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The Case of the World's Missing Money

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No country in today's world has an economy completely closed to international transactions, but the world as a whole is still a closed economy. (So far we have not opened any trading or financial relationships with other planets.) According to reported balance of payments statistics, however, the world runs a global deficit on the current account of its balance of payments that recently has approached $100 billion annually. This amount may not seem like much compared to many contemporary global economic magnitudes, but in recent years it has risen to a significant percentage of total balance of payments flows.

Some reason for concern exists in a world in which major policy decisions are made because of balance of payments pressures. Where can that missing money be found? Are there any particular accounts, countries, or groups of countries that are responsible? Can an overstated U.S. current account deficit explain the continued coexistence of the record deficit and the strong dollar?

Conceptually, one nation's export should be some other nation's import, and vice versa. Illegal international traffic goes unrecorded, of course, but the underground economy might not bias the numbers either way unless an illegal transaction were recorded on one side but not the other. In order for a global asymmetry to emerge, either recorded exports must go unrecorded as imports, or recorded imports must go unrecorded as exports. For example, an industrial country consultant to an oil-producing country may not report as income a recorded payment by that country.

The current account, the most widely used measure of the balance of payments today, includes not only international trade in goods, but also the so-called "invisible" items like services—such as shipment costs, travel and tourism, investment income, royalties, advertising and professional fees, worker remittances and pensions, and private and official unilateral transfers. In any period, by definition, a given country's current account is balanced by equal capital flows in the opposite direction. In the box, the chart of accounts illustrates the fundamentals of a typical balance of payments.

If the current and capital flows (official plus private) do not offset each other, a statistical discrepancy account exists to fill any gap. An individual country's statistical discrepancy may consist of either unrecorded capital or current account

Balance of Payments Structural Components

Accounts	Debits	Credits
Current account		
Merchandise exports and imports	a	a*
Services		
Net investment income	b	b*
Shipment and other transportation	c	c*
Travel	d	d*
Net military transactions	e	e*
Other services, net	f	f*
Remittances, pensions, and other transfers	g	g*
Capital account		
Official and private short- and long-term capital flows, net	h	h*
Statistical discrepancy	i	i*

Current account balance

$$(a^* + b^* + c^* + d^* + e^* + f^* + g^*) - (a + b + c + d + e + f + g)$$

Summation of balance of payments accounts

$$a + b + c + d + e + f + g + h + i = a^* + b^* + c^* + d^* + e^* + f^* + g^* + h^* + i^*$$

flows. Some of the global current account discrepancy can be traced to statistical discrepancies of individual countries, but other origins also exist.

This article identifies some of the sources of the discrepancy in the global balance of payments. But it is hard to use this information to adjust existing balance of payments numbers. Unless a marked improvement occurs in balance of payments reporting—which can take place only at the individual country level—it is not likely that this discrepancy will disappear.

Current account balances by country group

Recent International Monetary Fund data on current account balances for major country groupings are shown in Table 1.[1] For the United States, the impact of the 1981–82 recession can be detected as the shrinking domestic demand has cut back on U.S. purchases of foreign goods and services. But then an expanding economy combined with a strong dollar drove the U.S. current account increasingly into deficit. Movements in oil prices in 1979 and recent problems of the developing countries are also discernible in the data.

The other countries category in Table 1 includes some of the 148 member nations in the Fund. It is apparent, however, that this group does not contribute much to the global deficit sum and has even had a net current account surplus in recent years. The most important of these other countries are the Soviet Union and nonmember Eastern European countries. The figures for other countries in Table 1 are estimates based on incomplete information and represent only convertible or hard-currency transactions. The relative importance of the Soviet Union is illustrated by U.S. Central Intelligence Agency

Table 1
WORLD CURRENT ACCOUNT BALANCES
(In billions of U.S. dollars)[1]

Countries	1977	1978	1979	1980	1981	1982	1983	1984
Industrial countries	**-2.4**	**31.9**	**-5.6**	**-38.8**	**3.1**	**1.2**	**2.2**	**-34.2**
United States	-11.7	-12.3	2.6	6.6	10.7	-3.8	-35.5	-93.4
Other industrial countries	9.3	44.2	-8.2	-45.4	-7.7	5.0	37.7	59.1
Of which,								
Japan	11.3	17.0	-7.9	-9.5	6.2	8.1	22.2	36.4
West Germany	8.5	13.4	0.1	-8.3	0.8	10.2	10.0	13.1
Developing countries	**-0.1**	**-36.2**	**0.2**	**22.6**	**-56.3**	**-99.6**	**-70.5**	**-43.9**
By region								
Africa	-10.4	-15.4	-6.6	-5.3	-25.2	-24.4	-15.5	-10.9
Asia	-0.9	-8.9	-15.2	-21.8	-23.4	-19.8	-16.3	-7.9
Europe	-9.0	-7.1	-9.9	-12.5	-10.5	-6.7	-5.3	-3.3
Middle East	31.8	14.5	53.7	91.6	45.8	-6.5	-21.7	-16.3
Western Hemisphere	-11.6	-19.4	-21.7	-29.3	-43.1	-42.1	-11.7	-5.5
By analytical criteria								
Fuel exporters	25.0	-0.7	54.0	100.1	34.7	-23.4	-17.0	-5.7
Nonfuel exporters	-25.1	-35.5	-53.8	-77.5	-91.0	-76.2	-53.6	-38.2
Other countries[2]	**-6.9**	**-3.5**	**-2.1**	**-3.0**	**-2.8**	**2.6**	**4.9**	**6.7**
Total[3]	**-9.5**	**-7.8**	**-7.6**	**-19.1**	**-56.0**	**-95.8**	**-63.5**	**-71.4**

1. On goods, services, and private transfers.
2. Covers estimated balances reported by the International Monetary Fund on current transactions only in convertible currencies of the USSR and other nonmember countries of Eastern Europe.
3. Reflects errors, omissions, and asymmetries in reported balance of payments statistics on current account, plus balance of listed groups with countries not included.

SOURCE OF DATA: International Monetary Fund, *World Economic Outlook, April 1985* (Washington, D.C., 1985), Table 29, p. 236.

estimates in Table 2. Because the Soviet Union has run a trade and current account surplus in most recent years, the global current account deficit puzzle cannot be solved here. The Soviet current account surplus has been counterbalanced largely by sizable outflows that can be recorded only on the statistical discrepancy or errors and omissions account. This includes an estimate of Soviet trade credit extended to finance their exports—including arms—to non-Communist hard-currency trade partners.[2] In any case, given the relatively small scale of Soviet transactions with the rest of the world, any inaccuracies in reporting are unlikely to be that important.

The most interesting aspects of Table 1, however, are the counterintuitive total deficit, the fact that it has increased substantially in recent years, and its volatility.

Who else is out there? Since 1982, when developing country deficits peaked, the discrepancy has declined somewhat, but it has remained at high levels after that. Can the origin of this total deficit be traced?

The discrepancy in international merchandise trade

An IMF estimate of the composition of the total deficit is shown in Table 3. One striking aspect of this breakdown is that, despite the global current account deficit, the world balance of merchandise trade transactions usually has been recorded as a surplus. Two explanations commonly are offered for this surplus.

One explanation—purely statistical—can only be partial and washes out in the overall current account. It has been suggested that excessively large

Table 2
SOVIET UNION HARD-CURRENCY BALANCE OF PAYMENTS
(In millions of U.S. dollars)

Balances	1979	1980	1981	1982	1983	1984[1]
Trade balance	1,837	1,714	200	4,433	4,713	4,175
Current account balance	**2,177**	**1,904**	**-175**	**4,333**	**4,663**	**4,225**
Capital account balance	**340**	**1,630**	**5,840**	**-1,340**	**1,650**	**200**
Errors and omissions[2]	-2,517	-3,534	-5,635	2,993	-6,313	4,425

1. Estimated.
2. Includes Soviet hard-currency aid to, and trade with, other CEMA countries; and trade credits extended to finance Soviet exports—including arms—to non-Communist hard-currency trade partners.

SOURCES OF DATA: U.S. Central Intelligence Agency, *Handbook of Economic Statistics, 1980*, Report no. ER 80-10452 (Washington, D.C.: National Foreign Assessment Center, October 1980), Table 49, p. 72; and *Handbook of Economic Statistics, 1985*, Report no. CPAS 85-10001 (Washington, D.C.: Directorate of Intelligence, September 1985), Table 46, p. 72.

adjustment factors are used for converting imports from a cost, insurance, and freight shares (c.i.f.) basis to one which excludes these charges.[3]

When customs statistics are collected initially for imports on a c.i.f. basis, some conversion is necessary to make them comparable to exports in the overall balance of payments. But an offsetting entry is then necessary in the services component of the current account. If the adjustment factor is large, this would help explain both the global trade surplus and the global deficit in services. But the overall global current account deficit is no closer to being explained, because any overestimation is cancelled within the current account.

The other frequent explanation derives from the practice of recording exports and imports at the time they cross national frontiers rather than when ownership actually changes. This practice leaves a substantial portion of world trade continuously in transit because exports may be recorded when they leave one country but not be recorded until weeks or months later as imports by the receiving country.

