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**LIQUIDITY EFFECTS AND TRANSACTIONS TECHNOLOGIES**

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## 1. Introduction

Recently there has been renewed interest in using general equilibrium models to understand the effects of monetary policy on interest rates and real economic activity. This research effort has involved the search for models that will account for the liquidity effects--the decrease in short-term interest rates and the increase in output and employment--that are associated with expansionary monetary policy. Empirically, liquidity effects have been isolated by Cochrane (1989), Strongin (1991), Christiano and Eichenbaum (1991b), and Gordon and Leeper (1992). More informally, financial market participants usually interpret Federal Reserve engineered rises in short-term nominal interest rates as a tightening of monetary policy.

The theoretical impetus for this literature is found in Lucas (1990). No two papers use the exact same specification, but a common feature of the literature is the presence of cash-in-advance (CIA) constraints that limit the amount of money available for use in loan or securities markets.<sup>1</sup> Each change in specification involves various assumptions about financial structure that place infinite transactions costs on flows of funds across segmented markets. Most frequently, the differences in specification are motivated by the emphasis of the particular model: whether it is primarily concerned with asset pricing or with generating business cycles.

In fact, the assumption of infinite transactions costs across markets is most reasonable when applied to understanding the behavior of

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<sup>1</sup>For various examples see Fuerst (1992), Christiano (1991), Christiano and Eichenbaum (1991b, 1992), Coleman, Gilles, and Labadie (1992), Schlagenhaut and Wrase (1992).

asset prices on a daily or weekly basis. To study the effects of monetary policy at business cycle frequencies, however, assumptions of infinite transactions costs are less innocuous. In this paper we consider the effects of relaxing these extreme assumptions in the monetary business cycle model of Christiano and Eichenbaum (1991b).<sup>2</sup> We do this by generalizing their CIA constraints, allowing agents to rearrange their portfolios at a finite cost after observing the monetary disturbance. Given the quarterly periodicity of the model, it seems realistic that agents have access to such a transactions technology. Our ultimate goal is to study the interaction between the magnitude of the transactions costs and the presence of liquidity effects on a quarterly basis.

The CIA constraints in Christiano and Eichenbaum's model give rise to one of the model's principal implications, that "a disproportionately large share of monetary injections is absorbed by firms to finance variable inputs" (Christiano and Eichenbaum 1992, p.352). In the absence of detailed flow-of-funds data with which to test this implication, our generalized version of the Christiano-Eichenbaum model illuminates an alternative, but closely related, implication. Specifically, our transactions technology gives rise to a spread between loan and deposit rates that varies systematically with the size of the monetary shock. In essence, our framework reveals that prices (i.e., interest rates), rather than quantities (i.e., flows of funds), can be used to assess the empirical relevance of the Christiano-Eichenbaum model. In fact, we find that specifications for transactions costs that allow the

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<sup>2</sup>We choose the Christiano-Eichenbaum model as a starting point because it is the most developed and successful in this class of models.

model to match the behavior of key interest rate spreads either remove or greatly dampen the liquidity effects that are present in Christiano and Eichenbaum's original work.

The next section briefly reviews Christiano and Eichenbaum's model. Section 3 generalizes this model, as suggested above, by allowing agents more flexibility to adjust their financial portfolios in response to monetary disturbances. Section 4 describes the solution and parameterization of our generalized model, while section 5 presents the results and section 6 concludes.

## **2. The Christiano-Eichenbaum Model**

Here we briefly sketch out the main features of the Christiano and Eichenbaum model. Each period is broken into two parts--the justification being that production requires a sustained flow of labor input and that open market operations occur in the midst of ongoing productive activity. The division of the period into two parts is a tractable way of representing such an environment.

During the first part of the period, following the realization of the technology shock but prior to the realization of the monetary shock, households allocate their portfolios between a transactions medium and the liability of a financial intermediary. They also decide on their first-part-of-period labor effort. To finance their wage bill, firms decide how much to borrow from intermediaries at a first-part-of-period nominal interest rate. They also decide how much to invest. In this model,

payments to labor are subject to a cash-in-advance constraint while investment is a credit good.

In the middle of the period, a monetary disturbance occurs in the form of a lump-sum transfer to intermediaries. Households and firms then make their second-part-of-period labor decisions, with firms borrowing their wage payments from intermediaries at a second-part-of-period interest rate. Households make their consumption decisions subject to a cash-in-advance constraint that involves not only their initial holdings of the medium of exchange but their current period wage receipts as well. The liquidity effect arises because the lump sum transfer affects the quantity of loanable funds. It is augmented by the fact that certain production decisions, namely initial labor hours, are state variables from the perspective of second-part-of-period decisions. A more formal description of the economic environment follows.

a. The firm's problem

The firm's problem is to maximize  $E \sum_{t=0}^{\infty} \beta^{t+1} \left[ \frac{u_{c,t+1}}{P_{t+1}} F_t | \Omega_0^1 \right]$  where  $F_t$

is the flow of nominal profits,  $u_{c,t+1}$  is the representative agent's marginal utility of consumption at time  $t+1$ ,  $P_{t+1}$  is the price level at time  $t+1$ , and the information set  $\Omega_t^1$  includes all variables and disturbances dated  $t-1$  and earlier as well as the first-part-of-period wage rate  $W_{1t}$ , the first-part-of-period loan rate  $R_{1t}$ , the capital stock  $K_{t+1}$ ,

the technology shock  $z_t$ , and the first-part-of-period hours worked  $H_{1t}$ . The firm performs this maximization subject to

$$(1) \quad F_t = P_t[f(K_t, z_t H_t) - K_{t+1}] - R_{1t}N_{1t} - R_{2t}N_{2t} + (N_{2t} - W_{1t}H_{1t}) + (N_{2t} - W_{2t}H_{2t})$$

$$(2) \quad z_t = \exp(\mu t + \theta_t)$$

$$(3) \quad \theta_t = (1 - \rho_\theta)\theta + \rho_\theta\theta_{t-1} + \varepsilon_{\theta t}$$

$$(4) \quad W_{1t}H_{1t} \leq N_{1t}$$

$$(5) \quad W_{2t}H_{2t} \leq N_{2t}$$

$$(6) \quad K_{t+1} = I_t + (1 - \delta)K_t$$

$$(7) \quad H_t = [(1/2)H_{1t}^{1/\rho} + (1/2)H_{2t}^{1/\rho}]^\rho$$

where  $f(K_t, z_t H_t)$  describes output as a function of capital, labor, and the technology shock,  $N_{1t}$  and  $N_{2t}$  are the amounts of funds borrowed to finance wage payments, and  $H_t$  gives the effective amount of labor, which depends on both first and second-part-of-period hours. Thus, the marginal product of second-part-of-period labor depends on  $H_{1t}$ . The technology innovation  $\varepsilon_{\theta t}$  is normally distributed with mean zero and standard deviation  $\sigma_\theta$ .

