UNITED STATES DEPARTMENT OF LABOR
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The Making and Using of Index Numbers

By

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PREFACE

This study of the making and using of index numbers, by Wesley C. Mitchell, was originally published in 1915 as part of Bulletin 173 of the Bureau of Labor Statistics, dealing with wholesale-price index numbers in the United States and foreign countries. A revision of this bulletin, including a revision of Dr. Mitchell's section, was issued as Bulletin 284 in 1921, following the world-wide revolution in prices caused by the war.

Insofar as these bulletins dealt with current price-reporting methods they are, of course, long since obsolete. However, the section by Dr. Mitchell on the making and use of index numbers has been in continuing demand, particularly in colleges and universities, and, to meet this demand, is now being reprinted, without change, from the original plates.

ISADOR LUBIN,
Commissioner of Labor Statistics.

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CONTENTS.

LIST OF CHARTS.

PART I.—THE MAKING AND USING OF INDEX NUMBERS.

BY WESLEY C. MITCHELL.[1]

I.—THE HISTORY OF INDEX NUMBERS.

The honor of inventing the device now commonly used to measure changes in the level of prices probably belongs to an Italian, G. R. Carli. In an investigation into the effect of the discovery of America upon the purchasing power of money, he reduced the prices paid for grain, wine, and oil in 1750 to percentages of change from their prices in 1500, added the percentages together, and divided the sum by three, thus making an exceedingly simple index number. Since his book was first published in 1764, index numbers are over 150 years old.[2]

It was in England, however, where practically the same device had been hit upon by Sir George Schuckburg-Evelyn in 1798,[3] that the theory and practice of index numbers were chiefly developed. The generation that created the classical political economy was deeply interested in the violent price fluctuations that accompanied the Napoleonic wars and the use of an irredeemable paper currency from 1797 to 1821. Several attempts were made to measure these fluctuations, and in 1833 G. Poulett Scrope suggested the establishment of a "tabular standard of value."[4]

Interest in the study of price fluctuations lagged somewhat in the forties; but the great rise of prices after the Californian and Australian gold discoveries started fresh investigations. W. S. Jevons in England and Adolf Soetbeer in Germany gave a powerful impetus to the theoretical discussion and the practical computation of index numbers. The problem changed somewhat in form but received even more attention after 1873, when a prolonged fall of prices began. In the sixties the chief aim of investigation had been to discover the relations between the rise of prices and the increased production of gold; in the seventies and eighties the chief aim was to find the relations between the fall of prices and the restrictions placed upon the free coinage of

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[1] The writer has received generous help from Prof. Irving Fisher, Prof. Allyn A. Young, Dr. Royal Meeker, and Mr. C. H. Verrill, all of whom read the first draft of this paper and made many effective criticisms. In revising the paper the writer has made free use of the criticisms of the first edition published by Prof. F. Y. Edgeworth, Economic Journal, June, 1818, Vol. XXVIII, pp. 176-197, and by Prof. Fredrick R. Macaulay, American Economic Review, March, 1916, Vol. VI, pp. 203-209. He is indebted once more to Dr. Royal Meeker for critical and constructive suggestions, and to Prof. W. F. Ogburn for supervising certain computations and for reading the manuscript. Prof. Macaulay has considered the theoretical sections with care and suggested numerous improvements in both text and tables.

[2] Del Valore e della Proporzione de' Metalli Monetati con i generi in Italia prima delle Scoperte dell' Indie col confronto del Valore e della Proporzione de' Tempi nostri. Republished by Custodi in his Scrittori Italiani de Economia Politica. Parte Moderna, Vol. XIII, pp. 297-363, especially pp. 335-354.

[3] "An account of some endeavors to ascertain a standard of weight and measure," Philosophical Transactions of the Royal Society of London, 1798, Part I, Art VIII, pp. 133-182, especially pp. 175 and 176.

[4] Principles of Political Economy, London, 1833, pp. 405-408. It is interesting to note, however, that neither David Ricardo, who wrote several pamphlets on currency and prices during the "bank restriction," nor Thomas Tooke, who published an elaborate History of Prices in 1793-1847, made use of index numbers.

silver. The weightiest theoretical contributions of this period were made by Prof. F. Y. Edgeworth, who served as secretary of a committee appointed by the British Association for the Advancement of Science "for the purpose of investigating the best methods of ascertaining and measuring variations in the value of the monetary standard."[5]

The problem of price fluctuations entered upon another phase when the world-wide rise of prices which began in 1896–97 had been under way for several years. After 1900, and more insistently after 1910, complaints about the rising cost of living became common in all civilized countries. Efforts to measure this increase as well as efforts to explain it multiplied.

Index numbers are both troublesome and expensive to compile, yet now in the United States not less than seven wholesale-price series are currently maintained, four of them by financial papers. In England there are four important series; in France one; in Germany, before the beginning of the World War, there were three; while the Governments of Canada, Australia, South Africa, India, Netherlands, and New Zealand now publish official index numbers, and private investigators have made series for Italy, Japan, Belgium, Denmark, Norway, Austria, Spain, and Sweden, although not all of these were kept up during the war period. This list may well be incomplete at present, and is almost certain to require additions within a short time.

Most of the series just mentioned have been established but recently. The oldest—that of the London Economist—was begun in 1869.[6] Sauerbeck's English series dates from 1886, Conrad's German series from 1887 (though in a sense it continues investigations made by Laspeyres in 1864), and Bradstreet's American series from 1897. Of the remaining index numbers regularly published at present, all date from years since 1899, and the majority from years since 1909.[7]

With this increase in numbers there has come an improvement in quality. The early index numbers were made by private investigators, at irregular intervals, from such price quotations as chance had preserved. As public appreciation of the importance of measuring changes in price levels has developed, the work has more and more been assumed by financial journals and Government bureaus. This shift has produced a greater measure of continuity in the series, as well as greater frequency, regularity, and promptness in the publication of the results. Even more important is the improvement in the character and the scope of the price quotations from which the index numbers are made Whereas the individual investigator had to take what he could get in the way of data, financial journals and Government bureaus can collect those current prices that are best adapted for statistical treatment, and can give better assurance of the representative value of their quotations and the uniform quality of the commodities included in successive years.

[5] For the reports of this committee, see the Reports of the British Association, 1887, pp. 247–254; 1888, pp. 181–188; 1889, p. 133; 1890, pp. 485–488, See particularly the memoranda by Prof. Edgeworth subjoined to these reports.

[6] From 1864 to 1869 the Economist published the relative prices of commodities, but such separate figures without a sum or an average do not constitute an index number proper.

[7] The years mentioned are the dates of first publication, not the earliest dates for which relative prices are shown. In most cases the computers carried their investigations back into the past, frequently for a decade or more.

This improvement in the quantity and quality of index numbers is as marked in the United States as elsewhere. Price quotations had been published with more or less care and system by various newspapers and periodicals for many years before the first effort to compile an average of price variations was made. In 1881, Mr. H. C. Burchard, Director of the Mint, made an index number covering the years 1825 to 1880 from quotations that had been printed in certain reports of the Secretary of the Treasury, supplemented by quotations from a New York newspaper. But his data were of uncertain quality and his series was allowed to lapse after 1884.[8] After an interval of eight years, the Senate Committee on Finance authorized a more ambitious effort. Under the direction of Dr. Roland P. Falkner, the statistician of this committee, the (then) Department of Labor made a huge collection of price quotations, running back as far as 1840, and compiled an index number including more than 200 commodities for the years 1860 to 1891, and 85 commodities for 1840 to 1891.[9] But this also was a single investigation, and the United States did not have an index number regularly maintained year after year until the establishment of Bradstreet's series in 1897. A quasi continuation of the Senate Finance Committee's work, covering the years 1890–1899, was prepared by Dr. R. P. Falkner, and published by the Department of Labor in March, 1900.[10] Another short-lived series was begun by Prof. John R. Commons and Dr. N. I. Stone in the Quarterly Bulletin of the Bureau of Economic Research later in the same year.[11] In January, 1901, the second continuous American series was started by Dun's Review and gradually carried back to 1860; the third, covering the years 1890 to date, was added by the Federal Department of Labor in March, 1902. Other series of this type were begun by Thomas Gibson's weekly market letters in 1910, by the New York Times Annalist in 1913, and by the Federal Reserve Board in 1918.

This activity in the making of index numbers was accompanied by a rapid growth of the literature of the subject. Among the later contributions dealing with theoretical issues, the first place belongs to the work of an American scholar, Mr. C. M. Walsh. His great treatise upon the Measurement of General Exchange-Value, published in 1901, is still the most comprehensive book upon the subject. But the bibliographies that aim to cover the field now include hundreds of items, and to them must go the student who wishes a guide to further reading.[12]

Some of the more important new series known to have been established since the war are the series compiled by the Price Section of the War Industries Board and published in its "History of Prices

[8] See Finance Reports, 1881, pp. 312–321; 1882, pp. 252–254; 1883, pp. 316–318; Report of the Director of the Mint on the Production of the Precious Metals, 1884, pp. 497–502. Compare the criticism of this series by Prof. J. Laurence Laughlin, Quarterly Journal of Economics, April, 1887, pp. 397 and 398.

[9] See the description given on pp. 149–159.

[10] See Bulletin No. 27 of the Department of Labor, March, 1900.

[11] See the issues for July and October, 1900.

[12] For such bibliographies see Walsh, The Measurement of General Exchange-Value, pp. 553–574, and J. L. Laughlin, Principles of Money, pp. 221–224. The most important contributions of later date than Laughlin's entries are Prof. Irving Fisher's Purchasing Power of Money, pp. 385–429, Mr. C. M. Walsh's "The Problem of Estimation," Prof. Irving Fisher's paper on "The Best Form of Index Numbers," with discussion, in the Quarterly Publication of the American Statistical Association, March, 1921, and Mr. A. W. Flux's paper on "The Measurement of Price Changes," with discussion, in the Journal of the Royal Statistical Society, March, 1921.

During the War," the series compiled by the Federal Reserve Board from data gathered by the United States Bureau of Labor Statistics, the series designed by the same board for making international comparisons, the series published by the United States Food Administration in 1918 in a pamphlet entitled "General Index Numbers of Food Prices on a Nutritive Value Base," the series established by the London Times for Great Britain and by the Handelstidning for Sweden, the series for Italy compiled by Prof. Riccardo Bachi, the series compiled by the Bank of Japan, and those published by the Governments of South Africa and New Zealand.

II.—THE DIFFICULTIES OF MEASURING CHANGES IN THE LEVEL OF PRICES.

It is a curious fact that men did not attempt to measure changes in the level of prices until after they had learned to measure such subtle things as the weight of the atmosphere, the velocity of sound, fluctuations of temperature, and the precession of the equinoxes. Their tardiness in attacking that problem is the more strange because price changes had frequently been a subject of acrimonious debate among publicists and a cause of popular agitation. Long before the high development of the credit system and a class of permanent wage earners practical issues of grave importance were raised by the instability of prices, as the disturbances created in sixteenth-century Europe by the inflow of American silver and gold abundantly show. Perhaps disinclination on the part of "natural philosophers" to soil their hands with such vulgar subjects as the prices of provisions was partly responsible for the delay;[13] but after all a number of eminently "respectable" men wrote upon economic topics in every generation after the days of Columbus—to go no further back. Nor can the technical difficulties of the problem explain this tardiness; for the mathematical intricacy of index numbers, and even the necessity of allowing for changes in the pure silver content of coins, are obstacles far less formidable than those surmounted long before in other fields of research.

Probably the chief cause of delay was that averages of price fluctuations did not promise to command much confidence after they had been made. The quotations available for use by the early investigators were few in number and often of doubtful accuracy. Carli, for example, dealt with only three commodities; Shuckburg-Evelyn with 12. About the vastly greater number of unrecorded price fluctuations the one firmly established fact was that they exhibited bewildering diversity. Under these circumstances, could an average made from a few samples be accepted as a reliable measure of changes in the general level of prices? And if averages could not be trusted, why trouble to devise a plan of making them? So writers upon

[13] One of the early British writers on prices, Bishop Fleetwood, remarked: "* * * as the World now goes, the greatest (tho' I will not think the best) Part of Readers will be rather apt to despise than to commend the Pains that are taken in making Collections of so mean Things as the price of Wheat & Oats, of Poultry, and such like Provisions * * *."—*Chronicon Preciosum*, 1707, 2d ed., 1745, p. 6. Sir G. Shuckburg-Evelyn, in the paper referred to above, also felt himself on the defensive in presenting the first English index number: "* * * However, I may appear to descend below the dignity of philosophy, in such economical researches, I trust I shall find favour with the historian, at least, and the antiquary." Shuckburg-Evelyn's discussion of index numbers, indeed, was merely a minor appendix to his discussion of standards of weights and measures. But it has become his chief claim to remembrance.

prices long contented themselves with statements about the fluctuations of particular commodities, and with indefinite assertions that the purchasing power of money had changed little or changed much. So, also, when certain bold investigators did finally venture to make index numbers, no one was particularly impressed by the significance of their achievement.

This lack of faith in the validity of averages of price variations was overcome rather slowly, partly in consequence of improvements in business organization. The multiplication of commercial newspapers and the more systematic keeping of private and public records provided a larger and more accurate body of quotations. Improved means of transportation made wholesale prices in the larger cities basic for many local markets. The grading and standardizing of commodities increased the number of articles which could be accepted as substantially uniform in quality from one year to the next. More important still was the discovery by statisticians that social phenomena of most kinds, though seeming to result from the uncontrolled choice of individuals, yet reveal a striking regularity when studied in large numbers.[14] The demonstration that a formerly unsuspected regularity lay hidden in one set of numerical data after another encouraged economists to believe that the known price variations might after all be fair samples of the more numerous unknown variations. The general similarity of the results reached by different investigators using dissimilar data confirmed this faith. Thus emboldened, economic statisticians devoted much time to extending the scope and improving the technique of index numbers. And their growing confidence in the trustworthiness of their series was gradually imparted to the public.

To-day few, if any, competent judges doubt the validity of index numbers or the substantial accuracy of the results they show when properly constructed from carefully collected data. Indeed the danger at present is rather that the figures published will be taken too absolutely as a complete representation of the facts about price fluctuations. It is therefore well to begin a study of index numbers, not by analyzing the finished series, but by inspecting the actual changes in prices from which they are made, and which they purport to summarize. In no other way, indeed, can the value and the limitations of index numbers be learned.

III.—THE CHARACTERISTICS OF PRICE FLUCTUATIONS.

An excellent collection of materials for the study of changes in wholesale prices is found in Bulletin No. 149 of the Bureau of Labor Statistics, published in 1914. Here are given the average annual prices at wholesale of more than 230 commodities for a period of almost a quarter of a century. Comparison of the changes in these actual prices is facilitated by the publication of two series of relative prices for each commodity. One series reduces the quotations in dollars and cents to percentages of the average actual prices in the decade 1890–1899. The second series, which may be called "chain relatives," shows the percentage by which each article rose or fell in

[14] The Belgian statistician, Adolphe Quetelet, and Thomas Henry Buckle, author of the History of Civilization in England, 1857 and 1861, were perhaps the most effective demonstrators of this fact.

price each year as compared with the year before.[15] Since this section is concerned wholly with problems of method which have no connection with any given period of time, there is no reason for bringing all the illustrative materials down to date.

A survey of these relative figures for the 230 commodities throws the diversity of price fluctuations into high relief. (1) During the 24 years 1890–1913 no two of the commodities quoted underwent the same changes in price. Some articles rose rather steadily in price and fluctuated on a much higher level in 1913 than in 1890; for example, rosin and crude petroleum. Other articles fell much more than they rose and fluctuated on a much lower level at the end than at the beginning; for example, soda and wood alcohol. Some articles were steady in price, seldom changing from one year to the next; for example, bread and certain kinds of tools. Other articles changed in price every year; for example, cotton and pig iron. (2) In every year a considerable proportion of the commodities rose in price, a considerable proportion fell, and a somewhat smaller proportion remained unchanged. (3) The range covered even by the fluctuations from one year to the next was very wide. For example, in 1896 potatoes fell 54.6 per cent, while coke rose 41.5 per cent; in 1899 wheat flour fell 20.2 per cent, while steel billets rose 103.3 per cent; in 1913 onions fell 38.5 per cent, while cabbage rose 58.5 per cent.[16]

Such extreme diversities as have been cited, however, give a misleading impression of chaos among the fluctuations. A just impression can be had only from some scheme of presentation which takes account of all the commodities at once. Table 1 is a first rough approximation toward this end.[17] It shows for each year how many of the commodities quoted rose, remained unchanged, or fell in price, and divides those which rose and those which fell into six groups, according to the magnitude of their fluctuations.

[15] The reader may follow the discussion more easily if he runs over the following sample of the figures referred to.

Cotton, upland, middling.

Year.	Average price per pound.	Relative price.	Per cent of increase(+) or decrease(−) compared with preceding year.
Average, 1890–1899	$0.07762	100.0	....
1890	.11089	142.9	....
1891	.08606	110.8	−22.4
1892	.07686	99.0	−10.7
1893	.08319	107.2	+ 8.2
1894	.07002	90.2	−15.8
1895	.07298	94.0	+ 4.2
1896	.07918	102.0	+ 8.5
1897	.07153	92.2	− 9.7
1898	.05972	76.9	−16.5
1899	.06578	84.7	+10.1
1900	.09609	123.8	+46.1
1901	.08627	111.1	−10.2
1902	.08932	115.1	+ 3.5
1903	.11235	144.7	+25.8
1904	.12100	155.9	+ 7.7
1905	$0.09553	123.1	−21.0
1906	.11025	142.0	+15.4
1907	.11879	153.0	+ 7.7
1908	.10463	134.8	−11.9
1909	.12107	156.0	+15.7
1910	.15118	194.8	+24.9
1911	.13037	168.0	−13.8
1912	.11503	148.2	−11.8
1913	.12792	164.8	+11.2

[16] All of these figures show percentages of rise or fall from the average prices of the commodities in question in the preceding year.

[17] The figures in this table have been brought down to 1918 to harmonize with the material in Section V, on "A comparison of the leading American index numbers for the years 1890 to 1918."

TABLE 1.—CONSPECTUS OF THE CHANGES IN WHOLESALE PRICES IN THE UNITED STATES, BY YEARS, 1891 TO 1918.

[Based upon the percentages of increase or decrease in price from one year to the next, computed from Table 9 of Bulletin of the United States Bureau of Labor Statistics, No. 269, May, 1920.]

Year.	Total number of commodities quoted each year.	Number of commodities that fell in price.	Number of commodities that fell in price by—						Number of commodities that did not change in price.	Number of commodities that rose in price by—						Number of commodities that rose in price.
			50.0 per cent or more.	20.0 to 49.9 per cent.	10.0 to 19.9 per cent.	5.0 to 9.9 per cent.	2.0 to 4.9 per cent.	Less than 2.0 per cent.		Less than 2.0 per cent.	2.0 to 4.9 per cent.	5.0 to 9.9 per cent.	10.0 to 19.9 per cent.	20.0 to 49.9 per cent.	50.0 per cent or more.	
1891	232	106	....	13	26	30	19	18	44	17	17	15	16	16	1	82
1892	232	140	....	11	47	39	27	16	37	19	9	13	12	2	....	55
1893	234	114	....	5	25	44	25	15	42	15	17	21	10	14	1	78
1894	236	192	....	29	70	44	34	15	25	4	6	3	3	3	....	19
1895	237	138	....	10	35	41	40	12	22	15	18	17	12	13	2	77
1896	240	133	1	22	35	22	30	23	34	15	16	18	18	6	....	73
1897	241	118	1	9	22	35	27	24	31	12	20	30	11	17	2	92
1898	242	73	....	2	16	21	21	13	34	15	22	34	40	22	2	135
1899	242	46	....	1	7	12	16	10	27	17	28	45	39	26	14	169
1900	242	38	....	3	4	13	9	9	20	7	25	59	57	33	3	184
1901	242	128	....	10	40	32	35	11	25	19	22	16	21	9	2	89
1902	242	61	....	6	13	14	12	16	38	20	31	35	29	27	1	143
1903	242	92	....	9	23	21	22	17	22	16	29	44	29	10	....	128
1904	242	106	....	12	24	22	28	20	23	27	32	28	10	16	....	113
1905	242	89	....	3	13	26	24	23	22	22	26	37	31	15	....	131
1906	242	47	....	5	10	9	13	10	28	13	31	52	52	19	....	167
1907	242	48	....	2	5	9	18	14	32	27	25	43	45	20	2	162
1908	242	155	....	25	50	32	30	18	32	14	14	12	11	4	....	55
1909	253	98	....	2	15	21	28	32	31	24	24	40	17	16	3	124
1910	253	81	....	3	10	20	22	26	26	24	33	42	25	22	....	146
1911	253	147	....	18	31	43	30	25	31	19	16	14	13	10	3	75
1912	253	80	....	6	12	18	25	19	36	21	27	34	35	20	....	137
1913	252	84	....	12	14	16	23	19	35	28	39	30	27	7	2	133
1914	329	152	....	12	35	44	28	33	59	30	21	35	23	8	1	118
1915	342	137	2	9	35	29	27	35	36	22	38	39	38	28	4	169
1916	342	12	....	....	1	5	3	3	10	11	27	37	88	115	42	320
1917	337	9	....	2	2	....	3	2	2	2	7	15	30	172	100	326
1918	348	56	1	11	16	14	12	2	1	9	18	27	73	136	28	291

A more significant presentation of the same set of price fluctuations is given by Table 2. To make this table a tally sheet was drawn up for each year from 1891 to 1918, on which the changes from prices in the preceding year were entered in the order of their magnitude, beginning with the greatest percentage of fall and running up through "no change" to the greatest percentage of rise. Then the whole number of recorded fluctuations for each year was divided into 10 numerically equal groups, again beginning with the case of greatest fall and counting upward. Finally the nine dividing points between these 10 equal groups were marked off in the percentage scale of fall, "no change," or rise. For example, the tally sheet for 1913 showed how the average prices of 252 commodities in that year differed from their average prices in 1912. One-tenth of these 252 commodities showed a fall of prices ranging between 38.5 per cent and 10.4 per cent, the second tenth ranged between a fall of 10.4 per cent and one of 3.7 per cent; the third tenth ranged between a fall of 3.7 per cent and one of 1 per cent; the fourth tenth between a fall of 1 per cent and "no change;" the fifth tenth between "no change" and a rise of 0.5 per cent, and so on. These dividing points (−10.4 per cent, −3.7 per cent, −1 per cent, ±0 per cent, +0.5 per cent, etc.) between the successive tenths into which the data were split are called "decils." The midmost decil, which of course divides the whole number of observations into two equal groups, is called the "median." Table 2 presents the results drawn from the tally sheets—that is, the nine decils for each year, together with the percentages of greatest fall and of greatest rise from prices in the year before.

TABLE 2.—CHAIN INDEX NUMBERS OF PRICES AT WHOLESALE IN THE UNITED STATES, BY YEARS, 1891 TO 1918.

[The decils are those points in the percentage scale of rise or fall in price which divide the whole number of price changes recorded each year into 10 equal groups. Based upon the percentages of increase or decrease in price from one year to the next, computed from Table 9 of Bulletin of the United States Bureau of Labor Statistics, No. 269, May, 1920.]

(− indicates a fall; + indicates a rise; ±0 indicates "no change.")

Year.	Greatest fall.	1st decil.	2d decil.	3d decil.	4th decil.	Median.	6th decil.	7th decil.	8th decil.	9th decil.	Greatest rise.
	Per ct.	*Per ct.*	*Per ct.*	*Per ct.*	*Per ct.*	*Per ct.*	*Per ct.*	*Per ct.*	*Per ct.*	*Per ct.*	*Per ct.*
1891	−30.5	−13.2	− 8.0	− 4.8	− 1.4	± 0	± 0	+ 1.5	+ 5.0	+15.3	+ 53.0
1892	−41.2	−16.0	−11.2	− 8.5	− 5.4	− 3.1	− 0.5	± 0	+ 1.1	+ 5.5	+ 28.0
1893	−27.5	−11.9	− 8.0	− 5.5	− 2.4	± 0	± 0	+ 1.1	+ 4.8	+11.0	+ 59.1
1894	−44.3	−21.4	−15.8	−13.4	−10.8	− 7.1	− 5.0	− 3.3	− 1.3	± 0	+ 31.1
1895	−38.0	−14.0	− 9.6	− 6.5	− 4.1	− 2.4	± 0	+ .7	+ 4.2	+12.1	+ 61.9
1896	−54.6	−17.8	−11.3	− 7.5	− 3.0	− 1.2	± 0	+ .3	+ 4.3	+10.2	+ 41.5
1897	−50.9	−11.5	− 7.2	− 4.4	− 1.7	± 0	± 0	+ 2.9	+ 6.2	+12.7	+101.6
1898	−21.9	− 7.0	− 3.3	− .4	± 0	+ 1.8	+ 5.0	+ 8.3	+13.3	+19.8	+ 60.4
1899	−20.2	− 3.8	± 0	± 0	+ 2.6	+ 5.5	+ 7.6	+10.6	+16.4	+30.8	+103.3
1900	−29.2	− 3.6	± 0	+ 3.2	+ 5.1	+ 7.5	+ 9.6	+12.7	+17.4	+25.6	+ 72.8
1901	−42.6	−15.0	−10.2	− 6.1	− 3.7	− 1.5	± 0	+ 1.3	+ 4.9	+13.2	+ 53.0
1902	−40.6	− 7.4	− 1.6	± 0	± 0	+ 2.2	+ 4.7	+ 7.1	+12.1	+20.4	+ 58.9
1903	−33.7	−12.6	− 5.3	− 2.1	± 0	+ 1.3	+ 3.7	+ 5.3	+ 8.3	+14.1	+ 37.4
1904	−43.8	−15.0	− 7.6	− 3.5	− .6	± 0	+ 1.3	+ 3.0	+ 5.9	+11.7	+ 39.9
1905	−44.9	− 7.6	− 3.9	− 1.0	± 0	+ .7	+ 3.2	+ 5.9	+ 9.6	+15.9	+ 46.0
1906	−39.1	− 4.8	± 0	± 0	+ 2.8	+ 5.1	+ 6.4	+ 9.7	+14.5	+18.9	+ 40.7
1907	−43.0	− 3.2	± 0	± 0	+ 1.2	+ 3.9	+ 6.6	+ 8.9	+12.3	+17.6	+ 67.8
1908	−39.5	−21.3	−16.0	−10.8	− 5.8	− 3.8	− .9	± 0	+ .8	+ 6.2	+ 44.9
1909	−29.8	− 7.7	− 3.7	− 1.1	± 0	± 0	+ 1.7	+ 5.0	+ 8.1	+16.0	+ 70.1
1910	−37.7	− 6.1	− 2.4	− .4	± 0	+ 1.5	+ 3.6	+ 6.3	+ 9.2	+18.6	+ 49.5
1911	−47.4	−15.1	− 9.8	− 7.0	− 4.2	− .9	± 0	± 0	+ 2.9	+11.0	+ 86.1
1912	−36.1	− 6.8	− 2.9	− .5	± 0	+ 1.0	+ 3.6	+ 6.7	+11.0	+17.7	+ 46.2
1913	−38.5	−10.4	− 3.7	− 1.0	± 0	+ .5	+ 2.4	+ 4.5	+ 7.5	+12.7	+ 58.5
1914	−37.3	−12.0	− 7.4	− 4.1	− 1.3	± 0	± 0	+ 1.5	+ 5.0	+ 9.1	+ 76.4
1915	−60.4	−12.0	− 5.9	− 1.9	− .1	± 0	+ 2.7	+ 6.0	+10.1	+18.7	+172.9
1916	−19.1	+ 2.1	+ 6.7	+10.5	+14.4	+18.6	+24.0	+30.1	+38.7	+53.4	+155.1
1917	−34.1	+ 8.7	+19.4	+25.1	+28.6	+31.8	+42.1	+49.3	+57.5	+69.3	+154.2
1918	−51.0	− 6.0	+ 2.0	+ 8.6	+14.8	+18.5	+22.1	+28.6	+36.1	+46.3	+118.0
Average	**−31.9**	**−10.1**	**− 5.0**	**− 2.9**	**+ .9**	**+ 3.0**	**+ 5.1**	**+ 7.3**	**+11.5**	**+19.0**	**+ 71.0**

CHART 1.—CONSPECTUS OF YEARLY CHANGES IN PRICES, 1891-1918.

(Based on Table 2.)

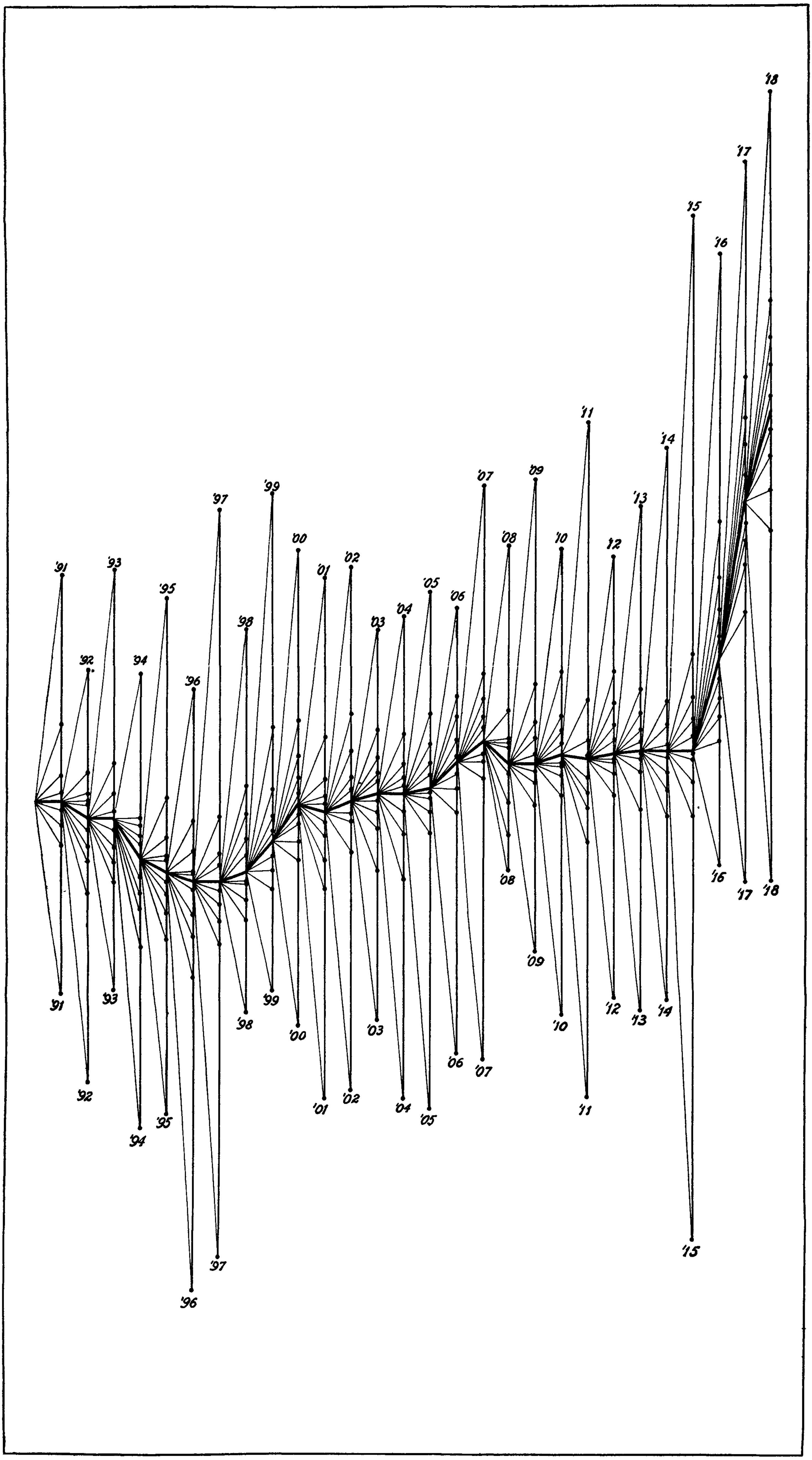

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Chart 1, based upon Table 2 and drawn to a logarithmic scale, gives a more vivid idea of these price fluctuations. It shows for each year the whole range covered by the recorded changes from prices in the preceding year by vertical lines, which connect the points of greatest rise with the points of greatest fall. These lines differ considerably in length, which indicates that price changes cover a wider range in some years than in others. The heavy dots upon the vertical lines show the positions of the decils. One-tenth of the commodities quoted in any given year rose above their prices of the year before by percentages scattered between the top of the line for that year and the highest of the dots. Another tenth fell in price by percentages scattered between the bottom of the line and the lowest of the dots. The fluctuations of the remaining eight-tenths of the commodities were concentrated within the much narrower range between the lowest and the highest dots. The dots grow closer together toward the central dot, which is the median. This concentration indicates, of course, that the number of commodities showing fluctuations of relatively slight extent was much larger than the number showing the wide fluctuations falling outside the highest and lowest decils, or even between these decils and the decils next inside them.

The middle dots or medians in successive years are connected by a heavy black line, which represents the general upward or downward drift of the whole set of fluctuations. To make this drift clear the median of each year is taken as the starting point from which the upward or downward movements in the following year are measured. Hence the chart has no fixed base line. But in this respect it represents faithfully the figures from which it is made; since these figures are percentages of prices in the preceding year, a price fluctuation in any year establishes a new base for computing the percentage of change in the following year. The fact that prices in the preceding year are the units from which all the changes proceed is further emphasized by connecting the nine decils, as well as the points of greatest rise and fall, with the median of the year before by light diagonal lines. The chart suggests a series of bursting bomb shells, the bombs being represented by the median dots of the years before and the scattering of their fragments by the lines which radiate to the decils and the points of greatest rise and fall.[18]

Time is well spent in studying this chart, because it is capable of giving a truer impression of the characteristics of price changes than can be given by any other device. The marked diversity of the fluctuations of different commodities in the same year—some rising, some falling, some remaining unchanged—the wide range covered by these fluctuations, the erratic occurrence of extremely large changes, and the fact that the greatest percentages of rise far surpass the greatest percentages of fall are strikingly shown; but so also are the much greater frequency of rather small variations, the dense concentration near the center of the field, the existence of a general drift in the whole complex of changes, and the frequent alterations in the direction and the degree of this drift. But if the chart is effective in giving these impressions, it leaves them rather vague. To render certain of them

[18] Owing to the constant shifting of the base line, no fixed scale of relative prices can be shown on the margin of the chart. Because of its intricacy, the chart had to be reproduced on a larger scale than in the other cases, but of course that fact does not alter the slant of the lines, and this slant is the matter of importance.

more definite, recourse must be had to the figures from which the chart was drawn.

These figures, already given in Table 2, enable us to measure the concentration of the mass of fluctuations about the center of the field. One way to measure this concentration is to compute the differences between the successive decils; that is, to find the range within which successive tenths of the fluctuations fall. This "range" is, of course, a certain number of points in the percentage scale of change from prices in the year before. When this computation is made for the whole period covered by the table, we get the results presented in Table 3. As heretofore, the successive tenths of the fluctuations represented are reckoned by starting with cases of greatest fall in price and counting upward to cases of greatest rise. The central division of the table shows that the average range covered by the fluctuations diminishes rapidly as we pass from the cases of greatest fall toward the cases of little change, and then increases still more rapidly as we go onward to the cases of greatest rise. The right-hand group of columns shows how the range increases if we start with the two middle tenths, take in the two-tenths just outside them, then the two-tenths outside the latter, and so on until we have included the whole body of fluctuations. The left-hand group of columns, on the other hand, combines in succession the two-tenths on the outer boundaries, then the two-tenths immediately inside them, and so on until we get back again to the two central tenths. Perhaps the most striking single result brought out by this table is that eight-tenths of all the fluctuations are concentrated within a range (29.1 per cent) slightly wider than that covered by the single tenth that represents the heaviest declines (21.8 per cent), and much narrower than that covered by the single tenth that represents the greatest advances (52 per cent).

TABLE 3.—AVERAGE CONCENTRATION OF PRICE FLUCTUATIONS AROUND THE MEDIAN, 1891 TO 1918.

[Based upon Table 2. The fluctuations represent percentage changes from average prices in the preceding year.]

Average range covered by the—										
1st and 10th tenths of the price fluctuations.	2d and 9th tenths of the price fluctuations.	3d and 8th tenths of the price fluctuations.	4th and 7th tenths of the price fluctuations.	5th and 6th tenths of the price fluctuations.	Successive tenths of the price fluctuations.	Central two tenths of the price fluctuations.	Central four tenths of the price fluctuations.	Central six tenths of the price fluctuations.	Central eight tenths of the price fluctuations.	Whole number of the price fluctuations.
					1st tenth, 21.8					
					2d tenth, 5.1					
					3d tenth, 2.1					
					4th tenth, 3.8					
73.8	12.6	6.3	6.0	4.2	5th tenth, 2.1	4.2	10.2	16.5	29.1	102.9
					6th tenth, 2.1					
					7th tenth, 2.2					
					8th tenth, 4.2					
					9th tenth, 7.5					
					10th tenth, 52.0					

Such results as these gain greatly in significance by being put beside corresponding results for other groups of statistical data. The best comparison to make, however, is one between the actual distribution of our price fluctuations about their average and a "normal" distribution of the same data—that is, a distribution according perfectly with the so-called "normal law of error." This law shows how phenomena are distributed about their average when the number of phenomena observed is very large, and when each phenomenon is the resultant of numerous independent factors, none of which is of preponderating importance. It has been found that many kinds of phenomena tend to conform rather closely to this normal distribution; for example, human heights, errors of observation, shots at a target, wage rates in different occupations, etc.[19] When it can be shown that phenomena are distributed approximately in this fashion, their average can safely be accepted as a significant measure of the whole set of variations, since even the deviations from the average are then grouped in a tolerably definite and symmetrical fashion about the average.

With such a comparison in view we may treat each recorded percentage of rise or fall in price as an observation of the degree and direction in which prices vary from one year to the next. Taking all the commodities and all the years up to 1913 covered by the bureau's chain relatives, we have 5,578 observations for analysis. Table 4 shows how these cases are distributed along a percentage scale of rise or fall in prices which jumps two points at a time. The columns headed "number of cases" show how many price variations of the given magnitude and directions occur, and the columns headed "proportion of cases" show the same numbers in the rather clearer form of percentages of their sum (5,578).

Such is the actual distribution of the phenomena under analysis. To compare it with the "normal" distribution, we put these figures on a chart, which presents the facts clearly to the eye. Here the horizontal scale represents percentages of rise or fall in price, and the vertical scale represents the number of times each percentage of change is observed. The dotted line shows how our 5,578 cases would have been distributed had they followed strictly the normal law of error. The areas included by the unbroken line and the dotted line are equal.[20]

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[19] See, for example, Prof. F. Y. Edgeworth's article "Probability," Part II, Encyclopædia Britannica, 11th ed., and the references there given.

