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Abstract

The analysis in Ball and Romer [1991] suggests that models with fixed costs of changing price may be rife with multiple equilibria; in their static model price adjustment is always characterized by strategic complementarity, a necessary condition for multiplicity. We extend Ball and Romer's analysis to a dynamic setting. In steady states of the dynamic model, we find only weak complementarity and no evidence of multiplicity, although nonexistence of a symmetric steady state with pure strategies does arise in a small number of cases.

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1. Introduction

Dynamic models with monopolistic competition and sticky prices are one of the leading areas of quantitative research aimed at understanding the real effects of monetary policy.[1] Such models have emerged from two strands of the literature: the New Keynesian approach, whose hallmark is imperfect competition, and the real business cycle approach, whose hallmark is dynamic general equilibrium. While most of the recent work does not model the primitives which cause prices to be sticky, fixed costs of price adjustment ("menu costs") are a common explanation. Models in which monopolistically competitive firms face such costs hold considerable promise for improving our understanding of business cycles, as well as for evaluating the effects of monetary policy.

Models with monopolistic competition and fixed costs of price adjustment may, however, exhibit multiplicity in the degree of equilibrium price rigidity. Ball and Romer [1991] argued that if a firm anticipates that other firms will have sticky prices, then sticky prices may be privately optimal, but if the firm expects others to have flexible prices, then price adjustment may be privately optimal. The extent of price stickiness – and hence the effect of aggregate demand shocks on prices and output – would then be indeterminate. Ball and Romer argue that multiple equilibria enable these models to better explain phenomena such as the varying degrees of nominal rigidity observed across countries. At the same time, however, multiplicities of equilibria might weaken the models' predictive power, and hence limit their usefulness for studying monetary policy. Because dynamic models with sticky prices and monopolistic competition have typically taken price stickiness as exogenous, there has been no role in them for the type of coordination failure described by Ball and Romer in a static setting.

In this paper, we therefore examine the issue raised by Ball and Romer using the Dotsey,

[1] See, for example, Yun [1996], Rotemberg and Woodford [1997], Chari, Kehoe and McGrattan [1996].

King and Wolman [1999] state-dependent pricing model. This is a dynamic monopolistic competition model in which firms face a distribution of menu costs, and there is thus an endogenous degree of price stickiness.[2] The experiment Ball and Romer analyze is a one time change in the money stock, in a model with only one decision period.[3] They find that in the face of such a change, there is complementarity in price-setting: the higher is the fraction of firms adjusting their prices, the more likely it is that an individual firm will adjust its price (the reaction function for price adjustment slopes up). Such complementarity is a necessary condition for multiple symmetric equilibria (Cooper and John [1988]).

In a dynamic setting, there are two possible counterparts to the multiple equilibria identified by Ball and Romer. First, a dynamic model might possess multiple steady states, so that economies identical in terms of their primitives might exhibit different degrees of price rigidity in steady state. Second, a dynamic model might exhibit multiplicity away from steady state, meaning that an unanticipated shock might be consistent with multiple paths back to steady state. We address the first kind of multiplicity in this paper, and return briefly to the second kind in the conclusion.

By restricting attention to steady states, we are able to derive a reaction function with

[2] The framework we use differs from the seminal menu cost models of Caplin and Spulber [1987] and Caplin and Leahy [1991] in that there is a nondegenerate distribution of fixed costs. In the framework of their earlier paper, Caplin and Leahy [1997] also discuss strategic complementarity when there is state-dependent pricing. Caplin and Leahy's model is in continuous time (we work in discrete time), and the extent of complementarity is described by a reduced-form parameter which they do not explicitly link to an underlying structure. Strategic complementarity does not lead to multiple equilibria in Caplin and Leahy's model, apparently because of rich heterogeneity in the initial relative price distribution.

Loyo (1998) uses a model that is closer to ours to study the effect of money shocks under the fiscal theory of the price level. Loyo's model mixes state- and time- dependent pricing; after one period, firms choose whether to incur a cost and adjust, but they can adjust costlessly after two periods.

[3] Our discussions of Ball and Romer [1991] will be referring specifically to their section 3.A, where heterogeneity is introduced in the form of random menu costs.

a similar interpretation to that in Ball and Romer [1991]. Our reaction function effectively gives the constant probability of price adjustment that an individual firm would choose, as a function of the common adjustment probability chosen by all other firms.[4] Although price adjustment can exhibit complementarity in this model – the reaction function can slope up – we find no evidence of multiple steady states except for implausibly low values of the discount factor. The complementarity in the model can be seen as working through two mechanisms. First, there is the effect of the aggregate adjustment probability on other economywide variables (the real wage, demand), and second there is the effect of those economywide variables on the inducement for an individual firm to adjust. Both of these mechanisms are weak in a dynamic setting, preventing multiplicity from occurring.

While multiple steady states do not occur for the large number of parameterizations we examine, we do find limited evidence of nonexistence of symmetric steady states with pure strategies. This nonexistence is symptomatic of a discontinuity in the reaction function, which occurs when feedback from a firm's adjustment probability to its value causes it to jump between local maxima of its steady state value function as the aggregate adjustment probability changes.

The paper proceeds as follows. Section 2 describes the model. Section 3 derives the equations characterizing steady states, and section 4 studies the steady state reaction function as extensively as possible using analytical methods. In section 5 we turn to computational analysis, describing the number of steady state equilibria for a range of parameterizations. Section 6 concludes.

[4] These probabilities do not represent mixed strategies. Rather, they correspond to cutoff values of the menu cost, below which a firm would choose to adjust its price.

2. The Model

Although our paper takes much of its motivation from Ball and Romer's insight that menu costs can be a source of complementarity, we do not simply present a dynamic version of their model. Our aim is to provide a model that fits directly into the current literature on dynamic models with monopolistic competition. Other than the fact that our model is dynamic, there are three significant differences between our framework and that of Ball and Romer. First, they peopled their economy with "yeoman farmers" who produced goods using their own labor; by contrast, we assume firms who hire labor in an economy-wide labor market. Ball and Romer also assumed that preferences were linear in consumption, whereas we assume more general preferences. Finally, Ball and Romer assumed that the cost of changing prices was a direct utility cost that was separable from the cost of production. We assume that changing prices has an explicit labor cost.

Our model is a simplified version of that in Dotsey, King and Wolman [1999]. There, nominal rigidity is introduced into a monopolist competition framework through the assumption of fixed costs of price adjustment, as in Blanchard and Kiyotaki [1987]. Because the menu costs are random, however, the model can be easily analyzed in a general dynamic, stochastic setting.[5] The size of the state space is limited by the number of different prices charged, and positive inflation together with bounded costs that are uncorrelated over time means that a finite number of prices are charged. Here, we make further assumptions that guarantee only two prices are charged in steady state; equivalently, an individual firm keeps its price fixed for no more than two periods. This additional simplification permits us to obtain some analytical insight into the likelihood of multiple equilibria.

[5] Random menu costs also enable the model to match qualitatively the empirical fact that while price changes are infrequent, they also vary widely in magnitude.

2.1. Consumers

An infinitely-lived representative consumer has preferences over consumption (c_t) and labor (n_t), where consumption is a nonlinear CES aggregate of differentiated products:

$$c_t = \left[\int_0^1 c_t(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\varepsilon/(\varepsilon-1)}, \varepsilon > 1. \tag{2.1}$$

The consumer maximizes the present discounted value of utility,

$$\sum_{j=0}^{\infty} \beta^j u(c_{t+j}, n_{t+j}), \tag{2.2}$$

where $u(c, n)$ is a standard utility function. The consumer sells labor in a competitive labor market for the real wage (w_t). The consumer's sources of income are labor earnings and dividend payments from firms ($\tilde{\Pi}_t$), and all income is used up each period purchasing consumption goods from firms:

$$c_t = w_t n_t + \tilde{\Pi}_t. \tag{2.3}$$

From the consumer's perspective, the model is non-dynamic, in that there is no vehicle for saving.[6] However, since the consumer owns the firms, the appropriate discount factor for firms' future profits is the marginal utility of consumption, and as will become clear below, firms do face a dynamic problem.

We introduce money demand in the simplest possible way, by assuming that velocity is unity. Thus,

$$\int_0^1 P_t(z) c_t(z) dz = M_t, \tag{2.4}$$

where $P_t(z)$ is the price of good z and M_t is the aggregate money stock. The advantage of this assumption is simplicity. Later, we will show that our main findings go through if we instead derive money demand from a cash-in-advance constraint.

[6] It is straightforward to allow for capital accumulation in this model (see Dotsey, King and Wolman [1997]). We study a model without capital in order to keep the analysis as tractable as possible.

2.2. Firms

Each firm, z, possesses a linear technology for producing output ($y_t(z)$):

$$y_t(z) = (n_t(z) - n_t^p(z)), \tag{2.5}$$

where $n_t(z)$ is total labor employed by the firm, and $n_t^p(z)$ is labor employed in price adjustment. Given the preferences specified for the consumer, each firm faces a demand curve

$$c_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} \cdot c_t, \tag{2.6}$$

where

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz\right]^{1/(1-\varepsilon)}. \tag{2.7}$$

Firms meet all demand at the price they set, so

$$y_t(z) = c_t(z) \; \forall z. \tag{2.8}$$

An individual firm's cost in labor units of changing its price, ξ, is assumed to be drawn from a continuous distribution, $F(\xi)$, on $[0, B]$. Firms receive independent draws from this distribution each period. In general, there will therefore be a distribution of firms distinguished by when they last adjusted prices. To keep the analysis as transparent as possible, we place restrictions on the parameters to guarantee that all firms adjust their price after at most two periods. Essentially, this entails assuming that the inflation rate is sufficiently high or that the highest possible cost of adjusting (B) is sufficiently low, so that even a firm that drew B every period would not hold its price fixed for more than two periods. We are not directly imposing a condition that firms cannot keep prices fixed for longer than two periods; rather, we are restricting parameter values so that it is optimal for firms to behave in this way.[7]

[7] There is no difficulty in principle in relaxing this restriction. It is computationally intensive, however, given that we characterize the behavior of the model in a relatively large parameter space.

Each firm, upon observing its cost of changing prices, either leaves its price at the level in effect in the previous period, or adjusts to its optimal price. Since a newly set price may remain in effect the next period, adjusting firms do not simply maximize current profits, and the gain from adjusting depends on the expected future state of the economy. If a firm last set its price two periods ago, our assumptions guarantee that it will choose to adjust regardless of its cost realization. Therefore we need focus our attention only on the adjustment decision of firms that adjusted their price last period; we let $\hat{\alpha}_t$ denote the ex-ante probability (i.e., before observing its menu cost) that such a firm chooses to adjust in period t.[8] All adjusting firms choose the same price, regardless of when they last adjusted, because other than price there are no firm-specific state variables.

Define real current profits and discounted present value at time t of a firm charging a price P^*_{t-j} set in period $t-j$ $(j = 0, 1)$ as $\pi_{j,t}$ and $v_{j,t}$, respectively. Profits are given by

$$\pi_{j,t} = c_t \cdot \left[\left(\frac{P^*_{t-j}}{P} \right)^{1-\varepsilon} - w_t \cdot \left(\frac{P^*_{t-j}}{P} \right)^{-\varepsilon} \right]. \tag{2.9}$$

Recalling that menu costs are incurred in the form of labor, a firm that set its price one period ago faces the following problem upon drawing cost $\tilde{\xi}$ in period t :

$$\max \left\{ v_{1,t}, \left(v_{0,t} - w_t \cdot \tilde{\xi} \right) \right\}, \tag{2.10}$$

where

$$v_{1,t} = \pi_{1,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \cdot \left[v_{0,t+1} - w_{t+1} E\left(\xi\right) \right], \tag{2.11}$$

$$v_{0,t} = \max_{P^*_t} \{ \pi_{0,t} + \\ \beta E_t \tfrac{\lambda_{t+1}}{\lambda_t} \left(1 - \hat{\alpha}_{t+1}\right) v_{1,t+1} + \hat{\alpha}_{t+1} \left(v_{0,t+1} - w_{t+1} E\left(\xi \mid \xi < F^{-1}(\hat{\alpha}_{t+1})\right) \right)] \}. \tag{2.12}$$

[8]Throughout the paper, $\hat{\alpha}_t$ will denote the adjustment probability of an individual firm, whereas α_t will denote the overall fraction of "one-period old" firms adjusting; in equilibrium these two objects will be the same.

