Marshallian Cross Diagrams and Their Uses before Alfred Marshall: The Origins of Supply and Demand Geometry

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INTRODUCTION

Undoubtedly the simplest and most frequently used tool of microeconomic analysis is the conventional partial equilibrium demand-and-supply-curve diagram of the textbooks. Economics professors and their students put the diagram to at least six main uses. They use it to depict the equilibrium or market-clearing price and quantity of any particular good or factor input. They employ it to show how (Walrasian) price or (Marshallian) quantity adjustments ensure this equilibrium: the first by eliminating excess supply and demand, the second by eradicating disparities between supply price and demand price. They use it to illustrate how parametric shifts in demand and supply curves induced by changes in tastes, incomes, technology, factor prices, and prices of related goods operate to alter a good's equilibrium price and quantity. They apply it to show how the shifting and incidence of a tax or tariff on buyers and sellers depends on elasticities of demand and supply. With it they demonstrate that price ceilings and price floors generate shortages and surpluses, respectively. Finally, they employ it to compare the allocative effects of competitive versus monopoly pricing and to indicate the welfare costs of market imperfections.

The diagram's applications are of course well known. Not so well known, however, are its origins and early history. Economists typically tend to associate the diagram with Alfred Marshall, its most persuasive and influential nineteenth century expositor. So strong is the association that economists have christened the diagram the Marshallian cross or Marshallian scissors after Marshall's analogy comparing the price-determining properties of a brace of demand and supply curves with the cutting properties of the blades of a pair of scissors.

The diagram itself, however, long predates Marshall. Antoine-Augustin Cournot originated it in 1838. And Karl Rau (1841), Jules Dupuit (1844), Hans von Mangoldt (1863), and Fleeming Jenkin (1870) thoroughly developed it years before Marshall presented it in his *Pure Theory of Domestic Values* (1879) and later in his *Principles of Economics* (1890). Far from merely introducing the diagram, these writers applied it to derive many of the concepts and theories often attributed to Marshall or his followers. The notions of price elasticity of demand and supply, of stability of equilibrium, of the possibility of multiple equilibria, of comparative statics analyses involving shifts in the curves, of consumers' and producers' surplus, of constant, increasing and decreasing costs, of pricing of joint and composite products, of potential benefits of price discrimination, of tax incidence analysis, of deadweight-welfare-loss triangles and the allocative inefficiency of monopoly: all find expression in early expositions of the diagram.

These expositions, however, have not always been fully appreciated. John Maynard Keynes ([1925] 1956: 24), for example, dismissed them as vastly inferior to "Marshall's diagrammatic exercises" which "went so far beyond the 'bright ideas' of his predecessors that we may justly claim him as the founder of modern diagrammatic economics." In the same vein, Michael Parkin (1990: 85) recently has claimed that of early graphical treatments of supply and demand only Marshall's is sufficiently modern to be recognized by textbook readers today. Such discounting of the pre-Marshall work has contributed to what Joseph Schumpeter (1954: 839 n. 13) complained of as economists' "uncritical habit of attributing to Marshall what should, in the 'objective' sense, be attributed to others (even the 'Marshallian' demand curve!)." Seen this way, Marshall's definitive contribution emerges as the *culmination* of the diagram's development. Economists, Schumpeter thought, misperceived it as the *origin*.

In an effort to correct such misperceptions and to give the earlier writers their due, this paper traces their pioneering contributions so as to counter the notion that the Marshallian cross diagram begins with Marshall. To paraphrase Frederick Lavington's famous remark that "it's all in Marshall, if you'll only take the trouble to dig it out," (Wright 1927: 504) the following paragraphs attempt to show, with respect to the diagram and its applications, that it's all in Marshall's predecessors too.

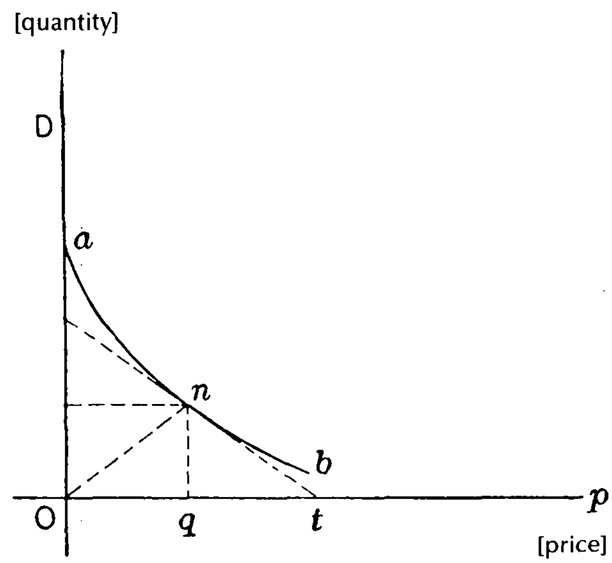
ANTOINE-AUGUSTIN COURNOT (1801-1877)

Though hardly the first to state that supply and demand determine price, Antoine-Augustin Cournot, in his 1838 *Recherches sur les principes mathématiques de la theorie des richesses (Researches into the Mathematical Principles of the Theory of Wealth)*, was the first to draw market demand and supply curves for a particular good.[1] Of the two curves, Cournot analyzed demand before introducing supply. Figure 1 shows his diagram of the demand function $D = F(p)$ where D is quantity sold at different prices p, which Cournot measured along the horizontal rather than the vertical axis as is customary today.[2] Cournot ([1838] 1971: 52-3) noted that corresponding to each price on the demand curve *ab* is a price-quantity rectangle whose area $R = pD$ represents total revenue R associated with that price. He sought to determine the particular price at which revenue is at a maximum. A profit-maximizing monopolist would charge this price if his costs were either zero or a fixed sum independent of quantity produced.

To find the revenue-maximizing price, Cournot differentiated the revenue function $R = pD = pF(p)$ with respect to price. He obtained the expression $pF'(p) + F(p)$, where F' is the derivative of the demand function F. Setting this expression equal to zero as required for a maximum and rearranging, he got $p = -F(p)/F'(p)$, which says that the revenue-maximizing price must equal the ratio of the quantity demanded to the slope of the demand curve at that price. To depict this solution diagrammatically, he rewrote the expression as $F(p)/p = -F'(p)$.

Figure 1

COURNOT'S DEMAND CURVE



Source: Cournot ([1838] 1971: Fig. 1 at end of book).

The left-hand side he represented, for any point n on the demand curve, by the slope $qn/0q$ of the ray $0n$ (see Figure 1). The right-hand side he portrayed by the slope qn/qt of the line segment nt. He noted that these slopes are equal only when $0q$ equals qt, which uniquely determines the revenue-maximizing price $0q$ as one-half of $0t$.

Price Elasticity of Demand

Cournot (53-4) also anticipated Marshall's concept of price elasticity of demand, defined as the percentage change in quantity demanded divided by percentage change in price: $(dD/D)/(dp/p)$ or pdD/Ddp. True, William Whewell had foreseen the idea before him in 1829. But Whewell, whose work was unknown to Cournot, did not use diagrams or draw a demand curve. Cournot, in drawing the curve, argued that one can determine the effect of a small change in price Δp on total revenue $pF(p)$ by comparing the curve's slope $\Delta D/\Delta p$ with the ratio of quantity demanded to price D/p. A small rise in price, he claimed, will cause total revenue to rise, fall, or remain unchanged at its peak level as $\Delta D/\Delta p$ is less than, greater than, or equal to D/p. Setting $\Delta D/\Delta p = D/p$ and dividing both sides by D/p yields $(p/D)(\Delta D/\Delta p) = 1$. Now the left-hand side of this expression is Marshall's measure of elasticity. Hence Cournot's statement that total revenue

[1] That supply and demand determine price must have been known for thousands of years. But the phrase "supply and demand" itself is of more recent vintage; Sir James Steuart originated it in 1767 (Thweatt 1983). Adam Smith ([1776] 1937: 56) in his *Wealth of Nations* used the notion of supply and demand to explain deviations of actual market price from natural price determined by long-run cost of production. And David Ricardo ([1817-21] 1951) gave the idea added prominence when he incorporated it into the title "On the Influence of Demand and Supply on Price" of Chapter 30 of his *Principles of Political Economy and Taxation*.

[2] On Cournot see Theocharis (1983: 138-46) and Ekelund, Furubotn, and Gramm (1972: 15-19).

achieves its stationary maximum when $\Delta D/\Delta p$ equals D/p is equivalent to saying that it does so at the point of unitary elasticity on the demand curve. Similarly, his contention that revenue rises or falls with a rise in price as $\Delta D/\Delta p$ is less than or greater than D/p is tantamount to declaring that it does so as elasticity is above or below one in value.

Supply Curve Introduced

After specifying the elasticity-revenue relationship, Cournot incorporated supply curves into his diagram. Assuming a regime of perfect competition, he (90-2) argued that profit-maximizing firms equate price with marginal cost. Since marginal cost after a certain point rises with the level of output, each firm produces along a schedule showing output as an increasing function of price. Summing over the individual firms' supply functions, Cournot obtained the total market supply function $S(p)$ expressing a schedule of quantities offered at all possible prices. Finally, he equated market supply and demand $S(p) = F(p)$ to determine equilibrium price and output. All this he depicted in Figure 2 in which the downward-sloping demand curve *MN* is combined with the upward-sloping supply curve *PQ* to yield the equilibrium price-quantity combination S.[3]

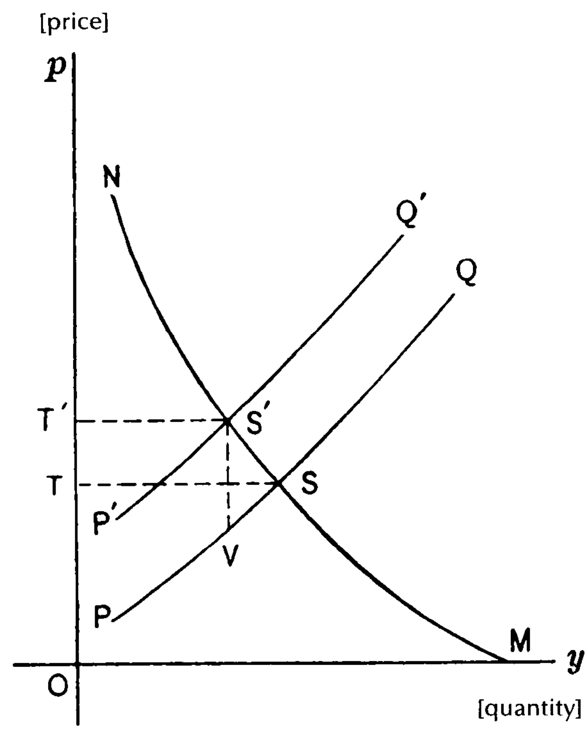
Effects of a Tax

Having depicted supply-demand equilibrium, Cournot (92-3) next used his curves to show the effect of a per-unit tax of amount *VS'* levied on a particular good. He argued that the tax, by adding *VS'* francs to the cost of supplying each unit of output, would shift the supply curve upward by that same amount. The result is a rise in the equilibrium price from *0T* to *0T'* and a fall in the equilibrium quantity from *TS* to *T'S'*. He noted, however, that with an upward-sloping supply curve the price increase *TT'* is always less that the tax *VS'* provided the demand curve is not vertical. Indeed, if the demand curve were horizontal the tax would not increase price at all. Cournot was thus the first to show that given a positively sloped supply curve the portion of an excise tax shifted to buyers in the form of higher prices varies inversely with demand elasticity, being nil at infinite and complete at zero values of that parameter. Cournot, not Marshall, invented geometrical tax-incidence analysis.

