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PORTFOLIO INSURANCE WITH STOCK INDEX FUTURES,

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ABSTRACT

In this paper, we analyze the use of stock index futures contracts as substitutes for cash market stock baskets in achieving stock portfolio insurance targets. We investigate the differences between the results from alternative simulated cash S&P500 portfolio insurance programs created through S&P500 futures trades. The focus of both our analytical and empirical results is on the impact of futures contract mispricings on portfolio insurance cost and effectiveness.

The empirical analysis reveals three basic results. First, Black's [1976] option on the futures model consistently generates greater policy premium valuations than those from the cash index option model simply because the volatility of the futures return is observed to be greater than that of the cash return. This futures-cash volatility difference is the result of intermediate date mispricings between the two markets. Second, Black's model is better specified than alternative futures-based models incorporating explicit cost of carry pricing restrictions from the viewpoint of delivering a hedging strategy that hits the end-of-period portfolio put target. Nevertheless, simple adjustments to the cost of carry models imply futures-based strategies which outperform the Black model on a cost-adjusted basis. Finally, we show how the initial futures mispricing provides significant predictive power for the deviation of the synthetic put strategy from its terminal date target value.

I. INTRODUCTION

Portfolio insurance refers to the strategy of augmenting an underlying risky securities portfolio with an insurance policy protecting the position against any losses experienced at the end of a specific time horizon. The insurance policy may be a specific security, such as an exchange-traded put option contract on a stock index, or it may be a synthetic option created through dynamic trading strategies. Such strategies might incorporate positions in the underlying asset portfolio and/or related futures and option contracts. The insurance may be written by another party or it may be "homemade." The growing popularity of portfolio insurance strategies attests to their usefulness in tailoring portfolio return distributions to the specific desires of institutional investors.¹

Like life, property and casualty insurance, portfolio insurance involves costs. In the simple case, where the insurance is provided by the purchase of traded option contracts with payoffs corresponding exactly to the desired insurance indemnity payments, the costs are determined by the option premiums as fixed by market supply and demand forces. Such premiums may appear fair, cheap or expensive.² Potential purchasers and/or writers of such options will continuously evaluate their

¹See Brennan and Solanki [1981], Leland [1980] and Benninga and Blume [1985] for formal critiques of portfolio insurance from the investor's perspective.

²Relative either to (1) the costs of synthetically replicating the option's payoff stream through dynamic cash or futures market strategies (if feasible) or (2) the expected value of the option's payoffs discounted at the appropriate risk-adjusted discount rate.

participation in portfolio insurance based upon the fluctuations of these option prices.

Unfortunately, listed options do not encompass the range of policy expiration dates or underlying asset definitions necessary to accommodate all potential users of portfolio insurance. In the more complex cases, where synthetic options are created through dynamic trading strategies, the costs are determined by the expenses incurred in maintaining the proper option replicating position. Because the trading costs of futures markets are significantly lower than those of cash securities markets, futures contract positions are natural vehicles through which to create the synthetic option positions that are fundamental to portfolio insurance.³

In principle, the substitution of futures contract trades for cash market transactions to achieve portfolio insurance should be innocuous. However, especially in the case of stock market portfolio insurance, the prices at which the relevant stock index futures positions are traded may deviate substantially from the futures prices that are fair relative to cash stock values. In some cases, overall returns to the portfolio insurance program will be enhanced because of the futures-cash mispricing. In other cases, returns will suffer--and perhaps more than negate the

³Of course, standardized exchange-traded option contracts could be employed instead as the low cost surrogates for the desired cash asset positions. Except for the specific details, the following discussion of the effects of futures contract mispricings on futures-based portfolio insurance hedges also apply to option-based insurance hedges.

direct trading cost advantages of using futures as cash market transaction substitutes. In general, slippages between the futures price and its fair value will cause portfolio insurance programs to be less than fully effective in eliminating end-of-horizon portfolio value risk.

Furthermore, the predictable reversion of a mispriced futures towards its theoretical value will imply higher or lower than fair expected returns on a given futures-based portfolio insurance strategy. Finally, the existence of futures mispricings has a nontrivial impact on the choice of the analytical model used to derive the "fair" portfolio insurance policy premium.

