An Introduction to Portfolio Insurance

Peter A. Abken

Portfolio insurance is distinguished from other types of hedging by its continuous adjustments of the investment position. Like other forms of hedging, however, portfolio insurance does not perform perfectly, as was amply demonstrated during the stock market crash of October 19.

The stock market crash on October 19, 1987, and subsequent market turmoil heated up debate over two relatively new trading techniques, stock-index arbitrage and portfolio insurance. While both are types of so-called program trading that involve use of stock-index futures contracts, stock-index arbitrage has come to be the better known of the two. This article attempts to demystify portfolio insurance by explaining this portfolio management technique and by illustrating its performance in recent market history. Consideration is also given to whether, as some market observers allege, portfolio insurance destabilized the stock market and exacerbated the crash.

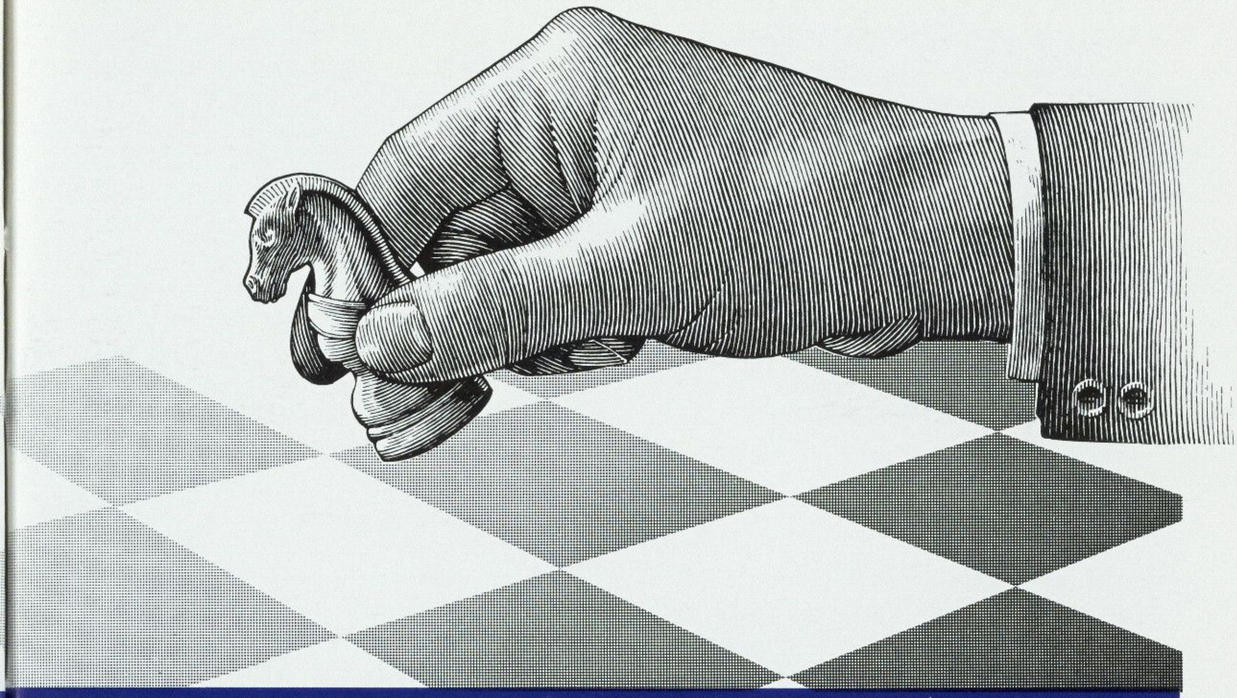
Portfolio insurance (PI) programs have been offered by major banks, brokerage firms, insurance companies, and specialized PI firms. These insurance programs have attracted large institutional users, primarily pension funds. Compared with the potential market, however, portfolio insurance is still relatively obscure. Before the stock market crash in October 1987, estimates of asset values covered by portfolio insurance programs ranged from $60 billion to $100 billion. Even so, insured portfolios constituted only a small percentage of total pension fund assets.

This article presents the basic theoretical and practical aspects of portfolio insurance. It also reports simulations of portfolio insurance using the Standard and Poor's (S&P) 500 index as the underlying portfolio. Two different kinds of PI implementations are considered: the index/Treasury bill and index/futures methods. The latter is of particular interest both because it is the actual method most commonly used and because the literature on portfolio insurance has not treated it in any depth. The article concludes with a discussion of the recent controversy surrounding portfolio insurance.

Hedging Instruments

A review of the underlying financial instruments facilitates discussion about portfolio

The author, a specialist in futures and options markets, is an economist in the financial section of the Atlanta Fed's Research Department. *He thanks Steven Feinstein for helpful comments and both Sharon Fleming and Allison Smith for excellent research assistance.*



insurance. The standard means for implementing portfolio insurance strategies uses stock-index futures contracts, usually the S&P 500 contract traded on the Chicago Mercantile Exchange (CME). An alternative to this approach would be simply to employ stock-index option contracts. Both futures and option contracts are referred to as derivative assets because their value depends on the value of an underlying asset, in this case a unit of the S&P 500 index.[1] A unit or "share" of the S&P 500 index is a portfolio of stocks that is identical in composition to the index. In general, a futures contract establishes a certain price at the time of purchase (sale) for deferred delivery of a specified quantity of a commodity or asset. By purchasing an S&P 500 futures contract, which is referred to as taking a *long* position in the contract, the buyer is obligated to take future delivery of the cash value of the S&P 500 index upon expiration of the contract.[2] Selling a contract, or taking a *short* position, binds the seller to pay the cash value. The obligation to make actual payment can be nullified at any time if an investor simply takes an opposite position in futures; for example, an investor could buy a futures contract if one had been previously sold, and vice versa. Gross profit on a long or short futures position solely depends upon the difference between the value of the initial futures position and its final value.

The usefulness of stock-index futures for hedging the value of a portfolio will be discussed below in detail. For now, suffice it to say that the essence of a hedging operation entails taking a futures position whose value is negatively correlated with the asset or commodity being hedged. Suppose, for instance, that a portfolio manager wants to protect a portfolio from a drop in value until some future date when the portfolio will be sold. The manager could sell stock-index futures that expire at the time he intends to liquidate the portfolio, so that for every dollar the portfolio loses (gains) in value, the short futures position gains (loses) a dollar. Obviously, although this procedure entails no risk of loss, it also presents no opportunity for gain. Using futures in this way sacrifices all "upside" potential for the portfolio. In fact, as will be demonstrated below, holding a short futures position against a portfolio is equivalent to liquidating the portfolio and holding only cash, or more precisely, holding a risk-free asset.

A portfolio's upside potential is retained by a stock-index put option, which gives the purchaser the *right* but not the *obligation* to sell the underlying units at a specified price upon expiration of the contract. (A call option gives the purchaser the corresponding right to buy.) The price specified in an option contract is known as the exercise or striking price. For an index option, the exercise price is a particular index value.

Unlike futures contracts, option contracts offer asymmetric payoffs. For example, the value of a portfolio of stocks held along with a stock-index put option may fall below the exercise price at the time of the option's expiration. This portfolio loss will result in an opposite, offsetting gain in the value of the put option, an outcome similar to that for a short index futures position. However, a rise in the portfolio's value above the exercise price is offset only by the cost of the option. Thus, this kind of option strategy, commonly known as a protective put, both "insures" a portfolio and permits it to participate in rising markets. The price of the option, or the option premium, represents the cost of the portfolio insurance.

The use of index puts in conjunction with an index portfolio is one means of creating an insured portfolio, but it has several limitations. Exchange-traded index options have a maximum maturity of nine months and are offered only for a limited number of exercise prices. Furthermore, such options may be exercised not only upon expiration, but at any time after purchase. This last feature of a so-called American option is not needed for the type of portfolio insurance generally practiced; the extra flexibility it offers to the option holder adds to the cost of the option. For these reasons, the portfolio insurance strategies considered below will involve the creation of European put options, which may be exercised only at expiration.