[3] This is Cournot's Figure 6 with the axes switched to conform to current practice.

Figure 2

COURNOT'S SCISSORS DIAGRAM AND TAX INCIDENCE ANALYSIS



Source: Cournot ([1838] 1971: Fig. 6 at end of book).

KARL HEINRICH RAU (1792-1870)

Cournot did not use his scissors diagram to expound the stability of market-clearing equilibrium. Although he employed stability analysis in his famous reaction-function model of duopoly, he never did so in his competitive supply and demand model. In 1841, three years after the publication of Cournot's *Researches* but with no knowledge of its contents, the German economist Karl Heinrich Rau rectified this oversight (Hennings 1979).[4] In so doing, he became the first to employ a Marshallian cross diagram to investigate the stability of market equilibrium and to indicate the forces that restore that equilibrium once it is disturbed.

Rau's diagram, which owed nothing to Cournot's and indeed differed from it by putting price on the vertical axis and quantity on the horizontal, first

[4] Hennings (1979: 6) argues that Rau never saw a copy of Cournot's *Researches* and thus was ignorant of its diagrams. He points out that neither Rau's library nor that of his university possessed Cournot's book.

appeared in a note in the 1841 volume of the *Bulletin de l'Académie des Sciences et des Belles-Lettres de Bruxelles*.[5] He published a more elaborate version in the appendix to section 154 of the fourth (1841) and all subsequent editions of his textbook *Grundsätze der Volkswirthschaftslehre* (see Figure 3). There he argued that, given the linear demand curve *gh* and the vertical supply curve *de*, a below-equilibrium price *ec* would produce an excess demand *cb*. Suppliers would take advantage of the resulting shortage of the good to raise price until the shortage disappeared and equilibrium *m* was restored.[6]

Rau thought this result important enough to state algebraically. His formula, $cm = (ab - ac) \tan w$, expresses the price rise *cm* as the product of the excess demand $(ab - ac)$ times the slope of the demand curve $\tan w$ where w is the angle formed by lines *cb* and *bm*. This trigonometric expression, of course, holds only for the linear demand-vertical supply case. Alternatively, if the demand curve were instead the concave schedule *fi*, the same excess demand would induce sellers to raise price to *p*. And if the supply schedule were the positively sloped curve *ek*, then excess demand would spur sellers to boost price to *l* or *n* depending on which demand curve they faced. Conversely, an above-equilibrium price would generate an excess supply that would put downward pressure on price until equilibrium was restored.

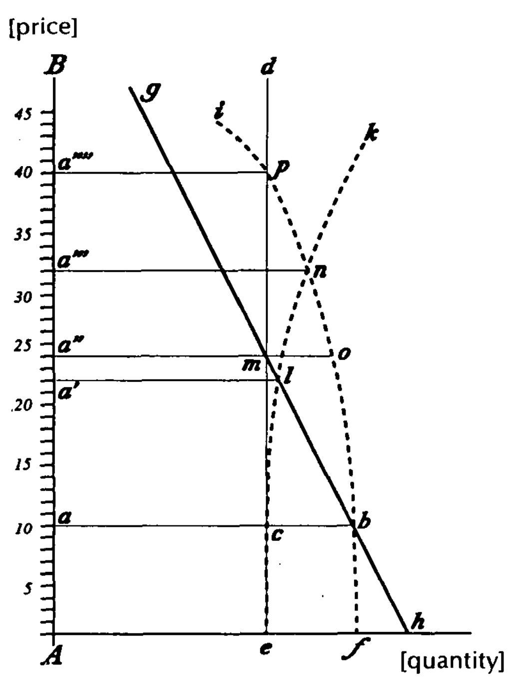
Rau's diagram seems to have influenced only one of his contemporaries, Hans von Mangoldt, who used it as the starting point for his own demand-and-supply-curve analysis in 1863, twenty-two years after Rau first drew it.[7] In the meantime, other writers, including the French engineer Jules Dupuit who published demand-curve diagrams in 1844, either ignored it or were unaware of its existence.

[5] On Rau's diagram and its applications see Hennings (1979) who provides an English translation of the relevant passages.

[6] Of course demanders too could take the initiative and bid up the price. But Rau did not mention this possibility.

Figure 3

RAU'S DIAGRAM



Source: Rau (1841b: 527).

JULES DUPUIT (1804-1866)

Despite their originality, neither Cournot nor Rau derived welfare propositions from their diagrams. Because they saw demand curves as empirical sales schedules rather than as replicas of theoretical marginal utility functions, they said nothing about the welfare implications of monopoly pricing, public utility rate setting, discriminatory pricing, or commodity taxation.[8]

Jules Dupuit, however, was not so hampered. Explicitly identifying demand curves with marginal utility schedules, he became, in 1844, the first to derive welfare theorems with the aid of a Marshallian diagram. True, he drew no supply curves. He merely assumed a constant supply price or one that varies independently of the level of output. But he made path-breaking use of the demand curve to define such Marshallian concepts as total utility, consumer surplus, and deadweight-welfare-loss triangles, not to mention Laffer-curve relationships between tax rates and revenues. In so doing he advanced demand theory far beyond Cournot and Rau, whose work was unfamiliar to him.[9]

[7] Hennings (1980: 670) notes than Mangoldt, in advertising his book, explicitly cited Rau as a precedent for employing diagrams.

[8] On Cournot's and Rau's view of demand functions as purely empirical schedules, see Ekelund, Furubotn, and Gramm (1972: 17) and Hennings (1979: 2).

[9] Ekelund and Hébert (1983: 260) note that Dupuit was ignorant of Cournot's work even though both writers once lived and worked in Paris at the same time.

Laws of Demand

Figures 4, 5 and 6 depict Dupuit's diagrams as presented in the appendix to his 1844 article "On the Measurement of the Utility of Public Works" in the *Annales des Ponts et Chaussées.*[10] The diagrams as shown illustrate Dupuit's two "laws" of demand.[11] Law number one says that demand curves—expressed by Dupuit as $y = f(x)$, where y denotes quantity demanded and x price—slope downward because of diminishing marginal utility: extra quantities of a good or service add less and less to total satisfaction and thus command lower demand prices. Law number two says that a given fall in price induces larger increases in quantity demanded the lower the price at which it occurs. This law Dupuit attributed to the pyramidal distribution of income; each price decrement activates the demands of a new group of buyers larger and poorer than the group above it on the income scale. The resulting new demands, added to those already existing at higher prices and responding to lower prices, give the curve its characteristically convex shape.

[10] These are Dupuit's Figures 1, 3 and 4 with the axes switched to facilitate inspection.

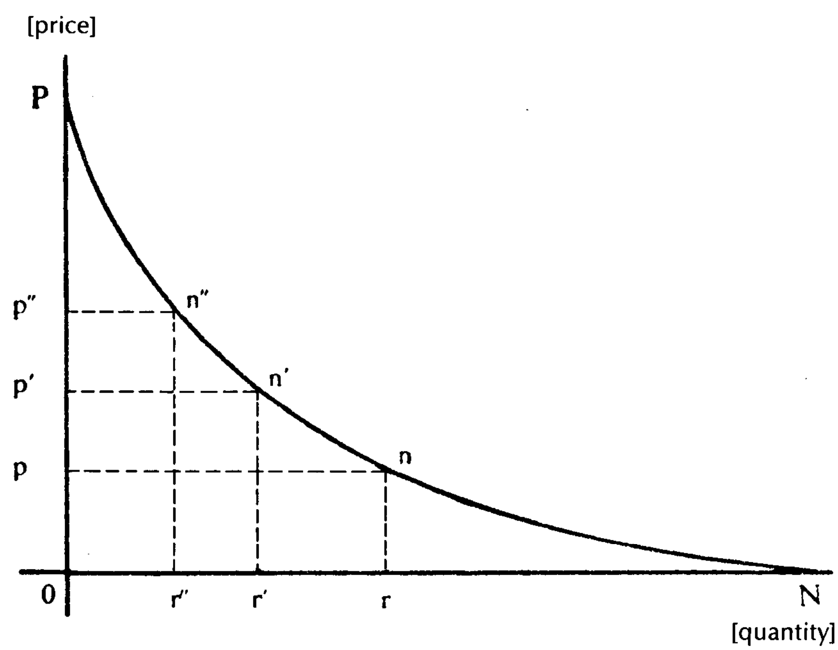
[11] On Dupuit's two laws of demand, see Houghton (1958: 50) and Ekelund and Thornton (1991).

Marginal Utility, Total Utility, Consumers' Surplus (Figure 4)

Having deduced the shape of the demand curve, Dupuit used his first diagram to refute J. B. Say's contention that a good's market price measures the utility of each unit consumed. Not so, said Dupuit ([1844] 1969: 280-1). Price *0p* measures the (marginal) utility *nr* of the last unit purchased only. Preceding or inframarginal units such as r' and r'' yield higher marginal utilities as indicated by the higher demand prices p' and p'' buyers would pay for those units rather than go without. Summing over these successive demand prices as one moves up the demand curve gives a measure of the total utility of the entire quantity consumed. This measure is represented by the roughly trapezoidal area *0Pnr* under the demand curve and not, as Say implied, by the price-times-quantity or total-expenditure rectangle *0pnr*. Say's measure understates utility by the amount of the consumers' surplus triangle *pPn*—called *utilité relative* by Dupuit. Taken to its extreme, Say's analysis erroneously implies that total

Figure 4

DUPUIT ON TOTAL UTILITY, MARGINAL UTILITY, AND CONSUMERS' SURPLUS



Source: Dupuit ([1844] 1969: 280).

utility is zero when price is zero. In fact, total utility would then be at its maximum equal to the entire area under the demand curve.

As for the triangular area *Nrn* in the lower right-hand corner of the diagram, Dupuit defined it as the utility lost when a positive market price *0p* constrains consumption short of the satiation point *N*.[12] Under competitive conditions this loss is the natural result of resource scarcity and cannot be avoided. Here price measures the satisfaction forgone on other goods whose production falls so that resources can be freed to produce an extra unit of the good in question. That the last *rN* units of this good possess marginal utilities falling below price indicates that they are not worth producing; their opportunity cost exceeds the extra satisfaction they would bring. Thus units *r* through *N* are forgone so that resources can be put to higher-valued uses elsewhere.