Based on  $\Omega_t^1$  (i.e., prior to the monetary disturbance  $X_t$ ) the firm chooses  $H_{1t}$ ,  $N_{1t}$ , and  $K_{t+1}$ . After observing  $X_t$  and all other variables

dated  $t$  and earlier, the firm chooses  $H_{2t}$  and  $N_{2t}$ . Thus, the information set  $\Omega_t^2$  contains all variables dated  $t$  and earlier.

b. The intermediary's problem

The financial intermediary accepts deposits  $B_t$  from households and makes loans  $N_{1t}$  and  $N_{2t}$  to firms. It also receives a lump sum transfer of  $X_t$  dollars halfway through the period. It maximizes  $E \sum_{t=0}^{\infty} \beta^{t+1} \left[ \frac{u_{ct+1}}{P_{t+1}} D_t \mid \Omega_0^1 \right]$

subject to

$$(8) \quad D_t = R_{1t}N_{1t} + R_{2t}N_{2t} - R_t^d B_t + (B_t + X_t - N_{1t} - N_{2t})$$

$$(9) \quad N_{1t} + N_{2t} \leq B_t + X_t$$

by choosing  $N_{1t}$  and  $B_t$  based on the information set  $\Omega_t^1$  and  $N_{2t}$  based on information contained in  $\Omega_t^2$ . In equilibrium, intermediaries also face a zero profit condition with respect to funds received from households

$$(10) \quad R_t^d B_t = R_{1t}N_{1t} + R_{2t}(N_{2t} - X_t).$$

The deposit rate  $R_t^d$  is determined after the realization of the monetary shock.

c. The household's problem



The household maximizes discounted expected lifetime utility

$E \sum_{t=0}^{\infty} \beta^t [u(C_t, J_t) | \Omega_0^1]$ , where  $C_t$  is consumption and  $J_t$  is leisure, subject to

$$(11) \quad J_t = 1 - L_{1t} - L_{2t}$$

$$(12) \quad P_t C_t \leq M_t - B_t + W_{1t} L_{1t} + W_{2t} L_{2t}$$

$$(13) \quad M_{t+1} \leq R_t^d B_t + D_t + F_t + (M_t - B_t + W_{1t} L_{1t} + W_{2t} L_{2t} - P_t C_t)$$

where  $L_{1t}$  is first-part-of-period labor supply,  $L_{2t}$  is second-part-of-period labor supply, and  $M_t$  is money holdings. The household chooses  $L_{1t}$  and  $B_t$  based on information in  $\Omega_t^1$  (i.e., before seeing the contemporaneous money shock). Recall that  $R_t^d$  does not belong to  $\Omega_t^1$ . In the second part of the period,  $L_{2t}$ ,  $C_t$ , and  $M_{t+1}$  are chosen.

#### d. Equilibrium

The model is closed with a description of the money supply process; it is governed by

$$(14) \quad x_t = (X_t/M_t) = (M_{t+1} - M_t)/M_t$$

$$(15) \quad x_t = (1 - \rho_x)x + \rho_x x_{t-1} + \varepsilon_{xt}$$

where the monetary innovation  $\varepsilon_{xt}$  is normally distributed with mean zero and standard deviation  $\sigma_x$ .

An equilibrium for this economy can now be defined as a set of prices and quantities such that (i) firms, households, and intermediaries are all optimizing and (ii) markets clear. Market clearing in the loan, labor, and goods markets is implied by the conditions

$$(16) \quad N_{1t} + N_{2t} = B_t + X_t$$

$$(17) \quad L_{jt} = H_{jt} \quad j=1,2$$

$$(18) \quad C_t + K_{t+1} = f(K_t, Z_t H_t) + (1-\delta)K_t.$$

With this economic environment, Christiano and Eichenbaum (1991b) are able to generate liquidity effects, although these effects lack the required persistence. Given their interpretation of the period length as one quarter, it seems natural to question the complete inability of economic agents to alter their portfolios in response to economic disturbances. Modern financial markets certainly offer a multitude of ways for easily and quickly transferring funds.<sup>3</sup> In the next section, therefore, we investigate the effects of embedding a costly transactions technology into this model.

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<sup>3</sup>The lack of significant welfare costs due to inflation in many of these models is used as a justification for ignoring transaction technologies. We do not find this argument persuasive since it may still be in an agent's interest to exercise portfolio rearrangement. This decision depends on marginal conditions not overall welfare considerations.

### 3. Transactions Costs Model

Our transactions technology allows agents to deposit or withdraw additional funds after observing the monetary disturbance. This requires us to distinguish between first-part-of-period deposits  $B_{1t}$ , which are made costlessly, and second-part-of-period deposits  $B_{2t}$ , which require the use of a costly transactions technology.

As in the Christiano and Eichenbaum model, the household divides its money holdings  $M_t$  between savings deposits  $B_{1t}$  and cash  $M_t - B_{1t}$ . The initial deposits earn  $R_{1t}^d$ , a rate that is determined after the monetary injection. Upon observing  $X_t$ , the household can transfer funds between its savings account and cash. The dollar value of this transfer,  $B_{2t}^d$ , earns  $R_{2t}^d$ ; its value can be positive (a deposit) or negative (a withdrawal). In order to make a transfer, the household must expend an amount of time given by  $T_H(B_{2t}^d/M_t)$ , where  $T_H$  is convex, continuously differentiable, and satisfies  $T_H(0)=0$ . Note that transactions costs are specified here as functions of the fraction of the total money supply that is moved. In this sense, these resource costs are invariant to changes in the nominal unit of account.

One interpretation of the convex transactions cost function  $T_H$  in our representative agent model is that it is obtained by aggregating over heterogeneous agents, each of whom faces a different fixed cost of

performing cash management activities (induced, perhaps, by different proximities to a bank). Agents with the lowest fixed cost are the first to adjust their deposits when interest rates change, while those with the highest costs require substantial interest rate movements before engaging in cash management. In the aggregate, total cash management costs are smooth and convex.

Intermediaries also incur transactions costs when altering their portfolio. Following the realization of  $X_t$ , there will be a change in the demand for loans, and intermediaries can respond by altering their supply of savings accounts,  $B_{2t}^s$ . This can be done at a cost  $T_B(B_{2t}^s/M_t)$ , where  $T_B$  is also convex, continuously differentiable, and satisfies  $T_B(0)=0$ . Specifically, intermediaries must hire  $T_B(B_{2t}^s/M_t)$  units of labor at a wage rate of  $W_{3t}$  in order to alter the level of intermediation. Like the household's cost function  $T_H$ , the representative intermediary's cost function  $T_B$  can be interpreted as an aggregate over heterogeneous financial institutions, each of which faces a different fixed cost of altering its portfolio.

Incorporating these changes into the Christiano and Eichenbaum model requires the following modifications. In the intermediary's problem, equations (8) and (9) are replaced with

$$(8') \quad D_t = R_{1t}N_{1t} + R_{2t}N_{2t} - R_{1t}^d B_{1t} - R_{2t}^d B_{2t}^s \\ + (B_{1t} + B_{2t}^s + X_t - N_{1t} - N_{2t} - W_{3t}T_B(B_{2t}^s/M_t))$$

$$(9a') \quad N_{1t} \leq B_{1t}$$

$$(9b') \quad N_{1t} + N_{2t} + W_{3t}T_B(B_{2t}^s/M_t) \leq B_{1t} + B_{2t}^s + X_t.$$

Equation (8') takes into account the effect of the wage payments  $W_{3t}T_B(B_{2t}^s/M_t)$  on dividends. Equations (9a') and (9b') reflect the balance sheet constraints on loans. The zero-profit condition for the intermediary's first-part-of-period deposit activity becomes

$$(10') \quad B_{1t}R_{1t}^d = N_{1t}R_{1t} + (B_{1t} - N_{1t})R_{2t}.$$

For the household, equations (11)-(13) become

$$(11') \quad J_t = 1 - L_{1t} - L_{2t} - L_{3t} - T_H(B_{2t}^d/M_t)$$

$$(12') \quad P_t C_t \leq M_t - B_{1t} - B_{2t}^d + W_{1t}L_{1t} + W_{2t}L_{2t} + W_{3t}L_{3t}$$

$$(13') \quad M_{t+1} \leq R_{1t}^d B_{1t} + R_{2t}^d B_{2t}^d + D_t + F_t$$

$$+ (M_t - B_{1t} - B_{2t}^d + W_{1t}L_{1t} + W_{2t}L_{2t} + W_{3t}L_{3t} - P_t C_t).$$

The household now has two additional uses of its time, the labor  $L_{3t}$  supplied to intermediaries and the cost  $T_H(B_{2t}^d/M_t)$  of performing second-part-of-period transactions.

