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# ECONOMIC REVIEW

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FEDERAL RESERVE BANK  
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# Money

by James Madison

James Madison was  
president of the United  
States from 1809 to 1817.\*

*Observations written<sup>1</sup> posterior to the circular Address of Congress in September 1779, and prior to their Act of March, 1780.<sup>2</sup>*

It has been taken for an axiom in all our reasonings on the subject of finance, that supposing the quantity and demand of things vendible in a country to remain the same, their price will vary according to the variation in the quantity of the circulating medium; in other words, that the value of money will be regulated by its quantity. I shall submit to the judgment of the public some considerations which determine mine to reject the proposition as founded in error. Should they be deemed not absolutely conclusive, they seem at least to shew that it is

liable to too many exceptions and restrictions to be taken for granted as a fundamental truth.

If the circulating medium be of universal value as specie, a local increase or decrease of its quantity, will not, whilst a communication subsists with other countries, produce a correspondent rise or fall in its value. The reason is obvious. When a redundancy of universal money prevails in any one country, the holders of it know their interest too well to waste it in extravagant prices, when it would be worth so much more to them elsewhere. When a deficiency happens, those who hold commodities, rather than part with them at an under-value in one country, would carry them to another. The variation of prices in these cases, cannot therefore exceed the expence and insurance of transportation.

Suppose a country totally unconnected with Europe, or with any other country, to possess specie in the same proportion to circulating

■ \*This essay was originally published in two parts, in the December 19 and 22, 1791, issues of Philip Freneau's *National Gazette* of Philadelphia. The edited, annotated version reprinted here is taken from *The Papers of James Madison*, vol. 1, 16 March 1751–16 December 1779, ed. William T. Hutchinson and William M. E. Rachal (Chicago: University of Chicago Press, 1962), pp. 302–10. The note referenced by a dagger (†) is James Madison's; the numbered notes are those of the University of Chicago Press editors. This version of the essay is reprinted with the permission of the University of Chicago Press. © 1962 by the University of Chicago. All rights reserved.

■ 1 The original manuscript of the essay is not known to be extant. In the Tracy W. McGregor Library, University of Virginia, is a transcript of about the first one-third of the article, which John C. Payne probably copied from the newspaper version of it.

property that Europe does; prices there would correspond with those in Europe. Suppose that so much specie were thrown into circulation as to make the quantity exceed the proportion of Europe tenfold, without any change in commodities, or in the demand for them: as soon as such an augmentation had produced its effect, prices would rise tenfold; or which is the same thing, money would be depreciated tenfold. In this state of things, suppose again, that a free and ready communication were opened between this country and Europe, and that the inhabitants of the former, were made sensible of the value of their money in the latter; would not its value among themselves immediately cease to be regulated by its quantity, and assimilate itself to the foreign value?

Mr. Hume in his discourse on the balance of trade supposes, “that if four fifths of all the money in Britain were annihilated in one night, and the nation reduced to the same condition, in this particular, as in the reigns of the Harrys and Edwards, that the price of all labour and commodities would sink in proportion, and every thing be sold as cheap as in those ages: That, again, if all the money in Britain were multiplied fivefold in one night, a contrary effect would follow.” This very ingenious writer

seems not to have considered that in the reigns of the Harrys and Edwards, the state of prices in the circumjacent nations corresponded with that of Britain; whereas in both of his suppositions, it would be no less than four fifths different. Imagine that such a difference really existed, and remark the consequence. Trade is at present carried on between Britain and the rest of Europe, at a profit of 15 or 20 per cent. Were that profit raised to 400 per cent. would not their home market, in case of such a fall of prices, be so exhausted by exportation—and in case of such a rise of prices, be so overstocked with foreign commodities, as immediately to restore the general equilibrium? Now, to borrow the language of the same author, “the same causes which would redress the inequality were it to happen, must forever prevent it, without some violent external operation.”<sup>3</sup>

The situation of a country connected by commercial intercourse with other countries, may be compared to a single town or province whose intercourse with other towns and provinces results from political connection. Will it be pretended that if the national currency were to be accumulated in a single town or province, so as to exceed its due proportion five or tenfold, a correspondent depreciation would ensue, and every thing be sold five or ten times as dear as in a neighboring town or province?

If the circulating medium be a municipal one, as paper currency, still its value does not depend on its quantity. It depends on the credit of the state issuing it, and on the time of its redemption; and is no otherwise affected by the quantity, than as the quantity may be supposed to *endanger or postpone* the redemption.

That it depends in part on the credit of the issuer, no one will deny. If the credit of the issuer, therefore be perfectly unsuspected, the time of redemption alone will regulate its value.

To support what is here advanced, it is sufficient to appeal to the nature of paper money. It consists of bills or notes of obligation payable in specie to the bearer, either on demand or at a future day. Of the first kind is the paper currency of Britain, and hence its equivalence to specie. Of the latter kind is the paper currency of the United States, and hence its inferiority to

■ 2 Pledging on September 1, 1779, not to increase its \$160 million of outstanding bills of credit by more than 25 percent, and that only in case of a dire emergency, the Continental Congress had John Jay draft a “Circular Address” to the states (adopted September 13) exhorting them to supply enough soldiers, money, and matériel to restore public credit and advance the common cause. And yet, by March 18, 1780, the gloomy situation obliged Congress to authorize the states to issue new bills of credit and declare that the old continental issues would be redeemed at only one-fortieth of their face value (*Journals of the Continental Congress*, XV, 1052–62; XVI, 262–67). Although in the prefatory note Madison declared that he wrote his essay during the six months intervening between these two actions by Congress, he probably could have narrowed the time to the period from late in December 1779 to early in March of the next year.

In his brief third-person autobiography, written long afterward, Madison mentioned his election to Congress on December 14, 1779, and then added: “To prepare himself for this service, he employed an unavoidable detention from it in making himself acquainted with the state of Continental affairs, and particularly that of the finances which, owing to the depreciation of the paper currency, was truly deplorable. The view he was led to take of the evil, and its causes, was put on paper, now to be found in several periodical publications, particularly Freneau’s *National Gazette*.” By “unavoidable detention” he most likely referred to his necessary preparations at Montpellier for his residence in Philadelphia and the heavy snow which delayed his departure for that city until March 6, 1780, or for some days after he had planned to begin the trip. The essay was printed as the fourth in Madison’s series of seventeen politically tinged articles appearing in Freneau’s newspaper late in President Washington’s first term. Even though Madison may have revised his original manuscript before releasing it for publication, it deals with a problem which was much less acute by 1791 than when he wrote the essay nearly twelve years earlier.

■ 3 Madison accurately reflects the thought, but does not always quote the exact words, of David Hume in his *Political Discourses* (Edinburgh, 1752), pp. 82–83.

specie. But if its being redeemable not on demand but at a future day, be the cause of its inferiority, the distance of that day, and not its quantity, ought to be the measure of that inferiority.

It has been shewn that the value of specie does not fluctuate according to local fluctuations in its quantity. Great Britain, in which there is such an immensity of circulating paper, shews that the value of paper depends as little on its quantity as that of specie, when

■ 4 The portion of the essay in the issue of the *National Gazette* for December 19, 1791, ends here. The remainder is in the issue of December 22, 1791.

■ †As the depreciation of our money has been ascribed to a wrong cause, so, it may be remarked, have effects been ascribed to the depreciation, which result from other causes. Money is the instrument by which men's wants are supplied, and many who possess it will part with it for that purpose, who would not gratify themselves at the expence of their visible property. Many also may acquire it, who have no visible property. By increasing the quantity of money therefore, you both increase the means of spending, and stimulate the desire to spend; and if the objects desired do not increase in proportion, their price must rise from the influence of the greater demand for them. Should the objects in demand happen, at the same juncture, as in the United States, to become scarcer, their prices must rise in a double proportion.

It is by this influence of an augmentation of money on demand, that we ought to account for that proportional level of money, in all countries, which Mr. Hume attributes to its direct influence on prices. When an augmentation of the national coin takes place, it may be supposed either, 1. not to augment demand at all; or, 2. to augment it so gradually that a proportional increase of industry will supply the objects of it; or, 3. to augment it so rapidly that the domestic market may prove inadequate, whilst the taste for distinction natural to wealth, inspires, at the same time, a preference for foreign luxuries. The first case can seldom happen. Were it to happen, no change in prices, nor any efflux of money, would ensue; unless indeed, it should be employed or loaned abroad. The superfluous portion would be either hoarded or turned into plate. The second case can occur only where the augmentation of money advances with a very slow and equable pace; and would be attended neither with a rise of prices, nor with a superfluity of money. The third is the only case, in which the plenty of money would occasion it to overflow into other countries. The insufficiency of the home market to satisfy the demand would be supplied from such countries as might afford the articles in demand; and the money would thus be drained off, till that and the demand excited by it, should fall to a proper level, and a balance be thereby restored between exports and imports.

The principle on which Mr. Hume's theory, and that of Montesquieu's before him, is founded, is manifestly erroneous. He considers the money in every country as the representative of the whole circulating property and industry in the country; and thence concludes, that every variation in its quantity must increase or lessen the portion which represents the same portion of property and labor. The error lies in supposing, that because money serves to measure the value of all things, it represents and is equal in value to all things. The circulating property in every country, according to its market rate, far exceeds the amount of its money. At Athens oxen, at Rome sheep, were once used as a measure of the value of other things. It will hardly be supposed, they were therefore equal in value to all other things.

the paper represents specie payable on demand. Let us suppose that the circulating notes of Great Britain, instead of being payable on demand, were to be redeemed at a future day, at the end of one year for example, and that no interest was due on them. If the same assurance prevailed that at the end of the year they would be equivalent to specie, as now prevails that they are every moment equivalent, would any other effect result from such a change, except that the notes would suffer a depreciation equal to one year's interest? They would in that case represent, not the nominal sum expressed on the face of them, but the sum remaining after a deduction of one year's interest. But if when they represent the full nominal sum of specie, their circulation contributes no more to depreciate them, than the circulation of the specie itself would do; does it not follow, that if they represented a sum of specie less than the nominal inscription, their circulation ought to depreciate them no more than so much specie, if substituted, would depreciate itself? We may extend the time from one, to five, or to twenty years; but we shall find no other rule of depreciation than the loss of the intermediate interest.

What has been here supposed with respect to Great Britain has actually taken place in the United States. Being engaged in a necessary war without specie to defray the expence, or to support paper emissions for that purpose redeemable on demand, and being at the same time unable to borrow, no resource was left, but to emit bills of credit to be redeemed in future. The inferiority of these bills to specie was therefore incident to the very nature of them. If they had been exchangeable on demand for specie, they would have been equivalent to it; as they were not exchangeable on demand, they were inferior to it. The degree of their inferiority must consequently be estimated by the time of their becoming exchangeable for specie, that is the time of their redemption.

To make it still more palpable that the value of our currency does not depend on its quantity, let us put the case, that Congress had, during the first year of the war, emitted five millions of dollars to be redeemed at the end of ten years; that, during the second year of the war, they had emitted ten millions more, but with due security that the whole fifteen millions should be redeemed in five years; that, during the two succeeding years, they had augmented the emissions to one hundred millions, but from the discovery of some

extraordinary sources of wealth, had been able to engage for the redemption of the whole sum in one year: it is asked, whether the depreciation, under these circumstances, would have increased as the quantity of money increased—or whether on the contrary, the money would not have risen in value, at every accession to its quantity?<sup>4</sup>

It has indeed happened, that a progressive depreciation of our currency has accompanied its growing quantity; and to this is probably owing in a great measure the prevalence of the doctrine here opposed. When the fact however is explained, it will be found to coincide perfectly with what has been said. Every one must have taken notice that, in the emissions of Congress, no precise time has been stipulated for their redemption, nor any specific provision made for that purpose. A general promise entitling the bearer to so many dollars of metal as the paper bills express, has been the only basis of their credit. Every one therefore has been left to his own conjectures as to the time the redemption would be fulfilled; and as every addition made to the quantity in circulation, would naturally be supposed to remove to a proportionally greater distance the redemption of the whole mass, it could not happen otherwise than that every additional emission would be followed by a further depreciation.

In like manner has the effect of a distrust of public credit, the other source of depreciation, been erroneously imputed to the quantity of money. The circumstances under which our early emissions were made, could not but strongly concur, with the futurity of their redemption, to debase their value. The situation of the United States resembled that of an individual engaged in an expensive undertaking, carried on, for want of cash, with bonds and notes secured on an estate to which his title was disputed; and who had besides, a combination of enemies employing every artifice to disparage that security. A train of sinister events during the early stages of the war likewise contributed to increase the distrust of the public ability to fulfill their engagements. Before the depreciation arising from this cause was removed by the success of our arms, and our alliance with France, it had drawn so large a quantity into circulation, that the quantity itself soon after begat a distrust of the *public disposition* to fulfill their engagements; as well as new doubts, in timid minds, concerning the issue of the contest. From that period, this cause of depreciation has been incessantly operating. It has first conduced to swell the

amount of necessary emissions, and from that very amount has derived new force and efficacy to itself. Thus, a further discredit of our money has necessarily followed the augmentation of its quantity; but every one must perceive, that it has not been the effect of the quantity, considered in itself, but considered as an omen of public bankruptcy.<sup>†5</sup>

Whether the money of a country, then, be gold and silver, or paper currency, it appears that its value is not regulated by its quantity. If it be the former, its value depends on the general proportion of gold and silver, to circulating property throughout all countries having free inter communication. If the latter, it depend[s] on the credit of the state issuing it, and the time at which it is to become equal to gold and silver.

