



# Working Paper Series



This paper can be downloaded without charge from:  
<http://www.richmondfed.org/publications/>



**THE FEDERAL RESERVE BANK OF RICHMOND**  
RICHMOND ■ BALTIMORE ■ CHARLOTTE

The Bond Rate and  
Actual Future Inflation

Yash P. Mehra\*  
Federal Reserve Bank of Richmond  
Richmond, VA 23261

804-697-8247

November 1996

Abstract

The long-term bond rate is cointegrated with the actual one-period inflation rate during two sample periods, 1961Q1 to 1979Q3 and 1961Q1 to 1995Q4. This result indicates that in the long run the bond rate and actual inflation move together. The nature of short-run dynamic adjustments between these variables has, however, changed over time. In the pre-1979 period, when the bond rate rose above the one-period inflation rate, actual inflation accelerated. In the post-1979 period, however, the bond rate reverted back and actual inflation did not accelerate. Thus, the bond rate signaled future inflation in the period before 1979, but not thereafter. The results here indicate that in the period after 1979 Fed policy prevented any pickup in inflationary expectations (evidenced by the rise in the bond rate) from getting reflected in higher actual future inflation.

\*Vice President and Economist. The views expressed are those of the author and not necessarily those of the Richmond Fed or the Federal Reserve System.

## 1. INTRODUCTION

It is widely recognized that the bond rate contains useful information about long-term expected inflation. In a series of papers Mishkin (1990a, 1990b, 1991) and Jorion and Mishkin (1991) investigate whether the slope of the term structure has any predictive content in forecasting future inflation. Their general finding is that at long horizons it does, whereas at short horizons it does not. Blough (1994) reports that one-year ahead changes in the one-year inflation rate are not predicted by the current prevailing relationship between one- and two-year interest rates, leading him to conclude that a steep yield curve is not a reliable forecast of accelerating inflation in the near term. Engsted (1995) investigates whether the spread between the long-term interest rate and the one-period inflation rate predicts future one-period inflation. For a number of countries this spread does help predict future inflation over the period 1962 to 1993. For the U.S., however, the results reported there are not very favorable to this hypothesis.<sup>1</sup>

In this paper, I provide new evidence on the predictive content of the bond rate for future inflation using cointegration and error-correction modeling. The empirical work here extends the previous research in two main directions. First, I relax the assumption made in previous studies that the ex ante real interest rate is constant. Hence the predictive content of the bond rate for future inflation is investigated conditioning on variables that capture movements in the real rate of interest. The empirical results here indicate that inferences regarding the predictive content of the bond rate for future inflation are sensitive to such conditioning. Second, recent research reported in McCallum (1994) and Rudebusch (1995) indicates that the predictive

---

<sup>1</sup>Though this spread does Granger-cause the U.S. inflation rate, the sum of coefficients that appear on lagged values of the spread in the inflation equation is small in magnitude. Engsted, however, does not test whether the sum of these coefficients is different from zero.

content of the slope of the term structure for future economic variables may be influenced by the way the Fed conducts its monetary policy.<sup>2</sup> Most economists would agree that since 1979 the Fed has made repeated attempts to bring down the trend rate of inflation and contain inflationary expectations. Hence the empirical work here examines the temporal stability of results over two sample periods, 1961Q1 to 1979Q3 and 1961Q1 to 1995Q4.

The empirical results that are presented here focus on the behavior of the nominal yield on 10-year U.S. Treasury bonds. The economic variables that appear in the cointegration and error-correction modeling are the bond rate, the actual inflation rate, the nominal funds rate and the output gap. The last two variables control for variations in the real component of the bond rate which are due to funds rate policy actions and the state of the economy. The cointegration test results indicate that the bond rate is cointegrated with the actual inflation rate during both the sample periods, implying that the bond rate and the inflation rate move together in the long run. The estimated error-correction model, however, indicates that a change has occurred in the way these two variables have adjusted in the short run. In the pre-1979 period, when the bond rate rose above the actual inflation rate, actual inflation accelerated. In the post-1979 period, however, the bond rate reverted back and actual inflation did not accelerate. Thus the bond rate signaled an acceleration in future inflation in the period before 1979, but not thereafter.

---

<sup>2</sup>In the context of rational expectations hypothesis tests, McCallum (1994) shows how the reduced form regression coefficients depend upon the Fed's policy rule when the Fed smooths interest rates and responds to movements in the long-short spread.

As noted before, many researchers have found that at long horizons the slope of the term structure does help predict future inflation, a finding that is in line with sub-sample, but not full-sample, results here. Although the approach followed here differs from the one used in previous studies, a potential explanation of these different results is that in previous studies the ex ante real rate of interest is assumed to be constant. In previous studies the predictive content of the spread for future inflation is investigated without conditioning on variables that capture movements in the real rate of interest. In order to illustrate whether results are sensitive to such conditioning, I also examine the predictive content of the spread between the bond rate and the inflation rate for long-horizon future inflation, using a framework similar to one used in other studies. Those results indicate that the spread does help predict future inflation even during the full sample period if the spread is not conditioned on other variables. In the presence of conditioning, however, the predictive content of the spread disappears. Those results also indicate that Fed policy prevented any pickup in inflationary expectations (evidenced by the rise in the bond rate spread) from getting reflected in higher actual future inflation.

The plan of this article is as follows. Section 2 presents the model and the method used in investigating the dynamic interactions among the variables. Section 3 presents empirical results. Section 4 contains concluding observations.

## **2. THE MODEL AND THE METHOD**

### **2.1 The Fisher Relation, the Bond Rate and Future Inflation**

In order to motivate the empirical work I first discuss what does the Fisher relation

imply about the predictive content of the bond rate for future inflation. The Fisher relation for the m-period bond rate is

$$BR_t^{(m)} = rr_t^{(m)} + \dot{p}_t^{e(m)} \quad (1)$$

where  $BR^{(m)}$  is the m-period bond rate,  $\dot{p}^{e(m)}$  is the m-period expected inflation rate, and  $rr^m$  is the m-period expected real rate of interest. The Fisher relation (1) relates the bond rate to expectations of inflation and the real rate over the maturity (m) of the bond.

