + 4\rho\sigma[E_t \Delta r_{t+1}(3)]\sigma[\phi_t(6, 3)]}, \quad (4)$$

where  $\sigma^2[E_t \Delta r_{t+1}(3)]$  is the variance of the expected change in the three-month interest rate,  $\rho$  is the correlation between  $E_t \Delta r_{t+1}(3)$  and  $\phi_t(6, 3)$ , and  $\sigma^2[\phi_t(6, 3)]$  is the variance of the term premium.<sup>3</sup>

Expression (4) is informative for our purposes. Notice that for nonstochastic term premia,  $\text{plim } \hat{\beta} = 1$ . Hence stochastic term premia are required for  $\text{plim } \hat{\beta} \neq 1$ . Also observe that as  $\sigma^2[\phi_t(6, 3)]$  increases,  $\text{plim } \hat{\beta}$  decreases. Further, note that  $\text{plim } \hat{\beta}$  is a complicated function of  $\sigma^2[E_t \Delta r_{t+1}(3)]$ , but as this term gets fairly large,  $\text{plim } \hat{\beta}$  goes to one. More generally,  $\hat{\beta}$ 's deviation from a value of one will depend on the ratio of the variance of the term premium to the variance of the expected change in interest rates.

It is this latter variance that Fed behavior may influence. In this regard, Mankiw and Miron (1986) document the variation over time in this variable and show that  $\sigma^2[E_t \Delta r_{t+1}(3)]$  was much larger prior to the creation of the Federal Reserve System. Mankiw and Miron attribute this finding to the Fed's concern for interest rate smoothing.

As Cook and Hahn (1990) point out, however, rate smoothing cannot be the total story since the regression coefficient on  $[r_t(2) - r_t(1)]$  is highly significant and close to one; moreover, for longer-term bonds the term structure does help predict future changes in interest rates. Regarding the short end of the yield curve, Cook and Hahn postulate that one must consider the discontinuous

<sup>3</sup> Rudebusch (1993) derives a similar expression with  $\rho = 0$ .

**Table 1 Coefficient Estimates from Literature**

Source	Short ( <i>m</i> ) Long ( <i>n</i> )	1 period 2 period	1 period 3 period	1 period 4 period	1 period 6 period	2 period 4 period	3 period 6 period
Campbell & Shiller Table 2, T-bills, 1952–87	coefficient standard error	0.5010 0.1190	0.4460 0.1990	0.4360 0.2380	0.2370 0.1670	0.1950 0.2810	−0.1470 0.2000
Roberds, Runkle & Whiteman, Table 6 F Fund, 1984–91	coefficient standard error	0.5925 0.0983	0.3935 0.1437	na na	0.2121 0.2822	na na	−0.1411 0.6079
Roberds, Runkle & Whiteman, Table 9 F Fund, 1984–91, SW*	coefficient standard error	0.7596 0.1359	0.2953 0.1399	na na	0.1557 0.1861	na na	−0.2971 0.3675
Roberds, Runkle & Whiteman, Table 11 F Fund, 1984–91, FOMC†	coefficient standard error	0.7119 0.1720	0.4104 0.1688	na na	0.0869 0.1878	na na	−0.3149 0.4553

\* Settlement Wednesday.

† FOMC meeting date.

Note: Roberds, Runkle, and Whiteman use daily data in their regressions.

and infrequent changes in policy. Therefore, economic information that will affect future policy is often known prior to actual policy reactions. This factor implies that movements in the short end of the term structure will anticipate policy and hence have predictive content. In terms of equation (4), the variance of  $\Delta E_t r_{t+1}(1)$  is likely to be greater than the variance of  $\Delta E_t r_{t+1}(3)$ .

Additional arguments supporting the relevance of monetary policy for tests of REHTS can be found in Goodfriend (1991) and McCallum (1994). McCallum shows that if the Fed reacts to movements in the term structure, then the strength of that reaction will influence estimates of  $\beta$  in tests of REHTS.

