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THE EFFECTS OF FISCAL POLICY IN A NEOCLASSICAL GROWTH MODEL

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1. INTRODUCTION

This paper studies the effects of fiscal policies--depicted as stochastic changes in government spending and distortionary tax rates--when the government cannot use lump sum taxes to achieve intertemporal budget balance. This framework contrasts the more standard analysis in which spending and taxes follow exogenous Markov process and where lump sum taxation is used to balance the government's budget. Although we also model tax rates and spending as following Markov processes, the transition probabilities of these processes depend on the ratio of government debt to gnp. The ratio of debt to gnp, will have consequences for the future choices of government spending and distortionary taxation and hence will affect real economic activity. The paper, therefore, is able to contribute to current public discussions over the economic effects of debt and deficits and to the effects of policies that attempt to reduce the deficit through cuts in government expenditures or increases in distortionary taxation.

Our depiction of fiscal policy gives bite to the restriction imposed by intertemporal budget balance since debt can not be viewed as a residual of policy that is dealt with via lump sum means. The results generated in our model can differ substantially from those in standard stochastic models. For example, the effects due to changes in the tax rate on capital depend on both the debt to gnp ratio and the persistence of the tax process. Even for processes that are fairly persistent, increases in the tax rate on capital can lead to increases in investment and this counterintuitive result is more likely to happen at very high or very low levels of the debt to gnp ratio. Thus the debt to gnp ratio has interesting qualitative effects on behavior.

Also, the economic effects of changes in government debt depend on the way that intertemporal budget balance is attained. If budget balance is primarily due to future changes in the tax rate on capital then debt crowds out investment. But unlike a standard Keynesian model higher debt ratios are associated with lower real interest rates. If on the other hand budget balance results from changes in the path of tax rates on labor, then investment is actually crowded in. It is only when government spending varies and taxes are held fixed that crowding out and higher interest rates are associated with higher ratios of debt.

Our model of fiscal policy implies that the debt to gnp ratio is mean reverting, which is consistent with evidence in Kremers (1989), King (1990), and Bohn (1991b). The model, despite its simplicity, also generates debt behavior that is reasonably consistent with U.S. data.

The paper also represents an extension and alternative method for analyzing the effects of fiscal policy from the perfect foresight models of Judd (1985, 1987) and Baxter and King (1993). We essentially take the central messages of Bizer and Judd (1989) and Judd (1985) seriously by both investigating a model that explicitly incorporates uncertainty and that also includes an elastic labor supply. The modeling strategy, as mentioned, allows us to incorporate the behavior of public debt in a meaningful way, which represents an extension of the literature on stochastic fiscal policy. The paper is thus most closely related to Dotsey (1994), but the model analyzed below is much richer than the one studied in that paper. The inclusion of elastic labor supply adds important behavioral elements to the model and allows us to more realistically investigate the effects that debt has on economic activity.

The paper proceeds as follows. In section 2 we present the basic model and in section 3 we describe the effects of stochastic taxation. Section 4 investigates stochastic government spending while section 5 analyzes the welfare implications of using capital taxation versus taxing labor. A notable feature of our model is that it is optimal to significantly tax capital. Section 6 compares the fiscal policy generated by our methodology with actual fiscal policy, and section 7 concludes the paper.

2. THE MODEL

The basic model is a standard neoclassical growth model into which we introduce distortionary taxation and government spending. These variables are modeled as Markov processes. To maintain intertemporal government budget balance the transition probabilities are functions of the debt to gnp ratio. The stochastic process characterizing fiscal policy is endogenous and the government debt is mean reverting as in (Dotsey (1994)). Empirically, neither Kremers (1989) nor King (1990) can reject mean reversion in U.S. government debt, and Bohn (1991b) finds evidence that debt levels are mean reverting. Bohn (1991a) also shows that historically U.S. deficits have been eliminated both by reductions in spending and increases in tax rates. Our model is consistent with these observations. Because all but the stochastic part of the model is standard, we give only a brief description of the model.

Firms

Firms maximize profits, d_t, which are remitted to households, by producing output via a constant return to scale technology that employs both capital, k, and labor, n. Both factors are rented from individuals. Capital is always supplied inelastically while we consider both inelastic and elastic labor supply. Formally,

PF:
$$\max_{\{k_t,n_t\}} \quad d_t = f(k_t,n_t) - r_t k_t - w_t n_t$$

where r is the rental rate on capital and w is the real wage. The first order conditions equate each factor's marginal product with its rental rate.

<u>Individuals</u>

Individuals maximize lifetime utility which depends on both consumption and leisure. They are endowed with one unit of time each period and an initial stock of capital. Individuals make their labor-leisure, consumption, and investment-saving decisions taking as given wage rates and rental rates. They also purchase one period government debt at a price p_t. Each bond pays one unit of consumption in the succeeding period. Consumers observe the current state of fiscal policy summarized by beginning of period per capita government debt, B_t, current tax rates on capital and labor income, τ^k and τ^n, and the current level of government spending. They also know current aggregate economic magnitudes such as output, the capital stock, employment, investment, and end of period debt B_{t+1}. Formally, the individual's problem, PI, is written

PI:
$$\max_{\{c_t,n_t,b_{t+1}\ k_{t+1}\}} \quad U = E_0 \sum_{t=0}^{\infty} \beta^t\ u(c_t,1-n_t)$$

subject to

$$c_t + i_t + p_t b_{t+1} \le (1-\tau_t^n)w_t n_t + (1-\tau_t^k)r_t k_t + b_t + TR_t + \tau_t^k \delta k_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

where TR is aggregate per capita transfers, $\tau_t^k \delta k_t$ is a depreciation allowance, and lower case variables indicate values at the individual level.

Maximization yields the following first order conditions

$$\text{(1a)} \quad u_2(c_t,\ 1-n_t) = u_1(c_t,\ 1-n_t)(1-\tau_t^n)w_t$$

$$\text{(1b)} \quad u_1(c_t, 1-n_t) = \beta E_t\{[(1-\tau_{t+1}^k)r_{t+1} + \tau_{t+1}^k\delta + (1-\delta)]u_1(c_{t+1}, 1-n_{t+1})\}$$

$$\text{(1c)} \quad p_t u_1(c_t,\ 1-n_t) = \beta E_t u_1(c_{t+1},\ 1-n_{t+1})$$

where u_j refers to the partial derivative with respect to the j^{th} argument.

Fiscal Policy

The government spends resources and finances its spending through taxes and debt. Debt evolves according to

$$\text{(2)} \quad p_t B_{t+1} = G_t + B_t - \tau_t^k r_t K_t - \tau_t^n w_t N_t + TR_t$$

where capital letters refer to per capita aggregate quantities. G is government spending, B is the stock of one-period bonds outstanding, and TR is the level of transfers. Tax rates on capital and labor income, τ^k and τ^n, and the ratio of government spending to gnp, $\tilde{g}$, depend on the debt to gnp ratio,

$\tilde{b}$.[1] Government budget balance is achieved through changes in distortionary taxation and government spending. Specifically, we model the elements of fiscal policy as a two-state Markov process with transition probabilities given by

$$\text{(3a)}\quad \text{prob}\ (\tau_{t+1} = \tau_\ell \mid \tau_t = \tau_\ell) = \min\ \{\max[(1-\gamma\tilde{b}_t)^{1/\mu},\ 0],\ 1\}$$
$$\text{(3b)}\quad \text{prob}\ (\tau_{t+1} = \tau_h \mid \tau_t = \tau_h) = \max\ \{\min[\gamma\tilde{b}_t^{\ 1/\mu},\ 1],\ 0\}$$

$$\text{(4a)}\quad \text{prob}\ (\tilde{g}_{t+1} = \tilde{g}_\ell \mid \tilde{g}_t = \tilde{g}_\ell) = \max\ \{\min[\gamma\tilde{b}_t^{\ 1/\eta},\ 1],\ 0\}$$
$$\text{(4b)}\quad \text{prob}\ (\tilde{g}_{t+1} = \tilde{g}_h \mid \tilde{g}_t = \tilde{g}_h) = \min\ \{\max[(1-\gamma\tilde{b}_t)^{1/\eta},\ 0],\ 1\}$$

where the subscripts ℓ, h refer to low and high values respectively. These transition probabilities imply that the debt to gnp ratio is bounded and only rarely lies outside the interval $[0, 1/\gamma]$. As $\tilde{b}$ approaches a value of $1/\gamma$, taxes will be high and spending will be low with probability one. As long as a combination of high taxes and low spending reduces debt, the debt to gnp ratio will be driven down. Similarly as $\tilde{b}$ approaches zero the economy will be in a low-tax, high-government-spending state and the debt will rise. Thus,

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[1] We focus on the ratio of government spending to gnp rather than the level of spending because the ratio is stationary making it easy to extend our analysis to economies with steady state growth. One could easily add growth to our model by including technical progress in labor productivity. In that case one could interpret our model as represening deviations from trend as in King, Plosser, and Rebelo (1988).

there is some tendency for debt to revert toward its mean.[2] In what follows we will call this policy a managed debt policy.

The parameters μ and η control the persistence of the tax and spending processes. As these parameters increase the probabilities of remaining in a given tax or spending state increase for any value of the debt to gnp ratio.

Equilibrium

Equilibrium is described by a set of functions representing quantities and prices that solve the firms and consumers maximization problems, do not let either consumers or the government borrow more than can be repaid, and obey the following aggregate equilibrium conditions.

(5) $C_t + I_t + G_t = f(K_t, N_t)$

(6) $b_t = B_t$

(7) $k_t = K_t$

(8) $n_t = N_t$

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[2]The debt to gnp ratio can temporarily move outside $[0, 1/\gamma]$ because next period's taxes and spending depend on this period's debt to gnp ratio. For example, the current state could be $\tau_t = \tau_\ell$, $\tilde{g}_t = \tilde{g}_h$, $\tilde{b}_t = (1/\gamma)-\varepsilon$. Given this state it is possible that next period's taxes and spending will not change. Thus tomorrow's debt/gnp could exceed $1/\gamma$ and the debt/gnp two periods hence could be larger still. However, since $\tilde{b}_{t+1} > 1/\gamma$ implies $\tau_{t+2} = \tau_h$ and $\tilde{g}_{t+2} = \tilde{g}_\ell$ the debt to gnp ratio will start to decline. Since a combination of τ_ℓ, $\tilde{g}_h$ can only increase $\tilde{b}$ by so much, $\tilde{b}$ is bounded above. Similarly, $\tilde{b}$ is bounded below. Further our process for fiscal policy rules out any Ponzi games. That is $\lim_{T\to\infty} E_t[P_T B_{T+1} / \prod_{s=t}^{T} (1/P_s)] = 0$ for equilibrium paths in this model.