This discrepancy should therefore get larger when world trade is increasing and smaller when world

Table 3
DECOMPOSITION OF CURRENT ACCOUNT ASYMMETRY
(In billions of U.S. dollars)[1]

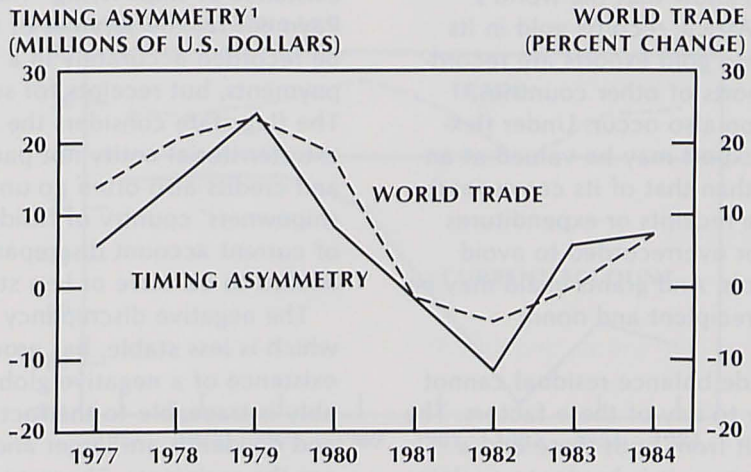
Countries	1977	1978	1979	1980	1981	1982	1983	1984
Total	**-9.5**	**7.8**	**-7.6**	**-19.1**	**-56.0**	**-95.8**	**-63.5**	**-71.4**
Trade balance	16.0	16.1	20.8	29.2	19.6	-0.9	15.7	19.7
Timing asymmetry[2]	5.6	13.9	23.0	7.9	-1.3	-11.8	4.8	6.5
Residual asymmetry	10.4	2.2	-2.2	21.3	20.8	10.9	10.9	13.2
Services and private transfers	-25.5	-23.9	-28.4	-48.3	-75.6	-94.8	-79.1	-91.2

1. On goods, services, and private transfers.
2. International Monetary Fund estimates of the difference between the beginning-of-year and end-of-year "float"; that is, the value of those exports that have not yet been recorded as imports (usually because the goods are in transit or because of delays in the processing of the documentation). The estimates should be viewed only as rough orders of magnitude.

SOURCE OF DATA: International Monetary Fund, *World Economic Outlook, April 1985* (Washington, D.C., 1985), Table 29, p. 236.

Chart 1

Global Timing Asymmetry and World Trade



1. See Table 3.
SOURCE OF PRIMARY DATA: International Monetary Fund.

trade declines.[4] In Chart 1, the IMF estimate of this timing asymmetry in Table 3 is compared to changes in the total exports of IMF member countries. A marked correspondence does exist between the timing asymmetry and changes in recorded world trade, but part of the estimation procedure for this component is based on world trade values.

In Table 3, the trade balance asymmetry is decomposed into the estimate of timing asymmetry and a residual. The residual, which includes effects of the first explanation above as well as any other factors, has been more of a random influence. The two years that do stand out, however—1980 and 1981—are not easily explained.

One possible explanation is an increase in oil inventories held at sea—a reduction in the average speed of the world tanker fleet or the use of idle tankers for storage. But if this is in fact the source of the 1980–81 discrepancy, then it should be reflected more correctly in the timing asymmetry category. The argument itself may not seem very convincing in retrospect, because 1980 was a year in which oil prices were rising, OPEC (Organization of Petroleum Exporting Countries) was in full control, and Saudi Arabia was at peak production. Incentives for the use of tankers as floating storage vehicles by oil producers did not occur very much until about 1983.

Some incentives for floating inventory buildup from the consumer side did exist, on the other hand, prior to the advent of the oil glut, so that this explanation is more plausible. Major oil importing countries such as Japan could have then been more responsible for the 1980–81 residual than taking into account a view only the producer side would accede to.

The price effect of the second oil shock could explain the 1980 experience partially, but not that for 1981, since prices had leveled off. If oil price effects dominated the residual item, moreover, the component in subsequent years would have become negative with falling prices.

Illegal traffic might contribute to the residual world trade discrepancy, but no hard evidence exists. Smuggling in drugs, and even gold and other commodities—which may be a significant part of trade for some developing economies—probably is not recorded as imports or exports. If goods are smuggled out of an exporting country but imported legally by another country, a negative discrepancy would result. This would not help explain the positive balance in the residual category. Only if

legally recorded exports were smuggled into the country of destination would an explanation be found.

Regarding gold, we do know that the world's largest producer, South Africa, records gold in its exports. Only part of these gold exports are recorded as corresponding imports of other countries.

Differences in valuation also occur. Under flexible exchange rates, an export may be valued at an exchange rate different than that of its corresponding import. Foreign trade receipts or expenditures may be underrecorded or overrecorded to avoid taxes or exchange controls. And grants-in-aid may be recorded differently by recipient and donor countries.

Unfortunately, the trade balance residual cannot be linked predominantly to any of these factors. The residual is likely to result from both these and a combination of other factors. It is harder to explain, however, why it runs largely in one direction. An allocation of the residual against particular countries or zones also cannot be based on very rigorous analysis.

The discrepancy in services and private international transfers

From Table 3, it is also quite clear that the world current account deficit can be traced to the deficit in the invisibles category, which more than offsets the usual surplus on the global trade account. This invisibles deficit has grown significantly in recent years.

Among the components of this category, shipment and investment income are by far the most important recent sources of the negative discrepancy. Other service components—travel and transportation, for example—show little substantial asymmetry, tending just to fluctuate between a positive and a negative balance.

The shipment component for IMF member countries has been a source of a large negative discrepancy for some time. As mentioned in the discussion of trade asymmetry, this category is linked to merchandise trade by its inclusion of items such as freight and insurance. When imports are recorded on a c.i.f. basis, however, a more reliable source for shipment payments usually is available in the customs records. Exports on an f.o.b. or f.a.s. basis do not include shipment costs. Typically, therefore, receipts from these services are understated relative to payments.

In addition, world shipment payments are likely to exceed world shipment receipts because of the existence of ships flying "flags of convenience." Payment for the services of these fleets likely will be recorded accurately in a country's balance of payments, but receipts for such services will not. The flag state considers the shipping line to be an extraterritorial entity not part of its own economy, and credits also often go unrecorded in the shipowners' country of residence. While this source of current account discrepancy is large, it has tended to be more or less stable over time.

The negative discrepancy on investment income, which is less stable, has grown during the 1980s. The existence of a negative global balance here probably is traceable to the fact that payers of interest and dividends are larger and more identifiable than are the recipients. Those on the receiving end often are paid through intermediaries and may not be reporting income in order to avoid tax or exchange control. In this case, some geographic identification is plausible. A downward bias on credits is likely in countries whose residents are net receivers of investment income—mainly industrial and oil-exporting countries. The growth in this category's asymmetry may be attributed simply to the growth in value of transactions in the category itself.

Another source of services asymmetry that has grown in recent years can be traced to relationships between oil exporters and the industrial countries. Foreign industrial-country consultants who provide a variety of services to oil-exporting countries also have incentives to underreport. In addition, payments are often made to multinational entities whose nationality is difficult to determine.

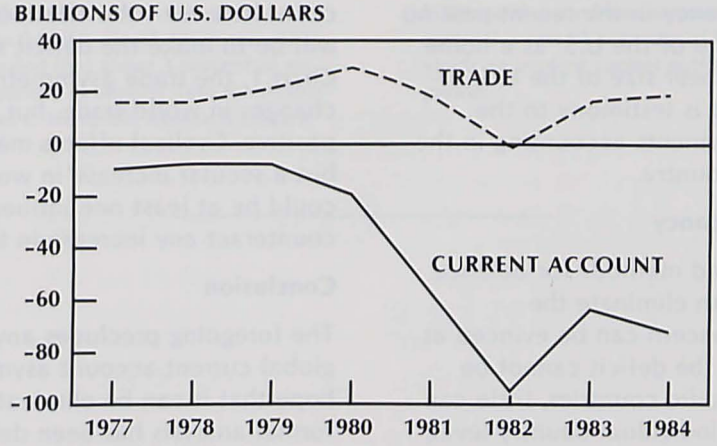
Country distribution of the global asymmetry

If it were possible to identify the global current account discrepancy relative to specific countries, some progress could be made in more accurate accounting, economic analysis, and policy decision making. While identification of individual countries as the source is difficult, it is possible to generalize to a certain extent about groups of countries. The deficit discrepancy is likely to come more from the industrial or oil-exporting countries than from the non-oil group of countries.

Part of this generalization is founded in reasoning along the lines of the investment income discussion

Chart 2

Global Trade and Current Account Imbalance[1]



1. See Table 3.
SOURCE OF PRIMARY DATA: International Monetary Fund.

above. If we can identify the kinds of transactions most likely to give rise to a discrepancy, then we can identify the countries that more frequently conduct those kinds of transactions. Because residents of industrial and oil-exporting countries are more often on the receiving end of investment income transactions, logically they would probably contribute more to the global deficit on the services component of the current account.

It may be less likely that recorded current account deficits of the non-oil developing countries are overstated. Most of those countries historically have had fewer net recipients of investment income. Moreover, they have generally run large current account deficits for some time. By accounting definition, these deficits have resulted in the growth of foreign debts so evident in recent years. Some analysis even indicates that the debt of these countries exceeds substantially their accumulated current account deficits. Thus, their deficits would need to have been all the higher to account for present debt levels. In Mexico, over the 1978–82 period, for example, the increase in gross debt was $58.4 billion, while the cumulated current account deficit was only $36.4 billion.[5]

All this should not be taken to mean that residents of the non-oil developing countries do not also have incentives to underreport investment income. And given the amount of capital flight from these countries, especially recently, one cannot ignore the investment income that may be accruing to residents of these countries also.