Taken together, these papers indicate that capturing Fed behavior is potentially important for understanding the term structure. We now attempt such an exercise.

## 2. A MODEL OF FED BEHAVIOR

Our model of Fed behavior is designed to capture the basic characteristics described by Goodfriend's (1991) analysis of Federal Reserve policy. In particular, we model the Federal Reserve's adjustment of its funds rate target as occurring at intervals and only in relatively small steps. Also, funds rate changes are often followed by changes in the same direction so that the Fed does not "whipsaw" financial markets. While the Fed is generally viewed as adjusting the funds rate to achieve various economic goals, for our purposes it is sufficient to let the Fed's best guess of an unconstrained optimal interest rate target follow an exogenous process. For simplicity, let

$$\Delta r_t^* = \rho \Delta r_{t-1}^* + u_t, \quad (5)$$

where  $r_t^*$  is the unconstrained optimal interest rate. That is, it is the interest rate the Fed would choose before the arrival of new information if it were not constrained to move the funds rate discretely. One could think of  $r_t^*$  as arising from a reaction function, but equation (5), along with additional behavioral constraints, is sufficient for the purpose of our investigation. To capture Fed behavior, we model changes in the funds rate according to the following criteria:

$$\begin{aligned} r_t^f &= r_{t-1}^f + 1/2 \text{ if } r_t^* - r_{t-1}^f \geq 1/2, \\ r_t^f &= r_{t-1}^f + 1/4 \text{ if } 1/4 \leq r_t^* - r_{t-1}^f < 1/2, \\ r_t^f &= r_{t-1}^f \text{ if } -1/4 < r_t^* - r_{t-1}^f < 1/4, \\ r_t^f &= r_{t-1}^f - 1/4 \text{ if } -1/2 < r_t^* - r_{t-1}^f \leq -1/4, \\ r_t^f &= r_{t-1}^f - 1/2 \text{ if } -1/2 > r_t^* - r_{t-1}^f. \end{aligned} \quad (6)$$

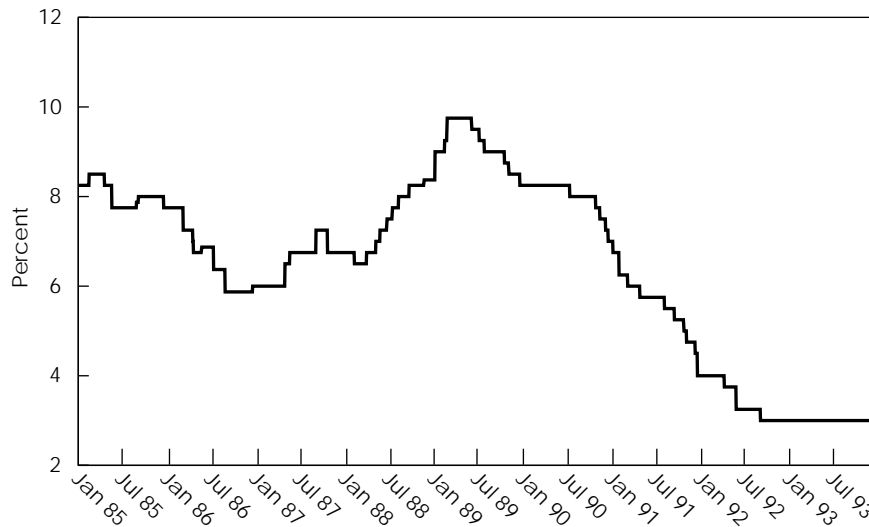
The behavior described by equation (6) implies that at each decision point the Fed is guided by its overall macroeconomic goals as depicted by the

behavior of  $r_t^*$ . It adjusts its instrument  $r_t^f$  incrementally and discretely. Thus, for a big positive shock to  $r_t^*$ , the Fed would be expected to raise the funds rate at a number of decision points until  $r_t^f$  approximated  $r_t^*$ . There would also be only a small probability that the Fed would ever reverse itself (i.e., raise the funds rate one period and lower it the next).

