

*B*eyond Duration: Measuring Interest Rate Exposure

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While many factors contributed to the savings and loan industry's extensive losses in the 1980s, the biggest losses, those that brought on the savings and loan crisis, resulted primarily from interest rate fluctuations during the late 1970s and early 1980s (see George J. Benston and George G. Kaufman 1990). Those losses demonstrated the importance of calculating and avoiding interest rate risk for financial practitioners who fund and manage all sizes of portfolios. They also focused the attention of financial regulators, the public, and, ultimately, Congress on potential losses from interest rate risk. In the aftermath, hedging instruments and techniques have been applied more broadly.¹ Congress, in the Federal Deposit Insurance Corporation Improvement Act of 1991 (FDICIA), has also instructed federal bank regulators to account for interest rate risk in their risk-based capital requirements.

A simple and potentially inadequate approximation of interest rate risk exposure results from the use of a technique called "modified duration." This technique is used to gauge the changes in the value of an asset or portfolio of assets that occur in response to a parallel shift in interest rates. It thus measures the portfolio's sensitivity to interest rate fluctuations. Modified duration gauges interest sensitivity by making equal interest rate shifts at all maturities of the current term structure and revaluing a portfolio under the new (parallel) term structure.

Acceptance of modified duration as a measure of interest rate exposure can be seen in federal bank regulators' recent proposal of the method for the purpose of integrating interest rate risk exposure into risk-based capital

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guidelines and in a modification of that proposal discussed by the Federal Reserve Board on March 31, 1993. The joint proposal seeks to approximate an institution's exposure to interest rate changes by measuring changes in its net economic value that would result from 100 basis point parallel shifts in interest rates over a three-month period. The change in net economic value would be measured as the change in the present value of its assets minus the change in the present value of its liabilities and off-balance-sheet positions. The more recent Federal Reserve proposal adds 200 basis point shifts and a nonparallel shift based on interest rate changes over the past five years to the proposed exposure measures.²

In addition, modified duration's simplicity has made it a common topic in textbooks. As useful as the method may be for teaching purposes, however, it is an insufficient measure for hedging interest rate exposure in the real world. This article identifies two major problems with using modified duration for this purpose. The discussion first presents the theory underlying modified duration and illustrates its benefits as a hedging model. For the analysis a simple mock portfolio was constructed and revalued using simulated term structures. The analysis points out some of the faults of modified duration, which failed to capture major elements of interest rate exposure, and suggests more accurate measures.

Understanding Duration

Duration is a term that is usually applied to bonds but can be used in reference to any cash-flow stream. The duration of a portfolio's cash flow may be thought of as the weighted average maturity of its securities' cash flows, where the weights are the proportion of the cash flows' present value in the current period over the total present value of the portfolio's future cash flows.

For example, consider the prices in Table 1 for \$100.00 default-free securities to be paid off at a specified time in the future. The price of a zero-coupon, \$100.00 face-value bond maturing in two years would be \$89.96. The duration of the bond would be $(2 \cdot 89.96)/89.96 = 2$.

Next, consider a \$100.00 face-value bond that pays 5 percent coupons semiannually. (The bond pays \$5.00 [or $.05 \cdot \$100.00$] in six months, one year, and one and one-half years. Additionally, the bond pays \$105.00 in two years, reflecting both interest and face-

value payments.) Assuming that the price of the bond is the sum of its individual payments, the price of this bond would be

$$\begin{aligned} \text{Price} &= (.05 \cdot 97.89) + (.05 \cdot 95.56) + (.05 \cdot 92.77) \\ &\quad + (1.05 \cdot 89.96) \\ &= 4.8945 + 4.778 + 4.6385 + 94.458 \\ &= 108.769. \end{aligned}$$

The duration of the bond would be

$$\begin{aligned} \text{Duration} &= [(0.5 \cdot 4.8945) + (1.0 \cdot 4.778) + (1.5 \cdot 4.6385) \\ &\quad + (2.0 \cdot 94.4586)]/108.769 \\ &= 1.87. \end{aligned}$$