[20] Table 4 and Chart 2 might be improved by a change in form. If the "price variations" in each year were computed as percentage deviations from their geometric mean in that year, the distribution of their variations would doubtless be more symmetrical than is the distribution here shown.

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TABLE 4.—DISTRIBUTION OF 5,578 CASES OF CHANGE IN THE WHOLESALE PRICES OF COMMODITIES FROM ONE YEAR TO THE NEXT, ACCORDING TO THE MAGNITUDE AND DIRECTION OF THE CHANGES.

[Based upon the chain relatives in Table II of Bulletin No. 149 of the Bureau of Labor Statistics.]

Rising prices.						Falling prices.		
Per cent of change from the average price of the preceding year.	Number of cases.	Proportion of cases.	Per cent of change from the average price of the preceding year.	Number of cases.	Proportion of cases.	Per cent of change from the average price of the preceding year.	Number of cases.	Proportion of cases.
102-103.9	1	0.018	46-47.9	11	0.197	Under 2.	[1] 405	7.261
100-101.9	1	.018	44-45.9	10	.179	2- 3.9	[1] 375	6.723
98- 99.9			42-43.9	6	.108	4- 5.9	329	5.898
96- 97.9			40-41.9	14	.251	6- 7.9	[1] 238	4.267
94- 95.9			38-39.9	17	.305	8- 9.9	200	3.585
92- 93.9			36-37.9	11	.197	10-11.9	173	3.101
90- 91.9			34-35.9	18	.323	12-13.9	[1] 120	2.151
88- 89.9			32-33.9	17	.305	14-15.9	107	1.918
86- 87.9	1	.018	30-31.9	22	.394	16-17.9	76	1.362
84- 85.9	1	.018	28-29.9	30	.538	18-19.9	71	1.273
82- 83.9	1	.018	26-27.9	29	.520	20-21.9	45	.807
80- 81.9	1	.018	24-25.9	47	.843	22-23.9	39	.699
78- 79.9			22-23.9	45	.807	24-25.9	32	.574
76- 77.9			20-21.9	65	1.165	26-27.9	17	.305
74- 75.9	1	.018	18-19.9	73	1.308	28-29.9	27	.484
72- 73.9	4	.072	16-17.9	[1] 102	1.828	30-31.9	16	.287
70- 71.9	1	.018	14-15.9	106	1.900	32-33.9	7	.125
68- 69.9	3	.054	12-13.9	115	2.062	34-35.9	10	.179
66- 67.9	4	.072	10-11.9	167	2.994	36-37.9	7	.125
64- 65.9			8- 9.9	[1] 237	4.249	38-39.9	5	.090
62- 63.9			6- 7.9	261	4.679	40-41.9	5	.090
60- 61.9	4	.072	4- 5.9	[1] 356	6.382	42-43.9	4	.072
58- 59.9	6	.108	2- 3.9	355	6.364	44-45.9	2	.036
56- 57.9	1	.018	Under 2.	[1] 410	7.350	46-47.9	1	.018
54- 55.9	3	.054				48-49.9	1	.018
52- 53.9	4	.072	No change.	[1] 697	12.494	50-51.9	1	.018
50- 51.9	1	.018				52-53.9		
48- 49.9	5	.090				54-55.9	1	.018

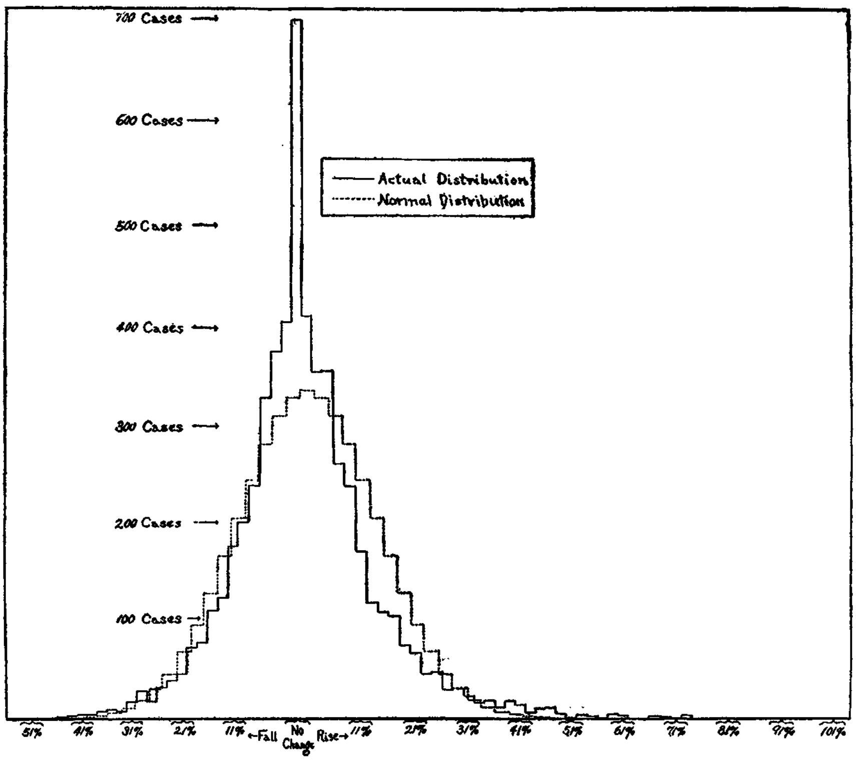
Summary.

	Number of cases.	Proportion of cases.
Rising prices	2,567	46.021
No change	697	12.494
Falling prices	2,314	41.485
Total	5,578	100.000

[1] Location of the decils.

There are several points to notice here. While the actual and the "normal" distributions look much alike, they are not, strictly speaking, of the same type. The actual distribution is much more pointed than the other, and has a much higher "mode," or point of greatest density. On the other hand, the actual distribution drops away rapidly on either side of this mode, so that the curve representing it falls below the curve representing the "normal" distribution. The actual distribution is "skewed" instead of being perfectly symmetrical. The outlying cases of a "normal" distribution extend precisely the same distance from the central tendency in both directions, whereas in the actual distribution the outlying cases run about twice as far to the right (in the direction of a rise of prices) as to the left (in the direction of a fall). This fact suggests that the actual distribution would be more symmetrical if it were plotted on a logarithmic scale, one which represents the doubling of one price by the same distance from zero as the halving of another price. Another aspect of the difference in symmetry is that the central tendency about which the variations group themselves is free from ambiguity in one case but not in the other. In the "normal" distribution this tendency may be expressed indifferently by the median, the arithmetic mean, or the mode; for these three averages coincide. In the actual distribution, on the contrary, these averages differ slightly; the median and the "crude" mode stand at ± 0, while the arithmetic

CHART 2.—DISTRIBUTION OF 5,578 PRICE VARIATIONS (PERCENTAGES OF RISE OR FALL FROM PRICES OF PRECEDING YEAR).

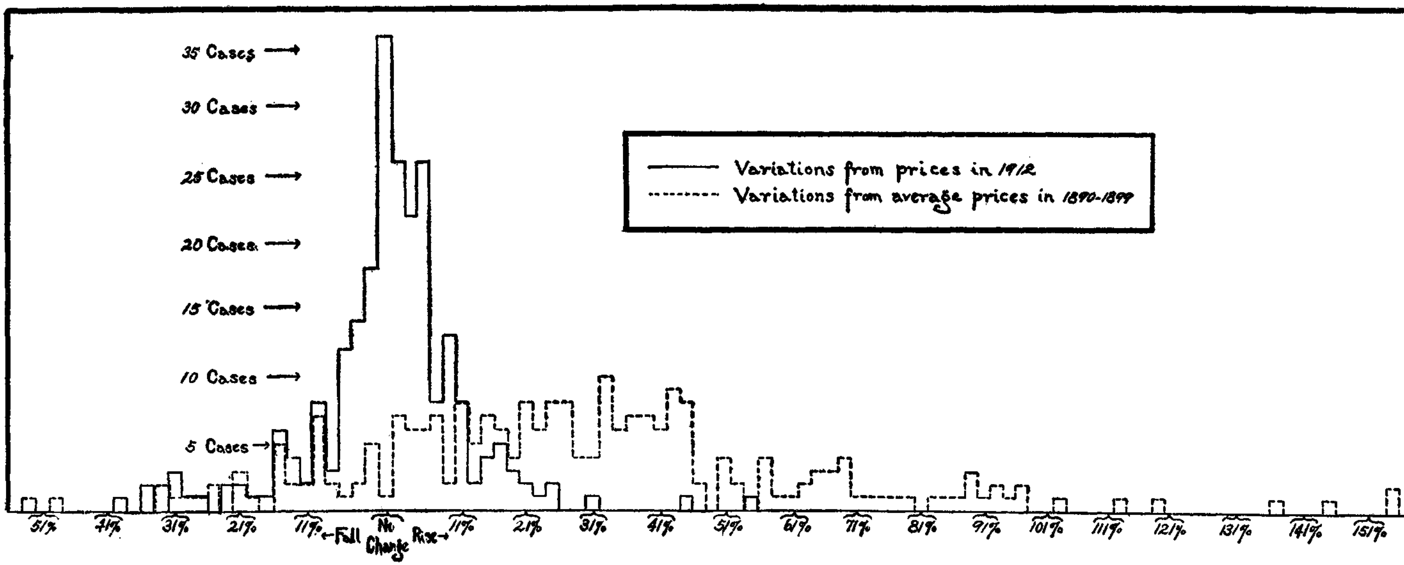


mean is +1.36 per cent.[21] These departures of the actual distribution from perfect symmetry possess significance; but the fact remains that year-to-year price fluctuations are highly concentrated about their central tendency.

This study of the actual distribution of price fluctuations from one year to the next will be found to throw light upon several problems presently to be faced in discussing the methods of making index

[21] That the arithmetic mean is slightly above zero arises partly from the fact that there are 33 percentages of rise greater than any percentage of fall. But it also arises partly from the fact that our data come from a period (1890–1913) when the trend of year-to-year fluctuations was more often upward than downward; there were 2,567 cases of advance in price against 2,314 cases of fall. The median is kept from rising above zero because the cases of "no change," 697 in number, more than offset the difference between the numbers of advances and of declines in price.

CHART 3.—DISTRIBUTION OF THE PRICE VARIATIONS OF 241 COMMODITIES IN 1913 (PERCENTAGES OF RISE OR FALL IN PRICES).



numbers. For the moment we have use primarily for the demonstration that these fluctuations are highly concentrated about a central tendency. This conclusion strengthens the hope that we may make measurements of price fluctuations that fairly represent the net resultant of all the changes, miscellaneous as they seem to be. For properly constructed averages have clearly a better chance of being representative and significant when the phenomena for which they stand have a strongly marked central tendency about which deviations are grouped than when the phenomena are irregularly scattered over their range.

But it must be remembered, and with the reminder doubt reenters, that the variations just analyzed are percentages of increase or decrease from the prices of the year before. Most index numbers, however, attempt to measure price fluctuations, not with reference to the preceding year, but with reference to a period considerably more remote. For example, in its old series, here used for illustration, the Bureau of Labor Statistics measured prices in 1913 in terms of average prices in the decade 1890–1899. Are price variations computed in this manner highly concentrated around their central tendency like the price variations with which we have been dealing?

Chart 3 answers this question emphatically in the negative. It represents the distribution of the price variations of 241 commodities quoted by the Bureau of Labor Statistics for the year 1913.[22] These variations are computed in two ways: (1) As percentages of rise or fall from the prices of 1912; (2) as percentages of rise or fall from the average prices of 1890–1899. Of course the first set of variations corresponds in character to the variations represented above in Chart 2. The distribution of these variations, shown by the area inclosed by the unbroken line, is similar in type to the actual distribution in Chart 2; although it is less regular—a difference to be expected, since the number of observations is only 241 here as against 5,578 there. But the distribution of the second set of variations (percentages of change from the average prices of 1890–1899) as represented by the area inclosed within the dotted line has no obvious central tendency; it shows no high degree of concentration around the arithmetic mean (+30.4 per cent) or median (+26 per cent) and it has a range between the greatest fall (52.2 per cent) and greatest rise (234.5 per cent) so extreme that two of the cases could not be represented on the chart.[23]

Price variations, then, become dispersed over a wider range and less concentrated about their mean as the time covered by the variations increases. The cause is simple: With some commodities the trend of successive price changes continues distinctly upward for years at a time; with other commodities there is a consistent downward trend; with still others no definite long-period trend appears. In any large collection of price quotations covering many years each of these types, in moderate and extreme form, and all sorts of crossings among them, are likely to occur. As the years pass by the commodities that

[22] The bureau quoted 252 commodities in 1913; but 11 could not be included in the present comparison because no quotations are given for them in 1890–1899.

[23] In commenting on this chart Prof. Edgeworth has shown that, despite appearances, the distribution of the price variations from the 1890–1899 base, may conform to the normal distribution as closely as the variation from the preceding year base. For, under the condition presented by prices, the quantity observed may move either up or down at each successive interval (here a year), and with a number of observations such as here used, an ideal distribution would appear more or less oblong (as does the dotted line in Chart 3) rather than bell shaped.—Economic Journal, June, 1918, Vol. XXVIII, pp. 183–185.

have a consistent trend gradually climb far above or subside far below their earlier levels, while the other commodities are scattered between these extremes. Thus the percentages of variation for any given year gradually get strung out in a long, thin, and irregular line, without a marked degree of concentration about any single point. Another factor in scattering the percentage variations is probably that the degree of scatter is a function of the degree of variation, and of course variations are likely to be larger between years far apart than between years close together.

The consequence is that the measurement of price fluctuations becomes difficult in proportion to the length of time during which the variations to be measured have continued. In other words, the farther apart are the dates for which prices are compared, the wider is the margin of error to which index numbers are subject, the greater the discrepancies likely to appear between index numbers made by different investigators, the wider the divergencies between the averages and the individual variations from which they are computed, and the larger the body of data required to give confidence in the representative value of the results.

From this preliminary survey of the characteristics of price fluctuations it appears (1) that year-to-year changes in the price level can be measured with good prospects of success, because such variations show a marked degree of concentration about their central tendency, but (2) that measurements of variations between years far apart have a more problematical value. The practical question whether the index numbers in current use can be trusted, then, may have two answers. Perhaps they give results that are reliable as between successive years, and at the same time doubtful for dates between which 50, 20, or even 10 years have intervened.

The best way to test the reassuring conclusion about index numbers for successive years and to resolve the disturbing doubt about index numbers covering long periods is to compare different series of index numbers that purport to measure price changes in the same country during the same time. If the results turn out to be consistent with one another, our faith will be confirmed. If the results are not consistent, we must find a valid reason for the discrepancies, or become skeptical about the present methods of measuring changes in the price level.

When this test is applied, the first impression is unfavorable. For example, the five currently published American index numbers show the following results for 1912 and 1913:

Year.	Bureau of Labor Statistics' index number (old series).	Bradstreet's index number.	Annalist index number.	Gibson's index number.	Dun's index number.
1912	133.6	$9.1867	143.25	62.6	$124.44
1913	135.2	9.2076	139.98	58.1	120.89
Changes	+1.6	+.0209	−3.27	−4.5	−3.55
Percentage changes	+1.2	+.2	−2.3	−7.2	−2.9

Here no two of the series are as closely consistent with each other as one could wish. On the contrary, the five series disagree not only as to the degree but also as to the direction of the change in prices. And this is a comparison between the same successive years, where measurements should be especially accurate.

Such offhand comparisons as the above, however, are not fair, and the conclusion they suggest as to the unreliability of index numbers can not be accepted without further study, for these various index numbers mean different things. They do not all undertake to measure the same quantity, hence they do not all employ the same methods, and hence the discrepancies among their results may reveal no real inconsistency. No valid comparison of index numbers can be made, indeed, without a careful examination of what is measured and how the measurement is made. Such an examination accordingly we must make before we can satisfy our minds upon the question whether index numbers yield trustworthy results.

IV.—VARIETIES OF METHODS USED IN MAKING INDEX NUMBERS.

Making an index number involves several distinct operations: (1) Defining the purpose for which the final results are to be used; (2) deciding the numbers and kinds of commodities to be included; (3) determining whether these commodities shall all be treated alike or whether they shall be "weighted" according to their relative importance; (4) collecting the actual prices of the commodities chosen, and, in case a weighted series is to be made, collecting also data regarding their relative importance; (5) deciding whether the form of the index number shall be one showing the average variations of prices or the variations of a sum of actual prices; (6) in case average variations are to be shown, choosing the base upon which relative prices shall be computed; and (7) settling upon the form of average to be struck, if averages are to be used.

At each one of these successive steps choice must be made among alternatives that range in number from two to thousands. The possible combinations among the alternatives chosen are indefinitely numerous. Hence there is no assignable limit to the possible varieties of index numbers, and in practice no two of the known series are exactly alike in construction. To canvass even the important variations of method actually in use is not a simple task.

1. THE RELATIONS BETWEEN METHODS AND USES.

The first step, framing a clear idea of the ultimate use of the results, is most important, since it affords the clue to guide the compiler through the labyrinth of subsequent choices. It is, however, the step most frequently omitted.

Mr. C. M. Walsh and Prof. Irving Fisher, indeed, hold that "an index number is itself a purpose." "In averaging price variations," Mr. Walsh explains, "the purpose or object is given: It is to measure variations in the exchange value or purchasing power of money." Hence they logically contend that there is one "best form of index number."[24] But this position is untenable. (1) As a statistical device, index numbers have found a wide range of application outside the field of prices. To deny the term index numbers to series which show average variations in municipal water supply, rainfall, railroad traffic, and the like conflicts with established usage. (2) Within the field of prices index numbers are needed which do not aim to measure

[24] See Walsh's The Problem of Estimation, p. 116, and Fisher's "Rejoinder" in Quarterly Publication of the American Statistical Association, March, 1921, p. 547. The merits of the formula which they consider "the best" are discussed below, in section 9, pp. 91–93.

the purchasing power of money. For example, some one should compile a special series for forecasting changes in business conditions. The compiler might select those commodities whose prices in the past have given the earliest and most regular indications of changes that subsequently occurred in the larger index numbers, he might weight these series in accordance with their past reliability as price "barometers," and he might use whatever method of averaging the fluctuations gave the best results for his purpose. Such a series probably would not be a reliable measure of variations in "the purchasing power of money," but it probably would be better adapted to its special purpose than a series made by the formula which Prof. Fisher and Mr. Walsh advocate as "the best." (3) To "measure variations in the exchange value or purchasing power of money" is not a clearly defined aim. For example, in explaining his new form of the British Board of Trade index number to the Royal Statistical Society Prof. A. W. Flux pointed out that he might have aimed either to find the change in the money cost of the things people buy, or to find the net effect of the general economic situation, and especially of currency and credit, on prices. In discussing this paper Prof. G. Udney Yule added a third aim, "To find the effect of price-changes on currency and credit."[25] These three aims, which at first sight seem much the same, turn out on closer scrutiny to differ and to call for the use of dissimilar formulas, as Prof. Flux and Prof. Yule argued. Nor is their list of aims in measuring the purchasing power of money exhaustive. (4) What does "the purchasing power of money" include? Merely the standardized wares of the wholesale markets which are sampled with varying thoroughness in the current index numbers? Or does it include also commodities at retail, stocks, bonds, labor of all sorts, farm lands and town lots, loans, transportation, insurance, advertising space, and all the other classes of goods that are bought and sold? As Mr. W. T. Layton remarked in discussing Prof. Flux's paper, "The wholesale price index number is not a measure of the general purchasing power of money, though all the wholesale price index numbers are constantly quoted as such."[26]

In fine, the problem of measuring the purchasing power of money has not yet been thoroughly explored. To insist that this problem has but one meaning and therefore one "best" solution obstructs progress. It is wiser to exploit all the significant interpretations of the problem and to consider what solution is appropriate to each. And in addition to this general problem we should devise "special-purpose" index numbers to solve particular problems with a view to learning all we can about the fluctuations of economic quantities, physical as well as pecuniary. The making of index numbers is still in the experimental stage, and it will progress by the differentiation of many types of series, each with its clearly defined uses.

The most systematic plan of treating the subject, then, would be to begin with the different uses of index numbers and to consider the methods appropriate to each. But that plan can not be followed in an interpretative study of the currently published series, because most of the wholesale price index numbers are "general-purpose" series designed with no aim more definite than that of "measuring changes in the price level." The only plan feasible

[25] Journal of the Royal Statistical Society, March, 1921, pp. 175–179 and 200. [26] Idem, p. 206.

for such a study at present is to invert the problem. Instead of studying methods in the light of uses, we must study uses in the light of methods. That is, we must analyze the effect of the different methods followed in practice and so determine what the resulting figures mean and the uses to which they may properly be put.

The following discussion proceeds upon this plan. It deals primarily with the popular general-purpose series and endeavors to show how the various methods used in constructing these index numbers determine the uses to which they are severally adapted.

2. COLLECTING AND PUBLISHING THE ORIGINAL QUOTATIONS.

The reliability of an index number obviously depends upon the judgment and the accuracy with which the original price quotations were collected. This field work is not only fundamental, it is also laborious, expensive, and perplexing beyond any other part of the whole investigation. Only those who have tried to gather from the original sources quotations for many commodities over a long series of years appreciate the difficulties besetting the task. The men who deal with data already published are prone to regard all this preliminary work as a clerical compilation requiring much industry but little skill. To judge from the literature about index numbers, one would think that the difficult and important problems concern methods of weighting and averaging. But those who are practically concerned with the whole process of making an index number from start to finish rate this office work lightly in comparison with the field work of getting the original data.

We commonly speak of *the* wholesale price of articles like pig iron, cotton, or beef as if there were only one unambiguous price for any one thing on a given day, however this price may vary from one day to another. In fact there are many different prices for every great staple on every day it is dealt in, and most of these differences are of the sort that tend to maintain themselves even when markets are highly organized and competition is keen. Of course varying grades command varying prices, and so as a rule do large lots and small lots; for the same grade in the same quantities, different prices are paid by the manufacturer, jobber, and local buyer; in different localities the prices paid by these various dealers are not the same; even in the same locality different dealers of the same class do not all pay the same price to everyone from whom they buy the same grade in the same quantity on the same day. To find what really was the price of cotton, for example, on February 1, 1920, would require an elaborate investigation, and would result in showing a multitude of different prices covering a considerable range.

Now the field worker collecting data for an index number must select from among all these different prices for each of his commodities the one or the few series of quotations that make the most representative sample of the whole. He must find the most reliable source of information, the most representative market, the most typical brands or grades, and the class of dealers who stand in the most influential position. He must have sufficient technical knowledge to be sure that his quotations are for uniform qualities, or to make the necessary adjustments if changes in quality have occurred in the markets and require recognition in the statistical office. He must be able to recognize anything suspicious in the data offered him and to get at the facts. He must know how commodities are made and must seek comparable information concerning the prices of raw materials and their manufactured products, concerning articles that are substituted for one another, used in connection with one another, or turned out as joint products of the same process. He must guard against the pitfalls of cash discounts, premiums, rebates, deferred payments, and allowances of all sorts. And he must know whether his quotations for different articles are all on the same basis, or whether concealed factors must be allowed for in comparing the prices of different articles on a given date.

Difficult as it is to secure satisfactory price quotations, it is still more difficult to secure satisfactory statistics concerning the relative importance of the various commodities quoted. What is wanted is an accurate census of the quantities of the important staples, at least, that are annually produced, exchanged, or consumed. To take such a census is altogether beyond the power of the private investigators or even of the Government bureaus now engaged in making index numbers. Hence the compilers are forced to confine themselves for the most part to extracting such information as they can from statistics already gathered by other hands and for other purposes than theirs. In the United States, for example, estimates of production, consumption, or exchange come from most miscellaneous sources: The Department of Agriculture, the Census Office, the Treasury Department, the Bureau of Mines, the Geological Survey, the Internal Revenue Office, the Mint, associations of manufacturers or dealers, trade papers, produce exchanges, traffic records of canals and railways, etc. The man who assembles and compares estimates made by these various organizations finds among them many glaring discrepancies for which it is difficult to account. Such conflict of evidence when two or more independent estimates of the same quantity are available throws doubt also upon the seemingly plausible figures coming from a single source for other articles. To extract acceptable results from this mass of heterogeneous data requires intimate familiarity with the statistical methods by which they were made, endless patience, and critical judgment of a high order, not to speak of tactful diplomacy in dealing with the authorities whose figures are questioned. The keenest investigator, after long labor, can seldom attain more than a rough approximation to the facts. Yet it is only by critical use of the data now available that current index numbers can be weighted, and the best hope of improving weights in the future lies in demonstrating not only the imperfections of our present statistics of production, consumption, and exchange, but also the importance of making them better.

When all this preliminary work has been done, the original quotations and the weights should be published at length. Unfortunately, many compilers of index numbers publish only the final results of their computations, upon the ground of expense or lack of interest in the detailed information. But much is sacrificed by taking this easy course. First, the reputation of the index number itself is compromised, and deservedly. No one can really test whether a series is accurately compiled from representative quotations unless the data and their sources are given in full. Second, and more important, the publication of actual quotations greatly extends the usefulness of an investigation into prices. Men with quite other ends in view than those of the original compilers can make index numbers of their own adapted to their peculiar purposes if provided with the original data.

Nor is the importance of such unplanned uses to be rated lightly. If we are ever to make the money economy under which we live highly efficient in promoting social welfare we must learn how to control its workings. What wares our business enterprises produce and what goods our families consume are largely determined by existing prices, and the production and consumption of goods are altered by every price fluctuation. What we waste and what we save, how we divide the burden of labor and how we distribute its rewards, whether business enjoys prosperity or suffers depression, whether debts of long standing become easier or harder to pay—all these and many other issues turn in no small measure upon what things are cheap and what are dear, upon the maintenance of a due balance within the system of prices, upon the upward or downward trend of the price changes that are always taking place. But if the prices of yesterday are powerful factors in determining what we shall do and how we shall fare to-day, what we do and how we fare to-day are powerful factors in determining what prices shall be to-morrow. If prices control us, we also control them. To control them so that they shall react favorably upon our economic fortunes we need more insight than we have at present. It is, then, one of the great tasks of the future to master the complicated system of prices which we have gradually developed—to find how prices are interconnected, how and why they change, and what consequences each change entails. For when men have learned these things they will be vastly more skillful in mending what they find amiss in economic life, and in reenforcing what they find good. As yet our knowledge is fragmentary and uncertain. But of all the efforts being made to extend it none is more certain to prove fruitful than the effort to record the actual prices at which large numbers of commodities are bought and sold. For such data are the materials with which all investigators must deal, and without which no bits of insight can be tested. Indeed, it is probable that long after the best index numbers we can make to-day have been superseded, the data from which they were compiled will be among the sources from which men will be extracting knowledge which we do not know enough to find.

3. MARKET PRICES, CONTRACT PRICES, INSTITUTION PRICES, AND IMPORT-EXPORT VALUES.

Most American index numbers are made from "market prices." These prices are usually obtained directly from manufacturers, selling agents, or wholesale merchants; from the records of produce exchanges and the like; or from trade journals and newspapers which make a specialty of market reporting in their respective fields.

Several of the important foreign index numbers are made wholly or partly from "import and export values"; that is, from the average prices of important articles of merchandise as officially declared by the importing or exporting firms, or as determined by governmental commissions. For example, Soetbeer's celebrated German series, and (until 1921) the British Board of Trade's official series were made mainly from such material, and the official French series was made wholly from import values until 1911.

A fourth source of quotations often drawn upon in Europe is the prices paid for supplies by such institutions as hospitals, normal schools, poorhouses, army posts, and the like. The official Italian series, Alberti's series for Trieste, and Levasseur's French series are examples.

These four classes of quotations—market prices, contract prices, import and export values, and institution prices—usually differ somewhat, not only with respect to the prices prevailing on a given date, but also with respect to the degree of change from time to time. Accordingly it is desirable to inquire into the several advantages possessed by each source of quotations.

Institution prices may be set aside promptly, because index numbers made from them have a limited range of usefulness. Though the institutions whose records are drawn upon often make purchases on a considerable scale, yet the common description of their contract rates as "semiwholesale" prices points to the peculiar and therefore unrepresentative character of such data. Moreover, there is often more doubt about the strictly uniform character of the supplies furnished to these institutions than about the uniformity of the standardized goods which are usually quoted in the market reports. If the aim of the investigation is to find the average variations in the cost of supplies to public institutions, doubtless the prices they pay are the best data to use. But if the aim is to measure the average variations in the wholesale prices paid by the business world at large, then market and contract prices are distinctly the better source. Indeed, institution prices are seldom used for the latter purpose except when well-authenticated market quotations can not be had.

So far as is known, the series of index numbers compiled by the Price Section of the War Industries Board for 1913–1918 is the only series in which free use has been made of contract prices, and even in this series contract prices were not obtained for some important articles handled largely on the contract basis—especially pig iron. Contract prices, indeed, seem more difficult to ascertain than open-market prices, and they are really less appropriate data than the latter when the purpose is primarily to ascertain in what direction prices are tending from one month to the next. But when it is desired to show the fluctuations in the prices at which the bulk of business is carried on, it is clear that the index numbers should be made from both contract and open-market prices and that the two sets of quotations should be weighted in accordance with the volume of transactions which each set represents. In the long run there may be little difference between the fluctuations in the contract and the open-market prices for the same commodity; but within short periods the difference is sometimes wide. In 1915–1918, for example, contract prices made at the beginning of a year were often far below the level attained by open-market prices by the end of the year. The collection of contract prices on a larger scale and the analysis of their relation with open-market prices are matters to which the makers of index numbers may profitably direct greater attention.[27]

[27] The best presentation of contract and open-market prices yet made is in The Prices of Coal and Coke, by Carl E. Lesher, War Industries Board Price Bulletin, No. 35.

The theory on which import and export values are sometimes preferred to market prices is that the former figures show more nearly the variations in the prices actually paid or received by a country for the great staples which it buys and sells than do market quotations for particular brands or grades of these commodities. For example, England buys several different kinds of cotton in proportions that vary from year to year. A price obtained by dividing the total declared values of all the cotton consignments imported by their total weight will show the average cost per pound actually paid by Englishmen for cotton with more certainty than will Liverpool market quotations for a single grade of cotton like "Middling American"—provided always that the "declared values" are trustworthy. Now, if the aim of the investigation is to find out the variations in the average prices paid or received for staples—irrespective of minor changes in their qualities—then the preference for import and export values is clearly justified, again granted the trustworthiness of the returns. But if the aim is to measure just one thing—the average variation in prices—market prices for uniform grades are clearly better data. For index numbers made from import and export values measure the net resultant of two sets of changes, and one can not tell from the published figures what part of the fluctuations is due to changes in prices and what part is due to changes in the qualities of the goods bought and sold.

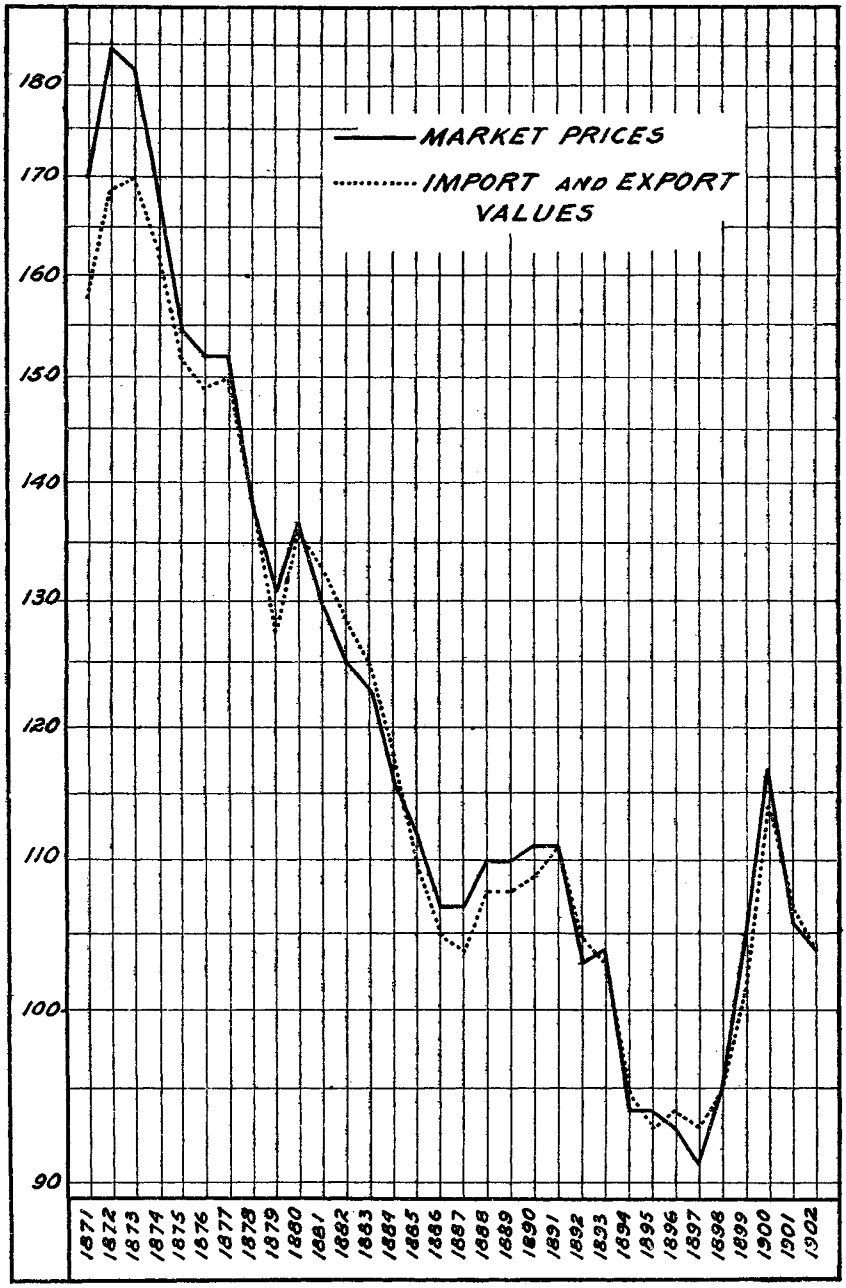
As might be expected, import and export series generally pursue a more even course than market-price series. But this difference may be due less to the sources from which the quotations are obtained than to differences in the lists of commodities used. Fortunately, we can arrange a more certain test than any of the common series provide. In 1903 the British Board of Trade published the average import or export prices of 25 commodities for which Mr. Sauerbeck has published market prices.[28] Index numbers made from these two

[28] Wholesale and Retail Prices. Return to an Order of the . . . House of Commons . . . for "Report on Wholesale and Retail Prices in the United Kingdom in 1902, with Comparative Statistical Tables for a Series of Years." For Sauerbeck's figures see his annual articles in the Journal of the Royal Statistical Society. The list of commodities in question is as follows:

Commodity.	Quotations given by Board of Trade.	Brands quoted by Sauerbeck.
Bacon	Average import values	Waterford.
Barley	do	English Gazette.
Coal	Average export values.	Wallsend, Hetton, in London.
Coffee	Average import values	Rio, good channel.
Copper	do	Chile bars.
Cotton	do	Middling American.
Flax	do	St. Petersburg.
Hides	do	River Plata, dry.
Iron, pig	Average export values.	Scotch pig.
Jute	Average import values	Good medium.
Lead	do	English pig.
Linseed	do	Linseed.
Maize	do	American mixed.
Oats	do	English Gazette.
Oil, olive	do	Olive oil.
Oil, palm	do	Palm oil.
Petroleum	do	Petroleum, refined.
Rice	do	Rangoon, cargoes to arrive.
Silk	do	Tsatlee.
Sugar, refined	do	Java, floating cargoes.
Tea	do	Congou, common.
Tin	do	Straits.
Wheat	do	English Gazette.
Wool	do	Merino, Adelaide, average grease.
Do	Average export values.	English, Lincoln, half hogs.

CHART 4.—INDEX NUMBERS MADE FROM THE MARKET PRICES AND FROM THE IMPORT AND EXPORT VALUES OF IDENTICAL LISTS OF COMMODITIES. ENGLAND, 1871-1902.

(Based on Table 5.)[1]



[1] This and the succeeding charts have been drawn on a logarithmic, instead of an arithmetic, scale in order that the per cent of change may easily be discerned.

sets of data for the same commodities for the years 1871 to 1902 are given in Table 5. The results confirm the expectation: As compared with the import and export index number, the market-price index number starts on a higher level in 1871, falls to a lower point during the middle nineties, rises to a higher level in 1900, and again drops to as low a level in 1902. But the differences are not wide.

TABLE 5.—COMPARISON OF INDEX NUMBERS MADE FROM IMPORT AND EXPORT VALUES WITH INDEX NUMBERS MADE FROM THE MARKET PRICES OF THE SAME COMMODITIES, BY YEARS, 1871 TO 1902.

[Data from the British Board of Trade and from Sauerbeck.]

(Arithmetic means of relative prices. Average prices in 1890-1899=100. 25 commodities.)

Year.	Import and export values.	Market prices.	Year.	Import and export values.	Market prices.
1871	158	170	1887	104	107
1872	169	185	1888	108	110
1873	170	182	1889	108	110
1874	162	168	1890	109	111
1875	152	155	1891	111	111
1876	149	152	1892	105	103
1877	150	152	1893	103	104
1878	139	138	1894	95	94
1879	128	131	1895	93	94
1880	136	137	1896	94	93
1881	133	130	1897	93	91
1882	129	125	1898	95	95
1883	125	123	1899	101	105
1884	118	116	1900	114	117
1885	110	112	1901	107	106
1886	105	107	1902	104	104

4. RELATIVE VERSUS ACTUAL PRICES.

In February, 1864, Hunt's Merchants' Magazine published the following statement to show how rapidly prices rose after the suspension of specie payments in December, 1861, and the issue of the irredeemable United States notes.[29] These figures are the total prices of 55 articles quoted by their customary commercial units.

Value of 55 leading articles of New York commerce.

January, 1862	$804
April, 1862	844
January, 1863	1,312
March, 1863	1,524
July, 1863	1,324
October, 1863	1,455
January, 1864	1,693

For example, in January, 1862, coal oil is entered as 30 cents per gallon and pig iron as $24 per ton; molasses is entered as 42½ cents per gallon and whalebone as $69 per ton; oats is entered as 38 cents per bushel and corn as $59.25 per hundred bushels, etc.[30]

Clearly, this simple method of measuring changes in the price level by casting sums of actual prices is not trustworthy. For a relatively slight fall in the quotation for whalebone would affect the total, as Hunt's Merchants' Magazine computes it, much more than a relatively enormous increase in the price of molasses. The fact that corn

[29] Vol. 50, p. 132.

[30] See vol. 48, p. 129.

happens to be quoted by the hundred bushels makes a 1 per cent change from its price in January, 1862, equal to a 43 per cent change in the price of wheat and to a 156 per cent change in the price of oats, both of which are quoted by the bushel.