If the expected real interest rate is constant and if expectations of inflation are rational, then the Fisher relation above can be expressed as in (2) or (3)

$$BR_t^{(m)} = rr + \dot{p}_{t+m} - \epsilon_{t+m} \quad (2)$$

$$BR_t^{(m)} - \dot{p}_t = rr + (\dot{p}_{t+m} - \dot{p}_t) - \epsilon_{t+m} \quad (3)$$

where  $rr$  is the constant real rate,  $\dot{p}_{t+m}$  is the m-period future inflation rate,  $\dot{p}_t$  is the one-period current inflation rate, and  $\epsilon_{t+m}$  is the m-period future forecast error that is uncorrelated with past information. Equation 2 indicates that the bond rate contains information about the (m-period) future inflation rate, and equation 3 similarly shows that the spread between the bond rate and the current inflation rate has information about a change in the future inflation rate.

Equations (2) and (3) above however assume that the expected real interest rate is constant. If this assumption is incorrect, then changes in the bond rate or the bond rate spread as

defined above will not necessarily signal changes in future inflation rates.

## 2.2 Testing the Predictive Content of the Bond Rate for Future Inflation

### 2.2.1 Previous Studies

Equations (2) and (3) above form the basis of empirical work in most previous studies of the predictive content of the bond rate for future inflation. Previous researchers have investigated the term structure's ability to predict future inflation by running regressions that are of the form (4) and (5)

$$(\dot{p}_{t+m} - \dot{p}_{t+n}) = a + b (BR_t^m - BR_t^{(n)}) + \epsilon_{1t} \quad (4)$$

$$(\dot{p}_{t+m} - \dot{p}_t) = c + d (BR_t^m - \dot{p}_t) + \epsilon_{2t} \quad (5)$$

where  $BR^n$  is the n-period bond rate,  $\dot{p}_{t+n}$  is the n-period future inflation rate, and other variables are as defined before. As can be seen, these regressions are just re-arranged versions of Fisher relations (2) and (3). In (4) the spread between the m-period and n-period nominal interest rates is used to predict the difference between the m-period and n-period inflation rates, and in (5) the spread between the m-period bond rate and the (one-period) inflation rate is used to predict change in future inflation. Regressions like (4) appear in Mishkin (1990, 1991) and those like (5) in Engsted (1994). If  $b \neq 0$  in (4) or  $d \neq 0$  in (5), then that result indicates that the slope of the term structure does help predict future inflation.

But, as noted before, equations (2) and (3) (or regressions (4) and (5)) embody the assumption that the expected real interest rate is constant. This is a questionable assumption.

Plosser and Rouwenhorst (1994) in fact present evidence that indicates that the long end of the term structure does seem to contain information about the real economic activity and hence about the real rate of interest. Therefore, in (4) or (5) inferences concerning the predictive content of the term structure for future inflation are likely to be suspect.

Another issue not investigated fully in previous research is that slope parameters in (4) and (5) are likely to be influenced by the way the Fed conducts its monetary policy. For example, if the Fed has in place a disinflationary policy, then a current increase in the bond rate spread (as in (5)) may not be followed by higher actual inflation. This could happen if current widening in the bond rate spread causes the Fed to raise the funds rate, leading to slower real growth and lower actual inflation in the future. In this scenario a current increase in the bond rate spread still reflects expectations of rising future inflation. However, the ensuing Fed behavior prevents those expectations from getting reflected in higher actual inflation. Hence in regressions like (5) the estimate of the slope parameter (b) may be small in periods during which the Fed has been vigilant. Those considerations suggest that parameters that measure the predictive content of the term structure for future inflation may not be stable during the sample period.

### **2.2.2 Cointegration and Error-Correction Modeling**

The empirical work here examines the predictive content of the bond rate using cointegration and error-correction modeling. This empirical procedure, as I illustrate below, yields regressions that are similar in spirit to those employed in some previous research but differ in including additional economic variables that control for potential movements in the real rate of interest.

As indicated before, the Fisher relation (1) for interest rates relates the bond rate to expectations of future inflation and the real interest rate. If one assumes that those expectations can be proxied by distributed lags on current and past values of actual inflation and other fundamental economic determinants, then the Fisher relation implies the following regression (6)

$$BR_t = a + \sum_{s=0}^k b_s \dot{p}_{t-s} + \sum_{s=0}^k c_s X_{t-s} + U_t \quad (6)$$

where  $\dot{p}_t$  is the actual inflation rate,  $X_t$  is the vector containing other economic determinants of the real rate, and  $U_t$  is the disturbance term. The presence of the disturbance term in (6) reflects the assumption that distributed lags on actual values of economic determinants may be good proxies for their anticipated values in the long run, but not necessarily in the short run.<sup>3</sup>

If levels of the empirical measures of these economic determinants including the bond rate are unit root nonstationary, then the bond rate may be cointegrated with these variables as in Engle and Granger (1987). Under those assumptions, regression (6) can be reformulated as in (7).

$$BR_t = d_0 + d_1 \dot{p}_t + d_2 X_t + e_t \quad (7)$$

Equation (7) is the cointegrating regression. The coefficients that appear on  $\dot{p}_t$  and  $X_t$  in (7) then measure the long-run responses of the bond rate to inflation and its other real rate determinants.

---

<sup>3</sup>The only assumption I make about the random disturbance term in (2) is that it has a zero mean.

Hence, I investigate the question whether the bond rate incorporates expectations of future inflation by testing whether the bond rate is cointegrated with the actual inflation rate. The analysis here thus views the positive relationship between the bond rate and actual inflation as a long-run phenomenon.

The cointegrating bond rate regression thus defines the long-run, equilibrium value of the bond rate. Should the bond rate rise above its long-run equilibrium value, then either the bond rate should fall, or the economic determinants including inflation should adjust in the direction needed to correct the disequilibrium, or both (Granger 1987). I examine such short-run dynamic adjustments by building a vector error-correction model. Thus, if the sum of coefficients that appear on the error-correction term or the bond rate is positive and statistically significant in the short-run inflation equation, then that evidence can be interpreted to mean that the bond rate signals future inflation.<sup>4</sup>

To illustrate, assume that the bond rate depends only on the inflation rate in the long run and that the expected real rate is mean stationary. The cointegrating regression is then defined by the relation

$$BR_t = a + b \dot{p}_t + U_t \quad (8)$$

where  $U_t$  is a stationary random disturbance. The presence of cointegration implies the following error-correction model in  $\Delta BR$  and  $\Delta \dot{p}$ .