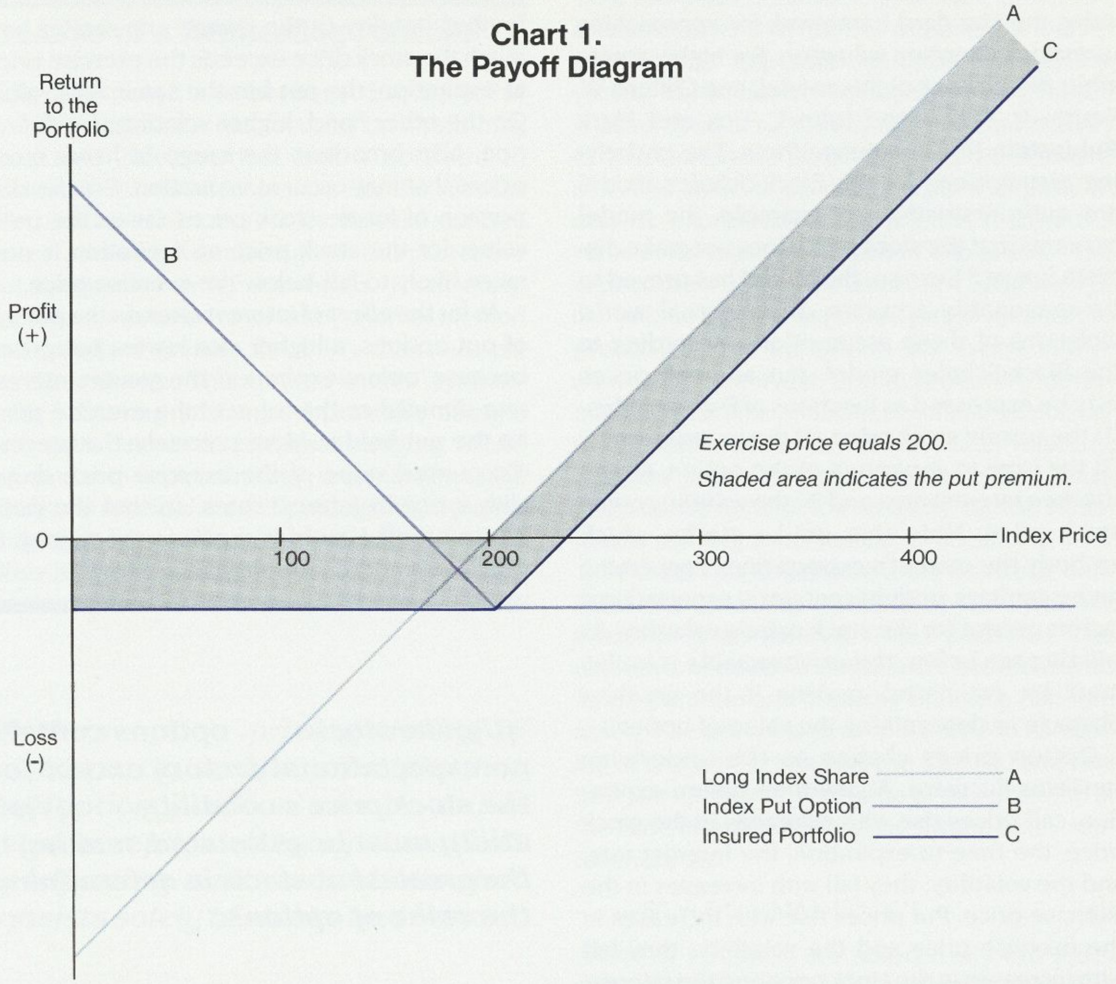
The Put Option as Insurance

As mentioned above, the combination of a long index put option with a unit of the S&P 500 index creates one type of portfolio insurance. For a better intuitive understanding of this relationship, the widely used payoff diagram may be helpful (see Chart 1). The diagram illustrates the range of returns available to an insured portfolio. Its horizontal axis indicates the index price at expiration of the put, and the vertical axis shows the return to the portfolio. A long position in the index is represented by line A, which intersects the horizontal axis at 200, the price at which the index was originally purchased. At expiration, every dollar rise above this purchase price corresponds to a dollar in capital appreciation by the portfolio; conversely, every dollar drop below 200 represents capital losses. Line A therefore reflects the returns to the uninsured S&P 500 portfolio. Line B depicts the returns to an index put option whose exercise price is 200. If the index price climbs to 200 or higher, the put expires unexercised, worthless. The option is said to have expired "at-the-money" (final index price equals the exercise price) or "out-of-the-money" (final index price exceeds the exercise price). The constant negative return represents the cost of the put, the put premium, which is the maximum loss that can be realized from a long put. On the other hand, if the index price is below 200, the put ends "in-the-money" and its return rises dollar for dollar with the drop in the index price.

By summing vertically the returns to the long index (line A) and long index put (line B) positions, the payoff line for the insured portfolio, line C, is derived. The maximum loss below 200 is limited to the put premium, while above 200 the portfolio rises dollar for dollar with a rise in the index price. Notice, however, that the return to the insured portfolio for index prices above 200 is shifted downward by the amount of the put premium—the cost of portfolio insurance. The "upside capture" on the insured portfolio is less than 100 percent of the return on its uninsured counterpart due to the initial investment in the index put.

In view of this cost, portfolio insurance should be seen as a way of trading off upside potential for downside protection. Electing to insure a portfolio therefore alters its return distribution. The decision to buy insurance depends on the portfolio's objectives. As reason would suggest, funds geared toward investors who are more risk-averse than average would choose insurance.[3] Besides the very serious practical questions about the effectiveness of these insurance strategies during turbulent markets, there is

Chart 1.
The Payoff Diagram



some controversy about whether simpler, traditional strategies are more cost-effective. This issue is complex and unresolved, and is beyond the scope of this article.

The index put option in Chart 1 was chosen to be "at-the-money"; that is, its exercise price equals the purchase price of the index portfolio. The insured portfolio finishes with a loss at expiration when the index itself ends unchanged from its initial value, as the cost of the put option must be subtracted. By choosing an index put option whose exercise price exceeds the purchase price of the index portfolio, the investor trades off greater downside protection—a higher floor for the portfolio return—against smaller potential upside returns. This approach accords with the insurance analogy since choosing a higher exercise price is like reducing the size of the insurance deductible, thus increasing the insurance and raising its cost. Similarly, reducing the exercise price raises the deductible and lowers the cost of insurance.

Factors Affecting the Cost of Insurance

The exercise price is but one of several determinants of the insurance cost. For the purposes of this analysis, there are two general kinds of factors that influence the cost of portfolio insurance: (1) those bearing on the value of put options and (2) those arising from the implementation of the insurance, namely, transactions costs. Option valuation factors are discussed here and the second category is deferred until later.

The Black-Scholes option pricing model provides the standard framework for approaching questions of option valuation. For highly accessible discussions of this model, see Clifford W. Smith, Jr. (1977) and John C. Cox and Mark Rubinstein (1985), among others. The underlying assumptions for the Black-Scholes model are quite restrictive; for example, the model assumes that the stock price does not make discrete jumps.[4] Even so, the model has proved to be reasonably accurate despite real world violations of those assumptions. According to the Black-Scholes model, put and call prices may be expressed as functions of five variables: (1) the current stock price, (2) the exercise price, (3) the time to expiration of the option, (4) the risk-free interest rate, and (5) the volatility of the stock price. Note that unlike stocks, which embody the market's expectations concerning future earnings, options contain no expectational factors except for the stock price's volatility. As will be seen below, this unobservable volatility must be estimated, making it the greatest obstacle in determining the value of options.

Option prices change as the underlying variables fluctuate. At any time before expiration, call prices rise with increases in the stock price, the time to expiration, the interest rate, and the volatility; they fall with increases in the exercise price. Put prices rise with increases in the exercise price and the volatility; they fall with increases in the stock price and the interest rate. A lengthening of the time to expiration has an ambiguous impact on put prices. In evaluating these effects, it is assumed that when a change in any particular variable is considered, all other variables remain constant.

An intuitive understanding of the relationships noted above facilitates discussion of option pricing and portfolio insurance. A higher stock price before the option expires reduces the put price because it lowers the probability that the put will end in-the-money. Assuming all other variables—including the volatility and the current stock price—remain unchanged, a higher exercise price increases the likelihood that the put will expire in-the-money. This greater probability raises the put price. A higher stock price volatility also increases the put price. On the one hand, given a fixed current stock price, greater dispersion of future stock prices cannot drive the put price any lower than zero. This is so because put prices, like stock prices, have limited liability to the owner: no matter how much the stock price exceeds the exercise price at expiration, the put has the same zero value. On the other hand, higher volatility, by definition, also broadens the range of lower stock prices that may occur at expiration. Greater dispersion of future stock prices raises the put's value, for the stock price at expiration is now more likely to fall below the exercise price.

As for the effect of interest rates on the pricing of put options, a higher rate lowers put prices because, before expiration, the greater interest rate diminishes the value of the exercise price for the put holder. More precisely, the present discounted value of the exercise price drops with a rise in interest rates, so that the put's future payoff, should it expire in-the-money, is worth less at the current time.

"(U)nlike stocks . . . options contain no expectational factors except for the stock price's volatility. . . . (V)olatility must be estimated, making it the greatest obstacle in determining the value of options."

Lengthening the time to expiration has an ambiguous effect on put prices. An increase in the time to expiration reduces put prices if the ratio of the stock price to the exercise price is small. In other words, the more in-the-money the put is, the less value the prospective receipt of the exercise price will have since the payment is further out in the future. This price-reducing effect may be dominated by a second effect, however. The longer the time to expiration, the greater will be the variability of the stock price over the life of the option, which, everything else being constant, boosts the probability that the put will expire in-the-money. For a sufficiently high ratio of stock price to exercise price, this second effect will overwhelm the first, so that lengthening the time to expiration raises put prices.

Creation of a Replicating Portfolio

To this point, the present discussion of portfolio insurance has focused on combining an actual index option with a portfolio to achieve the desired insurance. For the reasons cited earlier—short maturities, limited exercise prices, and early exercise rights—this protective put approach has its drawbacks. Fortunately, the same end can be reached by another method that provides greater flexibility than outright purchase of a put. This method entails "synthetically" creating an insured portfolio using a technique known as dynamic hedging. While the protective put strategy is generally quite familiar to financial market participants, dynamic hedging is less well understood and thus more likely to evoke an aura of mystery.

"Modern option pricing theory is based partly on the principle that any option position can be duplicated or replicated by systematically adjusting another portfolio that consists solely of stocks and bonds."

In a sense, the techniques of portfolio insurance are a by-product of theoretical advances in option pricing. Modern option pricing theory is based partly on the principle that any option position can be duplicated or replicated by systematically adjusting another portfolio that consists solely of stocks and bonds. The original Black-Scholes equation for the price of a European call option was derived in this way in their 1973 article. Only much later were the principles used in a practical implementation, notably by Mark Rubinstein and Hayne E. Leland (1981), which led to the development and widespread institutional use of portfolio insurance. To understand what synthetically created options are, one must first review some theoretical details, which again are discussed with emphasis on the underlying intuition.