An equilibrium for the economy is defined, as before, as a set of prices and quantities such that (i) firms, households, and intermediaries are optimizing and (ii) markets clear. The market clearing conditions are now given by equations (17), (18), and

$$(16') \quad N_{1t} + N_{2t} + W_{3t}T_B(B_{2t}/M_t) = B_{1t} + B_{2t} + X_t$$

$$(19) \quad L_{3t} = T_B(B_{2t}/M_t)$$

$$(20) \quad B_{2t}^d = B_{2t}^s = B_{2t}.$$

Equation (16') implies equilibrium in loan markets. Equation (19) provides for labor market clearing. Equation (20) is the market clearing condition for second-part-of-period deposits.

#### 4. Solution and Parameterization

##### a. Solution

For the transactions costs economy, the first order conditions of the firm, intermediary, and household as well as the market clearing

conditions form a system of 19 nonlinear equations in 19 unknowns that completely describes the behavior of equilibrium prices and quantities. The 19 equations can be reduced to a 5-equation system as outlined in the appendix. Fourteen of the equations are used to obtain expressions for the 14 unknowns  $C_t$ ,  $N_{1t}$ ,  $N_{2t}$ ,  $H_{2t}$ ,  $L_{1t}$ ,  $L_{2t}$ ,  $L_{3t}$ ,  $P_t$ ,  $W_{2t}$ ,  $W_{3t}$ ,  $R_{1t}^d$ ,  $R_{2t}^d$ ,  $R_{1t}$ , and  $R_{2t}$ . When these expressions are substituted into the remaining 5 equations, we are able to solve for  $B_{1t}$ ,  $B_{2t}$ ,  $H_{1t}$ ,  $K_{t+1}$ , and  $W_{1t}$ . These 5 equations are depicted by (21)-(25) where for ease of exposition we have refrained from substituting out  $R_{1t}^d$ ,  $R_{2t}^d$ ,  $R_{1t}$ , and  $R_{2t}$ :

$$(21) \quad E \left[ \frac{u_{ct+1} P_t}{P_{t+1}} - \frac{\beta u_{ct+2} f_{kt+1} P_{t+1}}{P_{t+2}} \mid \Omega_t^1 \right] = 0$$

$$(22) \quad E \left[ u_{jt} - u_{ct} \frac{W_{1t}}{P_t} \mid \Omega_t^1 \right] = 0$$

$$(23) \quad E \left[ \frac{u_{ct}}{P_t} - \frac{\beta u_{ct+1} R_{1t}^d}{P_{t+1}} \mid \Omega_t^1 \right] = 0$$

$$(24) \quad E \left[ \frac{u_{ct}}{P_t} + \frac{u_{jt} T_H'(B_{2t}/M_t)}{M_t} - \frac{\beta u_{ct+1}}{P_{t+1}} R_{2t}^d \mid \Omega_t^2 \right] = 0$$

$$(25) \quad E \left[ \frac{\beta u_{ct+1}}{P_{t+1}} (R_{1t} - R_{2t}) \mid \Omega_t^1 \right] = 0,$$

where  $u_{ct}$  and  $u_{jt}$  are the marginal utilities of consumption and leisure and  $f_{kt}$ ,  $f_{H_{1t}}$ , and  $f_{H_{2t}}$  are the marginal products of capital, first-part-of-period labor, and second-part-of-period labor.

Equation (21) is the firm's first order condition for capital. It reveals that the firm balances the benefits from (i) paying an extra dollar in dividends at time  $t$  and (ii) using the extra dollar to buy capital at time  $t$ , thereby producing and selling additional output in period  $t+1$ , and using the proceeds to pay a higher dividend in period  $t+1$ . Equation (22) describes the household's first-part-of-period labor supply. It indicates that the household equates the marginal utility of an extra unit of leisure to the marginal utility of an extra unit of real wages.

Equations (23) and (24) are the first order conditions for the household's deposit decisions. They show that in each sub-period, the household balances the utility cost of lower consumption in period  $t$  against the utility gain from higher consumption in period  $t+1$ . The return on second-part-of-period deposits is adjusted to take into account the



marginal transactions costs  $T_H'(B_{2t}/M_t)$ . Finally, equation (25) describes how the intermediary acts so as to equalize its expected rate of return on first-part-of-period and second-part-of-period loans.

It is not possible to obtain exact solutions to the system (21)-(25). Thus, we follow Christiano and Eichenbaum (1991b) in using numerical methods to construct an approximate solution. The five-equation system is linearized and solved by the method of undetermined coefficients described by Christiano (1991) and Christiano and Eichenbaum (1991a).

To apply numerical methods, preferences and technologies are specialized to

$$U(C_t, J_t) = \begin{cases} [C_t^{1-\gamma} J_t^\gamma]^\psi / \psi & -\infty < \psi < 1, \quad \psi \neq 0 \\ (1-\gamma)\ln(C_t) + \gamma\ln(J_t) & \psi = 0 \end{cases}$$

$$f(K_t, z_t H_t) = K_t^\alpha (z_t H_t)^{1-\alpha} + (1-\delta)K_t$$

$$T_H(B_{2t}/M) = a_H (B_{2t}/M_t)^2$$

$$T_B(B_{2t}/M) = a_B (B_{2t}/M_t)^2.$$

The functional forms for  $U$  and  $f$  are exactly those used by Christiano and Eichenbaum (1991b). The quadratic functional form for  $T_H$  and  $T_B$  is a

simple one that satisfies the requirements that costs are convex with  $T_i(0)=0$  for  $i=H$  and  $i=B$ .

b. Parameterization

We use the same parameter values to describe tastes, technology, and the money supply process that Christiano and Eichenbaum (1991b) choose by comparing the model's steady state implications to figures from the US time series data. These values are  $\beta=(1.03)^{-0.25}$ ,  $\psi=0$ ,  $\gamma=0.761$ ,  $\alpha=0.346$ ,  $\delta=0.0212$ ,  $\mu=0.0041$ ,  $\theta=1$ ,  $\rho=10/9$ ,  $\rho_\theta=0.9857$ ,  $\sigma_\theta=0.01369$ ,  $x=0.0119$ ,  $\rho_x=0.81$ , and  $\sigma_x=0.0041$ .

The critical parameters for our results are those of the transactions technologies. The intermediary's first order condition for its optimal supply of second-part-of-period deposits is

$$(26) \quad R_{2t} \frac{W_{3t}}{M_t} T_B'(B_{2t}/M_t) = 2a_B R_{2t} \frac{W_{3t}}{M_t} (B_{2t}/M_t) = R_{2t} - R_{2t}^d.$$

This first order condition links the intermediary's transactions cost parameter  $a_B$  to the behavior of the loan-deposit interest rate spread.

