The Origins of Velocity Functions

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Like any practical, policy-oriented discipline, monetary economics employs useful concepts long after their prototypes and originators are forgotten. A case in point is the notion of a velocity function relating money's rate of turnover to its independent determining variables.

Most economists recognize Milton Friedman's influential 1956 version of the function. Written $v = Y/M = v(r_b, r_e, 1/PdP/dt, w, Y/P, u)$, it expresses income velocity as a function of bond interest rates, equity yields, expected inflation, wealth, real income, and a catch-all taste-and-technology variable that captures the impact of a myriad of influences on velocity, including degree of monetization, spread of banking, proliferation of money substitutes, development of cash management practices, confidence in the future stability of the economy and the like.

Many also are aware of Irving Fisher's 1911 transactions velocity function, although few realize that it incorporates most of the same variables as Friedman's.[1] On velocity's interest rate determinant, Fisher writes: "Each person regulates his turnover" to avoid "waste of interest" (1963, p. 152). When rates rise, cashholders "will avoid carrying too much" money thus prompting a rise in velocity. On expected inflation, he says: "When . . . depreciation is anticipated, there is a tendency among owners of money to spend it speedily . . . the result being to raise prices by increasing the velocity of circulation" (p. 263). And on real income: "The rich have a higher rate of turnover than the poor. They spend money faster, not only absolutely but relatively to the money they keep on hand. . . . We may therefore infer that, if a nation grows richer per capita, the velocity of circulation of money will increase" (p. 167). Finally, with respect to the catch-all variable, Fisher cites all of the following

[1] Among the few is Boris Pesek (1976, pp. 857–58) who notes that Fisher's velocity function contains more variables than Friedman's.

as affecting velocity: "habits as to thrift and hoarding," "book credit," "use of checks," "frequency and regularity of receipts and payments," "density of population," and "extent and speed of transportation" (p. 79). A comprehensive list indeed.

The purpose of this article, however, is not to evaluate Friedman's and Fisher's velocity functions. Rather it is to correct the impression that such functions begin with Fisher. Thus J. S. Cramer (1992), in his authoritative article "Velocity of Circulation" in Volume 3 of the *New Palgrave Dictionary of Money and Finance*, traces the concept to the equation of exchange "which is due to Irving Fisher" (p. 757). Countless macro and money and banking textbooks echo this view. Even Merton Miller and Charles Upton's 1975 classic *Macroeconomics: A Neoclassical Introduction* categorically asserts that the term "velocity of circulation" is "associated with Irving Fisher" (1986, p. 231). Such statements overlook 250 years of monetary theorizing. For, as demonstrated below, the notion of a functional relationship between velocity and its determinants dates from the middle of the seventeenth century and received frequent restatement throughout the eighteenth and nineteenth centuries before being bequeathed to Fisher and his successors in the twentieth. Seen in this perspective, Fisher emerges not as the originator of velocity functions but rather as a particularly innovative recipient of them.

Before documenting the latter assertion, however, it should be noted that one era's velocity determinants become another's money-stock components. Changes in the definition of money ensure as much. Thus modern analysts define money to include coin, paper currency, and deposits subject to check. By contrast, most of the pre-Fisher velocity theorists discussed below defined money as consisting solely of gold and silver coin. They excluded bank notes and deposits on the ground that such instruments lack the unconditional power of specie to settle final transactions and thus are not money per se but rather devices to accelerate money's velocity. Consequently, they saw note and deposit expansions and contractions as velocity shifts rather than as money-stock shifts. Their view may seem strange to the modern reader accustomed to regarding notes and deposits as cash, but it was entirely consistent with their metallist conception of money.

1. THE FIRST VELOCITY FUNCTION

Sir William Petty (1623–1687) enunciated the first velocity function, albeit in verbal rather than algebraic form, in his *A Treatise of Taxes and Contributions* (1662) and *Verbum Sapienti* (1664). He did so in an effort to estimate the amount of money—defined by him as consisting solely of gold coin—necessary to support the commercial activity of a nation. This amount he saw as depending on velocity or its inverse, the ratio of money to trade. Unlike writers such as John Briscoe (1696), who identified the requisite money stock with national income and so assumed a ratio of unity, Petty treated it as a fractional magnitude.[2] His pathbreaking statistical studies of the economies of Ireland and England had convinced him that the money stock was but a small part of national expenditure, which meant that a velocity coefficient greater than 1 existed to adjust money to the needs of trade. Here is the origin of the notion of velocity as the multiplier that equates the stock of money with the flow of income.[3]

Petty's statistical studies also suggested certain institutional characteristics that determine velocity. His function embodies these characteristics in the form of five independent variables: (1) frequency of payments, (2) size of payments, (3) income, (4) its distribution among socioeconomic classes, and (5) banking.

Of these variables, the first enters the function with a positive sign, reflecting Petty's belief that the more frequently income recipients are paid, i.e., the shorter the pay period, the less cash per unit of income they need to hold between paydays and so the higher is velocity. Illustrating this point, Petty claimed that workers receiving wages once per week would spend a unit of money an average of 52 times a year whereas landlords receiving rents quarterly would spend the same monetary unit only 4 times per year.[4]

Unlike the payment-interval variable, Petty's size-of-payments variable bears a negative sign. He believed larger payments require a greater accumulation of cash in advance relative to income than do smaller payments. To him, many small payments at short intervals spelled a higher velocity than did a few large payments at long intervals.[5]

Like Fisher, Petty saw income entering the function with a positive sign. "The most thriving men," he said, "keep little . . . money by them, but turn and wind it into various commodities to their great profit" (quoted in Marshall [1923], p. 41). Evidently he believed that scale economies in cash holding permit the rich to hold smaller balances in relation to their incomes than do the poor so that velocity rises with incomes. Only such economies can explain why

[2] On Briscoe's formula *money stock* = *national income*, see Heckscher (1983), p. 224, Schumpeter (1954), pp. 314–15, and Viner (1937), p. 42.

[3] On Petty's contribution see Holtrop (1929), Roncaglia (1985), Schumpeter (1954), and Wu (1939).

[4] Petty's frequency-of-payments analysis launched a line of research leading to modern endogenous payment period models. See Grossman and Policano (1975) and the references cited there. In such models, rises in the cost of holding cash induce agents to shorten the pay interval and increase velocity. Such was the case in the German hyperinflation of 1923 when employers, to avoid the astronomical depreciation cost of holding marks to meet the wage bill, started paying workers daily rather than weekly.

[5] Recent theorizing on this point tends to support Petty. Thus Grossman and Policano (1975) model the case where households purchase some goods more frequently, and other goods less frequently, than they receive income. The model predicts that velocity will rise with purchasers' holdings of the first class of goods and fall with holdings of the second.

his "thriving men" hold so little money and spend it so fast. Certainly he did not see money as a luxury good whose velocity varies inversely with income. Pierre Boisguilbert (1704) enunciated the luxury-good hypothesis when, in his *Dissertation de la nature des richesses,* he declared that a coin spent by the poor has a velocity "a hundred times more" than one spent by the rich "in whose coffers large sums of money may remain useless for months and whole years at a time" (quoted in Hutchison [1988], p. 110).

The fourth velocity-determining variable in Petty's function is the distribution of income across socioeconomic classes. Because workers, landlords, and other income recipients have different pay periods, their transaction needs for cash per unit of income and thus their velocities differ. The economywide aggregate velocity figure, being a weighted average of the various velocities of the income groups, obviously depends on relative shares and the fraction of the money stock each group commands. Indeed, as mentioned below, Petty employed such weights to estimate aggregate velocity.

As for banks, Petty saw them as speeding up velocity. "Where there are banks," he wrote, "less money is necessary to drive a trade" (quoted in Wu [1939], p. 37). In his view, banks economize on the use of money—that is, gold coin—by issuing money substitutes in the form of notes. The notes effectuate transactions formerly mediated by gold, thus freeing the latter for other uses. With less money required to circulate trade, the velocity of the remaining stock increases. It is easy to understand why Petty regarded the spread of banks as a form of technological progress. Banks saved on scarce metallic reserves, thus enabling a given volume of transactions to be supported by a smaller gold stock or a larger volume of transactions by a given stock. "A bank," he wrote, "doth almost double the effect of our coined money" (quoted in Spengler [1954], p. 415). In doing so, banks helped reduce the real resource cost of effecting the nation's business.

Having specified velocity's determinants, Petty used the velocity concept, together with the exchange identity $MV = Y$, to estimate the minimum amount of money required to finance a given volume of income and trade. Assuming a national income of £40 million, he reckoned that, if money traveled a weekly circuit from employers to workers and back, annual velocity would be 52, thus rendering a money stock of £40/52 million sufficient to meet the needs of trade. If, instead, the income circuit involved quarterly rent and tax payments alone, then velocity would be 4. In this case, a money stock of £10 million would be required to accommodate trade. Finally, if money had to traverse both circuits at once, aggregate velocity V, the average of the individual circuit velocities V_1 and V_2 weighted by their circuit money shares $M_1/(M_1 + M_2)$ and $M_2/(M_1 + M_2)$ would be roughly 7.5 and the corresponding required money stock M would total approximately £5.5 million. That is, $V = V_1[M_1/(M_1 + M_2)] + V_2[M_2/(M_1 + M_2)] = 52\{[40\text{m}/52]/[(40\text{m}/52) + 10\text{m}]\} + 4\{10\text{m}/[(40\text{m}/52) + 10\text{m}]\} = 7.429 \approx 7.5$ and $M = Y/V = 40\text{m}/7.5 =$

£5.333 million ≈ £5.5 million.[6] In still another calculation, Petty, using agricultural income as a proxy for national income, estimated money's annual income velocity to be 10.

2. JOHN LOCKE'S FUNCTION

John Locke's (1632–1704) place in the history of velocity theory is secured by three contributions.[7] He was the first to explicitly relate velocity functions to the underlying money demand functions of cashholders, a point barely hinted at by Petty. In his *Some Considerations of the Consequences of the Lowering of Interest and Raising the Value of Money* (1691) he sought to "consider how much money [defined by him as gold coin] it is necessary to suppose must rest constantly in each man's hand, as requisite to the carrying on of trade" (quoted in Vickers [1959], p. 58). For laborers and their employers, he estimated this amount to be a fiftieth part of wages, for brokers (i.e., merchants and tradesmen) a twentieth part of their annual returns, and for landlords and their tenants one-fourth of the yearly revenue of land. Elsewhere, however, he halved these requisite amounts, presumably on the grounds that credit could substitute for money in driving trade.

Second, while retaining Petty's income, pay period, and distributional arguments, he introduced a new variable, the interest rate, into the velocity function. He viewed the interest rate as measuring the opportunity cost of holding money, a noninterest-earning asset, instead of assets yielding an explicit rate of return. A fall in the rate, he argued, lowers the cost of holding idle balances. In so doing it increases the quantity of such balances demanded. As a result, bankers and other monied men are, in his words, "content to have more money lie dead by them" when rates fall (quoted in Holtrop [1929], p. 506). The consequent rise in the quantity of money held per unit of income lowers velocity.

Motivating Locke's analysis of velocity's interest rate determinant was his strong opposition to contemporary English proposals for a legal 4 percent interest rate ceiling. As noted by Leigh (1974), he feared that the imposition of a below-equilibrium rate would depress output and employment in two ways. First, it would deprive the country of the money needed to drive trade. By precipitating capital outflows financed by corresponding drains of gold, the artificially low rate would create a shortage of money as investors moved their funds abroad to realize higher foreign yields. Second, it would lower velocity by reducing the cost of holding idle balances in the manner described above. Together, the velocity and money-stock reductions would constitute a contraction

[6] Petty actually expressed the required money stock M as half the sum of the individual circuit stocks M_1 and M_2. That his expression is equivalent to the ratio of income to aggregate velocity $M = Y/V$ can be seen by substituting into the latter equation his assumptions $V = (M_1V_1 + M_2V_2)/(M_1 + M_2)$ and $M_1V_1 = M_2V_2 = Y$ to obtain $M = (M_1 + M_2)/2$.

[7] On Locke, see Holtrop (1929), Leigh (1974), and Vickers (1959).

of aggregate demand. With English prices imperfectly flexible or exogenously given from world markets by purchasing power parity considerations, the aggregate demand contractions would cause corresponding contractions in real activity. For this reason, Locke advocated removal of rate ceilings so that money, velocity, spending, output, and employment could return to their equilibrium levels.

Third, Locke said that velocity could be speeded up if there were fewer middlemen standing between producers and consumers. Here is the origin of the notion that velocity varies inversely with the number of stages of production separating raw materials from finished product and so increases with the degree of vertical integration.

Like Petty, Locke regarded velocity increases as beneficial. Such increases either reduced the quantity of money required to support a given volume of trade or raised the volume of trade that could be supported by a given stock of money. To this end, he recommended a shortening of pay periods. By enhancing velocity, such shortening would be "better for trade, and consequently for everybody (for more money would be stirring and less would be necessary to do the business)" (quoted in Hutchison [1988], p. 65). He failed, however, to note the equivalence of velocity increases and money-stock increases in raising the price level in a closed economy. Not until 1755 was this equivalence articulated in published form. And the first economist to do so was Richard Cantillon (1680–1734), the foremost velocity theorist of the eighteenth century.

3. RICHARD CANTILLON

The prize for introducing the largest number of variables into an eighteenth-century velocity function goes to Cantillon.[8] Certainly his function, as presented in his 1755 *Essai sur la nature du commerce en général,* was the most elaborate to be found in the literature of that era. As the premier economist of his day, he possessed a profound understanding of the real forces shaping velocity. And as a banker and foreign exchange specialist who amassed two fortunes speculating on the South Sea Bubble and Mississippi System schemes, he also had a keen appreciation of the monetary and financial forces involved. Some of these forces—urbanization, monetization, growing financial sophistication, advent of new credit facilities and the like—pertained to France's emerging transition from a predominantly agricultural economy to a mercantile and manufacturing one. Others were stressed by his predecessors, Petty and Locke, whose metallist conception of money he also shared. All were assimilated into Cantillon's velocity analysis. Thus his velocity function contains the following arguments: (1) income, (2) frequency of payments, (3) size of payments, (4) stages of production, (5) interest rates, (6) distribution among social classes,

[8] On Cantillon, see Bordo (1983) and Murphy (1986).

(7) banking, (8) trade credit, (9) extent of barter, (10) urbanization (monetization), (11) hoarding, (12) uncertain expectations of the future, and (13) minimum denomination restrictions on asset purchases. Of these, the first seven he took from Petty and Locke. The last six, however, were original with him.

According to Cantillon, urbanization, hoarding, uncertainty, and minimum denomination restrictions all tend to reduce velocity. Trade credit and barter, on the other hand, enlarge it. Urbanization—the growth of cities and towns—expands the sphere of money transactions relative to barter transactions and production for one's own use. It does so because "all country produce is furnished by labour which may . . . be carried on with little or no actual money" whereas "all merchandise is made in cities or market towns by the labour of men who must be paid in actual money" (1964, p. 143). The resulting monetization of economic activity boosts the demand for cash per unit of income so that velocity falls. Hoarding likewise slows velocity as "many miserly and timid people bury and hoard cash for considerable periods" (p. 147). Similarly, uncertainty induces people to "keep some cash in their pockets or safes against unforeseen emergencies and not to be run out of money" (p. 147). The consequent rise in the precautionary demand for cash lowers velocity. Finally, minimum denomination restrictions, which establish lower limits or floors to the size of asset purchases, retard velocity by compelling agents to "keep out of circulation small amounts of cash until they have enough to invest at interest or profit" (p. 147).

Working in the opposite direction is the use of trade credit, clearing arrangements, and other substitutes for money.[9] These items, by allowing businessmen to dispense with money in financing ongoing commercial transactions and by permitting them to cancel claims against each other so that only net balances need be paid, "seem to economize much cash in circulation, or at least to accelerate its movement" (p. 141). Thus it "is not without reason that it is commonly said that commercial credit makes money less scarce." The same is true of barter which likewise reduces the need for cash and so raises velocity.

Taking these factors into account, Cantillon estimated income velocity to be 9. With all determinants of money demand considered, he calculated that a country's money stock M should be one-third of landowners' annual rent R. Since he reckoned rent to constitute one-third of the value of annual produce Y, he obtained velocity V as $V = (R/M)(Y/R) = Y/M = 3 \times 3 = 9$. His estimate, which like Petty's used farm income as a proxy for national income, was close to Petty's estimate of 10.

[9] Cantillon's analysis thus implies a U-shaped pattern over time for velocity in developing economies. At first, increasing monetization causes velocity to fall. Thereafter, increasing financial sophistication and the growth of money substitutes cause velocity to rise. Recent work in the Cantillon tradition offers strong empirical support for this hypothesis. See the studies of Ireland (1991) and Bordo and Jonung (1987).

4. INTRODUCTION OF INFLATIONARY EXPECTATIONS INTO THE FUNCTION

Cantillon, in his list of velocity determinants, had neglected to include inflationary expectations. This step was taken in the first three decades of the nineteenth century by economists who had witnessed the debacle of the French assignats (1794–1796). This episode, Western Europe's first hyperinflation, left a lasting impression. It revealed that excessive monetary expansion, by generating anticipations of future inflation, could precipitate a flight from cash and a corresponding rise in velocity such that prices would rise even faster than the money stock. It was an easy task to incorporate this lesson into velocity theory.

Henry Thornton (1760–1815), in his 1802 *Paper Credit of Great Britain,* was the first to do so. He said that when cashholders extrapolate observed current falls in the purchasing power of the currency into expected future falls, the expectations themselves will speed up velocity and quicken the currency's depreciation (p. 108). Using this insight, he explained how the excessive issue of French assignats had "operated on their credit, and became a very powerful cause of their depreciation" (p. 233).

J. B. Say (1776–1832), in his 1803 *Traité d'économie politique,* likewise attributed the assignats' "prodigious" rate of turnover to cashholders' attempts to rid themselves of a depreciating currency as fast as possible.[10] The same point was made by Simonde de Sismondi (1773–1842) in his 1827 *Nouveaux principes*. He contrasted (1) distrust of the future stability of the real economy with (2) distrust of the future value of the currency. The first type of distrust, he said, tends to lower velocity whereas the second type tends to raise it.[11]

But the most precise account of the impact of inflationary expectations on velocity and thus on the inflation rate itself came from Nassau Senior (1790–1864) in his 1830 *Three Lectures on the Cost of Obtaining Money*. Referring to the depreciation of the assignats stemming from the loss of confidence in their future value, he wrote: "The prices of commodities rose in proportion, not merely to the existing depreciation [true of course by definition], but to the well-founded apprehension of a still further depreciation" (quoted in Eshag [1963], p. 16). The result of such perceptions of the likely future depreciation of the currency was exactly what one would expect: "Everybody taxed his ingenuity to find employment for a currency of which the value evaporated from hour to hour. It was passed on as it was received, as if it burned everyone's hands who touched it" (quoted in Eshag, p. 16). After Senior's exposition, it would be hard indeed to claim that anticipated inflation had been left out of the velocity function.

[10] See Holtrop (1929), p. 519.
[11] See Holtrop (1929), p. 520.

5. OTHER NINETEENTH-CENTURY CONTRIBUTIONS

The preceding hardly begins to exhaust the wealth of pre-twentieth century writing on velocity functions. Holtrop (1929, pp. 518–20) notes that in the nineteenth century alone, writers John Stuart Mill, Thomas Tooke, Christian von Schlozer, Heinrich Storch, Karl Heinrich Rau, and Johann Karl Rodbertus all discussed velocity functions. By far the most important contributions, however, came from Henry Thornton and Knut Wicksell. Towering above the rest, their pathbreaking work constitutes the peak achievement of velocity-function analysis prior to Irving Fisher.

We have already met Thornton, the pioneer of inflation-expectations analysis. This contribution alone would warrant his mention in any survey of velocity theory. But he contributed much more to the theory than merely introducing an expectations argument into the velocity function. Advancing beyond his predecessors, he defined the relevant monetary aggregate as the total stock of circulating media rather than its narrow specie component. Moreover he was the first to specify how two variables, namely (1) the composition of the payments media and (2) the state of business confidence, influenced velocity. He had observed how these variables operated to produce the velocity swings of the turbulent 1790s and sought to correct the tendency of his predecessors to neglect them. In addition, as a banker and financial expert who had connections with correspondent banks throughout the country, he was particularly alert to the fundamental changes occurring in the English credit mechanism (Hayek [1939], p. 38). These changes, which included rapid growth in the number of country banks, the increasing use of checks, the establishment of the London Clearing House, and the emergence of the Bank of England as the central bank and lender of last resort, induced him to extend Cantillon's analysis of the velocity-enhancing effects of financial innovation.

His first task was to show how the composition of the payments media enters the velocity function. He argued that the total means of payment consists of coin, banknotes, and bills of exchange. Each circulates with a speed that varies inversely with the opportunity cost of holding it. This cost is measured as the differential between the instrument's own rate of return and the prevailing market rate. The lower the own rate relative to the prevailing rate, the greater the cost of holding the instrument and the stronger the incentive to spend it instead. Thus coin and banknotes, which yield no interest, circulate faster than interest-bearing bills of exchange. Add to this the fact that gold coins are hoarded more than notes and so circulate more slowly than the latter in times of panic and it becomes apparent that different instruments possess different velocities. It follows that aggregate velocity, the weighted average of the component velocities, depends on the composition of the payments media. When that composition changes, so does aggregate velocity.

Thornton next identified as a determinant of velocity the state of mercantile confidence arising from general business and financial conditions. Confidence refers to the certainty of agents' beliefs that receipts will match expenditures, thus obviating the need to hold emergency reserves. A high state of confidence produces a low demand for precautionary balances and a rapid velocity. Conversely, a low state of confidence stemming from distrust and alarm produces a high demand for precautionary balances and a slow velocity. Thornton summarizes:

> A high state of confidence serves to quicken [money's] circulation. . . . [It] contributes to make men provide less amply against contingencies. At such a time, they trust, that if the demand upon them for a payment, which is now doubtful and contingent, should actually be made, they shall be able to provide for it at the moment; and they are loth to . . . make the provision much before the period at which it shall be wanted. When, on the contrary, a season of distrust arises, prudence suggests, that the loss of interest arising from a detention of notes for a few additional days should not be regarded. . . . Every one fearing lest he should not have his notes ready when the day of payment should come, would endeavor to provide himself with them beforehand. (1939, pp. 96–98)

Thornton concluded that no single money stock always supports the same level of nominal activity. Since velocity fluctuates with the state of confidence, more money is required to effect a given volume of transactions when confidence is low than when it is high.