We solve for equilibrium by first using equation (5) to eliminate consumption. Equation (1a) together with the relationship $w_t = f_2[K_t, N_t]$, and equations (7) and (8) are then used to solve for labor $n_t = n(k_t, \tilde{b}_t, \tau^n_t, \tau^k_t, \tilde{g}_t, k_{t+1}) = n(s_t, k_{t+1})$ where the state $s_t = (k_t, \tilde{b}_t, \tau^n_t, \tau^k_t, \tilde{g}_t)$. We then substitute for labor in equation (1b) to yield an equation determining capital accumulation,

$$
\begin{aligned}
(9) \quad & u_1[f(k_t, n(s_t, k_{t+1})) + (1-\delta)k_t - g_t - k_{t+1}, 1-n(s_t, k_{t+1})] \\
& = \beta E_t[(1-\tau^k_{t+1})f_1(k_{t+1}, n(s_{t+1}, k_{t+2})) + \tau^k_{t+1}\delta + (1-\delta)] \\
& \quad \times u_1[f(k_{t+1}, n(s_{t+1}, k_{t+2})) + (1-\delta)k_{t+1} - g_{t+1} - k_{t+2}, 1-n(s_{t+1}, k_{t+2})].
\end{aligned}
$$

Equation (9) is a nonlinear second order stochastic difference equation. Given $n(s, k')$ where the "$'$" indicates next period's value of a variable, we solve for the function, $k' = h(s)$ which is the fixed point of (9). This equilibrium policy function for k' then yields the equilibrium policy function for labor n, because n was a function of arbitrary k'. At each step of the iteration we use equations (1c) and (2) to determine $\tilde{b}'$ based on the current state s and the policy functions n and h. The algorithm is similar to the discrete state space method described in Baxter (1991) and Dotsey and Mao (1992).

3. STOCHASTIC TAXES

We can highlight the effects of distortionary taxation by comparing an equilibrium generated by a policy with managed debt with the standard case in which taxes follow an exogenous Markov process. Our comparisons are based on an examination of policy functions, impulse response functions, and impact effects. To understand the effects of fiscal policy, we proceed sequentially by first taking the simplest case--a stochastic tax rate on capital and a fixed tax on labor with inelastic labor supply--and then proceed to the more general cases.

The experiments in this section are dynamic stochastic analogs to comparative static analysis. Our fundamental concern is understanding the workings of a fairly intricate fiscal policy process. We use post-Korean War U.S. data as a rough guide for calibrating the models. We fix the ratio of government spending to gnp at .18, which is the ratio reported in Christiano and Eichenbaum (1991). We also fix the level of transfers at 5% of gnp. In our experiments the debt to gnp ratio essentially lies between 0 and 1/2. Until recently, measured government debt/gnp has remained within this range. Picking a limited range also helps conserve on grid points.

Our remaining parameter values are within the realm of most real business cycle models. Labor's share of gnp is chosen to be .68, utility is logarithmic and separable in consumption and leisure, the discount factor is .97, and the depreciation rate on capital is .06. We parameterize the utility function so that individuals spend 20% of their time working.[3]

(a) <u>Fixed Labor Supply with the Variable Tax Rates on Income from Capital</u>

In this example we allow the tax rate on capital to vary and use a persistance parameter of μ=3. With this parameter, tax rates are unlikely to

[3]The parameterization lies within the ranges of a number of RBC models, in particular Kydland and Prescott (1982), Hansen (1985), Greenwood, Hercowitz and Krusell (1992), King, Plosser and Rebelo (1988), Rebelo and Stokey (1993), and Finn (1995).

change for most of the values for the debt/gnp ratio.[4] The tax rate on capital takes on the value of either .20 or .50. The mean of the tax rate is .38 with a standard deviation .168 and an AR1 coefficient of .57. This parameterization is roughly consistent with one of the series reported in Auerbach and Hines (1988) which has a mean of .40, a standard deviation of .141, and an AR1 coefficient of .82. We choose somewhat lower than actual persistence to illustrate an interesting result, that it can be optimal for agents to invest more when taxes are high even when tax rates on capital are persistent.

The policy functions for capital and consumption, and the equilibrium function for the real after-tax rate of interest are displayed in Figure 1. The policy functions are drawn for a capital value chosen from the middle of capital's ergodic set. As shown, the capital stock in the high tax state (dotted line) lies above the capital stock in the low tax state. This result implies that investment is higher when taxes are high even though a high tax rate today generally implies a high tax rate next period. This result is the same as the one in Dotsey (1994) for an economy using a linear technology and occurs for the same reason. A high tax rate today lowers the debt to gnp ratio implying that the future path of taxes will be lower and that investment is profitable. This response is only optimal if tax rates are not too persistent. If we set μ=4 implying an AR1 coefficient on taxes of .70, agents will invest less when taxes are high. Therefore, for a tax

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[4]For example, the probabilities of taxes remaining in the low-tax state for debt/gnp ratios of (-.10, -.063, -.026, .011, .047, .084, .121, .158, .195, .232, .268, .305, .342, .379, .416, .453, .489, .526, .563, .60) are (1.0, 1.0, 1.0, .99, .96, .93, .90, .87, .83, .79, .75, .71, .65, .59, .52, .42, .24, 0, 0, 0). It is not until the debt/gnp ratio reaches .49 that next period's tax rate is more likely to be high than low.

process displaying persistence that conforms more closely to the data investment will fall when the tax rate rises. Further, investment declines with debt because higher debt levels imply higher future taxes.

The above result stands in sharp contrast to the standard tax literature[5], where labor supply is typically fixed and taxes follow a Markov process. As long as tax rates are positively correlated the standard case implies that high taxes today result in higher future tax rates and less current investment.

The policy function for consumption is a mirror image of the policy function for capital. With inelastic labor supply investing more implies consuming less. The equilibrium function for interest rates is also shown in Figure 1 and its shape is related to the policy function for consumption. Interest rates are lower in the high tax state due to the upward slope of the consumption policy function. When taxes are high today, debt and consumption will fall next period, while if taxes are low, debt and consumption will rise. This implies that for any given debt level interest rates in the high tax state lie below those in the low tax state, a result that is contrary to that presented in the perfect foresight model of Judd (1987). The interest rate equilibrium functions are also downward sloping attaining their lowest value when debt is high. In the high tax-high debt state there is little probability that a low tax rate will occur tomorrow, hence the expected consumption decline is relatively large implying a low real interest rate. In the low tax state there is a reasonably high probability

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[5]For example see Coleman (1991) or Dotsey (1990). In a nonstochastic environment see Judd (1987), Abel (1982), Abel and Blanchard (1983), Becker (1985), Brock and Turnovsky (1981), Danthine and Donaldson (1985), and Hall (1981).

that high taxes will occur tomorrow, implying a relatively small expected increase in consumption and hence a lower real interest rate. Similarly rates are higher when the debt is low.

The extent to which debt is non-neutral in our model can be illustrated by the elasticity of the various policy functions with respect to debt around the steady state debt to gnp ratio (see Table 1) and by the correlations between debt and other endogous variables (see Table 2). An increase in debt crowds out investment and slightly increases consumption. The non-neutrality in this model differs from a standard Keynesian model because real rates in this model are negatively related to the level of debt. These features also appear in the correlation coefficients which show a negative correlation between debt and investment as well as a negative correlation between debt and the real interest rate.

(b) Variable Labor Supply with Variable Tax Rates on Income from Capital

For these experiments we keep the same parameter values but allow labor to vary, which creates another degree of freedom in the model.[6] With labor fixed, changes in investment must be offset one for one with changes in consumption. With variable labor that need not be the case since output can adjust contemporaneously. Variable labor allows consumption to be much smoother and at the same time allows investors to take advantage of low persistent marginal tax rates.

The policy functions for capital, labor, consumption, and the equilibrium function for the real after-tax interest rate are depicted in Figure 2. The policy functions for capital and consumption differ from those

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[6]Varying labor represents a significant extension over Dotsey (1994).

in the fixed labor case. With varying labor, agents now invest more, work more, and consume less in the low tax state over much of the debt space.

Persistence of the tax processes also plays a role in the shape of the policy functions. Reducing the persistence of the tax series by setting $\mu=2$, which implies $\rho=.46$ yields the same qualitative results as the fixed labor case. Crossovers in the policy functions occur because the expected duration of remaining in any particular state depends on the value of the debt to gnp ratio. For example, if debt were high and taxes were low, agents would expect taxes to rise and stay high for a greater number of periods than if taxes were currently high. Hence they invest less in the low tax state. As in the previous example, the policy functions for consumption imply that the real interest rate will be higher in the low tax state and negatively related to debt.

Evaluating the elasticities of the various policy functions with respect to debt and the correlation coefficients leads to the conclusion that only half of the standard Keynesian story occurs. Higher debt crowds out investment but reduces the interest rate.

(c) Variable Labor with a Varying Labor Tax and Fixed Tax on Capital Income

We next examine the effects of varying the tax on labor income rather than the tax on capital. Here we allow labor tax rates to vary between .23 and .31. With $\mu=7$, these rates have a mean of .28, a standard deviation of .039, and an AR1 coefficient of .79. Using post-Korean War data our tax process matches the one constructed by Barro and Sahasakul (1986), which has a mean of .278, a standard deviation of .039, and an AR1 coefficient for their detrended series of .78.

Intratemporal substitution effects in the labor-leisure decision dominate the results. Individuals substitute labor effort into low tax states, driving up the marginal productivity of capital and hence increasing investment demand. Greater labor effort results in more output and more is invested. As debt rises, the probability of high taxes next period increases thus inducing individuals to take even greater advantage of the current low tax rate. In the low tax state, high debt means that future taxes are more likely to be high so the incentive to work is greater than when debt is low. Thus the policy function for labor effort is upward sloping (see Figure 3).

Because the policy function for both labor and capital are now upward sloping (a non-Keynesian result) the policy function for consumption is downward sloping even though there is more output available at high levels of debt. Agents, however, consume and invest more in the low tax state due to increased labor effort and greater output. As in the previous case interest rates are higher when taxes are low. This is because capital and, therefore, next period's consumption increase when taxes are low. That is, shifts in the consumption policy function dominate movements along the function.

The variable tax on labor income creates crowding in rather than crowding out, just the opposite of the standard Keynesian story. The policy function for investment has a positive elasticity and positive correlation with respect to debt while the real interest rate is negatively correlated with debt.

