FEDERAL RESERVE BANK OF RICHMOND

MONTHLY REVIEW

Linear Programming: A New Approach To Bank Portfolio Management

Corporate Financing and Liquidity: 1968-1972

Volume 58
Number 11

NOVEMBER 1972

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LINEAR PROGRAMMING:
A New Approach to Bank Portfolio Management

Perhaps the most important and most difficult problem facing any commercial bank's senior management on a continuing basis is asset portfolio management. Portfolio decisions made at any given time directly affect a bank's current income and profits. Moreover, current decisions may significantly influence income and profit flows in future periods. What makes asset selection difficult is that alternative courses of action invariably present trade-offs between profits, liquidity, and risk. Evaluating and weighing these factors is an inherently complex task. The problem has been compounded during recent years by the pressure on commercial banks to maintain adequate profits in the face of increased competition for funds both from nonbank financial institutions and from various money market instruments.

As a result of this increased pressure, the commercial banking industry has begun to seek more sophisticated approaches to portfolio management. Management scientists are assisting the industry by devising improved decision techniques that can be understood and effectively employed by bankers.[1] One technique receiving considerable attention is linear programming. Linear programming is a basic analytical procedure, or "model," employed extensively in management science and operations research. Although the theory underlying the technique involves advanced mathematics, the model's structure is straightforward and can be understood by management personnel having only minimal training in mathematics. The purpose of this article is to describe the technique in a nonmathematical manner and to indicate how it can be used in the bank portfolio management process. Section I outlines two currently popular approaches to asset management and points out some of their principal deficiencies. Section II describes the linear programming model and uses a highly simplified numerical example to indicate the model's applicability to bank portfolio decisions. Section III discusses how banks might employ the model in practice and attempts to suggest the model's proper role in the overall portfolio decision process. Section IV summarizes the technique's advantages in banking applications and points out some of its limitations.

I. CURRENT APPROACHES

The typical bank's balance sheet lists a variety of assets and liabilities. Liabilities, such as demand and savings deposits, are sources of bank funds. Assets, such as business loans, consumer installment loans, and government securities, are uses of bank funds. The essence of the asset management problem is the need to achieve a proper balance between (1) income, (2) adequate liquidity to meet such contingencies as unanticipated loan demand and deposit withdrawals, and (3) the risk of default. The problem arises because assets carrying relatively high yields, such as consumer installment loans, are generally less liquid and riskier than assets having relatively low yields, such as short-term government securities.

The "Pooled-Funds" Approach During the early postwar period, funds were generally available to banks in ample supply at low cost. Consequently, most banks followed what has been termed a "pooled-funds" approach in deciding how to allocate funds among competing assets. Under the pooled-funds concept, a bank begins its asset selection procedure by arbitrarily defining a fixed liquidity standard, usually some target ratio of reserves and secondary reserve assets to total deposits. Using this standard, the bank then allocates each dollar it attracts, from whatever source, in identical proportions among various categories of assets. A principal deficiency of this procedure is its failure to take into account variations in liquidity needs that arise from variations in the structure of liability and loan accounts.[2]

[1] Two management scientists, Kalman J. Cohen and Frederick S. Hammer, have been instrumental in this effort. Their published work in this area, on which the present article draws extensively, is listed in the accompanying references.

[2] The "structure" of an individual bank's liabilities refers to the proportionate allocation of total funds among various liability categories such as demand deposits, savings deposits, and certificates of deposit. Similarly, the structure of a bank's loan accounts refers to the allocation of total loans among various classes of loans.

The "Asset Allocation" Technique The pooled-funds approach served most banks reasonably well during the late 1940's and early 1950's when funds were relatively plentiful and the majority of bank liabilities were noninterest-bearing demand deposits. Since that time, the financial environment in which banks operate has changed dramatically. Nonbank financial institutions, particularly savings and loan associations and mutual savings banks, began to compete vigorously with individual commercial banks for deposits during the 1950's. In addition, corporate treasurers, motivated by sharp increases in the yields of such money market instruments as Treasury bills and high-grade commercial paper, began to trim their working balances held in commercial bank demand deposits to bare minimums. The banking industry has responded to these deposit drains by developing new sources of funds, notably negotiable certificates of deposit, commercial paper issued through affiliates, and Eurodollar borrowings. While these innovations have permitted the banking industry to grow at an adequate rate, they have proved costly, resulting in increased pressure on bank profits. Therefore, a premium has been placed on efficient bank balance sheet management.

The management tool developed to meet the need for more sophisticated portfolio management was the so-called Asset Allocation technique.[3] The distinguishing feature of this procedure is that it takes explicit account of a bank's liability structure in guiding asset choice. More specifically, the Asset Allocation approach recognizes that the velocity of various types of liabilities differs systematically from one liability category to another.[4] The procedure specifies that funds obtained from liabilities with rapid turnover rates (such as demand deposits) should be invested relatively heavily in assets of short maturity, and, conversely, that funds obtained from low velocity liabilities (such as certificates of deposit) should be invested relatively heavily in long-term assets. In its most extreme form, the technique divides a bank into subsystems by liability classes: for example, a "demand deposit bank," a "time deposit bank," and a "Eurodollars bank." Funds flowing into each of these "banks," that is, funds obtained from each liability source, are then allocated proportionately among alternative assets using formulas that reflect liability velocities. For example, the demand deposit formula might specify relatively high proportions of short-term government securities and short-term business loans, while the time deposit formula might specify a relatively high proportion of mortgages.

Faced with an ever-widening range of diverse sources of funds, many bank portfolio managers have adopted the Asset Allocation approach because of its explicit attention to asset-liability linkages. But while the method represents an improvement over earlier procedures, it possesses several fundamental weaknesses.[5] First, velocity is a poor guide to the liquidity requirements imposed by a given class of liabilities. A far more relevant consideration is account stability, that is, the net daily variation of an account's total balance. It is widely recognized that no correlation necessarily exists between stability and velocity.[6] Second, the technique is arbitrary and inflexible. It is arbitrary because no clearly-defined bank goal (such as some form of constrained profit maximization) guides the determination of the various fund conversion formulas. It is inflexible because no systematic procedure is provided for altering the formulas in the face of changing external conditions, such as shifts in particular asset yields. Third, by compartmentalizing a bank into various subsystems, the method diverts attention from the overall goals of the bank's operations and fails to recognize important interactions between various bank activities. The linear programming approach described below avoids these difficulties.

