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# The Dynamic Effects of Government Spending Shocks on Employment and Work Hours \*

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## Abstract

In this paper, we analyze the dynamic behaviour of employment and hours worked per worker in a stochastic general equilibrium model with a matching mechanism between vacancies and unemployed workers. The model is estimated for the U.S. using the Generalized Methods of Moments (GMM) estimation technique. An increase in government spending raises hours worked per worker, and crowds out private consumption due to a negative wealth effect. On the path converging towards the steady state, private consumption is below its long run average and increases, which implies that the interest rate is above its long run average and declines. The interest rate effect dominates the pure economic rent effect on the capital value of a hired worker to the firm, causing a reduction of job openings and consequently a decrease in employment. These results are contrasted with the predictions of a version of Burnside, Eichenbaum and Rebelo's (1993) labor hoarding model.

*Key words:* Employment; Hours; Government Spending; Job Search

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# 1 Introduction

An important objective of fiscal policy is to influence the behavior of aggregate unemployment over the business cycle. Understanding the effect of fiscal policies, such as government spending on employment, is empirically and theoretically critical.

In the literature, the effects of government spending on total hours worked have been analyzed in Aiyagari, Christiano and Eichenbaum (1992), Christiano and Eichenbaum (1992), Burnside, Eichenbaum and Rebelo (1993), Burnside and Eichenbaum (1996), Baxter and King (1993), and Campbell (1994). However, given a demand shock, both the labor intensive margin, the number of hours worked, and the labor extensive margin, the number of employees of a firm, can be changed. The total hours, as a product of the hours worked per worker and the number of employees, alone cannot provide information about the changes in the two margins. If we want to examine the behavior of unemployment after an aggregate demand shock, a decomposition of total hours worked is necessary.

This paper intends to accomplish two tasks. First, we will investigate the historical facts about the impact of a temporary government spending shock on employment, hours per worker and output based on the U.S. data. Second, we will develop a search theoretical model which can generate similar impulse responses to those of the empirical studies especially the responses of the two labor market margins, and examine the model predictions on the relative effects of transitory versus persistent government shocks on the number of employees and hours worked per worker.<sup>1</sup> These results are compared with a reinterpreted version of Burnside, Eichenbaum and Rebelo (1993) (BER) model. The BER model enjoys success in replicating the impulse responses of total output and total hours worked, yet does poorly in

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<sup>1</sup>Following Aiyagari, Christiano and Eichenbaum (1992), we call a shock with zero persistence a transitory shock, and those with positive persistence persistent shocks.

capturing the different responses of the two margins.

In the first part of this paper, we examine the effects of government consumption on the labor market of the U.S. economy in the postwar period. The responses of multivariate vector autoregression (VAR) models show that a temporary government spending shock increases total hours worked and output. However, when total hours worked are decomposed into the number of employees and hours worked per worker, the effects are quite different. The shock increases hours worked per worker, but reduces employment. Furthermore, the hours worked per worker responds to the shock more quickly than that of employment. The changes in employment on the other hand are more persistent compared to that of the hours worked per worker.<sup>2</sup>

In the second part, we first consider a theoretical model which is built upon recent advances in general equilibrium theories where employment is determined through a mechanism which matches unemployed workers and vacancies. Pissarides (1990) incorporates this matching mechanism in a balanced growth model. Andolfatto (1996) and Merz (1995) consider a stochastic real business cycle growth model with matching and compares the model's responses to technology shocks with a standard RBC model. Different from the RBC models, in a search model the allocation of labor force is determined by a matching mechanism. The inflow of workers into employment is the outcome of successful matching between job openings and unemployed workers. A firm creates vacancies at some cost, and a hired worker brings some economic rent to the firm. The firm equalizes the cost of each job opening to the expected benefit of the opening in equilibrium. One implication of this model is that simultaneous vacancies and unemployed workers exist.<sup>3</sup>

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<sup>2</sup>Karras (1993) also observes negative employment effects of government spending in the following countries: Australia, Belgium, Canada, Denmark, Finland, Japan, Korea and Phillipines.

<sup>3</sup>For theoretical analysis of vacancies, see Pissarides (1985) and Blanchard and Diamond (1989). For the empirical analysis at aggregate level, see Abraham (1987). For the empirical analysis on the magnitude and

The parameters in the model are estimated by the generalized method of moments (GMM) estimation technique. The near steady state dynamics are obtained by using the log linear approximation method of King, Plosser and Rebelo (1990).

There are two main findings of this model. First, the parameterized model generates similar responses of employment and hours worked per worker to a shock in government spending to those of VAR models. Second, numerical experiments with different degrees of persistence of government spending shocks show that a transitory government spending shock lowers employment, while a persistent government spending shock may decrease or increase employment depending on the degree of persistence of the shock. While the BER model generates similar impulse response for total hours, it is incapable of capturing the different responses of hours worked per worker and employment. Both models predict that a persistent shock has a larger effect on total hours and output than a transitory one, the main reason is the larger wealth effect on labor supply of a more persistent shock, echoing the argument of Aiyagari, Christiano and Eichenbaum (1992).

Concerning the output effect of the shock, three elements need to be considered, the capital stock, hours worked per worker and the number of employees. Hours worked per worker increase after a shock in government consumption. The capital stock and the number of employees may increase or decrease depending upon the persistence of the shock. Although the capital stock and the number of employees may decrease, the positive hours effect dominates the negative effect on output and output still increases. The effect of government expenditure on total hours worked in the BER model is determined through intertemporal substitution. By assuming leisure is a superior good, a temporary shock to the government spending decreases household consumption and leisure due to a negative wealth effect. Thus

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determinants of job vacancy rates in the U.S. firms, see Holzer (1994).

total hours worked and output both increase.

In our model, as in BER model the effect of government spending on hours worked per worker is determined through intertemporal substitution. However, the shock affects employment through a matching mechanism between unemployed workers and vacancies. Assuming exogenous separations of workers from employment, the job creation decision is determined by the capital value of a hired worker to the firm. A temporary shock in government spending has two effects on the capital value of a hired worker, a negative interest rate effect and a pure economic rent effect. The shock leads to an increase in the interest rate because only a higher interest rate will clear the goods market given an increase in aggregate demand. At the same time, higher interest rates lower the expected capital value of a hired worker to the firm. A higher pure economic rent in each period a hired worker brings to the firm causes an increase of the capital value of a hired worker to the firm. Thus the overall effect of a shock on the capital value of a hired worker depends on the relative magnitudes of these two opposite forces. In the BER model, given a government consumption shock, the only contemporaneous labor margin that can be adjusted is hours worked per worker and it increases due to a negative wealth effect. In the following periods, employment would be increased and hours worked per worker would return to its steady state value. This is because increasing employment reduces an agent's expected utility less than increasing hours worked per worker (utility is linear in employment and convex in hours worked per worker).

The plan of this paper is as follows. Section 2 reports the empirical results from multivariate VAR models. In section 3, a theoretical model with a matching mechanism is set up and equilibrium conditions are derived. Section 4 discusses the parameter estimation procedure, GMM. Section 5 contains a discussion of the parameter estimates and comparisons

of the search model, the BER model and the empirical VAR models. In Section 6, some conclusions are drawn.

## 2 An Empirical Analysis of the Effects of Government Spending Shocks

This section investigates the effects of government spending on the labor market, the interest rate and output in the postwar U.S. economy based on the use of VAR models. The data used in this analysis are quarterly seasonally adjusted time series for the sample period 1948:1 - 1993:4. National account variables are in 1987 dollars. More details are given in Appendix 1.

The VAR model is specified as

$$Z_t = A_0 + \sum_{j=1}^q A_{t-j} Z_{t-j} + u_t$$

The disturbance vector,  $u_t$ , is assumed to be serially uncorrelated and to have variance-covariance matrix  $V$ . Furthermore,  $u_t$  is assumed to be related to the underlying shocks,  $\epsilon_t$ , by  $u_t = C\epsilon_t$  where  $C$  is a lower triangular matrix and  $\epsilon_t$  has covariance matrix equal to the identity matrix.  $V = CC'$ . The orthogonality conditions on  $\epsilon_t$  correspond to imposing a particular causal structure on the variables involved in the model. For example, the  $k^{th}$  element in  $Z_t$  is determined by  $Z_{t-j}$  for  $j = 1, \dots, q$  and  $Z_{it}$  for  $i = 1, \dots, k - 1$  and  $k > 1$ .  $q$  is set to 4.

In the VAR models, two measures of government spending are considered, government purchases of goods and services and federal defense spending. The reason to consider military spending is that it is usually regarded as an exogenous component in government spending, and also the effect of military spending on employment is a matter of considerable importance.

Besides the labor market activity variables, total hours, employment and hours worked per worker, we also consider the real interest rate and output which are closely related to labor market activities. Government consumption shocks will affect aggregate demand, which in turn will lead to changes in labor demand decisions by firms. In the first VAR model, the total hours variable is included, and the second model includes the number of employees to hire and the number of hours each employee works.

Denote the following variables:

$GC$  = the log of government purchases of goods and services;

$GFD$  = the log of federal defense spending;

$NLF$  = the log of employee hours in nonagricultural establishments;

$NF$  = the log of the number of civilian employed in nonagricultural industries;

$LF$  = the log of hours worked per worker in nonagricultural industries;

$GDPC$  = the log of GDP;

$RR$  = the real interest rates, calculated from 91-day T-bill yields and GDP deflator.<sup>4</sup>

#### *The Effect of A Government Spending Shock on Total Hours Worked*

To examine the effect of a government spending shock on total hours worked, we examine two alternative specifications of  $Z$ ,

$$Z = [GC \ RR \ GDPC \ NLF]'$$

and

$$Z = [GFD \ RR \ GDPC \ NLF]'$$

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<sup>4</sup>According to the augmented Dickey-Fuller test statistics,  $RR$  is stationary and the other variables are nonstationary I(1) processes. As suggested in Doan (1992), p8-3, the variables except  $RR$  are in logarithms and no time trend is included in the following VAR estimations.

