Working Paper Series

Endogenous Financial Innovation and the Demand for Money

WP 92-03

Peter Ireland
Federal Reserve Bank of Richmond

THE FEDERAL RESERVE BANK OF RICHMOND
RICHMOND ■ BALTIMORE ■ CHARLOTTE

Working Paper 92-3

ENDOGENOUS FINANCIAL INNOVATION AND THE DEMAND FOR MONEY

Peter N. Ireland*

Economist
Research Department
Federal Reserve Bank of Richmond
PO Box 27622
Richmond, VA 23261-7622
804-697-8262

December 1992

Abstract

This paper embeds two key ideas about the nature of financial innovation taken from the empirical literature into a familiar equilibrium monetary model. It provides formal support for several alternative econometric specifications for money demand that attempt to capture the effects of financial innovation and demonstrates that a popular theoretical model of money demand, when suitably modified, can account for some unusual monetary dynamics found in the data. Thus, it helps to establish both the theoretical relevance of recent empirical work and the empirical relevance of recent theoretical work on the demand for money.

I. Introduction

Over the last two decades, an enormous body of literature has documented the continuing instability of standard econometric money demand specifications and attributed the instability to innovation in the private financial sector.[1] In contrast, almost no theoretical work has considered the possibility that financial innovation may have important effects on the demand for money.[2] This paper seeks to fill this gap on the theoretical side of monetary economics by embedding two key ideas about the nature of financial innovation taken from the empirical literature into a familiar equilibrium monetary model. It provides formal support for several alternative econometric specifications for money demand that attempt to capture the effects of financial innovation and demonstrates that a popular theoretical model of money demand, when suitably modified, can account for some unusual monetary dynamics found in the data. Thus, it helps to establish both the theoretical relevance of recent empirical work and the empirical relevance of recent theoretical work on the demand for money.

As its starting point, this study takes two landmark pieces by Stephen M. Goldfeld (1973, 1976). Goldfeld's earlier work (1973) finds that a single-equation econometric model expressing the demand for real M1 as a stable function of real GNP and nominal interest rates does a remarkably good job of characterizing quarterly US data from 1952-1972, as judged both by the accuracy of its forecasts and by the inability of a Chow test to reject the hypothesis of parameter constancy across subsamples. In work published just three years later, however, Goldfeld

(1976) reports that by the same criteria of the accuracy of forecasts and the results of Chow tests, the performance of his money demand equation deteriorates markedly when the sample period is extended to 1976. In fact, money demand regressions continue to be plagued by instability when the sample runs through the present day, with their forecasts systematically overpredicting actual real M1 figures for the late 1970's and underpredicting actual figures for the 1980's (Goldfeld and Sichel 1990).[3]

The years during which standard money demand equations broke down also witnessed the proliferation of a number of assets that appear to be very close substitutes for demand deposits, including NOW accounts and security repurchase agreements, as well as the development of a variety of new cash management techniques used by firms to economize on their real balances. As a result, Goldfeld's findings launched an extensive research program directed at repairing the conventional specification by taking the effects of these financial innovations on the demand for money into account. Lieberman (1977), for example, includes a time trend in his money demand regression as a crude proxy for the improvement in cash management techniques made possible by the application of new technologies in the financial sector.

An alternative approach to modifying the standard equation, used by Goldfeld (1976) himself, as well as by Enzler, Johnson, and Paulus (1976), Simpson and Porter (1980), and Cagan (1984) includes a past peak, or ratchet, interest rate as an additional independent variable based upon an argument that can be traced back to Duesenberry (1963). If the process of financial innovation involves significant initial fixed costs because of the need for newly-trained personnel, newly-developed computer equipment, or a newly- established secondary market for a new security, then the decision to innovate might not be made unless the opportunity costs of continuing to hold higher money balances instead--as measured by the nominal interest rate--exceed some threshold level. Conversely, once these fixed costs have been incurred, the new product might not be immediately abandoned should interest rates fall. In addition, if the initial costs of bringing a new financial service on line are quite high, there may be a lag between the decision to innovate and the actual change in money demand as these costs are spread over time.[4] Thus, the current level of real balances will be found to depend not only on how high nominal interest rates are today, but also on how high they have been in the past.