We parameterize the variance of  $u_t$  and the parameter  $\rho$  so that the behavior of the funds rate target,  $r_t^f$ , is consistent with actual behavior over the period 1985:1 to 1993:12. The parameter  $\rho$  is set at 0.15, and  $u_t$  has a variance of 0.09. In particular,  $u_t$  is distributed uniformly on the interval  $[-0.525, 0.525]$ . A uniform distribution is used to facilitate the pricing of multiperiod bonds in the next section.

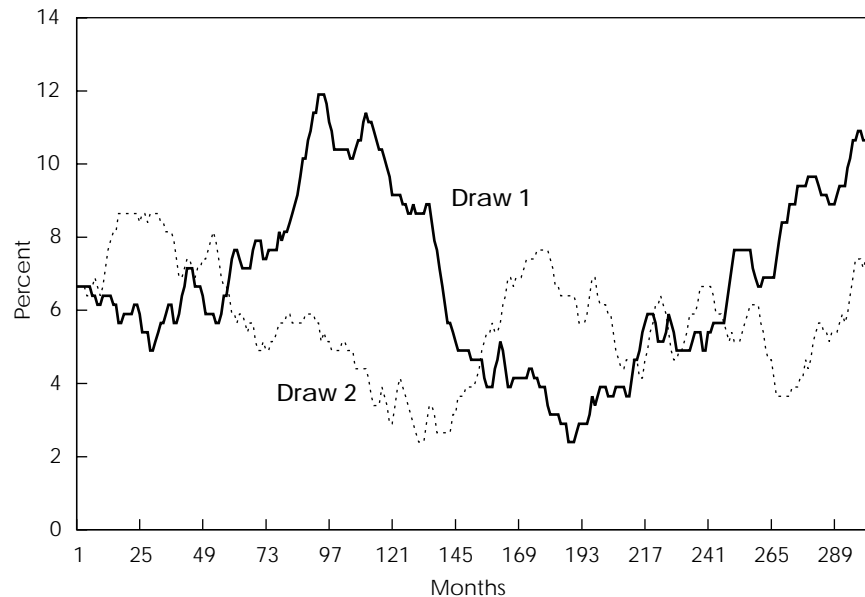
The behavior generated by a typical draw from our stochastic process and by the actual funds rate target are reasonably similar. These are depicted in Figures 1 and 2. Table 2 provides some additional methods of comparison.

**Figure 1 Federal Funds Rate Target**



The low p-values of Fisher's exact test indicate that both actual and model data are consistent with targeted interest rate changes not being independent of the sign of previous changes.<sup>4</sup> However, a somewhat smaller percentage of interest rate changes are of the same sign in the model. The Fed, as modeled here, is

<sup>4</sup> Model data are from 250 draws of a series with 300 observations.

**Figure 2 Representative Draws of Funds Rate Target**

more likely to reverse itself than the Fed actually did over this period. Also, the Fed of our model is more likely to leave the funds rate unchanged. Finally, the correlation coefficient between funds rate changes in the model is not significantly different from the actual correlation coefficient displayed by the data.

We thus feel that equations (5) and (6) jointly represent a reasonable and tractable model of Federal Reserve behavior, especially if  $r_t^*$  is thought of as depending upon underlying economic behavior.

### 3. MONETARY POLICY AND THE TERM STRUCTURE

As described by equations (5) and (6), the Federal Reserve determines the behavior of the one-period nominal interest rate. The FOMC meets formally eight times per year and informally via conference calls. Also, the chairman may act between FOMC meetings so that in actuality the term of the one-period rate is less than one month. Further, the timing between decision periods is stochastic and can be as little as one week or as long as an intermeeting period.<sup>5</sup> For simplicity, we model the decision period as monthly. Thus, the pricing of a

<sup>5</sup> For a more detailed modeling of behavior along these lines, see Rudebusch (1994).



