Research Department
Federal Reserve
Bank of
San Francisco

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FRB SF Weekly Letter

Risk and Duration

During the last year and a half, individuals have lived with considerably more interest-rate variability than they had experienced in the previous twenty years. The three-month Treasury bill rate in 1980 fluctuated between a high of 16.7 percent and a low of 6.5 percent—about a 10-percentage-point spread, compared with a high-low spread of less than 4 percentage points in 1979. Moreover, long-term interest rates were over four times more variable in 1980 than they were (on average) during the 1960-79 period, on the basis of rate movements in the representative 20-year Treasury bond (Chart 1).

The reasons for the increased interest-rate variability—especially for long-term rates—still remain somewhat of a puzzle. Rather than hold a postmortem on why interest rates were so variable in 1980, let us consider the relationship between interest-rate variability and "risk," and the relationship between risk and the maturity of U.S. government debt. The recent change in these relationships suggests that the average maturity of government debt may be a misleading indicator of the availability of long-term debt to the public. Secondly, the change suggests that the risk of holding government bonds in terms of price variability depends on an assessment of a bond's "duration," and not simply its maturity.

Risk factors

The first question facing an investor in a debt security is whether to purchase a short- or long-term asset. In answering this question, the individual must balance two types of risk—price risk and coupon risk. Price risk is simply the risk that the market value of the security will change while the individual holds the asset, given the chance that he may want/have to sell the security before maturity. Consider, for example, the individual who purchased a three-year Treasury security in July 1980 at 9.3 percent, and sold it in December when its yield rose to 13.7 percent. Since bond yield and price are inversely related, the investor in that case would have suffered a substantial capital loss.

Coupon risk, on the other hand, is the risk a short-term investor faces when reinvesting his money at some uncertain interest rate in the future. It would be to the individual's advantage to know what the trade-off is between coupon risk and price risk, since the two move inversely to one another (a rise in interest rates reduces current market price but increases the return on reinvested funds). This requires, however, that the investor estimate what interest rates might be in the future.

Yield curve

To develop such an estimate, economists often focus on the "term structure" of interest rates—the relationship between the market yield on a security and the number of years to maturity. Typically economists focus on Treasury debt, since its yield is not affected by the risk of default. An efficient market generally will provide approximately the same yield for a long-term bond as for a sequence of short-term securities held over the same time period, assuming that investors do not demand any premium for holding a less liquid long-term security rather than a more liquid short-term asset.

Some economists argue that the current long-term bond rate is an average of current and expected future short-term rates. Hence, a rising yield curve implies that the market expects short-term rates to rise in the future. That is, if long-term rates are higher than short-term rates, expected *future* short-term rates have to be higher than *current* short-term rates to preserve the equality of returns between the two investments. This so-called "expectation theory" of the term structure is widely employed by financial analysts and economists who compute "forward rates"—implied future short-term interest rates—from the relationship between current short-term

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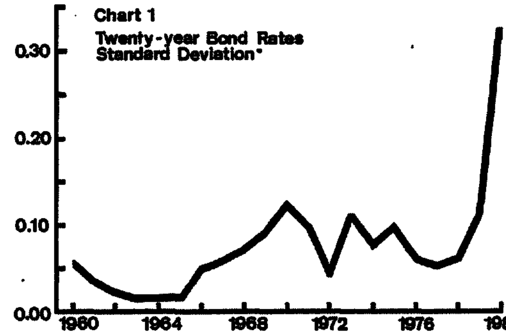


*Standard deviation in weekly average change in U.S. Government 20-year bond yield, calculated for each annual period 1960-1980.

and long-term interest rates. The yield curve thus contains information on the expected movement of short-term interest rates.

In evaluating the potential price risk of holding long- versus short-term securities, many economists argue that for a given yield change a bond price change generally will be greater, the greater is the time to maturity. This, however, is not always the case. Moreover, the change in price due to a change in yield, for a given maturity, will not always be the same. This means that the prices of longer-term securities are not always more sensitive than prices of shorter-term securities to a given change in interest rates. The relationship between price change and the movement in interest rates can be misunderstood, because the actual maturity of a bond only tells the investor when the final payment is due, and conveys little information on the time pattern of payments to be received over the life of the bond. We need to recall that the higher the market yield, the more important to the bond's present value are the coupon payments relative to the final payment at maturity. This information is ignored if we focus only on the yield-maturity relationship.