In these expressions, λ_t denotes the representative agent's marginal utility of consumption in period t. Because individuals own the firms, $\beta\frac{\lambda_{t+1}}{\lambda_t}$ is the appropriate discount factor for future profits.

Equations 2.11 and 2.12 are easily interpreted. A firm that does not adjust in the current period knows that it will adjust in the next period. Its value is therefore equal to its current profits, $\pi_{1,t}$, plus the discounted value of an adjusting firm next period, $\beta\left(\lambda_{t+1}/\lambda_t\right)v_{0,t+1}$, minus the discounted value of expected adjustment costs next period, $\beta\left(\lambda_{t+1}/\lambda_t\right)w_{t+1}E\left(\xi\right)$. A firm that does adjust in the current period will obtain the current profits from its new price, $\pi_{0,t}$. It also knows that with probability $1-\hat{\alpha}_{t+1}$ it will not adjust next period, in which case it will have the discounted value of a non-adjusting firm $\left(\beta\left(\lambda_{t+1}/\lambda_t\right)v_{1,t+1}\right)$, and with probability $\hat{\alpha}_{t+1}$ it will adjust, in which case it will have the discounted value of an adjusting firm $\left(\beta\left(\lambda_{t+1}/\lambda_t\right)v_{0,t+1}\right)$, but will also incur a menu cost with an expected value that is conditional on adjustment $\left(w_{t+1}E\left(\xi\mid\xi<F^{-1}(\hat{\alpha}_{t+1})\right)\right)$. As noted above, $\hat{\alpha}_{t+1}$ is the ex-ante probability that a firm that adjusted its price in period t will adjust again in period $t+1$. An optimal adjustment policy will involve the firm choosing to adjust when the benefit of adjusting exceeds the adjustment cost it draws; thus its adjustment probability will simply be the probability of drawing a cost less than the benefit of adjusting:

$$\hat{\alpha}_{t+1}=F\left(\frac{v_{0,t+1}-v_{1,t+1}}{w_{t+1}}\right). \tag{2.13}$$

The first order condition for (2.12) and (2.11) yields an explicit expression for P_t^*; this is (A.1) in the appendix.

Notice that the firm makes two interconnected decisions. Each period it faces a *price-adjustment decision* of whether to retain the price from the previous period, or instead to incur the menu cost and set a new price. It also faces a *price-setting decision*: if it adjusts, what price should it choose? When making the price-setting decision, the firm recognizes that the price it sets in the current period may also be in force next period; furthermore, it recognizes that the price it sets will affect its decision about whether or not to adjust in the future. The price-adjustment decision, meanwhile, depends upon the profits the firm will earn if it adjusts, which of course depend upon the price that it chooses to set.

2.3. Monetary Policy

The model is closed by adding a policy rule for the monetary authority. We assume that the monetary authority simply picks an exogenous process for the money growth rate μ_t:

$$\mu_t = \frac{M_{t+1}}{M_t}. \tag{2.14}$$

Our focus in this paper is on inflationary steady states, so we set $\mu_t = \mu > 1, \ \forall \ t$.

2.4. Equilibrium

A rational expectations equilibrium for this economy involves (i) consumers maximizing utility; (ii) firms choosing cost-minimizing amounts of labor; (iii) firms that adjust their price doing so optimally; (iv) firms adjusting their price if and only if the gain from adjustment exceeds the cost; (v), consistency of individual decisions with aggregate outcomes; (vi) goods and labor market clearing; (vii) rational expectations, and (viii) nonnegative firm values.

3. Steady State

To analyze steady states of the model, we derive a reaction function that expresses an individual firm's choice of adjustment probability given the aggregate adjustment probability. This reaction function should be interpreted as follows. Consider a firm that is adjusting its price and takes as given that the economy is in a steady state corresponding to some arbitrary α (that is, an arbitrary adjustment probability for all other firms). Precisely because α is arbitrary this steady state may not be an equilibrium, but conditional on α all other equilibrium conditions hold. Firms set optimal prices conditional on α, and individuals optimally consume and work conditional on α. Now ask the firm what individual adjustment probability ($\hat{\alpha}$) it most prefers. The answer to this question gives the value of the steady state reaction function at α. Because the state of the economy is unchanging, the firm will in fact choose a constant $\hat{\alpha}$ policy, and thus any steady state equilibrium is a fixed point of this mapping, and vice versa.

To derive the steady state reaction function, we first examine the implications of optimal behavior by an individual firm, taking as given economywide variables. We then use the assumption of symmetric behavior across firms, combined with optimal pricing by adjusting firms and optimal consumption and labor supply behavior to show how the adjustment probability pins down economywide variables. Returning to the individual firm, we can then express its optimal adjustment probability as an implicit function of aggregate adjustment. This is the steady state reaction function.

3.1. An Individual Firm's Behavior

An individual firm's behavior consists of choosing (i) whether or not to adjust its price, and (ii) a price if it does adjust. The firm takes as given the real wage (w) and aggregate demand (c), which are constant in steady state. We first describe the optimal choice of a price, conditional on a constant adjustment probability, and then the optimal choice of that adjustment probability.

3.1.1. *Price Setting*

Suppose that an individual firm's adjustment probability is fixed at $\tilde{\alpha}$ in periods immediately following adjustment, and at unity in other periods. That is, the firm adjusts with certainty if it last adjusted two periods ago, and adjusts with probability $\tilde{\alpha}$ if it last adjusted in the previous period. We use the notation $\tilde{\alpha}$ to indicate that this adjustment probability is arbitrary. If the price level is increasing at rate μ – as it must be in a steady state with money growing at rate μ – then it is straightforward to show using (2.11) and (2.12) that an adjusting firm will set its relative price at[9]

$$\frac{P^*}{P} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \cdot g\left(\tilde{\alpha}; \beta, \mu, \varepsilon\right) \cdot w, \tag{3.1}$$

where

$$g\left(\tilde{\alpha}; \beta, \mu, \varepsilon\right) \equiv \left(\frac{1 + \beta(1 - \tilde{\alpha})\mu^{\varepsilon}}{1 + \beta(1 - \tilde{\alpha})\mu^{\varepsilon - 1}}\right). \tag{3.2}$$

In a static model, the markup of an adjusting firm would simply be the familiar $(\varepsilon/(\varepsilon-1))$. This is also the value of the markup in a steady state with complete discounting ($\beta = 0$) or in a steady state with complete price flexibility ($\tilde{\alpha} = 1$), since in either case firms only consider current profits when setting their price, and $g\left(\right) = 1$. As long as $\beta > 0$ and $\tilde{\alpha} < 1$, however (and given that inflation is positive ($\mu > 1$)), it follows that $g\left(\tilde{\alpha}; \beta, \mu, \varepsilon\right) > 1$: an adjusting firm's markup exceeds the static markup. Firms set their price in the knowledge that it may be in effect not just in the current period, but also in the next period when the general price level will be higher.

An adjusting firm's markup is increasing in inflation ($\partial g/\partial \mu > 0$), because inflation erodes any relative price set in the past, and price-setters counteract this effect by setting a higher price when they adjust. It is increasing in β ($\partial g/\partial \beta > 0$), because a price set one period ago will have eroded in real terms so that it is below the level which maximizes current profits; higher β penalizes this deviation more severely, so adjusting firms raise their markup in order to lessen the deviation. Finally, the markup charged by an adjusting firm is decreasing in $\tilde{\alpha}$ ($\partial g/\partial \tilde{\alpha} < 0$), because in this context, the probability of *not* adjusting plays exactly the same role as the discount factor. There is thus a link between the price-setting decision of the firm and the price adjustment decisions that it will make in the future.

[9] This is the steady state version of (A.1) in the appendix.

3.1.2. *Price Adjustment*

Now we take as given that adjusting firms will behave according to (3.1), and study their choice of how often to adjust. In steady state, the benefit of adjusting its price for a firm that adjusted last period is $(v_0 - v_1)/w$, where v_0 and v_1 refer to the steady state versions of (2.12) and (2.11). From the standpoint of an individual firm, v_0 and v_1 are functions of c and w. Thus, given values for c and w, if we knew the functions $v_0(\cdot)$ and $v_1(\cdot)$ we could use the adjustment cutoff rule ($\hat{\alpha} = F((v_0 - v_1)/w)$) to determine a firm's ex-ante adjustment probability. The analysis is significantly complicated by the fact that the functions $v_0()$ and $v_1()$ also depend upon $\hat{\alpha}$ (see (2.12) and (2.11)). It follows that to find a firm's optimal adjustment policy, we must simultaneously solve for its value.

We begin by defining firms' values for arbitrary adjustment policies:

$$\tilde{v}_0(c, w, \tilde{\alpha}) = \pi_0(c, w, \tilde{\alpha}) +$$
$$\beta \cdot [\tilde{\alpha} \cdot (v_0(c, w, \tilde{\alpha}) - w \cdot E(\xi | \xi < F^{-1}(\tilde{\alpha}))) + (1 - \tilde{\alpha}) \cdot v_1(c, w, \tilde{\alpha})], \tag{3.3}$$

and

$$\tilde{v}_1(c, w, \tilde{\alpha}) = \pi_1(c, w, \tilde{\alpha}) + \beta \cdot [v_0(c, w, \tilde{\alpha}) - w \cdot E(\xi)]. \tag{3.4}$$

The functions $\pi_0(c, w, \tilde{\alpha})$ and $\pi_1(c, w, \tilde{\alpha})$ are given by[10]

$$\pi_j(c, w, \tilde{\alpha}) = c \cdot w^{1-\varepsilon} \times$$
$$\left\{ \left(\mu^{-j} \cdot \left(\tfrac{\varepsilon}{\varepsilon - 1} \right) \cdot g(\tilde{\alpha}; .) \right)^{1-\varepsilon} - \left(\mu^{-j} \cdot \left(\tfrac{\varepsilon}{\varepsilon - 1} \right) \cdot g(\tilde{\alpha}; .) \right)^{-\varepsilon} \right\} \tag{3.5}$$
$$j = 0, 1,$$

where we have used (3.1) to substitute out for $\frac{P^*}{P}$. Combining (3.3) and (3.4) yields an expression for the value of an adjusting firm in terms of economywide variables and the

[10] To derive (3.5), simply impose a steady state on (2.9), remembering that P^* must grow at the rate μ.

firm's arbitrary adjustment probability:

$$\tilde{v}_0(c,w,\tilde{\alpha}) = \frac{\pi_0(c,w,\tilde{\alpha})+\beta\cdot(1-\tilde{\alpha})\cdot\pi_1(c,w,\tilde{\alpha})}{(1-\beta)\cdot(1+\beta\cdot(1-\tilde{\alpha}))} - \frac{\beta w\cdot\left[\tilde{\alpha}\cdot E\left(\xi|\xi<F^{-1}(\tilde{\alpha})\right)+\beta\cdot(1-\tilde{\alpha})\cdot E(\xi)\right]}{(1-\beta)\cdot(1+\beta\cdot(1-\tilde{\alpha}))}. \tag{3.6}$$

The numerator of (3.6) consists of two terms. The first term is profits when adjusting plus discounted expected profits when not adjusting, and the second term – subtracted – is the sum of appropriately discounted adjustment costs when a price is one and two periods old. With complete discounting ($\beta = 0$), the value of an adjusting firm would consist only of the numerator. However in general there are present value considerations, and these are reflected in the denominator. The first factor in the denominator is the usual divisor in present value expressions $(1-\beta)$. The second factor can be thought of as a correction which accounts for the fact that $\tilde{v}_0(c,w,\tilde{\alpha})$ is the present value of a two-period expected return rather than a one-period return.[11]

We can likewise solve for the value of a non-adjusting firm with an arbitrary constant adjustment probability ($\tilde{v}_1(c,w,\tilde{\alpha})$) using (3.4), and combine the solutions to yield an expression for the benefit that a firm derives by adjusting its price:

$$\tilde{v}_0(c,w,\tilde{\alpha}) - \tilde{v}_1(c,w,\tilde{\alpha}) = \frac{[\pi_0(c,w,\tilde{\alpha})-\pi_1(c,w,\tilde{\alpha})]-\beta w\cdot\left[\tilde{\alpha}\cdot E\left(\xi|\xi<F^{-1}(\tilde{\alpha})\right)-E(\xi)\right]}{(1+\beta\cdot(1-\tilde{\alpha}))}. \tag{3.7}$$

[11] Notice that when $\tilde{\alpha} = 1$, which means that firms adjust every period,

$$\tilde{v}_0(c,w,1) = \frac{\pi_0(c,w,1)}{1-\beta} - \frac{\beta w\cdot E(\xi)}{1-\beta}$$

That is, the value of the firm is the sum of the discounted stream of current profits minus the sum of the discounted stream of expected adjustment costs. By contrast, when $\tilde{\alpha} = 0$, firms adjust their prices every two periods, and

$$\tilde{v}_0(c,w,0) = \frac{\pi_0(c,w,0)+\beta\cdot\pi_1(c,w,0)}{1-\beta^2} - \frac{\beta^2 w\cdot E(\xi)}{1-\beta^2}$$

In this case the value of the firm is the stream of two-period profits and costs, discounted at the rate β^2.