II. THE LINEAR PROGRAMMING MODEL: AN EXAMPLE

Linear programming is a general mathematical procedure for maximizing target variables subject to constraints.[7] The linear programming model has been extensively applied in industrial production analysis, where the objective typically is to maximize profits by achieving the proper product mix within the constraints imposed by technical production procedures, resource availability, and resource costs. This section presents a simple numerical example designed to illustrate how the model can be used by bank portfolio managers. The example employs a set of graphs to assist readers unfamiliar with the

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[3] The Asset Allocation or "conversion of funds" procedure was originally devised by Harold E. Zarker. See Harold E. Zarker, *Conversion of Commercial Bank Funds* (Cambridge, Massachusetts: Bankers Publishing Company, 1942).

[4] The velocity of a given liability account is the ratio of the dollar flow within that account during some specified time period to the average stock of dollars in the account during the same period. The reciprocal of velocity is then the length of time an average dollar remains in the account.

[5] For a more extensive critique, see Kalman J. Cohen and Frederick S. Hammer, ed., *Analytical Methods in Banking* (Homewood, Illinois: Richard D. Irwin, Inc., 1966), pp. 45-53.

[6] See George R. Morrison and Richard T. Selden, *Time Deposit Growth and the Employment of Bank Funds* (Chicago: Association of Reserve City Bankers, 1965), p. 12.

[7] A comprehensive treatment of linear programming is contained in G. Hadley, *Linear Programming* (Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1962).

model in grasping the essence of the technique's substantive content. While graphs are a useful explanatory device, their employment restricts the scope of the illustration. Consequently, the example is a necessarily artificial and unrealistic representation of the actual portfolio decision process. Nonetheless, the illustration conveys the flavor of the technique and demonstrates its applicability to bank balance sheet decisions.

Consider a hypothetical bank that holds two classes of liabilities, demand deposits (DD) and time deposits (TD), and that can choose between two classes of assets, loans (L) and securities (S). Hence, the bank's balance sheet takes the following form:

Assets	Liabilities
L	DD
S	TD
	Capital Accounts

Assume that the rate of return on loans is 10 percent during some relevant decision period, but that no loan matures and no loan can be sold during the period. Assume further that securities yield 5 percent during the period and can be liquidated at any time without the risk of capital loss. Total funds available to the bank are fixed at, say, \$100 million, distributed as follows: \$45 million in demand deposit accounts, \$45 million in time deposit accounts, and \$10 million in capital and surplus. Finally, assume for illustrative simplicity that the bank incurs no costs in attracting and maintaining deposits.

The bank would like to select an asset portfolio that maximizes its total return over the period. If this were all that were involved, the optimal asset selection decision would be obvious: channel all available funds into loans, the asset yielding the higher return. The bank recognizes, however, certain constraints upon its actions. In reality, the constraints are numerous. The present example will consider three.

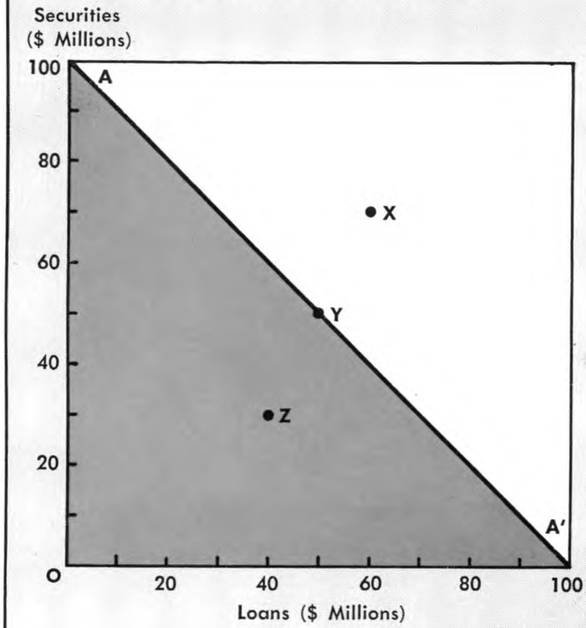
Total Funds Constraint As indicated above, the bank has \$100 million to allocate between loans and securities. Consequently, the sum of its loan and securities balances cannot exceed \$100 million. This constraint can be expressed mathematically as:

(1) $$L + S \leq 100 \text{ million}$$

where the symbol $\leq$ means "less than or equal to."[8] Chart 1 depicts this restriction graphically. Any point on the graph represents some combination of loans and securities. For example, point X corresponds to a loan balance of \$60 million and a securities balance of \$70 million. The diagonal line AA' (the graphical representation of the equation L + S = 100 million) is the locus of points at which loans and securities total \$100 million. At point Y, for example, the loan balance is \$50 million, the securities balance is \$50 million, and total assets are therefore \$100 million. At any point above and to the right of line AA', such as X, total assets exceed \$100 million. At any point below and to the left of AA', such as Z, total assets are less than \$100 million. The total funds constraint requires that the point representing the bank's asset portfolio either fall on AA' or within the shaded region below and to the left of AA'.[9]

Chart I

TOTAL FUNDS CONSTRAINT



Liquidity Constraint The bank recognizes that, because loans cannot be liquidated during the time period under consideration, some quantity of negotiable securities must be held to meet unanticipated deposit withdrawals. Therefore, the bank makes it a rule always to maintain some minimum ratio of securities to total assets. Assume that, with \$45 million of demand deposits and \$45 million of time deposits, the bank always maintains a securities balance equal to or greater than 25 percent of total

[8] The opposite symbol $\geq$ means "greater than or equal to."

[9] Strictly, with total funds equal to \$100 million, the balance sheet identity requires that L + S equal exactly \$100 million, that is, that the point representing the bank's asset portfolio fall *on* line AA'. For the purpose of illustrating the linear programming technique, it is helpful to treat the constraint as an inequality rather than an equality. This deviation will not affect the example's solution.

assets. The mathematical expression for this constraint is:

(2) $S \geq .25(L + S)$,

or, equivalently and more conveniently, as:

(3) $S \geq .33(L)$.

Constraint (3) is depicted graphically by Chart 2. It requires that the bank's asset portfolio fall on line OB or at some point in the shaded region above the line.

On the presumption that time deposits are generally more stable than demand deposits, the bank's management varies its liquidity ratio inversely with shifts in the ratio of time to total deposits. Hence, an increase in the ratio would cause line OB to rotate downward, thereby enlarging the shaded area of acceptable portfolio. Conversely, a reduction in the ratio would rotate OB upward, diminishing the area of acceptable portfolios. The effects of such shifts will be considered below.