The impulse responses of the VAR models are plotted in Figure 1. Solid lines represent the point estimates, while dashed lines denote plus and minus two standard deviation bands.<sup>5</sup>

[Insert Figure 1 here]

The graphs of the impulses for the two measures of government spending show that a positive innovation in government spending will increase output and total hours worked. Interest rate declines in the first period and then starts increasing. However, the changes of neither total hours worked nor interest rate are statistically significant within two standard deviations. The responses to different spending shocks are qualitatively similar. These responses are consistent with the theoretical results of Aiyagari, Christiano and Eichenbaum (1992).

*The Effect of A Government Spending Shock on Employment, Hours Worked per Worker*

In this section, total hours worked are decomposed into the number of employees and the hours worked per worker. Two specifications of  $Z$  are considered,

$$Z = [GC \ RR \ GDPC \ LF \ NF]'$$

and

$$Z = [GFD \ RR \ GDPC \ LF \ NF]'$$

[Insert Figure 2 here]

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<sup>5</sup>These estimates are computed using the Monte Carlo method described in Doan (1992), example 10.1, using 500 draws from the estimated asymptotic distribution of the VAR coefficients and the covariance matrix of the innovations,  $u_t$ .

The impulse response functions of VAR models are plotted in Figure 2. The responses of each variable to the shocks of different measures of government spending are qualitatively similar. Employment and hours worked per worker respond differently to a shock in government spending. Hours worked per worker respond to the shock quickly and the changes are statistically significant for the first several quarters under both measures of government expenditures. The time path increases for about 6 quarters and then declines. Employment responds slowly but the changes are more persistent. To an innovation in government spending, employment decreases in the first two quarters, increases in the second half of the third quarter, and then decreases again, but the changes are not statistically significant though the decrease after one year are near significant. The decrease in employment to an innovation in military spending is monotone and close to be statistically significant after two years.

These results are robust to two kinds of perturbations. One is changing the ordering of the variables in  $Z$ . The experiments show that the responses of each variable are very similar with different position of government spending in the ordering. Another is using a different measure of hours worked: data from household survey. The results are qualitatively similar. The responses using household survey data show a larger decrease in employment and a less increase in hours worked per worker than those from establishment data.

Negative employment effects of government spending shocks are observed in some other countries according to Karras (1993). Karras (1993) considers the effect of government spending on employment and output. The results show that a transitory increase in government spending lowers employment in eight countries in the sample of eighteen countries. A persistent increase in government spending lowers employment in eight countries.<sup>6</sup> The paper also finds that the multipliers of transitory changes in government spending are gen-

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<sup>6</sup>The countries in the sample are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Ireland, Italy, Japan, Korea, Netherland, New Zealand, Philippines, Spain, Sweden, United Kingdom.

erally small, for a representative country, a one percent transitory increase raises output by only 0.1 percent.

The employment effect of military spending of the VAR model is consistent with the findings of some other empirical works. Dunne (1991) provides an analysis of military spending and unemployment for 14 OECD countries. The results suggest that fears that cuts in military spending will lead to an increase in unemployment are unjustified, and he shows that disarmament presents an economic opportunity rather than problem. Abell (1990) brings U.S. time series evidence to bear on the relationship of defense spending and unemployment rates. The analysis indicates that the increase in defense spending during the 1970s was associated with the improvement in the overall unemployment rate. However, during the 1980s, such increases were associated with a worsening of the unemployment rate.

Summarizing the results, (1) if the variable, total hours worked, is included in the VAR model, it responds positively to the positive innovations to government spending; (2) if total hours worked are decomposed into two components, employment and hours worked per worker, a positive spending innovation leads to different responses of employment and hours worked per worker. Hours worked per worker respond to the shock more quickly than employment. Hours worked per worker increase in the first several periods and then decrease. Employment stays at the same level in the first several periods and then gradually decreases. The changes in employment are more persistent than hours worked per worker.

### **3 A Stochastic General Equilibrium Model with a Matching Mechanism**

The model economy includes households, firms and government. The household's employment status is determined by a lottery mechanism. It is assumed that there is an insurance

market in the economy such that agents can insure themselves fully against the idiosyncratic risks. This assumption makes households ex-ante identical and simplifies the analysis. Firms create vacancies in the labor market and some vacancies are filled through a matching mechanism. In each period, some existing jobs are destroyed exogenously. The difference between the filled vacancies and the job separations is the increment in employment.

### 3.1 Households

The economy has a large number of infinitely lived households. The population size is normalized to 1. Each household has capital good  $a_t$  which can be rented to a firm and a unit of time which can be divided into working hours  $h_t$  and leisure  $1 - h_t$ . The household gets utility from consuming consumption goods,  $c_t$ , and enjoying leisure,  $1 - h_t$ . The momentary utility function at  $t$  is given by

$$U(c_t) + \xi H(1 - h_t)$$

where  $\xi$  is a constant parameter and  $U$  and  $H$  are assumed to be increasing, concave and twice continuously differentiable.

A household's employment status is determined in each period via a lottery mechanism similar to the one described by Hansen (1985) and Rogerson (1988). Assume there exists a competitive and costless insurance market. At the beginning of each period, households may purchase  $b_t$  units of insurance at a price  $p_t$  per unit, where  $b_t$  is the quantity of the consumption good which is delivered to the household contingent on unemployment during that period.

In period  $t$ , there are  $n_t$  available jobs to be rationed among the population. For each individual, the probability of being employed equals  $n_t$ . Households rent their capital stock at  $u_t$ , provide their labor  $h_t$  if they are employed.

The budget constraints of agents are contingent on their employment status. In the following constraints, subscript 1 represents the status of being employed, and 2 represents the status of being unemployed. Denote  $\delta_k$  as the depreciation rate of capital stock,  $w_t$  as the wage rate,  $u_t$  as the rental rate of capital stock. If the agent is employed, his income is composed of wage income ( $w_t n_t$ ), rent ( $u_t a_t$ ), dividend payment ( $\pi_t$ ) and transfer payment from government ( $T_t$ ); the income is allocated among consumption ( $c_t$ ), investment ( $a_{t+1} - (1 - \delta_k)a_t$ ), and payment of insurance premium ( $p_t b_t$ ). Thus the budget constraint for an employed agent is

$$c_{1,t} + a_{1,t+1} - (1 - \delta_k)a_{1,t} + p_t b_t \leq w_t h_t + u_t a_t + \pi_t + T_t$$

Similarly, an unemployed agent's income is composed of receipt of insurance payment, interest, dividend payment and net transfer from government; and the income is allocated among consumption, investment and insurance premium. The budget constraint for an unemployed agent is

$$c_{2,t} + a_{2,t+1} - (1 - \delta_k)a_{2,t} + p_t b_t \leq b_t + u_t a_t + \pi_t + T_t$$

The household's problem is to maximize the expected discounted utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \{n_t [U(c_{1,t}) + \xi H(1 - h_{1,t})] + (1 - n_t) [U(c_{2,t}) + \xi H(1)]\}$$

with probability  $n_t$ , the household is employed, and with probability  $1 - n_t$ , the household is unemployed, subject to the above employment contingent budget constraints, where the agent takes  $\{w_t, u_t, \pi_t, n_t\}$  as given.

In the presence of a costless competitive insurance market, it can be shown that households choose to insure themselves fully in equilibrium. Consequently, the agents are ex ante identical, and  $c_{1,t} = c_{2,t}$ ,  $a_{1,t} = a_{2,t}$ ,  $b_t = w_t h_t$ ,  $p_t = (1 - n_t)$ .

Given that the agents are ex ante identical, the household's optimization problem can be rewritten as

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \{U(c_t) + n_t \xi H(1 - h_t) + (1 - n_t) \xi H(1)\}$$

subject to

$$c_t + a_{t+1} - (1 - \delta_k) a_t = u_t a_t + w_t n_t h_t + \pi_t + T_t$$

where the household takes  $\{n_t, w_t, u_t, \pi_t\}$  as given.

Two kinds of disturbances will be introduced later: a technology shock and a government spending shock. The government spending shock is of particular interest in this paper.

The Lagrangian associated with the optimization problem is

$$L_1 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [U(c_t) + n_t \xi H(1 - h_t) + (1 - n_t) \xi H(1)] \right. \\ \left. + \sum_{t=0}^{\infty} \beta^t \lambda_t [u_t a_t + w_t n_t h_t + \pi_t - c_t - a_{t+1} + (1 - \delta_k) a_t + T_t] \right\}$$

where  $a_0$  is given and  $\lambda_t$  is the multiplier attached to the t-period resource constraint.

The efficiency conditions are

$$\begin{aligned} U'(c_t) &= \lambda_t & (a) \\ \xi H'(1 - h_t) &= \lambda_t w_t & (b) \\ \lambda_t &= E_t \{ \beta \lambda_{t+1} [u_{t+1} + (1 - \delta_k)] \} & (c) \\ c_t + a_{t+1} &= (1 - \delta_k) a_t + T_t + u_t a_t + w_t n_t h_t + \pi_t & (d) \end{aligned} \tag{1}$$

for  $t = 0, 1, \dots, \infty$  and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t E_0 \{ \lambda_t a_{t+1} \} = 0$$

The household selects  $c_t$ ,  $h_t$  and  $a_t$  in a dynamically efficient manner. In (1.a),  $\lambda_t$  represents the shadow value of  $c_t$ , the equation states that the household equates marginal utility of date  $t$ 's consumption to its opportunity cost in terms of utils. Equation (1.b) equates the marginal utility of leisure to the value of foregone earnings. The opportunity cost of investment is equal to its future discounted returns in (1.c). Equation (1.d) is the budget constraint.

## 3.2 Firms

In each period, a firm's economic activities include renting capital service, hiring workers, creating job vacancies, organizing production and selling products.

A firm employs many workers; and on average, it is large enough to eliminate all uncertainty about the flow of its labor force. The firm's labor force is determined by the inflow, new hires, and outflow, separations. The separation rate,  $\delta_n$ , is exogenous and constant over time.

The firm creates vacancies ( $o_t$ ). The new hires ( $m_t$ ) are determined by the number of vacancies and the success rate ( $\theta_t$ ) at which vacancies become filled, specifically,

$$m_t = \theta_t o_t$$

The success rate is exogenous to the firm, and it is governed by the efficiency of the matching process in the labor market. For the firm, the law of motion of employment ( $n_t$ ) is

$$n_{t+1} = (1 - \delta_n)n_t + m_t$$

where  $m_t$  is the inflow of employment and  $\delta_n n_t$  is the outflow of employment, or the flow from employment to unemployment.