Another reason for assigning the discrepancy more to the industrial countries, however, is simply that this group conducts more transactions than the developing countries do. It can be assumed that the discrepancy is at least partially related to the volume of transactions. In 1984, those countries classified by the IMF as industrial economies accounted for over 70 percent of member-country exports.

Less can be said about the quality of balance of payments reporting across countries. It might be assumed that because industrial countries can devote more resources to the collection of balance of payments statistics, their accounts are more accurate. On the other hand, this advantage could be offset by the trade volume considerations above, and the errors and omissions accounts of industrial countries are frequently enormous.

As a case in point, the U.S. errors and omissions account shows quite large net inflows in recent years. In 1982, a record year, inflows on the U.S. errors and omissions account were over four times

the recorded U.S. current account deficit. It is still conventionally believed that the errors and omissions item derives mainly from capital flows rather than current account transactions. The role of the U.S. dollar as a haven currency in the recent past no doubt has fostered an image of the U.S. as a home for flight capital. But the sheer size of the U.S. errors and omissions account is testimony to the fallibility of balance of payments accounting in the world's largest industrial country.

The outlook for the discrepancy

It is not likely that improved methods for balance of payments accounting can eliminate the worldwide discrepancy. Concern can be evinced at the global level, but when the deficit cannot be assigned accurately to specific countries, little can be done to correct it. The individual country level, moreover, is the only place any correction can be made. Even if discrepancy could be allocated accurately among countries, it is not clear that greater resources devoted to the problem would solve it. This problem is illustrated by the probable role of the industrial countries, whose share may be greater than developing countries even though their methodological and statistical procedures are more sophisticated.

Moreover, regardless of the country and its statistical sophistication, the collection of balance of payments statistics faces at least one obstacle not present in collecting data on domestic transactions. In any international transaction, one party is foreign, and less information is available. Adding to the problems are different classifications of transactions across countries, as well as a host of other factors that make international commerce more complex than purely domestic economic activity.

Some components of the discrepancy seem to expand with international commerce in general. The total current account deficit has declined somewhat since 1982 but has trended upward throughout the 1970s. It might be assumed, therefore—barring any drastic improvement in data collection—that we must simply resign ourselves to a growing current account deficit asymmetry as other world economic magnitudes increase secularly over time.

This growth in the global deficit might not necessarily come to pass, however. As demonstrated in Chart 2, much of the fluctuation in the overall current account deficit derives from the positive trade asymmetry. To the extent that the trade asymmetry imparts volatility to the overall series—much as the merchandise trade account often imparts most of the volatility to a single nation's current account balance—the effect of growth in world trade will be to make the deficit smaller. As indicated in Chart 1, the trade asymmetry does tend to follow changes in world trade, but the relationship is positive. Cyclical effects may still be in evidence, but a secular increase in world trade over time could be at least one influence that might counteract any increase in the services deficit.

Conclusion

The foregoing precludes any single cause for the global current account asymmetry and leaves little hope that it can be eliminated. Relatively little formal analysis has been devoted to analyzing the conundrum, but it is more than just a statistical curiosity. If current account balances are off by substantial amounts, entire policies and attitudes toward international adjustment may be affected. Although the problem likely will not disappear, it is by no means clear, on the other hand, that the world deficit will increase to significantly higher than present proportions. With greater stability in world economic activity—and especially with lower interest rates and oil prices—the global deficit may even decline substantially. Forces within the global deficit also could pull in opposite directions, especially because the different components that contribute to the overall balance are related to different factors.

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1. Frequent revisions to this IMF data contribute to problems in analyzing the discrepancy. The data in Table 1 appeared in the International Monetary Fund's *World Economic Outlook, April 1985* (Washington, D.C., 1985), Table 29, p. 236.

2. Also included in the errors and omissions account of Table 2 is an estimate of Soviet hard-currency aid to, and trade with, other members of the Council for Economic Mutual Assistance (CEMA), which includes Poland, Bulgaria, East Germany, Czechoslovakia, Romania, and Hungary.

3. In trade terminology, here and later, c.i.f. indicates that "cost, insurance, and freight" are included; f.o.b. means "free on

board"; and f.a.s. is "free alongside ship." The value of imports frequently is quoted on a c.i.f. basis, but it may be on a customs basis. Both exports and imports—more frequently the former—can be quoted on an f.a.s. basis.

4. One estimate indicates that the length of the transport lag in recorded world trade averages 0.6 of a month. With respect to trade flows, it has been estimated that about 3 percent of exports are not received and counted as imports until the following calendar year. See William L. Hemphill, Research Department, International Monetary Fund, "Estimation of the Timing Asymmetry in International Trade," *Staff Papers* 27 (March 1980): 135–60.

5. Given a net direct and portfolio investment inflow of $10.2 billion and a combined decrease in international reserves plus external assets of commercial banks of only $0.3 billion, this yields an implicit capital outflow of $32.5 billion over the period.

Velocities of M1 and the Monetary Base: A Correction of Standard Formulas

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> Kenneth Mason, while exploring deep in the Pamirs, ran out of money and was lent some by a yak owner: "I wrote out on half a sheet of note-paper to Cox's, Karachi: 'Please pay bearer on receipt of this the sum of fifty pound sterling.' It must've been eight or nine months later that I heard from my bankers, Cox's at Karachi, that a greasy piece of paper had arrived and had been presented in the Peshawar bazaar and was said to be worth fifty pounds sterling. That piece of paper had gone from hand to hand all over Central Asia. It had marks of people that couldn't sign. It had thumb marks which had been dipped in ink. It had been to Samarkand and Kiva and God knows where, and it'd come over the Khyber Pass and was presented in Peshawar bazaar and was still said to be worth fifty pounds sterling."
>
> —From "Topees Overboard," in *Plain Tales of the Raj*, edited by Charles Allen

The purpose of this article is to explain velocities of circulation and equations of exchange clearly and correctly. It is intended to help students who are confused by the explanations in their textbooks and empirical researchers who wish to compute velocity correctly. The basic problem is this: the thing called velocity in nearly all explicit definitions is not the thing called velocity in further textbook discussions, in empirical research, or in official statistics. Alternatively, if the thing called velocity in official statistics and empirical research is intended to be the thing defined by the usual explicit definition, it is *never* computed correctly.[1]

The article proceeds as follows. Section 1 explains the fundamental concepts of the subject in a very simple example. The example is intended mainly for students, but it also serves as a base for further development. Section 2 explains the confusion stated above and shows what choices we have in clearing it up. Section 3 briefly discusses the importance of velocity in economic analysis and policy. Section 4 presents an accounting framework useful for deriving equations of exchange. Section 5 presents equations of exchange in M1 and formulas for computing the velocities of M1. Section 6 does the same for the monetary base. Section 7 gives formulas relating the velocities of M1 and those of the monetary base.

1. A very simple example

Consider a mythical economy with no credit, no securities, no middlemen, and no means of payment

except gold coins. Coins change hands for goods and services ("goods" hereafter) in every transaction. All transactions are "final" in the sense of being made by final consumers, so all transactions count in GNP. Finally, nobody eats the beans he grows but sells them to others and eats beans grown by someone else, so there is no imputed income and all of GNP is generated by monetary transactions.

There is only one kind of velocity in this mythical economy, and its relation to certain other macroeconomic magnitudes may be shown in the economy's equation of exchange. Like all correct equations of exchange, the equation for this economy represents a double classification of all transactions that occur during a certain arbitrary period of time—the "period of analysis." One classification groups transactions according to *what is paid*, and the other groups them according to *what is paid for*.

Let there be k coins, all existing throughout the period, and let m_j ($j = 1,...,k$) be the value (e.g., \$1) of the jth one. The total value of all k coins, $m_1 + ... + m_k$, defines the *money stock*, M.

$$M := m_1 + ... + m_k. \quad (1)$$

Let coin j change hands—that is, be paid—v_j times during the period ($v_j = 0$ if coin j never changes hands during the period). Changes of hands in "making change," such as the surrender of a \$1 coin for ten dimes, are called conversions, not payments, and do not count in v_j. By definition, v_j is the *velocity*, or *rate of turnover, of coin j* during the period. The *velocity of money*, V, is the weighted average velocity of all coins, weights being the values of the coins:

$$V := \frac{m_1v_1 + ... + m_kv_k}{m_1 + ... + m_k} = \frac{m_1v_1 + ... + m_kv_k}{M}. \quad (2)$$

Classified according to *what is paid*, total payments $= m_1v_1 + ... + m_kv_k = MV$, or the stock of money times its velocity. MV is the left-hand side of the equation of exchange for this economy.