As for financial innovations, Thornton saw them as boosting money's turnover rate. He explained how the invention of the clearinghouse, with its mutual cancellation of claims, economized on the amount of money required to settle transactions. And he cited still other developments—correspondent banking arrangements, improved communications, and the like—that had the same effect. Like Cantillon, he drew the conclusion that such devices economize on the use of money and speed up velocity.

Thornton's analysis of financial innovations influenced his contemporaries. Classical quantity theorists, notably David Ricardo (1772–1823) and the authors of the 1810 *Bullion Report,* endorsed it. But so too did anti-quantity theorists. Thornton's work initiated the notion that monetary contraction stimulates the very financial innovation and compensating rise in velocity that offsets the initial monetary contraction. Indeed, nobody stated this idea better than Thornton himself. Let such a contraction occur, he said, and the resulting "great limitation of the number of bank notes would, therefore, lead . . . to some new modes of economy in the use of the existing notes: the effect of which economy on prices would be the same, in all respects, as that of the restoration of the usual quantity of bank notes" (p. 119). Coming from a leading classical quantity theorist, this was a startling admission indeed.

6. KNUT WICKSELL

Thornton's work illustrates the flourishing of velocity analysis at the century's beginning. Knut Wicksell's work illustrates its vitality at the century's end. Thus Wicksell (1851–1926) devoted the entire 30-page Chapter 6 of his 1898 volume *Interest and Prices* to a discussion of the determinants of "The Velocity of Circulation of Money."[12]

He began by defining money as consisting solely of gold coin. His definition rules out notes and deposits, which he treated as credit instruments that raise the "virtual velocity" of money. He explained that such instruments, when used in payment, free an equivalent amount of coin to facilitate purchases elsewhere. In so doing, they effect a virtual turnover of coin and thus raise the velocity of money. Having made this point, he next defined velocity as the inverse of the "average period of idleness" or "interval of rest" of coin. In so doing, he evoked the notion of money demand as velocity's reciprocal. Finally, he identified at least five determinants of velocity.

The first is a transactions demand for cash to bridge the gap caused by the lack of synchronization between receipts and expected payments. The second consists of a precautionary demand to meet unexpected payments. Although Cantillon and Thornton had incorporated these demands into the velocity function before Wicksell, they had not derived them from probability theory. Wicksell, however, did so. Inspired by Francis Edgeworth's (1888) application of probability theory to banking, he argued that the frequency with which cash shortfalls of various amounts are likely to occur could be described by a probability distribution whose mean represents expected shortfalls and whose dispersion or spread measures the risk that actual shortfalls will be larger than expected.

For his dispersion parameter, Wicksell used a statistic called the probable deviation. Equaling 0.6745 times the standard deviation, this statistic has the following property. When positioned on both sides of the mean, it includes half of the elements of the distribution. That is, half the elements lie within, and half without, one probable deviation of the mean. It follows that cashholders wishing to secure themselves against a 50–50 chance of an unexpected shortage of cash will hold precautionary balances equivalent to one probable deviation. And cashholders with still greater degrees of risk aversion will hold even more. Wicksell explained:

> Suppose that experience has shown that . . . the excess of payments over simultaneous receipts . . . tends to oscillate from year to year about a certain mean value, a. Let the "probable deviation" be b: this means that the odds are even . . . in favor of the payments over the period in question lying between

[12] On Wicksell's velocity analysis, see Laidler (1991), pp. 123–29, and Uhr (1960), pp. 220–24.

> $a + b$ and $a - b$. If the business man is satisfied with this so-called simple margin of safety, he must have by him a cash holding of $a + b$. But if he demands a greater degree of security against the possible exhaustion of his till, his cash holding must of course be somewhat larger. With a cash holding of as little as $a + 2b$, the betting on the total exhaustion of his till . . . would, according to the laws of probability, be more than 9 to 1; with a cash holding of $a + 3b$ it would be more than 44 to 1; and with one of $a + 5b$ it would be more than 2600 to 1, *i.e.* the till would be exhausted only about *once in three thousand five hundred years.* (1936, pp. 57–58)

From this analysis it follows that the distribution's mean and probable deviation parameters a and b constitute arguments of the velocity function.

Wicksell also entered into his velocity function what he called simple trade credit between businessmen. This variable bears a positive sign since the availability of trade credit reduces the amount of cash businessmen need to hold relative to income to finance regular recurring transactions. By far the most important determinant, however, is "organized credit" involving the operations of commercial banks. Banks, Wicksell argued, boost velocity by multiplying the volume of credit instruments—notes and deposits—erected on a given money base. Once created, the notes and deposits mediate additional exchanges. In so doing, they raise the volume of transactions per unit of money (gold) and so enhance velocity.

To illustrate how banks evolved to raise money's efficiency in supporting more transactions, Wicksell sketched the following hypothetical sequence of events. First, the emergence of banks allows agents to dispense with money (gold) holdings by converting them into credit instruments instead. The resulting flow of gold into banks continues until those institutions eventually hold the entire stock of the precious metal as reserves.

At the same time, bankers discover that three considerations—(1) the regularity of chance or law of large numbers, (2) the interdependence of firms such that payments of one set of bank customers are the receipts of another, and (3) the practice of settling offsetting claims of different customers of the same bank through bookkeeping transfers from one account to another rather than through the use of money—permit them to operate with fractional reserves. These same inducements spur banks to form clearinghouse associations. Scale and settlement economies also provide incentives to consolidate the banking system's reserve holdings in a centrally located bank.

Together, these developments tend to reduce the fractional reserve ratio to negligible proportions. The ensuing potentially unlimited expansion of the stock of credit instruments mediates a much larger volume of trade than would the gold itself if it were used directly in making payments. Here is the essence of Wicksell's doctrine that bank notes and deposits raise the "virtual" velocity of gold reserves resting in bank vaults with an actual physical velocity of zero.

Finally, Wicksell saw velocity as a function of the difference between the market (loan) and natural (equilibrium) rates of interest. In his famous cumulative process analysis of price-level movements, he argued that excesses of the natural rate over the market rate produce corresponding excesses of desired investment spending over desired saving. As a result, the demand for loanable funds to finance investment exceeds the amount of such funds voluntarily supplied by savers. Banks supply the remainder through credit (i.e., note and deposit) creation. The consequent rise in the volume of bank credit erected on a given money stock constitutes a rise in the virtual velocity of that stock. This rise in turn puts upward pressure on prices. Thus price-level movements emanate from rate differentials—more precisely from natural rate movements given the market rate—operating through the velocity function.

This conclusion—that velocity rises with the natural rate-market rate differential—is entirely the result of Wicksell's definition of money to exclude notes and deposits. Had he included those items in his definition, he would have seen the rate differential as boosting the money stock rather than its velocity. This point notwithstanding, he provided the most complete analysis of velocity and its determinants since Thornton. His work is proof positive that a sophisticated literature on the subject existed before Fisher.

7. CONCLUSION

The preceding discussion has concentrated exclusively on major landmarks in the evolution of velocity functions. In so doing, it has no doubt neglected other milestones. For example, nothing was said about Alfred Marshall's work on velocity in the 1870s and 1880s. D. P. O'Brien (1981, pp. 58–59) notes that Marshall (1824–1924) followed Thornton and the *Bullion Report* in attributing velocity's movements to fluctuations in the state of confidence and economic activity, to financial innovation and the growth of money substitutes, to technical progress in production, and to changes in transportation, communications and the like.[13] Much like Wicksell, Marshall viewed bank deposits not as money but rather as a device for economizing on its use and speeding up velocity.

Nor was anything said about Thomas Attwood's 1817 distinction between income velocity and transaction velocity. The distinction between the two velocity concepts is often traced to Arthur Cecil Pigou, who discussed it in his 1927 book *Industrial Fluctuations.* It originates, however, with Attwood, who estimated income velocity at 4 and transaction velocity at 50 per annum.[14]

Nor was mention made of the pathbreaking 1895 statistical work of Pierre des Essars. His cross-country time-series estimates of the deposit turnover rates

[13] On Marshall, see also Eshag (1963), pp. 2–18, and Whitaker (1975), pp. 172–73.
[14] On Attwood, see Marget (1938), p. 358.

of continental European banks for the period 1884–1894 anticipated all later empirical work on velocity. In essence, he computed deposit velocity as the ratio of bank debits to average balances in deposit accounts.[15] Irving Fisher (1963, pp. 63 and 87) cited his findings as evidence that population density and anticipated inflation act to raise velocity.

Also unmentioned was E. W. Kemmerer's 1907 attempt to verify the Thornton-Marshall hypothesis that velocity varies directly with the state of business confidence. Not the least of Kemmerer's achievements was his construction, from data on business failure rates and the dollar liabilities of failed firms, of an index of business distrust. Movements of the index, he thought, accounted for corresponding movements in velocity.

Finally, nothing was said about early versions of the $MV = Py$ equation of exchange. The pre-Fisher literature boasts at least 14 such equations.[16] All contain at least one velocity variable and two contain separate velocity terms for each component of the payments media.

Nevertheless, enough has been said to document the main contention of the article, namely that velocity functions long predate Irving Fisher and his recent counterparts. This is not to say, however, that older and modern versions of the function are identical. On the contrary, modern versions tend to be stated mathematically, often in the form of least-squares regression equations yielding numerical estimates of the equation's coefficients.[17] By contrast, older versions of the function tended to be expressed verbally rather than algebraically.

Still, the basic notion of a stable functional relationship between velocity and its independent determining variables has remained unchanged since the time of Petty. So too has the practice of specifying the function's arguments. Thus Petty's successors in the eighteenth and nineteenth centuries completed his list of velocity determinants and bequeathed it to twentieth-century writers. Seen in this perspective, the work of Fisher, Friedman, and other modern velocity theorists constitutes the culmination of a long tradition rather than the beginning of a new one.

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[15] On Des Essars' estimates, see Kemmerer (1907), pp. 115–16.

[16] On pre-Fisher versions of the equation of exchange, see Humphrey (1984) and the references cited there.

[17] For examples, see Bordo and Jonung (1987), pp. 32–39, and Goldfeld (1973), p. 633.

REFERENCES

Attwood, T. *Letter to the Right Honorable Nicholas Vansittart, on the creation of money, and on its action upon national prosperity.* Birmingham, 1817.

Boisguilbert, Pierre de. *Dissertation de la nature des richesses, de l'argent et des tributs*. 1704.

Bordo, Michael D. "Some Aspects of the Monetary Economics of Richard Cantillon," *Journal of Monetary Economics,* vol. 12 (August 1983), pp. 235–58.

__________, and Lars Jonung. *The Long-Run Behavior of the Velocity of Circulation.* Cambridge: Cambridge University Press, 1987.

Briscoe, John. *A Discourse of Money.* London, 1696.

Cantillon, Richard. *Essai sur la nature du commerce en général*. 1755. Edited with English translation and other material by Henry Higgs, London: Macmillan, 1931, reprinted, New York: A. M. Kelley, 1964.

Cramer, J. S. "Velocity of Circulation," in Peter Newman, Murray Milgate, and John Eatwell, eds., *The New Palgrave Dictionary of Money and Finance,* Vol. 3. London: Macmillan, 1992.

Des Essars, Pierre. "La vitesse de la circulation de la monnaie," *Journal de la Société de Statistique de Paris,* vol. 36 (1895), pp. 143–51.

Edgeworth, Francis Y. "The Mathematical Theory of Banking," *Journal of the Royal Statistical Association,* vol. 51 (March 1888), pp. 113–26.

Eshag, Eprime. *From Marshall to Keynes; An Essay on the Monetary Theory of the Cambridge School*. Oxford: B. Blackwell, 1963.

Fisher, Irving. *The Purchasing Power of Money*. 1911. New and revised edition, New York: Macmillan, 1913, reprinted, New York: A. M. Kelley, 1963.

Friedman, Milton. "The Quantity Theory of Money—A Restatement," in Milton Friedman, ed., *Studies in the Quantity Theory of Money*. Chicago: University of Chicago Press, 1956.

Goldfeld, Stephen M. "The Demand for Money Revisited," *Brookings Papers on Economic Activity,* 3:1973, pp. 577–638.

Grossman, Herschel I., and Andrew J. Policano. "Money Balances, Commodity Inventories, and Inflationary Expectations," *Journal of Political Economy,* vol. 83 (December 1975), pp. 1093–1112.

Hayek, Friedrich A. von. Introduction to H. Thornton, *An Enquiry into the Nature and Effects of the Paper Credit of Great Britain.* 1802. Reprinted, New York: Farrar & Rinehart, 1939.

Heckscher, Eli F. *Mercantilism*, Vol. 2. London: George Allen and Unwin, 1935. Revised 2d ed. translated by Mendel Shapiro and edited by E. F. Söderlund, 1955, reprinted, New York and London: Garland Publishing, 1983.

Holtrop, Marius W. "Theories of the Velocity of Circulation of Money in Earlier Economic Literature," *Economic History* (Supplement to the *Economic Journal*), vol. 1 (January 1929), pp. 503–24.

Humphrey, Thomas M. "Algebraic Quantity Equations Before Fisher and Pigou," Federal Reserve Bank of Richmond *Economic Review,* vol. 70 (September/October 1984), pp. 13–22.

Hutchison, Terence W. *Before Adam Smith: The Emergence of Political Economy, 1662–1776*. New York: B. Blackwell, 1988.

Ireland, Peter N. "Financial Evolution and the Long-Run Behavior of Velocity: New Evidence from U.S. Regional Data," Federal Reserve Bank of Richmond *Economic Review,* vol. 77 (November/December 1991), pp. 16–26.

Kemmerer, Edwin W. *Money and Credit Instruments in their Relation to General Prices.* New York: Henry Holt, 1907.

Laidler, David. *The Golden Age of the Quantity Theory*. New York: Philip Allan, 1991.

Leigh, Arthur H. "John Locke and the Quantity Theory of Money," *History of Political Economy,* vol. 6 (Summer 1974), pp. 200–219.

Locke, John. *Some Considerations of the Consequences of the Lowering of Interest and Raising the Value of Money.* 1691. In Vol. 5 of *The Works of John Locke*, pp. 3–116. London, 1823, reprinted, Aalen (Germany), 1963.

Marget, Arthur W. *The Theory of Prices,* Vol. 1. New York: Prentice Hall, 1938.

Marshall, Alfred. *Money, Credit and Commerce*. London: Macmillan, 1923.

Miller, Merton H., and Charles W. Upton. *Macroeconomics: A Neoclassical Introduction*. Chicago: University of Chicago Press, 1974 and 1986.

Murphy, Antoin E. *Richard Cantillon: Entrepreneur and Economist*. New York: Oxford University Press, 1986.

O'Brien, Denis P. "A. Marshall, 1842–1924," in Denis P. O'Brien and John R. Presley, eds., *Pioneers of Modern Economics in Britain*. Totowa, N.J.: Barnes and Noble Books, 1981.

Pesek, Boris P. "Monetary Theory in the Post-Robertson 'Alice in Wonderland' Era," *Journal of Economic Literature,* vol. 14 (September 1976), pp. 856–84.

Petty, William. *Treatise of Taxes and Contributions*. 1662. In Charles H. Hull, ed., *The Economic Writings of Sir William Petty,* Vol. 1. Cambridge, 1899, reprinted, New York: A. M. Kelley, 1964.

__________. *Verbum Sapienti*. 1664. In Charles H. Hull, ed., *The Economic Writings of Sir William Petty,* Vol. 1. Cambridge, 1899, reprinted, New York: A. M. Kelley, 1964.

Pigou, Arthur C. *Industrial Fluctuations*. London: Macmillan, 1927.

Rodbertus, Johann K. *Die Preussische Geldkrisis*. 1845.

Roncaglia, Alessandro. *Petty: The Origins of Political Economy*. Armonk: M. E. Sharpe, Inc., 1985.

Say, Jean-Baptiste. *Traité d'économie politique*. Paris: Degerville, 1803.

Schumpeter, Joseph A. *History of Economic Analysis*. New York: Oxford University Press, 1954.

Senior, Nassau W. *Three Lectures on the Cost of Obtaining Money.* London: John Murray, 1830.

Sismondi, Simonde de. *Nouveaux principes d'économie politique*, 2d ed. Paris: Delauny, 1827.

Spengler, Joseph J. "Richard Cantillon: First of the Moderns," *Journal of Political Economy*, vol. 62 (August and October 1954), pp. 281–95 and 406–24.

Thornton, Henry. *An Enquiry into the Nature and Effects of the Paper Credit of Great Britain.* 1802. Edited with an introduction by F. A. von Hayek, New York: Farrar & Rinehart, Inc., 1939.

Uhr, Carl G. *Economic Doctrines of Knut Wicksell*. Berkeley: University of California Press, 1960.

Vickers, Douglas. *Studies in the Theory of Money, 1690–1776.* Philadelphia: Chilton, 1959.

Viner, Jacob. *Studies in the Theory of International Trade*. New York: Harper and Bros., 1937.

Whitaker, John K., ed. *The Early Economic Writings of Alfred Marshall, 1867–1890*, Vol. I. New York: Free Press, 1975.

Wicksell, Knut. *Interest and Prices*. 1898. Translated by R. F. Kahn, with an introduction by Bertil Ohlin, London: Macmillan, 1936.

Wu, Chi-Yuen. *An Outline of International Price Theories*. London: George Routledge & Sons, 1939.

Public Investment and Economic Growth

William E. Cullison

In the years following World War II, the papers of any major city . . . told daily of the shortages and shortcomings in the elementary municipal and metropolitan services. The schools were old and overcrowded. The police force was under strength and underpaid. The parks and playgrounds were insufficient. Streets and empty lots were filthy, and the sanitation staff was underequipped and in need of men. . . . Internal transportation was overcrowded, unhealthful, and dirty. . . . The discussion of this public poverty competed, on the whole successfully, with stories of ever-increasing opulence in privately produced goods.

J. K. Galbraith (1958), p. 253

After a lively debate in the late 1950s and early 1960s about the merits of John Kenneth Galbraith's theory of social balance (*The Affluent Society*), the economics profession dismissed (or forgot) Galbraith's admonitions about the perils of neglecting the public infrastructure. David Aschauer, however, rekindled a great deal of interest in the efficiency of public capital spending by showing that additional spending by governments for nondefense capital goods apparently had a very large positive effect on private productivity and, hence, output.

Although economists were not surprised that public infrastructure spending could promote private output growth, the magnitude of the effect found by Aschauer was startling to most. Aschauer estimated that additional public capital spending would increase the output of private firms by more than 1 1/2 times as much as would an equivalent dollar increase in the firms' own capital stock.

A Congressional Budget Office (CBO) study of the effects of public infrastructure spending concluded that Aschauer's results merited some skepticism because "the statistical results are not robust [and] there is a lack of corroborating evidence" (CBO 1991, p. 25). The CBO observed that other empirical

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research, including cost-benefit studies, found private output to be more responsive to investments in private capital than to investments in public capital. There were a number of other studies in response to Aschauer.[1] Some of the studies found the effects of public investment on economic growth to be smaller than Aschauer found them to be.

Alicia Munnell, formerly of the Federal Reserve Bank of Boston, tried a different statistical approach to measuring the productivity of government spending. Although Munnell (1990), like Aschauer, used a production function approach to evaluate the effects of government infrastructure spending, she approached the problem by estimating her production functions from cross-sectional state-by-state data.

Munnell (1990) used estimates of gross state product and of private inputs of capital to develop estimates of public capital stocks for 48 states over the 1970–86 time period. She then used the state-by-state data to estimate the production functions, concluding that "the evidence seems overwhelming that public capital has a positive impact on private output, investment, and employment" (p. 94).

Munnell's (1990) estimates of the relative effects of public investment were smaller than those made by Aschauer. Hulten (1990), commenting on Munnell, observed that her findings of smaller relative effects were consistent with other studies that analyzed state data but that her findings differed sharply from the results of studies that were based upon time series.

The CBO (1991), in summarizing the results of cost-benefit studies, noted that there has been little support for the view that across-the-board increases in public capital programs have remarkable effects on economic output. Rather, they concluded that "cost-benefit analysis paints a fairly consistent picture of high returns to maintaining the existing stock of physical infrastructure and to expanding capacity in congested urban highways and runway traffic and air traffic control at major airports" (p. 40).

Indeed, it seems clear that a sensible approach to spending for government infrastructure would not include across-the-board increases in government investment spending as a means of stimulating economic growth. Rather, any project should stand on its own merits and be able to withstand a cost-benefit analysis. Given this caveat, however, there is interest in what sorts of public investment spending would tend to have the most impact on economic growth. If the most important sectors can be isolated, project proposals within those categories can be given priority in setting governmental budgetary goals.

The types of government infrastructure spending evaluated by Aschauer and Munnell and commented upon by the Congressional Budget Office fall

[1] The many evaluations of Aschauer's results include Aaron (1990), Hulten (May 1990), Hulten and Schwab (1991), Jorgenson (1991), Rubin (1991), and Tatom (1991).

into the category of physical capital investment, but government also invests in its people. This latter type of investment produces human capital if it improves the job skills (potential and actual productivity) of its citizens.

This article examines the effects on economic growth of government investment in both physical (nonhuman) and human capital, paying particular attention to the relative social returns of investments in human capital. As noted earlier, Aschauer, Munnell, and others use the aggregate production function approach to evaluating the effects of government spending. While such a method works well for evaluating the effects of government spending for physical capital, it is not clear that it is equally appropriate for human capital.

Investments in human capital may affect aggregate production possibilities in ways that are far more complicated than investments in physical capital. In the case of physical capital, it seems reasonable to assume that the government stock of physical capital enters an aggregate production function in a manner that is symmetric to, or at least quite similar to, private capital. It is far more difficult to isolate, a priori, the role played by government spending for human capital in an aggregate production function.

Fortunately, other statistical techniques are available to evaluate the effects of government spending on economic growth (see Cullison [1993]). The methodologies used for this article are Granger-causality tests and simulations from a vector autoregressive (VAR) model. These techniques also have the advantage of requiring data only on investment flows rather than on stocks of capital. The data on investment flows are readily available in disaggregated form, thus facilitating the article's research plan of evaluating the effects of government spending by functional component.