Every circumstance which has been found to accelerate the depreciation of our currency naturally resolves itself into these general principles. The spirit of monopoly hath affected it in no other way than by creating an artificial scarcity of commodities wanted for public use, the consequence of which has been an increase of their price, and of the necessary emissions. Now it is this increase of emissions which has been shewn to lengthen the supposed period of their redemption, and to foster suspicions of public credit. Monopolies destroy the natural relation between money and commodities; but it is by raising the value of the latter, not by debasing that of the former. Had our money been gold or silver, the same prevalence of monopoly would have had the same effect on prices and expenditures; but these would not have had the same effect on the value of money.

The depreciation of our money has been charged on misconduct in the purchasing departments: but this misconduct must have operated in the same manner as the spirit of monopoly. By unnecessarily raising the price of articles required for public use, it has swelled the amount of necessary emissions, on which has depended the general opinion concerning the time and the probability of their redemption.

■ 5 Madison's entire footnote is in italics in the *National Gazette*. In the last paragraph of the footnote, he refers to Book XXII of Montesquieu's *De l'esprit des lois*, first published in Geneva in 1748 and soon thereafter translated into English. Madison's daring in challenging the correctness of this redoubtable authority is noted by Paul Merrill Spurlin in his *Montesquieu in America, 1760–1801* (Baton Rouge, La., 1940), pp. 175–76.

The same remark may be applied to the deficiency of imported commodities. The deficiency of these commodities has raised the price of them; the rise of their price has increased the emissions for purchasing them; and with the increase of emissions, have increased suspicions concerning their redemption. Those who consider the quantity of money as the criterion of its value, compute the intrinsic depreciation of our currency by dividing the whole mass by the supposed necessary medium of circulation. Thus supposing the medium necessary for the United States to be 30,000,000 dollars, and the circulating emissions to be 200,000,000 the intrinsic difference between paper and specie will be nearly as 7 for 1. If its value depends on the time of its redemption, as hath been above maintained, the real difference will be found to be considerably less. Suppose the period necessary for its redemption to be 18 years, as seems to be understood by Congress; 100 dollars of paper 18 years hence will be equal in value to 100 dollars of specie; for at the end of that term, 100 dollars of specie may be demanded for them. They must consequently at this time be equal to as much specie as, with compound interest, will amount, in that number of years, to 100 dollars. If the interest of money be rated at 5 per cent. this present sum of specie will be about 41 1-2 dollars. Admit, however the use of money to be worth 6 per cent. about 35 dollars will then amount in 18 years to 100. 35 dollars of specie therefore is at this time equal to 100 of paper; that is, the man who would exchange his specie for paper at this discount, and lock it in his desk for 18 years, would get 6 per cent. for his money. The proportion of 100 to 35 is less than 3 to 1. The intrinsic depreciation of our money therefore, according to this rule of computation, is less than 3 to 1; instead of 7 to 1, according to the rule espoused in the circular address,<sup>6</sup> or 30 or 40 to 1, according to its currency in the market.

I shall conclude with observing, that if the preceding principles and reasoning be just, the plan on which our domestic loans have been obtained, must have operated in a manner directly contrary to what was intended. A loan-office certificate differs in nothing from a common bill of credit, except in its higher

denomination, and in the interest allowed on it; and the interest is allowed, merely as a compensation to the lender, for exchanging a number of small bills, which being easily transferable, are most convenient, for a single one so large as not to be transferable in ordinary transactions. As the certificates, however, do circulate in many of the more considerable transactions, it may justly be questioned, even on the supposition that the value of money depended on its quantity, whether the advantage to the public from the exchange, would justify the terms of it. But dismissing this consideration, I ask whether such loans do in any shape, lessen the public debt, and thereby render the discharge of it less suspected or less remote? Do they give any new assurance that a paper dollar will be one day equal to a silver dollar, or do they shorten the distance of that day? Far from it: The certificates constitute a part of the public debt no less than the bills of credit exchanged for them, and have an equal claim to redemption within the general period; nay, are to be paid off long before the expiration of that period, with bills of credit, which will thus return into the general mass, to be redeemed along with it. Were these bills, therefore, not to be taken out of circulation at all, by means of the certificates, not only the expence of offices for exchanging, re-exchanging, and annually paying the interest, would be avoided; but the whole sum of interest would be saved, which must make a formidable addition to the public emissions, protract the period of their redemption, and proportionally increase their depreciation. No expedient could perhaps have been devised more preposterous and unlucky. In order to relieve public credit sinking under the weight of an enormous debt, we invest new expenditures. In order to raise the value of our money, which depends on the time of its redemption, we have recourse to a measure which removes its redemption to a more distant day. Instead of paying off the capital to the public creditors, we give them an enormous interest to change the name of the bit of paper which expresses the sum due to them; and think it a piece of dexterity in finance, by *emitting loan office certificates*, to elude the necessity of *emitting bills of credit*.

# James Madison's Monetary Economics

by Bruce D. Smith

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## Introduction

James Madison's essay, "Money" (pp. 2–6 as reprinted here), considers issues that are as timely and important today as they were when it was first written. While his concern with an eighteenth-century economy and his focus on an ultimate return to a gold standard may seem to relegate his writings to the history of economic thought, he was in fact wrestling with questions that have been central to monetary theory and policymaking for more than two centuries. Even now, his way of framing these issues creates a wonderful opportunity for contrasting two fundamentally different views of monetary policy's role and function.

Madison, of course, wrote as a member of a Revolutionary government that faced profound fiscal and monetary policy problems. With no

established tax base and little ability to borrow in the conventional "capital markets" of the day, it confronted huge wartime expenditures against a great power. With no alternative but to run large deficits and monetize them, this government accomplished the astounding feat of financing 82 percent of its expenditures by printing money.<sup>1</sup>

Needless to say, this achievement had its cost. The massive printing of money was associated with a major inflation, which accelerated over time. In January 1777, \$1.25 in Continental currency could purchase \$1.00 in gold, reflecting a relatively modest depreciation of 25 percent during the first year and a half of the war. By January 1781, however, \$100 Continental was required to purchase \$1.00 in gold.

What was the source of this inflation? The conventional answer—with which Madison takes issue—is the quantity of money printed. Madison (p. 6) makes a fairly standard guess that puts the prewar money supply of the 13 colonies at \$30,000,000. During the war, the Federal government issued \$226,000,000 in Continental currency, and the states issued a similar amount.<sup>2</sup> Was this the "cause" of the

■ 1 This figure, which refers to the period 1775–79, is from Ferguson (1961, pp. 43–44). It contrasts markedly with the fraction of expenditures financed through seignorage revenue in any modern economy.

■ 2 See Ferguson (1961) and Nevins (1927, p. 481).

inflation? Or was it rather that the size of the deficit, combined with the behavior of prices, forced that much money to be printed? And could such a large inflation have been averted?

In analyzing Madison's response, it is important to note carefully how the wartime currency was created. In 1775, the money supply of the 13 colonies consisted of paper printed by the individual colonies plus gold and silver coin.<sup>3</sup> Colonial governments did not redeem their paper currencies in specie, but they did sometimes promise to do so in the future, and they typically made taxes payable in either paper money or specie, accepting paper at a fixed rate in terms of specie. When the Continental Congress began issuing its own money in 1775, it followed the same system. Continental dollars were not redeemable in gold or silver, but the Congress promised postwar redemption of paper into gold on a one-for-one basis, and it also made paper money acceptable for taxes on the same basis as coin.

Madison's writings presumed that the government would honor its promise of a one-for-one redemption. While that faith eventually proved unfounded,<sup>4</sup> one could ask a counterfactual question: What would have happened if the government's promise of redemption had been believed and had been honored?

This question is motivated by much more than pure intellectual curiosity. Over and over again, governments facing large wartime expenditures have suspended gold standards, issued then-irredeemable paper money, and promised to resume gold convertibility at some date after the war's end. Some examples of particular importance in monetary history include Britain during and after the Napoleonic Wars and World War I and the United States during and after the Civil War. In each case, there was some wartime inflation, accompanied either by contemporary or later historical debate over whether "better"—that is, less inflationary—policies were available. Invariably, there was

also a postwar deflation before resumption of a gold standard. And indeed, there were substantial deflations in many parts of the United States after the Revolution; in some places, these were just as pronounced as wartime inflation. For example, by 1786 prices in Pennsylvania had returned to their 1773 levels (Bezanson [1951, p. 174]).

The fact that the policies followed by the Revolutionary government—while no doubt necessary for it—have been so widely adopted elsewhere suggests that Madison's concerns are of interest in a far broader context than just that of the Revolutionary War. In fact, he addressed a number of issues that remain basic in monetary theory to this day, including:

- i. To what extent is inflation determined by money growth? Is this all that matters, as is often asserted, or does it matter almost not at all, as Madison argued?
- ii. If money growth is not all that matters, does the degree of inflation depend on the nature of the government's promises about "backing" its money in the future through gold redemption or some other scheme?
- iii. Madison's argument about the lack of inflation resulting from money growth is not based on any commitment to reduce the money stock at some future date. Thus, he asserts that some future redemption in gold—essentially, a commitment to future price-level stabilization or "targeting"—is adequate to prevent money growth from raising the price level today. This seems in conflict with the quantity theory of money, namely, that inflation is always and everywhere a monetary phenomenon.<sup>5</sup> What is the theoretical foundation for Madison's view?
- iv. Wisdom that was, at least until recently, "conventional" asserts that it is always less inflationary to finance a deficit by borrowing (issuing bonds) than by printing money. At the end of his essay, Madison denies the validity of this wisdom.

The first three of these issues relate to one of the oldest debates in monetary economics: Does a permanent increase in the money supply necessarily raise the price level? Most adherents of the quantity theory would argue that the answer is yes, and their viewpoint currently prevails in the formulation of monetary policy. However, a competing school of thought argues that permanent increases in the money supply need not be inflationary

■ 3 See Smith (1988) and Rolnick, Smith, and Weber (1994) for a discussion of this period.

■ 4 The ultimate redemption rate was 3 cents on the dollar (Ferguson [1961]).

■ 5 For discussions of the quantity theory of money, see Friedman (1956) and Lucas (1980). Also, while I will follow Madison in using the term "money," note that his arguments apply equally to any expansion in the stock of government liabilities. Viewed from this perspective, Madison's assertions are less obviously in conflict with the quantity theory than they may first appear to be.



if accompanied by appropriate “backing” of the newly created money. The modern intellectual foundations of this idea, which has ancient antecedents,<sup>6</sup> appear in Tobin (1963), Wallace (1981), Sargent (1982), Sargent and Wallace (1982), and Sargent and Smith (1987a,b). Most of the backing discussed in this literature assumes that increases in the money supply are accompanied by government asset acquisitions of equal value; hence, this reasoning can be applied in only a limited way even to temporary deficit monetization. Madison argues that the government can print money to finance temporary deficits, backed by a promise of future redemption (but not retirement), and that the only resulting inflation will be due to the delay in redemption. Moreover, he strongly denies that the behavior of the price level should depend in any way on the quantity of money printed, unless this delays the time to redemption.<sup>7</sup> He also asserts that delays in future redemption put upward pressure on the current price level.

The remainder of this article constitutes a modern theoretical attempt to formalize and evaluate Madison’s views. To a large extent, my conclusions are favorable to his line of argument. The commitment to a future redemption, which is little more than a pledge to stabilize prices in the future, is enough to break the link between money growth and inflation. In addition, his assertion that delayed redemption puts upward pressure on the price level is also strongly supported. However, the analysis does not suggest that price-level behavior is fully independent of changes in the money stock. In this respect, Madison seems to have gone too far.

To be fair to him, a formal theoretical analysis should capture the main economic aspects of his reasoning. I take these to be as follows:

- i. Money is held primarily as an asset. If it constitutes a future claim to specie, its current value should be just the discounted present value of that claim.

- ii. As a corollary, if other assets earn a higher return than money, this happens only because they involve some “inconvenience,” such as being issued in excessively large denominations.
- iii. An alternative asset to money exists. The real rate of interest on it is unaffected by government policy (although it need not always be held in positive quantities).

In the subsequent section, I build a model incorporating these features and apply it to some of the issues that concerned Madison. In doing so, I gloss over some other issues that he took up, but the concluding section offers comments on them as well.