Other studies employ more direct measures of financial innovation. Kimball (1980) and Dotsey (1984, 1985) point out that since many cash management procedures used by firms to economize on their demand deposit balances involve the transfer of idle funds by wire into overnight interest-bearing accounts, the number of electronic funds transfers is likely to be highly correlated with the use of innovative financial techniques. Dotsey (1984) notes that in contrast to a time trend, which captures only changes in the costs of financial innovation from technological change, and in contrast to the ratchet variable, which proxies only for changes in the potential benefits of financial innovation from peaks in nominal interest rates, the wire transfer approach recognizes that the rate of innovation depends jointly on changes in costs and benefits. In equilibrium, the extent to which resources are devoted to the process of financial innovation is determined by agents who weigh the costs of computer and telecommunications services against the benefits of recapturing the interest income foregone by holding cash, just as the extent to which resources are devoted to any other investment project is based on an assessment of both costs and benefits. Dotsey (1984) reports that while trend and ratchet variables both aid in explaining changes in the demand for money, equations with the wire transfer variable perform best.

This paper takes two key ideas from the empirical work on financial innovation. First, as in Dotsey (1984), the process of financial innovation is regarded as an investment project. The decision to allocate resources to this investment project is made by agents who balance its costs against its benefits, so that in equilibrium, the level of financial innovation is endogenous. Second, as in the ratchet variable literature, the process of financial innovation is assumed to involve a significant initial fixed cost, the presence of which may complicate the relationship between money demand and interest rates when rates are high and volatile. These key ideas are embedded here into the specification of a general equilibrium model of financial innovation.

The model extends Lucas and Stokey's (1983) interpretation of the cash-in-advance framework to account for the effects of financial innovation.[5] The model features a pure exchange economy. Hence, it focuses on how financial innovation affects consumers's demand for money, although its implications are compared to results from empirical studies that aggregate household and firm behavior. Presumably, a more elaborate version of the model with production opportunities would allow innovations that facilitate firms's cash management activities to be considered explicitly as well.

In the pure exchange economy, agents buy and sell a large number of differentiated goods in a large number of spatially distinct markets. Improvements over time in communications and record-keeping technologies, brought about by irreversible investment in financial capital, enable shoppers to purchase goods on credit in markets where money was once required. Thus, the model's financial sector resembles a credit card network; financial innovation allows credit cards to be used in a wider range of transactions. White (1976), Garcia (1977), and Dotsey (1984, 1985) all present evidence that increases in credit card use have been associated with decreases in money demand in the United States economy; the model's implications are consistent with this evidence.

The model is specified at the level of preferences, endowments, and technologies in the next section. Competitive equilibria for the model economy are characterized analytically in section III and numerically in section IV, both to provide theoretical support for the empirical specifications surveyed above and to demonstrate that the model is capable of generating artificial series that share some of the features of the data uncovered by the empirical work. Section V concludes by pointing to some implications for model-building and for policy-making.

II. A Model of Endogenous Financial Innovation and the Demand for Money

A discrete time, infinite horizon, perfect foresight economy is imagined to consist of a continuum of markets arranged around the boundary of a circle having unit circumference.[6] By arbitrarily selecting one of these markets as market 0, each is given a name $i \in [0,1)$ corresponding to its distance, moving clockwise around the circle, from market 0. A unique perishable consumption good is traded in each market, so goods are also indexed by $i \in [0,1)$, corresponding to the locations at which they are bought and sold. The commodity space in this economy is then defined as $L = \prod_{t=1}^{\infty} L_t$, where

$$L_t = \{ c_t \mid c_t : [0,1) \mapsto \mathbb{R} \text{ piecewise continuous } \}.$$