In this paper, we analyze the use of stock index futures contracts as substitutes for cash market stock baskets in achieving stock portfolio insurance targets. Using data for S&P500 index futures contracts, we investigate the differences between the results from alternative simulated cash S&P500 portfolio insurance programs created through futures trades. The empirical analysis reveals three basic results. First, contrary to well-known equivalence theorems, European options on the cash and like options on the futures are not equally valued. Black's [1976] option on the futures model consistently generates greater policy premium valuations than those from the cash index option model because the volatility of the futures return is observed to be greater than that of the cash return. This futures-cash volatility difference is the result of mispricings between the two markets. Second, Black's model is better specified than alternative futures-based models incorporating explicit cost of carry pricing restrictions from the viewpoint of delivering a hedging strategy that hits the end-of-period portfolio put target. Nevertheless, simple

adjustments to the cost of carry models imply futures-based strategies which outperform the Black model on a cost-adjusted basis. Finally, we show how the initial futures mispricing provides significant predictive power for the deviation of the synthetic put strategy from its terminal date target value.

II. FUTURES-BASED PORTFOLIO INSURANCE STRATEGIES: AN OVERVIEW

The basic portfolio insurance strategy is the "protective put."⁴ Combining a stock portfolio with an at-the-money synthetic put option will create a position which shares in any stock value appreciation, but which is protected against stock market declines. In its classic form, the synthetic put is produced by a dynamic trading strategy involving short stock and long bond positions. Substitution of short stock futures contract positions for the short stock positions is straightforward if the futures price can be related to the cash stock price in an exact manner. This substitution also changes the amount of the bond position required to complete the synthetic put strategy.

A. A Simple Two-period Binomial Example

Consider the following illustrative example of a futures-based protective put strategy for an underlying stock portfolio weighted to reflect a specific cash market stock index. Assume that in each of two periods, the cash stock index follows a binomial process with known "up" and "down" state gross returns. Furthermore, assume that a futures

⁴An alternative plan is the "fiduciary call."

contract on the underlying stock that expires at the end of the second time period is traded. Assume both that the one-period interest rate is constant over both periods and also that the stock pays no dividend. Under the nonstochastic interest rate assumption the theoretical futures price is the forward price of the stock (even though futures entail marked-to-market daily settlement). Here, the forward price equals the stock price multiplied by the interest factor compounded over the number of periods remaining until contract expiration. Finally, assume that there are no transactions costs.

The creation of a synthetic two-period, at-the-money European put on the stock index portfolio via futures contract positions is feasible under these assumptions as long as the futures price always equals its theoretical value.⁵ The solution strategy is to study the standard dynamic program which exploits the put's terminal condition, the stock index "tree" and the exact relation of the futures price to the stock index at each node of that tree. The solution yields paths for the short futures and long bond positions that replicate the terminal date payoffs of the put. The hedge ratio solution path consists of the known hedge ratio at date 0, h_0 , and a hedge ratio function for date 1, h_1 . The latter takes on value h_1^u if the up state is observed and value h_1^d if the

⁵ Alternatively, Black [1976] posits a futures price distribution directly, without regard for cost of carry pricing restrictions and the distribution of the underlying cash index.

down state is observed.⁶ The fair date 0 value of the synthetic put, P_0 , also is solved once the strategy's initial bond position is determined.

B. The Effects of Mispricings

The cash flows of the synthetic put strategy as of date 2 can be expressed as

$$P_2 = P_0 r r + h_0(F_1 - F_0)r + h_1(F_2 - F_1), \quad (1)$$

where F_i is the actual futures price at dates $i = 0, 1$ and 2 .⁷ Under our assumptions, as long as the futures prices exactly equal their theoretical values as given above, the strategy's cash flows, P_2 , equal the necessary put value, $\text{Max}(0, S_0 - S_2)$. However, suppose, as the empirical literature suggests, that the futures is sometimes mispriced. Let F_i^* denote the theoretical futures price at date i and $W_i = F_i - F_i^*$ denote the futures mispricing. Then, given the expiration date restriction that $W_2 = 0$ and substituting for the option replicating value of the (theoretically priced) synthetic put strategy, the futures strategy's date 2 cash flows can be rewritten as

⁶In a typical example of an initially at-the-money option, the up state will involve buying back some futures and the down state will involve shorting more contracts. Thus, synthesizing a portfolio put entails "buying on strength and selling on weakness."

⁷Interpret this cash flow equation as follows: P_0 , the fair time 0 value of the put, also equals the initial long bond position which is compounded at interest rate r over two periods. The marking-to-market of the futures contract at date 1 entails an adjustment of the bond position equal to $h_0(F_1 - F_0)$, which is compounded for one period at rate r . The third term in the cash flow expression represents the final marking-to-market of the updated futures position.

$$P_2 = P_0 r r + h_0(F_1^* - F_0^*)r + h_1(F_2^* - F_1^*) + h_0(W_1 - W_0)r + h_1(W_2 - W_1), \quad (2)$$

$$= \text{Max}(0, S_0 - S_2) + h_0(W_1 - W_0)r + h_1(-W_1).$$

In the presence of futures mispricings, this futures-based synthetic put strategy will not deliver the cash flows necessary to match the promised $\text{Max}(0, S_0 - S_2)$ value. The hedge ratio solution path was derived under the assumption that the futures price always equalled its cost of carry value. Thus, this strategy only hedges the synthetic put against fluctuations in the theoretical futures price. However, trades must take place at actual market prices, which may deviate substantially from theoretical prices. These mispricings will cause the actual returns to the strategy to differ from their planned values.