The Granger-causality tests are used to determine what types of government investment spending are correlated with economic growth. The VAR model is a modified version of the model that Ireland and Otrok (1992) used to test the effects on economic growth of reducing the federal debt by cutting defense spending 20 percent over six years. The attractive feature of the VAR model is that it is atheoretical, imposing no structure on the data. As a result, it is not necessary to know exactly how government-provided human capital enters into the aggregate production function.

1. DATA ON GOVERNMENT SPENDING BY FUNCTION

The Department of Commerce publishes annual data on total government expenditures by function. The functions include the following: (1) expenditures for central executive, legislative, and judicial activities; (2) international affairs; (3) space; (4) national defense; (5) civilian safety; (6) education; (7) health and hospitals; (8) income support, social security, and welfare; (9) veterans benefits and services; (10) housing and community activities; (11) recreational and cultural activities; (12) energy; (13) agriculture; (14) natural resources; (15)

transportation; (16) postal service; (17) economic development, regulation, and services; (18) labor training and services; (19) commercial activities; (20) net interest paid; and (21) other.

When government investment is defined broadly, including both human and nonhuman capital, some items in most of the 21 categories denoted above probably would be classified as investment. Examples discussed below include government expenditures for space, national defense, civilian safety, education, health and hospitals, income support, veterans benefits, housing, agriculture, transportation, economic development, labor training, and commercial activities.

Government spending for space and national defense are likely to result in innovations useful for private production. In addition, much spending for space and national defense is contracted from private business. Government spending for civilian safety (police protection) provides an environment in which the private economy can operate efficiently. Government spending for education enhances human capital directly. One must at least be able to read, write, and cipher to hold even menial jobs in the current job market. Higher education is necessary to hold better jobs.

Government spending for health and hospitals also enhances human capital by curing maladies and injuries that can impair the productivity of individuals in the labor force. Income support programs such as aid to families can help to keep families together so that the children can become productive members of the labor force. Veterans benefits can help veterans reenter society as productive members by improving their physical and mental abilities. Housing expenditures, by providing housing for those who otherwise might not be able to afford it, can also enhance human capital by providing better-quality workers as well as providing the homeless an entry into the labor force (by providing them with an address).

Government spending for agriculture has for decades provided for basic agricultural research through the land grant college system and other arms of the Department of Agriculture. The fruits of such research are distributed throughout the country by the county agricultural extension system. Government spending for transportation enhances the productivity of the private economy by providing roads and other methods of getting products from producers to purchasers. Economic development programs can bring modern technology to less developed areas of the United States, thus putting formerly underutilized resources to work. Labor training programs can enhance human capital by improving the job skills of recipients of the program. Government commercial activities increase GDP in and of themselves and provide job experience to the work force.

Given that there are so many conceivable ways in which government spending can affect the private economy, this article will start by evaluating all 21 categories mentioned above to determine which actually had empirically observable effects. Intuitively, it would seem that education, space, national defense, civilian safety, transportation, agriculture and labor training would have the more pronounced effects on the growth of the private economy. As a preliminary procedure, a simplified version of the so-called Granger-causality test is used to determine those categories of government spending that seem most likely to have promoted economic growth.

2. GRANGER-CAUSALITY TESTS

A Granger-causality test examines whether the variable to be tested adds explanatory power to an existing relationship between one (or more) other variable(s) and its (their) lags. For example, if Z_t is a dependent variable and Z_{t-1} is the variable lagged one period, then $Z_t = f(Z_{t-1}, v_t)$ would represent a statistical relation between the two, when v_t is some unknown source of variation in the functional relation between them. For the Granger test, a known variable would be put into the functional relation of Z_t and Z_{t-1} with various lags and leads to determine whether it helped to reduce v_t.

The Granger-causality tests and the VAR simulations reported in this article are consistent in using only one lagged value of the relevant variables. The tests are restricted to one lagged value because the short span of the available annual data necessitates economizing on degrees of freedom—the shortage of degrees of freedom being especially acute for the VAR analysis.

Table 1 shows the results of a Granger-causality test run on each of the various classes of government expenditures. The equation used for the test is

$$\Delta \ln(Y_t) = a + b_1 \cdot \Delta \ln(Y_{t-1}) + b_2 \cdot \Delta \ln(X_{t-1}), \tag{1}$$

where Y is private gross domestic product, X is the government spending variable to be tested, and a, b_1, and b_2 are parameters to be estimated.[2] The notations "Δ" and "ln" represent, respectively, one-year first differences and natural logarithms, and the "t" subscripts are time indexes (in years). All variables are calculated in real (1987) dollars.

As the table shows, when X = ALL GOVERNMENT (total government spending), the t-statistic for the coefficient b_2 is 0.24, which is not statistically significant. However, education spending and spending for labor training, both of which enhance human capital, are statistically significant at the 5 percent level. Spending for income support, agriculture, civilian safety and net interest (negatively signed) are significant at the 15 percent level.[3]

[2] All estimations in this article use ordinary least squares (OLS).

[3] Transportation spending was not statistically significant, according to the Granger-causality tests. This result was somewhat surprising because the Finn analysis in this issue of the *Economic Quarterly* found highway capital to have a significant, if imprecise, effect on productivity. Finn's analysis, however, deals with the *stock* of highway capital, while this article deals with the *flow* of

Table 1 Granger-Causality Test Results, Government Purchases, 1955 to 1992

Equation: $\Delta \ln(Y_t) = a + b_1 \cdot \Delta \ln(Y_{t-1}) + b_2 \cdot \Delta \ln(X_{t-1})$,

where

Y = private gross domestic product in 1987 dollars,

X = government spending variable, measured in 1987 dollars, and

Δ = an operator designating the year-to-year first difference.

***X* Equals**	***b_2***	**"t" Value**	**Corrected R^2**
ALL GOVERNMENT	0.037	0.24	0.00
AGRICULTURE	0.03	1.87	0.05
CIVILIAN SAFETY	0.295	1.64	0.03
COMMERCIAL ACTIVITY	0.002	1.25	0.00
ECON. DEVELOPMENT	0.028	0.85	0.00
EDUCATION	0.269	2.33†	0.10
ENERGY	−0.011	−0.35	0.00
EXECUTIVE, LEGISLATIVE & JUDICIAL	0.03	0.26	0.00
HEALTH & HOSPITAL	−0.05	−0.54	0.00
HOUSING	0.007	0.179	0.00
INCOME SUPPORT	0.151	1.71	0.04
INTERNATIONAL AFFAIRS	0.004	0.10	0.00
LABOR TRAINING	0.080	2.74†	0.14
NATIONAL DEFENSE	−0.039	−0.60	0.00
NATURAL RESOURCES	0.018	0.36	0.00
NET INTEREST PAID	−0.12	−1.69	0.04
POSTAL SERVICE	0.003	0.28	0.00
RECREATION & CULTURE	−0.007	−0.121	0.00
SPACE*	0.020	1.12	0.06
TRANSPORTATION	0.080	0.745	0.00
VETERANS' BENEFITS	0.094	0.95	0.00
OTHER	0.082	1.28	0.00

* The effects of space spending are estimated over the 1961–92 period because space spending was zero in 1955–60.
† Statistically significant at the 5 percent level.

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government transportation spending. The Finn article also uses quite different statistical methodology. In addition, the transportation spending category used in this article includes expenditures for air, rail, water, and transit as well as highways. In deference to Finn's results, however, transportation spending was also examined with the VAR model, explained below. While the F-statistic for transportation with one lag indicated that transportation had a significant effect on real private GDP, the 95 percent confidence interval for the impulse-response function was practically symmetrical around zero, indicating no clear direction of the resulting change in the level of real private GDP.

Surprisingly, neither government spending for national defense nor space spending had statistically significant effects on the growth of the real private economy. In the case of space, the results may have been influenced by the shorter span of available data (1961–92).[4]

The results of the tests shown in Table 1 lead to the conclusion that the types of government spending most likely to have a statistically significant effect on economic growth are education and labor training. Thus, the analysis implies that the most efficient way to increase economic growth by increasing government spending would be to channel expenditures to well-thought-out education or labor training projects without ignoring projects in agriculture, civilian safety, and income support and policies designed to reduce government interest payments. The analysis, however, gives little information about the relative effectiveness of the different types of government spending. For that, it is necessary to move to the simulations from the VAR model mentioned earlier.

3. SIMULATIONS FROM A VAR MODEL

The Ireland-Otrok VAR model can be modified to test the effects of various types of government spending on economic growth. Since the analysis in Section 2, above, provides evidence that government expenditures for education and labor training have statistically significant impacts on private economic growth, the analysis that follows will examine those variables. In addition and for completeness, the economic effects of spending on agriculture, civilian safety, and income support will also be considered.[5]

The following VAR model is estimated over the 1953–91 time period.

$$X_t = \sum_{s=1}^{k} B_s \cdot X_{t-s} + u_t, \tag{2}$$

where

$$X_t = [RDEF_t, GSF_t, RDEBT_t, \mathrm{M2}_t, Y_t]. \tag{3}$$

RDEF is the growth rate of real defense spending, *RDEBT* is the growth rate of real government debt, *GSF* is defined as the growth rates of the various types of real government spending, *Y* is the growth rate of real private gross domestic product, and M2 is the growth rate of money.

[4] When space spending is combined with other government spending data and the resulting sums are evaluated for Granger-causality, the addition of space spending usually improves the statistical results. That cannot be said of defense spending, the addition of which usually lowers the statistical significance of the resulting aggregate.

[5] Since the Ireland-Otrok model includes a federal debt variable, net interest paid will not be evaluated separately.

Empirical Results from the Model

Table 2 reports some results of estimating the system of equations with one-, two-, and three-year lags. F-statistics were computed to evaluate the effects on real private GDP growth of spending on education, labor training, agriculture, income support, and civilian safety with one-, two-, and three-year lags. The table shows agriculture not to have been a statistically significant factor at any of the three lag lengths. The other four types of spending showed statistical significance at 5.5 percent or less. For the subsequent analysis/forecasts from the VAR, the lag length $k = 1$ was chosen to conserve degrees of freedom.

The estimates for the parameters of the model with one lag were used to develop impulse-response functions outlining the effects on real economic growth of cuts in defense spending and the federal debt and increases in the government spending categories noted above. The cuts in defense spending and

Table 2 F-Statistics for Government Spending and Real Private GDP, 1952 to 1991

Lags	Variable	F-Statistics for Combined Lags	Significance Levels	Degrees of Freedom
	(F-statistics calculated as a part of VAR system)			
1	EDUCATION	20.86	0.00007*	32
1	LABOR TRAINING	12.69	0.001*	32
1	AGRICULTURE	0.26	0.613	32
1	CIVILIAN SAFETY	3.98	0.055*	32
1	INCOME SUPPORT	6.61	0.015*	32
1	ED + L TRAIN + C SAF	27.00	0.00001*	32
2	EDUCATION	7.76	0.002*	26
2	LABOR TRAINING	2.46	0.105	26
2	AGRICULTURE	0.94	0.404	26
2	CIVILIAN SAFETY	0.42	0.661	26
2	INCOME SUPPORT	1.69	0.205	26
2	ED + L TRAIN + C SAF	7.88	0.002*	26
3	EDUCATION	7.07	0.002*	20
3	LABOR TRAINING	1.43	0.263	20
3	AGRICULTURE	0.85	0.484	20
3	CIVILIAN SAFETY	0.21	0.891	20
3	INCOME SUPPORT	2.12	0.130	20
3	ED + L TRAIN + C SAF	6.76	0.002*	20

* Six percent or smaller probability that the variable's effect on GDP growth was due to chance.

Note: All variables are in 1987 dollars and measured as changes in natural logarithms.

the federal debt are reported because the next step in the analysis will be to perform a policy experiment similar to that done in the Ireland-Otrok study (1992) in which both defense spending and the debt were reduced.[6]

Figures 1-A through 1-G depict impulse-response functions that show what might happen to the level of real private GDP if there were a one-time one-standard-deviation shock to the growth rate of a particular type of government spending. It is customary in the literature for the researcher to apply a shock of the magnitude of one standard deviation of the variation in the series to be tested. Limiting the shock to one standard deviation ensures that it will be within the purview of the data from which the model is estimated.

Figure 1-A shows the effect of a one-time $7.95 billion (one standard deviation of the growth rate) cut in defense spending, while Figure 1-B shows the effect of a one-time $26.8 billion (one standard deviation of the growth rate) reduction in the federal debt. The dotted lines represent 95 percent confidence limits for the impulse-response predictions. Since the areas between the dotted lines in each figure include 0.0, the results do not conclusively show even the direction of the effect on real private GDP of cutting defense or the debt.

Figures 1-C through 1-G show the responses of real private GDP to one-standard-deviation shocks to spending for agriculture, civilian safety, education, labor training, and income support. As might be expected, the magnitudes of the one-standard-deviation shocks vary considerably. One standard deviation for education spending, for example, is $3.1 billion in 1987 dollars, while one standard deviation for labor training is only $0.6 billion. The magnitudes of the standard deviations of the various series are reported in Table 3.

As the impulse-response figures show, shocks to spending for education and labor training might be expected to result in a cumulative increase in the level of real private GDP, an expectation predicted with 95 percent confidence. The impulse-response analysis for income support payments, on the other hand, not only showed the 95 percent confidence band to be practically symmetrical around zero, but the prediction itself to be for no change in the level of real GDP. Income support payments, therefore, were dropped from consideration as possible sources of economic growth, while education and labor training expenditures were considered likely sources worthy of further examination.

A Policy Experiment with the Model

In 1991, the Bush Administration presented a proposal entitled “The Future Years Defense Program” (popularly known as the “1991 plan”) that called for a 20 percent reduction in real defense spending between 1992 and 1997. Ireland

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[6] For the purpose of generating the impulse-response functions, the ordering of the variables assumes that policy decisions that change defense spending and the distribution of its proceeds are made before contemporaneous values of money and output are observed.

Figure 1 Response of Growth Rate of Private GDP

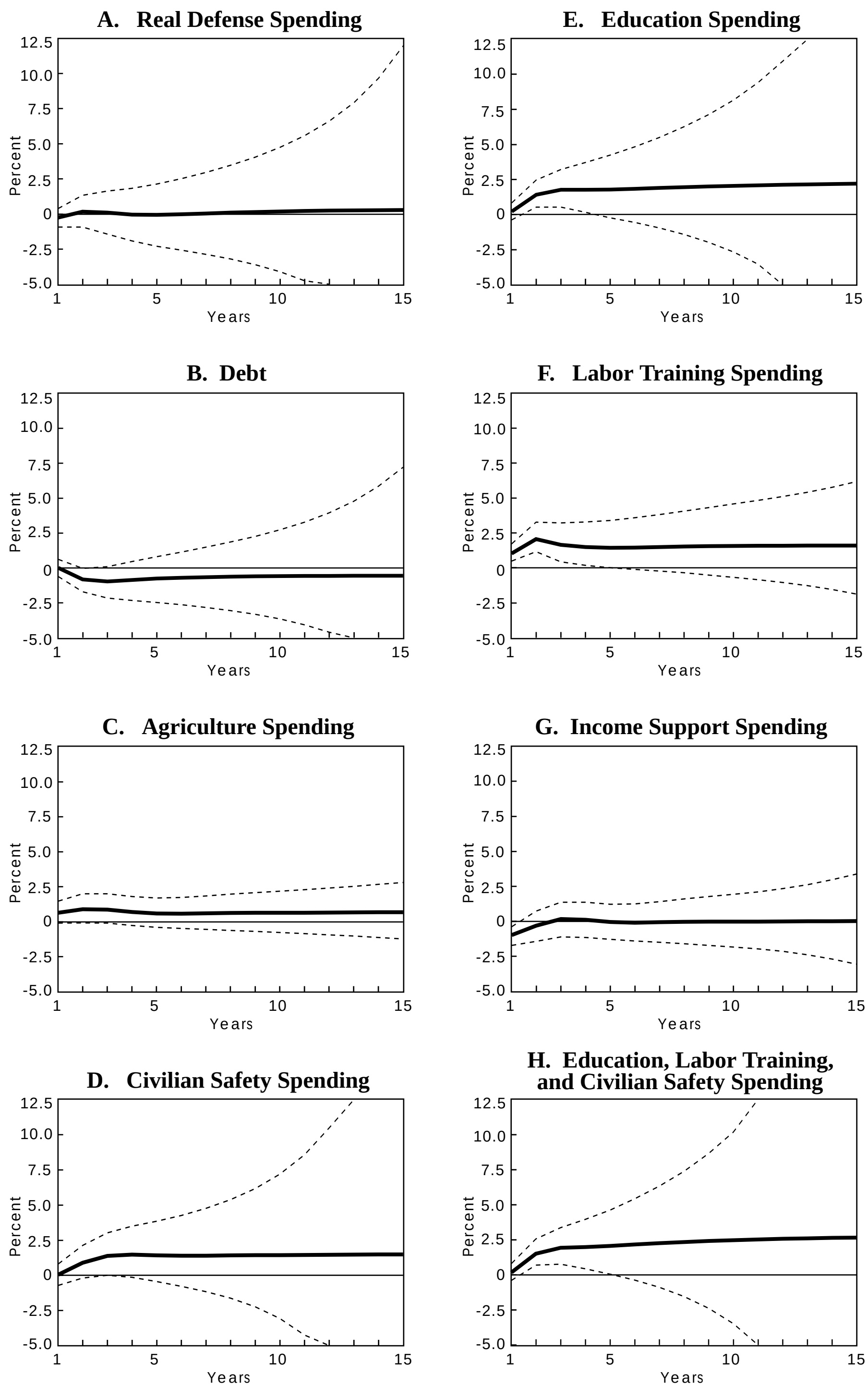


Table 3 Standard Deviations of the Growth Rates of Selected Data Series, 1952 to 1991

Data Series	Standard Deviation
(converted into billions of 1987 dollars)	
Agriculture	$ 2.8
Civilian Safety	0.6
Education	3.1
Federal Debt	26.7
Income Support	9.5
Labor Training	0.6
National Defense	8.0

and Otrok (1992) evaluated the 1991 plan with their VAR model. They found, using data from 1931 to 1991, that implementation of the 1991 plan with the proceeds going to federal debt reduction would be likely to reduce private GNP in the short run but increase it slightly after 13 or more years.

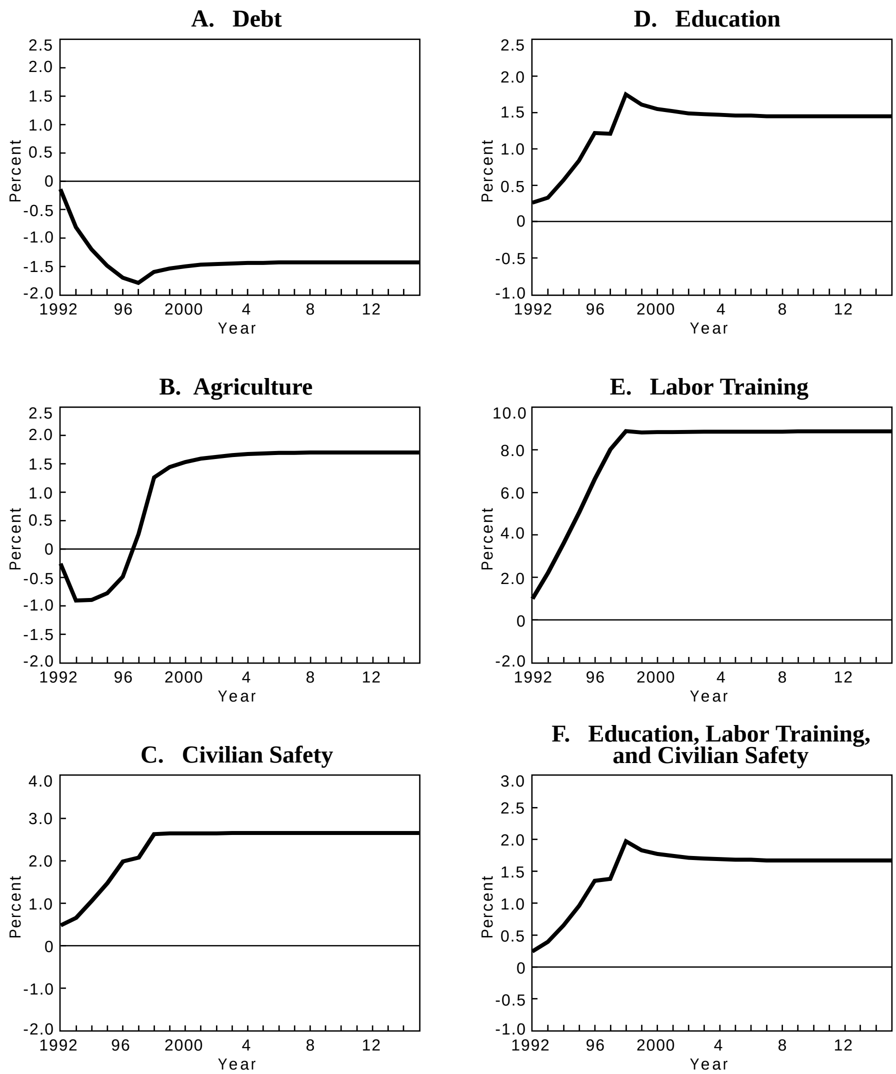
As a complement to the Ireland-Otrok study, the policy experiment reported here will also evaluate the 1991 plan. The new simulations, however, will assume that only a portion of the proceeds of the defense cuts are used for federal spending reductions. The remainder will be used to raise government spending on a specified function. An implicit assumption in the policy experiment is that any new spending programs would be as cost-benefit effective as has been average government spending for each function tested over the past 40 years.

Six simulations were made assuming the defense cutbacks of the 1991 plan, but with differing uses of the proceeds. The goal of the 1991 plan, recall, was to cut defense spending by 20 percent between 1992 and 1997. In 1987 dollars, this meant cutbacks of $17 billion in 1992, $21.7 billion in 1993, $10.2 billion in 1994, $9.0 billion in 1995, $6.5 billion in 1996, and $7.0 billion in 1997.