Loan Balance Constraint Because the bank considers lending its most important activity, it imposes certain restrictions on its loan balance. Specifically, the bank attempts to satisfy all of the requests for loans submitted by its principal customers. Assume that the aggregate demand of these customers totals $30 million during the period. This constraint is depicted by Chart 3. The restriction requires the bank's portfolio to fall on or to the right of line CC′. The mathematical statement of the constraint is:

(4) $L \geq 30$ million.

The Feasible Region The three constraints just outlined are all relevant when the bank's management meets to allocate available funds between loans and securities. Chart 4 shows how the constraints taken as a group restrict the bank's range of choice. Any asset portfolio represented by a point outside the shaded region EFG violates one or more of the constraints. Conversely, any portfolio represented by a point within or on one of the boundaries of this region satisfies all of the constraints. Therefore, the portfolio selected must lie within the region or on one of its boundaries. For this reason, the area is called the "feasible region."

The Objective Function The reader will recall the assumption that loans yield 10 percent and securities 5 percent during the relevant time period. Consequently, the bank's total income during the period equals 10 percent of its loan balance plus 5 percent of its securities balance.[10] Mathematically:

(5) $\text{Income} = .10(L) + .05(S)$.

[10] For simplicity, the possibility of loan default is ignored.

Chart 2
LIQUIDITY CONSTRAINT

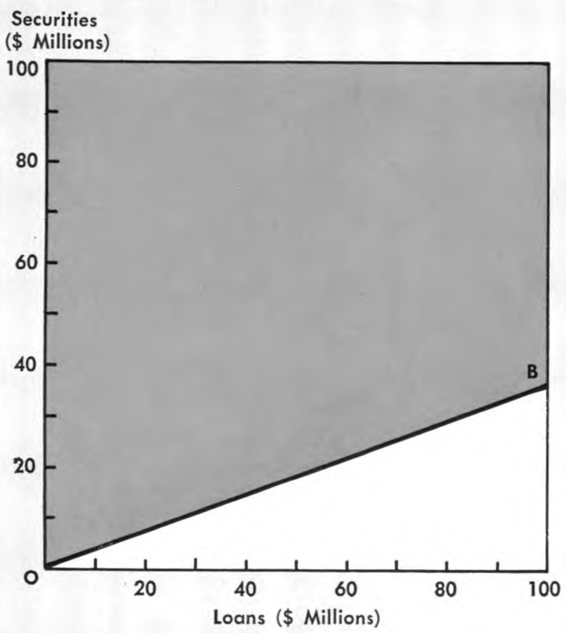
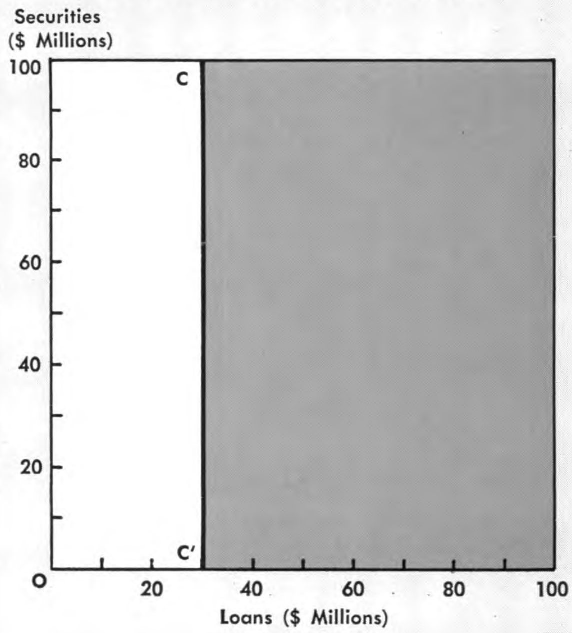


Chart 3
LOAN BALANCE CONSTRAINTS



Expression 5 is called the objective function of the linear programming problem. Chart 5 depicts the "family" of objective functions represented by equation 5. Each member of the family, that is, each of the parallel lines on the graph, corresponds to some unique income level. On the graph, the line closest to point O corresponds to income of \$1 million, the middle line to income of \$3 million, and the outermost line to income of \$5 million.[11] Hence, the bank's income increases as the objective function shifts upward and to the right.

The Optimal Asset Portfolio All of the elements relevant to the bank's portfolio decision have now been developed. The linear programming problem is summarized by the following mathematical statement:

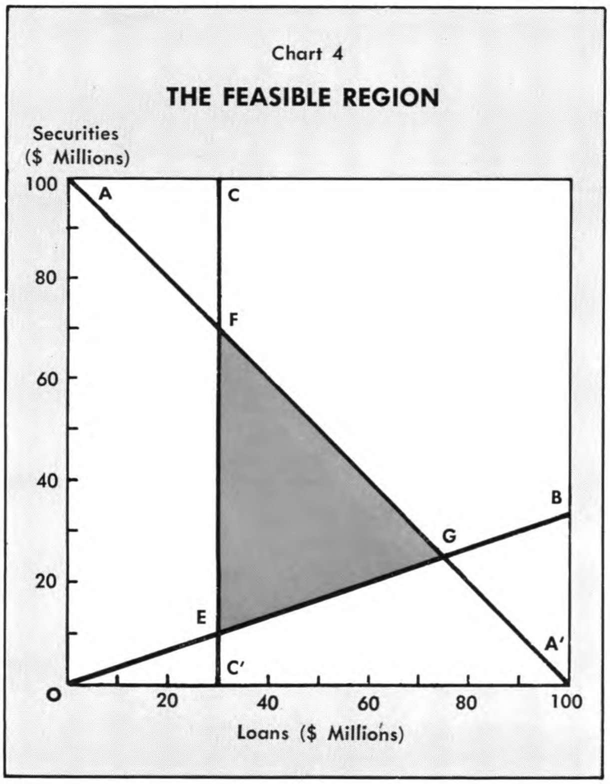
(6) Maximize income $= .10(L) + .05(S)$
Subject to:

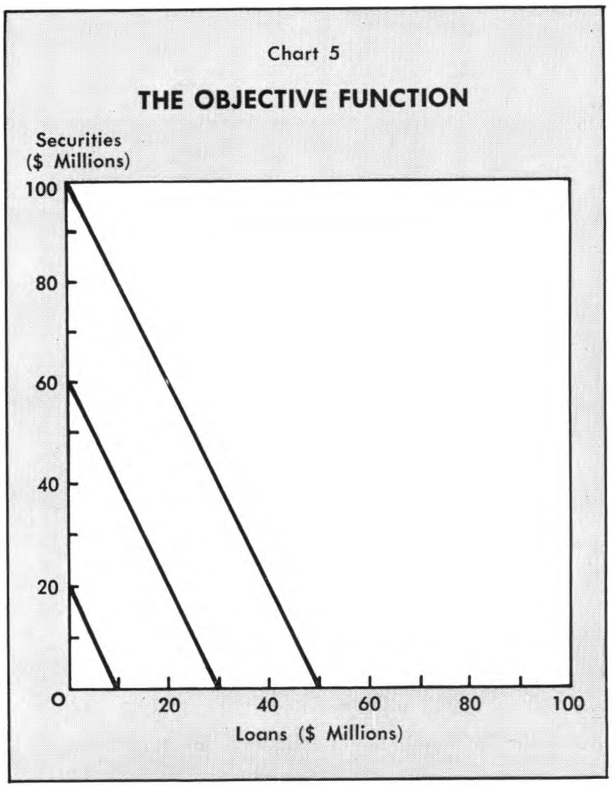
$$L + S \leq 100 \text{ million}$$
$$S \geq .33\ (L)$$
$$L \geq 30 \text{ million.}$$

The solution to the problem is depicted graphically by Chart 6, which reproduces the feasible region of Chart 4 along with several members of equation 5's family of objective functions. From what has been said, it should be clear that the bank can find its income-maximizing portfolio by pushing the objective function outward as far as possible without going beyond the point where some part of the function lies within the feasible region. Clearly, the income-maximizing objective function in this case is line NN'. This line barely touches the feasible region at point G. Any line to the right of NN', such as PP', lies entirely outside of the feasible region. Lines to the left of NN', such as MM', may contain points within the feasible region but correspond to income levels less than that represented by NN'. The solution to the problem is given by point G. The bank can maximize its income, while observing all constraints, by choosing the combination of loans and securities represented by point G: that is, by allocating \$75 million to loans and \$25 million to securities.[12] This portfolio would yield \$8.75 million of income during the period. The linear programming model has provided the bank an objective procedure for determining its optimal portfolio. The model has taken explicit and simultaneous account of the various factors assumed relevant to the decision.

[11] The reader can easily confirm that *any* point on one of these lines represents a portfolio that yields the designated income.

[12] For simplicity, the solution values are rounded to the nearest million.





Analytical Uses of the Model The linear programming model can perform a number of useful analytical tasks for the bank in addition to suggesting reasonable approximations to income-maximizing portfolios. In particular, the model can specify how the bank's optimal portfolio changes when one of the constraints changes. Through analysis of this sort, the bank can determine the costs, in terms of foregone income, of the various constraints under which it operates. Knowledge of these costs, in turn, can assist the bank in such diverse tasks as deciding how much interest to pay depositors, determining the rate of return on capital, and deciding whether to borrow or lend in the Federal funds market. A simple extension of the above example will serve to illustrate.

It will be recalled that the bank's deposits total \$90 million: \$45 million of demand deposits and \$45 million of time deposits. Imagine that the bank gain access to an additional \$10 million of *time* deposits. These additional time deposits affect two of the constraints in problem (6). First, the total funds constraint is eased to:

(7) $$L + S \leq 110 \text{ million.}$$

Second, it will be recalled that, by assumption, the bank's management varies the minimum ratio of securities to total assets inversely with the ratio of time to total deposits. Assume that, with \$55 million of time deposits and \$45 million of demand deposits, management considers a 20 percent liquidity ratio constraint adequate. Under these conditions, the restriction becomes:

(8) $$S \geq .20(L + S),$$

or:

(9) $$S \geq .25(L).$$

With these modifications, the mathematical statement of the bank's problem is changed from (6) to:

(10) Maximize income $= .10(L) + .05(S)$
Subject to:

$$L + S \leq 110 \text{ million}$$
$$S \geq .25(L)$$
$$L \geq 30 \text{ million.}$$

Chart 7 depicts the altered situation graphically. EFG is the feasible region of the preceding problem. E'F'G' is the extended feasible region of the new problem that results from the easing of the total funds and liquidity constraints attendant upon the \$10 million increase in time deposits. Point G' represents the solution to the new problem, with the objective function in position QQ'. As indicated by G', the bank's new income-maximizing portfolio contains \$88 million of loans and \$22 million of securi-

Chart 6
THE OPTIMAL ASSET PORTFOLIO

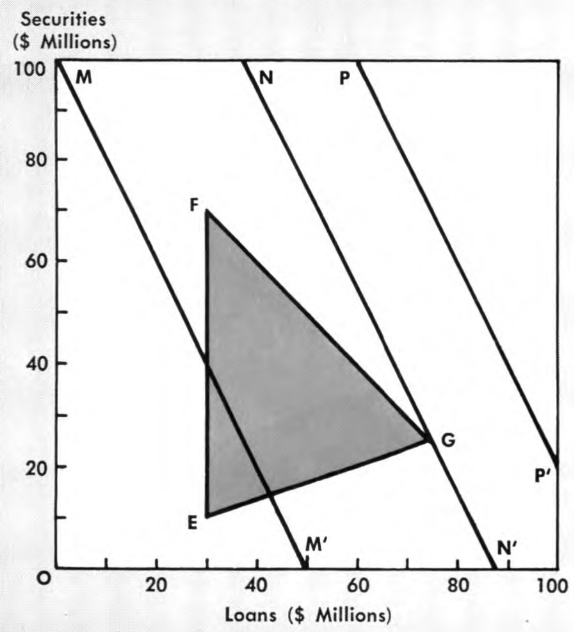
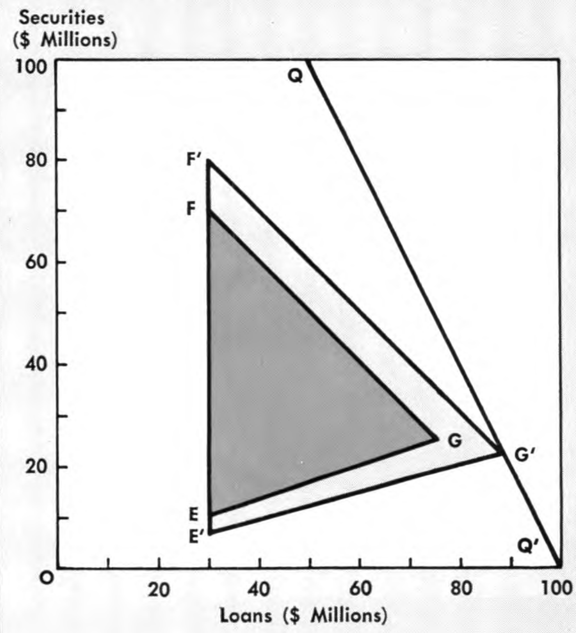


Chart 7
SOLUTION TO PROBLEM (10)



ties. Since yields have not changed, the bank's income is now $9.9 million.