The impact of futures contract mispricings on futures-based portfolio insurance depends on the paths of the mispricing variable itself and the appropriate hedge ratio. From (2), the excess cash flow from the strategy relative to the put's true terminal value is

$$P_2 - \text{Max}(0, S_0 - S_2) = h_0(W_1 - W_0)r + h_1(-W_1). \quad (3)$$

Since $h_0 \leq 0$, the strategy will gain (lose) from a fall (rise) in W from time 0 to time 1. Recent research has documented a significant mean reversion pattern in S&P500 index futures mispricings.⁸ Thus, a fall in

⁸See Merrick [1987]. On average, 25-30% of any given day's mispricing at market close will be eliminated by the end of the next day.

the mispricing will occur most often with an initially overpriced futures ($W_0 > 0$). However, at time 1, the futures position is changed to reflect the change in the synthetic option's delta. This change could involve either buying back or selling more futures contracts. Importantly, the expiration date convergence property of the futures contract ensures that the mispricing at time 1 will be completely eliminated at time 2. Thus, the strategy will gain (lose) from buying (selling) underpriced futures ($W_1 < 0$) or from selling (buying) overpriced futures ($W_1 > 0$).

In sum, the impact of mispricings on the dynamic trading strategy's return depends in a complex way on the change in the mispricing variable and the path of the hedge ratio. However, the mean-reversion and expiration day convergence properties of mispricings yield some clues as to the probable sign of the profit impact. For example, in the special case in which (i) the mispricings and the put strategy hedge ratios are stochastically independent and (ii) the initial mispricing is eliminated on average by date 1, the expected difference between the date 2 strategy value and the terminal put value as of time 0 information is

$$E_0[P_2 - \text{Max}(0, S_0 - S_2)] = -h_0 W_0 r.$$

The simpler problem of hedging a cash stock portfolio with mispriced stock index futures has been analyzed extensively by Merrick [1987]. That analysis identifies both hedge ratio and expected return consequences of mispricings in the S&P500 stock index futures market for risk-minimizing hedges of the cash S&P500 portfolio. Like the put option hedge example above, cash-futures hedge returns contain a predictable "mispricing

return" component since mispricings tend to be eliminated by arbitrage trading after a given period of time. Furthermore, mispricings tend to be associated with overshootings of the futures relative to the cash ($\text{Cov}[W, F] > 0$).⁹ Thus, a futures to cash hedge ratio less than the theoretical cost of carry ratio typically is the risk-minimizing solution. This overshootings result has an important implication for the synthetic put-futures hedge strategy that underlies portfolio insurance here. In particular, it suggests that the theoretical futures-cash hedge incorporated into the derivation of the futures-based synthetic put hedge ratio should be revised downward.

III. FUTURES MISPRICINGS AND PORTFOLIO INSURANCE: SIMULATION EVIDENCE

The distribution of returns to futures-based portfolio insurance strategies is closely tied to the pricing performance of the arbitrage sector. In this section, we provide an empirical analysis of the implications of arbitrage mispricing for synthetic portfolio put creation using the S&P500 stock index futures market. In particular, we present simulation evidence aimed at determining the importance of contract mispricing effects on the performance of futures-based portfolio insurance schemes.

Our plan is to analyze a sequence of three-month synthetic puts on the S&P500 cash stock portfolio created by dynamic S&P500 stock index futures trading strategies. The cash flows of synthetic puts created by

⁹ Thus, the first condition underlying the example from the paragraph above may be violated in practice.

trading at actual market prices will be compared with those created by trading at theoretical prices. The latter counterfactual exercise creates a benchmark against which to evaluate mispricing-related effects.

We also examine the performance of nonstandard synthetic put trading rules specified so as to account for changes in the conditional joint distribution of cash and futures returns. In particular, we analyze hedges which incorporate adjustments for the overshootings effect.