Under monopoly pricing, however, the loss may be due to contrived restriction of output and is a true measure of social harm. A costless monopoly charging price *0p*, for example, needlessly deprives consumers of satisfaction equivalent to the area *Nrn*.

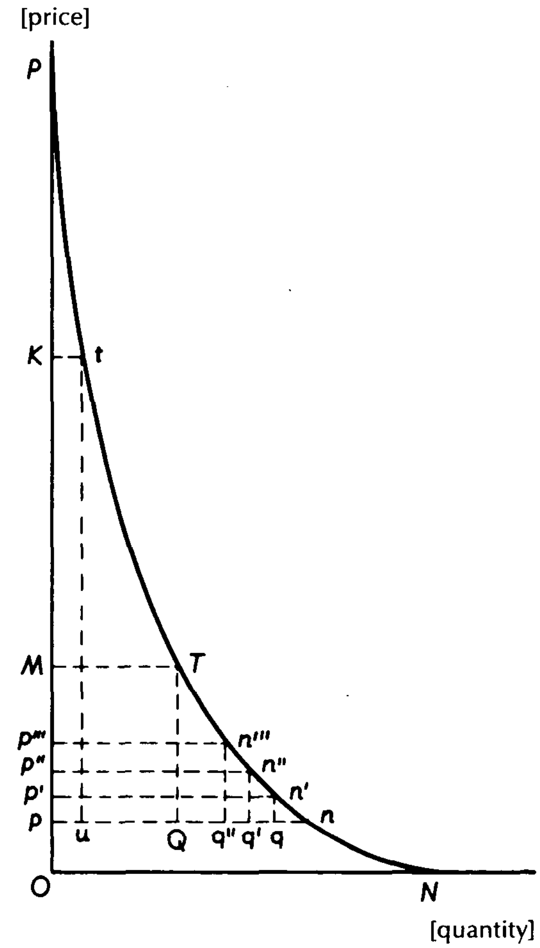
Here is the first diagram in the history of economic thought to depict a deadweight-loss-from-monopoly triangle and a total utility trapezoid under the demand curve. And it is the first to partition the trapezoid into a price-quantity rectangle showing buyer expenditure on the good and a consumers' surplus triangle showing the excess of what consumers would pay over what they actually do pay. In short, Dupuit, not Marshall, was the first to demonstrate diagrammatically that consumers get more utility than they pay for when they buy a good or service at a single market price. He was also the first to extend this insight to the evaluation of public works. In particular, he noted that the potential benefits of such projects cannot be measured by their costs. One needs to estimate the area under the demand curve.

Dupuit's Tax Theorems (Figure 5)

Armed with the utility concepts developed in his first diagram, Dupuit (281-2) used them in his second diagram to derive key propositions concerning the welfare effects of commodity taxes. His first proposition states that the imposition of a tax results in a loss in consumers' surplus that exceeds the yield of the levy. His diagram shows how a per unit tax

Figure 5

DUPUIT'S TAX THEOREMS



Source: Dupuit ([1844] 1969: 282).

of *qn'* raises price by *pp'*, thus reducing purchases to *p'n'* and consumers' surplus to *p'Pn'*. On the *p'n'* units still bought, consumers pay a total tax of *pp'n'q* to the government, which Dupuit assumed puts it to socially productive uses. But, on the *qn* units no longer bought, buyers lose consumers' surplus *qnn'* with no corresponding gain to the government. Hence the tax causes a loss of consumers' surplus that exceeds the tax yield by the roughly triangular area *n'qn*: the deadweight loss of the tax. This loss, consisting of the tax-induced distortion of relative prices and consumption patterns, persists even if the government returns the proceeds to the taxpayers.

Dupuit's second theorem states that the deadweight loss is proportional to the square of the tax rate. As mentioned above, the welfare loss ΔU is the area of the triangle *n'qn* whose height is the

[12] On Dupuit's concept of lost utility (*utilité perdue*) see Ekelund (1970: 271-3).

tax rate t and whose base is the reduction in quantity bought ΔQ caused by the duty. Since the area of a triangle is half its height times its base, one sees that the net loss in utility is half the tax rate times the fall in amount purchased or $\Delta U = \frac{1}{2}t\Delta Q$. Now ΔQ, the change in amount bought, can by definition be expressed as $\Delta Q = kt$, where k is the inverse $\Delta Q/t$ of the slope of the triangle's hypotenuse. Substituting this expression into its predecessor yields Dupuit's taxation theorem: $\Delta U = \frac{1}{2}kt^2$ or, in his (281) words, "the loss of utility increases as the square of the tax."[13] It follows that the government minimizes the welfare burden of a given total tax collection by charging a low rate on a great many goods rather than a high rate on a few. For the welfare loss, which shrinks as the square of a lowered rate, approaches zero as the tax becomes general or diffused and its rate correspondingly small.

Dupuit's third theorem posits an inverted U-shaped or Laffer-curve relationship between tax rates and tax revenue. Like Arthur Laffer in the 1980s, Dupuit in 1844 saw tax revenues rising from zero with small increases in the rate, reaching a maximum *pMTQ* at rate *pM*, then falling with further rate increases, and eventually returning to zero when the rate becomes prohibitive. This rate-revenue relationship together with his second tax theorem led him (282) to conclude "that the yield of a tax is no measure of the loss which it causes society to suffer." For the same yield can be obtained from two different rates entailing markedly different deadweight losses. Rates *pK* and *pp*", for example, yield the same revenue; yet the first rate's welfare-loss triangle is more than ten times the size of the second's. Likewise, zero and prohibitive tax rates both yield zero revenue. The zero rate, however, produces no welfare loss while the prohibitive rate produces a loss equal to the whole area under the demand curve.

Pricing Policies and Price Discrimination (Figure 6)

Dupuit (282-3) employed his last diagram to specify appropriate pricing policies for private and public monopolies having identical fixed costs *Opnr*. A private monopoly would charge the price *OM* that maximizes its receipts *OMTR* and, with costs given and independent of output, its profits too. By contrast, a public utility would charge the lowest price *Op* that maximizes consumer satisfaction subject to meeting the cost constraint. Consumers' surplus would be larger and deadweight loss smaller by the amounts *pMTn* and *TRrn*, respectively.

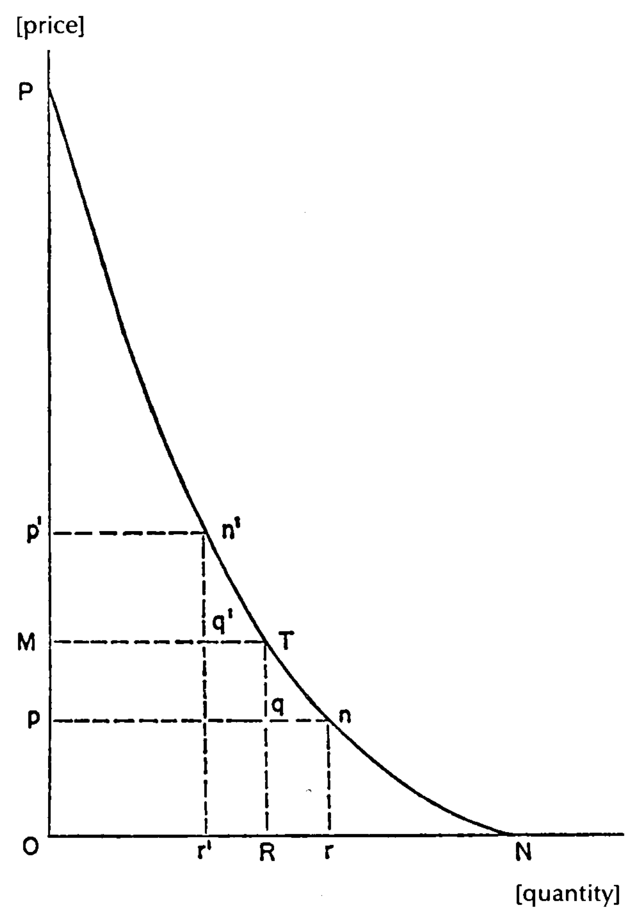
Dupuit also analyzed price discrimination—the practice of charging separate customers different prices for the same product—with the aid of his third diagram.[14] He argued that discriminatory pricing could render profitable a firm that would suffer losses if it charged a single price. Dupuit examined the case of a monopolist whose fixed costs exceed his receipts *OMTR* at the revenue-maximizing price *OM*. The monopolist, by dividing his market *MT* into two groups, one paying price *Op'* for quantity *p'n'* and the other price *OM* for quantity *q'T*, could expand

[13] See Hébert and Ekelund (1984: 62) for an alternative derivation of this formula.

[14] On Dupuit's analysis of price discrimination see Ekelund (1970: 271-5).

Figure 6

DUPUIT ON PRICING POLICIES AND PRICE DISCRIMINATION



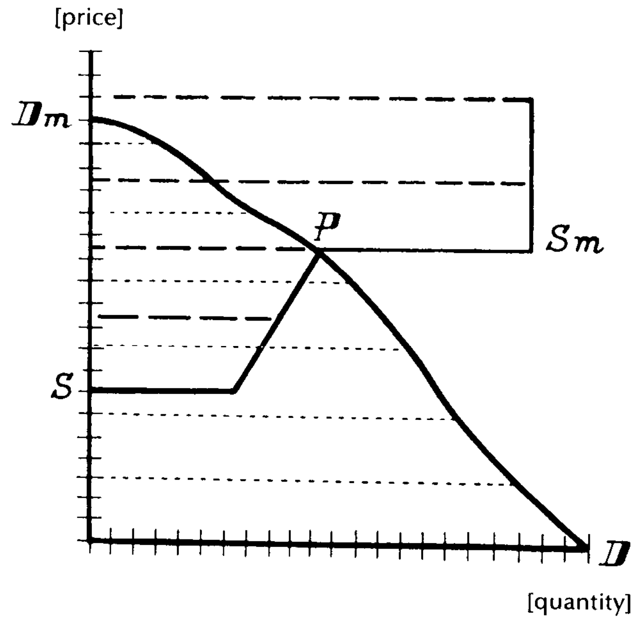
Source: Dupuit ([1844] 1969: 283).

his receipts by an amount $Mp'n'q'$ sufficient to defray his costs. Further discrimination would yield even larger receipts. For example, were the monopolist able to charge the maximum price for each successive unit of the *MT* quantity sold, he could effectively redistribute consumers' surplus to himself and capture revenue equal to the entire area *OPTR*.