## I. The Environment

To illustrate Madison’s points, I consider a two-period-lived, overlapping-generations model of a particularly simple variety.<sup>8</sup> At each date  $t = 0, 1, \dots$ , a set of  $N$  identical agents is born. They are endowed in both periods with some of a single perishable consumption good; let  $w_j$  ( $j = 1, 2$ ) denote the age  $j$  endowment of a representative agent. I assume throughout that  $w_1 > 0$  and  $w_2 > 0$ .

In addition, agents have access to a *reversible* linear technology which allows one unit of current consumption to be converted into  $\phi > 0$  ounces of gold (and back again, if desired). One possible interpretation of the technology is that this is a small open economy operating in a world on a gold standard, so that the consumption good can always be bought or sold abroad at a fixed rate for some amount of gold. Such an interpretation would obviously be fairly appropriate to the economy in which Madison lived.

Once obtained, gold can either be stored in raw form or—if the domestic economy is on a gold standard—it can be coined. In either case, it depreciates at the rate  $\delta \in (0,1)$ .

Each young agent values age  $j$  consumption, denoted  $c_j$ , according to the (common) utility function  $\ln c_1 + \beta \ln c_2$ . Note that gold is then not held by agents for its consumption value; it is held—if at all—only as an asset.

It will be necessary in what follows to allow for the possibility that agents born at certain dates face a government-levied tax. Any direct taxes that the government does levy are lump-sum in nature. I also assume that agents pay these taxes (if at all) only when old.

The following notation will prove useful: Suppose an agent, born at date  $t$ , pays a lump-

■ 6 See Mints (1945) for a discussion of this idea, as well as for criticisms.

■ 7 Note that one could also cast this as an increase in the stock of outstanding government debt without affecting the market value of the debt, unless it delayed the time to redemption.

■ 8 The model closely resembles that of Sargent and Wallace (1983).

sum tax of  $\tau_{t+1}$  when old and faces a gross rate of return on a single asset of  $r_{t+1}$  between  $t$  and  $t+1$ . Then, let  $s(w_1, w_2 - \tau_{t+1}, r_{t+1})$  denote the savings function of this agent. Given the assumed form of the utility function, clearly

$$(1) \quad s(w_1, w_2 - \tau_{t+1}, r_{t+1}) = \frac{\beta w_1}{1 + \beta} - \frac{w_2 - \tau_{t+1}}{(1 + \beta)r_{t+1}}.$$

I will assume that agents are willing to save, even if they must do so by storing gold in raw form; I therefore impose an assumption:<sup>9</sup>

Assumption 1.  $s(w_1, w_2, 1 - \delta) > 0$ .

## II. The Government

The government that Madison contemplated as he wrote had large wartime spending needs and very limited powers of taxation. The result was a massive government budget deficit that was financed by printing paper money.

In Madison's vision—which essentially eventuated in practice—the war would be followed by a period of peace accompanied by a relatively balanced government budget, or perhaps even one in surplus. During this time, wartime paper money would continue to circulate.<sup>10</sup> After a transitional period, the paper money would be retired, having been converted into gold at some specified rate. Thereafter, the economy would remain on a gold standard, and Madison (as well as others) probably envisioned a purely metallic currency from that point onward.

In consonance with this scenario, I consider a government confronting the following circumstances. For dates  $t = 0, 1, \dots, T_1$ , the government has a real per capita expenditure (and deficit) level of  $g > 0$ .<sup>11</sup> It finances expenditures solely by printing paper currency that is not then redeemable, but that it promises to convert into gold at some future date.

■ 9 Assumption 1 is equivalent to  $\beta w_1(1 - \delta) > w_2$ .

■ 10 Of course, this did not happen after the Revolution, as the phrase "not worth a Continental" indicates. However, a good deal of new paper money was created, by both state and federal governments, from 1783 to 1789. For an overview of this period, see Rolnick, Smith, and Weber (1994).

■ 11 As noted above, the Revolutionary government financed about 82 percent of its expenditures by printing money. Thus, the implied abstraction from tax revenue is well founded.

During this period, the government confronts the budget constraint

$$(2) \quad g = (M_t - M_{t-1})/p_t; \quad t = 0, \dots, T_1$$

where  $M_t$  is the stock of paper currency outstanding at  $t$ , and  $p_t$  is the time  $t$  price level. The initial money stock  $M_{-1}$  is given as an initial condition. For simplicity—and consistency with the realities of a Revolutionary government—I set  $M_{-1} = 0$ .

The government has no direct expenditures for dates  $t > T_1$ . I assume that for  $t = T_1 + 1, \dots, T - 1$ , it engages in no activity whatsoever, and neither adds to nor subtracts from the existing stock of money. A gold standard has not yet been established, and the money is not yet redeemable. Thus, for  $t = T_1, \dots, T - 1$ ,  $M_t = M_{T_1}$  holds. This corresponds to the transitional period prior to establishment of a full gold standard.

At date  $T$ , the government "calls in" the existing stock of paper currency and replaces it dollar for dollar with gold coins which it mints—at its own expense—at that date. Thereafter, it stands ready to coin freely any gold brought to the mint by private agents. The government coins gold dollars and establishes a mint ratio  $b$  stating the number of ounces of gold in a newly minted gold dollar. Any subsequent change in the money supply is purely the result of minting and melting activity by the private sector. There are no policy-induced changes in the money supply, nor is there any further government expenditure. I also assume that there is no uncertainty or lack of commitment, so that the transitional dates  $T_1$  and  $T$  are known in advance by everyone.

At  $T$ , the government must mint enough new coins to redeem the existing money stock and must raise some resources for this purpose. Let  $n_T^g$  be the number of new gold coins, in dollars, created by the government at  $T$ . Clearly  $n_T^g = M_{T_1}$  must hold. Moreover, to mint  $n_T^g$  new gold dollars, the government requires  $n_T^g(b/\phi)$  units of the consumption good, which it obtains by levying a lump-sum tax on old agents at  $T$ . Since there is no other taxation at any date, under the policy described,

$$(3) \quad \tau_T = n_T^g(b/\phi) = M_{T_1}(b/\phi)$$

$$\tau_t = 0; \quad t \neq T.$$

After date  $T - 1$ , the entire stock of money consists of gold dollars. I assume that gold

coins circulate by weight,<sup>12</sup> and I let  $G_t$  denote the stock of gold dollars—by weight—at  $t$ . As before,  $p_t$  continues to denote the time  $t$  price level.

### III. The Behavior of Agents

In this section, I describe the behavior of agents before, during, and after the implementation of a gold standard.

#### The Paper Money Regime ( $t < T - 1$ )

For all  $t < T$ , paper currency is in circulation (although the promise of ultimate redemption is understood and believed). I also focus on the situation where paper money is accepted voluntarily in exchange for private assets.<sup>13</sup> Since agents can choose between holding money and holding raw gold as an asset,<sup>14</sup> clearly money will be held only if it earns a real return as great as that on raw gold.<sup>15</sup> The gross real return on paper currency between  $t$  and  $t + 1$  is given by  $p_t/p_{t+1}$ , and the gross real return on storage of raw gold is  $1 - \delta$ . Thus,

$$(4) \quad p_t/p_{t+1} \geq 1 - \delta; \quad t = 0, \dots, T-1$$

must hold.

Let  $g_t$  denote the storage of raw gold by a young agent at  $t$ , and let  $m_t$  denote the accumulation of real balances. Then  $g_t$ ,  $m_t$ , and a consumption profile  $(c_{1t}, c_{2t})$  are chosen to maximize  $\ln c_{1t} + \beta \ln c_{2t}$ , subject to

$$(5) \quad c_{1t} + m_t + (g_t/\phi) \leq w_1$$

and

■ 12 I could assume equally well that coins circulate according to their face value ("by tale"). Each outcome is an equilibrium, and it makes no qualitative difference to the results which situation obtains. For a discussion of coins that circulate by tale, see Sargent and Smith (1997).

■ 13 In practice, during the Revolution, the army often seized what it needed, offering a choice of paper liabilities or nothing in exchange. Clearly here, when paper money was taken, it was not taken voluntarily. Madison obviously conceived of a situation where agents take money for goods of their own volition.

■ 14 And since there is no uncertainty.

■ 15 Parenthetically, Madison's argument implies that he regarded money as an asset, earning a return competitive with that on relatively close substitutes.

$$(6) \quad c_{2t} \leq w_2 + m_t(p_t/p_{t+1}) \\ + (1 - \delta)(g_t/\phi).$$

The solution to this problem sets

$$(7) \quad m_t + g_t = s(w_1, w_2, p_t/p_{t+1})$$

and

$$(8) \quad [(p_t/p_{t+1}) - (1 - \delta)]g_t = 0.$$

Equation (8) asserts that, if the return on money exceeds that on the storage of raw gold, no raw gold will be stored.

#### The Transition ( $t = T - 1$ )

Young agents born at  $T - 1$  will live through the transition to a gold standard; when old, they will bear the costs of this transition. They thus bear the lump-sum tax,  $\tau_T$ , when old.

In addition, when old, these agents will have the opportunity to coin or melt gold. Let  $n_t^j$  be the coinage (or melting, if negative) of gold by a representative agent of age  $j$  ( $j = 1, 2$ ) in period  $t$ . Clearly this coinage can be nonzero only for  $t \geq T$ . Young agents born at  $T - 1$  choose a level of real balances,  $m_{T-1}$ , a quantity of raw gold storage,  $g_{T-1}$ , a consumption profile  $(c_{1T-1}, c_{2T-1})$ , and a minting/melting strategy when old  $n_T^2$ , to maximize  $\ln c_{1T-1} + \beta \ln c_{2T-1}$ , subject to

$$(9) \quad c_{1T-1} + m_{T-1} + (g_{T-1}/\phi) \leq w_1$$

and

$$(10) \quad c_{2T-1} \leq w_2 - \tau_T + m_{T-1}(p_{T-1}/p_T) \\ + (1 - \delta)(g_{T-1}/\phi) + n_T^2 [(1/p_T) - (b/\phi)],$$

where the last term in (10) represents the profit from using  $(b/\phi)$  units of resources to obtain  $n_T^2$  gold dollars, which then have a purchasing power of  $n_T^2/p_T$ .

An absence of arbitrage opportunities requires that

$$(11) \quad p_T = \phi/b.$$

When (11) holds, as it must in equilibrium, the total savings of a young agent at  $T - 1$  must satisfy

$$(12) \quad m_{T-1} + g_{T-1} = s(w_1, w_2, p_{T-1}/p_T).$$

In addition,  $g_{T-1} = 0$  holds if  $p_{T-1}/p_T > 1 - \delta$ .

### A Gold Standard ( $t \geq T$ )

For  $t \geq T$ , the economy is on a gold standard. No further taxes are levied, and all agents, old and young, have the opportunity to mint and melt coins at all dates. As before, agents can select a level of real balances,  $m_t$  (now held in the form of gold coins), a quantity of raw gold to store,  $g_t$ , a consumption profile,  $(c_{1t}, c_{2t})$ , and a minting/melting strategy,  $(n_t^1, n_t^2)$ , to maximize  $\ln c_{1t} + \beta \ln c_{2t}$ , subject to

$$(13) \quad c_{1t} + m_t + (g_t/\phi) \leq w_1 + n_t^1[(1/p_t) - (b/\phi)]$$

and

$$(14) \quad c_{2t} \leq w_2 + m_t(1 - \delta)(p_t/p_{t+1}) + (g_t/\phi)(1 - \delta) + n_{t+1}^2[(1/p_{t+1}) - (b/\phi)].$$

The real balance term in (14) must now be multiplied by  $1 - \delta$ , since gold coins circulate by weight and depreciate at the rate  $\delta$ .

As before, an absence of arbitrage opportunities associated with minting and melting requires that

$$(15) \quad p_t = \phi/b; \quad t \geq T.$$

In addition, the price stability revealed in (15) implies that there is never any reason for agents to store raw gold rather than hold gold coins. Hence, without loss of generality, we can take  $g_t = 0$ ;  $t \geq T$ . Then, agents save entirely in the form of gold, which earns a gross real return of  $(1 - \delta)(p_t/p_{t+1}) = (1 - \delta)$ , where the equality follows from (15). Real balances per capita are then given by

$$(16) \quad m_t = s(w_1, w_2, 1 - \delta); \quad t \leq T.$$

## IV. A General Equilibrium

For  $t \geq T$ , it is clear what must happen in equilibrium. Equation (15) gives the price level. The nominal per capita gold stock at  $G_t$ , must then obey

$$(17) \quad G_t = p_t s(w_1, w_2, 1 - \delta) = (\phi/b) s(w_1, w_2, 1 - \delta); \quad t \geq T.$$

Since the nominal gold stock (in ounces) is constant, private minting/melting in each period must just replace the depreciated gold stock:

$$(18) \quad (n_t^1 + n_t^2)/2 = \delta G_{t-1}; \quad t \geq T + 1.$$

In periods before the advent of the gold standard, there is a much richer variety of possible equilibrium outcomes.<sup>16</sup> Here I construct an equilibrium having certain features and then display the restrictions on parameters required for those features to emerge. The equilibrium features I consider are chosen for two reasons. First, they seem illustrative of what Madison had in mind. Second, for much of history, economies have abandoned gold standards in time of war and financed their deficits by printing paper money. With the cessation of hostilities, the government budget is roughly balanced (or even in surplus), although gold convertibility is not immediately resumed. A postwar deflation occurs during this period, terminating with the resumption of gold convertibility. Indeed, such a pattern was observed through much of the United States following both the Revolutionary War and the Civil War, and in the United Kingdom after World War I. For these reasons, I focus on equilibria which display inflation for  $t = 0, 1, \dots, T_1$ , followed by a deflation for  $t = T_1 + 1, \dots, T$ . This deflation ends with conversion to a gold standard.