To get the right-hand side, we classify all the transactions according to *what is paid for*. Let there be n goods and let q_i ($i = 1,...,n$) be the total amount of good i sold during the period ($q_i = 0$ if good i is never sold during the period). Let p_i be the average price at which good i is sold. Total payments for good i are p_iq_i, and total payments for all goods are $p_1q_1 + ... + p_nq_n$. This sum is the scalar product of the vectors p and q, where $p = (p_1,...,p_n)$ and $q = (q_1,...,q_n)$; I shall denote the sum by G. Thus total payments, classified according to what is paid for, are

$$G := p_1q_1 + ... + p_nq_n. \quad (3)$$

Most economists write $p_1q_1 + ... + p_nq_n$ as PQ (or sometimes PT), where P is a price index intended to represent the average price of all goods and Q (or T) is a quantity index supposed to represent the average quantity of goods sold. But neither index can be defined satisfactorily except under uselessly rare conditions, and I will not perpetuate their formal use.[3]

Clearly, MV equals G identically, no matter what values m_j, v_j, p_i, or q_i take during the period. This fact is expressed by the *equation of exchange* for this economy,

$$MV = G. \quad (4)$$

Note carefully how velocity is defined in equation 2, as the weighted average of velocities which are themselves defined as "number of times paid." Velocity is not defined by equation 4 or as the ratio of G to M. Yet the equation

$$V = G/M \quad (5)$$

always holds because equation 4 always holds.

Although equation 5 does not *define* velocity, it is very useful in *computing* it. The individual velocities v_j are not easily recorded, even in this simple economy, and the computation of V directly from its definition would not be feasible. Total sales and the money stock are somewhat more easily recorded. In practice, therefore, V would be computed by dividing M into G. This does not mean that velocity is a residual, determined by p, q, and M but not influencing these variables in any way. Velocity is *computed* residually but not *defined* residually.

If velocity were defined residually, equation 4 would contain only three independently defined variables and would therefore be a useless identity. Take any three variables x, y, and z; define a fourth variable w as the ratio yz/x; then the equation $wx = yz$ holds identically but imparts no information not already contained in the definition of w: it is a useless identity. The equation of exchange is an identity, but a very useful one, for it expresses a relation between four independently defined

variables and imparts information not contained in any of the four definitions; it follows from the *conjunction* of the four definitions, not from any proper subset of them.[4]

Velocity is not a behavioral residual, either, taking whatever value is necessary to satisfy the equation of exchange when M, p, and q vary in whatever manner we might imagine. Velocity is the inverse of the average "resting period" of money; if velocity is 12 during some year, for instance, each dollar's worth of coins is "at rest" for a month on the average. The resting period of money depends on the stock of money, the demand of traders to hold money, and institutional arrangements in the payments system—just as in the real world.[5]

If this mythical economy had intermediate-goods transactions, so that not all transactions counted in GNP, it would have two kinds of velocity—income velocity and transaction velocity. Both velocities would be defined as weighted-average rates of money turnover, but only final-goods transactions would count in the former while all transactions would count in the latter. Let us now open this possibility but, for the time being, continue to interpret G as nominal GNP and V as income velocity.

2. Current practice

When we return to the real world, we find velocity, either income or transaction velocity, used in two distinct senses, often by the same writer and sometimes on the same page. Income velocity is nearly always defined explicitly in terms of turnover, just as above. But fast on the heels of the definition there usually follows a discussion of equation 4, with the right-hand side interpreted as nominal GNP (and, of course, with the definition of M appropriately modified for modern conditions). Since the writer has not explicitly introduced a new definition of velocity, we naturally infer that the V of equation 4 is the one he defined earlier on his page. But if this inference is correct, equation 4 cannot be true of any economy that, like ours, has much consumer credit or imputed income. Nominal GNP exceeds total spending on final goods by that part of national income sold on credit plus the part not sold at all but consumed directly by its producers. Therefore, if the explicit definitions of the variables are to be taken seriously, equation 4 is just a mistake.

But equation 4 is widely regarded as an identity in the real world (just as it is correctly regarded in the mythical economy) and an elementary one at that. Since the possibility that economists cannot get their elementary identities right is too gruesome to contemplate, the V in equation 4 must not represent velocity in the sense of turnover. If equation 4 really is an identity in the real world, V cannot be understood in the sense of turnover but only in the sense implied by equation 4 and expressed more directly as

$$V := G/M, \tag{6}$$

that is, as a residual variable that takes on whatever value is required for the truth of equation 4. Velocity is just along for the ride.

In its sense as a residual, velocity has no explicit definition but is defined implicitly by the way it is discussed. Writers who discuss the velocity of M2 or of some larger credit aggregate such as total debt are obviously not using the word in the sense of turnover—even if they verbally defined it that way a few sentences earlier—because these aggregates contain financial instruments that cannot be spent. Only things that can be spent—media of exchange or means of payment—have velocity in the sense of turnover. But anything can have a velocity in the sense of a residual. Evidently, the behavior of such a velocity can never be explained.

We can end this confusion in one of two ways. Either we stop explicitly defining velocity in the sense of turnover and start calling it the *GNP-multiple*, as equation 6 defines it, or we start *using* it in the same sense of turnover in which we explicitly define it. The first choice would make current practice respectable; the second requires a change in that practice. All in favor of the first choice may stop reading.

Complaints about velocity-as-residual go back at least to Knut Wicksell.[6] Such complaints seem even more pertinent today because of recent developments in monetary theory that pay attention to payment arrangements (see Kohn, for example). These developments require the use of velocity in its original sense of turnover and need equations of exchange that are correct when velocity is used in this sense. Unfortunately, the derivation of such equations is much more complicated in our world than it was in the mythical economy described in section 2. Before embarking on this project, let us see why

velocity gets so much attention.

3. Some questions concerning velocity and equations of exchange

Equations of exchange, such as equation 4, are usually associated with the Quantity Theory of Money (in fact, they are often called "quantity equations"). There is no good reason for this association, for every correct equation of exchange is an identity; as such, it is independent of all theories and must fit into any useful theory, not just the Quantity Theory. Neither the Quantity Theory nor any other monetary theory has special claims on equations of exchange, and no useful theory can contradict them. Moreover, the Quantity Theory is not confined to an equation of exchange but consists of substantive propositions about the behavior of M, V, p, and q. According to the simplest version, both V and q depend on forces that are independent of M, so that only p varies when M does: prices vary directly with the quantity of money. More sophisticated versions do not imply this proportionality, especially in the short run. But all versions regard money and prices as far more changeable than real transactions.

Today all carefully specified monetary theories accept that velocity depends on institutional arrangements and payment practices that evolve slowly but are subject to the influence of inflation (even though the calculations make sense only when it is a residual). Only the most naive versions of Keynesian theory regard velocity as a behavioral residual that tends to vary inversely with the money stock, leaving real macroeconomic magnitudes unchanged. Such a theory cannot explain the data, which show that large changes in money tend to cause fairly large changes in velocity *in the same direction*.[7]

Velocity, then, is by no means the exclusive concern of Quantity Theorists, and it figures prominently in current issues of monetary theory and policy. The *stability* of velocity is an especially lively issue, for it relates to the stability of money demand and (according to some writers) to the choice among monetary policies.

Stability—referring either to the magnitude of velocity or, more often, to its rate of change—bears a one-to-one relation to the resting period of money: both velocity and the resting period are stable or unstable together. The resting period depends on the demand for money, among other things. If this demand is unstable, velocity ought to be too, so one way to evaluate the stability of money demand is to study the stability of velocity. As money demand is widely thought to have been unstable during much of the past decade, it accounts for some of the recent interest in velocity.[8]

Two things, however, should be noticed about the relation between the stabilities of money demand and velocity. First, the relation is not one-to-one. Although velocity and the resting period are related one-to-one, the resting period and money demand are not. The resting period also depends on money supply. If supply is unstable, velocity can be unstable even if demand is stable. It is the stability of *excess demand* for money (demand minus supply) that bears most directly on the stability of velocity. If supply has been as unstable as some scholars maintain,[9] it might well have destabilized velocity.

Second, the velocity that reflects the resting period is transaction velocity—the rate of money turnover in transactions of all kinds, not just the transactions counted in GNP. Desired holdings of money depend on planned purchases of used cars as well as new ones, of barber shops as well as haircuts, of securities as well as apples. An unstable excess demand for money would cause an unstable transaction velocity but would bear no necessary relation to income velocity. This fact is overlooked in much of the recent literature, which tends to dwell on income velocity.

The role of velocity in monetary policy is very complex but can be indicated in a rough sort of way by equation 7,

$$\frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{\Delta G}{G}, \tag{7}$$

which follows from equation 4. Equation 7 does not apply to our economy, but it does serve to highlight some controversies by expressing the relation between the percentage changes, or growth rates, implied by equation 4. The right-hand side represents the growth rate of GNP,[10] which, according to the equation, always equals the sum of the growth rates of money and velocity. (The appropriate version of equation 7 for our economy would account for credit sales, imputed income, and intermediate transactions.)

If velocity is unstable, so that $\Delta V/V$ fluctuates appreciably from period to period, steady growth in

Table 1
LIST OF TRANSACTIONS

Transactions	
Involving the public	Between banks
The public	*Banks*
1. Buys goods for cash.	19. Buy securities from other banks in secondary markets.
2. Buys new bank securities for cash.	20. Lend reserve balances to other banks.
3. Repays debt to banks in cash.	21. Repay loans of reserve balances to other banks
4. Deposits cash in banks.	22. Transfer reserve balances to other banks in clearing checks.
5. Buys goods for deposits.	
6. Lends deposits to the public.	
7. Repays debt with deposits.	
8. Buys securities in secondary markets from the public with deposits.	
9. Buys securities in secondary markets from banks with deposits.	
10. Buys new securities from banks with deposits.	
11. Repays debt to banks with deposits.	
12. Buys goods on credit.	
13. Withdraws cash from deposits.	
Banks	
14. Retire securities with cash.	
15. Issue deposits for goods.	
16. Issue deposits as loans to the public.	
17. Issue deposits to retire their securities.	
18. Issue deposits to buy securities from the public in secondary markets.	