It was to avoid such patent absurdities that Carli threw his actual prices of grain, wine, and olives in 1750 into the form of percentages of rise or fall from their prices in 1500, and then struck the average of the three percentages. When this operation is performed it makes no difference whether the commodities are quoted by large or by small units. The obvious common sense of this precedent has caused it to be followed or reinvented by most makers of index numbers to this day—with one slight modification. To avoid the awkwardness of the plus and minus signs necessary to indicate whether prices have advanced or receded, it is usual to substitute for percentages of rise or fall relative prices on the scale of 100. For example, a rise of 10 per cent and a fall of 10 per cent are expressed by relatives of 110 and 90, respectively. Occasionally, however, percentages of rise or fall are still used as by Carli; as, for instance, in the chain relatives published by the Bureau of Labor Statistics in Bulletin No. 149 and averaged in Table 4 of this bulletin. A second unimportant variant, long practiced by the London Economist, but now seldom used, is to publish as the final result the sums of relative prices, instead of their averages.[31]

In recent years a few statisticians have gone back from the use of relative to the use of actual prices, adopting various devices to avoid such crude errors as those perpetrated in the figures cited from Hunt's Merchants' Magazine. In 1897 Bradstreet's began reducing all its original quotations by the gallon, ton, dozen, square yard, etc., to prices by the pound, and presenting as its index number the aggregate prices per pound of 98 articles.[32] Four years later, Dun's Review followed this lead with an important difference. Instead of reducing actual quotations to quotations by the pound, it multiplied the actual quotation for each article included by the quantity of that article supposed to be consumed in the course of a year by the average individual. These products were then cast up, and the sums, in dollars and cents, were presented as an index number purporting to show the changes in the per capita cost of a year's supplies.[33]

Still later (1912), the method practiced by Dun was adopted by the Commonwealth statistician of Australia as the basis of his official series. However, after he had calculated the aggregate expenditure of Australians upon his bill of goods in terms of pounds sterling, he threw these pecuniary sums back into the form of relative numbers on the scale of 1,000. In 1914 the United States Bureau of Labor Statistics dropped its former practice of averaging relative prices on the 1890–1899 base, and began to use aggregates of actual prices, weighted by quantities entering into exchange and thrown into the form of relatives to facilitate comparison.

Accordingly, three types of index numbers are now in general use: (1) Averages of relative prices or average percentages of change in

[31] Gibson's index number is such a sum. See pp. 172 to 175. The difference between sums of relative prices and these sums divided by the number of articles included is, of course, purely formal. Averages have displaced sums in current use mainly because it is easier to make comparisons on the scale of 100 than on the scale of 2,200, or whatever number is given by the addition of relative prices.
[32] For a criticism of this method, see p. 110.
[33] The confidence merited by this index number is discussed in Section V.

prices; (2) sums in dollars and cents showing changes in the aggregate cost of certain definite quantities of certain commodities; (3) relative figures made from series of the second sort. The first type shows average variations, the second type shows the variations of an aggregate, the third type turns these variations of an aggregate into percentages of the aggregate itself as it stood at some selected time. The differences between these types, it is true, are differences of form, not differences of kind. As will later be shown, by using a certain scheme of weights an aggregate of actual prices can be made to give precisely the same results when turned into relatives that will be given by an average of relative prices computed from the same data. But it will also be shown that the differences of form are important. The advantages and shortcomings of the several types will appear as the various problems encountered in making index numbers are discussed.

5. THE NUMBERS AND KINDS OF COMMODITIES INCLUDED.

Since the earlier makers of index numbers had to use such price quotations as they could find, the problems how many and what kinds of commodities to include were practically solved for them. As Prof. Edgeworth remarks, "Beggars can not be choosers."

Paucity of data still hampers contemporary efforts to measure variations of prices in the past; but the compilers of index numbers for current years have a wider range of choice. The scope of their data is limited not by the impossibility but by the expense of collecting quotations. And in the case of governmental bureaus or financial journals the limits set by expense are neither narrow nor rigid. Such organizations can choose many commodities if they will or content themselves with few.

One principle of choice is generally recognized. Those commodities are preferable that are substantially uniform from market to market and from year to year. Often the form of quotation makes all the difference between a substantially uniform and a highly variable commodity. For example, prices of cattle and hogs are more significant than prices of horses and mules, because the prices of cattle and hogs are quoted per pound, while the prices of horses and mules are quoted per head.

It is often argued that the application of this common-sense principle rules out almost all manufactured goods, because such articles are continually altered in quality to suit the technical exigencies of new industrial processes or the varying tastes of consumers. But minor changes in quality, provided their occurrence is known, do not necessarily unfit a commodity for inclusion. When the brand formerly sold is replaced by a variant it is usually possible to get overlapping quotations for the old and new qualities during the time of transition. Then the new series may be spliced upon the old by means of the ratio borne by the price of the new grade to the price of the old grade in the years when the substitution is made. Statisticians willing to take the extra precautions and trouble involved by such operations can legitimately include not only a large number of staple raw materials and their simplest products, but also an even larger number of manufactured goods.

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Some of the modern index numbers, accordingly, have long lists of commodities. Dun's index number seems to be built up from about 300 series of quotations, the official Canadian index number includes 271, the Bureau of Labor Statistics' index number for 1919 has 328, and the index number compiled by the Price Section of the War Industries Board has 1,366 price series. On the other hand, many of the best-known index numbers use less than 50 series of quotations. Forty-five is a favorite number, largely because of the high reputation early established by Sauerbeck's English series. The British Board of Trade's series to 1921, the official French series, the New Zealand series, Von Jankovich's Austrian series, and Atkinson's series for British India all have just 45 commodities, while the new series of the London Economist and the relative prices published by the former Imperial Statistical Office of Germany include 44 articles. Even shorter lists are often used. For example, Schmitz's German series has only 29 commodities, the New York Annalist series 25, and Gibson's series 22. Private investigators working with limited resources sometimes confine themselves to a bare dozen commodities, or even less.[34]

These differences of practice raise important questions of theory. Does it make any substantial difference in the results whether 25 or 50 or 250 commodities be included—provided always that the lists be well chosen in the three cases? If differences do appear in the results, are they merely haphazard, or are they significant differences? If there are significant differences, which set of results is more valuable, that made from the long or from the short lists? And what does the proviso that the lists be well chosen mean? In short, do the index numbers including hundreds of commodities possess advantages over those including 50 or 25 sufficient to compensate for the greater trouble and expense of compiling them?

The best way to answer these questions is to experiment with large and small index numbers, made on a strictly uniform plan for the same country and the same years. Table 6 presents six such index numbers which differ only in respect to the number and kind of commodities included. The first column includes all the commodities quoted by the Bureau of Labor Statistics in 1913 except the 11 whose prices do not run back of 1908.[35] Many of the commodities in this list are merely different varieties of the same article; for example, there are two kinds of corn meal, four kinds of leather, six kinds of women's dress goods, eleven kinds of steel tools, etc. The second column gives an index number in which all such groups are represented by single averages, so that the number of series which enter directly into the final results is cut down to 145.[36] The third column, which includes 50 commodities, is made up from the list adopted for

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[34] These statements refer to the number of series of relative prices averaged to get the final results as now presented. Often two or more different varieties of an important article are counted as separate commodities, and, on the other hand, the relative prices of slightly different articles are sometimes averaged to make one of the series which enters into the final averages. In view of the diversity of practice in this respect, a perfectly consistent counting of the number of distinct "commodities" included in the general series is impossible. Moreover, the figures are often published with such imperfect explanations as to make the counting of the commodities included doubtful or impossible on any interpretation of that term. In 1921 the number of price series used in the British Board of Trade index was increased to 150.

[35] To facilitate comparison, decimals have been dropped and the index for each year rounded off to the nearest whole number. Regarding the changes in the number of commodities included, see Bulletin No. 149, p. 11. The reader may be reminded once more that this is the Bureau's old index number, made before the improved method of compilation was introduced.

[36] This experimental list of 145 commodities is given below. When the relative prices of closely related articles are averaged to make a single series, the number of these articles quoted by the Bureau and included in the group is indicated. Most of the bureau's series which do not cover the whole period,

the Gibson index number in its original form.[37] The fourth series is made from the prices of 20 pairs, each commodity being given in two forms, raw and manufactured, e. g., barley and malt, cattle and beef, copper ingots and copper wire, etc.[38] The last two columns contain

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1890-1913, are dropped altogether. As the basis of a general-purpose index number, this revised list is worse than the bureau's list in certain respects and better in others. See Section V.

FARM PRODUCTS.

1. Barley.
2. Cattle, 2.
3. Corn.
4. Cotton.
5. Flaxseed.
6. Hay.
7. Hides.
8. Hogs, 2.
9. Hops.
10. Oats.
11. Rye.
12. Sheep, 2.
13. Wheat.

FUEL AND LIGHTING.

1. Candles.
2. Coal, anthracite, 4.
3. Coal, bituminous, 3.
4. Coke.
5. Matches.
6. Petroleum, crude.
7. Petroleum, refined, 2.

FOOD, ETC.

1. Apples, evaporated
2. Beans.
3. Bread, crackers, 2.
4. Bread, loaf, 3.
5. Butter, 3.
6. Cheese.
7. Coffee.
8. Currants.
9. Eggs.
10. Fish, 4.
11. Flour, buckwheat.
12. Flour, rye.
13. Flour, wheat.
14. Lard.
15. Meal, corn, 2.
16. Meat, beef, 3.
17. Meat, pork, 4.
18. Meat, mutton.
19. Milk.
20. Molasses.
21. Onions.
22. Potatoes.
23. Prunes.
24. Raisins.
25. Rice.
26. Salt.
27. Soda.
28. Spice, pepper.
29. Starch, corn.
30. Sugar, 3.
31. Tallow.
32. Tea.
33. Vinegar.

CLOTHS AND CLOTHING.

1. Bags.
2. Blankets, 3.
3. Boots and shoes, 3.
4. Broadcloths.
5. Calico.
6. Carpets, 3.
7. Cotton flannels, 2.
8. Cotton thread.
9. Cotton yarns, 2.
10. Denims.
11. Drillings, 2.
12. Flannels.
13. Ginghams, 2.
14. Horse blankets.
15. Hose.
16. Leather, 4.
17. Linen thread.
18. Overcoatings, 2.
19. Print cloths.
20. Sheetings, 7.
21. Shirtings, 5.
22. Silk, 2.
23. Suitings.
24. Tickings.
25. Underwear, 2.
26. Women's dress goods, 6.
27. Wool, 2.
28. Worsted yarns, 2.

METALS AND IMPLEMENTS.

1. Bar iron, 2.
2. Barb wire.
3. Builders' hardware, 3.
4. Copper, ingot.
5. Copper, wire.
6. Lead, pig.
7. Lead pipe.
8. Nails, 2.
9. Pig iron, 4.
10. Quicksilver.
11. Silver.
12. Spelter.
13. Steel billets.
14. Steel rails.
15. Tin, pig.
16. Tools, 11.
17. Wood screws.
18. Zinc.

DRUGS AND CHEMICALS.

1. Alcohol, grain.
2. Alcohol, wood.
3. Alum.
4. Brimstone.
5. Glycerine.
6. Muriatic acid.
7. Opium.
8. Quinine.
9. Sulphuric acid.

LUMBER AND BUILDING MATERIALS.

1. Brick.
2. Carbonate of lead.
3. Cement.
4. Doors.
5. Hemlock.
6. Lime.
7. Linseed oil.
8. Maple.
9. Oak, 2.
10. Oxide of zinc.
11. Pine, white, 2.
12. Pine, yellow.
13. Plate glass, 2.
14. Poplar.
15. Putty.
16. Rosin.
17. Shingles, 2.
18. Spruce.
19. Tar.
20. Turpentine.
21. Window glass, 2.

HOUSE-FURNISHING GOODS.

1. Earthenware, 3.
2. Furniture, 4.
3. Glassware, 3
4. Table cutlery, 2.
5. Woodenware, 2.

MISCELLANEOUS.

1. Cottonseed meal.
2. Cottonseed oil.
3. Jute.
4. Malt.
5. Paper, 2.
6. Proof spirits.
7. Rope.
8. Rubber.
9. Soap.
10. Starch, laundry
11. Tobacco, 2.

[37] The list is as follows: Wheat, wheat flour (two kinds), barley, oats, corn, corn meal, potatoes, rye, sugar 89°, sugar 96°, coffee, tea, steers, fresh beef, salt beef, sheep, mutton, hogs, bacon, hams, butter, cotton, cotton yarns (two kinds), jute, wool (two kinds), worsted yarns, raw silk (two kinds), pig iron, bar iron, cement, copper ingots, copper sheets, lead, anthracite coal, bituminous coal (two kinds), hides, leather, cottonseed oil, linseed oil, petroleum (crude and refined), rubber, spruce lumber, yellow-pine lumber, and paper. See J. P. Norton, "A revised index number for measuring the rise in prices," Quarterly Journal of Economics, August, 1910, vol. 24, pp. 750-758.

[38] The remaining 17 pairs are corn and corn meal, cotton and cotton textiles, flaxseed and linseed oil, window glass and glassware, hides and leather, hogs and pork, lead (pig) and lead pipe, milk and cheese, petroleum (crude and refined), pig iron and nails, pine boards and pine doors, rye and rye flour, sheep and mutton, spelter and zinc, steel billets and steel tools, wheat and wheat flour, wool and woolen textiles.

index numbers each made from the prices of 25 important articles selected at random, the two lists having no items in common.[39]

TABLE 6.—SIX INDEX NUMBERS FOR THE UNITED STATES MADE FROM QUOTATIONS FOR DIFFERENT NUMBERS OF COMMODITIES, BY YEARS, 1890 TO 1913.

[Data from the Bulletin of the Bureau of Labor Statistics, No. 149.]

(Arithmetic means. Average prices in 1890-1899=100.)

Year.	242 to 261 commodities.	145 commodities.	50 commodities.	40 commodities.	25 commodities, first list.	25 commodities, second list.
1890	113	114	114	113	115	113
1891	112	113	114	114	112	118
1892	106	106	105	105	103	112
1893	106	105	105	101	103	107
1894	96	96	94	93	92	96
1895	94	93	94	95	95	93
1896	90	89	87	88	88	85
1897	90	89	89	89	90	84
1898	93	93	95	95	96	90
1899	102	103	103	108	107	103
1900	111	111	112	115	113	109
1901	109	110	109	116	111	107
1902	113	114	116	122	116	117
1903	114	114	115	118	118	117
1904	113	114	116	118	122	110
1905	116	116	118	122	123	115
1906	123	122	123	128	130	122
1907	130	130	132	138	132	132
1908	122	121	125	129	124	122
1909	125	124	132	135	133	128
1910	130	131	135	141	133	134
1911	126	130	129	135	129	131
1912	130	134	138	142	140	138
1913	130	131	138	139	142	133
Averages 1890-1899	100	100	100	100	100	100
1900-1909	118	118	120	124	122	118
1910-1913	129	132	135	139	136	134
Number of points by which prices rose (+) or fell (−) in—						
1890-1896	−23	−25	−27	−25	−27	−28
1896-1907	+40	+41	+45	+50	+44	+47
1907-1908	− 8	− 9	− 7	− 9	− 8	−10
1908-1912	+ 8	+13	+13	+13	+16	+16
Difference between highest and lowest relative prices	40	45	51	54	54	54
Average change from year to year	4.0	4.1	4.9	5.5	5.0	6.2

[39] The first list includes cotton, corn, wheat, hides, cattle, hogs, coffee, wheat flour, salt, sugar, tea, potatoes, wool, silk, anthracite coal, bituminous coal, crude petroleum, pig iron, steel billets, copper ingots, lead (pig), brick, average of nine kinds of lumber, jute, and rubber.

The second list includes hay, oats, rye, eggs, sheep, lard, beans, corn meal, butter, rice, milk, prunes, cotton yarns, worsted yarns, coke, cement (Rosendale 1890-1899, Portland domestic 1900-1913), tallow, spelter, bar iron, tin (pig), quicksilver, lime, tar, paper, proof spirits.

Number of points by which the selected index numbers were greater (+) ***or less*** (—) ***than the Bureau of Labor Statistics' series.***

Year.	145 commodities.	50 commodities.	40 commodities.	25 commodities, first list.	25 commodities, second list.
1890	+ 1	+ 1	± 0	+ 2	± 0
1891	+ 1	+ 2	+ 2	± 0	+ 6
1892	± 0	− 1	− 1	− 3	+ 6
1893	− 1	− 1	− 5	− 3	+ 1
1894	± 0	− 2	− 3	− 4	± 0
1895	− 1	± 0	+ 1	+ 1	− 1
1896	− 1	− 3	− 2	− 2	− 5
1897	− 1	− 1	− 1	± 0	− 6
1898	± 0	+ 2	+ 2	+ 3	− 3
1899	+ 1	+ 1	+ 6	+ 5	+ 1
1900	± 0	+ 1	+ 4	+ 2	− 2
1901	+ 1	± 0	+ 7	+ 2	− 2
1902	+ 1	+ 3	+ 9	+ 3	+ 4
1903	± 0	+ 1	+ 4	+ 4	+ 3
1904	+ 1	+ 3	+ 5	+ 9	− 3
1905	± 0	+ 2	+ 6	+ 7	− 1
1906	− 1	± 0	+ 5	+ 7	− 1
1907	± 0	+ 2	+ 8	+ 2	+ 2
1908	− 1	+ 3	+ 7	+ 2	± 0
1909	− 1	+ 7	+10	+ 8	+ 3
1910	+ 1	+ 5	+11	+ 3	+ 4
1911	+ 4	+ 3	+ 9	+ 3	+ 5
1912	+ 4	+ 8	+12	+10	+ 8
1913	+ 1	+ 9	+ 8	+12	+ 3
Arithmetic sums	23	60	129	97	70
Algebraic sums	+ 9	+44	+105	+73	+22
Average differences computed from the—					
Arithmetic sums	1.0	2.5	5.4	4.0	2.9
Algebraic sums	+ .4	+ 1.8	+ 4.4	+ 3.0	+ .9
Maximum differences	+ 4	+ 8	+12	+12	+ 8
Minimum differences	± 0	± 0	± 0	± 0	± 0

Number of points by which each index number rose (+) ***or fell*** (—) ***in each successive year.***

Year.	242 to 261 commodities.	145 commodities.	50 commodities.	40 commodities.	25 commodities, first list.	25 commodities, second list.
1891	− 1	− 1	± 0	+ 1	− 3	+ 5
1892	− 6	− 7	− 9	− 9	− 9	− 6
1893	± 0	− 1	± 0	− 4	± 0	− 5
1894	−10	− 9	−11	− 8	−11	−11
1895	− 2	− 3	± 0	+ 2	+ 3	− 3
1896	− 4	− 4	− 7	− 7	− 7	− 8
1897	± 0	± 0	+ 2	+ 1	+ 2	− 1
1898	+ 3	+ 4	+ 6	+ 6	+ 6	+ 6
1899	+ 9	+10	+ 8	+13	+11	+13
1900	+ 9	+ 8	+ 9	+ 7	+ 6	+ 6
1901	− 2	− 1	− 3	+ 1	− 2	− 2
1902	+ 4	+ 4	+ 7	+ 6	+ 5	+10
1903	+ 1	± 0	− 1	− 4	+ 2	± 0
1904	− 1	± 0	+ 1	± 0	+ 4	− 7
1905	+ 3	+ 2	+ 2	+ 4	+ 1	+ 5
1906	+ 7	+ 6	+ 5	+ 6	+ 7	+ 7
1907	+ 7	+ 8	+ 9	+10	+ 2	+10
1908	− 8	− 9	− 7	− 9	− 8	−10
1909	+ 3	+ 3	+ 7	+ 6	+ 9	+ 6
1910	+ 5	+ 7	+ 3	+ 6	± 0	+ 6
1911	− 4	− 1	− 6	− 6	− 4	− 3
1912	+ 4	+ 4	+ 9	+ 7	+11	+ 7
1913	± 0	− 3	± 0	− 3	+ 2	− 5

Now, these six index numbers, large and small, certainly have a strong family likeness. The great movements of American prices from 1890 to 1913 stand out boldly in them all—the heavy fall of prices in 1890–1896, the distinctly greater rise in 1896–1907, the sharp decline in 1908, the recovery in 1909, and the wavering course

CHART 5.—GENERAL-PURPOSE INDEX NUMBERS, INCLUDING 25, 50, AND 242 COMMODITIES, BY YEARS, 1890 TO 1913.

(Based on Table 6.)

in 1910–1913. If index numbers could pretend to nothing more than to show roughly the trend of price fluctuations, then it would indeed matter little which of these series were used. Either of the sets including only 25 commodities would serve that limited purpose as well as the set containing nearly ten times as many commodities, though doubtless the longer lists would command more confidence.

But the very success with which index numbers, even when made from scanty and dissimilar data, bring out the broader features of price movements encourages one to hope, from this device, for more than an indication of the direction and a rough approximation to the degree of change. Instead of concluding that an easy compilation, based on a few series of quotations "will do," we may hope that careful work covering a wide field will enable us to improve upon our first results and attain measurements that have a narrow margin of error.

When we make these more exacting demands upon our six index numbers we attach importance to the fact that their general similarity does not preclude numerous differences of detail. For example, two series indicate that prices rose in 1891, one indicates that prices did not change, and three indicate a fall; three put the lowest point in 1896, one in 1897, and two make the price level the same in these years; one series shows a rise in 1901, five show a fall; in 1913 again one series indicates a rise of prices, three indicate a fall, and two indicate no change; the general level of prices in the final year is made to vary between an average rise of 30 per cent and one of 42 per cent above the level of 1890–1899; there is also a difference in steadiness, the small series fluctuating through a wider range than the large ones, etc.

To what are these discrepancies due? Are they discreditable to the large series, or to the small ones, or to neither set? Can they be accounted for except as the results of random differences in sampling?

If an index number made from the wholesale prices of 25, or 50, or 250 commodities can measure approximately the changes in all wholesale prices, it must be because the known fluctuations in the prices of these selected commodities are fair samples of the unknown fluctuations in the prices of the vastly larger number of other commodities for which quotations are not collected. Now if (1) the price fluctuations of each commodity that is bought and sold were strictly independent of the price fluctuations of every other commodity, and if (2) each commodity had just the same importance as an element in the general system of prices as every other commodity, then any series of price quotations collected at random would be a fair sample for determining the average changes in the wholesale prices of commodities in general. Of course, the larger the number of commodities included, the more trustworthy would be the index number. In Table 6, for example, the first index number would be adjudged the best, and the divergencies between it and its fellows would be held to result from the scantier material from which the latter are made.

In fact, however, the situation is by no means so simple, because neither of the above-mentioned conditions holds true. Commodities are far from being all of the same importance as elements in the whole system of prices. With the complications arising from this fact the section on the problems of weighting will deal. Neither are the price fluctuations of different commodities independent of each other. On the contrary, the price changes of practically every commodity in the markets of the whole country are causally related to the changes in the prices of a few or of many, perhaps in the last resort of all, other commodities that are bought and sold. Most of these relations are so slight that they can not be traced by statistical methods. But certain bonds are so close and so strong that they establish definite groups of related prices which fluctuate in harmony with one another and which differ in definable ways from the fluctuations of other such groups. The present task is to show the existence of these groups and the effects which they exercise upon index numbers.

First, the price fluctuations of a raw material are usually reflected in the prices of its manufactured products. Hence to quote in some cases both the raw material and several of its finished products, and to quote in other cases the raw material alone, assigns certain groups of related prices a larger influence upon the results than is assigned the other groups. When the aim is to secure a set of samples which fairly represent price fluctuations as a whole, the existence of these groups must be taken into account. Neglect on this score may give a misleading twist to the final index numbers. A celebrated case in point is that of the Economist index number in 1863–1865. Out of the 22 commodities included in the Economist's list as then constituted 4 consisted of cotton and its products. Hence when the blockade of Southern ports during the Civil War raised the price of cotton, the Economist index numbers grossly exaggerated the average rise in the price level, as appears from the following comparison between the Economist's results for 1860–1865 and the corrresponding English figures compiled by Sauerbeck:[40]

Year.	Economist index number (prices in 1860=100).	Sauerbeck's index number (prices in 1860=100).
1860	100	100
1861	102	100
1862	109	106
1863	136	109
1864	145	112
1865	136	106

Directly opposing the relations which unite the prices of finished goods with the prices of their raw materials is a second set of influences which make the price fluctuations of manufactured goods considered as a group characteristically different from the price fluctuations of their raw materials considered as a separate group. Table 7 presents several sets of index numbers designed to throw these characteristic differences into high relief. The first two columns compare the relative prices of the 49 raw materials quoted by the Bureau of Labor Statistics in 1913 and of the 183 to 193 more or less manufactured commodities in its list.[41] The second pair of columns contains index numbers made from the prices of 20 raw materials and of 20 products manufactured from these same materials.[42] Then

[40] To make the comparison as fair as possible, both series are here given, not in their original form, but recomputed on a common basis. See Wholesale Prices, Wages, and Transportation, report by Mr. Aldrich from the Committe on Finance, Mar. 3, 1893, 52d Cong., 2d sess., Senate Report No. 1394, Part I, pp. 226 and 255.

[41] See Bulletin No. 149, pp. 13 and 14. The differences between the original figures and those given here are due (1) to the dropping of decimals, (2) to the exclusion of 11 commodities which the Bureau of Labor Statistics quotes in the years 1908–1913 only, (3) to the computation of the arithmetic means in these years by the method applied in 1890–1907 in place of the Bureau's roundabout method.

[42] The articles included here are those from which the index number of 40 commodities in Table 6 was made. For the list, see p. 35 and note.

come three columns giving index numbers made from the prices of five great staples at three successive stages of manufacture: Wheat, flour, and bread; cotton, cotton yarns, and cotton textiles; wool, worsted yarns, and woolen textiles; pig iron, steel billets, and steel tools; hides, leather, and shoes.[43] The later sections of the table give the data for each of these last-mentioned groups separately. These several comparisons establish the conclusion that manufactured goods were steadier in price than raw materials. The manufactured goods fell less in 1890–1896, rose less in 1896–1907, again fell less in 1907–1908, and rose less in 1908–1913. Further, the manufactured goods had the narrower extreme range of fluctuations, the smaller average change from year to year, and the slighter advance in price from one decade to the next.[44] It follows that index numbers made from the prices of raw materials, or of raw materials and slightly manufactured products, must be expected to show wider oscillations than index numbers including a liberal representation of finished commodities.

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[43] For the list of textiles and of tools, see Bulletin No. 99 of the Bureau of Labor, March, 1912, pp. 554–556 and 682–683.

[44] Like most generalizations about price changes, these statements are strictly valid only in the case of averages covering several commodities, but the exceptions are not numerous, even in the case of single commodities, as detailed study of the wheat, cotton, wool, iron, and leather groups will show.

TABLE 7.—INDEX NUMBERS MADE FROM THE PRICES OF RAW MATERIALS

[Data from Bulletin No. 149 of the

(Arithmetic means. Average

Year.	49 raw materials.	183 to 193 manufactured products.	Twenty pairs.		Five triplets.			Wheat group.		
			Raw materials.	Manufactured goods.	Raw materials.	Intermediate products.	Finished goods.	Wheat.	Wheat flour.	Bread.
Number of commodities included	......	......	......	......	......	......	......	1	2	2
1890	115	112	113	112	125	119	108	119	121	101
1891	116	111	114	114	117	116	107	128	126	101
1892	108	106	104	105	103	109	106	105	104	101
1893	104	106	99	103	95	100	105	90	89	101
1894	93	97	91	94	79	86	98	74	78	101
1895	92	94	94	96	89	89	95	80	84	98
1896	84	92	85	92	87	88	95	85	91	97
1897	88	90	88	89	94	90	94	106	110	101
1898	94	93	98	92	101	95	95	118	109	101
1899	106	101	114	103	111	107	98	95	88	101
1900	112	110	118	111	120	110	105	94	88	101
1901	111	108	120	113	110	102	102	96	87	101
1902	122	111	127	118	123	110	103	99	90	101
1903	123	112	122	114	125	114	106	105	97	101
1904	120	111	123	113	128	115	110	138	125	106
1905	121	115	127	117	132	115	114	135	122	110
1906	127	122	135	120	136	119	121	106	97	110
1907	133	129	146	131	145	126	125	121	109	110
1908	124	121	135	124	130	117	120	132	119	113
1909	131	123	143	127	149	126	121	160	139	116
1910	135	129	149	132	149	125	124	146	126	118
1911	135	124	144	127	135	115	120	131	112	118
1912	145	127	151	132	141	119	124	140	122	122
1913	139	128	149	128	143	122	127	127	109	123
Averages, 1890-1899	100	100	100	100	100	100	100	100	100	100
1900-1909	122	116	130	119	130	115	113	119	107	107
1910-1913	139	127	148	130	142	120	124	136	117	120
Number of points by which prices rose (+) or fell (−) in—										
1890-1896	−31	−20	−28	−20	−38	−31	−13	−34	−30	− 4
1896-1907	+49	+37	+61	+39	+58	+38	+30	+36	+18	+13
1907-1908	− 9	− 8	−11	− 7	−15	− 9	− 5	+11	+10	+ 3
1908-1913	+15	+ 7	+14	+ 4	+13	+ 5	+ 7	− 5	−10	+10
Difference between highest and lowest relative prices.	61	39	66	43	70	40	33	86	61	26
Average change from year to year	5.5	4.0	6.4	4.9	8.4	5.5	3.1	13.6	11.6	1.3

AND OF MANUFACTURED GOODS, BY YEARS, 1890 TO 1913.

Bureau of Labor Statistics.]

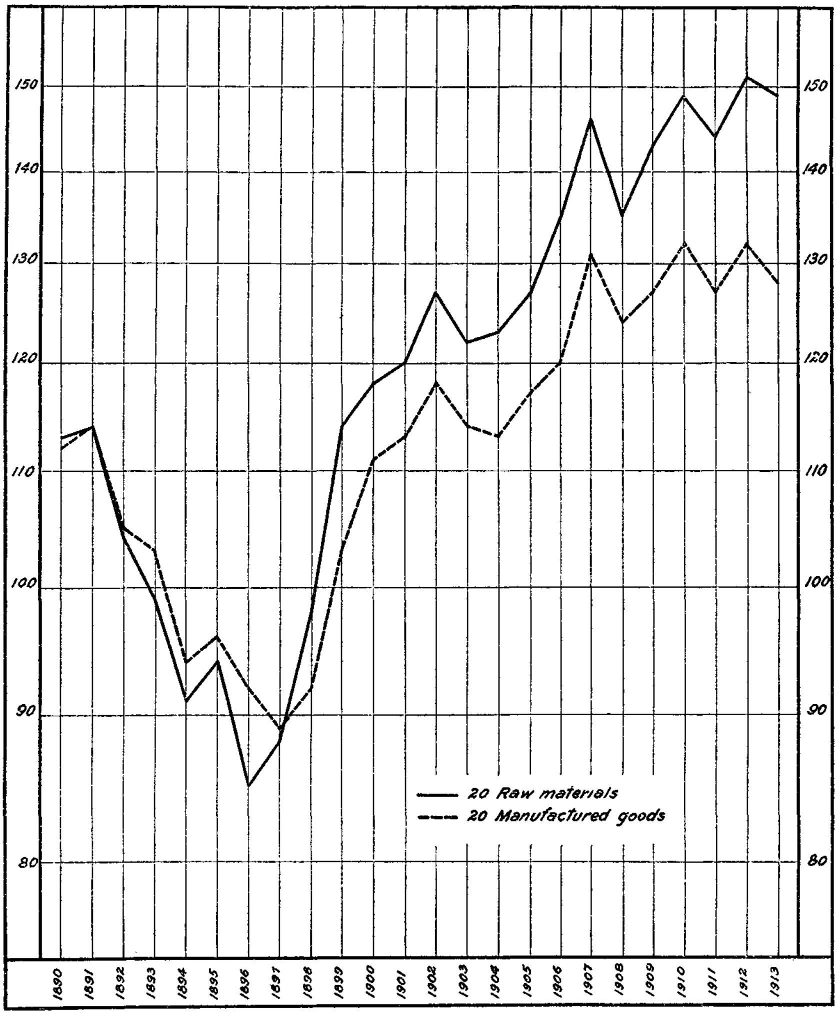
prices in 1890-1899=100.)

Cotton group.			Wool group.			Iron group.			Leather group.			Year.
Raw cotton.	Cotton yarns.	Cotton textiles.	Raw wool.	Worsted yarn.	Woolen textiles.	Pig iron.	Steel billets.	Steel tools.	Hides.	Leather.	Shoes.	
1	2	24	2	2	16	4	1	11	1	4	3	Number of commodities included.
143	112	117	132	122	111	131	142	107	100	101	106	1890.
111	113	112	126	123	112	116	118	106	102	101	104	1891.
99	117	111	113	117	112	106	110	105	93	97	103	1892.
107	111	109	102	110	109	96	95	103	80	97	101	1893.
90	93	98	79	91	96	83	77	99	68	92	99	1894.
94	92	94	70	74	88	91	86	95	110	108	100	1895.
102	93	95	71	73	87	88	88	96	87	95	101	1896.
92	91	90	89	83	90	78	70	95	106	96	96	1897.
77	91	85	108	101	98	77	71	94	123	104	94	1898.
85	89	91	111	107	100	134	145	101	132	109	95	1899.
124	116	103	118	118	111	140	116	112	127	113	98	1900
111	98	99	97	102	105	112	112	110	132	111	96	1901.
115	94	100	101	112	106	155	142	115	143	113	96	1902.
145	113	105	110	118	111	141	130	118	125	112	96	1903.
156	120	114	116	117	112	104	103	118	124	109	98	1904.
123	106	107	127	125	119	124	112	128	153	112	106	1905.
142	121	117	121	129	125	145	128	134	165	120	119	1906.
153	134	133	122	128	124	175	136	138	155	124	120	1907.
135	109	116	118	118	121	125	122	134	143	119	114	1908.
156	119	117	127	130	122	127	114	129	176	127	121	1909.
195	133	127	116	124	124	124	118	131	165	125	118	1910.
168	125	125	108	116	120	112	100	123	158	121	116	1911.
148	120	122	111	119	123	118	104	124	188	129	127	1912.
165	132	126	105	113	123	122	120	126	196	139	137	1913.
100	100	100	100	100	100	100	100	100	100	100	100	Averages, 1890-1899.
136	113	111	116	120	116	135	122	124	144	116	106	1900-1909.
169	128	125	110	118	123	119	111	126	177	129	125	1910-1913.
												Number of points by which prices rose (+) or fell (−) in—
−41	−19	−22	−61	−49	−24	−43	−54	−11	−13	− 6	− 5	1890-1896.
+51	+41	+38	+51	+55	+37	+87	+48	+42	+68	+29	+19	1896-1907.
−18	−25	−17	− 4	−10	− 3	−50	−14	− 4	−12	− 5	− 6	1907-1908.
+30	+23	+10	−13	− 5	+ 2	− 3	− 2	− 8	+53	+20	+23	1908-1913.
118	45	48	62	57	38	98	75	44	128	47	43	Difference between highest and lowest relative prices.
18.1	9.8	6.1	9.1	8.1	3.9	17.5	16.0	3.7	14.7	5.0	3.7	Average change from year to year.

Third, there are characteristic differences among the price fluctuations of the groups consisting of mineral products, forest products, animal products, and farm crops. Table 8 presents index numbers for these four groups. Fifty-seven commodities are included, all of them raw materials or slightly manufactured products.[45] Here the

CHART 6.—INDEX NUMBERS OF THE PRICES OF 20 RAW MATERIALS AND 20 PRODUCTS MANUFACTURED FROM THEM.

(Based on Table 7.)



[45] The lists of commodities are as follows:

Farm crops: Cotton, flaxseed, barley, corn, oats, rye, wheat, hay, hops, beans, coffee, rice, pepper, tea, onions, potatoes, cottonseed meal, and jute—18 articles.

Animal products: Hides, cattle, hogs, sheep, eggs, lard, milk, tallow, silk, and wool—10 articles.

Forest products: Hemlock, maple, oak, white pine, yellow pine, poplar and spruce lumber, together with turpentine, tar, and rubber—10 articles.

Mineral products: Salt, anthracite coal, bituminous coal, coke, crude petroleum, copper ingots, lead (pig), pig iron, bar iron, steel billets, quicksilver, silver bars, tin (pig), spelter, zinc, brick, cement, lime, and brimstone—19 articles.

striking feature is the capricious behavior of the prices of farm crops under the influence of good and bad harvests. The sudden upward jump in their prices in 1891, despite the depressed condition of business, their advance in the dull year 1904, their fall in the year of revival 1905, their failure to advance in the midst of the prosperity of

CHART 7.—INDEX NUMBERS OF THE PRICES OF WOOL, COTTON, HIDES, WHEAT, AND PIG IRON IN THEIR RAW, PARTIALLY MANUFACTURED, AND FINISHED FORMS.

(Based on Table 7.)

1906, their trifling decline during the great depression of 1908, and their sharp rise in the face of reaction in 1911 are all opposed to the general trend of other prices. The prices of animal products are distinctly less affected by weather than the prices of vegetable crops, but even they behave queerly at times, for example in 1893. Forest-product prices are notable chiefly for maintaining a much higher level of fluctuation in 1902–1913 than any of the other groups, a level on which their fluctuations, when computed as percentages of the much lower prices of 1890–1899, appear extremely violent. Finally, the prices of minerals accord better with alternations of prosperity, crisis, and depression than any of the other groups. And the anomalies that do appear—the slight rise in three years (1896, 1903, and 1913) when the tide of business was receding—would be removed if the figures were compiled by months. For the trend of mineral prices was downward in these years, but the fall was not so rapid as the rise had been in the preceding years, so that the annual averages were left somewhat higher than before.[46] An index number composed largely of quotations for annual crops, then, would be expected at irregular intervals to contradict capriciously the evidence of index numbers in which most of the articles were mineral, forest, or even animal products.

TABLE 8.—INDEX NUMBERS MADE FROM PRICES OF MINERAL, FOREST, ANIMAL, AND FARM PRODUCTS, BY YEARS, 1890 TO 1913.

[Data from the Bulletin of the Bureau of Labor Statistics, No. 149.]

(Arithmetic means. Average prices in 1890–1899=100.)

