Working Paper Series

Money, Banking, and Capital Formation

WP 94-05

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Money, Banking, and Capital Formation*

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May 1994

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Abstract

We consider a monetary growth model in which banks arise to provide liquidity. In addition, there is a government that issues not only money, but interest-bearing bonds: these bonds compete with capital in private portfolios. When the government fixes a constant growth rate for the money stock, we show that there can exist multiple nontrivial monetary steady states. One of these steady states is a saddle, while the other can be a sink. Paths approaching these steady states can display damped endogenous fluctuations, and development trap phenomena are common. Across different steady states, low capital stocks are associated with high nominal interest rates; the latter signal the comparative inefficiency of the financial system. Also, increase in the steady state inflation rate can easily reduce the steady state capital stock.

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Financial markets perform numerous allocative functions that affect economic development by influencing capital accumulation, rates of return on savings, and the level of real activity. Many of these functions have been formally analyzed in the context of general equilibrium models of growth and capital accumulation. [1] Yet, until now, the class of models that integrates explicit financial market structures into neoclassical growth models has been purely real in nature. In particular, these models have abstracted both from the presence of money or other government liabilities and from the determination of price levels and nominal rates of interest. These are substantive omissions.

Such omissions, for example, preclude use of these models to analyze how monetary policy affects the behavior of financial markets and, through such markets, the development process. Thus these models are silent on whether various methods of conducting monetary policy can significantly aid or interfere with the financial system's efficient operation. Moreover, in many economies, the banking system is a large-scale investor in capital, while in others — particularly developing economies — it largely holds government debt. Why do some financial systems (directly or indirectly) hold large quantities of capital in their portfolios while others hold mostly government debt? An answer to this question clearly requires the analysis of an economy in which government liabilities are present.

Yet another issue that merits renewed consideration is the effect of inflation on capital accumulation and growth. Economists widely believe, and the existing empirical evidence strongly suggests, that sustained high rates of inflation interfere with the allocative functions of financial

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[1] For early discussions of the importance of financial markets in economic development see Patrick (1966), Cameron (1967), Gurley and Shaw (1967), Goldsmith (1969), McKinnon (1973), and Shaw (1973). Early theoretical analyses of this topic include Greenwood and Jovanovic (1990) and Bencivenga and Smith (1991); see Greenwood and Smith (1993) for a survey of more recent developments.

There is also a formal empirical literature demonstrating the importance of financial market factors for growth and capital accumulation. Prominent examples include Antje and Jovanovic (1993) and King and Levine (1993 a,b).

markets. [2] Existing models, however, do not permit an analysis of whether and, if so, why this is the case.

Finally, numerous authors [including Keynes (1936)] have argued that financial market activity is a source of multiple equilibria and endogenous fluctuations that cannot be observed in economies with only primitive financial systems. Again, an investigation of this possibility should incorporate monetary considerations since monetary policy might significantly affect the potential for both indeterminacies and endogenous fluctuations. Moreover, suppose that the coexistence of money and financial markets can result in the existence of multiple monetary steady state equilibria. Such a finding would have important implications for economic development because it would raise the possibility that an economy might be "stuck" with low levels of real activity even though higher levels of real activity are feasible — and consistent with — a competitive equilibrium outcome. Even more significantly, the scope for these kinds of development trap phenomena might be influenced by the choice of monetary policy. This possibility further magnifies the importance of monetary policy activity.

To date the existing models that incorporate money into neoclassical growth contexts have little to say on these topics. [3] In particular, such models typically predict that monetary steady states are unique and have unique dynamical equilibrium paths approaching them. Along these paths, capital stocks, output levels, and other endogenous variables monotonically approach their steady state levels. Thus monetary policy cannot influence the number of steady state equilibria, and money market activity cannot endogenously induce economic fluctuations or lead to development trap phenomena. In addition, monetary growth models typically predict that higher steady state rates of money growth — which lead, of course, to higher steady state rates of

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[2] See, for instance, Levine and Renelt (1992) or Wynne (1993).

[3] Examples of the monetary growth models we have in mind include Diamond (1965), Sidrauski (1967a,b), Shell, Sidrauski, and Stiglitz (1969), Brock (1974, 1975), and Tirole (1985). For attempts to formalize the notion of instability in growth models with financial markets see Shell and Stiglitz (1969) or Azariadis and Smith (1993).

inflation— are not detrimental to, and may actually enhance, capital formation and real activity. [4] Such an implication is wildly at variance with observation.

This paper considers a monetary growth model in the spirit of Diamond (1965) and introduces into it a role for banks that provide liquidity in the sense of Diamond and Dybvig (1983). [5] In addition, there is a government that issues interest-bearing, but illiquid, bonds and noninterest-bearing currency. Policy is conducted by having the government make a once-and-for-all choice of the rate of money creation. The results we then obtain are as follows. First, if the rate of money growth is sufficiently large, there will be exactly two nontrivial steady state equilibria with valued fiat money and positive nominal interest rates. One of these equilibria has a high nominal interest rate, which here as elsewhere is a signal of financial market inefficiency. This steady state equilibrium has a low per capita capital stock and banks with large holdings of government bonds relative to their holdings of capital. This resembles the observed situation in economies with poorly developed banking systems. The other steady state equilibrium has a relatively low nominal interest rate—signaling the comparatively efficient operation of its banking system—and a relatively high capital stock. Moreover, the real value of government bonds held by the banking system is low in comparison to its capital investments. Indeed, the government's net position in bond markets, which is endogenous in the policy regime under consideration, makes it a net lender to the banking system. When this occurs, monetary expansions fund lending to the financial system, which is conducive to capital formation. On the other hand, when the value of outstanding government bonds is positive, which occurs in the high nominal interest rate steady state, government bonds "crowd out" capital in investors' portfolios. When this occurs monetary expansions have adverse consequences for capital formation. Thus, in high interest rate steady states, an increase in the money growth rate interferes with the financial system's efficiency. In

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[4] Examples of monetary growth models yielding this implication are those of Diamond (1965) and Tirole (1985), Mundell (1965), Sidrauski (1967a,b), Shell, Sidrauski, and Stiglitz (1969), and Brock (1974, 1975). Models yielding different implications include Stockman (1981), Azariadis and Smith (1993), and Boyd and Smith (1994).

[5] The formal analysis of banks that provide liquidity in a model explicitly incorporating money draws heavily on Champ, Smith, and Williamson (1992).

this situation, higher rates of money growth lead to lower steady state capital stocks and reduced economic development.[6]

In addition, we investigate in some detail the properties of dynamical equilibria in the economy at hand. Under conditions that we describe, which amount to requiring that the rate of money creation not be too large, the steady state equilibrium with a low nominal rate of interest (and a high capital stock) is a sink. If production is Cobb-Douglas, the steady state equilibrium with a high nominal interest rate (and a low capital stock) is necessarily a saddle. Since our economy has only one initial condition (the initial capital stock), there are therefore often many dynamical equilibrium paths. Thus not only are financial markets a source of indeterminacy of dynamical equilibria, but with multiple steady states, development trap phenomena are ubiquitous. In particular, economies with very similar—or even identical—initial capital stocks may, depending on their (endogenous) initial nominal interest rates, follow the saddle path to the low capital stock steady state or one of the many paths approaching the high capital stock steady state. Economies that have high initial nominal interest rates will suffer the former fate, while economies that have low initial nominal interest rates will avoid it.

Finally, dynamical equilibria approaching the steady state with a high nominal interest rate display monotonic approaches to this steady state. For a wide variety of parameter values, this is not the case for the steady state equilibrium with a low nominal rate of interest. Indeed, we examine a variety of numerical examples, all of which have the feature that, if this steady state is a sink, paths approaching it display damped endogenous fluctuations as they do so. Thus not only are monetary and financial market factors a source of multiple equilibria here, but they are also a source of enhanced economic volatility that cannot be observed in versions of our model that, like Diamond (1965) and Tirole (1985), lack a role for banks.

Finally, we note that while our model is, like Diamond's and Tirole's, an overlapping generations model, it also has a kind of spatial separation that generates a transactions role for

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6 This result is in the spirit of Azariadis and Smith (1993), which shows how an increase in the rate of money creation can exacerbate incentive problems in credit markets, thereby reducing the efficiency of the financial system and interfering with capital accumulation.

money and liquidity-providing banks.[7] Thus none of our results depend on money lacking a transactions function.

The remainder of the paper proceeds as follows. Section I describes the economic environment we consider, while section II discusses the nature of trading and the role for banks in the model. In Section III we examine the existence and number of steady state equilibria and analyze the comparative static consequences of monetary policy activities. Section IV takes up dynamical issues and discusses development traps and endogenous fluctuations, while Section V concludes.

I. Environment

We consider an economy consisting of an initial old generation and an infinite sequence of two-period-lived overlapping generations. At each date $t = 0, 1, \ldots$, young agents are assigned to one of two locations, with locations indexed by $j = 1, 2$. Each location contains a continuum of young agents with unit mass, and our assumptions will imply that locations are always symmetric.

All young agents are identical ex ante. They are endowed with one unit of labor when young, which they supply inelastically, and are retired when old. Young agents have no other endowments of goods or assets at any date.

At each date in each location there is a single final good that is produced using capital and labor as inputs. Agents care only about old-age consumption, which we denote simply by c, and their lifetime utility is given by $\ln(c)$.[8]

Production in each location occurs according to the commonly available, constant returns to scale technology $F(K,L)$, where K and L denote capital and labor inputs respectively. Let $f(k) \equiv F(k,1)$ denote the intensive production function, where $k \equiv K/L$ is the capital-labor ratio. We assume that $f'(k) > 0 > f''(k) \forall k$, that $f(0) = 0$ holds, and that f satisfies the usual Inada conditions.

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[7] In these respects our model resembles Townsend (1987) and Champ, Smith, and Williamson (1992).

[8] Schreft and Smith (1994) consider more general preferences.