The duration of a two-year zero-coupon bond is two years, and the duration of a two-year 10 percent coupon bond (5 percent semiannually) is 1.87 years, illustrating that the duration of a zero-coupon bond is the maturity of the bond and that duration decreases as the coupon rate increases. (Duration declines as the proportion of the total income stream paid early increases.) The box on page 30 shows that a cash flow's duration is the sensitivity of its present value to a parallel shift in interest rates. The implication, therefore, is that the price of the zero-coupon bond is more sensitive to parallel shifts in interest rates than the price of the 10 percent bond is. This concept is important in hedging interest rate exposure. Moreover, if the cash flow's duration is zero, the cash flow will not change value in response to a small parallel change in interest rates. In other words, when the duration of a cash flow is zero, the present value of the cash flow is hedged against small parallel movements in the term structure. (See the box for a more complete discussion of duration.)

Table 1
Prices of \$100.00 Default-Free Securities

Years until Maturity	Price of \$100.00 Bond
0.5	\$97.59
1.0	\$95.56
1.5	\$92.77
2.0	\$89.96

Hedging with Duration

A simple example will illustrate the process of hedging with duration. Consider a portfolio on July 1, 1992, that consisted of receiving \$100.00 on July 1 of each year from 1993 through 1996 (face value: \$400.00). According to the term structure constructed from the July 1, 1992, *Wall Street Journal*, this portfolio would have the price and duration depicted in Table 2.

The first column shows the date of payment, and the second column lists its present value. The sum of the second column is the portfolio's price. The third column is the time (years) remaining until the payment date. The fourth column weights the time into the future, multiplying it by the payment's price and dividing that figure by the total portfolio price. The sum of the weighted times is the duration of the portfolio. The fifth column is the new price of the payments if the term structure were shifted up by 1 basis point.³

Given that the portfolio's price changed with the shift in interest rates, is it possible to find a single cash

flow that would hedge the portfolio's present value to this shift? One hedging instrument would be a single cash flow with a duration of 2.42 years (for simplicity approximated as 2.5 years) and a face value of \$350.51. Because the duration of a single cash flow is the maturity of the cash flow, this security would be one that would mature on January 1, 1994. According to the term structure on July 1, 1992, a \$398.81 face-value security maturing January 1, 1994, would be priced at \$350.51. If the term structure were shifted 1 basis point higher, the new price of the cash flow would decrease to \$350.43. The two asset prices change by the same amount with the shift in interest rates. Thus, the present value of the cash flow of the four-year portfolio can be hedged for small parallel movements of the term structure by shorting, or selling, the single cash-flow security that would mature in two and one-half years. Table 3 illustrates the benefits of using duration as a hedging tool.

A second example of hedging with duration involves a portfolio with a greater duration. In the interest of simplicity the example analyzes only default-free, fixed-income securities. A security is constructed to

Table 2
The Cash Flow Portfolio of a Four-Year Security*

Date of Payment	Price	Years until Payment	Weighted Time	Adjusted Price (+1 basis point)
July 1993	\$96.03	1.0	0.27	\$96.02
July 1994	\$90.87	2.0	0.52	\$90.85
July 1995	\$84.94	3.0	0.73	\$84.92
July 1996	\$78.67	4.0	0.90	\$78.64
Total	\$350.51		2.42	\$350.43

* The portfolio receives \$100.00 on each July 1 from 1993 through 1996. The term structure is constructed from the July 1, 1992, *Wall Street Journal*.

Table 3
A Portfolio Hedged with a Single Cash Flow

Asset	Current Price	Adjusted Price	Difference
Long 4-Year Security	\$350.51	\$350.43	+\$0.08
Short 2.5-Year Security	-\$350.51	-\$350.43	-\$0.08
Combined Portfolio	0.0	0.0	0.0

resemble a thirty-year mortgage. However, again for simplicity, the prepayment option and default risk are not included and only biannual payments are considered. Specifically, at time July 1, 1992 (the beginning of the third quarter), a cash flow is considered that consists of \$100.00 payments on January 1 and on July 1 in the years from 1993 through 2022 (a face value of \$6,000.00). Using a term structure of interest rates constructed from the prices of stripped Treasury bonds as reported in the *Wall Street Journal* on July 1, 1992, this security had a market price of \$2,316.38 and a duration of 9.54 years.