Creating a vacancy is costly, the firm has to spend resources on each job opening. The recruiting cost embodies the cost incurred when the firm advertises the job opening, recruits candidates, trains the successful candidate and organizes his job. In the dynamic equilibrium, vacancies reflect recruiting effort and move in response to expectations about profitability.

The firm's output  $q_t$  is determined by a Cobb-Douglas production technology:

$$q_t = f(k_t, n_t h_t; z_t) = k_t^\alpha (z_t n_t h_t)^{1-\alpha}$$

where

$k_t$  = capital stock at period  $t$ ;  
 $h_t$  = hours provided by a worker;  
 $z_t$  = an aggregate exogenous shock to technology;  
 and  $0 < \alpha < 1$ .

We assume the  $z_t$  process is a logarithmic random walk with drift:

$$\ln(z_t) = \ln(z_{t-1}) + \ln(z) + \epsilon_{z,t}$$

where the innovation,  $\epsilon_{z,t}$ , is assumed to be identically and independently distributed through time with zero mean. The growth rate of this economy is  $\ln(z)$ .

A firm's profit is the difference between the revenue from the sale of output and the cost of hiring labor, renting capital services and creating vacancies. We treat output as the numeraire, all factor prices are relative prices. The firm's pure profit in period  $t$  is

$$\pi_t = q_t - u_t k_t - w_t n_t h_t - \kappa_t o_t$$

where

$u_t$  = rental rate of capital service at  $t$ ;  
 $w_t$  = wage rate at  $t$ ;  
 $\kappa_t$  = recruiting cost per vacancy at  $t$ .

Given the prices  $\{(u_t, w_t, \kappa_t)\}_{t=0}^{\infty}$ , the separation rate  $\delta_n$ , the success rate  $\theta_t$  and the initial labor force  $n_0$ , the problem faced by a firm is to choose the amount of capital services, the number of vacancies and output  $\{(k_t, o_t, q_t)\}_{t=0}^{\infty}$  that maximize present value of profits. Thus its decision problem is

$$\max E_0 \sum_{t=0}^{\infty} \prod_{j=0}^t R_j^{-1} \pi_t$$

subject to the following constraints:

$$\begin{aligned} q_t &= f(k_t, n_t h_t; z_t) \\ n_{t+1} &= n_t(1 - \delta_n) + \theta_t o_t \end{aligned}$$

where  $R_t = u_t + 1 - \delta_k$ , the firm takes  $k_0, o_0, \delta_n, u_t, w_t, n_t, h_t, \kappa_t, \theta_t$  as given.

The Lagrangian of this problem is

$$L_2 = E_0 \sum_{t=0}^{\infty} \Pi_{j=0}^t R_j^{-1} \{ f(k_t, n_t h_t; z_t) - u_t k_t - w_t n_t h_t - \kappa_t o_t - \{ \eta_t (n_{t+1} - (1 - \delta_n) n_t - \theta_t o_t) \} \}$$

The efficiency conditions are the following:

$$\begin{aligned} f_1(k_t, n_t h_t; z_t) &= u_t & (a) \\ \kappa_t &= \eta_t \theta_t & (b) \\ \eta_t &= ER_{t+1}^{-1} [f_2(k_{t+1}, n_{t+1} h_{t+1}; z_{t+1}) h_{t+1} - w_{t+1} h_{t+1} + (1 - \delta_n) \eta_{t+1}] & (c) \\ n_{t+1} &= (1 - \delta_n) n_t + \theta_t o_t & (d) \end{aligned} \quad (2)$$

The act of job creation is a decision by the firm to fill a vacant job at some cost. In equilibrium, the aggregate number of vacancies adjusts to eliminate any rent attributable to holding a job vacancy. Equation (2.b) is a free-entry condition. It equates the recruiting cost of a vacancy to the expected present value of holding a vacancy. The variable  $\eta_t$  can be explained as the capital value of a hired worker to the firm.<sup>7</sup> Equation (2.c) defines the shadow value  $\eta_t$  as the profit the new worker will make to the firm at  $t + 1$  plus the expected shadow value which is 0 with probability  $\delta_n$  if the worker separates from the firm, and is  $\eta_{t+1}$  if he remains to work in the following period.

The Euler conditions satisfied by the optimal sequences of  $k_t$  and  $o_t$  are:

$$\begin{aligned} f_1(k_t, n_t h_t; z_t) &= u_t & (a) \\ \kappa_t / \theta_t &= E_t R_{t+1}^{-1} \{ f_2(k_{t+1}, n_{t+1} h_{t+1}; z_{t+1}) h_{t+1} - w_{t+1} h_{t+1} + (1 - \delta_n) \kappa_{t+1} / \theta_{t+1} \} & (b) \end{aligned} \quad (3)$$

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<sup>7</sup>This becomes clear if we write out the complete expression from (2.c)

$$\eta_t = E_t \sum_{i=1}^{\infty} \Pi_{j=0}^i R_j^{-1} (1 - \delta_n)^{(i-1)} (f_2(k_{t+i}, n_{t+i} h_{t+i}; z_{t+i}) h_{t+i} - w_{t+i} h_{t+i}).$$

In each period, the firm realizes some economic rent,  $f_2 h - w h$ , from a retained worker. The worker hired at  $t$  will remain in this firm at  $t + 1$  and has probability  $\delta_n$  to leave the firm in the periods after. Thus  $\eta_t$  is the summation of the expected economic rent induced from a job filled in at  $t$ .

We observe that if  $\kappa_t = 0$ , equation (3.b) would reduce to the standard marginal productivity condition for employment.

### *Job Creation, Job Destruction and the Matching Process*

According to Blanchard and Diamond (1989), the labor market in the U.S. is highly effective in allocating workers to jobs. The flows are large in proportion to stocks. The average duration of unemployment rarely exceeds 3 months; and the average duration of vacancies does not exceed a month. This implies the simultaneous coexistence of unemployment and vacancies. The study of worker flows to and from employment has generated a considerable literature.<sup>8</sup> The theoretical foundation of the matching process arose out of search and matching theory (see Pissarides (1990)). The basic idea is that the recruiting effort of employers and the search effort of workers serve as inputs in a market matching function that generates new hires. The job vacancies and unemployed workers that are matched at any point in time are randomly selected from the sets  $o_t$  and  $1 - n_t$ .  $(1 - n_t)/o_t$  is a measure of labor market tightness.

In this paper, we assume that all the unemployed workers search for jobs.<sup>9</sup> The rate at which vacant jobs and searching workers match is determined by an increasing, concave, and homogeneous of degree one function  $m(o_t, 1 - n_t)$  where  $o_t$  and  $1 - n_t$  respectively represent the number of jobs that employers are attempting to fill and the number of workers seeking those

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<sup>8</sup>Devine and Kiefer (1991) include an extensive review of panel-based studies. Mortensen and Pissarides (1993) and Mortensen (1994) study the interaction of job creation and job destruction in a dynamic stochastic equilibrium framework.

<sup>9</sup>This assumption could be relaxed if we consider the situation with discouraged workers. More people participate when wages are higher and labor market tightness is higher. The dependence of the participation rate on labor market tightness is called the “discouraged worker effect”. When the rate at which unemployed workers find jobs falls, fewer workers participate.

jobs. Under the assumptions of random matching and constant returns, the probabilistic rate at which vacancies are filled is  $\theta_t = m(o_t, 1 - n_t)/o_t = m(1, (1 - n_t)/o_t)$ . The process that changes the state of a vacant job is Poisson with rate  $\theta_t$ . The mean duration of a vacant job is  $1/\theta_t$ . Unemployed workers move into employment according to a Poisson process with rate  $m(o_t, 1 - n_t)/(1 - n_t) = \theta_t o_t/(1 - n_t)$ . The mean duration of unemployment is  $d_t = (1 - n_t)/m_t$ .

In this paper, we assume the matching process is governed by a well behaved Cobb-Douglas matching function,<sup>10</sup>

$$m(o_t, 1 - n_t) = \chi o_t^\psi (1 - n_t)^{(1-\psi)} \leq \min\{o_t, 1 - n_t\}$$

### *Wage Rate Determination and Recruiting Cost*

Wage is determined by decentralized bargaining between workers and firms. The match between the worker and the firm creates a surplus that must be bargained over. The wage rate is given by implicit bargaining at the individual level. The outcome of the bargaining is simply assumed as:

$$w_t = (1 - \gamma)f_2(k_t, n_t h_t; z_t)$$

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<sup>10</sup>The matching function in this paper is assumed to be constant returns to scale. According to Blanchard and Diamond (1989), there is an empirical evidence suggesting constant or mildly increasing return to scale. The assumption of the inequality implies that the mean duration of vacancy  $o_t/h_t$  is more than one month and also the mean duration of unemployment  $(1 - n_t)/h_t$  is more than one month. There is no question about the assumption that the mean duration of unemployment is more than one month, according to Blanchard and Diamond (1989), it is about three months. However, according to the same study, the average duration of vacancies is less than one month. But van Ours and Ridder (1992) point out that a distinction should be made between the time a help-wanted advertisement is removed and the time it actually takes to fill a vacant position. They report that while seventy-five percent of all vacancies are filled by applicants who arrive in the first two weeks, it takes on average forty-five days to select a suitable employee from the pool of applicants. In this paper, the help-wanted advertising index is used as the proxy of vacancies.

where  $\gamma$  is a constant ( $0 < \gamma < 1$ ) and a measure of bargaining power of the firm. Thus, the wage rate is proportional to hourly productivity. The standard model equates wage rate to marginal product of labor. Thus the wage equation in the standard model can be viewed as a special case of the wage equation in this paper ( $\gamma = 0$ ).

Firms spend resources on hiring, and this activity is an economic one just like production. To be consistent with balanced growth, recruiting cost per vacancy  $\kappa_t$  is assumed to have the same growth rate as the technological level  $z_t$ . In a detrended economy, the recruiting cost is constant,  $\kappa_t = \kappa$ .