The simulations distributed the proceeds of the defense cutbacks either (1) all to federal debt reduction as in Figure 2-A or (2) a portion to a one-standard-deviation increase in one of the four types of government spending with the remainder going to federal debt reduction (Figures 2-B, 2-C, 2-D, and 2-E). The simulation with all of the proceeds of the defense cutbacks going to debt reduction shows the resulting level of real private GDP to be a persistent 1.5 percent below what it would have been with no change in defense spending.[7]

[7] This result differs from the result found by Ireland and Otrok using 1931–91 data and their slightly different model. However, when their model was reestimated over the 1955–91 and 1947–91 time periods, the results were quite similar to the results found here.

Figure 2 Forecasts of Difference in Output Between Base Case and 1991 Plan



As Figures 2-C, 2-D, and 2-E show, this outlook changes considerably when a portion of the proceeds of the defense cuts are used to increase spending on civilian safety, education, or labor training. These simulations put the level of real private GDP persistently above what it otherwise would have been even though most of the proceeds of the defense cuts are still used to reduce the federal debt. For example, the simulation channeling $3.1 billion per year of the defense cuts (25 percent of the total defense reduction) to education raises the level of real GDP 1.5 percent.

Surprisingly, the simulation with $0.6 billion per year of the defense cuts going to labor training has the level of real private GDP rising a whopping 9 percent above what it otherwise would have been. The magnitude of this result is not credible. It probably indicates that the model has been affected by some kind of spurious correlation with respect to labor training, which is a very small part (0.5 percent) of government spending.

The predicted effects of civilian safety spending also seem suspiciously large. The simulation has $0.6 billion per year in additional spending for civilian safety raising the level of private GDP almost 3 percent higher than it otherwise would have been. Given the error structure of the impulse-response function depicted in Figure 1-D, however, the forecast errors on the civilian safety simulation would undoubtedly be relatively large, were they available.[8]

The policy experiment was run with a variable that combined government spending for civilian safety, education, and labor training (Figure 2-F). The simulation using this variable, which was significant for the F-test reported in Table 2, and which had an impulse-response function (Figure 1-H) that was significantly greater than zero, predicts that the level of real private GDP will be persistently 1.8 percent larger with the policy experiment than without it. The increase in real GDP comes about as a result of $3.47 billion per year apportioned among civilian safety, labor training, and education during the years of the defense cuts. Over the six-year period, this experiment results in a cumulative $20.8 billion increase in civilian safety, education, and labor training and a $50.6 billion reduction in the federal debt.

4. CONCLUSIONS AND POLICY IMPLICATIONS

First some caveats. The analysis in this article uses past data to simulate future events. Although that approach is the only one available for empirical studies, it is always subject to question. One should have good reason to believe that past trends will continue if one is to put much credence in simulations of the type reported in this article. Moreover, while one can find certain correlations between past events and guess that one event may cause another, it is virtually impossible for an economist to *prove* that one economic occurrence in the real world caused another. Thus, the results of this study cannot be considered to

[8] The effects of spending on labor training and civilian safety were examined further to find whether or not they were likely to have been the result of reverse causation. Reverse Granger-causality tests were run to determine whether GDP determined labor training or civilian safety spending. Lagged GDP did not have a statistically significant effect on either.

be conclusive.

The results of the study, however, imply that government spending on education and labor training (and perhaps also civilian safety) have statistically significant, and numerically significant, effects on future economic growth. It is noteworthy that spending for education, civilian safety, and labor training directly affect human capital rather than physical capital. The VAR simulations with education, labor training, and civilian safety spending show effects so strong, in fact, that policies to reduce defense spending 20 percent and apportion the proceeds between debt reduction and one or all of those three spending types were estimated to result in higher levels of real private GDP than would have resulted with no reductions in defense spending.

As noted above, however, the results reported here are based upon correlations of past events and the correlations may or may not continue in the future. Thus, programs to increase government spending for, say, education or labor training should not be undertaken willy-nilly, justified by the promotion of economic growth. Rather, any such program should stand up to a cost-benefit analysis and prove itself worthy on its own merits.

REFERENCES

Aaron, Henry J. “Discussion,” in Alicia H. Munnell, ed., *Is There a Shortfall in Public Capital Investment?* Boston: Federal Reserve Bank of Boston, 1990, pp. 51–63.

Aschauer, David A. “Why Is Infrastructure Important?” in Alicia H. Munnell, ed., *Is There a Shortfall in Public Capital Investment?* Boston: Federal Reserve Bank of Boston, 1990, pp. 21–48.

__________. “Is Government Spending Stimulative?” *Contemporary Policy Issues*, vol. 8 (October 1990), pp. 30–46.

__________. “Public Investment and Productivity Growth in the Group of Seven,” Federal Reserve Bank of Chicago *Economic Perspectives*, vol. 13 (September/October 1989), pp. 17–25.

__________. “Does Public Capital Crowd Out Private Capital?” *Journal of Monetary Economics*, vol. 24 (September 1989), pp. 171–88.

__________. “Is Public Expenditure Productive?” *Journal of Monetary Economics*, vol. 23 (March 1989), pp. 177–200.

__________. “Tax Rates, Deficits, and Intertemporal Efficiency,” *Public Finance Quarterly*, vol. 16 (July 1988), pp. 374–84.

__________. “Government Spending and the ‘Falling Rate of Profit,’ ” Federal Reserve Bank of Chicago *Economic Perspectives*, vol. 12 (May/June 1988), pp. 11–17.

Cullison, William E. “Saving Measures as Economic Growth Indicators,” *Contemporary Policy Issues,* vol. XI (January 1993), pp. 1–8.

Hulten, Charles R. “Discussion,” in Alicia H. Munnell, ed., *Is There a Shortfall in Public Capital Investment?* Boston: Federal Reserve Bank of Boston, 1990, pp. 104–7.

__________. “Infrastructure: Productivity, Growth, and Competitiveness.” Hearing statement before the House Committee on Banking, Finance, and Urban Affairs, Subcommittee on Policy Research and Insurance, May 8, 1990.

__________, and Robert M. Schwab. “Is There Too Little Public Capital?” Paper presented at the American Enterprise Institute, Washington, D.C., February 1991.

Galbraith, John Kenneth. *The Affluent Society*. Boston: The Houghton Mifflin Company, 1958.

Ireland, Peter, and Christopher Otrok. “Forecasting the Effects of Reduced Defense Spending,” Federal Reserve Bank of Richmond *Economic Review,* vol. 78 (November/December 1992), pp. 3–11.

Jorgenson, Dale W. “Fragile Statistical Foundations: The Macroeconomics of Public Infrastructure Investment.” Paper presented at the American Enterprise Institute, Washington, D.C., February 1991.

Munnell, Alicia H. “How Does Public Infrastructure Affect Regional Economic Performance,” in Alicia H. Munnell, ed., *Is There a Shortfall in Public Capital Investment?* Boston: Federal Reserve Bank of Boston, 1990, pp. 69–103.

Rubin, Laura S. “Productivity and the Public Capital Stock: Another Look,” Economic Activity Section Working Paper Series, no. 118. Washington: Board of Governors of the Federal Reserve System, 1991.

Tatom, John. “Public Capital and Private Sector Performance,” Federal Reserve Bank of St. Louis *Review,* vol. 73 (May/June 1991), pp. 3–15.

U.S. Congress, Congressional Budget Office. *How Federal Spending for Infrastructure and Other Public Investments Affects the Economy.* Washington: U.S. Government Printing Office, July 1991.

Unit Labor Costs and the Price Level

Yash P. Mehra

A popular theoretical model of the inflation process is the expectations-augmented Phillips-curve model. According to this model, prices are set as markup over productivity-adjusted labor costs, the latter being determined by expected inflation and the degree of demand pressure.[1] It is assumed further that expected inflation depends upon past inflation. This model thus implies that productivity-adjusted wages and prices are causally related with feedbacks running in both directions.

In this article, I investigate empirically the causal relationship between prices and productivity-adjusted wages (measured by unit labor costs) using cointegration and Granger-causation techniques.[2] In my recent paper, Mehra (1991), I used similar techniques[3] to show that inflation and growth in unit

[1] This version has been closely associated with the work of Gordon (1982, 1985, 1988) and differs from the original Phillips-curve model. The latter was formulated as a wage equation relating wage inflation to the unemployment gap.

[2] Let X_{1t}, X_{2t}, and X_{3t} be three time series. Assume that the levels of these time series are nonstationary but first differences are not. Then these series are said to be cointegrated if there exists a vector of constants $(\alpha_1, \alpha_2, \alpha_3)$ such that $Z_t = \alpha_1 X_{1t} + \alpha_2 X_{2t} + \alpha_3 X_{3t}$ is stationary. The intuition behind this definition is that even if each time series is nonstationary, there might exist linear combinations of such time series that are stationary. In that case, multiple time series are said to be cointegrated and share some common stochastic trends. Moreover, if series are cointegrated, then some series must adjust in the short run so as to maintain equilibrium among multiple series. That implies the presence of short-run feedbacks (and hence Granger-causality) among these series.

[3] The statistical inference in most of the empirical work prior to Mehra (1991) has often been conducted under the assumption that wage and price series contain deterministic trends. Recent evidence has called this assumption into question and has shown that the trend components of several of these time series also contain stochastic components (Nelson and Plosser 1982). A misspecification of trend components can lead to incorrect tests of hypotheses. Mehra (1991) therefore employed recent techniques to investigate trends in wage and price series and used the analysis to determine the nature of causal structure between prices and unit labor costs.

labor costs are correlated in the long run and that the presence of this correlation appears to be due to Granger-causality running from inflation to growth in unit labor costs, not the other way around. The results presented there indicate that the "price markup" hypothesis is inconsistent with the data and that growth in unit labor costs does not help predict the future inflation rate.

This article examines the robustness of the conclusions in Mehra (1991) to changes in the measure of the price level, the sample period, and unit root-cointegration test procedures used there. In particular, the price series used in Mehra (1991) is the fixed-weight GNP deflator that covered the sample period 1959Q1 to 1989Q3; the test for cointegration used is the two-step procedure given originally in Engle and Granger (1987); and the stationarity of data is examined using Dickey-Fuller unit root tests. This article considers an additional price measure, the consumer price index, which covers consumption goods and services bought by urban consumers. In contrast, the implicit GNP deflator, the other price measure used here, covers prices of consumption, investment, government services, and net exports. Since the consumer price index is also a widely watched measure of inflation pressures in the economy, the article examines whether the causal relationships found between the general price level and unit labor costs carry over to consumer prices.

In my earlier empirical work (1991), I used Dickey-Fuller unit root tests to determine whether the relevant series contain stochastic or deterministic trends. Recently, some authors including Dejong et al. (1992) have shown that Dickey-Fuller tests have low power in distinguishing between these two alternatives. These studies suggest that economists should supplement unit root tests by tests of trend stationarity. Thus, a series now is considered having a unit root if two conditions are met: (1) the series has a unit root by Dickey-Fuller tests and (2) it is not trend stationary by tests of trend stationarity. Furthermore, the test for cointegration recently proposed by Johansen and Juselius (1990) overcomes several pitfalls associated with the Engle-Granger test for cointegration.[4] This article employs these additional, refined cointegration-stationarity tests to determine the stationarity of data and to study the nature of the causal structure between the general price level and unit labor costs.

[4] The Engle-Granger test for cointegration is implemented by regressing one series on the other remaining series and then testing whether the residuals from that regression are stationary or not. If the residuals are stationary, then the multiple time series are said to be cointegrated. This test has several shortcomings: (1) the test results are sensitive to the particular series chosen as the dependent variable; (2) the test cannot tell whether the number of cointegrating relationships is one or more than one; and (3) tests of hypotheses in the cointegrating vectors cannot be carried out because estimated coefficients have unknown nonstandard distributions. In contrast, the test proposed in Johansen and Juselius (1990) does not have any of the aforementioned problems. Their test procedure enables one to test directly for the number of cointegrating vectors and provides at the same time the maximum likelihood estimates of the cointegrating vectors. Tests of hypotheses in such estimated cointegrating vectors can be easily carried out. Lastly, the test results are not sensitive to the particular normalization chosen.

The empirical evidence reported here indicates that wage and price series contain stochastic, not deterministic, trends and that long-run movements in prices are correlated with long-run movements in unit labor costs. That is, the wage and price series used here are cointegrated as discussed in Engle and Granger (1987). This result holds whether the particular price series used is the implicit GDP deflator or the consumer price index.

Tests of Granger-causality presented here indicate that short-run movements in prices and unit labor costs are also correlated, with Granger-causality running one way from prices to unit labor costs when the price series used is the implicit GDP deflator. Test results with the consumer price index, however, are consistent with the presence of bidirectional feedbacks between prices and unit labor costs.

The empirical work here supports and extends the results in Mehra (1991). Though the cointegration test procedures, the sample periods, and the general price-level series used in these studies differ, both studies indicate that the "price markup" hypothesis is inconsistent with the data when the price series used measures the general price level. The additional results here, however, indicate such is not the case when the price series used is less broadly measured by the consumer price index. Thus, movements in unit labor costs help predict movements in consumer prices, but not in the general price level.

The plan of this article is as follows. Section 1 presents a Phillips-curve model of the inflation process and discusses its implications for the relationship between wages and prices. It also discusses how tests for cointegration and Granger-causality can be used to examine such wage-price dynamics. Section 2 presents the empirical results, and Section 3 contains concluding observations.

1. THE MODEL AND THE METHOD

The Phillips-Curve Model

The view that systematic movements in wages and prices are related derives from the expectations-augmented Phillips-curve model of the inflation process. Consider the price and wage equations that typically underlie such Phillips-curve models described in Gordon (1982, 1985) and Stockton and Glassman (1987):

$$\Delta P_t = h_0 + h_1 \Delta(w - q)_t + h_2 \chi_t + h_3 Sp_t \tag{1}$$

$$\Delta(w - q)_t = k_0 + k_1 \Delta P_t^e + k_2 \chi_t + k_3 Sw_t \tag{2}$$

$$\Delta P_t^e = \sum_{j=1}^{n} \lambda_j \Delta P_{t-j}, \tag{3}$$

where all variables are in natural logarithms and where P_t is the price level, w_t is the wage rate, q_t is labor productivity, χ_t is a demand-pressure variable, P_t^e is the expected price level, Sp_t represents supply shocks affecting the price equation, Sw_t represents supply shocks affecting the wage equation, and Δ is the first-difference operator. Equation (1) describes the price markup behavior. Prices are marked up over productivity-adjusted labor costs $(w - q)$ and are influenced by cyclical demand (χ) and the exogenous relative price shocks (Sp). This equation implies that productivity-adjusted wages determine the price level, given demand pressure. Equation (2) is the wage equation. Wages are assumed to be a function of cyclical demand (χ) and expected price level, the latter modeled as a lag on past prices as in equation (3). The wage equation, together with equation (3), implies that wages depend upon past prices, ceteris paribus.[5]

The price and wage behavior described above suggests that long-run movements in wages and prices must be related. In fact, some formulations of (1) and (2) predict that these two variables would grow at similar rates in the long run.[6] Furthermore, if one allows for short-run dynamics in such behavior, the analysis presented above would also suggest that past changes in wages and prices should contain useful information for predicting future changes in those same variables, ceteris paribus. These implications can be examined easily using tests for cointegration and Granger-causality between wage and price series.

Tests for Cointegration and Granger-Causality

If wage and price series have stochastic trends that move together, then the two time series should be cointegrated as discussed in Granger (1986). Thus, the long-run comovement of wages and prices is examined using the test for cointegration given in Johansen and Juselius (1990). The test procedure,

[5] The price and wage equations used here should be viewed as the reduced form equations. Price behavior as characterized in equation (1) is based on a markup model of pricing by firms. Nordhaus (1972) shows such pricing could be derived from optimizing behavior in which the technology is characterized by a Cobb-Douglas production function. Gordon (1985), on the other hand, derives a wage equation like (2) from an explicit model of labor demand and supply in which the wage rate adjusts in response to any change in the size of the gap between the two.

[6] For example, as indicated in footnote 5, the markup model of pricing behavior characterized in equation (1) is consistent with optimizing behavior in which the technology is characterized by a Cobb-Douglas production function. Given the additional assumptions of constant returns and the constant relative price of capital, the production environment implies a long-term coefficient of unity on unit labor costs in the price equation (1), $h_1 = 1$. That result indicates that prices and wages would grow at the same rate in the long run. Alternatively, the natural rate hypothesis, if valid in the long run, would indicate that the sum of the coefficients on past prices in (2) should be one, $k_1 \sum_{j=1}^{n} \lambda_j = 1$. That result also would indicate wages and prices grow at similar rates in the long run.

denoted hereafter as the JJ procedure, consists of estimating a VAR model that includes differences as well as levels of nonstationary time series. The matrix of coefficients that appear on levels of these time series contains information about the long-run properties of the model.

To explain the model, let X_t be a vector of time series on prices and wages. The VAR model is

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \Pi_k X_{t-k} + \epsilon_t, \tag{4}$$

where $\Pi_i, i = 1, \dots k$, is a matrix of coefficients that appear on X_{t-i}. Under the hypothesis that the series in X_t are difference stationary, it is convenient to transform (4) in a way that it contains both levels and first differences of the time series in X_t. That transformation is shown in (5).

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_{k-1} \Delta X_{t-k-1} + \Pi X_{t-k} + \epsilon_t, \tag{5}$$

where $\Gamma_i, i = 1, \dots k-1$, and Π are matrices of coefficients that appear on first differences and levels of the time series in X_t. The component ΠX_{t-k} in (5) gives different linear combinations of levels of the time series in X_t. Thus, the matrix Π contains information about the long-run properties of the model. When the matrix's rank[7] is zero, equation (5) reduces to a VAR in first differences. In that case, no series in X_t can be expressed as a linear combination of other remaining series. That result indicates that there does not exist any long-run relationship between the series in the VAR. On the other hand, if the rank of Π is one, then there exists only one linearly independent combination of series in X_t. That result indicates that there exists a unique, long-run (cointegrating) relationship between the series. When the rank is greater than one, then there is more than one cointegrating relationship among the elements of X_t.

Two test statistics can be used to evaluate the number of the cointegrating relationships. The trace test examines the rank of Π matrix and the hypothesis that rank $(\Pi) \leq r$ is tested, where r represents the number of cointegrating vectors. The maximum eigenvalue test tests the null that the number of cointegrating vectors is r given the alternative of $r + 1$ vectors. The critical values of these test statistics have been reported in Johansen and Juselius (1990).

Granger (1988) points out that if two series are cointegrated, then there must be Granger-causation in at least one direction. Assume that the JJ test procedure indicates that wage and price series are cointegrated and that the estimated cointegrating relationship is

$$P_t = \delta(w - q)_t + U_{1t}, \delta > 0,$$

where U_1 is the random disturbance term. Equation (5) then implies that there exists an error-correction representation of price and wage series of the form

[7] The rank of a matrix is the number of linearly independent columns (or rows) in that matrix.

$$\Delta P_t = a_0 + \sum_{s=1}^{k} a_{1s}\Delta P_{t-s} + \sum_{s=1}^{k} a_{2s}\Delta(w-q)_{t-s} + \lambda_1[P_{t-1} - \delta(w-q)_{t-1}] + \epsilon_{1t} \tag{6.1}$$

$$\Delta(w-q)_t = b_0 + \sum_{s=1}^{k} b_{1s}\Delta(w-q)_{t-s} + \sum_{s=1}^{k} b_{2s}\Delta P_{t-s} + \lambda_2[P_{t-1} - \delta(w-q)_{t-1}] + \epsilon_{2t}, \tag{6.2}$$

where all variables are as defined before and where one of $\lambda_1, \lambda_2 \neq 0$.[8] Equation (6) indicates that whenever the price level P_{t-1} deviates from the long-run value $\delta(w-q)_{t-1}$, then either prices or wages or both adjust so as to keep these two series together in the long run. Lagged levels of the variables now enter the VAR via the error-correction term $P_{t-1} - \delta(w-q)_{t-1}$. Test of the hypothesis that wages do not Granger-cause prices is that all $a_{2s} = 0$ and/or $\lambda_1 = 0$. Hence, the presence of Granger-causality is also examined by testing whether one or both of $\lambda_1, \lambda_2 \neq 0$.

Estimation and Tests of Hypotheses in Cointegrating Vectors

Suppose that the JJ test procedure indicates that price and wage series are cointegrated. In order to examine the nature of long-term correlations between price and wage series, the cointegrating wage and price regressions are estimated using the dynamic OLS procedure described in Stock and Watson (1993).[9] The dynamic versions of these regressions are

$$P_t = a_0 + a_1(w-q)_t + \sum_{s=-k}^{k} a_{2s}\Delta(w-q)_{t-s} + U_{1t} \tag{7.1}$$

$$(w-q)_t = b_0 + b_1 P_t + \sum_{s=-k}^{k} b_{2s}\Delta P_{t-s} + U_{2t}, \tag{7.2}$$

where all variables are as defined before and where U_1 and U_2 are random disturbance terms. Equation (7) includes, in addition, past, current and future values of first differences of the right-hand variables that appear in the cointegrating regression. Since the random disturbance terms, U_1 and U_2, may be

[8] If $\lambda_1 = \lambda_2 = 0$, then the matrix Π in (5) has a zero rank, indicating the absence of any long-run relationship between wage and price series.

[9] The JJ test procedure also provides maximum likelihood estimates of the cointegrating price and wage regressions. These estimates, though superior asymptotically, do not behave well in small samples. In contrast, Stock and Watson's (1993) dynamic OLS behaves well in small samples.

serially correlated, standard test statistics corrected for the presence of serial correlation are used to test hypotheses in (7). Thus, wages are not significantly correlated with the price level in the long run if the hypothesis $a_1 = 0$ or $b_1 = 0$ is not rejected.

Testing for Unit Roots and Mean Stationarity

The cointegration test requires that the time series in X_t be integrated of order one.[10] That is, the data should be stationary in their first differences but not in levels. To determine the order of integration, I use the test procedure suggested by Dickey and Fuller (1979). In particular, the unit root tests are performed by estimating the Augmented Dickey-Fuller regression of the form

$$y_t = a_0 + \rho y_{t-1} + \sum_{s=1}^{k} a_{2s} \Delta y_{t-s} + \epsilon_t, \tag{8}$$

where y_t is the pertinent variable; ϵ the random disturbance term; and k the number of lagged first differences of y_t necessary to make ϵ_t serially uncorrelated. If $\rho = 1$, y_t has a unit root. The null hypothesis $\rho = 1$ is tested using the t-statistic. The lag length (k) used in tests is chosen using the procedure[11] given in Hall (1990), as advocated by Campbell and Perron (1991).