**Table 2 Actual and Model Data Comparison**

	Actual	Model
Percent of Changes of Same Sign	0.833	0.648
Fisher's Exact Test p-value (standard error)	2.6E-06	0.003600 (0.011)
Corr( $\Delta r_t^f, \Delta r_{t-1}^f$ ) (standard error)	0.2661	0.3242 0.04948
Std( $E_t r_{t+1} - r_t$ ) (standard error)	0.2409	0.1234 (0.0034)
Std[ $E_t r_{t+1}(2) - r_t$ ] (standard error)	0.2562	0.1457 (0.0039)
Std[ $E_t r_{t+1}(3) - r_t$ ] (standard error)	0.2687	0.1565 (0.0040)
Std[ $E_t r_{t+3}(3) - r_t(3)$ ]*	0.1858	0.1306 (0.0052)

\* Goldsmith-Nagan yields 0.2011.

Notes: Model data are from 250 draws of a series with 300 observations. The null of Fisher's exact test is that the sign of the change in the funds rate is independent of the sign of the previous change.

two-month bond or, more accurately, a two-month federal funds contract will obey

$$\begin{aligned}
2r_t(2) = & r_t^f + \frac{1}{2}(\text{Prob}[r_{t+1}^* - r_t^f \geq \frac{1}{2}]) \\
& + \frac{1}{4}(\text{Prob}[\frac{1}{4} \leq r_{t+1}^* - r_t^f < \frac{1}{2}]) \\
& - \frac{1}{4}(\text{Prob}[-\frac{1}{2} < r_{t+1}^* - r_t^f \leq -\frac{1}{4}]) \\
& - \frac{1}{2}(\text{Prob}[r_{t+1}^* - r_t^f \leq -\frac{1}{2}]) + 2\phi_t(2, 1). \tag{7}
\end{aligned}$$

In calculating the expectation of interest rates further than one period ahead, say, for example, two periods ahead, one needs to form time  $t$  expectations of terms such as  $\text{Prob}[r_{t+2}^* - r_{t+1}^f > \frac{1}{2}]$ . Assuming that  $u_t$  is uniformly distributed, the expressions we obtain for the various probabilities are linear in  $r_{t+j}^*$  and  $r_{t+j-1}^f$ . Thus, one can pass the expectations operator through the respective cumulative distribution functions.

For pricing three-, four-, five-, and six-month term federal funds, we use expressions analogous to equation (7). In order to examine the effect that our model of monetary policy has on tests of the rational expectations hypothesis of the term structure, we generate 250 simulations, each containing 300 values of each rate. The results are presented in Table 3, where standard errors have been corrected using the Newey-West (1987) procedure. We report results when

**Table 3 Coefficient Estimates Using Model-Generated Data**

Independent Variable	$r(2) - r$	$r(3) - r$	$r(4) - r$	$r(5) - r$	$r(6) - r$	$r(4) - r(2)$	$r(6) - r(3)$
	(a) $\sigma(\phi) = 0$						
Coefficient	1.03	1.00	1.00	.99	.99	.97	.95
Standard Error	(.005)	(.118)	(.109)	(.158)	(.179)	(.22)	(.42)
	(b) $\sigma(\phi) = .10$						
Coefficient	.28	.49	.57	.62	.64	.35	.31
Standard Error	(.055)	(.084)	(.105)	(.122)	(.138)	(.083)	(.114)

there is no term premium (row 1) and when there is a white-noise term premium with standard deviation 0.10 (row 2).

The results indicate that in the absence of time-varying term premia, there is no departure of estimates of  $\beta$  from one. This essentially serves as a check on our calculations, since all interest rates are calculated using REHTS. With a time-varying term premia, REHTS is rejected. However, the rejection of the model's data is not in keeping with the result on actual data. The estimates of  $\beta$  are increasing in  $k = n/m$  rather than decreasing. Also, the results for  $k = 2$  and  $m =$  one month, two months, and three months, respectively, are almost identical for the model, while they are strikingly different for the data. Looking at Table 2 and equation (4) shows why. Table 2 indicates that  $\sigma^2[E_t \Delta r_{t+1}(m)]$  is approximately the same for  $m = 1$  and  $m = 3$ . (When  $m = 2$ , its value is 0.151.) With  $\sigma^2[\phi(n, m)]$  equivalent by construction, the estimate of  $\beta$  will not vary much across experiments. For a model with white-noise term premia to replicate actual empirical results, it must generate  $\sigma^2(E_t \Delta r_{t+1}) > \sigma^2[E_t \Delta r_{t+2}(2)] > \sigma^2[E_t \Delta r_{t+3}(3)]$ , which does not happen in our particular model.

