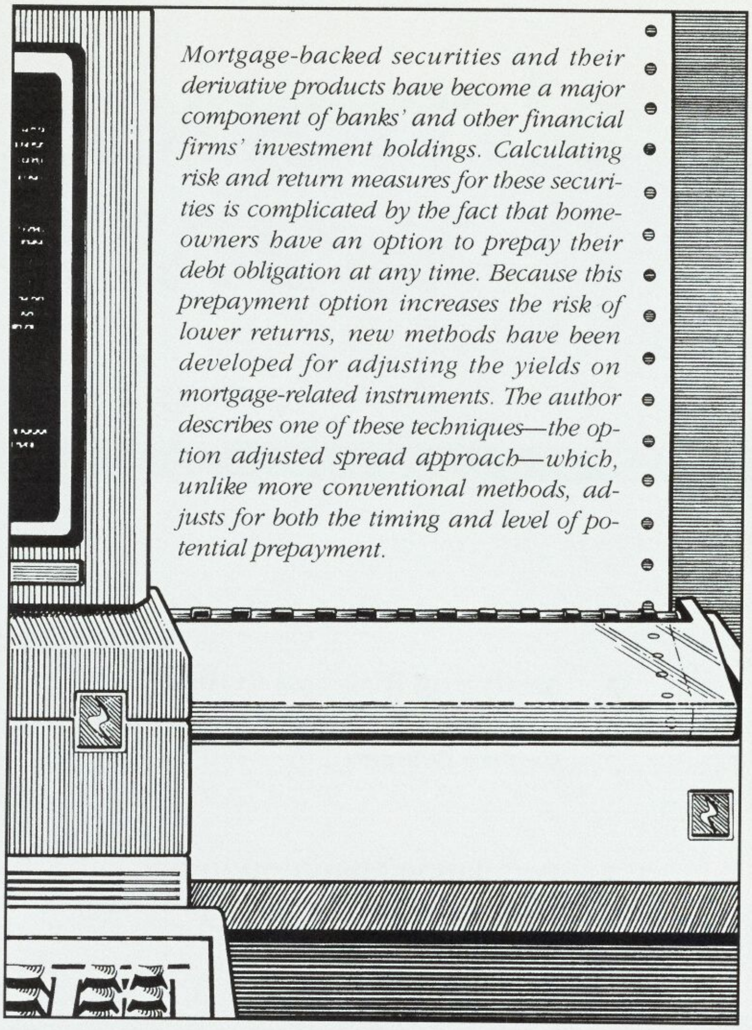
Analyzing Risk and Return for Mortgage-Backed Securities



Mortgage-backed securities and their derivative products have become a major component of banks' and other financial firms' investment holdings. Calculating risk and return measures for these securities is complicated by the fact that homeowners have an option to prepay their debt obligation at any time. Because this prepayment option increases the risk of lower returns, new methods have been developed for adjusting the yields on mortgage-related instruments. The author describes one of these techniques—the option adjusted spread approach—which, unlike more conventional methods, adjusts for both the timing and level of potential prepayment.

Stephen D. Smith

The growth of an active secondary market for home mortgages was one of the many important innovations in financial markets over the past decade. Although organizations such as the Federal National Mortgage Association (FNMA or "Fannie Mae") have been buying securities backed by the Veterans Administration and the Federal Housing Administration for decades, only recently have mortgage-related securities become an integral component of financial statements for a number of banks and other intermediaries.[1] The growth of these securities has led to a dazzling array of derivative and hybrid products produced by repackaging the basic cash flows from a pool of fixed-rate mortgages.[2] Equally bewildering to potential investors in these products is the technology invented to calculate adjusted yields or, equivalently, adjusted spreads over Treasury yields.

Yield adjustments for mortgage-backed securities, or MBSs, are necessary primarily because of the law allowing homeowners to prepay the principal balance on their mortgages without penalty.[3] Since such prepayments occur primarily when market rates fall substantially below existing coupon rates (that is, contract rates) on the mortgages, investors in the mortgages face the risk that, after having paid a premium for a high coupon security, they will be saddled with money that must be reinvested at lower (current market) rates. Investment banks and other financial firms have developed methods for adjusting the yields on mortgage-related instruments to reflect this possibility of prepayments and the corresponding lower yields.

Regulators are becoming cognizant of these issues as they build a framework for analyzing the risk profiles of an increasingly large pool of securities with unconventional cash flow characteristics. Indeed, the Comptroller of the Currency has recently provided some specific guidelines for the holdings of collateralized mortgage obligations (see William B. Hummer 1990).

The purpose of this article is to provide a nontechnical introduction to the methods used to analyze the risks and returns associated with investing in mortgage-related securities. This information should help potential investors better compare the cash flow and yield measures for mortgage-backed securities with those on alternative investments.