The unit root tests in (8) test the null hypothesis of unit root against the alternative that y_t is mean stationary (the alternative is trend stationary if a linear trend is included in [8]). Recently, some authors including DeJong et al. (1992) have presented evidence that the Dickey-Fuller tests have low power in distinguishing between the null and the alternative. These studies suggest that in trying to decide whether the time series data are stationary or integrated, it would also be useful to perform tests of the null hypothesis of mean stationarity (or trend stationarity). Thus, tests of mean stationarity are performed using the procedure advocated by Kwiatkowski, Phillips, Schmidt, and Shin (1992). The test, hereafter denoted as the KPSS test, is implemented by calculating the test statistic

$$\hat{n}_u = \frac{1}{T^2} \sum_{t=1}^{T} S_t^2 / \hat{\sigma}^2(k),$$

where $S_t = \sum_{i=1}^{t} e_i, t = 1, 2, \ldots T$; e_t is the residual from the regression of y_t on an intercept; S_t is the partial sum of the residuals e; $\hat{\sigma}(k)$ is a consistent

[10] The series is said to be integrated of order one if it is stationary in first differences.

[11] The procedure is to start with some upper bound on k, say k max, chosen a priori (eight quarters here). Estimate the regression (8) with k set at k max. If the last included lag is significant (using the standard normal asymptotic distribution), select $k = k$ max. If not, reduce the order of the estimated autoregression by one until the coefficient on the last included lag (on Δy in [8]) is significant. If none is significant, select $k = 0$.

estimate of the long-run variance[12] of y; and T is the sample size. The statistic $\hat{n}_u$ has a nonstandard distribution and its critical values have been provided by Kwiatkowski et al. (1992). The null hypothesis of mean stationarity is rejected if $\hat{n}_u$ is large. Thus, a time series y_t is considered unit root nonstationary if the null hypothesis that y_t has a unit root is not rejected by the Augmented Dickey-Fuller test and the null hypothesis that it is mean stationary is rejected by the KPSS test.

2. EMPIRICAL RESULTS

This section presents empirical results. In particular, I examine the long- and short-term interactions between wages and prices in a trivariable system consisting of the price level, productivity-adjusted wage, and a demand pressure variable. The price level is measured either by the log of the implicit GDP deflator ($\ln P$) or by the log of the consumer price index (ln CPI); productivity-adjusted wage by the log of the index of unit labor costs of the nonfarm business sector (ln ULC); and demand pressure variable by the log of real over potential GDP (denoted as GAP). Unit labor costs are measured as compensation per hour divided by output per hour. Since supply shocks could have important short-run effects on wages and prices, tests of Granger-causality are conducted including some of these in the trivariable system. The supply shocks considered here include relative prices of energy and imports. Dummy variables for the period of President Nixon's wage and price controls and for the period immediately following the wage and price controls are also included.[13] The data used are quarterly and cover the sample period 1955Q1 to 1992Q4.

Test Results for Unit Roots and Mean Stationarity

In order to determine first whether linear trend is present in the data, Table 1 presents t-statistics on constant and time variables from regressions of the form

[12] The residual e_t is from the regression $y_t = a + b\text{Time} + e_t$. The variance of y_t is the variance of the residuals from this regression and is estimated using the Newey and West's (1987) method as

$$\hat{\sigma}(k) = \frac{1}{T}\sum_{t=1}^{T} e_t^2 + \frac{2}{T}\sum_{s=1}^{T} b(s,k) \sum_{t=s+1}^{T} e_t e_{t-s},$$

where T is the sample size; the weighing function $b(s,k) = 1 + \frac{s}{1+k}$; and k is the lag truncation parameter. The lag parameter was set at $k = 8$. For another simple description of the test procedure, see Ireland (1993).

[13] The relative price of energy is the ratio of the producer price index for fuels, petroleum, and related products to the producer price index for all commodities, and the relative price of imports is the ratio of the implicit deflator for imports to the implicit GNP deflator. The dummy variable for the period of price controls is 1 for 1971Q3 to 1974Q1 and 0 otherwise. The dummy variable for the period immediately following price controls is 1 for 1974Q2 to 1974Q4 and 0 otherwise. The data on prices, unit labor costs, and real GDP are from the Citibase data bank and that on potential GDP from the Board of Governors of the Federal Reserve System.

$$\Delta X_t = a + b\text{Time}_t + \sum_{s=1}^{k} C_s \Delta X_{t-s} + U_t,$$

where X_t is the pertinent variable; U_t a random disturbance term; and k the number of lagged first differences of X_t needed to make U_t serially uncorrelated. If the t-statistic on the constant is large, then X_t has linear trend. In addition, if the t-statistic on the time variable is large, then X_t has quadratic trend. As can be seen, the t-statistics presented in Table 1 are not large for $\ln P$, ln CPI, ln ULC, and GAP, indicating that linear or quadratic trends are not present in any of these time series. Hence, linear trend is not included in tests of unit roots where the alternative hypothesis now is that of mean, not trend, stationarity.

Table 1 Tests for Trends, Unit Roots, and Mean Stationarity

Series X	**Panel A** t-statistics for a Regression of ΔX on:			**Panel B** Tests for Unit Roots				**Panel C** Tests for Mean Stationarity
	Constant	Trend	k	$\hat{\rho}$	$t_{\hat{\rho}}$	k	Confidence Interval for ρ	$\hat{n}_u$
$\ln P$	1.5	.1	3	.99	−0.3	3	(1.00, 1.03)	1.28**
ln CPI	1.2	.1	8	1.00	−0.5	8	(1.00, 1.03)	1.27**
ln ULC	1.5	.3	3	.99	0.0	3	(0.99, 1.03)	1.19**
GAP	−0.4	.2	8	.93	−2.8*	1	(0.83, 1.00)	.23
$\Delta \ln P$				.88	−2.3	2	(0.88, 1.00)	.42
Δ ln CPI				.88	−2.5	8	(0.85, 1.01)	.50
Δ ln ULC				.72	−3.5**	2	(0.77, 0.96)	.35

* Significant at the 10 percent level.
** Significant at the 5 percent level.

Notes: P is the implicit GNP deflator; CPI is the consumer price index; ULC is the unit labor cost; and GAP is the logarithm of real GDP to potential GDP. ln is the natural logarithm and Δ the first-difference operator. The sample period studied is 1955Q1–1992Q4. The t-statistics in Panel A above are from regressions of the form $\Delta X_t = a_0 + a_1 \text{TREND} + \sum_{s=1}^{k} a_s \Delta X_{t-s}$, where X is the pertinent series. ρ and t-statistics ($t_{\hat{\rho}}$) for $\rho = 1$ in Panel B above are from the Augmented Dickey-Fuller regression of the form $X_t = a_0 + \rho X_{t-1} + \sum_{s=1}^{k} a_s \Delta X_{t-s}$. The 5 and 10 percent critical values for $t_{\hat{\rho}}$ are −2.9 and −2.6. The number of lagged first differences (k) included in these regressions are chosen using the procedure given in Hall (1990), with maximum lags set at eight quarters. The confidence interval for ρ is constructed using the procedure given in Stock (1991).

The test statistic $\hat{n}_u$ in Panel C above is the statistic that tests the null hypothesis that the pertinent series is mean stationary. The 5 and 10 percent critical values for $\hat{n}_u$ given in Kwiatkowski et al. (1992) are .463 and .574.

Tests for unit roots and mean stationarity are also presented in Table 1. As can be seen, the t-statistic ($t_{\hat{\rho}}$) that tests the null hypothesis that a pertinent time series has a unit root is small for $\ln P$, ln CPI, and ln ULC, but large for GAP. On the other hand, the statistic $\hat{n}_u$ that tests the null hypothesis that a pertinent time series is mean stationary is large for $\ln P$, ln CPI, and ln ULC, but small for GAP. These results indicate that the time series $\ln P$, ln CPI, and ln ULC have a unit root by the ADF test and are not mean stationary by the KPSS test. The GAP variable, on the other hand, does not have a unit root by the ADF test and is mean stationary by the KPSS test. Thus, the wage and price series used here are nonstationary in levels, whereas the demand pressure variable GAP is stationary in levels.

As indicated before, the series has a unit root if $\rho = 1$. Table 1 contains estimates of ρ and their 95 percent confidence intervals.[14] As can be seen, the estimated intervals contain the value $\rho = 1$ and are very tight for $\ln P$, ln CPI, and ln ULC. In contrast, the estimated interval for ρ is fairly wide for the GAP series (.83 to 1.0 for GAP vs. .99 to 1.03 for others). These results further corroborate the evidence above that $\ln P$, ln ULC, and ln CPI each have a unit root whereas the GAP series does not.

The unit root and mean stationary tests using first differences of $\ln P$, ln CPI, and ln ULC are also presented in Table 1. The results here are mixed. The inflation series, $\Delta \ln P$ and Δ ln CPI, have a unit root by the ADF test but are mean stationary by the KPSS test. The 95 percent confidence interval for ρ is (.88, 1.0) for $\Delta \ln P$ and (.85, 1.0) for Δ ln CPI. These confidence intervals are quite wide, indicating that ρ could as well be below unity (say, $\rho = .8$) and thus the inflation series could as well be stationary. The wage growth series Δ ln ULC, on the other hand, does not have a unit root by the ADF test and is mean stationary by the KPSS test. These results indicate that the wage growth series is mean stationary. The empirical work presented hereafter also treats the inflation series as mean stationary.[15]

Cointegration Test Results

The results presented in the previous section indicate that the price and wage series used here have stochastic, not deterministic, trends. I now examine

[14] The confidence interval for ρ is constructed using the procedure given in Stock (1991). The 95 percent confidence interval provides the range which may contain the true value of ρ with some probability (.95).

[15] These results differ somewhat from those given in Mehra (1991). The unit root tests given in Mehra (1991) were performed including a linear trend and indicated that gap, inflation ($\Delta \ln P$), and growth in unit labor costs (Δ ln ULC) series are nonstationary. The additional test results—such as those on linear trend, mean stationarity, and the confidence intervals for ρ—presented here, however, indicate that inflation, the output gap, and growth in unit labor costs could as well be stationary.

Table 2 Cointegration Test Results

System	k^a	Trace Test	Maximum Eigenvalue Test
($\ln P$, ln ULC)	3	17.2*	17.2*
(ln CPI, ln ULC)	5	20.2**	19.8**

[a] The lag length k was selected using the likelihood ratio test procedure described in footnote 16 of the text.
* Significant at the 10 percent level.
** Significant at the 5 percent level.

Notes: Trace and maximum eigenvalue tests are tests of the null hypothesis that there is no cointegrating vector in the system. The 5 percent and 10 percent critical values are 17.8 and 15.6 for the Trace statistic and 14.6 and 12.8 for the maximum eigenvalue statistic. Critical values are from Johansen and Juselius (1990).

whether there exists a long-run equilibrium relationship between $\ln P$ and ln ULC or between ln CPI and ln ULC, using the test of cointegration.

Table 2 presents cointegration test results using the JJ procedure.[16] As can be seen, trace and maximum eigenvalue test statistics that test the null that there is no cointegrating vector are large and significant, indicating that the wage and price series are cointegrated. The cointegrating price and wage regressions estimated using the dynamic OLS procedure are reported in Table 3. χ_1^2 is the Chi-square statistic that tests the null hypothesis that the coefficient on ln ULC in the price regression is zero. Similarly, χ_2^2 tests the null that the coefficient on $\ln P$ (or on ln CPI) is zero in the wage regression. As can be seen, χ_1^2 and χ_2^2 take large values and are significant, indicating that prices and wages are significantly correlated in the long run.[17] Furthermore, the estimated coefficients that appear on price and wage variables in these cointegrating regressions are positive and not far from unity. This indicates that wage and price series may grow at similar rates in the long run.

[16] The lag length parameter (k) for the VAR model was chosen using the likelihood ratio test described in Sims (1980). In particular, the VAR model initially was estimated with k set equal to a maximum of eight quarters. This unrestricted model was then tested against a restricted model where k is reduced by one, using the likelihood ratio test. The lag length finally selected in performing the JJ procedure is the one when the restricted model is rejected.

[17] The relevant statistics have a Chi-square, not an F, distribution because standard errors have been corrected for the presence of moving average serial correlation. The order of the moving average correction was determined by examining the autocorrelation of the residuals at various lags.

Table 3 Cointegrating Vectors; Dynamic OLS

Price Regressions	Wage Regressions
$\ln P_t = -.29 + 1.03 \ln \mathrm{ULC}_t$ $\chi_1^2 = 41.3$	$\ln \mathrm{ULC}_t = .31 + .96 \ln P_t$ $\chi_2^2 = 83.9$
$\ln \mathrm{CPI}_t = -.23 + 1.05 \ln \mathrm{ULC}_t$ $\chi_1^2 = 111.1$	$\ln \mathrm{ULC}_t = .23 + .94 \ln \mathrm{CPI}_t$ $\chi_2^2 = 196.8$

Notes: All regressions are estimated by the dynamic OLS procedure given in Stock and Watson (1993), using eight leads and lags of first differences of the relevant right-hand side explanatory variables. χ_1^2 is the Chi-square statistic that tests the null hypothesis that ln ULC is not significant, whereas χ_2^2 tests the null that $\ln P$ or ln CPI is not significant. Both statistics are distributed Chi-square with one degree of freedom. The standard errors in these regressions were corrected for the presence of moving average serial correlation.

Granger-Causality Test Results

The presence and nature of short-term interactions between wage and price series are investigated by estimating regressions of the form

$$\Delta \ln P_t = a_0 + \sum_{s=1}^{k1} a_{1s} \Delta \ln P_{t-s} + \sum_{s=1}^{k2} a_{2s} \Delta \ln \mathrm{ULC}_{t-s} + \sum_{s=1}^{k3} a_{3s} \mathrm{GAP}_{t-s} + \lambda_1 \hat{U}p_{t-1} + \epsilon_{1t} \tag{9}$$

$$\Delta \ln \mathrm{ULC}_t = b_0 + \sum_{s=1}^{k1} b_{1s} \Delta \ln \mathrm{ULC}_{t-s} + \sum_{s=1}^{k2} b_{2s} \Delta \ln P_{t-s} + \sum_{s=1}^{k3} b_{3s} \mathrm{GAP}_{t-s} + \lambda_2 \hat{U}w_{t-1} + \epsilon_{2t}, \tag{10}$$

where P is the price level measured either by the implicit GDP deflator or by the consumer price index; $\hat{U}p$ the residual from the cointegrating price regression; $\hat{U}w$ the residual from the cointegrating wage regression;[18] and $k1$, $k2$, and $k3$ the lag lengths on various variables needed to make random disturbances (ϵ_1, ϵ_2) serially uncorrelated. Wages do not Granger-cause prices if all $a_{2s} = 0$ and/or $\lambda_1 = 0$, and prices do not Granger-cause wages if all $b_{2s} = 0$ and/or $\lambda_2 = 0$.

[18] The appearance of error-correction terms in (9) and (10) follows directly from equation (5). If wage and price series are cointegrated, then the term ΠX_{t-k} in equation (5) captures coefficients that appear on the linear combination of wage and price variables. That is also demonstrated in equations (6.1) and (6.2).

Table 4 Error-Correction Coefficients and F Statistics for Granger-Causality; Implicit GDP Deflator

	Statistics from Price Regressions				Statistics from Wage Regressions			
Sample Period	Lag Lengths ($k1$, $k2$, $k3$)	$\lambda 1$ (t-value)	F1	d.f.	$\lambda 2$ (t-value)	F2	d.f.	χ^2_w
1956Q1–1992Q4	(4,0,0)	.02(1.6)			−.14(3.9)			
	(8,0,0)	.03(1.9)			−.17(4.5)			5.8
	(4,4,4)	−.01(0.4)	0.3	4,116	.05(1.1)	7.1**	4,124	
	(8,8,8)	−.01(0.2)	1.6	8,104	.06(1.2)	1.9*	8,112	
	(7,8,2)[a]	−.00(0.1)	1.2	8,111				
	(0,4,1)[a]				−.02(0.5)	22.4**	4,131	
1956Q1–1979Q3	(4,0,0)	.03(1.5)			−.17(3.9)			
	(8,0,0)	.04(1.7)			−.18(4.1)			7.0
	(4,4,4)	.04(0.9)	0.1	4,63	.06(1.0)	4.8**	4,71	
	(8,8,8)	.05(0.9)	0.8	8,51	.14(1.3)	0.8	8,59	
	(7,0,0)[a]	.03(1.6)						
	(4,4,1)[a]				.03(0.5)	7.3**	4,74	

[a] Lag lengths chosen using the procedure given in Hall (1990).
* Significant at the 10 percent level.
** Significant at the 5 percent level.

Notes: The price regressions are of the form

$$\Delta \ln P_t = a_0 + \sum_{s=1}^{k1} a_{1s} \Delta \ln P_{t-s} + \sum_{s=1}^{k2} a_{2s} \Delta \ln \mathrm{ULC}_{t-s} + \sum_{s=1}^{k3} a_{3s} \mathrm{GAP}_{t-s} + \lambda_1 U_{1t-1}$$

and wage regressions are of the form

$$\Delta \ln \mathrm{ULC}_t = b_0 + \sum_{s=1}^{k1} b_{1s} \Delta \ln \mathrm{ULC}_{t-s} + \sum_{s=1}^{k2} b_{2s} \Delta \ln P_{t-s} + \sum_{s=1}^{k3} b_{3s} \mathrm{GAP}_{t-s} + \lambda_2 U_{2t-1}.$$

U_1 is the residual from the cointegrating price regression and U_2 from the cointegrating wage regression, both reported in Table 3. F1 tests all $a_{2s} = 0$, F2 tests all $b_{2s} = 0$, and d.f. is the degree-of-freedom parameter for the F statistic given in the relevant row. The price regressions also included eight past values of the relative prices of energy and imports and dummies for President Nixon's price controls. The wage regressions included eight past values of the relative price of imports and price control dummies. χ^2_w is the Lagrange multiplier test for the hypothesis that eight lags of the relative price of energy do not enter the wage regression (the 5 percent critical value is 15.5).

Tables 4 and 5 report estimates of λ_1 and λ_2 (with t-statistics in parentheses) from regressions of the form (9) and (10). In Table 4 the price series used is the implicit GDP deflator and in Table 5 it is the consumer price index. The regressions are estimated using some arbitrarily chosen lag lengths ($k1$, $k2$, $k3$) as well as those chosen on the basis of the procedure given in Hall (1990). In addition, the results are presented for the subperiod 1956Q1 to 1979Q3. The price regression (9) estimated here also included eight past values of relative

Table 5 Error-Correction Coefficients and F Statistics for Granger-Causality; Consumer Price Index

		Statistics from Price Regressions			Statistics from Wage Regressions			
Sample Period	**Lag Lengths ($k1$, $k2$, $k3$)**	**$\lambda1$ (t-value)**	**F1**	**d.f.**	**$\lambda2$ (t-value)**	**F2**	**d.f.**	χ^2_w
1956Q1–	(4,0,0)	−.05(2.2)			−.10(1.8)			
1992Q4	(8,0,0)	−.05(2.2)			−.12(2.3)			18.1
	(4,4,4)	−.05(2.1)	0.70	4,116	−.04(0.8)	4.4**	4,132	
	(8,8,8)	−.02(0.6)	1.58	8,104	−.04(0.8)	0.5	8,120	
	(8,0,2)[a]	−.05(2.7)		8,104				
	(0,1,1)[a]				−.00(0.4)	68.7**	1,142	
1956Q1–	(4,0,0)	−.05(1.4)			−.01(0.4)			
1979Q3	(8,0,0)	−.04(1.4)			−.24(2.5)			
	(4,4,4)	−.10(2.4)	1.4	4,63	−.08(1.0)	4.9**	4,63	
	(8,8,8)	.01(0.2)	1.5	8,51	−.15(1.1)	1.1	8,51	
	(8,4,1)[a]	.11(2.9)	2.2*	4,62				
	(0,1,1)[a]				−.05(0.8)	14.5**	1,73	

Notes: See notes in Table 4. The regressions are estimated using the consumer price index.

prices of energy and imports, and "on" and "off" dummies for President Nixon's price controls. The wage regression (10) included, in addition, eight past values of the relative price of imports and price control dummies. Coefficients for the relative price of energy were not significant in such regressions.[19]

If we focus on results for the general price level presented in Table 4, it is clear that λ_1 is generally not statistically significant whereas λ_2 is significant (see t-statistics in Table 4). Moreover, other lags of $\Delta \ln$ ULC when included in price regressions are not statistically significant, whereas other lags of $\Delta \ln P$ when included in wage regressions are statistically significant (compare F1 and F2 statistics in Table 4). These results are consistent with the presence of Granger-causality, not from wages to prices, but from prices to wages.

The results using consumer prices are somewhat different from those using the general price level. As can be seen from Table 5, λ_1 is generally statistically significant, even though other lags of $\Delta \ln$ ULC when included in price regressions are not (see t- and F1 statistics in Table 5). On the other hand, λ_2

[19] Whether or not some of these supply shocks enter price and wage regressions was first tested using the Lagrange-multiplier (LM) test for omitted variables (Engle 1984). An LM test for k omitted variables is constructed by regressing the equation's residuals on both the original regressors and on the set of omitted variables. If the omitted variables do not belong in the equation, then multiplying the R^2 statistic from this regression by the number of observations will produce a statistic asymptotically distributed χ^2 with k degrees of freedom.

or other lags of $\Delta \ln$ CPI when included in wage regressions are statistically significant (see t- and F2 statistics in Table 5). These results are consistent with the presence of Granger-causality between prices and wages with feedbacks in both directions.

3. CONCLUDING OBSERVATIONS

A central proposition in the expectations-augmented Phillips-curve model of the inflation process is that prices are marked up over productivity-adjusted labor costs. If that proposition is correct, then long-run movements in prices and labor costs must be correlated. Moreover, we should find that short-run movements in labor costs help predict short-run movements in the price level. The evidence reported here indicates that these implications are consistent with the data when prices are narrowly measured by the consumer price index but not when they are broadly defined by the implicit GDP deflator. For the latter measure, short-run movements in labor costs have no predictive content for the future price level. The general price level and unit labor costs are still correlated in the long run. But the presence of this correlation appears to be due to Granger-causality running from the general price level to labor costs, not the other way around. Huh and Trehan (1992) report similar results using business sector price and wage data.