Macaulay's duration

In his monumental study of interest rates, Frederick R. Macaulay (1938) attempted to distill a bond time structure from a comparison of bonds with different maturities, coupon payments and market yields. To accomplish this task, Macaulay introduced the concept of "duration"—the average life of a debt instrument where the time between the present and each future payment is weighted by the present value of the respective coupon or principal payments, relative to the bond's current price. Duration thus measures the weighted average of the future periods during which the bond generates coupon and principal payments, where the weights are the marginal present-value contributions of all payments received.

Consider, as an example, the case where interest rates rise. In that case, the present value of a future payment falls, and the present value of a payment far in the future falls relatively more than a near-term payment. Hence the bond's duration falls. Similarly, when interest rates fall, the bond's duration lengthens. Analysts miss this information on a bond's time profile if they consider simply the bond's maturity or term structure of rates.

The advantage of focusing on *duration* rather than *maturity* can be summarized with one simple point. For a given change in interest rates, the price of a bond will vary proportionately (and inversely) with its duration but not proportionately with its maturity. For example, if a bond has a duration of five years, a one-percentage point increase in market yield from 10 to 11 percent will result in a decline in market price equal to the duration divided by one plus the initial market yield, or about 4.5 percent.* Thus, price risk and duration are intimately linked—as economists and financial analysts are only now beginning to appreciate.

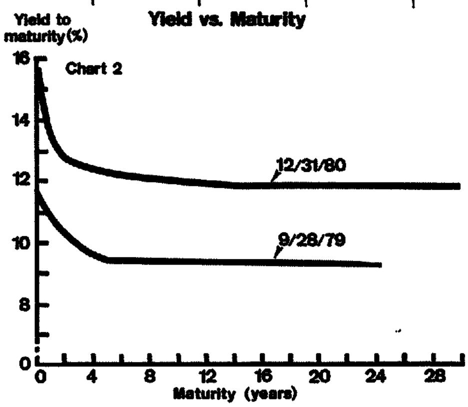
Since duration lengthens when interest rates fall and shortens when interest rates rise, we may paraphrase Gertrude Stein's comment on roses to say that a one-percentage point change in interest rates is *not* a one-percentage point change in interest rates. That is, the same change in interest rates generates a larger change in market price when interest rates are low than when they are high, because a given bond will have a longer duration at low than at high interest rates. Thus to understand the riskiness of a bond, an investor must know its duration and not simply its maturity.

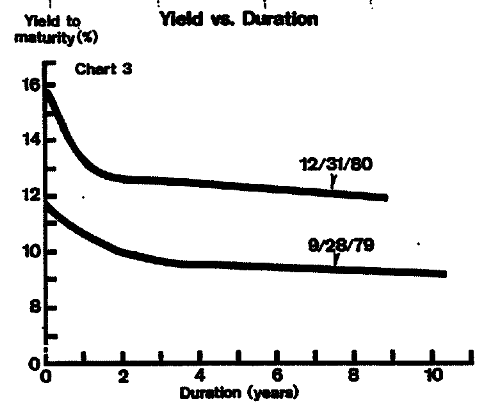
Macaulay's duration concept is valuable because it tells an investor how to convert a risky asset into essentially a much less risky

*Algebraically this may be written as:

$$\text{Change in bond price (\%)} = -\frac{\text{Duration}}{(1 + r)} \times \text{(Change in market yield)}$$

where r is the initial market yield; change in market yield in basis points divided by 100.





asset, by balancing price risk against coupon reinvestment risk. Abstracting from default risk and taxes, an investor can "immunize" his rate of total return from effects of unexpected interest-rate movements if he chooses a portfolio whose duration (and not maturity) exactly equals his desired holding period. The investor with an immunized portfolio balances the cost of suffering a decline in the prices of his assets with the benefit of being able to reinvest in higher earning assets—or inversely, if interest rates decline. By acting in this way, the investor guarantees that the yield expected at the time of the initial investment is actually obtained when the portfolio is liquidated.