The solutions to problems (6) and (10) can assist the bank in determining how much to pay depositors for the $10 million increment of time deposits. Comparing incomes in the two cases, it is clear that the additional deposits produce $1.15 million of additional income ($9.9 million — $8.75 million), or $.115 per additional time deposit dollar. Consequently, the bank can afford to pay up to a rate of 11.5 percent for each additional time deposit dollar.[13] At first glance, management might consider it ridiculous to contemplate incurring additional deposit costs at a rate that exceeds the available return on either loans or securities. The reason it is profitable to do so is that the additional time deposits have both a direct and a secondary effect on the bank's income. The direct effect in this case is the additional income resulting from the investment of the extra funds. The secondary effect is the additional income generated by the reallocation of the bank's original $100 million of funds to a higher proportion of loans made possible by the eased liquidity constraint. The linear programming technique takes account of such secondary effects automatically. This illustration demonstrates the potential usefulness of the comprehensive decision framework that characterizes the model.[14]

III. APPLYING THE MODEL IN PRACTICE

The example presented in the preceding section has conveyed something of the flavor of the linear programming technique. This section builds on the example to describe more fully how the model might be applied to portfolio management in practice. The section concludes with a few remarks regarding actual use of the technique at one large commercial bank.

Decision Variables and Constraints The example developed above considered only two decision variables: that is, only two variables over which the bank had direct control. These were the bank's loan and securities balances. In reality, of course, bank balance sheets break assets down into far more detailed categories. (They also show a much wider variety of liabilities than the twofold deposit classification used in the example.) To exploit the model fully, a bank should define as many asset decision variables as there are assets of significantly different yield, liquidity, and risk in its portfolio. The model is capable of handling any number of decision variables. Problems containing more than two or three variables cannot be solved using graphs. Several standardized solution procedures (known as algorithms) exist, however, for solving large problems.[15]

In addition to handling as many decision variables as necessary, the linear programming model can accommodate as many constraints as bank managers consider relevant to the portfolio decision process. Specifically, detailed and realistic sets of liquidity constraints can be built into the model reflecting liability and capital structures, cash flow patterns, seasonal fluctuations in loan demand, and miscellaneous restrictions imposed by management on the basis of experience.[16] A variety of other constraints are conceivable, taking account of such operating factors as legal reserve requirements, corresponding balances, and the use of certain assets as collateral to support government deposits.

Dynamic Considerations The Section II illustration was static. That is, the bank's decision process was cast in terms of a single time period. Actual portfolio management is anything but static, and no rational portfolio manager can confine his attention myopically to the present. For example, current portfolios should provide adequate liquidity to accommodate anticipated future loan demand. As a second example, loan decisions in the current period may affect deposit levels in future periods. One of the distinct advantages of the linear programming framework is its capacity to treat such inter-temporal linkages explicitly. In portfolio decision applications, the model can be designed in such a way that it takes account of anticipated future conditions and generates optimal portfolios for several future periods as well as for the current period. The reader should not infer that management would, at some point, use the model to suggest desirable portfolios for, say, the next five quarters, and then slavishly follow the prescriptions for each quarter as time passes. Obviously, the model should be updated and solved again as fore-

[13] It should be emphasized that this conclusion applies only to *additional* time deposits, not to deposits already held. A bank could pay a higher rate for additional deposits by, for example, issuing a new certificate of deposit.

[14] In actual linear programming applications, questions of the sort just discussed are analyzed in a more sophisticated manner, using the so-called "dual" linear program. For an elementary treatment of duality in linear programming, see William J. Baumol, *Economic Theory and Operations Analysis*, 2nd ed. (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965), pp. 103-28.

[15] The most widely used algorithm is the so-called "simplex" method. See Baumol, *op. cit.*, pp. 82-97.

[16] In their pioneering application of the linear programming method to bank portfolio management, Chambers and Charnes developed a detailed system of capital adequacy-liquidity constraints using some of the bank examination criteria employed by the Federal Reserve System. See D. Chambers and A. Charnes, "Inter-Temporal Analysis and Optimization of Bank Portfolios," *Management Science*, 7 (July 1961), 393-410.

casts are superseded by knowledge of actual events. Rather, the value of explicit attention to the future lies in the resulting clarification of the factors relevant to current decisions.

Bank Goals It was assumed in the illustration that the banks objective was to maximize gross income during the single time period considered. Obviously, actual banks must define more refined objectives. First, deposit interest and other operating expenses have to be considered. In the terminology of the model, the variable maximized should be net income in some form. The model can easily meet this requirement by treating bank expenses as negative increments in the objective function. Second, if, as suggested earlier, a multiperiod time framework is employed, management must select a means of discounting future income to present value equivalents. A number of alternative procedures are available, any of which can be explicitly incorporated in the model.[17] The model cannot itself select an objective; however, the model forces management to define some operating goal. Moreover, the model is structured in such a way that each specific portfolio decision has a definite quantitative effect on the goal variable and can be evaluated on this basis.

Use of the Model at Bankers Trust Company During the 1960's, a group of management scientists developed a linear programming model at Bankers Trust Company in New York to assist that bank's management in reaching portfolio decisions.[18] The model is quite detailed. It employs a multiperiod decision framework, a large number of balance sheet categories as decision variables, and numerous constraints of the type described above.

Perhaps the most interesting aspect of the Bankers Trust experiment is the role played by the model in the overall decision process. The model has not served in any sense as a substitute for the judgment of management. Rather, its principal function has been to clarify the consequences of alternative decisions. An excellent example is provided by management's use of the model to analyze liquidity ratio constraints.

When the consulting team initially formulated the model, they included no constraint on the ratio of government securities to total assets. The bank's executive management was troubled by this omission. They feared possibly adverse consequences in the market for the bank's stock should the Bankers Trust balance sheet show a much lower ratio than the balance sheets of other large New York banks. Informed of this criticism, the consulting team reformulated the model to include a minimum ratio of 16 percent. Subsequently, the scientists used the model to specify the effects on profits of small reductions in the ratio. The model indicated that quite small reductions could increase profits significantly. Management was unaware of this sensitivity. On the basis of this information, a more flexible policy was adopted.

This experience demonstrates the kind of informative dialogue that can develop between a bank's executives and a team of management scientists using a relatively sophisticated linear programming model. It is precisely in such interchanges that the model's value to management lies.

IV. CONCLUSIONS

This article has described the linear programming technique and has indicated how it can be applied to bank balance sheet management decisions. A few cautionary remarks and a brief summing up are now in order.