In our model, second-part-of-period deposits are the intermediary's marginal source of funds from the non-financial sector. Similarly, second-part-of-period loans are the firm's marginal source of funds from the financial sector. Natural analogs to the deposit rate  $R_{2t}^d$  and the loan rate  $R_{2t}$  in the US economy, therefore, are the rate on small

time deposits (which consist mostly of small CD's) and the commercial paper rate.

Since the monetary shock  $\varepsilon_{xt}$  is distributed symmetrically about zero and since the quadratic transactions costs function is symmetric about zero, equation (26) indicates that the loan-deposit rate spread will be approximately zero on average in the model. This contrasts with the US data, where the commercial paper rate-small CD rate has been positive on average since CD rates were deregulated in 1984. Thus, it is not possible to choose the parameter  $a_b$  to match the average interest rate spread from the data.<sup>4</sup> It is possible, however, to choose  $a_b$  to match the standard deviation of the spread, equal to 0.000838, that is in US quarterly data, 1984:1-1992:3. This is the approach that we take.

Ideally, the household's transactions cost parameter  $a_H$  could be chosen based on the household's first order condition for  $B_{2t}^d$ , which is given above by equation (24). In fact, equation (24) implies that

$$(27) \quad 2a_H \frac{W_{2t}}{M_t} (B_{2t}/M_t) = \beta E \left[ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} R_{2t}^d \mid \Omega_t^2 \right] - 1,$$

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<sup>4</sup>The positive average spread found in the data could be matched by the model simply by assuming that the intermediary faces constant marginal costs associated with making loans or accepting deposits. Adding this assumption to the model would not change its implications for the presence or absence of liquidity effects, and we would still require the standard deviation of the loan-deposit spread to parameterize the cost function  $T_H$ .

which shows how the presence of transactions costs alters the standard consumption-based asset-pricing Euler equation. Again, the symmetry of the distribution of  $\varepsilon_{xt}$  and the function  $T_H$  imply that the left-hand-side of (27) will be approximately zero on average in the model economy. Hence  $a_H$  cannot be chosen to match the sample average of  $\beta(C_t/C_{t+1})(P_t/P_{t+1})R_{2t}^d - 1$  from the US data, which is negative using quarterly figures from 1984:1 through 1992:3. Moreover, since the standard deviation of the conditional expectation on the right-hand-side of (27) cannot be easily estimated using US data,  $a_H$  cannot be chosen to match a sample standard deviation either.

Thus, in the absence of observable data with which to choose  $a_H$ , it is simply assumed that the household's transactions costs are some multiple of the intermediary's costs, so that  $a_H = \lambda a_B$ . This strategy reduces the problem of parameterizing transactions costs to one of choosing a value for  $a_B$  to match the standard deviation of the commercial paper-small CD rate spread found in the US data, for any given value of  $\lambda$ . Below, we present results for a wide range of values for the free parameter  $\lambda$ .

## 5. Results

Insight into the complicated mechanism by which monetary surprises affect real activity in the model can be gained by examining Table 1, which describes the contemporaneous response of our economy to an unanticipated doubling of the money supply when transactions costs are zero ( $a_H = a_B = 0$ ). With zero transactions costs, the economy is similar to more

conventional cash-in-advance models in which there are no liquidity effects. Column 2 shows results for the case where there is no serial correlation in monetary disturbances, so that  $\rho_x=0$ . Agents deposit roughly 89 percent of their money holdings in a savings account, and banks lend half of that amount out in first-part-of period loans. If there was no disturbance, the other half of savings would be lent out in the second-part-of period, and each half of the period would look identical.

With a permanent doubling of the money supply half way through the period, agents exactly offset the real effects of additional money through cash management. In order for real quantities to remain unchanged in the new equilibrium, second-part-of-period wages must double, as must prices. With the same steady state real wage, hours worked do not change. To meet their second-part-of-period wage bill, firms require twice as much funds as in the steady state. In particular, they need 0.89. But banks have 1.43 to lend. A transfer by consumers of 0.56 from savings to transaction accounts allows the loan market to clear at the steady state nominal interest rate and preserves the steady state equilibrium.

In the presence of serial correlation in money growth, firms reduce their demand for labor in response to an inflation tax (column 3). Output falls and nominal interest rates rise as in a standard cash-in-advance model. With no transactions costs, agents fully offset the liquidity effects, and only the inflation tax effects are left. Offsetting liquidity effects requires more than a one-for-one movement in savings deposits since firms require smaller second-part-of-period loans than in the case of white noise monetary disturbances.

If transactions costs are infinite (as they are in the original Christiano-Eichenbaum model), on the other hand, consumers cannot react to the monetary shock. In this case, a doubling of the price level cannot be an equilibrium since the loan market no longer clears at the steady state nominal interest rate. Interest rates must fall, inducing firms to demand more labor. Real wages and output increase in equilibrium.

Figure 1 shows the effect of a one standard deviation positive shock to the money supply on the nominal interest rate and hours worked in the original Christiano-Eichenbaum (1991b) model. The impulse response functions are computed by starting the economy in its nonstochastic steady state and tracing out the model's response to a monetary injection at  $t=10$ .<sup>5</sup> The graphs show that, indeed, the Christiano-Eichenbaum model gives rise to liquidity effects associated with the policy shock: the nominal interest rate falls and hours worked increases in response to a monetary easing. The liquidity effect does not persist, however. Because the money supply process displays positive serial correlation, the surprise injection raises inflationary expectations in periods following the shock. Thus, for  $t \geq 11$ , the model displays the usual effects found in cash-in-advance models: the interest rate increases and hours worked declines. Eventually, the effects of the shock die out and the economy returns to its steady state.

Figure 2 shows the impulse response functions for economies in which, for a given value of  $\lambda$ ,  $a_b$  is chosen so that the standard deviation of the loan-deposit rate spread in the model matches the standard deviation

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<sup>5</sup>The interest rate in figures 1 and 2 is  $(R_{2t})^4$ : the second-part-of-period deposit rate, expressed in annualized terms. Hours worked are expressed as a fraction of steady state hours worked.

of the commercial paper-small CD rate spread in the US data. With  $\lambda$  held fixed, the standard deviation of  $R_{2t} - R_{2t}^d$  is strictly increasing as a function of  $a_b$ ; the interest rate spread becomes increasingly variable as the intermediary's costs increase. Thus, for each value of  $\lambda$ , there is a unique value of  $a_b$  that allows the model to match the standard deviation of the spread from the data. For any fixed value of  $a_b$ , the standard deviation of the spread in the model is strictly decreasing as a function of  $\lambda$ ; the spread becomes more variable as the household's costs decrease. Thus, the value of  $a_b$  that allows the model to match the data increases as  $\lambda$  becomes larger. For  $\lambda=1$ ,  $a_b=0.03$ ; for  $\lambda=3$ ,  $a_b=0.04$ ; for  $\lambda=5$ ,  $a_b=0.09$ ; and for  $\lambda=7$ ,  $a_b=0.45$ . Note that these figures imply that as  $\lambda$  increases, so do the total transactions costs of intermediaries and households combined.

For the first two cases, in which the household's transactions costs are 1 or 3 times the magnitude of the intermediary's costs, figure 2 reveals that the added financial flexibility that our model offers leads to the complete elimination of the dominant liquidity effect seen in figure 1. In response to a surprise monetary injection, the interest rate rises and hours worked fall, both reflecting the effects of higher expected inflation. For the third case, in which the household's costs are 5 times those of the intermediary, the liquidity effect returns. Relative to the benchmark case shown in figure 1, however, the decline in interest rates and the increase in hours worked is significantly dampened by the cash management efforts of intermediaries and households. Only for the final

case, where household costs are 7 times those of the intermediary, are the liquidity effects similar in magnitude to those in figure 1.