To hedge the price of this security, a bond with a single payment on January 1, 2002 (duration 9.5 years), was selected. Using the same term structure, a face value of \$4,714.52 maturing on January 1, 2002, was calculated as having a market price of \$2,316.38. Thus, this security was chosen as the hedging instrument. Imagine a portfolio that is long the thirty-year security and short the nine-and-one-half-year security.⁴ Such a portfolio would have a face value of zero and a duration of approximately zero. To demonstrate the usefulness of matching duration, a 1 basis point parallel shift increase to the entire term structure was implemented, and the securities were repriced. After the shift, the thirty-year security has a market price of \$2,314.17 and the nine-and-one-half-year security has a market price of \$2,314.17. Thus, even though the securities' prices have changed by \$2.21 (.1 percent), the price of the portfolio is unchanged. Table 4 illustrates how matching the duration of a portfolio can hedge the portfolio to small parallel shifts of the term structure.

term structure. However, interest rates in the United States may become very volatile in relatively short periods of time. To capture a more realistic measure of parallel movement interest rate exposure over three months, many practitioners simulate larger parallel shift movements. This study continues the previous example of a portfolio that is long the thirty-year security and short the nine-and-one-half-year security, altering the July 1 term structure by plus and minus 100 basis points throughout the curve (as in the interagency proposal cited earlier), and revaluing the securities with this new term structure. For a 100 basis point parallel shift increase in interest rates the portfolio price was +\$4.99. For a 100 basis point decrease in rates the portfolio price was +\$8.29. This analysis indicates that the portfolio faces little interest rate exposure. In fact, for any significant parallel shift in the term structure, the price of the portfolio increases. Thus, modified duration indicates that there should be no concern about losses from interest rate fluctuation.

As a test of this measure's accuracy, the portfolio price was recalculated using the actual term structure constructed from the stripped Treasury bond prices reported three months later, on October 1, 1992—and the difference in the price of the portfolio was -\$54.75 (see Table 5). Modified duration would have grossly underestimated the actual interest rate exposure of the simplest portfolio during the third quarter of 1992. There are two important possible sources of such results: mismatched convexity and nonparallel term structure movements.

Adjusting for Convexity. While duration is the amount the price of a portfolio will change for small parallel movements in the term structure, convexity is how much duration will change for small parallel shifts in the term structure.⁵ Thus, if durations are matched and convexities are not, the portfolio prices are hedged only to small changes in the term structure. After a small shift the durations would no longer be

Testing Parallel Shift Simulations

Users of duration-based models realize that the models are useful only for small movements in the

Table 4
A Portfolio Hedged with Matching Durations

Asset	Current Price	Adjusted Price	Difference
Long 30-Year Security	\$2,316.38	\$2,314.17	+\$2.21
Short 9.5-Year Security	-\$2,316.38	-\$2,314.17	-\$2.21
Combined Portfolio	0.0	0.0	0.0

matched, and in the event of a larger parallel shift the portfolio prices would no longer be hedged.

The examples discussed demonstrate the results of unmatched convexity. Recall that the portfolios were perfectly hedged for a 1 basis point increase in the term structure but that their prices differed for a 100 basis point shift. Unmatched convexity is clearly evident in Table 6, in which the portfolio is priced for a 200 basis point shift. Compared with the price changes for a 100 basis point shift (+\$4.99 to +\$8.27), the price changes for a 200 basis point shift (+\$19.21 to +\$35.09) seem to indicate a nonlinear increase in the magnitude of the differences with the size of the parallel movement increases.

Eliminating convexity errors would be the first suggested improvement in simulating 100 basis point parallel shifts. This step is taken in the Federal Reserve's revised proposal, where simulations of 200 basis point shifts are included. Such shifts approximate two standard deviations of historical volatility. Because convexity errors can be large, at least two standard deviations should be simulated.⁶

Incorporating convexity clearly improves the accuracy of duration-based models. However, in the example above convexity was not a problem. Movements exceeding 100 basis points would have shown profits

in the portfolio. Recall that the portfolio had a large positive price difference for both a 200 basis point increase and a 200 basis point decrease.