Year.	Mineral products.	Forest products.	Animal products.	Farm crops.
Number of commodities included	19	10	10	18
1890	119	107	106	119
1891	111	105	108	126
1892	105	99	109	110
1893	98	98	116	105
1894	87	95	94	101
1895	91	96	95	92
1896	92	94	82	76
1897	88	95	88	83
1898	92	99	97	92
1899	117	112	105	96
1900	120	121	111	105
1901	113	113	112	114
1902	119	123	128	120
1903	124	137	117	116
1904	115	142	113	124
1905	123	149	121	116
1906	135	163	128	146
1907	137	169	135	125
1908	118	151	126	124
1909	121	164	144	130
1910	120	181	152	134
1911	120	172	131	151
1912	132	168	146	158
1913	136	169	150	135
Averages, 1890–1899	100	100	100	100
1900–1909	123	143	124	119
1910–1913	127	173	145	145
Number of points by which prices rose (+) or fell (−) in—				
1890–1896	−27	−13	−24	−43
1896–1907	+45	+75	+53	+49
1907–1908	−19	−18	−9	−1
1908–1913	+18	+18	+24	+11
Difference between highest and lowest relative prices	50	87	70	82
Average change from year to year	7.0	7.4	8.9	8.2

Fourth, there are characteristic differences between the price fluctuations of manufactured commodities bought by consumers for family use and the price fluctuations of manufactured commodities bought by business men for industrial or commercial use. Such at

[46] Compare the monthly figures compiled by the Bureau of Labor Statistics for its group of "Metals and implements," Bulletin No. 149, p. 18. These figures are largely influenced by the relatively stable prices of 11 different kinds of tools. Monthly data for the 19 mineral products of Table 8 would probably show even more decline between January and December in these years.

least is the story told by Table 9. The data employed here are quotations for 28 articles from the Bureau of Labor Statistics' list that rank distinctly as consumers' goods and 28 that rank as producers' goods.[47] Though consisting more largely of the erratically fluctuating farm products, the consumers' goods are steadier in

CHART 8.—INDEX NUMBERS OF THE PRICES OF 19 MINERAL PRODUCTS AND OF 18 FARM CROPS.

(Based on Table 8.)

[47] The consumers' goods are bread, crackers, butter, cheese, salt fish, evaporated apples, prunes, raisins, beef, mutton, pork, molasses, cornstarch, sugar, vinegar, shoes, cotton textiles, woolen textiles, candles, matches, quinine, furniture, earthenware, glassware, woodenware, table cutlery, soap, and tobacco. The producers' goods are bags, cotton yarns, leather, linen shoe thread, worsted yarns, refined petroleum, barbed wire, builders' hardware, copper wire, lead pipe, nails, steel rails, tools, wood screws, pine doors, plate glass, window glass, carbonate of lead, oxide of zinc, putty, rosin, shingles, muriatic acid, sulphuric acid, malt, paper, proof spirit, and rope.

It will be noticed that a large proportion of the consumers' goods are subject to very slight manufacturing processes, notably the foods. Hence the difference between the two index numbers can scarcely be regarded as merely a fresh contrast between the fluctuations of finished goods and of intermediate products.

price than the producers' goods, because the demand for them is less influenced by changes in business conditions.

TABLE 9.—INDEX NUMBERS MADE FROM THE PRICES OF CONSUMERS' GOODS AND OF PRODUCERS' GOODS, BY YEARS, 1890 TO 1913.

[Data from Bulletin of the Bureau of Labor Statistics, No. 149.]

(**Arithmetic means. Average prices in 1890-1899=100.**)

Year.	Consumers' goods.	Producers' goods.
1890	112	115
1891	109	111
1892	104	107
1893	108	102
1894	100	92
1895	95	91
1896	91	93
1897	90	89
1898	94	93
1899	98	107
1900	106	117
1901	105	113
1902	108	114
1903	105	114
1904	103	114
1905	106	117
1906	110	124
1907	114	133
1908	112	119
1909	114	118
1910	118	126
1911	119	125
1912	118	125
1913	121	123
Averages, 1890-1899	100	100
1900-1909	108	118
1910-1913	119	125
Number of points by which prices rose (+) or fell (−) in—		
1890-1897	−22	−26
1897-1907	+24	+44
1907-1908	− 2	−14
1908-1913	+ 9	+ 4
Difference between highest and lowest relative prices	31	44
Average change from year to year	3.4	4.7

Other groups of related prices having specific peculiarities of fluctuation doubtless exist,[48] but the analysis has been carried far enough for the present purpose. That purpose is to show how the existence of groups of prices which fluctuate in harmony with each other and at variance with other groups affects index numbers in general and in particular the six index numbers for the United States given in Table 6. To apply the knowledge gained from the preceding analysis to the explanation of the differences among these six index numbers is not difficult when once the commodities included in each index number have been classified on the basis of the groups which have been examined.

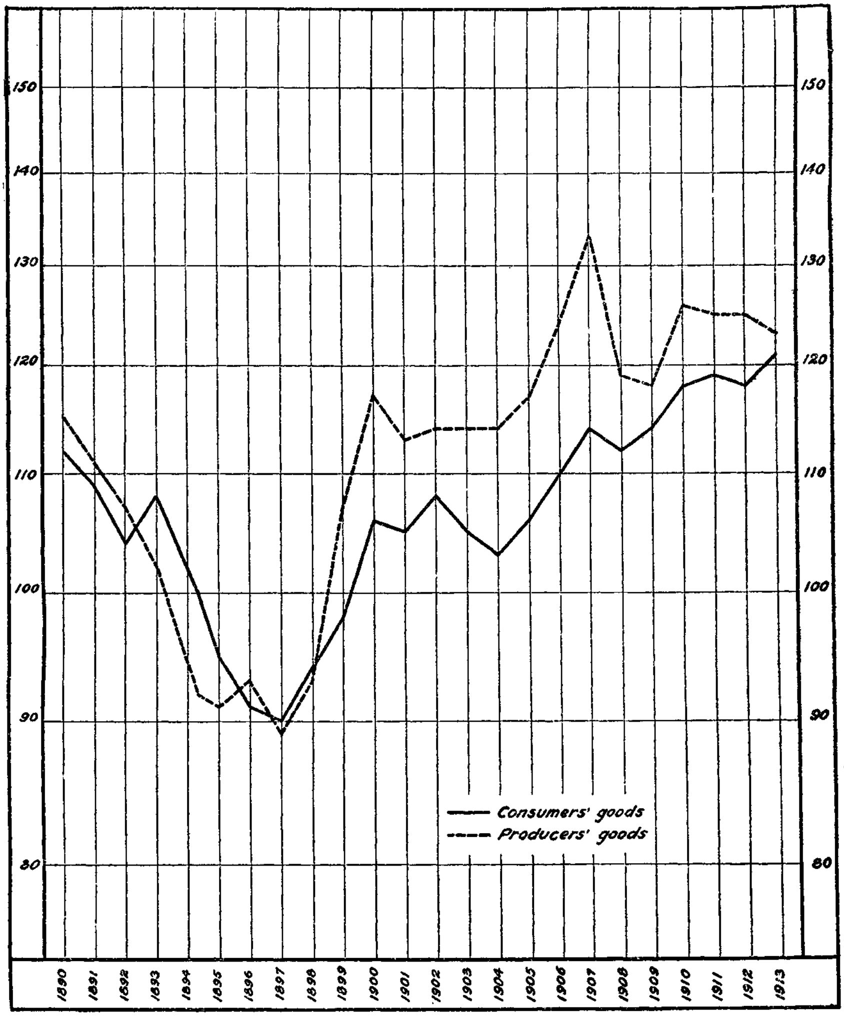
First, the list of commodities used by the Bureau of Labor Statistics includes 29 quotations for iron and its products, 30 quotations for cotton and its products, and 18 for wool and its products, besides 8 more quotations for fabrics made of wool and cotton together. On the other hand, it has but 7 series for wheat and its products, 8 for coal and its products, 3 for copper and its products, etc. The iron,

[48] For example, there is evidence that the products of industries characterized by a large measure of concentration in business control are steadier in price than products of industries characterized by unhampered competition.—See W. C. Mitchell, Business Cycles, pp. 462-464.

cotton and wool groups together make up 85 series out of 242, or 35 per cent of the whole number. The same three groups furnish 36 (or 25 per cent) of the 145 series in the second index number in Table 8.

CHART 9.—INDEX NUMBERS OF THE PRICES OF MANUFACTURED GOODS USED FOR FAMILY CONSUMPTION AND FOR INDUSTRIAL PURPOSES.

(Based on Table 9.)



Does this large representation of three staples distort these index numbers—particularly the bureau's series where the disproportion is greatest? Perhaps, but if so the distortion does not arise chiefly from the undue influence assigned to the price fluctuations of raw cotton, raw wool, and pig iron. For, contrary to the prevailing impression, the similarity between the price fluctuations of finished products and their raw materials is less than the similarity between the price fluctuations of finished products made from different materials. Such at least is the testimony of Table 7. As babies from different families are more like one another than they are like their respective parents, so here the relative prices of cotton textiles, woolen textiles, steel tools, bread, and shoes differ far less among themselves than they differ severally from the relative prices of raw cotton, raw wool, pig iron, wheat, and hides.[49] Hence the inclusion of a large number of articles made from iron, cotton, and wool affects an index number mainly by increasing the representation allotted to manufactured goods. What materials those manufactured goods are made from makes less difference in the index number than the fact that they are manufactured. To replace iron, cotton, and woolen products by copper, linen, and rubber products would change the result somewhat, but a much greater change would come from replacing the manufactured forms of iron, cotton, and wool by new varieties of their raw forms.[50]

This similarity among the price fluctuations of manufactured goods arises from the fact demonstrated by Table 7 that such articles are relatively steady in price. Does knowledge of this steadiness assist in explaining the differences among the six American index numbers of Table 6? To answer we must find the proportions of raw and manufactured commodities included in each index number. Classification along this line is rather uncertain in many cases, but the results shown in the following schedule, if not strictly correct, are at least uniform in their errors.

Table 10.—NUMBER AND PER CENT OF RAW AND MANUFACTURED COMMODITIES INCLUDED IN THE SIX INDEX NUMBERS OF TABLE 6.

Index number.	Total number of commodities.	Number of—		Percentage of—	
		Raw commodities.	Manufactured commodities.	Raw commodities.	Manufactured commodities.
First	242	49	193	20	80
Second	145	36	109	25	75
Third	50	26	24	52	48
Fourth	40	17	23	43	57
Fifth	25	19	6	76	24
Sixth	25	10	15	40	60

[49] A compilation of the differences among the relative prices in question taken seriatim for each of the 24 years 1890–1913 yields the following results:

Average differences between the relative prices of—

Raw cotton and cotton textiles	20.7 points.
Raw wool and woolen textiles	8.9 points.
Pig iron and steel tools	14.0 points.
Wheat and bread	15.0 points.
Hides and shoes	31.6 points.
Average	18.0 points.
Cotton textiles and woolen textiles	5.3 points.
Cotton textiles and steel tools	7.8 points.
Cotton textiles and bread	6.9 points.
Cotton textiles and shoes	6.7 points.
Woolen textiles and steel tools	6.1 points.
Woolen textiles and bread	7.3 points.
Woolen textiles and shoes	8.1 points.
Steel tools and bread	9.4 points.
Steel tools and shoes	9.6 points.
Bread and shoes	4.7 points
Average	7.2 points.

[50] While the fluctuations in the prices of manufactured goods are generally slighter than those in the prices of raw materials, they are nevertheless violent at times, as in the case of cotton yarns and cotton textiles during the Civil War. (See p. 40.)

On this showing the Bureau of Labor Statistics series ought to be the steadiest, and the second series the next steadiest—and so they are, as the summaries at the bottom of the columns in Table 6 show. With the smaller index numbers, however, the rule does not work well, for the most variable of all—the sixth—has a larger per cent of manufactured goods than the other three. Moreover, number four is more variable than number three, though it has relatively more manufactured goods. But the preceding studies of different groups throw further light upon the matter.

It has been found that among manufactured commodities those bought for family consumption are steadier in price than those bought for business use. To take account of this factor the manufactured goods in the several series are classified as primarily consumers' goods, primarily producers' goods, or as bought in large measure by both classes of purchasers.

TABLE 11.—CLASSIFICATION OF THE MANUFACTURED COMMODITIES INCLUDED IN THE SIX INDEX NUMBERS OF TABLE 6.

Index number.	Number of—				Per cent of—			
	Manufactured articles.	Consumers' commodities.	Producers' commodities.	Both consumers' and producers' commodities.	Manufactured articles.	Consumers' commodities.	Producers' commodities.	Both consumers' and producers' commodities.
First	193	108	73	12	80	45	30	5
Second	109	51	47	11	75	35	32	8
Third	24	11	12	1	48	22	24	2
Fourth	23	10	12	1	57	25	30	2
Fifth	6	3	3	....	24	12	12	....
Sixth	15	4	11	....	60	16	44	....

Here it does turn out that the two series (numbers four and six) which are highly variable despite the inclusion of many manufactured goods have relatively more of those manufactured goods which as a group are most variable. So far as this factor counts, then, it counts toward clearing up the contradiction pointed out in the preceding paragraph. It also brings out a further reason for the comparative stability of the first two series.

The one remaining form of analysis suggested above seems easy to apply. In the schedule below, raw and slightly manufactured commodities like those used in Table 8 are distributed among four groups according as their constituents come chiefly from mines, forests, animal sources, or cultivated fields. There is little doubt about the classification here, but there is much doubt about the significance of the results as applied to our six index numbers. The figures in the schedule are either such small percentages of the whole number of series that they can not exercise much influence upon the results, or such small numbers that they can not claim to be typical of their groups. Further, the second part of the schedule shows that there is less difference among the six index numbers than appears at first sight in the proportions of the raw and slightly manufactured commodities which consist of mineral, forest, animal, and farm products. Hence it is not surprising that efforts to account for the divergences in Table 6 by appealing to this schedule and to Table 8 accomplish little, especially for the smaller index numbers. This much does appear regarding the first two series: Whenever mineral products and farm crops move sharply in opposite directions the Bureau of Labor Statistics' index diverges from its mate in harmony with mineral products, while the series of 145 commodities bends toward the agricultural products—which is what should happen according to the schedule.

TABLE 12.—FARM, ANIMAL, FOREST, AND MINERAL PRODUCTS IN RAW OR SLIGHTLY MANUFACTURED FORM, INCLUDED IN THE SIX INDEX NUMBERS OF TABLE 6.

Index number.	Total number of commodities.	Number of—					Per cent of the whole number consisting of—				
		Raw and slightly manufactured goods.	Farm crops.	Animal products.	Forest products.	Mineral products.	Raw and slightly manufactured goods.	Farm crops.	Animal products.	Forest products.	Mineral products.
First	242	74	18	15	12	29	30	7	6	5	12
Second	145	57	18	10	10	19	39	12	7	7	13
Third	50	30	10	8	3	9	60	20	16	6	18
Fourth	40	19	6	6	1	6	48	15	15	3	15
Fifth	25	23	7	5	2	9	92	28	20	8	36
Sixth	25	18	5	5	1	7	72	20	20	4	28

Index number.	Per cent of the raw and slightly manufactured commodities consisting of—			
	Farm crops.	Animal products.	Forest products.	Mineral products.
First	25	20	16	39
Second	31	18	18	33
Third	33	27	10	30
Fourth	32	32	4	32
Fifth	30	22	9	39
Sixth	28	28	5	39

Two practical conclusions of moment to both the makers and the users of index numbers are established by this section. (1) To make an index number that measures the changes in wholesale prices at large, samples must be drawn from all the various groups that behave in peculiar ways. (2) In using an index number made by others, one must study the list of commodities included critically with these groups in mind to know what it really does measure.

The first conclusion seems to contradict a rule often practiced and sometimes preached. Most of the middle-sized index numbers are confined to raw materials and slightly manufactured goods. Most of the small index numbers are confined to foods alone. The makers of both sets argue that their series are more "sensitive" and therefore better measures of price changes than the larger series, which are "loaded down" with a mass of miscellaneous manufactured goods. And many users of index numbers seem to prefer a series like Sauerbeck's with only 45 commodities, or even one like the Annalist's with only 25 commodities, to one like that of the Bureau of Labor Statistics with five or ten times the number.

Critics who take this stand usually assume tacitly that the purpose of an index number is to serve as a "business barometer," or to measure changes in "the cost of living." If these aims were always clearly realized by the critics and clearly stated for their readers the room left for differences of opinion would be narrow. In Table 6 the index number with 145 commodities shows itself a more sensitive and on the whole more faithful barometer of changing business conditions during the 24-year period from 1890 to 1913 than the official series with 242 commodities,[51] and the preceding analysis shows that the sluggishness of the larger index number is due chiefly to its proportion of manufactured goods. For this particular purpose, then, a series modeled after Sauerbeck's has strong claims to preference over one including a larger number of commodities. Indeed, in the light of the preceding discussion one might carry the process of exclusion much further and throw out of the business barometer not only manufactured goods but also all farm crops, on the ground that their prices depend on the eccentricities of the weather, and most forest products, on the ground that their prices in the period covered by Table 6 were rising so fast as to obscure the effects of bad times, etc. But clearly such exclusions, while they might make the resulting figures more responsive to changes in business conditions, would also make the figures less acceptable as a measure of changes in prices as a whole. The sluggish movements of manufactured goods and of consumers' commodities in particular, the capricious jumping of farm products, etc., are all part and parcel of the fluctuations which the price level is actually undergoing. Consequently, an index number which pretends to measure changes in the general level of prices can not logically reject authentic quotations from any of these groups. Every restriction in the scope of the data implies a limitation in the significance of the results.

As for the small series made from the prices of foods alone or from the prices of any single group of commodities, it is clear that however good for special uses they may be, they are untrustworthy as general-purpose index numbers. Table 13 shows what differences are likely to appear at any time between series confined to foods and series covering a wider field. The general-purpose indexes are taken from Table 6, two of the food indexes include the commodities quoted by the Annalist index number and by the Gibson index number as now constituted; the third food index is the bureau's old series for foods, with decimals dropped and new arithmetic means for 1908–1913.

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[51] Compare p. 36.

TABLE 13.—INDEX NUMBERS OF THE PRICES OF FOODS, AND GENERAL-PURPOSE INDEX NUMBERS, BY YEARS, 1890 TO 1913.

[Data from Bulletin of the Bureau of Labor Statistics, No. 149.]

(Arithmetic means. Average prices in 1890-1899=100.)

Year.	General-purpose index number from Table 6.		Index numbers of the prices of foods.		
	242 to 261 commodities.	25 commodities, first list.	25 commodities, Annalist, list.	22 commodities, Gibson list.	48 commodities, Bureau of Labor Statistics list.
1890	113	115	109	109	112
1891	112	112	119	121	116
1892	106	103	108	108	104
1893	106	103	116	110	110
1894	96	92	102	98	100
1895	94	95	95	94	95
1896	90	80	81	81	84
1897	90	90	84	87	88
1898	93	96	92	96	94
1899	102	107	93	96	98
1900	111	113	99	100	104
1901	109	111	105	106	106
1902	113	116	117	118	111
1903	114	118	107	107	107
1904	113	122	109	115	107
1905	116	123	110	114	109
1906	123	130	115	111	113
1907	130	132	120	121	118
1908	122	124	126	128	122
1909	125	133	134	127	125
1910	130	133	137	137	129
1911	126	129	131	134	127
1912	130	140	143	147	135
1913	130	142	139	139	131
Averages, 1890-1899	100	100	100	100	100
1900-1909	118	122	114	115	112
1910-1913	129	136	138	139	131
Number of points by which prices rose (+) or fell (−) in—					
1890-1896	−23	−27	−29	−28	−28
1896-1907	+40	+44	+40	+40	+34
1907-1908	−8	−8	+6	+7	+4
1908-1912	+8	+16	+17	+19	+17
1912-1913	±0	+2	−4	−8	−4
Difference between highest and lowest relative prices	40	54	63	66	51
Average change from year to year	4.0	5.0	7.1	7.3	5.0

The three index numbers for foods agree better than might have been expected in view of the dissimilarity of the lists of commodities which they quote and the brevity of two of the lists.[52] The bureau

[52] Of the 56 articles included altogether, only 11 are common to all three lists. The Gibson list has 8 commodities and the Annalist list has 4 commodities classified by the bureau with farm products instead of with foods, while the bureau has 34 foods not quoted by Gibson and 27 not quoted by the Annalist. Even the two short lists have only 15 articles in common, while Gibson has 7 articles not quoted by the Annalist, and the Annalist has 10 articles not quoted by Gibson.

For the Bureau's list see Bulletin No. 149, pp. 90-107.

The Annalist list runs—oats, cattle, fresh beef, salt beef, hogs, bacon, salt pork, lard, sheep, mutton, butter (two kinds), cheese, coffee, sugar, wheat flour (two kinds), rye flour, corn meal, rice, beans, potatoes, prunes, evaporated apples, and codfish.

The Gibson list is—barley, corn, oats, rye, wheat, cattle, hogs, sheep, butter, coffee, wheat flour (two kinds), corn meal, bacon, fresh beef, salt beef, hams, mutton, sugar (two kinds), tea, and potatoes.

series is rather steadier than the others, because of the larger proportion of manufactured products included in it; but this series and that of the Annalist invariably agree about the direction in which prices

CHART 10.—INDEX NUMBERS OF THE PRICES OF 25 FOOD PRODUCTS AND OF 25 MISCELLANEOUS COMMODITIES.

(Based on Table 13.)

are moving,[53] and the Gibson figures agree with the other two series in 19 years out of the 24. On the other hand, the three food indexes

[53] Even in 1903–4 the bureau's figures record a slight advance of prices in harmony with the Annalist figures, though this advance is confined to the decimal columns and disappears when the decimals are rounded off.

often contradict the evidence of the two general-purpose index numbers in a striking fashion. Such contradictions occur in 1890–1891, 1892–1893, 1900–1901, 1902–1903, 1907–1908, and 1912–1913. These differences are due chiefly to a contrast in the years mentioned between business conditions and harvest conditions. They parallel the differences in Table 8 between the index numbers of mineral products and those of farm crops, or farm crops and animal products taken together; for the food indexes are made up almost wholly from the pieces of vegetable crops, food animals, and their derivatives.[54] A food index number, then, is likely at any time to give a wrong impression regarding the shifting of prices in general and is especially treacherous as a business barometer. Nor can such an index when made from wholesale prices be trusted to show changes in the "cost of living"; for living expenses are made up of retail prices, and fluctuations in retail prices do not always follow closely those in the wholesale markets.

But while it is clear that an index number intended to measure fluctuations in "the general level of prices" should grant due representation to the various groups of prices that behave in specific ways, it is not possible to give a definitive list of these groups. For our knowledge of the interrelations among prices even in the recent past is very limited. Moreover, a change in the social conditions under which business is done may at any time produce new groupings of commodities important to the maker of index numbers, or may cause old groups to fluctuate in novel ways. For example, the distinction between commodities over which the Government assumed some form of price control and commodities whose prices were left unrestricted became of first importance in the summer of 1917. After July the controlled prices dropped, and while they advanced again in the latter part of 1918, they did not again attain the high level at which they stood when the price control began. Uncontrolled prices, on the contrary, which stood lower than the other group in July, 1917, advanced month by month until the armistice was signed.[55] Forest products in 1915–1918 illustrate the way in which a group may change its characteristic price behavior. The demand for lumber has been declining jerkily in the United States since 1909, primarily because of the increased use of cement for building. Further, the terms on which many large lumber holdings are financed compel the owners to cut and market their timber as fast as possible. Finally, in 1917–18 the War Industries Board discouraged the construction of buildings that were not called for by the military program. Under these circumstances, the price of forest products lagged behind most classes

[54] The exceptions are salt and soda, and of these articles the Annalist and Gibson quote neither.

[55] See the tables in Government Control Over Prices, by Paul W. Garrett, War Industries Board Price Bulletin, No. 3.

The following index numbers, while not covering the whole ground, bring out the main point. One series shows the fluctuations of 586 commodities that were subjected to price control at some time during American participation in the war; the second series shows the fluctuations of 780 commodities that were left uncontrolled in price. Since the practice of "fixing" prices did not begin until several months after the declaration of war (April, 1917), and was extended gradually month by month until the signing of the armistice (November, 1918), the "controlled" list contains many articles that remained uncontrolled until late in 1918. The two series therefore minimize rather than exaggerate the differences between the behavior of prices that were controlled earlier in the war and prices that were left to find their own levels. That this understatement is not more serious arises from the fact that the Government naturally took the most important (and therefore most heavily weighted) commodities under control at an early date. It may be of commodities in the wartime rise.[56] To give another illustration, rubber is rapidly passing from the group of forest products to the group of cultivated crops. These cases give force to the warning that the groupings with which the economic statistician deals do not always rest on permanent foundations. It would be as unwarranted to draw up a list of groups that should be represented in index numbers for all periods as to draw up a list of groups to be represented for all purposes. In every case in which an investigator plans to measure changes in the general level of prices he should canvass his particular field to see whether there are not hitherto unrecognized groups of commodities that fluctuate in similar ways, and then try to represent each group in the due measure of its importance. Such investigations may add much, not only to the accuracy of index numbers but also to our knowledge of the interrelations among price fluctuations.

In most large index numbers the commodities quoted are divided into several classes; but these classes seldom have economic significance or even logical consistency. Among the nine groups recognized by the Bureau of Labor Statistics, for example, one group, "Farm products," emphasizes the place of production; four groups, "Food, etc.," "Fuel and lighting," "Lumber and building materials," and "House-furnishing goods," emphasize the use to which commodities are put; three groups apply a double criterion, use and physical character of the goods, namely, "Cloths and clothing," "Metals and implements," and "Drugs and chemicals"; the remaining group is frankly styled "Miscellaneous." Such a classification is not without usefulness, for there doubtless are readers especially interested in the prices of, say, all things that are raised on farms, and others who care especially about the prices of things used to furnish houses, or things that can be classed together as drugs and chemicals whether they are used chiefly as medicines or to make farm fertilizers. But if a classification of this empirical character is maintained, it might with advantage be accompanied by a classification that throws more light upon the workings of the complex system of prices.

pointed out also that the commodities early brought under control were articles that, as a group, had risen more than the average in price before we entered the war.

Index numbers of commodities that were and of commodities that were not subjected to price control by the Government during the war with Germany.

[From War Industries Board Price Bulletin No. 3.]

(Relatives made from weighted aggregates of actual prices. Average prices in July, 1913, to June, 1914=100.)

Year and month.	Controlled commodities.	Uncontrolled commodities.	Year and month.	Controlled commodities.	Uncontrolled commodities.
1917			1918		
January			January	195	178
February			February	198	180
March			March	197	182
April	183	146	April	196	187
May	192	149	May	192	189
June	201	152	June	189	191
July	209	160	July	195	194
August	204	162	August	199	195
September	205	163	September	204	199
October	198	167	October	201	201
November	200	172	November	200	200
December	193	174	December	204	197

[56] See R. B. Bryant, The Prices of Lumber, War Industries Board Price Bulletin, No. 43, and Homer Hoyt, The Prices of Building Materials, War Industries Board Price Bulletin, No. 6.

Another interesting experiment has recently been made by the Price Section of the War Industries Board. This section was able to collect quotations for so large a number of price series (1366 in form to be used in the index number) that it attempted to classify its commodities according to industries by which they are manufactured. The advantage of this arrangement is that many users of index numbers desire to follow the fluctuations of the prices that are paid for materials and received for products in different lines of business and to compare fluctuations in one line with those in others. There are many industries in which the plan works well, because the demarcation between industries follows, at least roughly, commodity lines; for example, in the cotton, woolen, silk and leather trades. But many commodities are used in such a variety of industries, and many industries use such a variety of commodities, that the classifier is forced to resort at times to other criteria, such as the physical characteristics of commodities, their uses, or their sources of supply.

Probably the most illuminating way of presenting an index number that aspires to cover the whole field of prices at wholesale would be to publish separate results for the groups that have characteristic differences of price fluctuations, and then to publish also a grand total including all the groups. The groups to be recognized and the distribution of commodities among them is a difficult matter to decide. But, as matters stand, the most significant arrangement seems to be (1) a division of all commodities into raw and manufactured products; (2) the subdivision of raw commodities into farm crops and animal, forest, and mineral products; (3) the subdivision of manufactured products according as they are bought mainly for personal consumption, mainly for business use, or largely for both purposes.[57]

This classification is based upon differences among the factors affecting the supply of and the demand for commodities that belong to the several groups—that is, upon differences among the factors which determine prices. If we wish our index numbers to help toward an understanding of changes in the price level, a classification along such causal lines seems to be the most promising line of progress.

Where means permit, it is desirable to supplement this general scheme by a series of special indexes for classes of commodities that possess interest for whatever reason. These supplementary indexes would not rest on classifications which include all the commodities, and they might, therefore, employ many different criteria and employ each one only in those cases in which it was significant. Some commodities might appear in several of the special indexes, and others might appear in none. There need, then, be no artificial forcing of a criterion upon facts which it does not fit, and no hesitation about presenting any classes that merit separate attention.

Large index numbers are more trustworthy for general purposes than small ones, not only in so far as they include more groups of related prices, but also in so far as they contain more numerous samples from each group. What is characteristic in the behavior of the prices of farm crops, of mineral products, of manufactured wares, of consumers' goods, etc.—what is characteristic in the behavior of any group of prices—is more likely to be brought out and to exercise its due effects upon the final results when the group is represented by

[57] Since the first edition of this bulletin appeared, the Federal Reserve Board has adopted this suggestion with interesting results. In its monthly bulletin the board publishes the index number compiled by the Bureau of Labor Statistics recast into the six groups mentioned.

10 or 20 sets of quotations than when it is represented by only one or two sets. The basis of this contention is simple: In every group that has been studied there are certain commodities whose prices seldom behave in the typical way, and no commodities whose prices can be trusted always to behave typically. Consequently, no care to include commodities belonging to all the important groups can guarantee accurate results, unless care is also taken to get numerous representatives of each group.

Even here the matter does not end. The different groups that have been discussed, the other groups that might have been discussed, and the commodities that are included within the several groups differ widely in importance as elements in the system of prices. To these differences, and to the methods of making them count in index numbers, we must now turn.

6. PROBLEMS OF WEIGHTING.

It is customary to distinguish sharply between "simple" and "weighted" index numbers. When an effort is made to ascertain the relative importance of the various commodities included, and to apply some plan by which each commodity shall exercise an influence upon the final results proportionate to its relative importance, the index number is said to be weighted. When, on the contrary, no such effort is made, but every commodity is supposedly allowed just the same chance to influence the result as every other commodity, the index number is said to be unweighted, or simple.

This expression, however, that "every commodity has just the same chance to influence the result as every other commodity" conveys no clear meaning. It is better to think of all index numbers as weighted, for so they are whether their maker knows it or not, and to ask whether the scheme of weights is good or bad. For example, in Bradstreet's index the influence of every article upon the result varies as its price per pound happens to be large or small.[58] Again, the decisive objection to making index numbers by merely adding the ordinary commercial quotations for different articles is that these nominally simple series are in fact viciously weighted series.[59] Nor does the substitution of relative prices for actual prices assure an "equal chance" to every article. For instance, in its famous report of 1893, the Senate Committee on Finance presented three wholesale-price index numbers—one simple and two weighted; but in the simple series it included relative prices for 25 different kinds of pocketknives, giving this trifling article more than eight times as many chances to influence the results as they gave wheat, corn, and coal put together. Finally, even if one series of relative prices, and only one, be accorded each commodity, it does not follow that equal percentages of change in the price of every article will always exercise equal influence upon the results. For when relative prices are computed upon a fixed base and averaged by the use of arithmetic means, those commodities that have a long period upward trend in price will presently for outweigh in influence those commodities whose prices are declining.

Lack of attention to weighting, then, does not automatically secure a fair field and no favor to every commodity; on the contrary, it

[58] For details, see pp. 161-168. [59] See p. 31.

results in what Walsh happily termed haphazard weighting.[60] Perhaps "unconscious weighting" would be an even better expression. The real problem for the maker of index numbers is whether he shall have weighting to chance or seek to rationalize it.

There are two excuses for neglect of weighting. First, as has been shown in another connection, to collect satisfactory statistics showing the relative importance of different commodities is extremely laborious and extremely difficult.[61] Second, there are high authorities who hold that the results turn out much the same whether or not formal weights are used.[62] Certainly "the weights are of * * * less importance in determining an index number of prices than the prices themselves."[63] But whether their importance is negligible is a question best answered by a study of actual cases such as are shown in the next table.[64]

The discrepancies here revealed between the averages with haphazard and with systematic weights seldom amount to 10 per cent of the results, except under the chaotic price conditions created by the greenback standard in 1862–1873. In many kinds of statistics a 10 per cent margin of error is not accounted large. But in making wholesale-price index numbers for current years we may reasonably try to get not two, but three, significant figures; and the third figure is

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[60] C. M. Walsh, The Measurement of General Exchange-Value, pp. 81 and 82. Haphazard weighting is not necessarily the worst weighting; indeed, it may be better than the weighting which results from some systematic calculations. For example, Bradstreet's plan of using actual prices per pound is certainly systematic, but the weighting which this system involves is probably less defensible than the haphazard weighting involved in most averages of the relative prices of commodities selected at random. See p. 78.

[61] See p. 26. When the (then) Department of Labor started its former index number it canvassed the subject of weighting, but decided to use a simple average, because of the "impossibility of securing even approximately accurate figures for annual consumption in the United States of the commodities included." (Bulletin No. 39, of the Department of Labor, p. 234, March, 1902.) It did, however, allot two or more series to certain commodities, and thus introduced a rough system of weights. Unfortunately the number of series allotted to each commodity seems to have been determined quite as much by the ease of securing quotations as by the importance of the articles. For criticism of the weighting which resulted, see pp. 48 and 49.

[62] Compare A. L. Bowley, Elements of Statistics, 2d ed., pp. 113 and 220–224.

[63] Irving Fisher, The Purchasing Power of Money, revised edition, p. 406. For further details see the papers by Edgeworth, to which Fisher refers in his footnote.

[64] Details concerning the first three sets of simple and weighted averages can be found in the documents referred to in the table. But the fourth set of comparisons is based upon hitherto unpublished data and requires description.

The "unweighted" index numbers in this set are arithmetic means of the relative prices given in the bulletins of the Bureau of Labor Statistics for the commodities listed below. But where two or more series of relative prices are shown in the bulletins for different grades of the same articles, as in the case of cattle, hogs, bacon, butter, corn meal, pig iron, etc., they were replaced by a single average series for the article in question before the arithmetic means of the group were computed.

The "weighted" index numbers were made from these same relative prices in the following way: (1) For each commodity included the Bureau of Labor Statistics made a careful estimate, based upon a critical study of the best available sources of information, of the physical quantity of it entering into exchange in the year 1909. By "quantity entering into exchange" is meant the quantity bought and sold, irrespective of the number of times it changed hands. (See pp. 63 and 64.) (2) These physical quantities were multiplied by the average prices in 1909 of the respective commodities. (3) The resulting sums of money were used as weights to multiply the relative prices of the respective commodities on the 1890–1899 base. (4) The sums of the products were cast up for each year, and finally these sums were divided by the sums of the weights, i. e., the value in exchange for 1909.

The average prices of the commodities in 1909 may be found in any of the recent wholesale-price bulletins, e. g., No. 149. The commodities included, and the estimated physical quantity of each entering into exchange in 1909, are as follows:

Farm products: Barley, 75,300,538 bu.; cattle, 124,346,349 cwt.; corn, 460,778,251 bu.; cotton, 5,409,760,011 lbs.; flaxseed, 20,106,433 bu.: hay, 10,685,804 tons; hides, 922,243,894 lbs.; hogs, 76,438,923 cwt.; hops, 48,076,921 lbs.; oats, 267,859,660 bu.; rye, 29,520,508 bu.; sheep, 11,498,090 cwt.; wheat, 683,416,528 bu.

Food, etc.: Beans, 8,468,385 cwt.; butter, 1,042,709,708 lbs.; cheese, 353,641,892 lbs.; coffee, 1,038,439,285 lbs.; eggs, 926,690,112 doz.; codfish, 684,692 cwt.; herring, 428,804 bbls.; mackerel, 190,565,bbls.; salmon, 18,431,003 doz. cans; buckwheat flour, 2,009,599 cwt.; rye flour, 1,594,346 bbls.; wheat flour, 107,306,408 bbls.; currants, 32,163,998 lbs.; prunes, 138,795,607 lbs.; raisins, 12,438.044 boxes; glucose, 7,701,223 cwt.; lard 1,243,572,129 lbs.; corn meal, 53,353,466 cwt.; bacon, 741,354,500 lbs.; beef, fresh, 4,209,196,748 lbs.; beef, salt, 632,388 bbls.; hams, 789,861,744 lbs.; mutton, 495,458,067 lbs.; pork, salt, 4,760,690 bbls.; milk, 7,749,070,256 qts.; molasses, 55,689,983 gals.; rice, 1,042,538,693 lbs.; salt, 22,136,489 bbls.; soda, bicarbonate, 165,600,000 lbs.; pepper, 36,241,462 lbs.; sugar, raw, 6,316,033,669 lbs.: sugar, granulated, 7,366,818,210 lbs.; tallow, 203,209,103 lbs.; vinegar, 98,403,927 gals.; potatoes, 397,491,062 bu.; onions, 4,972,947 cwt.; tea, 113,547,647 lbs.

Metals and implements: Bar iron, 2,166,529,067 lbs.; barbed wire, 6,471,300 cwt.; copper, ingot, 1;312,437,919 lbs.; copper wire, 278,964,000 lbs.; lead, pig, 732,152,538 lbs.; lead pipe, 1,058,280 cwt.; nails, wire, 13,916,097 kegs: pig iron, 9,896,248 tons: tin (pig), 94,248,471 lbs.; silver, 151,969,144 ozs.; speiter, 464,903,059 lbs.; steel billets, 4,972,179 tons; steel rails, 3,025,009 tons; tin plate, 12,968,174 cwt.

usually altered in appreciable degree by the substitution of systematic for haphazard weights. Even the large Canadian series, with its 272 commodities, is shifted 9.5 points, or more than 7 per cent, in 1912 by weighting.

Table 14.—COMPARISONS OF WEIGHTED AND UNWEIGHTED INDEX NUMBERS.

[1. From the report of the Senate Committee on Finance, Mar. 3, 1893. By years, 1860 to 1891.]

(Arithmetic means. Prices in 1860=100.)