The Prepayment Problem

The problem of prepayment on a mortgage (an asset) is in some ways the reverse of the problem of early withdrawal of a fixed-rate certificate of deposit (CD; a liability). Imagine that a banker has issued a fixed-rate CD for some period of time, and suppose the depositor has the right to withdraw his or her funds at any time before maturity, without penalty. The depositor might withdraw early for two general reasons. If market rates on CDs rose substantially above the current rate on the CD, the CD holder might choose to withdraw early and reinvest the funds in a higher-yielding account. Whether funds are actually removed or simply rolled over into a new account at the current bank, the banker will be replacing this relatively low-cost CD with funds that will cost substantially more than the old deposit. The second reason for early withdrawal would fall into a "catch-all" category that includes noninterest factors like the depositor's moving or developing an unexpected need for funds. In either case, the bank suffers a cost if it imposes no early withdrawal penalty.

Prepayment on a mortgage is analagous to the CD example and may occur because rates fall substantially below the mortgage rate the homeowner is paying. Prepaying the mortgage for this reason is called "rational exercise" of the option. Exercising the prepayment option in other cases (such as moving for a new job) is called "irrational exercise" because such behavior is not tied directly to interest savings.[4]

Rational exercise of prepayment options forces mortgage holders to reinvest their funds at rates substantially below those they would have earned if prepayment had not occurred. Moreover, since mortgages typically have maturities much longer than those of most other assets or liabilities, the earnings loss is felt over a longer period of time. Uncertainty concerning repayment of principal makes conventional yield measures unreliable indicators of the return to be expected from holding mortgage-backed securities, as discussed below.

Shortcomings of Static Yield

Assuming that payments are guaranteed against default by a government agency such as the Government National Mortgage Association (GNMA or "Ginnie Mae"), a standard fixed-rate mortgage is, in the absence of the prepayment clause, nothing more than an annuity contract. Given a remaining life, a market price, and the promised payments per period, it is possible to find the contract's yield to maturity (YTM), or "static" yield. Static is used to denote the fact that an investor will earn the yield to maturity per period if all of the promised payments are made when due and are reinvested at the same rate (that is, rates do not change over the life of the loan). The latter condition is a well-known shortcoming of using the yield-to-maturity method to calculate the expected return on any security. It is the first condition that makes the yield-to-maturity approach particularly unattractive for analyzing mortgage-backed securities. In short, the static yield treats the payments from a mortgage-backed security as a sure thing over time, which they clearly are not.

The static yield approach will also distort calculations commonly used to measure the interest-rate risk of a security. Duration and convexity are two such measures. However defined, the duration of a fixed-income security is basically a measure of the percentage change in a security's price if interest rates change by a small amount.[5] Securities with shorter durations experience smaller price decreases, in percentage terms, for a small increase in rates than do securities with longer duration. Likewise, smaller increases occur for shorter duration securities when rates decrease.

However, duration is itself a function of the level of interest rates. In fact, duration declines as interest rates rise (and vice versa) for standard fixed-income securities. This relationship is simply a result of the fact that price changes are not symmetric. The percentage change in bond prices as rates increase is smaller than the price changes associated with equal rate decreases. Thus, the risk measure (duration) is

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inversely related to the interest-rate level. *Convexity* is the term usually applied to this "drift" in the duration. However, unlike fixed-income securities, mortgage-related securities' prepayment option makes cash flows a function of interest rates. Risk measures such as duration need to be adjusted to reflect this fact.

To summarize, the standard yield-to-maturity approach for calculating risk and return is inadequate when analyzing mortgage-related securities primarily because prepayment risk causes cash flows to be a function of interest rates and other factors. By treating the promised cash flows as certain, an investor is likely to overstate seriously the return from holding mortgage-backed securities. The option adjusted spread (OAS) approach discussed in the following section is an attempt to adjust the cash flows to reflect prepayment risk.[6]

The Logic of the Option Adjusted Spread Approach

The basic premise of the option adjusted spread approach is that prepayments, and therefore cash flows, will be a function of both the evolution of interest rates and other (for example, demographic) factors that could cause irrational prepayments on pools or portfolios of mortgages. A distribution of future cash flows (or prices) is generated by assigning probabilities to plausible alternative future interest-rate scenarios. Finally, in a step analogous to finding the discount rate (the static yield) that equates the present value of the promised cash flows to the current price, a yield measure can be found that equates the average present value of these option adjusted cash flows to the current price. The difference between this adjusted yield and that on a base security—a comparable duration Treasury bond, for example—is considered the option adjusted spread. Although this analogy is not exactly correct unless the yield curve is flat (see the appendix for general definitions of option adjusted spread), the general idea is that similar calculations result in a yield measure for mortgage-backed securities that has been adjusted for the expected level (and timing) of prepayments over the life of the mortgage pool.[7]

The critical steps to be taken in the option adjusted spread process appear in Chart 1. Although each practitioner is faced with a number of specific choices (some of which are discussed below), the steps outlined in Chart 1 must be followed for almost all of the option adjusted spread models currently in use.

Raw input is provided from a number of sources. Interest rate information is gathered from the current Treasury term structure of interest rates or yield curve. Typically, the implied one-period future interest rates (or forward rates) from the Treasury curve are used as the mean, or expected value, around which a distribution of future short-term interest rates is constructed.[8] Future mortgage rates are either constructed as a markup over the short-term rates or, in more complex models, a markup over a long-term Treasury rate that does not move exactly in concert with short-term rates. A volatility (or variance) estimate is also needed to construct a distribution of future interest rates. This parameter restricts the degree to which rates may deviate from the current term structure (the mean). Estimated prepayments are critically dependent on the volatility estimates, which may come from historical data or more exotic forms, such as implied volatilities from options contracts.