---

<sup>4</sup>Miller (1991) has used this methodology to investigate short-run monetary dynamics.

$$\Delta BR_t = c_o + \sum_{s=1}^k c_{1s} \Delta BR_{t-s} + \sum_{s=1}^k c_{2s} \Delta \dot{p}_{t-s} + \lambda_1 U_{t-1} + \epsilon_{1t} \quad (9.1)$$

$$\Delta \dot{p}_t = d_o + \sum_{s=1}^k d_{1s} \Delta BR_{t-s} + \sum_{s=1}^k d_{2s} \Delta \dot{p}_{t-s} + \lambda_2 U_{t-1} + \epsilon_{2t} \quad (9.2)$$

where  $U_{t-1}$  is the lagged residual from (8) and where all other variables are as defined above. The presence of cointegration between  $BR_t$  and  $\dot{p}_t$  implies that in (9) either  $\lambda_1 \neq 0$ , or  $\lambda_2 \neq 0$ , or both.

Thus, if  $\lambda_2$  is positive and statistically significant, then that implies that a rise in the spread

( $U_t = BR_t - a - b \dot{p}_t$ ) signals higher actual future inflation. Since the real interest rate is assumed to

be mean stationary, not constant, the error-correction equations should be estimated including

other (stationary) short-run determinants of the real interest rate.<sup>5</sup>

<sup>5</sup>It is worth pointing out that Engsted (1995) uses an equation like (9.2) to investigate whether the spread between the bond rate and the actual inflation rate ( $U_{t-1}$  in (8) here) helps predict future inflation. He, however, derives this implication of the Fisher hypothesis under the assumptions that expectations of inflation are rational and forward looking and that the expected real interest rate is constant. To see it, consider the following version of the Fisher hypothesis (1) for the long-term bond rate

$$BR_{(t)} = rr + (1 - b) \sum_{j=1}^{\infty} b^j E_t \dot{p}_{t+j} \quad (a)$$

where  $rr$  is the constant real rate and  $b = \bar{e}^i \approx (1 + rr)$  is the discount factor (Engsted 1995). That is, the long bond rate is given as the constant real rate plus a weighted average of expected future one-period inflation rates ( $E_t \dot{p}_{t+j}$ ,  $j \geq 1$ ). If  $BR_t$  and  $\dot{p}_t$  are non-stationary and expectations are rational, then the above equation can be reformulated as

$$BR_t - b \dot{p}_t \equiv S_t = rr + \sum_{j=1}^{\infty} b^j E_t \Delta \dot{p}_{t+j} \quad (b)$$

Equation b implies that  $BR_t$  and  $b \dot{p}_t$  are cointegrated and that the spread

(continued...)

### 2.3 Data, and Definition of Economic Determinants in the Multivariable Analysis

The empirical work here examines the dynamic interactions between the bond rate and the inflation rate within a framework that allows for movements in the real component of the bond rate. The descriptive analysis of monetary policy in Goodfriend (1993) and the error-correction model of the bond rate estimated in Mehra (1994) indicate that the real component of the bond rate is significantly influenced by monetary policy actions and the state of the economy. Hence the economic variables that enter the analysis here are the bond rate, the actual inflation rate, the nominal federal funds rate, and the output gap that measures the state of the economy.

The empirical work uses quarterly data that spans the period 1959Q1 to 1995Q4. The bond rate is the nominal yield on 10-year U.S. Treasury bonds (BR). Inflation as measured by the behavior of the consumer price index (excluding food and energy) is the actual, annualized quarterly inflation rate ( $\dot{p}$ ). The measure of monetary policy used is the nominal federal funds rate (NFR), and the output gap (gap) is the natural log of real GDP minus the natural log of potential GDP, the latter is generated using the Hodrick-Prescott filter (1980). The interest rate data are the last month of the quarter.

### 2.4 Tests for Unit Roots and Cointegration

Cointegration and error-correction modeling involves four steps. First, determine the stationarity properties of the empirical measures of economic determinants suggested above

---

<sup>5</sup>(...continued)

$S_t = BR_t - b \dot{p}_t$  is an optimal predictor of future changes in inflation. Engsted (1995) examines the second implication by estimating a VAR in  $S$  and  $\Delta \dot{p}$  and then testing whether  $S$  Granger-causes  $\Delta \dot{p}$ .

Second, test for the presence of cointegrating relationships in the system. Third, estimate the cointegrating regression and calculate the residuals. Fourth, construct the short-run error-correction equations.

In order to determine whether the variables have unit roots or are mean stationary, I perform both unit root and mean stationarity tests. The unit root test used is the augmented Dickey-Fuller test and the test for mean stationarity is the one advocated by Kwiatkowski, Phillips, Schmidt, and Shin (1992). Thus a variable  $X_t$  is considered unit root nonstationary if the hypothesis that  $X_t$  has a unit root is not rejected by the augmented Dickey-Fuller test and the hypothesis that it is mean stationary is rejected by the mean stationarity test.

The test for cointegration used is the one proposed in Johansen and Juselius (1990), and the cointegrating relations are identified imposing restrictions as in Johansen and Juselius (1994). The cointegrating regressions are also estimated using an alternative estimation methodology, Stock and Watson's (1993) dynamic OLS procedure.

### **3. EMPIRICAL RESULTS**

#### **3.1 Unit Root and Mean Stationarity Test Results**

As indicated before, the economic variables that enter the analysis are the bond rate (BR), the inflation rate ( $\dot{p}$ ), the nominal funds rate (NFR) and the output gap (gap). The output gap variable by construction is mean stationary. Table 1 reports test results for determining whether other variables have a unit root or are mean stationary. As can be seen, the t-statistic ( $t_p$ ) that tests the null hypothesis that a particular variable has a unit root is small for BR,

$\dot{p}$ , and NFR. On the other hand, the test statistic ( $\hat{\eta}_0$ ) that tests the null hypothesis that a particular variable is mean stationary is large for all these variables. These results thus indicate that BR,  $\dot{p}$  and NFR have a unit root and are thus nonstationary in levels.

### 3.2 Cointegration Test Results

Table 2 presents test statistics for determining the number of cointegrating relations in the system (BR,  $\dot{p}$ , NFR, gap). Trace and maximum eigenvalue statistics presented there indicate that there are three cointegrating relations in the system.<sup>6</sup> This result holds in both the sample periods, 1961Q1 to 1995Q4 and 1961Q1 to 1979Q3.