The managed debt case also yields somewhat greater impact effects than the standard exogenous Markov case because of the stronger intertemporal substitution effects on labor effort (see Table 3). With debt management, lower current taxes imply a higher future path of taxes making agents work even harder today. The greater impact on effort feeds over into output and investment.

(d) Taxing Both Labor and Capital

In this example both labor and capital are taxed. The capital tax rates take values of .18 and .53 with a persistence parameter of μ=9. This degree of persistence implies an empirically relevant value for the AR1 coefficient of .80. Taxes on labor again vary between .23 and .31 with a persistence parameter of μ=9. The AR1 coefficient on labor taxes is, therefore, .79. The results are a hybrid of the results in the last two sections. The large divergence in policy functions (Figure 4) between high and low tax states reflects the responsiveness of labor to a tax on wage income. The negative slope of the capital and labor policy functions as well as the positive slope of the consumption policy function reflect the influence of the tax on capital. Because this case is hybrid of the previous two experiments, the elasticity of investment with respect to debt is greatly diminished from the case when only τ^k varies. Thus when both factors of production are taxed there is much less crowding out than in the case where only income from capital is taxed. The interest rate, however, varies indirectly with government debt and thus only half of the traditional Keynesian story holds.

4. GOVERNMENT SPENDING

This section examines the effects of government spending. To highlight the differences from standard models, we first keep tax rates constant throughout and allow lump sum taxes to balance the budget when spending follows an exogenous two state Markov process. When there are no lump sum taxes government spending must adjust so that the debt to gnp ratio is bounded. We allow government spending relative to gnp to vary between .14 and .22. Its mean is .174 in the following experiments and its standard deviation is .04.. The parameter η is varied between 6 and 1 implying AR1 coefficients of .74 and .11. This allows us to explore the effects that persistence has on economic activity. Thus our spending process with η=6 matches the key features of the government spending series reported by Christiano and Eichenbaum (1992). The government taxes production at the constant rate of 26%. After isolating the effects of government spending, we allow tax rates and spending to vary simultaneously.

(a) <u>Persistent Government Spending</u>

We assume that government spending is useless. The economic response to changes in government spending, therefore, mainly arise through wealth and crowding out effects. The policy functions in Figure 5, show that agents work harder and consume less when spending is high. Although high government spending causes high output through increased labor effort, output rises by less than government spending. Hence next period's capital stock falls.

As debt rises the expected future path of government spending falls. The policy function for labor is, therefore, downward sloping with respect to debt while the consumption policy function is upward sloping. As labor hours decrease, output and the capital stock fall. Hence debt crowds out investment. High government spending raises interest rates motivating agents to work harder and consume less. As the debt rises, implying less future government spending, labor effort, capital, and consumption growth decline. Thus the equilibrium function for interest rates is downward sloping with respect to debt.

Even though the equilibrium function for the interest rate is negatively related to debt, the correlation between interest rates and debt is positive. The intuition can be seen by examining the economy's response to a high government spending shock, which is displayed in Figure 6. Debt rises when spending is above its average value causing spending to eventually fall below its steady state expected value. This mild oscillatory behavior in spending sets up oscillatory behavior in the other variables. As spending falls and debt rises, labor effort declines. However, declining government spending allows agents to increase consumption and investment even though output mimics the behavior of labor. The real rate is generally above its steady state value as a result of consumption growth, so the correlations between debt and investment and debt and interest rates resemble the predictions of standard Keynesian models. Investment is below average when the debt is relatively high while interest rates are above average.

With the exception of labor (and as a result output), the behavior of the other endogenous variables is not strikingly different from what occurs when spending follows an exogenous Markov process. The impact effects in Table 4 show that labor responds with more vigor to an increase in government spending when spending follows a Markov process. In the debt management case higher spending raises the level of debt implying that future spending must be lower than it otherwise would have been. The wealth effects are, therefore, smaller than when spending is exogenous.[7]

[7]We calculated the present value of government spending to be about 10% less for the managed debt policy in this example.

(b) The Effects of Lowering Persistence

When the persistence in government spending is greatly reduced by setting η=1 implying an AR1 coefficient on spending of .10, the results for the exogenous Markov process and the managed debt process are very similar (see Figure 7). Changes in government spending are transitory and have smaller wealth effects. Thus the impact effects of a rise in spending are much smaller (see Table 4 and Figure 7). These results are consistent with those in Aiyagari, Christiano, and Eichenbaum (1991) and Baxter and King (1993). Also, because government spending changes states so frequently the debt doesn't fluctuate very much and the path of shocks generated by each process are almost identical. As a result all endogenous variables behave in a like manner.

(c) The Effects of Very High Persistence

In this experiment we examine McGrattan's (1992) suggestion that very high persistence in government spending can lead to increased investment in the high spending state. To generate high persistence we set η=100 which corresponds to an AR1 coefficient of .92. We find that with log utility and hence a relative risk aversion parameter of σ=1 it is possible for investment to be higher when spending is high, but only over a narrow range of the debt space. With an exogenous Markov process for spending, investment is higher when spending is high, but this result is sensitive to the degree of relative risk aversion. With increased risk aversion (σ=2) investment is lower when spending is high in both the managed debt and exogenous Markov process cases.

The reason for the disparity in results is that with debt mangement the wealth effects of high or low government spending are almost identical near the boundaries of the debt space. If, for example, debt levels are very high the probability that next period's government spending will be low and stay low is high no matter what the current state. Therefore, labor effort and consumption do not differ by very much across spending states and the major difference across the two states is in investment. In particular, investment is lower in the high spending state. An analogous argument indicates that investment is lower in the high spending state when debt is very low. It is only in the middle of the debt space that the wealth effects of high spending can cause enough of an increase in labor effort and decline in consumption that investment is higher. The large increase in labor effort also increases the marginal product of capital reinforcing the wealth effects on consumption and investment. When government spending follows an exogenous Markov process the persistence of the process is independent of debt levels. Therefore, wealth effects and the accompanying substitution effects are either strong enough to encourage investment when spending is high or they are not.

An increased persistence in government spending and the accompanying higher investment in the high spending state results in greater consumption variability as well. With CRRA utility, an increase in relative risk aversion implies a reduction in the elasticity of intertemporal substitution of consumption. With agents less willing to substitute intertemporally, investment becomes less variable, and therefore it is less likely that investment will rise in response to high government spending.

(d) Taxes and Spending Both Vary

In this case we now add persistent taxes and compare how simultaneously varying taxes and spending affects behavior. These comparisons are done by examining the impulse response functions in figures 8 and 9 which are responses to a high spending-low tax shock and a high spending-high tax shock. The impulse responses are generated by averaging over 2000 realizations of 50 periods each.

The combination of low taxes and high spending is more expansionary than just lowering taxes or increasing spending. The tax induced substitution effects augment the wealth effects of government spending implying that labor effort increases by a large amount. This increases output by enough so that the impact effect on both consumption and investment is positive.

When the initial impulse to taxes is high, (Figure 9) the impact effect of fiscal policy is reversed. With an increase in the tax rate substitution effects outweigh wealth effects and labor effort falls. The fall in labor effort results in lower output, consumption, investment, and a drop in the real rate of interest. Thus the expansionary effect on output of government spending programs can be totally overturned if they are financed out of current tax revenue. This latter result is consistent with the analysis in Baxter and King (1993).

5. WELFARE COMPARISONS BETWEEN CAPITAL AND LABOR TAXATION

The model also allows us to check the relative efficiency of using capital taxation versus labor taxation. In particular, we analyze if it is more costly to vary the tax rate on labor, or capital, or both. The tax processes evaluated are similar to those in section 3d, and hence represent processes that are representative of actual U.S. tax rates. The experiment, therefore, answers the question of which tax rate should be the primary instrument for maintaining budget balance conditional on the mean of the other tax rate being set at its optimal value.[8]

To perform this experiment, we fix g at .18 and the transfer to gnp ratio at .05. We then compare the discounted utility of the representative individual when τ^k and τ^n are set at their optimal values with the discounted utility that arises when only τ^k varies, when only τ^n varies, and when both τ^k and τ^n vary. In the cases where tax rates vary around their optimal values we parameterize the processes so that the standard deviations and AR1 coefficients are approximately equal to what one observes in the actual data.[9]

The derivation of the optimal tax rates follow the methodology in Zhu (1990). For the case of no transfers, the social planners first-order condition for efficient capital accumulation is:

$$(11)\quad u'(c_t) = \beta E_t\{[(1-g_{t+1})f_1(k_{t+1}, n_{t+1}) + (1-\delta)]u'(c_{t+1})\},$$

while for the representative agent it is:

$$(12)\quad u'(c_t) = \beta E_t\{[(1-\tau^k_{t+1})f_1(k_{t+1}, n_{t+1}) + (1-\delta) + \delta\tau^k_{t+1}]u'(c_{t+1}()\}.$$

Setting $\tau^k_t = \dfrac{g_t f_1(k_t,n_t)}{f_1(k_t,n_t)-\delta}$ will result in the path for capital under a competitive equilibrium being identical to that chosen by the planner.

Further setting $\tau^n_t = g_t$ will result in an equivalence between the marginal

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[8]A full depiction of optimal taxation under uncertainty can be found in Zhu (1990).

[9]Recall for τ^k: σ = .14 and ρ = .82, and for τ^n: σ = .039 and ρ = .79. The values for τ^k are (.34, .68) and for τ^n are (.24, .34).

conditions that determine labor-leisure choices for the representative agent and the planner. The solution is first best and is analogous to the solution presented in Jones, Manuelli, and Rossi (1991).

When transfers are also involved and the inital capital tax is constrained to its mean, then these transfers will be optimally financed by taxing labor[10] Thus the optimal tax on labor income is $\tau_t^n = g_t + tr/(1-\alpha)$, where tr is the percent of output transfered by the government. Given our parameterization the optimal tax rate on labor should be .254 and that on capital should be .392. The latter value is quite high and substanially differs from the steady state value of zero found in models that treat the level of government spending as exogenous. Also, these values are very close to their actual means of .28 and .40.

When τ_n = .254 and τ_k = .392 the discounted utility of the representative agent is -18.09. Allowing τ_n to vary around its optimal value so that its standard deviation is .038 and its first-order autoregressive parameter is .76 yields a utility value of -18.18. If instead one lets the tax on capital fluctuate around its optimal value with a standard deviation of .135 and first-order autoregressive parameter of .75 the agent's discounted utility is -18.13. Allowing both tax rates to vary yields a discounted utility of -18.19. Thus variation in one tax or the other around its optimal value has very little effect on welfare. It may, therefore, be a matter of indifference which tax is used for budget balancing purposes so long as its mean is set correctly.