The finding that consumer prices and unit labor costs are Granger-causal with feedbacks in both directions differs from the one in Barth and Bennett (1975), which found Granger-causality running one way from consumer prices to wages. The empirical work in Barth and Bennett, however, does not test for the presence of Granger-causality occurring via the error-correction term. If we were to ignore this channel, other test results presented here are also consistent with the presence of Granger-causality running one way from consumer prices to wages (compare F1 and F2 statistics in Table 5).

REFERENCES

Barth, James R., and James T. Bennett. “Cost-Push Versus Demand-Pull Inflation: Some Empirical Evidence,” *Journal of Money, Credit, and Banking*, vol. 7 (August 1975), pp. 391–97.

Campbell, J. Y., and P. Perron. “Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots,” in O. J. Blanchard and S. Fischer, eds., *NBER Macroeconomics Annual, 1991*. Cambridge, Mass.: MIT Press, 1991, pp. 141–200.

DeJong, David N., John C. Nankervis, N. E. Savin, and Charles H. Whiteman. "Integration Versus Trend Stationarity in Time Series," *Econometrica,* vol. 60 (March 1992), pp. 423–33.

Dickey, D. A., and W. A. Fuller. "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association,* vol. 74 (June 1979), pp. 427–31.

Engle, Robert F. "Wald, Likelihood Ratio and Lagrange Multiplier Tests in Econometrics," in Zvi Griliches and Michael D. Intriligator, eds., *Handbook of Econometrics,* Vol. 2. New York: Elsevier, 1984, pp. 775–826.

__________, and C. W. Granger. "Cointegration and Error-Correction: Representation, Estimation and Testing," *Econometrica,* vol. 55 (March 1987), pp. 251–76.

Fuller, W. A. *Introduction to Statistical Time Series*. New York: Wiley, 1976.

Gordon, Robert J. "The Role of Wages in the Inflation Process," *American Economic Review,* vol. 78 (May 1988, Papers and Proceedings), pp. 276–83.

__________. "Understanding Inflation in the 1980s," *Brookings Papers on Economic Activity,* 1:1985, pp. 263–99.

__________. "Price Inertia and Policy Ineffectiveness in the United States, 1890–1980," *Journal of Political Economy,* vol. 90 (December 1982), pp. 1087–1117.

Granger, C. W. J. "Some Recent Developments in a Concept of Causality," *Journal of Econometrics,* vol. 39 (September/October 1988), pp. 199–211.

__________. "Developments in the Study of Cointegrated Economic Variables," *Oxford Bulletin of Economics and Statistics,* vol. 48 (August 1986), pp. 213–28.

Hall, A. "Testing for a Unit Root in Time Series with Pretest Data Based Model Selection." Manuscript. North Carolina State University, 1990.

Huh, Chan Guk, and Bharat Trehan. "Modelling the Time Series Behavior of the Aggregate Wage Rate," Working Paper 92–04. San Francisco: Federal Reserve Bank of San Francisco, 1992.

Ireland, Peter. "Price Stability Under Long-Run Monetary Targeting," Federal Reserve Bank of Richmond *Economic Quarterly,* vol. 79 (Winter 1993), pp. 25–45.

Johansen, Soren, and Katarina Juselius. "Maximum Likelihood Estimation and Inference on Cointegration—With Applications to the Demand for Money," *Oxford Bulletin of Economics and Statistics,* vol. 52 (May 1990), pp. 169–210.

Kwiatkowski, Denis, Peter C. B. Phillips, Peter Schmidt, and Yongcheol Shin. "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure Are We that Economic Time Series Have a Unit Root?" *Journal of Econometrics,* vol. 54 (October-December 1992), pp. 159–78.

Mehra, Yash P. "Wage Growth and the Inflation Process: An Empirical Note," *American Economic Review,* vol. 81 (September 1991), pp. 931–37.

Nelson, Charles R., and Charles I. Plosser. "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications," *Journal of Monetary Economics,* vol. 10 (September 1982), pp. 139–62.

Newey, Whitney K., and Kenneth D. West. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica,* vol. 55 (May 1987), pp. 703–8.

Nordhaus, William D. "Recent Developments in Price Dynamics," in Otto Eckstein, ed., *The Econometrics of Price Determination*. Washington: Board of Governors of the Federal Reserve System, 1972.

Sims, Christopher A. "Macroeconomics and Reality," *Econometrica,* vol. 48 (January 1980), pp. 1–49.

Stock, James H. "Confidence Intervals for the Largest Autoregressive Root in U.S. Macroeconomic Time Series," *Journal of Monetary Economics,* vol. 28 (December 1991), pp. 435–59.

__________, and Mark W. Watson. "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica,* vol. 61 (July 1993), pp. 783–820.

Stockton, David J., and James E. Glassman. "An Evaluation of the Forecast Performance of Alternative Models of Inflation," *Review of Economics and Statistics,* vol. 69 (February 1987), pp. 108–17.

Is All Government Capital Productive?

Mary Finn

The early 1970s witnessed dramatic change in per-capita output and labor-productivity growth rates in the United States. These growth rates averaged 2.2 percent and 2.0 percent, respectively, for the 1950–1969 period compared to 1.3 percent and 0.8 percent for the 1970–1989 period. Aschauer (1989) advances the idea that an important explanatory factor in this productivity slowdown is the government's stock of capital. Estimating a production function that relates private sector output to private sector labor and capital and to total government capital for the aggregate U.S. economy (1949–1985), Aschauer finds the output elasticity of government capital to be 0.39.[1] That is, for every 1 percent change in government capital, output responds by 0.39 percent. This productivity coefficient, coupled with the sharp fall in the average growth rate of government capital from 4.1 percent for 1950–1970 to 1.6 percent for 1971–1985, constitutes the evidence underlying Aschauer's view.[2]

The Aschauer (1989) study is innovative and important. His evidence suggests that government capital plays a significant role in economic growth. His findings are, however, surprising and somewhat unconvincing. The evidence is surprising because the output elasticity of government capital is relatively high and because government capital contains many different types of stocks (e.g., museums, hospitals, airports, prisons, seawalls, and wildlife preservation

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[1] See Aschauer (1989), Table 1, equation (1.1). Also, more exactly, Aschauer (1989) estimates a generalized, constant-returns-to-scale, Cobb-Douglas production function. The estimation method is least-squares and applies to the functional relationship in level form. The measure of total government capital is the net, consolidated-government, nonmilitary, nonresidential stock.

[2] These numbers are from Aschauer (1989), Table 7.

facilities), some of which are highly unlikely to make a direct productive contribution to output.[3] Aschauer's evidence is unconvincing not only because it fails to distinguish the growth rate of the productive component of government capital from the growth rate of total government capital, but also because the output elasticity of government capital may be inflated from reverse-causation bias.[4] That is, the productivity-coefficient estimate may be capturing the effect of output on government investment spending and hence on government capital instead of the effect of government capital on output. Output could affect government investment spending because government investment decisions possibly depend on output performance—higher output can lead to more tax revenue to finance such investment.

Aschauer's (1989) work raises many questions. What is unique to government capital that could be so productive? Which components of government capital play a role in production? What is the nature of the production channel through which they exercise this role? Do these channels differ across components? What are the magnitudes of the associated productivity coefficients, controlling for possible reverse causation? How do these magnitudes explain output and labor-productivity growth rates in the post-World War II United States? Finally, are the real returns to investing in productive government capital components high?

This article addresses these questions. The answers provide guidance for government investment policies by elucidating how components of government capital influence output production and by quantifying their effects on economic growth. Lucas (1987) underscores the importance of these questions by showing that changes in economic growth as small as 1 percentage point can have huge social welfare effects.

The article proceeds as follows. Section 1 describes the components of total government capital and considers their possible production roles. The resulting analysis suggests that only government-owned, privately operated capital (GOPO), government enterprise capital (ENTP), and government highway capital (HGWY) directly contribute to private production. GOPO's and ENTP's contribution to private production stems from the measurement of private sector output. One possible way that GOPO and ENTP enter the production function is through the same channel as most private sector capital; i.e., GOPO and ENTP perfectly substitute for private sector capital.[5] To capture this effect, the present study retains standard production function theory but changes the

[3] The figure 0.39 is large relative to 0.30, the output elasticity of private capital found in other studies (see Lucas [1990]).

[4] Aschauer's use of the total stock, rather than the productive component, of government capital also may affect the estimate in unknown ways.

[5] "Most private sector capital" here means total private (business) sector capital less its transportation-vehicle component.

standard measure of capital by adding private capital, GOPO, and ENTP together. HGWY affects production because of transportation services. In particular, HGWY and the transportation-vehicle component of private (business) sector capital together yield services that facilitate the transportation of both final and intermediate goods and, in turn, help produce final delivered output. Capturing this idea, Section 2 extends standard production function theory so as to model transportation services and distinguish them from those of the other factors of production—labor and the augmented stock of private capital mentioned above.

Section 2 also embeds the production theory in a general equilibrium model of the economy that allows derivation of mathematical statements of private firms' investment and capital-utilization decision rules. These rules are useful in the estimation exercise, undertaken here, by bringing more information to bear on the parameter values of the production function. Section 3 outlines the data and method for estimating the production function and firms' decision rules. For this method, Generalized-Method-of-Moments, the possible reverse-causation phenomenon does not distort the coefficient estimates.

The highlights of the empirical findings of Section 4 pertain to the productivity coefficient of highway capital. The point estimate is 0.16; it is statistically significant but imprecise. For example, a 95 percent confidence interval around this point estimate implies that the true productivity coefficient could be as much as 0.32 or as little as 0.001. Using the point estimate 0.16, highway capital reduced output growth by 0.1 percent during the 1970–1989 period. These results support, but strongly moderate, Aschauer's (1989) claim that government capital is an important explanatory factor in the productivity slowdown. Section 5 concludes with thoughts on the policy implications of the empirical findings.

1. COMPONENTS OF GOVERNMENT CAPITAL

This section describes the components of total government capital, listing the main types of capital goods in each component and summarizing some quantitative features. The key quantitative features include the components' average shares in total government capital (see Table 1) and the characteristics of the component shares' trends. The sample period is 1950–1989. The appendix provides further detail on all data underlying this discussion and indicates data sources and measurement caveats. Also, this section considers the possible production role for each component.

Highway Capital (HGWY)

HGWY includes highways, streets, bridges, tunnels, overpasses, viaducts, and association lighting and erosion control structures. It is the largest component of government capital, with an average share of 0.36. From 1965 to 1989, this share exhibits a downward trend.

Table 1 Component Average Shares in Total Government Capital

Component	Share
Highway Capital	0.361
Government Enterprise Capital	0.248
Educational and Hospital Capital	0.190
Fire and Natural Resource Stocks	0.081
Equipment Capital	0.048
Administrative, Judicial, Police, and Research and Development Stocks	0.040
Government-Owned Privately Operated Capital	0.032
Total	1.000

Notes: (1) The entries are average annual shares over the period 1950–1989.
(2) See the appendix for data definitions, caveats, and the reason that equipment capital is a separate component in this table.

HGWY influences output production in the private sector through the provision of transportation services. HGWY and the stock of private (business) sector transportation vehicles are necessary stocks for the transportation of both intermediate and final goods. The flow of transportation services from these stocks directly contributes to the production of final delivered goods.

The ratio of HGWY to private sector output is small but not insignificant; its average value is 0.18.[6] This ratio trends down from 1975 to 1989.

Government Enterprise Capital (ENTP)

Government enterprises include various credit and insurance corporations (e.g., Commodity Credit Corporation, FDIC, and FSLIC); the U.S. Post Office; gas and electric utilities; water and sewerage utilities; public transit agencies; airport and maritime terminal operators; and miscellaneous service-producing agencies (e.g., agencies that administer lotteries, parking, highway tolls, and housing and urban renewal). Their capital consists of office buildings, electrical transmission facilities, gas structures, parking structures, sewer systems, water supply facilities, public transit stations (bus, streetcar, subway, and rail), railroad structures, airport facilities, maritime buildings, harbors, amusement structures,

[6] Compare it to the average value of the private capital to private output ratio of about one (see Table 5).

and associated equipment.[7,8] This component is the second-largest component of government capital; on average, its share in government capital is 0.25. For the period 1968–1989, this share shows an upward trend.

The measure of private sector production in the national income accounts includes the output of government enterprises (see Department of Commerce [1988]).[9] The underlying national income accounting rationale is that the output of enterprises is very similar to the output of private firms (one can, for example, compare electricity or postal services across the two). This measurement, combined with the fact that government enterprises use their capital stocks to produce their output, implies that enterprise capital directly contributes to the production of private sector output. Given that outputs are similar across enterprises and private firms, presumably so too are the associated production techniques/methods, suggesting that the production functions of enterprises and private firms should be treated the same. It follows that the aggregate production function relates the private sector product measure to the sum of private and enterprise capital. That is, private and enterprise capital are perfect substitutes for one another in the production process.

Government-Owned, Privately Operated Capital (GOPO)

GOPO includes research and development facilities, atomic energy facilities, nuclear weapon factories, arsenals, shipyards, and associated equipment. It is the smallest component, with an average share of only 0.03. This share trends downward from 1955 to 1989. During the two world wars, GOPO was quantitatively significant (see Braun and McGrattan [1993]).

GOPO directly enters the production process of private sector output. Since GOPO contains capital goods similar to privately owned capital goods, one possible way of capturing its productive contribution is to treat it as a perfect substitute for private capital in the production function.

The ratio of ENTP and GOPO to private sector output is small but not negligible, with an average value of 0.14. This ratio does not exhibit a noticeable trend over the sample period.

[7] The term *harbors* refers to harbors, piers, canals, docks, and dredging and drainage equipment.

[8] Harbors and some airport facilities (primarily those on national parks and Indian reservations) included in this component are not owned (or operated) by enterprises. They are owned and operated by state and local government and by federal government, respectively. These are included here because they cooperate with, in the case of harbors, or are very similar to, in the case of federal airports, some of the enterprise capital stocks. Since both stocks are small, their inclusion/exclusion is not of quantitative importance. See the appendix for more detail.

[9] It does not include the output of the rest of government. The rest of government is referred to as "general government" in Department of Commerce (1988).

Educational and Hospital Capital (EDHS)

This component consists of primary, secondary, and university-level educational buildings, associated buildings (laboratories, libraries, student unions, and dormitories), and equipment; stocks that serve an educational purpose (e.g., public libraries, museums, art galleries, observatories, archives, and botanical and zoological gardens); and health care and institutional facilities (e.g., hospitals, clinics, and infirmaries). Its average share in government capital is 0.19. This share evolves like an inverted "v," trending upwards to peak in 1975 and trending downwards thereafter.

Without doubt, EDHS influences output production by promoting the knowledge and well-being of labor input. But, if the measure of labor input accounts for both labor input's quantity, in terms of the number of manhours, and labor input's quality, in terms of, for example, each worker's educational level and age, then it is difficult to see why EDHS should have a separate productive effect. This study uses a labor input measure that incorporates many quality adjustments; therefore, EDHS is not included as a direct factor of production in the quantitative analysis.

Administrative, Judicial, Police, and Research and Development Stocks (ADMN)

Office buildings, customs houses, courthouses, prisons, police buildings, research and development facilities, and associated equipment comprise this category. Its share, averaging only 0.04, sharply trends up from 1963 to 1989.

Could ADMN affect production? It could since it is linked to the setting of rules and regulations governing the conduct of business and to research and development that affects technology. For rules, regulations, and technology determine the amount of output that can be produced from any given quantity of inputs (see Hansen and Prescott [1993]). But linkages such as these are subtle and indirect.

Fire and Natural Resource Stocks (NATR)

NATR consists of structures on government land that are intended for water, land and animal protection (e.g., reservoirs, irrigation facilities, seawalls, erosion control systems, fish hatcheries, and wildlife preservation facilities), housing for forest rangers and national park employees, fire buildings, and associated equipment.[10] As a share of government capital, it averages 0.08 and shows a downward trend during 1953–1989.

[10] This component includes housing for forest rangers and national park employees even though the total government capital stock under review is classified as nonresidential by the Department of Commerce.

Some of the capital goods in NATR contribute to the output production process. Specifically, stocks such as fire buildings, fire equipment, reservoirs, and seawalls mitigate or prevent the destruction of other capital stocks that are directly employed in production, for example, private capital stocks. But, the productive role of these government stocks, while valuable, is merely supportive. Therefore, they do not qualify for the present analysis as a direct factor of production.

The foregoing considerations suggest that only HGWY, ENTP, and GOPO directly contribute to private production. Accordingly, they are the only capital components entering the quantitative part of this study's investigation. The channels of their contribution, suggested above, require an extension of standard production function theory to allow an explicit role for HGWY and an expanded measure of private capital to include ENTP and GOPO. The upshot of this for the quantitative questions motivating the study is that they will pertain to HGWY only. That is, those quantitative questions will not apply to ENTP and GOPO since their effects cannot be separated from that of private capital.[11,12]

2. THE MODEL ECONOMY

The model economy presented here provides a mathematical framework to address the quantitative questions raised above. The model specifies economic agents' objectives and constraints, including the production function, the market structure, and the stochastic exogenous processes. From it firms' investment and capital-utilization decision rules can be derived, which are useful for estimating the parameters of the production function.

Consider an economy with a large number of identical firms and households and a government. Since all firms and all households are identical, one can focus on the behavior of any one representative firm and any one representative

[11] Aschauer (1989) undertakes one estimation differing from that described in the beginning of the article by breaking the total government capital stock into separate components. These components are more comprehensive than those considered in this article. All of them are included in his estimation. See Aschauer (1989), Table 6, for further details.

[12] Other studies estimating production functions involving government capital for the United States include Munnell (1990a, 1990b), Garcia-Mila and McGuire (1992), Hulten and Schwab (1991), Holtz-Eakin (1992), Tatom (1992), Fernald (1992), and Lynde and Richmond (1993). In most cases the stock of government capital is measured by a total (or comprehensive) capital stock. Garcia-Mila and McGuire (1992) and Fernald (1992) use the highway component of government capital. Their production function specifications differ from that of this article. See footnote 17 for more detail. Also, Garcia-Mila and McGuire (1992) and Fernald (1992) use state and industrial-level data, respectively, in contrast to the aggregate economy-level data employed here.

household.[13] Therefore, most quantity variables will be expressed in per-capita terms. The goods and factor markets in which firms, households, and the government interact are competitive, with all agents viewing prices as beyond their control; i.e., the agents are price takers. There are three stochastic, exogenous variables in the economy: technology, energy prices, and government investment spending. In the following discussion, unless otherwise indicated, most variables are current-period variables; variables with a prime ($'$) attached are next-period variables. The notation is explained in Table 2.

The representative firm maximizes profit:

$$\Pi = y - wl - r_v(vu_v) - r_k(ku_k) - p(e_v + e_k). \tag{1}$$

Profit is the difference between the revenue from the sale of output and the cost of labor, capital services, and energy. Output is the numeraire, so its price is normalized at one. All factor prices are relative prices. Notice that the utilization rate of a given amount of any one capital stock determines the flow of capital services. Interpret a utilization rate as the number of hours worked per period and/or the intensity of work per hour of the capital stock. The firm's choice variables are $y, l, v, k, u_v, u_k, e_v$, and e_k and are subject to the following technical constraints. The production function

$$y = (zl)^{\theta 1}(ku_k)^{\theta 2}s^{\theta 3}, \quad 0 < \theta_i < 1 \quad (i = 1, 2, 3) \\ \theta_1 + \theta_2 + \theta_3 = 1 \tag{2}$$

states that output positively depends on technology, labor, services from capital (k), and transportation services.[14] Transportation services directly contribute to output production by facilitating the transportation of both final and intermediate goods associated with the production process.[15] Transportation services are an increasing function of the services from vehicles and the effective highway stock:

$$s = (vu_v)\overline{g}^{\psi}, \psi > 0. \tag{3}$$

The effective highway stock is defined as the aggregate highway stock adjusted for, or divided by, its aggregate usage:

$$\overline{g} = \frac{G}{(VU_v)}, \tag{4}$$

[13] In this model economy, no distinction is drawn between a private firm and a government enterprise. An implicit assumption, therefore, is that enterprises are profit maximizers.

[14] Placing the exponent parameter, θ_1, on z is for algebraic convenience only. With this specification, the steady-state growth rate of the economy is given by the growth rate of z rather than that of z/θ_1 (see King, Plosser, and Rebelo [1988]).

[15] Although intermediate goods underlie this production process, they do not enter the production function (2), since implicitly it is derived by averaging across all firms' production functions for all goods (final and intermediate). Intermediate-good output of one firm cancels with the intermediate-good input of another. Alternatively expressed, (2) is the value-added production function for the economy.

Table 2 Notation

Π	= per-capita profit
y	= per-capita output
w	= wage rate for labor
l	= per-capita labor hours
v	= per-capita stock of private transportation vehicles, in place at the beginning of the period
k	= per-capita stock of other private capital, in place at the beginning of the period
$r_v(r_k)$	= rental rate for capital services from $v(k)$
$u_v(u_k)$	= utilization rate of $v(k)$
p	= exogenous energy price
$e_v(e_k)$	= per-capita energy use required for the utilization of $v(k)$
z	= exogenous technology
s	= per-capita transportation services
θ_i $(i = 1, 2, 3)$	= parameters
$\bar{g}$	= per-capita effective stock of government highway capital
G	= aggregate stock of government highway capital, in place at the beginning of the period
V	= aggregate stock of private transportation vehicles, in place at the beginning of the period
U_v	= economy-wide average value of u_v
ω_i $(i = 1, 2, 3, 4)$	= parameters
g	= per-capita stock of government highway capital, in place at the beginning of the period
E_t	= expectations operator conditioned on information available in time period t
β	= subjective discount factor
c	= per-capita consumption
γ	= a parameter
t	= time period t
log	= natural logarithm
τ	= the tax rate on income from capital services due to v and k
n	= per-capita lump-sum transfer payment from government
$i_v(i_k)$	= per-capita private gross investment in $v'(k')$
x	= per-capita lump sum tax paid to government
$\delta_v, \delta_k, \delta_g$	= depreciation rates of v, k, and g
i_g	= per-capita exogenous government gross investment in g'
$\bar{z}$	= mean gross growth rate of z
ϵ	= innovation/disturbance term
E	= unconditional expectations operator

where VU_v measures the aggregate usage. So, equations (3) and (4) capture the notion that there is congestion of aggregate highway capital. That is, the higher the total use of vehicles in the economy (VU_v), the lower the contribution of aggregate highways (G) to each firm's transportation services (s). For this reason, $\bar{g}$ is referred to as the effective highway stock: it is the highway stock effectively contributing to transportation services. Since $\bar{g}$ depends on the aggregate stocks, G and V, and the economy-wide average utilization rate, U_v, each of which is beyond the representative firm's control, it follows that $\bar{g}$ is exogenous to the firm. Barro and Sala-i-Martin (1992) and Glomm and Ravikumar (1992) model congestion of public goods in a similar fashion to that modeled here.[16]

The remaining technical constraints are the energy relationships:

$$\frac{e_v}{v} = \omega_1 (u_v)^{\omega 2}, \omega_i > 0 \ (i = 1, 2) \tag{5}$$

and

$$\frac{e_k}{k} = \omega_3 (u_k)^{\omega 4}, \omega_i > 0 \ (i = 3, 4). \tag{6}$$

These equations specify that energy is essential for the utilization of capital, k and v, with an increase in utilization increasing energy use per unit of capital at an increasing rate. These specifications follow those in Finn (1993). Their presence here serves the purpose of forming cost margins for utilization decisions and of explicitly according a role to energy in the production process. Finn (1993) shows that the latter is important when addressing questions involving productivity.