Year.	Simple arithmetic means, all articles.	All articles averaged according to importance, certain expenditures being uniform.	All articles averaged according to importance: 68.6 per cent of total expenditure.	Difference between simple and first weighted averages.	Difference between simple and second weighted averages.	Difference between first and second weighted averages.
1860	100.0	100.0	100.0	....	....	....
1861	100.6	95.9	94.1	4.7	6.5	1.8
1862	117.8	102.8	104.1	15.0	13.7	1.3
1863	148.6	122.1	132.2	26.5	16.4	10.1
1864	190.5	149.4	172.1	41.1	18.4	22.7
1865	216.8	190.7	232.2	26.1	15.4	41.5
1866	191.0	160.2	187.7	30.8	3.3	27.5
1867	172.2	145.2	165.8	27.0	6.4	20.6
1868	160.5	150.7	173.9	9.8	13.4	23.2
1869	153.5	135.9	152.3	17.6	1.2	16.4
1870	142.3	130.4	144.4	11.9	2.1	14.0
1871	136.0	124.8	136.1	11.2	.1	11.3
1872	138.8	122.2	132.4	16.6	6.4	10.2
1873	137.5	119.9	129.0	17.6	8.5	9.1
1874	133.0	120.5	129.9	12.5	3.1	9.4
1875	127.6	119.8	128.9	7.8	1.3	9.1
1876	118.2	115.5	122.6	2.7	4.4	7.1
1877	110.9	109.4	113.6	1.5	2.7	4.2
1878	101.3	103.1	104.6	1.8	3.3	1.5
1879	96.6	96.6	95.0	....	1.6	1.6
1880	106.9	103.4	104.9	3.5	2.0	1.5
1881	105.7	105.8	108.4	.1	2.7	2.6
1882	108.5	106.3	109.1	2.2	.6	2.8
1883	106.0	104.5	106.6	1.5	.6	2.1
1884	99.4	101.8	102.6	2.4	3.2	.8
1885	93.0	95.4	93.3	2.4	.3	2.1
1886	91.9	95.5	93.4	3.6	1.5	2.1
1887	92.6	96.2	94.5	3.6	1.9	1.7
1888	94.2	97.4	96.2	3.2	2.0	1.2
1889	94.2	99.0	98.5	4.8	4.3	.5
1890	92.3	95.7	93.7	3.4	1.4	2.0
1891	92.2	96.2	94.4	4.0	2.2	1.8

[2. From Bulletin of the Department of Labor, No. 27, March, 1900. January of the years, 1890 to 1899.]

(Arithmetic means. Averages of 9 quarterly quotations, January, 1890, to January, 1892=100.)

Year and month.	All articles simply averaged.	All articles averaged according to importance, certain expenditures being considered uniform.	All articles averaged according to importance, comprising 68.6 per cent of total expenditure.	Difference between simple and first weighted averages.	Difference between simple and second weighted averages.	Difference between first and second weighted averages.
1890, January	102.0	100.1	100.2	1.9	1.8	0.1
1891, January	100.6	102.2	103.2	1.6	2.6	1.0
1892, January	96.5	100.0	100.1	3.5	3.6	.1
1893, January	97.2	103.4	105.0	6.2	7.8	1.6
1894, January	89.6	97.5	96.4	7.9	6.8	1.1
1895, January	84.7	93.5	90.5	8.8	5.8	3.0
1896, January	85.2	92.8	89.5	7.6	4.3	3.3
1897, January	82.0	90.3	85.9	8.3	3.9	4.4
1898, January	83.3	91.0	86.8	7.7	3.5	4.2
1899, January	86.5	91.0	86.8	4.5	.3	4.2

TABLE 14.—COMPARISONS OF WEIGHTED AND UNWEIGHTED INDEX NUMBERS—Con.

[3. From Wholesale Prices, Canada, 1913. Report by R. H. Coats. By years, 1890 to 1913.]

(Arithmetic means. Average prices in 1890–1899=100.)

Year.	Weighted index number.	Unweighted index number.	Differences.	Year.	Weighted index number.	Unweighted index number.	Differences.
1890	112.0	110.3	1.7	1902	109.6	109.0	0.6
1891	111.3	108.5	2.8	1903	109.7	110.5	.8
1892	104.9	102.8	2.1	1904	110.6	111.4	.8
1893	103.9	102.5	1.4	1905	113.8	113.8	....
1894	97.2	97.2	....	1906	120.1	120.0	.1
1895	95.6	95.6	....	1907	129.2	126.2	3.0
1896	90.6	92.5	1.9	1908	125.1	120.8	4.3
1897	89.9	92.2	2.3	1909	126.3	121.2	5.1
1898	95.5	96.1	.6	1910	128.0	124.2	3.8
1899	99.0	100.1	1.1	1911	131.1	127.4	3.7
1900	105.8	108.2	2.4	1912	143.9	134.4	9.5
1901	106.0	107.0	1.0	1913	139.6	135.5	4.1

[4. From computations by the Bureau of Labor Statistics.[1]]

(Arithmetic means. Average prices in 1890–1899=100.)

Year.	13 farm products.			37 food products.			14 metallic products.		
	Unweighted.	Weighted by estimated expenditures upon each article in 1909.	Differences.	Unweighted.	Weighted by estimated expenditures upon each article in 1909.	Differences.	Unweighted.	Weighted by estimated expenditures upon each article in 1909.	Differences.
1890	113	109	4	114	114	0	128	131	3
1891	124	117	7	116	114	2	118	116	2
1892	112	105	7	105	103	2	110	107	3
1893	106	107	1	112	111	1	102	98	4
1894	96	94	2	99	97	2	88	84	4
1895	93	95	2	95	94	1	88	88	0
1896	78	86	8	83	86	3	93	91	2
1897	84	93	9	87	90	3	82	80	2
1898	97	97	0	93	96	3	83	81	2
1899	99	98	1	98	96	2	124	124	0
1900	109	109	0	108	100	8	124	123	1
1901	117	115	2	110	102	8	114	113	1
1902	130	129	1	114	108	6	114	114	0
1903	120	120	0	110	104	6	114	113	1
1904	130	128	2	113	110	3	105	102	3
1905	125	123	2	110	109	1	116	113	3
1906	122	124	2	115	106	9	131	130	1
1907	139	136	3	120	112	8	138	140	2
1908	135	135	0	122	119	3	103	108	5
1909	150	154	4	124	126	2	109	107	2
1910	161	165	4	129	127	2	111	108	3
1911	166	150	16	128	125	3	111	103	8
1912	173	164	9	137	137	0	120	114	6
1913	152	161	9	133	127	6	119	115	4

[1] See explanations in footnote, p. 60.

If rational weighting is worth striving after, then by what method shall the weights of the different commodities be arrived at? That depends upon the object of the investigation. If, for example, the aim be to measure changes in the cost of living, and the data be retail quotations of consumers' commodities, then the proportionate expenditures upon the different articles as represented by collections of family budgets make appropriate weights. If the aim be to study changes in the money incomes of farmers, then the data should be "farm prices," the list of commodities should be limited to farm products, and the weights should be proportionate to the total money receipts from the several products. If the aim be to construct a "business barometer," the data should be prices from the most representative wholesale markets, the list should be confined to commodities whose prices are most sensitive to changes in business prospects and least liable to change from other causes, and the weights may logically be adjusted to the relative faithfulness with which the quotations included reflect business conditions. If the aim be merely to find the differences of price fluctuation characteristic of dissimilar groups of commodities, or to study the influence of gold production or the issue of irredeemable paper money upon the way in which prices change, it may be appropriate to strike a simple arithmetic average of relative prices. If, on the other hand, the aim be to make a general-purpose index number of wholesale prices, the question is less easy to answer.

One proposition, however, is clear. The practice of weighting wholesale-price index numbers by figures drawn from family budgets is to be deprecated; for family budgets do not show the importance of wheat and cotton, of petroleum and spelter, of tar and lime, of pig iron and hides, of brick and lumber; indeed, to apply budget weights to half or more of the articles in any wholesale list is nonsensical. And to pretend that wholesale-price index numbers when weighted on the basis of family expenditures show fluctuations in the cost of living is to overtax the credulity of those who know and to abuse the confidence of those who do not.

Allied to the family-budget method of weighting and yet vastly better for wholesale-price index numbers is the "aggregate expenditure" method.[65] Here an attempt is made to ascertain the aggregate sums of money laid out by the people of a whole country upon the articles quoted and to adjust the weights upon this basis. Of course the country as a whole buys raw materials, as single families do not, and of course consumers' commodities can be taken at their aggregate values in wholesale markets. Similar in net effect is the weighting on the basis of consumption practiced by the British Board of Trade. For "consumption is taken to mean any process by which the commodity is substantially changed in character. In other words, consumption in manufacture is recognized as well as consumption by an individual."[66] Somewhat different weights would result if quantities or values produced were taken in place of quantities or values consumed. Mr. Walsh thinks it best to combine these two criteria—that is, to take "either the total product or the total consumption according as the one or the other is the greater."[67] Prof. Irving Fisher prefers "an index number in which every article or service is weighted according to the value of it exchanged at base prices in the year whose level of prices it is desired to find."[68] On this system the weight assigned to each article would be affected by the number of times it changed hands on its way from producer to

[65] See G. H. Knibbs, Prices, Price Indexes, and Cost of Living in Australia. Commonwealth Bureau of Census and Statistics, Labour and Industrial Branch, Report No. 1, pp. 11–14.

[66] Report on Wholesale and Retail Prices in the United Kingdom in 1902. London, 1903, p. 441. The accuracy of the statistics upon which the Australian and British index numbers are based may be open to question. Not the data, but the method is of interest here.

[67] C. M. Walsh, The Measurement of General Exchange-Value. New York, 1901, p. 95.

[68] Irving Fisher, The Purchasing Power of Money, revised edition. New York, 1913, pp. 217 and 218.

final consumer. A variation of his plan is therefore represented by the proposal to weight each article according to the quantity of it which enters into the country's commerce, irrespective of the frequency with which it changes hands.

The practical consequences of adopting these different systems of weighting may be illustrated by considering their application to cotton, corn, and coffee in the United States. Production weights would give cotton much greater importance than consumption or aggregate-expenditure weights, because so large a part of the American crop is exported and consumed abroad. Exchange weights would be practically equivalent to production weights, because practically all the cotton grown is sold by the planters and enters into the commerce of the country, and relatively little cotton is imported. On Prof. Fisher's plan, however, the exchange weights would be some multiple of the production weights, depending upon the average number of American hands through which the cotton passed. In the case of corn, production and consumption weights would substantially agree, for we import little corn and export but a small percentage of the production. On the other hand, exchange weights would be much less than either production or consumption weights, because a large part of the corn crop is never sold, but is consumed on the farms where it is grown. In the case of coffee, production weights would be zero, while consumption and exchange weights would correspond closely.

We are helped toward a choice among these rivals by common agreement upon a slightly different point. In arranging any system of weights except Prof. Fisher's, double counting is to be avoided so far as possible. For example, if cotton is counted at its full importance as a raw material, then cotton yarns and later cotton fabrics made of the yarn can not be counted at their full importance without assigning triple weight to the raw cotton which is represented at these two successive stages of manufacture. Now, if this sensible observation be applied to cases like those of corn, hay, etc., it casts the die in favor of exchange weights. For if these articles, which are used largely by the original producers in making things quite different from corn and hay (for instance, pork and beef) are counted at the full amount produced or consumed, and if their products (the pork and beef) are also counted at the full amount produced or consumed, there will be a great deal of double counting. Not all but much of this duplication can be eliminated by counting only the amount of corn and hay sold by the producers and letting the rest of these articles produced and consumed get their proper representation under the caption of pork, beef, etc.[69]

If for this reason exchange appears a rather better criterion of importance than production, consumption, or a combination of the two, it remains only to decide whether the number of times a thing is exchanged should be recognized. Prof. Irving Fisher had good cause to propose multiple counting, for he wanted an index number of prices for constructing the "equation of exchange," a mathematical expression of the necessary equivalence between the total volume of

[69] Of course, this same end might be attained without surrendering the production or consumption basis if the rule against double counting of raw materials and products were made broad enough to include corn, for example, as the raw material of pork; but needless to say there is little liklihood that the common meaning of terms will be stretched to such an extent.

business done in a country and the total volume of payments effected by means of money and credit instruments. Of course the oftener an article is sold and paid for the more important it is as a factor in this equation. But it does not follow that the economic importance of an article is greatly changed by reorganizing the chain of business enterprises that deal in it. "Integration of industry," as expressed in our trusts, does not make pig iron less significant as an item in the country's economic life, except in the sense that it reduces the average number of transfers of ownership. The quantity of the article that enters into exchange, then, irrespective of the number of turnovers, is probably the most satisfactory gauge of importance to apply in making general-purpose index numbers. Anyone experienced in the search for statistical information will need no warning that in the working out of weights along this line many puzzling cases will arise in which consistency will be difficult to maintain, to say nothing of the wide gaps and weak places that will be revealed among the available data. That this system of weighting is feasible in practice as well as desirable in theory, however, was proved by the Bureau of Labor Statistics in 1914, when it gave up averaging relative prices and began multiplying actual prices by the quantities of commodities that entered into trade in the base year 1909.[70]

Three interesting questions remain: Should the weights be sums of money or physical quantities? Should the weights be changed from year to year or kept constant? Should the weights be adjusted to the importance of the commodities as such, or should there be taken into account also the importance of the commodities as representing certain types of price fluctuations?

When relative prices are being used the weights should be reduced to a common denominator. As multipliers, of course, weights may be regarded as merely abstract numbers; but in studying the weights themselves it is necessary to have some common standard by which the relative importance assigned to various commodities can be accurately compared. The only common denominator for all commodities that is significant for economic ends and capable of quantitative expression is money value. But it is ill advised to weight by money values when actual prices are being used, for the common denominator is already present in the quotations themselves. These price quotations are best multiplied by the physical quantities of the goods produced, exchanged, or consumed, as the case may be.

Like most of the issues on which authorities differ, the question whether it is desirable to change weights at frequent intervals depends upon the precise end in view. Most makers of index numbers have wished to isolate the price factor from other changes in the economic complex. Hence they have preferred to keep their weights as nearly constant as possible. For when the weights are altered the index number becomes a measure of two sets of changes, and no one can tell what part of the net results is due to variations in prices and what to variations in weights.[71] Yet it is clear that a system of fixed weights applied over a long period is certain to become inaccurate for most of the years, however carefully it is adjusted to conditions prevailing at some base period. Practically, then, a compiler who wishes to ascertain how prices have changed must choose between

[70] For details see Bulletin No. 181 of the Bureau of Labor Statistics.
[71] See the criticism of index numbers made from import-export values, pp. 29-31.

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two evils—inaccurate weights and ambiguous price measures. Sometimes he can minimize the first evil by collecting data showing the average importance of his commodities over a period of years, for these averages are less likely to go awry than figures for a single year. In other cases the least objectionable compromise is probably to revise the scheme of weights, say, once a decade, and to show the effect of this change by computing overlapping results for a few years with both the old and new weights.[72] A further practical reason in favor of this compromise is found in the heavy expense in time and labor required for frequent revisions of the weights.

Writers like Mr. Walsh, Prof. Pigou, and Prof. Fisher, who urge the adoption of a formula in which the weights are changed every year, put another aim in the foreground. Their primary purpose is to secure the utmost possible nicety in measuring the rise or fall of prices in each pair of years treated. Of course an index number made with these changing weights "measures neither the varying cost of a constant amount of goods nor the varying amount of goods which a dollar will buy."[73] But, since the importance of price fluctuations depends largely upon the accompanying changes in the quantities of goods bought, there is use for index numbers that do not attempt to measure the price factor in isolation. By changing weights each year it is possible to make these constantly occurring changes in quantities bought influence the price index, and therefore to secure results better fitted for certain uses than the results of an unambiguous measure of fluctuations in prices.[74]

To the third question, whether weights should be adjusted to the importance of the commodities as such, or whether there should also be taken into account the importance of these commodities as representatives of certain types of price fluctuations, little attention has been paid. But the preceding section shows that this neglected problem is both important and difficult. The prices of raw materials behave differently from the prices of manufactured goods; among the raw materials the prices of farm crops, of forest, animal, and mineral products behave differently; there are also differences of behavior between the prices of manufactured goods bought by producers and by consumers, etc. Is an accurate measure of changes in the level of all wholesale prices obtained unless all of the different types of fluctuation, doubtless including types not yet definitely recognized, are represented in accordance with the relative importance of the commodities exhibiting each type?

How can such representation be attained? If all the commodities bought and sold could be included on a strictly uniform basis in the index number, it would suffice to weight each by the criterion of its own individual importance. Since that is out of the question, it is desirable to draw from each part of the whole system of prices samples sufficient to determine its characteristic fluctuations, and then to make sure that each part of the whole system counts for the proper amount in determining the final result. On this plan commodities would be weighted simply as commodities in making the subtotals

[72] Compare G. H. Knibbs, Prices, Price Indexes, and Cost of Living in Australia. Commonwealth Bureau of Census and Statistics, Labour and Industrial Branch, Report No. 1, pp. xxiv and xlix.

[73] Prof. Warren M. Persons: "Fisher's formula for index numbers," Review of Economic Statistics, May, 1921, p. 115, note.

[74] See the discussion of the "ideal" formula in section 9, p. 91.

for each recognized group, and these subtotals would be weighted again in making up the grand totals.

Such a plan was adopted by the Price Section of the War Industries Board in making their index number of prices in 1913–1918. As noted above, the subdivisions used by the Price Section were 50 classes of commodities based, so far as possible, on the organization of industries. Within each class, raw materials were weighted according to quantities used by the industry represented, and products were weighted according to the quantities produced. A separate index number was made in this way for each of the 50 classes. These indexes and the materials from which they were made, both price quotations and weights, seemed fairly satisfactory as such matters go; but before the aggregates of the commodity prices times commodity weights for these 50 classes were assembled to make aggregates for "all commodities," it was clear that there would be wide differences in the fullness with which prices in the various industries had been covered. In some industries 75 to 90 per cent of the value of the transactions was represented by the prices multiplied by the weights; in other industries the percentage sank below 25. Again, there were industries in which it had been possible to quote commodities at three stages—raw materials, partly manufactured goods, and finished products—while in other industries the available data represented only raw materials or only finished products. That is, while the weights within each class had been systematized, and while the plan of systematizing the weights was uniform in all classes systematized, the weights as between different classes were haphazard to a degree. To overcome the difficulty, the Price Section prepared a second set of weights. It estimated the value of the products sold by each industry represented, divided these estimates by the aggregate of commodity prices times commodity weights, and so obtained a set of factors which when applied to the class aggregates give each class an influence upon the index for "all commodities" proportioned to its estimated importance.[75]

Professor Edgeworth has pointed out a yet further desideratum in weighting. Most index numbers are made from samples of the data which logically fall within the field investigated; and the task is to make from these samples the best approximation to a measure of the unknown whole. Now "the theory of errors-of-observation shows that in the combination of the given observations, 'less weight should be attached to observations belonging to a class which are subject to a wider deviation from the mean. Such would be prices of articles which, exclusive of the common price movement of all the selected articles, are liable to peculiarly large *proper* fluctuations.'"[76]

Perhaps it is a counsel of perfection to urge such refinements in systems of weighting. Certainly the difficulties to be encountered are very great. Statistical knowledge is not complete enough to supply accurate data for weighting all the different parts of the system of prices that are known to have characteristic peculiarities of fluctuation. Nor have these different types and the commodities exhibiting

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[75] See History of Prices During the War, Summary, War Industries Board Price Bulletin, No. 1.

[76] Economic Journal, June, 1918 (Vol. XXVIII, p. 188). The quotation within the quotation is from the British Association Memorandum, 1887 (p. 36). To make his point clearer, Prof. Edgeworth adds in a footnote this remark from the corresponding memorandum of 1889 (p. 157): "If more weight attaches to a change of price in the article rather than another, it is not on account of the importance of that article to the consumer or the shopkeeper, but on account of the importance to the calculators of probabilities as affording an observation which is peculiarly likely to be correct."

each been adequately studied. And puzzling difficulties are raised by overlapping among the types—there are commodities that belong in two places at once. But here is certainly a promising lead for future efforts to improve present measurements of changes in the price level. Even now it might be feasible by taking pains to secure rough justice as between raw and manufactured commodities, and as between raw vegetable, animal, forest, and mineral products. One modest step in the right direction can readily be taken by any compiler of index numbers: He can make clear that his results do not measure changes in the general level of wholesale prices accurately when they are obtained without an effort to represent each part of the field according to its due importance.

7. AVERAGES AND AGGREGATES.

Among all the problems involved in the making of index numbers the one that has been most discussed is the best form of average to strike. Most of these discussions have come from men interested in the mathematical side of statistics rather than in the problem of ascertaining what changes have actually occurred in prices. The practical makers of index numbers, on the contrary, have seldom troubled themselves greatly about theoretical considerations. Indeed, the two problems of finding out how prices have actually changed, and finding the best method of measuring changes, appeal to two types of interest, which are seldom strongly developed in the same mind. The mathematical statistician is likely to know little and care less about the field work of collecting price quotations. To the practical statistician this field work is of overshadowing importance, and the subsequent manipulation of his data is a matter of secondary interest. Hence, a study of index numbers as they are made need not carry one into long mathematical flights.[77]

First, it should be recalled that certain compilers of index numbers do not strike averages at all. The old form of the Economist index and Gibson's present index, for example, are sums of relative prices. More important are the series which dispense with the use of relative prices for each commodity, and give results in the form of sums of actual prices, or such sums thrown back into a series of relative numbers. These cases are still exceptional, however, and most index numbers are made by finding some sort of average from the relative prices of the commodities included.

The sort of average struck is usually the arithmetic mean—that is, the sum of the relative prices divided by their number. Occasionally medians are used—that is, the midmost relative prices which divide the whole number of cases into two equal groups, half above the median and half below. In one famous investigation,[78] geometric means were employed—that is, all the relative prices for a given date were multiplied together and the nth roots of the products were extracted, n standing for the number of commodities included. But Jevons has had few imitators, though Mr. A. W. Flux has just adopted

[77] The best systematic discussions of averaging for the purpose in hand are to be found in Prof. Edgeworth's papers referred to in the footnote on p. 8; Irving Fisher's The Purchasing Power of Money, revised edition, 1913, pp. 385–429; and C. M. Walsh's The Measurement of General Exchange-Value, 1901, and his new treatise, The Problem of Estimation, 1921.

[78] W. S. Jevons, "A serious fall in the value of gold ascertained," 1863. Reprinted in his Investigation in Currency and Finance, 1884, pp. 13–118

the geometric mean for the new form of the British Board of Trade index number. The other standard forms of averages—the mode and the harmonic mean—have been discussed frequently, but so far as is known they have never been consciously used in making index numbers.[79]

For the geometric mean two great merits are claimed. First, unlike the arithmetic mean, it is not in danger of distortion from the asymmetrical distribution of price fluctuations. Chart 2 shows that in a large collection of percentage variations from the prices of the preceding year, the extreme cases of rise run about twice as far up the scale as the extreme cases of fall run down. Such a distribution is characteristic of relative prices in general. Indeed, the case cited is distinctly moderate; most collections of variations covering many years would show a greater difference. There is indeed no limit to the possible percentage of rise in prices, while the possible percentage of fall can not exceed 100.[80] The cases of extraordinary advance, accordingly, tend to raise the arithmetic mean more than the cases of extraordinary decline tend to depress it. If, for example, one commodity rose tenfold in price and another commodity fell to one-tenth of the old price, the arithmetic mean would show an average rise of 505 per cent $(1{,}000 + 10 \div 2)$, while the geometric mean would show no change in the average, since $\sqrt{1{,}000 \times 10} = 100$.

This favorite imaginary case of 10 and 1,000 seems extreme, but contrasts approximately as violent as that actually occurred in the recent war. The Price Section of the War Industries Board has computed the relative prices of 1,437 commodities in 1918 on the basis of their average prices in the twelve months, July, 1913, to June, 1914. These figures are reproduced in somewhat condensed form in Table 15. Here the array of relative prices is far more elongated in one direction than in the other, and the highest relative price is upwards of 100 times as great as the lowest relative price.[81] Accordingly, the arithmetic mean (217) stands high above the geometric mean (194) and median (191).[82]

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[79] Concerning the properties of these averages see, for example, F. Zižek, Statistical Averages (translated by W. M. Persons), and G. U. Yule, Introduction to the Theory of Statistics, pp. 120–123, 128–129. The "crude mode" is that relative price which occurs most frequently in the data under examination, e. g., in Chart 2 it is "no change." The true mode is "the value of the variable corresponding to the maximum of the ideal frequency-curve which gives the closest possible fit to the actual distribution." "The harmonic mean of a series of quantities is the reciprocal of the arithmetic mean of their reciprocals."

[80] Negative prices are conceivable of course; but do they ever occur in the sources which the maker of index numbers uses? Suppose that some kind of factory waste, which usually commands a low price, should fail of its market, and accumulate so as to become a nuisance. The factory manager might logically set it down at a negative price; but he is much more likely to offer a positive price for another commodity—the removal of the waste.

[81] This ratio of 100 to 1 was indeed surpassed in some months. The highest relative price found was 5,081 (acetiphenetidin, November, 1916).—See History of Prices during the War (War Industries Board Price Bulletin, No. 54, p. 18).

[82] From the skewed distribution characteristic of relative prices when arranged on the ordinary arithmetic scale, Prof. Frederick R. Macaulay has developed an ingenious argument in favor of the geometric mean.

He puts the matter in this way: "What is the *most probable value* for the general percentage movement? If the 'errors' (variation due to the influence of particular commodity factors) were distributed *arithmetically* according to the normal law, the arithmetic mean—least mean square deviation—would certainly seem indicated. But if the logarithms of the percentages and not the percentages themselves follow more closely the curve of error, is not the geometric mean indicated? From that point the curve of the squares of the *logarithms* of the percentage deviations will be a minimum; and is not this what sound theory should demand?" American Economic Review, March, 1916, Vol. VI, p. 207.

The answer to Prof. Macaulay's final question is that what sound theory demands depends upon the precise magnitude one desires to measure. It is argued hereafter in the text that if the purpose be to measure the average ratio of change in prices, the geometric mean is in strictness the only proper average to employ. Those who can utilize measures of average change for their purposes will be gratified to know that the arrays from which their averages are made usually conform better to the normal law of distribution than the arrays from which arithmetic means of relative prices are derived. As Prof. Edgeworth humorously admits, "it is a merit in a statistical group to conform to the normal law." (Economic Journal, June, 1918, Vol. XXVIII, p. 182). But, of course, the symmetry of the distribution of data from which different averages are derived is but one, and generally a subordinate, consideration in the choice of averages.

TABLE 15.—DISTRIBUTION OF THE RELATIVE PRICES OF 1,437 COMMODITIES IN 1918.

(Average prices in July, 1913, to June, 1914=100.)

Relative prices.	Number of cases.	Relative prices.	Number of cases.	Relative prices.	Number of cases.	Relative prices.	Number of cases.
36	1	250–269	76	490–509	4	848	1
49	1	270–289	54	510–529	5	900	1
50–69	4	290–309	42	530–549	3	1,165	1
70–89	17	310–329	30	550–569	4	1,356	1
90–109	61	330–349	31	587	1	1,585	1
110–129	64	350–369	16	627	1	1,764	1
130–149	130	370–389	13	727	1	2,049	1
150–169	212	390–409	7	730	1	2,863	1
170–189	219	410–429	7	743	1	3,009	1
190–209	164	430–449	8	761	1		
210–229	135	450–469	4	784	1		
230–249	104	470–489	4	826	1		

The second merit claimed for geometric means is that they can be shifted from one base period to another without producing results that seem to be inconsistent. Suppose, for example, that the price of wheat falls from $1 per bushel in 1913 to 50 cents in 1914, while the price of corn remains unchanged at 40 cents. Then the relative prices are—

(1) On the basis, prices in 1913=100:

	1913	1914
Wheat	100	50
Corn	100	100

(2) On the basis, prices in 1914=100:

	1913	1914
Wheat	200	100
Corn	100	100

The arithmetic and geometric means of these relative prices are—

(1) On the basis of prices in 1913:

	Arithmetic means.	Geometric means.
1913	(100+100)÷2=100	$\sqrt{100\times100}=100.00$
1914	(50+100)÷2= 75	$\sqrt{50\times100}=$ 70.71+

(2) On the basis of prices in 1914:

	Arithmetic means.	Geometric means.
1913	(200+100)÷2=150	$\sqrt{200\times100}=141.42-$
1914	(100+100)÷2=100	$\sqrt{100\times100}=100.00$

Here the arithmetic means can not, but the geometric means can, be shifted from the 1913 base to the 1914 base or vice versa by simply dividing the index number for one year by that for the other. That is, $100 \div 75 = 133\frac{1}{3}$, not 150; but $100 \div 70.71 = 141.42$.[83] By shifting the base in this simple fashion geometric means can be made to give direct comparisons between the price levels at any two dates covered by the investigation, whereas with arithmetic means comparisons not made in terms of prices at the original base period give results that may present formal inconsistencies and results whose meaning is difficult to grasp and put into words.

A third advantage of geometric means is that they are likely to be nearer the modes of the distributions which they represent than are arithmetic means. The importance of this point will be more generally appreciated as statisticians come to study the whole array of the price fluctuations with which they deal, instead of concentrating their attention merely upon averages.

The chief objection to geometric means in an index number intended for general use is that this form of average is unfamiliar and therefore more likely to be misinterpreted than arithmetic means. Further, geometric means do not have any direct bearing upon changes in the purchasing power of money as do arithmetic means and weighted aggregates of actual prices.[84] Finally, geometric means are somewhat more laborious to compute than arithmetic means or medians. Instead of adding the relative prices just as they stand and dividing the sums by their number, the computer must convert the relative prices of every commodity into their logarithms, add these logarithms, divide the sum by the number of logarithms, and look up the natural numbers corresponding to the quotients.[85] Statisticians are the more loath to incur the extra labor of this process since the special merits of the geometric mean are shared in part by certain

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[83] See the discussion of shifting bases, pp. 83–90.

[84] This point is more fully explained on pp. 76 and 77.

[85] If relative prices are not needed for any other purpose, it is quicker to compute the geometric mean from the logarithms of the successive actual prices and then to find the ratios between the results. But even that is a somewhat longer process than calculating relative prices, casting them up, and dividing by their number.

That geometric means can be computed either with or without the use of relative prices is readily shown.

Let $\left.\begin{matrix} p_o, & p_x \\ p'_o, & p'_x \\ p^n_o, & p^n_x \end{matrix}\right\}$ stand for the actual prices of n commodities in the two years o and x.

Then the relative prices of these articles in the year x on the basis of actual prices in the year o are

$$\frac{p_x}{p_o}, \frac{p'_x}{p'_o} \quad . \quad . \quad . \quad \frac{p^n_x}{p^n_o}$$

The geometric mean of these relatives is

$$\sqrt[n]{\left(\frac{p_x}{p_o}\right)\left(\frac{p'_x}{p'_o}\right) \cdot \cdot \cdot \left(\frac{p^n_x}{p^n_o}\right)}$$

But this expression is equal to

$$\frac{\sqrt[n]{(p_x)\ (p'_x)\ \cdot \cdot \cdot\ (p^n_x)}}{\sqrt[n]{(p_o)\ (p'_o)\ \cdot \cdot \cdot\ (p^x_o)}}$$

And the latter expression, of course, is the ratio between the geometric means of the actual prices in the two years.

other forms of index numbers. Like geometric means, aggregates of actual prices, or relatives made directly from them, can be shifted to any base desired without raising difficult problems of interpretation. Like geometric means, again, medians are not more affected by cases of exceptionally great advances in price than by cases of exceptionally great declines. Hence in practice most makers of index numbers who distrust arithmetic means abandon the practice of averaging relative priees or use medians instead of geometric means.

Medians, indeed, have several distinguished champions among theoretical statisticians.[86] It is generally claimed that of all averages medians are the easiest to compute, for a quick ordering of the data, followed by a counting of the items, takes the place of casting sums and dividing by the proper number. But in this day of adding machines the palm for ease of computation has shifted to the arithmetic mean and the aggregate of actual prices. More important is the fact demonstrated by Prof. Edgeworth that the median is safer than the arithmetic mean when, as in the case of index numbers, the items to be averaged are samples drawn from a larger field. For, according to the theory of probabilities, the probable error of the median can not in any case be much greater than that of the arithmetic mean and in other cases it may be very much less.[87]

But medians have their drawbacks. (1) They are not perfectly reversible; that is, they can not be shifted from one base to another by simple division without ambiguity. (2) Medians of different groups can not be combined, averaged, or otherwise manipulated with ease as can arithmetic means. For example, in making up its old form of index number the Bureau of Labor Statistics could add the sums used for making arithmetic means of the relative prices of farm products, foods, cloths and clothing, etc., and from the sum of these sums strike the grand average for all commodities. It could not handle medians in this convenient fashion; instead of combining the sums from the groups it would have to reexamine and rearrange the relative prices of those commodities which fell near the respective medians. Similarly, a reader who finds arithmetic means of two groups in different sources can compute the arithmetic mean of these means, provided the number of items in each group be stated, with no greater error than that arising from the dropping of fractions in the published data; but he can not approximate except in the vaguest way the median of two medians.[88] (3) When the number of items to be averaged is small, medians are erratic in their behavior. For in such groups there is often a considerable interval between the midmost relative price and the relative prices standing next above it and next below. No change in any of the items, large or small, can alter the position of the median unless it shifts an item from the

[86] Compare, for example, F. Y. Edgeworth, "Index numbers," Dictionary of Political Economy, Vol. II, p. 386; Irving Fisher, The Purchasing Power of Money, revised edition, p. 425; A. L. Bowley, Elements of Statistics, second edition, p. 224. Walsh, however, does not recognize the median as a mean. See Quarterly Publication of the American Statistical Association, March, 1921, p. 542, and the numerous references to medians in his Problem of Estimation.

[87] See his paper "On the use of analytical geometry to represent certain kinds of statistics," Journal of the Royal Statistical Society, June, 1914, Vol. LXXVII, p. 733.

[88] It is a convenient feature of arithmetic means computed from relatives based on average prices over a period of years that the mean of these means for the base period must be 100—again barring discrepancies caused by dropping fractions. For example, the arithmetic means of the Bureau of Labor Statistics old-style index numbers for the 10-year period 1890–1899 would always add up to 1,000.0, had all the fractions been kept and had all commodities been quoted in every year of the decade. If medians made from these figures add up to 1,000.0 in 1890–1899, it is accidental.

upper half of the list to the lower half, or vice versa. But any change of this character, large or small, will make the median jump over the whole interval between its former position and that of the next highest or next lowest relative price, unless the change happens to place a new item within these limits. In large groups such erratic jumps are less likely to occur, because the intervals between the median and its nearest neighbors are usually slight.[89] (4) If the number of commodities included in an index number is even, the position of the median may be indeterminate, though within a determinate range.

Most of the advantages and defects of arithmetic means have been mentioned incidentally, but it is well to list them all together: (1) Arithmetic means (and aggregates of actual prices) stand first in ease of computation, when an adding machine is available, especially when the items are to be averaged first in small and later in large groups. (2) Their familiarity to all readers is supposed to be an advantage in work intended for wide reading though perhaps this familiarity means a dangerous lack of curiosity rather than clear understanding of the figures. (3) They can themselves be averaged and manipulated algebraically in various other ways.[90] On the other side of the score it must be said (4) that arithmetic means are liable to distortion from the occurrence of a few extremely high relative prices, (5) that arithmetic means of relative prices can not consistently be shifted from one base to another without recomputation in full,[91] and (6) that they may conceivably give contradictory results concerning the direction in which prices are moving, according as relative prices are computed on one base or on another.[92]

Concerning the numerical value of the three averages under discussion, it can be proved that the geometric mean is always less than the arithmetic. On the other hand, the median may be either above or below the arithmetic mean, and likewise either above or below the

[89] "This objection is met," says Prof. Edgeworth, "by denying that the interval between two adjacent observations at the middle of the group is likely to be 'considerable'; large relatively to the magnitude with which it is proper to compare that interval—that is, the *minimum mensurable*, as we may say—that interval which is equal to (or of the same order as) the smallest degree which the compared method of measurement is capable of distinguishing with accuracy. For this minimum we may take at the least the 'probable error' incident to the arithmetic mean. That the interval between adjacent observations is likely to be small compared with this minimum is sufficiently evidenced by the following proposition: When the number of observations (n) is large the interval at the middle of the group, which is as likely as not vacant, within which it is an even chance that no observation falls, is most probably very small compared with the probable range of the arithmetic mean (in the ratio of about $1: \sqrt{n}$). When the number of observation is not large the proposition is less accurate. But it remains roughly true, as the number can not be supposed very small consistent with the applicability of the theory of probabilities." Economic Journal, June, 1918, Vol. XXVIII, p. 193.

Granting the justness of these general remarks, the writer has found several cases in his own work where the medians of groups numbering 25 or more items moved in a way not representative of the whole array. For examples see "A critique of index numbers of the prices of stocks," Journal of Political Economy, July, 1916, Vol. XXIV, pp. 674, 675. It may, indeed, be set down as an advantage of medians that working with them may bring the full array of fluctuations under the eye and lead to the detection of peculiarities which would have escaped notice had arithmetic means been employed. When medians are used in averaging small groups the practice of scrutinizing the whole set of data is strongly recommended as a means of guarding against the occasional cases of erratic movement.

[90] See, for example, G. U. Yale, Introduction to the Theory of Statistics, pp. 114–116.

[91] See section 8 below.

[92] Take, for example, the following data:

	1913	1914
Wheat, per bushel	$0.50	$1.00
Corn, per bushel	.48	.24

geometric mean. For example, if the relative prices of the 145 commodities represented in the second index number of Table 6 be averaged in these three ways, the results are as follows for 1913:

Geometric mean, 125.7; median, 126.9; arithmetic mean, 131.3.

A more striking example of differences among the averages was incidentally remarked above. The relative prices of Table 15 yield the following figures:

Geometric mean, 194; median, 191; arithmetic mean, 217.

A fuller study of the relations between medians and arithmetic means is provided for by the following table.[93] In the chain index [94] the two forms of average never quite coincide; the median is smaller in 20 cases and larger in 3; it is also steadier than the arithmetic mean in the sense that it indicates an average annual change of 2.22 per cent from prices in the preceding year, as against 3.64 per cent for the arithmetic mean. In the fixed-base series for 1890–1913, including 145 commodities, the median is likewise steadier than the arithmetic mean, showing a smaller percentage of change, except during the middle nineties, when the price level was at its lowest. The second series for these years illustrates the behavior of medians and arithmetic means when used to average small groups. Here the median is greater than the arithmetic mean in 13 years, the same in 1 year, and less in 10 years. Moreover, it shows a greater average change from one year to the next than the arithmetic mean. Finally, the median drops 9 points in 1913 while the arithmetic mean rises 2 points. Scrutiny of the full array of relative prices in this year as compared with 1912 shows that this violent drop is not an apt

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Then compute index numbers on the basis 1913=100:

	1913	1914
Wheat, relative prices	100	200
Corn, relative prices	100	50
	200	250
Index numbers	100	125

Also, compute index numbers on the basis 1914=100:

	1913	1914
Wheat, relative prices	50	100
Corn, relative prices	200	100
	250	200
Index numbers	125	100

Thus it appears that prices were 25 per cent higher in 1913 than in 1914 and also that they were 25 per cent higher in 1914 than in 1913.

Much stress is often laid upon illustrations of this sort, but they are not seriously damaging to the good repute of arithmetic means when properly interpreted. What they really say is: The arithmetic mean variation of prices from 1913 to 1914 may conceivably be upward in percentages of prices in 1913, and at the same time be downward in percentages of prices in 1914. No real inconsistency is involved in that statement to one who can keep the meanings of the two results in mind. It should be added that cases in which such apparent inconsistency occurs, while common in theoretical discussions, seldom if ever occur in the practical computation of wholesale-price index numbers. In retail-price indexes they are not unknown. An example has been pointed out in the British Board of Trade's reports upon cost of living of the working classes. See the reviews by J. M. Keynes in the Economic Journal, September and December, 1908.