### The Deflation ( $t = T_1 + 1, \dots, T$ )

In this section, I state conditions under which there is an equilibrium satisfying

$$(19) \quad p_t \geq p_{t+1}; \quad t = T_1, \dots, T - 1.$$

Note that (19) implies that no agent will wish to store raw gold during the period in question.

At date  $T - 1$ , young agents understand that they will be required to pay for the transition to a gold standard. In addition, since they store no gold when young, the time  $T - 1$  equilibrium condition in the money market is that

$$(20) \quad M_{T-1}/p_{T-1} = M_{T_1}/p_{T-1} = s(w_1, w_2 - \tau_T, p_{T-1}/p_T) = \beta w_1/(1 + \beta) - (w_2 - \tau_T)/(1 + \beta)(p_{T-1}/p_T).$$

■ 16 This is not to say that any given economy has multiple possible equilibrium outcomes. Rather, different economies may have equilibria that look quite different from one another.

Substituting (3) and (11) into (20), and solving for  $M_{T_1}/p_{T-1}$  yields the equilibrium level of real balances at  $T-1$ :

$$(21) \quad M_{T_1}/p_{T-1} = w_1 - w_2(\phi/b)/\beta p_{T-1}.$$

Equation (21) implies that the time  $T-1$  price level is given by

$$(22) \quad p_{T-1} = (M_{T_1}/w_1) + (\phi/b)(w_2/\beta w_1).$$

Notice that  $p_{T-1} \geq p_T = (\phi/b)$  holds iff

$$(23) \quad M_{T_1} \geq (\phi/b)[w_1 - (w_2/\beta)] \\ = (\phi/b) \left\{ \frac{1+\beta}{\beta} \right\} s(w_1, w_2, 1)$$

is satisfied. Below I derive restrictions on parameters implying that (23) holds.

For  $t = T_1 + 1, \dots, T-2$ , young agents will experience no regime transitions and will bear no taxation. In addition, (19) implies that they will store no gold. Hence the equilibrium level of real balances at these dates is given by

$$(24) \quad M_t/p_t = M_{T_1}/p_t = s(w_1, w_2, p_t/p_{t+1}) \\ = \beta w_1/(1+\beta) - w_2 \\ + (1+\beta)(p_t/p_{t+1}).$$

Equation (24) can be solved for  $p_t$  in terms of  $p_{t+1}$ ; the implied solution is

$$(25) \quad p_t = M_{T_1}[(1+\beta)/\beta w_1] + (w_2/\beta w_1)p_{t+1}; \\ t = T_1 + 1, \dots, T-2.$$

Equations (22) and (25) describe the evolution of the price level during the transitional period prior to the establishment of a gold standard. Equation (25) implies that  $p_t \geq p_{t+1}$  is satisfied iff

$$(26) \quad M_{T_1}/p_{t+1} \geq \beta w_1/(1+\beta) - w_2/(1+\beta) \\ = s(w_1, w_2, 1)$$

holds. Thus, by induction, if

$$(27) \quad M_{T_1}/p_{T_1+1} \geq s(w_1, w_2, 1)$$

obtains, so does equation (19). Thus, (27) is sufficient for a sustained postwar deflation to be observed.

The following proposition fully describes the behavior of the price level during this deflation, given the inherited money supply  $M_{T_1}$ :

**PROPOSITION 1.** For  $t = T_1 + 1, \dots, T-1$ , the price level satisfies

$$(28) \quad p_t = (\phi/b)(w_2/\beta w_1)^{T-t} \\ + M_{T_1}\{w_1^{-1}(w_2/\beta w_1)^{T-(t+1)} \\ + [(1+\beta)/\beta w_1][1 - (w_2/\beta w_1)^{T-(t+1)}] \\ \div [1 - (w_2/\beta w_1)]\}.$$

Proposition 1 is easily verified by a comparison of equations (22), (25), and (28).

### The Inflation ( $t \leq T_1$ )

It remains to describe the evolution of the money supply and the price level during the wartime period of positive government expenditure, which was obviously the issue that concerned Madison. In addition, he believed that there was an alternative asset to money that was relevant during this period, that money and this other asset were closely substitutable, and that government policy could not influence the rate of return on the alternative asset. Motivated by Madison's thinking, I proceed as follows in this section: If agents store raw gold, then gold competes with money in agents' portfolios. Moreover, the return on gold,  $1 - \delta$ , is exogenously given. Thus, if agents store gold at any date  $t \leq T_1$ , this serves the role of Madison's alternative asset.

Of course, gold and paper money can both be held voluntarily at  $t$  iff

$$(29) \quad p_t/p_{t+1} = 1 - \delta.$$

I now construct an equilibrium where (29) holds for all  $t = 0, 1, \dots, T_1 - 1$ . In addition, raw gold is (at least potentially) stored at these dates. I also impose

$$(30) \quad p_{T_1} > (1 - \delta)p_{T_1+1}.$$

Equation (30) is consistent with inflation occurring between  $T_1$  and  $T_1 + 1$ , but raw gold is not stored between these periods. Allowing (29) to be violated at  $t = T_1$  eases the construction of the desired equilibrium.

Equation (29) implies that

$$(31) \quad p_t = (1 - \delta)^{T_1-t} p_{T_1}; \quad t = 0, \dots, T_1$$

In addition, the government budget constraint (2) requires the money supply to evolve according to

$$(32) \quad M_t = M_{t-1} + gp_t \\ = M_{t-1} + gp_{T_1}(1 - \delta)^{T_1-t}; \quad t \leq T_1,$$

with  $M_{-1} = 0$  given as an initial condition. The following proposition then describes the evolution of the real and nominal money supplies:

**PROPOSITION 2.**

a) For  $t = 0, 1, \dots, T_1$ , the nominal money supply satisfies

$$(33.a) \quad M_t = M_0 + (g/\delta)(1 - \delta)^{T_1-t} \\ \times [1 - (1 - \delta)^t]p_{T_1}$$

with

$$(33.b) \quad M_0 = gp_0 = g(1 - \delta)^{T_1}p_{T_1}.$$

b) For  $t = 0, 1, \dots, T_1$ , the real money supply satisfies

$$(34) \quad M_t/p_t = (g/\delta)[1 - (1 - \delta)^{t+1}].$$

Part a of the proposition can be verified directly by substituting (33.a) into (32). Part b is immediate from (31) and (33.b).

Of course, the construction of equilibrium just undertaken is predicated on gold being stored at all dates prior to  $T_1$  and on (30). Raw gold is stored for  $t \leq T_1 - 1$  if

$$(35) \quad M_t/p_t = (g/\delta)[1 - (1 - \delta)^{t+1}] \\ < s(w_1, w_2, 1 - \delta)$$

is satisfied for all such dates. Clearly this condition is equivalent to

$$(35') \quad M_{T_1-1}/p_{T_1-1} = (g/\delta)[1 - (1 - \delta)^{T_1}] \\ < s(w_1, w_2, 1 - \delta).$$

Also, in order for (30) to hold,

$$(36) \quad M_{T_1}/p_{T_1} = (g/\delta)[1 - (1 - \delta)^{T_1+1}] \\ > s(w_1, w_2, 1 - \delta)$$

must obtain.

It remains to determine the price level and money stock at time  $T_1$ . Since there is no raw gold storage at  $T_1$ , money market clearing

requires that

$$(37) \quad M_{T_1}/p_{T_1} = s(w_1, w_2, p_{T_1}/p_{T_1+1}) \\ = \beta w_1/(1 + \beta) \\ - w_2/(1 + \beta)(p_{T_1}/p_{T_1+1}).$$

Solving (37) for  $p_{T_1}$  yields

$$(38) \quad p_{T_1} = M_{T_1}[(1 + \beta)/\beta w_1] \\ + (w_2/\beta w_1)p_{T_1+1}.$$

It is then immediate from (38), (25), and proposition 1 that

$$(39) \quad p_{T_1} = (\phi/b)(w_2/\beta w_1)^{T-T_1} \\ + M_{T_1}\{w_1^{-1}(w_2/\beta w_1)^{T-(T_1+1)} \\ + [(1 + \beta)/\beta w_1] \\ \times [1 - (w_2/\beta w_1)^{T-(T_1+1)}] \\ \div [1 - (w_2/\beta w_1)].$$

Equation (36) can be rewritten as

$$(40) \quad M_{T_1} = (g/\delta)[1 - (1 - \delta)^{T_1+1}]p_{T_1}.$$

Equations (39) and (40) then determine  $M_{T_1}$  and  $p_{T_1}$ . Once those values have been obtained, all other equilibrium price levels can be deduced from (28) and (31).

It will now be useful to introduce some notation. Define  $x$  by the relation

$$(41) \quad s(w_1, w_2, x) \equiv (g/\delta)[1 - (1 - \delta)^{T_1+1}].$$

A comparison of (36), (37), and (41) will indicate that  $x = p_{T_1}/p_{T_1+1}$ , and  $x$  is clearly an exogenous variable. Condition (30) requires that  $x > 1 - \delta$  hold. In addition, define  $\psi_1$  and  $\psi_2$  by

$$(42) \quad \psi_1 \equiv s(w_1, w_2, x)(w_2/\beta w_1)^{T-T_1}$$

and

$$(43) \quad \psi_2 \equiv (\beta/w_2)\psi_1\{1 + [(1 + \beta)/\beta] \\ \times [(\beta w_1/w_2)^{T-(T_1+1)} - 1] \\ \div [1 - (w_2/\beta w_1)]\}.$$

The following result is then immediate:

**PROPOSITION 3.** *Suppose that*

$$(44) \quad \psi_1 \geq (1 - \psi_2)(1 + \beta)s(w_1, w_2, 1)/\beta > 0$$

and

$$(45) \quad g(1 - \delta)^{T_1 - 1} \geq w_2[x - (1 - \delta)]/(1 + \beta)x \\ > \delta\beta w_1/(1 + \beta).$$

Then, an equilibrium satisfying (19), (29), and (30) exists. This equilibrium has

$$(46) \quad M_{T_1} = (\phi/b)\psi_1/(1 - \psi_2)$$

and

$$(47) \quad p_{T_1} = (\phi/b)(w_2/\beta w_1)^{T - T_1}/(1 - \psi_2).$$

The proof of proposition 3 appears in appendix A. The first inequality in (44) implies that (23) is satisfied and hence that  $p_{T-1} \geq p_T$  holds. The second inequality in (44) is required for  $M_{T_1} > 0$  and  $p_{T_1} > 0$  to hold. Finally, (45) implies that (35), (36),  $x > 1 - \delta$ , and (27) are satisfied. Satisfaction of (27), of course, implies that  $p_t \geq p_{t+1}$  holds for all  $t = T_1 + 1, \dots, T - 1$ .

It remains to describe conditions under which the inequalities in (45) are satisfied. These conditions are stated in the following:

**PROPOSITION 4.**

a) *The relations in (44) hold iff*

$$(48) \quad [(w_1 + w_2)/w_1](w_2/\beta w_1)^{T - (T_1 + 1)} \\ > w_2(x - 1)/xs(w_1, w_2, x) \\ \geq (w_2/\beta w_1)^{T - (T_1 + 1)}$$

is satisfied.

b) *Suppose that  $w_1, w_2, \beta, T_1$  and  $T$  satisfy  $\beta w_1 > w_2, T \geq T_1 + 2$ , and*

$$(49) \quad 1/(1 + T_1) > w_2/\beta w_1.$$

Then, there exists a nonempty interval,  $[\underline{x}, \bar{x}]$ , with  $\underline{x} > 1$ , such that (48) holds iff  $x \in [\underline{x}, \bar{x}]$ . In addition, for all  $x \in [\underline{x}, \bar{x}]$ , (1.a.) and (45) hold if  $\delta$  is sufficiently close to zero.

Proposition 4, which is proved in appendix B, asserts that parameter values can always be chosen so that the construction of equilibrium performed here is valid. In the next section, I examine some properties of this equilibrium.