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[10]Note, if utility was not separable in consumption and leisure, then financing transfers solely through the tax on labor would no longer be optimal (see Zhu (1990)).

6. EMPIRICAL RELEVANCE OF THE MANAGED DEBT POLICY

In this section we investigate the empirical relevance of the managed debt policy by examining if this policy can account for the behavior of debt, and if it is consistent with the behavior of tax rates and government spending. Because some of our empirical work will use frequency domain techniques we prefer a fairly long date set. We, therefore, test if our model is consistent with the actual post-1916 data set given in Bohn (1991a).[11]

Because we are concerned with a more detailed investigation of our methodology's ability to replicate actual data we require some essential modifications. First we extend the range of the admissable debt to gnp ratio to [-.1, 1.1] so that it is in accord with actual experience. Second, because the managed debt policy as described by (3a-4b) produces excessive oscillitory behavior, we allow taxes and government spending to follow exogenous markov processes on the interior of the debt space, but respond to debt when near the boundary. We also use three states for the tax rate and we set $\tau^k = \tau^n$ since Bohn's data only includes average tax rates.

The model generates tax rates that have a mean of .16, a standard deviation of .04, and an AR1 coefficient of .87, while it generates government spending that has a mean of .17, a standard deviation of .12, and an AR1 coefficient of .78. The comparable statistics for the data are .14, .04, and

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[11]We use his data because it doesn't net out any components of government spending. If we are to have any chance of matching the series on debt we must either use inclusive measures or model the different components of spending separately. We start in 1916 because that is the inception of income taxes, and the data over the entire sample, 1800-1988, does not appear to be generated by the simple model in this paper (i.e. the mean of government spending and tax revenue vary greatly over the last two centuries). To match the data we would need more than one fiscal policy regime. As it is the model is forced to confront two major wars in order to get enough data points for the spectra to have any meaning. What we would like is 100 years of post-Korean war data.

.89 for tax rates and .16, .08, and .80 for government spending. Thus we start out by replicating some essential features of the two fiscal policy processes.

(a) The Behavior of Debt

To compare the behavior of debt generated by our modified fiscal policy process with the behavior of actual government debt, we examine both the spectrum of actual and model generated government debt as well as the coherence between the two sets of data.[12] In generating model data on debt we use the same tax rate and government spending series as Bohn (1991a) and then derive model behavior by linearly interpolating between the theoretical policy functions. Thus the two data sets are comparable.

The results of this exercise are displayed in figure 10. The spectrum for the model has less power at low frequencies than the actual data, but the general shape of the spectra are fairly comformable (i.e. both spectra peak at low frequencies). The coherence between the model and the data is generally fairly high. The lowest value of the coherence occurs at a periodicity of 20 years, which roughly corresponds to intervals between major wars. Thus our model of debt does not accurately reflect war time behavior. At business cycle frequencies, however, the coherence exceeds .90 which is much higher than that displayed by real business cycle models for many relevant economic magnitudes (see Watson (1993)). We, therefore, find the overall results of this exercise encouraging.

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[12]The spectra were estimated using linearly detrended data. Since the model data do not display any trend the model data is in deviation from mean form.

(b) The Behavior of Tax Rates and Government Spending

In this experiment we work with filtered actual data.[13] For actual data, the behavior of taxes and government spending are depicted by the following two regressions (t-statistics are in parenthesis).

$$\text{(13a)}\quad \tau_t = \underset{(.22)}{.03} + \underset{(7.39)}{.81}\,\tau_{t-1} + \underset{(4.26)}{.59}\,\tau_{t-2} + \underset{(1.75)}{.05}\,b_{t-1} \qquad [R^2=.50]$$

$$\text{(13b)}\quad g_t = \underset{(.30)}{-.12} + \underset{(13.5)}{.89}\,g_{t-1} - \underset{(8.42)}{.42}\,b_{t-1} \qquad [R^2=.74]$$

The regression results from filtered model data are obtained from a sample of 1000 observations. These results are depicted by

$$\text{(14a)}\quad \tau_t = \underset{(.09)}{.004} + \underset{(25.5)}{.75}\,\tau_{t-1} + \underset{(2.09)}{.016}\,b_{t-1} \qquad [R^2=.56]$$

$$\text{(14b)}\quad g_t = \underset{(.08)}{-.02} + \underset{(25.9)}{.78}\,g_{t-1} + \underset{(3.72)}{.12}\,g_{t-2} - \underset{(10.86)}{.36}\,b_{t-1} \qquad [R^2=.74]$$

Comparing the two sets of regressions, one notices some important similarities. Debt affects fiscal policy in the data similarly to the way it affects policy in the model. Also, the coefficients on the first-order lags are approximately the same across the two sets of regressions. One important difference, however, is the number of significant lags in the data versus the model. Taxes in the data appear to be generated by an AR2, while model data is depicted as an AR1. The opposite appears to be true for government spending. Since government spending in the model is generated by a first-order Markov process the significant coefficient on the second lag must be due to the filter. Although the model and the data do not match exactly, the

[13]The data are filtered using Harvey's and Jaeger's (1993) procedure.

results of this analysis are encouraging and indicate that the theoretical methodology of this paper can be used to capture empirically relevant behavior.

(c) Forecasting Tax Rates and Government Spending

As a final experiment, we analyze the theoretical models one step ahead forecasting ability. We do this by taking actual data over the period 1916-1988 (from Bohn's data) and using our probability model to derive expected values for each succeeding period's taxes and government spending. The results of this exercise are depicted in Figure 11.

Regarding government spending the model does quite well, the RMSE is .07 and the only serious forecasting errors occur during World War II, although spending during the depression is also somewhat overpredicted. This latter result occurs because .08 is the lowest expected value of the ratio of government spending to gnp produced by our calibrated statistical model when the debt-gnp ratio is the interior of [-.1, 1.1]. For similar reasons the expected value of the tax rate is overestimated in the early portion of the sample and is largely responsible for the RMSE of .035. Overall, the forecasting performance, especially over the post World War II period, leads us to conclude that our methodology is flexible enough to capture important features of the data.

6. CONCLUSION

This paper has examined an alternative methodology for studying the effects of fiscal policy. Our model of fiscal policy takes the consequences of intertemporal budget balance seriously and at the same time allows for uncertainty in the fiscal policy process. The combination of these two elements is able to generate behavior that is, in some instances, strikingly different from standard results. Namely debt is non-neutral, the expansionary effects of government spending are dampened, and the taxation of capital can have surprising and counterintuitive results. The model generates cases where debt crowds in investment and the behavior of the real interest rate differs from behavior portrayed in standard Keynesian models. The model is also consistent with empirical evidence on U.S. fiscal policy as well as with the behavior of U.S. government debt. We feel, therefore, that our methodology represents a promising alternative for investigating the effects of fiscal policy in a dynamic stochastic general equilibrium framework.

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TABLE 1

Elasticity of Policy Functions Around Steady State Debt/GNP Ratios*

	Case 1		Case 2		Case 3		Case 4		Case 5	
	τ^k_{low}	τ^k_{high}	τ^k_{low}	τ^k_{high}	τ^n_{low}	τ^n_{high}	$\tau^k_{low}, \tau^n_{low}$	$\tau^k_{high}, \tau^n_{high}$	g_{low}	g_{high}
c	.048	.047	.036	.035	-.014	-.012	.015	.014	.020	.022
n	.000	.000	-.063	-.061	.024	.023	-.026	-.025	-.036	-.039
i	-.178	-.174	-.310	-.309	.105	.138	-.110	-.160	-.171	-.242
y	.000	.000	.0004	.0004	.000	.000	.0001	.0001	.000	.000
r	-.131	-.171	-.213	-.233	-.010	-.007	-.101	-.146	-.046	-.044

Note:

Case 1: labor fixed, τ^k varies ($\mu^k = 3$), τ^n fixed, g fixed.

Case 2: labor varies, τ^k varies ($\mu^k = 3$), τ^n fixed, g fixed.

Case 3: labor varies, τ^k fixed, τ^n varies ($\mu^n = 7$), g fixed.

Case 4: labor varies, τ^k varies ($\mu^k = 9$), τ^n varies ($\mu^n = 9$), g fixed.

Case 5: labor varies, τ^k fixed, τ^n fixed, g varies ($\eta = 6$).

TABLE 2

Correlation Coefficients with Respect to the Debt to GNP ratio
(3000 observations)

Case	i	r	c	n	y
τ^k varies, n fixed ($\mu^k = 3$)	-.97	-.20	.56	na	-.57
τ^k varies, n varies ($\mu^k = 3$)	-.93	-.31	-.01	-.89	-.92
τ^n varies ($\mu^n = 7$)	-.10	-.81	.40	-.20	.04
τ^k and τ^n vary ($\mu^k = 9$, $\mu^n = 9$)	-.52	-.57	.53	-.37	-.20
g varies ($\eta = 6$)	-.42	.96	-.79	-.61	-.82
g varies ($\eta = 1$)	-.33	.99	-.99	-.44	-.74
g varies ($\eta = 6$), τ^k and τ^n vary ($\mu^k = 3$, $\mu^n = 3$)	-.67	-.61	.12	-.32	-.33
g varies ($\eta = 15$), τ^k and τ^n vary ($\mu^k = 9$, $\mu^n = 9$)	-.75	-.54	.33	-.33	-.28

TABLE 3

Impact Effects for a Decline in Taxes
(measured as minus the ratio of the percent deviation from steady state values to the percent deviation in the decline in taxes)

Case — Managed Debt

	y	n	c	i	r	w
τ^k varies, n fixed ($\mu^k = 3$)	0	0	.001	-.003	.339	0
τ^k varies, n varies ($\mu^k = 3$)	.003	.004	-.002	.024	.316	-.001
τ^n varies ($\mu^n = 7$)	.380	.561	.070	1.494	.532	.210
τ^k and τ^n vary ($\mu^k = 9$, $\mu^n = 9$)	.131	.194	.011	.567	.688	.059

Case — Markov

τ^k varies, n fixed ($\rho_{\tau^k} = .57$)	0	0	-.017	.060	.301	0
τ^k varies, n varies ($\rho_{\tau^k} = .59$)	.018	.026	-.015	.135	.327	-.008
τ^n varies ($\rho_{\tau^n} = .79$)	.331	.486	.091	1.182	.433	.212
τ^k and τ^n vary ($\rho_{\tau^k} = .80$, $\rho_{\tau^n} = .79$)	.139	.205	-.004	.651	.523	.047