As mentioned earlier, options are derivative assets, and so they do not depend on market participants' expectations concerning future values. Furthermore, the Black-Scholes model is developed in such a way that it is unnecessary to consider market participants' attitudes toward risk. The value of an option before expiration may be expressed as the present discounted value of its expected payoff at expiration.[5] The expectation is a mathematical, not a psychological, one, so that the expected payoff reflects the probability that the option will expire in-the-money. The relevant probabilities are determined by the five variables enumerated above: the current stock price, interest rate, volatility, exercise price, and time to expiration. Thus, given an estimate of the volatility, option valuation is entirely a mechanical process.

To make this discussion more concrete, consider the following example of the synthetic creation of an insured index portfolio. The elements needed for this task are the uninsured index portfolio and Treasury bills (the risk-free asset). The object is to form a portfolio consisting of one unit or share of the S&P 500 and one S&P 500 index put. The value of this portfolio at the time of expiration can be symbolically represented as:

$$S^* + P^* = \text{MAX}\,[S^*, K^*], \tag{1}$$

where S, P, and K are respectively the index price, the put price, and the exercise price. The asterisks denote the value of these variables at the time of expiration. Omission of the asterisks indicates that these variables take on their current values. This equation simply states that the insured portfolio will be worth the greater of S^* or K^*. That is, if S^* exceeds K^*, the put expires worthless and the portfolio value is S^*; otherwise the put expires in-the-money, offsetting the decline in S^* below K^*, and the portfolio value is K^*.

Things are much more complicated prior to expiration. The formula for the *current* insured portfolio value is:

$$S + P = DF \cdot E\,(S^* + P^*), \tag{2}$$

where DF represents a discount factor, which converts the future expected value of the insured portfolio into a current value, and E symbolizes the current expected value of the insured portfolio at the expiration date. DF depends on the interest rate and the time to expiration. The expectation denoted by E may be thought of as an averaging of all the values that the insured portfolio can have at the time of expiration. Some values have higher probabilities than others and thus have greater weight in the computed average. Again, it bears emphasizing that the probabilities depend only on the current values of the five underlying "inputs" into the option pricing equation. In other words, the range of possible insured portfolio values is constrained by these input variables.

Thanks to Black and Scholes's ingenuity, equation (2) can be written in terms of the five underlying variables:[6]

$$S + P = S \cdot N_1 + K \cdot DF \cdot N_2 . \quad (3)$$

Only currently observed variables and the volatility appear in the equation, which is expressed in a simplified notation to emphasize the key variables. The protective put (long index share/long index put) on the left-hand side of the equation has the same value as the replicating portfolio of the index and Treasury bills on the right-hand side. $S \cdot N_1$ represents the dollar amount of the index held and $K \cdot DF \cdot N_2$ the dollar amount of T-bills. Adjustments to the composition of the index/T-bill portfolio over time are made so that the value of the replicating portfolio matches the value of the protective put. This adjustment process is referred to as dynamic hedging, which contrasts with traditional static hedging strategies such as taking a fixed position in short futures or holding a fixed proportion of a portfolio in bonds or bills.

$K \cdot DF$ is the present discounted value of the exercise price. It has an alternative interpretation as the floor for the value of an insured portfolio any time before expiration; K is the floor at expiration.[7] Any desired index put on the left-hand side can be created by an appropriate choice of the exercise price K. The first term on the right-hand side, $S \cdot N_1$, represents the present discounted expected value of the index S^* at the expiration date, given that S^* exceeds the exercise price K, that is, the put expires out-of-the-money. This so-called conditional expectation is a weighted average of all possible future values of the index that are greater than K. Analogously, the second term on the right-hand side, $K \cdot DF \cdot N_2$, is the present discounted value of K, given that S^* turns out to be less than or equal to K, that is, the put finishes in-the-money.

A more immediate sense of how the replicating portfolio mimics a protective put can be developed through a consideration of extreme values that N_1 and N_2 can take. The variables N_1 and N_2 are complicated functions of the five underlying variables; they embody terms for conditional expectations and probabilities. As N_1 approaches zero, N_2 approaches one, and vice versa. Technically, these variables are known as cumulative normal distribution functions, which take values that range from zero to one. As the underlying variables change—for example, as the stock price or its volatility changes—the values of N_1 and N_2 vary, resulting in shifts in the composition of the replicating portfolio that maintain its equality with the protective put. If

"Adjustments to the composition of the index/T-bill portfolio over time are made so that the value of the replicating portfolio matches the value of the protective put. This adjustment process is referred to as dynamic hedging."

the current index price S is very much greater than K, the put is likely to expire out-of-the-money (worthless), and so N_1 is approximately equal to one and N_2 approximately zero. Hence, the current values of the insured portfolio and the index are close. Conversely, if the current index price is very much less than K, the put is "deep" in-the-money and the current insured portfolio value is close to $K \cdot DF$.

The value of N_1 plays a pivotal role in portfolio insurance. It will henceforth be referred to as the option delta, which is the standard name given to this variable. As the index price rises, so does delta; the probability that the synthetic put will expire out-of-the-money increases. The option delta has another important interpretation: it indicates what fraction of the index to hold in the replicating portfolio. Similarly, N_2

indicates the fraction of the maximum current investment in Treasury bills, that is, K•DF. Thus, as delta increases and the index rises, the replicating portfolio is shifted out of Treasury bills and into the index by selling bills and using the proceeds to buy the index. Conversely, as delta decreases and the index falls, the replicating portfolio is shifted the other way by selling the index and using the proceeds to buy bills. At expiration, the portfolio is invested entirely in the index or entirely in T-bills.

Although the composition of the replicating portfolio changes over time as the five underlying variables fluctuate, its current value equals the current value of a long index/long put portfolio because both offer the same payoff at the expiration date. It is in this respect that the replicating portfolio is equivalent to the protective put. The two portfolios are exactly equivalent only if the replicating portfolio is adjusted continuously over time. The continuously-adjusted replicating portfolio is said to be self-financing because once the initial investment is made, no further cash flows come from outside the portfolio. The protective put is also obviously self-financing in that, after the initial purchase of the index share and the index put, no further cash flows arise until the expiration date. The complications that occur due to *discontinuously adjusted* replicating portfolios are discussed below.

"For the futures version, the portfolio holds the entire share of the index and has an appropriate number of short futures contracts in order to create the . . . synthetic bill position."

For the insured portfolio, part of the initial investment goes toward the purchase of a put, necessarily reducing the portfolio's upside potential. The upside capture for the replicating portfolio will be the same as that for an index-put portfolio. The cost is not an actual payment for the put, but the opportunity cost of holding part of the portfolio in Treasury bills. The greater the level of the floor, the higher are the exercise price and put premium and the smaller is the upside capture.

Using Index Futures. Although theoretically feasible, index/T-bill replicating portfolios are not used in practice. Instead, most insured portfolios are implemented using S&P 500 index futures contracts. The transactions costs for using the latter are about one-third less than for Treasury bills. Furthermore, the futures market is more liquid than the stock market, as order executions to buy or sell futures are transacted faster than those for the underlying S&P 500 itself.

The key to understanding the futures implementation is to see that a portfolio combining one share of the index with a short future is theoretically equivalent to holding a Treasury bill. In fact, a short future/long index portfolio is sometimes referred to as a synthetic money market instrument. This equivalence is based on what is known as a cost-of-carry model for futures. Buying an index share and simultaneously selling a futures contract short creates a hedged, (nearly) riskless portfolio because the futures contract fixes the selling price. What should the rate of return on a riskless asset be? By arbitrage, any assets of equal risk and maturity should earn the same rate of return. Thus, the hedged index portfolio should earn the T-bill rate. Further details about creating synthetic money market instruments are presented in the appendix to this article.

Using futures instead of T-bills simply involves an additional step in setting up and adjusting a replicating portfolio. The initial index/T-bill portfolio contains some proportion of bills, based on equation (3). For the futures version, the portfolio holds the entire share of the index and has an appropriate number of short futures contracts in order to create the bill position. In the event that the index rises after the portfolio is established, the short futures position is reduced by buying futures contracts, resulting in a smaller synthetic bill position. On the other hand, a falling index induces more futures sales to lessen the portfolio's exposure to the market, thus placing more of the portfolio in synthetic bills.

Practical Considerations

This section develops in greater depth the background needed to understand the index/T-bill and index/futures implementations of portfolio insurance. Before examining simulations run on actual historical data, some additional preliminary topics are discussed. These are intended to give a more detailed view of the mechanics of running an insured portfolio and to help in interpreting the simulation results. These topics will also help to illuminate the channels through which portfolio insurance may have affected the underlying stock market, particularly during the October 19 crash.

Sources of Error and Risk in Portfolio Replication. In practice, the floor provided by portfolio insurance can be rather soft. Inaccuracies in protective put replication arise from a number of sources. Nonconformity of real-world stock and stock-index price movements with those assumed in an option pricing model, as well as the possible inadequacy of the model itself, presents a source of error and risk to the insured portfolio owner.

The adaptation of theory to practice gives rise to replication errors, the differences between the values of the actual replicating portfolio and the theoretical insured portfolio (that is, long index/long put). Discontinuous adjustment to the composition of the insured portfolio necessarily produces replication errors, both gains and losses, which add to the uncertainty about insured portfolio performance. This adjustment process is referred to as rebalancing the portfolio. There are many possible rebalancing criteria that offer different trade-offs between more accurate replication of a long index/long put portfolio and the transactions costs that accrue due to rebalancing. According to the Black-Scholes model, one necessary condition for perfect replication is that rebalancing must occur *continuously* over time; however, even if this were possible, transactions costs would render such rebalancing prohibitively expensive. Thus, actual replicating portfolios will not match their theoretical potentials. For the purpose of this exposition, the simulated insured portfolios were rebalanced daily.