Table 3 presents estimates of the cointegrating relations found in the system. I first test the hypothesis that the three-dimensional cointegration space contains cointegrating relations that are of the form (10) through (12).

$$BR_t = a_o + a_1 \dot{p}_t + u_{1t}; a_1 = 1 \quad (10)$$

$$NFR_t = b_o + b_1 \dot{p}_t + u_{2t}; b_1 = 1 \quad (11)$$

$$gap_t = c_o + u_{3t} \quad (12)$$

---

<sup>6</sup>The lag length parameter ( $k$ ) for the VAR model was chosen using the likelihood ratio test described in Sims (1980). In particular, the VAR model initially was estimated with  $k$  set equal to a maximum number of eight quarters. This unrestricted model was then tested against a restricted model, where  $k$  is reduced by one, using the likelihood ratio test. The lag length finally selected in performing the Johansen-Juselius procedure is the one that results in the rejection of the restricted model.

Equation (10) can be interpreted as the Fisher relation for the bond rate and equation (11) as the Fed reaction function. Equation (12) simply states that the output gap variable is stationary. As shown in Johansen and Juselius (1994), these cointegrating relations can be identified imposing restrictions on long-run parameters in the cointegrating space.

In the full sample period, the hypotheses that cointegrating relations are of the form (10) through (12) and that  $a_1 = b_1 = 1$  are consistent with data (the  $x_1^2$  statistic that tests those restrictions is small; see Panel A, Table 3). However, in the subsample 1961Q1 to 1979Q3 the restrictions that  $a_1 = b_1 = 1$  are rejected by data. Hence for the subperiod 1961Q1-1979Q3 cointegrating relations are estimated without such restrictions. As can be seen, estimates indicate that the bond rate is cointegrated with the inflation rate. Hence inflation is the only source of the stochastic trend in the bond rate.

The estimation procedure in Johansen and Juselius (1990, 1994) is a system estimation method. In order to check the robustness of estimates, I also present estimates of the cointegrating relations (10) and (11) using a single equation estimation method.<sup>7</sup> Panel B in Table 3 presents results using the dynamic OLS procedure given in Stock and Watson (1993). As can be seen, this procedure yields estimates that are remarkably close to those reported above.

### **3.3 Results on the Error-Correction Coefficient in the Error-correction Model**

---

<sup>7</sup>Several single equation methods have been proposed for the estimation of cointegrating vectors. All these methods generate estimates that have the same asymptotic distribution as the full information maximum likelihood estimates. See Inder (1993) for a comparison of some of these methods.

The cointegration test results described in the previous section are consistent with the presence of cointegrating relations that are of the form

$$BR_t = a_o + a_1 \dot{p}_t + U_{1t} \quad (13)$$

$$NFR_t = b_o + b_1 \dot{p}_t + U_{2t} \quad (14)$$

where  $U_1$  and  $U_2$  are stationary disturbance terms. I now examine the behavior of the error-correction term  $U_{1t} = BR_t - a_o - a_1 \dot{p}_t$  in short-run equations of the form

$$\begin{aligned} \Delta BR_t = & b_o + \sum_{s=1}^{k1} b_{1s} \Delta BR_{t-s} + \sum_{s=1}^{k2} b_{2s} \Delta \dot{p}_{t-s} + \sum_{s=1}^{k3} b_{3s} \Delta NFR_{t-s} \\ & + \sum_{s=1}^{k4} b_{4s} gap_{t-s} + \lambda_1 U_{1t-1} + \delta_1 U_{2t-1} \end{aligned} \quad (15.1)$$

$$\begin{aligned} \Delta \dot{p}_t = & c_o + \sum_{s=1}^{k1} c_{1s} \Delta BR_{t-s} + \sum_{s=1}^{k2} c_{2s} \Delta \dot{p}_{t-s} + \sum_{s=1}^{k3} c_{3s} \Delta NFR_{t-s} \\ & + \sum_{s=1}^{k4} c_{4s} gap_{t-s} + \lambda_2 U_{1t-1} + \delta_2 U_{2t-1} \end{aligned} \quad (15.2)$$

where all variables are as defined before. The short-run equations include first differences of the bond rate, inflation, and the funds rate and level of the output gap, even though the last two variables do not enter the long-run bond equation (13). These variables capture the short-run impacts of monetary policy and the state of the economy on the bond rate and other variables. As

indicated before, the parameters of interest are  $\lambda_1$ ,  $\lambda_2$  and the sums of coefficients that appear on the bond rate in equation (15.2). The expected signs of the error-correction term  $U_{1t-1}$  are positive for  $\Delta\dot{p}$  and negative for  $\Delta BR$ .

A major decision emerges in the choice of the lag lengths used in the error-correction model. I chose lag-lengths using the procedure given in Hall (1990), as advocated by Campbell and Perron (1991). This procedure starts with some upper bound on lags, chosen a priori for each variable (eight quarters here) and then drops all lags beyond the lag with a significant coefficient. I however do present tests of the hypothesis that excluded lags are not significant. In addition I present results including only own lags in the error-correction model.

Table 4 reports the error-correction coefficients (t-values in parentheses) when the long-run bond equation is (13). In addition it also reports the sums of coefficients that appear on (first differences of) the bond rate in the inflation equation. Parentheses that follow contain t-statistics for the sum of coefficients, whereas brackets contain Chi-squared statistics for exclusion restrictions. Panel A reports results for the full sample 1961Q1 to 1995Q3 and Panel B for the subsample 1961Q1 to 1979Q3.<sup>8</sup> As can be seen, in full sample regressions the error-correction coefficient is negative and statistically significant in the bond ( $\Delta BR$ ) equation, but in inflation ( $\Delta\dot{p}$ ) equations it is generally small and not statistically different from zero.<sup>9</sup> Furthermore, individual

---

<sup>8</sup>Inflation equations include dummies for President Nixon's price and wage controls.

<sup>9</sup>The error-correction coefficients is in fact negative in the inflation equation that includes other determinants of the real rate. In the inflation equation that includes only lagged values of inflation, the coefficient that appears on the error-correction term is positive, small in magnitude and not statistically different from zero. The later result is similar in spirit to the one in Engsted (1995).

coefficients that appear on two lagged values of the bond rate in the inflation equation are .50 and -.33. These coefficients are individually significant but their sum is not statistically different from zero, indicating that ultimately increases in the bond rate have not been associated with accelerations in actual inflation.<sup>10</sup> Together, these results indicate that the short-run positive deviations of the bond rate from its long-run equilibrium values were corrected mainly through reversals in the bond rate. Actual inflation did not accelerate.