(2.b) is then rewritten as

$$-\frac{\kappa_t o_t}{m(o_t, 1 - n_t)} + ER_{t+1}^{-1} \left\{ f_2(k_{t+1}, n_{t+1} h_{t+1}; z_{t+1}) h_{t+1} \gamma + \frac{(1 - \delta_n) \kappa_{t+1} o_{t+1}}{m(o_{t+1}, 1 - n_{t+1})} \right\} = 0$$

### 3.3 Government

Government spending is exogenous. The government finances its consumption solely by lump-sum taxation. This paper does not consider distortionary taxes. The government faces the following budget constraint

$$g_t + T_t = 0$$

### 3.4 A Competitive Equilibrium

**Definition 1** *A competitive equilibrium is a set of prices  $\{u_t, w_t\}_{t=0}^{\infty}$ , an allocation  $\{(c_t, a_t, h_t, \pi_t)\}_{t=0}^{\infty}$  for a typical household and an allocation  $\{(k_t, o_t, \pi_t)\}_{t=0}^{\infty}$  for a representative firm, given exogenous sequences of technology shocks,  $\{z_t\}_{t=0}^{\infty}$  and government spending,  $\{g_t\}_{t=0}^{\infty}$ , such that*

(1) given  $\{u_t, w_t, \pi_t, T_t\}$ ,  $\{c_t, a_t, h_t\}$  solve the household's problem

(2) given  $\{u_t, w_t\}$ ,  $\{o_t, k_t\}$  solve the firm's problem

(3)  $w_t$  satisfies the bargaining solution

(4) government's budget constraints are satisfied

(5) all markets clear,

$$a_t = k_t, \theta_t = m(o_t, 1 - n_t)/o_t.$$

The equilibrium conditions are summarized as follows:

$$\begin{aligned}
U'(c_t) &= \beta E_t \{ (1 + f_1(k_{t+1}, n_{t+1} h_{t+1}; z_{t+1}) - \delta_K) U'(c_{t+1}) \} & (a) \\
\xi H'(1 - h_t) &= (1 - \gamma) f_2(k_t, n_t h_t; z_t) U'(c_t) & (b) \\
n_{t+1} &= (1 - \delta_n) n_t + m(o_t, 1 - n_t) & (c) \\
\kappa_t o_t / m(o_t, 1 - n_t) &= E_t R_{t+1}^{-1} \{ f_2(k_{t+1}, n_{t+1} h_{t+1}; z_{t+1}) h_{t+1} \gamma + (1 - \delta_n) \kappa_{t+1} o_{t+1} / m(o_{t+1}, 1 - n_{t+1}) \} & (d) \\
f(k_t, n_t h_t; z_t) &= c_t + k_{t+1} - (1 - \delta_k) k_t + g_t + \kappa_t o_t & (e)
\end{aligned} \tag{4}$$

These equations are used in GMM to get the estimates of the parameters.

## 4 Estimation Method (GMM) and Data Measures

The estimation method is Generalized-Method-of-Moments (GMM) developed by Hansen (1982) and Hansen and Singleton (1982). Like Christiano and Eichenbaum (1992), Burnside, Eichenbaum and Rebelo (1993) and Burnside and Eichenbaum (1996), we use an exact identified GMM estimator.

The GMM criterion is set up so that the estimated model exactly matches the sample analog of certain unconditional moments of the data generating process.

Government spending  $g_t$  is assumed to be exogenous and follows

$$\ln(g_t) = \ln(z_t) + \mu_g + \ln(\tilde{g}_t)$$

where  $\tilde{g}_t$  is zero mean, trendless component of  $g_t$  and follows

$$\ln(\tilde{g}_t) = \rho_g \ln(\tilde{g}_{t-1}) + \epsilon_{gt}$$

where  $|\rho_g| < 1$ ,  $\epsilon_{gt}$  is the innovation in  $\ln(\tilde{g}_t)$  with zero mean and standard deviation  $\sigma_g$ . This specification implies that government spending grows at the same rate as that of total output so that a balanced growth path exists.

The structure parameters in the models are:

*Preference:*  $\beta, \xi$

*Technology:*  $\alpha, \delta_k, z, \sigma_z$

*Labor Search:*  $\kappa, \gamma, \delta_n, \chi, \psi$

*Government:*  $\mu_g, \rho_g, \sigma_g$

The parameters  $\beta, \kappa, \psi$  are not estimated. Instead  $\beta$  is set at  $1.03^{-0.25}$ .  $\kappa$  is set to 2 such that the share of recruiting cost is around 4%. Alternatively,  $\kappa$  can be estimated by setting the value of  $\gamma$ , which reflects the bargaining power of a firm. Larger  $\gamma$  indicates bigger bargaining power of a firm.  $\gamma$  and  $\kappa$  are closely related, higher  $\gamma$  means more profit to the firm and then the firm can compensate more cost in recruitment, thus  $\kappa$  is bigger.  $\psi$  is set to 0.6, a value estimated by Blanchard and Diamond (1989) based on 1968-81 sample period.

### *The Moment Restrictions Underlying the GMM estimator*

Denote

$$\Psi = \{\delta_k, \alpha, \xi, \gamma, \delta_n, \ln(z), \sigma_z, \mu_g, \rho_g, \sigma_g, \chi\}$$

The time series used in GMM include private consumption,  $c_t$ , gross investment,  $i_t$ , capital stock,  $k_t$ , government spending,  $g_t$ , employment rate,  $n_t$ , hours per worker,  $h_t$ , vacancy rate,  $o_t$ , average duration of unemployment,  $d_t$ . Appendix 1 contains a detailed explanation and construction of these series.

*Remark on the construction of the series  $q_t$ :* different from previous studies, output in

this model equals aggregate demand plus the resource costs in labor market search, i.e. the summation of  $c_t$ ,  $i_t$ ,  $g_t$  and  $\kappa_t o_t$ . The data on  $q_t$  is not available because the data on recruiting costs are not available. We adopt the following strategy to solve this problem. As assumed earlier, recruiting cost per vacancy has to grow at the same rate as the technology level  $z_t$  to meet the requirement of balanced growth. In this model,  $q_t$  also grows at the same rate. So we assume  $\kappa_t$  is proportional to  $q_t$ ,  $\kappa_t = \kappa q_t$ . Then output can be written in terms of the available time series and a parameter  $\kappa$ , that is  $q_t = (c_t + i_t + g_t)/(1 - \kappa o_t)$ .

The parameter  $\delta_k$  is identified by a condition

$$E[\ln(\delta_{kt}) - \ln(\delta_k)] = 0 \quad (5)$$

where  $\delta_{kt} = 1 + (i_t - k_{t+1})/k_t$ .

The parameter  $\alpha$  is identified by the intertemporal Euler equation

$$E[1/\beta - (\alpha q_{t+1}/k_{t+1} + (1 - \delta_{kt}))c_t/c_{t+1}] = 0 \quad (6)$$

By the assumption of  $z_t$  process,

$$\ln(z_{t+1}) = \ln(z_t) + \ln(z) + \epsilon_{z,t+1}$$

and the production function, the technology shock is derived as

$$\ln(z_t) = \frac{1}{1 - \alpha} [\ln(q_t) - \alpha \ln(k_t)] - \ln(n_t h_t)$$

$\ln(z)$  can be identified by the balanced growth restriction which says that the mean growth rate of output coincides with that of technology.

$$E[\ln(q_t) - \ln(q_{t-1}) - \ln(z)] = 0, \quad (7)$$

also  $\sigma_z$  can be identified from the condition,

$$E[(\ln(z_t) - \ln(z) - \ln(z_{t-1}))^2 - \sigma_z^2] = 0 \quad (8)$$

The parameter  $\xi$  is identified through the intratemporal efficiency condition,

$$E\left[\xi - (1 - \gamma)(1 - \alpha)\frac{1 - h_t q_t}{n_t h_t c_t}\right] = 0 \quad (9)$$

Now we identify  $\kappa$ . From the Cobb-Douglas production function, we have

$$f_2(k_t, n_t h_t; z_t) = (1 - \alpha)q_t/(n_t h_t)$$

$$R_t = 1 - \delta_k + \alpha q_t/k_t$$

Rewrite the Euler equation (3.b) of  $n_t$  as

$$E\left\{-\kappa o_t/n_{t+1} - (1 - \delta_n)n_t(1 - \delta_K + \alpha q_{t+1}/k_{t+1}) + q_{t+1}/q_t((1 - \alpha)\gamma/n_{t+1} + (1 - \delta_n)\kappa o_{t+1}/n_{t+2} - (1 - \delta_n)n_{t+1})\right\} = 0 \quad (10)$$

Now consider government spending. The stationary component of  $g_t$ ,  $\tilde{g}_t$ , can be written as

$$\ln(\tilde{g}_t) = \ln(g_t) - \frac{1}{1 - \alpha}(\ln(q_t) - \alpha \ln(k_t)) + \ln(n_t h_t) - \mu_g$$

$$E[\ln(\tilde{g}_t)] = 0 \quad (11)$$

$$E[\ln(\tilde{g}_t) - \rho_g \ln(\tilde{g}_{t-1})] \ln(\tilde{g}_{t-1}) = 0 \quad (12)$$

$$E[(\ln(\tilde{g}_t) - \rho_g \ln(\tilde{g}_{t-1}))^2 - \sigma_g^2] = 0 \quad (13)$$

where  $T$  is the number of observations. These moment restrictions can be used to estimate  $\mu_g$ ,  $\rho_g$ , and  $\sigma_g$ .

From the matching function

$$m_t = \chi o_t^\psi (1 - n_t)^{1-\psi}$$

The following two moment conditions are used to estimate  $\delta_n$  and  $\chi$ .

$$E(n_{t+1} - (1 - \delta_n)n_t - m_t) = 0 \quad (14)$$

$$E[(n_{t+1} - (1 - \delta_n)n_t - m_t)t/T] = 0 \quad (15)$$

We have 11 parameters in  $\Psi$  to be estimated and 11 moments conditions (equation 5-15) forming an exact identification system. The application of GMM requires that each equation include only stationary variables. The moment conditions relating to  $\Psi$  already satisfy the requirement, because  $c_t$ ,  $q_t$ ,  $k_t$  and  $g_t$  all grow at rate  $\ln(z)$ , and  $n_t$ ,  $h_t$  and  $o_t$  are stationary, then the variables involved  $\delta_{kt}$ ,  $q_t/k_t$ ,  $c_t/c_{t+1}$ ,  $q_t/c_t$ ,  $(1 - h_t)/(n_t h_t)$ ,  $o_t/n_t$ ,  $q_t/q_{t-1}/n_t$  are stationary.