In order to offset the effects of monetary shocks in our model, agents must transfer money across markets in spite of transactions costs. For each of the four parameterizations shown in figure 2, table 2 summarizes the contemporaneous response of the example economy to a one standard deviation positive money shock. For comparison, the table also includes figures for the economy with no transactions costs ( $a_B=a_H=0$ ) and the original Christiano-Eichenbaum economy with infinite transactions costs ( $a_B=a_H=\infty$ ). Column 3 of the table shows that the contemporaneous response of the interest rate to the surprise monetary injection is positive in the economy with no transactions costs and in the economies with  $\lambda=1$  and  $\lambda=3$ , where the liquidity effects are dominated by the expected inflation effects. The liquidity effects return in the remaining examples and are largest, of course, in the original Christiano-Eichenbaum model.

Columns 4 and 5 of table 2 report the size of the transfer  $B_{2t}$  as a ratio of the total money supply and the money shock, respectively. Since the money shock is positive, households wish to withdraw funds from their savings accounts after the shock; hence  $B_{2t}$  is negative. With no transactions costs, the model reverts to a standard cash-in-advance model and agents actually over compensate for the shock, so that  $B_{2t}/\epsilon_{xt} < -1$ . They do this in an effort to smooth consumption, which requires additional transaction balances to offset lower second-part-of-period wage payments. Wage payments fall because firms hire less labor in the face of the inflation tax. As transaction costs increase, agents transfer a smaller fraction of the monetary innovation from their savings balances. Transfers



are precluded altogether in the original Christiano-Eichenbaum model, so  $B_{2t}=0$  in this case.

Column 6 of table 2 shows the household's marginal time cost  $-T_H'(B_{2t}/M_t)$  of cash management following the surprise injection, expressed in minutes per quarter year.<sup>6</sup> It is assumed, following Christiano (1991), that the model's time endowment of 1 unit per period represents 1460 hours per quarter in real time. These figures indicate that with  $\lambda=1$ , the representative household would have to spend an additional 17.5 minutes to withdraw an additional unit of money following its optimal response to the monetary shock. With  $\lambda=3$ , the household would require an additional 50 minutes per quarter to withdraw an extra unit of money. With  $\lambda=5$ , the marginal cost is 91 minutes per quarter, and with  $\lambda=7$ , the marginal cost is 123 minutes per quarter.

With a disaggregated interpretation of the transactions cost functions, the results for  $\lambda=1$  imply that only agents who face a fixed time cost of less than 17.5 minutes per quarter will engage in cash management after the money shock. With this parameterization, most of the agents do adjust their portfolios in response to the shock, since most of the open market operation is offset. Introspection indicates that this parameterization implies quite low transactions costs. On the other hand, when  $\lambda=7$  the marginal household requires over two hours per quarter to adjust its portfolio. Almost no households find it worthwhile to transfer

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<sup>6</sup>Since  $T_H'(B_{2t}/M_t)$  is the household's marginal cost of making an additional deposit,  $-T_H'(B_{2t}/M_t)$  is its marginal cost of making an additional withdrawal.

funds between accounts in this case. Table 2 suggests, therefore, that liquidity effects continue to dominate in our generalized version of the Christiano-Eichenbaum model only when the household's marginal costs of cash management are very large. With small or moderate marginal transactions costs, the liquidity effect is either eliminated or significantly dampened.

## 6. Conclusion

In this paper we have experimented with relaxing the extreme restrictions imposed by CIA constraints on flows of funds across markets. These constraints may be appropriate in models where the period length is interpreted as one day or one week, but at the quarterly horizon agents are likely to have access to a more flexible transactions technology.

Introducing such a transactions technology into the model of liquidity effects developed by Christiano and Eichenbaum (1991b) reveals that this model has a implication not previously considered in the literature: it predicts that the spread between interest rates on loans and deposits should be systematically related to shocks to the nominal money supply. When the transaction technologies are parameterized so that the model matches the behavior of interest rate spreads in the US data, the ability of the Christiano-Eichenbaum model to explain liquidity effects is substantially reduced. Only when marginal transactions costs are quite high does the model continue to predict that the interest rate will fall and hours worked will rise in response to a surprise monetary injection. For reasonable parameterizations, the liquidity effects vanish completely.

We believe that our results cast doubt on the usefulness of this class of models for studying liquidity effects at business cycle frequencies. Research efforts should return to the more careful methodology of building financial structure from microfoundations or, alternatively, be directed toward extending other classes of models that can also generate negative correlations between nominal interest rates and money. Fuhrer and Moore (1992), for instance, alter the Phelps-Taylor contracting model by specifying staggered contracts that are negotiated in real rather than nominal terms. In this setting they can generate inflationary persistence that is consistent with the data as well as generate liquidity effects.

The model in Goodfriend (1987), which has no nominal rigidities, can also produce correlations consistent with liquidity effects. In that model, purposeful behavior by the Fed can set up negative correlations between the federal funds rate and money. If the Fed wishes to reduce inflation, it can do so by reducing the future money supply. Due to anticipated inflation effects, the nominal interest rate would then fall, increasing the demand for money. If the Fed were also concerned with price level surprises, it could supply money today in order to prevent price level movements. The outcome is a negative correlation between interest rates and money. Of course, this mechanism is not what is thought of as a liquidity effect, but the example shows how Fed behavior rather than the presence of financial rigidities can be largely responsible for any negative correlations between money and nominal interest rates that can be found in the data.