There are three assets in this economy: money, government bonds, and capital. The per capita supplies of money and bonds in each location at date t are given by M_t and B_t, which are nominally denominated. We let p_t denote the time t price level, and $m_t \equiv M_t / p_t$ and $b_t \equiv B_t / p_t$ denote the per capita supplies of money and bonds at time t, respectively, in real terms. Obviously k_t is the time t per capita capital stock. All bonds are of one-period maturity, and one dollar held in bonds at t constitutes a sure claim to I_t dollars at $t + 1$. Thus I_t is the gross nominal rate of interest.

Capital is produced as follows. One unit of current consumption foregone at t becomes one unit of capital at $t + 1$ with probability $q \in (0,1]$. With probability $1 - q$ no capital is received, and the investment becomes worthless. The capital obtained can be used in production, and for simplicity we assume that it depreciates completely in the production process.

The nature of interlocation interaction is as follows. At the beginning of each period, agents cannot move between or communicate across locations. Goods can never be transported between locations. Thus goods, labor, and asset market transactions occur autarkically within each location at the beginning of each period. After this trade is concluded at t, some randomly selected fraction $\pi \in (0,1)$ of young agents is chosen to be moved to the other location. We assume that currency is the only asset that can be transported between locations and that limited communication prevents the cross-location exchange of privately issued liabilities.[9] Therefore relocated agents will seek to liquidate their holdings of bonds and capital to obtain currency. Relocation thus plays the role of a "liquidity preference shock" in the Diamond-Dybvig (1983) model, and it is natural to assume that banks arise to insure agents against these shocks. Our timing assumptions are depicted in figure 1. We assume that π is constant across periods and known by all agents. In addition, the probability of being relocated is iid across young agents. Thus there is no aggregate uncertainty here.

[9] These assumptions follow Townsend (1987), Mitsui and Watanabe (1989), Hornstein and Krusell (1993), and most specifically, Champ, Smith, and Williamson (1992). The notion that bonds cannot be used in interlocation exchange could be motivated by the assumption that they are issued in large denominations.

The initial old agents at each location are endowed with the initial per capita capital stock, $k_0 > 0$, and the initial per capita money supply, $M_{-1} > 0$. $B_{-1} = 0$ and $M_0 > 0$ also are given as initial conditions; thereafter the money supply evolves according to

(1) $$M_{t+1} = \sigma M_t \ ; \ t \geq 0,$$

with $\sigma > 0$ exogenously given. The sequence $\{B_t\}_{t=0}^{\infty}$ is endogenous.

II. Trade

A. Factor Markets

Factor market trade at each date occurs autarkically within each location. We assume that factor markets are perfectly competitive; therefore factors of production are paid their marginal product in each period. Let r_t denote the time t rental rate for capital, and let w_t denote the time t real wage rate. Then

(2) $$r_t = f'(k_t) \ ; \ t \geq 0,$$

(3) $$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) \ ; \ t \geq 0.$$

Note that $w'(k) > 0$, $\forall k$.

An additional assumption on technology will prove convenient. Specifically, define

$$\Omega(k) \equiv k/qw(k).$$

We then assume that, $\forall k \geq 0$,

(a.1) $\Omega'(k) > 0.$

It is straightforward to verify that the assumption (a.1) is equivalent to

(a.1') $kw'(k)/w(k)<1; k\geq 0$.

Assumption (a.1) [or (a.1')] holds, for instance, if f is any CES production function with elasticity of substitution no less than one.

B. Banks and Asset Markets

As noted previously, the possibility of stochastic relocation for us plays the same role as a "liquidity preference shock" in the Diamond-Dybvig (1983) model. Thinking of banks as arising to insure against the random liquidity needs associated with relocation is therefore natural. As in Diamond-Dybvig, if banks exist all savings will be intermediated. Thus each young agent simply deposits his entire savings, here w_t at t, in a bank.

Banks use deposits to acquire primary assets: money, bonds, and capital. In addition, banks promise to pay agents who are (are not) relocated, that is, "movers" ("nonmovers"), a gross real return of d_t^m (d_t^n) per unit on their deposits. We assume that there is free entry into banking and that banks are competitive in the sense that they take the real return on assets as given. On the deposit side we assume that intermediaries are Nash competitors; that is, banks announce deposit return schedules (d_t^m, d_t^n), taking the announced return schedules of other banks as given. Of course, announced return schedules must satisfy a set of balance sheet constraints, which we now describe.

As already noted, at time t a young depositor will deposit his entire savings, w_t, with a bank. The bank acquires an amount m_t of real balances and an amount b_t of real bond holdings, and it makes an investment in capital of i_t per depositor. Thus the bank's balance sheet requires that

(4) $$m_t+b_t+i_t\leq w_t;\ \ t\geq 0.$$

Announced deposit returns must satisfy the following constraints. First, relocated agents (of whom there are π per depositor) must be given currency, since this is the only asset that can be transported between locations. This is accomplished using the bank's holdings of real balances. In addition, each unit of real balances paid to a "mover" at t returns p_t / p_{t+1} between t and $t + 1$ (that is, while the mover holds it). Thus

$$(5) \qquad \pi d_t^m w_t \leq m_t(p_t / p_{t+1}); \;\; t \geq 0$$

must hold.

If $I_t > 1$, then the bank will never choose to carry real balances between t and $t + 1$.[10] "Nonmovers" (of whom there are $1-\pi$ per depositor) therefore will be repaid from the return on the bank's bond holdings and capital investments. Bonds earn the gross real return $R_t \equiv I_t p_t / p_{t+1}$ between t and $t + 1$. In addition, we assume that the bank can diversify its capital investments, thereby guaranteeing itself q units of capital at $t + 1$ per unit invested at t. Since capital rents for r_{t+1} at $t + 1$ (and then depreciates completely), capital investments return qr_{t+1}. Thus d_t^n must satisfy (if $I_t > 1$)

$$(6) \qquad (1-\pi)d_t^n w_t \leq R_t b_t + qr_{t+1} i_t; \;\; t \geq 0.$$

We focus throughout on equilibria satisfying $I_t > 1 \; \forall t$.

It will prove convenient to transform these constraints as follows. Let $\gamma_t \equiv m_t / w_t$ be the currency-deposit ratio of the bank at time t, and let $\beta_t \equiv b_t / w_t$ be the ratio of bonds to deposits. This implies that $1 - \gamma_t - \beta_t$ is the ratio of capital investments to deposits. Constraints (5) and (6) can now be rewritten as

[10] Rather than do so, the bank could replace its real balances with bonds, which dominate money in rate of return.

(5') $\quad d_t^m \leq \gamma_t(p_t / p_{t+1})/\pi;\ t \geq 0,$

(6') $\quad d_t^n \leq [R_t\beta_t + qr_{t+1}(1 - \gamma_t - \beta_t)] / (1\text{-}\pi)\ ;\ t \geq 0.$

In addition, currency holdings and capital holdings are constrained to be nonnegative; thus $\gamma_t \geq 0$ and $\gamma_t + \beta_t \leq 1$ must hold, $\forall\, t \geq 0$. We do not similarly constrain bond holdings; we thus in effect allow for the possibility that banks might borrow from the government (that is, that the government might operate a discount window or other loan program).

Competition among banks for depositors will, in equilibrium, force banks to choose return schedules and portfolio allocations to maximize the expected utility of a representative depositor, subject to the constraints we have described. Thus banks choose — in equilibrium — values for d_t^m, d_t^n, γ_t, and β_t to solve the problem

$$max\ \pi\, ln(d_t^m w_t) + (1-\pi)\, ln(d_t^n w_t)$$

subject to (5'), (6'), $\gamma_t \geq 0$, and $\gamma_t + \beta_t \leq 1$. The solution to this problem sets

(7) $\quad \gamma_t = \pi\ ;\ \ t \geq 0.$

In addition, an absence of arbitrage opportunities in an economy with positive capital investment requires that

(8) $\quad R_t = qr_{t+1};\ t \geq 0.$

In equilibrium, of course, all primary asset holdings are intermediated. Thus the money market clears if the demand by banks for cash reserves equals the supply of money at each date. This, in turn, requires that

(9) $m_t = \pi w_t = \pi w(k_t);\ t \geq 0.$

Similarly, the time $t+1$ per capita capital stock, k_{t+1}, must equal qi_t. This requires that

(10) $k_{t+1} = q[w(k_t) - m_t - b_t]\ ;\ t \geq 0.$

Finally, from (2) and (8), an equilibrium with a nonzero capital stock at all dates must have

(11) $R_t = I_t p_t / p_{t+1} = qf'(k_{t+1});\ t \geq 0.$

C. The Government

We assume that the government has no direct (that is, noninterest) expenditures and that it levies no direct taxes at any date. Thus, satisfaction of the government budget constraint requires that

(12) $R_{t-1} b_{t-1} = (M_t - M_{t-1}) / p_t + b_t\ ;\ t \geq 0.$

Equations (9) - (12) constitute the complete set of equilibrium conditions for this economy.

III. Steady State Equilibria

A. Existence

We begin by analyzing the steady state equilibria of this economy with $I > 1$. First, note that, by definition, $p_t / p_{t+1} \equiv (m_{t+1} / m_t)(M_t / M_{t+1}) \equiv m_{t+1} / \sigma m_t$ for all $t \geq 0$. Hence, in a steady state equilibrium, $p_t / p_{t+1} = 1/\sigma$. Using this fact in (11) gives a relationship between the steady state nominal interest rate and the steady state capital stock:

(13) $I = \sigma qf'(k).$

Evidently, equation (13) defines a downward-sloping locus in figure 2. Since our interest is in equilibria with $I > 1$, we focus only on capital stocks satisfying $\sigma q f'(k) > 1$.