To Dupuit, however, price discrimination could accomplish more than merely redistributing a fixed sum of welfare from buyers to sellers. It could *increase* total welfare if it led to increased output. Let a monopolist initially charging price *OM* on output *OR* find it profitable to sell extra output *Rr* at discriminatory price *Op*. Then total utility, Dupuit claimed, would rise by *RTnr* at the expense of a corresponding shrinkage in deadweight loss to *nrN*. Discriminatory pricing, in other words, would yield a net social benefit. Later, in the 1920s and 1930s, Marshall's students A.C. Pigou and Joan Robinson would echo Dupuit's declaration of the welfare superiority of output-increasing price discrimination over simple monopoly pricing.

Figure 7

MANGOLDT'S CROSS DIAGRAM



Source: Mangoldt ([1863] 1962: 33).

HANS VON MANGOLDT (1824-1868)

Dupuit had shown how much one could accomplish working with the demand curve alone. It was time, however, to reintroduce the supply curve into the diagram and to examine the role of cost in price determination. Hans von Mangoldt took this step in his 1863 *Grundriss der Volkswirtschaftslehre (Outline of Political Economy)*.

Taking his cue from Karl Rau, whose work he cited, Mangoldt began his chapter on "The Exchange Ratio of Goods" with a scissors diagram similar to Rau's (Figure 7). Like Rau, he identified the market-clearing equilibrium price *P* and described the adjustment process that restores it once it is disturbed. His stability analysis, like Rau's, highlights the price-equilibrating role of excess demand or supply. Let price fall below equilibrium, he ([1863] 1962: 32) said, and the resulting excess of demand over supply bids it back to equilibrium. Likewise, an above-equilibrium price activates an excess of supply over demand that puts downward pressure on price until it returns to equilibrium.

Demand Curve (Figure 8)

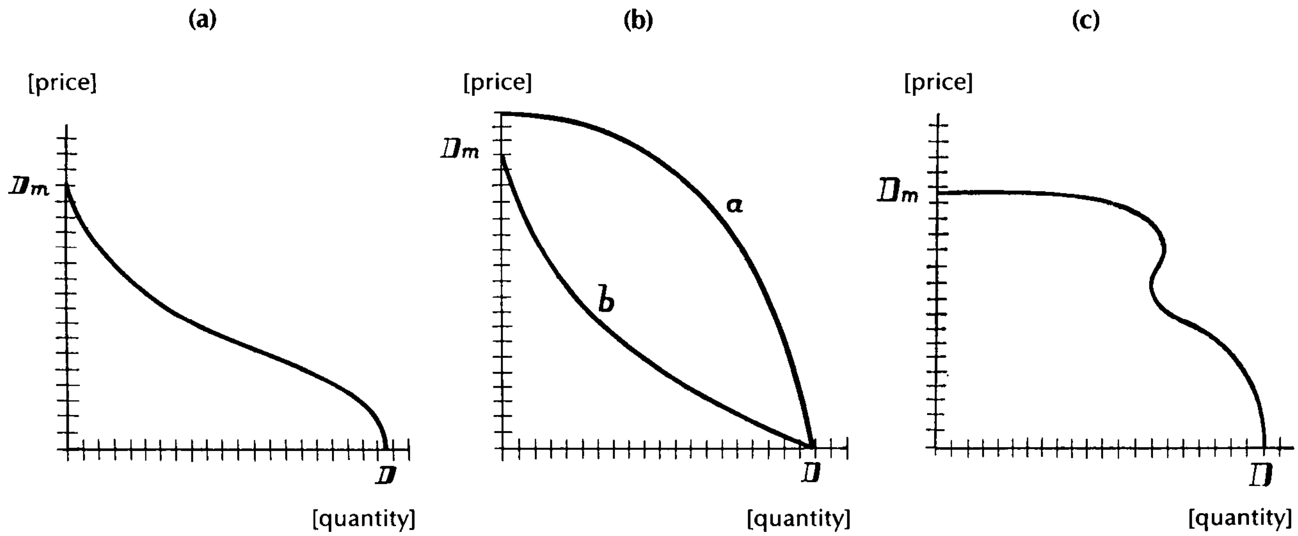
Following his stability analysis, Mangoldt (33-5) proceeded to examine the demand curve in great detail. He argued that (1) the height of each point on the curve represents the marginal utility of the corresponding quantity, (2) the curve slopes downward because of diminishing utility of additional units and the resulting reduction in prices buyers are willing to pay, and (3) a rise in price induces buyers to cut back their purchases until the marginal utility of the last unit bought rises to match the higher price. The demand curve cuts the vertical axis, he said, at a price which just exceeds the marginal utility of the good's first unit (point *Dm* on Figure 8a). Conversely, demand reaches its satiation point *D* on the quantity axis when price is zero.

As for shifts in the curve, Mangoldt attributed them to population growth, to changes in tastes and knowledge, and to economic development and the resulting rise in income and wealth. Unlike Dupuit, who believed that demand curves must be of convex shape, Mangoldt held that they could be either convex or concave (see Figure 8b) depending on the type of good (luxuries or necessities), on the degree of inequality of income distribution, and on the availability of close substitutes.

Finally, he noted certain exceptions to the law of demand. Demand curves, he argued, could possess upward-sloping segments (see Figure 8c) if tastes for conspicuous consumption ("vanity") or expectations of further price hikes ("fear") motivated consumers

Figure 8

MANGOLDT'S DEMAND CURVES



Source: Mangoldt ([1863] 1962: 34-35).

to buy larger quantities at higher prices. And rising demand curves, he realized, could intersect supply curves more than once, giving rise to the possibility of multiple equilibria.

Supply Curves (Figure 9)

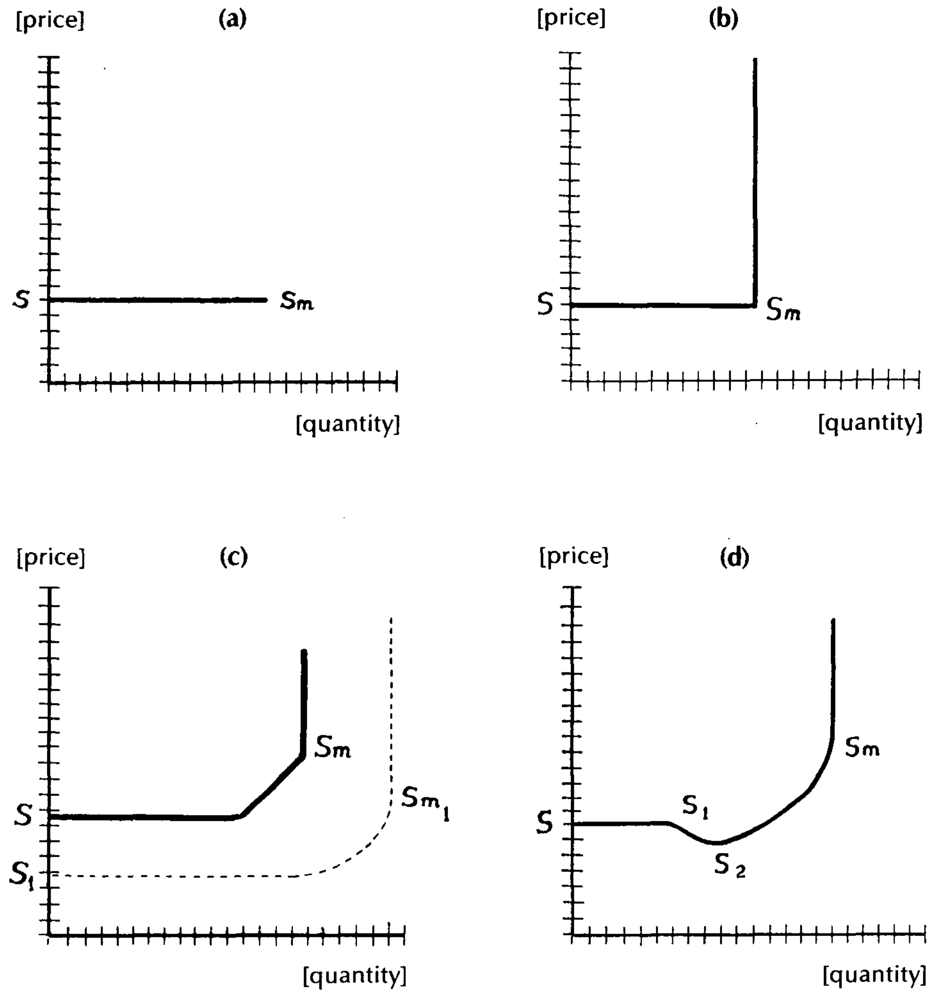
Turning his attention to supply curves, Mangoldt (35-7) made his most enduring contribution. He was the first to draw such curves with different shapes depending on the behavior of costs of production. Constant unit costs yield a horizontal or perfectly elastic curve (Figure 9a). Constant costs up to the limit of a rigidly fixed capacity yield a reverse L-shaped curve possessing horizontal and vertical segments (Figure 9b). Constant costs that give way to increasing costs and then to rigidly fixed limits yield a curve with perfectly elastic, relatively elastic, and perfectly inelastic components (Figure 9c) . Finally, decreasing costs owing to economies of scale over a certain range of output followed by increasing costs due to diseconomies of scale yield a roughly U-shaped curve that falls before it subsequently rises (Figure 9d). As for outward secular shifts in supply curves, Mangoldt ascribed them to technological progress and resource discovery—forces tending to lower the cost of producing any level of output.

Comparative Statics Exercises (Figure 10)

Today Marshall's name is associated with the partial equilibrium, comparative statics method. But it was Mangoldt, not Marshall, who pioneered the technique. Having presented curves of demand and supply, Mangoldt (38-40) put them through a series of exercises designed to show how shifts in the curves affect equilibrium price and quantity. Rightward shifts of the demand curve along a perfectly elastic supply curve raise quantity but not price (Figure 10a). Price, that is, is supply-determined in the constant cost case. The same demand shifts occurring along a vertical or perfectly inelastic segment of the supply curve raise price but not quantity (Figure 10b). Price is demand-determined in this case. Similarly, leftward shifts in demand produce only price falls when supply is perfectly inelastic (vertical curve PS_m in Figure 10c) and only quantity reductions when supply is perfectly elastic (curve SS_m). In the typical case of relatively elastic supply, however, demand-curve shifts change both price and quantity (Figure 10d). Finally, simultaneous rightward shifts in both demand and supply curves can cause equilibrium price to rise, fall, or remain unchanged depending upon which shift, if either, predominates (Figure 10e).

Figure 9

MANGOLDT'S SUPPLY CURVES



Source: Mangoldt ([1863] 1962: 36-37).