Interestingly enough, as shown in Table 2, the required behavior of  $\sigma^2[E_t r_{t+1}(m) - r_t(1)]$  does not occur in the data either. We are therefore forced to conclude that our description of monetary policy, along with white-noise term premia, is insufficient to explain the empirical results in Roberds, Runkle, and Whiteman (1993) as well as in Campbell and Shiller (1991). Our failure could be due primarily to an insufficient model of policy or to an inadequate model of term premia. In the next section we modify our model of term premia and reexamine REHTS on data generated by our modified model.

#### 4. A DESCRIPTION OF TERM PREMIA

To generate term premia that potentially resemble the stochastic processes of actual term premia, we need some way of estimating term premia. For this we

turn to the multivariate ARCH-M methodology described in Bollerslev (1990). We use a multivariate model since the term premia generated from a univariate model are highly correlated. In essence, we estimate a multivariate ARCH-M model of excess holding period yields then use the estimated process to simulate time-varying term premia. The simulated processes, along with the model in Section 2, are used to generate data on interest rates. This simulated data is then used to estimate regressions like (3').

In estimating term premia (for the case in which  $k = 2$ ), first define the excess holding period yield,  $y_t(n, m)$ , as

$$2r_t(n) - r_{t+m}(m) - r_t(m).$$

From equation (1) we see that this is merely  $E_t r_{t+m}(m) - r_{t+m}(m) + 2\phi_t(n, m)$ , which is the sum of an expectational error and twice the actual term premium as defined in (1).

The multivariate ARCH-M specification that we estimate over the sample period 1983:1–1993:12 is given by

$$y_t = \beta + \delta \log h_t + \varepsilon_t, \quad (8)$$

where  $\varepsilon_t$  conditioned on past information is a normal random vector with variance-covariance matrix  $H_t$ . The elements of  $H_t$  are given by

$$\begin{aligned} h_{jj,t}^2 &= \gamma_j + \alpha_j \sum_{i=1}^{12} w_i \varepsilon_{j,t-i}^2 \\ h_{ij,t} &= \rho_{ij} h_{ii,t} h_{jj,t}, \end{aligned} \quad (9)$$

where  $y_t$  is a 3 by 1 vector of the ex-post excess holding period yields on Treasury bills that includes the two-month versus one-month bill, the three-month versus one-month bill, and the six-month versus three-month bill.<sup>6</sup> The  $w_i$  are fixed weights given by  $(13 - i)/78$ . In this specification of the model, the covariances  $h_{ij}$  are allowed to vary but the correlation coefficients,  $\rho_{ij}$ , between the errors are constant. The coefficient estimates are reported in Table 4. Almost all the coefficients are highly significant.

The term premia derived from this model are depicted in Figure 3 and are labeled T2, T3, and T6. Recall that T2 and T6 are twice  $\phi(2, 1)$  and  $\phi(6, 3)$ , respectively, while T3 is three times  $\phi(3, 1)$ . One notices the term premia spike upward in 1984, in late 1987, and in early 1991. The term premium on two-month bonds also spikes in late 1988 and early 1989. The 1987 episode is associated with the October stock market crash. Interestingly, the 1984 and

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<sup>6</sup> We use T-bills rather than term federal funds because coefficient estimates using the federal funds rate are insignificant. One possible explanation for this result is that the excess holding period yield on federal funds involves both a term premia derived from a consumption-based asset pricing model as well as default risk that may be uncorrelated with the term premia. This default risk may add sufficient noise that it is difficult to estimate the term premia using ARCH-M type regressions.