From (3.7), the total benefit to adjusting has two components. First, there is the difference between profits from a changed price and profits from an unchanged price (below we will refer to this term as π_{DIF}). In addition, there is a term (to be referred to as C_{DIF}) reflecting the saving in expected adjustment costs next period that a firm achieves if it adjusts in the current period. The denominator of (3.7) reflects present value considerations as discussed above. Note that when firms adjust their prices every period ($\tilde{\alpha} = 1$), the cost difference is zero and the gain from adjusting is simply the profit difference; firms expect to incur the same adjustment cost next period irrespective of their adjustment this period. When firms only adjust their prices every two periods ($\tilde{\alpha} = 0$), the cost difference is positive – but both the profit difference and the cost difference are deflated by a factor of $(1 + \beta)$.

Because all of the functions on the right-hand side of (3.6) are known, it is straightforward to use (3.6) to find $\hat{\alpha}(c, w)$ and $v_0(c, w, \hat{\alpha}(c, w))$ by searching over a grid of $\tilde{\alpha} \in [0, 1]$:

$$\hat{\alpha}(c, w) = \arg\max_{\tilde{\alpha} \in [0,1]} \tilde{v}_0(c, w, \tilde{\alpha}). \tag{3.8}$$

$$v_0(c, w, \hat{\alpha}(c, w)) = \max_{\tilde{\alpha} \in [0,1]} \tilde{v}_0(c, w, \tilde{\alpha}). \tag{3.9}$$

Later – once we have expressed c and w as functions of the adjustment probability chosen by all firms – we will use this method to construct reaction functions numerically. Until then it will be more fruitful to focus on the necessary condition given by (2.13):

$$\hat{\alpha} \equiv F\left(\frac{v_0(c, w, \hat{\alpha}) - v_1(c, w, \hat{\alpha})}{w}\right), \tag{3.10}$$

which implicitly expresses the firm's adjustment decision (its choice of $\hat{\alpha}$) as a function of the aggregate real wage and the aggregate level of demand. The next step is to use the price index and the consumer's first-order condition to express these variables in terms of α, the economy-wide adjustment probability.

3.2. Aggregate Outcomes Conditional on α

Thus far we have concentrated on how an individual firm chooses its price and its pattern of adjustment, taking as given economy-wide variables – specifically demand (c) and the real wage (w). We now work from the other direction. Suppose that all firms have the same constant adjustment probability α, and that they set their prices accordingly, as in (3.1). The price index and the consumer's labor supply curve then determine the real wage and the level of aggregate demand as a function of α.

3.2.1. The Real Wage

Combining the steady state price index with individual firms' behavior allows us to derive a solution for the real wage in terms of the aggregate adjustment probability α. The steady state version of the price index (2.7) is given by

$$P = \left[\left(\frac{1}{2-\alpha}\right)(P^*)^{1-\varepsilon} + \left(\frac{1-\alpha}{2-\alpha}\right)\left(\frac{P^*}{\mu}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \tag{3.11}$$

Of those firms that did adjust last period, a fraction α also choose to adjust this period, so the total fraction of firms that adjust is given by $1/(2-\alpha)$, and the fraction of firms that do not adjust is given by $(1-\alpha)/(2-\alpha)$.[12] Those that do adjust set their price equal to P^*, while the remaining firms leave their price at P^*/μ. The general price level is therefore given by (3.11).[13]

[12] To see this, let ω be the total fraction of firms that adjust each period. Then

$$\omega = \alpha\omega + (1-\omega), \tag{3.12}$$

which yields the expressions in the text. Because all firms adjust after at most two periods, the fraction that adjust in a given period must lie between 1/2 and 1.

[13] To derive this expression, use (2.7), and note that $P_t(z) = P^*$ for $z \in (0, 1/(2-\alpha))$, and $P_t(z) = P^*/\mu$ for $z \in (1/(2-\alpha), 1)$.

Because firms *raise* their prices when they adjust, the ratio of the price level to the price chosen by adjusting firms is increasing in the fraction of firms adjusting. Another way of seeing this is to note that the price level must be growing at rate μ in steady state. Thus, if more firms are adjusting their prices, they must also be adjusting by a smaller amount. Rearranging (3.11),

$$P = P^{*} \cdot h(\alpha; \varepsilon, \mu), \tag{3.13}$$

where

$$h(\alpha; \varepsilon, \mu) = \left[\frac{1 + (1-\alpha)\mu^{\varepsilon-1}}{2-\alpha} \right]^{1/(1-\varepsilon)} < 1, \tag{3.14}$$

and $\partial h()/\partial\alpha > 0$, as was just noted.

In equilibrium, $\frac{P^*}{P}$ must be consistent with firms' price-setting decisions, as given by (3.1). The price index tells us that a higher aggregate adjustment fraction will reduce $\frac{P^*}{P}$ in steady state; if more firms are adjusting, then the aggregate price level will be closer to the price chosen by adjusting firms. From (3.1), we know that firms *choose* $\frac{P^*}{P}$ to be decreasing in their own adjustment probability and increasing in the real wage. Combining (3.13) with (3.1), which is now assumed to hold for all firms, we see that the steady state wage is an explicit function of the aggregate adjustment probability:

$$w(\alpha) = \left(\frac{\varepsilon - 1}{\varepsilon} \right) \cdot \frac{1}{g(\alpha) \cdot h(\alpha)}. \tag{3.15}$$

We summarize some of the properties of $w(\alpha)$ in the following lemma.

Lemma 3.1. *For $\alpha \in [0,1]$,(i) When β is sufficiently small, $w(\alpha)$ is decreasing in α;(ii) When β is sufficiently large, $w(\alpha)$ is first decreasing in α and then increasing in α.*

Lemma 3.2. *(see appendix for proof)*

When β is small, the real wage is decreasing in the adjustment probability. The intuition is as follows. When $\beta \simeq 0$, $g(\alpha) \simeq 1$; firms take no account of future periods when setting their current price, and so simply choose the static markup. The adjustment probability therefore affects the real wage only through its role as a weight in the price index. The price index requires that more frequent adjustment correspond to a lower ratio of $\frac{P^*}{P}$, but firms will only reduce their choice of $\frac{P^*}{P}$ if the wage is lower. When $\beta > 0$, that effect is still present (the $h(\cdot)$ function), but there is also an indirect effect that acts in the opposite direction. The relative price charged by adjusting firms is decreasing in the adjustment probability ($\partial g/\partial \alpha < 0$). When α rises, the real wage needs to fall less to induce firms to decrease P^*/P enough to make (3.13) hold. When β is sufficiently large, this effect more than offsets the price index effect, so that $w(\alpha)$ is increasing over some of its range.[14]

It is noteworthy that we are able to characterize $w(\alpha)$ purely by utilizing the equations describing firm behavior, together with the steady-state restrictions on the behavior of the price level. We now close the model by looking at the consumer's problem, and investigate how preferences over consumption and labor affect the level of aggregate demand.

3.2.2. The Consumer's Problem

Utility maximization by consumers will determine aggregate demand as a function of the adjustment probability α. The consumer's intratemporal first-order condition is

$$w = -u_n(c,n)/u_c(c,n); \tag{3.16}$$

because the consumer's problem is static, there are no additional optimality conditions. As just shown, the real wage is given as a function of α by (3.15). Labor input is the sum of labor used in price adjustment and labor used in final goods production. For both adjusting firms and non-adjusting firms, the demand curve and the technology together determine the amount of labor hired for final goods production, and the economywide aggregate is a

[14] The behavior of $w(\alpha)$ for intermediate values of β depends upon the other parameters of the model. Under some circumstances, there are values of β for which the wage is first increasing, then decreasing in α.

weighted sum:

$$n^y = \left(\frac{1}{2-\alpha}\right)c_0 + \left(\frac{1-\alpha}{2-\alpha}\right)c_1 \tag{3.17}$$

$$= c \cdot h(\alpha)^{\varepsilon} \cdot \left(\frac{1+(1-\alpha)\mu^{\varepsilon}}{2-\alpha}\right); \tag{3.18}$$

(3.18) is derived from (3.17) by replacing c_0 and c_1 with the appropriate demand curves, and eliminating the relative prices using (3.13).

Labor used in price adjustment has two components. First, all firms that did not adjust in the previous period will adjust in the current period, and the average adjustment cost for such firms – who make up a fraction $\left(\frac{1-\alpha}{2-\alpha}\right)$ of firms –is simply the mean of the distribution. Second, a fraction α of those $\left(\frac{1}{2-\alpha}\right)$ firms that did adjust in the previous period will also adjust in the current period, and their average adjustment cost is $\alpha^{-1} \cdot \int_0^{F^{-1}(\alpha)} \xi dF(\xi)$:

$$n^p(\alpha) = \left(\frac{1}{2-\alpha}\right) \cdot \int_0^{F^{-1}(\alpha)} \xi dF(\xi) + \left(\frac{1-\alpha}{2-\alpha}\right) \cdot \int_0^{B} \xi dF(\xi). \tag{3.19}$$

With $n = n^y(\alpha, c, w(\alpha)) + n^p(\alpha)$, and with $w(\alpha)$ given by (3.15),(3.20) implicitly characterizes $c(\alpha)$:

$$w(\alpha) = -\frac{u_n(c, n(\alpha, c, w(\alpha)))}{u_c(c, n(\alpha, c, w(\alpha)))}. \tag{3.20}$$

It is straightforward to differentiate (3.20) implicitly and thereby arrive at an expression for $\frac{\partial c}{\partial \alpha}$, but without specifying preferences one cannot sign $\frac{\partial c}{\partial \alpha}$. In an appendix we show how $\frac{\partial \ln c}{\partial \alpha}$ can be expressed in terms of $\frac{\partial w}{\partial \alpha}, \frac{\partial n}{\partial \alpha}, \frac{\partial n}{\partial w}, \frac{\partial n}{\partial c}$ and derivatives of the utility function, and we will look at an example in section 4.

3.3. The Steady State Reaction Function

For an individual firm that takes as given the constant adjustment probability of all other firms (α), a necessary condition for optimal choice of adjustment probability in steady state

$(\hat{\alpha})$ is that it be a solution to (3.10), where we can now incorporate the dependence of c and w on α :[15]

$$\hat{\alpha} = F\left(\frac{v_0\left(w(\alpha), c(\alpha), \hat{\alpha}\right) - v_1\left(w(\alpha), c(\alpha), \hat{\alpha}\right)}{w(\alpha)}\right). \tag{3.21}$$

Tracing out the solution to (3.21) for $\alpha \in [0, 1]$ yields the reaction function, and a steady state equilibrium is a fixed point of this reaction function. In their analysis of multiplicity in the static case, Ball and Romer emphasized the complementarity in price adjustment in their model: if more firms choose to adjust their price, then the incentive for an individual firm to adjust its price increases. In cases where the reaction function is continuous, a positive slope – that is complementarity – is a necessary condition for multiple steady states in our dynamic model, and it is clear that whether or not complementarity occurs depends in a complicated way on the cdf and on the functions $w(\alpha)$ and $c(\alpha)$. In the next section we will study the reaction function analytically, assuming differentiability, which is equivalent to assuming that (3.21) is a necessary *and* sufficient condition. This analysis will be suggestive as to whether there is widespread multiplicity of steady states. Then in section 5 we will turn to numerical analysis, using (3.8) and (3.9) to determine the number of steady state equilibria for a large number of examples. This method will be robust to discontinuities in the reaction function.

4. Feedback From Aggregate to Individual Adjustment

Our purpose in this section is to characterize the steady state reaction function (3.21) and the channels by which the aggregate adjustment probability (α) affects individual adjustment $(\hat{\alpha})$, assuming that the conditions for application of the implicit function theorem to (3.21) are satisfied.

[15] We have not proved that every solution to (3.21) is a solution to (3.8). For now we proceed as though (3.21) were also a sufficient condition.