For $t \geq 1$, equation (12) can be rewritten as

$$(14) \quad qf'(k_t)b_{t-1} = b_t + [(\sigma - 1)/\sigma]m_t.$$

Thus, in a steady state equilibrium,

$$b = (\sigma - 1)m/\sigma[qf'(k) - 1] = (\sigma - 1)m/(I - \sigma).$$

It follows that $m + b = [(I-1)/(I-\sigma)]m$ in a steady state. Using this fact, along with the steady state relation $m = \pi w(k)$ [see equation (9)] in (10), gives a second steady state equilibrium condition:

$$(15) \quad k/qw(k) \equiv \Omega(k) = 1 - \pi[(I-1)/(I-\sigma)].$$

To characterize the locus described by (15) in figure 2, we define the following two values of the capital-labor ratio. Let k' satisfy $\Omega(k') = 1 - \pi$, and let $\hat{k}$ be given by $\Omega(\hat{k}) = 1$. Assumption (a.1) implies that, if k' and $\hat{k}$ exist, they are unique. In addition, values k' and $\hat{k}$ exist, under our assumptions, iff

$$(a.2) \quad qw'(0) > 1/(1-\pi),$$

an assumption we henceforth maintain.

Evidently, the locus defined by (15) passes through the point ($\hat{k}$, 1). In addition, along (15),

$$\lim_{I \to \infty} \Omega(k) = 1 - \pi .$$

Hence, as $k \to k', I \to \infty$. Finally, straightforward differentiation establishes that the slope of the locus defined by (15) is given by

$$dI / dk|_{15} = (I - \sigma)^2 \Omega'(k) / \pi(\sigma - 1).$$

Hence (15) is positively (negatively) sloped if $\sigma > (<) 1$.

Regarding the configuration of (15) in figure 2 there are two possibilities.

Case 1. $\sigma \leq 1$. In this case $I > 1$ implies $I > \sigma$, and hence $k > k'$ must hold for all finite values of I. Moreover, (15) is negatively sloped and therefore has the configuration depicted in figure 2a. If $\sigma q f'(\hat{k}) > 1$, then there exists a nontrivial steady state equilibrium with $I > 1$.[11] This equilibrium may or may not be unique.

Case 2. $\sigma > 1$. In this case the locus (or loci) defined by (15) are positively sloped. Moreover, the vector ($\hat{k}$, 1) satisfies (15), and for all $I \in (1, \sigma)$, (15) yields a unique value of $k > \hat{k}$. As $I \uparrow \sigma$, it is easy to show that $\Omega(k)$, and hence k, must go to infinity to satisfy (15).

Among the values of I that are greater than or equal to σ, only those with $I \geq (\sigma - \pi)/(1 - \pi) > \sigma$ imply a nonnegative value for the right-hand side of (15). If $w'(0) = \infty$ (as would be the case with Cobb-Douglas production), then the vector $(0, (\sigma - \pi) / (1 - \pi))$ satisfies (15). Equation (15) then yields a positive value of k for all $I > (\sigma - \pi) / (1 - \pi)$, and as $k \uparrow k'$, $I \to \infty$. Thus (15) has the configuration depicted in figure 2b.

If $\sigma q f'(\hat{k}) > 1$, there are then exactly two nontrivial steady state equilibria with

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[11] While $\sigma q f'(\hat{k}) > 1$ is sufficient for the existence of such an equilibrium with $\sigma \leq 1$, it is not necessary. However, if σ is too small, there may be no steady state equilibrium with a positive nominal interest rate.

$I > 1$. This condition will hold if σ is sufficiently large.

1. A Class of Examples

Suppose that $f(k) = Ak^{\alpha}$, with $\alpha \in (0,1)$, so that production is Cobb-Douglas. Then

$$\Omega(k) = k^{1-\alpha} / qA(1-\alpha).$$

Substituting this expression into (15), solving for I, and inserting the result into (13) yields the following steady state equilibrium condition, which is a function of I alone:

$$(16) \quad (1-\pi)I^2 - \{[\sigma - \pi(1-\alpha)]/(1-\alpha)\}I + \alpha\sigma^2/(1-\alpha) = 0.$$

Equation (16) has at least one positive solution iff

$$\sigma \geq \pi(1-\alpha)/\{1-2[\alpha(1-\alpha)(1-\pi)]^{0.5}\},$$

that is, iff σ is sufficiently large. If $\sigma > 1$, then $\sigma q f'(\hat{k}) > 1$ holds iff $\sigma \geq (1-\alpha)/\alpha$ in which case there are at least two nontrivial steady state equilibria with $I > 1$.

Example 1. Let $\sigma = 4$, $\pi = 0.5$, and $\alpha = 0.25$. Then the values $I = 8.396$ and $I = 1.27$ satisfy (16).

Example 2. Let $\sigma = 0.9$, $\alpha = 0.39$, and $\pi = 0.9$. Then the values $I = 4.6374$ and $I = 1.1167$ satisfy (16). This example illustrates that there can be multiple nontrivial steady state equilibria with $I > 1$ when $\sigma \leq 1$.

2. Discussion

Multiple nontrivial steady state equilibria in which the value of money is positive are uncommon in models of money and capital accumulation.[12] The possibility of multiple equilibria arises here because the government's net lending position is endogenous. This is most clearly seen in the case of $\sigma > 1$. If σ is sufficiently large there are two nontrivial steady state equilibria: one with $I < \sigma$ and one with $I > \sigma$. The former has $b < 0$, so that the government becomes a net lender in bond markets. Thus the creation of money constitutes a subsidy to capital formation as the government effectively lends the money created to banks; this lending aids banks in funding their capital investments. The equilibrium with $I > \sigma$ has $b > 0$; here government debt competes with private capital in banks' portfolios, crowding out investment and reducing the steady state capital stock.

This argument might seem to suggest that the existence of multiple (steady state) equilibria could be avoided by having the government pursue a different kind of policy. It could, for example, fix the ratio of money to bonds via open market operations. When agents have logarithmic utility this would indeed prevent the existence of multiple steady state equilibria. However, when agents have more general utility functions the possibility of multiple steady state equilibria still arises. This situation is considered by Schreft and Smith (1994).

The preceding discussion suggests that the comparative static effects of an increase in σ depend on which steady state equilibrium obtains. This is indeed the case, as we now demonstrate formally.

B. Comparative Statics

Straightforward inspection indicates that an increase in the rate of money growth (from σ to σ') shifts the locus defined by (13) up and to the right and the locus defined by (15) to the left in figures 3 and 4. We now consider three cases.

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[12] Consider, for example, the monetary growth models of Diamond (1965), Sidrauski (1967a, b), Brock (1974, 1975), or Stockman (1981).

Case 1. $\sigma > 1$. Here an increase in σ has differential effects, depending on which steady state equilibrium obtains. If there are two steady state equilibria at the initial value of σ, there will continue to be two such equilibria when σ is increased. One will have $I > \sigma$; in this situation an increase in σ shifts the steady state equilibrium from point A to point B in figure 3. The other will have $I \in (1,\sigma)$; here an increase in σ causes the steady state equilibrium position of the economy to shift from point C to point D.

In each case, the nominal rate of interest will rise along with an increase in σ. The same increase may appear to have ambiguous consequences for the steady state capital stock; but as we show below, this is not so. Indeed, for equilibria with $I > \sigma$, an increase in the rate of money growth reduces the steady state capital stock, while when $I < \sigma$, an increase in σ results in an increase in the steady state capital stock.

Case 2. Suppose that $\sigma < 1$ (and $\sigma' < 1$) hold and that there is a unique steady state equilibrium. This situation is depicted in figure 4a. The effect of an increase in σ is to shift the economy's steady state equilibrium from point A to point B. A higher rate of money growth leads to a higher nominal rate of interest and a lower steady state capital stock. Since the capital stock falls, the real rate of interest $[qf'(k)]$ rises, so that the nominal rate of interest increases by more than the increase in the rate of inflation.

Case 3. In this case $\sigma < 1$ (and $\sigma' < 1$) hold, but there are multiple steady state equilibria. Here the effect of an increase in σ depends even more heavily than in case 1 on which equilibrium obtains, as indicated by figure 4b. Comparing points A and C, an increase in the rate of money creation leads to an increase in the steady state capital stock and a *reduction* in the nominal rate of interest. This conclusion is reversed when comparing points B and D, reproducing the outcome in figure 4a.

It remains to demonstrate that an increase in σ has unambiguous effects on each of the steady state equilibrium capital stocks in case 1. We now state

Proposition 1. Suppose that $\sigma > 1$ and $\sigma q f'(\hat{k}) > 1$ hold. Then there are two steady state equilibria: one with $I \in (1,\sigma)$, and one with $I > (\sigma-\pi)/(1-\pi)$. The former has $dk/d\sigma > 0$, while the latter has $dk/d\sigma < 0$.

Proof. The existence of the two steady states follows from our earlier discussion. To obtain an expression for $dk/d\sigma$ differentiate (13) and (15) with respect to σ. Some algebra then yields

$$(17) \qquad (dk/d\sigma)\{\sigma q f''(k) - [(I-\sigma)^2 \Omega'(k)/\pi(\sigma-1)]\} = (I-\sigma)/\sigma(\sigma-1).$$

Assumption (a.1) and $\sigma > 1$ imply that $dk/d\sigma$ is opposite in sign to $I-\sigma$, establishing the result.

1. Discussion

A wide variety of money and growth models have the implication that an increase in the rate of money creation either increases the steady state capital stock or leaves it unchanged.[13] Such an implication is wildly at variance with observation.[14] Expanding the class of models in which higher money creation can be detrimental to capital accumulation and real activity is therefore of value.

Adverse consequences of higher money creation for capital accumulation can occur here in all three cases. Indeed, they must occur in case 2, and they occur in one of the steady state equilibria in cases 1 and 3. Why is it that higher rates of money creation can lead to a reduction in the per capita capital stock here? The answer is that, in our formulation, an increase in the money growth rate can increase the ratio of outstanding government liabilities to total saving [that is, raise

[13] Examples of models where increased money creation raises the steady state equilibrium capital stock include Diamond (1965) [see Azariadis (1992) for a discussion], Mundell (1965), Tobin (1965), Sidrauski (1967a), and Shell, Sidrauski and Stiglitz (1969). Examples of models where money growth rates have no effect on the steady state capital stock include those in the tradition of Sidrauski (1967b). There are some models where increased money creation leads to reduced capital formation; prominent examples include Stockman (1981), Azariadis and Smith (1993), and Boyd and Smith (1994).