FIGURE 1

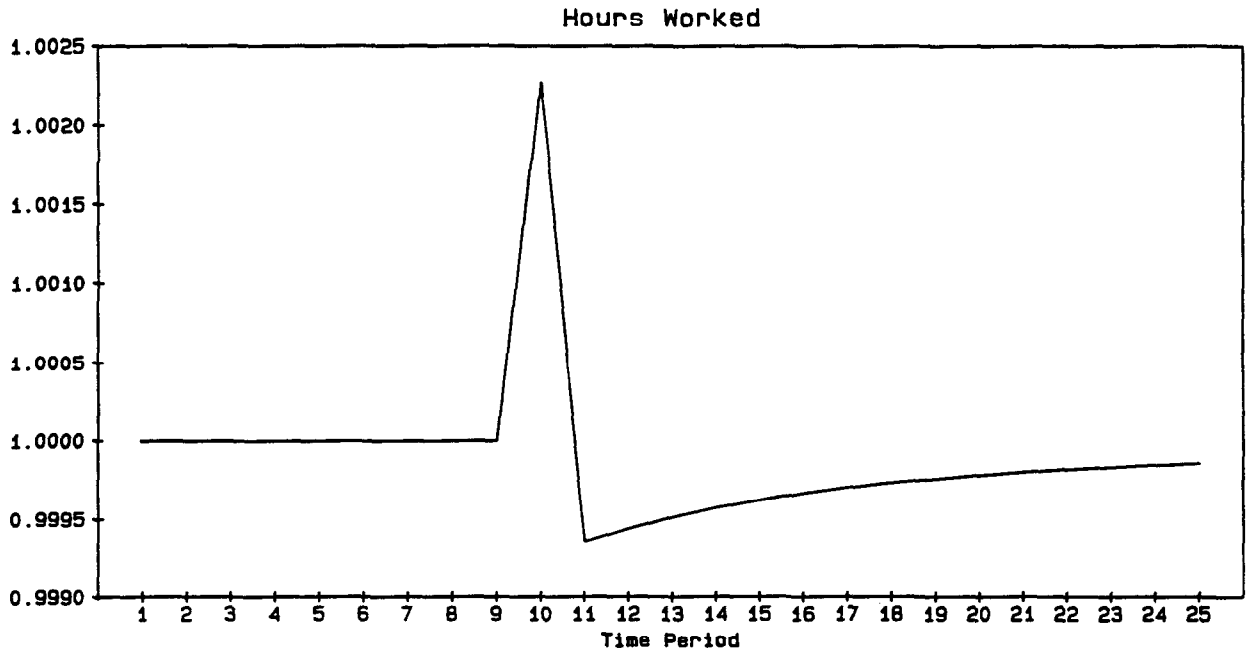
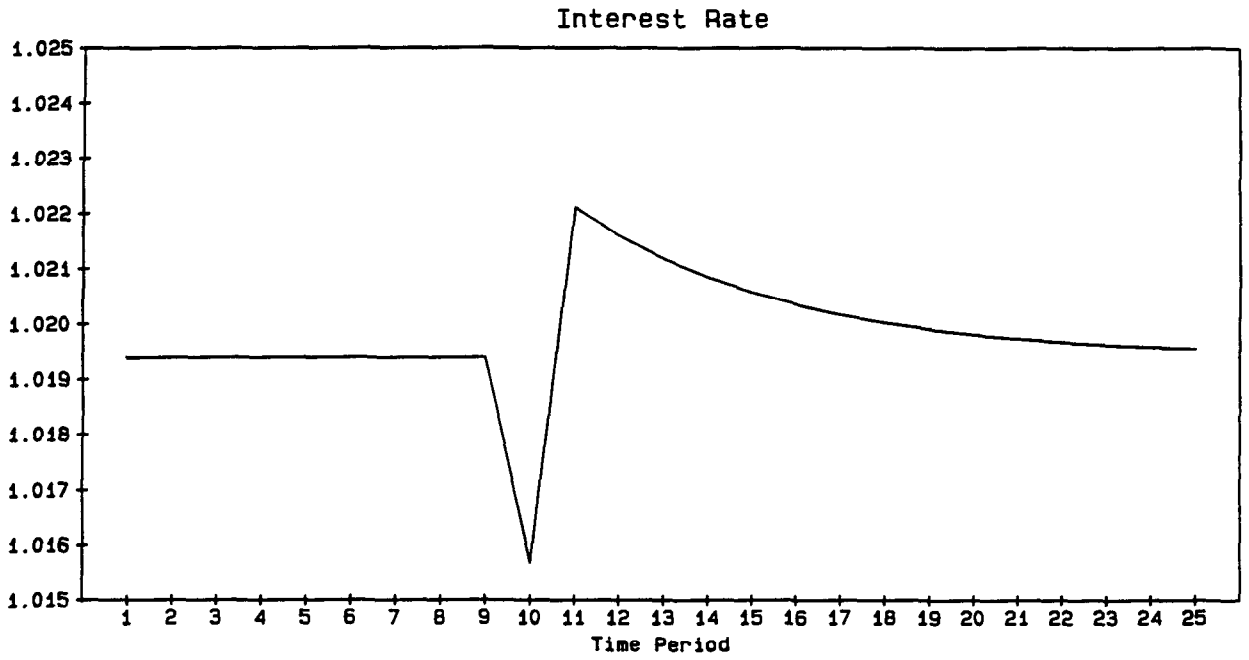


FIGURE 2

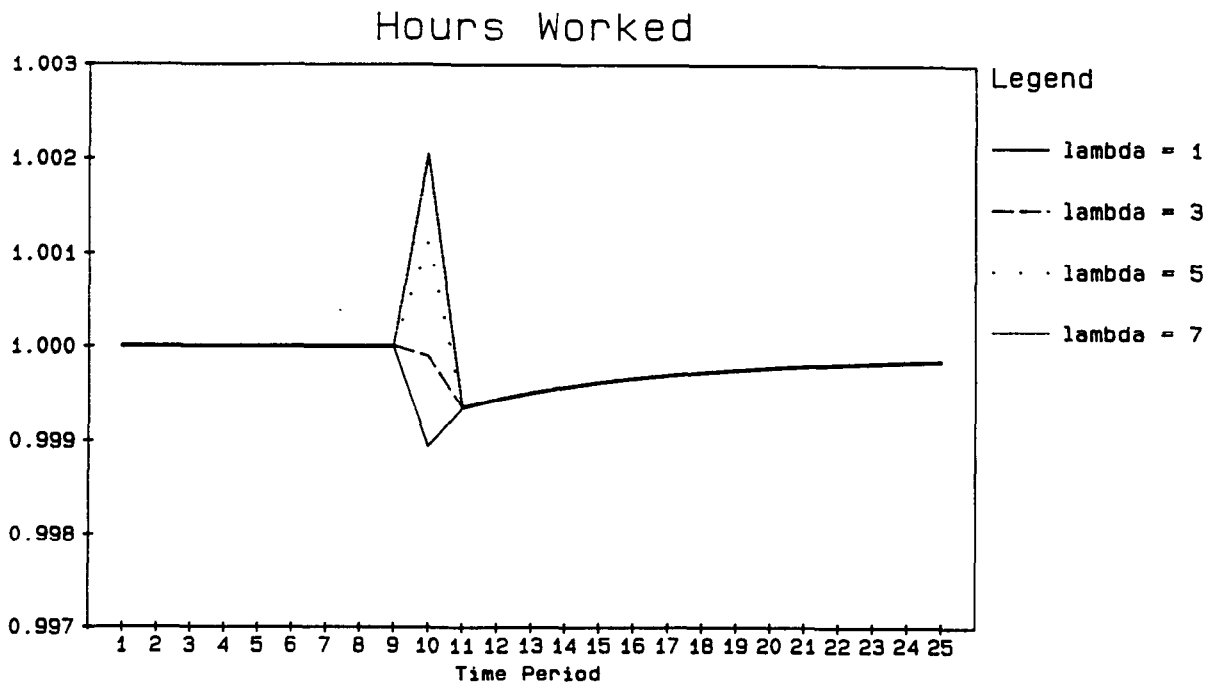
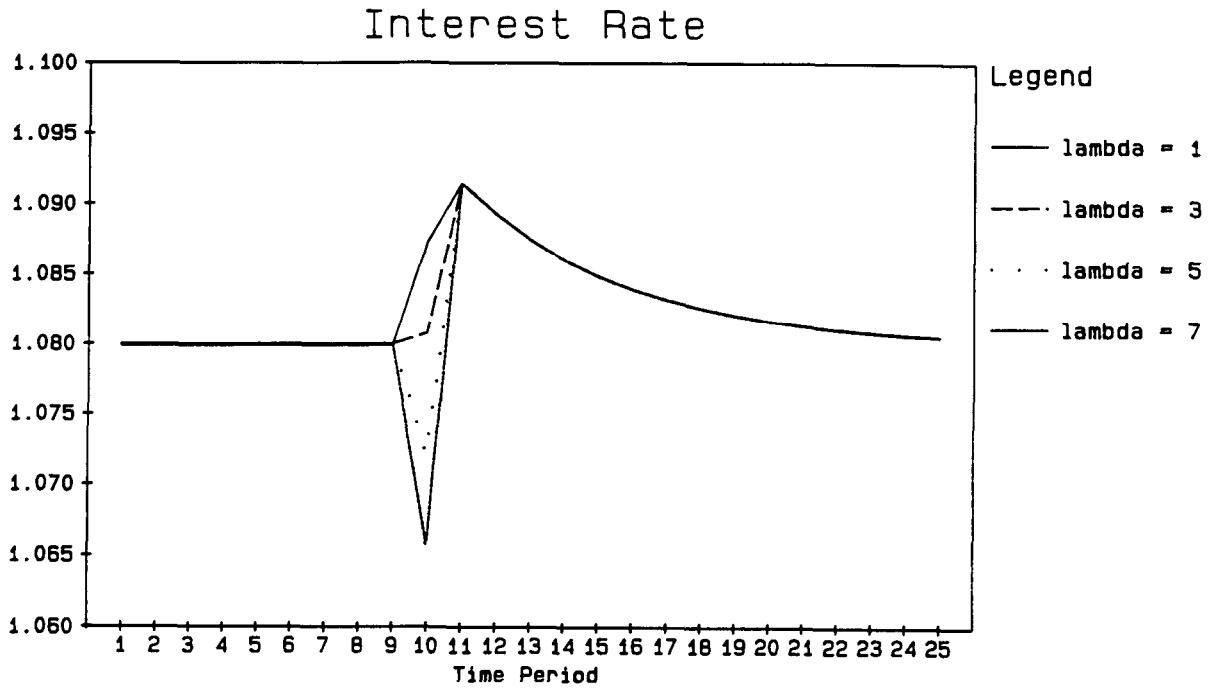