**Table 4 Coefficient Estimates for ARCH-M Model**

Log likelihood = 149.72

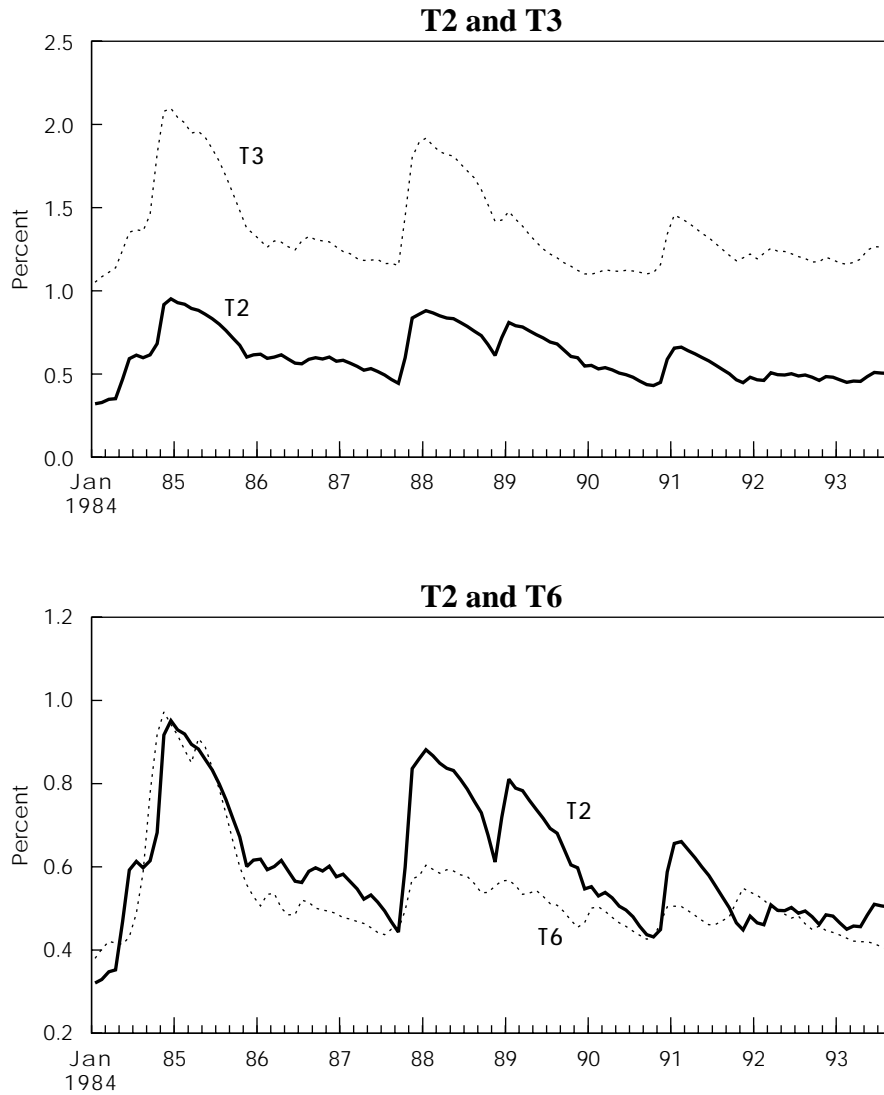
Coefficient	Estimate	Standard Error	Significance Level
$\beta_2$	.98	.13	.0000
$\delta_2$	.46	.13	.0005
$\gamma_2$	.058	.018	.0009
$\alpha_2$	.92	.20	.0000
$\beta_3$	1.46	.13	.0000
$\delta_3$	1.25	.47	.0076
$\gamma_3$	.52	.089	.0000
$\alpha_3$	.44	.13	.0010
$\beta_6$	.94	.45	.0389
$\delta_6$	.79	.82	.331
$\gamma_6$	.25	.08	.0011
$\alpha_6$	.23	.17	.0601
$\rho_{23}$	.92	.016	.0000
$\rho_{26}$	.48	.082	.0000
$\rho_{36}$	.67	.060	.0000

1988–89 episodes correspond to the inflation-scare episodes documented in Goodfriend (1993). The last spike in the term premia occurs around the time of the Gulf War and a recession.