Nonparallel Shifts in the Term Structure. The biggest problem with using modified duration and parallel shift simulations is that term structure movements historically have rarely been parallel. Unfortunately, portfolios hedged for parallel movements of the term structure may have considerable exposure to nonparallel movements. A statistical technique called principal component analysis is a useful tool for illustrating this point. Principal component analysis breaks down a sequence of random motions into its most dominant independent components, with the first principal component being the most dominant, or most often occurring, component in the random sequence. The second principal component is the next dominant component after removing the first one. Chart 1 shows the two largest principal components of historical forward interest rate volatility.⁷ In the chart the first principal component of forward interest rate fluctuation is similar to a parallel shift in that the entire curve moves in the same direction. Observe, however, that short-term rates are more volatile than long-term rates (a point missed by parallel shift simulation). This characteristic is similar to the nonparallel

Table 5
Simulating a Portfolio under a 100 Basis Point Shift

Simulation	Price of the 30-Year Security	Price of the 9.5-Year Security	Difference
+100 Basis Points	\$2,111.24	\$2,106.25	+\$4.99
-100 Basis Points	\$2,555.74	\$2,547.47	+\$8.27
Actual Outcome	\$2,475.81	\$2,530.56	-\$54.75

Table 6
Simulating a Portfolio under a 200 Basis Point Shift

Simulation	Price of the 30-Year Security	Price of the 9.5-Year Security	Difference
+200 Basis Points	\$1,934.39	\$1,915.18	+\$19.21
-200 Basis Points	\$2,836.71	\$2,801.62	+\$35.09

shift that the revised proposal discussed by the Federal Reserve Board uses for monitoring interest rate risk. The second principal component of historical forward rate movement is fundamentally different from parallel shifts. It involves "twists" of the curve, or short-term and long-term rates moving in different directions. Combined, these two principal components account for more than 98 percent of the historical interest rate fluctuation (see Robert Litterman and Jose Scheinkman 1991).

Given that historically the most likely changes in the term structure are the independent movements of its principal components, a useful measure of interest rate exposure would be the change in the portfolio price in relation to the movements resulting from possible combinations of historical principal components of term structure fluctuation. Table 7 recalculates the market price of the portfolio for simulated term structures. The term structures are the result of 0, 1, 2, and 3 standard deviation movements of the historical principal components. The row number is the number of standard deviations of the first principal component. (For example, +1 in the row means that the term structure was raised by one standard deviation of the first principal component, and -2 means that the term structure was lowered by two standard deviations of

the first principal component.) The column number is the number of standard deviations of the second principal component (so that +1 in the column means that the term structure was steepened by one standard deviation of the second principal component and -2 means that it was flattened by two standard deviations of the second principal component, assuming the curve was initially steep). All standard deviations are for a three-month period. For every simulated term structure the profit/loss of the portfolio is calculated.

The greatest portfolio loss arising from the combinations of the first two historical principal components is -\$59.18. This simulated loss is close to the actual loss of -\$54.75 (see Table 5). Simulating more than parallel shifts indicates that the actual loss should not have been unexpected. Use of only parallel shift simulations was misleading as to the size of, and even the existence of, possible losses.⁸ It is important to note that using historical interest rate fluctuations does not require any reporting information about the securities beyond what is required for modified duration; it simply requires the user to simulate more than parallel shift scenarios. Thus, better information is available at no additional cost.