TABLE 4

Impact Effects for a Rise in Government Spending/GNP

Case	Managed Debt					
	y	n	c	i	r	w
τ^k and τ^n fixed, $\eta = 6$	.040	.058	-.032	-.674	.103	-.018
τ^k and τ^n fixed, $\eta = 1$	.015	.022	-.013	-.859	.064	-.007
τ^k and τ^n vary (μ^k =3, μ^n =3), $\eta = 6$	.255	.378	.014	.096	.904	.107
τ^k and τ^n vary (μ^k =9, μ^n =9), $\eta = 15$	.268	.395	.016	.165	1.318	.113

Case	Markov					
τ^k and τ^n fixed, $\rho_g = .74$	.078	.113	-.065	-.428	.171	-.036
τ^k and τ^n fixed, $\rho_g = .11$	.032	.047	-.026	-.770	.092	-.015
τ^k and τ^n vary ($\rho_{\tau^k} = .62$, $\rho_{\tau^n} = .59$), $\rho_g = .70$	.364	.538	-.052	.778	1.193	.083
τ^k and τ^n vary ($\rho_{\tau^k} = .77$, $\rho_{\tau^n} = .75$), $\rho_g = .75$	.403	.596	-.077	1.032	1.584	.073

Policy Functions (τ^k varies: μ^k=3, Labor fixed)

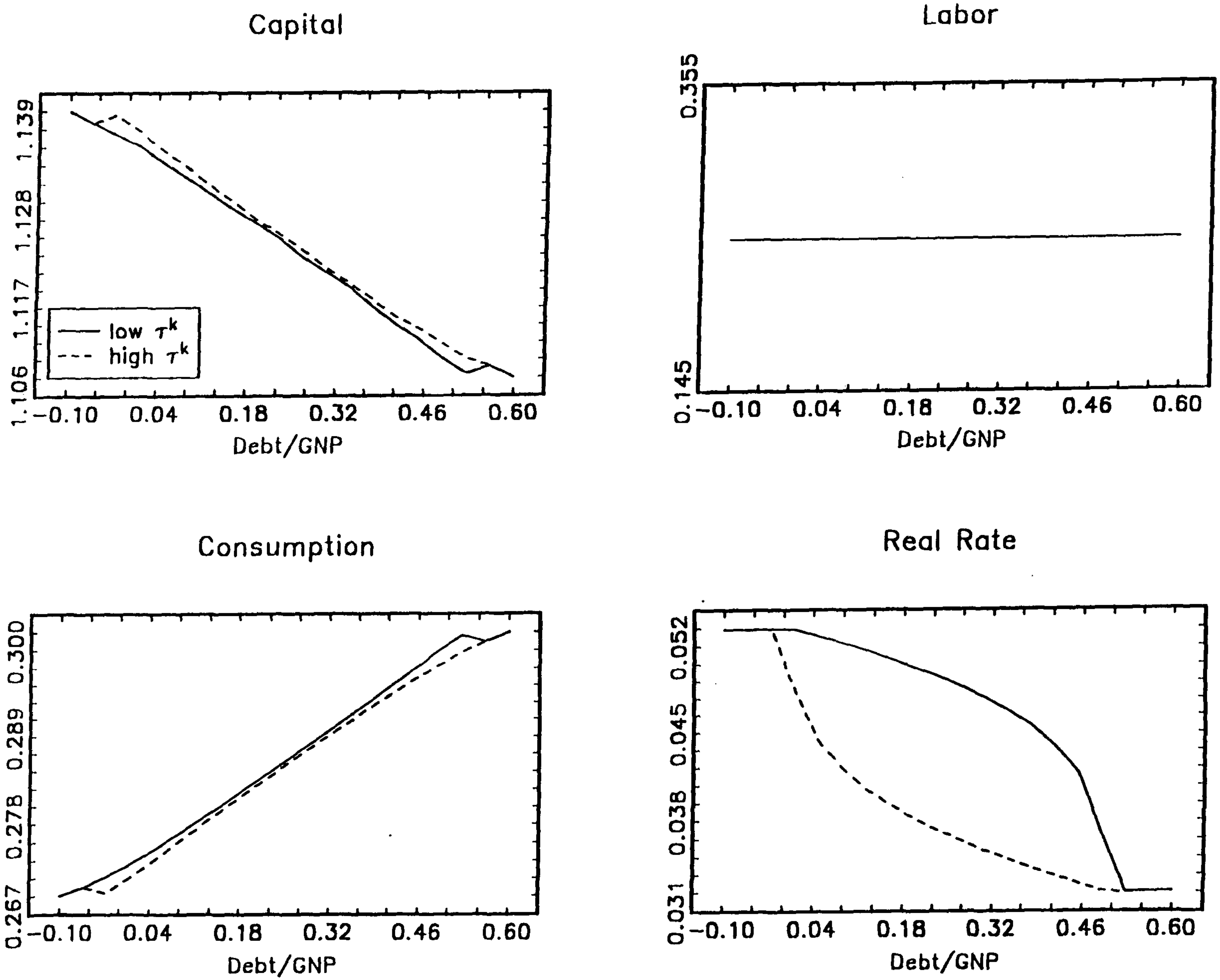
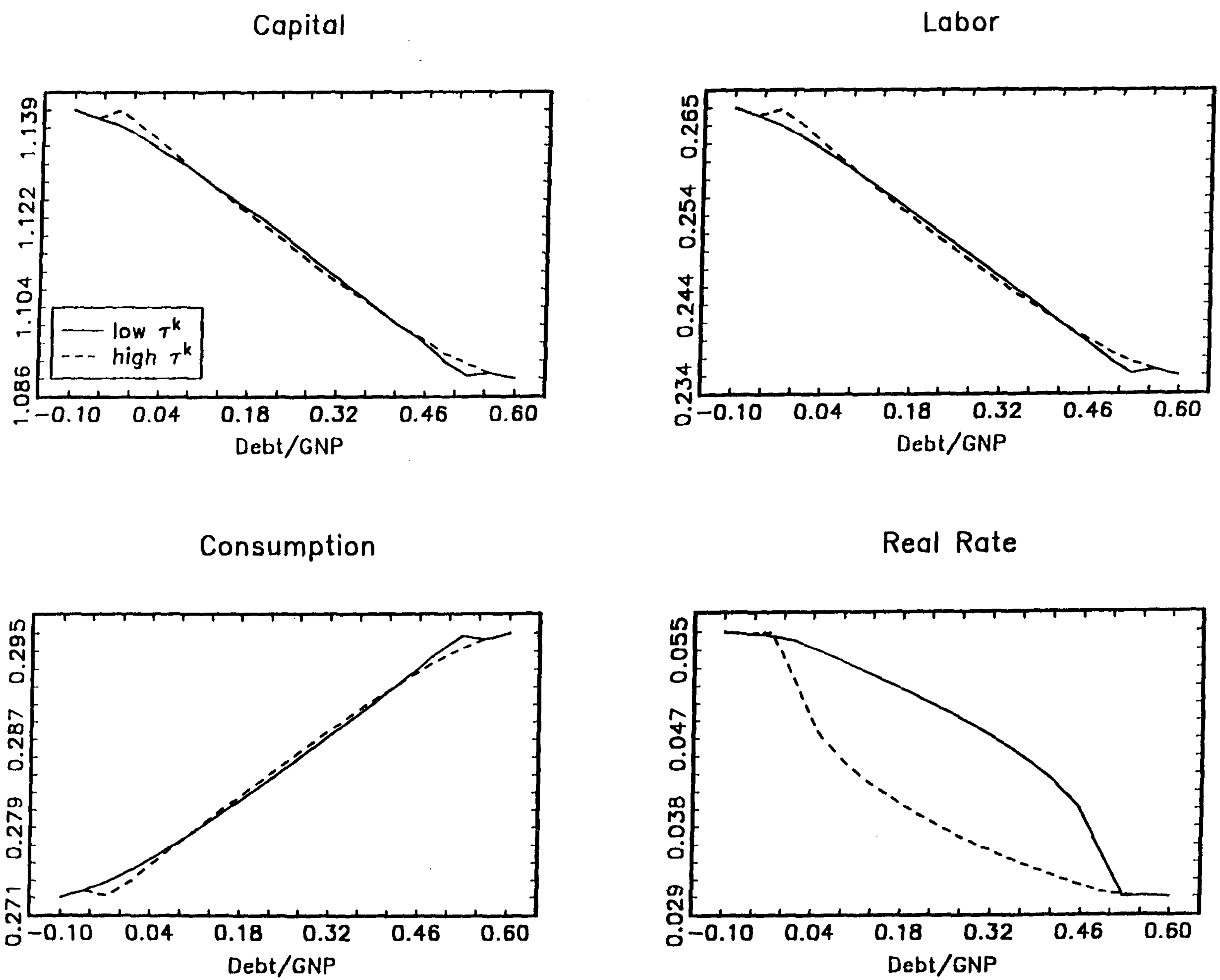


Figure 2

Policy Functions (τ^k varies: μ^k=3, Labor varies)



Policy Functions (τ^n varies: μ^n=7, Labor varies)