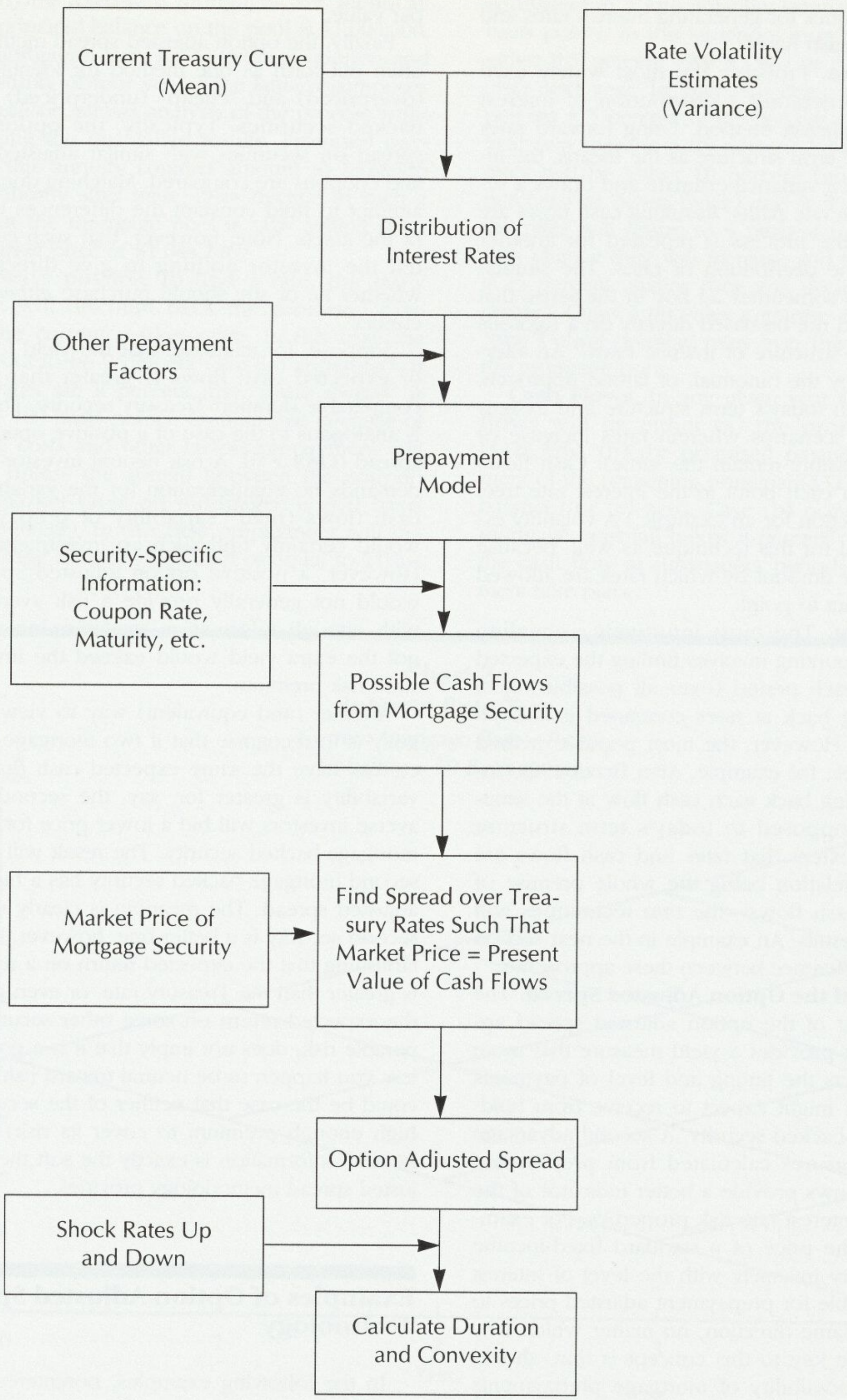
Prepayments are estimated as a function of the deviation of current coupon rates in the mortgage pool from estimated market rates and other currently available information such as the average age of the mortgage pool and other known factors (for example, the region of the country in which the mortgages originated). Future cash flows are then generated as a function of the evolution of interest rates and the demographic factors. The fact that future cash flows are dependent on the entire interest rate process is commonly referred to as *path dependency*.

Consider the case in which a downward movement in mortgage rates will prompt a prepayment. If rates increase next period and return to their original level in period two, no prepayment will occur. By the same token, if rates should fall next period and then increase to their original level, prepayment will occur. The level of rates in period two is the same in both scenarios, but the cash flow in period two is not. In this case the period two payment is either the promised payment or zero and is clearly a function of earlier interest rates (in this case interest rates in period one). Therefore, each path of rates can generate a different cash flow pattern for the mortgage.