## V. Madison's Assertions

It is now possible to use the construction of sections III and IV to investigate the validity of some of Madison's main assertions, which I take to be as follows:

- i. For a government following the kind of policy outlined in section II, the simple quantity theory of money fails, even before the transition to a gold standard.
- ii. That is, the rate of inflation and the growth rate of the money stock are not the same, and it is easy for the money growth rate to substantially exceed the inflation rate.
- iii. More strongly, the behavior of the price level is independent of the quantity of money printed, so long as there is no uncertainty about the date of transition to a gold standard.
- iv. While this may reflect my own reading, Madison seems to suggest that the behavior of the price level does not depend on the size of the government deficit, so long as there is no uncertainty about  $T$ . Anything that delays the transition to a gold standard acts to raise prices, at least up to date  $T_1$ .

I now investigate each of these propositions.

It is the case here that the rate of growth of the money stock does exceed the rate of inflation in all periods prior to  $T$ . Indeed, in some periods the difference can be quite substantial. I now state the following result:

**PROPOSITION 5.** *For all  $t < T$ , it is true that*

$$M_{t+1}/M_t > p_{t+1}/p_t \text{ holds.} \\ \text{Indeed,}$$

a) *for  $t \leq T_1 - 1$ ,*

$$(50) \quad p_{t+1}/p_t = (M_{t+1}/M_t)[1 - (1 - \delta)^{t+1}] \\ \div [1 - (1 - \delta)^{t+2}] < (M_{t+1}/M_t).$$

b) *For  $t = T_1, \dots, T - 1$ ,*

$$(51) \quad p_{t+1}/p_t \leq (1/x)(M_{t+1}/M_t) \\ < M_{t+1}/M_t.$$

The proof of proposition 5 appears in appendix C. For small values of  $\delta$ ,  $[1 - (1 - \delta)^{t+1}] \div [1 - (1 - \delta)^{t+2}]$  is approximately equal to  $(t+1)/(t+2)$ . Thus, early in the period of deficit finance, the price level can rise far more slowly than the money supply. In addition, since the money supply is constant for  $t = T_1, \dots, T-1$ , and since deflation is under way during this time, the money supply grows faster than the price level here as well. Indeed, since  $x$  can be fairly large, equation (51) implies that the difference between the rate of inflation and the rate of money creation can again be quite great. Thus, Madison's first assertion is borne out.

As is apparent from proposition 1 and equation (31), however, the model supports Madison's second assertion less well. The price level at all dates can be viewed as depending—and, moreover, depending proportionally—on  $M_{T_1}$ . Since  $M_{-1} = 0$ ,  $M_{T_1}$  is the total quantity of paper money printed during the period of deficit finance. The entire time path of prices, up to date  $T$ , depends on  $M_{T_1}$ , although the rate of inflation does not.

Similarly, the analysis suggests that the total size of the deficit financed through date  $T_1$  affects the *price level* at all dates and the rate of inflation/deflation at all dates  $t = T_1, \dots, T-1$ . However, it does *not* affect the rate of inflation for  $t < T$ , which is simply  $1/(1 - \delta)$ . To see the first point, notice from equations (41), (42), and (43) that a higher government budget deficit raises  $\psi_2$  and hence—by equation (47)—raises  $p_{T_1}$ . From equation (41), an increase in  $g$  also raises  $x$ ; this clearly raises  $p_{T_1}/p_{T_1+1}$ ; indeed, it raises  $p_t/p_{t+1}$  for all  $t = T_1, \dots, T-1$ . Thus, a larger deficit raises the price level for all dates up to and including  $T_1$ ; but a larger deflation also ensues when deficit spending ceases.

It remains to investigate Madison's last assertion, namely, that a delay in the transition to a gold standard implies a higher price level for all  $t \leq T_1$ . This is, in fact, accurate, as the next proposition asserts. Its proof is given in appendix D.

**PROPOSITION 6.** *Consider two economies that are identical in all respects except their dates of transition*

*to a gold standard. Let  $T(T)$  denote the transition date in the first (second) economy, and let  $\tilde{p}_t(p_t)$  be the date  $t$  price level in the first (second) economy. Suppose that  $\tilde{T} > T$  and  $x \geq \tilde{x}$  hold. Then,  $\tilde{p}_t > p_t$  for all  $t \leq T_1$ .*

Thus, other things equal, a more rapid movement to a gold standard implies less upward pressure on the price level, exactly as Madison argued. It is also easy to show that it implies less money will be printed.

## VI. Conclusion

Madison's essay, "Money," challenges the belief in a necessary connection between money growth and inflation that underlies much of the quantity theory of money. He obviously considered the circumstances of a government that was engaged in monetizing a temporary budget deficit, issuing inconvertible paper money, and promising to establish a gold standard and redeem its paper currency at some future time. If honored, as this (and Madison's) analysis assumes, such a promise would constitute a type of future "backing" of money issues that he thought would limit inflation and break the connection between inflation and the rate of money growth. Moreover, his concerns have universal application; many other governments at other times have confronted similar circumstances and conducted similar policies.

The model constructed here can—under circumstances that have been described—give rise to equilibria that mimic general observations about what occurs when governments follow these kinds of policies. There is inflation during wars, but deflation begins when the government's wartime spending ceases. This deflation permits resumption to begin as scheduled, even if the government does nothing to contract the money supply. The latter point is of some interest: Friedman and Schwartz (1963), for example, argue that it was a purely "accidental" consequence of the postwar deflation that the United States was able to resume gold convertibility after the Civil War and that very little active policy was conducted to restore it. The analysis here, however, suggests that resumption of convertibility was no accident.



I have argued that Madison's views have much theoretical validity. Indeed, the kind of policy he describes allows the inflation rate to be very different from the rate of money growth, and the time to redemption has potentially great importance in determining the behavior of the price level. However, the link between the behavior of the money supply and the behavior of the price level is not completely broken, as he asserts it should be.

But why isn't this link broken? Tobin (1963), Wallace (1981), and Sargent and Smith (1987a,b) describe circumstances under which appropriately conducted increases in the money supply—that is, increases which are appropriately “backed”—have no price level consequences. Madison's policy backs current money creation with a promise of future gold redemption; here, this promise requires that the government run future surpluses to raise the resources required for redemption. Why don't these resources constitute the backing required by Tobin, Wallace, and Sargent and Smith? The answer is that Madison's scheme assigns the redemption cost to a specific generation; that is, it redistributes resources among generations. This prevents the kind of policy he discusses from being irrelevant to price-level behavior.

Madison's analysis nonetheless raises a host of fascinating issues which remain unaddressed here. For example, is there an “optimal” speed of transition to a gold standard? Mitchell (1897) maintained that the United States took too long to resume gold convertibility after the Civil War; Keynes argued that Britain resumed too quickly after World War I, causing an excessively large postwar deflation. An analysis of the “correct” length of time to redemption would definitely be interesting in light of these discussions.

Madison also challenged the common notion that borrowing to finance a deficit is less inflationary than monetizing the same deficit. His particular concern was that the implied interest payments on the government debt simply add to the government's financial burden, exacerbating inflation. The same concern is reflected in Sargent and Wallace's (1981) work on “unpleasant monetarist arithmetic,” which describes conditions under which Madison's reservations are well founded. Indeed, the Sargent–Wallace conditions can be weakened substantially, as shown by Bhattacharya, Guzman, and Smith (1995).

However, Madison's analysis of money versus bond financing of a government

budget deficit raises an even subtler issue. His final paragraph discusses government bonds that bear interest only because their large denominations make them costly to use in many transactions. Adding this feature to the others implicit in his description would yield a model similar to that of Bryant and Wallace (1979), in which bond finance is always more inflationary than money finance because it increases the costs of trade. However, Bryant and Wallace did not consider a government confronting some of the other conditions that concerned Madison. An integration of these considerations would also be extremely interesting.

## Appendix A

*Proof of Proposition 3.* Equations (46) and (47) are immediate from the equilibrium conditions (39) and (40), and from the definitions of  $\psi_1$  and  $\psi_2$ . It then remains to verify that the solution sequence  $\{p_t\}$  implied by (46), (47), (38), (31), and (28) satisfies the maintained hypotheses of the construction. These hypotheses are that equations (19), (29), and (30) are satisfied, as are (35) and (36). Equation (29) is clearly satisfied by construction for  $t \leq T_1 - 1$ .

The first equality in (44) implies that (23) is satisfied; as noted in the text, satisfaction of (23) is equivalent to  $p_{T-1} \geq p_T \cdot p_{T_1} > (1 - \delta)p_{T_1+1}$  is equivalent to  $x > 1 - \delta$ . If (27) is satisfied,  $p_t \geq p_{t+1}$  holds for  $t = T_1 + 1, \dots, T - 1$ . In view of (30), a sufficient condition for (27) is that

$$\begin{aligned} \text{(A.1)} \quad & (1 - \delta)M_{T_1}/p_{T_1} = (1 - \delta)(g/\delta) \\ & \times [1 - (1 - \delta)^{T_1 - 1}] \\ & = (1 - \delta)s(w_1, w_2, x) \\ & \geq s(w_1, w_2, 1) \end{aligned}$$

be satisfied. (A.1) is easily shown to be equivalent to the second inequality in (45). This inequality also implies that  $x > 1 - \delta$ . Thus, the first inequality in (44) and the second inequality in (45) imply that (19) and (30) are satisfied.

As already noted, (36) holds iff  $x > 1 - \delta$ , which is implied by the second inequality in (44). Moreover, by the definition of  $x$ , (35') is equivalent to

$$\begin{aligned}
\text{(A.2)} \quad & (g/\delta)[1 - (1 - \delta)^{T_1+1} - 1 + (1 - \delta)^{T_1}] \\
& = g(1 - \delta)^{T_1} \geq s(w_1, w_2, x) \\
& - s(w_1, w_2, 1 - \delta) \\
& = w_2[x - (1 - \delta)] / (1 + \beta)(1 - \delta)x.
\end{aligned}$$

But this is obviously the first inequality in (45), establishing the proposition. ■

## Appendix B

*Proof of Proposition 4.* The second inequality in (44) obviously holds iff  $\psi_2 < 1$ . It is easy to verify that

$$\begin{aligned}
\text{(A.3)} \quad \psi_2 & \equiv [s(w_1, w_2, x) / s(w_1, w_2, 1)] \\
& \times \{1 - (w_2/\beta w_1)^{T - (T_1+1)} \\
& \times [1 - s(w_1, w_2, 1) / w_1]\}.
\end{aligned}$$

Then,  $\psi_2 < 1$  holds iff

$$\begin{aligned}
\text{(A.4)} \quad & [s(w_1, w_2, x) - s(w_1, w_2, 1)] \\
& \div s(w_1, w_2, x) < [1 - s(w_1, w_2, 1) / w_1] \\
& \times (w_2/\beta w_1)^{T - (T_1+1)}
\end{aligned}$$

is satisfied. Rearranging terms in (A.4) yields the first inequality in (48).

To obtain the second inequality in (48), note that the first inequality in (44) holds iff

$$\begin{aligned}
\text{(A.5)} \quad & [\beta / (1 + \beta)] s(w_1, w_2, x) (w_2/\beta w_1)^{T - T_1} \\
& \geq s(w_1, w_2, 1) - s(w_1, w_2, x) \\
& + s(w_1, w_2, x) [1 - s(w_1, w_2, 1) / w_1] \\
& \times (w_2/\beta w_1)^{T - T_1 - 1}.
\end{aligned}$$

Rearranging terms in (A.5) yields the second inequality in (48).

To establish part (b) of the proposition, notice that

$$\begin{aligned}
\text{(A.6)} \quad & w_2(x - 1) / x s(w_1, w_2, x) \\
& = (1 + \beta) w_2(x - 1) / \beta w_1 [x - (w_2/\beta w_1)].
\end{aligned}$$

and

$$\begin{aligned}
\text{(A.7)} \quad & \lim_{x \rightarrow \infty} w_2(x - 1) / x s(w_1, w_2, x) \\
& = (1 + \beta) w_2 / \beta w_1
\end{aligned}$$

are satisfied. Moreover,  $T \geq T_1 + 2$  implies that

$$\begin{aligned}
& \lim_{x \rightarrow \infty} w_2(x - 1) / x s(w_1, w_2, x) \\
& > (w_2/\beta w_1)^{T - (T_1+1)}
\end{aligned}$$

holds. Thus, the condition

$$\begin{aligned}
\text{(A.8)} \quad & w_2(\underline{x} - 1) / \underline{x} s(w_1, w_2, \underline{x}) \\
& \equiv (w_2/\beta w_1)^{T - (T_1+1)}
\end{aligned}$$

has a solution  $\underline{x} > 1$ . Likewise, if the condition

$$\begin{aligned}
\text{(A.9)} \quad & w_2(x - 1) / x s(w_1, w_2, x) \\
& = [(w_1 + w_2) / w_1] (w_2/\beta w_1)^{T - (T_1+1)}
\end{aligned}$$

has a solution, let  $\bar{x}$  denote it. If (A.9) has no solution, let  $\bar{x} = \infty$ . Then, the inequalities in (48) are clearly satisfied iff  $x \in [\underline{x}, \bar{x}]$ .