4.1. The Slope of the Reaction Function

Without assuming functional forms for preferences or the distribution of adjustment costs, we can make significant progress toward simplifying (3.21). As a first step, define the *profit difference (π_{DIF})*,

$$\begin{aligned}\pi_{DIF}\left(c\left(\alpha\right),w\left(\alpha\right),\hat{\alpha}\right) &\equiv \pi_{0}\left(c\left(\alpha\right),w\left(\alpha\right),\hat{\alpha}\right)-\pi_{1}\left(c\left(\alpha\right),w\left(\alpha\right),\hat{\alpha}\right) \\ &= c\left(\alpha\right)\cdot\left(w\left(\alpha\right)g(\hat{\alpha})\right)^{1-\varepsilon}\left(\tfrac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}\left[\left(\tfrac{\varepsilon-1}{\varepsilon}\right)\left(\tfrac{1}{g(\hat{\alpha})}\right)\left(\mu^{\varepsilon}-1\right)-\left(\mu^{\varepsilon-1}-1\right)\right].\end{aligned} \tag{4.1}$$

and the *cost difference (C_{DIF})*,

$$C_{DIF}\left(\hat{\alpha}(c,w)\right)\equiv\hat{\alpha}(c,w)\cdot E\left(\xi|\xi<F^{-1}\left(\hat{\alpha}(c,w)\right)\right)-E\left(\xi\right). \tag{4.2}$$

The profit difference is the difference between the profits of an adjusting firm and the profits of a firm that set its price one period ago. The cost difference is the difference between expected price adjustment costs next period if a firm adjusts its price today and expected price adjustment costs next period if a firm does not adjust its price today.

Lemma 4.1. *: For $\hat{\alpha}\in[0,1]$,(i) When β is sufficiently small, $\pi_{DIF}(\hat{\alpha})>0$; (ii) When β is sufficiently large, there is a critical value $\hat{\alpha}=\hat{\alpha}^{*}$ such that for $\hat{\alpha}<\hat{\alpha}^{*}$, $\pi_{DIF}(\hat{\alpha})<0$ and for $\hat{\alpha}>\hat{\alpha}^{*}$, $\pi_{DIF}(\hat{\alpha})>0$; (iii) $C_{DIF}<0$.*

(for proof see appendix)

The behavior of the profit difference is intuitive. When firms pay little attention to the future, they care only about current profits. Hence, when they adjust their price, they increase their current profits. (In the limiting case where $\beta=0$, they choose the price which maximizes their current (one-period) profits, and so it is evident that profits increase.) As β approaches 1, however, firms care equally about current profits and profits one period ahead. If $\hat{\alpha}$ is relatively large, then firms expect to adjust their price with high probability next period, and so will none the less set their price close to the current optimum, implying a positive profit difference. But when $\hat{\alpha}$ is relatively small, firms expect the price they set this period will probably be in place next period. In these circumstances, they actually find it worthwhile to adjust their prices so as to reduce their current profits, knowing that this will be offset by higher expected profits next period. Finally, the cost difference reflects the fact that a firm that does not adjust this period adjusts next period with certainty, but a firm that adjusts this period will only adjust next period if costs are sufficiently low.

From (3.7), the benefit to a firm of adjusting its price one period after its last adjustment can be written in terms of the profit difference and the cost difference:

$$v_0 - v_1 = \frac{\pi_{DIF}(\alpha, \hat{\alpha}) - \beta w C_{DIF}(\hat{\alpha})}{1 + \beta \cdot (1 - \hat{\alpha})}. \tag{4.3}$$

Recall that the denominator reflects the fact that the firm's choice of adjustment probability influences its effective discount rate (see the discussion in Section 3.1 above). The reaction function can be written as

$$\hat{\alpha} = F\left(\frac{\pi_{DIF}/w - \beta C_{DIF}}{1 + \beta \cdot (1 - \hat{\alpha})}\right). \tag{4.4}$$

(Notice that the profit difference ends up being deflated by the wage in order to convert it to units of labor, since menu costs are a labor cost.) The slope of the reaction function is[16]

$$\frac{d\hat{\alpha}}{d\alpha} = \frac{\left(\frac{F'(\cdot)}{1+\beta\cdot(1-\hat{\alpha})}\right) \cdot \frac{\pi_{DIF}}{w} \cdot \left\{\frac{\partial \ln c}{\partial \alpha} - \varepsilon \cdot \frac{\partial \ln w}{\partial \alpha}\right\}}{1 - \left(\frac{F'(\cdot)}{1+\beta\cdot(1-\hat{\alpha})}\right) \frac{1}{w} \cdot \frac{\partial \pi_{DIF}}{\partial \hat{\alpha}}}. \tag{4.5}$$

This expression displays the channels through which the aggregate adjustment probability affects individual adjustment.

The denominator represents the effect of a marginal change in the firm's own adjustment probability on the difference between its adjustment probability and the probability that it draws a cost of adjustment less than the benefit to adjusting. The denominator will be

[16] An intermediate step in the derivation of the slope of the reaction function is provided in the appendix; (4.5) uses the fact that $\frac{\partial \pi_D}{\partial w} = (1 - \varepsilon) \frac{\pi_D}{w}$ and $\frac{\partial \pi_D}{\partial c} = \frac{\pi_D}{c}$.

positive at a firm's optimal choice of $\hat{\alpha}$. Intuitively, if a marginal increase in adjustment probability were to lead to a *larger* increase in the probability of it being optimal to adjust, then the initial adjustment probability could not have been an optimum.[17]

The numerator of (4.5) represents the direct, partial effect of a marginal increase in the aggregate adjustment probability (α) on the probability that an individual firm finds it optimal to adjust. From the previous analysis, changes in α affect both the equilibrium wage and the equilibrium level of aggregate demand, and both of these terms affect the profit difference.

The $F'()$ term that shows up in both the numerator and denominator reflects the effect of the distribution of menu costs. If the cdf is relatively steep, then, other things equal, small changes in the gain from adjusting translate into relatively large changes in the individual firm's adjustment probability; this is captured by the $F'()$ in the numerator. At the same time, though, a steep cdf magnifies the impact of the firm's own adjustment probability on its gain from adjusting, which is why $F'()$ also shows up in the denominator.

Lemma 3.1 revealed that, in general, $\frac{\partial \ln w}{\partial \alpha}$ can be of either sign. As noted earlier, it is also not possible to sign $\frac{\partial \ln c}{\partial \alpha}$ unambiguously, since aggregate demand depends upon assumptions about preferences. Thus, at the level of generality we have pursued so far, we cannot make any definitive statements about complementarity in price adjustment. We proceed by making an additional assumption on preferences that allows us to make further analytical progress, and then we will turn to the computer.

4.2. A Simplification and a Limiting Result

With a common specification of the period utility function, (4.5) simplifies significantly. Suppose preferences are given by

$$u(c, n) = \ln c - \chi n,$$

[17] The derivation of the denominator uses the fact that $\frac{\partial C_{DIF}}{\partial \hat{\alpha}} = F^{-1}(\hat{\alpha})$.

which implies infinite labor supply elasticity. Then the consumer's optimality conditions imply $w = \chi c$, so $\frac{\partial \ln c}{\partial \alpha} \equiv \frac{\partial \ln w}{\partial \alpha}$. Hence, from (4.5) the slope of the reaction function reduces to

$$\frac{d\hat{\alpha}}{d\alpha} = \frac{\left(\frac{F'(\cdot)}{1+\beta\cdot(1-\hat{\alpha})}\right)(1-\varepsilon)\cdot\frac{\pi_{DIF}}{w}\cdot\frac{\partial \ln w}{\partial \alpha}}{1-\left(\frac{F'(\cdot)}{1+\beta\cdot(1-\hat{\alpha})}\right)\frac{1}{w}\cdot\frac{\partial \pi_D}{\partial \hat{\alpha}}}.$$

In this case, whether or not there is complementarity in price adjustment depends only on the sign of the profit difference and the sign of the real wage semi-elasticity with respect to the adjustment probability. In particular, there is strategic complementarity whenever these two objects are of opposite sign.

Proposition 4.2. *Assume $u(c,n) = \ln c - \chi n$. Assume also that the reaction function is continuous. (i) For small β, the reaction function exhibits strategic complementarity; (ii) As $\beta \to 1$, the reaction function displays strategic substitutability at any equilibrium; (iii) If $\beta \simeq 1$, there are no multiple equilibria.*

Proof: From Lemmata 3.1 and 4.1, we know that when the discount factor is small, the profit difference is positive and the reaction function has negative slope. Part (i) follows immediately. We also know that when $\beta \simeq 1$, the real wage is decreasing for $\alpha < \alpha^*$, and increasing for $\alpha > \alpha^*$ (where α^* is defined in the proof of Lemma 3.1 in the appendix). Meanwhile the profit difference is negative for $\hat{\alpha} < \hat{\alpha}^*$, and positive for $\hat{\alpha} > \hat{\alpha}^*$ (where $\hat{\alpha}^*$ is defined in the proof of Lemma 4.1 in the appendix). When $\beta = 1, \hat{\alpha}^* = \alpha^*$. It thus follows that, for $\beta \simeq 1$, the profit difference and the slope of the real wage function are of the same sign for all $\alpha = \hat{\alpha}$. Thus, whenever the reaction function crosses the 45 degree line, it must be downward sloping, which establishes part (ii). Part (iii) of the proposition follows immediately from part (ii) together with the assumption that the reaction function is continuous. The reason is that a necessary (not sufficient) condition for multiple equilibria is that the reaction function cuts the 45 degree line with positive slope. ◦

Proposition 4.2 is our main analytical result. It states that under a common specification of preferences, multiple degrees of price rigidity are impossible when $\beta = 1$. The assumption that $\beta = 1$ is of course an approximation, since if β were truly unity, there would be no equilibria with finite utility. However, since the equations describing a steady state are continuous in β, the behavior of these equations for $\beta = 1$ reveals the approximate behavior of the reaction function for β near unity. Thus Proposition 4.2 suggests that we are unlikely to observe multiple equilibria for the values of β that are relevant for business cycle analysis.[18] Note that the result is independent of the distribution of menu costs.

Proposition 4.2 also reveals a striking contrast between the case of low and high discount factors. When firms take little or no account of the future in their price-setting decisions, there is strategic complementarity in price adjustment. The basic mechanism is as follows. Suppose firms in the economy have a high probability of adjustment. They will then choose a smaller change in their price – if they are adjusting frequently, they will adjust by a smaller amount, given that the overall level of inflation is pinned down by money supply growth. As a consequence, the aggregate price level is close to the price chosen by firms that adjust, which means that adjusting firms must be choosing a low relative price. Adjustment of the real wage ensures that those firms desire a low relative price. The equilibrium wage is relatively low when firms adjust prices frequently, and from (3.1) we know that when $\beta = 0$ firms simply choose a fixed markup over the real wage. The profit difference is, in turn, positive and decreasing in the real wage, so lower real wages increase the benefit of adjustment for an individual firm. Thus, there is strategic complementarity.

Ball and Romer's model bears some resemblance to the case of $\beta = 0$, because in their static setting, firms by definition did not consider any future consequences of their actions.

[18] Loosely speaking, when β is close to 1, there is only a small region of the 45 degree line where it is possible to observe an upward-sloping reaction function, which is a necessary (but of course not sufficient) condition for multiplicity.

The current model differs, however, in that it imposes a steady-state requirement even in the case where $\beta = 0$. The fact that the general level of prices must rise at the rate of inflation means that a greater *frequency* of price adjustment must be associated with a smaller *magnitude* of price adjustment. As Proposition 4.1, part (i) reveals, however, the imposition of steady state alone does not eliminate the complementarity from the model.

The critical difference in our dynamic model comes from the requirement that the optimal relative price for an adjusting firm must be consistent with the steady state evolution of the price level. The accounting of the price level (3.13) determines the steady-state equilibrium value of $\frac{P^*}{P}$. When $\beta = 0$, adjustment of the real wage ensures that this is consistent with individual firms' optimization. But when $\beta > 0$, a firm's choice of its relative price depends on its adjustment probability as well as on the real wage. More frequent adjustment means that forward-looking firms will tend to choose a lower relative price, because they do not expect their current price to be in force in the future. The wage then needs to fall less in order for firms to choose the relative price dictated by (3.13). And if the real wage changes little, then we have broken the key linkage between aggregate adjustment probabilities and the adjustment probability of an individual firm.