[14] See Azariadis and Smith (1993) for a discussion.

(m + b) / w(k)]. This crowds out private capital formation. On the other hand, an increase in the money growth rate *increases* the steady state capital stock when b is negative. In this event an increase in σ raises the government's seigniorage revenue, which in turn permits the government to expand its lending to banks. This allows an increase in the capital stock to occur. Hence, an increase in the rate of money creation is conducive to capital formation for us only when the government is effectively using its seigniorage revenue to finance capital investments (here by the private sector).

IV. Dynamical Equilibria

When multiple steady state equilibria exist, understanding the stability properties of the various steady states is essential. In this section we investigate dynamical equilibria. To simplify the discussion, we consider only the case $\sigma > 1$. We begin by deriving the phase diagram describing the behavior of dynamical equilibria; after doing so we consider local dynamics in a neighborhood of the steady state equilibria. We assume throughout that $\sigma q f'(\hat{k}) > 1$ holds, so that there are exactly two steady state equilibria of this economy.

A. Equilibrium Laws of Motion

Equation (9) can be used to eliminate m_t from equations (10) and (14); doing so yields

$$(18) \quad k_{t+1} = q(1-\pi)w(k_t) - qb_t \ ; \ t \geq 0$$

$$(19) \quad b_{t+1} = -[(\sigma-1)/\sigma]\pi w(k_{t+1}) + qf'(k_{t+1})b_t \ ; \ t \geq 0.$$

Obviously $k_0 > 0$ is given, and from the government budget constraint [and (9)], b_0 must satisfy

$$(20) \quad b_0 = M_{-1} / p_0 - \pi w(k_0).$$

Hence b_0 is endogenous.

B. A Phase Diagram

Equation (18) implies that $k_{t+1} \geq k_t$ holds iff

$$(21) \quad q(1-\pi)w(k_t) - k_t \geq qb_t .$$

The locus defined by (21) at equality is depicted in figure 5; points below this locus have $k_{t+1} > k_t$, while points above it have $k_t > k_{t+1}$.

Substituting (18) into (19) and rearranging terms yields

$$(22) \quad b_{t+1} = qf'\left[q(1-\pi)w(k_t) - qb_t\right]b_t - \pi\left[(\sigma-1)/\sigma\right]w\left[q(1-\pi)w(k_t) - qb_t\right] \; ; \; t \geq 0.$$

Hence $b_{t+1} \geq b_t$ holds iff

$$(23) \quad \left\{qf'\left[q(1-\pi)w(k_t) - qb_t\right] - 1\right\}b_t \geq \pi\left[(\sigma-1)/\sigma\right]w\left[q(1-\pi)w(k_t) - qb_t\right].$$

To depict the loci defined by (23) at equality in figure 5, we proceed as follows. Define the locus (BB) by

$$(\mathrm{BB}) \quad qb_t = q(1-\pi)w(k_t) - (f')^{-1}(1/q).$$

Combinations (k_t, b_t) that lie above the locus (BB) in figure 5 have $qf'\left[q(1-\pi)w(k_t) - qb_t\right] > 1$ and hence yield a positive value for b_t in (23) (at equality). Combinations (k_t, b_t) that lie below (BB), on the other hand, yield negative values for b_t in (23) (at equality). Notice that the locus (BB) intersects the horizontal axis at the unique value k_t satisfying $q(1-\pi)w(k_t) = (f')^{-1}(1/q)$.

We now show that (23) at equality defines at least two loci in figure 5: one that lies entirely in the positive orthant and one with $b_t < 0$. Specifically, we state

Proposition 2. Suppose that k_t satisfies $q(1-\pi)w(k_t) > (f')^{-1}(1/q)$. Then there exist at least two values of b_t that satisfy (23) at equality. One of these is positive, and one is negative.

Proposition 2 is proved in the appendix.

It remains to derive the slopes of the continuous loci defined by (23) at equality. Differentiating (23) at equality yields the expression

$$(24) \quad (db_t / dk_t)\left[qf'(\cdot) - 1 - q^2 f''(\cdot)b_t + \pi(\sigma - 1)qw'(\cdot)/\sigma\right] =$$
$$\pi(\sigma - 1)w'(\cdot)q(1-\pi)w'(k_t)/\sigma - qb_t f''(\cdot)q(1-\pi)w'(k_t).$$

When $b_t \geq 0$, and hence for the branch of (23) in the positive orthant, $qf'(\cdot) > 1$ holds. Thus this branch is unambiguously upward sloping. The branch of (23) with $b_t < 0$ has an ambiguous slope in general; however it is possible to show the following.

Proposition 3. At the steady state equilibrium with $I \in (1,\sigma)$, and hence with $b < 0$, db_t / dk_t as given by (24) is positive.

(Proposition 3 is proved in appendix B.) Finally, equation (23) (at equality) passes through the origin. Thus (23) (at equality) has the general configuration depicted in figure 5.

It is straightforward to verify that, above (BB), (k_t, b_t) combinations above (below) the locus defined by (23) at equality have $b_{t+1} > (<) b_t$. Below the locus (BB) the relationship between b_{t+1} and b_t is less transparent; however we can establish the following result.

Proposition 4. In a neighborhood of the steady state with $b_t < 0$, points above (below) the locus defined by (23) at equality have $b_{t+1} < (>)\ b_t$.

Proposition 4 follows immediately from lemma A.1 in the appendix.

We also know that, under the assumptions of this section, there are exactly two steady state equilibria: one has $I \in (1,\sigma)$, $b < 0$, and a relatively high capital stock; the other has $I > \sigma$, $b > 0$, and a relatively low capital stock. The local behavior of the economy near these steady state equilibria is depicted in figure 5.

Figure 5 suggests, and the local analysis below confirms, that the steady state equilibrium with $I > \sigma$ and $b > 0$ is a saddle.[15] Paths approaching this steady state display monotonic increases or decreases in k_t and b_t over time. Figure 5 also suggests that the steady state equilibrium with $I \in (1,\sigma)$ and $b < 0$ may be a sink; we establish below conditions under which this is the case. In addition, for a broad range of parameter values, nonstationary equilibria approaching this steady state display damped oscillation.

C. Development Traps and Endogenous Fluctuations

When an economy has two steady state equilibria, with one potentially being asymptotically stable and one being a saddle, there is the possibility that two economies with very similar— or even identical — initial capital stocks will converge to very different steady state equilibria. This possibility is depicted for a particular initial capital stock in figure 5. Thus, in economies with money, capital, and a government allowing its net lending position to be endogenous, development traps are ubiquitous whenever the money growth rate is sufficiently high.[16]

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[15] If $\sigma > 1$ but $\sigma q f'(\hat{k}) < 1$, then there is only a single steady state equilibrium here. This has $I > \sigma$, and hence is a saddle. This case of a unique steady state equilibrium that is a saddle, of course, closely resembles the situation in the Diamond (1965) model. [See Azariadis (1992)].

[16] This conclusion does *not* require that the government let the value of its outstanding bonds be endogenous. See Schreft and Smith (1994) for an analysis of policies that fix the money-bond ratio.

Additionally, for a very large set of parameter values, dynamical equilibrium paths approaching the high-capital-stock steady state display damped oscillation en route. Thus the presence of banks, plus an array of government liabilities, is a source of endogenous fluctuations that are not possible in the Diamond (1965) model. Economies that are fortunate enough to avoid development traps will experience these fluctuations along transition paths to the steady state.

D. Local Dynamics

In this section we examine the behavior of a linear approximation to the dynamical system defined by (18) and (19) in a neighborhood of *any* steady state equilibrium with $I > 1$. Thus we replace (18) and (19) with

$$(25) \qquad (k_{t+1}-k,\ b_{t+1}-b)' = J(k_t-k,\ b_t-b)' \ ;\ t \geq 0,$$

where (k, b) are any steady state values of the capital stock and outstanding government bonds and where J is the Jacobian matrix

$$J = \begin{bmatrix} \partial k_{t+1}/\partial k_t & \partial k_{t+1}/\partial b_t \\ \partial b_{t+1}/\partial k_t & \partial b_{t+1}/\partial b_t \end{bmatrix}.$$

From (18) and (19) we have that [evaluated at the steady state (k,b)],

$$(26) \qquad \partial k_{t+1}/\partial k_t = q(1-\pi)w'(k) = (1-\pi)[kw'(k)/w(k)]/\Omega(k)$$

$$(27) \qquad \partial k_{t+1}/\partial b_t = -q$$

$$(28) \qquad \partial b_{t+1}/\partial k_t = [qbf''(k) - \pi(\sigma-1)w'(k)/\sigma]\partial k_{t+1}/\partial k_t$$

$$(29) \quad \partial b_{t+1} / \partial b_t = qf'(k) + \left[bqf''(k) - \pi(\sigma - 1)w'(k)/\sigma\right](\partial k_{t+1} / \partial b_t) =$$
$$(I/\sigma) + \pi\left[(\sigma - 1)/\sigma\right]\left[kw'(k)/w(k)\right] \{1 + \left[\sigma / (I - \sigma)\right] / \Omega(k)\} / \Omega(k).$$

In addition, we note that a steady state equilibrium has

$$(30) \quad \Omega(k) \equiv k / qw(k) = 1 - \pi(I - 1)/(I - \sigma) = \left[(1 - \pi)I + \pi - \sigma\right] / (I - \sigma)$$

and

$$(31) \quad b = (\sigma - 1)\pi w(k) / \sigma\left[qf'(k) - 1\right] = (\sigma - 1)\pi w(k) / (I - \sigma).$$

Substituting (30) into (29) gives

$$(29') \quad \partial b_{t+1} / \partial b_t = (I/\sigma) + \pi\left[(\sigma - 1)/\sigma\right]\left[kw'(k)/w(k)\right] \cdot$$
$$\left[(1 - \pi)I + \pi\right] / \left[(1 - \pi)I + \pi - \sigma\right]\Omega(k).$$

Now let D and T denote the determinant and the trace, respectively, of J. Straightforward algebraic manipulation establishes that

$$(32) \quad D = (I/\sigma)(1 - \pi)\left[kw'(k)/w(k)\right] / \Omega(k)$$

$$(33) \quad T = (I/\sigma) + \left[kw'(k)/w(k)\right]\{1 - \pi + \pi\left[(\sigma - 1)/\sigma\right] \cdot$$
$$\left[(1 - \pi)I + \pi\right] / \left[(1 - \pi)I + \pi - \sigma\right]\} \Omega(k).$$

We now consider these expressions evaluated at each of the two steady state equilibria.