### *Data Measures*

The following time series are expressed in quarterly real per capita terms. Detailed data description is given in Appendix 1.

Private consumption,  $c_t$ , is measured as personal expenditures on nondurable goods plus services. Government spending,  $g_t$ , is measured by real government purchases of goods and services. The capital stock,  $k_t$ , is measured as a net, end-of-period stock consisting of non-residential, fixed capital owned by the private sector and government enterprises and that owned by government but privately operated plus government (federal, state, and local), net, end-of-period, fixed, nonresidential capital plus government and private residential capital, and consumer durable goods. Gross investment,  $i_t$ , is measured as the sum of private investment and government investment corresponding to the above capital stock. Employment rate,  $n_t$ , is measured as the ratio of persons employed to non-institutional population, aged

16 and over. Average working hours,  $h_t$ , is measured as the ratio of aggregate hours of wage and salary workers in non-agricultural establishments to the number of persons employed. Vacancy rate,  $o_t$ , is measured as the ratio of help-wanted advertising to non-institutional population.

### *GMM Estimation Results*

For the purposes of comparison, a version of Burnside, Eichenbaum and Rebelo’s (1993) labor hoarding model is also considered and referred as the *benchmark model*.<sup>11</sup> In the original BER model, effort is flexible and the shift length is fixed. To facilitate comparison, we reinterpret effort to be hours worked per worker and set shift length to be one. Appendix 2 presents a more detailed description of the BER model. The parameters in the BER model to be estimated include  $\{\delta_k, \alpha, \xi, \ln(z), \sigma_z, \mu_g, \rho_g, \sigma_g, e\}$ . where  $e$  is a fixed cost to go to work in terms of hours of foregone leisure. The model developed in Section 2 is referred as the *search model*. The parameters of the benchmark model and the search model are estimated using the same data set. Model parameter estimates and standard errors are reported in Table 1.

[Insert Table 1 here]

The estimate of the capital depreciation rate for both is 0.02. The capital share of output,  $\alpha$ , is estimated to be around 0.30. These estimates are similar to those in Christiano and Eichenbaum (1992) and Burnside and Eichenbaum (1996).

The separation rate,  $\delta_n$ , is estimated to be 0.11. In steady state, the average duration of a job advertisement is equal to  $o/\delta_n n = 0.29$  month.

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<sup>11</sup>We thank an anonymous referee for this suggestion.

The matching efficiency coefficient in the matching function,  $\chi$ , is estimated to be 0.96.

## 5 Near Steady State Dynamics of the Model

In order to examine the dynamic responses of the economy to different persistence levels of government spending shocks, we adopt the method by King, Plosser and Rebelo (1990) to get the near steady state dynamics of the model.

In this model, there are two state variables, the capital stock  $k_t$  and employment  $n_t$ , and also two co-state variables,  $\lambda_t$  in the household's problem and  $\eta_t$  in the firm's problem. For control variables, the households decide on consumption  $c_t$  and hours worked per worker  $h_t$ , and the firm decides on vacancies  $o_t$ . By the law of motion of the labor force, vacancies can be expressed in terms of employment, thus the problem can be rewritten to have 4 state, co-state variables and 2 control variables.

In the following analysis based on the search model, four kinds of shocks with different degrees of persistence are considered, transitory shock ( $\rho_g = 0$ ), persistent shocks, ( $\rho_g = 0.93$ ), ( $\rho_g = 0.95$ ) and ( $\rho_g = 0.97$ ), where ( $\rho_g = 0.95$ ) corresponds to the GMM estimate in the last section, and ( $\rho_g = 0.93$ ) and ( $\rho_g = 0.97$ ) correspond to minus-one and plus-one standard deviations in the persistence level. Also we will compare the VAR model with the search model which has ( $\rho_g = 0.95$ ).

In the benchmark model, we also consider four kinds of shocks with different degrees of persistence, transitory shock ( $\rho_g = 0$ ), persistent shocks, ( $\rho_g = 0.98$ ), ( $\rho_g = 0.97$ ) and ( $\rho_g = 0.99$ ), where ( $\rho_g = 0.98$ ) corresponds to the GMM estimate in the last section, and ( $\rho_g = 0.97$ ) and ( $\rho_g = 0.99$ ) are close to minus-one and plus-one standard deviations.

Figures 3 to 6 plot the dynamic responses of selected variables in the benchmark and search models to a one-standard-deviation shock to government consumption.

### *Total Hours Worked*

In the benchmark model with a positive income effect on leisure, persistent changes in government spending always have an effect on total hours worked and output that is larger than the effect of transitory changes. The response functions in Figure 3(b) show that a transitory shock ( $\rho_g = 0$ ) increases hours by 0.02%, while persistent shocks ( $\rho_g = 0.98$ ) increase hours by 0.32%.<sup>12</sup> The reason is well explained in Aiyagari, Christiano and Eichenbaum (1992). Holding constant private investment, the effect of government spending on hours worked is positive due to a negative wealth effect. A transient increase in government spending reduces investment, but a persistent increase in government spending either increases investment or does not reduce it by as much as in the transient case. Thus persistent changes in government spending generate larger contemporaneous effects on both hours worked and output than transient changes.

Figure 3(a) plots the impulse responses of total hours worked in the search model. These responses are the combination of the responses of employment and hours worked per worker which we will explain in the next section. The time paths of total hours worked are similar to those in the benchmark model, but with smaller magnitudes.

[Insert Figure 3 here]

### *Employment and Hours Worked per Worker*

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<sup>12</sup>These magnitudes of the responses are similar to Campbell (1994). In the model with indivisible labor (Table 7  $\sigma_n = \infty$ ), the hours worked elasticities with government spending are 0.04, 0.06, 0.27, 0.43, corresponding to the persistence of government spending, 0.00, 0.50, 0.95, and 1.00. If  $g = 0.2$  and  $n = 1/3$ , the changes are 0.06, 0.1, 0.45, 0.72 base points, given a base point change in government spending. The output changes are 0.1, 0.2, 0.9 and 1.45 base points, given a base point change in  $g$  and  $q = 1$ .

In the benchmark BER model, the hours worked per worker increases in the first quarter and then returns to the steady state level, the increase in total employment starts in the second quarter and returns to the steady state gradually from above (Figure 3(d),4(b)). The reason is well explained in Burnside, Eichenbaum and Rebelo (1993) and Hall (1996). Given a positive government expenditure shock, the only contemporaneous labor margin that can be adjusted is the hours worked per worker. It would increase due to a negative wealth effect. In the second quarter, employment starts to increase so that the hours worked returns to its steady-state level since in the steady state, increasing employment reduces an agent's expected utility less than increasing the hours worked per worker (utility is linear over employment while curved over effort).

In the search model, hours worked per worker and employment respond differently to a government spending shock (Figure 3(c),4(a)). The shock affects them through different mechanisms. Hours worked per worker are determined by the household through intertemporal substitution. Employment is determined by job destruction and creation in the labor market. Changes in employment in this model depend crucially on the decision by the firm to create a vacant job at some cost. In the dynamic equilibrium, vacancies reflect recruiting effort and move in response to the expectation of the profitability of a successful match.

[Insert Figure 4 here]

Either a transient or persistent government spending shock increases hours worked per worker in the first period. More persistent shocks lead to larger increases in hours worked per worker. In the following periods, hours decrease and gradually return to the steady state value.

Compared to the response of hours worked per worker, employment responds to the shock slowly. The transitory shock ( $\rho_g = 0$ ) decreases employment, and employment reaches its

lowest point 5 periods after the shock, then employment increases gradually and eventually returns to its steady state value.<sup>13</sup>

The explanation of the employment behavior is as follows. In this model, job creation is determined by the job openings of the firm. The firm increases or decreases job openings based on the expected capital value of a hired worker to the firm. A higher value of a hired worker to the firm encourages the firm to create more vacancies. The capital value of a hired worker to the firm is the expected discounted economic rent that a worker brings to the firm. There are two factors that affect the capital value. One is the real interest rates and the other is pure economic rent in each period. Higher interest rates lower the expected capital value of a hired worker. Higher economic rent increases the capital value.

Negative wealth effects bring private consumption below its steady state value in the impact period, and then consumption gradually returns to its steady state value (Figure 5(a)). The convergence is monotonic.<sup>14</sup> Under the demand shock, there exists an excess demand in the goods market at the prevailing interest rate, and the interest rate must go up to clear the goods market (Figure 5(c)). The model shows that an increasing private consumption time path from below the steady state is accompanied by an above steady state average interest rate.

[Insert Figure 5 here]

[Insert Figure 6 here]

A worker's economic rent is determined by his/her productivity and hours. A shock may either increase or decrease the economic rent, because the response of the productivity of

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<sup>13</sup>A persistent shock may decrease or increase employment depending on the degree of the persistence. Numerical experiments show that a random walk government spending shock with ( $\rho_g = 1$ ) increases employment.

<sup>14</sup>A shock may crowd in or crowd out private investment depending on the value of  $\rho_g$ . Numerical experiments show that a temporary shock with persistence  $\rho_g$  close enough to one crowds in private investment.

labor may be opposite to that of hours (Figure 6(a),(b)). The figures show that even though the economic rent increases to a shock with  $0 < \rho_g \leq 0.97$  (Figure 3(a) and 6(a)), the negative interest rate effect on the capital value still dominates the positive economic rent effect. Thus the expected capital value decreases and vacancies are less (Figure 4(c),(d)).<sup>15</sup> In Figure 4(a), the employment levels to shocks with higher persistence levels ( $\rho_g = 0.91, 0.95$ ) are lower than that to a transitory shock ( $\rho_g = 0$ ). We also observe that the employment levels with ( $\rho_g = 0.95, 0.97$ ) are above that of ( $\rho_g = 0.93$ ).