TABLE 1

Response of the Economy to a Doubling of the Money Supply  
When Transaction Costs are Zero

|         | Nonstochastic Steady State | Doubling Money-<br>( $\rho_x=0$ ) | Doubling Money-<br>( $\rho_x=.81$ ) |
|---------|----------------------------|-----------------------------------|-------------------------------------|
| $B_1$   | .886                       | .886                              | .886                                |
| $B_2$   | 0                          | -.557                             | -1.063                              |
| $H_1$   | .109                       | .109                              | .109                                |
| $H_2$   | .109                       | .109                              | .050                                |
| $W_1$   | 4.070                      | 4.070                             | 4.070                               |
| $W_2$   | 4.070                      | 8.140                             | 7.573                               |
| P       | 1.325                      | 2.650                             | 3.574                               |
| $W_2/P$ | 3.071                      | 3.071                             | 2.119                               |
| C       | .754                       | .754                              | .560                                |
| $R_1$   | 1.007                      | 1.007                             | 1.007                               |
| $R_2$   | 1.007                      | 1.007                             | 1.707                               |

Notes: The table presents equilibrium prices and quantities for the economy with zero transactions costs ( $a_B=a_H=0$ ), which is similar to a basic cash-in-advance model. Column 1 describes the nonstochastic steady state. Columns 2 and 3 describe the economy after an unanticipated doubling of the money supply with  $\rho_x=0$  and  $\rho_x=0.81$ .

TABLE 2

Cash Management Response to a One Standard Deviation  
Monetary Shock  
( $\varepsilon_x=0.00410$ )

| 1         | 2        | 3               | 4            | 5                         | 6                   |
|-----------|----------|-----------------|--------------|---------------------------|---------------------|
| $\lambda$ | $a_B$    | $\Delta R_{2t}$ | $B_{2t}/M_t$ | $B_{2t}/\varepsilon_{xt}$ | $-T'_H(B_{2t}/M_t)$ |
| 0         | 0        | 0.799           | -0.00435     | -1.06                     | 0                   |
| 1         | 0.03     | 0.399           | -0.00333     | -0.821                    | 17.5                |
| 3         | 0.04     | 0.0339          | -0.00239     | -0.583                    | 50.3                |
| 5         | 0.09     | -0.447          | -0.00115     | -0.282                    | 91.0                |
| 7         | 0.45     | -0.808          | -0.000224    | -0.0545                   | 123.4               |
| $\infty$  | $\infty$ | -0.895          | 0            | 0                         | $\infty$            |

Notes: The table describes each example economy's contemporaneous response to a positive, one-standard deviation monetary shock. The parameters  $\lambda$  and  $a_B$  describe the household and intermediary's transactions technologies as indicated in the text. The example with  $\lambda=0$  and  $a_B=0$  is similar to a basic cash-in-advance model; the example with  $\lambda=\infty$  and  $a_B=\infty$  is equivalent to the original Christiano-Eichenbaum model.  $\Delta R_{2t}$  is the percentage change in the quarterly loan rate, expressed as a fraction of the monetary shock.  $B_{2t}/M_t$  and  $B_{2t}/\varepsilon_{xt}$  are second-part-of-period deposits, expressed as a fraction of the total money supply and the monetary shock.  $-T'_H(B_{2t}/M_t)$  is the household's marginal transactions costs following the shock, expressed in minutes per quarter.

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## Appendix: Derivation of Equilibrium Conditions

This appendix shows that the behavior of equilibrium prices and quantities is completely described by equations (21)-(25). Let  $u_{ct}$ ,  $u_{jt}$ ,  $f_{kt}$ ,  $f_{H_1t}$ , and  $f_{H_2t}$  be as defined in the text. The firm's first order conditions for  $K_{t+1}$ ,  $H_{1t}$ , and  $H_{2t}$  are given by

$$(A.1) \quad E \left[ \frac{u_{ct+1}}{P_{t+1}} P_t - \frac{\beta u_{ct+2}}{P_{t+2}} f_{kt+1} P_{t+1} ; \Omega_t^1 \right] = 0$$

$$(A.2) \quad E \left[ \frac{u_{ct+1}}{P_{t+1}} (W_{1t} R_{1t} - P_t f_{H_1t}) ; \Omega_t^1 \right] = 0$$

$$(A.3) \quad W_{2t} R_{2t} - P_t f_{H_2t} = 0.$$

The household's first order conditions for  $L_{1t}$ ,  $L_{2t}$ ,  $L_{3t}$ ,  $B_{1t}$ , and  $B_{2t}^d$  are

$$(A.4) \quad E \left[ u_{jt} - W_{1t} \frac{u_{ct}}{P_t} ; \Omega_t^1 \right] = 0$$

$$(A.5) \quad u_{jt} - W_{2t} \frac{u_{ct}}{P_t} = 0$$

$$(A.6) \quad u_{jt} - W_{3t} \frac{u_{ct}}{P_t} = 0$$

$$(A.7) \quad E \left[ \frac{u_{ct}}{P_t} - \frac{\beta u_{ct+1}}{P_{t+1}} R_{1t}^d ; \Omega_t^1 \right] = 0$$

$$(A.8) \quad E \left[ \frac{u_{ct}}{P_t} + \frac{u_{jt} T'_H (B_{2t}^d / M_t)}{M_t} - \frac{\beta u_{ct+1}}{P_{t+1}} R_{2t}^d ; \Omega_t^2 \right] = 0,$$

The intermediary's first order conditions for  $N_{1t}$  and  $B_{2t}^s$  are

$$(A.9) \quad E \left[ \begin{array}{c} \frac{\beta u_{ct+1}}{P_{t+1}} (R_{1t} - R_{2t}) \quad ; \quad \Omega_t^1 \end{array} \right] = 0$$

$$(A.10) \quad R_{2t}^d + R_{2t} \frac{W_{3t}}{M_t} T'_B(B_{2t}/M_t) - R_{2t} = 0.$$

Note that the market clearing condition  $B_{2t}^d = B_{2t}^s = B_{2t}$  has been substituted into both (A.8) and (A.10).