A. The Data

Our data span the period from June 16, 1982 to March 21, 1986. We calculate our futures position returns using daily closing price data for the nearest-to-expiration Chicago Mercantile Exchange S&P500 stock index futures contract. In order to construct each day's theoretical futures price, we adjust the daily closing S&P500 cash stock price index for its financing costs (interest less dividends) over the time period until the near futures contract's expiration.¹⁰ Daily series on one- and three-month certificate of deposit rates were used to infer a daily linear term structure from which each day's implicit horizon- τ discount rate was calculated. The effective dividend adjustment for each pricing date was constructed from the actual daily cash dividend inflows for the S&P500

¹⁰ Matching the theoretical futures price, constructed from the closing cash index, with the closing futures price slightly misaligns these price data. The S&P500 index futures market closes at 4:15 p.m. New York time, fifteen minutes after the 4:00 p.m. NYSE cash market close. Furthermore, due to the inclusion of the prices of some infrequently traded stocks, the closing cash index will contain some price components sampled earlier in the day. The exact expression for the theoretical stock index futures price is given below.

portfolio. All of these dividend, interest rate, and cash and futures price data were obtained from the Interactive Data Corporation.

B. Alternative Synthetic Put Strategies

To evaluate the effects of futures mispricing on portfolio insurance performance, we attempt to create a series of synthetic three-month European puts on the S&P500 index portfolio using positions in near S&P500 stock index futures contracts. The base analysis incorporates both the diffusion-based Black-Scholes European put option approach on the cash S&P500 index and the cost of carry forward pricing model which relates the cash index to the futures. The proper short futures and long bond positions in the dynamic trading strategy are determined by substituting the "cash-equivalent" value of the futures into the equation wherever the cash index should appear.

Assume that the forward pricing relation always holds exactly for futures such that

$$F = S \exp(r\tau) - \sum_{s=1}^{\tau} D_{t+s} \exp(r\tau_s), \quad (4)$$

where: the right hand side is the theoretical date t price of the futures contract maturing a date T ; S is the date t cash index underlying both the portfolio insurance target and the futures contract; r is the time t interest rate; D_{t+s} is the assumed nonstochastic daily dividend inflow (measured in index units) of the underlying cash market stock portfolio on date $t+s$; $\tau = (T-t)/365$; $\tau_s = (T-t-s)/365$; and $\exp(.) \equiv$ the exponential function.

Define the "cash-equivalent" futures price as

$$F_{\exp}(-r\tau) = S - \sum_{s=1}^{\tau} D_{t+s} \exp(-r[\tau - \tau_s]).$$

The form of the put pricing equation used is derived by substituting the cash-equivalent futures price for the cash price in the Black-Scholes model.¹¹ Thus, the time t value of an index put written with a strike price of K and τ periods to expiration is

$$P = \exp(-r\tau) \{K[N(-d_2)] - F[N(-d_1)]\}, \quad (5)$$

where:

$$d_1 = \{\ln(F/K) + .5\sigma^2\tau\}/\sigma\sqrt{\tau},$$

$$d_2 = d_1 - \sigma\sqrt{\tau},$$

$F \equiv$ the date t price of the index futures contract expiring in τ periods (at date T),

$r \equiv$ the constant instantaneous interest rate,

$\sigma \equiv$ the standard deviation of the cash stock return,

$N(.) \equiv$ the cumulative standard normal distribution function, and

$\ln(.) \equiv$ the natural logarithmic function.

The synthetic put option strategy is implemented in the baseline simulation by forming an initial position of $\exp(-r\tau)N(-d_1)$ short index

¹¹Note that we have adjusted the stock price for dividend payouts over the life of the option (i.e., only the capital gains portion is relevant for the option).

futures contracts and long $K[N(-d_2)]$ pure discount bonds. The position is updated at the end of each trading day. At that time, the futures position is marked-to-market (with the gains or losses used to adjust the bond position) and the number of short futures contracts is altered to reflect the new appropriate hedge ratio. Index points are used as the numeraire.

We attempt to synthesize fifteen at-the-money three-month S&P500 index puts. The first put policy begins on June 16, 1982 (the day after the June 1982 contract expired) and entails a put on an S&P500 weighted stock portfolio. This at-the-money put has a strike price of 107.28 and starts with 90 days to expiration. The put's expiration date coincides with the expiration date of the September 1982 S&P500 index futures contract.¹²

The initial synthetic September put position consists of .563 short September futures and 5.27 (index points) invested in bonds. At expiration, the value of the final position determined by this trading strategy is compared to the terminal value of the true put. The remaining fourteen hedges repeat this strategy using later data. The last covers the December 23, 1985 to March 21, 1986 period using March 1986 futures contract positions.

¹²Note that all of our futures-based policies entail the forced convergence between actual and theoretical futures prices at expiration (i.e., $W_2 = 0$ as in equation [2]).