1. Steady States with $I \in (1,\sigma)$.

For steady state equilibria with $I \in (1,\sigma)$, conditions implying that such equilibria are sinks are easy to state. We begin with the following lemma.

Lemma 1. Consider any steady state equilibrium with $I \in (1,\sigma)$. Then $D \in (0,1-\pi)$ and

$$(1/\sigma)-\pi < T < 1+\{[\sigma(1-\pi)-\pi]/(1-\pi)\}D$$

hold.

Lemma 1 is proved in the appendix.

Because $\sigma > 1$ and $D > 0$ hold here, the steady state equilibrium with $I < \sigma$ is a sink if $T < 1 + D$ holds. From lemma 1, a sufficient condition for $T < 1 + D$ to hold is that

(34) $\sigma \leq 1/(1-\pi)$.

Thus, if (34) holds, the steady state equilibrium with $I < \sigma$ is necessarily a sink. Excessively high rates of money growth, however, may overturn this result.

1.a. An Example

If $f(k) = Ak^{\alpha}$, with $\alpha \in (0,1)$, it was shown in section III.A that two steady state equilibria with $I > 1$ exist if $\sigma > max[(1-\alpha)/\alpha, 1]$. Thus, if $1-\pi < \alpha/(1-\alpha)$ and $1/(1-\pi) \geq \sigma >$ $max[(1-\alpha)/\alpha, 1]$ both hold, there necessarily exists a steady state equilibrium with $I \in (1,\sigma)$, which is a sink. Of course while these conditions are sufficient for the existence of such an equilibrium, they are not necessary for the equilibrium to be asymptotically stable.

2. Steady States with $I > \sigma$.

To make progress with this case, we specialize to the situation of Cobb-Douglas production. That is, in this section, we assume that $f(k) = Ak^{\alpha}$, with $\alpha \in (0,1)$.

Under this assumption, $kw'(k)/w(k) = \alpha$ holds, as does

$$(35) \quad \Omega(k) = k^{1-\alpha} / qA(1-\alpha).$$

In addition, the steady state equilibrium condition $I = \sigma q f'(k)$ takes the form

$$(36) \quad k^{1-\alpha} = \sigma q \alpha A / I.$$

Notice that (35) and (36) imply that

$$(37) \quad \Omega(k) = \sigma\alpha / (1-\alpha)I.$$

Finally, for two steady state equilibria to exist with $I > 1$, $\sigma > max[(1-\alpha)/\alpha, 1]$ must hold. When it does, the steady state equilibrium values of I are given by the two solutions to equation (16): the largest of these (the solution with $I > \sigma$) is

$$(38) \quad I = [\sigma - \pi(1-\alpha)]/2(1-\alpha)(1-\pi) + \\ [\{[\sigma - \pi(1-\alpha)]/(1-\alpha)\}^2 - 4(1-\pi)\alpha\sigma^2/(1-\alpha)]^{0.5}/2(1-\pi).$$

Substituting (37) and $kw'(k)/w(k) = \alpha$ into (32) and (33) yields

$$(39) \quad D = (1-\alpha)(1-\pi)(I/\sigma)^2$$

$$(40) \quad T=(I/\sigma)+(1-\alpha)(I/\sigma)\{1-\pi+\pi[(\sigma-1)/\sigma]\cdot$$
$$[(1-\pi)I+\pi]/[(1-\pi)I+\pi-\sigma]\}.$$

We now state

Lemma 2. $T>1+D$ holds when $I > \sigma$.

Lemma 2 is proved in the appendix. It implies that the steady state equilibrium with $I > \sigma$ is a saddle [see Azariadis (1992), chapter 6.4]. Paths approaching the steady state equilibrium display monotonic increases or decreases in k_t and b_t.

3. Numerical Examples

Each of the examples that follows has $f(k)=Ak^{\alpha}$. The parameters of the model are then A, α, q, π, and σ.

Table 1 reports features of steady state equilibria for the parameters $A = 1$, $q = 0.3$, $\pi = 0.5$, and $\sigma = 1.5$. The parameter α is allowed to vary between 0.4 and 0.8. For each set of parameter values, table 1 reports the values of the gross nominal interest rate and the per capita capital stock at the low capital stock and the high capital stock steady states. Table 1 also reports the trace and the determinant of J, evaluated at the steady state with $I \in (1,\sigma)$. As is apparent, each entry of this table has $T^2 < 4D$. This implies that the eigenvalues of J are complex conjugates. Thus entries in the table with $D < 1$ correspond to steady state equilibria that are sinks. Dynamical equilibrium paths approaching these steady states display damped oscillation. For entries in table 1 with $D > 1$, the steady state with $I \in (1,\sigma)$ is a source. Development traps do not exist for these combinations of parameter values.

Table 2 examines steady state equilibria when $A = 1$, $\alpha = 0.5$, $q = 0.3$, $\sigma = 1.5$, and π is free to vary between 0.1 and 0.9. As before, each entry in the table has $T^2 < 4D$ at the steady state with $I \in (1,\sigma)$. Thus these steady state equilibria corresponding to very low values of π are sources; for values of π above 0.2 they are sinks. Dynamical equilibria approaching these steady states display damped oscillation.

Table 3 considers the effects of variations in the money growth rate, σ. For the set of parameter values $A=1$, $\alpha = 0.6$, $\pi = 0.5$, and $q = 0.3$, σ is varied from 1.5 to 20. As before, for each entry in table 3, $T^2 < 4D$ holds. Thus each steady state equilibrium with $I \in (1,\sigma)$ in table 3 is a sink, and transient paths approaching these equilibria display damped oscillation.

Discussion

For a wide variety of parameter values our economy has two steady state equilibria: one is a sink; the other, a saddle. For combinations of parameter values producing this outcome, the development trap phenomenon depicted in figure 5 arises. In particular, economies with similar — or even identical — initial capital stocks may follow dynamical equilibrium paths that approach different steady state equilibria. Economies that are unfortunate enough to have high initial nominal interest rates will end up with low capital stocks, while economies with relatively low initial nominal interest rates will converge to values of the capital stock that are relatively high. For the parameter values that we examine, economies approaching the higher-steady-state capital stock will experience endogenous fluctuations in their capital stocks, output levels, real and nominal rates of interest, and inflation rates as they do so.

V. Conclusions

We have examined an economy where monetary and financial market factors determine the level of capital accumulation and real development. In this economy, sufficiently high rates of money growth necessarily lead to the existence of multiple, nontrivial steady state equilibria with valued fiat money and positive nominal interest rates. One of these has a relatively high nominal rate of interest and a relatively low capital stock. In this equilibrium, government bonds compete with productive capital in the portfolios of banks. As a result, banks hold relatively little capital, and increases in the rate of money growth will induce them to hold even less. The other steady state equilibrium has a relatively low nominal interest rate and a relatively high capital stock. This equilibrium has the feature that the government actually contributes to the financing of capital investment by the banking system and, when this is the case, increases in the rate of money creation are conducive to enhanced capital formation.

Under conditions that are easily satisfied, the first of these equilibria is a saddle and the second is a sink. As a result development trap phenomena are observed. In addition, economies can readily display damped endogenous fluctuations in all endogenous variables as they approach the high-capital-stock steady state .

These results, admittedly, have been obtained under some special assumptions. We have, for example, confined our attention to the case where agents have logarithmic utility. We also have considered only the situation where monetary authorities fix the rate of money growth. Both assumptions are relaxed by Schreft and Smith (1994), which examines a monetary authority that manipulates the ratio of money to government bonds and allows agents to have general CRRA utility functions. (The former assumption is arguably a more plausible description of monetary policy activity than a constant rate of money growth, at least for more-developed economies.) We show there that some version of all of our results carries over if agents are sufficiently risk averse. The formulation of that paper also allows us to analyze how open market operations by a central bank affect real activity and rates of inflation. The second issue is not one that we can address in an interesting way here.

We also have examined a government that engages in no fiscal activity (has no expenditures and engages in no direct taxation) and that imposes no regulations on financial market participants. The analysis of a government that has a deficit to finance and that can regulate its financial system is left as a topic for future investigation. Indeed, the fact that development trap phenomena are associated with (unnecessarily) high nominal interest rates is suggestive that a government might be tempted to impose ceilings on interest rates offered by banks to prevent falling into a development trap, even in the absence of a government deficit. Whether or not such a policy can be effective is an interesting question since financial market regulation can lead to some activity taking place outside the regulated sector. Such regulation is common in developing countries; interest rate controls are a prominent example. Issues related to regulatory policies also would be interesting to analyze.

Appendix

A. Proof of Proposition 1.

To establish the existence of a positive solution to (23) at equality, begin by considering the value $b_t = (1-\pi)w(k_t)$. Then, by construction, the left-hand side of (23) is infinite, while the right-hand side of (23) is zero. In contrast, if we set $b_t = \{(1-\pi)w(k_t) - [(f')^{-1}(1/q)]\}/q > 0$ (by the hypothesis of the proposition), the left-hand side of (23) is zero (by construction). The right-hand side is given by $\pi[(\sigma-1)/\sigma]w[(f')^{-1}(1/q)] > 0$. Therefore, since both sides of (23) are continuous in b_t, the existence of a value $b_t > 0$ satisfying (23) at equality is implied by the intermediate value theorem.