The subsample results reported in panel B of Table 4 are however strikingly different. As can be seen, the error-correction coefficient is negative and significant in the bond rate equation, but is positive and significant in the inflation equation. These results suggest that positive deviations of the bond rate from its long-run equilibrium value were eliminated partly through declines in the bond rate and partly through increases in actual inflation. Actual inflation did accelerate when the spread between the bond rate and the one-period inflation rate rose.<sup>11</sup>

### 3.4 Comparison with Previous Studies

The full-sample results discussed in the previous section indicate that the spread between the bond rate and the one-period inflation rate does not help predict one-quarter ahead changes in the rate of inflation. Since inflation is a unit root process, the results above also imply that the spread has no predictive content for long-horizon forecasts of future inflation. The latter

---

<sup>10</sup>This result, of course, means that the bond rate Granger-causes inflation.

<sup>11</sup>I get similar results if cointegrating regressions (13) and (14) are estimated without restrictions  $b_1 = a_1 = 1$ . In particular, over the sample period 1961Q1 to 1979Q3, the error-correction variable  $U_{1t-1}$  has a positive coefficient in the inflation equation, indicating that actual inflation did accelerate following an increase in the bond rate spread.

implication is in contrast with the finding in Mishkin (1990, 1991) that at long horizons the slope of the term structure does help predict future inflation.

As indicated before, an important assumption implicit in the regressions used by Mishkin is that long-horizon term spreads contain mostly information about inflation and not about the real rate of interest, because the ex ante real rate of interest is assumed to be constant. This is a debatable assumption. Hence the predictive content of the spread for future inflation should be investigated by conditioning on variables that may provide information about the real rate of interest.

In order to illustrate whether results are sensitive to such conditioning, I also investigate the predictive content of the spread between the bond rate and the (one-period) inflation rate for future inflation by estimating regressions of the form<sup>12</sup>

$$(\ln(P_{t+m}/P_t)/m) - \ln(P_t/P_{t-1}) = a_o + \lambda_c U_{1t} + V_{1t} \quad (16)$$

$$\begin{aligned} (\ln(P_{t+m}/P_t)/m) - \ln(P_t/P_{t-1}) = & a_o + \lambda_d U_{1t} + \sum_{s=1}^{k1} a_{1s} \Delta \dot{p}_{t-s} \\ & + \sum_{s=1}^{k2} a_{2s} \Delta NFR_{t-s} + \sum_{s=1}^{k3} a_{3s} \Delta BR_{t-s} + \sum_{s=1}^{k4} a_{4s} gap_{t-s} + V_{2t} \end{aligned} \quad (17)$$

---

<sup>12</sup>These regressions differ from those reported in Mishkin (1990, 1991). Mishkin uses zero coupon bond data, derived from actually traded coupon-bearing bonds. So, he is able to match the horizon of the inflation forecast with that of the term spread. The empirical work here instead uses yield-to-maturity data on coupon bonds and the inflation forecast horizon does not match with that of the term spread. These differences, however, do not reduce the importance of examining the potential role of additional variables that may provide information about movements in the real rate of interest.

$$\begin{aligned}
 (\ln(P_{t+m}/P_t)/m) - \ln(P_t/P_{t-1}) = & a_o + \lambda_e U_{1t} + \delta U_{2t} + \sum_{s=1}^{k1} a_{1s} \Delta \dot{p}_{t-s} \\
 & + \sum_{s=1}^{k2} a_{2s} \Delta NFR_{t-s} + \sum_{s=1}^{k3} a_{3s} \Delta BR_{t-s} + \sum_{s=1}^{k4} a_{4s} gap_{t-s} + V_{2t}
 \end{aligned} \tag{18}$$

where

$$U_{1t} = BR_t - a_o - a_1 \dot{p}_t$$

$$U_{2t} = NFR - b_o - b_1 \dot{p}_t$$

where m is the number of quarters, and other variables are as defined.  $U_1$  measures the spread between the bond rate and the (one-period) inflation rate and  $U_2$  the spread between the nominal funds rate and the inflation rate. Regression (16) examines the predictive content of the spread for long-horizon forecasts of future inflation without controlling for variations in the spread due to real growth, monetary policy actions, and inflation. Regressions (17) and (18), however, control for such variations. Regression (18) is similar to regression (17) except in that it also includes the current stance of short-run monetary policy measured by the funds rate spread ( $U_{2t}$ ). The regressions are estimated over two sample periods, 1961Q1 to 1979Q3 and 1961Q1 to 1995Q4 and for horizons up to four years in the future.

Table 5 presents estimates of the coefficient (t-values in parentheses) that appears on the bond rate spread variable ( $\lambda_c$  in (16),  $\lambda_d$  in (17),  $\lambda_e$  in (18)).<sup>13,14</sup> I also report the coefficient on

---

<sup>13</sup>The t-values have been corrected for the presence of moving-average serial correlation  
(continued...)

the funds rate spread variable ( $\delta$  in (18)). If we focus on sub-sample regression estimates, they indicate that the bond rate spread does help predict future inflation (see t-values on  $\lambda_c$ ,  $\lambda_d$ ,  $\lambda_e$  in Panel B, Table 5). This result holds at all forecast horizons and is not sensitive to the inclusion of other variables in regressions. Furthermore, the funds rate spread variable that controls for policy-induced movements in the real component of the bond rate is never significant in those regressions, indicating that the current stance of monetary policy had no predictive content for future inflation. Hence during this sub-period the widening in the bond rate spread was followed by higher actual future inflation.

The full-sample regression estimates, however, suggest strikingly different results. The coefficient that appears on the bond rate spread variable is now about one third the size estimated in sub-sample regressions.<sup>15</sup> For forecast horizons up to 2 years in the future, this coefficient is not statistically significant, and for somewhat longer-horizons it is marginally significant at the 10 percent level (see t-values on  $\lambda_c$ ,  $\lambda_d$ , or  $\lambda_e$  Panel A, Table 5). Those estimates

---

<sup>13</sup> (...continued)

generated due to overlap in forecast horizon. The degree of the moving-average serial correlation correction was determined by examining the autocorrelation function of the residuals. This procedure generated the order of serial correlation correction close to the value given by  $(m-1)$ , where  $m$  is the number of quarters in the forecast horizon. Furthermore, the use of realized multi-period inflation in these regressions led to the loss of observations at the end of sample, so that the effective sample sizes are 1961Q1 to 1995Q3- $m$  and 1961Q1 to 1979Q3- $m$ .