We analyze a number of different simulations. As discussed above, strategy 1 computes the synthetic option strategy's cash flows using actual futures market price data. In strategy 2, we substitute the theoretical futures price (the right-hand-side of (4)) for F in (5) to determine the proper hedge ratio and initial bond position. However, the cash flows throughout the course of the dynamic trading strategy are computed by marking the futures position to actual market prices. Strategy 3 is based upon the counterfactual simulation in which theoretical prices are assumed always to have ruled the market. In particular, strategy 3 uses the hedge ratios and initial bond position as in strategy 2 (i.e., those evaluated at theoretical prices), but also marks the futures positions to market at theoretical prices.

The differences among the terminal payoffs of these three dynamic trading strategies will highlight the impact of futures contract mispricings on futures-based portfolio put strategy performance. Specifically, the difference between the results of strategies 1 and 2 will indicate the impacts of choosing a hedge ratio path upon the basis of implied cash-equivalent prices indicating the put to be deeper in or out of the money than actually is the case. Furthermore, since strategy 3 yields terminal payoffs identical to those from a cash market based hedge, the deviation between these "counterfactual" results and the target terminal put option value will isolate the extent to which the Black-Scholes model itself succeeds in synthesizing S&P500 puts. Certainly, there is room for slippage between the put's terminal value and this trading strategy because the strategy updates only once a day (instead of the continuous rebalancing assumed in the model) and because

of the failure of the model's maintained hypotheses.¹³ Nevertheless, because we expect a reasonably good put tracking performance from a cash-based strategy, strategy 3 will serve as a useful benchmark for comparisons of the futures-based synthetic put hedges.

The next set of simulations studied uses actual market prices, but offers two alternative adjustments for the relevant short futures position in response to observed differences between actual and theoretical futures prices. In strategy 4, the hedge ratio used equals the solution of the option model evaluated at the theoretical futures price multiplied by $(1-\beta_t^{wf})$, where β_t^{wf} is the estimate of the derivative of the futures mispricing with respect to the futures return as developed in Merrick [1987].¹⁴ This derivative is constructed from regression estimates of the slope relation between the change in the mispricing on the futures price change (both scaled by the cash index), where the slope is allowed to depend upon the time to expiration on the contract and the initial value of the futures mispricing.¹⁵ Thus, strategy 4 is a mispricing slope-adjusted version of strategy 2.

¹³For example, the S&P500 stock index may not follow a constant variance diffusion process and interest rates do fluctuate in our data.

¹⁴In particular, the hedge ratio is $\exp(-r\tau)N(-d_1)(1-\beta_t^{wf})$.

¹⁵Estimates of these functions are reported in Table 3 of Merrick [1987]. The estimated coefficient of the time to expiration variable is statistically significant and suggests a decrease in the overshooting adjustment by about .001 per day as the contract approaches expiration (i.e., β_t^{wf} falls, so $[1-\beta_t^{wf}]$ rises as time to expiration decreases).

Finally, strategy 5 is that implied by replacing the cash stock return volatility input in the option pricing equation (5) with the volatility of the futures return. Here, the futures return volatility is treated as a constant parameter, but is allowed to differ in value from that of the cash. Under this assumption, the synthetic option is priced as a European option on the futures as in Black [1976]. No attempt is made to relate its valuation to explicit futures-cash cost of carry restrictions except incidentally in the specification of the terminal condition.

In all of these simulations, the current daily interest rate and effective dividend yield values are used as inputs for both construction of the theoretical futures price and the put option models.¹⁶ In order to abstract from some of the problems associated with employing the wrong volatility input, the estimated standard deviation of the daily close-to-close cash index returns for each simulation period is used as an "ex post correct" cash index volatility measure. Similarly, for the final Black model simulation, an "ex post correct" estimate of the futures return volatility for each hedging period is used as the volatility input.

C. The Simulation Results

Table 1 summarizes the results of the five simulated portfolio put strategies for the fifteen historical periods. For each period, we report

¹⁶This ensures proper measurement of the theoretical futures price, but is inconsistent with the constant interest rate assumption invoked by the option pricing models.

the initial conditions--the S&P500 futures price, its theoretical value, the S&P500 cash index (strike price), the cash index and futures return volatilities and the initial interest rate--along with the beginning-of-period portfolio put values derived from each of the models.¹⁷ These data reveal two patterns that will be important for interpreting our later results. First, futures prices sometimes deviate substantially from their theoretical cost of carry values.¹⁸ This suggests that the mispricing bias effect on a futures-based strategy's put target deviation will be an important issue. Furthermore, the initial portfolio insurance premium valuations given by, say, strategies 1 and 3 will differ. When the futures is overpriced (underpriced), strategy 1 attributes less (more) initial value to the put than does strategy 3.