It remains to show that  $\delta$  can be selected to satisfy (a.1) and (45). To begin, rewrite (45) as

$$\begin{aligned}
\text{(A.10)} \quad & g(1 - \delta)^{T_1 - 1} \equiv \delta(1 - \delta)^{T_1 - 1} \\
& \times s(w_1, w_2, x) / [1 - (1 - \delta)^{T_1 + 1}] \\
& \geq w_2[x - (1 - \delta)] / (1 + \beta)x \\
& > \delta \beta w_1 / (1 + \beta).
\end{aligned}$$

Clearly,  $\beta w_1 > w_2$  implies that, for  $\delta$  near zero, (a.1) and the second inequality in (A.10) are satisfied. Moreover, the first inequality in (A.10) can be written in the form

$$\begin{aligned}
\text{(A.11)} \quad & \delta(1 - \delta)^{T_1 - 1} / [1 - (1 - \delta)^{T_1 + 1}] \\
& \geq (w_2/\beta w_1)[x - (1 - \delta)] \\
& \div [x - (w_2/\beta w_1)].
\end{aligned}$$

For  $\delta$  satisfying assumption 1 and for all  $x > 1$ , we clearly have

$$\begin{aligned}
& (w_2/\beta w_1)[x - (1 - \delta)] / [x - (w_2/\beta w_1)] \\
& < (w_2/\beta w_1).
\end{aligned}$$

Moreover, by L'Hopital's rule,

$$\begin{aligned} & \lim_{\delta \rightarrow 0} \delta (1 - \delta)^{T_1 - 1} / [1 - (1 - \delta)^{T_1 + 1}] \\ &= 1 / (T_1 + 1). \end{aligned}$$

Thus, condition (49) implies that, for all  $x \in [\underline{x}, \bar{x}]$ , both inequalities in (45) are satisfied whenever  $\delta$  is chosen sufficiently small. ■

## Appendix C

### *Proof of Proposition 5.*

a) For  $t \leq T_1 - 1$ , equation (50) follows immediately from (34). Moreover, by L'Hopital's rule,

$$\begin{aligned} & \lim_{\delta \rightarrow 0} [1 - (1 - \delta)^{t+1}] / [1 - (1 - \delta)^{t+2}] \\ &= (t+1) / (t+2). \end{aligned}$$

Hence, the assertion in the text following the proposition.

b) Equation (34), the definition of  $x$ , and  $M_t/p_t = s(w_1, w_2, p_t/p_{t+1})$ ,  $t = T_1, \dots, T-1$ , imply that  $p_{T+1}/p_{T_1} = x = x(M_{T_1+1}/M_{T_1})$ , since the money supply is constant for all  $t \geq T_1$ . Moreover, for  $t = T_1+1, \dots, T-1$ , the conditions  $p_{t+1} \leq p_t$  and  $M_t/p_t = M_{T_1}/p_{T_1} = s(w_1, w_2, p_t/p_{t+1})$  are satisfied. Therefore,  $M_t/p_t \geq M_t/p_{t-1} \geq \dots \geq M_{T_1}/p_{T_1} = s(w_1, w_2, x)$  must hold. It is then immediate that, for all such  $t$ ,  $p_t/p_{t+1} > x$ , and  $p_{t+1}/p_t < (1/x)(M_{t+1}/M_t) = 1/x$  obtain. ■

## Appendix D

*Proof of Proposition 6.* Define  $\bar{\psi}_2(\psi_2)$  by

$$\begin{aligned} \text{(A.12)} \quad \bar{\psi}_2 &\equiv (g/\delta)[1 - (1 - \delta)^{T_1 + 1}] \\ & \{w_1^{-1}(w_2/\beta w_1)^{\bar{T} - (T_1 + 1)} \\ & + [(1 + \beta)/\beta w_1][1 - (w_2/\beta w_1)^{\bar{T} - (T_1 + 1)}] \\ & \div [1 - (w_2/\beta w_1)]\} \end{aligned}$$

and

$$\begin{aligned} \text{(A.13)} \quad \psi_2 &\equiv (g/\delta)[1 - (1 - \delta)^{T_1 + 1}] \\ & \times \{w_1^{-1}(w_2/\beta w_1)^{T - (T_1 + 1)} \\ & + [(1 + \beta)/\beta w_1][1 - (w_2/\beta w_1)^{T - (T_1 + 1)}] \\ & \div [1 - (w_2/\beta w_1)]\}, \end{aligned}$$

respectively. Then,

$$\text{(A.14)} \quad \bar{p}_{T_1} = (\phi/b)(w_2/\beta w_1)^{\bar{T} - T_1} / (1 - \bar{\psi}_2)$$

$$\text{(A.15)} \quad p_{T_1} = (\phi/b)(w_2/\beta w_1)^{T - T_1} / (1 - \psi_2)$$

hold, and  $\bar{p}_{T_1} > p_{T_1}$  iff

$$\text{(A.16)} \quad (w_2/\beta w_1)^{\bar{T} - T}(1 - \psi_2) > 1 - \bar{\psi}_2$$

is satisfied.

Now, straightforward manipulation establishes that

$$\begin{aligned} \text{(A.17)} \quad (w_2/\beta w_1)^{\bar{T} - T} \psi_2 &= \bar{\psi}_2 - (g/\delta) \\ & \times [1 - (1 - \delta)^{T_1 + 1}][(1 + \beta)/\beta w_1] \\ & \times [1 - (w_2/\beta w_1)^{\bar{T} - T}] / [1 - (w_2/\beta w_1)]. \end{aligned}$$

Substituting (A.17) into (A.16) and rearranging terms, one obtains that  $\bar{p}_{T_1} > p_{T_1}$  holds iff

$$\begin{aligned} \text{(A.18)} \quad \bar{\psi}_2 - (w_2/\beta w_1)^{\bar{T} - T} \psi_2 & \\ &= (g/\delta)[1 - (1 - \delta)^{T_1 + 1}][(1 + \beta)/\beta w_1] \\ & \times [1 - (w_2/\beta w_1)^{\bar{T} - T}] / [1 - (w_2/\beta w_1)] \\ & > 1 - (w_2/\beta w_1)^{\bar{T} - T}. \end{aligned}$$

When  $\bar{T} > T$  obtains, (A.18) is equivalent to

$$\begin{aligned} \text{(A.18')} \quad (g/\delta)[1 - (1 - \delta)^{T_1 + 1}][(1 + \beta)/\beta w_1] \\ & \div [1 - (w_2/\beta w_1)] \equiv s(w_1, w_2, x) \\ & \div s(w_1, w_2, 1) > 1. \end{aligned}$$

Since  $\underline{x} > 1$ , (A.18') clearly holds for all  $x \geq \underline{x}$ . ■

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# Beneficial “Firm Runs”

by Stanley D. Longhofer

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## Introduction

Recent research in law and corporate finance suggests that existing bankruptcy rules have evolved to eliminate inefficiencies that result when lenders rush to retrieve their assets from a firm in financial distress. In contrast, first-come, first-served (FCFS) rules, often considered a benchmark in the absence of other bankruptcy rules, are commonly thought to be inefficient because they reduce the value of the defaulting firm's assets. This, however, may not always be the case. Moral hazard problems associated with the choice of project may make the act of running on a firm desirable, since it can help align investment incentives.

This paper looks at the problem of an entrepreneur who must raise outside funds to finance one of two investment alternatives. One of them is risky, and the bankruptcy costs expected to result from this project make it less desirable socially than the alternative, riskless project. Nevertheless, the firm is unable to commit to the less risky enterprise.

FCFS rules act to diminish this moral hazard problem. I derive a mixed-strategy equilibrium in which lenders monitor the firm with some positive probability. When the firm is caught

investing in the risky project, it is liquidated; otherwise, it is allowed to continue. Although this mixed-strategy equilibrium may exist under both a FCFS rule and a proportionate priority rule (PPR), I demonstrate that it is less likely to exist under the PPR, and that when it does, the FCFS equilibrium is Pareto superior.

The fact that lenders can run on the firm when they observe that it has chosen the risky project helps keep the firm honest. The FCFS aspect of asset distribution keeps lenders from wanting to free ride on the monitoring efforts of others because the lenders who monitor are first in line to receive their claim on the firm's assets and are thus likely to be paid in full. Lenders who wait to observe the monitoring of others are less likely to receive anything if the firm goes under. This process is similar to that described in Calomiris and Kahn (1991), where demandable debt is used to control the banker's moral hazard problem, while sequential service prevents depositors from free riding on the monitoring efforts of others.

The key idea here is that *bankruptcy institutions should reward monitors when and only when they have performed their duties*. A similar argument has been made by Rajan and Winton (1995), who analyze how the choice of

different priority and term structures in loan contracts affects lenders' incentives to monitor the firm. They argue that information conditions determine which structures provide the best monitoring incentives, meaning that the firm's capital structure can be used to achieve outcomes that are not directly contractible. In other words, *ex ante* efficiency is improved by choosing a capital structure that properly rewards monitors. This paper differs from Rajan and Winton, however, in that it focuses directly on the structure of the bankruptcy institution, outside of which private agents are not allowed to contract by law.

Although FCFS rules are beneficial in my model, there are other factors I have ignored that work in the opposite direction. Longhofer and Peters (1997) examine the coordination problem discussed above and show how lenders might fail to coordinate their liquidation decisions, even when doing so would be a Pareto-superior outcome. As a result, mandatory bankruptcy procedures (which implicitly enforce a PPR) can be socially desirable, since they enable lenders to coordinate in states of the world where they would not otherwise do so.

In many respects, the work of Longhofer and Peters represents the opposite side of this analysis. Notably, their paper assumes that FCFS rules entail a deadweight social cost, while ignoring the potential impact of such rules on a lender's monitoring incentives. This paper does exactly the opposite, focusing on how FCFS rules can ameliorate the firm's moral hazard problem. A more complete model would attempt to capture both effects of a FCFS rule: the benefits associated with improved monitoring incentives and the costs associated with inefficient default.

The next section summarizes traditional bankruptcy analyses. Here, I outline some of the standard arguments in favor of an alternative to a FCFS rule in bankruptcy law and question whether they are valid in all circumstances. I then use this background to analyze other studies of bankruptcy. I introduce my model in section II and show that under certain conditions, a firm may be unable to obtain financing because it cannot commit *ex ante* to a low-risk project; possible solutions to this problem are analyzed. In particular, I show that there exists a mixed-strategy equilibrium in which the firm is able to find lenders. Section III looks at the effect different bankruptcy rules may have on the equilibrium of this game. I show that a PPR reduces lenders' incentives to monitor the firm, thus raising the social cost of these contracts. I conclude in section IV, relegating all proofs to the appendix.

## I. Justifications for PPRs

Most discussions of bankruptcy institutions start with the assumption that a formal procedure is needed for distributing an insolvent firm's assets, and then focus on the specific form such a procedure should take. It is not clear, however, that this assumption is valid in all cases. To see this, consider some of its standard justifications.

In the absence of bankruptcy laws, assets are distributed to creditors in the order in which they have staked their claims. Thus, the first lender to request repayment is generally the first to receive it. Lenders who end up last in line are paid last and quite possibly receive nothing.<sup>1</sup> For this reason, these default bankruptcy proceedings are typically called FCFS rules.

Traditional rationales for a more orderly mechanism cite several potential problems with FCFS rules. First, lenders may wish to protect their position by expending excessive resources to monitor the firm's condition. If a lender does not do this, the argument goes, he will certainly be the last to know when the firm is about to default, and consequently be the last in line to collect his claim. Furthermore, since all lenders are engaged in this monitoring, no one will get a better place in line than he would if none of them monitored, so these resources are spent in vain. This game looks much like the classic prisoners' dilemma, in which the Pareto-superior outcome with no monitoring is not a Nash equilibrium. It is argued that an orderly bankruptcy procedure allows lenders to avoid these costs, making all of them better off.

A second argument against FCFS rules is the classic "common pool" problem. Here, it is claimed that in their rush to be paid, lenders might reduce the total liquidation value of the firm by separating assets that would be more valuable together.<sup>2</sup> An orderly liquidation, on the other hand, would ensure that the firm's assets are put to their most productive uses, maximizing their value to the creditors. Worse yet, lenders might actually run too soon and foreclose on illiquid but otherwise viable firms. Again, formal bankruptcy rules should help prevent these inefficient liquidations.

■ 1 This can be true even when some lenders insert seniority covenants into their loan agreements. Once the first lender removes his assets from the firm, a later lender's seniority provision no longer carries much benefit. Effectively, under a FCFS rule, lenders can assign seniority to themselves *ex post* by being the first to request repayment.