[93] For numerical examples of geometric and arithmetic means computed from the same data, see F. Y. Edgeworth, "A defense of index numbers," Economic Journal, Vol. VI (1896), p. 137, and A. W. Flux, "Modes of constructing index numbers," Quarterly Journal of Economics, Vol. XXI (1907), p. 627.

[94] On the character of chain indexes, see the following section (pp. 81 to 91).

summary of the combined movements.[95] The figures for prices during the period of irredeemable paper money (1862–1878, inclusive) show how far arithmetic means may depart from the medians when a few commodities attain very high relative prices. The maximum difference occurs in July, 1864, when the arithmetic mean exceeds the median by 42 points, or more than 20 per cent. This excessive difference is due to the high prices of cotton, tar, and other southern products. It is precisely in cases such as this that the median is distinctly safer to trust than the arithmetic mean.

TABLE 16.—COMPARISONS OF MEDIANS AND ARITHMETIC MEANS AS AVERAGES OF RELATIVE PRICES.

[Data from Bulletin No. 149 of the Bureau of Labor Statistics.]

Year.	Chain index number (prices in preceding year=100).[a]		Relative prices of 145 commodities (average prices in 1890–1899=100) [b]		Relative prices of 25 commodities (average prices in 1890–1899=100).[c]	
	Medians.	Arithmetic means.	Medians.	Arithmetic means.	Medians.	Arithmetic means.
1890			112	114	116	115
1891	±0	− 0.2	111	113	109	112
1892	−3.1	− 4.4	107	106	106	103
1893	±0	− .2	104	105	102	103
1894	−7.1	− 8.7	96	96	90	92
1895	−2.4	− 1.5	94	93	94	95
1896	−1.2	− 2.8	90	89	89	88
1897	±0	+ .2	91	89	92	90
1898	+1.8	+ 4.8	94	93	99	96
1899	+5.5	+10.4	100	103	108	107
1900	+7.5	+ 9.4	109	111	117	113
1901	−1.5	− 1.1	107	110	112	111
1902	+2.2	+ 4.6	110	114	115	116
1903	+1.3	+ 1.2	111	114	112	118
1904	±0	− .1	112	114	124	122
1905	+ .7	+ 2.9	114	116	126	123
1906	+5.1	+ 5.8	119	122	131	130
1907	+3.9	+ 6.0	129	130	133	133
1908	−3.8	− 5.6	119	121	125	124
1909	±0	+ 3.2	121	124	130	133
1910	+1.5	+ 4.1	124	131	126	133
1911	− .9	− 1.9	125	130	131	129
1912	+1.0	+ 3.4	127	134	136	140
1913	+ .5	+ 1.2	127	131	127	142
Averages, 1890–1899			100	100	101	100
1900–1909			115	118	123	122
1910–1913			126	132	130	136
Average change from one year to the next	2.22	3.64	3.61	4.13	5.70	5.09

[a] Compare Tables 2 and 17. [b] Compare Table 6, second series. [c] Compare Table 6, fifth series.

[95] Of the 25 commodities 13 rose in price and 12 fell; the median percentage of change from prices in the year before is +1.0.

TABLE 16.—COMPARISONS OF MEDIANS AND ARITHMETIC MEANS AS AVERAGES OF RELATIVE PRICES—Concluded.

[From W. C. Mitchell, Gold Prices and Wages under the Greenback Standard, pp. 59, 60.]

92 commodities at wholesale (prices in 1860=100).

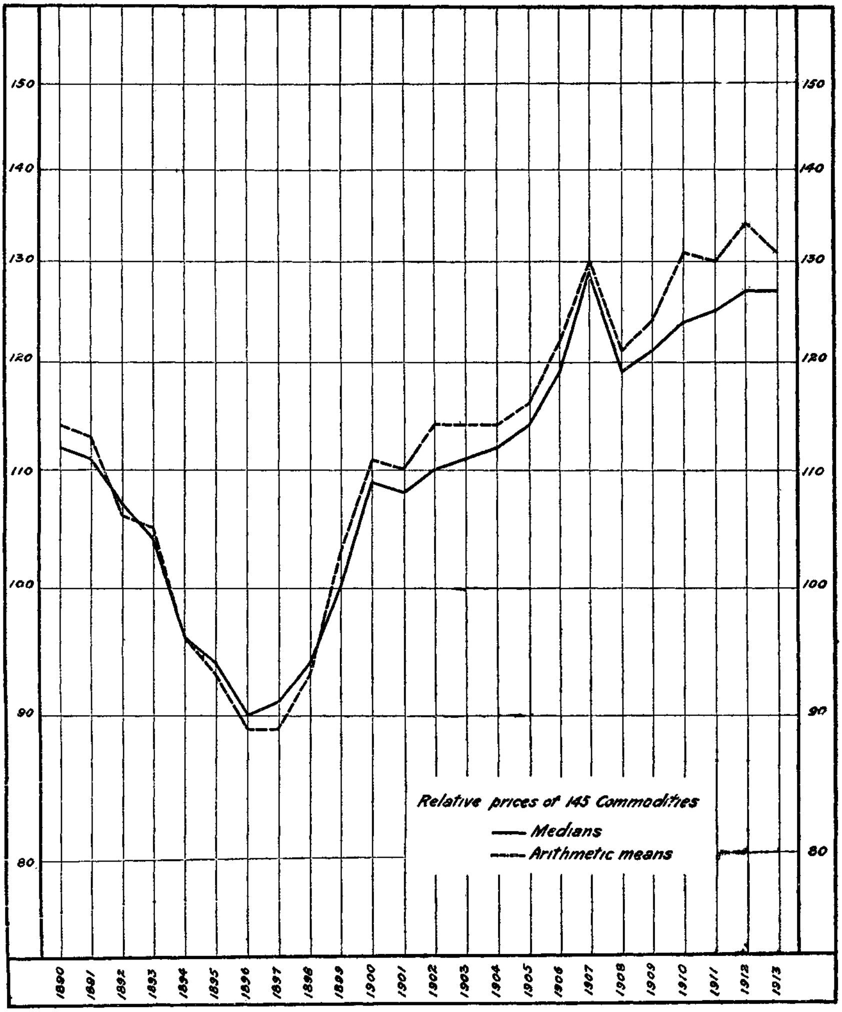
Year.	Medians.	Arithmetic means.	Year.	Medians.	Arithmetic means.	Year.	Medians.	Arithmetic means.
1860, January	100	102	1867, January	169	179	1874, January	130	140
April	100	102	April	166	175	April	129	141
July	100	100	July	150	170	July	130	138
October	100	102	October	162	172	October	130	138
1861, January	100	100	1868, January	158	171	1875, January	127	138
April	96	98	April	162	176	April	125	132
July	96	95	July	154	165	July	121	129
October	97	103	October	159	166	October	120	127
1862, January	100	115	1869, January	159	165	1876, January	117	122
April	100	112	April	159	165	April	115	122
July	100	120	July	158	158	July	110	118
October	111	126	October	153	157	October	108	117
1863, January	125	142	1870, January	147	152	1877, January	114	121
April	137	160	April	140	146	April	108	118
July	134	155	July	132	145	July	100	114
October	135	155	October	135	143	October	102	110
1864, January	156	179	1871, January	133	142	1878, January	99	107
April	169	197	April	131	140	April	96	105
July	194	236	July	130	137	July	90	99
October	200	239	October	129	139	October	94	102
1865, January	216	248	1872, January	133	141	1879, January	88	100
April	190	206	April	140	145	April	84	99
July	158	183	July	130	139	July	85	98
October	175	205	October	133	143	October	95	103
1866, January	182	199	1873, January	135	142	1880, January	108	114
April	173	186	April	137	144	April	107	116
July	181	191	July	130	140	July	102	110
October	173	188	October	131	140	October	101	111

Average change from one quarter to the next: Medians, 5.66 points; arithmetic means, 5.65 points.

Wise choice of the average to use in making an index number, then, involves careful consideration of the materials to be dealt with and of the purpose in view. (1) If that purpose be to measure the *average ratio of change* in prices, the geometric mean is the best; indeed, in strictness, it is the only proper average to employ—on one interpretation of that somewhat indefinite problem. For, alone among our averages, the geometric mean always allows equal influence to equal ratios of change in price, quite irrespective of the previous levels of the prices in question, the amounts of money represented by the changes themselves, or any other factor. As has been said already, in a geometric mean the doubling of one price is precisely offset by the halving of another price—though if the two prices were originally the same the rise amounts in money to twice the fall. And further changes of 10 per cent from the two new prices will again be precisely equal in their influence upon a geometric mean, although 10 per cent of the price that has doubled represents a sum of money four times as great as 10 per cent of the price that has been halved. (2) But these same examples show that geometric means are not proper averages for measuring alterations in the amount of money that a given bill of goods costs. And as a rule our interest does center in the money cost of goods rather than in the average ratio of changes in price. For example, when we are investigating the increased cost of living, the doubling of one item in the family budget may well be twice as important as its halving; and when we are studying the "relation of prices to the currency, a

CHART 11.—A COMPARISON OF MEDIANS AND ARITHMETIC MEANS OF THE RELATIVE PRICES OF 145 COMMODITIES.

(Based on Table 16.)



large upward variation should count for more than a small downward variation, for it requires more currency;"[96] provided always that the changes in prices are not offset or more than offset by contrary changes in quantities bought. For such purposes the arithmetic mean is the logical average to use. (3) Frequently, however, the very fact that an article has advanced greatly in price cuts down its market, so that the increase in money cost represented by the arithmetic mean exists on paper rather than in fact.[97] When such cases of extreme advance are numerous among the relative prices to be averaged, the median may give more significant results than the arithmetic mean. (4) When the number of commodities included in the index number is small, however, medians may occasionally prove erratic, representing less the general trend of prices than the peculiarities of the data from which they are made. (5) If the index number is designed for the public at large, the familiarity of arithmetic means is an argument in their favor; but it counts for nothing in the case of figures intended for specialists. (6) Often the usefulness of a new index number may be enhanced without detriment to its special purpose by throwing it into a form directly comparable with that of index numbers already in existence. Then, of course, not only the form of average but also the base period employed in making the existing series has special claims for imitation. (7) Finally, the desirability of making index numbers that can be shifted from one base to another without raising difficult problems of interpretation, deserves more consideration than is commonly accorded it. On this count the score is in favor of the geometric mean. If geometric means were invariably used, all index numbers could readily be compared with one another, whatever the bases on which they were originally computed. And that would be a great gain to all students of prices.

No single form of average made from relative prices, then, is without its merits and its defects. Can we not escape the necessity of relying upon any one of them by giving up the use of relative prices and falling back upon aggregates of actual prices?

Index numbers made on this latter plan practically compel the compiler to adopt a systematic scheme of weighting. This should constitute a great safeguard against crude work, though in view of Bradstreet's method of weighting one can not claim that it always is effective. For the haphazard weighting involved in merely adding up the raw quotations for different commodities in terms of their ordinary commercial units is far more dangerous than the haphazard weighting involved in using the same materials after reduction to relative prices.[98] It is true that sums in dollars and cents are likely to run in amounts awkward for comparison; but these sums can quickly be turned into a series of relatives on the scale of 100. The same device

[96] Irving Fisher, The Purchasing Power of Money, revised edition, p. 426, note 2. Mr. Flux and Mr. Yule hold that to measure changes "in the money cost of the things we buy" is "the retail-prices problem," and is not the appropriate aim of a wholesale-price index; but they do not consider the arguments which Prof. Fisher advances. Journal of the Royal Statistical Society, March, 1921, pp. 175–9, and 200, 201.

[97] Such cases might be met by reducing the weight allowed the article in question; but we have seen that revising weights blurs the meaning of the index number, by making it impossible to say how far the finat results measure the change in prices and how far they measure the change in weights. (See p. 65.)

[98] See the example from Hunt's Merchant's Magazine given on pp. 31 and 32. However, a very rough system of weights based upon guesswork may give quite as good results as the haphazard weighting of relative prices. Prof. Irving Fisher suggests to the writer a "lazy man's index number," made by adding actual prices for ordinary commercial units, with their decimal points shifted forward or backward, or left unchanged, according to the estimated importance of each article.

meets the objection that the introduction of new commodities, necessary at intervals in any large index number that is kept up to date, disturbs a sum of actual prices more than it disturbs an average of relative prices. This statement is valid because the quotations for new commodities, however adjusted, are just so much added to the old sum; while the relative prices of new commodities may be either above or below the old average, and often exercise but a trifling net effect upon its value. But by noting the ratios between the sums of actual money which include and which exclude the new commodities, and by using these ratios to adjust successive aggregates, the compiler meets this difficulty quite as well as if he were averaging relatives from the start.

The technical difficulties attending the construction of index numbers made of actual prices, then, can be surmounted. Offsetting these difficulties are numerous and substantial advantages. Aggregates of money prices weighted according to the importance of the several articles are even easier to understand than arithmetic means of relative prices. They are less laborious to compute than any other form of weighted series, for no relative prices are used; the original quotations are multiplied directly by the physical quantities used as weights, and the products added together. They are not tied to a single base period; but from them relative prices can quickly be made upon the chain system or any fixed base that is desired, and these relative prices themselves can be shifted about at will as readily as geometric means.[99] Hence they are capable of giving direct comparisons between prices on any two dates in which an investigator happens to be interested. Hence, also, they can be compared with any index numbers covering the same years, on whatever base the latter are computed. They can not be made to give apparently inconsistent results like arithmetic means. When published as sums of money, they can be added, subtracted, multiplied, divided, or averaged in any way that is convenient. When comprehensive in scope and weighted on a sound system, they are not likely to be unduly distorted by a very great advance in the price of a few articles, and yet, unlike medians, they allow every change in the price of every article

[99] The legitimacy of shifting these relatives by the "short" method is best shown by the use of symbols.

Let $\left.\begin{matrix} p_o, p_x, p_y, \\ p'_o, p'_x, p'_y \end{matrix}\right\}$ represent the money prices of the two commodities p and p' in three years o, x, and y.

Then the sums of these actual prices will be—

$p_o+p'_o$ in the year o.

$p_x+p'_x$ in the year x.

$p_y+p'_y$ in the year y.

Relative prices in the year x computed from these sums will be—

$\frac{p_x+p'_x}{p_o+p'_o}$ on the basis of prices in the year o, and

$\frac{p_x+p'_x}{p_y+p'_y}$ on the basis of prices in the year y.

Relative prices in the year y will be—.

$\frac{p_y+p'_y}{p_o+p'_o}$ on the basis of prices in the year o.

Now the relative price in the year x, computed on the basis of prices in the year o, can be turned into the relative price for the year x on the basis of prices in the year y, by dividing the relative for the year x on the basis of prices in the year o by the relative for the year y on the basis of prices in the year o. For

$$\frac{p_x+p'_x}{p_o+p'_o} \div \frac{p_y+p'_y}{p_o+p'_o} = \frac{p_x+p'_x}{p_y+p'_y}$$

The reason why ordinary arithmetic means of relative prices can not be consistently shifted to another base by this simple method is explained on p. 83.

to influence the result. In fact, they combine most of the merits and few of the defects characteristic of the various methods of averaging relative prices.

But the main issue has still to be faced. Do we wish to know how certain sample prices have changed on the average, or do we wish to know how the total cost of a sample bill of goods has changed? This is practically the same question we considered on pages 76 to 78 in discussing how best to average relative prices. And the answer given there is valid here. If our interest really lies in measuring average ratios of change, then geometric means are best. But (1) the unfamiliarity of this average outside technical circles is itself an objection to measuring average changes in an index number designed for wide use, and (2) a measure of change in the money cost of goods probably serves more uses than a measure of average ratios of change in prices. Now, the weighted aggregate of prices is the best measure of change in the money cost of goods; it is better in several ways than the simple arithmetic mean of relative prices, and in addition it has all the merits of the latter form of average. For the relatives which can be computed from these aggregates with little trouble are identical with arithmetic means of relative prices, when the latter are weighted by the money value of the physical quantities used as weights for the corresponding actual prices.

This identity of the variations of a weighted aggregate of actual prices and the arithmetic-mean variations of similarly weighted relative prices can readily be demonstrated. Suppose that we have collected the price quotations and the quantities to be used as weights in an index number, and have decided what period to make the base for comparisons. Then if we want an aggregate of actual prices, we merely multiply the quotations of each commodity at each date by the physical quantities used as weights, and add these products. To measure the variations of these aggregates in terms of prices at the base period, we have only to divide the aggregate for each period by the aggregate for the base period. But if we plan to make a weighted arithmetic mean of price variations, we begin by turning the quotations into relative prices. That is, we divide the actual price of each commodity at each date by its price in the base period. Then we weight these relatives, not by physical quantities as in the first case, but by the money values of the physical quantities at the prices of the base year. But in this step the prices of the base year, which were just used as divisors to get relative prices, are used again as factors by which the relative prices are multiplied. Hence our results are the same as if we had neither multiplied nor divided by the prices of the base year; in other words, the same as if we had multiplied the quotations of each commodity in each year by the physical quantities used as weights. But that is just what we did when we set out to make an aggregate of actual prices. So far, then, the two processes are identical in their outcome. And the remaining steps are also the same. The products must be added, and the sums divided by the physical quantities used as weights times the actual prices of the base year. Therefore, to make relative prices from aggregates of actual prices is a shorter way of getting the same results as are obtained by making similarly weighted arithmetic means of relative prices.[1]

But while an arithmetic mean of relative prices is always equivalent to some aggregate of actual prices turned into relatives, this fact does not mean that the arithmetic mean of relatives is as desirable a form of general-purpose index number as its rival. For the particular aggregate of actual prices to which a given arithmetic mean of relatives corresponds is one difficult to grasp. It is that aggregate in which the price of each commodity included, quoted in terms of its ordinary commercial unit, has been multiplied by the number of commercial units which is necessary to make its price in the base period some predetermined multiple of 100. Now this is a much more complicated idea to carry in mind and to make clear to readers than the idea of the price of the commodity multiplied by the number of units that are ordinarily produced, exchanged, or consumed. In other words, the arithmetic average of relatives has the same relation to its corresponding aggregate of actual prices that a complicated mathematical expression has to the same expression reduced to a simpler form. The difference is one of form, but simplicity of form greatly increases the efficiency of thinking.

8. BASE PERIODS, CHAIN INDEX NUMBERS, AND FIXED-BASE SERIES.

When relative prices are used it is necessary to select the quotations of some given period as a base. The actual prices in this base period are called 100; all antecedent and subsequent prices are divided by the base prices, and the quotients, multiplied by 100, make the relatives which are usually summed and divided by the number of commodities to get the final index number. When aggre-

[1] The explanation given in the text may be put in the form of algebraic formulæ for readers willing to study symbols.

Let $\left.\begin{matrix} p_o, p_x \\ p'_o, p'_x \\ p^n_o, p^n_x \end{matrix}\right\}$ represent the prices of the commodities from which an index number is to be made in the base year o and in some other year designated by the subscript x.

Let q, q' and q^n respectively represent the physical quantities of these commodities to be used as weights. Then an unweighted arithmetic mean of relative prices is represented by the following formula, in which n stands for the number of commodities included:

$$\frac{\frac{p_x}{p_o}+\frac{p'_x}{p'_o}+\ \cdot\ \cdot\ \cdot\ \frac{p^n_x}{p^n_o}}{n}$$

A weighted aggregate of prices reduced to relatives is represented by the following formula:

$$\frac{p_x\,q+p'_x\,q'+\ \cdot\ \cdot\ \cdot\ p^n_x\,q^n}{p_o\,q+p'_o\,q'+\ \cdot\ \cdot\ \cdot\ p^n_o\,q^n}$$

A weighted arithmetic mean of relative prices with money weights corresponding to the physical weights of the expression immediately above is represented by the following formula:

$$\frac{\frac{p_x}{p_o}\,(q\,p_o)+\frac{p'_x}{p'_o}\,(q'\,p'_o)+\ \cdot\ \cdot\ \cdot\ \frac{p^n_x}{p^n_o}\left(q_n\,p^n_o\right)}{p_o\,q+p'_o\,q'+\ \cdot\ \cdot\ \cdot\ \cdot\ p^n_o\,q_n}$$

But in the numerator of this fraction, p_o, p'_o, and p^n_o cancel out. Then formula (3) becomes identical with formula (2). That is, the weighted aggregate of prices gives the same results when turned into relative as the weighted arithmetic mean of relative prices, and gives them with less work.

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gates of actual prices are first made and then turned into relatives the problem of selecting a proper base period has to be faced at the end of the computation.

In some cases the prices of a single day have been used as the base, but as a rule average prices for a year, five years, a decade, or an even longer period have been preferred. For this preference there is a simple justification when arithmetic means are used as averages of the relative prices.[2] If the price of any commodity happens to be unusually high or unusually low in the base period, its relative prices at other periods will be correspondingly low or high, and the very high relative prices will exercise much more influence upon arithmetic means than the very low ones. If an appreciable proportion of the commodities in the list be very high or very low, the final index number may be distorted. Though numerically correct, the results have less significance than if they showed changes in terms of prices that men consider "normal."[3] Of course exceptionally high or exceptionally low quotations are less likely to last for a year than for a day, and less likely to last for a decade than for a year.

The period chosen as base for the relative prices should be that period with which accurate comparisons are most significant for the purpose in hand. Probably most users of general-purpose index numbers prefer to make their comparisons with recent dates. Hence the case for "chain" indexes is very strong—that is, for indexes like the medians of Table 2, which show the average rise or fall of prices on the basis of prices in the preceding year.[4] Hence, also, any index number with a fixed base becomes in one respect less significant the longer it is maintained. For example, when the Bureau of Labor Statistics series was established in 1902, the public was interested to know how much prices in that year had changed in terms of average prices in the decade 1890–1899. In 1918, however, when people cared less about knowing changes in terms of what prices had been 19 to 28 years earlier, the Bureau shifted its base to 1913. Similarly, Sauerbeck's index number, which uses prices in 1867–1877 as a base, suffers in significance for recent comparisons because it forces one to make all comparisons in terms of prices in a period that ended before most of the people now living were old enough to know the meaning of prices.

Index numbers made on a base many years in the past, moreover, encounter all the difficulties that inhere in the problem of measuring price variations through long periods of time. As was shown in Section III of this bulletin (pp. 11 to 23), price variations become dispersed over a wider range and less concentrated about their mean as the time covered by the variations increases. That is, the longer a fixed-base series is maintained, the more scattered as a rule become the relative prices. The difficulty is particularly serious when arithmetic means are used. The commodities that have a con-

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[2] If geometric means are used the ratios between the index numbers for different dates are not influenced at all by the selection of the base, and if medians are used they are likely to be affected but slightly, provided the number of commodities included be large.

[3] The selection of a proper base period, however, does not guarantee immunity from the exercise of undue influence by certain articles. More important than the base is the choice of proper weights. Or, to speak with more precision, the choice of base is itself part of the problem of weighting in its inclusive sense.

[4] This form of index number was invented by Prof. Alfred Marshall. See Contemporary Review, March, 1887.

sistent long-period trend gradually climb far above or fall far below the average relative price. Then the high relative prices of the commodities that have risen exercise a much stronger pull upon the position of the arithmetic mean than do the low relative prices of the commodities that have fallen. For most purposes this constitutes a defect, since commodities that have increased greatly in price are likely to have become scarce, and commodities that have become cheaper are likely to be more abundant. The changes in the influence exercised on the mean by the relative prices are likely to be in inverse ratio to the changes in the importance of the commodities. In other words, the use of the distant base itself introduces a surreptitious set of weights into the figures to be averaged, and a set which may well counteract in large measure the formal set of weights which the investigator uses to show the importance of his articles.

It is not uncommon, of course, to shift fixed-based index numbers from a remote to a recent base. For example, Sauerbeck's index as continued by the Statist was 85 in 1913 on the 1867–1877 base. If one wishes to find how much English prices rose in 1914–1918 as compared with their prewar level, he may put 85 = 100, and recast the indexes for the years of war on that scale. But this is a purely formal manipulation of the results. It does not diminish the scattering of the relative prices from which the averages are computed, and it does not give the same result that recomputing the relative prices of the 45 commodities on the 1913 base and averaging them afresh would give. The first point is obvious; the second requires explanation.

Averages of relative prices on a given base may be regarded as averages of actual prices made with a peculiar scheme of haphazard weights. That is, the quotation of every commodity is in effect multiplied by the factor necessary to make its price in the base period equal 100.[5] To change the base is of course to change this set of implicit haphazard weights for another set, which may be better or worse—the computer is unlikely to know which—but which will be different unless the ratio of change in prices between the old and new base periods has been precisely identical for all the commodities included. Of course, different sets of weights applied to the same set of price quotations will probably alter the average variations somewhat. Hence, if one really wants to know how a given set of prices have varied with reference to their standing at any given time, the only way to find out accurately is to weight the variations of each commodity by the factors which the chosen base determines; that is, in practice, to compute new relative prices article by article. But if the purpose in hand is such that one set of haphazard weights will serve as well as another, then there is no objection to shifting the base by the short method of manipulating merely the averages, provided the results are properly explained.

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[5] Compare F. R. Macaulay, "Index numbers for retail prices," American Economic Review, December, 1915, Vol. V, pp. 928, 929.

It is easy to arrange examples in which wide discrepancies appear between the results of the two methods of shifting the base.[6] But the difficult and the important thing is to find out how serious the discrepancies are in actual practice. For to use index numbers effectively, it is often necessary to shift the base, and sometimes the short method must be followed, either because recomputation in full requires a prohibitive amount of labor, or because the original data necessary for recomputation have not been published. The next table gives three pertinent examples. In the first case when Sauerbeck's index is shifted from 1867–1877 = 100 to 1890–1899 = 100 the discrepancies are fairly regular and rather small both absolutely and relatively. In the last case, when the same series is shifted to 1860 = 100, the discrepancies are highly irregular from year to year, and are rather large both absolutely and relatively—several times exceeding 5 per cent of the recomputed figures. In the remaining case the discrepancies are small absolutely, though often large relatively to the recomputed figures, and also highly variable from year to year.[7] The conclusion which these experiments suggest is that the two methods almost always give different results; that the discrepancies are by no means constant from year to year in a given case, and that their magnitude both absolutely and relatively differs much from one case to another. Hence it is well to avoid the short method of

[6] For example, suppose that an index number includes only wheat and corn, and that their prices are as follows:

	1913	1914
Wheat, per bushel...	$1.00	$0.50
Corn, per bushel.....	.40	.40

If 1913 be made the base, the relative prices and index numbers will be:

	1913	1914
Wheat, relative prices.	100	50
Corn, relative prices..	100	100
Sums...........	200	150
Index numbers......	100	75

If now the base be shifted from 1913 to 1914 by the short method, the index number for 1913 will be (100÷75) 100=133⅓. But if the figures be recomputed on the basis of prices in 1914, the result is an index number of 150 in 1913:

	1913	1914
Wheat, relative prices.	200	100
Corn, relative prices..	100	100
Sums...........	300	200
Index numbers......	150	100

[7] The discrepancies shown in the table do not result wholly from the mathematical inconsistency of the short method, but partly from the fact that when an index number is shifted to a new base by recomputation in full it is commonly impossible or undesirable to utilize all the original data. Some commodity, for example, may not be quoted for the dates used as the new base, and therefore has either to be dropped or introduced at a later date by means of some doubtful assumption as to what its price would have been had it been quoted for the full period. Of course this observation makes the objection to using the short method stronger rather than weaker. It means that this method often leads the statistician into uses of the original data which he would have avoided had he undertaken the recomputation of the index number.

shifting bases whenever possible; and when that method must be used, its results should not be treated as showing what the index number would have been had it been made originally on the new base.

TABLE 17.—EXAMPLES OF DISCREPANCIES BETWEEN THE RESULTS OF TWO METHODS OF SHIFTING THE BASES ON WHICH INDEX NUMBERS ARE COMPUTED.

(Arithmetic means.)

Year.	Sauerbeck's index number, 1890–1913.				Bureau of Labor Statistics index number (old series).				Year.	Sauerbeck's index number, 1860–1891.			
	Original form, 1867–1877=100.	Shifted to 1890–1899=100, by short method.	Recomputed on basis 1890–1899=100, by long method.	Discrepancies.	Bureau's series on basis 1890–1899=100.	Chain index made by short method.	Chain index made by long method.	Discrepancies.		Original form, 1867–1877=100.	Shifted to 1860=100, by short method.	Recomputed on basis 1860=100.	Discrepancies.
1890..	72	109	109	....	112.9	....	....	....	1860.	99	100.0	100.0	....
1891..	72	109	109	....	111.7	−1.1	− 0.2	0.9	1861.	98	99.0	99.6	0.6
1892..	68	103	103	....	106.1	−5.0	− 4.4	.6	1862.	101	102.0	105.5	3.5
1893..	63	103	103	....	105.6	− .5	− .2	.3	1863.	103	104.0	109.3	5.3
1894..	63	95	95	....	96.1	−9.0	− 8.7	.3	1864.	105	106.1	112.3	6.2
1895..	62	94	94	....	93.6	−2.6	− 1.5	1.1	1865.	101	102.0	105.8	3.8
1896..	61	92	92	....	90.4	−3.4	− 2.8	.6	1866.	102	103.0	106.5	3.5
1897..	62	94	93	1	89.7	− .8	+ .2	.4	1867.	100	101.0	103.9	2.9
1898..	64	97	97	....	93.4	+4.1	+ 4.8	.7	1868.	99	100.0	103.1	3.1
1899..	68	103	104	1	101.7	+8.9	+10.4	1.5	1869.	98	101.0	101.9	2.9
1900..	75	114	115	1	110.5	+8.7	+ 9.4	.7	1870.	96	99.0	100.3	3.3
1901..	70	106	107	1	108.5	−1.8	− 1.1	.7	1871.	100	97.0	102.6	1.6
1902..	69	105	106	1	112.9	+4.1	+ 4.6	.5	1872.	109	101.0	112.5	2.4
1903..	69	105	106	1	113.6	+ .6	+ 1.2	.6	1873.	111	112.1	116.6	4.5
1904..	70	106	108	2	113.0	− .5	− .1	.4	1874.	102	103.0	107.0	4.0
1905..	72	109	111	2	115.9	+2.6	+ 2.9	.3	1875.	96	97.0	100.3	3.3
1906..	77	117	119	2	122.5	+5.7	+ 5.8	.1	1876.	95	96.0	97.5	1.5
1907..	80	121	123	2	129.5	+5.7	+ 6.0	.3	1877.	94	95.0	97.4	2.4
1908..	73	111	112	1	122.8	−5.2	− 5.6	.4	1878.	87	87.9	91.2	3.3
1909..	74	112	114	2	126.5	+3.0	+ 3.2	.2	1879.	83	83.8	86.7	2.9
1910..	78	118	120	2	131.0	+4.0	+ 4.1	.1	1880.	88	88.9	91.8	2.9
1911..	80	121	123	2	129.2	−1.8	− 1.9	.1	1881.	85	85.9	88.5	2.6
1912..	85	129	130	1	133.6	+3.4	+ 3.4	....	1882.	84	84.9	88.0	3.1
1913..	85	129	130	1	135.2	+1.2	+ 1.2	....	1883.	82	82.8	86.0	3.2
....	....	....	....	....	....	....	....	....	1884.	76	76.8	79.3	2.5
....	....	....	....	....	....	....	....	....	1885.	72	72.7	75.4	2.7
....	....	....	....	....	....	....	....	....	1886.	69	69.7	72.4	2.7
....	....	....	....	....	....	....	....	....	1887.	68	68.7	70.7	2.0
....	....	....	....	....	....	....	....	....	1888.	70	70.7	73.9	3.2
....	....	....	....	....	....	....	....	....	1889.	72	72.7	76.7	4.0
....	....	....	....	....	....	....	....	....	1890.	72	72.7	76.0	3.3
....	....	....	....	....	....	....	....	....	1891.	72	72.7	75.4	2.7

Chain index numbers on the base, prices in the preceding year = 100, have the advantage pointed out in Section III, that the variations which they represent are highly concentrated and therefore apt for averaging. That is, year-to-year variations are relatively easy to measure with approximate accuracy. It is true that makers of index numbers find chain relatives more troublesome to compute than fixed-base series, since most of the prices used as divisors change every year; but that fact weighs lightly with such laborious folk in comparison with an improvement in their results. Why, then, should they not make successive averages of year-to-year variations covering as long a period as desired and weld the successive links together by multiplication to form a continuous chain?

For example, in Table 17 it is shown that the old Bureau of Labor Statistics index in 1890 on the 1890–1899 base was 112.9 and that prices fell 0.2 per cent in 1891. On multiplying, we get 112.9×0.998

=112.7. In 1892 the average change of prices was a fall of 4.4 per cent. $112.7 \times 0.956 = 107.7$. Once more, in 1893 prices fell 0.2 per cent on the average. Adding this new link to the chain, we have $107.7 \times 0.998 = 107.5$. The next table shows this process carried through to 1913. The result is a new index number covering 24 years, in which each successive step is taken by averaging relatives which are probably better fitted for averaging, since they are more highly concentrated, than the corresponding relatives on the 1890–1899 base. Is it not better than the old index on the fixed base?

One may answer, first, that while each successive step in the chain index may be taken with confidence, any errors which do inhere in the steps are likely to accummulate. There is no magic in the year-by-year computation which makes the final comparison between prices in 1913 and 1890 more reliable on the one basis than on the other. Second, the interpretation of the final result is certainly simpler in the case of the fixed-base than in the case of the chain index. The figures say in the first case that between 1890 and 1913 there was an average net increase of prices equal to 22.3 per cent of average prices in 1890–1899. The chain index says that there was an increase between these two years of 37.1 per cent; but when one asks, "Per cent of what?" the answer is complicated. Third, the chain index which was begun arbitrarily on a par with the fixed-base series drifts away from it upward, and by the end of the period has opened a gap of nearly 15 points, or more than 11 per cent—a notable discrepancy. Stated in another way, the chain series makes the percentage increase in prices from 1890 to 1913 more than half again as great as the fixed-base series makes it.

TABLE 18.—A FIXED-BASE INDEX NUMBER, A CHAIN INDEX NUMBER MADE FROM THE SAME DATA, AND THE CHAIN INDEX MADE INTO A CONTINUOUS SERIES.

[Data from Bulletin No. 149 of Bureau of Labor Statistics.]

(Arithmetic means.)

Year.	Bureau's index number on basis prices in 1890–1899=100.	Chain index number, on basis prices in preceding year=100.	Chain index number made into a continuous series.	Year.	Bureau's index number on basis prices in 1890–1899=100.	Chain index number, on basis prices in preceding year=100.	Chain index number made into a continuous series.
1890	112.9	....	112.9	1902	112.9	104.6	123.4
1891	111.7	99.8	112.7	1903	113.6	101.2	124.9
1892	106.1	95.6	107.7	1904	113.0	99.9	124.8
1893	105.6	99.8	107.5	1905	115.9	102.9	128.4
1894	96.1	91.3	98.2	1906	122.5	105.8	135.9
1895	93.6	98.5	96.7	1907	129.5	106.0	144.1
1896	90.4	97.2	94.0	1908	122.8	94.4	136.0
1897	89.7	100.2	94.2	1909	126.5	103.2	140.3
1898	93.4	104.8	98.7	1910	131.6	104.1	146.1
1899	101.7	110.4	109.0	1911	129.2	98.1	143.3
1900	110.5	109.4	119.3	1912	133.6	103.4	148.2
1901	108.5	98.9	118.0	1913	135.2	101.2	150.0

Why should the annual shifting of the base on which relatives are computed make such a difference in the results? On looking at the figures in Table 17 from which the continuous chain in Table 18 is forged, we see that when prices are falling the percentage of change on the preceding-year base is generally smaller than the corresponding change on the fixed base. On the contrary, when prices are rising the preceding year base gives the larger percentage of change. In two years the percentages are the same (1912 and 1913), and in two other years the rule is reversed (1908 and 1911); but the rule holds in 19 cases out of 23.[8] The problem is to account for the fact that chain relatives usually rise more than fixed-base relatives when prices are rising and fall less when prices are falling.

The following numerical examples give the clue to the solution. We have in the first two columns of each example two relatives on a fixed base, for two successive years. First the larger of the two relatives is made to increase 25 per cent in the second year, and then to fall 25 per cent in the second year, the smaller relative remaining constant. Afterwards the smaller of the two relatives is made to rise and then to fall by 25 per cent in the second year, the larger relative being constant. In the third column the figures for the second year are turned into chain relatives. Index numbers are computed for both sets of relatives and the percentages of change on the two bases are given.

1. When a relative above the average of the relatives rises, its rise makes a smaller percentage addition to the chain than to the fixed-base index.

Fixed base.		Preceding-year base—
First year.	Second year.	Second year.
240 160 2)400 200	300 160 2)460 230	125 100 2)225 112.5
Per cent of change..... +15		Per cent of change.. +12.5

2. When a relative above the average of the relatives falls, its fall makes a smaller percentage subtraction from the chain than from the fixed-base index.

Fixed base.		Preceding-year base—
First year.	Second year.	Second year.
240 160 2)400 200	180 160 2)340 170	75 100 2)175 87.5
Per cent of change..... −15		Per cent of change...−12.5

[8] The fact was pointed out and the explanation of it suggested by Professor F. R. Macaulay, in American Economic Review, March, 1916, Vol. VI, pp. 207, 208.

3. When a relative below the average of the relatives rises, its rise makes a larger percentage addition to the chain than to the fixed-base index.

Fixed base.		Preceding-year base—
First year.	Second year.	Second year.
240 160 2)400 200	240 200 2)440 220	100 125 2)225 112.5
Per cent of change..... +10		Per cent of change.. +12.5

4. When a relative below the average of the relatives falls, its fall makes a larger percentage subtraction from the chain than from the fixed-base index.

Fixed base.		Preceding-year base—
First year.	Second year.	Second year.
240 160 2)400 200	240 120 2)360 180	100 75 2)175 87.5
Per cent of change..... −10		Per cent of change.. −12.5

All that these figures show is that in certain cases the fluctuations will be greater in the chain relatives and in other cases greater in the fixed-base relatives. The vital point is, however, that cases 2 and 3 occur in price quotations much more frequently than cases 1 and 4. Relative prices above the average seem more likely to fall than to rise further; relative prices below the average seem more likely to rise than to fall further. That is, the prices of individual commodities tend to conform to the average movement, and when they have already diverged from this average they move back toward it more often than they move away. These cases that occur more frequently than the others are those that make the chain relatives rise more (case 3) or fall less than the fixed-base relatives (case 2).[9]

The net difference to be expected on this ground in a large body of quotations between the movements of the relatives on the two bases

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[9] Of course this argument can be more generally, as well as more compactly, stated in algebraic terms. Prof. W. M. Ogburn contributes the following formulation:

Let $p'_1, p''_1, \ldots$ stand for relative prices of commodities during the first year, and $p'_2, p''_2, \ldots$ stand for relative prices of commodities during the second year. Let n be the number of commodities and m_1 the arithmetic mean of the relative prices during the first year.