The next step in the process is to find a constant discount factor which, when applied to every path of future short-term Treasury rates, equates the cash outflow's present value (the current market price of the mortgage) to the average present value of the cash inflows. This constant discount factor is the option adjusted spread.

The final step in Chart 1 involves shocking interest rates up and down by some amount. Combined with the current price, the new prices provide sufficient information to calculate option adjusted duration and convexity measures.

Chart 1
Steps in Option Adjusted Spread Calculation



Computational Choices

The procedure outlined in Chart 1 has at least two different versions, depending on the practitioners' choice of techniques for generating interest rates and discounting the cash flows.

Interest Rates. Probably the most widely used approach for generating a distribution of interest rates is the simulation method. Using forward rates embodied in the term structure as the means, the investigator inputs a variance estimate and draws a series of short-term rate paths. Resulting cash flows are generated and the process is repeated for another drawing from the distribution of rates. The simulation approach is sometimes ad hoc in the sense that the method need not be based directly on a rigorous link to the term structure of interest rates.[9] An alternative is given by the binomial, or lattice, approach, which starts with today's term structure and assigns probabilities to scenarios wherein rates increase or decrease (or possibly remain the same). Cash flows are calculated at each point in the interest rate tree. (See the next section for an example.) A volatility estimate is needed for this technique as well, because it determines the amount by which rates are allowed to vary from point to point.

Discounting. The most intuitively appealing method for discounting involves finding the expected cash flow for each period (over all possible rates) and discounting back at rates contained in today's Treasury curve. However, the most popular method in use today (see, for example, Alan Brazil 1988) involves discounting back each cash flow at the simulated rate (as opposed to today's term structure rates). To the extent that rates and cash flows are correlated—correlation being the whole premise of rate-sensitive cash flows—the two techniques will yield different results. An example in the next section illustrates the difference between these approaches.

Properties of the Option Adjusted Spread. The foremost benefit of the option adjusted spread approach is that it provides a yield measure that more accurately reflects the timing and level of payments that an investor might expect to receive from holding a mortgage-backed security. A second advantage is that risk measures calculated from prepayment adjusted cash flows provide a better indicator of the security's true interest-rate risk properties. For example, although the price of a standard fixed-income security will vary inversely with the level of interest rates, it is possible for prepayment adjusted prices to change in the same direction, no matter which way rates move. The key to this concept is that, should rates fall, the possibility of mortgage prepayments may go up, in which case investors may bid *down* the mortgage-backed security's price. This action is, of course, the opposite of what would happen with a truly fixed-income security like a Treasury bond. This "whipsaw" effect is particularly evident in mortgage-backed securities that are selling at a premium from par value.

Finally, the option adjusted spread methodology is often put forth as one method for identifying "rich" (overpriced) and "cheap" (underpriced) mortgage-backed securities. Typically, the option adjusted spread on securities with similar adjusted durations and coupons are compared. Matching durations is an attempt to hold constant the differences in the risks of the assets. Note, however, that such comparisons tell the investor nothing to give direction about whether he or she should purchase *either* of the securities.

Suppose, for example, that the yield for a stream of expected cash flows is greater than that for a comparable duration Treasury security. This situation is analogous to the case of a positive option adjusted spread (OAS > 0). A risk neutral investor—one who demands no compensation for the variability of the cash flows (read "variability of prepayments")—would certainly find such an investment attractive. However, a positive option adjusted spread alone would not generally provide a risk averse investor with enough information to determine whether or not the extra yield would exceed the investor's desired risk premium.

Another (and equivalent) way to view this ambiguity is to recognize that if two mortgage-backed securities have the same expected cash flows but the variability is greater for, say, the second one, risk-averse investors will bid a lower price for the second mortgage-backed security. The result will be that the second mortgage-backed security has a higher option adjusted spread. The meaning is clearly not that the second security is a better one, however. In short, establishing that the expected return on a risky security is greater than the Treasury rate, or even greater than the expected return on some other security of comparable risk, does not imply that it is a good buy unless you happen to be neutral toward risk (because it could be the case that neither of the securities has a high enough premium to cover its risk). Such "risk neutral" information is exactly the sort the option adjusted spread methodology provides.

Examples of Option Adjusted Spread Technology

In the following examples, noninterest rate "irrational" factors that might influence prepayments are ignored for simplicity. Moreover, for simplicity the examples will deal with a case where the term structure is flat.

Consider a mortgage-backed security for which the underlying fixed-rate mortgages are identical. The total principal balance on the pool is $1,000,000, and the mortgages carry an 11 percent coupon and have a maturity of four years. The latter assumption, while unrealistic, allows analysis of the process without changing the qualitative results. Promised payments on this annuity contract amount to $322,326 per year. Shortly after issue, rates decline in such a fashion that the mortgage pool is now selling for a current market price of $1,044,246.