There is also a continuum of infinitely-lived households in the economy, with names $j \in [0,1)$. Household j's endowment of good i at time t is denoted by $e_t^j(i)$, its consumption of good i at time t by $c_t^j(i)$. Household j is imagined to inhabit a region on the boundary of the circle including markets $i \in [j, j+\varepsilon)$, where $1 > \varepsilon > 0$, and is endowed at the beginning of every period with positive amounts of each of the goods traded in those markets.[7] For simplicity, it is assumed that household j's endowment is distributed uniformly on $[j, j+\varepsilon)$, so that

$$e_t^j(i) = \begin{cases} e_t > 0 \text{ for } i \in [j, j+\varepsilon) \\ 0 \text{ otherwise} \end{cases}$$

where e_t does not depend on j. Thus, for each t, the aggregate endowment is a constant function $\bar{e}_t(i) = \bar{e}_t$.

Households have identical preferences defined on the consumption sets $X^j = \prod_{t=1}^{\infty} X_t^j$, where

$$X_t^j = \{ c_t^j : c_t^j : [0,1) \mapsto [0,\infty) \text{ piecewise continuous } \},$$

as represented by the additively time separable utility function

$$U(\{c_t^j\}_{t=1}^{\infty}) = \sum_{t=1}^{\infty} \beta^t \left\{ \int_0^1 u[c_t^j(i)]di \right\}. \tag{1}$$

It is assumed that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $\lim_{c \to 0} u'(c)=\infty$; $\beta \in (0,1)$ is the discount factor.

For all $i \in [0,1)$ and for all $t \geq 1$, there is an uncountable number of households having an endowment that includes positive amounts of good i at time t. Thus, it is assumed that all markets are competitive. In addition, given the strong symmetry that has been imposed on preferences and endowments, attention is confined to competitive equilibria in which at each date, all goods trade for the same relative prices. Opportunities and objectives are identical across households in such equilibria, so that the behavior of a representative household with endowment on $[0,\varepsilon)$ can be studied with the understanding that all other households will behave symmetrically. Accordingly, the j superscripts are now dropped and equilibrium conditions are expressed in terms of quantities for the representative household.

The assumption $\lim_{c \to 0} u'(c)=\infty$ implies that although the representative household is endowed with goods $i \in [0,\varepsilon)$ only, it will in general demand positive quantities of all goods $i \in [0,1)$ and must therefore obtain goods $i \in [\varepsilon,1)$ through trade with other households. To describe the household's opportunities for trade it is assumed, following Lucas and Stokey (1983), that each household consists of two members: a buyer and a seller. In each period, while the buyer travels around the circle to purchase each of the different consumption goods, the seller remains at home to sell the endowment to buyers from other households. When visiting a market close to home, in the interval $[\varepsilon,x)$, the representative household's buyer is known to the sellers there and is able to make his purchases on credit. Farther from home, in the interval $[x,1)$, the buyer is not known to the sellers and must pay for all purchases with government-issued noninterest-bearing money. Symmetrically, the seller from the representative household is willing to extend credit to buyers he knows, with names on $(1-x,1-\varepsilon]$, but insists on receiving cash from everyone else.

The Lucas-Stokey interpretation of the cash-in-advance framework is extended here by assuming that through a costly process of financial innovation, it is possible for each buyer to become known in more distant markets and thereby make purchases on credit where cash was once required. Formally, this process is modeled by indexing the variable x defined above by time and allowing the representative household to choose x_t at each date t subject to the constraints

$$f(k_t) \geq x_t, \qquad t=1,2,\ldots, \tag{2}$$

where k_t is its stock of financial capital at date t and where the financial production function $f:[0,\infty)\mapsto[0,1)$ is strictly increasing, strictly concave, and twice continuously differentiable. The representative household can increase its stock of financial capital between periods t and t+1 by choosing to invest, rather than consume or sell, $s_t(i)$ units of any good $i\in[0,\varepsilon)$ with which it is endowed during period t; the stock evolves according to

$$(1-\delta)k_t + b_t\left[\int_0^{\varepsilon} s_t(i)di\right] \geq k_{t+1}, \qquad t=1,2,..., \tag{3}$$