Replication errors are particularly serious when stock prices "jump" to a new level. During such sharp market moves, there is no opportunity to rebalance the insured portfolio. Due to the trade-offs involved in managing an insured portfolio, achieving a firm floor is difficult in practice. This obstacle poses a serious risk to the holder of an insured portfolio.

Mentioned above were the problems associated with estimating volatility.[8] Furthermore, the volatility may change over time, adding to uncertainty about the cost of portfolio insurance. The volatility estimate is usually revised whenever the insured portfolio is rebalanced.

For portfolios whose composition is not identical to the index, there also exists a basis risk. This risk refers to the less than perfect positive correlation between movements in the values of the portfolio and (discounted) long futures. Even for exact index portfolios, a basis risk is still present if the put option expiration date does not coincide with the futures expiration. Nonsynchronous expiration dates also gives rise to an interest rate risk. The S&P 500 index futures contracts can be readily traded only in contract maturities of no longer than four months. Furthermore, these short-maturity contracts, being more liquid, are traded with lower transactions costs. Portfolio insurance programs, however, typically run from one to five years. Hence, the futures position must be "rolled over" at relatively frequent intervals. The difference in the expiration dates for the PI program and futures contracts creates a mismatch between the interest rate appropriate for the replicating portfolio and the short-term interest rate implied by the futures position. Thus, using futures generally entails both basis risk and interest rate risk.

"Nonconformity of real-world stock and stock-index price movements with those assumed in an option pricing model, as well as the possible inadequacy of the model itself, presents a source of error and risk to the insured portfolio owner."

The simulation runs discussed below are intended as an illustration of portfolio insurance, not as an evaluation of its efficacy. A more sophisticated modeling effort would be needed to capture many of the subtleties involved in running real-world insured portfolios.[9] Actual portfolio insurance providers do not simply let their programs run on automatic. Instead, much judgment comes into play, particularly in estimating the unobservable option volatility. Judgment is likewise instrumental when an insurer wishes to profit from futures mispricing and timing of rollovers, to decide on the timing and criteria for rebalancing, and, for some insurers, to switch among different methods for creating the insurance. These complications make actual implementation of portfolio insurance as much an art as a science.

"Actual portfolio insurance providers do not simply let their programs run on automatic. Instead, much judgment comes into play. . . . These complications make actual implementation of portfolio insurance as much an art as a science."

Some Refinements. To simplify the exposition thus far, it has been assumed that the index does not pay dividends. Because the simulations presented below use actual index data, the option pricing model needs to account for dividend payments. The insured portfolios to be examined will create European, payout-unprotected puts, that is, puts that protect only the capital appreciation component of the index return, not the dividend component. To value a dividend-unprotected put, it was assumed that the future dividend payments and their timing are known with certainty. Each index price was adjusted by subtracting the present discounted value of all future dividend payments remaining between the current date and the expiration date. In theory, stock prices fall by the amount of the dividend payment on their ex-dividend dates; the same is true of the index, which experiences dividend distributions almost daily from its constituent stocks. The dividend-adjusted index was used in the Black-Scholes option formula to evaluate the value of delta. Both insured and uninsured index portfolios receive dividends, which are reinvested in T-bills. Bear in mind that variations in the dividend flow from the index account for only a minor proportion of the variations in the total returns on the insured and uninsured portfolios.

The simulations reported below compare insured with uninsured index portfolios. In order to make the two comparable, both were constructed to start with the same initial investment, which was the current index value for each simulation period. (Establishing an insured portfolio of the index and T-bills, or the index with short futures, is conceptually equivalent to allocating part of the funds toward paying the put premium and the remainder toward buying the index). Details concerning the method of equating the initial values of both portfolios are discussed in Mark Rubinstein (1984) and in F.J. Gould and C.B. Garcia (1987). All portfolios used daily closing prices for the S&P 500 index and futures. Unless otherwise noted, daily interest rates were computed from the outstanding T-bill that matured immediately after the S&P 500 futures expired. The expiration dates used in the simulations were taken to be the first trading day during the delivery month for the expiring S&P 500 futures contract. Transactions costs were incorporated in the Black-Scholes model using the Leland (1985) method of augmenting the estimated option volatility. The assumed transactions costs of adjusting the insured portfolio are taken to be 1 percent of the volume of transactions for the index/T-bill version and 0.33 percent for the index/futures. These figures are consistent with those reported in Rubinstein (1984), Ethan S. Etzioni (1986), and Garcia and Gould (1987). The relative cost advantage of futures assumed here is fairly conservative.

Another detail that deserves mention is that the cash flows associated with a futures position are ignored in the simulations. Futures contracts are marked to market daily, which means that gains (or losses) to the futures position are received (or paid) daily. In managing an actual insured portfolio, some provision must be made to handle these cash flows, particularly the out-

Table 1.
An Example of an Index/T-Bill Insured Portfolio, September 3, 1986 - September 16, 1986*

Date	Uninsured Portfolio	Percent Change	Insured Portfolio	Percent Change	Delta	Synthetic Put Price**	Stock	Bills	Interest Rate
9/3	250.08	.00	250.08	.00	.278	11.38	66.60	183.48	5.25
9/4	253.86	1.51	250.90	.33	.377	9.09	91.58	159.53	5.22
9/5	250.54	.18	250.00	-.03	.302	11.44	72.27	177.63	5.28
9/8	248.24	-.73	249.51	-.23	.248	13.08	58.95	190.31	5.28
9/9	247.81	-.91	249.45	-.25	.221	13.12	52.31	196.87	5.27
9/10	247.23	-1.14	249.39	-.28	.204	13.56	48.13	200.96	5.22
9/11	235.38	-5.88	247.50	-1.03	.119	24.94	26.69	220.10	5.28
9/12	230.90	-7.67	247.05	-1.21	.083	29.11	18.42	227.90	5.26
9/15	232.20	-7.15	247.20	-1.15	.092	27.94	20.47	225.99	5.23
9/16	232.02	-7.22	247.22	-1.14	.088	28.13	19.43	227.06	5.20

* *The portfolio is rebalanced daily.*

** *The expiration date for the synthetic put is December 1, 1986.*

flows. Either a separate fund of cash instruments (T-bills) is set aside for this purpose, or part (say, 5 percent) of the insured portfolio is held in T-bills. A somewhat more complicated accounting scheme would have been needed in the simulations to keep track of the interest earned (or forgone) on the gains (or losses) to the futures position. However, including this accounting would have only a minor effect on the simulation results.

Two Examples of Insured Portfolios

The Index/T-Bill Version. Table 1 provides a detailed comparison of the daily changes in sample insured and uninsured S&P 500 index portfolios. The insured portfolio uses the index/T-bill implementation; its transactions costs were set to zero for this example. Both portfolios are initiated on September 3, 1986, and have initial index values of 250.08. The insured portfolio contains a synthetic put that expired on December 1, 1986, and that was constructed to insure to a maximum (theoretical) loss of zero percent of the initial portfolio value. The zero percent floor applies to the insured portfolio value as of the expiration date. Prior to that date, as can be seen, the portfolio can fall below the floor level, although capital losses will be less than those on the uninsured portfolio.

During the ten-trading-day period reported in the table, the S&P 500 fell sharply, experiencing a very large 4.8 percent decline on Thursday, September 11, from the previous day's close. This produced a 42 percent drop in delta, which triggered a 45 percent reduction in the insured portfolio index holdings. The proceeds from the partial index liquidation were used to increase bill holdings. Due to this gradual daily shifting of index holdings to bills, the insured portfolio had lost 1.0 percent of its value from September 3 as compared with a 5.9 percent loss on the uninsured index itself. Another way to view this process is to note that the synthetic put value rose over this period as the index fell, thus providing partial insurance during the market's decline.

Over the full insurance period, the actual index turned out to be almost unchanged, finishing on December 1 at 249.05. The uninsured portfolio (which includes accumulated dividends and interest) was up 0.41 percent, while the insured portfolio was down 0.60 percent. As expected, the insured portfolio was less volatile than the index: the maximum loss for the uninsured portfolio was 7.8 percent as opposed to a

Table 2.
An Example of an Index/Futures Insured Portfolio, September 3, 1986 - September 16, 1986*

Date	Uninsured Portfolio	Percent Change	Insured Portfolio	Percent Change	Delta	Synthetic Put Price**	Stock	Bills	Synthetic Interest Rate	Futures Basis
9/3	250.08	.00	250.08	.00	.278	11.38	66.60	183.48	8.54	2.27
9/4	253.86	1.51	251.17	.44	.377	9.09	91.58	159.68	8.10	2.07
9/5	250.54	.18	250.83	.30	.302	11.44	72.27	178.46	5.71	.98
9/8	248.24	-.73	249.25	-.33	.248	13.08	58.95	190.18	8.93	2.31
9/9	247.81	-.91	249.46	-.25	.221	13.12	52.31	197.02	8.12	1.93
9/10	247.23	-1.14	249.63	-.18	.204	13.56	48.13	201.35	7.37	1.59
9/11	235.38	-5.88	249.07	-.40	.119	24.94	26.69	220.02	2.68	-.28
9/12	230.90	-7.67	249.25	-.33	.083	29.11	18.42	230.47	0.74	-.97
9/15	232.20	-7.15	249.19	-.35	.092	27.94	20.47	228.36	1.21	-.79
9/16	232.02	-7.22	249.28	-.32	.088	28.13	19.43	229.48	0.94	-.87

* *The portfolio is rebalanced daily.*

** *The expiration date for the synthetic put is December 1, 1986.*

maximum loss of 1.5 percent for the insured; the maximum gains were 1.5 percent and 0.3 percent, respectively. Again, downside protection comes at the expense of upside performance.