In order to compare the different portfolios' exposure, the simulated portfolio values can be combined

Chart 1
Principal Components of Historical Volatility

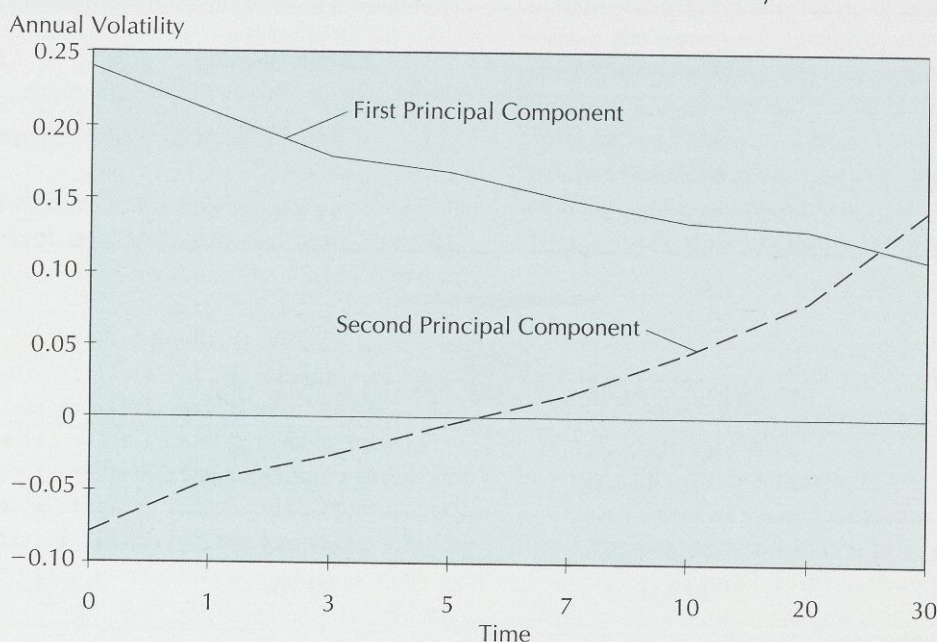


Table 7
Simulated Portfolio Values*

Standard Deviations Of First Principal Component	Standard Deviations of Second Principal Component						
	-3	-2	-1	0	1	2	3
-3	+53.99	+35.30	+16.79	-1.54	-19.70	-37.69	-55.53
-2	+46.73	+28.69	+10.82	-6.90	-24.47	-41.90	-59.18
-1	+45.59	+28.26	+11.07	-5.97	-22.89	-39.68	-56.35
0	+49.34	+32.77	+16.33	0.00	-16.22	-32.33	-48.35
1	+56.98	+41.20	+25.53	+9.95	-5.54	-20.94	-36.27
2	+67.63	+52.67	+37.79	+22.98	+8.25	-6.42	-21.03
3	+80.58	+66.44	+52.37	+38.35	+24.39	+10.47	-3.41

* In dollars.

into different test statistics. For example, the loss of -\$59.18—the worst-case scenario—would be a useful statistic for determining margin (or capital) for the portfolios. However, this method may still yield errors. First, although primary principal components capture more than 98 percent of the historical movements, term structures do not move exactly as historical patterns predict. Thus, there is additional “noise” that does not get simulated. Second, it is possible (although unlikely) for interest rates to move more than three standard deviations during the three-month period. For this reason, some may argue that caution calls for more than three standard deviations to be included in the simulation. Third, any number of historical principal components can be used in the simulation. Clearly, including more components reduces the amount of unmonitored interest rate risk. Performing simulations with these dimensions in mind permits a more realistic assessment of the portfolio’s actual interest rate exposure and results in a statistic with a greater degree of accuracy than modified duration.⁹

In the example discussed, one may question why the zero and one standard deviation movements were included in the simulations when the big gains and losses occurred in the two and three standard deviation movements. The smaller movements were included because, when options are part of a set of securities, portfolios may exist that make money for all large movements of the term structure but lose money when the term structure is relatively stable. It is, therefore, necessary to simulate more than just the extreme outcomes. For in-

stance, consider a portfolio consisting of long positions in a far, out-of-the-money call and put options on Treasury bond futures contracts. If interest rates fluctuate by only small amounts, all options in this portfolio would expire out-of-the-money and the original cost of the options would be lost. However, if interest rates fluctuate by a large amount in either direction, the portfolio has options that will finish in-the-money.

Conclusion

Both Hugh Cohen (1991) and James H. Gilkeson and Stephen D. Smith (1992) show that the nature of cash flows is important in evaluating prices and risks. This article shows that the evolution of interest rate movements is also important in these evaluations. Modified duration and parallel shift simulations give useful rough approximations of interest rate exposure. However, because of the very simplicity that makes them attractive, these models have restrictions that affect their accuracy, especially over long or volatile periods of time.