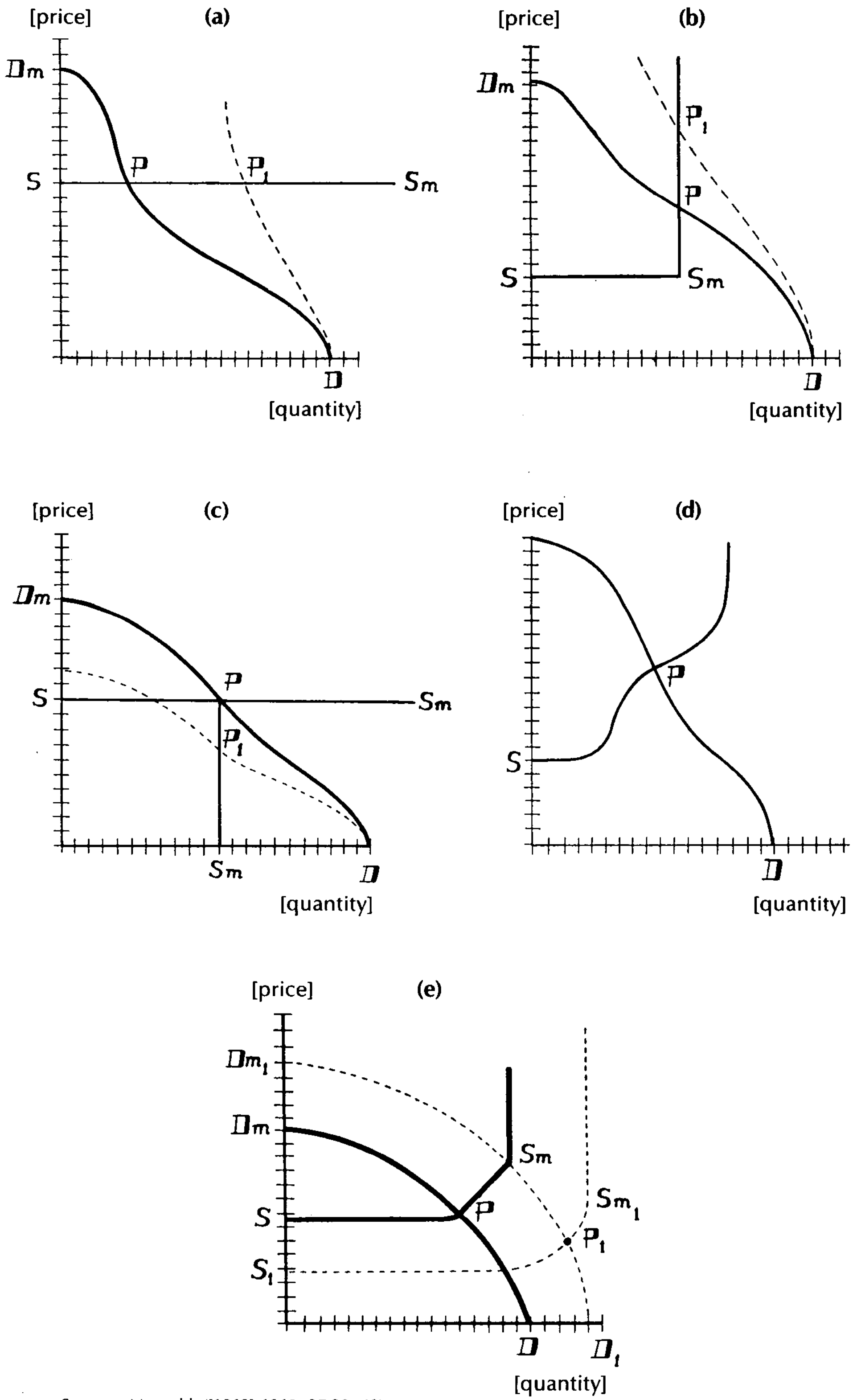
These exercises alone are sufficient to ensure Mangoldt's place in the history of economic thought. But he went beyond them to describe a three-step adjustment process by which price and quantity move from one equilibrium position to another. Years before Marshall he (51) posited a Marshallian mechanism. First comes an outward shift in either the demand curve or supply curve. This produces a positive gap between demand price and supply price at the existing level of output. The resulting rise in profits induces producers to expand output until the price differential is eliminated and the new equilibrium is attained. All this, Mangoldt noted, refers to unanticipated shifts in the curves. Should the shifts be anticipated, price immediately jumps to its new equilibrium, thus avoiding the sequential adjustment process.

Multiple Equilibria

Anticipating Marshall, Mangoldt (50) discussed the possibility of multiple equilibria of demand and supply. He noted that such phenomena cannot occur when the two curves slope in opposite directions and so intersect no more than once. But they can occur when both curves slope in the same direction. Here Mangoldt cited demand curves that rise with price because of desires for conspicuous consumption or expectations of even higher future prices. Likewise he cited supply curves that fall because of

Figure 10

COMPARATIVE STATICS EXERCISES



Source: Mangoldt ([1863] 1962: 37-38, 40).

increasing returns to scale. In such cases, multiple intersections are possible and there may be several equilibrium prices. Having said this, however, Mangoldt said nothing about which equilibria are stable and which unstable. He failed to apply the analysis he had used before in the case of a single unique equilibrium. With respect to multiple equilibria, he did not recognize the stability problems involved.

Pricing of Joint Products (Figure 11)

Nevertheless, Mangoldt's analysis of intersecting supply and demand curves must be judged an outstanding performance that in many respects exceeded those of his predecessors. Equally impressive was his application of the diagram to the problem of price determination when goods are jointly demanded or supplied in fixed proportions. True, John Stuart Mill had briefly discussed this problem in his 1848 *Principles of Political Economy*. But Mangoldt's geometric and algebraic analysis eclipsed Mill's purely verbal treatment and was not superseded until Marshall's *Principles*. A "great achievement" and "Mangoldt's most significant contribution to price theory," Eric Schneider (1960: 380, 384) called it. A "brilliant theoretical contribution" concurred Jürg Niehans (1990: 128). What follows sketches the geometric part of Mangoldt's contribution. Readers can find treatments of the algebraic portion in the appendix to this article as well as in Schneider (1960) and Creedy (1992: 38-46).

Mangoldt first examined the "joint demand" case of a pair of goods A and B purchased in fixed proportions under a given spending constraint.[15] He (41-4) showed how demand and supply determine the equilibrium price and quantity of both goods consistent with the constraint. He also showed that a fall in A's supply price—i.e., a rightward shift in its supply curve (curve ff_5 in Figure 11a)—raises B's price and explained why. The cheapening of A induces buyers to take more of it and, because of fixed proportions, to demand more B too. The resulting upward shift in the demand curve for B (curve gg_5) raises its price. In this way, a cost reduction that increases the supply of A raises the price of B.

Turning to the "joint supply" case of a pair of goods such as beef and cowhides produced in fixed proportions subject to a given cost constraint, Mangoldt showed how equilibrium is established in both markets. He (46-8) also showed that a rise in the demand for beef must lower the price of hides and gave the rationale. With fixed proportions, the increased demand for beef leads to a rise in its output as well as that of hides and so, given the demand for hides, to a fall in their price. In this way an upward shift in the demand curve for beef (curve ee_5 in Figure 11b) produces a downward shift in the supply curve of hides (curve dd_5) that lowers their price.

Having examined joint demand and supply, Mangoldt for completeness considered composite demand and supply. Composite demand refers to the case where two competing uses (e.g., furniture and firewood) vie for one fixed input (timber). Here Mangoldt (48-50) showed that a fall in the demand for furniture would, by making more timber available for firewood, increase the latter's supply and lower its price. Composite supply refers to the case where two substitute goods (e.g., flax and cotton) satisfy a single need (for cloth). Here he (44-6) showed that a rise in supply and hence fall in the price of flax lowers the demand for and so the price of cotton. These topics were not further developed until Marshall took them up in his *Principles*.

Mangoldt's Influence

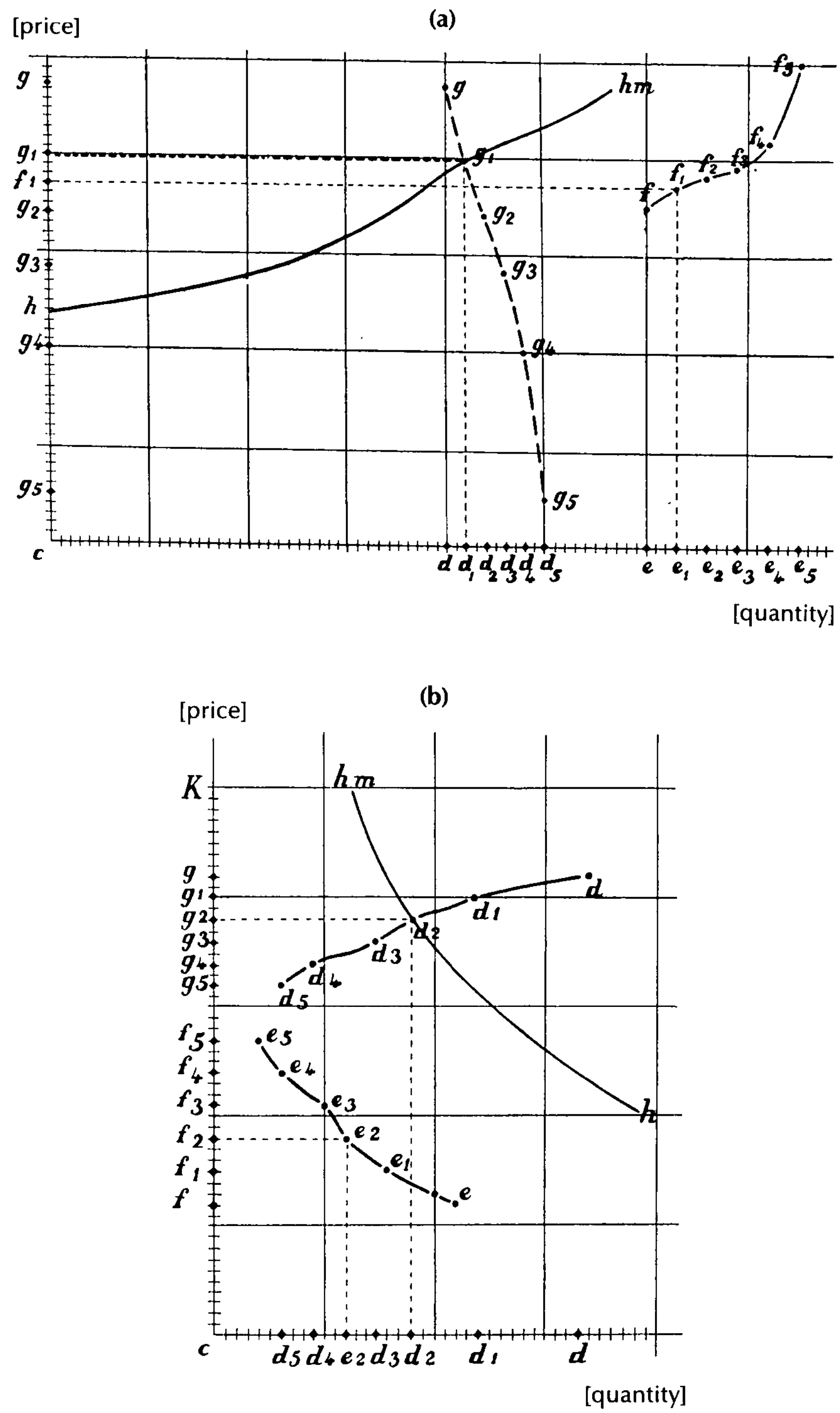
Mangoldt's diagrammatic analysis should have become common property to all economists by the 1870s. That it did not is attributable to one Friedrich Kleinwächter who, upon publishing a reprint of Mangoldt's book in 1871 shortly after his death, deleted the diagrams on the grounds that "it is utterly inconceivable to me that graphs or mathematical formulae could facilitate the understanding of economic laws" (Creedy 1992: 46; see also Schneider 1960: 392). Mangoldt's contribution fell into oblivion for twenty-three years until Francis Edgeworth ([1894] 1925: 53) rediscovered it in 1894 and proclaimed its author "one of the independent discoverers of the mathematical theory of Demand and Supply." Edgeworth might well have said the same thing about Henry Charles Fleeming Jenkin, the distinguished electrical engineer and inventor, who, with no formal training in economics and no acquaintance with the work of Mangoldt or his predecessors, introduced demand and supply curves—indeed the technique of diagrammatic analysis—into the English economic literature circa 1870.[16]

[15] Examples include (1) scotch and soda, and (2) copper and zinc used in making brass.