<sup>14</sup>All regressions are estimated including four lagged values of other information variables. The results, however, are similar when instead eight lagged values are used.

<sup>15</sup>Mishkin (1990b) also finds that in full-sample regressions the coefficients that appear on term spreads are generally smaller in size than those in pre-1979 regressions. However, his regressions pass the conventional test of parameter stability. The regressions estimated here, however, do not depict such parameter constancy.

suggest there had been a significant deterioration in the predictive content of the bond rate spread for future inflation in the period since 1979. Furthermore, results are now sensitive to variables included in the conditioning set. If we ignore the current stance of Fed policy measured by the funds rate spread, then the bond rate spread has no predictive content for actual future inflation at any forecast horizon (see  $\lambda_d$  in Panel A, Table 5). However, when the funds rate spread variable is included in the conditioning set, then in long-horizon inflation regressions the bond rate spread variable appears with a positive coefficient. But in those same regressions the coefficient that appears on the funds rate spread is negative and statistically significant (see  $\delta$  in Panel A, Table 5). This result is consistent with the presence of significant policy-induced movements in the real component of the funds rate and their subsequent negative effects on future inflation rates. In fact, the coefficients that appear on the bond rate and the funds rate spreads are equal in size but opposite in signs ( $\lambda_e + \delta$  in (18) sum to zero). Those estimates suggest that increases in the bond rate spread that are accompanied by increases in the funds rate spread have had no effect on actual future inflation rates.<sup>16</sup>

The descriptive analysis of monetary policy in Goodfriend (1992) indicates that since 1979 the Fed has in force a disinflationary policy to reduce the trend rate of inflation and contain inflationary expectations. Hence this Fed behavior may be at the source of the disappearance of the predictive content of the bond rate for actual future inflation. To the extent that rising long-run inflationary expectations evidenced by the rise in the bond rate were triggered in part by news of strong actual or anticipated real growth, the Fed may have calmed those

---

<sup>16</sup>This result is similar in spirit to the finding reported using cointegration and error-correction methodology.

expectations by raising the funds rate. The induced tightening of monetary policy may have reduced inflationary expectations by reducing actual or anticipated real growth, thereby preventing any pickup in actual inflation. Given such Fed behavior, observed increases in the bond rate do not necessarily indicate that actual inflation is going to accelerate in the near term.

#### **4.0 CONCLUDING OBSERVATIONS**

This paper views the Fisher hypothesis as a long-run relationship, with short run variation in the real interest rate. The results here indicate that the bond rate is cointegrated with the inflation rate during two sample periods, 1961Q1 to 1979Q3 and 1961Q1 to 1995Q4. This result indicates that during these sample periods permanent movements in actual inflation have been associated with permanent movements in the bond rate.

The short-run error-correction equations provide information about the sources of the long-run co-movements of the bond rate with inflation. The empirical work here indicates that in the pre-1979 period increases in the bond rate were followed by an acceleration in actual inflation, whereas that did not happen in the post-1979 period. In the latter period short-run increases in the bond rate have usually been reversed, with no follow up in actual inflation.

In the period since 1979 the Fed has made serious attempts to reduce the trend rate of inflation and contain inflationary expectations. Such Fed behavior has prevented the short-run increases in inflationary expectations as evidenced by increases in the bond rate from finally resulting in higher actual inflation.

#### **REFERENCES**

- Blough, Stephen. "Yield Curve Forecasts of Inflation: A Cautionary Tale," Federal Reserve Bank of Boston, New England Economic Review, May/June 1994, pp. 3-15.
- Campbell, John Y., and Pierre Perron. "Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots," in Olivier Blanchard and Stanley Fisher (eds.), *NBER Macroeconomics Annual* (1991), pp. 141-201.
- Engle, Robert F., and C. W. Granger. "Cointegration and Error-Correction: Representation, Estimation and Testing," *Econometrica*, vol. 55 (March 1987), pp. 251-76.
- Engsted, Tom. "Does the Long-Term Interest Rate Predict Future Inflation? A Multi-Country Analysis," *The Review of Economics and Statistics*, (February 1995), pp. 42-54.
- Goodfriend, Marvin. "Interest Rate Policy and the Inflation Scare Problem: 1979 to 1992," Federal Reserve Bank of Richmond *Economic Quarterly*, vol. 79 (Winter 1993), pp. 1-24.
- Hall, A. "Testing for a Unit Root in Time Series with Pretest Data Based Model Selection." Manuscript. North Carolina State University, 1990.
- Hockrick, R. And E. C. Prescott. "Post-war U.S. Business Cycles: An Empirical Investigation." Mimeo. Carnegie-Mellon University, Pittsburg, PA, 1980.
- Inder, Brett. "Estimating Long-run Relationships in Economics: A Comparison of Different Approaches," *Journal of Econometrics*, 57 ((1993), pp. 53-68.
- Johansen, Soren, and Katarina Juselius. "Identification of the long-run and the Short-run Structure: An Application to the ISLM Model," *Journal of Econometrics*, vol. 63, (1994), pp. 7-36.
- \_\_\_\_\_. "Maximum Likelihood Estimation and Inference on Cointegration--With Applications to the Demand for Money," *Oxford Bulletin of Economics and Statistics*, vol. 52 (May 1990), pp. 169-210.
- Jorion, Philip, and Frederick Mishkin, "A Multi-country Comparison of Term Structure Forecasts at Long Horizons," *Journal of Financial Economics*, vol. 29 (January 1991), pp. 59-80.
- Kwiatkowski, Denis, Peter C. B. Phillips, Peter Schmidt, and Yoncheol Shin. "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root," *Journal of Econometrics*, vol. 54 (October-

December 1992), pp. 159-78.

McCallum, Bennett T. "Monetary Policy and the Term Structure of Interest Rates," Manuscript. Carnegie-Mellon University, June 1994.

Mehra, Yash P. "An Error-Correction Model of the Long-Term Bond Rate," Federal Reserve Bank of Richmond *Economic Review*, vol. 80, Number 4, (Fall 1994), pp. 49-68.