Second, the volatility of futures returns tends to be significantly higher than that of cash index returns. If futures were always priced exactly off the cash, these two return volatilities would be equal. Thus, our measured mispricings account for the excess relative volatility of futures returns. This mispricing effect on relative volatilities has very significant impacts on the initial put premium of the Black model (strategy 5) versus the four other cash volatility based models. Black model values are always higher than strategy 1 values since both evaluate the put at market futures prices. However, in 2 cases (the March and June

¹⁷ By construction, strategies 2, 3 and 4 have the same initial bond position (option value).

¹⁸ For a recent analysis of mispricing patterns found in intraday S&P500 index futures and cash data, see Mackinlay and Ramaswamy [1987].

1985 expiration cycles), the Black model valuation is lower than the benchmark model value (strategy 3) because the positive volatility difference effect is more than offset by a negative bias due to highly overpriced futures.¹⁹

Table 2 reports the deviations of end values of each of the five strategies from the target put value as of the terminal date, along with summary statistics for the entire sample. These deviations are calculated in raw form for all five strategies and also in "cost-adjusted" form for strategies 1 and 5. The raw deviation is defined as a strategy's end payoff minus the corresponding terminal date put value. The cost-adjusted deviation is the raw deviation minus the (compounded) initial difference between the strategy's initial value and strategy 3's benchmark initial value.²⁰ The raw results are the more insightful for examining model specification (i.e., how well does each synthetic strategy perform in attaining its put value target?). However, the cost-adjusted results are most meaningful from the viewpoint of a prospective issuer of portfolio put insurance. Relative initial cost of achieving a given degree of insurance protection clearly is an issue in choosing the technology with which to produce the synthetic put. Nevertheless, if the cash index portfolio is easily traded, we must continue to emphasize that the

¹⁹The average initial cost of each strategy 1, 3 and 5 (the Black model) position was 3.07, 3.00 and 3.84 index points, respectively.

²⁰Since strategies 2 and 4 have initial values equal to that of 3 by construction, their cost-adjusted deviations are identical to their raw deviations.

fundamental reason why all five policies do not share a single value is the futures cash mispricing phenomenon.²¹

As reported in Table 2, the benchmark strategy 3 generates terminal values which are reasonably close to the put value targets. The average deviation from target equals -0.18 index points. This difference is statistically significant from zero at standard levels of significance (t-value of -2.41). The average absolute deviation from target for strategy 3 equals 0.25 index points.

Strategies 1 and 2--the two cost of carry plans--generate significantly worse results in terms of matching the promised terminal put option value. There is a tendency for both strategies to undershoot the target (e.g., their mean deviations are -.79 and -.54, respectively). Furthermore, their negative biases are statistically significant. The average absolute deviations from target of strategies 1 and 2 are about triple the size of the benchmark case (0.79 and 0.72 index points, respectively). The largest target shortfall a writer of portfolio insurance hedging with strategy 1 would have experienced was 2.40 index points. The corresponding extreme shortfall for a user of futures strategy 2 was 2.98 points.

²¹See the results of Brenner, Courtadon and Subrahmanyam [1985] and Ramaswamy and Sundaresan [1985] and the more recent discussion of Brenner, Courtadon and Subrahmanyam [1987].

Strategies 4 and 5, the two plans not based solely upon the cost of carry model's guide to hedge ratio selection, performed much better in terms of meeting the terminal date put value target. Strategy 4, the overshootings-adjusted version of strategy 1, had a target deviation bias smaller than that of the benchmark case (-0.16 versus -0.18). Clearly, though, the Black-based strategy 5 produced the least bias (-0.05 points). Both mean deviations from target are statistically indistinguishable from zero. Furthermore, the Black model's average absolute deviation from target (.26) essentially matches that of the benchmark strategy and is noticeably smaller than that of strategy 4 (.58).

In cost-adjusted terms, however, strategy 5's apparent advantages disappear. All three other futures-based strategies outperform the Black model in terms of minimizing both the target deviation bias and put hedge value variability. The overshootings-adjusted strategy 4 emerges as the best futures-based model on a cost-adjusted basis: it has the smallest bias, smallest standard deviation, smallest average absolute error, and the smallest maximum single hedge loss. However, since the overshootings-adjustment function was estimated using the same data set as the hedge simulations, a comparison of this strategy with the others is not strictly valid. Nevertheless, the evidence indicates that adjustment of the cost of carry futures-cash relation for the observed overshootings pattern in daily data will be useful for index futures portfolio insurance applications.