■ 2 See Longhofer and Peters (1997).

Jackson (1986, p. 10) summarizes the intuition behind these arguments: “The basic problem that bankruptcy law is designed to handle ... is that the system of individual creditor remedies may be bad for the creditors *as a group* when there are not enough assets to go around.”

With these (often implicit) assumptions, modern studies of bankruptcy rules investigate what shape formal liquidation rules should take. For example, many authors have looked at the relative efficiency of the absolute priority rule (where the order of repayment is determined *ex ante* by assigning each lender a priority level) and PPRs.<sup>3</sup> Under various assumptions, they all conclude that these rules are inefficient with regard to both the liquidation/continuation decision and the decision to make new investments. Numerous other studies analyze Chapter 11 of the Bankruptcy Code and show that, in general, it does not provide efficient investment or liquidation incentives either. None of these studies, however, examines the *relative* efficiency of various bankruptcy rules compared to the natural default—FCFS rules—a necessary starting place for bankruptcy analyses.

In addition, most models analyze the effects of bankruptcy *ex post*. They begin with a firm whose existing capital structure cannot meet its current debt obligations. These models focus on whether different bankruptcy rules provide proper incentives so that creditors will foreclose if and only if the firm is insolvent, and will extend new credit to the firm for and only for positive net present value projects. It is certainly interesting to ask whether bankruptcy rules provide for decisions that are efficient *ex post*. But debt contracts are designed to resolve *ex ante* uncertainty, and their efficiency must therefore be measured from the viewpoint of the initial contracting problem. The proper question, then, is how different bankruptcy rules affect the social cost of debt contracts *at the time they are written*.

Boyes, Faith, and Wrase (1991) is one of only a few papers that address both these issues.<sup>4</sup> Its authors compare the *ex ante* social cost of debt contracts under PPRs and under FCFS rules, concluding that the PPR found in Chapter 7 is more efficient than FCFS rules, since it reduces the cost of contracting. Their result depends on their assumption that rushing to liquidate the firm is costly, whereas formal bankruptcy proceedings are not. In a FCFS world, lenders must pay to enter a queue to obtain the firm’s assets. If they allow a firm to continue despite the fact that its expected return is negative, they will avoid these queuing costs some of the time

(when the firm does well). Thus, lenders have an incentive to allow some firms with negative net present value to continue.<sup>5</sup>

The model of Boyes, Faith, and Wrase differs from this one in several important respects. First, they assume that a FCFS rule is costlier to implement than is a PPR.<sup>6</sup> More important, in my model the firm chooses between two different investment projects. This choice is the firm’s private information, creating a moral hazard problem that requires lenders to monitor the firm. When there are many lenders, they may wish to free ride on one another’s monitoring efforts. I propose that FCFS rules can serve to ameliorate this problem.

## II. The Model

Consider a two-period world in which a risk-neutral firm has the opportunity to invest in one of two projects in period 0, either of which will mature in period 2. The first, project *B*, has a random return, paying  $x_h$  in period 2 with probability  $p$  and  $x_l$  with probability  $(1 - p)$ . In contrast, project *G* is a safe project, returning  $\bar{x} = px_h + (1 - p)x_l$  in period 2 with certainty.<sup>7</sup>

Either project requires an investment of  $I$  to undertake. Because the firm has no resources of its own, it must borrow these funds from outside investors. I assume the loan market is composed of a large number of identical, risk-neutral agents. In equilibrium, competition will always drive down the interest rate,  $R$ , to ensure that all lenders earn zero profits. Assume that  $\bar{x}$  is high enough always to enable the firm with the riskless project to make its promised payments in period 3. In contrast,  $I > x_l$ , so that if the firm chooses project *B*, it can meet its obligations only when the project is successful. When the firm is unable to repay its loans, default costs of  $d$  are incurred.

■ 3 See, for example, Bulow and Shoven (1978), White (1980, 1983), and Gertner and Scharfstein (1991).

■ 4 See also Longhofer and Peters (1997).

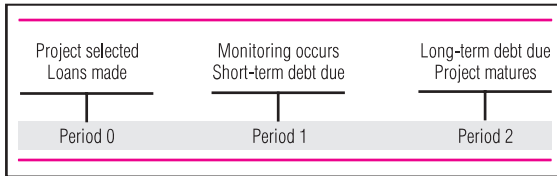
■ 5 The authors also acknowledge that FCFS rules may result in inefficient liquidations of firms that have a positive net present value and claim that this further supports their argument that PPRs are more efficient. They ignore, however, the possibility that these two effects may offset each other, reducing the net inefficiency of FCFS rules.

■ 6 If they were to assume that both types of rules entailed the same costs, their model would indicate a preference for FCFS rules, which involve these costs only a fraction of the time, rather than PPRs, which always do.

■ 7 More generally, I could assume that *G* is second-order stochastic dominant over *B*.

FIGURE 1

## Sequence of Events



SOURCE: Author.

Assume that the choice of project is costlessly observable by the firm. Outsiders, however, must monitor the firm in period 1 in order to discover its project choice. Let  $c$  denote the cost of doing so. Although the results of this monitoring provide a perfect signal of the firm's project choice, I assume that this information cannot be verified in court, making it impossible for contracts to depend on the choice of project.<sup>8</sup>

To analyze the effects of priority on the efficiency of financial contracting, I assume that the firm must borrow from multiple lenders.<sup>9</sup> For simplicity, assume that the firm borrows  $I/2$  from each of two lenders. Figure 1 shows the order of events in this economy.

### Long-term Debt

To finance either of these projects, the firm could issue long-term debt—that is, debt that comes due in period 2. On the basis of their beliefs about the firm's project choice, prospective lenders will demand a default premium commensurate with that project's anticipated risk. If they believe the firm will choose to invest in the riskless project  $G$ , each lender will simply charge the zero-profit interest rate,  $R_G^* = I/2$ . On the other hand, if they anticipate the firm will choose project  $B$ , each lender's expected return is

$$(1) \quad pR + (1-p) \frac{x_B - d}{2} - \frac{I}{2},$$

implying a zero-profit interest rate,  $R_B^* = [I - (1-p)(x_B - d)]/2p$ ; it is straightforward to verify that  $R_B^* > R_G^*$ .

Of course, the firm is concerned about the interest rate it pays only to the extent that its profits are affected. Given any promised payment,  $R$ , the firm's period-2 profit from project  $G$  is  $\bar{x} - 2R$  with certainty. In contrast, its profit

from project  $B$  is  $p(x_B - 2R)$ . Because there are deadweight costs associated with default, it is certain that, if both projects were priced competitively, the firm would earn a higher expected return from project  $G$ . Nevertheless, it is easy to show:

**PROPOSITION 1.** *Given any fixed promised repayment  $R$ , the firm will always choose to invest in project  $B$ .*

This proposition implies that long-term debt prevents the firm from credibly promising to invest in the riskless project in equilibrium. Once it receives the (relatively low) interest rate associated with project  $G$ , it would like to go ahead and invest in  $B$ , since it suffers none of the losses associated with the project's increased variability. If long-term debt is the only option, no lender will accept any interest rate below  $R_B^*$ , and the firm will invest in project  $B$ .

This inability to commit to the riskless project obviously entails social costs. Because lenders earn zero profits in equilibrium, these costs can be measured by comparing the profits the firm would have earned had it been able to commit to project  $G$  with those it earns from project  $B$ . This ends up equaling the expected deadweight default costs associated with project  $B$ :  $d(1-p)$ .

### Short-term Debt

Is it possible to avoid such costs? One solution to this moral hazard problem is a maturity mismatch with short-term debt.<sup>10</sup> Suppose the firm must make a payment to its lenders in period 1. Since it has no revenues until period 2, it must either default or convince the lenders to roll over its debt. Before renewing the debt, however, lenders can monitor the firm and determine which project has been selected.

If lenders could credibly commit to monitoring the firm in period 1, short-term debt would give the firm an incentive to invest in project  $G$ .

■ 8 If contracts could depend on the specific contract chosen, the first-best outcome would occur, in which the firm always chooses project  $G$ .

■ 9 I do not formally motivate this assumption here. See Bolton and Scharfstein (1996) for a formal model motivating the use of multiple creditors.

■ 10 It would be formally identical if we continued to consider long-term contracts with covenants that allow lenders to demand repayment in period 1. Note, however, that such covenants would not explicitly depend on the choice of project, since we have assumed this choice is not verifiable in court.



I assume, however, that such commitment is not possible. As a result, once project  $G$  is chosen, lenders no longer have any incentive to monitor the firm. Of course, the firm can anticipate that this will happen, and will once again choose project  $B$ . Thus, the pure-strategy equilibrium is the same with short- and long-term debt contracts; the firm chooses project  $B$ , while lenders charge  $R_B^*$  and never monitor the firm.

Thus, to find an equilibrium in which the firm invests in the riskless project, we must focus on mixed strategies, in which the firm chooses each of the projects with some positive probability, and lenders randomly monitor this choice.

After financing is provided in period zero, the firm must decide how often it will invest in each of the two projects, and the bank must decide how often it will monitor. Let  $\pi \in (0, 1)$  be the probability that the firm selects project  $B$ , and  $\alpha \in (0, 1)$  be the probability that each lender monitors the firm in period 1. Since I am looking for a symmetric equilibrium, the total probability that the firm is monitored is  $1 - (1 - \alpha)^2$ .<sup>11</sup> Finally, let  $R$  denote the face value of the debt owed to each lender, so that the firm's total debt is  $2R$ .

Conditional on the results of its monitoring in period 1, each lender must decide whether to roll over its debt or to demand immediate repayment of its loan. Assume that the firm has no cash assets in period 1, so that it must be liquidated whenever either lender demands repayment, and that all debt contracts contain cross-default clauses, stipulating that the loan is in default whenever another creditor demands early repayment of its debt. Let  $z$  be the post-bankruptcy-cost, period-1 liquidation value of the firm. I assume that  $z < I/2$ , so that this value is insufficient to pay off either of the firm's creditors. Lenders who monitor the firm are first in line for its assets when it is liquidated in period 1, since they are the first to be aware that the firm has cheated. Thus, under a FCFS rule, the firm's assets,  $z$ , are distributed only to lenders who actually monitor the firm; since  $z < I/2$ , nothing remains for a nonmonitoring lender. In contrast, under the PPR, each lender receives  $z/2$  when the firm is liquidated in period 1, whether it monitored the firm or not.

Finally, if the firm is not liquidated in period 1, its project matures and revenues are received in period 3. If the firm's project is successful, it pays off its lenders and keeps the balance of its revenues as profit; otherwise, it is liquidated and its assets ( $x_l - d$ ) are divided equally between the two creditors.

## Derivation of an Equilibrium

Since we are looking for a symmetric, mixed-strategy Nash equilibrium,  $\alpha$  and  $\pi$  must be chosen so as to make the firm and the lenders, respectively, willing to randomize. That is, each lender's probability of monitoring,  $\alpha$ , must be such that the firm earns the same expected return regardless of which project it chooses:

$$(2) \quad \bar{x} - 2R = (1 - \alpha)^2 p (x_h - 2R)$$

or

$$(3) \quad \alpha^* = 1 - \sqrt{\frac{\bar{x} - 2R}{p(x_h - 2R)}}.$$

Direct differentiation of  $\alpha^*$  shows that it is increasing in  $R$ . In other words, the firm's moral hazard problem worsens as  $R$  gets larger, implying that more monitoring is required to keep it indifferent between the two projects when the interest rate is high.

Similarly,  $\pi$  must be chosen to ensure that lenders are indifferent between monitoring and not monitoring the firm. The competitive loan market imposes the additional constraint that each lender must earn zero expected profit. It follows that  $\pi^*$  and  $R^*$  must be jointly chosen to solve

$$(4) \quad (1 - \pi)R + \pi \left[ \alpha \frac{z}{2} + (1 - \alpha)\lambda_1 \right] - c - \frac{I}{2} = 0$$

and

$$(5) \quad (1 - \pi)R + \pi \left\{ \alpha \lambda_2 + (1 - \alpha) \left[ pR + (1 - p) \frac{x_l - d}{2} \right] \right\} - \frac{I}{2} = 0,$$

where  $\lambda_1$  and  $\lambda_2$  are a lender's payoffs from early liquidation when it is the only one to monitor and when it does not monitor, respectively. These two payoffs depend on the bankruptcy rules in effect.