Statistical data for the in-sample residuals, the estimated term premia, and the ex-post holding period yields are depicted in Table 5. In attempting to ascertain the joint importance of Fed behavior and time-varying term premia in explaining the regression results of Campbell and Shiller (1991) as well as Roberds, Runkle, and Whiteman (1993), we perform three experiments. First we generate a funds rate that is stationary and thus does not display the interest rate smoothing or discrete interest rate changes that are embodied in our model of Fed behavior. Longer-term interest rates are then derived using equation (1) and the rational expectations hypothesis. We do this to see if our model of term premia by itself can account for the actual regression results. Next we examine an interest rate process that includes a greater degree of smoothing but does not require discrete changes in the funds rate. Finally, we combine our model of term premia with our depiction of Fed behavior and investigate whether this model of interest rate determination can explain the regression results obtained using actual data. The results we analyze involve the cases in which  $n = 2$ ,  $m = 1$ , and  $n = 6$ ,  $m = 3$  (i.e., the term spread between the two-month and one-month bills and the six-month and three-month bills).

To begin, we model the one-period interest rate as  $r_t^* = 0.75r_{t-1}^* + u_t$ . As in our actual model of Fed behavior,  $u_t$  is distributed uniformly on the interval  $[-0.525, 0.525]$ . Combining this behavior with term premia generated from our estimated ARCH-M model, we generate data on longer-term interest

**Figure 3 Interest Rate Term Premia**



Notes: T2 is the term premium between the two-month and one-month bonds. T3 is the term premium between the three-month and one-month bonds. T6 is the term premium between the six-month and three-month bonds.

**Table 5 Statistical Data from ARCH-M Model**

Residuals		Standard Error	Correlation Matrix		
$\varepsilon_2$		.45	1.0		
$\varepsilon_3$		.99	.92	1.0	
$\varepsilon_6$		.64	.50	.70	1.0
Estimated Term Premia	Mean	Standard Error	Correlation Matrix		
T2	.60	.15	1.0		
T3	1.35	.25	.90	1.0	
T6	.53	.13	.77	.85	1.0
Actual Ex-post Yields	Mean	Standard Error	Correlation Matrix		
y2	.65	.47	1.0		
y3	1.51	1.04	.93	1.0	
y6	.62	.68	.55	.74	1.0

rates using equation (1). The regression results based on 500 simulations of 125 observations are

$$r_{t+1}(1) - r_t(1) = a_0 + 0.43 [r_t(2) - r_t(1)] + \varepsilon_{1t}, \quad (0.16)$$

$$r_{t+3}(3) - r_t(3) = \alpha_0 + 1.00 [r_t(6) - r_t(3)] + \varepsilon_{3t}, \quad (0.18)$$

where standard errors are in parentheses. REHTS is not rejected by the second regression, and the results of this regression are consistent with those documented in Mankiw and Miron (1986) and Choi and Wohar (1991) for the period prior to the founding of the Fed. One must therefore conclude that our model of term premia is not sufficient for generating data that are capable of replicating regression results using actual post-Fed data.

Next we model the short-term interest rates as  $\Delta r_t^* = 0.15 \Delta r_{t-1}^* + u_t$ , which is consistent with our modeling of  $r_t^*$  in equation (5). Thus, the only element lacking from our complete model of Fed behavior is the discrete nature of funds rate behavior given by equation (6). Generating data using this nonstationary model of  $r_t^*$ , along with our model of term premia, we obtain the following regression results:

$$r_{t+1}(1) - r_t(1) = b_0 + 0.10 [r_t(2) - r_t(1)] + e_{1t}, \quad (0.19)$$

$$r_{t+3}(3) - r_t(3) = \beta_0 + 0.12 [r_t(6) - r_t(3)] + e_{3t}. \quad (1.24)$$

Here both coefficients are insignificantly different from zero. Thus, this experiment does not generate the statistically significant coefficient commonly found when using actual data on two- and one-month interest rates.