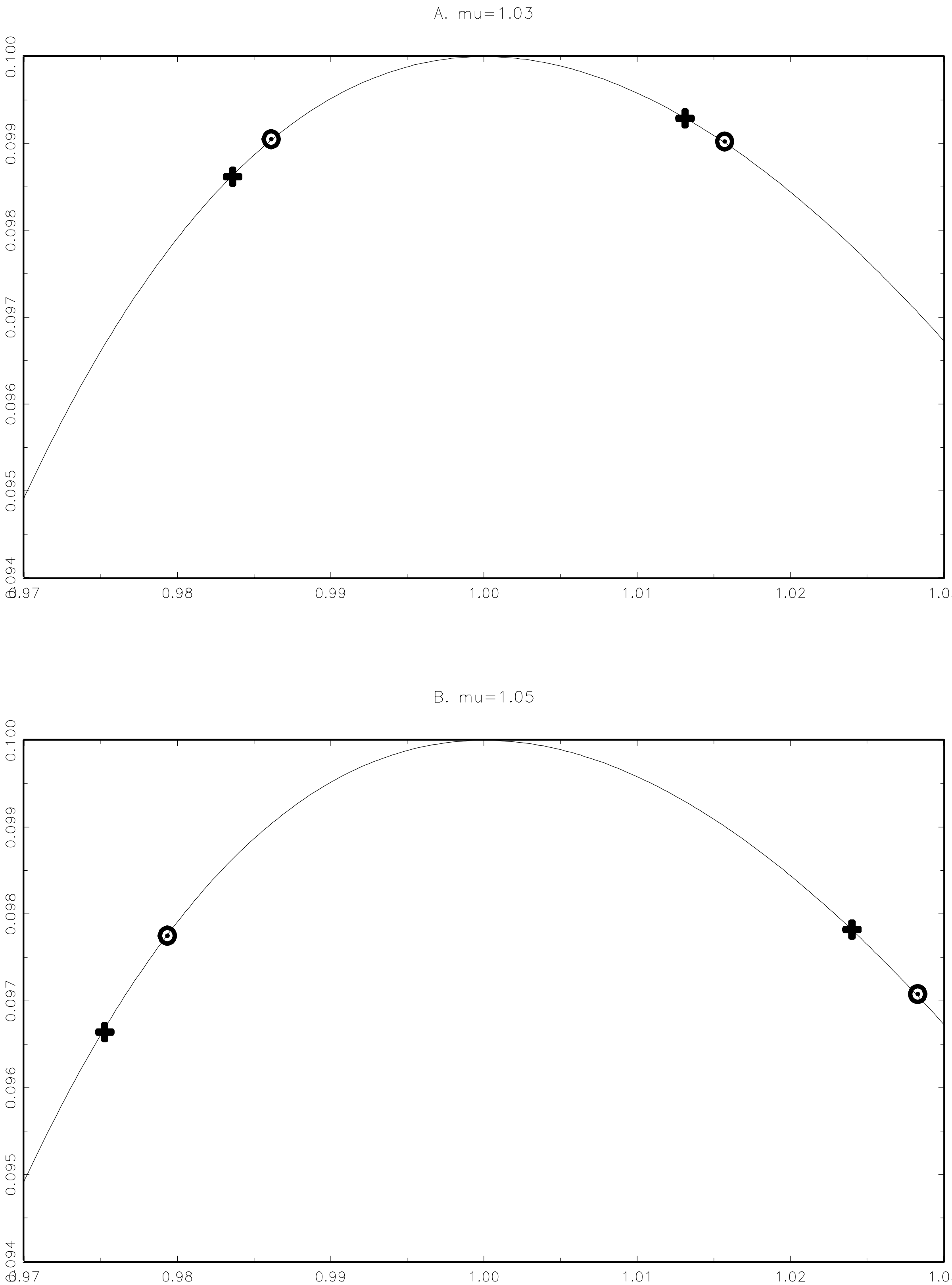
In the general case of $\beta > 0$, the profit difference need not be positive, and both the real wage and aggregate demand may be increasing or decreasing in the adjustment probability. A negative profit difference means that profits are lower in periods when a firm adjusts its price than when one period of inflation has eroded its relative price. Holding ε and μ constant, a negative profit difference will be associated with a high β or a low $\hat{\alpha}$. With a higher weight assigned to the future – or a higher probability that the price set today will be in effect in the future – it becomes costlier to allow tomorrow's price to deviate too far from the static optimum, and hence more likely that profits will be higher in those periods when no price adjustment occurs. From another perspective, at an optimum a marginal change in the relative price of an adjusting firm yields an identical decrease (increase) in current profits and increase (decrease) in expected discounted future profits. With a higher β, discounted and undiscounted future marginal profits are closer together, so the firm will choose to set the marginal decrease in current profits closer to the marginal increase in *undiscounted* future profits. This implies that the profit difference will be relatively lower, and possibly negative. Figure 1 illustrates profits for adjusting and non-adjusting firms for two different values of β when $\mu = 1.03$ (panel A) and $\mu = 1.05$ (panel B).

While complementarity can occur for $\beta << 1$, when it does occur it tends to be weak. At the high values of the discount factor relevant for business cycle analysis, the difference between profits if a firm adjusts and if it does not is small; from (4.5), it is clear that this will work against a steep reaction function. More intuitively, the reason that the profit difference is crucial for the slope of the reaction function is that profits of adjusting and non-adjusting firms – and hence the profit difference – have a constant elasticity with respect to demand and the real wage. Thus, the greater in absolute value is the profit difference, the greater in absolute value will be the effect of a given change in aggregate adjustment on an individual firm's incentive to adjust. When the discount factor is much lower than the value of 0.975 used here, the profit difference can be quite large (refer back to Figure 1), and for low enough values of the discount factor we have found parameterizations that generate multiple steady states. In ongoing work, we pursue in detail the case where $\beta = 0$.

To summarize, changes in the aggregate adjustment probability affect the steady state wage rate in the economy, and changes in the wage rate affect the adjustment probability of an individual firm. Both of these linkages are weak in our dynamic setting, and hence we do not observe the degree of complementarity necessary to generate multiple equilibria.

Figure 1. Current profits vs. relative price,

highlighting periods 0 and 1 when beta=.7(+) and .99(o)



5. Numerical Analysis of the Steady State Reaction Function

The analytical approach of the previous section provides some insight into the steady state reaction function, and gives us some reason to doubt the existence of multiple steady states. We have thus far ignored one possible complication, however – namely discontinuities in the reaction function. Moreover, our analytical findings also assumed an infinite labor supply elasticity. In order to account for discontinuities and also to know whether or not there are multiple steady states for a broader range of preferences, there is no substitute for going to the computer. In this section we provide information on how the number of steady state equilibria varies with the demand elasticity (ε), the inflation rate (μ), the distribution of fixed costs ($F\,()$), and the labor supply elasticity. We also ask how the results are affected by determining money demand from a cash-in-advance constraint rather than imposing money demand arbitrarily.

The distribution function for fixed costs of price adjustment is not something for which there are sharp estimates available: we assume that ξ is drawn from a beta distribution on $(0, B)$, and experiment with the mean and variance (equivalently, the exponents commonly referred to as ALPHA and BETA in the beta c.d.f.), while keeping B at 0.01, a value that is small relative to the labor input used in producing final goods. The beta distribution is flexible, and allows us to examine the robustness of our results to different assumptions about the costs of changing prices. The class of preferences we consider is $u(c, n) = \ln c - \chi n^{1+\psi}$, with $\psi = 0$ or 1. Throughout, we fix the preference parameter β at 0.975, which makes it natural to interpret one period in the model as six months.

For fixed values of β, $F\,(\xi)$, ψ, and χ, we use (3.6) and (3.8) to find the number of equilibria at each point in a 30 x 30 grid of ($\frac{\varepsilon}{\varepsilon-1} = 1.01, 1.02, ..., 1.30$) and ($\mu = 1.005, 1.01, 1.015, ..., 1.15$).[19]

[19] We utilize $\frac{\varepsilon}{\varepsilon-1}$ instead of ε because $\frac{\varepsilon}{\varepsilon-1}$ is the average markup when there is no price stickiness, and hence is easier to interpret than the demand elasticity.

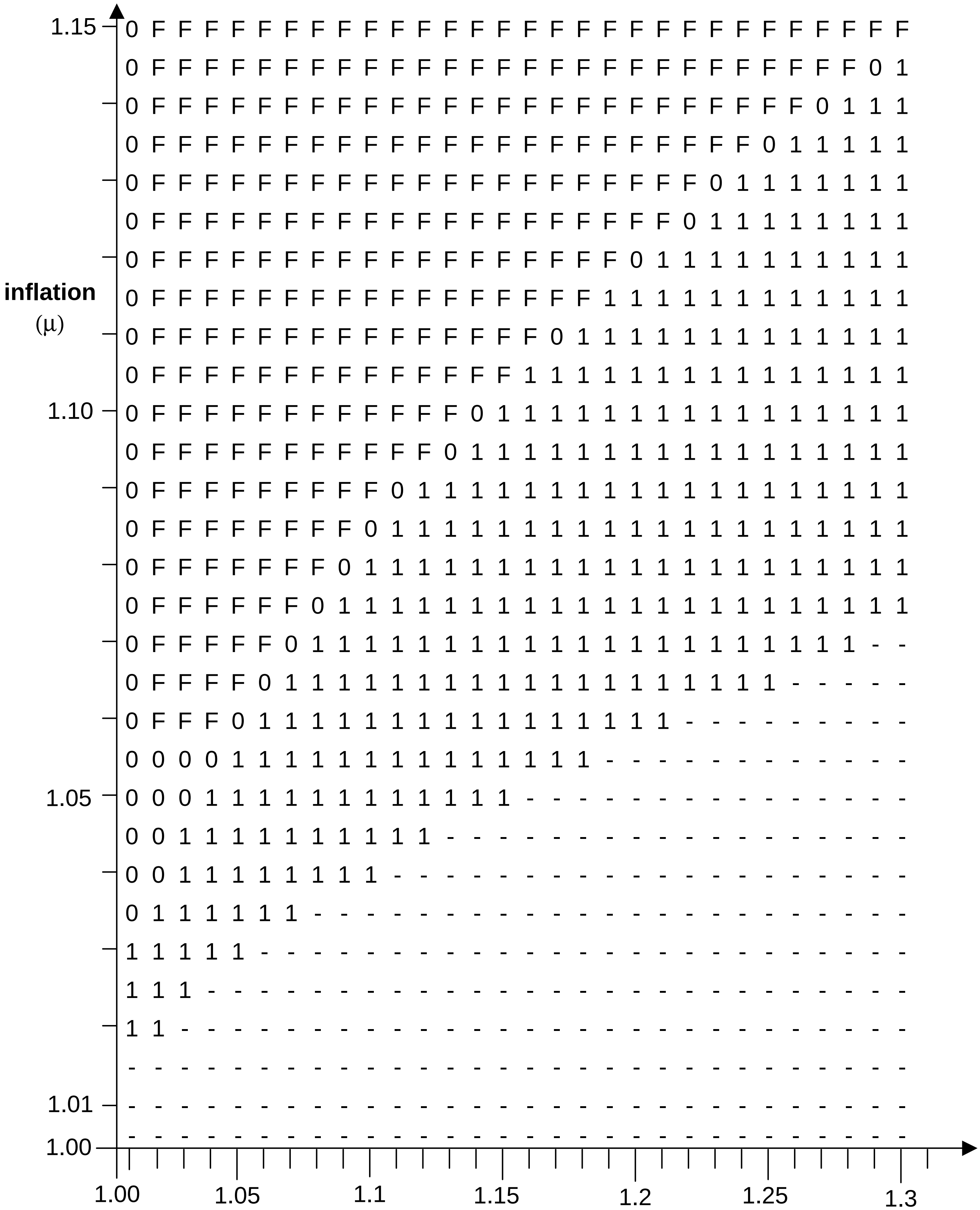
That is, for each point in the grid we substitute the solutions for $w(\alpha)$ and $c(\alpha)$ from (3.15) and (3.20) into (3.6) and then compute the reaction function using (3.8). The fixed points of the reaction function are the steady state equilibria for that point in the parameter space. Our grid thus covers specifications of market power and inflation levels that are reasonable for developed low-inflation economies, such as those of the United States or Europe.