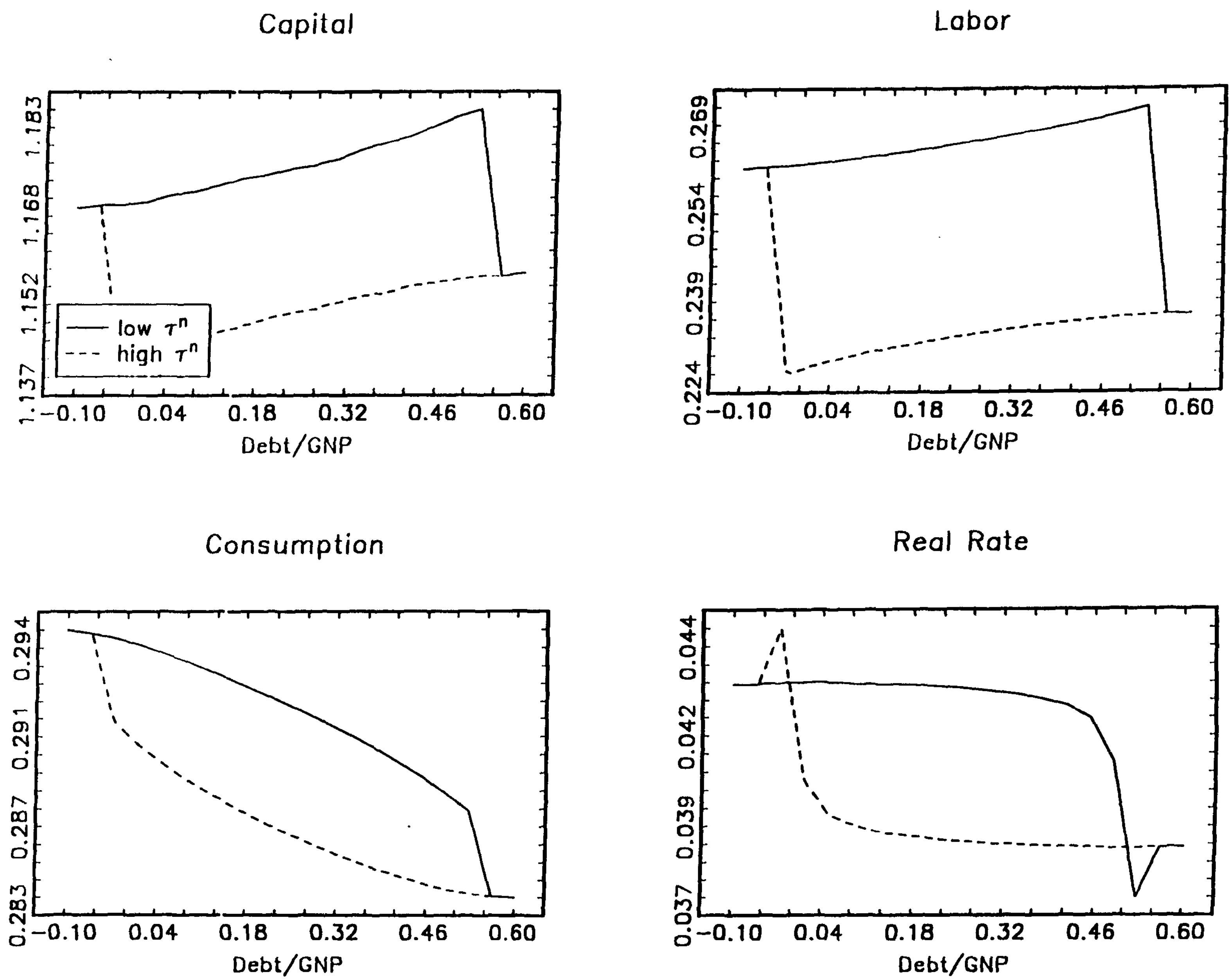


Figure 4

Policy Functions (g fixed, τ^k varies: $\mu^k=9$, τ^n varies: $\mu^n=9$)

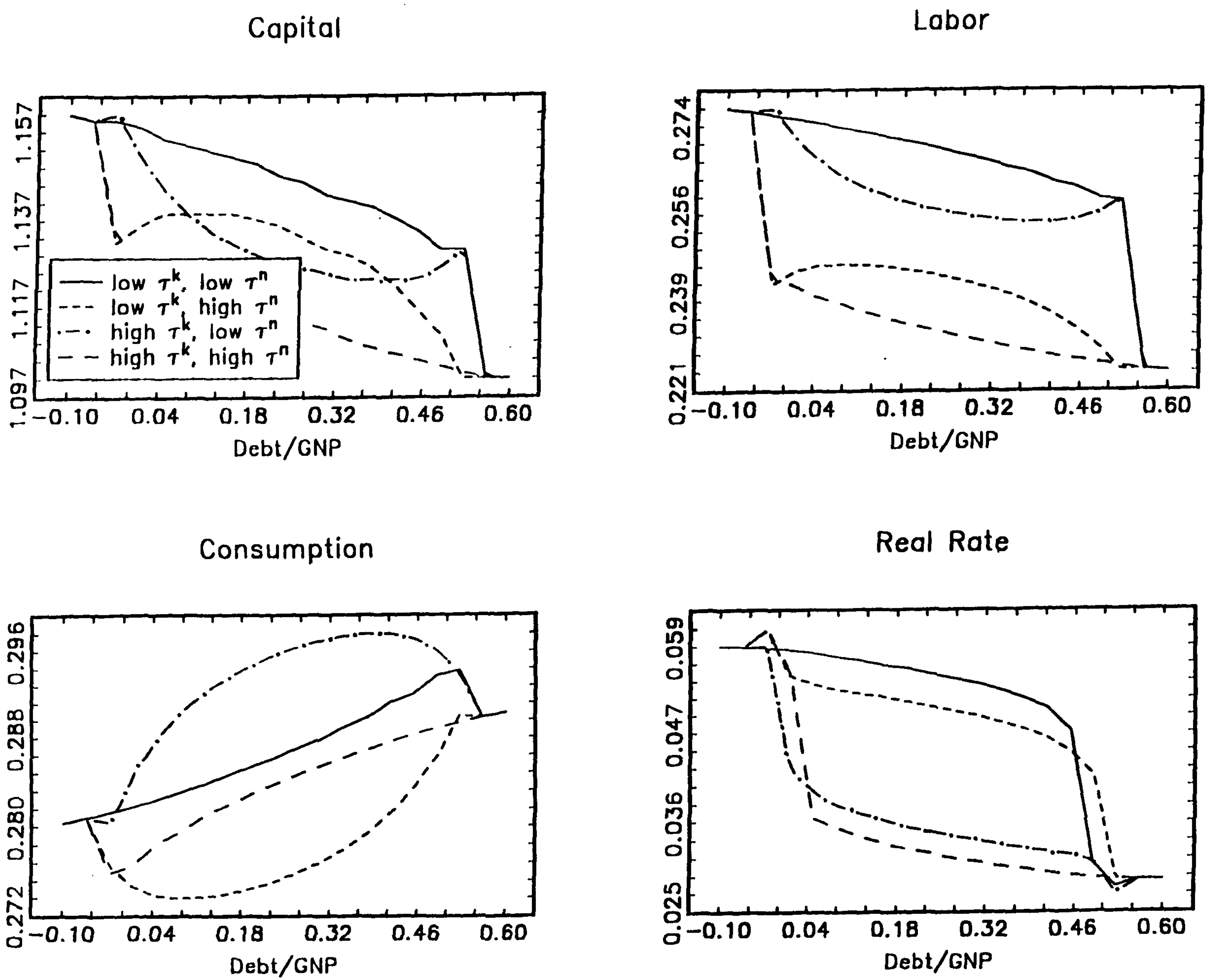


Figure 5

Policy Functions (g varies: η=6, τ^k and τ^n fixed)

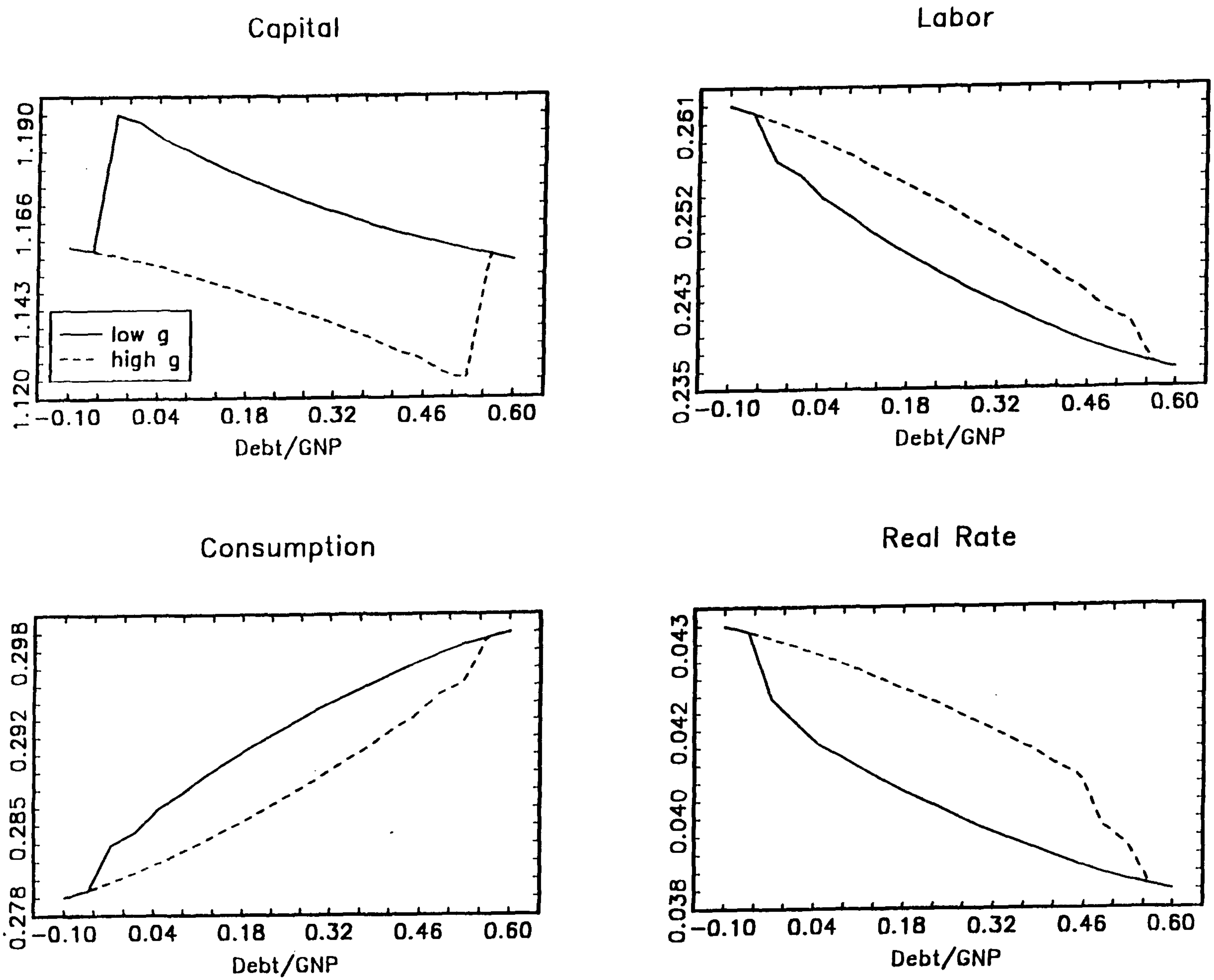
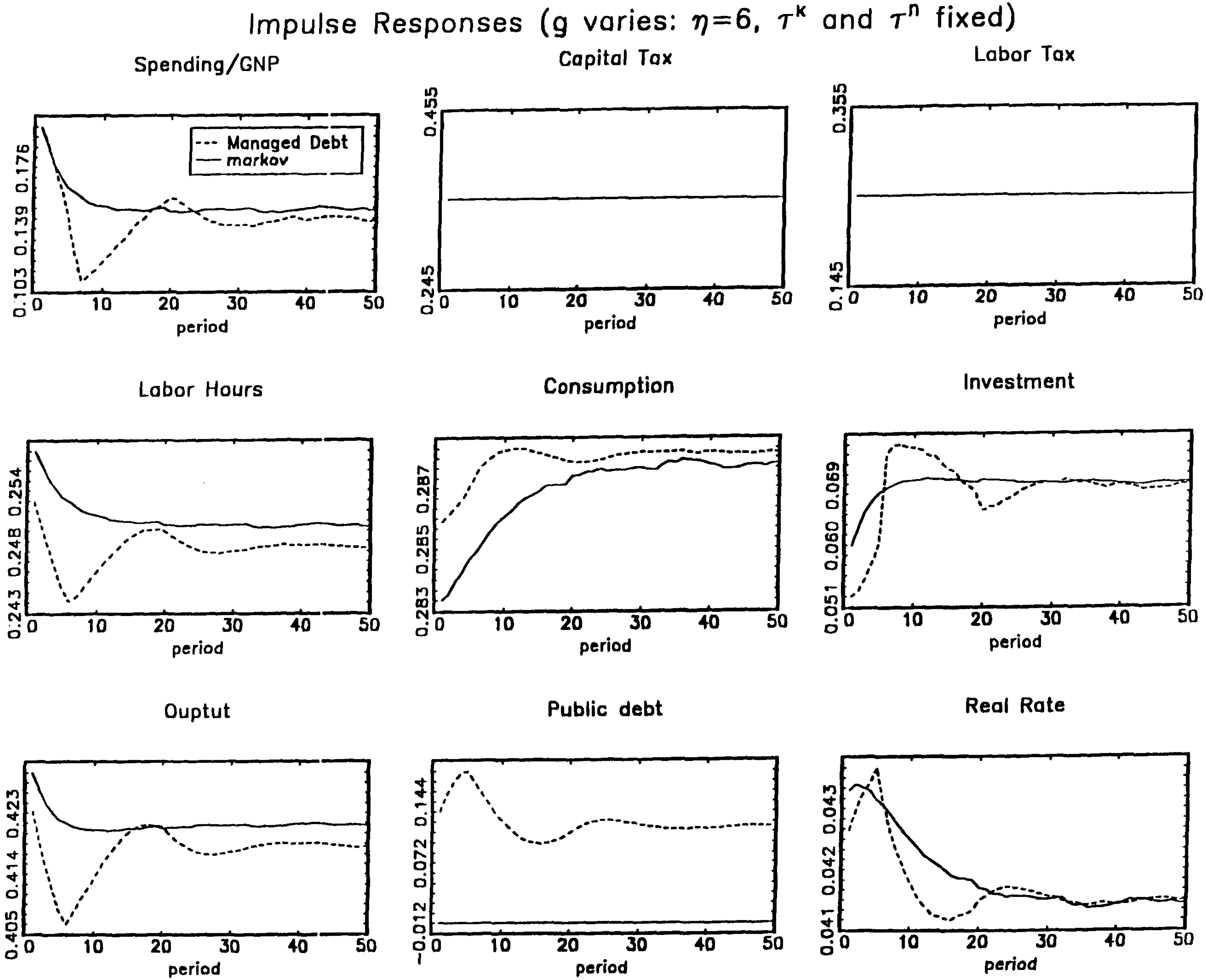