To establish the existence of a solution to (23) at equality with $b_t < 0$, begin by considering the value $b_t = 0$. Evidently, when $b_t = 0$, the right-hand side of (23) is positive, while the left-hand side of (23) is zero.

Now rewrite (23), for $b_t < 0$, as

$$\text{(A.1)} \quad 1 - qf'[q(1-\pi)w(k_t) - qb_t] \geq \pi(\sigma-1)w[q(1-\pi)w(k_t) - qb_t]/(-\sigma b_t).$$

As $b_t \to -\infty$, the left-hand side of (A.1) approaches one by the Inada conditions. It is easy to show that, as $b_t \to -\infty$, the right-hand side of (A.1) approaches

$$\lim_{b_t \to \infty} -\pi(\sigma-1)f[q(1-\pi)w(k_t) - qb_t]/\sigma b_t.$$

Since we are holding k_t fixed, we can use L'Hospital's Rule to show that this limit equals

$$\lim_{b_t \to -\infty} [\pi(\sigma-1)/\sigma]qf'[q(1-\pi)w(k_t) - qb_t] = 0,$$

where again the equality follows from the Inada condition.

Thus, as $b_t \to -\infty$, the left-hand side of (23) exceeds the right-hand side of (23). The intermediate value theorem then establishes the existence of a value $b_t \in (-\infty, 0)$ satisfying (23) at equality. □

B. Proof of Proposition 2.

Rearranging terms in (24) gives

(A.2) $db_t / dk_t =$

$$\frac{q(1-\pi)w'(k_t)\{\pi(\sigma-1)w'(\cdot)/\sigma - qb_t f''(\cdot)\}}{[qf'(\cdot)-1]+q\{\pi(\sigma-1)w'(\cdot)/\sigma - qb_t f''(\cdot)\}}.$$

Below the locus (BB), $qf'(\cdot) < 1$ holds. Hence, if we can establish that

(A.3) $\pi(\sigma-1)w'[q(1-\pi)w(k_t)-qb_t] < \sigma qb_t f''[q(1-\pi)w(k_t)-qb_t]$

at the steady state with $b < 0$, we will have the desired result. We now state

Lemma A1. At the steady state with $I \in (1, \sigma)$, (A.3) holds.

Proof. Since $w'(k) \equiv -kf''(k)$, equation (A.3) is equivalent to

(A.4) $\pi(\sigma-1)[q(1-\pi)w(k)-qb] > \sigma qb$,

where k and b denote the appropriate steady state values.

Now, at this steady state, equation (19) implies that

(A.5) $b = (\sigma-1)\pi w(k)/\sigma[qf'(k)-1] = (\sigma-1)\pi w(k)/(I-\sigma)$.

Substituting (A.5) into (A.4) and rearranging terms gives the equivalent condition

(A.6) $I/(I-\sigma)<\pi(I-1)/(I-\sigma)$.

Since, at the relevant steady state, (A.6) is equivalent to $I>\pi(I-1)$, (A.6) obviously holds. This establishes lemma A.1, and hence proposition 3.

C. Proof of Lemma 1.

a) $D\in(0,1-\pi)$. From (32) it is apparent that $D > 0$. Moreover, $I < \sigma$ holds by hypothesis, and $kw'(k)/w(k)< 1$ holds by (a.1'). Finally, from (30) and $I\in(1,\sigma)$, it is clear that $\Omega(k)>1$. Thus (32) implies that $D<1-\pi$.

b) $T>(1/\sigma)-\pi$. We have that

$$(d/dI)\{[(1-\pi)I+\pi]/[(1-\pi)I+\pi-\sigma]\} = -\sigma(1-\pi)/[(1-\pi)I+\pi-\sigma]^2<0.$$

Thus, since $I<\sigma$,

(A.7) $$[(1-\pi)I+\pi]/[(1-\pi)I+\pi-\sigma]\geq[(1-\pi)\sigma+\pi]/[(1-\pi)\sigma+\pi-\sigma]=$$
$$[(1-\pi)\sigma+\pi]/\pi(1-\sigma).$$

But equations (A.7) and (33) imply that

(A.8) $T>(I/\sigma)-\pi[kw'(k)/w(k)]/\Omega(k)>(1/\sigma)-\pi$, where the second inequality in (A.8) follows from $I > 1 > kw'(k)/w(k)$ and $\Omega(k) > 1$ [at any steady state with $I \in (1,\sigma)$].

c) $T < 1 + \{[\sigma(1-\pi)-\pi]/(1-\pi)\}D$. It follows from our previous observations that

(A.9) $[(1-\pi)I+\pi]/[(1-\pi)I+\pi-\sigma] < 1/(1-\sigma)$.

From (A.9) and (33) it follows that

(A.10) $T < (I/\sigma) + [kw'(k)/w(k)][\sigma(1-\pi)-\pi]/\sigma\Omega(k)$.

Then, from (32) and (A.10),

(A.11) $T < (I/\sigma) + \{[\sigma(1-\pi)-\pi]/(1-\pi)I\}D$.

Then $1 < I < \sigma$ and (A.11) imply that

(A.12) $T < 1 + \{[\sigma(1-\pi)-\pi]/(1-\pi)\}D$.

This completes the proof. □

D. <u>Proof of Lemma 2</u>.

Suppose to the contrary that $1+D \geq T$ holds. From (39) and (40), this condition is equivalent to

(A.13) $1+(1-\pi)(1-\alpha)(I/\sigma)^2 \geq (I/\sigma)+(1-\pi)(1-\alpha)(I/\sigma)+$

$(I/\sigma)(1-\alpha)\pi[(\sigma-1)/\sigma][(1-\pi)I+\pi]/[(1-\pi)I+\pi-\sigma]$.

Rearranging terms in (A.13) and using the fact that $I > \sigma$, we obtain the equivalent condition

(A.13') $(1-\pi)(1-\alpha)(I/\sigma) \geq 1+$
$$\pi I(1-\alpha)(\sigma-1)[(1-\pi)I+\pi]/\sigma(I-\sigma)[(1-\pi)I+\pi-\sigma].$$

Moreover it is evident from (38) that

(A.14) $I/\sigma < 1/(1-\pi)(1-\alpha)$

holds. But (A.13') and (A.14) imply that

(A.15) $1 > 1+\pi I(1-\alpha)(\sigma-1)[(1-\pi)I+\pi]/\sigma(I-\sigma)[(1-\pi)I+\pi-\sigma]$.

However, $I > (\sigma-\pi)/(1-\pi) > \sigma$ holds (see figure 2b), and therefore the second term on the right-hand side of (A.15) is positive. But this is the desired contradiction. □

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Table 1

α	High Capital k	High Capital I	Low Capital k	Low Capital I	High Capital D	High Capital T
.4	.0574	1.00	.0092	3.00	.36	.85
.45	.0464	1.09	.0061	3.36	.46	.87
.5	.0365	1.18	.0035	3.82	.56	.87
.55	.0277	1.24	.0016	4.42	.67	.84
.6	.0198	1.30	.0006	5.20	.77	.78
.7	.0073	1.38	.00002	7.62	.98	.63
.8	.0010	1.43	-------	-------	1.19	.42

Parameters: $A = 1$, $q = 0.3$, $\pi = 0.5$, $\sigma = 1.5$

Table 2

π	High Capital k	High Capital I	Low Capital k	Low Capital I	High Capital D	High Capital T
.1	.0299	1.30	.0137	1.92	1.24	2.00
.2	.0324	1.25	.0100	2.25	1.01	1.64
.3	.0341	1.22	.0073	2.64	.84	1.36
.4	.0355	1.19	.0051	3.14	.70	1.10
.5	.0365	1.18	.0035	3.82	.56	.87
.6	.0374	1.16	.0022	4.84	.44	.65
.7	.0382	1.15	.0012	6.52	.32	.44
.8	.0389	1.14	.0005	9.86	.21	.23
.9	.0395	1.13	.0001	19.87	.10	.03

Parameters: $A = 1$, $q = 0.3$, $\alpha = 0.5$, $\sigma = 1.5$

Table 3

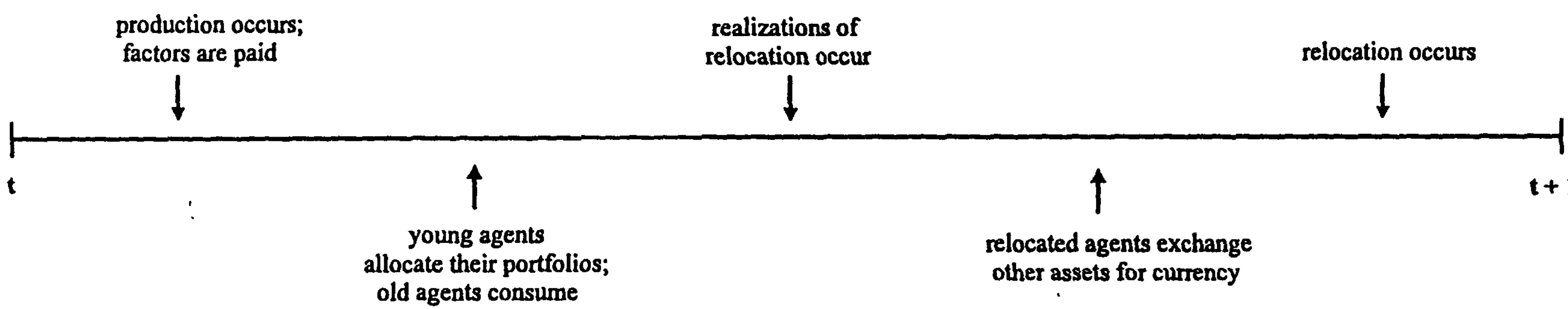
σ	High Capital k	High Capital I	Low Capital k*	Low Capital I	High Capital D	High Capital T
1.5	.0198	1.30	.0006	5.20	.77	.78
2.0	.0230	1.63	.0005	7.37	.68	.85
4.0	.0282	3.00	.0004	16.00	.58	.94
6.0	.0301	4.39	.0004	24.61	.55	.97
8.0	.0310	5.78	.0004	33.22	.54	.99
10.0	.0316	7.17	.0004	41.83	.53	1.00
15.0	.0323	10.66	.0004	63.34	.52	1.01
20.0	.0327	14.14	.0004	84.86	.51	1.01