Figure 2 displays the number of steady state equilibria for what will serve as a benchmark case: $\psi = 0$ and a uniform cost distribution with $B = .01$ (this is a special case of the beta distribution with ALPHA= 1 and BETA= 1). The various characters in the grid should be interpreted as follows. Nonnegative integers indicate the number of steady state equilibria in which no price is ever fixed for more than two periods; an "F" (for Flexible prices) means that there is a unique symmetric, pure strategy steady state equilibrium (hereafter SPSSE) with $\alpha = 1$, and a "–" means that some firms with one-period old prices would choose to keep their prices fixed for another period (in this region we do not say anything about the number of equilibria; there may or may not be equilibria with prices fixed for more than two periods).

Certain aspects of Figure 2 are easy to interpret. First, at higher rates of inflation, prices become less sticky. This implication of state-dependent pricing models has previously been discussed by Ball, Mankiw and Romer [1988] and Dotsey, King and Wolman [1996]. (Similarly, at very low rates of inflation firms would want to keep their prices fixed for more than two periods.) Second, price stickiness tends to increase with market power. At low values of $\frac{\varepsilon}{\varepsilon-1}$, which correspond to high values of ε, a firm whose price is significantly below the price level would be swamped with demand, and would have to meet that high demand at a suboptimal price. Such a firm would be willing to incur a relatively high menu cost to adjust its price.

The one feature of Figure 2 which is not self-explanatory is the significant number of zeros present. Nonexistence of SPSSE can occur for two reasons. First, firms may simply be

Figure 2.
Benchmark case
(β=.975,ψ=0,χ=3,B=.01)



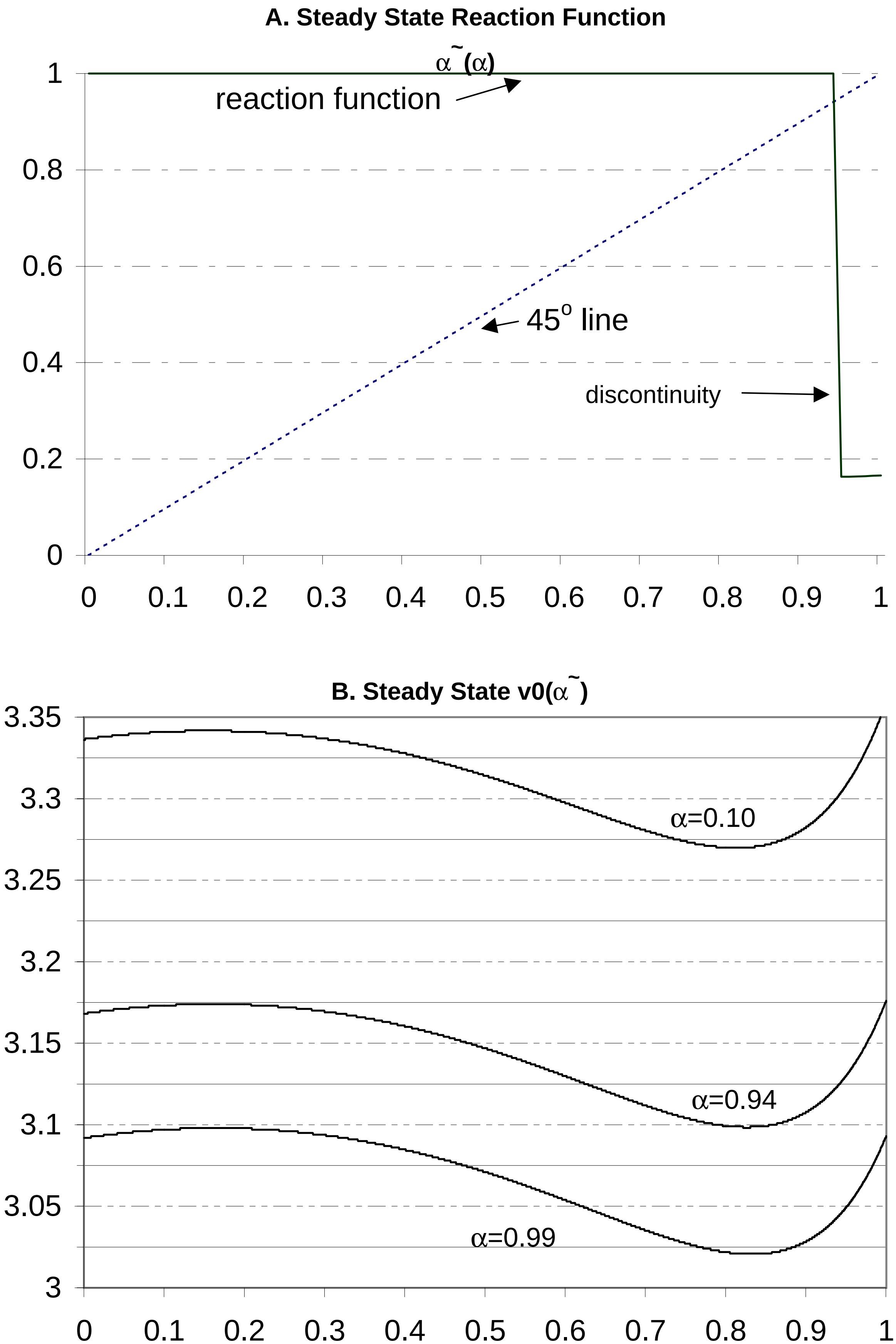
Key: 0, no symmetric steady state equilibrium (SSE)
1, unique SSE with some prices fixed for two periods
F, unique SSE with flexible prices
-, some prices fixed for more than two periods

unable to earn high enough profits to cover their fixed costs of changing price. This explains the zeros that show up in the far left of the figure, because that is a region where firms have little market power and consequently earn low profits. The line of zeros that separates sticky prices from flexible prices is associated not with negative profits, but with a discontinuity in the steady state reaction function. To illustrate and help us understand these discontinuities, Figure 3 displays the steady state reaction function for $\frac{\varepsilon}{\varepsilon-1} = 1.10, \mu = 1.085$ in panel A, and three corresponding value functions (3.6) in panel B (note that the reaction function is $\hat{\alpha}(\alpha)$, whereas the value functions are $v_0(\tilde{\alpha})$ for particular values of α).[20] For a given value of α, the steady state value function has two *local* maxima with respect to $\tilde{\alpha}$. One of these maxima is at $\tilde{\alpha} = 1$, and is associated with high discounted profits gross of adjustment costs, that is the first term in (3.6); when a firm adjusts every period it maximizes gross profits in each period, and thus $\tilde{\alpha} = 1$ maximizes the discounted gross profit term. The second term in (3.6) represents discounted adjustment costs, and they are minimized at an interior value of $\tilde{\alpha}$, not at $\tilde{\alpha} = 0$, which would be the obvious candidate to minimize adjustment costs.

While $\tilde{\alpha} = 0$ entails adjusting with the least frequency, this does not translate into the smallest discounted costs of adjustment. The explanation is that a marginal increase in $\tilde{\alpha}$ from zero has a first-order negative effect on adjustment costs in "second periods" (that is in periods when a firms price was last changed two periods ago), but only a second-order positive effect on adjustment costs in "first periods." Intuitively, a firm can incur lower average adjustment costs by taking advantage of those occasions when it draws a low cost, even if it adjusted in the previous period, than by mechanically adjusting every two periods. When the aggregate adjustment fraction is low, the profit term dominates, and an individual firm chooses $\hat{\alpha} = 1$, whereas when the aggregate adjustment fraction is high, the cost term dominates and $\hat{\alpha}$ is roughly 0.18. At $\alpha = 0.94$, the two effects are exactly offsetting, resulting

[20] As earlier, we use $\hat{\alpha}(\alpha)$ to denote a firm's optimal choice of adjustment probability, and $\tilde{\alpha}$ to denote an individual firm's arbitrary (suboptimal) choice of adjustment probability.

Figure 3
details on benchmark case when
markup=1.1, inflation=1.085



in a firm being indifferent between the two adjustment probabilities.

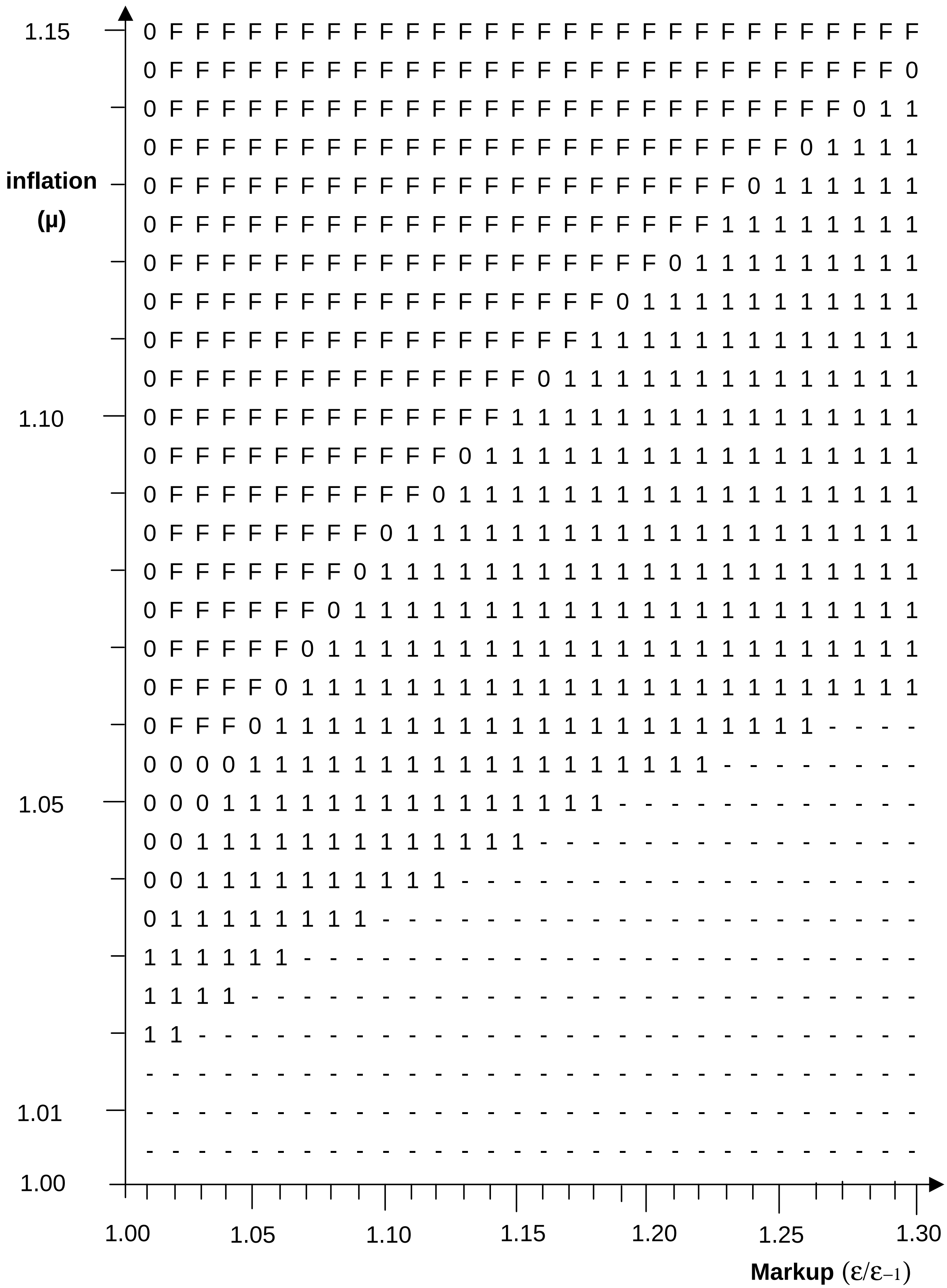
At the point of discontinuity the firm is actually indifferent between two strategies, one entailing a low frequency of adjustment, and the other entailing adjustment every period. At first glance this seems counterintuitive – after all, whenever a firm draws a menu cost, it either finds it worthwhile to adjust, or else it does not. The explanation is that at the point of discontinuity, an adjusting firm is indifferent between two different prices that it could set, each of which implies a different *future* probability of adjustment.[21]

The existence of discontinuities in the reaction function raises the possibility of an additional source of multiplicity: the reaction function might jump from below to above the 45 degree line. It turns out that this is indeed sometimes observed for low values of β. We have found no evidence, however, of such discontinuities or multiplicities at values of β that are relevant for business cycle analysis. Note, finally, that the nonexistence associated with these discontinuities is only nonexistence of SPSSE; there is likely a steady state with randomizing near the point of discontinuity.