Miller, Stephen M. "Monetary Dynamics: An Application of Cointegration and Error-Correction Modeling," *Journal of Money, Credit, and Banking* (May 1991), pp. 139-154.

Mishkin, Frederick S. "What Does the Term Structure Tell Us About Future Inflation?" *Journal of Monetary Economics*, Vol. 25 (1990a), pp. 77-95.

\_\_\_\_\_. "The Information in the Longer-maturity Term Structure about Future Inflation," *Quarterly Journal of Economics*, vol. 55 (1990b), pp. 815-828.

\_\_\_\_\_. "A Multi-country Study of the Information in the Term Structure about Future Inflation," *Journal of International Money and Finance*, vol. 10 (1991), pp. 2-22.

Plosser, Charles I., and K. Geert Rouwenhorst. "International Term Structure and Real Economic Growth," *Journal of Monetary Economics*, 33, (February 1994), pp. 133-55.

Rudebusch, Glenn D. "Federal Reserve Interest Rate Targeting, Rational Expectations and the Term Structure," *Journal of Monetary Economics*, 35 Vol. 2, (April 1995), pp. 245-274.

Sims, Christopher A. "Macroeconomics and Reality," *Econometrica*, vol 48 (January 1980), pp. 1-49.

Stock, James H., and Mark W. Watson. "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica*, vol. 61 (1993), pp. 783-820.

**Table 1**

**Tests for Unit Roots and Mean Stationarity**

Series X	Panel A Test for Unit Roots					Panel B Test for Mean Stationarity
	$\rho$	$t_{\hat{\rho}}$	k	$x^2(2)$	$x^2(4)$	$\hat{\eta}_u$
BR	.96	-1.7	5	2.1	1.5	.86*
$\dot{p}$	.89	-2.3	2	1.7	1.6	.42**
NFR	.89	-2.8	5	1.1	.56	.49*

Notes: BR is the bond rate;  $p$  is the annualized quarterly inflation rate measured by the behavior of the consumer price index excluding food and energy ; and NFR is the nominal federal funds rate. The sample period studied is 1961Q1 to 1995Q4.  $\rho$  and t-statistics ( $t_{\hat{\rho}}$ ) for  $\rho = 1$  in Panel A above are from the augmented Dickey-Fuller regressions of the form

$$X_t = a_0 + \rho X_{t-1} + \sum_{s=1}^k a_s \Delta X_{t-s},$$

where  $X$  is the pertinent series. The series has a unit root if  $\rho = 1$ . The 5 percent critical value is 2.9. The lag length  $k$  is chosen using the procedure given in Hall (1990), with maximum lag set at eight quarters.  $x^2(2)$  and  $x^2(4)$  are Chi-squared statistics that test for the presence of second-order and fourth-order serial correlation in the residual of the augmented Dickey-Fuller regression, respectively. The test statistics  $\hat{\eta}_u$  in Panel B is the statistic that tests the null hypothesis that the pertinent series is mean stationary. The 5 percent critical value for  $\hat{\eta}_u$  given in Kwiatkowski et. al (1992) is .463 (10 percent critical value is .347).

- \* significant at the 5 percent level.
- \*\* significant at the 10 percent level.

**Table 2**

**Cointegration Test Results**

Panel A: 1961Q1 - 1995Q4

<u>System</u>	<u>Trace Test</u> Ho:	<u>Maximum Eigenvalue Test</u> Ho vs H1	<u>k</u>
(BR, p̂, NFR, gap)	r = 0	67.5**	8
	r ≤ 1	38.4**	
	r ≤ 2	14.3**	
	r ≤ 3	3.6	
		r = 0 vs r ≤ 1 : 29.1**	
		r = 1 vs r ≤ 2 : 24.1**	
		r = 2 vs r ≤ 3 : 10.7**	
		r = 3 vs r ≤ 4 : 3.6	

Panel B: 1961Q1 - 1995Q4

<u>System</u>	<u>Trace Test</u> Ho:	<u>Maximum Eigenvalue Test</u> Ho vs H1	<u>k</u>
(BR, p̂, NFR, gap)	r = 0	73.8**	5
	r ≤ 1	38.9**	
	r ≤ 2	16.3**	
	r ≤ 3	3.1	
		r = 0 vs r ≤ 1 : 34.8**	
		r = 1 vs r ≤ 2 : 22.6**	
		r = 2 vs r ≤ 3 : 13.2**	
		r = 3 vs r ≤ 4 : 3.12	

---

Notes: Trace tests the null hypothesis that the number of cointegrating vectors (r) is less than and equal to a chosen value, and maximum eigenvalue tests the null that the number of cointegrating vectors is r, given the alternative of r + 1 vectors. The VAR lag-length (k) was chosen using the likelihood ratio test in Sims (1980).

\*\* significant at the 10 percent level. The critical values used are from Tables given in RATS CATS manual.

**Table 3**

**Estimates of Restricted Cointegrating Vectors**

Panel A: Johansen- Juselius Procedure

	Sample period <u>1961Q1 to 1995Q4</u>	Sample Period <u>1961Q1 to 1979Q3</u>
A1	$BR_t = 3.0 + \dot{p}_t + U_{1t}$	$BR_t = 3.2 + .69 \dot{p}_t + U_{1t}$
A2	$NFR_t = 2.2 + \dot{p}_t + U_{2t}$	$NFR_t = 2.6 + .70 \dot{p}_t + U_{2t}$
	$\chi^2_1(3) = 1.55$	$\chi^2_2(1) = .20$

Panel B: Dynamic OLS

	<u>1961Q1 to 1995Q4</u>	<u>1961Q1 to 1979Q3</u>
A1	$BR_t = 2.9 + 1.0 \dot{p}_t + U_{1t}$	$BR_t = 3.2 + .66 \dot{p}_t + U_{1t}$
A2	$NFR_t = 2.2 + 1.0 \dot{p}_t + U_{2t}$	$NFR_t = 2.5 + .67 \dot{p}_t + U_{2t}$

---

Notes: Panel A above reports two of the three cointegrating vectors that lie in the cointegration space spanned by the 4-variable VAR (BR,  $\dot{p}$ , NFR, gap). The cointegrating vectors A1 and A2 are the Fisher relation for the bond rate and the funds rate.  $\chi^2_1(3)$  and  $\chi^2_2(1)$  are Chi-squared statistics (degrees of freedom in parentheses) that test the null that the identifying restrictions imposed are consistent with data (Johansen and Juselius 1994).