***For $\sigma > 4$, steady state values of *k* vary only in the fifth decimal place**

Parameters: $A = 1$, $\alpha = 0.6$, $q = 0.3$, $\pi = 0.5$

Figure 1

Timing of Events



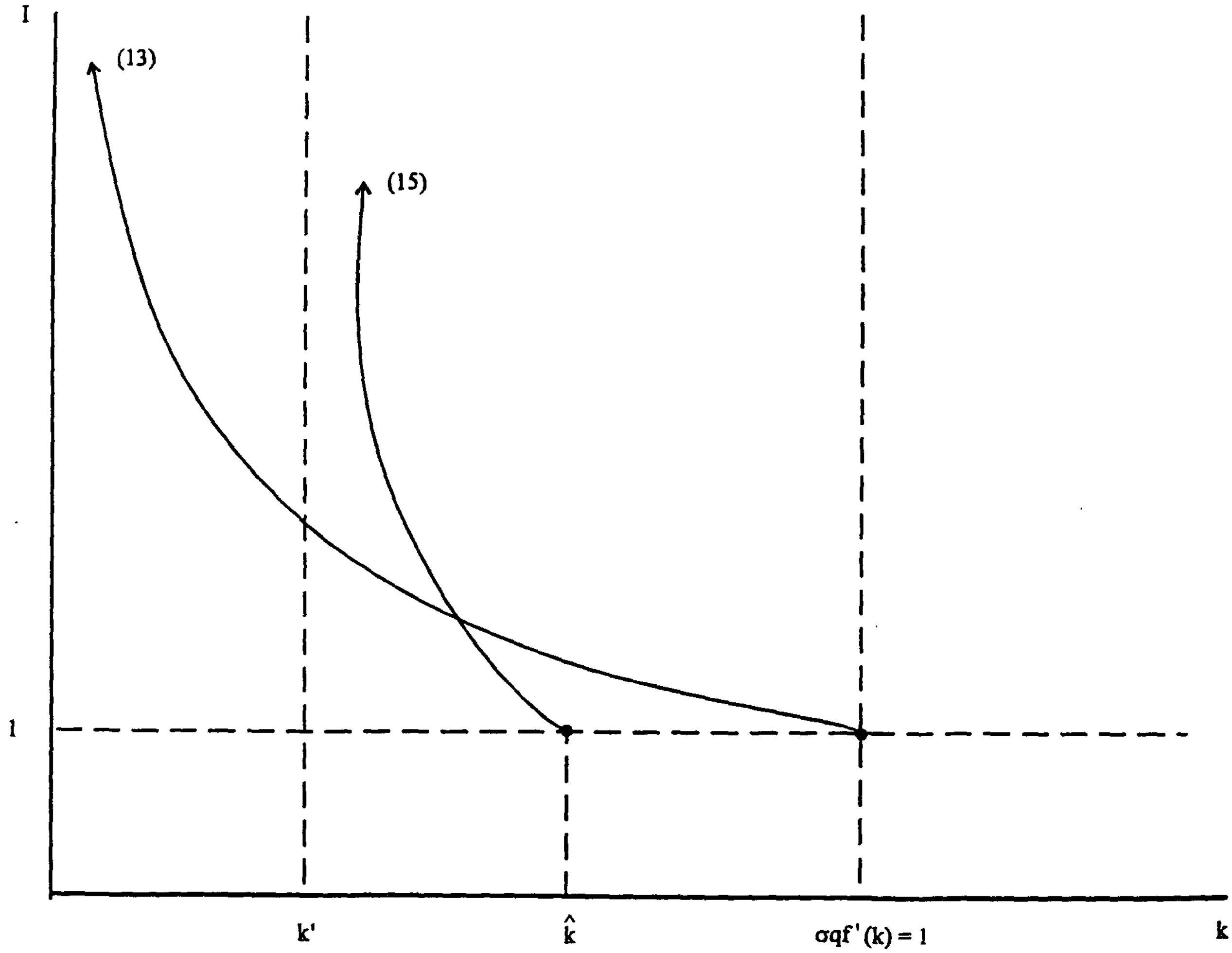


Figure 2a

Steady State Equilibria

$\sigma \leq 1$

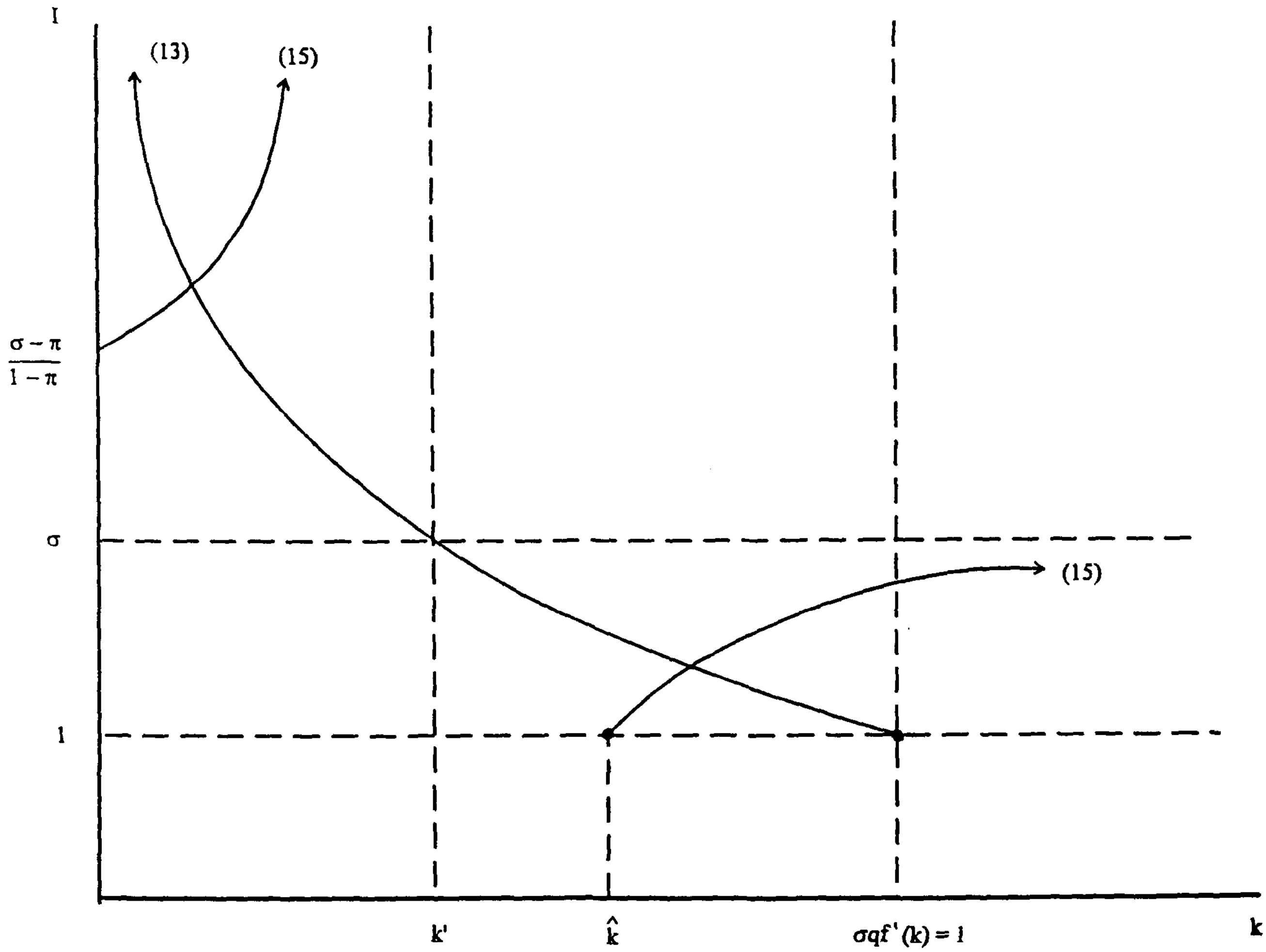


Figure 2b

Steady State Equilibria

$\sigma > 1$

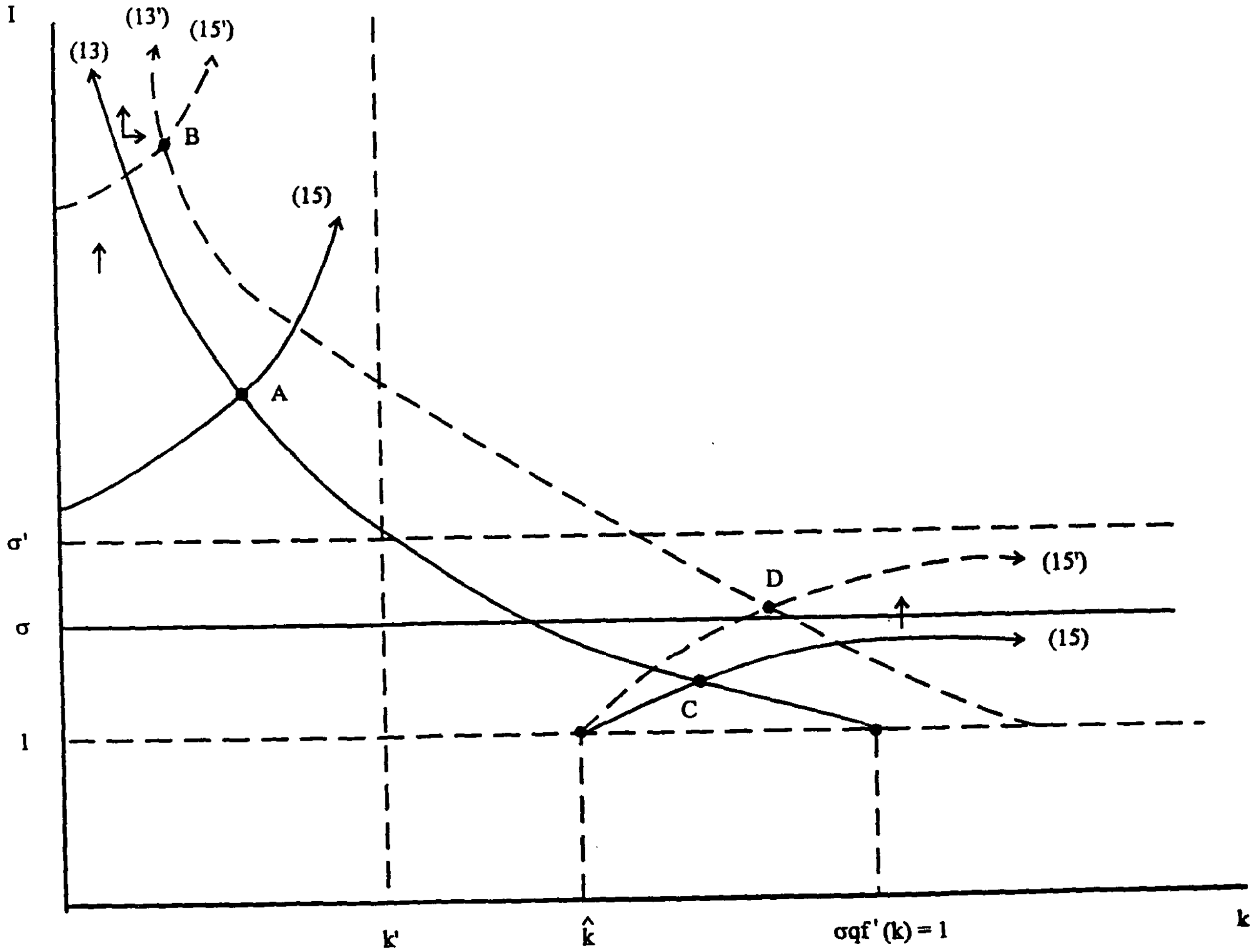