The fixed-base index is obtained by getting the average of the relative prices; the fixed-base index for the first year is:

$$\frac{p_1'+p_1''+\ \cdot\ \cdot\ \cdot}{n}$$

And that for the second year is:

$$\frac{p_2'+p_2''+\ \cdot\ \cdot\ \cdot}{n}$$

is small in any one year. A glance at the figures in Table 18 will show that the observed differences are generally less than 1 per cent. But though small the differences are tolerably constant in direction, and therefore when cumulated by multiplication they become significant in 10 or 20 years.

The conclusion is that close agreement is not to be expected between efforts to measure the change of prices between years far apart when the measures are made first on a fixed base and then by the chain method. The chain method is perfectly legitimate, of course, when its results are carefully interpreted; but, as remarked above, the interpretation is difficult to put into words. Where means permit it is well to make from the original quotations two series of index numbers, one a chain index, the other a fixed-base series, and then to call attention to the differences between the two.

The per cent increase, or the rise, is the ratio of the second to the first, or

$$Rf=\frac{p_2'+p_2''+\ \cdot\ \cdot\ \cdot}{p_1'+p_1''+\ \cdot\ \cdot\ \cdot} \qquad (1)$$

Let the per cent increase from first year to second in the prices of individual commodities be r', r'' . . . then the relation between the prices during the first and second year can be expressed by the following equations:

$$p_2'=p_1'\ (1+r')$$

$$p_2''=p_1''\ (1+r''), \text{ etc.}$$

By substitution in (1):

$$Rf=\frac{p_1'(1+r')+p_1''(1+r'')+\ \cdot\ \cdot\ \cdot}{p_1'+p_1''+\ \cdot\ \cdot\ \cdot}$$

$$Rf=\frac{p_1'+p_1''+\ \cdot\ \cdot\ \cdot\ +p_1'r'+p_1''r''+\ \cdot\ \cdot\ \cdot}{p_1'+p_1''}$$

$$Rf=\frac{\Sigma p_1+\Sigma p_1r}{\Sigma p_1}=1+\frac{\Sigma p_1r}{\Sigma p_1}$$

Putting $p_1-m_1=x_1$ where x_1 is the size of the relative, we have:

$$Rf=1+\frac{\Sigma(m_1+x_1)r}{\Sigma p_1}$$

$$Rf=1+\frac{m_1\Sigma r}{\Sigma p_1}+\frac{\Sigma x_1r}{\Sigma p_1}$$

$$\Sigma p_1=nm_1$$

$$Rf=1+\frac{m_1\Sigma r}{nm_1}+\frac{\Sigma x_1r}{\Sigma p_1}=1+\frac{\Sigma r}{n}+\frac{\Sigma x_1r}{\Sigma p_1} \qquad (2)$$

The chain index is obtained by averaging the ratios of the individual prices of commodities and is expressed in the following manner:

$$Rc=\frac{\frac{p_2'}{p_1'}+\frac{p_2''}{p_2'}\ \cdot\ \cdot\ \cdot}{n}$$

$$Rc=\frac{\frac{p_1'(1+r')}{p_1'}+\frac{p_1''(1+r'')}{p_1''}+\ \cdot\ \cdot\ \cdot}{n}=\frac{(1+r')+(1+r'')+\ \cdot\ \cdot\ \cdot}{n}$$

$$Rc=\frac{n+\Sigma r}{n}=1+\frac{\Sigma r}{n} \qquad (3)$$

by subtracting (3) from (2):

$$Rf-Rc=\frac{\Sigma x_1r}{\Sigma p_1}$$

In other words the fixed-base index number will not equal the chain index number unless $\Sigma x_1r_1=o$ (which is true when r is constant). When Σx_1r_1 is negative the chain-index number will be larger and when positive the fixed-base index will be larger. Σx_1r_1 is positive when x (the size of the relative) is correlated (positively) with r (the percentage of increase), which is rarely if ever the case. The exact difference can be measured by $\frac{\Sigma x_1r_1}{\Sigma p_1}$.

Even this combination, however, is far from meeting all the needs of users of index numbers. For certain users may require for special purposes accurate measurements of price fluctuations in terms of the price level in any given month or year, or any given stretch of time in the whole period covered by the investigation. If such users are few as compared with all the people who note or quote the popular index numbers, they are precisely the few most interested in price fluctuations and most likely to increase knowledge by their use of the figures. But of course compilers can not foresee what base periods would serve best all these special purposes, and they can not be expected to work out index numbers on all the bases made possible by their original data. It is therefore highly desirable to have index numbers that can be shifted from one base to another readily and without involving difficulties of interpretation.

It is this desideratum, in large part, that has led to the recent reaction against index numbers made by striking arithmetic means of relative prices and in favor of index numbers made by adding actual prices. For the latter form of index, being a sum of dollars and cents, with an explicit scheme of weights, can be thrown into the form of a series of relative prices on any base that is desired, with slight labor and with no ambiguity. Geometric means, of course, possess the same advantage.

Another problem in base periods has recently been developed by Prof. Fisher. Should the period to which the weights refer be the same as the period used as the base for computing relative prices, or should the weights be taken from a different period? Suppose that the index number is to be an arithmetic mean of relative prices weighted by the values of the commodities exchanged in some year. Then "if the weights used are the values of the *base* year (that is, the base year for the relative prices) they impart a *downward bias* to all the index numbers of any given year calculated thereby, while, on the other hand, if the weights used are the values of the *given year* itself, they impart an *upward bias*."

To understand this effect one must note that the commodities which have unusually high market prices in the base year will tend to have both high values (prices multiplied by quantities) in that year and low relative prices in other years. Vice versa, the commodities which have unusually low market prices in the base year will tend to have both low values in that year and high relatives in other years. Then the multiplication of the low relatives by the high values and of the high relatives by the low values will tend to reduce the index numbers for all other years in comparison with the base year. Changing the weights from values in the base year to values in any other year will tend to reverse these combinations. For commodities that have unusually low market prices in the base year and therefore high relatives in other years will tend to have higher values in the latter years, and the commodities with high market prices in the base year and low relatives in other years will tend to have lower values in the latter years. The index number with "given-year" weights will therefore tend to combine high relatives heavily weighted and low relatives lightly weighted, and so give figures that run high for all other years in comparison with the base year.

How considerable this "biasing" of the results by the choice of the period to which the weights refer will prove in practice depends upon whether the prices and quantities of commodities usually fluctuate in the same or in opposite directions, for the influence of high and low prices on the values as weights may be offset, or more than offset, by contrary changes in the quantities. Little is positively known concerning the run of these facts. Prof. Fisher believes, however, that the quantity factor is almost as likely to influence the weights in one direction as in the other. If so, the price factor has a fair field to influence the values used as weights and the above argument holds good.

On this basis Prof. Fisher advises that in making arithmetic means of relative prices the weights be taken from the base year, in order that the downward bias of these weights may run counter to the upward bias of the arithmetic mean (caused by the greater influence exercised by high than by low relatives upon this form of average). Harmonic means, on the contrary, have a downward bias (are more influenced by low than by high relatives) and should therefore be weighted by values taken from some other year than the base. Geometric means, medians, and modes, which have no inherent bias, he holds, should be weighted by values both in the base and in the given year; for otherwise they will be affected by the bias of the weights.[10]

9. THE "IDEAL" FORMULA.

A more complicated formula for making index numbers than those heretofore discussed has recently been invented independently by three high authorities and recommended as the best for making general-purpose series. It may be written thus:

$$P_n = \sqrt{\frac{\Sigma p_n q_n}{\Sigma p_o q_n} \cdot \frac{\Sigma p_n q_o}{\Sigma p_o q_o}}$$

where Σ indicates "the sum of such terms as"

p_n = the price of any commodity in a given year (or period).
q_n = the quantity of that commodity in the given year.
p_o = the price of that commodity in the base year.
q_o = the quantity of that commodity in the base year.[11]

To use this formula it is necessary to have data concerning the prices and the quantities of every commodity in every year covered by the index number. From these data four sets of aggregates of actual prices multiplied by quantities are made for each year: (1) Prices in the given year times quantities in the given year, (2) the same prices times quantities in the base year, (3) prices in the base

[10] Irving Fisher: "The best form of index number." Quarterly Publication of the American Statistical Association, March, 1921, pp. 535, 536.

Prof. W. M. Persons has tested Prof. Fisher's contention that a geometric mean weighted by prices in the base year will have a downward bias. He finds that "Indices of quantity or of prices of agricultural products of the United States for the period 1880–1920 when measured relative to a fixed base (1910 in this case) show the same general movement whether the Fisher method or the geometric average is used . . . no cumulative divergence of the two indices is evident."—Review of Economic Statistics, May, 1921, p. 111.

[11] Mr. Walsh mentioned this formula in a footnote in his Measurements of General Exchange Value, 1901, but did not then exploit its merits. In 1912 Prof. A. C. Pigou published the same formula in Wealth and Welfare (p. 46); but failed to note that the square root of the product should be extracted. This oversight he remedied in his Economics of Welfare, 1920 (p. 78). In 1921 Prof. Irving Fisher having invented the formula in his turn, presented it before the American Statistical Association. Meanwhile Mr. Walsh in reviewing his earlier work had concluded that his footnote formula was perhaps the best of all. (See Quarterly Publication of the American Statistical Association, March, 1921, pp. 536, 539, and "The Problem of Estimation," p. 102.)

I have adopted Prof. Persons's notation as clearer than that of the inventors.—Review of Economic Statistics, May, 1921, p. 107, note.

year times quantities in the given year, and (4) the latter prices times quantities in the base year. Then the first and second aggregates (prices in the given year weighted in two ways) are reduced to relatives by dividing them respectively by the third and fourth aggregates (prices in the base year weighted in the same two ways). Finally these relatives are multiplied together and the square root of their product extracted.

What advantages does this formula possess to compensate for the great amount of labor it entails?

Prof. Pigou uses it in an index of changes in the volume of "real income." He finds it necessary to use weights for two periods because of "The root fact . . . that in the first period our group expends its purchasing power upon one collection of commodities, and in the second period it expends it on a second and different collection." The change in real income can not be accurately measured unless these alterations in the quantities of goods bought are represented in the index of prices used in reducing money income to real income.[12]

Prof. Fisher wants this formula for use in his equation of exchange. It serves admirably there, because an index number of prices made by it when multiplied by a similarly constructed index number of quantities will show the changes in the total values of goods exchanged.

Mr. Walsh's purpose is more general, "to measure variations in the exchange value or purchasing power of money," and his argument concerning its merits is more technical. The first of the two ratios included in the formula is equivalent to an harmonic mean of relative prices weighted by values in the given year, while the second ratio is equivalent to an arithmetic mean of price relatives weighted by values in the base year. By using imaginary examples covering four years, in which the last year has the same prices and quantities as the first year, Mr. Walsh tests arithmetic and harmonic means weighted in his way. He finds that they yield different results which "lie on opposite sides of the truth, and apparently equally above and below it proportionately." This result suggests the propriety of taking the geometric mean between the two averages. That step yields the "ideal" formula. Mr. Walsh adds: "Note that it involves the arithmetic average, the harmonic average, the weightings of the first and second periods, and the geometric mean. . . . It seems to contain everything that could be desired."[13]

We may agree with Prof. Pigou that this formula is well adapted to use in a measure of change in real income and with Prof. Fisher that it is well adapted to use in the equation of exchange. Can we agree with Mr. Walsh that it is the best formula for making general-purpose index numbers?[14]

If the end in view is to compare the change in prices between any two years, then this formula is more desirable than an aggregate of actual prices weighted by quantities in either year alone. That proposition holds true of every year-to-year comparison however far extended. Hence the "ideal" formula is admirably adapted for making chain index numbers, whenever it is possible to secure the

[12] Economics of Welfare, p. 72.
[13] The Problem of Estimation, p. 102.
[14] Mr. Walsh is explicit upon this point. (See The Problem of Estimation, p. 118.)

necessary annual data for quantities as well as prices and to meet the necessary expense of computation.

But can the separate links in such a chain index be welded together to make an equally admirable index covering long periods? Two objections lie against it on this score. (1) The ideal formula changes weights in each successive link in the chain. The quantities for 1920 and 1921 used in computing the link for that year are not likely to be the same as the quantities for 1921 and 1922 used in computing the latter link. As pointed out in section 6 above a change in the weights makes it uncertain what part of the net result is due to price fluctuations and what part to fluctuations in quantities. Whenever the purpose in view requires that the price factor shall be isolated, it is therefore undesirable to use the "ideal" formula for any comparisons except those between two specified years.[15] (2) It has been shown in section 8 that an arithmetic mean of relatives on the preceding-year base when forged into a continuous chain drifts away upward from the corresponding fixed-base series made from the same data. Now the ideal formula does not use relative prices, but is made from aggregates of actuals which can not drift in this fashion, provided they are made with constant weights. Does the annual change of weights required by the "ideal" formula introduce errors that cumulate and so cause the chain index to part company from a fixed-base series? Prof. Persons has answered that question by an actual trial. Taking the prices and quantities of 12 leading crops of each year of the decade of 1910–1919, he has made first for the quantities and second for the prices two index numbers, one using the "ideal" formula computed directly to the fixed base 1910, another using the "ideal" formula chain fashion. Both of the chain indices are found to diverge from their fixed-base mates by a distance that is rather wide considering that the errors are cumulated for no more than nine years. The chain index for quantities drifts *upward* and the chain index for prices drifts *downward*. In both cases the discrepancies reach 4 per cent in 1919.[16] Hence the "ideal" formula is ill-fitted for making index numbers covering a long period of years, when it is applied in the way which its logic strictly requires, namely, year-by-year comparisons. And a fixed-base series made by this formula—that is, one in which the index for each year is made by compounding the weights of that year with some base year (instead of the year before)—yields accurate comparisons only between the base year and any given year and not comparisons that are accurate as between any two given years. If it is desired to make possible comparisons between any years of a period longer than two years aggregates of actual prices or geometric means, both made with constant weights, seem better than the "ideal" formula, as well as far easier to compute.[17]

[15] This objection is reduced but not removed if the indices for each year are computed directly to a fixed base, say 1913. Then the prices for the year 1920 would be weighted by quantities in 1913 and 1920, the prices in 1921 by the quantities in 1913 and 1921, etc. The weights would still change, but not so much as in the chain index.

[16] Review of Economic Statistics, May, 1921, pp. 113, 114.

[17] Concerning the difference in labor of computing Prof. Persons gives an interesting note. The relative times required to compute the "ideal" index numbers and the geometric means in his test of the two were as follows:

	Relative times required.
Geometric means, constant weights	27
"Ideal" index number, fixed base	51
"Ideal" index number, chain series	100

Of course the difference would be much larger if the time were counted in that is spent in collecting yearly data concerning quantities called for by the "ideal" formula. A sum of actual prices made with fixed weights takes still less time for computation than a weighted geometric mean.

V.—A COMPARISON OF THE LEADING AMERICAN INDEX NUMBERS FOR THE YEARS 1890 TO 1918.

Many of the threads running through the preceding sections can be woven into a comparison of the best-known index numbers currently published in the United States—a comparison having intrinsic interest of its own, as well as making a fitting summary of Part I of this bulletin.

1. ANALYSIS OF THE SIMILARITIES AND DIFFERENCES BY YEARS, 1890 TO 1918.

Three general-purpose index numbers are available for the critical study proposed, the latest form of the Bureau of Labor Statistics series, Bradstreet's index, and Dun's index. It seems hardly worth while to include in the comparison index numbers made solely of the prices of foods, because they do not profess to measure changes in the commodity markets at large. It has been shown that these special indexes are not in close agreement with series containing not only foods but also minerals, forest products, textiles, chemicals, etc.; and that demonstration need not be repeated.[18]

The first step toward comparing index numbers is to throw them into similar form and establish them upon a common base. The new series of the Bureau of Labor Statistics is a weighted sum of actual prices, turned into relatives on the base, prices in 1913 = 100. This series can be shifted to any base desired without appreciable loss in accuracy. Dun's and Bradstreet's series are sums of actual prices, and have no base of their own. Accordingly they may be recast into relatives on the base, the average of the original figures for 1890–1899 = 100. Dun's figures for this decade average $84.32. By dividing the published figures by this sum and multiplying the results by 100 we can make a new series strictly comparable with the rest of our material. Shifting Bradstreet's series is less satisfactory, because it does not begin until 1892. The best that can be done is to equate Bradstreet's average for 1892–1899 with the average made from the Bureau's figures for these years—that is, to put $6.7785 = 97.1—and then to apply the rule of three.[19]

These three series in comparable form are assembled in Table 19.[20]

[18] See subdivision 5, "The numbers and kinds of commodities included," especially pp. 52–56.

[19] No violence is done by this procedure to Bradstreet's series; but the comparision is not quite satisfactory, because our other series were not worked out on the basis, prices in 1892–1899 = 97.1, and would probably have shown slightly different results if they had been.

[20] The annual averages, made from the original figures published by Dun and Bradstreet's, run as follows:

Year.	Dun's.	Bradstreet's.	Year.	Dun's.	Bradstreet's.
1890	$90.9	....	1907	$111.8	$8.90
1891	92.2	....	1908	109.9	8.01
1892	90.0	$7.78	1909	117.8	8.52
1893	92.4	7.53	1910	119.2	8.99
1894	84.7	6.68	1911	116.8	8.71
1895	81.3	6.43	1912	124.4	9.19
1896	76.0	5.91	1913	120.9	9.21
1897	74.0	6.12	1914	122.2	8.90
1898	78.9	6.57	1915	126.4	9.85
1899	82.8	7.21	1916	148.8	11.83
1900	93.4	7.88	1917	204.1	15.66
1901	95.9	7.57	1918	229.2	18.73
1902	100.4	7.88	Averages:		
1903	99.0	7.94	1890–1899	84.3	[a] 6.78
1904	100.2	7.92	1900–1909	103.4	8.11
1905	100.6	8.10	1910–1914	120.7	9.00
1906	105.3	8.42	1915–1918	177.1	14.02

[a] Average of 1892–1899.

The second and third divisions of the table bring out certain differences among the figures, and the summaries in the latter part show the average or net movements in various periods.

TABLE 19.—A COMPARISON OF THE CHIEF AMERICAN INDEX NUMBERS FOR THE YEARS 1890 TO 1918.

Year	The three index numbers shifted to the 1890-1899 base.			Percentage differences among the three index numbers.			Percentage by which each of the three index numbers rose (+) or fell (−) each year.		
	Bradstreet's.	Bureau of Labor Statistics.	Dun's.	Bradstreet's greater (+) or less (−) than Bureau of Labor Statistics.	Dun's greater (+) or less (−) than Bureau of Labor Statistics.	Bradstreet's greater (+) or less (−) than Dun's.	Bradstreet's.	Bureau of Labor Statistics.	Dun's.
Period of decline.									
1890	....	111	108	....	−2.7	....	....	....	....
1891	....	111	109	....	−1.8	....	....	± 0	+ .9
1892	111	103	107	+7.8	+3.9	+ 3.7	....	− 7.2	+ 1.8
1893	108	106	110	+1.9	+3.8	− 1.8	− 2.7	+ 2.9	+ 2.8
1894	96	95	100	+1.1	+5.3	− 4.0	−11.1	−10.4	− 9.1
1895	92	95	96	−3.2	+1.1	− 4.2	− 4.2	± 0	− 4.0
1896	85	90	90	−5.6	±0	− 5.6	− 7.6	− 5.3	− 6.3
Period of gradual rise.									
1897	88	91	88	−3.3	−3.3	± 0	+ 3.5	+ 1.1	− 2.2
1898	94	95	94	−1.1	−1.1	± 0	+ 6.8	+ 4.4	+ 6.8
1899	103	101	98	+2.0	−3.0	+ 5.1	+ 9.6	+ 6.3	+ 4.3
1900	113	109	111	+3.7	+1.8	+ 1.8	+ 9.7	+ 7.9	+13.3
1901	108	108	114	±0	+5.6	− 5.3	− 4.4	− .9	+ 2.7
1902	113	116	119	−2.6	+2.6	− 5.0	+ 4.6	+ 7.4	+ 4.4
1903	114	117	117	−2.6	±0	− 2.6	+ .9	+ .9	− 1.7
1904	113	117	119	−3.4	+1.7	− 5.0	− .9	± 0	+ 1.7
1905	116	117	119	− .9	+1.7	− 2.5	+ 2.7	± 0	± 0
1906	121	121	125	±0	+3.3	− 3.2	+ 4.3	+ 3.4	+ 5.0
1907	127	128	133	− .8	+3.9	− 4.5	+ 5.0	+ 5.8	+ 6.4
1908	115	125	130	−8.0	+4.0	−11.5	− 9.4	− 2.3	− 2.3
1909	122	133	140	−8.3	+5.3	−12.9	+ 6.1	+ 6.4	+ 7.7
1910	129	136	141	−5.1	+3.7	− 8.5	+ 5.7	+ 2.3	+ .7
1911	125	130	139	−3.8	+6.9	−10.1	− 3.1	− 4.4	− 1.4
1912	132	138	148	−4.3	+7.2	−10.8	+ 5.6	+ 6.2	+ 6.5
1913	132	137	143	−3.6	+4.4	− 7.7	± 0	− .7	− 3.4
1914	128	136	145	−5.9	+6.6	−11.7	− 3.0	− .7	+ 1.4
1915	141	139	150	+1.4	+7.9	− 6.0	+10.2	+ 2.2	+ 3.4
Period of accelerated rise due to war.									
1916	169	170	176	− .6	+3.5	− 4.0	+19.9	+22.3	+17.3
1917	224	241	242	−7.1	+ .4	− 7.4	+32.5	+41.8	+37.5
1918	268	269	272	− .4	+1.1	− 1.5	+19.6	+11.6	+12.4

TABLE 19.—A COMPARISON OF THE CHIEF AMERICAN INDEX NUMBERS FOR THE YEARS 1890 TO 1918—Concluded.

Item.	The three index numbers shifted to the 1890–1899 base.			Percentage variations among the three index numbers.			Percentage variations of the yearly rise and fall of each of the three index numbers.		
	Bradstreet's.	Bureau of Labor Statistics.	Dun's.	Bradstreet's compared with Bureau of Labor Statistics.	Dun's compared with Bureau of Labor Statistics.	Bradstreet's compared with Dun's.	Bradstreet's.	Bureau of Labor Statistics.	Dun's.
Averages by 5-year periods:									
1890–1894	105	105	107	3.6	3.5	3.2	6.9	5.1	3.7
1895–1899	92	94	93	3.0	1.7	3.0	6.3	3.4	4.7
1900–1904	112	113	116	2.5	2.3	3.9	4.1	3.4	4.8
1905–1909	120	125	129	3.6	3.6	6.9	5.5	3.6	4.3
1910–1914	129	135	143	9.5	5.8	10.0	3.5	2.9	2.7
1915–1918	201	205	210	2.4	3.2	4.7	20.6	19.5	17.7
Averages by 10-year periods:									
1890–1899	(100)	100	100	3.3	2.6	3.1	6.5	4.2	4.2
1900–1909	116	119	123	3.0	3.0	5.4	4.8	3.5	4.5
1910–1918	161	166	173	3.6	4.6	7.5	11.1	10.2	9.3
Maxima and minima:									
1890–1914—									
Maxima	132	138	148	8.3	7.2	12.9	11.1	10.4	13.3
Minima	85	90	88	0	0	0	0	0	0
Differences	47	48	60	8.3	7.2	12.9	11.1	10.4	13.3
1914–1918—									
Maxima	268	269	272	7.1	7.9	11.7	32.5	41.8	37.5
Minima	128	136	145	.4	.4	1.5	3.0	.7	1.4
Differences	140	133	127	6.7	7.5	10.2	29.5	41.1	36.1
Net rise (+) or fall (−):									
1890–1896	−26	−21	−18						
1896–1907	+42	+38	+43						
1907–1908	−12	−3	−3						
1908–1914	+13	+11	+15						
1914–1918	+140	+133	+127						
Algebraic averages:									
1890–1894				+3.6	+1.7	−.7	−6.9	−3.7	−1.8
1895–1899				−2.2	−1.3	−.9	+1.6	+1.3	−.3
1900–1904				−1.0	+2.3	−3.2	+2.0	+3.1	+4.1
1905–1909				−3.6	+3.6	−6.9	+1.7	+2.7	+3.4
1910–1914				−4.5	+5.8	−10.0	+1.0	+.5	+.8
1915–1918				−1.7	+3.2	−4.7	+20.6	+19.5	+17.7
1890–1914				−2.0	+2.4	−4.6	+.8	+1.0	+1.4
1890–1918				−2.0	+2.5	−4.6	+3.9	+3.6	+3.7

A cursory examination of this table, or a glance at Chart 12, shows that these index numbers made by three independent organizations have a marked family resemblance. They all agree that prices fell heavily in 1890–1896, rose still more sharply in 1896–1900, wavered uncertainly in 1901–1904, rose rapidly again in 1905–1907, fell in 1908, more than recovered their lost ground in 1909–1910, dropped back in 1911, rose to a new high record in 1912, receded somewhat in 1912–1914, and finally shot up at an extraordinary rate during the war. Further, the three index numbers agree that the general level about which the yearly oscillations clustered was higher in 1910–1914 than in 1900–1909, and higher in 1900–1909 than in 1890–1899. About the major facts of price history, in short, the testimony of the leading American index numbers is unanimous.

On the other hand, Table 19 shows that the series differ in many details. For example, not once in the 29 years covered by the present record are all three index numbers identical, and in only six years

CHART 12.—INDEX NUMBERS OF THE BUREAU OF LABOR STATISTICS, DUN, AND BRADSTREET, 1890 TO 1918.

(Based on Table 19.)

†311739 O—41 (To face page 96.)

are any two indexes precisely the same. On the average of the whole period the Bureau of Labor Statistics series varies from Bradstreet's by 3.3 per cent, from Dun's by 3.4 per cent, while Bradstreet's index varies from Dun's by 5.4 per cent. The maximum differences in any one year rise to 8.3 per cent between the bureau's index and Bradstreet's (1909), 7.9 per cent between the bureau's and Dun's (1915), and 12.9 per cent between Dun's and Bradstreet's (1909). Concerning the direction in which prices move from one year to the next, the bureau's series contradicts Bradstreet's in one year (1893) and Dun's series in four years, while Dun's and Bradstreet's indexes contradict each other in six years. If we count cases in which one index remains the same for two successive years while another series rises or falls, we find four years of partial contradiction when we compare the bureau's index with Bradstreet's, three years when we compare the bureau's index with Dun's, and two years when we set Bradstreet's against Dun's. In general, the bureau's index steers a middle course between the other two, averaging 2 per cent higher than Bradstreet's and 2.5 per cent lower than Dun's, while the margin by which Dun's index exceeds Bradstreet's averages 4.6 per cent.[21]

Most of the detailed differences among the annual figures of the three index numbers may be regarded as resulting from differences in respect to (1) secular trend and (2) degree of variability from one year to the next.

1. Chart 12 and the averages by decades in Table 19 show that on the whole Dun's index number has risen more than the bureau's, and the bureau's more than Bradstreet's. This long-period shifting of the level about which the monthly and yearly oscillations occur is technically called the secular trend. Graphically it may be represented by a straight line. Two turning points occur in the 29 years covered by the table. The great fall of prices which began in 1873 ended in 1896 or 1897, and a rise began. In 1915 the rate of this rise was violently accelerated by the war, so that the slope of the straight line representing the direction of the secular trend was suddenly made steeper. Of the three periods marked off by these turning points in the first half of Table 19, the middle one, 1896–1914, alone is long enough to make the computation of the secular trend significant.

The secular trends of the three index numbers during this period of 19 years, given in Table 20, are represented collectively on Chart 13 and are shown separately with their respective curves on Charts 14, 15, and 16. They are summarized in the following table:

TABLE 20.—SECULAR TRENDS OF INDEX NUMBERS OF BUREAU OF LABOR STATISTICS, BRADSTREET, AND DUN, 1896 TO 1914—SUMMARY.

Index numbers.	Annual geometric increment of secular trend in 1896–1914.	Geometric mean in 1896–1914.	Ratio of annual increment to geometric mean (per cent).	Terminal points of the straight line representing the secular trend.		Net per cent of rise in lines of secular trend, 1896–1914.
				1896	1914	
Bradstreet's	1.0230	113.7	0.90	92.7	139.6	15.1
Bureau of Labor Statistics	1.0232	117.1	.87	95.3	144.0	15.1
Dun's	1.0269	120.3	.85	94.7	152.6	16.1

[21] These averages are made, of course, from algebraic sums of the yearly percentage differences.

†311739 O—41——7

It is primarily these differences in secular trend that make the bureau's index number follow a course intermediate between Bradstreet's and Dun's indexes.

CHART 13.—SECULAR TRENDS OF INDEX NUMBERS OF BUREAU OF LABOR STATISTICS, DUN, AND BRADSTREET, 1896-1914.

(Based on Table 21.)

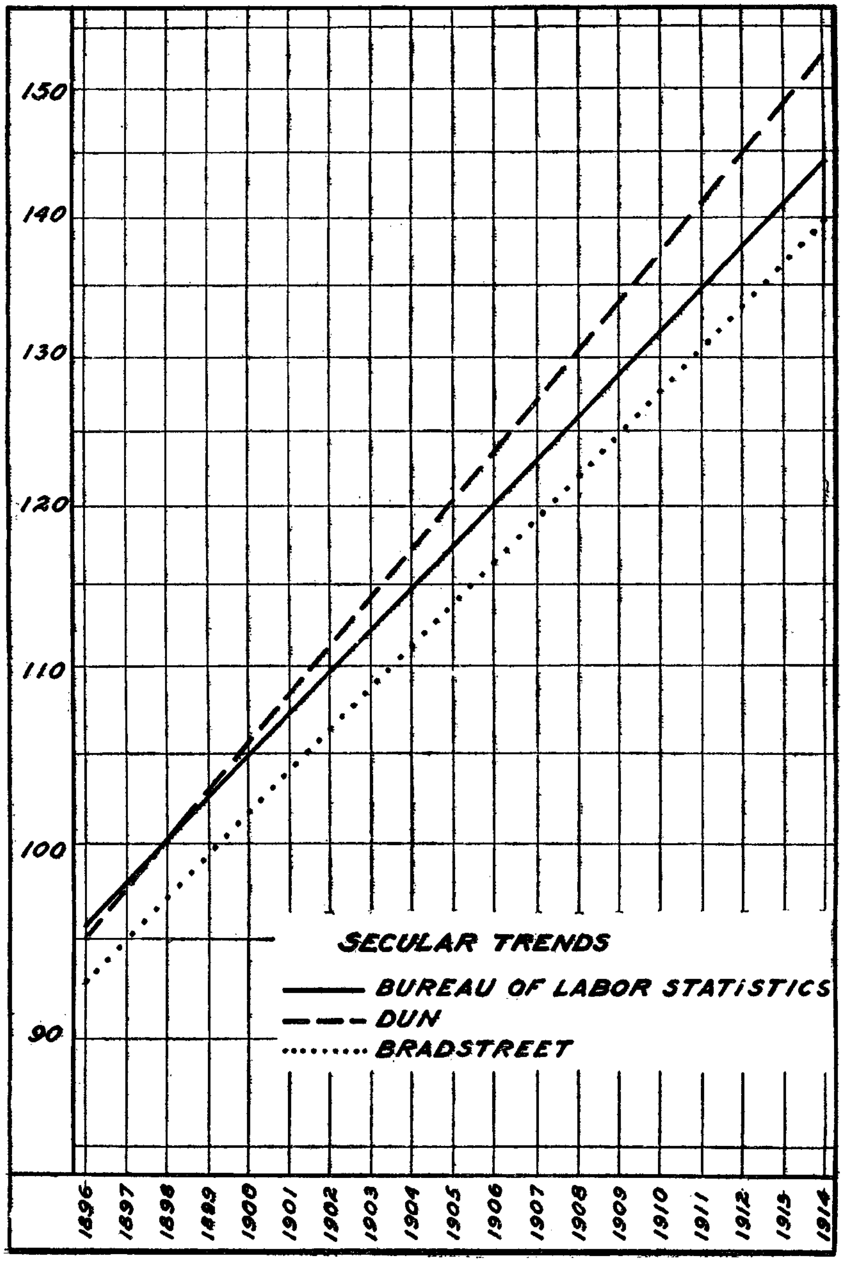


CHART 14.—INDEX NUMBERS OF BRADSTREET, COMPARED WITH THEIR SECULAR TREND, 1896-1914.

(Based on Table 21.)

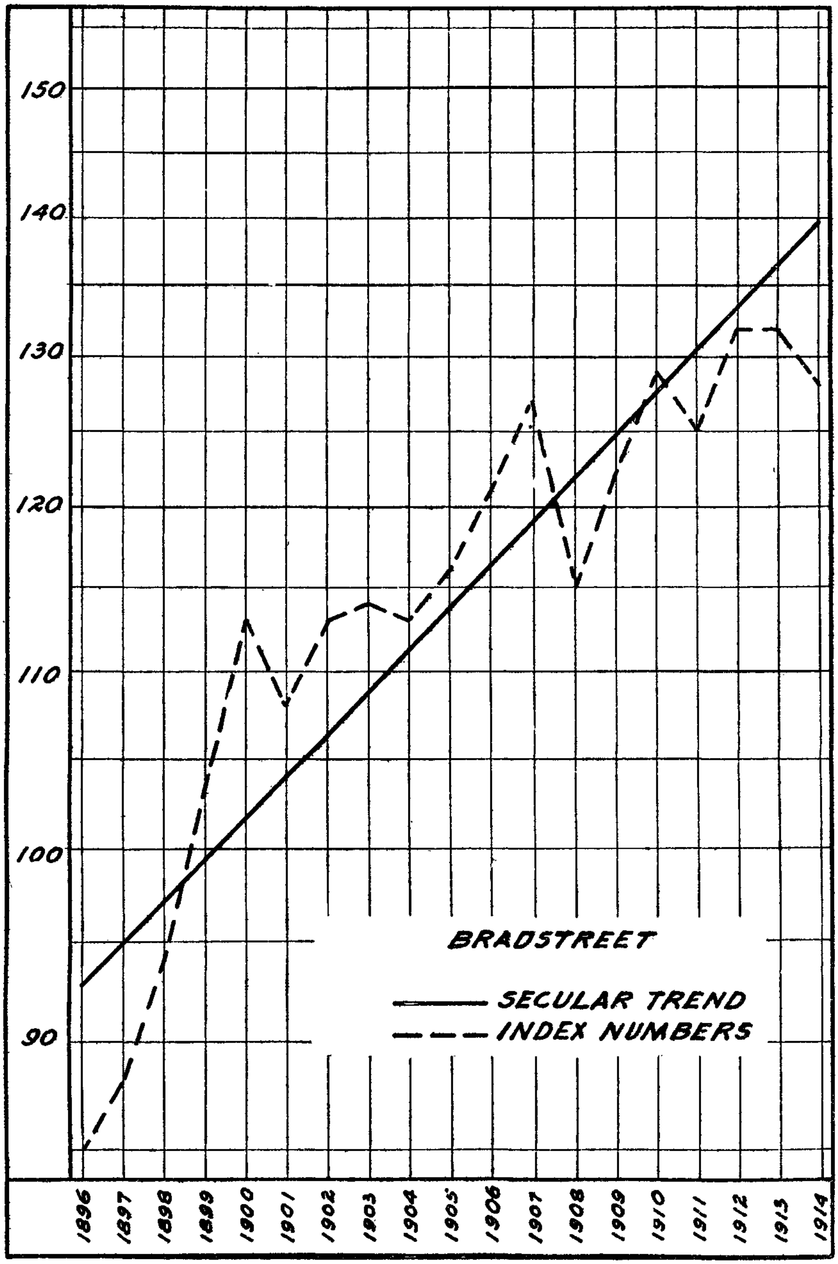


CHART 15.—INDEX NUMBERS OF BUREAU OF LABOR STATISTICS, COMPARED WITH THEIR SECULAR TREND, 1896–1914.

(Based on Table 21.)

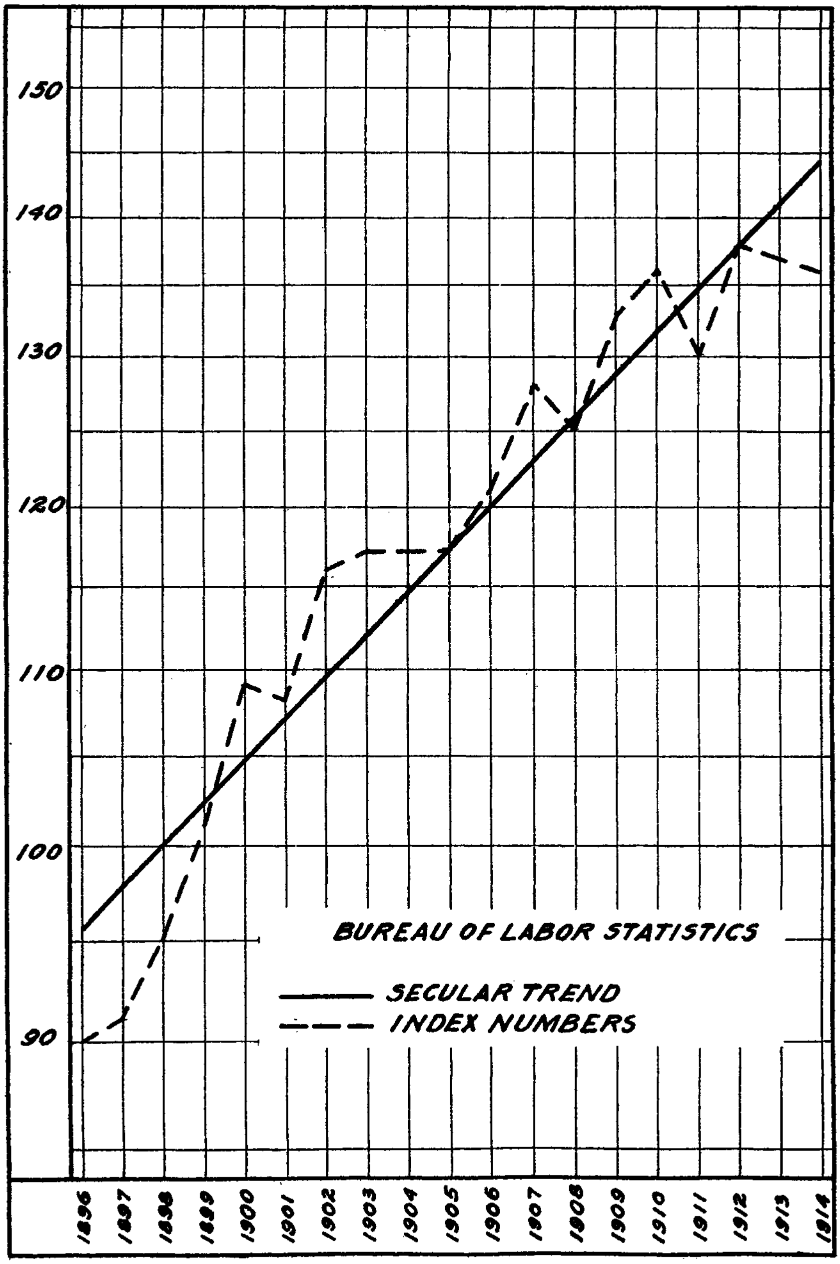


CHART 16.—INDEX NUMBERS OF DUN, COMPARED WITH THEIR SECULAR TREND, 1896–1914.