Finally, we combine the joint modeling of term premia using the ARCH-M process and Fed behavior given by equations (5) and (6). These regression results are the following:

$$r_{t+1}(1) - r_t(1) = c_0 + 0.46 [r_t(2) - r_t(1)] + w_{1t}, \quad (0.10)$$

$$r_{t+1}(3) - r_t(3) = \gamma_0 + 0.64 [r_t(6) - r_t(3)] + w_{3t}. \quad (0.59)$$

Here the joint modeling of term premia and Fed behavior is capable of explaining a statistically significant coefficient that is less than one in the shorter-maturity regression, whereas the coefficient in the regression involving longer maturities is insignificantly different from zero. An explanation for the increased significance of the coefficient in the first regression from that estimated in the previous experiment goes as follows. Due to Fed behavior, the standard deviation of the expected change in the one-month rate has risen from a value of 0.095 to 0.123, while the standard deviation of the term premia has remained unchanged. However, there are only 35 episodes in which the coefficient in the first regression is greater than 0.5 while the coefficient in the second regression is also less than zero. Thus, the coefficient estimates that are consistent with the results presented in Roberds, Runkle, and Whiteman (1993) occur in approximately 7 percent of the trials.

The results presented above are not entirely satisfactory because the generated term premia do not exactly match the fitted term premia of the model (perhaps because the correlation coefficients are constrained to be time invariant). The standard deviations of the generated term premia are somewhat less than those depicted in Table 5, whereas the correlation coefficients are appreciably less. With generated data,  $\sigma_{T2} = 0.17$ ,  $\sigma_{T3} = 0.14$ , and  $\sigma_{T6} = 0.06$  while  $\rho_{23} = 0.66$ ,  $\rho_{26} = 0.20$ , and  $\rho_{36} = 0.41$ .

To remedy this situation, we generate data by also allowing the correlation coefficients,  $\rho_{ij}$ , to vary intertemporally. We do this by allowing them to depend on the  $h_{ji,t}$ s in equation (9), producing standard deviations of  $\sigma_{T2} = 0.14$ ,  $\sigma_{T3} = 0.44$ , and  $\sigma_{T6} = 0.12$  and correlation coefficients of  $\rho_{23} = 0.88$ ,  $\rho_{26} = 0.73$ , and  $\rho_{36} = 0.83$ . In a regression using data generated by this mechanism, the coefficient on the 2,1 term is 0.93(0.16) and on the 6,3 term is 0.79(0.63), where standard errors are in parentheses. Also, in 10 percent of the cases the 6,3 coefficient is less than zero, while the 2,1 coefficient is greater than 0.5. When there is no discretization of movements in the funds rate, these coefficients are 0.54(0.47) and 0.11(1.95). Both coefficients differ insignificantly from zero.

While the term premia in the last simulation do not come from any estimated model, the experiment at least shows that regression results that are in accord with those obtained in practice can be generated by the combination of (1) Fed behavior that both smooths the movements in interest rates and only moves interest rates discretely and (2) time-varying term premia that are calibrated to match data moments.

## 5. CONCLUSION

This article explores the linkage between Federal Reserve behavior and time-varying term premia and analyzes what effect these two economic phenomena have on tests of the rational expectations hypothesis of the term structure. Adding both these elements to a model of interest rate formation produces simulated regression results that are reasonably close to those reported using actual data. We thus feel that a deeper understanding of interest rate behavior will be produced by jointly taking into account the behavior of the monetary authority along with a more detailed understanding of what determines term premia. Reconciling theory with empirical results probably does not require abandonment of the rational expectations paradigm.

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