[16] On these points see Brownlie and Lloyd Prichard (1963: 211, 216) who note that Jenkin had read little economics other than J. S. Mill's *Principles*.

Figure 11

PRICING OF JOINT PRODUCTS



Source: Mangoldt ([1863] 1962: 43, 47).

FLEEMING JENKIN (1833-1885)

Jenkin presented his analysis in three papers: his 1868 *North British Review* article "Trade-Unions: How Far Legitimate?," his 1870 *Recess Studies* piece "The Graphic Representation of the Laws of Supply and Demand, and Their Application to Labour," and his 1872 contribution to the *Proceedings of the Royal Society of Edinburgh 1871-2* "On the Principles Which Regulate the Incidence of Taxes." In his 1870 paper he ([1931]: 77) drew intersecting curves representing equations which he ([1931]: 17-8) had stated in his 1868 piece, namely $D = f(A+1/x)$ and $S = F(B+x)$ where D and S denote quantities demanded and supplied, respectively, x denotes price, and A and B denote shift parameters that determine the location or height of the curves on the diagram.

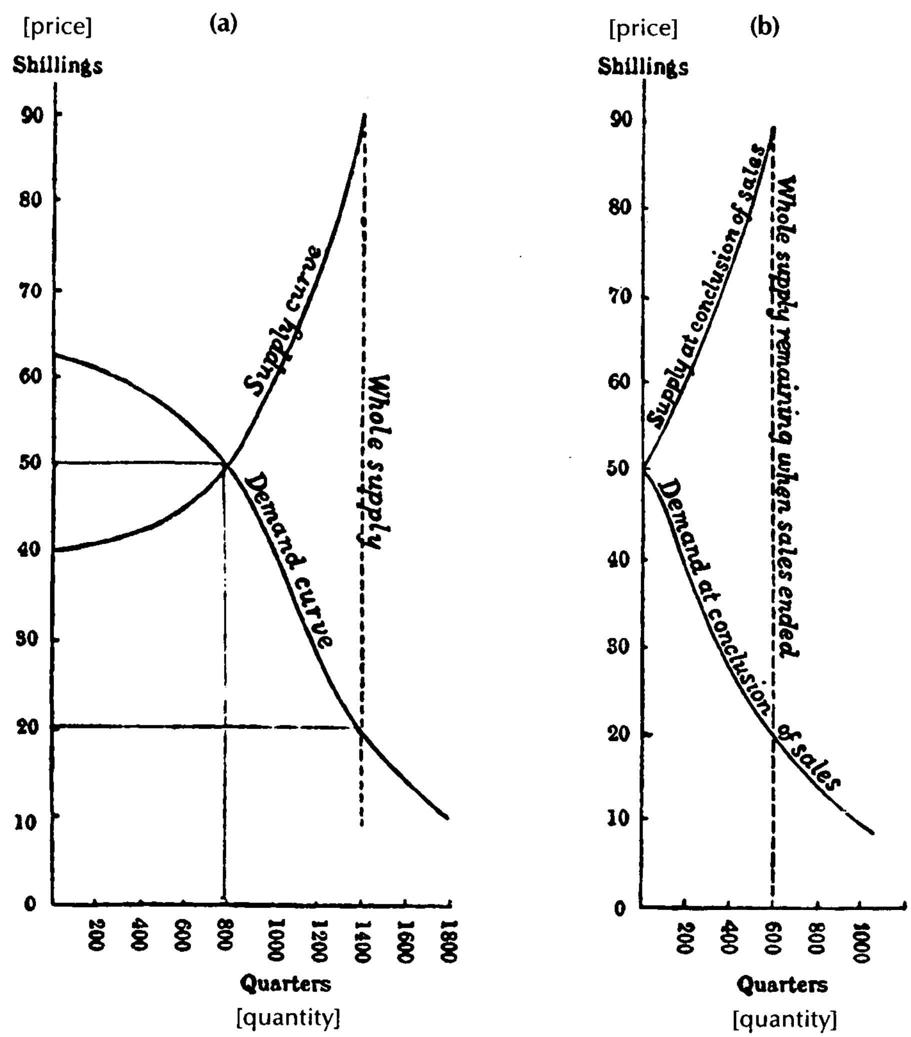
He (17-8) noted that the curves' intersection point depicts the equilibrium price and quantity that solve the market-clearing equation $D=S$ or $f(A+1/x) = F(B+x)$. To ensure stability of equilibrium he relied on excess supply or demand triggered by price deviations from equilibrium. These excess supplies or demands, he said, act immediately to restore price to its market-clearing level.

Market-Period Price Determination (Figures 12 and 13)

Anticipating Marshall's assumption of separate operational time periods, Jenkin (78, 89) conceived two hypothetical intervals—market period and long run—to which his analysis applied. In the market period the stock of goods is fixed and cost of production plays no role in price determination. Quantity supplied equals the stock on hand times the fraction offered for sale. This fraction varies directly with the differential between actual market price and traders' reservation prices, i.e., prices below which stocks are held for future sale and above which they are marketed immediately. Different traders possess different reservation prices stemming from their expectations of future prices (99, 109). Those expecting low future prices have low reservation prices. Those expecting high future prices have high reservation prices. A rising market price surpasses a growing number of reservation prices, thus enlarging the fraction of the

Figure 12

JENKIN ON MARKET-PERIOD PRICE DETERMINATION



Source: Jenkin ([1870] 1931: 77, 79).

stock marketed. Accordingly, quantity supplied rises with price and the supply curve slopes upward until it turns vertical when the entire stock or "whole supply" (Figure 12a) is marketed.[17]

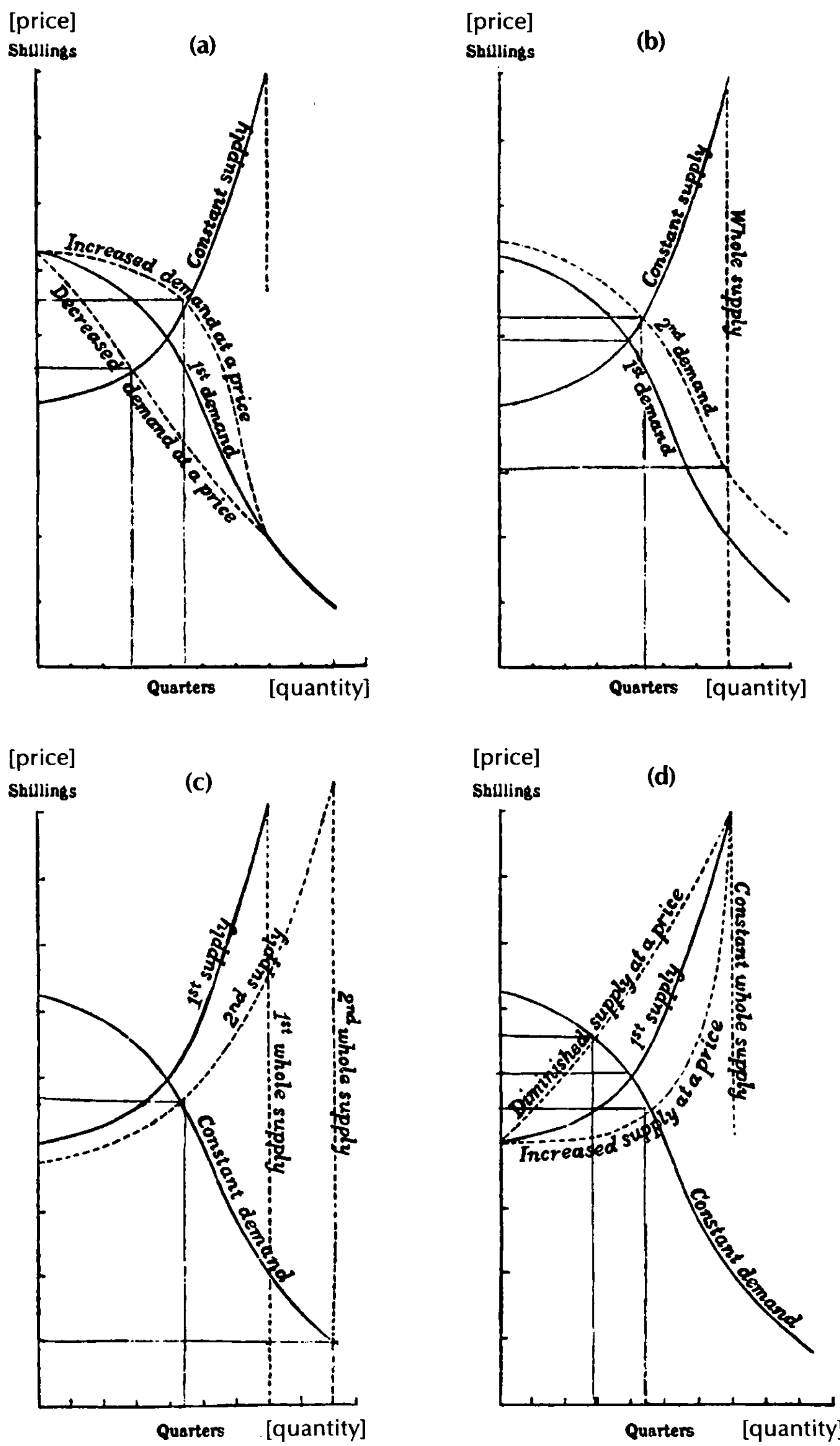
As for market-period demand curves, Jenkin drew them with a negative slope indicating that lower prices are required to compensate for diminishing marginal utility of additional units bought. Intersection of demand and supply curves yields an equilibrium (Figure 12a) characterized by zero excess supply or demand as the market clears (Figure 12b). This equilibrium, however, is extremely volatile. It changes with every event, real or imagined, that shifts the curves. Demand curves shift with variations in buyers' whims, desires, and expectations (Figure 13a) as well as with changes in incomes (Figure 13b). Supply curves shift with variations in the size of stocks on hand (Figure 13c) and with changes in traders' expectations and thus the reservation prices they set (Figure 13d).