Panel B above reports the same cointegrating vectors estimated using the dynamic OLS procedure (the number of leads and lags used is 8).

**Table 4**

**Granger-Causality Results From Error-Correction Equations: General-to-Specific Using Hall Approach**

Panel A: 1961Q1 - 1995Q4

Cointegrating Regressions (Dynamic OLS)

$$BR_t = 2.9 + \dot{p}_t + U_{1t}; NFR_t = 2.2 + \dot{p}_t + U_{2t};$$

<u>Equation</u>	<u>U<sub>1t-1</sub></u>	<u><math>\sum_{s=1}^{kl} \Delta BR_{t-s}</math></u>	<u>(k<sub>1</sub>, k<sub>2</sub>, k<sub>3</sub>, k<sub>4</sub>)</u>	<u>x<sup>2</sup>(sl)</u>
$\Delta BR_t$	-0.06 (1.9)		(5,0,0,0)	4.1 (.25)
$\Delta BR_t$	-0.20 (3.6)		(7,7,8,1)	9.4 (.39)
$\Delta \dot{p}_t$	.08 (.9)		(0,2,0,0)	6.5 (.37)
$\Delta \dot{p}_t$	-0.16 (1.6)	.17(.6) [10.6]*	(2,8,8,8)	5.8 (.44)

Panel B: 1961Q1 - 1979Q3

Cointegrating Regressions (Dynamic OLS)

$$BR_t = 1.7 + \dot{p}_t + U_{1t}; NFR_t = 1.0 + \dot{p}_t + U_{2t};$$

<u>Equation</u>	<u>U<sub>1t-1</sub></u>	<u><math>\sum_{s=1}^{kl} \Delta BR_{t-s}</math></u>	<u>(k<sub>1</sub>, k<sub>2</sub>, k<sub>3</sub>, k<sub>4</sub>)</u>	<u>x<sup>2</sup>(sl)</u>
$\Delta BR_t$	-0.01 (.2)		(0,0,0,0)	9.8 (.27)
$\Delta BR_t$	-0.24 (3.5)		(8,7,6,1)	6.5 (.77)
$\Delta \dot{p}_t$	.32 (3.2)		(0,0,0,0)	6.6 (.58)
$\Delta \dot{p}_t$	.32 (3.2)		(0,0,0,0)	37.1 (.28)

Notes: The coefficients reported are from error-correction regressions that include the bond rate, the inflation rate, the nominal federal funds rate, and the output gap (see equation 15 of the text). In addition the model has two error-correction variables (U<sub>1t</sub> and U<sub>2t</sub>). (k<sub>1</sub>, k<sub>2</sub>, k<sub>3</sub>, k<sub>4</sub>) refers to lag lengths that are chosen for the bond rate (BR), the inflation rate (ṗ), the funds rate (NFR), and the output gap (gap). Parentheses contain t-statistics for the error-correction variable (U<sub>1t-1</sub>) or for the sum of coefficients that appear on the bond rate  $\left( \sum_{s=1}^{kl} \Delta BR_{t-s} \right)$ . For the latter, brackets contain the Chi-squared statistic for the null that every coefficient in this sum is zero. x<sup>2</sup>(sl) tests the null that remaining lags are not significant (significance levels are in parentheses that follow).

**Table 5**

**Long-horizon Inflation Equations**

Panel A: 1961Q2 - 1995Q4

Cointegrating Regressions:  $BR_t = 2.8 + \dot{p}_t + U_{1t}$  ;

$NFR_t = 2.2 + \dot{p}_t + U_{2t}$

Horizons in Quarters (m)	<u>Equation C</u>	<u>Equation D</u>	<u>Equation E</u>	
	$\lambda_c$ (t-value)	$\lambda_d$ (t-value)	$\lambda_e$ (t-value)	$\delta$ (t-value)
4	.16 (1.6)	.09 (.9)	.04 (.4)	-.06 (.8) <sup>a</sup>
8	.20 (1.6)	.05 (.5)	.18 (1.0)	-.15 (1.0) <sup>a</sup>
12	.23 (1.7)	-.01 (.1)	.26 (1.7)	-.32 (1.7) <sup>a</sup>
16	.25 (2.9)	-.07 (.6)	.32 (1.8)	-.45 (2.4) <sup>a</sup>

Panel B: 1961Q2 - 1979Q3

Cointegrating Regressions:  $BR_t = 1.7 + \dot{p}_t + U_{1t}$  ;

$NFR_t = 1.0 + \dot{p}_t + U_{2t}$

Horizons in Quarters (m)	<u>Equation C</u>	<u>Equation D</u>	<u>Equation E</u>	
	$\lambda_c$ (t-value)	$\lambda_d$ (t-value)	$\lambda_e$ (t-value)	$\delta$ (t-value)
4	.58 (9.6)	.60 (5.8)	.65 (.4)	-.04 (.8) <sup>b</sup>
8	.85 (8.2)	.81 (5.4)	1.10 (4.3)	-.32 (1.1) <sup>b</sup>
12	1.00 (7.8)	.91 (14.1)	1.00 (3.9)	-.15 (.6) <sup>b</sup>
16	1.00 (10.8)	1.0 (15.1)	.91 (3.6)	.17 (.6) <sup>b</sup>

Notes: The coefficients reported are from regressions of the form

$$p(t,m) = f_o + \lambda_c U_{1t} \quad (C)$$

$$p(t,m) = g_o + \lambda_d U_{1t} + \sum_{s=1}^{k1} g_{1s} \Delta BR_{t-s} + \sum_{s=1}^{k2} g_{2s} \Delta \dot{p}_{t-s} + \sum_{s=1}^{k3} g_{3s} \Delta NFR_{t-s} + \sum_{s=1}^{k4} g_{4s} gap_{t-s} \quad (D)$$

$$p(t,m) = \lambda_e U_{1t} + \delta U_{2t} + \text{other variables as in (D)} \quad (E)$$

where  $p(t, m)$  is  $(\log(P_{t+m}/P_t))/m - \log(P_t/P_{t-1})$ ,  $m$  is the number of quarters in the forecast horizon and the rest of variables are as defined before. All regressions are estimated setting  $k_1 = k_2 = k_3 = k_4 = 4$ .

a The restriction  $\lambda_e + \delta = 0$  is consistent with data

b The restriction  $\lambda_e + \delta = 0$  is not consistent with data