Figure 3

An Increase in the Rate of
Money Growth $(\sigma > 1)$

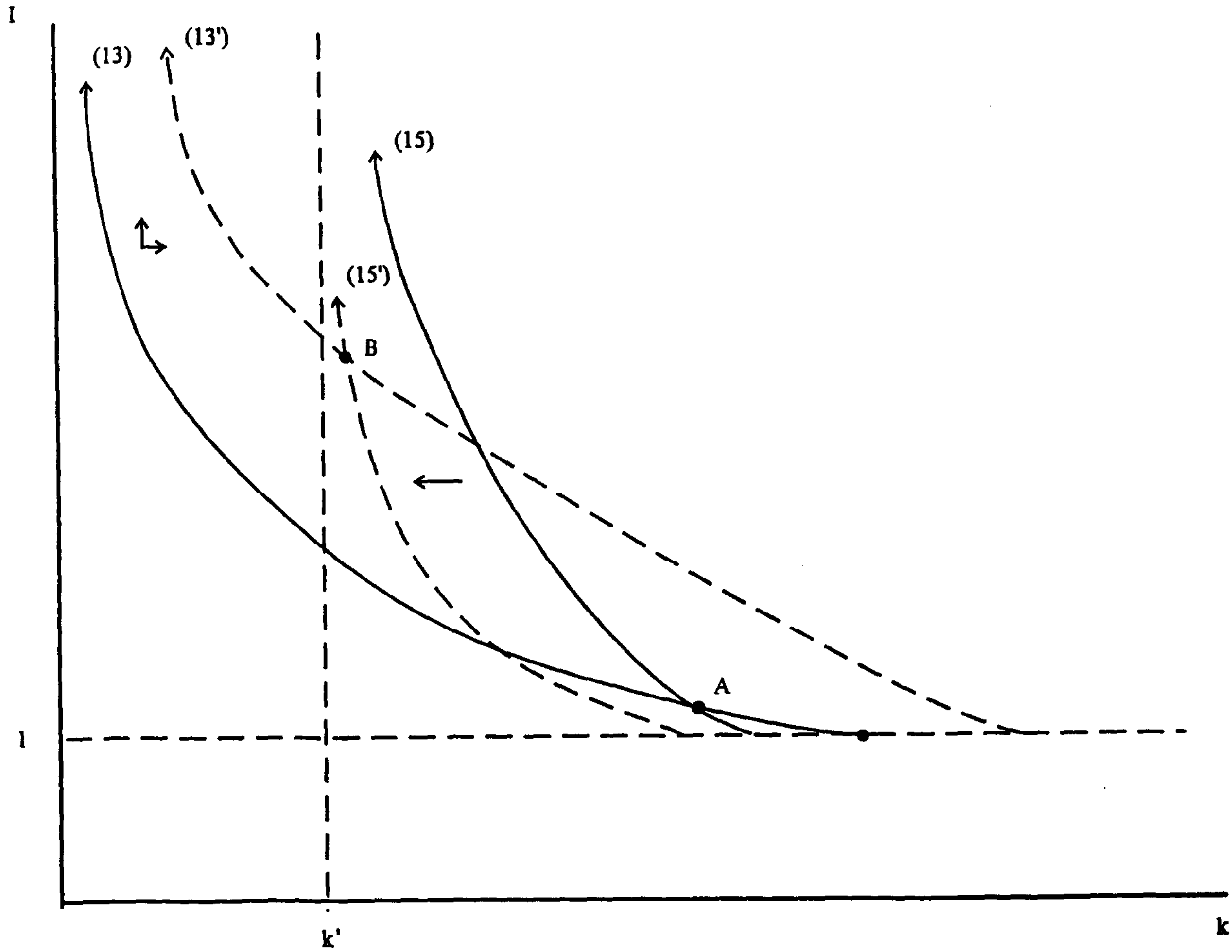


Figure 4a

An Increase in the Rate of
Money Growth $(\sigma < 1)$:

A Unique Steady State Equilibrium

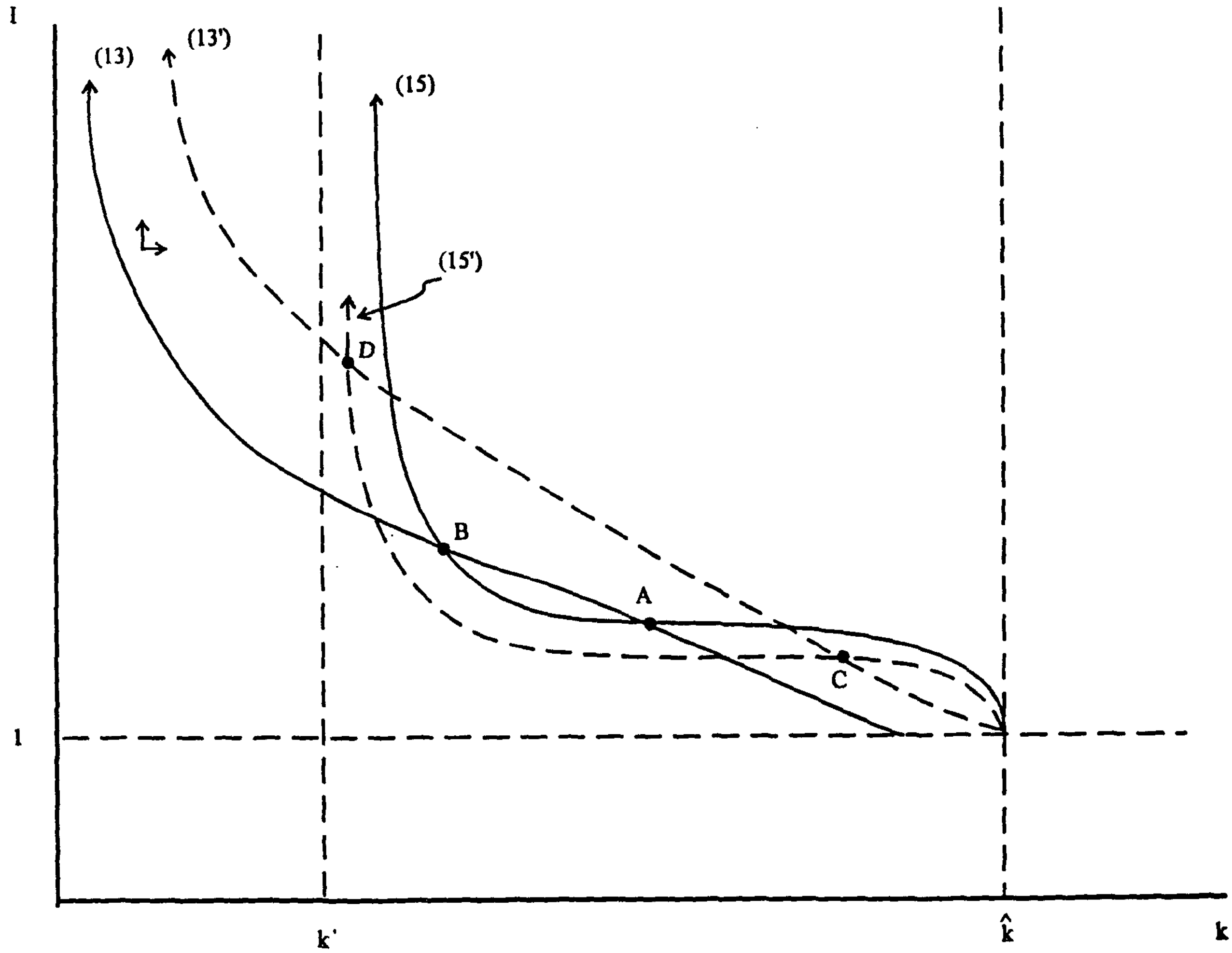


Figure 4b

An Increase in the Rate of Money Growth ($\sigma < 1$):

Multiple Steady State Equilibria

Figure 5
Dynamical Equilibria