To investigate a final consequence of mispricing-related bias, Table 3 reports regressions of each strategy's raw terminal date deviation from

target put value on an intercept and the negative of the product of the starting date futures position ($-h_0$) and the compounded value of the starting date futures mispricing ($W_0 r$). As foreshadowed by the discussion of section II, the weighted initial futures mispricing provides substantial predictive power for the subsequent terminal date deviation from target put value.²² The R^2 s for these cases range from a high of .61 (strategy 2) to a low of .26 (strategy 5). Thus, there appears to be predictable strategy return consequences to entering standard futures-based synthetic portfolio put strategies at unfair initial futures prices. One consequence of this pattern is that writers and buyers of futures-based portfolio insurance should adjust the initial policy premium to reflect the predictable effect of the inception date futures contract mispricing bias.

IV. CONCLUSIONS

Portfolio insurance relies on dynamic trading strategies aimed at synthetically producing payoffs to specific untraded portfolio options. In this paper, we have analyzed stock index futures-based strategies designed to replicate at-the-money three-month European put options on the S&P500 cash stock portfolio. Specifically, we focused on the effects that futures contract mispricings have on the performance of such portfolio insurance strategies.

²²Results for (the counterfactual) strategy 3 are not presented since mispricings play no role in its cash flow determination.

Futures contract mispricings affect both model choice and model performance. Mispricings cause a divergence between cash index and futures return volatilities. Alternative models which are equivalent under strict cost of carry futures pricing lead to different valuations because of this volatility divergence. Furthermore, our empirical results indicate that futures-based strategies which select hedge ratio paths guided solely by the joint theoretical relations among the index put, the cash index and the cash index futures are suboptimal. In particular, adjusting theoretical hedge ratio paths for the observed tendency of futures prices to overshoot cash index values produces a significant decrease in variance of strategy errors.

Mispricings contribute to increased variance of the deviation between terminal date synthetic put strategy values and target put values--even for put policies which end on futures contract expiration dates. Thus, *ceteris paribus*, futures contract mispricings decrease the attractiveness of futures-based strategies. Interestingly, however, a substantial component of the overall variance of terminal date strategy deviations is predictable given the state of the futures contract's mispricing as of the strategy starting date. Such predictable mispricing biases have obvious implications for determining the negotiated portfolio insurance policy premium.

As we noted earlier, the attractiveness of futures-based portfolio insurance strategies largely rests on their low transactions costs and the liquidity of their markets (relative to the cash markets for index component stocks). In our formal analysis, we have ignored the

transactions costs dimension.²³ Analysis of the trade-off between differential cash and futures transactions costs and futures-cash basis fluctuations would be useful. However, it is clear that mispricing effects will still be important. Writing index portfolio puts against futures-based hedges will always be more (less) attractive when the futures is overpriced (underpriced) relative to the cash index. Only the definition of a true mispricing would change.

Of course, the more fundamental question raised concerns the reasons why index futures mispricings occur in the first place. Along this line, some of the more interesting questions concern the degree to which the index portfolio can be reasonably thought of as tradable, the extent to which program early unwinding and rollover options contain value, and the causes of the "wild" days in the stock market in which futures market arbitragers appear to have only marginal impact as the "marginal" investors. These questions are left for future work.

²³ See Leland (1985) and Figlewski (1987) for recent analyses of the impacts of transactions costs on cash-based call option hedging strategies.

Table 1

Futures-Based S&P500 Portfolio Put Simulations: Initial Conditions and Model Values for Alternative Strategies.

<u>Exp.</u>	<u>Futures</u>	<u>Theor.</u>	<u>Cash</u>	<u>Volatility</u>		<u>Interest</u>	<u>Option Model Values</u>		
			<u>(Strike)</u>	<u>Cash</u>	<u>Futures</u>	<u>Rate</u>	<u>Str.1</u>	<u>Str.2-4</u>	
<u>Str.5</u>									
8209	104.65	109.82	107.28	.192	.275	.153	5.27	2.86	6.91
8212	122.65	124.38	122.55	.223	.292	.111	5.21	4.45	6.84
8303	138.55	139.01	137.49	.167	.216	.089	3.96	3.76	5.28
8306	150.35	151.65	149.90	.118	.150	.090	3.22	2.67	4.16
8309	171.45	171.56	169.13	.134	.142	.093	3.37	3.32	3.62
8312	168.95	168.54	166.24	.099	.116	.096	2.05	2.20	2.58
8403	165.75	164.86	162.39	.122	.137	.101	2.45	2.77	2.90
8406	160.30	159.88	157.78	.117	.123	.103	2.46	2.62	2.63
8409	154.95	154.39	151.73	.142	.174	.113	2.91	3.12	3.86
8412	169.40	168.09	165.28	.116	.144	.113	2.02	2.47	2.87
8503	171.15	168.27	166.76	.113	.143	.084	1.77	2.79	2.64
8506	180.35	179.26	176.88	.092	.101	.095	1.85	2.23	2.16
8509	191.80	190.91	189.15	.088	.104	.077	2.08	2.41	2.64
8512	185.80	185.91	184.30	.105	.117	.081	3.03	2.98	3.45
8603	210.45	210.59	208.57	.131	.148	.079	4.39	4.33	5.08

Notes: Strategy 1 - Evaluates both model and cash flows at market prices.