We can understand the above observation by viewing the responses of the shadow values of a hired worker and vacancies (Figure 4(c),(d)). The shadow value to a shock with ( $\rho_g = 0$ ) is higher than those of ( $\rho_g = 0.95$ ) and ( $\rho_g = 0.97$ ), the ones with ( $\rho_g = 0.95$ ) and ( $\rho_g = 0.97$ ) are higher than the one with ( $\rho_g = 0.93$ ). Correspondingly, the case with transitory shock has more vacancies than the cases with ( $\rho_g = 0.93$ ) and ( $\rho_g = 0.95$ ), and the cases with ( $\rho_g = 0.95$ ) and ( $\rho_g = 0.97$ ) have more vacancies than the case with ( $\rho_g = 0.93$ ).

### *Output Effects*

The output effects are similar in these two models (Figure 6(c), (d)). A shock in government spending always increases output with more persistent shocks leading to greater increases. We observe the multiplier effects with persistent government spending shocks. In the search model, an increase in government spending unambiguously raises hours worked per worker, but may increase or decrease the capital stock and employment. The results indicate that the positive hours effect dominates the other two effects regardless of the persistence of the shock.

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<sup>15</sup>However, for shock with  $\rho_g$  close enough to 1, the positive rent effect dominates the negative interest rate effect, resulting in an increase to the expected capital value.

## *Comparison of Hours and Employment Effects of the Search Model and VAR Model*

Another comparison is between the VAR models in section 2 and the search model with a shock ( $\rho_g = 0.95$ ) which is the GMM estimate. In the search model, a one standard deviation increase in government spending increases hours worked per worker by 0.3% in the impact period, and then hours worked per worker gradually return to the steady state. In the VAR models, hours worked per worker increase by 0.24% during the first 5 quarters and then decreases.

The employment time paths of these two models are similar, employment gradually decreases in the two models after a shock, and reaches its lowest level in the 15<sup>th</sup> to 20<sup>th</sup> quarters. But the quantitative decrease in the search model is much smaller than that in the VAR model. However the employment response in the VAR model is not significantly different from zero. Thus qualitatively, the search model can generate similar responses of employment and hours worked per worker to those in the empirical VAR studies.

## **6 Conclusions**

This paper focuses on the effects of temporary government spending shocks in the U.S. economy on employment, hours worked per worker and output. Several VAR models demonstrate that, according to the U.S. data, a temporary innovation in government spending raises both hours worked per worker and output, but lowers the employment level. We construct a stochastic general equilibrium model in which employment is determined by a matching mechanism. The results show that in contrast to a reinterpreted Burnside, Eichenbaum and Rebelo's (1993) labor hoarding model, the search model can generate similar responses of

hours worked per worker and employment to those of VAR models.

# Appendix 1: Data Source Description and Transformation

## *Raw Data Source and Series*

(1) U.S. National Income and Product Accounts Tables, Bureau of Economic Analysis, U.S. Department of Commerce. Quarterly data. In 1987 dollars (Billions). Sample periods: 1948:1-1993:4.

CC - Personal Consumption Expenditures  
GC - Government Purchases of Goods and Services  
GFD- Federal National Defence(deflated by GDP deflator)  
IFC - Fixed Investment  
XNETC - Net Exports: Goods and Services  
VC - Change in Business Inventories  
GDPC - Gross Domestic Product  
JGDP - Implicit Deflator: Gross Domestic Product

(2) Fixed Reproducible Tangible Wealth in the U.S. tables. Bureau of Economic Analysis, U.S. Department of Commerce. Annual data. Sample periods: 1948-1993. In 1987 dollars (Millions).

NBNTIC - Nonres Pvt Cap, by Leg Org, Tot All Ind:Net Stock, Eq & Str  
IBNTIC - Nonres Pvt Cap, by Leg Org, Tot All Ind: Invest., Eq & Str  
NBNGOAMC - Nonres Gvt-Owned,Pvt Oper Cap, All Agen-Mfg:Net Stk, Eq & Str  
IBNGOAMC - Nonres Gvt-Owned,Pvt Oper Cap, All Agen-Mfg: Invest., Eq & Str  
NENTOXMC - Nonres Cap, All Owners, exc Military: Net Stock, Eq  
NSNTOXMC - Nonres Cap, All Owners, exc Military: Net Stock, Str  
NEDGTC - Consumer-Total Durable Goods: Net Stock, Eq  
IEDGTC - Consumer-Total Durable Goods: Investment, Eq  
NBRTOTGC - Res Cap, by Legal Org, All Own, inc Gvt:Net Stock, Eq & Str  
IBRTOTGC - Res Cap, by Legal Org, All Own, inc Gvt: Invest., Eq & Str  
NBNGFC - Nonres Gvt-Owned Capital, Federal: Net Stock, Eq & Str  
IBNGFC - Nonres Gvt-Owned Capital, Federal: Invest., Eq & Str  
NBNSLC - Nonres Total State & Local Govt, Net Stock, Eq & Str

IBNGSC - Nonres Govt Capital: State & Local, Investment, Eq & Str

(3) Bureau of Labor Statistics, U.S. Department of Labor, seasonal adjusted monthly data.

LE - Civilians Employed (Thous)

LQ - Employment Rate (%)

LHTNAGRA - Aggregate Hours of Wage and Salary Workers in Nonagr Estab (Bil. Hrs)

LENA - Civilians Employed: Nonagricultural Industries (SA, Thous.)

LR - Civilian Unemployment Rate (%)

LP - Civilian Participation Rate (%)

LNN - Civilian Noninstitutional Population (NSA, Thous.)

RATADVHW - The Ratio of Help-wanted advertisings to Persons Unemployed(%)

(4) CANSIM

TBR - 91-Day Treasury Bill yield (%)

#### *Data Transformation*

The following series are in per capita terms:

$$c = \log((CC/LNN) * 1000)$$

$$g = \log((GC/LNN) * 1000)$$

$$i = \log((IBNTIC + IBNGOAMC + IEDGTC + IBRTOTGC + IBNGFC + IBNSLC)/LNN)$$

$$k = \log((NBNTIC + NBNGOAMC + NEDGTC + NBRTOTGC + NBNGFC + NBNSLC)/LNN)$$

$$nl = (LHTNAGRA/LNNN) * 1000 * 1000 / (1369 * 4)$$

$$n = LE/LNN$$

$$l = nl/n$$

$$v = RATADVHW * LP * LR * 0.0001$$

## Appendix 2: A Version of Burnside, Eichenbaum and Rebelo's (1993) Labor Hoarding Model

The model economy is populated with a large number of infinitely lived individuals. To go to work, an individual must incur a fixed cost,  $e$ , denominated in terms of hours of forgone leisure. Once at work, an individual chooses hours worked  $h_t$ . The time endowment is normalized to 1. The momentary utility at time  $t$  is given by

$$\ln(c_t) + \xi n_t \ln(1 - e - h_t) + \xi(1 - n_t) \ln(1). \quad (16)$$

Output,  $y_t$ , is produced via the Cobb-Douglas production function

$$y_t = k_t^\alpha (z_t n_t h_t)^{1-\alpha} \quad (17)$$

where  $0 < \alpha < 1$ ,  $n_t$  denotes the total number of individuals going to work at time  $t$ ,  $k_t$  denotes the beginning of period of capital stock,  $z_t$  represents the growth rate of exogenous labor-augmenting technological progress and it evolves according to

$$\ln z_t = \ln z_{t-1} + \ln z + \varepsilon_{zt}, \quad (18)$$

$\varepsilon_{zt}$  is the innovation to  $\ln z_t$  with a standard deviation of  $\sigma_z$ . Firms commit to the number of workers employed before observing any shocks to the economy, after observing the shocks, firms can adjust the work hours of their employees.

The aggregate resource constraint is given by

$$c_t + k_{t+1} - (1 - \delta_k)k_t + g_t \leq y_t. \quad (19)$$

The parameter  $\delta$  represents the depreciation rate on capital. The random variable  $g_t$  denotes time  $t$  government consumption which evolves according to

$$\ln g_t = \ln z_t + \mu_g + \ln \tilde{g}_t \quad (20)$$

where  $\tilde{g}_t$  has the law of motion

$$\ln \tilde{g}_t = \rho_g \ln \tilde{g}_{t-1} + \varepsilon_{gt}, \quad (21)$$

$\varepsilon_{gt}$  is the innovation to  $\ln(g_t)$  with standard deviation  $\sigma_g$ .

The social planner chooses a set of stochastic processes  $\{k_{t+1}, n_{t+1}, h_t\}_{t=0}^{\infty}$  to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \xi n_t \ln(1 - e - h_t) + \xi(1 - n_t) \ln(1)] \quad (22)$$

subject (17)-(21) and  $k_0$  and  $n_0$ . Except some notations, the only difference between this model and the one presented in BER is that instead of having agents choose effort and shift length fixed, we treat the product of the two as the hours worked per worker which is a choice variable of the households.

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Parameter	Benchmark Model		Search Model	
	Estimate	St. Error	Estimate	St. Error
$\delta_k$	0.0202	0.0001	0.0204	0.0001
$\alpha$	0.3027	0.0056	0.2958	0.0049
$\ln(z)$	0.0044	0.0008	0.0034	0.0008
$\sigma_z$	0.0122	0.0009	0.0128	0.0009
$\xi$	4.0955	0.0440	4.6151	0.0404
$\mu_g$	-2.3773	0.0257	-2.4709	0.0225
$\rho_g$	0.9829	0.0142	0.9531	0.0225
$\sigma_g$	0.0193	0.0021	0.0127	0.0009
$e$	0.0974	0.0007		
$\delta_n$			0.1140	0.0045
$\gamma$			0.0657	0.0022
$\chi$			0.9602	0.0404