Although the linear programming model is a powerful analytical tool, it is in no sense an automatic procedure for generating optimal portfolio decisions. The complex and continually changing conditions faced by banks cannot be fully specified by a set of equations. It is unlikely that any bank will ever know, precisely and definitively, its optimal portfolio at a point in time. At best, techniques such as linear programming can only suggest a range within which the "best" portfolio is likely to fall.

Nor is the model a substitute for the judgment of experienced portfolio managers. While it is unnecessary for executives to understand in detail the mathematical theory underlying the model or its computational procedures, management must be directly involved in the construction and application of any operational model. Specifically, management must define the objectives of the bank's operations so that the model can reflect these objectives. Further, management must specify the constraints it considers relevant to asset selection decisions in order that these constraints can be built into the model. Finally, management must determine the specific questions that the model is used to analyze. In short, the model does not reduce the need for managerial judgment. On the contrary, it challenges that judgment in a very comprehensive manner.

[17] For a comparative discussion of these alternatives, see Kalman J. Cohen and Frederick S. Hammer, "Linear Programming and Optimal Bank Asset Management Decisions," *The Journal of Finance*, 22 (May 1967), 159-62.

[18] Kalman J. Cohen served as a principal consultant in the Bankers Trust project. The following remarks summarize his description of the model and its application. See Cohen, "Dynamic Balance Sheet Management: A Management Science Approach," *Journal of Bank Research*, 2 (Winter 1972), 11-18.

With due attention to the proper role of the model in the decision process, it seems clear that the linear programming approach has several distinct advantages over many alternative asset management tools, such as the Asset Allocation method described earlier. First, the structure of the model forces a bank's management to establish a definite operational objective and provides a convenient framework for considering factors relevant to portfolio choice. Second, the model simultaneously determines each element of a bank's optimal portfolio, given the particular goals and constraints specified by management. Because of its simultaneous approach, the model automatically takes account of trade-offs between decisions with respect to one element of the portfolio and decisions with respect to another element of the portfolio. Third, the model provides a convenient tool for evaluating (1) the comparative consequences of alternative decisions, (2) the effect of alternative constraints on bank profits, and (3) how portfolios should be adjusted when economic and financial conditions change.

The application of linear programming to asset management appears to be one of the more important recent developments in banking.[19] Small banks may find the costs of constructing and operating linear programming models prohibitive. If the technique becomes widespread among larger banks, however, small banks may find themselves exposed to the procedure through the portfolio management services provided by correspondents. Consequently, all bankers should be aware of the technique and its implications.

Alfred Broaddus

[19] In this regard, it should be pointed out that linear programming is only one, and by no means the most advanced, of the modern quantitative models currently being employed in private industry. It is quite possible that in the future one or several of the other techniques may prove more useful in banking applications than linear programming.

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CORPORATE FINANCING AND LIQUIDITY: 1968-1972

The financial positions of most U. S. corporations have undergone some significant structural changes since 1968. Over this period, considerable fluctuations have occurred in the total quantity of credit market funds obtained, as well as in the composition of these funds. New financing, net of repayments, by nonfinancial corporate business was as low as $33 billion in 1968 and 1970 but rose to nearly $63 billion in 1969 and 1972, according to Federal Reserve Flow of Funds data shown in Chart 1. Also evident from Chart 1 are the marked shifts between permanent (i.e., equity and long-term debt) and short-term sources of funds at various times between 1968 and 1972. Largely as a result of these financing policies, corporate liquidity positions, as measured by the ratio of financial assets[1] to short-term liabilities, showed a net decline between 1966 and 1972.

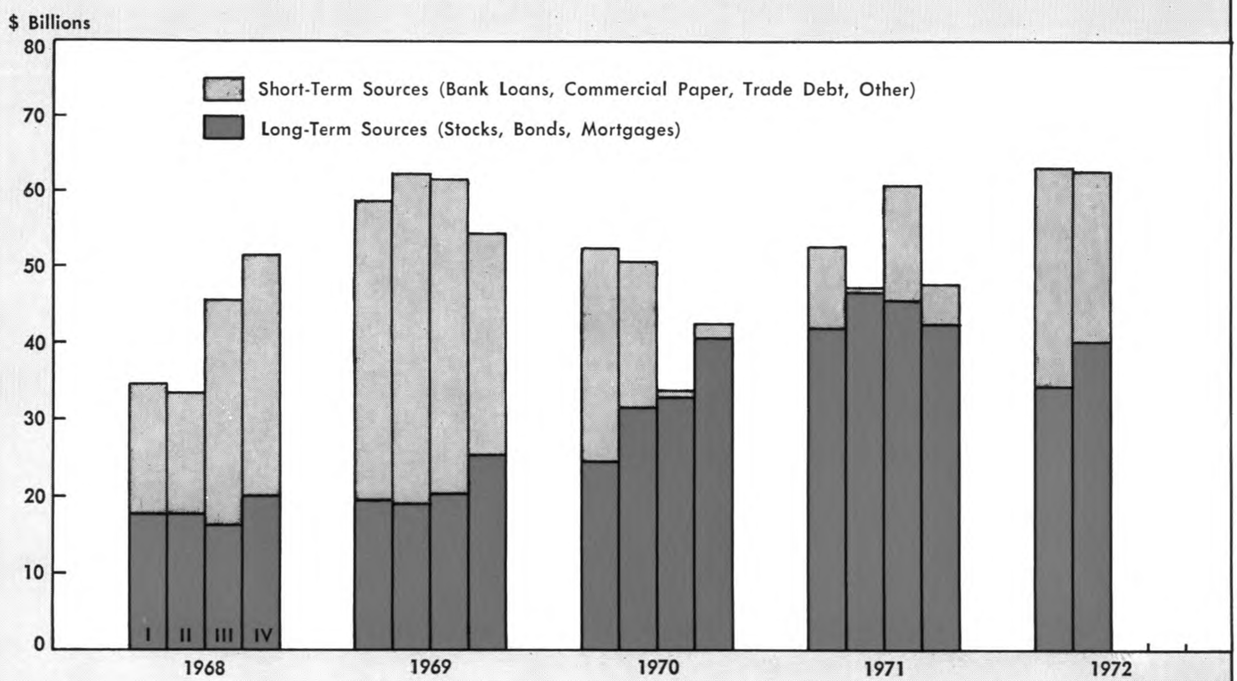
Total credit market funds obtained by corporations closely reflect the overall pace of economic activity and conditions in the credit markets. In 1968 and 1969, the great business expansion of the 1960's was

[1] Financial assets as defined here are composed of the following: demand deposits and currency, time deposits, U. S. Government securities, open-market paper, state and local obligations, consumer credit, and trade credit.