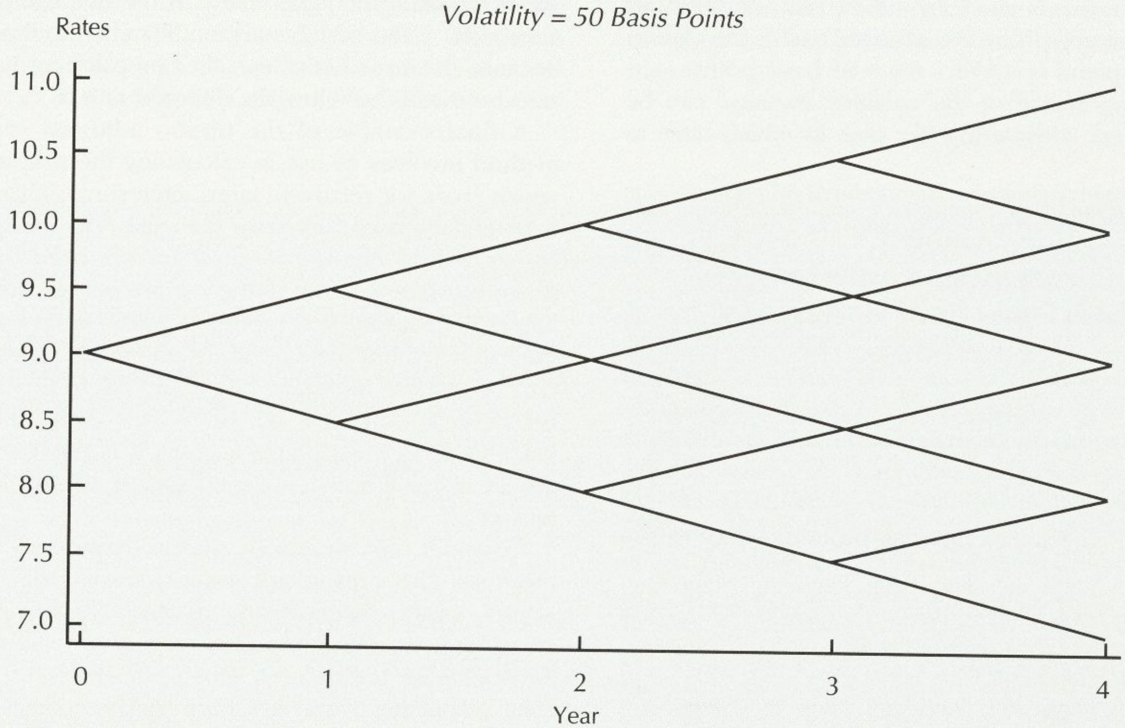
It is always possible to find a rate, the static yield, that will discount back the promised cash flows to the current market price. In this case the rate turns out to be 9 percent (see the appendix). If Treasury rates are 8 percent, the "static spread" is 9% – 8% = 100 basis points. Assume that because of refinancing costs and other factors prepayment will not occur unless the spread between the coupon rate and market rates on mortgages is 300 basis points (3 percent). In this case prepayment will occur if mortgage rates are less than or equal to 11% – 3% = 8%.

Interest Rates. A simple binomial (or two-state) process is used to model changes in interest rates.[10] Rates can move up and down with equal likelihood from their current levels. The assumed change is equivalent to a rate volatility estimate. Let this be 50 basis points. In this situation, mortgage rates will be either 9.5 percent (9% + .5%) or 8.5 percent (9% – .5%) next year. In year two rates will be either 9.5 percent ± .5 percent or 8.5 percent ± .5 percent. So in year two there is a 25 percent [(.5)(.5)] chance that rates will be either 10 percent (go up twice) or 8 percent (go down twice) and a 50 percent chance the rates will return to 9 percent (that is, a 25 percent chance they will increase and then decline and a 25 percent chance they will decline and then increase). Chart 2 provides a graphic representation (a "tree") of all possible rates over the four-year life of the mortgage pool.

Cash Flows. In any given year the realized cash flows will be either (a) the promised payment ($322,326), (b) the promised payment plus prepayment of the remaining principal ($322,326 + principal balance) or (c) zero. Case (b) occurs if and when mortgage rates fall below 8 percent, while case (c) is the cash flow in subsequent periods should prepayment take place.

Chart 2
Interest Rate Tree

Current Mortgage Rate = 9.0%
Volatility = 50 Basis Points



Given the volatility estimate of 50 basis points there is no chance of prepayment in period one because rates can only fall as low as 8.5 percent. The cash flow is certain to be $322,326. However, in period two there is a 25 percent chance that rates will drop to 8 percent and prepayment will occur. Should this happen, the cash flow will be $322,326 + $551,992 = 874,318, where $551,992 is the remaining principal balance on the mortgage. Once prepayment has occurred, the cash flow in years three and four will be zero.

The expected (or average) cash flow from the pool can also be calculated by multiplying each possible cash flow by the probability that it will occur. In year one, for example, there is a 0 percent chance of prepayment because rates will always be above 8 percent. Alternatively, in year two there is a 25 percent chance the mortgage rate will be 8 percent and the corresponding cash flow $874,318. There remains a 75 percent chance the rates will be above 8 percent and the cash flow will be $322,326. Thus, the expected cash flow is $874,318 (.25) + $322,326 (.75) = $460,324. Table 1 provides the possible and expected cash flows by period.