The intuition behind each of these expressions is clear. Equation (4) is a lender's expected return when it monitors the firm. If it discovers that the firm selected the riskless project (which happens with probability  $1 - \pi$ ), it allows the firm to continue until period 2 and receives its

■ 11 I focus on a symmetric equilibrium because of its analytical tractability. The same basic conclusions would follow from a model in which one lender was designated to monitor more frequently than the other.

promised payment  $R$  at that time. On the other hand, if it finds that the firm chose project  $B$ , it demands immediate repayment.<sup>12</sup> If the other lender also monitored (which happens with probability  $\alpha$ ), the two lenders divide  $z$  equally. On the other hand, if the other lender did not monitor the firm, then the monitoring lender's payoff is  $\lambda_1$  and depends on the bankruptcy rule in effect. Under FCFS rules, the sole monitor is entitled to all of the firm's liquidation value, while these assets are divided equally under a PPR. Thus,  $\lambda_1^{FCFS} = z$ , and  $\lambda_1^{PPR} = z/2$ . Finally, note that the lender's total costs in this case include the monitoring expenses it incurs,  $c$ , and its original investment in the firm,  $I/2$ .

A similar intuition is behind equation (5), the lender's expected return when it does not monitor. With probability  $1 - \pi$ , the firm chooses the riskless project, and the lender earns  $R$  with certainty. With probability  $\pi$ , the firm chooses project  $B$ . With probability  $\alpha$ , the other lender monitors the firm and demands immediate repayment. In this last case, the *non*monitoring lender receives  $\lambda_2$ , where  $\lambda_2^{FCFS} = 0$  and  $\lambda_2^{PPR} = z/2$ . On the other hand, if neither lender monitors the firm, its project is allowed to mature. With probability  $p$ , the project succeeds, paying each lender  $R$ . With probability  $1 - p$ , it fails, and the two lenders split  $x_j - d$  between them. Finally, since no monitoring costs are incurred in this case, the interest rate must simply recoup the firm's investment,  $I/2$ .

Given this setup, we have the characterization of equilibrium in

**PROPOSITION 2.** *The following strategies constitute a mixed-strategy, sequential Nash equilibrium with short-term debt:*

- a) *The firm chooses project  $B$  with probability  $\pi^*$  and project  $G$  with probability  $1 - \pi^*$ ;*
- b) *Each lender chooses to monitor the firm with probability  $\alpha^*$  and refuses to renew its loan only after observing the firm has chosen project  $B$ ; and*
- c) *Lenders never liquidate the firm in period 1 when they do not monitor.*

Proposition 2 tells us that short-term debt may be one device for mitigating the firm's incentive to invest in the risky project,  $B$ , and that it holds regardless of which bankruptcy rules are in effect. The possibility that it might be monitored and liquidated by one of its lenders gives the firm an incentive to invest in the safe project with some positive probability.

Since the deadweight costs of default associated with project  $B$  are incurred less often, the firm's ex ante profits are higher.

### III. The Relative Efficiency of Bankruptcy Rules

In the last section, I showed how short-term debt with monitoring can be used to lessen a firm's moral hazard problem, improving the efficiency of financial contracting. In this section, I focus on how the institutional structure used to divide the assets of a financially distressed firm can affect the efficiency of these contracts.

The equilibrium derived in the last section was equally consistent with FCFS rules and PPRs. My goal in this section is to show that this mixed-strategy equilibrium is less likely to exist under PPRs, and that when it does exist, the total social cost of the contract will be higher with PPRs. I do this by examining the interest rate in the problem under each of these rules.

For a mixed-strategy equilibrium to exist,  $R^*$  and  $\pi^*$  must jointly solve (4) and (5). In addition, the following conditions must be satisfied:  $\alpha^* \in (0,1)$ ,  $\pi^* \in (0,1)$ , and  $R^* \leq \bar{x}/2$ .<sup>13</sup>

Because I have assumed that the loan market is perfectly competitive, it is straightforward to measure the relative efficiency of bankruptcy rules by calculating the difference between the firm's expected profits under each of them. Because  $\alpha^*$  is chosen to make the firm indifferent between the two projects, the firm's expected profit is simply equal to

$$(6) \quad \bar{x} - 2R.$$

Expression (6) makes it clear that the preferred bankruptcy rule will be the one that minimizes the total face value of the firm's debt.

I am now able to prove the primary result of this paper:

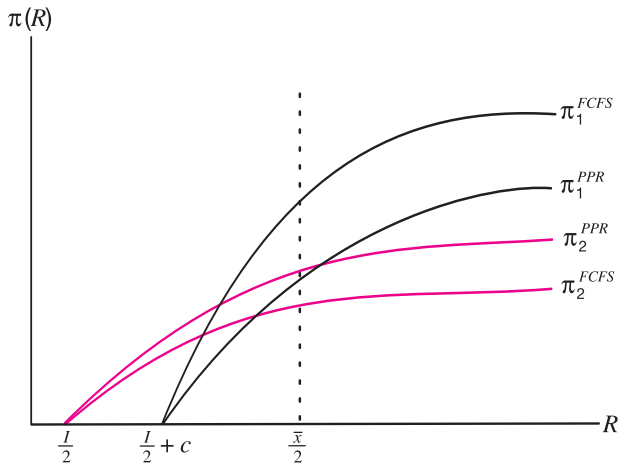
**PROPOSITION 3.** *The total face value of the firm's debt is lowest under the FCFS rule, meaning that social welfare is highest under this bankruptcy rule.*

■ 12 This is simply assumed in the formulation of expression (4) and is formally proven in proposition 2.

■ 13 If this final condition is violated in any candidate equilibrium, the firm will have no incentive to choose the riskless project ( $\pi^* = 1$ ), giving the lender no incentive to monitor the firm. As the previous section showed, this degenerates to the (inefficient) long-term debt solution.

FIGURE 2

## Advantage of FCFS Rules



SOURCE: Author.

Proposition 3 is illustrated in figure 2. Let  $\pi_1$  be the locus of  $(\pi, R)$  pairs that solve (4), and  $\pi_2$  the locus solving (5). The leftmost intersection of these loci is the equilibrium.<sup>14</sup>

When the firm is caught investing in project  $B$ , a FCFS rule gives more to lenders who monitor than does a PPR. Thus,  $\pi_1^{FCFS}$  is everywhere *above*  $\pi_1^{PPR}$ . Similarly, when the firm is caught cheating, lenders who monitor have a higher expected return under a PPR than under a FCFS rule. Hence,  $\pi_2^{FCFS}$  is everywhere *below*  $\pi_2^{PPR}$ . Together, these two facts imply that the first intersection of the two curves under a PPR must be to the right of their first intersection under a FCFS rule. Consequently, the equilibrium under a FCFS rule must entail a lower interest rate.

Proposition 3 affirms that, contrary to the generally accepted view, a bankruptcy institution in which lenders may run on a firm in default to collect their assets can actually be socially preferred to an institution prohibiting such runs. Essentially, PPRs encourage lenders to free ride on the monitoring efforts of others, since these rules give each lender—whether it monitors or not—the same claim on the firm's assets. With FCFS rules, lenders have more incentive to monitor because they get first call on the defaulting firm's assets. This reduces the interest rate needed to give lenders zero expected profits, letting the firm earn a higher return.

Proposition 3 has an immediate corollary:

**PROPOSITION 4.** *A mixed-strategy equilibrium is less likely to exist under a PPR than under a FCFS rule.*

Proposition 4 implies that equilibrium under PPRs is more likely to degenerate to the pure-strategy, long-term debt equilibrium in which the firm always invests in project  $B$ . To understand this proposition, note that the largest value that  $R$  can take in any mixed-strategy equilibrium is  $\bar{x}/2$ ; for any larger  $R$ , the firm would never choose to invest in project  $G$ , since doing so would provide it with a negative return. Basically, the shifts in  $\pi_1$  and  $\pi_2$  resulting from a move to a PPR make it less likely that the intersection between these two curves will occur within this relevant range.

To summarize, FCFS rules can be beneficial for two reasons. First, socially desirable debt contracts are more likely to be feasible under FCFS rules than when PPRs govern default. Second, the total cost of this debt is lower under FCFS rules, increasing the firm's ex ante expected profit. In short, allowing lenders to run on the firm can be beneficial because it improves lenders' monitoring incentives by compensating them when, and only when, they perform this socially desirable activity.

#### IV. Concluding Thoughts

This paper questions the standard assumption that preventing lenders from running on a firm is always necessary in bankruptcy. In the model presented here, a moral hazard problem makes the act of monitoring a socially beneficial public good. As a result, the total cost of debt contracting is reduced when the bankruptcy procedure compensates those lenders who monitor a misbehaving firm. Allowing creditors to run on a financially distressed firm to retrieve their assets serves to implement just such a compensation mechanism.

Lately, there has been extensive debate about whether bankruptcy laws should be reformed, and if so, how. One proposal receiving significant attention is by Aghion, Hart, and Moore (1992).<sup>15</sup> They suggest that each of a

■ 14 Since any such intersection provides lenders with zero expected profit, the leftmost intersection results in the lowest possible interest rate for the borrower, making it the equilibrium.

■ 15 See also Roe (1983) and Bebchuk (1988).

firm's creditors should be given an option to purchase the firm's assets from more senior claimants at the value of their claims. This system would guarantee that a distressed firm's assets end up with the individual or group that values them most, and would ensure that economically viable firms will continue. While this proposal would do much to eliminate the ex post inefficiencies associated with modern bankruptcy proceedings, it does not resolve the basic concerns addressed in this paper. Like the PPR I discuss above, their proposal does not consider the impact a proper compensation scheme can have on the probability that bankruptcy will occur in the first place.

The main point of this model with respect to the debate over bankruptcy reform is that policymakers should consider the impact of bankruptcy rules not only on the distribution of a financially distressed firm's assets, but also on the terms of debt contracts. It is this latter influence that has the largest effect on social welfare.

## Appendix

**Proof of Proposition 1.** Holding the interest rate constant, the difference between the firm's profit from project  $B$  and project  $G$  is  $(1-p)(2R-x_j)$ . In any equilibrium,  $2R \geq 1 > x_j$ , showing that this difference must be positive. ■

**Proof of Proposition 2.**  $R^*$  and  $\pi^*$  are defined in the text so as to ensure that lenders are willing to randomize, and  $\alpha^*$  is defined to ensure that the firm is willing to randomize. Thus, it remains to be shown only that 1) lenders will liquidate the firm after monitoring and observing project  $B$ ; 2) lenders will not liquidate after monitoring and observing project  $G$ ; and 3) lenders will not liquidate the firm in period 1 when they do not monitor.

1) Upon observing project  $G$ , the lender's expected return from liquidating the firm is  $z/2$  under a PPR. Thus, for a lender to be willing to liquidate a risky firm under a PPR, it must be the case that

$$(7) \quad \frac{z}{2} > \alpha \frac{z}{2} + 1 - \alpha \left[ pR + (1-p) \frac{x_1 - d}{2} \right] \\ = \frac{I/2 - (1-\pi)R^*}{\pi} \quad (\text{by expression [5]}) \\ = \frac{z}{2} - \frac{c}{\pi} \quad (\text{by expression [4]}),$$

which must hold since  $c$  is positive. An analogous argument shows that the lender will always want to liquidate the risky firm under FCFS rules.

2) The lender's expected return from allowing the riskless project to mature is  $R^* > I/2 > z$ , verifying that lenders will not force early liquidation in this case.

3) The most a lender can hope to earn from liquidating a firm that it does not monitor is  $z$ , which is (by assumption) less than  $I/2$ , the expected return from allowing this firm's project to mature (by expression [5]). Thus, lenders will allow the firm's project to mature in this case as well. ■

**Proof of Proposition 3.** Solving (4) and (5) for  $\pi$  as functions of  $R$  gives us

$$(8) \quad \pi_1(R) = \frac{R - c - I/2}{R - \alpha z/2 - (1-\alpha)\lambda_1}$$

and

$$(9) \quad \pi_2(R) = \frac{R - I/2}{R - \alpha \lambda_2 - (1-\alpha)[pR + (1-p)(x_1 - d)/2]}$$

The intersection of these two functions in the positive orthant gives the  $(\pi, R)$  pairs that simultaneously solve (4) and (5). If these curves intersect more than once, the first such intersection is the candidate for equilibrium, since it entails the lowest interest rate.

Now,  $\pi_1(R) = 0$  when  $R = I/2 + c$ , and  $\pi_2(R) = 0$  when  $R = I/2$ . As  $R$  gets larger, each of these must move into the positive orthant, since  $\pi$  is a convex weight. As noted in the text,  $\lambda_1$  is smaller under a PPR than under a FCFS rule. Thus,  $\pi_1^{PPR}$  minorizes  $\pi_1^{FCFS}$ .<sup>16</sup> Similarly,  $\lambda_2$  is larger under a PPR than under a FCFS rule, implying that  $\pi_2^{FCFS}$  minorizes  $\pi_2^{PPR}$ . This implies that the first intersection of  $\pi_1^{PPR}$  and  $\pi_2^{PPR}$  must lie to the right of the first intersection of  $\pi_1^{FCFS}$  and  $\pi_2^{FCFS}$  (see figure 2). Compared to a FCFS rule, then, a PPR must entail a higher interest rate. ■

■ 16 That is, for every  $R$ ,  $\pi_1^{PPR}(R) < \pi_1^{FCFS}(R)$ .

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