Equations (2), (3), (5), and (6) together show that output exhibits constant returns to scale in l, k, v, e_k, and e_v. Therefore, this production structure is consistent with the assumed competitive market structure.

The assumption of identical firms implies that in equilibrium the economy's per-capita amount of V is the same as each firm's choice variable v. Furthermore, U_v coincides with u_v. Noting these equilibrium results and dividing G and V on the right-hand side of (4) by the population size leads to:

$$\bar{g} = \frac{g}{(vu_v)}. \tag{7}$$

Substitute (7) into (3), and the result into (2) to obtain:

$$s = (vu_v)^{(1-\psi)} g^{\psi} \tag{8}$$

and

$$y = (zl)^{\theta 1} (ku_k)^{\theta 2} (vu_v)^{\theta 3(1-\psi)} g^{\theta 3 \psi}. \tag{9}$$

[16] In Barro and Sala-i-Martin (1992), a total government spending flow entering the production function is subject to congestion. The government spending flow is divided by the economy's aggregate private capital stock. In Glomm and Ravikumar (1992), an aggregate stock of government capital entering the production function is congested. The government capital stock is divided by a Cobb-Douglas function of the economy's aggregate amounts of private labor and capital. See Glomm and Ravikumar (1992) for references to other, earlier studies advancing the congestion idea.

Equation (8) shows that s satisfies constant returns to scale in v and g, while equation (9) specifies y as a constant-returns-to-scale function of l, k, v, and g. To this list one may add e_k and e_v, by noting (5) and (6). These constant-returns-to-scale features are important. They imply that the production function is consistent with steady-state or balanced growth. Therefore, in the absence of temporary innovations to the exogenous variables, output, all three capital stocks, and energy use will grow at the constant rate of technology growth. (See the technical appendix of King, Plosser, and Rebelo [1988] for an explanation.) This result is important because steady-state growth is a characteristic of many developed market economies (see King, Plosser, and Rebelo [1988]).[17]

The exponents in (9) are the output elasticities or productivity coefficients of the corresponding factors of production. Of particular interest is $\theta_3\psi$, the productivity coefficient of highway capital. Equation (9) differs from standard production functions by distinguishing vu_v from ku_k and by including g. These differences stem from the objective of explicitly accounting for the role of highway capital in production.

The representative household maximizes its expected, discounted lifetime utility:

$$\mathop{E}_{0} \sum_{t=0}^{\infty} \beta^t[\log c_t + \gamma \log(1 - l_t)], 0 < \beta < 1, \gamma > 0. \tag{10}$$

Here, utility in any one period positively depends on consumption and leisure. The time endowment in each period is normalized at one. As part of its maximizing behavior, the household engages in market activities that involve purchasing consumption and investment goods from and selling labor and capital services to the firm. The household pays taxes to and receives transfer payments from the government. Therefore, its choice variables in any one period are c, i_v, i_k, and l and are subject to the following budget and technical constraints. Total income must equal total spending:

$$wl + (1 - \tau)[r_v vu_v + r_k ku_k] + n = c + i_v + i_k + x. \tag{11}$$

The sum of wage and after-tax capital income and transfer payments constitutes total income. Total spending is the sum of consumption, investment, and lump-sum taxes. The reason for the presence of taxes and transfers is

[17] The production function specifications of Garcia-Mila and McGuire (1992) and Fernald (1992) differ from equation (9). Garcia-Mila and McGuire specify a production function of private structures, private equipment, labor, the highway stock, and government educational spending. Fernald's specification comes closer to that of the current study. He specifies a production function in which, like here, highway capital and private business sector vehicle capital affect production through transportation services. His specification assumes increasing returns to scale to labor, private capital, and highway capital. By contrast, here the specification assumes constant returns to scale to all of the inputs, making the production theory consistent with the balanced-growth facts.

explained below. Also, for any one type of capital good, its one-period future value depends on the current undepreciated quantity of and investment in that capital good:

$$v' = (1 - \delta_v)v + i_v \tag{12}$$

and

$$k' = (1 - \delta_k)k + i_k. \tag{13}$$

Ensuring internal consistency of the model economy requires a description of the behavior of government. This behavior is kept as simple as possible given the existence of highway capital and distortional taxation. More exactly, highway capital evolves over time according to a technical constraint analogous to those of the household capital stocks:

$$g' = (1 - \delta_g)g + i_g. \tag{14}$$

Government investment spending is exogenous and must be balanced each period by lump-sum tax revenue:

$$i_g = x. \tag{15}$$

Government revenue from distortional capital-income taxation is rebated each period through lump-sum transfers:

$$n = \tau[r_v v u_v + r_k k u_k]. \tag{16}$$

The above description of government behavior is the simplest possible one given the internal consistency requirement, since the government's budget is balanced each period and government investment spending is both exogenous and independent of distortional-taxation effects on the household. (See Aschauer and Greenwood [1985], Baxter and King [1993], and Dotsey and Mao [1993] for analyses of more realistic fiscal policy.) But, the description is adequate for addressing the quantitative questions of this study. In particular, an explanation of government investment, which could involve the reverse-causation phenomenon, would have no effect on the production function or firms' investment and capital-utilization decision rules that are to be used in the econometric investigation below.[18] However, because distortional capital-income taxation does influence the decision rules and is quantitatively important for the behavior of private capital, such taxation is included in this model economy (see Greenwood and Huffman [1991] and Finn [1993] for some supporting evidence).

Regarding the stochastic, exogenous shock structure, i.e., the z, p, and i_g processes, only that pertaining to z needs detailed description here. The z process is a logarithmic random walk with drift:

$$\log\ z' = \log\ z + \log\ \bar{z} + \epsilon'. \tag{17}$$

[18] The econometric investigation will take account of the real world possibility of reverse causation.

Assume that the innovation, ϵ, is identically and independently distributed through time with zero mean. Suppose the p process is stationary. The i_g process is assumed to have a trend component, due to z, and a stationarity component. These assumptions ensure that there is only one source of growth in the economy, technology growth. Furthermore, i_g will grow at that rate along the steady-state growth path.

The competitive equilibrium of the economy obtains when the representative firm and household solve their maximization problems and the government satisfies its constraints. Of the equations implicitly defining this competitive equilibrium, only the production function, (9), the technology process, (17), and the following four equations are used in the estimation exercise. The intertemporal efficiency conditions:

$$1 = \beta E_t \left\{ \frac{c_t}{c_{t+1}} \left[(1-\tau)\theta_2 \frac{y_{t+1}}{k_{t+1}} + 1 - \delta_k - p_{t+1}\omega_1 (u_{kt+1})^{\omega_2} \right] \right\} \tag{18}$$

and

$$1 = \beta E_t \left\{ \frac{c_t}{c_{t+1}} \left[(1-\tau)\theta_3 \frac{y_{t+1}}{v_{t+1}} + 1 - \delta_v - p_{t+1}\omega_3 (u_{vt+1})^{\omega_4} \right] \right\} \tag{19}$$

govern the firm's investment decisions in k_{t+1} and v_{t+1}, respectively. The firm sets the marginal cost of investing an additional unit at time t equal to the time t expected discounted marginal benefit of the return from that investment at time $t+1$. For example, in equation (18), the marginal cost is the foregone marginal utility of consumption at time t, or $1/c_t$. The marginal benefit is the product, at time $t+1$, of the marginal utility of consumption, or $1/c_{t+1}$, and a term including the after-tax marginal product of k less its depreciation and marginal energy costs. So, the marginal benefit is:

$$(1/c_{t+1}) \left\{ \left[(1-\tau)\theta_2 \frac{y_{t+1}}{k_{t+1}} + 1 - \delta_k - p_{t+1}\omega_1 (u_{kt+1})^{\omega_2} \right] \right\}.$$

The intratemporal efficiency conditions

$$\omega_1 \omega_2 (u_{kt})^{\omega_2} p_t = (1-\tau)\theta_2 y_t / k_t \tag{20}$$

and

$$\omega_3 \omega_4 (u_{vt})^{\omega_4} p_t = (1-\tau)\theta_3 y_t / v_t \tag{21}$$

determine the firm's capital-utilization decisions, u_{kt} and u_{vt}, respectively. They equate, at time t, the marginal benefits and costs of increasing utilization rates. In equation (20), for example, the marginal benefit is the after-tax marginal product of u_{kt}, or $(1-\tau)\theta_2 y_t/u_{kt}$. The marginal cost is the marginal energy cost of u_{kt}, or $\omega_1\omega_2(u_{kt})^{\omega 2-1} p_t k_t$.

Notice that any corresponding pair of intertemporal and intratemporal efficiency conditions includes the productivity coefficient of the relevant capital stock (e.g., equations [18] and [20] each include θ_2, the productivity coefficient of k_t). This is the reason for including these equations in the estimation exercise. That is, these equations bring more information to bear on the values of the parameters of the production function.[19]

Also, the estimation exercise includes one of the model's balanced-growth restrictions, mentioned earlier:

$$E \log\,(y_{t+1}/y_t) = \log\ \bar{z}, \tag{22}$$

which states that the mean growth rate of output coincides with that of technology.

3. THE ESTIMATION METHOD AND DATA MEASURES

The estimation method is Generalized-Method-of-Moments (GMM) due to Hansen (1982) and Hansen and Singleton (1982). Those studies explain the method and show how the GMM estimator is consistent and asymptotically normal. Ogaki (1992, 1993) also explains GMM and provides practical guidance on its implementation.

Here, GMM is used to estimate the parameters of equations (9) and (18) through (22). There are two reasons it is particularly appropriate for this task. First, GMM is an instrumental-variables procedure and so avoids the possible reverse-causation bias noted at the beginning of the article.[20] Second, it is applicable to equations that are nonlinear in both parameters and variables.

In the remainder of this section, some key features and requirements of the estimation method are outlined with reference to the estimation equations of this study. Also, the data are briefly described.

The application of GMM requires that each equation include only stationary variables. Equations (18) through (22) already satisfy this requirement. Their variables are growth rates of consumption and output, output-capital ratios, utilization rates, and the relative price of energy, all of which are stationary.

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[19] This approach is similar in spirit to using the factor profit-share equations, derived from a profit function, in estimating production function parameters. Lynde and Richmond (1993) take this related approach.

[20] Aschauer (1989) obtains least-squares estimates of the production function expressed in a form that is not a cointegrating relationship in the sense of Engle and Granger (1987). Because of Aschauer's method, the possible two-way interaction between output and government capital, described at the beginning of the article, may affect his estimates. See Engle and Granger (1987) for further discussion of these econometric concepts.

By taking first differences of the logarithm of equation (9), it may be transformed into a stationary form, in which all of its variables are growth rates:

$$\hat{y}_{t+1} = \theta_1[\hat{z}_{t+1} + \hat{l}_{t+1}] + \theta_2[\hat{k}_{t+1} + \hat{u}_{kt+1}] + \theta_3(1-\psi)[\hat{v}_{t+1} + \hat{u}_{vt+1}] + \theta_3\psi[\hat{g}_{t+1}], \tag{9$'$}$$

where ˆ denotes the percentage rate of change. Using equation (17) to eliminate the unobservable $\hat{z}_{t+1}$ from equation (9′) and noting the constant-returns-to-scale restriction $\theta_1 = 1 - \theta_2 - \theta_3$ gives the estimation form of the production function

$$\hat{y}_{t+1} = (1-\theta_2-\theta_3)\hat{l}_{t+1} + \theta_2(\hat{k}_{t+1} + \hat{u}_{kt+1}) + \theta_3(1-\psi)(\hat{v}_{t+1} + \hat{u}_{vt+1}) + \theta_3\psi\hat{g}_{t+1} + (1-\theta_2-\theta_3)\ \log\bar{z} + \bar{\epsilon}_{t+1}, \tag{9$''$}$$

where $\bar{\epsilon}_{t+1} = (1-\theta_2-\theta_3)\epsilon_{t+1}$.

Next, each equation is used to generate or specify, as the case may be, a disturbance term. For equations (18) and (19), the disturbances are one-period expectational/forecast errors. The disturbances for equations (20) and (21) take the form of combinations of any omitted variables. In the case of equation (22), the deviation of output growth from its mean is the disturbance. For equation (9′′), $\bar{\epsilon}_{t+1}$ is the disturbance term.

GMM requires that the instrumental variables for any one equation belong to an information set of variables that are independent of the equation disturbance. Also, the instrumental variables must be stationary. There is no requirement that the instruments be econometrically exogenous. That is, candidate instruments, appropriately dated and transformed to satisfy the information set and stationarity requirements, include endogenous variables such as output and consumption growth. Also, instruments may include a constant term.

The instrumental variables and the corresponding disturbance terms are used to create a set of orthogonality conditions.[21] These conditions form the basis of GMM's criterion function, denoted by J here for convenience. The GMM estimator of the vector of parameters, b, is the parameter vector that minimizes the criterion function.

Recalling equation (9′′), if reverse causation is present in the data, $\hat{g}_{t+1}$ will be correlated with $\hat{y}_{t+1}$ and hence with $\bar{\epsilon}_{t+1}$. But, this possibility will not invalidate the orthogonality conditions used in the GMM procedure. It follows that possible reverse causation will not distort or bias the GMM estimates of the parameters in equation (9′′).

[21] No instrumental variables are chosen for equation (22) because it is already in the form of an orthogonality condition.

When the number of orthogonality conditions equals (exceeds) the number of parameters, the estimation system is exact (overidentified). In the case of an overidentified system, Hansen (1982) shows that the minimized value of J, multiplied by sample size, is asymptotically distributed as a Chi-square whose degrees of freedom equal the number of overidentifying restrictions. This Chi-square, therefore, provides a measure of the model's fit.

Ogaki (1992) shows that one way of testing coefficient restrictions (such as constant returns to scale) is based on the test statistic:

$$T[J(b^r) - J(b^u)], \tag{23}$$

where $b^r(b^u)$ is the GMM estimator imposing (relaxing) the coefficient restrictions and T is the sample size. This test statistic is asymptotically distributed as a Chi-square whose degrees of freedom equal the number of coefficient restrictions. Ogaki (1992) also describes various methods for correcting for serial correlation when using GMM.[22]

Some of the parameters entering into the estimation equations are not estimated in this article. They are β, τ, δ_k, and δ_v. Since they do not appear in the production function, these parameters are not central to the current exercise. Also, the existing literature provides guidance to their values, in the cases of β and τ, or to a simple method of obtaining them, in the cases of δ_k and δ_v. Therefore, β and τ are set equal to the values used in many other studies (see Finn [1993] for references), while δ_k and δ_v are set equal to the U.S. sample average depreciation rates implied by equations (12) and (13) (see Greenwood and Hercowitz [1991] for an example). The resultant values are the following: $\beta = 0.96$, $\tau = 0.35$, $\delta_k = 0.08$, and $\delta_v = 0.17$.

The data are annual, real, per-capita data for the United States during the period 1950–1989. Full details of these data, their sources, and caveats with respect to the capital-stock measures are presented in the appendix. Since there are no exact empirical counterparts for u_k and u_v, the total-industry total-private-capital utilization rate, denoted by u, proxies for both. Once estimates of the production function parameters are obtained, they are used in equation (9) to solve for a data measure of technology, z. This series is then used for one purpose in Section 4.

[22] The present article undertakes a diagnostic test for first-order serial correlation. It is a t-test and is described as follows. First, obtain the equation residuals, for each equation, using the GMM point-coefficient estimates reported in Table 3. Second, conduct an Ordinary-Least-Squares regression of the equation residuals on their one-period lagged values and a constant. The t-test pertains to the regressor's coefficient in this regression equation.

4. EMPIRICAL FINDINGS

This section discusses the empirical findings reported in Tables 3–7.

At the outset note that there is evidence of first-order autocorrelation in the residuals of equations (18) through (21), providing evidence of missing dynamic elements from these equations.[23,24] This autocorrelation is taken into account in the estimation and selection of the instruments.[25]

Consider the GMM estimation results in Table 3 for equations (18) through (22) and (9″). These results obtain for the particular choice of instruments indicated there. The productivity coefficients, θ_2 and θ_3, are statistically significant at the 5 percent level and have small standard errors. Their values seem plausible in view of the U.S. average share of total private capital in output, 0.30, found in many studies (e.g., Lucas [1990]) and the relative magnitudes of k and v.[26] The coefficients of the energy relationships, the ω_i $(i = 1, 2, 3, 4)$, are generally insignificant and very imprecise, presumably because of the omitted dynamics from equations (20) and (21). But, it is important to note that ω_i $(i = 1, 2, 3, 4)$ do not, it turns out, interfere much with the estimation of θ_2 and θ_3. That is, the estimates of θ_2 and θ_3 are essentially determined by equations (18) and (19), with little influence stemming from the marginal energy-cost terms.[27] This point is important because θ_2 and θ_3, not the ω_i $(i = 1, 2, 3, 4)$, enter the production function estimation equation, (9″). Furthermore, given the plausibility of the θ_2 and θ_3 estimates, it leads to the judgment here that the omitted dynamics from equations (18) and (19) are not that serious, at least for the purpose of this study. The mean annual rate of output growth, $\log\ \bar{z}$, is statistically significant, precise, and reasonable. The productivity coefficient of the capital stock v, $\theta_3(1 - \psi)$, is insignificant and imprecisely determined. Highway capital's productivity coefficient, $\theta_3\psi$, is 0.16. It is significant but imprecise. Highlighting the latter, note that a 95 percent confidence interval

[23] Autocorrelation was detected using the diagnostic test described in footnote 22.

[24] This finding may be related to the finding in Canova, Finn, and Pagan (1993) that the dynamic restrictions imposed by many real business cycle models are empirically rejected.

[25] The autocorrelation correction is the modified-Durbin method described in Ogaki (1992). First-order serial correlation in the residuals of equations (18) through (21) implies that the instruments for equations (18) through (21) must be lagged two periods relative to the dates of the variables appearing in those equations. The instruments for equation (9″) must be lagged only one period relative to the variables appearing in the equation, unless they are capital-stock growth rates. The latter need not (but can) be lagged because the dating of the stocks is such that they already are lagged one period relative to, say, output. (This makes the dating of the empirical stocks conform with that of the model's stocks.)

[26] The average values of k/y and v/y are 1.06 and 0.05, respectively.

[27] The evidence supporting this assertion is that similar estimation results for θ_2 and θ_3 obtain when the capital-utilization energy-cost margins are entirely ignored, i.e., when equations (20) and (21) and the terms involving utilization in equations (18) and (19) are dropped. The results for this experiment (for the same choice of instruments as in Table 3) are: $\theta_2 = 0.242(0.003)$, $\theta_3 = 0.018(0.001)$, $\psi = 8.533(3.602)$, $\log\ \bar{z} = 0.015(0.003)$, and $\chi^2_1 = 0.039(0.844)$.

Table 3 GMM Estimation Results

θ_2	$= 0.267\ (0.015)$	ω_1	$= 0.121\ (0.063)$	ω_2	$= 10.769\ (6.773)$
θ_3	$= 0.020\ (0.002)$	ω_3	$= 0.179\ (0.108)$	ω_4	$= 9.745\ (9.300)$
ψ	$= 7.963\ (3.590)$				
$\log\ \overline{z}$	$= 0.015\ (0.004)$				
$\theta_3\psi$	$= 0.158\ (0.077)$			$\theta_3(1-\psi)$	$= -0.138\ (0.075)$
		χ^2_1	$= 0.045\ (0.832)$		
Instruments:					
I(18)	$= \{\text{constant}\}$	I(20)	$= \{\text{constant}, u_{t-2}\}$	I(9′′)	$= \{\text{constant}, \hat{y}_{t-1}\}$
I(19)	$= \{\text{constant}\}$	I(21)	$= \{\text{constant}, u_{t-2}\}$	I(22)	$= \{\text{constant}\}$

Notes: (1) Coefficient standard errors are in parentheses.
(2) χ^2_1 denotes the Chi-squared statistic with one degree of freedom. Its probability value is in parentheses.
(3) I(x) denotes the instrument set for equation x.
(4) Equations (18) through (21) were estimated subject to correction for first-order serial correlation.

for $\theta_3\psi$ implies that the true value of $\theta_3\psi$ could be as high as 0.32 or as low as 0.001. The Chi-square measure of fit, χ^2_1, indicates that the model's overidentifying restrictions are not rejected at a high level of confidence.

These findings, especially regarding the productivity coefficients, are robust to a wide range of instrument sets.[28] Also, tests of the constant-returns-to-scale restrictions, i.e., $\theta_1 = 1 - \theta_2 - \theta_3$, and the constant-returns-to-scale restriction from the transportation-services equation, (8), are neither individually nor jointly rejected at high levels of confidence.

In short, the model specification finds a good deal of empirical support. The key finding is that highway capital is significantly productive, with a productivity coefficient of 0.16. However, the estimate is imprecise, which must be borne in mind when assessing the implications for growth and real returns to government investment.