The Index/Futures Version. Table 2 repeats the portfolio comparisons given in Table 1, but instead of index/bill implementation the insured portfolio uses index/futures. The synthetic put prices and option deltas are identical to those in Table 1, because the T-bill rate was still used in the Black-Scholes equation. Transactions costs are again assumed to be zero for this example. Differences between the two simulations arise because of mispricing of the futures. Both Etzioni (1986) and John J. Merrick, Jr. (1987b) discuss the empirically observed tendency for index futures price changes to "overshoot" index price changes. In other words, when the index price is rising (falling), the futures price tends to increase (decrease) more than proportionately. As a result of mispricing, the value of the synthetic bill position on a given day will differ from the value of the T-bill position on that day. The reported daily interest rates on the synthetic bill are clearly more volatile than the corresponding T-bill rates. This is true not only for this small sample but also over the entire history of the S&P 500 index futures. The volatility tends to be greatest during the contract's delivery month, which is why the expiration date for the insurance period was chosen to be the first trading day of the delivery month.

Underpriced futures contracts imply that bill yields are lower and bill prices are higher. Additional futures sales during a market decline are therefore equivalent to purchasing low (and possibly negative) yielding synthetic bills. During the market drop on September 11, the synthetic rate dropped from 7.37 percent the previous day to 2.68 percent, and continued to fall on the next day to 0.74 percent. As can be seen in the "Futures Basis" column, the decline in yield corresponds to a dipping of the S&P December futures price below the S&P 500 index (that is, a negative basis). This occurrence is not uncommon, despite the fact that it represents a stock-index arbitrage opportunity.[10]

The value of the synthetic bill component of the insured portfolio on September 11 exceeded the value of the corresponding actual bill component reported in Table 1. This disparity appears simply because the synthetic bills bought before September 11 were cheaper than the actual T-bills; that is, the synthetic yield was greater than the actual bill yield. After September 11, the pricing relationship reversed so that if the insured portfolio had been liquidated at the close of business on September 11, capital gains would have accrued to the synthetic bills. The superior performance of the index/futures portfolio over the index/T-bill portfolio resulted because futures were initially overpriced and later underpriced during the first half of September 1986. From September 3 to the close on September 11, the index/futures insured portfolio was down only 0.40 percent as compared with a decline of 1.03 percent for the index/T-bill portfolio.

During the course of the insurance period, it turned out that the synthetic rate was more often than not below the actual bill rate. The cumulative impact of the futures mispricing caused the index/futures insured portfolio to finish below the index/T-bill insured portfolio. At expiration on December 1, 1986, the index/T-bill insured portfolio was 0.60 percent below its initial value, while the index/futures insured portfolio was down 1.06 percent. Discontinuous trading results in cumulative errors in the replicating portfolio, so that, as in these cases, the portfolio performances will virtually always deviate from their theoretical potentials.

Simulations Using Historical Data

Tables 3 through 5 report simulation results for insured and uninsured S&P 500 index portfolios spanning different periods and using different implementations and insurance floors. Table 3 displays the results for portfolio simulations that ran one-year insured portfolios with starting dates from January 1974 to January 1986. The daily yield on the current one-year T-bill was used in the daily option pricing. Most portfolios were simulated over 253 trading days, and the insured portfolios, with - 5, 0, and 3 percent floors, were rebalanced daily. Most items in the table are self-evident. For any time period, the uninsured S&P 500 index and insured portfolio data and statistics are read by row. The final S&P 500 index value is less than the S&P portfolio value (the "Final Portfolio" column) because the latter includes accumulated dividends and interest on those dividends. The uninsured S&P portfolio is directly comparable with the various insured portfolios.

All percentage changes are taken relative to the initial portfolio values, which by construction are the same for all portfolios. The "Portfolio Percent Change" column gives the total change from the initial to final dates. The "Maximum Percent Change" column gives the maximum cumulative change in portfolio that occurred during the life of the portfolio. Similarly, the "Minimum Percent Change" column indicates the lowest cumulative percentage change from the initial portfolio value.

Finally, the "Cost" column reports the difference between a given insured portfolio's final return (under "Portfolio Percent Change") and the uninsured portfolio's final return (in the same column). Again, the cost can be thought of as being analogous to funds allocated to purchasing a put, so that less remains for investment in the index. The cost actually arises because part of an insured portfolio's value is placed in T-bills, which necessarily results in forgone capital and dividend returns when the uninsured portfolio appreciates faster than T-bills. The cost, therefore, is actually an opportunity cost associated with creating an insured portfolio. On the other hand, when the insured portfolio loses value relative to the floor level, the cost of holding an insured portfolio will be negative, that is, the insurance pays off.

As an example of insured and uninsured portfolio performance during a rising market, consider the January 2, 1986, to December 31, 1986, holding period in Table 3. All portfolios started at the initial index value of 209.59. The uninsured portfolio appreciated by 19.65 percent by December 31, whereas all insured portfolios underperformed this rate of appreciation, as expected. The - 5 percent floor portfolio had, in effect, the largest deductible and consequently had the next best return of 12.67 percent, 6.99 percentage points less than the uninsured portfolio's cumulative change. Reducing the deductible, by lifting the floor, raised the opportunity cost of insurance considerably. The cumulative returns on the 0 percent and 3 percent floor portfolios were, respectively, 9.39 and 5.08 percent. The synthetic creation of a protective put also dampened fluctuations in insured portfolio values over the lives of the portfolios, as is readily seen in the maximum and minimum cumulative percentage change columns.

The January 3, 1977, to December 30, 1977, period gives an example of portfolio performance during a declining market. Because the

Table 3.
Insured vs. Uninsured Index Portfolios*
(Transactions Costs Included)

Index/T-Bill Version

Portfolio	Floor Level	Initial Index	Final Index	Final Portfolio	Portfolio Percent Change	Maximum Percent Change	Minimum Percent Change	Cost
			January 3, 1974 - December 31, 1974 253 Trading Days					
S&P 500	***	97.68	68.56	72.39	-25.90	2.83	-33.32	***
-5%	92.80	***	***	94.45	-3.31	1.13	-5.44	-22.60
0%	97.68	***	***	99.79	2.16	2.16	-1.64	-28.06
+3%	100.61	***	***	101.84	4.26	4.26	-0.81	-30.16
			January 2, 1975 - December 31, 1975 253 Trading Days					
S&P 500	***	70.23	90.19	94.08	33.93	39.04	-0.19	***
-5%	66.72	***	***	84.99	21.02	28.25	-0.15	12.91
0%	70.23	***	***	80.59	14.74	21.83	-0.08	19.20
+3%	72.35	***	***	76.00	8.21	13.97	-0.01	25.72
			January 2, 1976 - December 31, 1976 253 Trading Days					
S&P 500	***	90.90	107.46	111.51	22.65	22.65	1.86	***
-5%	86.36	***	***	104.77	15.26	15.65	1.32	7.39
0%	90.90	***	***	101.73	11.91	12.24	0.88	10.74
+3%	93.63	***	***	97.30	7.04	7.20	0.46	15.62
			January 3, 1977 - December 30, 1977 252 Trading Days					
S&P 500	***	107.00	95.10	99.83	-6.72	-1.20	-11.59	***
-5%	101.65	***	***	100.80	-5.80	-0.89	-6.42	-0.92
0%	107.00	***	***	106.94	-0.06	-0.06	-2.29	-6.66
+3%	110.21	***	***	110.16	2.96	2.96	-0.46	-9.67
			January 3, 1978 - December 29, 1978 252 Trading Days					
S&P 500	***	93.82	96.11	101.43	8.09	17.91	-6.47	***
-5%	89.13	***	***	95.11	1.37	12.93	-4.73	6.71
0%	93.82	***	***	93.31	-0.55	10.49	-2.60	8.64
+3%	96.64	***	***	95.03	1.28	6.92	-1.07	6.80
			January 2, 1979 - December 31, 1979 253 Trading Days					
S&P 500	***	96.73	107.94	113.91	17.74	19.63	0.25	***
-5%	91.89	***	***	106.93	10.54	13.87	-0.58	7.20
0%	96.73	***	***	105.66	9.24	12.35	-0.14	8.51
+3%	99.63	***	***	103.75	7.26	10.10	0.35	10.48
			January 2, 1980 - December 31, 1980 253 Trading Days					
S&P 500	***	105.76	135.76	142.42	34.63	38.54	-5.73	***
-5%	100.47	***	***	134.19	26.88	31.32	-5.53	7.75
0%	105.76	***	***	132.10	24.90	29.26	-4.79	9.73
+3%	108.93	***	***	129.59	22.53	26.80	-4.18	12.10