The duration concept is readily applicable to a financial institution's risk sensitivity. The institution whose assets have a longer duration than its liabilities will lose when interest rates rise and gain when interest rates fall. The institution's risk thus depends on the time profile—duration—of its balance sheet. An institution can insulate itself from movements in interest rates if its asset- and liability-payment streams have the same weighted-average duration.

Duration and term structure

Consider now the effect of interest-rate movements on the *yield-maturity* (Chart 2) and *yield-duration* (Chart 3) relationships for Treasury securities. (These charts plot data for all Treasury securities outstanding except bills and "flower bonds," special Treasury securities with estate-tax benefits.) Chart 2 shows a similar-shaped yield-maturity structure, both downward sloping, for the dates September 28, 1979, and December 31, 1980. The term structure between those two dates appears to have been "lifted" by about 3½ percentage points. Also, the term-structure relationship for December 1980 appears sharply downward sloping, possibly suggesting an expectation of a rapid downturn in short-term interest rates. In contrast, the yield-duration relationship (Chart 3) shows a much shorter and slightly steeper yield curve, particularly in late 1980. Between September 1979 and December 1980, the longest duration bond went from about 10.4 to 9.0 years. This reduction in duration implies that, for long-term bonds, a one-percentage point change in interest rates would have resulted in about 16 percent less bond-price variability in December 1980 than in September 1979.

Higher interest rates in late 1980 thus essentially shrunk the yield curve in the yield-duration relationship. On the other hand, the yield-maturity relationship could be misleading, since by late 1980 it displayed an extension of the term-to-maturity structure, as well as a flatter yield curve than that shown in the yield-duration relationship. Government securities with longer maturities were available in late 1980, but the true availability of "long-term" government securities was *less* then than in the fall of 1979. Moreover, forward rates—expected future short-term rates—could be significantly different in the two different relationships.

The weighted average *duration* of privately-held Treasury debt (excluding Treasury bills) was about three years in late 1980, compared with a weighted average *maturity* of about five years. (The weights were volumes outstanding measured at face value.) This fact reinforces the argument about the short-term nature of the Federal government's debt. Thus, previous studies of the effects of Treasury debt management on interest rates may be in serious error, because of their focus on the average maturity rather than average duration of Treasury debt. Duration and not maturity gives a much truer picture of the time profile of Treasury debt obligations—and of the riskiness of potential interest-rate changes.

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BANKING DATA—TWELFTH FEDERAL RESERVE DISTRICT

(Dollar amounts in millions)

Selected Assets and Liabilities Large Commercial Banks	Amount Outstanding 3/18/81	Change from 3/11/81	Change from year ago Dollar	Change from year ago Percent
Loans (gross, adjusted) and investments*	146,834	416	8,294	6.0
Loans (gross, adjusted) — total#	124,400	473	7,986	6.9
Commercial and industrial	36,641	351	2,123	6.2
Real estate	51,318	– 2	6,086	13.5
Loans to individuals	23,399	– 25	– 1,076	– 4.4
Securities loans	1,409	– 37	381	37.1
U.S. Treasury securities*	6,768	– 53	43	0.6
Other securities*	15,666	– 4	265	1.7
Demand deposits — total#	40,969	– 439	– 2,506	– 5.8
Demand deposits — adjusted	28,813	– 945	– 1,885	– 6.1
Savings deposits — total	30,008	100	2,687	9.8
Time deposits — total#	77,252	217	16,172	26.5
Individuals, part. & corp.	68,219	290	15,757	30.0
(Large negotiable CD's)	29,904	137	8,303	38.4

Weekly Averages of Daily Figures	Week ended 3/18/81	Week ended 3/11/81	Comparable year-ago period
Member Bank Reserve Position			
Excess Reserves (+)/Deficiency (–)	n.a.	n.a.	20
Borrowings	29	40	263
Net free reserves (+)/Net borrowed(–)	n.a.	n.a.	– 243

* Excludes trading account securities.
# Includes items not shown separately.

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