In this special case, the option adjusted yield can be calculated by finding a discount factor that equates the discounted value of the expected cash flows (see Table 1) to the current price. The rate that solves this discounting problem is about 8.85 percent, somewhat less than the static yield of 9 percent. Finally, the option adjusted spread in this simple example is given by subtracting the Treasury rate (8 percent) from the adjusted yield; the option adjusted spread is 8.85% – 8% = 85 basis points.

This importance of the volatility measure can be seen by first considering the case in which rates in each period can change by only 25 basis points rather than 50. The mortgage rates now have no chance of falling to 8 percent before the mortgage matures. Therefore, the expected cash flow is $322,326 in every period and the option adjusted spread will be the same as the static spread (100 basis points). Alternatively, the option adjusted spread is actually negative (–12 basis points) if the volatility estimate is doubled to 100 basis points.

Option adjusted spread estimates are also extremely sensitive to the discounting method used by the investigator. The calculation procedure described above can be viewed as a "discounted average cash flow" approach. Alternatively, as mentioned earlier, many investment houses prefer what could be called an "average discounted cash flow" (or average price) approach. This method differs from the discounted average cash flow approach because the cash flows are discounted back at the realized interest rates in Chart 1, as opposed to today's rate. In this case, low cash flows are, on average, associated with lower discount rates (the prepayment problem) and vice versa for higher rates. The net result is that the option adjusted spread will be different (typically lower). For this particular example the option adjusted spread is actually a negative 2 percent (– 200 basis points) when using the average price method.

The appendix contains equations (2) and (3), which show the mathematical distinction between the approaches.[11] This rather large difference in results comes from the fact that the high cash flows are discounted at higher rates in the average price approach.[12] The benefits are not symmetric, though, because the zero cash flow after prepayment is still zero no matter how low the discount rate is.

A final example of the option adjusted spread method involves its use in calculating the risk an investor faces for relatively large interest-rate changes. A potentially useful exercise is to see how mortgage prices would change if rates are shocked up or down by some amount in the current period. For example, if Treasury and mortgage rates increased immediately by 100 basis points, there would be a zero chance that the mortgage rate could fall to 8 percent before year four (that is, 9% + 1% = 10% and the volatility is 50 basis points a year). It is possible, using the original option adjusted spread, to calculate a new price.

Consider, for simplicity, the average cash flow method. The option adjusted spread is 85 basis points, while current mortgage and Treasury rates are 9 percent and 8 percent, respectively. Suppose rates increase immediately to 10 percent and 9 percent, respectively, and volatility remains constant at 50 basis points. Then, using the average cash flow method, the new price is $1,025,057, which is less

Table 1
Possible and Expected Cash Flows from Mortgage by Period

Period	Possible Cash Flow	Expected Cash Flow
1	$322,326	$322,326
2	$322,326 or $874,318	$460,324
3	$322,326 or $0	$241,745
4	$322,326 or $0	$241,745

than \$1,044,246 because the present value of the future cash flows has been reduced.

Alternatively, if rates should immediately fall by 100 basis points, prepayment will occur as soon as possible (probably in period one), because the new mortgage rate is 8 percent. In this case the new price is \$1,029,208, which is also lower than the original price. This whipsaw effect occurs because the lower interest rate causes earlier prepayment, more than offsetting the present value added from having a lower rate with which to discount back the cash flows. These examples should make clear that, while it is conceptually an averaging technique, the actual option adjusted spread is extremely sensitive to a variety of input and methodological decisions practitioners make.

Conclusion

The calculation of yields on mortgage-backed securities is complex primarily because of the difficulty of valuing an owner's option to prepay the mortgage. Despite their computational complexities, current approaches to mortgage valuation are still just averaging techniques. As shown in this article, cash flows, and therefore returns, depend on the entire path of interest rates; that is, they are path dependent. Therefore, the interest-rate history over the life of a mortgage-backed security is an important piece of information. An additional complicating factor arises from the difficulty in determining whether a mortgage that looks "cheap" is really undervalued or whether it is selling at a discount because of above-average risk. Two other caveats merit attention. It is possible, especially for mortgage securities selling at premium above par, to encounter interest-rate changes that "whipsaw" the investor; that is, prices may fall if rates either decline or increase. The adjusted yield measures can be very sensitive to the inputs used—for example, the assumed volatility of interest rates.

The option adjusted spread has become a favored technology for dealing with this problem because it adjusts for both the timing and level of potential prepayments. In fact, the approach can be used in a variety of settings because many assets and liabilities have options of some sort embedded in their structure.[13]

While potentially useful, the option adjusted spread is not a panacea for investors hoping to find undervalued assets on a risk-adjusted basis. No formal model currently provides a basis for decomposing the option adjusted spread into compensation for risk and excess returns. Viewed properly as a yield (or yield spread over Treasury), however, the option adjusted spread measures are probably clearer indicators of the likely return than conventional static yield calculations. The trade-off here involves the option adjusted spread's sensitivity to inputs (such as volatility and the prepayment model) and the valuation framework the investigator employs. Potential purchasers, as well as regulatory personnel, should be aware of these facts. Asking potential sellers for information concerning the actual risk/return profiles of previously analyzed mortgage-backed securities would be useful in this regard.[14]

Appendix

This appendix contains the general formulas used for calculations in the text. As noted below, the actual examples usually involve simplified versions of these equations (for example, the term structure is flat).