Figure 4 modifies the distribution of fixed costs by setting the parameters of the beta distribution to ALPHA= BETA= 10. This maintains symmetry, and makes the distribution s-shaped (the p.d.f. bell-shaped), with low variance. Superficially, we might expect an s-shaped distribution to generate multiple SPSSE, because over a certain range, an s-shaped distribution has the property that $F'()$ is very high, and from (4.5) we know that other things equal, if the reaction function is upward sloping, it is steeper the higher is $F'()$. In fact, Figure 4 shows that modifying the distribution in this way has little effect, and we still do not observe any cases of multiple SPSSE. This is a significant result, because it gives us some confidence that our broad findings are not heavily dependent on the form of the menu cost distribution. As there is little empirical evidence on the appropriate form for this

[21] This raises a minor caveat to the claim we made at the beginning of Section 3 to the effect that a firm facing an unchanging economy will choose a constant $\hat{\alpha}$ policy.

Figure 4.
Concentrated distribution
(a=b= 10)



distribution, it is reassuring that our conclusions seem to be reasonably robust to different assumptions about the distribution.[22]

How is it that making the distribution much steeper has so little effect on the pattern of steady state equilibria? Figure 5 displays reaction functions that correspond to points in the middle of Figure 2 and Figure 4. In both cases the reaction function is almost completely flat. In the uniform case, however, it is shifted up so that the steady state has $\alpha \approx 0.26$ as compared to $\alpha \approx 0.005$ when the distribution is concentrated. Behind these reaction functions lies the fact that in equilibrium, the benefit to adjusting price is in large part pinned down by factors other than the distribution of fixed costs. Assuming this benefit is significantly less than the expected cost of adjustment, it follows that when mass is heavily concentrated about the mean, a smaller fraction of firms will draw costs that make it worthwhile to adjust. Figure 5 thus reveals that while modifying the distribution of fixed costs does not change the *number* of equilibria, it does affect the degree of price stickiness.

Figure 6 deviates from Figure 2 by changing the preference parameter ψ from zero to unity. This corresponds to a labor supply elasticity of unity instead of infinity. As with changing the distribution, changing the labor supply elasticity does not significantly alter the pattern of SPSSE, although it does lead to somewhat greater price rigidity.

Finally, Figure 7 replicates Figure 2, except that money demand is explicitly motivated by a cash-in-advance constraint. Again, there is little qualitative difference between the two figures. Minor differences are attributable to the fact that in a cash in advance framework with $\psi = 0$, the steady state ratio of consumption to the real wage is given by $(\chi \cdot (1 + R))^{-1}$, where R is the nominal interest rate. From (3.21), the argument of the steady state reaction function is then $(\chi \cdot c \cdot (1 + R))^{-1} \cdot (v_0 - v_1)$, instead of $(\chi \cdot c)^{-1} \cdot (v_0 - v_1)$. Higher inflation corresponds to a higher nominal rate, and this expression shows that by itself the higher nominal rate decreases the incentive to adjust prices, explaining why the region of price

[22] We have experimented with other parameterizations of the beta distribution and obtained similar figures.

Figure 5. Typical Reaction Functions

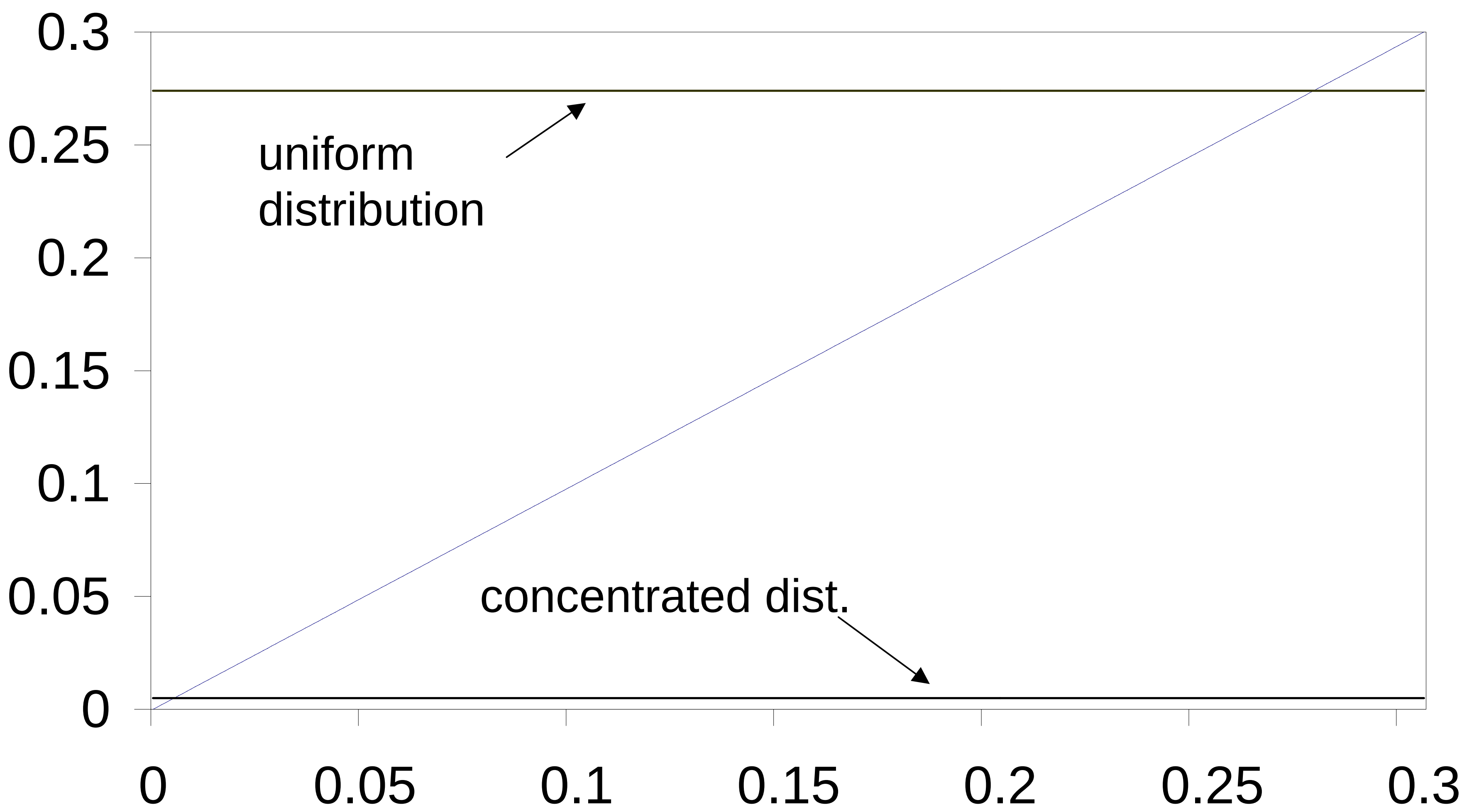


Figure 6.
Low labor-supply elasticity
(ψ=1, χ=10)

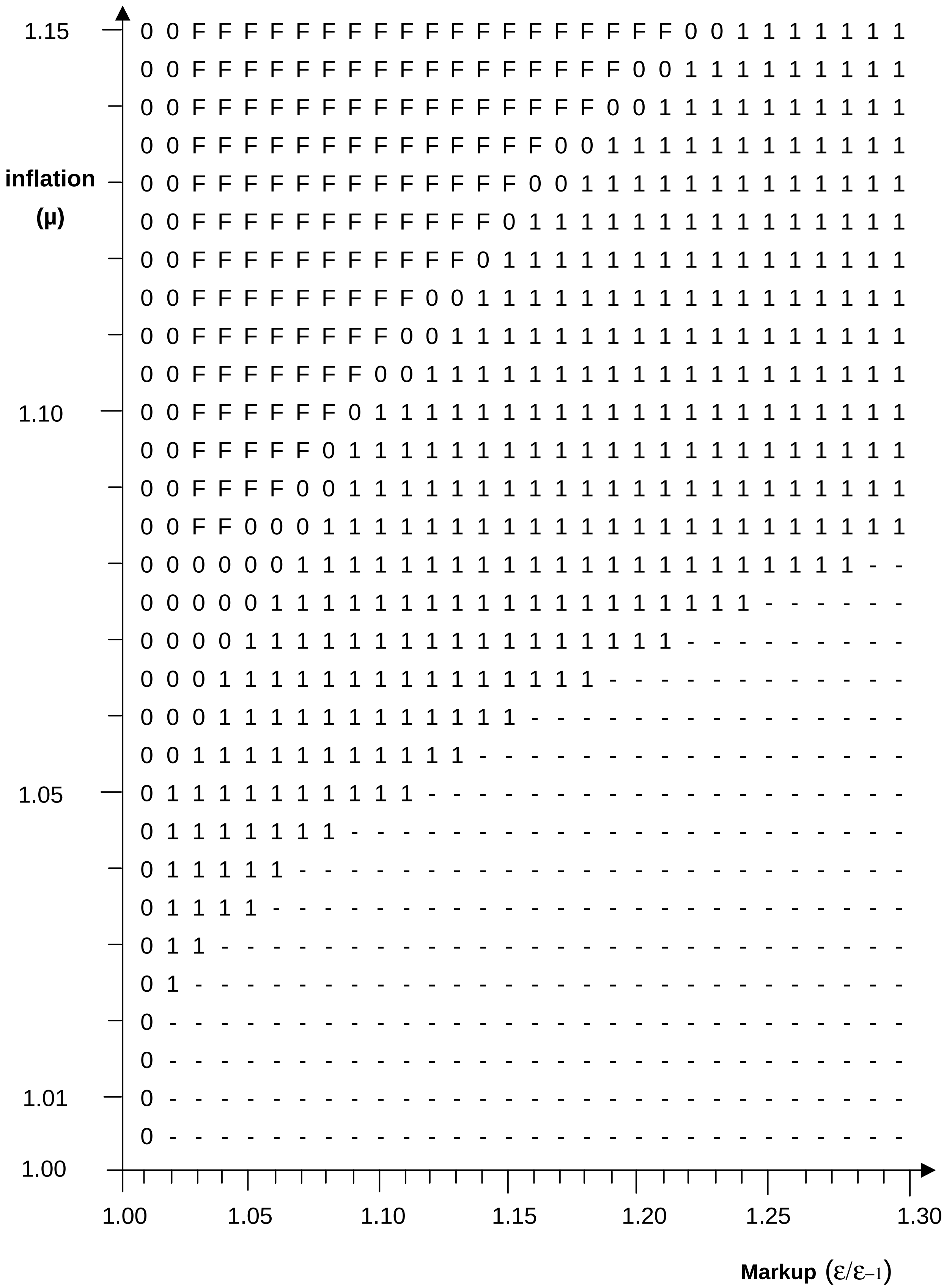
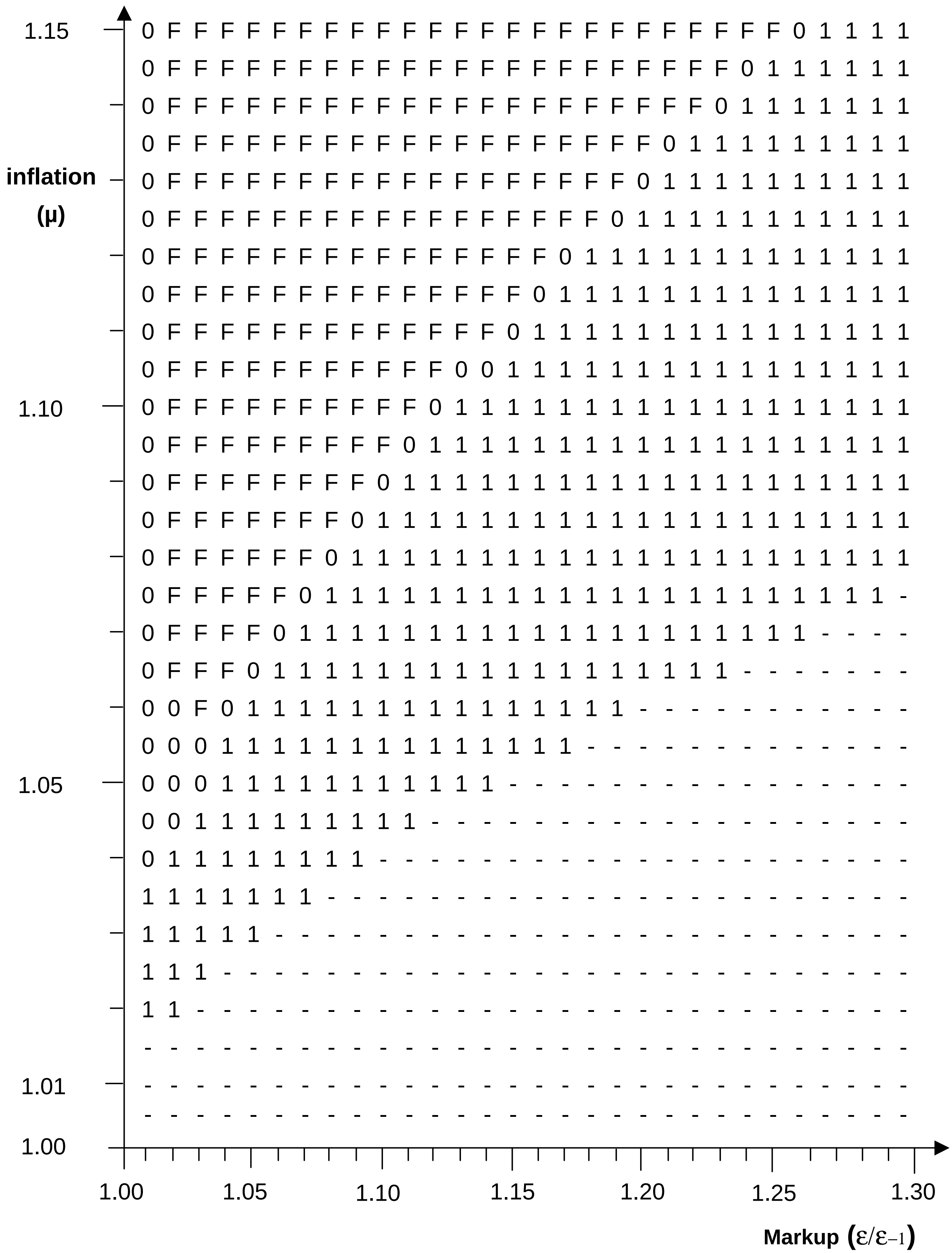