Table 1: GMM Estimates of Parameters in the Benchmark and Search Models

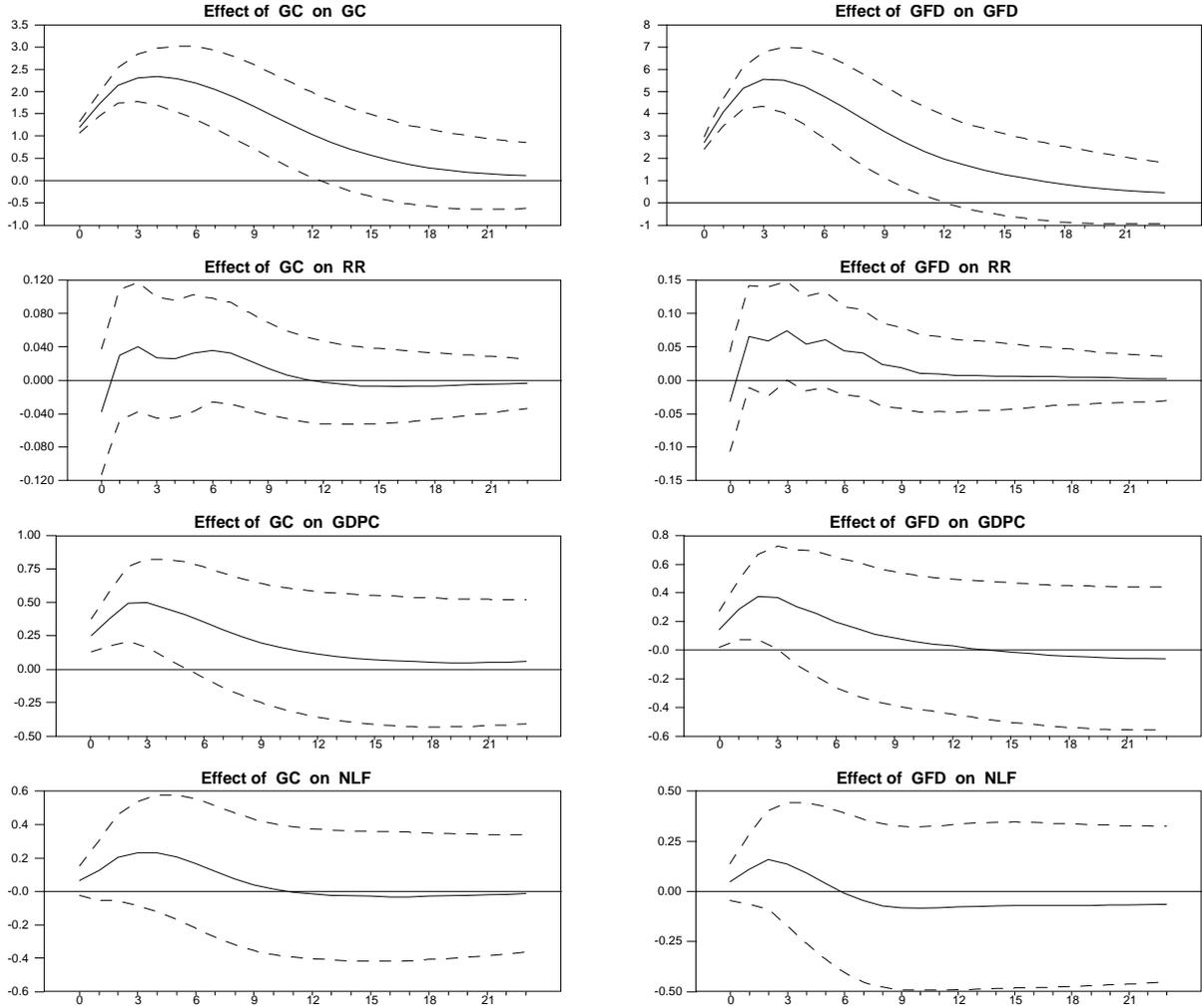


Figure 1. VAR: The Effect of Government Spending Shocks on Total Hours Worked

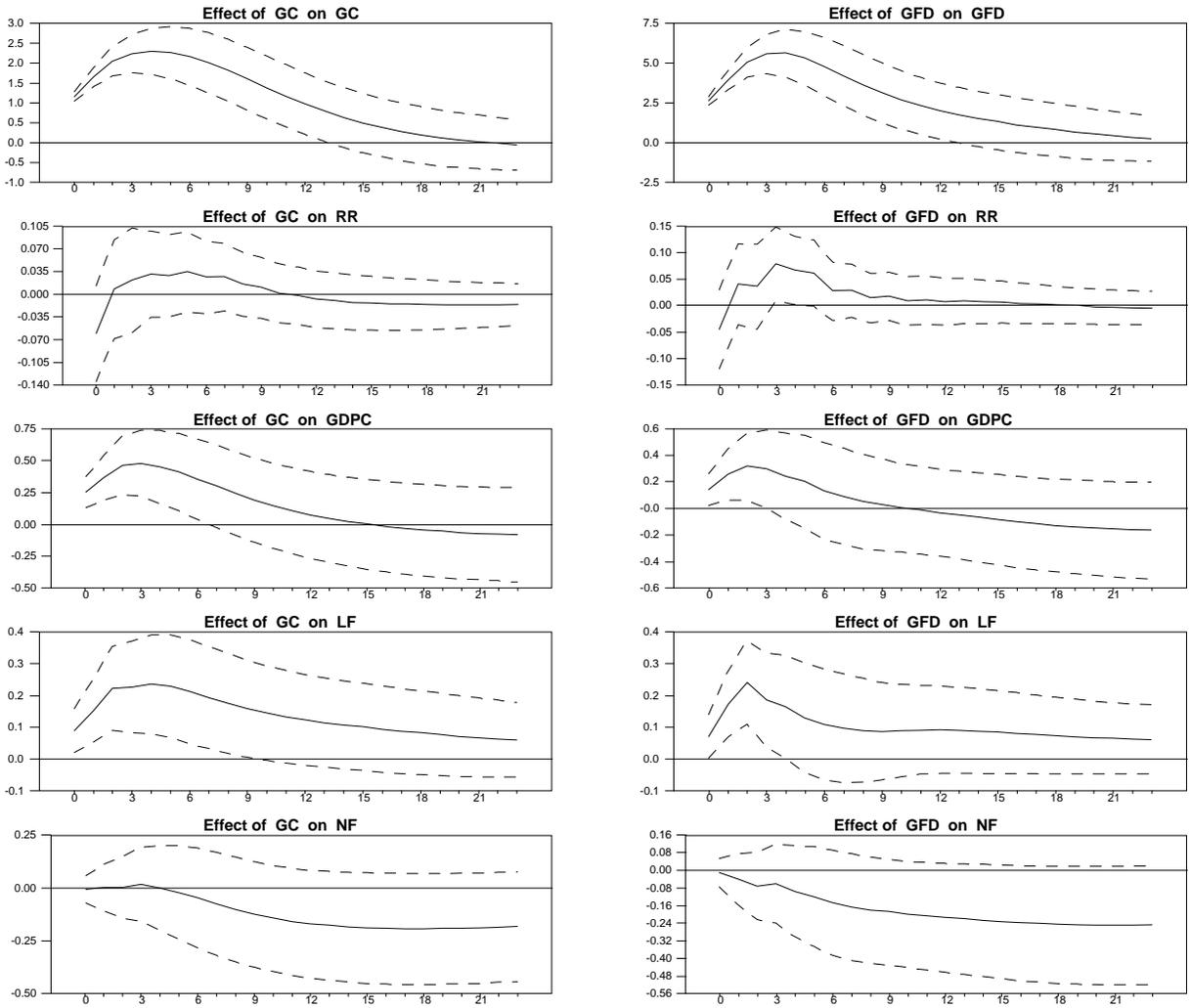


Figure 2. VAR: The Effect of Government Spending Shocks on Employment and Hours Worked per Worker