M = market price of the mortgage-backed security
C = promised payment per period on a fixed-rate contract
Y = static yield to maturity
N = remaining maturity of the contract

The static yield is the rate that equates the present value of the promised cash inflows (Cs) to the current market price. That is, Y solves

$$M = \frac{C}{(1+Y)} + \frac{C}{(1+Y)^2} + \ldots + \frac{C}{(1+Y)^N}. \qquad (1)$$

If G is the yield on a comparable maturity (more specifically, duration) Treasury security, the static spread is just $Y - G = S$. For the example in the text, C = \$322,126; N = 4; M = \$1,044,246, so Y = 9% and S = 9% – 8% = 1%.

More notation and calculations are needed to calculate the option adjusted spread. Let f_t = one-period forward rate for government securities in period t, $f_t = (1 + R_t)^t/(1 + R_{t-1})^{t-1}$, where R_t is the current spot rate for a government security that makes one payment in period t and zero otherwise. Define r_t = actual interest rate on one-period government securities in period t. Note that this number is unknown today if $t > 1$. With this notation, $f_1 = r_1 = R_1$. Finally, let C_t = actual cash flow realized from the mortgage in period t. Notice that there are two general cases: $C_t = C$ (promised payment) if there is no prepayment, or $C_t = C$ + remaining principal balance $> C$ if there is prepayment. In this case $C_{t+1} = C_{t+2} = \ldots = C_N = 0$. For notational purposes let $E(\bullet)$ denote expectation (or expected value) of the term inside the parenthesis. The expected value of x, for example, is given by the sum of the possible outcomes for x, multiplied by the probability that each outcome will occur. There are two approaches to calculating the option adjusted spread.

Expected Cash Flow Approach

Search for a discount factor, O_c, such that O_c solves

$$M = \frac{E[C_1]}{(1+f_1+O_c)} + \frac{E[C_2]}{(1+f_1+O_c)(1+f_2+O_c)} \qquad (2)$$

$$+ \ldots + \frac{E[C_N]}{(1+f_1+O_c)(1+f_2+O_c)\ldots(1+f_N+O_c)}.$$

The term $E[C_t]$, $t = 1, \ldots 4$ is given by the right-hand column of Table 1. These terms can be calculated by multiplying the possible cash flows by the probability that rates will be above or below the cutoff rate for prepayment. In particular, with equal probabilities (.5) of an increase or decrease in rates and volatility = 50 basis points, $E[C_1]$ = \$322,326 (1). Because there is a 25 percent chance [(.5)(.5) = .25] that the Treasury rate will fall to 7 percent in year two (mortgage rate = 8%), the expected cash flow in period two is given by multiplying the possible cash flows in Table 1 by their respective probabilities. This calculation gives the expected value, or $E[C_2]$ = \$322,326 (.75) + \$874,318(.25) = \$460,324. Likewise, because the cash flow in period three is zero if prepayment occurs, the expected cash flow in this case is $E[C_3]$ = \$322,326 (.75) + \$0 (.25). The expected cash flow in period four will be the same, so $E[C_4]$ = \$322,326 (.75) + \$0 (.25). Solving (2) for the option adjusted spread yields O_c = 85 basis points. Notice that when the term structure is flat, the forward rates will be the same. Thus, solving (2) is the same, in this case only, as the simpler approach used in the text—solving for the yield first and then subtracting the (constant) Treasury rate.

Expected Price Approach

Find a discount factor, O_p, that solves the following equation:

$$M = E\left[\frac{C_1}{(1+r_1+O_p)} + \frac{C_2}{(1+r_1+O_p)+(1+r_2+O_p)} \right. \qquad (3)$$

$$\left. + \ldots + \frac{C_N}{(1+r_1+O_p)(1+r_2+O_p)\ldots(1+r_N+O_p)}\right]$$

In this case the rs are the realized one-period Treasury rates from the tree in Chart 2. For this example, the current Treasury rate is 8 percent and the term structure is flat; therefore, r_1 = 8%. However, next-period rates will be either 7.5 percent or 8.5 percent, so r_2 = 7.5% with probability (.5) and r_2 = 8.5% with probability .5. Interest rates for future periods are calculated in a similar fashion. The possible Cs are found in the first column of Table 1. Using (3), O_p can be calculated as O_p = –2.0% or –200 basis points.