GLOSSARY OF TERMS

Banks. All issuers of deposits.

Bank securities. All liabilities, other than deposits, of banks to the public.

Cash. Federal Reserve notes and U.S. coins outstanding.

Cash reserve. Cash held by banks.

Currency. Cash held by the public.

Currency issued. Cash paid to the public by banks.

Currency retired. Cash paid to or deposited in banks by the public.

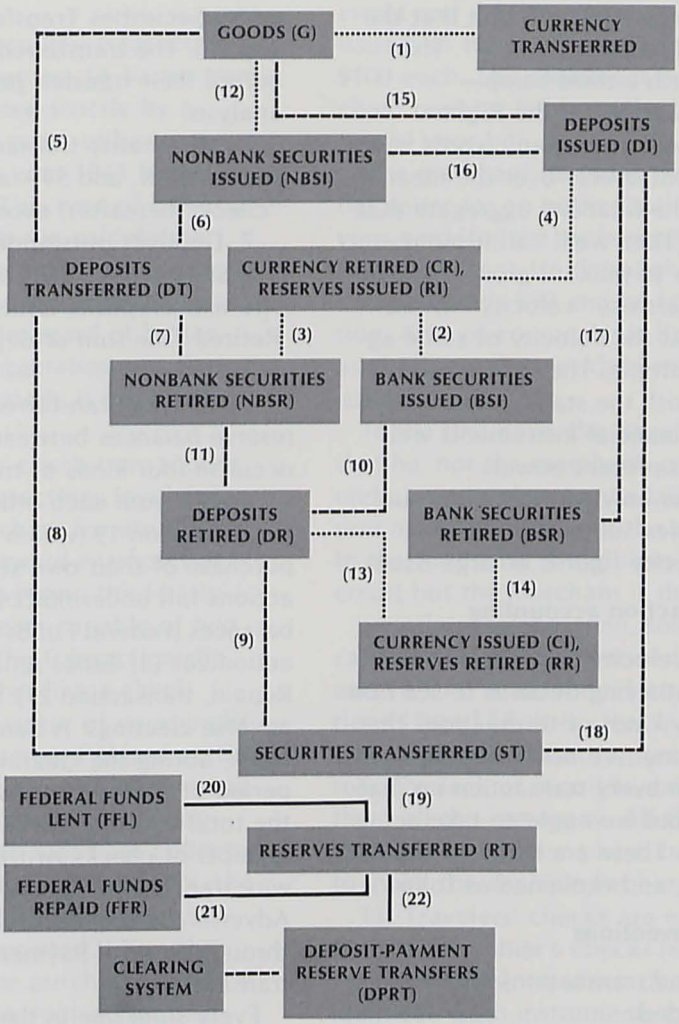
Deposits. Demand liabilities used as media of exchange.

Nonbank securities. Liabilities of the public to other members of the public or to banks, whether represented by the issuance of paper securities or entries in account books.

Reserves. Cash reserves plus banks' reserve balances with the Federal Reserve.

Reserves issued. Currency retired.

Reserves retired. Currency issued.



GNP requires compensating changes in the money stock. The appropriate growth rate for money might be 10 percent one quarter, 0 percent the next, and −5 percent the next, all depending on the fluctuations of velocity. One way to conclude that velocity is unstable is to regard it as a residual and to believe that private income-producing transactions are unstable; a steady value of *M* would then cause fluctuating values of *V*. Another way is to believe that the resting period of money is unstable because the demand for money is unstable. Either way leads to calls for active management of the money stock and probably for other kinds of government action as well. Unstable velocity thus seems to call for "fine tuning."

If velocity is stable, however, fine tuning might destabilize the economy. According to some scholars, the appropriate policy is steady growth (perhaps negative) of the money stock. If income can sustain 2-percent growth and velocity grows steadily at 3 percent, just keep money shrinking steadily at 1 percent.[11]

A controversial issue that cuts across the differences between the Fine Tuning and Steady Money camps is the identification of money. If you identify money as M1, you are not likely to be im-

pressed by figures purporting to show an unstable velocity of M2—even disregarding the fact that the only part of M2 possessing a velocity is M1. This controversy has called forth a third camp—sometimes called the Seekers After the Right Aggregate—who say, almost in so many words,[12] "Let us end this sterile controversy over the meaning of money. Let's find the financial aggregate that has the stablest velocity. Then we'll call it money and urge the central bank to make it grow at the appropriate rate." The Seekers use "velocity" in the sense of a residual, so that the velocity of some aggregate is just its GNP-multiple. They show no concern that the aggregate with the stablest multiple might consist mainly of financial instruments well beyond the central bank's present powers.

This brief discussion has only scratched the surface of a large and complex subject, but it is enough to show that velocity figures in large issues.

4. A framework for transaction accounting

The best way to explain velocity and equations of exchange in intuitively satisfying detail is to see how they depend on the many kinds of transactions that occur in a modern economy. We need not aim at utter realism, tracking down every transaction no matter how rare or obscure, but we ought to take account of the major types. These are listed in Table 1, diagrammed in the chart, and explained as follows.

4.1. Assumptions and conventions

1. Gifts, taxes, theft, and transactions with the central bank are disregarded.

2. The sum of currency and reserves—that is, the monetary base—is constant during the period of analysis. (This sum also equals the sum of reserve balances and cash.) Currency, reserve balances, reserves, and deposits may vary.

3. Banks issue *securities* only to the public. All interbank borrowing occurs in transaction 20. The public issues securities to itself (as when one member of the public borrows from another member) and to banks. All borrowing, except between banks, falls under security issuance; it is "gross," because gross flows are the relevant variables in velocity analysis.

4. All repayment of debt, except between banks (which occurs in transaction 21), falls under security retirements. The retired securities were issued before or during the period of analysis.

5. All secondary transactions in securities fall under Securities Transferred (transactions 8, 9, 18, and 19). The transferred securities were issued before their transfer, possibly during the period of analysis.

6. All security transactions—that is, *BSI*, *BSR*, *NBSI*, *NBSR*, and *ST*—are by check or wire (just "check" hereafter) except transactions 2, 3, and 14.

7. Deposits outstanding are invariant to Deposits Transferred (just their ownership changes), but they rise with Deposits Issued and fall with Deposits Retired. The sum of deposits and currency—that is, M1—may vary.

8. Reserves Transferred represents transfers of reserve *balances* between banks. These transfers occur in four kinds of transactions: (1) Banks buy securities from each other in secondary markets, as in transaction 19 (which does not include banks' purchase of their own securities, because such transactions fall under *BSR*). (2) Banks lend reserve balances (Federal Funds Lent) to other banks (transaction 20). (3) Banks repay such loans (Federal Funds Repaid, transaction 21). (4) Banks cover their adverse clearings. A bank has adverse clearings when, during the clearing period (a fraction of the period of analysis, a business day in most places), the total value of checks written on it exceeds its receipts of checks written on other banks. (I include wire transfers under the heading of "checks.") Adverse clearings cause a bank to lose reserves through Deposit-Payment Reserve Transfers in transaction 22.

Every solid line in the chart represents transactions involving reserve transfers; every dashed line represents transactions that *might* induce simultaneous or subsequent reserve transfers. In transaction 15, for example, a bank issues deposits in payment for goods. If the seller of goods is also a depositor at the bank, the transaction remains within the bank and has no effect on the bank's reserve position (if the seller subsequently withdraws the payment in cash, it is *then* counted in transaction 13); if the seller holds his deposits at another bank, the transaction potentially induces a reserve transfer. For another example, consider transaction 10, where bank A sells its security to someone who pays by check. If the check is on bank A, no reserve transfer occurs—just a change in the composition of the bank's liabilities; but if the check is on bank B, reserves will move from B to A

unless the check is offset in clearing. In either case, deposits are "retired."

9. Only one type of barter occurs—transaction 12, the exchange of goods for securities. (A barter transaction is one that is not executed strictly by a medium of exchange.[13]) Other types of barter occur in practice, as when you trade your 1941 Harley 74 for a 1959 Chevrolet Apache. This type of barter really ought to be accounted for in calculating transaction velocity (and income velocity in some cases, as when I paint your house in exchange for financial advice). The virtual disregard of barter leads to a possibly serious exaggeration of calculated velocities. The formal remedy is explained below.

10. Credit card and travelers'-check transactions deserve special comment. Transactions involving credit cards issued by the merchant for use in the merchant's store are straightforward examples of transaction 12. Transactions involving third-party cards are more complex and seem capable of two treatments. On the one hand, the transaction slip with your signature on it is rather like a check drawn on a bank, ordering the issuer of your credit card to pay the merchant the amount stated on the slip. The merchant will deposit the slip in his credit card account at his bank. He can write ordinary checks on this account once the funds are collected by the bank via the credit card clearing system, the only difference being that he gets only about 97 percent of the amount thus deposited.[14] You then owe the *bank* the amount of the purchase, just as if you had written an overdraft on your deposit account. Therefore, we could treat a $100 transaction of this sort as if the bank had lent you $100 by increasing your deposit account and you then wrote a $100 check on this account to the merchant. Under this treatment, transactions 16 and 5 increase by $100 each, but transaction 12 does not enter the picture—that is, there is no "sale on credit." Sales on credit occur only when the merchant grants the credit; according to this treatment, it is not the merchant but the bank that grants the credit.