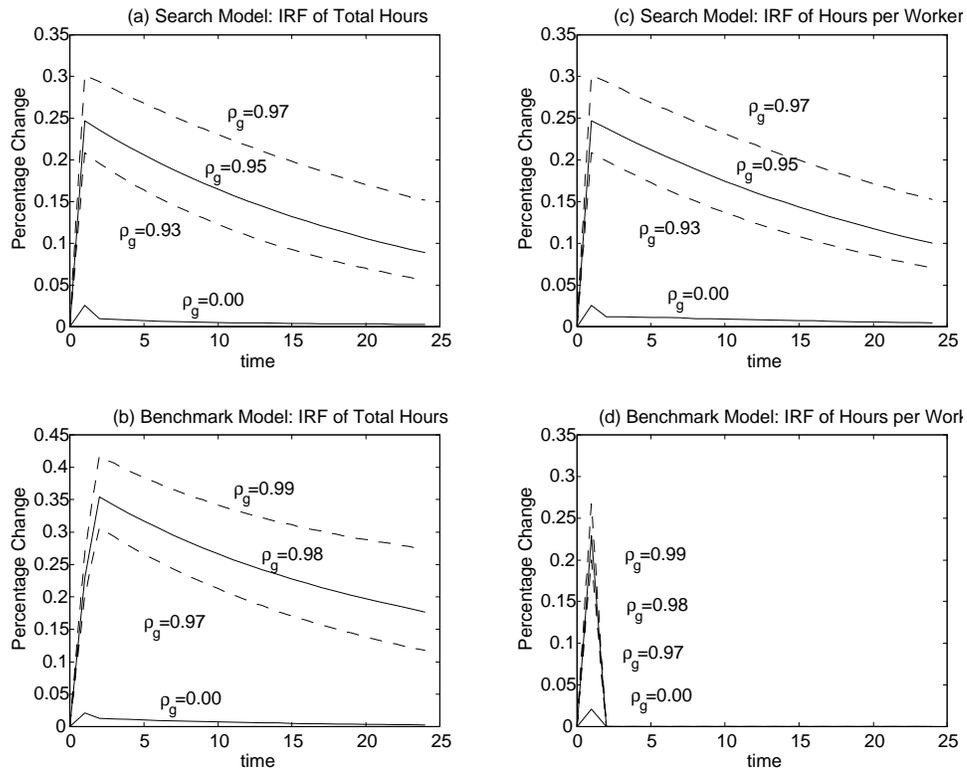


Figure 3. Impulse Response Functions of Total Hours and Hours Worked per Worker

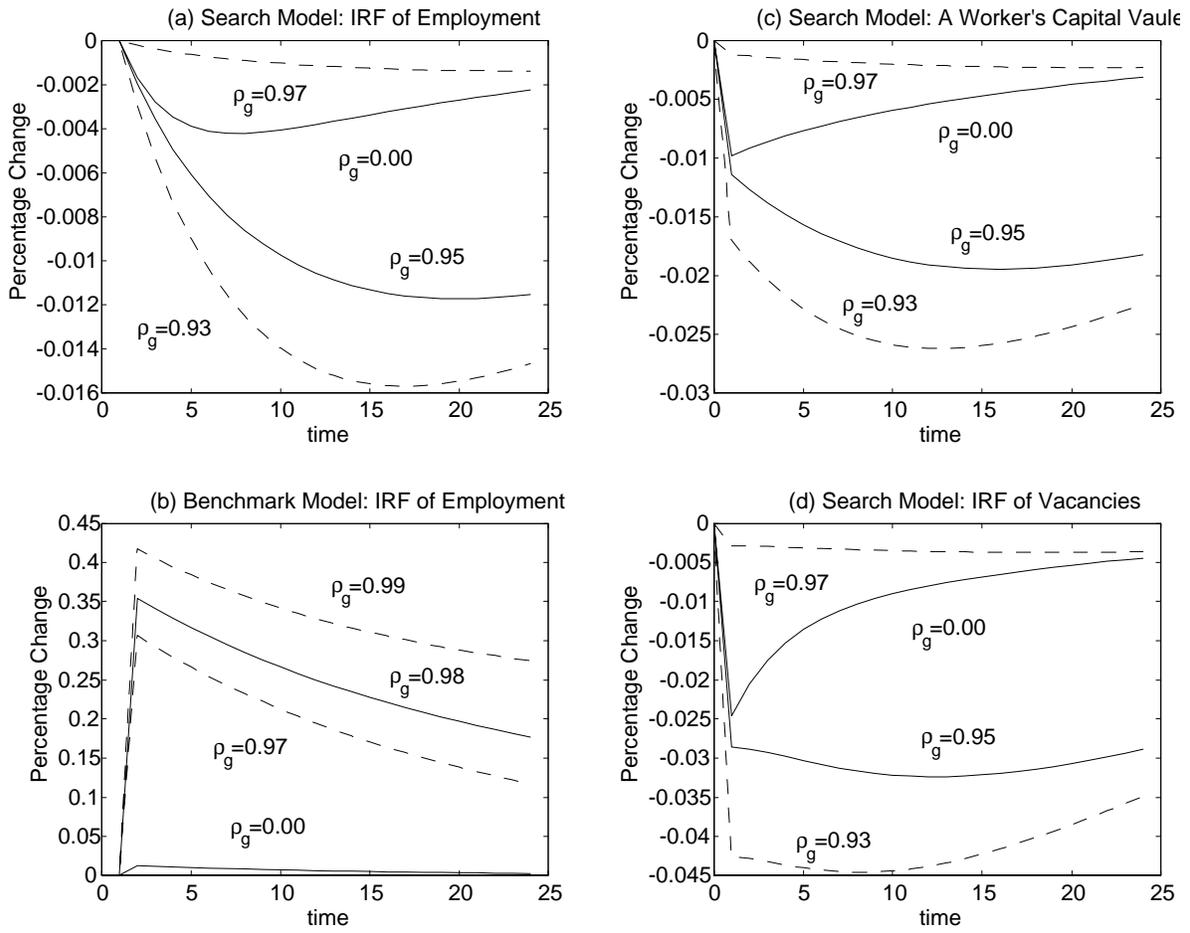


Figure 4. Impulse Response Functions of Employment

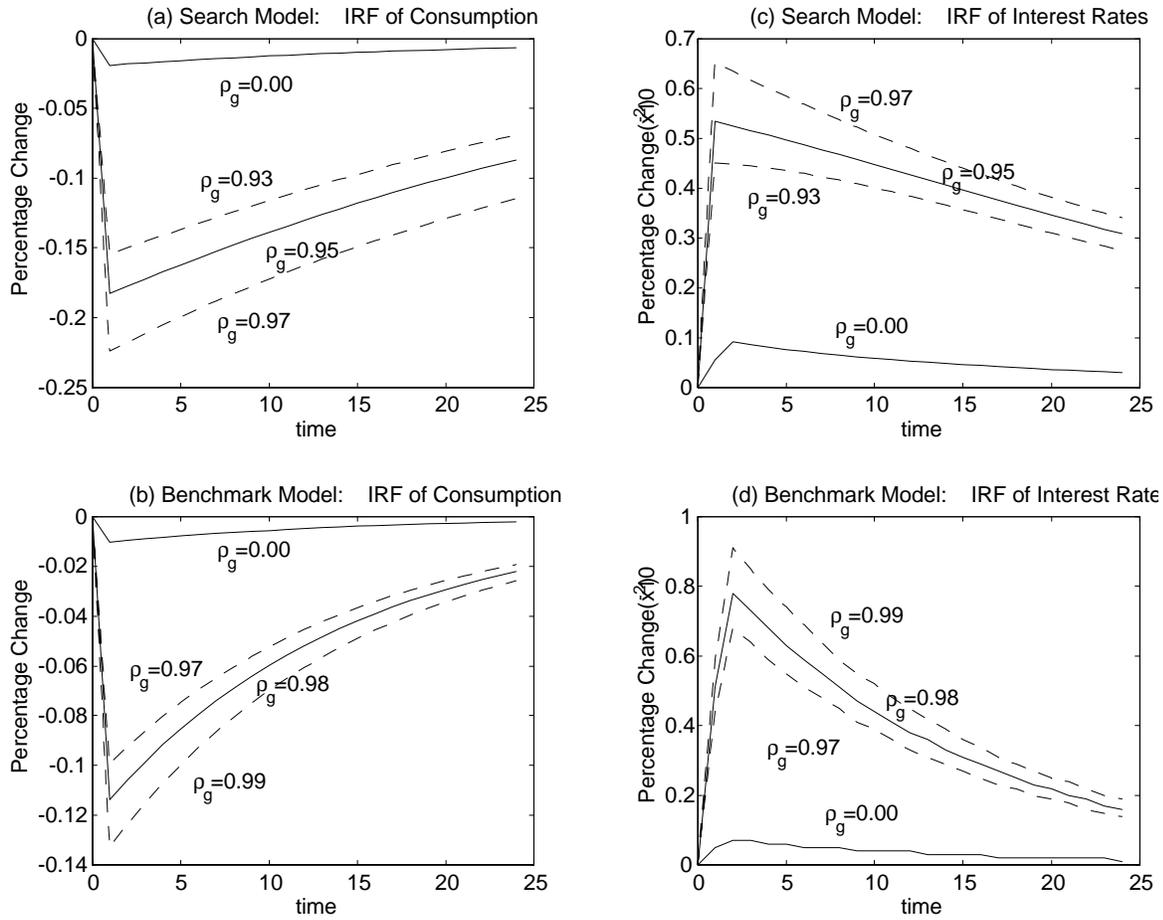


Figure 5. Impulse Response Functions of Consumption and Interest Rate

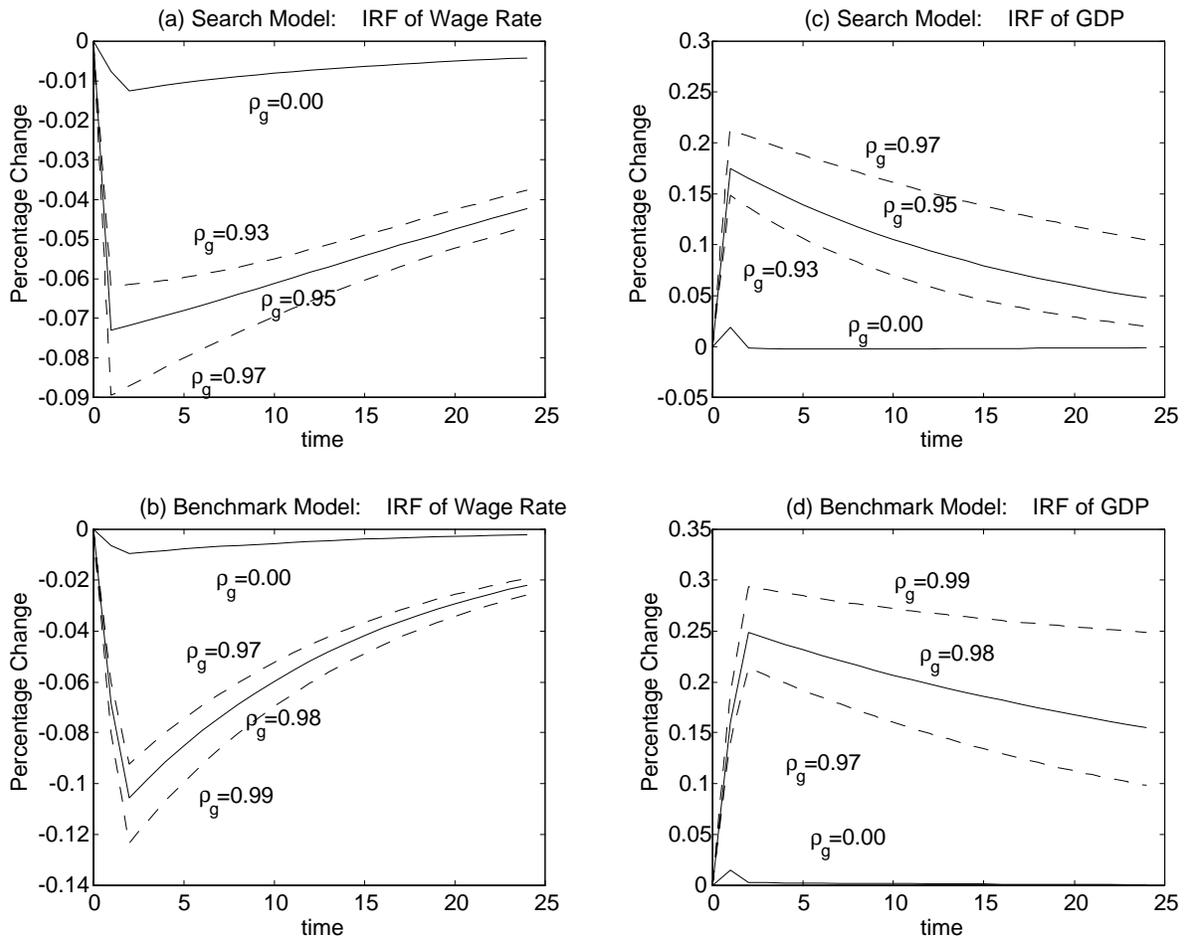


Figure 6. Impulse Response Functions of Wage Rate and Total Output