Highway capital growth has implications for output and labor-productivity growth, working through its productivity coefficient. These implications are summarized in Tables 4 and 5. The contribution of highway capital growth $\hat{g}_t$ to output growth $\hat{y}_t$ is measured by $\theta_3\psi\hat{g}_t$. Regarding labor-productivity growth,

[28] In checking robustness, the instrument set for any one equation always included a constant and possibly the appropriately lagged variables appearing in that equation. The total number of instruments was kept small, following Tauchen's (1986) advocacy of a small number of instruments for small samples. Only occasionally, when using some two-period lagged variables as instruments for equations (18) through (21) or lagged capital-stock growth rates as instruments for equation (9′′), the estimation algorithm failed to converge or sensitivity of the estimates was detected. This result stemmed from the absence of strong correlation of those instruments with the equation variables.

Table 4 Output Growth Accounting

			contribution of $\hat{g}_t$ (to $\hat{y}_t$) evaluated at		
			point estimate	**upper estimate**	**lower estimate**
	$\hat{y}_t$	$\hat{g}_t$	$(\theta_3\psi = 0.158)$	$(\theta_3\psi = 0.315)$	$(\theta_3\psi = 0.001)$
1950–1969	0.022	0.030	0.005	0.009	0.00004
1970–1989	0.013	−0.005	−0.001	−0.002	−0.000008
1950–1989	0.018	0.013	0.002	0.004	0.00002

	contribution of $\hat{g}_t$ (to $\hat{y}_t$) relative to $\hat{y}_t$ evaluated at		
	point estimate	**upper estimate**	**lower estimate**
	$(\theta_3\psi = 0.158)$	$(\theta_3\psi = 0.315)$	$(\theta_3\psi = 0.001)$
1950–1969	0.218	0.434	0.002
1970–1989	−0.064	−0.127	−0.001
1950–1989	0.115	0.229	0.001

Notes: (1) The entries are average annual growth rates of the indicated variables over the time period shown. These entries have been rounded.
(2) Upper (lower) estimate of $\theta_3\psi$ is the estimate at the upper (lower) bound of the 95 percent confidence region for $\theta_3\psi$.

Table 5 Labor Productivity Growth Accounting

			contribution of $(\hat{g}_t - \hat{l}_t)$ (to $[\hat{y}_t - \hat{l}_t]$) evaluated at		
			point estimate	**upper estimate**	**lower estimate**
	$(\hat{y}_t - \hat{l}_t)$	$(\hat{g}_t - \hat{l}_t)$	$(\theta_3\psi = 0.158)$	$(\theta_3\psi = 0.315)$	$(\theta_3\psi = 0.001)$
1950–1969	0.020	0.028	0.005	0.009	0.00004
1970–1989	0.008	−0.010	−0.002	−0.003	−0.00001
1950–1989	0.014	0.010	0.002	0.003	0.00001

	contribution of $(\hat{g}_t - \hat{l}_t)$ (to $[\hat{y}_t - \hat{l}_t]$) relative to $(\hat{y}_t - \hat{l}_t)$ evaluated at		
	point estimate	**upper estimate**	**lower estimate**
	$(\theta_3\psi = 0.158)$	$(\theta_3\psi = 0.315)$	$(\theta_3\psi = 0.001)$
1950–1969	0.222	0.443	0.002
1970–1989	−0.203	−0.405	−0.002
1950–1989	0.105	0.209	0.001

Notes: (1) The entries are average annual growth rates of the indicated variables over the time period shown. These entries have been rounded.
(2) Upper (lower) estimate of $\theta_3\psi$ is the estimate at the upper (lower) bound of the 95 percent confidence region for $\theta_3\psi$.

$\hat{y}_t - \hat{l}_t$, the contribution of growth in the highway capital-to-labor ratio, $(\hat{g}_t - \hat{l}_t)$, is measured by $\theta_3\psi(\hat{g}_t - \hat{l}_t)$.

First, look at the output growth accounting. Using the point estimate $\theta_3\psi = 0.16$, the contribution of $\hat{g}_t$ is always small but important. During the 1950–1969 period, it contributes 0.5 percent to the output growth rate of 2.2 percent, representing 22 percent of that rate. In the productivity slowdown period, 1970–1989, $\hat{g}_t$ has reduced the output growth rate of 1.3 percent by 0.1 percent, amounting to 6 percent of that output growth rate. Second, examine the labor-productivity growth accounting. At the point estimate $\theta_3\psi = 0.16$, the contribution of $(\hat{g}_t - \hat{l}_t)$ is again always small but not negligible. In the period 1950–1969, the contribution is 0.5 percent to the labor-productivity growth rate of 2.0 percent, which is 22 percent of labor-productivity growth. During the productivity slowdown, $(\hat{g}_t - \hat{l}_t)$ has reduced the labor-productivity growth rate of 0.8 percent by 0.2 percent, amounting to 20 percent of labor-productivity growth.

This accounting picture changes quite substantively when the upper and lower bound estimates, 0.32 and 0.001, of $\theta_3\psi$ are used. The contributions of $\hat{g}_t$ (or $\hat{g}_t - \hat{l}_t$) become much more important or negligible, as the case may be.

What is the real return to government investment in highway capital? How does it compare with the real returns to private investment in the private capital stocks, k and v? Table 6 summarizes the answers to these questions, pertaining to average annual real returns over the 1950–1989 period.

The real return to government investment is measured by the marginal product of g: $\theta_3\psi y/g$. The private marginal products of k and v, $\theta_2 y/k$ and $\theta_3 y/v$, give the real returns to private investment in those stocks.

Using the point productivity-coefficient estimates, the real returns from investments in k and v are 25 percent and 41 percent, respectively. These returns may seem high, but of course are consistent with the corresponding point-coefficient estimates and output-capital ratios. If they were compared to other returns (e.g., Treasury bill returns), it would be important to measure their net returns (net of taxes, depreciation, and marginal energy costs) and to note any differences in risk characteristics.

At the point estimate $\theta_3\psi = 0.16$, the real return to government investment of 87 percent is considerably higher than the above private real returns. The upper and lower bound estimates of $\theta_3\psi$ imply that the true real returns could be 174 percent or 0.8 percent, respectively.

Recall that three components of government capital, EDHS, ADMN, and NATR, do not enter the quantitative analysis. Section 1 suggests that influences on the production process could stem from EDHS if labor were inaccurately measured, and from ADMN because of its association with rules and regulations as well as with research and development. If these effects exist, then the technology measure, z, will embody them. In addition, if they are of quantitative importance, then a systematic correlation between the growth rates of technology and each of EDHS and ADMN will be evident. On the other hand, Section 1

Table 6 Average and Marginal Products

(y_t/k_t)	marginal product of k_t evaluated at		
0.942	**point estimate** ($\theta_2 = 0.267$)	**upper estimate** ($\theta_2 = 0.297$)	**lower estimate** ($\theta_2 = 0.236$)
	0.251	0.280	0.223
(y_t/v_t)	**marginal product of v_t evaluated at**		
20.360	**point estimate** ($\theta_3 = 0.020$)	**upper estimate** ($\theta_3 = 0.030$)	**lower estimate** ($\theta_3 = 0.010$)
	0.405	0.607	0.203
(y_t/g_t)	**marginal product of g_t evaluated at**		
5.507	**point estimate** ($\theta_3\psi = 0.158$)	**upper estimate** ($\theta_3\psi = 0.315$)	**lower estimate** ($\theta_3\psi = 0.001$)
	0.872	1.736	0.008

Notes: (1) The entries are annual averages of the indicated variables over the period 1950–1989.
(2) Upper (lower) estimates refer to the estimates of the relevant parameter at the upper (lower) bound of its 95 percent confidence region.

suggests that NATR, in and of itself, does not influence the production process, which implies that the technology measure, z, does not incorporate productivity effects stemming from NATR. Therefore, if there is no reason that the growth rates of NATR and z should be systematically linked together, then a significant correlation between the two will not be detected.

It is interesting, therefore, to compute these correlations. Table 7 reports the results. The only significant correlation is that involving NATR.[29,30] One possible interpretation of this correlation is that changes in z, by causing changes in output, affect changes in government investment in NATR. Or perhaps both z and NATR are jointly responding to movements in some other variable such as the weather. While it would be interesting to explore these interpretations further, note that the correlation between the growth rates of z and NATR is only marginally significant.

[29] Significance is judged at the 5 percent level. The critical value for the one-sided t-test statistic, at the 5 percent significance level and with 40 degrees of freedom, is 1.68.

[30] Another government capital component, EQIP, was also omitted from the quantitative analysis. This variable is defined and explained in the appendix. The correlation between the growth rates of EQIP and technology is 0.357, with a t-statistic of 2.321. But, it is difficult to interpret this correlation since EQIP is a component that should be split across the EDHS, ADMN, and NATR categories.

Table 7 Correlations Between Growth Rates of Technology and Omitted Government Capital Stocks

Fire and Natural Resource Capital	0.288	(1.826)
Educational and Hospital Capital	0.248	(1.557)
Administrative, Judicial, Police, and Research and Development Capital	0.150	(0.924)

Notes: The entries (not in parentheses) are correlations between the growth rates of technology and the indicated capital stock over the period 1950–1989. The numbers to the right of these entries (in parentheses) are corresponding t-statistics.

5. CONCLUSION

The key empirical finding is that highway capital is significantly productive. The point estimate of its productivity coefficient is 0.16, meaning that for every 1 percent change in highway capital, output responds by 0.16 percent. But, there is much uncertainty surrounding this estimate. To highlight this uncertainty, consider that the true productivity coefficient could be as high as 0.32 or as low as 0.001. Further work achieving more precise estimation of the productive effect of highway capital would be worthwhile.

Using the productivity-coefficient estimate, 0.16, the implications for output growth accounting are as follows. During the 1950–1969 period, highway capital growth contributes 0.5 percent to the output growth of 2.2 percent, representing 22 percent of the output growth. In the productivity slowdown period, 1970–1989, highway capital growth has reduced the output growth of 1.3 percent by 0.1 percent, amounting to 6 percent of that output growth. These effects are small but significant. They imply that government investment in highway capital matters for output growth. However, the uncertainty surrounding the productivity-coefficient estimate of 0.16 must qualify this assessment of the magnitude of the contribution of highway capital growth to output growth. That contribution could be much larger or smaller than the numbers just mentioned suggest.

Over the period 1950–1989 the real return to government investment in highway capital, when evaluated at the productivity coefficient 0.16, averages 87 percent per year. While, again, there is much uncertainty about this estimate, suppose for discussion purposes that it is reliable. The real return, 87 percent, is high. Compare it to, say, the real return to private investment in private capital that averages 25 percent per year over the same period. Does this imply that government investment in highway capital should be increased up to the point that ensures equality across the two returns? It is difficult to answer such a question about the optimal level of government investment. Much will depend on the financing of government investment. Suppose, for example, increases in government investment are financed by increases in the tax rates on labor and/or private capital income. Increases in these tax rates will work to reduce labor and private capital, thereby leading to output losses. On the other hand, the increase in government investment, by increasing government capital, will cause output to increase. The optimal level of government investment is that level which carefully balances these opposing output effects (see Glomm and Ravikumar [1992] for an analysis of these issues in a deterministic endogenous growth model). It is not clear that the optimal level occurs exactly at the point of equality between the real returns to private and government capital. Further complications arise if uncertainty is factored into the analysis. In the presence of uncertainty, real returns to investing in different assets are generally not equated, even in an expected sense, reflecting the differential roles that different assets play in hedging consumption risk (see Finn [1990]). Further exploration of optimal government investment that addresses the considerations just raised is an important task for future research.

APPENDIX: DATA SOURCES, DEFINITIONS, AND CAVEATS

The data sources are the following: (1) Citibase, (2) National Income and Wealth Division, Bureau of Economic Analysis, U.S. Department of Commerce, denoted by DC, (3) Federal Reserve Bulletin, denoted by FRB, and (4) Dale W. Jorgenson, Harvard University, denoted by DWJ. Unless otherwise indicated, the source is Citibase.

Population (thousands of persons): civilian non-institutional population aged 16 and over.

Output (billions of 1987 dollars): gross domestic product less gross government product.

Labor Hours (real index): aggregate domestic private quality-adjusted labor hours index, where the quality adjustment is based on a cross-classification by age, sex, education, class of worker, and occupation. The index is described in Jorgenson, Gallop, and Fraumeni (1987), Chap. 8. Source: DWJ.

Utilization Rate (real index): manufacturing sector utilization rate (1950–1953) and total industrial sector utilization rate for the remainder of the sample. Source: FRB.

Aggregate Price Deflator (1987=100): gross domestic product deflator.

Energy Price Index (1987=100): producer price index for fuels and related products (covering petroleum, natural gas, coal, and electricity).

Relative Price of Energy (1987=1): ratio of the energy price index to the aggregate price deflator.

Consumption (billions of 1987 dollars): personal consumer expenditures on nondurable goods plus services.

Private Transportation Vehicle Capital and Investment (billions of 1987 dollars): The capital is the net, end-of-period stock of transportation vehicles (automobiles, trucks, trailers, and buses) owned by the private (business) sector and government enterprises. The latter is proxied by taking one-tenth of enterprise equipment capital (see the discussion on caveats below, part [e], for an explanation). The investment is the corresponding gross investment. Source: DC.

Private Capital and Investment, excluding that pertaining to transportation vehicles (billions of 1987 dollars): The capital is a net, end-of-period stock consisting of nonresidential, fixed capital owned by the private (business) sector and government enterprises and that owned by general government but privately operated, plus federal government airport facilities, plus state and local government harbors. The terms *general government* and *harbors* are explained in footnotes 9 and 7, respectively. The latter two components of capital are proxied by three-quarters of the stocks in the DC "federal other structures" and "state and local conservation and development" categories (see the discussion on caveats, part [b], for more information). The investment series is the corresponding gross investment. Source: DC.

Total Government Capital (billions of 1987 dollars): government (federal, state, and local), net, end-of-period, fixed, nonresidential, nonmilitary capital. Source: DC.

Government Capital Components

In what follows, the mnemonics correspond to the paragraph titles in the text (pp. 55–58), except for EQIP, which denotes equipment capital. The components are defined with reference to the DC categories, the titles of which are in italics. Unless otherwise indicated, those categories are the sum of federal, state, and local government categories. Source: DC.

HGWY: *highways and streets*.

ENTP: government *enterprises* plus federal government airport facilities plus state and local government harbors. The latter two components of capital are approximated as described above.

GOPO: *government-owned and privately operated*.

EDHS: *educational and hospital buildings*.

NATR: federal *conservation and development* plus one-quarter of state and local *conservation and development* (i.e., the residual after measuring ENTP as described above).

ADMN: The structures component of total government capital less the structures components of ENTP and GOPO less HGWY, EDHS, and NATR (these last three components are entirely composed of structures).

EQIP: The equipment component of total government capital less the equipment components of ENTP and GOPO. Note that HGWY, EDHS, NATR, and ADMN do not have equipment components.

Some Caveats and Comments on the Capital-Stock Data

(a) The government *enterprises* category includes but does not isolate toll highways. *Highways and streets* also includes but does not isolate these. So there is unavoidable double-counting of toll highways. Because toll highways are a small part of government *enterprises* and *highways and streets,* this mismeasurement is probably not significant.

(b) Measures of federal government airport facilities and state and local government harbors are not published. Given the DC description of federal *other structures* and state and local *conservation and development,* it seems reasonable to get approximate measures of these variables by taking three-quarters of federal *other structures* and state and local *conservation and development,* respectively. Federal *other structures* are small relative to both total government capital and government *enterprises,* averaging 0.4 percent and 1.7 percent, respectively, over the sample period. Also, state and local *conservation and development* is small in relation to total government capital and government *enterprises,* averaging 1.2 percent and 5.1 percent, respectively, over the sample period. Therefore, any mismeasurement arising from the use of the indicated approximations in this article is not likely to be quantitatively significant.

(c) The fire capital-stock series described in the text on p. 58 is not separately available, nor is there useful information for forming an approximate measure. It is only because of the discussion in the text as opposed to the estimation task of the article, that the fire capital stock should be included in NATR and excluded from ADMN and EQIP, both of which, recall, are derived as residual series. But, this mismeasurement is probably immaterial since the fire capital stock is surely small.

(d) The only measures of equipment capital published by DC, relevant here, are the equipment components of total government capital, government *enterprises* and *government-owned and privately operated*. These components are used to form the EQIP series. No information is available for allocating EQIP among the EDHS, NATR, and ADMN components, which would have been interesting for the purpose of the discussion in the text.

(e) A measure of government enterprise vehicle capital is not available. Given the existence of the series on the equipment component of government *enterprises* and the fact that some enterprises undertake much transportation (public transit enterprises and the U.S. Post Office), it is desirable to get some proxy for enterprise vehicles. A proxy of one-tenth of the government *enterprises* equipment component seems reasonable in view of the list of government enterprises. Any mismeasurement arising from the use of this proxy in this study is not likely to be important since the equipment component of government *enterprises* is a small stock, averaging 0.8 percent of total government capital and 3.3 percent of government *enterprises* over the sample period.

REFERENCES

Aschauer, David. "Is Public Expenditure Productive?" *Journal of Monetary Economics,* vol. 23 (March 1989), pp. 177–200.

__________, and Jeremy Greenwood. "Macroeconomic Effects of Fiscal Policy," *Carnegie-Rochester Conference Series on Public Policy,* vol. 23 (Autumn 1985), pp. 91–138.

Barro, Robert, and Xavier Sala-i-Martin. "Public Finance in Models of Economic Growth," *Review of Economic Studies,* vol. 59 (October 1992), pp. 645–61.

Baxter, Marianne, and Robert King. "Fiscal Policy in General Equilibrium," *American Economic Review,* vol. 83 (June 1993), pp. 315–34.

Braun, Tony, and Ellen McGrattan. "The Macroeconomics of War and Peace." Manuscript. Federal Reserve Bank of Minneapolis, April 1993.

Canova, Fabio, Mary Finn, and Adrian Pagan. "Evaluating a Real Business Cycle Model," in C. Hargraves, ed., *Nonstationary Time Series Analyses and Co-integration.* Oxford: Oxford University Press, forthcoming 1993.

Department of Commerce, Bureau of Economic Analysis. Government Transactions Methodology Paper Series MP-5. Washington, D.C., 1988.

Dotsey, Michael, and Ching Mao. "Stochastic Fiscal Policy in a Neoclassical Growth Model." Manuscript. Federal Reserve Bank of Richmond, April 1993.

Engle, Charles, and Clive Granger. “Co-integration and Error Correction: Representation, Estimation and Testing,” *Econometrica*, vol. 55 (March 1987), pp. 257–76.

Fernald, John. “How Productive Is Infrastructure?” Manuscript. Harvard University, November 1992.

Finn, Mary. “Variance Properties of Solow’s Productivity Residual.” Manuscript. Federal Reserve Bank of Richmond, March 1993.

__________. “On Savings and Investment Dynamics in a Small Open Economy,” *Journal of International Economics*, vol. 29 (August 1990), pp. 1–21.

Garcia-Mila, Teresa, and Therese McGuire. “The Contribution of Publicly Provided Inputs to States’ Economies,” *Regional Science and Urban Economics*, vol. 22 (June 1992), pp. 229–41.

Glomm, Gerhard, and B. Ravikumar. “Public Investment in Infrastructure in a Simple Growth Model.” Manuscript. University of Virginia, October 1992.

Greenwood, Jeremy, and Zvi Hercowitz. “The Allocation of Capital and Time over the Business Cycle,” *Journal of Political Economy*, vol. 99 (December 1991), pp. 1188–1214.

Greenwood, Jeremy, and Gregory Huffman. “Tax Analysis in a Real Business Cycle Model: On Measuring Harberger Triangles and Okun Gaps,” *Journal of Monetary Economics*, vol. 27 (April 1991), pp. 167–90.

Hansen, Gary, and Edward Prescott. “Did Technology Shocks Cause the 1990–1991 Recession?” *American Economic Review*, vol. 83 (May 1993), pp. 280–86.

Hansen, Lars. “Large Sample Properties of Generalized Method of Moment Estimators,” *Econometrica*, vol. 50 (July 1982), pp. 1029–54.

__________, and Kenneth Singleton. “Generalized Instrumental Variable Estimation of Nonlinear Rational Expectations Models,” *Econometrica*, vol. 50 (September 1982), pp. 1269–86.

Holtz-Eakin, Douglas. “Public-Sector Capital and the Productivity Puzzle,” Working Paper 4122. Cambridge, Mass.: National Bureau of Economic Research, July 1992.

Hulten, Charles, and Robert Schwab. “Public Capital Formation and the Growth of Regional Manufacturing Industries.” Manuscript. University of Maryland, 1991.

Jorgenson, Dale, Frank Gollop, and Barbara Fraumeni. *Productivity and U.S. Economic Growth*. Oxford: Oxford University Press, 1987.

King, Robert, Charles Plosser, and Sergio Rebelo. “Production, Growth and Business Cycles: I,” *Journal of Monetary Economics*, vol. 21 (March/May 1988), pp. 195–232.

Lucas, Robert E. "Supply-Side Economics: An Analytical Review," *Oxford Economic Papers*, vol. 42 (April 1990), pp. 293–316.

__________. *Models of Business Cycles.* New York: Basil Blackwell, 1987.

Lynde, Catherine, and John Richmond. "Public Capital and Total Factor Productivity," *International Economic Review,* vol. 34 (May 1993), pp. 401–14.

Munnell, Alicia. "How Does Public Infrastructure Affect Regional Economic Performance?" in Alicia Munnell, ed., *Is There a Shortfall in Public Capital Investment?* Boston: Federal Reserve Bank of Boston, 1990a.

__________."Why Has Productivity Growth Declined? Productivity and Public Investment," *New England Economic Review,* January/February 1990b, pp. 3–22.

Ogaki, Masao. "GMM: A User Guide," Working Paper 368. Rochester: Rochester Center for Economic Research, April 1993.

__________. "Generalized Method of Moments: Econometric Applications." Manuscript. University of Rochester, March 1992.

Tatom, John. "Public Capital and Private Sector Performance," Federal Reserve Bank of St. Louis *Review,* vol. 73 (May/June 1992), pp. 3–15.

Tauchen, George. "Statistical Properties of Generalized Method of Moment Estimators of Structural Parameters Obtained from Financial Market Data," *Journal of Business and Economic Statistics*, vol. 4 (October 1986), pp. 397–425.