Table 3 continued

Portfolio	Floor Level	Initial Index	Final Index	Final Portfolio	Portfolio Percent Change	Maximum Percent Change	Minimum Percent Change	Cost
			January 2, 1981 - December 31, 1981 253 Trading Days					
S&P 500	***	136.34	122.55	129.71	-4.88	1.66	-13.53	***
-5%	129.52	***	***	128.67	-5.63	1.02	-8.39	0.75
0%	136.34	***	***	136.95	0.45	0.85	-3.58	-5.32
+3%	140.43	***	***	141.91	4.08	4.08	-2.58	-8.96
			January 4, 1982 - December 31, 1982 253 Trading Days					
S&P 500	***	122.74	150.64	148.06	20.60	21.87	-12.93	***
-5%	116.60	***	***	138.92	13.18	15.08	-9.09	7.42
0%	122.74	***	***	138.26	12.64	14.45	-6.41	7.96
+3%	126.42	***	***	137.05	11.66	13.42	-4.79	8.95
			January 3, 1983 - December 30, 1983 253 Trading Days					
S&P 500	***	138.34	164.93	172.40	24.59	28.91	1.48	***
-5%	131.42	***	***	159.01	14.94	20.18	0.55	9.65
0%	138.34	***	***	153.75	11.14	16.05	0.43	13.45
+3%	142.49	***	***	148.07	7.03	10.92	0.37	17.56
			January 3, 1984 - December 31, 1984 253 Trading Days					
S&P 500	***	164.04	167.24	175.18	6.77	7.93	-7.28	***
-5%	155.84	***	***	166.25	1.35	3.23	-7.10	5.42
0%	164.04	***	***	166.08	1.25	2.91	-4.39	5.52
+3%	168.96	***	***	166.21	1.32	2.88	-2.38	5.45
			January 2, 1985 - December 31, 1985 253 Trading Days					
S&P 500	***	165.37	211.28	219.67	32.81	33.03	-0.99	***
-5%	157.10	***	***	207.71	25.60	26.04	-0.74	7.21
0%	165.37	***	***	203.62	23.13	23.55	-0.52	9.28
+3%	170.33	***	***	198.39	19.97	20.37	-0.32	12.85
			January 2, 1986 - December 31, 1986 253 Trading Days					
S&P 500	***	209.59	242.17	250.82	19.65	24.95	-2.70	***
-5%	100.11	***	***	236.14	12.67	18.18	-2.05	6.99
0%	209.59	***	***	229.28	9.39	14.74	-1.39	10.26
+3%	215.88	***	***	220.23	5.08	10.60	-0.76	14.58

* *The portfolios are rebalanced daily.*

uninsured portfolio ended 6.72 percent below its initial value, all synthetic puts finished in-the-money. The - 5, 0, and 3 percent floor portfolios had cumulative final returns of - 5.80, - 0.06, and 2.96 percent respectively. These returns happen to be quite close to their floor values. Due to replication errors, returns for these portfolios during other down-market years are sometimes greater or smaller than their targeted floors. Notice that even before the insured portfolios' expiration dates, some degree of protection was obtained from market declines, since the minimum cumulative percentage changes are not as large as the -11.59 percent drop for the uninsured portfolio.

Tables 4 and 5 give simulation results for three-month insurance periods (see pp. 20-23). The two tables are identical in all respects, except that Table 4 represents the index/T-bill version of portfolio insurance while Table 5 represents the index/futures version. Each insurance period coincides with the final three months for each of the S&P 500 index futures contracts issued, beginning with the March 1983 contract. The T-bill expiring immediately after the futures contract was matched with the futures in doing each simulation, and the daily yield on that bill was used in pricing the option in both tables. The tables include insured portfolios with 0 and - 5 percent floors. The full simulation results for Tables 1 and 2 are given in the last block of entries in Tables 4 and 5.

In the sample considered in Tables 4 and 5, futures mispricing turns out to be substantial. In 14 out of 36 simulated insured portfolios, the cost of the index/futures version exceeded that of the index/T-bill version, despite the transactions cost advantage of using futures. These simulations were also recomputed setting transactions costs to zero, as in Tables 1 and 2. Differences between the two implementations now arise solely because of mispricing.[11] The results (not included in the tables) indicate that 18 out of 36 index/futures simulations are higher-cost compared with their index/T-bill counterparts. The simulations reveal that index futures mispricing is empirically important and can offset the cost advantage of using futures. This conclusion is tempered by the caveats offered above concerning simulations and by the conservative estimate for the futures cost advantage.

To the extent that the mispricing is systematic and predictable, the replication strategy can be adjusted to compensate for the mispricing. Merrick (1987b) discusses a procedure to correct for predictable mispricings. To some degree, judgment and discretion exercised in insuring a portfolio using futures would be expected to mitigate the costs arising from disadvantageous mispricing.

Insurers particularly want to protect against catastrophic market declines, and this is precisely where the usefulness of portfolio insurance is problematic. The mispricing phenomenon became acute during the October 19, 1987, stock market crash. One prominent portfolio insurer hesitated in selling futures as the market declined because of the steep discount on the futures below the index. The firm hoped for a realignment of futures and index prices, which did not occur due to the breakdown in arbitrage, and eventually sold less than half the futures contracts that their programs called for.[12] Other insurers were probably in a similar bind at the time. As a result, insured portfolios fell below their floor levels. The accompanying box discusses the issues and presents currently available information on the role of portfolio insurance in the October crash (see p. 19).

Conclusion

The term portfolio insurance is a misnomer, as the recent market crash has made abundantly clear. This article has shown how the most common implementation of portfolio insurance is a specialized form of hedging using stock-index futures. As is well known to practitioners, hedging is generally not without risk, and portfolio insurance strategies are no exception.

The dynamic adjustments associated with portfolio insurance distinguish it from other types of hedging. Frequent changes to the short futures position, or alternatively to the index and T-bill positions, are made to replicate synthetically a portfolio insured by an index put (a protective put). The success of this hedging strategy in providing downside protection depends on a host of factors, which have been discussed in this article. Actual insured portfolio performance may fail to achieve the prespecified floor rate of return. One reason, highlighted above, is that futures prices frequently differ substantially from their theoretically predicted values. Futures mispricing contributes to the uncertainty regarding insured portfolio performance and cost.

The mispricing that occurred during the October 19 stock market crash was unprecedented, as were practically all aspects of that financial collapse. As critics were quick to point out, portfolio insurance did not perform as expected. However, the partial failure of the insurance was a consequence of structural frictions in both the stock and futures markets, not of the insurance technique per se. Since the market collapse, the number of clients using portfolio insurance has shrunk, reportedly by half of the pre-October level, in terms of asset values covered.[13] Whether portfolio insurance recovers its appeal remains an open question. What is clear is that major institutional changes that transform trading into a more highly automated process would improve the effectiveness of PI strategies. Over recent years, advances in computer technology have revolutionized trading in traditional as well as in new securities and financial instruments and will surely continue to do so. Portfolio insurance is an outgrowth of progress in financial theory and practice, and is but one example of the evolutionary development of the marketplace.

Appendix

The relationship between the index price and the futures price is determined by the net cost of holding a hedged long position in the S&P 500 index. The opportunity cost of this investment is assumed to be the risk-free rate, that is, the interest rate on Treasury bills of comparable maturity. Selling an S&P 500 futures contract against a share of the S&P 500 index renders the long index position riskless because at expiration, due to the convergence of futures and index prices, the gain (or loss) on the long index position will be exactly offset by the loss (or gain) on the short futures position. In equilibrium, investors will be indifferent between holding a perfectly hedged position in the index and holding an equivalent position in T-bills.

The cost-of-carry relationship may be expressed as follows:

$$\frac{365}{\tau}\left[\left(\frac{F - I}{I}\right) + \frac{D}{I}\right] = r,$$

where F is the current futures price, I the current index price, D the present discounted value of anticipated dividends, r the annualized risk-free rate, and τ the time to expiration of the futures contract. The first term in brackets is the futures basis, expressed as a fraction of the index. The second term is the expected dividend yield. The annualized sum of these two yields is equal to the annualized risk-free rate. In other words, the holder of the hedged index position receives the capital appreciation locked in by the futures contract and the dividends paid by the stocks contained in the index up until expiration of the futures contract.

Given the values of the other variables, the equilibrium value of the futures price is determined. A futures basis greater than the equilibrium basis implies a risk-free arbitrage opportunity which entails selling the relatively overpriced future and buying the underpriced index. Conversely, a futures basis smaller than the equilibrium basis induces arbitrage, which involves buying the underpriced future and selling (or selling short) the overpriced index. See John J. Merrick, Jr. (1987a) for an introduction to stock-index arbitrage, and Hans R. Stoll and Robert E. Whaley (1985) for a discussion of practical aspects of carrying out the arbitrage.

Portfolio Insurance and the Crash of October 1987

The stock market crash on Monday, October 19, 1987, has raised questions both about how effectively portfolio insurance limited downside risk and about its possible systemic repercussions to the underlying stock market. On October 19, "Black Monday," the Dow Jones Industrial Average plunged a record 508 points (22 percent) and the S&P 500 Index dropped 57.6 points (20.5 percent), proportionately almost as much. In the following weeks, both stock-index arbitrage and portfolio insurance were widely blamed for exacerbating the market's turmoil.

Some critics have raised a well-founded concern that the *interaction* of portfolio insurance and stock-index arbitrage may be destabilizing. Stock-index arbitrage should be thought of as a trading link between the futures and stock markets that aligns index futures and stock index prices. Stock-index arbitrage is a straightforward form of arbitrage: buying a good or asset in a market where it is cheap and selling it in a market where it is dear. If the futures price is sufficiently below (above) the index price, arbitrageurs buy (sell) the futures and sell (buy) the index. In theory, ensuring that the "law of one price" holds cannot be destabilizing; in practice, however, the volume and timing of stock-index arbitrage could conceivably contribute to intraday volatility. Coupling index arbitrage with portfolio insurance may create destabilizing price movements. The critics' argument goes as follows: A large market decline triggers futures selling by portfolio insurers, which drives the futures price down relative to the index price. This in turn sets off arbitrage trading because the futures become underpriced relative to the index. Stock-index arbitrageurs buy the futures and sell short a basket of stocks that replicates the current composition of the index. Stock sales by arbitrageurs drive the index price down. Thus, stock-index arbitrage transmits the selling pressure from futures to the stock market. Arbitrage-induced price declines in the stock market then induce further portfolio-insurance futures selling, setting off a downward price spiral between the stock and futures markets.