This article illustrates that at the beginning of the third quarter of 1992, parallel shift simulations failed to detect the possibility of any losses to a simple portfolio, which in actuality sustained significant losses over the quarter. However, simulations based on historical term structure fluctuations, requiring no additional reporting information, would have warned the user that losses of the magnitude actually sustained were possible.

Using Duration to Hedge Interest Rate Exposure

Hedging with Constant Interest Rates

Consider at time 0 a default-free bond that pays \$1.00 at time T in the future. Assuming a constant interest rate and continuous compounding, the result is the relationship

$$b(T) = \exp(-RT), \quad (1)$$

where R is the constant interest rate per unit of time, T is the time in the future when the bond matures, and $b(T)$ is the price of the bond. For a coupon-paying bond,

$$\text{Price of the bond} = \sum_{i=1}^{i=n} CF_i \exp(-RT_i), \quad (2)$$

where n is the total number of cash flows contained in the bond and CF_i is the i th cash flow at time T_i . Duration, a well-known function of a bond, is defined as

$$\text{Duration} = \frac{\sum_{i=1}^{i=n} T_i CF_i \exp(-RT_i)}{\text{Price of the bond}}. \quad (3)$$

In words, duration is the weighted average maturity of the cash flow of the bond. Differentiating the price of a bond with respect to R finds that

$$\frac{d(\text{Price of the bond})}{dR} = \sum_{i=1}^{i=n} -T_i CF_i \exp(-RT_i), \quad (4)$$

which leads to the well-known relationship

$$\frac{\frac{d(\text{Price of the bond})}{dR}}{\text{Price of the bond}} = -\text{Duration}. \quad (5)$$

In words, the percent change in a bond's price in response to an infinitesimal positive change in the constant interest rate is minus the duration. Thus, under the assumption of a flat term structure, the duration of a bond is a single number that indicates the sensitivity of the bond price to a small change in interest rates. This result can be extended for more than constant interest rates.

Hedging with a Term Structure

Replace the assumption of a constant interest rate, R , with a forward interest rate curve denoted by $f(T)$. The forward interest rate is the interest rate agreed upon now at time 0 for an instantaneous default-free loan at time T .

For example, if $f(30) = 8\%$, it is implied that the annualized interest rate on a default-free loan agreed upon today that will mature thirty years in the future and will be instantaneously repaid is 8 percent. The forward interest rate curve is the forward rate, $f(T)$, for all $T \geq 0$. The forward interest rate curve can be used to price default-free cash flows. Again, let $b(T)$ be the time 0 price of a default-free bond that pays \$1.00 at time T , and then

$$b(T) = \exp \left[- \int_0^T f(t) dt \right]. \quad (6)$$

For a coupon-paying bond,

$$\text{Price of the bond} = \sum_{i=1}^{i=n} CF_i \exp \left[- \int_0^{T_i} f(t) dt \right]. \quad (7)$$

Duration is similarly defined as the weighted average maturity of the cash flows:

$$\text{Duration} = \frac{\sum_{i=1}^{i=n} T_i CF_i \exp \left[- \int_0^{T_i} f(t) dt \right]}{\text{Price of the bond}}. \quad (8)$$

If the price of the bond is differentiated with respect to a parallel shift in the forward rate curve [substitute $f(t) + R$ for $f(t)$ in equation 7 and differentiate with respect to R], the result as R approaches 0 is

$$\frac{d(\text{Price of the bond})}{dR} = \sum_{i=1}^{i=n} -T_i CF_i \exp \left[- \int_0^{T_i} f(t) dt \right]. \quad (9)$$

Substituting,

$$\frac{\frac{d(\text{Price of the bond})}{dR}}{\text{Price of the bond}} = -\text{Duration}. \quad (10)$$

This equation demonstrates the advantages of using duration as a measure of interest rate exposure. For any forward interest rate curve, the duration of a cash flow is the sensitivity of that cash flow to a small parallel shift in the term structure. The examples in the text illustrate the benefits and limitations of hedging with duration. For small parallel fluctuations in the term structure, the portfolios are well hedged. However, for larger parallel movements or nonparallel movements, the portfolios may sustain severe losses.