(Based on Table 21.)

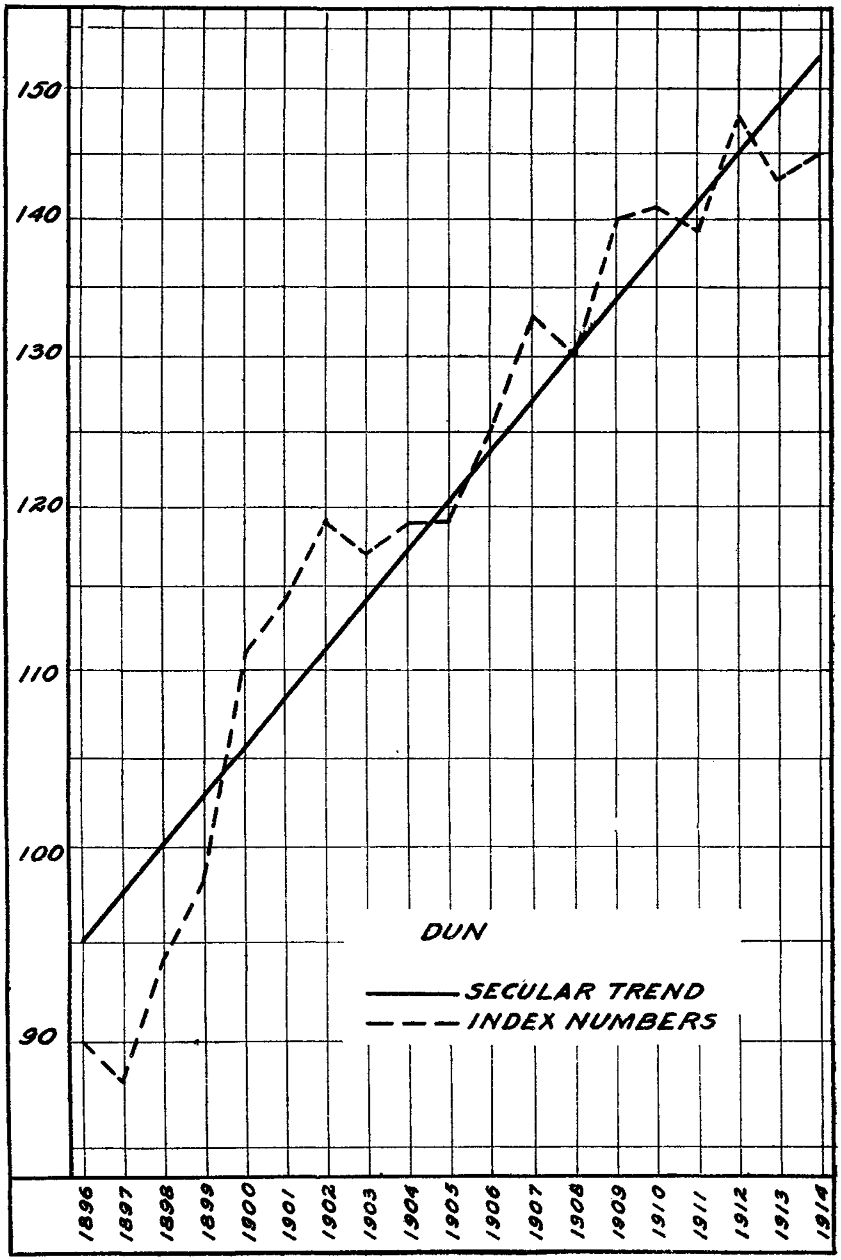


TABLE 21.—INDEX NUMBERS OF BRADSTREET, THE BUREAU OF LABOR STATISTICS, AND DUN, COMPARED WITH THEIR SECULAR TRENDS, BY YEARS, 1896 TO 1914.

Year.	Bradstreet's.				Bureau of Labor Statistics.				Dun's.			
	Secular trend.	Index number.	Excess of— Secular trend over index number.	Excess of— Index number over secular trend.	Secular trend.	Index number.	Excess of— Secular trend over index number.	Excess of— Index number over secular trend.	Secular trend.	Index number.	Excess of— Secular trend over index number.	Excess of— Index number over secular trend.
			Per ct.	*Per ct.*			*Per ct.*	*Per ct.*			*Per ct.*	*Per ct.*
1896	92.7	85	9.1	....	95.3	90	5.8	....	94.7	90	5.2	....
1897	94.9	88	7.8	....	97.5	91	7.1	....	97.3	88	10.6	....
1898	97.1	94	3.3	....	99.7	95	5.0	....	99.9	94	6.3	....
1899	99.3	103	....	3.7	102.0	101	1.0	....	102.5	98	4.6	....
1900	101.6	113	....	11.3	104.4	109	....	4.4	105.3	111	....	5.4
1901	103.9	108	....	3.9	106.8	108	....	1.1	108.1	114	....	5.5
1902	106.3	113	....	6.3	109.3	116	....	6.1	111.0	119	....	7.2
1903	108.7	114	....	4.8	111.9	117	....	4.6	114.0	117	....	2.6
1904	111.2	113	....	1.6	114.5	117	....	2.2	117.0	119	....	1.7
1905	113.7	116	....	2.0	117.1	117	.1	....	120.3	119	1.1	....
1906	116.3	121	....	4.0	119.8	121	....	1.0	123.5	125	....	1.2
1907	119.0	127	....	6.7	122.6	128	....	4.4	126.8	133	....	4.9
1908	121.8	115	5.9	....	125.5	125	.4	....	130.2	130	.2	....
1909	124.6	122	2.1	....	128.4	133	....	3.6	133.7	140	....	4.7
1910	127.0	129	....	1.2	131.3	136	....	3.5	137.3	141	....	2.7
1911	130.4	125	4.3	....	134.4	130	3.4	....	141.0	139	1.4	....
1912	133.4	132	1.0	....	137.5	138	....	.4	144.8	148	....	2.2
1913	136.4	132	3.3	....	140.7	137	2.7	....	148.6	143	3.9	....
1914	139.6	128	9.0	....	144.0	136	5.9	....	152.6	145	5.2	....

2. While steadier over a considerable period of time, Bradstreet's index changes more from one year to the next than does either the bureau's or Dun's series. Dun's index, further, is more variable than the bureau's.

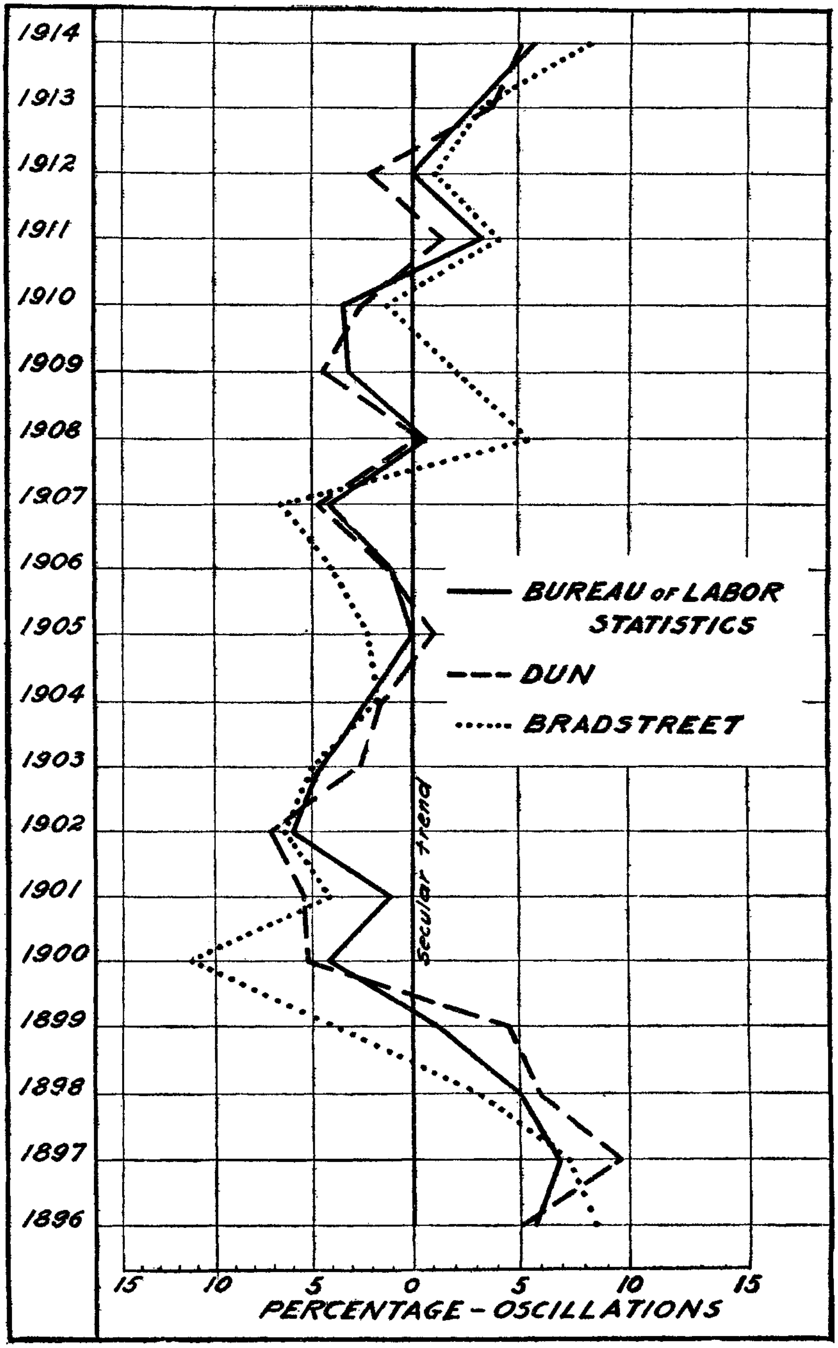
Several different ways of measuring year-to-year variations all support this conclusion: (1) If the "percentage by which each of the three index numbers rose or fell each year" as shown in Table 19 be averaged from 1892 to 1914, the results are Bradstreet's 5.15 per cent, Dun's 4.37 per cent, and the Bureau of Labor Statistics's 3.71 per cent. (2) The standard deviations of these annual percentages of rise and fall are, Bradstreet's 5.79, Dun's 5.06, and the bureau's 4.46. (3) If the figures showing the excess of the secular trend over the index number or the excess of the index number over the secular trend in Table 21 be averaged for 1896–1914, the results are, Bradstreet's 4.0 per cent, Dun's 4.0 per cent, the bureau's 3.3 per cent. (4) If the yearly deviations from the secular trend are plotted as in Chart 17, it appears that Bradstreet's fluctuates through the widest and the bureau's series through the narrowest range, Dun's index being intermediate.

To show that these index numbers differ in detail, however, means little. The significant problem is whether these differences are due to the inherent difficulty of measuring changes in the price level, to the crudity of the method of measurement in vogue, or to technical differences in the construction of the particular index numbers in question.

Unfortunately it is not possible to attack this problem effectively on the lines of analysis suggested in the preceding sections. For the compilers of Bradstreet's and Dun's index numbers do not give suffi-

CHART 17.—YEARLY DEVIATIONS FROM SECULAR TREND OF INDEX NUMBERS OF BUREAU OF LABOR STATISTICS, DUN, AND BRADSTREET, 1896–1914.

(Based on Table 21.)



cient data concerning the sources of information drawn upon for quotations, the commodities included and the weights employed for each commodity to make possible a close comparison with the bureau's series. Bradstreet's publishes quotations for 106 commodities, but bases its index number on the prices of 96, and does not say which 10 are omitted. Its prices per pound, which are added up to give the index number, were published for a short time in 1897, but are not disclosed in recent years. Dun's Review does not publish its list of commodities, to say nothing of their prices, and explains merely that it weights by per capita consumption, allowing 50 per cent of the total for foods, 18 per cent for textiles, 16 per cent for minerals, and 16 per cent for other commodities.[22] With such scanty information about these two series, statements concerning the reasons for the relatively slight differences between each of them and the bureau's index number would be subject to a relatively wide margin of error.[23]

After all, the important fact is that the three index numbers agree with one another very closely. The divergencies which do appear are smaller than those which result from most attempts to measure economic quantities. For example, two sets of experts employed upon a valuation case are likely to arrive at results farther apart than the maximum differences shown in Table 19. Again it is doubtful whether the margin of error in the average balance sheets of business enterprises, or in cost computations is as narrow as the average margin between Bradstreet's and Dun's index numbers, to say nothing of the narrower margins between the official series and either of these commercial indexes.

To sum up the comparisons in the most definite form the coefficient of correlation must be used. This coefficient is the standard statistical device for measuring the degree of agreement or difference between two variables. Its extreme limits are −1.0 and +1.0, the latter expressing perfect agreement.[24] When such coefficients are computed for the annual index numbers in 1892–1914, inclusive, the following results are obtained:

	Coefficients of correlation.
Bureau of Labor Statistics index number and Bradstreet's	+0.964
Bureau of Labor Statistics index number and Dun's	+ .992
Bradstreet's index number and Dun's	+ .959

High coefficients of correlation are to be expected, of course, when the variables compared are different measurements of the same quantity—in this case the general level of wholesale prices through a period of 23 years. To get such high coefficients as the preceding indicates that the measurements made by different hands are in close agreement and therefore presumably reliable.

A severer test may be applied by computing the coefficients of correlation between the percentage changes in the three index num-

[22] Compare J. P. Norton's article in the Quarterly Journal of Economics, Aug., 1910, Vol. XXIV, p. 754.

[23] Most of the analytic comparisons among various American index numbers in Bulletin No. 173 dealt with series much more perfectly known than Dun's or Bradstreet's. The reader who turns back to that discussion will probably share the writer's belief that were all the necessary data available, the differences among the three series dealt with would be found to result primarily from differences in the lists of commodities and in the systems of weighting. But that belief will remain a mere probability so long as the construction of Bradstreet's and Dun's indexes is not fully disclosed.

[24] Most statistical text books explain the method of computing the coefficient of correlation in detail. See for example, G. Udney Yule, Introduction to the Theory of Statistics, 2d edition, 1912, chs. IX and X.

bers from one year to the next. The results of this operation are as follows:

	Coefficients of correlation.
Bureau of Labor Statistics index number and Bradstreet's	+0.882
Bureau of Labor Statistics index number and Dun's	+ .873
Bradstreet's index number and Dun's	+ .788

Here the coefficients, though less than in the preceding case, are still high. Bradstreet's agrees a bit better with the bureau's series than does Dun's, whereas in the former comparison Dun's had distinctly the higher correlation. In both comparisons, the bureau's series makes the best showing. Other things being equal, among different measures of a given quantity, that measure has the best claim tō acceptance which is nearest the mean of all the measures. In the present case, however, other things are not equal. The bureaus's series includes more commodities than either of its rivals, its system of weighting is better, and its method of construction from start to finish is disclosed with a fullness which justifies confidence. On these grounds its superiority is clear. The fact that it agrees better with both the commercial indexes than they agree with each other merely confirms the choice which would be made on a priori grounds.

2. COMPARISON OF FOUR LEADING AMERICAN INDEX NUMBERS, BY MONTHS, JULY, 1914, TO DECEMBER, 1918.

The peculiar interest attaching to the revolution in prices during the World War makes desirable a more detailed comparison of the leading American index numbers in 1914–1918. For this period, there are available besides the three series discussed in the preceding section, the index number compiled by the Price Section of the War Industries Board.

Table 22 and chart 18 present the four series on a common base—namely, average prices in the twelve months preceding the outbreak of war (July, 1913–June, 1914) = 100, giving by months first the index numbers themselves, and then the percentage by which each of the four index numbers rose or fell as compared with the month preceding.

Study of the table and of the chart based upon it shows at once a closeness of agreement for which even the results of the preceding comparison scarcely prepare one. And this impression of close agreement is abundantly justified when the coefficients of correlation are worked out. These coefficients, shown on page 108, approach even more closely to the limit of perfect agreement (+1.0) than the remarkably high coefficients we have found for the yearly index numbers in times of peace.

TABLE 22.—A COMPARISON OF FOUR LEADING AMERICAN INDEX NUMBERS, BY MONTHS, JULY, 1914, TO DECEMBER, 1918.

Year and month.	The four index numbers shifted to the base, July, 1913–June, 1914=100.				Percentage by which each of the four index numbers rose (+) or fell (−) each month.			
	War Industries Board.	Bureau of Labor Statistics.	Bradstreet's.	Dun's.	War Industries Board.	Bureau of Labor Statistics.	Bradstreet's.	Dun's.
1914.								
July	97	99	97	99	±0	+0.8	+0.5	−0.1
August	101	103	103	102	+4.1	+3.2	+6.3	+3.0
September	101	103	106	103	±0	+ .5	+2.9	+1.1
October	99	99	101	102	−2.0	−4.2	−4.7	−1.1
November	98	98	100	102	−1.0	− .9	−1.2	+ .3
December	98	97	102	102	±0	− .8	+1.6	− .1
1915.								
January	100	98	105	103	+2.0	+1.1	+3.5	+0.6
February	100	100	108	103	±0	+1.7	+2.5	±0
March	100	99	109	103	±0	−1.0	+ .6	− .2
April	100	100	110	104	±0	+ .7	+ .9	+1.0
May	100	101	109	104	±0	+1.0	− .2	+ .4
June	100	99	110	103	±0	−1.6	+ .4	− .7
July	102	101	110	103	+2.0	+1.8	+ .4	− .4
August	102	100	110	103	±0	− .7	− .3	− .1
September	102	99	111	103	±0	−1.3	+ .8	+ .6
October	104	101	114	106	+2.0	+2.4	+2.9	+2.3
November	107	103	118	108	+2.9	+1.5	+3.3	+2.5
December	111	106	120	111	+3.7	+2.9	+2.6	+2.7
1916.								
January	115	110	123	115	+3.6	+4.2	+2.3	+3.4
February	118	111	126	117	+2.6	+1.2	+2.1	+1.6
March	121	114	129	118	+2.5	+2.3	+2.8	+1.2
April	123	116	132	120	+1.7	+2.0	+1.6	+1.4
May	123	118	131	120	±0	+1.6	− .3	− .1
June	122	118	130	119	− .8	+ .4	− .9	− .4
July	123	119	129	119	+ .8	+ .4	−1.1	− .5
August	125	123	130	122	+1.6	+3.5	+1.1	+2.4
September	127	127	133	125	+1.6	+3.3	+2.6	+2.8
October	132	134	139	130	+3.9	+5.0	+4.3	+4.2
November	141	143	148	137	+6.8	+7.2	+6.5	+5.0
December	144	146	153	139	+2.1	+2.0	+3.5	+1.4
1917.								
January	148	150	155	142	+2.8	+2.8	+1.0	+2.4
February	151	155	157	149	+2.0	+3.4	+1.5	+4.8
March	156	160	161	155	+3.3	+3.2	+2.3	+3.8
April	170	172	166	164	+9.0	+7.2	+3.4	+5.9
May	178	181	171	173	+4.7	+5.4	+3.0	+5.7
June	183	185	176	175	+2.8	+2.0	+3.1	+ .8
July	189	185	182	177	+3.3	+ .4	+3.0	+1.5
August	187	185	185	178	−1.1	− .4	+1.8	+ .7
September	186	182	188	179	− .5	−1.4	+1.6	+ .2
October	182	180	190	181	−2.2	−1.1	+1.3	+1.3
November	183	183	194	181	+ .5	+1.4	+2.0	+ .1
December	182	182	199	182	− .5	− .5	+2.6	+ .3
1918.								
January	185	186	202	190	+1.6	+2.2	+1.4	+4.3
February	187	187	202	187	+1.1	+ .7	+ .3	−1.4
March	188	188	204	188	+ .5	+ .4	+1.1	+ .7
April	191	191	209	188	+1.6	+1.9	+2.3	− .3
May	190	191	212	186	− .5	− .1	+1.5	−1.2
June	189	193	214	188	− .5	+1.0	+ .7	+1.3
July	193	199	215	191	+2.1	+2.9	+ .6	+1.6
August	196	203	214	191	+1.5	+2.2	− .6	+ .1
September	201	207	213	192	+2.5	+2.1	− .3	+ .3
October	201	204	212	191	±0	−1.5	− .4	− .5
November	201	207	212	190	±0	+1.3	+ .1	− .6
December	203	207	210	189	+1.0	±0	−1.0	− .1

Since both Bradstreet's and Dun's index numbers are computed from prices as of the first of the month while the Bureau of Labor Statistics and War Industries Board use average prices for the month or prices at various dates within the month, it is not quite accurate to compute coefficients of correlation from the figures as they stand

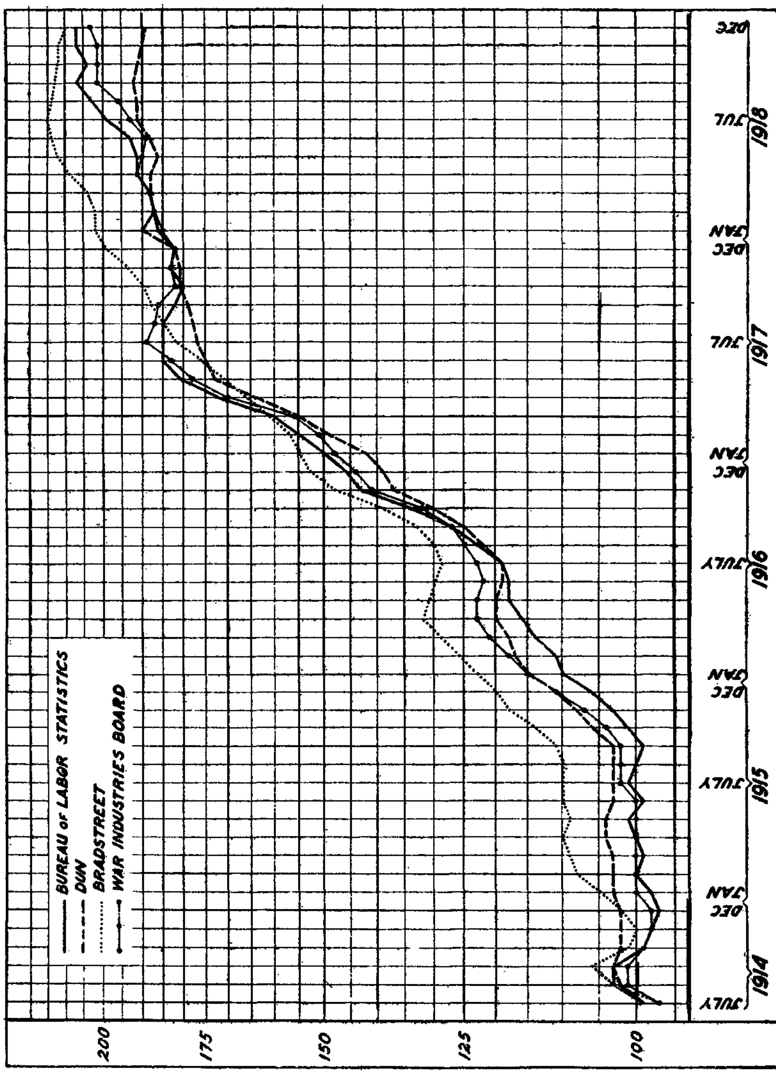


CHART 18.—INDEX NUMBERS OF THE BUREAU OF LABOR STATISTICS, DUN, BRADSTREET, AND THE WAR INDUSTRIES BOARD, JULY, 1914, TO DECEMBER, 1918.

(Average prices July, 1913, to June, 1914=100.)

(Based on Table 22.)

after shifting to a common base. To overcome this difficulty as well as may be, new monthly figures for Dun's and Bradstreet's have been made by averaging the index for July and August to get a new figure for July, then averaging the figures for August and September to get a new figure for August, and so on.

Coefficients of correlation among four American index numbers in the 54 months July, 1914, to December, 1918.

A. Coefficients of correlation computed from the monthly index numbers.

	Coefficients of correlation.
(1) Bureau of Labor Statistics' and War Industries Board's series	+0.997
(2) Bureau of Labor Statistics' and Bradstreet's series	+ .988
(3) Bureau of Labor Statistics' and Dun's series	+ .994
(4) War Industries Board's and Bradstreet's series	+ .986
(5) War Industries Board's and Dun's series	+ .995
(6) Bradstreet's and Dun's series	+ .991

B. Coefficients of correlation computed from the percentage change in prices from one month to the next.

(1) Bureau of Labor Statistics' and War Industries Board's series	+0.866
(2) Bureau of Labor Statistics' and Bradstreet's series	+ .633
(3) Bureau of Labor Statistics' and Dun's series	+ .801
(4) War Industries Board's and Bradstreet's series	+ .640
(5) War Industries Board's and Dun's series	+ .761
(6) Bradstreet's and Dun's series	+ .616

Taking both sets of coefficients into account, we find that the Bureau of Labor Statistics' index number has the closest agreement with the other three series. Then, in order, come the War Industries Board's series, Dun's, and Bradstreet's—which is the most divergent of the four. But there is a better test of reliability. In view of its very comprehensive list of commodities (1,366 in number) and its use of class in addition to commodity weights, the War Industries Board's series is probably the nearest approximation to a true measurement of the changes in the wholesale price level during the war. Accepting it as the standard, we may ask which of the three index numbers currently published is in closest agreement with it. Once more the answer is in favor of the bureau's series, when one considers the correlation either of the monthly index numbers themselves or of the monthly percentages of change. Dun's comes second and Bradstreet's again ranks lowest.

3. CRITICAL EVALUATION OF THE BUREAU OF LABOR STATISTICS', BRADSTREET'S, AND DUN'S INDEX NUMBERS.

A few additional remarks are called for on the relative merits of the three general-purpose index numbers now regularly published in the United States.

In the publication of actual prices, the Bureau of Labor Statistics' and Bradstreet's stand foremost. The contribution they have thus made to the knowledge of prices possesses great and permanent value over and above the value attaching to their index numbers. For, it is well to repeat, all efforts to improve index numbers, all investigations into the causes and consequences of price fluctuations, and all possibility of making our pecuniary institutions better instruments of public welfare depend for their realization in large measure upon the possession of systematic and long-sustained records of actual prices. And much of this invaluable material would be lost if it were not recorded month by month and year by year.

Critical users of statistics justly feel greater confidence in figures which they can test than in figures which they must accept upon faith. Hence the compilers of index numbers who do not publish their original quotations inevitably compromise somewhat the reputations of their series. They compromise these reputations still further when they fail to explain in full just what commodities they include, and just what methods of compilation they adopt. Bradstreet's index number suffers a bit in comparison because readers are not told which 96 commodities out of the 106 for which prices are published are included in the index number, and because the method of reducing prices by the yard, the dozen, the bushel, the gallon, etc., to prices per pound is not fully explained. Dun's index number is more mysterious still, because neither the list of commodities nor the weights applied to each commodity are disclosed.

The number of commodities now included in the three series is given as follows by the compilers: Bureau of Labor Statistics' 328, Dun's "about 300," Bradstreet's 96. Provided the commodities are equally well chosen, of course the longer the list of commodities included the better claim has an index number to acceptance as a measure of changes in the general level of commodity prices.

The preceding study of the relations among the leading American index numbers was made in the winter of 1919–20, just before the great fall in prices began. Early in the course of this fall marked discrepancies appeared between the Bureau's series and both the commercial indexes. These discrepancies presently became wider than any that had appeared in the preceding 30 years. By December, 1920, Bradstreet's index was 22.4 per cent lower than the Bureau's index and Dun's was 10.9 per cent lower.[25]

[25] The following table continues, by months, from January, 1919, to May, 1921, the index numbers of the Bureau of Labor Statistics, Bradstreet, and Dun in the form given in Table 22:

Comparison of three American index numbers, by months, January, 1919, to May, 1921.

Year and month.	Bradstreet.	Dun.	Bureau of Labor Statistics.
1919:			
January	202.35	185.13	202.89
February	195.02	179.74	197.24
March	193.03	179.72	200.90
April	193.11	181.83	203.43
May	197.64	185.12	206.85
June	206.92	189.85	206.91
July	217.62	195.48	218.74
August	220.84	197.38	226.42
September	218.15	195.01	220.55
October	220.56	195.10	222.98
November	224.22	198.71	229.89
December	226.80	202.34	238.28
1920:			
January	230.68	206.08	243.59
February	233.09	208.39	249.19
March	232.22	210.10	253.97
April	231.87	214.34	268.80
May	227.19	216.09	272.14
June	219.46	214.89	268.62
July	213.60	214.95	263.20
August	205.89	209.95	249.89
September	195.16	199.69	241.93
October	182.30	191.03	225.36
November	163.93	180.45	207.33
December	147.08	168.70	189.44
1921:			
January	140.05	158.09	177.92
February	135.58	151.23	167.41
March	130.02	146.53	161.83
April	124.17	140.26	153.72
May	119.93	136.80	151.09

These wide discrepancies mean, not that the index numbers had become suddenly worse, but that the diversity among price fluctuations had become greater, so that differences among index numbers in respect to the number of commodities included and methods of weighting produced wider differences in the results. In other words, we have here the demonstration of a significant fact about price fluctuations: The great drop of prices in 1920–21 was characterized by much more irregularity in the promptness and degree of readjustment of different markets to the new situation than was the great rise of prices in 1915–1919. Presumably these great irregularities will prove to be a feature of the transition period only, and the three index numbers will approach one another again as the readjustments are gradually worked out in all markets.

With reference to weighting, Bradstreet's index number takes low rank, for the plan of reducing all quotations to prices per pound grossly misrepresents the relative importance of many articles. That figures made thus should give results in close agreement with the Bureau of Labor Statistics' series is really remarkable and proves that if prices, the raw materials from which index numbers are made, are accurate the particular method used in computing the index number is of secondary importance. Dun's system of weighting is distinctly better than Bradstreet's in theory. Whether the practice is as good as the theory is doubtful, for anyone familiar with the deficiencies of American statistics of consumption must wonder whence the compilers derive their estimates of the quantities of "about 300" commodities "annually consumed by each inhabitant." Moreover, what little is known concerning the actual weights is not unobjectionable. Fifty per cent of the total is too large a weight to allow to foods in a wholesale-price series. Even in the great collection of budgets of workingmen's families made by the Commissioner of Labor in 1901 the average expenditure for food was less than 45 per cent of total family expenditure, and in 1918 it was found to be only 38.2 per cent.[26] The bureau's practice of weighting wholesale prices by the quantities of commodities that enter into trade is preferable to weighting by consumption. Moreover, the bureau publishes its weights, and shows each year the percentage which each weighted price makes of the total for the group in which the commodity is put, as well as of the total for all commodities.

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[26] Eighteenth Annual Report of the Commissioner of Labor, 1903, p. 66. The data represented 25,400 families and 124,108 persons, both natives and immigrants. Also the Monthly Labor Review of the Bureau of Labor Statistics, August, 1919, p. 118. The data represented 12,096 white families in 92 industrial centers.

In the form of presenting results, Bradstreet's set an admirable example, which was wisely followed by Dun's. Their sums of actual prices can readily be turned into relatives on any base desired, and hence can be made to yield direct comparisons between any two dates. The bureau's series shares this advantage, since it too is made by adding actual prices multiplied by weights; but it is presented in a form more convenient for comparison than the other two series. The relatives on the scale of 100, into which the bureau throws its figures in the last step of compilation, are easier to use than the awkward sums of dollars and cents which Dun's Review and Bradstreet's publish.

It is interesting, finally, to test the reliability of the several index numbers as "business barometers." Monthly figures would be better for this purpose than our yearly averages, but since they are not available for all three series in the 1890's, we must do the best we can with the rougher gauge. In 17 of the 26 years since 1892 (when Bradstreet's index in its present form begins), the three series agree concerning the direction in which prices were moving; they differ in nine years. In the following schedule these nine years are represented by columns in which each index number is credited with +1 when its change accords with the character of the alteration in business conditions, debited with −1 in case of disagreement, and marked ±0 when it recognizes no appreciable change in the price level.[27] The net scores made by casting up the plus and minus entries indicate roughly the relative faithfulness with which these series have reflected changes in business conditions in the past quarter of a century.

Index number.	1893	1895	1897	1901	1903	1904	1905	1913	1914	Net score.
Bradstreet's	+1	−1	+1	+1	−1	+1	+1	±0	+1	+4
Bureau of Labor Statistics'	−1	±0	+1	+1	−1	±0	±0	+1	+1	+2
Dun's	−1	−1	−1	−1	+1	−1	±0	+1	−1	−4

Of the three indexes, Bradstreet's makes the best showing. Presumably the poor quality of Dun's index as a business barometer is due chiefly to the heavy weight (50 per cent) which it ascribes to foods. For foods are largely farm crops whose prices in a given year depend at least as much upon the weather as upon the condition of business. The bureau's series in this respect stands intermediate between the two commercial series, giving a lighter weight to foods than Dun's and a heavier weight than Bradstreet's. Probably that is why it is a better business barometer than the one and not so good as the other.

Of course this conclusion that Bradstreet's index number is a better business barometer than the bureau's series does not invalidate the preceding conclusion that the bureau's series is the best measure of changes in the general level of prices. For when farm crops are given their due weight in an index number, it is not to be expected that the index will always rise with business prosperity and decline

[27] For a summary of the changes in business conditions during these years, see Business Cycles, by Wesley C. Mitchell, p. 88.

with business depression. In making a wholesale price index number for use as a business barometer, indeed, one should exclude altogether commodities whose price fluctuations are determined largely by the weather. We have no such series at present, and it is high time that this lack should be supplied. But if some one does make a wholesale price index that is a nearly infallible business barometer, it will not be as reliable a measure of changes in the general level of prices as the present Bureau of Labor Statistics series.

VI.—CONCLUSIONS.

1. Variations in the level of wholesale prices from one year to the next are capable of being measured by a close approximation to accuracy, for these variations are highly concentrated about a central tendency. There are two American chain index numbers which for a quarter of a century never differ by more than 5 per cent, and differ on the average by only 2 per cent, although they were compiled from start to finish quite independently of each other, based upon dissimilar sets of price variations, constructed by unlike methods, and covered a period of violent fluctuations.[28]

2. Variations in prices that have been cumulating through several or many years show much less concentration about a central tendency than variations from one year to the next. Hence, index numbers become less accurate the greater the time over which they are extended. Nevertheless, the discrepancies observed between the two series just referred to (Dun's and the Bureau of Labor Statistics' new series of index numbers) do not reach 8 per cent in a period of 26 years, and average 3.4 per cent. The coefficient of correlation between these two series in 1892–1914 is +0.992, a close approach to +1.0, the expression of perfect agreement.

3. The choice of methods to be employed in making an index number should be guided by the purpose for which the results are to be used. These purposes are so numerous and so diverse that it is impossible to make a single series well adapted to them all. Probably the time is near when certain uses will be so standardized that several divergent types of index numbers will be regularly compiled to serve the needs of various groups of users. Even now we have special index numbers of the prices of foods, of farm products, of metals, etc. To this list there might well be added a series especially designed to throw changes in business conditions into high relief, and assist in the bettering of business forecasts. Most of the currently

[28] These figures are computed from Table 19, by turning the percentages by which each index number rose or fell each year into relatives on the preceding-year base and computing the percentage differences between the resulting indexes. The results for three series are as follows:

Index numbers.	Average difference.	
	1893–1914.	1893–1918.
	Per cent.	*Per cent.*
Bureau of Labor Statistics and Bradstreet's	2.26	2.82
Bureau of Labor Statistics and Dun's	1.95	2.00
Bradstreet's and Dun's	2.92	3.15

published index numbers, however, are what may be called general-purpose series, which undertake to measure changes in the wholesale price level at large.

4. The best form for these general-purpose series is a weighted aggregate of actual prices or a weighted geometric mean. The latter is preferable for measuring average ratios of change in prices; the former is preferable for measuring average change in the amount of money required to buy goods.

5. The more commodities that can be included in such an index number the better, provided that the system of weighting is sound. Certainly, each of the following classes of commodities should be represented, and represented as fully as is feasible: Raw mineral, forest, animal, and farm products, and manufactured products in various stages of elaboration, bought for family consumption and for business use.

6. Probably the best weights to apply are the average physical quantities of the commodities bought and sold over a period of years without reference to the number of times their ownership is changed. In making an aggregate of actual prices these weights should be applied directly to the quotations of each commodity in making up the totals for the several groups that have been mentioned, and then, if the necessary data can be secured the totals for the several groups should be weighted again in making up the grand totals for "all commodities."

7. In presenting such an index number, it is well to publish the aggregate actual prices, both for the several groups and for the grand totals. But it is highly desirable to publish also relatives made from these actual prices on a percentage scale, since comparisons can be made more easily from such figures than from the aggregates of actual prices, which are likely to run in awkward quantities. Indeed, several sets of these relatives, computed on the basis of actual prices at different times, can readily be provided for readers interested in knowing how prices have changed with reference to recent or to past years. Among the relatives of greatest significance is the set which shows the annual percentage of rise or fall as compared with prices in the preceding year. In such chain index numbers it is usually possible to include some commodities for which quotations are lacking in certain of the years covered by the whole investigation.

8. Chain index numbers are best made by the "ideal" formula, when the chief aim is to attain the greatest possible accuracy in measuring fluctuations from one year to the next. But when the annual percentages of rise or fall in prices made in this way are forged into a continuous series, their errors cumulate and vitiate comparisons between the earlier and the later years. Such series are also faulty for some purposes in that one can not tell what part of the net results is due to changes in prices and what part to changes in the quantities used as weights. When the chief aim is to forge a chain which will yield reliable comparisons between prices in any two years it is best to use constant weights and make aggregates of actual prices or geometric means of price fluctuations, the choice turning once again upon the specific purpose in mind.

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9. While index numbers are a most convenient concentrated extract of price variations, they are far from being a competent representation of all the facts which they summarize. Most "consumers of statistics" lack the time to go back of the finished products to the data from which they are made. But the increase of knowledge concerning the causes and consequences of price variations depends much more upon intensive study of the ultimate data than upon the manipulation of averages or aggregates. Upon the extension of knowledge in this field depend in turn large issues of public welfare. Hence it is highly important to collect and to publish in full the actual prices of as many commodities as possible, even though some of the quotations may not now be available for use in making an index number.