Chart I

COMPOSITION OF CORPORATE FINANCING

SEASONALLY ADJUSTED ANNUAL RATES



Source: Federal Reserve Flow of Funds—Nonfinancial Corporate Business.

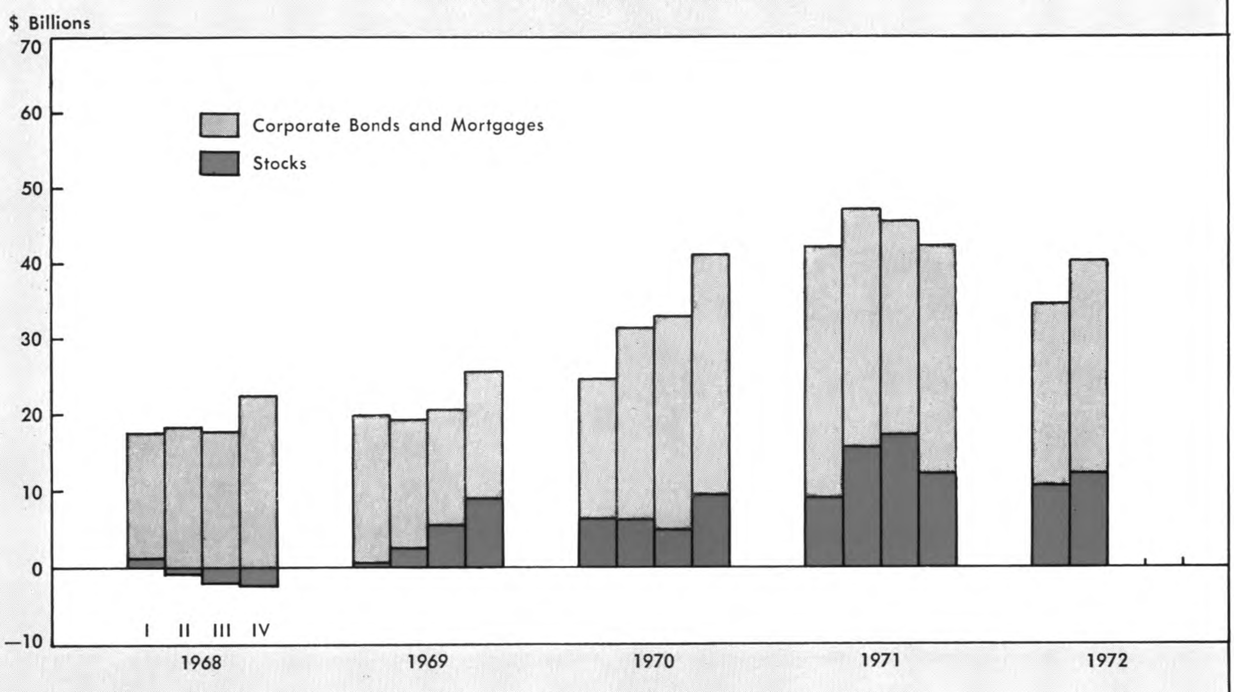
still underway. Demand for goods and services was high, and corporations were expanding working capital and plant and equipment. Although a large portion of the necessary funds came from internal sources, many corporations also obtained funds in the money and capital markets. The composition of the funds borrowed in these two markets began to change dramatically by the third quarter of 1968, when corporate treasurers increasingly relied on various short-term forms of financing—such as bank loans, trade debt, and commercial paper. This trend, which continued through the second quarter of 1970, was spurred by a combination of forces. The heavy demand for funds by these corporations and other economic units to finance the prevailing economic expansion again was putting strong upward pressure on interest rates in 1968 after the brief downturn experienced in 1967. Further increases in interest rates occurred in 1969 when the Federal Reserve was tightening credit in order to stem the rising tide of inflation. Both short-term and long-term rates rose to historically high levels in 1969. Even though short-term rates were higher than long-term rates in some cases, many corporations concentrated their borrowing in the short-term sector of the market. In this manner, many firms held down the length of their commitment to high interest costs.

In mid-1970, when economic activity had slowed and credit conditions had eased, the balance of corporate borrowing shifted back toward longer-term sources of funds. The impetus for this shift in financing policy was provided when the Penn Central filed for bankruptcy in June 1970 and defaulted on its commercial paper. Immediately, lenders became very cautious about making short-term, unsecured loans. Thus, when short-term obligations matured, corporate treasurers paid them off with proceeds from sales of stocks and bonds. This shift back to long-term sources of funds continued during the second half of 1970 and all of 1971, as shown in Chart 1. The widespread slowdown in economic activity experienced during 1970 is reflected by the small quan-

Chart 2

PERMANENT SOURCES OF FUNDS

SEASONALLY ADJUSTED ANNUAL RATES



Source: Federal Reserve Flow of Funds—Nonfinancial Corporate Business.

tity of credit market funds obtained by corporations at that time. With the improvement in economic activity since late 1971, total financing has increased; and the mix between short-term and long-term funds appears to have assumed a more traditional pattern.

With only a few exceptions, corporations obtained a steadily increasing quantity of funds from the capital markets between 1968 and 1972, as shown in Chart 2. Equity funds, in the form of either common or preferred stocks, experienced net decreases in 1968 but were used much more frequently in 1971 and 1972. During 1968, the merger movement was still strong, which tended to reduce the net amount of stock outstanding. Deteriorating stock prices in 1969 made stock sales expensive and stock repurchases comparatively attractive for most firms.

As prices in the stock market rebounded in 1970, sales of equity instruments also increased. Since late 1970, corporations, as part of an overall effort to improve liquidity positions, have issued new stock at a faster rate than at any time during the last 12 years. Also, the massive amounts of long-term debt incurred during 1970 and 1971 required additional equity funds to achieve a more desirable capital structure. Although some debt financing lowers the overall cost of capital to a firm, too much debt in the capital structure increases financial risk, which tends to raise the cost of capital.