Long-Run Price Determination (Figure 14)

Turning to the long period when output can vary, Jenkin (89-93) showed that the latter adjusts to equilibrate demand and supply prices. Long-period supply price consists of average cost of production, which includes the sum of the costs of factor inputs per unit of output

Figure 13

SHIFTS IN MARKET-PERIOD CURVES

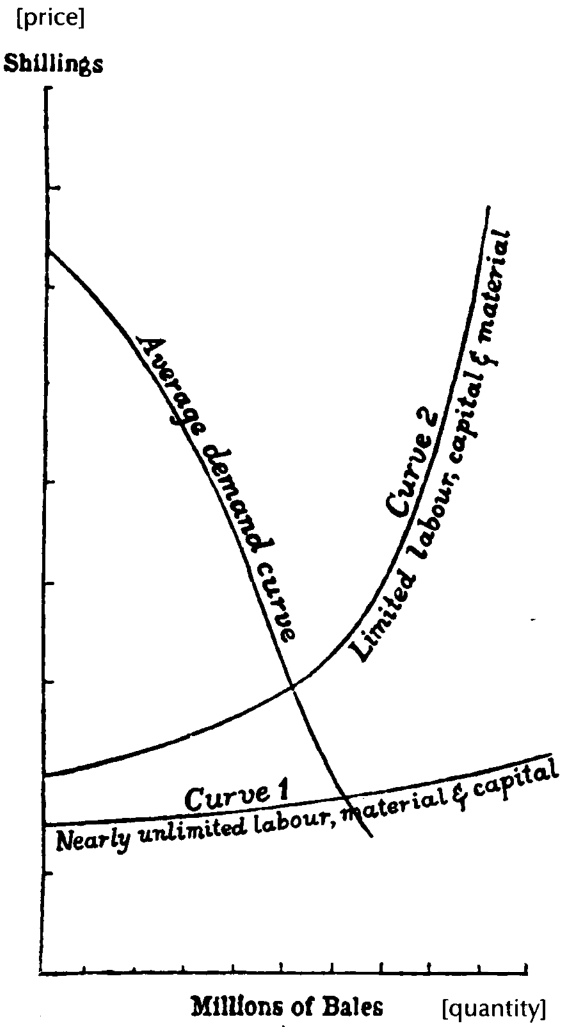


Source: Jenkin ([1870] 1931: 80-82).

[17] Jenkin's diagrams which, like Cournot's and Dupuit's, measured price horizontally and quantity vertically are shown here with their axes transposed.

Figure 14

LONG-PERIOD PRICE DETERMINATION



Source: Jenkin ([1870] 1931: 90).

plus normal profits of producers. Unit or average cost generally rises with output. The reason: productive factors are in limited supply and must be paid higher prices to bid them away in greater quantities from alternative uses. Consequently, if a good requires specialized inputs that are extremely scarce and thus increasingly costly to obtain, unit costs rise rapidly with output and the supply curve is steeply sloped (curve 2 of Figure 14). Conversely, if inputs are so plentiful that their prices are virtually invariant to increases in the demand for them, unit costs will be nearly constant and the supply curve relatively flat (curve 1 of Figure 14). In this latter case, cost of production approximately determines price while demand approximately determines output. Here is Jenkin's version of the Marshallian cases of long-run increasing and constant cost industries, respectively.

Application to Labor Unions (Figure 15)

As is evident from the titles of his papers, Jenkin developed his supply and demand diagrams for two main purposes: to examine the impact of trade unions on wage determination and to elucidate the welfare effects of taxes. With respect to trade unions, he (94-106) argued that they could, by accumulating a strike fund to support workers during walkouts, raise equilibrium wage rates in both the market period and the long run.

In the market period the labor force is given and, if non-unionized, will work for whatever wage it can get. Labor's supply curve is a vertical line whose intersection with employers' demand-for-labor curve determines the equilibrium wage (Figure 15a). Enter the trade union. With its strike fund the union allows its members to enjoy a reservation wage below which they withhold labor from employers rather than selling it for what it will fetch. The resulting labor supply curve, instead of being a vertical straight line, becomes a right-angled or reverse L-shaped curve at the reservation wage set by the union (Figure 15a). The labor demand curve will cut the supply curve in its horizontal segment such that equilibrium wages will be higher and equilibrium employment lower than in the non-union case. Unions raise wages at the expense of employment.

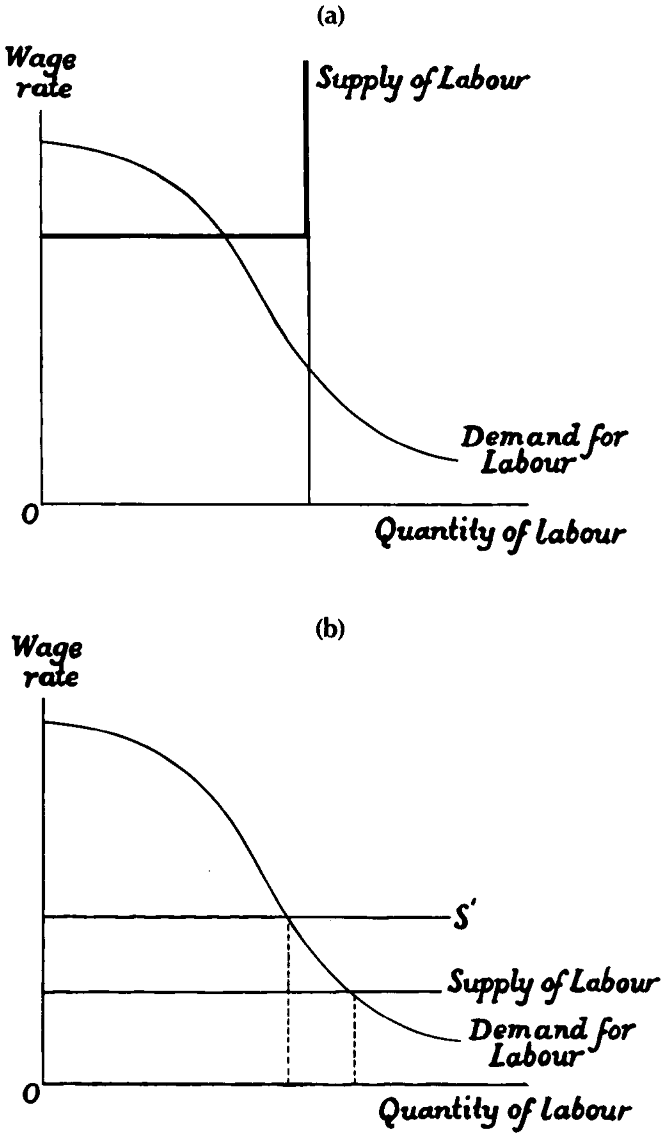
So much for the market period. In the long run the workforce is variable and labor's supply curve horizontal. Labor is produced at constant cost consisting of the expense of rearing and maintaining workers at some expected standard of comfort. Trade unions, by setting a reservation wage and so raising the standard of comfort, act to raise labor's cost of production. The resulting upward shift in the labor supply curve causes it to cut the demand curve at a higher equilibrium wage and a lower equilibrium labor force (Figure 15b). Unions, by raising the standard of comfort, influence the equilibrium size of the population. Here is Jenkin's most original contribution: his extension of the scissors diagram to the analysis of the labor market.

Welfare Effects of Taxes (Figure 16)

Jenkin also employed his diagrams to examine the welfare effects of excise taxes, exhibiting originality in conception if not establishing temporal priority in

Figure 15

JENKIN ON WAGE DETERMINATION



Source: Jenkin ([1870] 1931: 83).

publication in doing so. First, without having seen Cournot's tax incidence analysis, he showed that who bears the tax depends on the slopes of the demand and supply curves. According to Jenkin (114), the steeper the demand curve or the flatter the supply curve the greater the share of the tax shifted to demanders. Conversely, the steeper the supply curve and the flatter the demand curve the greater the share borne by suppliers. In the limiting case of a perfectly vertical demand curve or perfectly horizontal supply curve, all of the tax is shifted to demanders. But when supply is perfectly inelastic or demand perfectly elastic, the entire burden falls on suppliers.

Second, Jenkin, in his tax analysis, derived the Marshallian concepts of consumers' and producers' surplus. Being unaware of Dupuit's invention of the former idea, he (110) thought both were novel. Only the latter concept, however, was new with him. Like Marshall, he (109) defined it as the excess of sellers' actual receipts from supplying a good over the minimum necessary to induce them to do so. And, like Marshall, he measured it by the roughly triangular area lying between the price line and the supply curve (area *2* of Figure 16a). He (110) also pointed out that consumers' and producers' surplus triangles *1* and *2* are larger the more steeply sloped the demand and supply curves, respectively.