Figure 6


Impulse Responses (g varies: η=6, τ^κ and τ^n fixed)
Spending/GNP
Managed Debt
markov
0.103 0.139 0.176
Capital Tax
0.245 0.455
Labor Tax
0.145 0.355
Labor Hours
0.243 0.248 0.254
Consumption
0.283 0.285 0.287
Investment
0.051 0.060 0.069
Ouptut
0.405 0.414 0.423
Public debt
−0.012 0.072 0.144
Real Rate
0.041 0.042 0.043
0 10 20 30 40 50
period

Impulse Responses (g varies: $\eta=1$, τ^k and τ^n fixed)

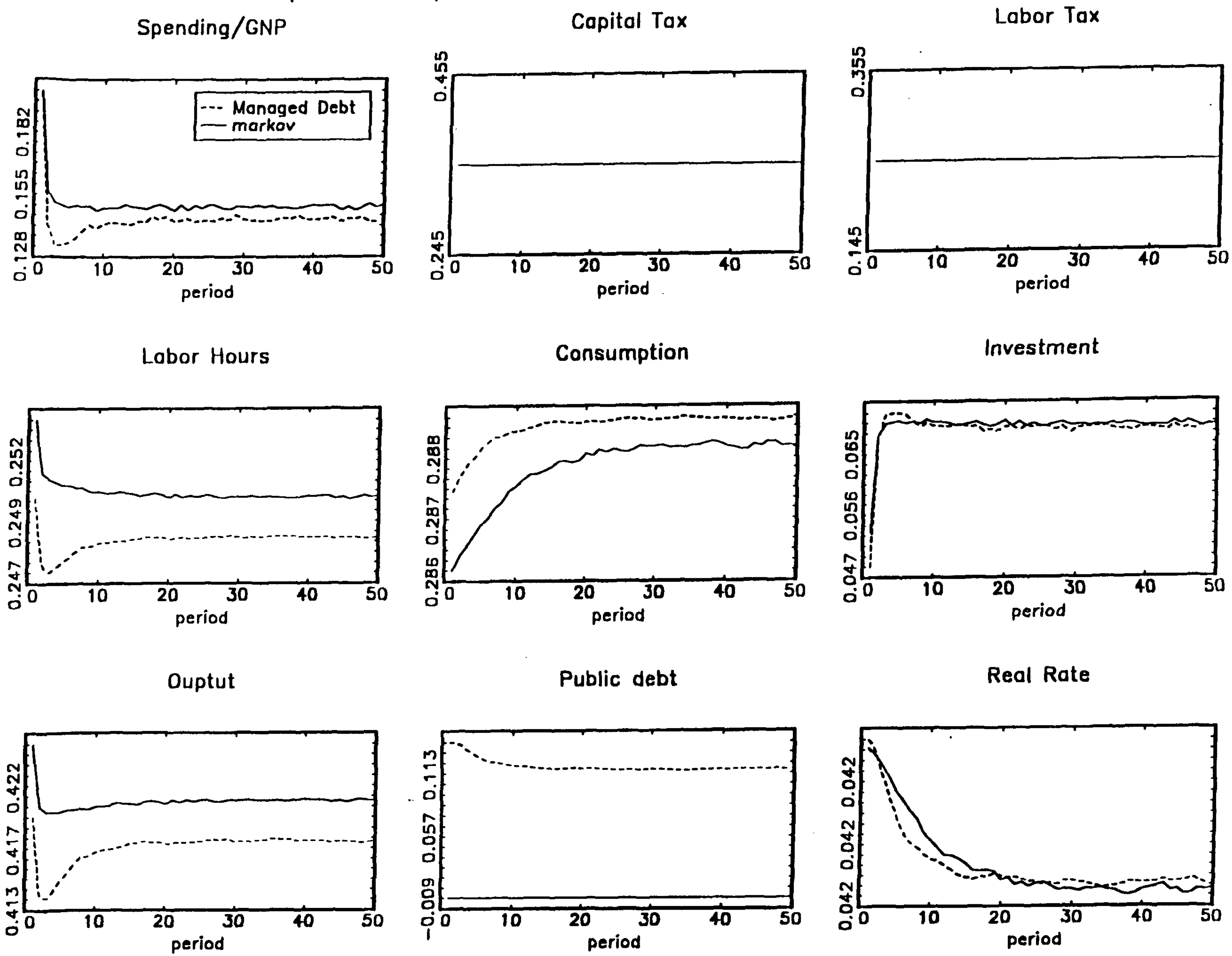
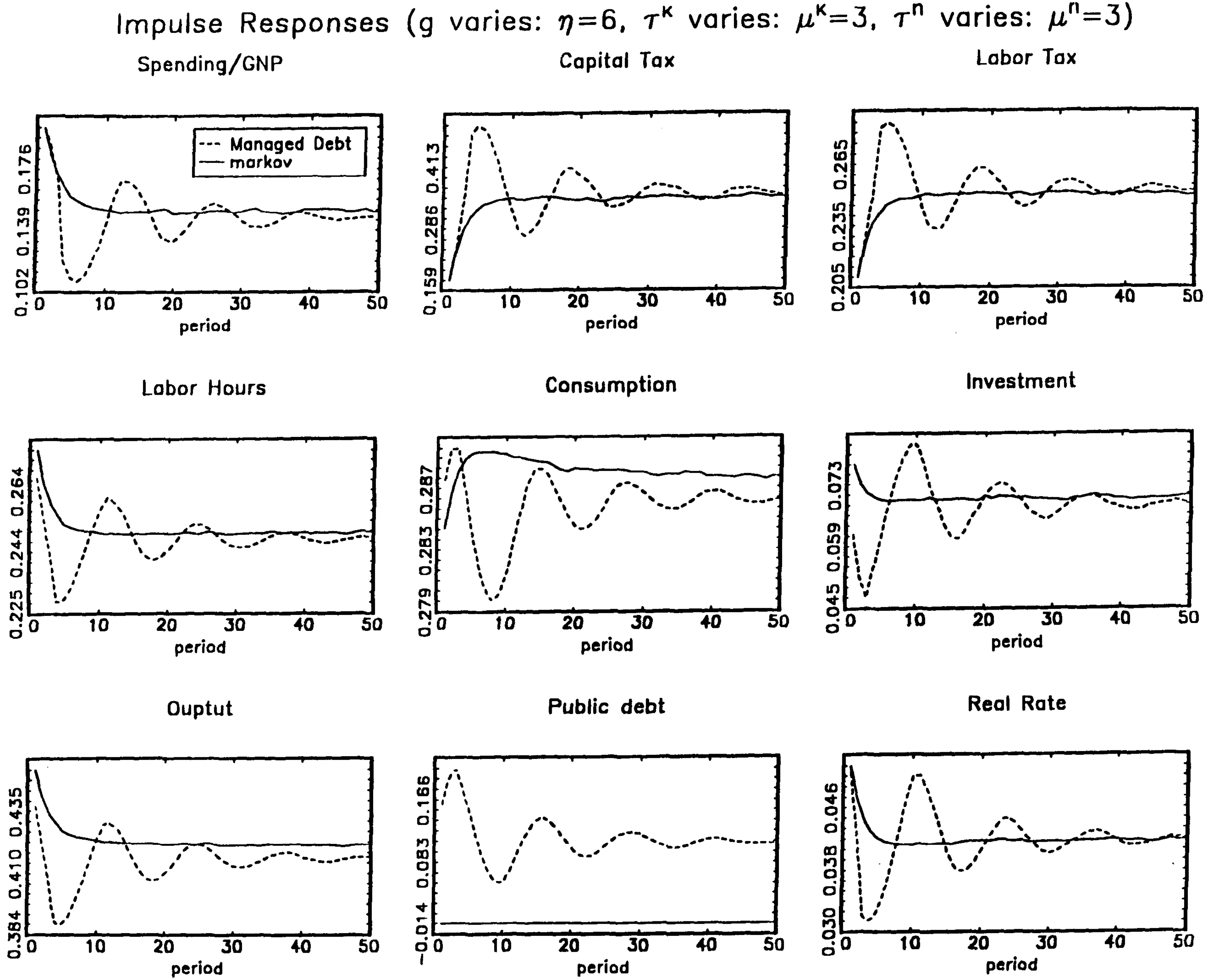