The fact that 100 and 200 basis point parallel shifts failed to detect that the mock portfolio could sustain any loss owing to interest rate exposure, or that a single-factor model detected only the possibility of small losses, should be alarming for those who depend solely upon these measures to determine their interest rate exposure. Furthermore, the mock portfolio constructed is the most straightforward sort of portfolio possible, consisting of only deterministic default-free cash flows. In contrast, the set of securities available to investors in interest rate contingent claims contains extremely complex securities. Even a "simple" fixed-rate mortgage contains a complicated prepayment option. In addition, caps, floors, swaps, futures, options on futures, and countless embedded options add to the com-

plexity of the problem. The failure to capture the true interest rate exposure of this relatively simple mock portfolio illustrates that a large amount of interest rate exposure is undetected by these measures.

The findings reported here should serve as a warning to both investors and regulators interested in determining interest rate exposure. It is important to know that oversimplified approaches to measuring interest rate exposure can be misleading, even for simple securities. Given the complex nature of securities that are common within interest rate contingent claims, the results of parallel shift and single-factor simulations should not, by themselves, be viewed as accurately reflecting interest rate exposure.

Notes

1. One indication of this development has been the increase in the open interest of the Treasury bond futures contract. (Open interest is the number of futures contracts in existence.) Over the period from March 31, 1981, to March 31, 1993, the open interest of the nearest June futures contract increased from 51,847 to 317,804.
2. See Docket R-0764, an interagency proposal of the Federal Deposit Insurance Corporation, the Office of the Comptroller of the Currency, and the Board of Governors of the Federal Reserve System. The modified proposal presented to the Federal Reserve Board was reported in the *American Banker*, April 1, 1993, 1. It was not available in the *Federal Register* at the time of publication.
3. A basis point is 1/100 of 1 percent. If interest rates were 3 percent, a 1 basis point increase would raise them to 3.01 percent.
4. Selling a security short is equivalent to borrowing the security and selling it at its current market price with the intention of repurchasing the security at a future date and returning it to its original owner. A short seller profits when the price of the underlying security declines. Longing a security is equivalent to purchasing the security.
5. If duration is considered the first derivative of the portfolio price with respect to parallel interest rate movements, convexity would be the second derivative. For a discussion of the "convexity trap" in pricing mortgage portfolios see Gilkeson and Smith (1992).
6. The actual deviation of interest rates would lie within one standard deviation approximately 65 percent of the time. It would lie within two standard deviations approximately 95 percent of the time.
7. These components were supplied by a large financial institution in 1991.
8. Note that a one-factor historical model similar to the regulators' nonparallel shift would not have worked much better. The 0 column in Table 7 simulates only the first historical factor shifts, and the worst loss is -\$6.90. Thus, two factors are the minimum number necessary for an adequate measure of this portfolio over this period.
9. If options were included in the portfolio, one would also want to simulate the effects of changes in the market's implied volatility of interest rates to the term structure simulation.

References

- Benston, George J., and George G. Kaufman. "Understanding the Savings and Loan Debacle." *Public Interest* 99 (Spring 1990): 79-95.
- Cohen, Hugh. "Evaluating Embedded Options." *Federal Reserve Bank of Atlanta Economic Review* 76 (November/December 1991): 9-16.
- FDIC Improvement Act of December 19, 1991, Pub. Law 102-242, 105 Stat. 2236.
- Gilkeson, James D., and Stephen D. Smith. "The Convexity Trap." *Federal Reserve Bank of Atlanta Economic Review* 77 (November/December 1992): 14-27.
- Interagency proposal on revising risk-based capital standards as prescribed by Section 305 of FDICIA (Docket R-0764), Press release, *Federal Register*, July 31, 1992.
- Litterman, Robert, and Jose Scheinkman. "Common Factors Affecting Bond Returns." *Journal of Fixed Income* (June 1991): 54-61.