What actually happened on October 19 is more complicated than the above scenario. Right at the opening of trade on "Black Monday," the S&P 500 futures market was exposed to great selling pressure. After the previous Friday's 106 point decline on the Dow, portfolio managers and others may have anticipated further futures selling by insurers and tried to get their own futures and stock sales in ahead of them.

The chaotic market conditions on Black Monday led to a breakdown of stock-index arbitrage because it became very risky. The volatility in both the futures and stock markets made it difficult to know what the current futures and index prices were. Trades based on incorrect prices could translate into large losses on what theoretically are riskless transactions. The record trading volume of 605 million shares on the New York Stock Exchange (NYSE) also compounded the risk, as orders could not be executed immediately and simultaneously in the two markets. The NYSE "uptick" rule restricted opportunities to sell stock short during the huge market decline on October 19. Arbitrageurs who executed their stock market trades by short selling had to wait for component stock prices to rise before having their sell orders executed. Severe order backlogs developed on the NYSE.

Preliminary survey data collected by the regulatory agency that oversees stock-index futures trading, the Commodity Futures Trading Commission (CFTC), indicate that index arbitrage constituted only 9 percent of total NYSE volume on that day. On the following day, after the Chicago Mercantile Exchange temporarily suspended trading in stock-index futures, the NYSE effectively banned arbitrage by prohibiting brokerage houses from executing orders through direct computer links to the exchange floor; arbitrage trading dropped to 2 percent of volume.[1]

According to preliminary CFTC trader position data, futures selling by institutional investors accounted for a greater volume of trades in the S&P 500 futures than stock-index arbitrage: their futures sales on October 19 represented between 12 and 24 percent of that day's total volume in the S&P 500 contract and between 19 and 26 percent on October 20.[2] Portfolio insurance-related futures sales were a portion of that hedging-related activity. Only careful study of market events surrounding the crash may uncover what role portfolio insurance played in the market turmoil.

Notes

[1] U.S. Commodity Futures Trading Commission, *Interim Report on Stock Index Futures and Cash Market Activity During October 1987*, November 9, 1987, p. 74.

[2] Ibid.

Table 4.
Insured vs. Uninsured Index Portfolios*
(Transactions Costs Included)
Index/T-Bill Version

Portfolio	Floor Level	Initial Index	Final Index	Final Portfolio	Portfolio Percent Change	Maximum Percent Change	Minimum Percent Change	Cost
			January 31, 1983 - March 1, 1983 22 Trading Days					
S&P 500	***	144.51	150.88	151.49	4.83	4.83	-1.03	***
-5%	137.28	***	***	150.16	3.91	3.91	-0.88	0.92
0%	144.51	***	***	145.08	0.39	0.48	-0.15	4.43
			March 3, 1983 - June 1, 1983 64 Trading Days					
S&P 500	***	152.30	162.55	164.36	7.92	10.23	-1.58	***
-5%	144.68	***	***	161.12	5.79	8.14	-1.22	2.13
0%	152.30	***	***	154.28	1.30	2.75	-0.25	6.62
			June 3, 1983 - September 2, 1983 66 Trading Days					
S&P 500	***	163.98	165.00	166.86	1.75	4.54	-2.12	***
-5%	155.78	***	***	164.06	0.05	3.67	-2.85	1.70
0%	163.98	***	***	163.41	-0.35	1.72	-0.92	2.10
			September 7, 1983 - December 1, 1983 62 Trading Days					
S&P 500	***	167.89	166.49	168.25	0.21	3.24	-2.89	***
-5%	159.50	***	***	166.42	-0.88	2.42	-2.95	1.09
0%	167.89	***	***	168.77	0.52	0.99	-0.40	-0.31
			December 5, 1983 - March 1, 1984 62 Trading Days					
S&P 500	***	165.54	158.19	160.01	-3.34	2.68	-5.79	***
-5%	157.26	***	***	157.66	-4.76	2.00	-5.27	1.42
0%	165.54	***	***	166.16	0.37	0.96	-0.58	-3.71
			March 5, 1984 - June 1, 1984 64 Trading Days					
S&P 500	***	159.24	153.24	155.14	-2.57	2.46	-4.48	***
-5%	151.28	***	***	152.94	-3.95	1.38	-4.66	1.38
0%	159.24	***	***	160.05	0.51	0.54	-0.78	-3.08
			June 5, 1984 - September 4, 1984 65 Trading Days					
S&P 500	***	154.34	164.88	166.85	8.11	9.82	-3.53	***
-5%	146.62	***	***	163.87	6.17	8.07	-3.38	1.93
0%	154.34	***	***	158.51	2.70	4.48	-0.93	5.41
			September 6, 1984 - December 3, 1984 63 Trading Days					
S&P 500	***	164.29	162.82	164.73	0.27	4.53	-1.16	***
-5%	156.08	***	***	162.05	-1.36	3.13	-1.44	1.63
0%	164.29	***	***	166.00	1.04	1.38	0.05	-0.77

Table 4 continued

Portfolio	Floor Level	Initial Index	Final Index	Final Portfolio	Portfolio Percent Change	Maximum Percent Change	Minimum Percent Change	Cost
			December 5, 1984 - March 1, 1985 61 Trading Days					
S&P 500	***	163.38	183.23	185.17	13.34	13.34	-0.83	***
-5%	155.21	***	***	182.18	11.51	11.58	-0.77	1.83
0%	163.38	***	***	176.08	7.78	7.83	-0.25	5.56
			March 5, 1985 - June 3, 1985 64 Trading Days					
S&P 500	***	182.06	189.32	191.36	5.11	5.22	-2.88	***
-5%	172.96	***	***	189.37	4.02	4.23	-2.30	1.09
0%	182.06	***	***	184.44	1.31	1.54	-0.55	3.80
			June 5, 1985 - September 3, 1985 64 Trading Days					
S&P 500	***	190.04	187.91	189.97	-0.04	3.47	-2.36	***
-5%	180.54	***	***	187.45	-1.36	2.61	-2.22	1.33
0%	190.04	***	***	190.54	0.26	1.22	-0.56	-0.30
			September 5, 1985 - December 2, 1985 63 Trading Days					
S&P 500	***	187.37	200.46	202.46	8.05	9.11	-3.33	***
-5%	178.00	***	***	199.66	6.56	7.66	-2.92	1.49
0%	187.37	***	***	194.52	3.82	4.88	-0.74	4.24
			December 4, 1985 - March 3, 1986 62 Trading Days					
S&P 500	***	200.86	225.42	227.49	13.25	13.98	1.11	***
-5%	190.82	***	***	223.82	11.43	12.17	0.82	1.82
0%	200.86	***	***	216.05	7.56	8.28	0.07	5.69
			March 5, 1986 - June 2, 1986 63 Trading Days					
S&P 500	***	224.38	245.04	247.12	10.13	11.41		
-5%	213.16	***	***	242.35	8.01	9.31	-0.01	2.12
0%	224.38	***	***	232.17	3.47	4.71	0.01	6.66
			June 4, 1986 - September 2, 1986 64 Trading Days					
S&P 500	***	245.51	248.52	250.65	2.09	3.99	-4.42	***
-5%	233.23	***	***	246.66	0.47	2.39	-3.67	1.63
0%	245.51	***	***	245.47	-0.02	0.87	-0.75	2.11
			September 4, 1986 - December 1, 1986 63 Trading Days					
S&P 500	***	250.08	249.05	251.12	0.41	1.51	-7.83	***
-5%	237.58	***	***	246.01	-1.63	1.10	-5.10	2.04
0%	250.08	***	***	249.38	-0.28	0.33	-1.21	0.69

* *The portfolios are rebalanced daily.*

Table 5.
Insured vs. Uninsured Index Portfolios*
(Transactions Costs Included)

Index/Futures Version

Portfolio	Floor Level	Initial Index	Final Index	Final Portfolio	Portfolio Percent Change	Maximum Percent Change	Minimum Percent Change	Cost
				January 31, 1983 - March 1, 1983 22 Trading Days				
S&P 500	***	144.51	150.88	151.53	4.83	4.83	-1.03	***
-5%	137.28	***	***	150.05	3.83	3.83	-0.91	0.99
0%	144.51	***	***	143.46	-0.73	0.16	-0.85	5.55
				March 3, 1983 - June 1, 1983 64 Trading Days				
S&P 500	***	152.30	162.55	164.36	7.92	10.23	-1.58	***
-5%	144.68	***	***	161.81	6.25	8.61	-1.07	1.67
0%	152.30	***	***	154.54	1.47	3.60	-0.26	6.45
				June 3, 1983 - September 2, 1983 66 Trading Days				
S&P 500	***	163.98	165.00	166.86	1.75	4.54	-2.12	***
-5%	155.78	***	***	163.96	-0.01	3.87	-3.14	1.77
0%	163.98	***	***	161.77	-1.35	1.97	-2.15	3.10
				September 7, 1983 - December 1, 1983 62 Trading Days				
S&P 500	***	167.89	166.49	168.25	0.21	3.24	-2.89	***
-5%	159.50	***	***	166.04	-1.10	2.47	-3.47	1.31
0%	167.89	***	***	167.78	0.06	0.91	-1.00	0.28
				December 5, 1983 - March 1, 1984 62 Trading Days				
S&P 500	***	165.54	158.19	160.01	-3.34	2.68	-5.79	***
-5%	157.26	***	***	156.65	-5.37	2.06	-6.09	2.03
0%	165.54	***	***	166.26	0.43	1.21	-1.08	-3.77
				March 5, 1984 -June 1, 1984 64 Trading Days				
S&P 500	***	159.24	153.24	155.14	-2.57	2.46	-4.48	***
-5%	151.28	***	***	152.65	-4.14	1.39	-5.24	1.57
0%	159.24	***	***	160.07	0.52	0.83	-1.13	-3.10
				June 5, 1984 - September 4, 1984 65 Trading Days				
S&P 500	***	154.34	164.88	166.85	8.11	9.82	-3.53	***
-5%	146.62	***	***	164.12	6.34	8.24	-3.72	1.77
0%	154.34	***	***	159.60	3.41	5.25	-1.69	4.70
				September 6, 1984 - December 3, 1984 63 Trading Days				
S&P 500	***	164.29	162.82	164.73	0.27	4.53	-1.16	***
-5%	156.08	***	***	162.00	-1.39	3.17	-1.68	1.66
0%	164.29	***	***	165.22	0.57	1.37	-0.36	-0.30