On the other hand, the signed transaction slip could be regarded as your IOU to the *merchant*, who then sells the IOU to the bank that maintains his credit card account. The bank buys this transferred security by issuing deposits to the merchant, just as if it bought a two-name bill of exchange (your name and the merchant's). Thus interpreted, the transaction would be treated as a $100 sale on credit and a $100 security transfer against deposits issued, so transactions 12 and 18 would increase by $100 each. This treatment is appropriate if the merchant remains liable to the bank in case you refuse to pay your bill.

The merchant definitely is *not* liable if he phones the issuer for an authorization before accepting your card (for the authorization commits the issuer to acceptance). He definitely *is* liable if the purchase exceeds $50 and he fails to obtain authorization. And, of course, he is liable if the card is listed as "bad" on the weekly memo sent out by the issuer.

In the first case, the issuer's authorization means that he, not the merchant, grants credit. In the second case, the merchant's failure to obtain authorization means that he, not the issuer, grants the credit. In the third case (fraudulent use), no one grants credit but the merchant is defrauded.

Strictly speaking, then, some third-party credit card sales should be entered under transactions 5 and 16, and some should be entered under transactions 12 and 18. It is clear in principle which ones should go where, but in practice it is impossible to tell. The lack of requisite data forces us to treat them all the same way. All things considered, the first treatment seems best, but the reader will have to make the decision for himself.

11. Travelers' checks are easier. They should be treated as cashier's checks (even when issued by nonbank firms), to which they are economically identical. Both instruments commit the issuer to pay cash upon presentment and both disappear afterwards. Thus a purchase of goods for travelers' checks goes under transaction 5. A purchase of travelers' checks for cash goes under transaction 4. A purchase for deposits is not treated as a transaction but as a conversion.

4.2. Classification of transactions according to what is paid for

The *goods* transactions are 1, 5, 12, and 15. Transactions 1, 5, and 15 are payments of M1, but only the first of these is a payment of the monetary base. Transaction 12 is not a payment of anything; it is included so that the total goods transactions will equal total sales of goods.

The set of *financial transactions* depends on how we identify money. On the M1 identification, the

financial transactions are 2 and 10 (*BSI*); 14 and 17 (*BSR*); 6, 12, and 16 (*NBSI*); 3, 7, and 11 (*NBSR*); and 8, 9, 18, and 19 (*ST*). (The last one could be left off this list, because it is a transaction between banks and does not involve M1. I include it so that *ST* will include all secondary security transactions.) Transactions 4 and 13 are not financial on the M1 identification of money because they merely convert one form of M1 into another; transactions 20, 21, and 22 are not financial because they just redistribute reserve balances among banks and do not fall under a subheading (such as *ST*) that is recognized as financial from the M1 point of view.

Every transaction mentioned in the preceding paragraph is financial from the standpoint of the monetary base. Transactions 4 and 13 do not just convert one form of money into another but involve the issuance or retirement of bank debt. Transactions 20, 21, and 22 represent major uses of the monetary base in creating and settling debts; that these debts lie wholly within the banking system is neither here nor there.[15]

Notice that this classification is not a partition. Transaction 12 is both a goods and a financial transaction; 4, 13, 20, 21, and 22 are neither goods nor financial transactions on the M1 identification. The classification according to what is paid is not a partition, either.

4.3. Classification of transactions according to what is paid

Payments of M1 occur in transactions 1–3, 5–11, and 15–18. Transactions 4 and 13 are not payments but conversions of M1; transaction 14 is a cash payment, but the cash is not counted in M1 because it is in banks. The M1 payments are divided into currency payments (transactions 1–3) and deposit payments (transactions 5–11 and 15–18).

Payments of the monetary base are payments of currency[16] (transactions 1–4) or reserves (transactions 13, 14, and 19–22). The total value of transaction 22 depends on the flows of checks between banks caused by deposit payments. Thus deposit payments figure implicitly in payments of the base.

5. Velocities of M1

We shall consider five velocities of M1: transaction, income, intermediate-goods, goods (the sum of the preceding two), and financial velocities. Each velocity is a weighted average of the corresponding velocities of currency and deposits, where the weights are the average quantities of currency and deposits, respectively, outstanding during the period.

Let C_1 denote average currency, D denote average deposits, C_1P denote total payments of currency (from the M1 point of view), and DP denote total payments of deposits.[17] The (transaction) velocities of currency and deposits, V_{c_1} and V_d, are defined by

$$V_{c_1} := C_1P/C_1$$
$$V_d := DP/D.$$

The transaction velocity of M1, V_1, is then defined by

$$V_1 := (C_1V_{c_1} + DV_d)/(C_1 + D). \tag{8}$$

The other velocities of M1 are defined in a similar manner (e.g., its goods velocity is defined as the same weighted average of the goods velocities of currency and deposits, where the latter velocities are defined as ratios of total cash (or deposit) payments for goods to C_1 (or D). As it is obvious how these definitions are expressed, I shall not write them down.

The average quantity of M1 outstanding during the period is denoted by M_1; it equals $C_1 + D$. The total value of all payments of M1 therefore equals M_1V_1.

As noted above, M1 payments occur in transactions 1–3, 5–11, and 15–18. They are shown in rows 3–6 of Table 2, where a_i denotes the total value of transaction i during the period.

Column 12 of the table shows $a_1 + a_2 + a_3$ as C_1P (row 3 of column 12); it shows the sums of rows 4, 5, and 6 as *DT*, *DR*, and *DI* (these and other abbreviations are defined in the chart). Column 13 shows the sum of *DT*, *DR*, and *DI* as *DP*, and the sum of *DP* and C_1P as M_1V_1, which equals total payments of M1.

Columns 1–6 of the table show what M1 is paid for. Row 8 of the table shows the sums of these columns. Row 9 shows aggregates of things paid for. The aggregates relative to M1 are G and F_1, where

$$F_1 := BSI + BSR + NBSI + NBSR + ST, \tag{9}$$

representing total financial transactions from the standpoint of M1. Note that G represents total sales

Table 2
TRANSACTIONS MATRIX

Paid \ Paid for	(1) Goods	(2) New bank securities	(3) Bank securities retired	(4) New nonbank securities	(5) Nonbank securities retired	(6) Securities transferred	(7) New deposits	(8) Deposits retired	(9) Federal funds lent	(10) Federal funds repaid	(11) Deposit-payment reserve transfers	(12) Sums of rows	(13) Aggregates
(1) Reserve balances						a_{19}			a_{20}	a_{21}	a_{22}	RBP	RP; $RP+CP=MV$
(2) Cash reserves			a_{14}					a_{13}				CRP	
(3) Currency	a_1	a_2			a_3		a_4					$C_1P+a_4=CP$	
(4) Existing deposits	a_5			a_6	a_7	a_8						DT	DP; $C_1P+DP=M_1V_1$
(5) Existing deposits, retiring them		a_{10}			a_{11}	a_9						DR	
(6) New deposits	a_{15}		a_{17}	a_{16}		a_{18}						DI	
(7) New nonbank securities	a_{12}			a_{12}								$2a_{12}$	
(8) Sums of columns	G	BSI	BSR	$NBSI$	$NBSR$	ST	a_4	a_{13}	a_{20}	a_{21}	a_{22}		
(9) Aggregates	G	F_1											
		F											

of goods, as in section 1, and is considerably larger than GNP.[18]

Evidently, $G + F_1$ exceeds M_1V_1 by $2a_{12} + a_{14} + a_{19}$. If we take this discrepancy into account, we get the Transaction Version of the equation of exchange in M1:

$$(10) \qquad M_1V_1 = G + F_1 - 2a_{12} - a_{14} - a_{19}.$$

The *goods velocity* of M1, V_1^g, is the weighted average of the goods velocities of currency and deposits; it always obeys the relation

$$(11) \qquad V_1^g = (G - a_{12})/M_1.$$

The *financial velocity* of M1, V_1^f, is the weighted average of the financial velocities of currency and deposits (definitions of which should be obvious), and always obeys the relation

$$(12) \qquad V_1^f = (F_1 - a_{12} - a_{14} - a_{19})/M_1.$$

Goods and financial velocities sum to transaction velocity.[19] These velocities are not *defined* by equations 10–12, but they satisfy those equations *identically*.

Goods velocity may be divided into *income velocity*, V_1^y, and *intermediate-goods velocity*, V_1^i. For $j = 1, 5, 12, 15$, write

$$(13) \qquad a_j = a_j^y + a_j^i,$$

where superscript y denotes "final" sales (sales counted in national income) and superscript i denotes intermediate sales. Similarly, write

$$(14) \qquad G^y = a_1^y + a_5^y + a_{12}^y + a_{15}^y$$
$$G^i = a_1^i + a_5^i + a_{12}^i + a_{15}^i.$$

The Income Version of the equation of exchange in M1 is

$$(15) \qquad M_1V_1^y = G^y - a_{12}^y.$$

The income velocity, V_1^y, is defined as the weighted average of the income velocities of currency and deposits (definitions obvious) and satisfies identically the equation

$$(16) \qquad V_1^y = (G^y - a_{12}^y)/M_1.$$

Income velocity is almost universally computed as GNP divided by M1, thus being wrong by the amount of imputed income and credit sales relative to M_1. If imputed income or credit sales are less stable than national-income sales generally, the error produces an artificial instability in velocity as usually computed.