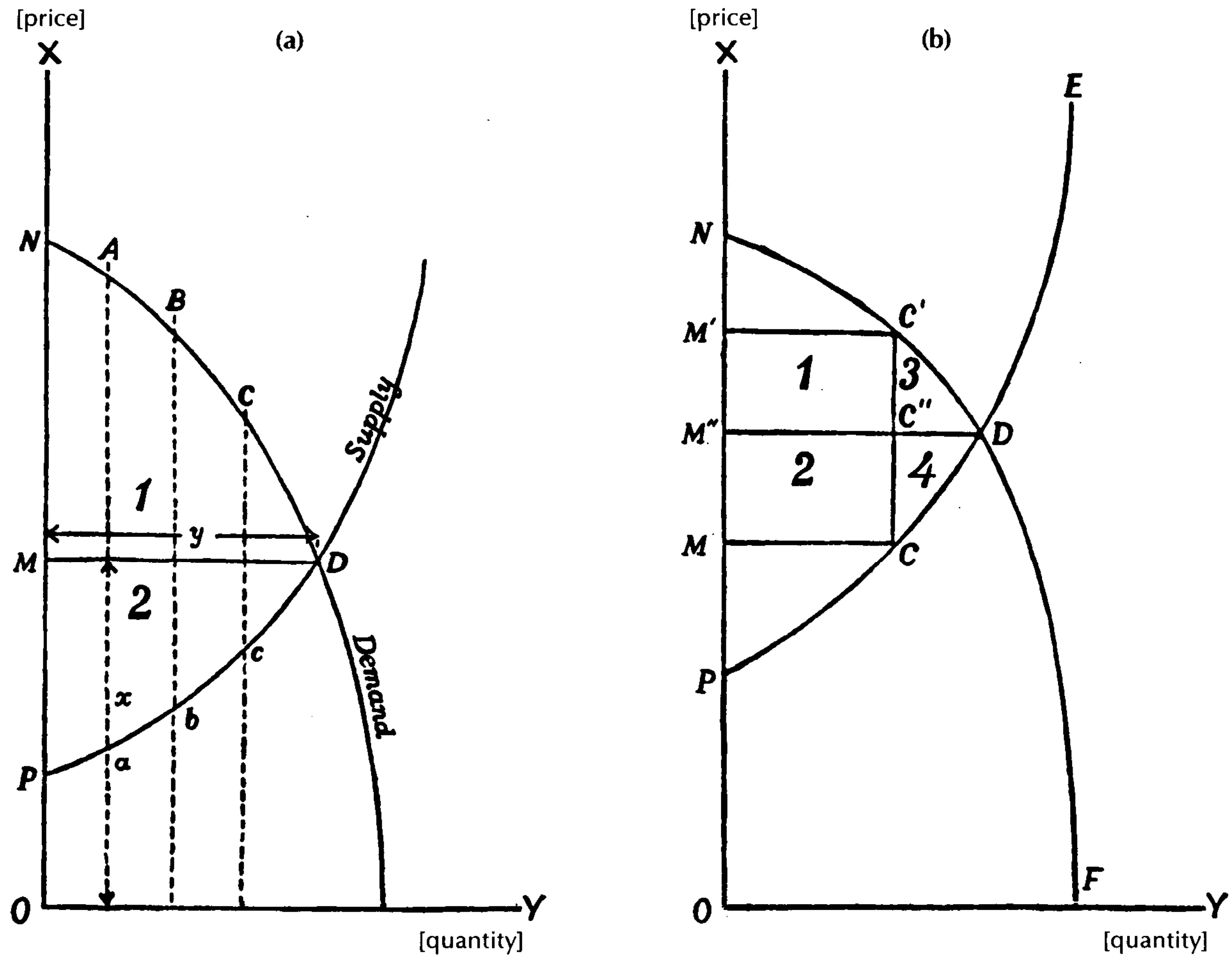
Finally, Jenkin (113-4) used these triangles to show that (1) the welfare losses of consumers and producers always exceed the tax they pay and (2) these losses rise with the rate of the tax. Thus, starting from pre-tax equilibrium D in Figure 16b, a tax of $C'C$ or MM' per unit of output drives a wedge between sellers' supply price $0M$ and buyers' demand price $0M'$ thus reducing output by $C''D$. The government obtains tax revenue equal to the sum of rectangular areas *1* and *2*, of which buyers pay area *1* and sellers area *2*. But buyers lose consumers' surplus equal to the sum of areas *1* and *3*; this loss exceeds their tax payment by the amount of area *3*. Similarly, sellers lose producers' surplus equal to the sum of areas *2* and *4*, which exceeds their tax liability by the amount of area *4*. These excess-burden triangles form constituent parts of the deadweight-loss triangle *3* + *4* whose size increases with the tax wedge. At very high tax rates the deadweight-loss triangle dwarfs the tax-yield rectangle and is a powerful argument for keeping rates low.

Influence and Recognition

Jenkin's work, especially his 1870 paper, fully foreshadowed Marshall's. Nobody saw this more clearly than Marshall himself. Marshall had lectured on demand and supply curves since 1868, but, at the time of Jenkin's writings, had published nothing. He realized that Jenkin had "scooped" him, as the following passage (Whitaker 1975: 45) from H. S.

Figure 16

CONSUMERS' AND PRODUCERS' SURPLUS AND THE WELFARE EFFECTS OF A TAX



Source: Jenkin ([1872] 1931: 108, 113).

Foxwell's letter of 24 April 1925 to J. M. Keynes confirms: "I happened to come across [Jenkin's 1870 article] in the Easter vacation of 1870, when I was attending Marshall's lectures on diagrammatic economics, & I shall never forget his chagrin as he glanced through the article when I showed it to him. There was nothing in Cournot which so closely agreed with Marshall's general approach to the Theory of Value & particularly to his statement of the equation of supply & demand."

Nevertheless, Marshall continued to insist that he, partly under the tutelage of Cournot, had invented the scissors diagram and pioneered its applications independently of Jenkin. William Stanley Jevons likewise dismissed Jenkin's contribution with the claim that he (Jevons) had used intersecting curves to depict price determination in lectures at Owens College as early as 1863, seven years before Jenkin published his diagrams. Such disparaging comments, together with Jenkin's lack of formal training in economics, caused his innovations to go unnoticed (Brownlie and Lloyd Prichard, 1963: 215-6). Even today his name is unfamiliar to most economists who instinctively think of Marshall when supply-and-demand analysis is mentioned.[18]

[18] Thus Blaug and Sturges (1983: 186) remark that Jenkin's work "was little noticed . . . and had little effect on the subsequent course of economic thought despite its striking quality and originality."

Conclusion

Economists typically consider Alfred Marshall the father of the Marshallian cross or scissors diagram. The pre-Marshall literature, however, reveals a somewhat different picture. In particular:

1. The Marshallian cross diagram did not originate with Alfred Marshall. At least five economists—Cournot, Rau, Dupuit, Mangoldt, and Jenkin—employed it in print before Marshall published it. And the first four did so before Marshall began his career as an economist.

2. Of the diagram's five originators, all but Mangoldt, who knew of Rau's contribution, were ignorant of the work of the others. The diagram was a multiple independent discovery.

3. Besides conceiving the diagram itself, its originators supplied all its components and pioneered most of its applications. Cournot contributed the original curves. Rau and Mangoldt provided the stability analysis. Mangoldt furnished the comparative statics exercises which Jenkin applied to the analysis of price determination in the market period and long run, respectively. The tax incidence analysis stems from Cournot, Dupuit, and Jenkin. The consumers' surplus and deadweight-loss triangles are Dupuit's and Jenkin's. Jenkin devised the idea of producers' surplus. Mangoldt applied the diagram to the problem of the pricing of joint and composite goods. Jenkin extended it to the labor market to explain wage determination. Cournot formulated the elasticity concept and Dupuit analyzed price discrimination. There is little in Marshall's use of the diagram that was not anticipated by his predecessors.

4. The diagram thus illustrates Stigler's (1980) Law of Eponymy according to which no scientific discovery is named for its original discoverer. The Marshallian cross diagram bears Marshall's name because he gave it its most complete, systematic, and persuasive statement, not because he was the first to invent it. His account was definitive, not pathbreaking. For this he received—and deserved—credit.

5. Later economists could have obtained their demand-and-supply analysis from Marshall's predecessors as well as from his *Principles*. Had they done so, today's microeconomics textbooks probably would be little changed. Given the existence of a well-formulated geometry of supply and demand before Marshall, it follows that his contribution was a sufficient but hardly a necessary condition for the diagram's subsequent dissemination. Had he never published, later economists probably would have discovered the work of his predecessors or invented the diagram themselves.

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APPENDIX

Mangoldt's Algebraic Model of Joint Demand and Supply

As stated in the article, Mangoldt ([1863] 1962: 43-50) analyzed price determination of interrelated goods algebraically as well as graphically. His first algebraic model refers to two goods A and B jointly demanded in fixed proportions under a given expenditure constraint.

Joint Demand Model

Letting D denote quantity demanded, S quantity supplied, p price, E total expenditure, f a functional relationship between two variables, n the fixed ratio in which the two goods (identified by subscript) are jointly demanded, Mangoldt's first model consists of the following equations:

$$
\begin{aligned}
&(1) \quad D_A = nD_B \\
&(2) \quad E = p_A D_A + p_B D_B \\
&(3) \quad p_A = f_A(S_A) \\
&(4) \quad p_B = f_B(S_B) \\
&(5) \quad D_A = S_A \\
&(6) \quad D_B = S_B.
\end{aligned}
$$

Equation (1) states the fixed-proportion assumption and equation (2) the expenditure or budget constraint. Equations (3) and (4) are supply functions, while equations (5) and (6) are market-clearing conditions.

To solve the system, Mangoldt first substituted (1) into (2) and solved for p_B to obtain

(7) $p_B = E/D_B - np_A$.

This expression he interpreted as good B's demand function containing good A's price as a shift parameter. He then eliminated p_A from the expression by using (3), (5), and (1) to obtain

(8) $p_B = E/D_B - nf_A(nD_B)$,

which he equated with supply function (4) to determine the market-clearing price and quantity of good B and thus equilibrium of the system as a whole.

Finally, from equation (7) he demonstrated that a fall in good A's supply price p_A raises the demand price p_B of good B. He explained why: As good A becomes less expensive, buyers take more of it. Because the two goods are consumed in fixed proportions, however, buyers necessarily take more of good B too. The resulting increased demand for B bids up its price. In short, a movement down the demand curve for A is accompanied by an outward shift in the demand curve for B. In this way cost reductions that increase the supply of A raise the price of B.

Joint Supply Model

Mangoldt's second model refers to two goods A and B jointly supplied in fixed proportions under a given cost constraint. Letting K and k denote total and unit costs, respectively, and using the same variables defined above, his model appears as follows:

(9) $S_B = nS_A$

(10) $p_AD_A + p_BD_B = K$

(11) $K = kS_A$

(12) $D_A = f_A(p_A)$

(13) $D_B = f_B(p_B)$

(14) $S_B = D_B$

(15) $S_A = D_A$.

Equation (9) states the fixed-proportions assumption that the goods are supplied in the ratio n. Equation (10) says that revenues must cover total cost. Equation (11) defines total cost K as the product of the unit cost c of a complex unit of output $S_A + S_B = (1+n)S_A$ times the number of units produced, or $K = c(1+n)S_A = kS_A$ where $k = c(1+n)$. Equations (12) and (13) constitute the goods' demand functions, while equations (14) and (15) state the market-clearing condition that supplies equal demands.

To solve this system, Mangoldt substituted equations (9), (11), (13), (14), and (15) into (10) to obtain an expression for B's price in terms of A's:

(16) $p_B = (k-p_A)/n$.

Further substitution yielded the expression

(17) $S_A = (1/n)f_B[(k-p_A)/n]$

which Mangoldt interpreted as A's supply function given equilibrium in the market for good B. Equating (17) with (12) allowed him to solve for A's market-clearing price and quantity and thus for equilibrium of the system as a whole.

He concluded that a rise in the demand for A reduces B's price and explained why. As A's output rises to match the increased demand so too, via the fixed-proportions assumption, does B's output. With the demand for B given, however, the extra output of that good constitutes an excess supply that puts downward pressure on its price. The price of B varies inversely with the demand for A.