Strategy 2 - Evaluates model at theoretical prices and cash flows at market.

Strategy 3 - Evaluates both model and cash flows at theoretical prices.

Strategy 4 - Strategy 2 cum overshooting adjustment as in Merrick [1987].

Strategy 5 - Black's European option on futures model.

Table 2

Raw and Cost-Adjusted Differences Between Terminal Date Strategy Values and Put Values For Alternative Strategies.

Terminal Date Strategy Value Minus Put Value								
Exp.	Put	Raw					Cost-Adjusted	
		Str.1	Str.2	Str.3	Str.4	Str.5	Str.1	Str.5
8209	0	-2.40	-2.98	-0.47	-1.89	-0.43	-4.90	-4.64
8212	0	-1.24	-1.32	-0.32	0.11	-0.18	-2.02	-2.64
8303	0	-1.57	-1.32	-0.34	0.04	-0.47	-1.77	-2.02
8306	0	-0.79	-0.36	-0.05	0.42	-0.26	-1.35	-1.78
8309	4.75	-0.67	-0.80	-0.20	-0.96	-0.30	-0.72	-0.61
8312	4.57	-0.19	0.55	0.37	-0.01	0.65	-0.04	0.26
8403	4.98	-0.31	0.57	-0.09	-0.11	0.20	0.02	0.07
8406	8.75	-0.19	-0.16	0	-1.38	0.01	-0.03	-0.00
8409	0	-0.51	-0.11	-0.33	0.73	0.11	-0.29	-0.65
8412	0	-1.57	-0.95	-0.90	-0.19	0.10	-1.11	-0.31
8503	0	-0.35	0.12	-0.27	0.71	0.08	0.69	0.23
8506	0	-0.18	0.07	0.08	0.55	0.01	0.21	0.08
8509	7.10	-0.32	-0.41	0.05	-0.92	0.38	0.02	0.15
8512	0	-0.34	0.06	-0.05	0.60	-0.10	-0.39	-0.58
8603	0	-1.22	-1.05	-0.20	-0.05	-0.58	-1.28	-1.34
Sum		-11.85	-8.09	-2.72	-2.35	-0.78	-12.97	-13.79
Average		-0.79	-0.54	-0.18	-0.16	-0.05	-0.86	-0.92
Std. Dev.		0.64	0.88	0.28	0.77	0.32	1.32	1.32
t-value		-4.62	-2.30	-2.41	-0.78	-0.58	-2.44	-2.61
Avg. Abs.								
Value		0.79	0.72	0.25	0.58	0.26	0.99	1.02
Min.		-2.40	-2.98	-0.90	-1.89	-0.58	-4.90	-4.64
Max.		-0.18	0.57	0.37	0.73	0.65	0.69	0.26

Notes: Strategy 1 - Evaluates both model and cash flows at market prices. Strategy 2 - Evaluates model at theoretical prices and cash flows at market. Strategy 3 - Evaluates both model and cash flows at theoretical prices. Strategy 4 - Strategy 2 cum overshooting adjustment as in Merrick [1987]. Strategy 5 - Black's European option on futures model. t-value tests the hypothesis that the mean deviation of the strategy's terminal date value from the put target equals zero.

Table 3

Effects of Initial Futures Mispricing on Expected Portfolio Put Strategy Tracking Error. OLS Regressions of the Form:

$$P_T^i - \text{Max}(0, K - S_T) = a + b(-h_0^i W_0 r) + \varepsilon_T,$$

where: P_T^i is the terminal date T value of strategy i; $\text{Max}(0, K - S_T)$ is the date T portfolio put value; h_0^i is the initial hedge ratio for strategy i; W_0 is the initial difference between market and theoretical futures prices; and r is the interest compounding factor. Sample: 15 replications for each strategy with quarterly terminal dates from September 1982 through March 1986.

Strategy	a	b	R ²	SER
1	-0.72	0.55 (0.14)	.55	.47
2	-0.52	1.01 (0.22)	.62	.59
4	-0.14	0.74 (0.34)	.27	.70
5	-0.03	0.19 (0.09)	.26	.30

Notes: Strategy 1 - Evaluates both model and cash flows at market prices. Strategy 2 - Evaluates model at theoretical prices and cash flows at market. Strategy 4 - Strategy 2 cum overshooting adjustment as in Merrick [1987]. Strategy 5 - Black's European option on futures model. Coefficient standard errors in parentheses. SER is the standard error of the equation's residual.

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