Other equilibrium conditions in the costly household transactions model include the household's cash-in-advance constraint

$$(A.11) \quad P_t C_t = M_t - B_{1t} - B_{2t} + W_{1t} L_{1t} + W_{2t} L_{2t} + W_{3t} L_{3t}$$

and the firm's cash-in-advance constraints,

$$(A.12) \quad N_{1t} = W_{1t} H_{1t}$$

$$(A.13) \quad N_{2t} = W_{2t} H_{2t},$$

all of which will hold with equality as indicated so long as net nominal interest rates are positive, the market clearing conditions

$$(A.14) \quad N_{1t} + N_{2t} + W_{3t} T_B(B_{2t}/M_t) = B_{1t} + B_{2t} + X_t$$

$$(A.15) \quad C_t + K_{t+1} = K_t^\alpha (z_t H_t)^{1-\alpha} + (1-\delta)K_t = f(K_t, H_{1t}, H_{2t}, z_t)$$

$$(A.16) \quad L_{1t} = H_{1t}$$

$$(A.17) \quad L_{2t} = H_{2t}$$

$$(A.18) \quad L_{3t} = T_B(B_{2t}/M_t),$$

and the zero-profit condition

$$(A.19) \quad R_{1t}^d B_{1t} = R_{1t} N_{1t} + R_{2t} (B_{1t} - N_{1t}).$$

Equations (A.1)-(A.19) represent 19 nonlinear equations in the 19 unknowns

$B_{1t}, B_{2t}, C_t, N_{1t}, N_{2t}, H_{1t}, H_{2t}, L_{1t}, L_{2t}, L_{3t}, K_{t+1}, P_t, W_{1t}, W_{2t}, W_{3t}, R_{1t}^d, R_{2t}^d, R_{1t},$  and  $R_{2t}.$

Equations (A.5) and (A.6) imply that  $W_{2t} = W_{3t}$ , which can be used to eliminate  $W_{3t}$  from the system. Equation (A.10), which implies that

$$R_{2t}^d = R_{2t} - R_{2t} \frac{W_{2t}}{M_t} T'_B(B_{2t}/M_t),$$

can be used to eliminate  $R_{2t}^d$  from the system. Equation (A.19) implies that

$$R_{1t}^d = \omega_{1t} R_{1t} + (1-\omega_{1t}) R_{2t},$$

where  $\omega_{1t} = N_{1t}/B_{1t}$ , which can be used to eliminate  $R_{1t}^d$  from the system.

Equations (A.16)-(A.18) can be used to eliminate  $L_{1t}$ ,  $L_{2t}$ , and  $L_{3t}$ :

$$L_{1t} = H_{1t} \quad L_{2t} = H_{2t} \quad L_{3t} = T_B(B_{2t}/M_t).$$

Equations (A.2) and (A.3) can be used to eliminate  $R_{1t}$  and  $R_{2t}$ :

$$R_{1t} = \frac{E \left[ (u_{ct+1}/P_{t+1}) P_t f_{H_1t} \quad ; \quad \Omega_t^1 \right]}{E \left[ (u_{ct+1}/P_{t+1}) W_{1t} \quad ; \quad \Omega_t^1 \right]}$$

$$R_{2t} = P_t f_{H_2t} / W_{2t}.$$

Equation (A.5) implies that

$$W_{2t} = u_{jt} P_t / u_{ct},$$

which can be used to eliminate  $W_{2t}$ , while equations (A.12) and (A.13) can be used to eliminate  $N_{1t}$  and  $N_{2t}$ .

We have used the 11 equations (A.2), (A.3), (A.5), (A.6), (A.10), (A.12), (A.13), and (A.16)-(A.19), to eliminate the 11 unknowns  $R_{1t}$ ,  $R_{2t}$ ,  $W_{2t}$ ,  $W_{3t}$ ,  $R_{2t}^d$ ,  $N_{1t}$ ,  $N_{2t}$ ,  $L_{1t}$ ,  $L_{2t}$ ,  $L_{3t}$ , and  $R_{1t}^d$ . Substituting these results into the remaining equations yields a system of 8 equations in the 8 unknowns  $B_{1t}$ ,  $B_{2t}$ ,  $C_t$ ,  $H_{1t}$ ,  $H_{2t}$ ,  $K_{t+1}$ ,  $P_t$ , and  $W_{1t}$ :

$$(A.1) \quad E \left[ \begin{array}{c} \frac{u_{ct+1}}{P_{t+1}} P_t - \frac{\beta u_{ct+2}}{P_{t+2}} f_{kt+1} P_{t+1} \quad ; \quad \Omega_t^1 \end{array} \right] = 0$$

$$(A.4) \quad E \left[ \begin{array}{c} u_{jt} - W_{1t} \frac{u_{ct}}{P_t} \quad ; \quad \Omega_t^1 \end{array} \right] = 0$$

$$(A.7') \quad E \left[ \begin{array}{c} \frac{u_{ct}}{P_t} - \frac{\beta u_{ct+1}}{P_{t+1}} \frac{u_{ct}}{u_{jt}} f_{H2t} \quad ; \quad \Omega_t^1 \end{array} \right] = 0$$

(A.8')

$$E \left[ \begin{array}{c} \frac{u_{ct}}{P_t} + \frac{u_{jt} T'_H (B_{2t}/M_t)}{M_t} - \frac{\beta u_{ct+1}}{P_{t+1}} f_{H2t} \left[ \frac{P_t T'_B (B_{2t}/M_t)}{M_t} - \frac{u_{ct}}{u_{jt}} \right] \quad ; \quad \Omega_t^2 \end{array} \right] = 0$$

$$(A.9') \quad E \left[ \begin{array}{c} \frac{\beta u_{ct+1}}{P_{t+1}} \left( \frac{P_t}{W_{1t}} f_{H1t} - \frac{u_{ct}}{u_{jt}} f_{H2t} \right) \quad ; \quad \Omega_t^1 \end{array} \right] = 0$$

$$(A.11') \quad P_t = \frac{M_t + X_t}{C_t}$$

$$(A.14') \quad H_{2t} = - \frac{u_{ct}}{u_{jt} P_t} [W_{1t} H_{1t} - B_{1t} - B_{2t} - X_t] - T'_B (B_{2t}/M_t)$$

$$(A.15') \quad C_t = f(K_t, H_{1t}, H_{2t}, z_t) - K_{t+1}.$$

Equations (A.11'), (A.14'), and (A.15') can now be used to eliminate the variables  $P_t$ ,  $H_{2t}$ , and  $C_t$  from the system. Substituting these variables out of (A.1), (A.4), (A.7'), (A.8'), and (A.9') reduces the 8-equation system to a five equation system in the unknowns  $B_{1t}$ ,  $B_{2t}$ ,  $H_{1t}$ ,

$K_{t+1}$ , and  $W_{1t}$ . Equations (21)-(25) in the text express these 5 equations in their original forms (A.1), (A.4), (A.7), (A.8), and (A.9). The 5-equation system can be linearized and solved by the undetermined coefficient method outlined by Christiano (1991) and Christiano and Eichenbaum (1991a).