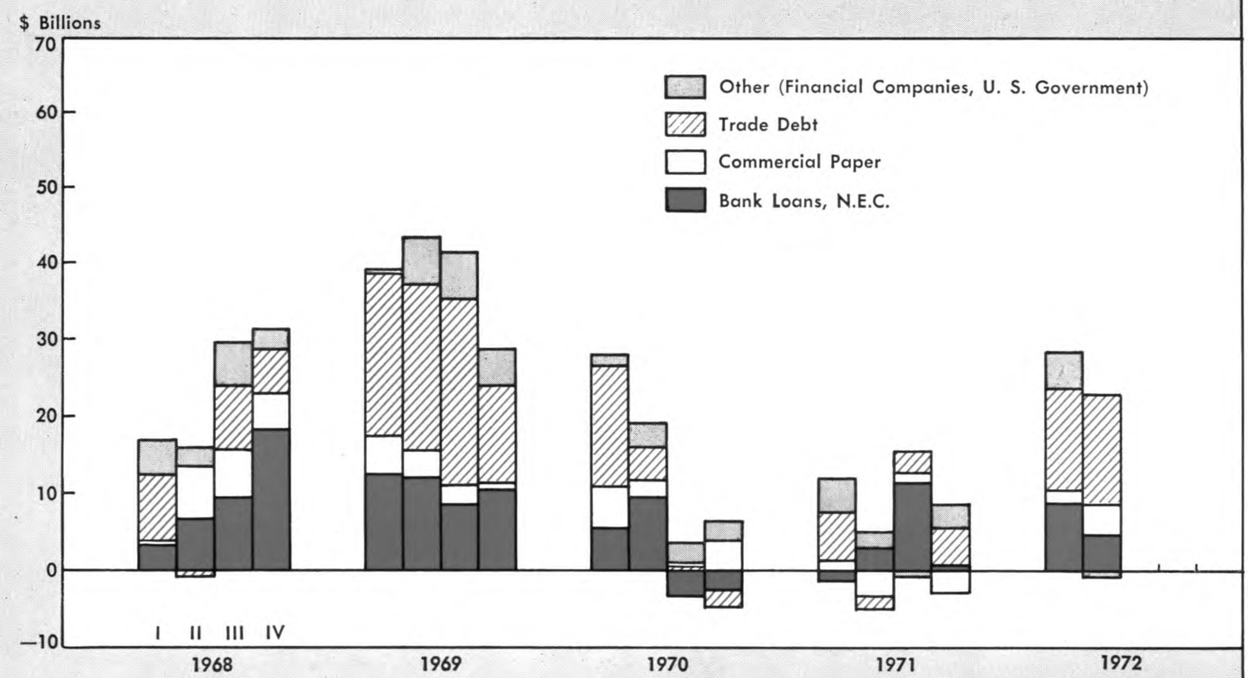
Between 1968 and 1972, short-term sources of funds were used extensively in expansionary periods but sparingly in recessionary periods. As business improved and operations expanded for most firms during the latter 1960's, increased working capital was needed. Seeking the most readily available and least expensive (over the long run) source of financing, corporations assumed large quantities of short-term debt. After demand slackened and interest rates receded, the short-term debt was paid off with newly acquired permanent funds.

The single largest source of short-term funds used during the expansionary periods was trade debt, or funds supplied by selling (primarily larger) firms to

Chart 3

SHORT-TERM SOURCES OF FUNDS

SEASONALLY ADJUSTED ANNUAL RATES



Source: Federal Reserve Flow of Funds—Nonfinancial Corporate Business.

buying (primarily smaller) firms in the form of accounts payable. Unable to obtain other types of financing during tight money periods, many small firms are forced to use trade debt even though it is often extremely expensive.

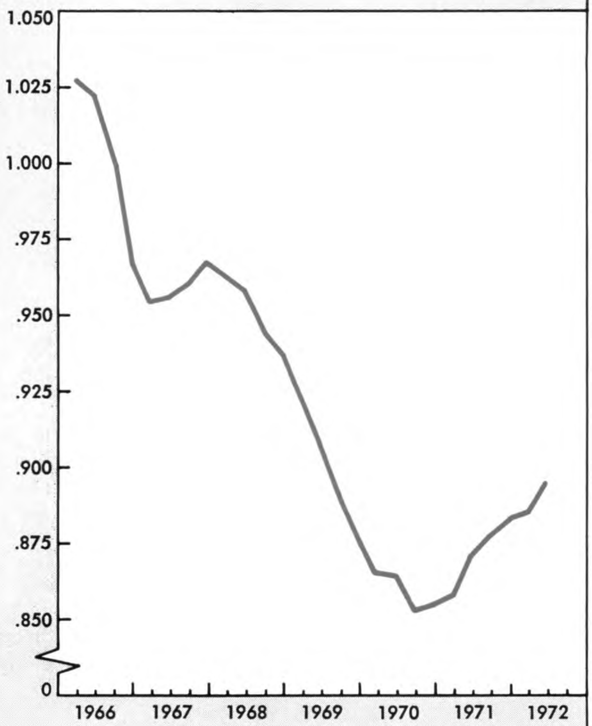
Since early 1972, business activity has accelerated, causing corporations to use short-term funds to finance rising accounts receivable and inventory. If the economy continues to expand as expected in the coming months, short-term funds will surely remain an important source of financing.

These shifts in the quantity and composition of financing by corporations have significantly altered the relationship between their holdings of financial assets and short-term liabilities between 1966 and 1972. This measure, as shown in Chart 4, is sometimes referred to as the "acid-test" ratio and is used to evaluate corporate liquidity. With the acceleration of economic activity in 1966, and the rapid assumption of short-term debt by many corporations, the acid-test ratio fell sharply, suggesting reduced liquidity. It continued to do so until mid-1967, when corporations turned their short-term debt into long-term debt. With the acceleration of economic activity in 1968, liquidity was reduced even further. The ratio hit a trough in mid-1970, about the same time that the Penn Central filed for bankruptcy. The resulting turmoil in the commercial paper market made both borrowers and lenders acutely aware of the impaired position of corporate liquidity. Although corporations began repairing their weakened liquidity positions in 1970, it was not until 1971 that actual improvements were discernible.

Chart 4

CORPORATE LIQUIDITY POSITIONS

RATIO OF
FINANCIAL ASSETS AND SHORT-TERM LIABILITIES



Source: Federal Reserve Flow of Funds—Nonfinancial Corporate Business.

Corporate liquidity positions will probably not return to their 1966 levels in the near future. Continued economic expansion in 1973 will most likely result once again in the extensive use of short-term debt, which tends to decrease liquidity.

Philip H. Davidson

PUBLICATIONS OF THE FEDERAL RESERVE BANK OF RICHMOND

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KEYS FOR BUSINESS FORECASTING A booklet containing broad statistical measures that have gained widespread recognition as key business indicators. Relates the behavior of these indicators to changes in the level of business. Describes statistical techniques for distinguishing normal seasonal changes in business data from changes associated with cyclical movements and underlying growth trends. 24 pages. 1970.

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YOU AND YOUR MONEY A 14-page, cartoon-style booklet dealing with the causes of inflation and deflation and some of the available remedies. Suitable for high schools. 1954.