Table 5 continued

Portfolio	Floor Level	Initial Index	Final Index	Final Portfolio	Portfolio Percent Change	Maximum Percent Change	Minimum Percent Change	Cost
			December 5, 1984 - March 1, 1985 61 Trading Days					
S&P 500	***	163.38	183.23	185.17	13.34	13.34	-0.83	***
-5%	155.21	***	***	182.58	11.75	11.82	-0.85	1.59
0%	163.38	***	***	177.48	8.63	8.70	-0.42	4.71
			March 5, 1985 - June 3, 1985 64 Trading Days					
S&P 500	***	182.06	189.32	191.36	5.11	5.22	-2.88	***
-5%	172.96	***	***	189.50	4.09	4.30	-2.72	1.02
0%	182.06	***	***	186.66	2.53	2.68	-1.38	2.59
			June 5, 1985 - September 3, 1985 64 Trading Days					
S&P 500	***	190.04	187.91	189.97	-0.04	3.47	-2.36	***
-5%	180.54	***	***	187.78	-1.19	2.89	-2.19	1.15
0%	190.04	***	***	191.79	0.92	2.08	-0.75	-0.96
			September 5, 1985 - December 2, 1985 63 Trading Days					
S&P 500	***	187.37	200.46	202.46	8.05	9.11	-3.33	***
-5%	178.00	***	***	199.80	6.64	7.74	-3.16	1.42
0%	187.37	***	***	196.48	4.86	5.94	-1.03	3.19
			December 4, 1985 - March 3, 1986 62 Trading Days					
S&P 500	***	200.86	225.42	227.49	13.25	13.98	1.11	***
-5%	190.82	***	***	224.42	11.73	12.47	0.87	1.52
0%	200.86	***	***	217.44	8.25	8.98	0.08	5.00
			March 5, 1986 - June 2, 1986 63 Trading Days					
S&P 500	***	224.38	245.04	247.12	10.13	11.41		
-5%	213.16	***	***	243.10	8.34	9.65	-0.09	1.79
0%	224.38	***	***	232.76	3.73	4.99	0.33	6.40
			June 4, 1986 - September 2, 1986 64 Trading Days					
S&P 500	***	245.51	248.52	250.65	2.09	3.99	-4.42	***
-5%	233.23	***	***	246.26	0.30	2.30	-4.16	1.79
0%	245.51	***	***	245.11	-0.16	1.31	-1.10	2.26
			September 4, 1986 - December 1, 1986 63 Trading Days					
S&P 500	***	250.08	249.05	251.12	0.41	1.51	-7.83	***
-5%	237.58	***	***	245.40	-1.87	1.21	-5.49	2.29
0%	250.08	***	***	247.83	-0.90	0.44	-0.90	1.32

* *The portfolios are rebalanced daily.*

Notes

[1]Rubinstein (1987, p. 73) defines a derivative asset as "an asset whose payoffs are completely determined by the prices or payoffs of other underlying assets." The underlying asset discussed in the article is the S&P 500 index, which is a value-weighted index of 500 stocks selected by the Standard and Poor's Corporation. The weight of each stock in the index is the ratio of the market value of outstanding shares for that stock to the market value of all outstanding shares for the 500 stocks.

[2]The actual cash value of the S&P 500 futures contract is 500 times the index value. For expositional convenience, it is assumed that the underlying asset size for either futures or option contracts is equal to one index unit.

[3]See Leland (1980).

[4]In addition to the assumption cited as an example in the text, other important assumptions of the model that will be discussed in more detail are that trading in stock and options takes place continuously, that the stock volatility is constant, and that the stock pays no dividends.

[5]The rate of interest used in discounting future values is the risk-free rate. Technically, the choice of the risk-free rate is only appropriate for a world of risk-neutral investors, in which equilibrium expected rates of return on *all* assets equal the risk-free rate. However, the Black-Scholes call option pricing equation is valid for any degree of risk aversion because the equation's derivation is based on the valuation of a riskless hedge portfolio of stock and calls. Smith (1976, pp. 22-23) and Jarrow and Rudd (1983, chapters 7 and 8) discuss the so-called risk neutrality argument. Although the interpretations regarding present discounted values offered in this section of the article are strictly correct only for a risk-neutral world, the value of the insured portfolio in terms of the underlying variables is correct for any degree of risk aversion.

[6]Smith (1976) contains an excellent exposition of the solution technique for call options.

[7]K•DF dollars invested in T-bills will increase to K dollars by the expiration date due to the accumulation of interest.

[8]There is no one method for estimating volatility. All existing techniques are ad hoc. The volatility calculations used for the simulations employed a 30-trading-day moving average of the squared log (dividend-adjusted) index price relatives.

[9]A study by Garcia and Gould (1987) is a comprehensive simulation that attempts to evaluate the cost of portfolio insurance. Ad hoc procedures are used to ensure a firm floor. They conclude that "the evidence does not indicate that a dynamically balanced, insured portfolio will over the long run outperform a static mix portfolio" (p. 44). They claim that their method is biased in favor of portfolio insurance, but certain aspects of their procedure, particularly their stop-out rule, may bias the results the other way.

[10]There has been concern expressed in the financial press about the apparent inadequate liquidity of the S&P 500 futures contract. See Falloon (1987, p. 63). Addressing a related issue, Rubinstein (1987, p. 84) considers various hypotheses for the apparent mispricing of index futures, and states: "I am forced to the conclusion that even today the growth in index futures trading continues to outstrip the amounts of capital that are available for arbitrage."

[11]Because, in fact, a long index/short futures portfolio is not riskless, the implied interest rate will usually exceed the T-bill rate. See Kawaller (1987). This interest rate differential may partly explain the apparent mispricing.

[12]See Anders (1987).

[13]See Wallace (1987).

References

Anders, George. "Portfolio Insurance Provided Cold Comfort." *Wall Street Journal*, October 28, 1987.

Black, Fischer, and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (May-June 1973): 637-59.

Bookstaber, Richard. "The Use of Options in Performance Structuring." *Journal of Portfolio Management* 11 (Summer 1985): 36-50.

Boyle, Phelim P., and David Emanuel. "Discretely Adjusted Option Hedges." *Journal of Financial Economics* 8 (September 1980): 255-82.

Cox, John C., and Mark Rubinstein. *Options Markets*. Englewood Cliffs, N.J.: Prentice-Hall, 1985.

Etzioni, Ethan S. "Rebalance Disciplines for Portfolio Insurance." *Journal of Portfolio Management* 13 (Fall 1986): 59-62.

Falloon, William. "Will PI Be Strangled by Illiquidity?" *Intermarket* 4 (November 1987): 63.

Garcia, C.B., and F.J. Gould. "An Empirical Study of Portfolio Insurance." *Financial Analysts Journal* 43 (July-August 1987): 44-54.

Jarrow, Robert A., and Andrew Rudd. *Option Pricing*. Homewood, Ill.: Richard D. Irwin, 1983.

Kawaller, Ira G. "A Note: Debunking the Myth of the Risk-Free Return." *Journal of Futures Markets* 7 (1987): 327-31.

Leland, Hayne E. "Who Should Buy Portfolio Insurance." *Journal of Finance* 35 (May 1980): 581-94.

__________. "Option Pricing and Replacement with Transaction Costs." *Journal of Finance* 60 (December 1985): 1283-301.

Merrick, John J., Jr. "Fact and Fantasy about Stock Index Futures Program Trading." Federal Reserve Bank of Philadelphia *Business Review* (September/October 1987a): 13-25.

__________. "Portfolio Insurance with Stock Index Futures." Working Paper No. 434, Salomon Brothers Center for the Study of Financial Institutions, New York University, August 1987b.

Rubinstein, Mark. "Alternative Paths to Portfolio Insurance." *Financial Analysts Journal* 41 (July-August 1984): 42-52.

__________. "Derivative Assets Analysis." *Journal of Economic Perspectives* 1 (Fall 1987): 73-93.

__________, and Hayne E. Leland. "Replicating Options with Positions in Stock and Cash." *Financial Analysts Journal* 37 (July-August 1981): 63-72.

Smith, Clifford W., Jr. "Option Pricing: A Review." *Journal of Financial Economics* 3 (January-March 1976): 3-51.

Stoll, Hans R., and Robert E. Whaley. "Expiration Day Effects of Index Options and Futures." *Monograph Series in Finance and Economics*, No. 1986-3, Salomon Brothers Center for the Study of Financial Institutions, New York University, 1986.

U.S. Commodity Futures Trading Commission. *Interim Report on Stock Index Futures and Cash Market Activity During October 1987 to the U.S. Commodity Futures Trading Commission*. The Division of Economic Analysis, The Division of Trading and Markets, November 9, 1987.

Wallace, Anise C. "A Suspect in Market's Plunge." *New York Times*, December 1, 1987.