Calculating New Prices

The new prices are calculated by fixing O_c (or O_p), adding a constant amount ($\pm$ 100 basis points) to each f_t (or r_t), and finding the new cash flows associated with these rates. The new M is then given by simply using the discount formula. For the expected cash flow approach, a 100-basis-point increase in rates will result in a zero probability of prepayment, so

$$E[C_1^+] = E[C_2^+] = E[C_3^+] = E[C_4^+] = \$322{,}326,$$

where C_t^+ is the cash flow for a rate increase. The discount rates in equation (2) are $(1 + f_1 + .01 + .0085)$ for period one, $(1 + f_1 + .01 + .0085)(1 + f_2 + .01 + .0085)$ for period two, and so forth. The new price can be calculated using equation (2). The rate decrease is the same problem except that, if rates should fall by 100 basis points, prepayment occurs immediately, and

$$E[C_1^-] = \$1{,}110{,}000 \text{ and } E[C_2^-] = E[C_3^-] = E[C_4^-] = O.$$

The discount factors are $(1 + f_1 - .01 + .0085)$ and so on. An analogous approach would also be used if one were applying the expected price methodology.

Notes

1. The book value of domestic bank holdings of guaranteed and nonguaranteed mortgage-backed securities was about $200 billion at the end of 1989. About $35 billion of these assets was held by banks in Alabama, Florida, Georgia, Louisiana, Mississippi, and Tennessee.
2. See Sullivan and Lowell (1988) for an introduction to the mechanics of the mortgage securities market and the major participants.
3. For convenience the terms *mortgage* and *mortgage-backed security* will, when there is no ambiguity, oftentimes be used interchangeably.
4. See, for example, Hendershott and Van Order (1987) for a formal options approach to modeling rational exercise.
5. Interestingly, duration can also be viewed as a weighted average time to maturity, where the weights for each period t are equal to the present value of period t's cash flow divided by the total present value (the market value, or price). Shorter-term securities are those that have relatively large cash flows in earlier periods. For fixed-rate mortgages, however, the promised cash flows are a level annuity, so the weights are simply the present value factor for each period relative to the present value interest factor for an annuity.
6. It should be noted that some adjustment for early prepayments is typically incorporated into the static yield framework. For example, yield quotes provided by investment banks will often incorporate a constant prepayment rate per month for the remaining mortgage balance.
7. Those familiar with options pricing theory will realize that this method is only loosely based on conventional option pricing models. Although it is true, for example, that an option's value can, under certain circumstances, be viewed as the expected payoff over a transformed (or "risk neutral") probability distribution, the approach used here is based on the original probabilities. In essence, such a formulation amounts to assuming that investors are, in some sense, truly indifferent to risk. See notes 8 and 11 for additional comments on this point.
8. Such a formulation is equivalent to assuming that investors demand no risk premium for holding longer-term bonds. See Abken (1990) for a review of this "expectations" hypothesis of the term structure.
9. Schwartz and Torous (1989) provide a good example of a method for explicitly linking interest rate processes to a theory of the term structure in the context of valuing mortgage-backed securities.
10. See Abken (1990) for a discussion of binomial processes in the context of the term structure of interest rates.
11. From a somewhat more technical perspective this choice of assumptions may be viewed as one of choosing between "global" and "local" risk neutrality on the part of investors. With local risk neutrality investors expect to earn the same return from all securities over any one period of time. It can be shown by repeated substitution that this results in the expected price approach discussed in the text. See Cox, Ingersoll, and Ross (1985) for a mathematical discussion of the "local" expectations hypothesis.
12. Part of the difference may also come from the fact that in this example the average realized rate is set equal to today's rate. This approach is different from the more mathematically correct one, which would be to make the average realized one-period bond price equal to today's one-period bond price.
13. See, for example, Ayaydin, Richard, and Rigsbee (1989) for a discussion.
14. For example, Toevs (1990) provides evidence on the return characteristics of various mortgage-backed securities when compared to duration-matched Treasury securities.

References

Abken, Peter. "Innovations in Modeling the Term Structure of Interest Rates." Federal Reserve Bank of Atlanta *Economic Review* 75 (July/August 1990): 2-27.

Ayaydin, Sirri, Charles Richard, and Stephen Rigsbee. "Applying an OAS Model Consistently." *Financial Managers Statement* 11 (November/December 1989): 65-75.

Brazil, Alan. "Citicorp's Mortgage Valuation Model: Option Adjusted Spreads and Option Based Durations." *Journal of Real Estate Finance and Economics* 1 (June 1988): 151-62.

Cox, John C., Jonathan E. Ingersoll, Jr., and Stephen A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica* 53 (March 1985): 385-407.

Hendershott, Patric, and Robert Van Order. "Pricing Mortgages: An Interpretation of the Models and Results." *Journal of Financial Services Research* 1 (September 1987): 77-111.

Hummer, William B. "Say Hello to the CMO." *Bankers Monthly* 107 (December 1990): 54.

Schwartz, Eduardo S., and Walter N. Torous. "Prepayment and the Valuation of Mortgage-Backed Securities." *Journal of Finance* 44 (June 1989): 375-92.

Sullivan, Kenneth, and Linda Lowell. "Mortgage Pass-Through Securities." In *The Handbook of Mortgage-Backed Securities*, edited by Frank J. Fabozzi, 69-114. Chicago: Probus Publishing, 1988.

Toevs, Alden. "Laser Brains Rejoice: Analytical Methods Can Help Shape Market Equilibrium Prices." *Financial Analysts Journal* 46 (November/December 1990): 8-10.