Figure 8



Impulse Responses (g varies: η=6, τ^K varies: μ^K=3, τ^n varies: μ^n=3)
Spending/GNP
Managed Debt
markov
0.102 0.139 0.176
Capital Tax
0.159 0.286 0.413
Labor Tax
0.205 0.235 0.265
Labor Hours
0.225 0.244 0.264
Consumption
0.279 0.283 0.287
Investment
0.045 0.059 0.073
Ouptut
0.384 0.410 0.435
Public debt
-0.014 0.083 0.166
Real Rate
0.030 0.038 0.046
0 10 20 30 40 50
period

Figure 9

Impulse Responses (g varies: $\eta=6$, τ^k varies: $\mu^k=3$, τ^n varies: $\mu^n=3$)

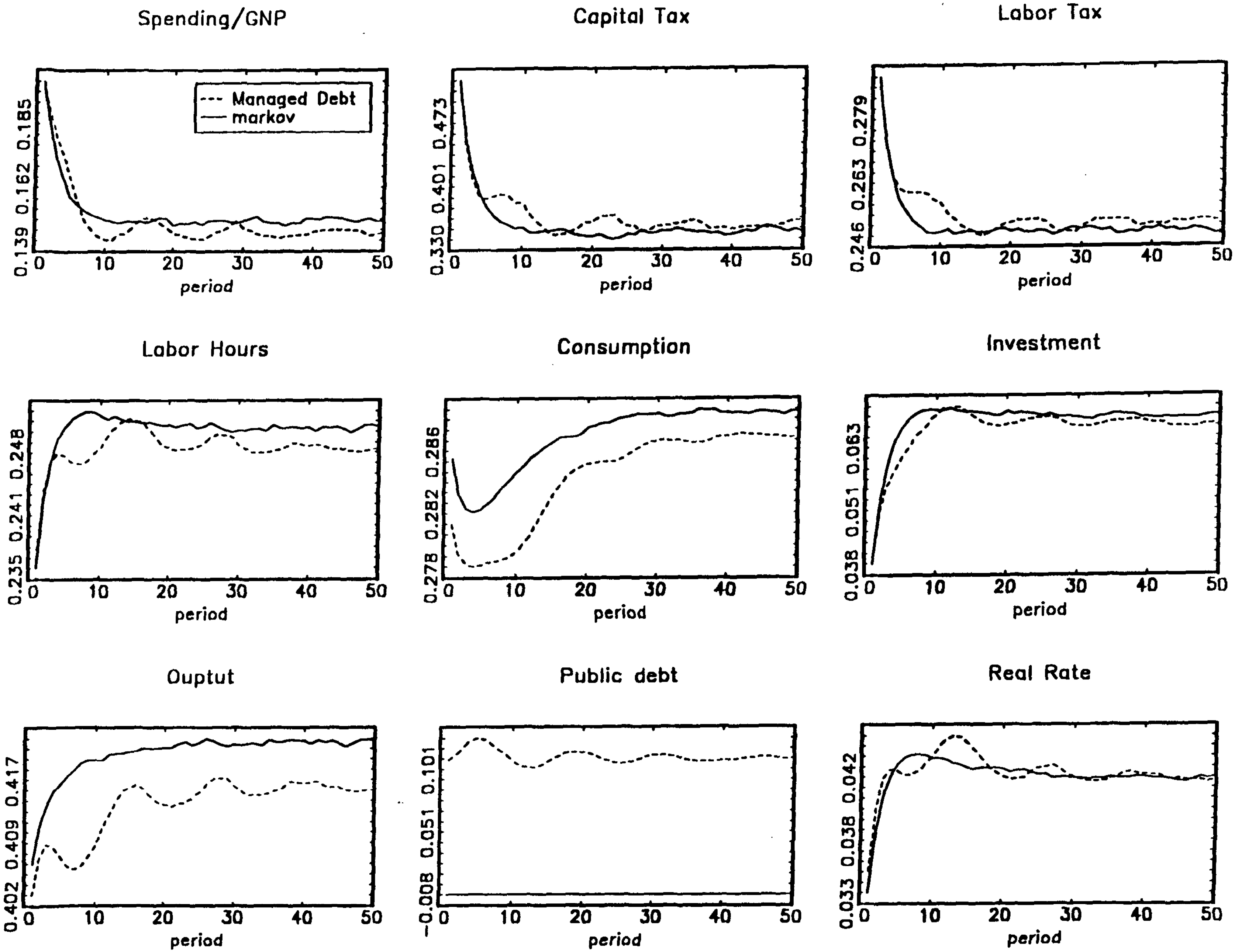
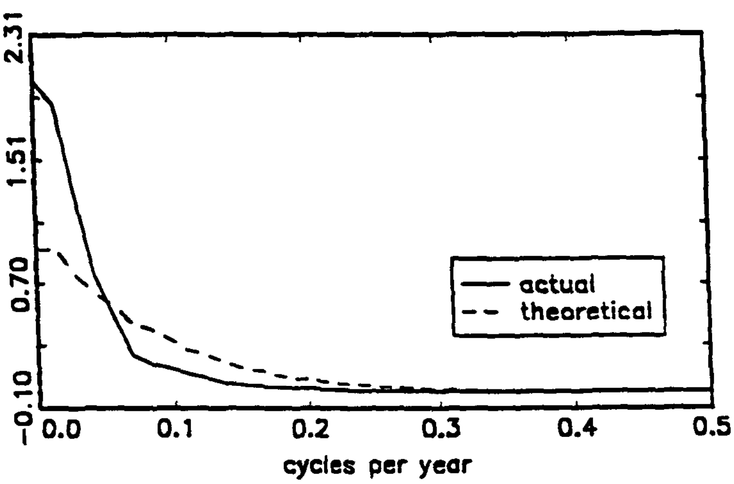


Figure 10

Frequency Domain Comparison (Actual: 1916–1988 vs. Theoretical)

Spectrum: Debt/GNP



Coherence: Debt/GNP

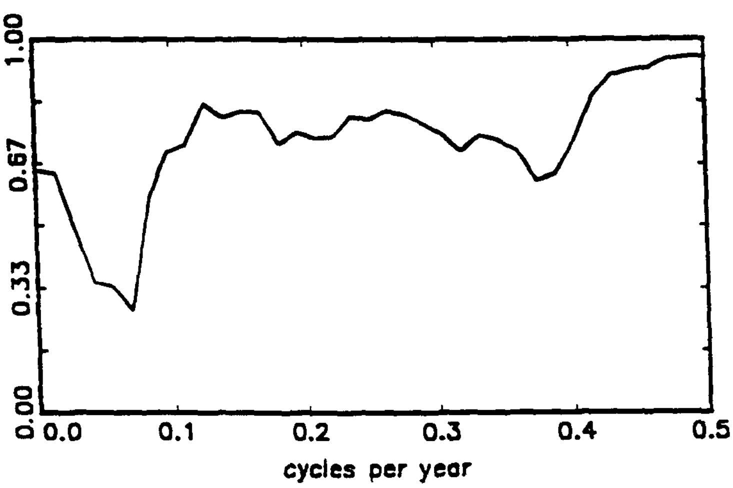
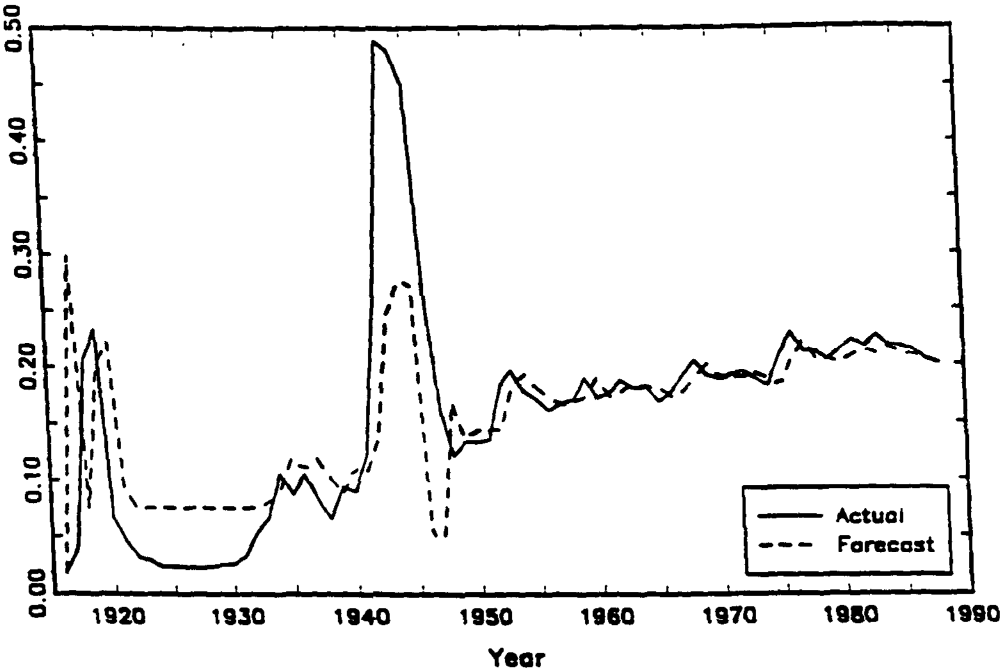


Figure 11

One–Step–Ahead Forecast: Spending/GNP



One–Step–Ahead Forecast: Tax Rate