Figure 7.
Cash in advance



stickiness in Figure 7 extends to higher levels of inflation than when money demand is modeled in an ad-hoc manner.

Our numerical analysis thus uncovers no evidence of multiple steady states for standard calibrations of the model. The absence of multiplicity is a consequence of the weakness or absence of steady state complementarity in price setting, as uncovered in our analytical work.

6. Conclusions

It is an intuitively appealing idea that price-setting is characterized by complementarity and hence can be a source of multiple equilibria. That intuition was given a foundation by Ball and Romer [1991], whose analysis was nonetheless limited by being essentially static. We have extended Ball and Romer's analysis to an explicitly dynamic framework, in which there is a competitive labor market. By so doing, we have considered whether or not complementarities and multiplicities are likely to arise in the context of modern business cycle models.

The model that we develop can potentially have multiple steady states and indeterminate local dynamics. Our objective here has been to study the model's steady states. For high (business cycle) values of the discount factor, the model lacks strong enough complementarity in price adjustment to generate multiple steady states. However, nonexistence of a steady state does occur in a small region of the parameter space. This nonexistence is associated with discontinuities in the reaction function that links aggregate to individual adjustment. Our results suggest that research using state-dependent pricing models is unlikely to be hindered by the presence of multiplicity, and that, contrary to the speculations of Ball and Romer, multiplicity will not be central to explaining the range of price stickiness experienced by economies with apparently similar fundamentals.

Of necessity, our results cannot be definitive. The search for multiplicity in this class of models requires assumptions on preferences, technology, and the distribution of menu costs, and it is of course impossible to rule out the possibility that multiple steady states arise for some configuration of parameters that we have not examined. Nevertheless, we have conducted a fairly extensive search, and have found no cases of multiplicity for values of the discount factor that are reasonable for business cycle analysis. Our analytical results also suggest that we are unlikely to find the complementarities that are necessary for multiple equilibria.

There are at least two important avenues that are as yet unexplored in this class of model. The first involves non-steady state equilibria. As mentioned in the introduction, this paper has pursued only one of the two natural dynamic counterparts to the type of complementarity discussed by Ball and Romer. Even if the model possesses a unique steady state, the local dynamics around that steady state may still exhibit multiplicity; Dotsey, King and Wolman [1997] provide examples of state-dependent pricing models where the nature of the local dynamics is sensitive to the distribution of adjustment costs.[23] Other possible equilibria involve deterministic cycles and their limiting case, equilibria with synchronized price adjustment. Synchronization versus staggering has been discussed previously in Fethke and Policano [1986] for wage-setting, Ball and Cecchetti [1988] and Ball and Romer [1989] for price-setting, and Lau [1996] for a general game-theoretic environment. However, in each of those papers decisions are time-dependent. Instead of assuming a fixed cost of adjustment, the authors assume that the choice variable is adjusted every n periods and ask whether synchronization or staggering is an equilibrium. Ball and Romer [1991] consider synchronization versus staggering in a non-inflationary state-dependent environment, whereas the natural extension of our analysis would involve positive inflation. If synchronized and

[23] For early analyses of indeterminacies in business cycle models, see Benhabib and Farmer (1994) and Gali (1994).

staggering equilibria coexist, it will be straightforward to Pareto-rank them, although it is unclear *a priori* which type of equilibrium would have higher welfare. Synchronized and staggering equilibria would involve the same amount of adjustment costs. Two other effects are present, however, which work in opposite directions. Other things equal, welfare will be higher with less relative price dispersion, and a synchronized equilibrium has zero relative price dispersion. On the other hand, agents in the model value smooth consumption over time, and on this score a staggering (steady state) equilibrium dominates.

Another avenue for research takes off from Bergin and Feenstra's [1998] discussion of a more general class of preferences which make a firm's demand elasticity vary with its relative price. The variable elasticity corresponds to firms caring directly about the prices charged by others. For the Dixit-Stiglitz case we consider, firms care about the prices charged by others only to the extent that other firms' prices affect nominal marginal cost. Bergin and Feenstra argue that this restriction unreasonably limits the persistence of nonneutrality under time-dependent pricing, but it also clearly works against complementarity under state-dependent pricing. Given the approximations that appear necessary in this case however, progress along these lines may be difficult.

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A. Appendix

A.1. Optimal Pricing

The solution to (2.12) and (2.11) yields the following expression for the optimal price of an adjusting firm relative to the price level:

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{w_t + \beta \cdot E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{c_{t+1}}{c_t} \left\{ (1 - \alpha_{t+1}) \cdot w_{t+1} \cdot (P_{t+1}/P_t)^{\varepsilon} \right\}}{1 + \beta \cdot E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{c_{t+1}}{c_t} \left\{ (1 - \alpha_{t+1}) \cdot (P_{t+1}/P_t)^{\varepsilon - 1} \right\}}. \tag{A.1}$$

A.2. Proofs

Proof of Lemma 3.1: By differentiation of $w(\alpha)$, it can be shown that

$$\frac{d\ln w}{d\alpha} = \frac{\beta\mu^{\varepsilon-1}(\mu-1)}{(1+\beta(1-\alpha)\mu^{\varepsilon-1})(1+\beta(1-\alpha)\mu^{\varepsilon})} + \frac{1-\mu^{\varepsilon-1}}{(\varepsilon-1)(2-\alpha)(1+(1-\alpha)\mu^{\varepsilon-1})}. \quad \text{(A.2)}$$

After considerable manipulation, it can be shown that

$$\begin{aligned} & sgn\left[\frac{d\ln w}{d\alpha}\right] \qquad \text{(A.3)} \\ = \; & sgn\left[\left(\frac{\mu^{\varepsilon-1}+1}{\mu^{\varepsilon-1}}-\alpha\right)(\alpha-\alpha^*)+\left(\frac{1-\beta}{\beta}\right)\left(\frac{\mu^{\varepsilon-1}-1}{\mu^{2(\varepsilon-1)}}\right)\left(\frac{(1-\alpha)^2\beta\mu^{2\varepsilon-1}-1}{(\mu^{\varepsilon}-1)-\varepsilon(\mu-1)}\right)\right], \end{aligned}$$

where

$$\alpha^* = 1-\frac{\varepsilon(\mu-1)-\frac{(\mu^{\varepsilon}-1)}{\mu^{\varepsilon-1}}}{(\mu^{\varepsilon}-1)-\varepsilon(\mu-1)}$$

As preliminaries, we note that $(\mu^{\varepsilon}-1)-\varepsilon(\mu-1)>0$, since this expression equals 0 when $\mu=1$, and is increasing in μ. Similarly, $(\mu^{\varepsilon}-1)-\varepsilon(\mu-1)>\varepsilon(\mu-1)-\frac{(\mu^{\varepsilon}-1)}{\mu^{\varepsilon-1}}>0$, so $\alpha^*\in(0,1)$. As $\beta\to 0$, the second term in (A.3) tends to $-\infty$, and so $w'(\alpha)<0\forall\alpha\in[0,1]$. This proves part (i). As $\beta\to 1$, the second term tends to 0, and so $w'(\alpha)<0$ for $\alpha<\alpha^*$ and $w'(\alpha)>0$ for $\alpha>\alpha^*$, proving part (ii). ∘

Proof of Lemma 4.1: Rearranging the expression in square brackets in (4.1), it can be shown that

$$sgn[\pi_{DIF}(\hat{\alpha})] = sgn[(1-\alpha^*)-\beta(1-\hat{\alpha})],$$

where, as established in Lemma 3.1,

$$\alpha^* = 1-\frac{\varepsilon(\mu-1)-\frac{(\mu^{\varepsilon}-1)}{\mu^{\varepsilon-1}}}{(\mu^{\varepsilon}-1)-\varepsilon(\mu-1)}\in(0,1)$$

Note that while the *magnitude* of the profit difference depends upon α and $\hat{\alpha}$, the *sign* of the profit difference depends only on $\hat{\alpha}$. Part (i) is immediate. The critical value of $\hat{\alpha}$ is defined by

$$\hat{\alpha}^* = \frac{\alpha^*-(1-\beta)}{\beta};$$

hence $\hat{\alpha}^*\to\alpha^*$ as $\beta\to 1$, and part (ii) follows. Part (iii) is immediate.∘

A.3. Characterizing the function $\mathbf{c}(\alpha)$

From (3.20) one can show that

$$\frac{\partial c}{\partial \alpha} = \frac{u_c \frac{\partial w}{\partial \alpha} + \left(u_{nn} - \frac{u_n u_{cn}}{u_c}\right)\left(\frac{\partial n}{\partial w}\frac{\partial w}{\partial \alpha} + \frac{\partial n}{\partial \alpha}\right)}{\left(\frac{u_n u_{cc}}{u_c} - u_{cn}\right) - \frac{\partial n}{\partial c}\left(u_{nn} - \frac{u_n u_{cn}}{u_c}\right)} \tag{A.4}$$

A.4. The slope of the reaction function

Implicitly differentiating (4.4) yields

$$\frac{d\hat{\alpha}}{d\alpha} = \frac{\frac{F'}{1+\beta\cdot(1-\hat{\alpha})} \cdot \frac{1}{w} \cdot \left\{\left(\frac{\partial \pi_D}{\partial w} - \frac{\pi_D}{w}\right) \cdot \frac{\partial w}{\partial \alpha} + \frac{\partial \pi_D}{\partial c} \cdot \frac{\partial c}{\partial \alpha}\right\}}{1 - \frac{F'}{1+\beta\cdot(1-\hat{\alpha})} \cdot \frac{1}{w} \cdot \frac{\partial \pi_D}{\partial \hat{\alpha}}}. \tag{A.5}$$

Using the fact that the profit difference is

$$\begin{aligned} \pi_D(c, w, \hat{\alpha}) &= c \cdot w^{1-\varepsilon} \cdot \left(\left(\frac{\varepsilon}{\varepsilon - 1}\right) \cdot g(\hat{\alpha}; \beta, \varepsilon, \mu)\right)^{-\varepsilon} \cdot \\ &\quad \left\{(\mu^{\varepsilon} - 1) - \left(\frac{\varepsilon}{\varepsilon - 1}\right) \cdot g(\hat{\alpha}; \beta, \varepsilon, \mu) \cdot \left(\mu^{\varepsilon - 1} - 1\right)\right\}, \end{aligned} \tag{A.6}$$

(A.5) simplifies to (4.5) in the text.