GNP does not appear in any of the preceding equations. If we want to express income velocity in terms of GNP, Y, we can use the identity

$$Y = G^y + Y_N,$$

where Y_N denotes nonmarket (or imputed) income, and write

$$(17) \qquad V_1^y = (Y - a_{12}^y - Y_N)/M_1.$$

Therefore, to correct the published statistics on the income velocity of M1, we must subtract the M1 multiples of income sales on credit and imputed income from the published figures.

We have one more M1 velocity, the intermediate-goods velocity V_1^i. This is defined in an obvious way and always obeys the equation

$$(18) \qquad V_1^i = (G^i - a_{12}^i)/M_1.$$

The velocities V_1, V_1^y, V_1^i, and V_1^f are defined independently of each other and of the variables G, G^y, G^i, and F_1; yet they always obey the relation

$$(19) \qquad V_1 = V_1^y + V_1^i + V_1^f.$$

6. Velocities of the monetary base

The size of the monetary base, or its quantity, M, is constant throughout the period but the quantities of its components may vary. Let C denote the average quantity of currency outstanding, CP denote total payments in currency, R denote average reserves outstanding, and RP denote the total payments of these reserves. The (transaction) velocities of currency and reserves are V_c and V_r, respectively,

$$V_c := CP/C$$
$$V_r := RP/R.$$

The transaction velocity of the monetary base, V, is defined by the weighted average

$$(20) \qquad V := (CV_c + RV_r)/(C + R).$$

All velocities of the base are defined as weighted averages of the corresponding velocities of currency and reserves.

As $M = C + R$, total payments of the base equal MV. Column 13 of Table 2 shows MV as the sum of rows 1–3.

Since MV contains elements from every column but one, the simplest way to derive an equation of

exchange in the monetary base is to use all the transactions and then correct for double counting. The sum of all transactions, in the sense of what is paid for, is $G + F + A_{22}$, where, as shown in row 9 of the table, F is the sum of the sums of columns 2–10 and represents total financial transactions from the standpoint of the monetary base:

$$F := F_1 + a_4 + a_{13} + a_{20} + a_{21}. \tag{21}$$

(We keep a_{22} out of F for later convenience.) Proceeding as indicated, we get the Transaction Version of the equation of exchange in the monetary base,

$$MV = G + F - DP - 2a_{12} + a_{22}. \tag{22}$$

This equation holds identically for all possible values of all transactions.

The transaction velocity of the base may be computed from the equation

$$V = (G + F - DP - 2a_{12} + a_{22})/M, \tag{23}$$

but a possibly more useful formula is shown in the next section.

The transaction velocity of the base could be formally expressed as the sum of goods and financial velocities, but the definitions of goods and financial velocities would be artificial. It is true that by subtracting a_5 and a_{15} (deposit payments for goods) from G we could write the formal expression $V^g = (G - a_5 - a_{12} - a_{15})/M$, which looks like a goods velocity comparable to the velocity expressed in equation 11. But the numerator of this expression does not really equal total base payments for goods, because it does not include the reserve transfers (counted in a_{22}) caused by checks written for goods. Not only are these transfers not distinguished from the rest of a_{22} in clearing, but they cannot be distinguished in principle. Checks offset each other in clearing according to the banks they are written on and deposited in, not the things they pay for. Neither the amounts offset nor the amounts cleared by reserve transfers can be traced to the goods or financial category. The above expression for V^g thus has only a formal significance.

If the reserves transferred in clearing cannot be segregated into goods and financial categories, they surely cannot be classified by their effect on GNP. This means that there is no such thing (except formally) as the income velocity of the monetary base.

There is indeed an Income Version of the equation in the monetary base, but it is stated in terms of "virtual velocity." Let us write deposit payments, DP, as

$$DP = DPRT + DPCT, \tag{24}$$

where $DPRT$ represents that part of deposit payments equal to the reserves transferred in clearing (which equals a_{22}) and $DPCT$ ("Deposit Payments Clearing Themselves") represents the rest of deposit payments. We cannot associate any particular deposit payment with either part, but we can compute both parts at least in principle. Each bank's contribution to $DPRT$ in any given clearing period equals the excess (if any) of its *on us* checks over its *to us* checks (checks written on the bank to another depositor of the bank are both *on us* and *to us*). The sum of these excesses, for all the banks where they are positive, for all the clearing periods in the period of analysis, equals $DPRT$ (equals a_{22}). The sum of *min* {*on us, to us*} for all banks and clearing periods equals $DPCT$. Now if the banks had not invented offset clearing, and if checks remaining *within* banks had passed *between* banks but were otherwise identical (e.g., if your landlord banked elsewhere than at your bank), the banks could get together each day and exchange reserves check by check until they had cleared every check. They would be a lot busier, but obviously they could accomplish the job with the same total quantity of reserves that support the actual transfers made under modern clearing arrangements. In other words, clearing practices save potential reserve transfers in the amount $DPCT$. Although this saving economizes on time and trouble, it does not reduce the net value of transfers. The same net transfers would occur either with or without clearing offsets.[20] Given the interbank distribution of checks, reserves could be transferred back and forth in the whole amount DP without causing any other changes in financial or goods transactions (beyond the additional time and trouble of the unnecessary transfers). These are the considerations behind the concept of virtual velocity.

Separating MV into its components, using equation 24, and recalling that $DPRT = a_{22}$, we can write

$$MV + DP = CV_c + R(V_r + DPCT/R) + a_{22}. \tag{25}$$

Let us call $V_r + DPCT/R$ the *virtual velocity of reserves* and denote it by W_r (think of two V's run together).[21] The virtual velocity is what the actual

velocity could be without affecting any other aspect of economic activity (beyond those noted). Let us call the weighted average of the velocity of currency and the virtual velocity of reserves the *virtual velocity of the monetary base*, W:

(26) $$W := (CV_c + RW_r)/(C + R).$$

Then equation 25 becomes

(27) $$MV + DP = MW + a_{22},$$

and the equation of exchange can be expressed in terms of W:

(28) $$MW = G + F - 2a_{12}.$$

Now the Income Version follows readily,

(29) $$MW^y = G^y - a_{12}{}^y,$$

where W^y is the *virtual income velocity of the monetary base*, defined in terms of the actual payments of the base in national-income purchases and the payments that are saved by clearing practices. The virtual income velocity of the base satisfies identically the equation:

(30) $$W^y = (G^y - a_{12}{}^y)/M.$$

The definitions of virtual intermediate-goods (W^i) and financial (W^f) velocities ought to be obvious by now. These velocities satisfy identically the equations

(31) $$W^i = (G^i - a_{12}{}^i)/M$$

and

(32) $$W^f = (F - a_{12})/M.$$

The relation between all the velocities of the base can be expressed simply once we introduce a name and symbol for the part of virtual velocity that is saved by clearing offsets. We can call this part the "unrealized velocity," V^u; from equations 24 and 27,

$$\begin{aligned} V &= W - (DP - a_{22})/M \\ &= W - DPCT/M \\ &= W - V^u, \end{aligned}$$

where

(33) $$V^u := DPCT/M.$$

Then

$$W = V + V^u,$$

and all together,

(34) $$W = V + V^u = W^y + W^i + W^f.$$

7. Relations between the velocities

The relation between income velocities is simple. From equations 16 and 30, $MW^y = M_1V_1{}^y$, that is,

$$W^y = (M_1/M)\, V_1{}^y.$$

The ratio M_1/M is usually called the money multiplier.[22] I will denote it by m:

(35) $$m := M_1/M,$$

so that

(36) $$W^y = mV_1{}^y.$$

Similarly, from equations 18 and 31, it follows that

(37) $$W^i = mV_1{}^i.$$

The relation between financial velocities is not so simple. From equations 12, 21, and 32,

$$MW^f = M_1V_1{}^f + (a_4 + a_{13} + a_{14} + a_{19} + a_{20} + a_{21}).$$

The term in parentheses represents transactions that are monetary from the standpoint of the base but not from that of M1. From the former standpoint, these transactions are payments of money to create, transfer, or retire bank liabilities; from the latter standpoint, they are either mere conversions of one form of money to another (transactions 4 and 13) or uses of reserves unaccompanied by uses of money (transactions 14, 19, 20, and 21). Indeed, these transactions are the only ones, apart from reserve transfers through clearing, that bear conflicting interpretations. Denoting their value by B, and their M-multiple by b,

(38) $$b := (a_4 + a_{13} + a_{14} + a_{19} + a_{20} + a_{21})/M,$$

we can write the equation

(39) $$W^f = mV_1{}^f + b.$$

The relation between transaction velocities now follows from equations 19, 34, 36, 37, and 39:

(40) $$W = mV_1 + b.$$

Alternatively,

(41) $$V = mV_1 + b - V^u.$$

In this equation, the transaction velocity of the base equals the multiplier times the transaction velocity

of M1 plus a correction factor; this factor is the M-multiple of all transactions, other than clearings, that are monetary from the standpoint of the base but not of M1, minus the unrealized velocity of the base, which is the M-multiple of all transactions that move deposits but not reserves. Thus the correction factor adds (the M-multiple of) purely base transactions and subtracts purely M1 transactions.

8. Summary

The velocity of money, in the sense of turnover, has interested economists for many years. Yet conventional modern treatments make sense only if velocity is interpreted as a residual, even though it is still conventionally defined in terms of turnover. This paper, about velocity in the sense of turnover, has presented an accounting framework for the derivation of equations of exchange and the computation of transaction, goods, financial, and income velocities of both M1 and the monetary base.

Although the framework accounts for most of the transactions in our economy, it disregards transactions involving the central bank and the federal government. The incorporation of such transactions would be conceptually straightforward but computationally tedious.

In the context of current discussions of velocity, the most important formulas are probably equations 17 and 36, which show how to compute the income velocity of M1 and the virtual income velocity of the monetary base in terms of GNP. Unfortunately, the credit-sales data needed for these computations are not regularly published. A research project under way is attempting to estimate the required figures. In the meantime, it is impossible to say whether or not the inaccurate conventional computations of velocity give a fair picture of its stability.

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1. This charge is scarcely credible in view of the long history of the subject. Thomas M. Humphrey has traced equations of exchange involving a velocity term all the way back to 1804 ("Algebraic Quantity Equations before Fisher and Pigou," *Economic Review*, Federal Reserve Bank of Richmond, September 1984, 13–22). Arthur W. Marget wrote a whole book about the subject and its history up to the middle 1930s (*The Theory of Prices*, vol. 1 [New York: Prentice-Hall, Inc., 1938]). This encyclopedic but barely readable book addresses nearly every question ever raised about equations of exchange.

2. Throughout this paper, the symbol ":=" indicates that the variable to the left of it is being defined by the expression to the right of it. The subject of velocity calls for especially careful observance of the distinctions between definitions, identities, and other equations. All symbolic definitions are identities, but not all identities are definitions. All identities are equations, but not all equations are identities.

3. See Irving Fisher's famous tournament of index-number formulas in *The Purchasing Power of Money*, rev. ed. (New York: The Macmillan Co., 1920 [1911], 198–233, 385–429). This tournament had no winners and could never have any. Its rules consist of a set of criteria that index-number formulas ought to meet, but no formula can meet all the criteria.

4. Unfortunately, many economists regard equations of exchange as "useless tautologies," or "truisms." This attitude shows in the following remark by J. R. Hicks in "A Suggestion for Simplifying the Theory of Money," *Economica* (New Series) 2 (February 1935): 1:

> To anyone who comes over from the theory of value to the theory of money, there are a number of things which are rather startling. Chief of these is the preoccupation of monetary theorists with a certain equation, which states that the price of goods multiplied by the quantity of goods equals the amount of money which is spent on them. This equation crops up again and again, and it has all sorts of ingenious little arithmetical tricks performed on it.

This attitude probably derives from the neoclassical way of thinking, which really allows no role for money in making exchanges. For criticism, see Robert W. Clower, "A Reconsideration of the Microfoundations of Monetary Theory," *Western Economic Journal* 6 (December 1967): 1–8, and Douglas Gale, *Money: In Equilibrium* (Cambridge: Cambridge University Press, 1982). A more direct answer to snobbishness about equations of exchange was given by Marget, p. 98:

> Viewed...in the light of the actual historical development of...the subject,...the importance of the "truistic" character of [useful equations of exchange]...is that the gradual attainment of this "truistic" character, instead of providing ground for criticism...,becomes a record of slow achievement, over centuries, of precisely the kind that is represented by the advance of knowledge in any branch of science. An earlier proposition, regarded in its own day as a "truism," is shown by later investigation to be true in fact only under certain specific conditions of which not even the nature was at first recognized ....

5. Howard S. Ellis gave a clear account of the role of convention and institutional arrangements in "Some Fundamentals in the Theory of Velocity," *The Quarterly Journal of Economics* 52 (May 1938): 431–72, and there cited much of the useful earlier literature. Meir Kohn further advanced the subject in "In Defense of the Finance Constraint," *Economic Inquiry* 19 (April 1981): 177–95.

6. See Knut Wicksell, *Lectures on Political Economy*, vol. 2, *Money*, published in Sweden in 1906, trans. E. Classen, ed. Lionel Robbins (London: Routledge and Kegan Paul, 1935), 60, 61.

7. This has been known at least since Cantillon. For a discussion, see Charles Rist, *History of Monetary and Credit Theory*,

published in France in 1938, trans. Jane Degras (New York: The Macmillan Co., 1940), chap. 2.

8. The conventional theory and econometrics of money demand are in a sorry state. See Thomas F. Cooley and Stephen F. LeRoy, "Identification and Estimation of Money Demand," *American Economic Review* 71 (December 1981): 825–44. The study of demand stability, via velocity, is therefore quite natural.

9. See especially Milton Friedman, "Monetary Policy: Theory and Practice," *Journal of Money, Credit, and Banking* 14 (February 1982): 98–118; "Monetary Variability: United States and Japan," 15 (August 1983): 339–43; and "The Fed Hasn't Changed Its Ways," *Wall Street Journal*, 20 August 1985, 24.

10. In this paper I do not distinguish between real and nominal GNP, but the right-hand side of equation 7 can be expressed as

$$\frac{\Delta G}{G} = (q \cdot \Delta p)/(p \cdot q) + (p \cdot \Delta q)/(p \cdot q),$$

where dots denote scalar products. The first term represents the contribution of price changes (the inflationary component) had quantities remained constant, and the second represents the real component had prices remained constant. These terms are related to Laspeyres indexes of price and quantity *changes*. Most of the interest in steady income growth centers on the second term.

11. The twentieth-century leader of the Steady Money camp is Milton Friedman, whose views rest partly on the belief that velocity would be stable if the money stock were stable and partly on the wish to reduce the role of central banks and other government arms in private affairs. See, for example, *A Program for Monetary Stability* (New York: Fordham University Press, 1959), and *Capitalism and Freedom* (Chicago: The University of Chicago Press, 1962). Most of the economists who are very well known to the public, however, are Fine Tuners.

12. See, for example, William A. Barnett, "The Optimal Level of Monetary Aggregation," *Journal of Money, Credit, and Banking* 14 (November 1982, pt. 2): 687–710. This paper, like many others by this author, energetically argues the case for what is called Divisia aggregation. It serves pretty well as a manifesto of the Divisia brotherhood, which is a splinter group among the Seekers. See, for example, William A. Barnett and Paul A. Spindt, Board of Governors of the Federal Reserve System, *Divisia Monetary Aggregates: Compilation, Data, and Historical Behavior*, Staff Studies no. 116 (Washington, D.C.: Board of Governors, 1982).

13. Some writers define barter more broadly, as any transaction not involving *money*. William Stanley Jevons, in *Money and the Mechanism of Exchange* (New York: D. Appleton and Co., 1882 (1879), 189, expressed it like this:

> No sooner have a people fully experienced the usefulness of a good system of money, than they begin to discover that they can dispense with it as a medium of exchange, and return to a method of traffic closely resembling barter. With barter they begin and with barter they end; but the second form of barter...is very different from the first.

Jevons's "second form of barter" involves circulating credit (bills of exchange in earlier days, bank deposits today). His useful book, intended for a lay audience before the days of universal schooling, received timely reviews in *The Hartford Post*, *The Boston Saturday Evening Gazette*, and *Popular Science*. The 1879 edition cited some of these reviews at the back of the book.

14. About 3 percent is kept by the bank as its fee for maintaining the account and guaranteeing payment on all credit card purchases up to $50; it is as if the bank remitted at 97 percent of par. I have disregarded this complication.

15. Strictly speaking, transactions 5 and 15, too, should be regarded as financial from the standpoint of the monetary base. If transaction 12 is financial because it exchanges debt, why are not transactions 5 and 15? In strict logic, they are; I do not count them as such in order to minimize the corrections for double counting.

16. Notice that currency payments are defined differently in the M1 and the monetary-base identifications of money.

17. Using Stieltjes integrals (because quantities change abruptly), C_1 and D are found by integrating $C_1(t)$ and $D(t)$ over the period and dividing by the length of the period, where $C_1(t)$ and $D(t)$ are currency and deposits at instant t.

18. In the mythical economy of section 2, G represented both GNP and total sales of goods.

19. It should be noted that the goods, but not the financial, velocity of M1 is affected by our treatment of credit card purchases. The first treatment makes the goods velocity—and therefore the transaction and income velocity—greater than does the second one.

20. Here, for simplicity, I am disregarding the fact that checks remaining within the bank create no potential reserve transfers. If such checks were randomly reassigned so that they traveled between banks and thus reached the clearinghouse, they would induce a statistically small increment to the actual amount of reserve transfers. Such additional transfers might, in turn, induce additional borrowing of federal funds.

21. Or think of velocity pronounced with a Swedish accent. This is how Knut Wicksell would have pronounced it, and he introduced the term "virtual velocity" (though the concept had long been known). See, for example, Wicksell's *Lectures on Political Economy: Money*, 67ff. Note, however, that I define virtual velocity somewhat differently than Wicksell. I think Wicksell would have reserved the term for what I will later call "unrealized velocity."

22. The multiplier is not constant. For an interesting and informative study of its determinants, see S. C. Tsiang, "The Diffusion of Reserves and the Money Supply Multiplier," *Economic Journal* 88 (June 1978): 269–84.