$k_1 \geq 0$ given,

where $\delta\in[0,1]$ is the depreciation rate for financial capital and where b_t is a technological parameter governing the rate of transformation between consumption and investment. Increases in b_t capture technological progress exogenous to the financial sector, such as improvements in computer and telecommunications technologies, which make financial innovation less costly over time.[8] The investment process is assumed to be irreversible, so that s_t must be nonnegative for all t. Since all goods trade for the same price, $s_t(\cdot)$ can without loss of generality be restricted to be a constant function $s_t(i)=s_t$ on $i\in[0,\varepsilon)$ and (3) simplifies to

$$(1-\delta)k_t + b_t s_t \varepsilon \geq k_{t+1}, \qquad t=1,2,..., \tag{4}$$

$k_1 \geq 0$ given.

Thus, as suggested by Dotsey (1984), the process of financial innovation is modeled here as an investment project that involves paying an initial cost at time t to purchase goods without money in more distant markets beginning in period t+1. As suggested in the ratchet variable literature, the initial cost is a fixed cost, since it is independent of the dollar volume of goods purchased in each market and since once incurred, it cannot be recovered should the fruits of innovation no longer seem necessary.

The model specification is completed with a description of what happens at the end of each period $t\geq1$ when, after consuming their purchases as well as the fractions of their endowments that remain unsold and uninvested, households convene in a centralized asset market to settle outstanding debts and to accumulate the money balances needed to make cash purchases in the following period. The government participates in this market by making a lump-sum transfer H_t of money to each household (if H_t is negative, this is instead a lump-sum tax). The representative household leaves the asset market at the end of time t with cash holdings denoted M_{t+1}.

Households are assumed to borrow and lend among themselves in the end-of-period asset market by trading in one-period nominally denominated discount bonds. The representative household purchases bonds paying B_{t+1} units of money in the time t+1 asset market for B_{t+1}/R_t units of money in the time t asset market, where R_t is the gross nominal interest rate between t and t+1. The asset market is also open in period 0, when each household receives an initial transfer H_0 of money from the government. Bonds are traded at this time as well; the representative household's initial bond holdings are denoted B_0 and the prevailing interest rate is R_0. Since bonds are available in zero net supply, $B_t=0$ must hold in equilibrium for all $t\geq 0$, as must the market clearing condition $M_{t+1}=M^s_{t+1}$, where the per-household money supply M^s_{t+1} is defined as

$$M^s_{t+1} = \sum_{k=0}^{t} H_k$$

for all $t\geq 0$.

It is now possible to state formally the problem facing the representative household and to define a competitive equilibrium for this economy. In the time 0 asset market, the representative household faces the budget constraint

$$B_0 + H_0 \geq \frac{B_1}{R_0} + M_1. \tag{5}$$

As sources of funds at time $t \geq 1$, the representative household has the income from selling the fraction of its endowment that it chooses not to either consume or invest, the money and bonds carried over from the previous period, and the end-of-period government transfer. As uses of funds, it has purchases of consumption goods as well as the money and bonds to be carried into the next period. It therefore faces the budget constraints

$$\frac{B_t+M_t+H_t}{p_t} + \int_0^{\varepsilon} [e_t(i)-c_t(i)-s_t(i)]di \geq \int_{\varepsilon}^{1} c_t(i)di + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{p_t R_t}, \quad t=1,2,\ldots,$$

where p_t is the nominal price of every consumption good at time t. Since $e_t(i)=e_t$ and $s_t(i)=s_t$ by assumption, these constraints may be rewritten as

$$\frac{B_t+M_t+H_t}{p_t} + (e_t-s_t)\varepsilon \geq \int_0^1 c_t(i)di + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{p_t R_t}, \quad t=1,2,\ldots. \tag{6}$$

The household's money balances at time t must be sufficient to cover its purchases of the goods $i \in [\max\{f(k_t),\varepsilon\},1)$ that must be bought with cash. This requirement gives rise to the cash-in-advance constraints

$$\frac{M_t}{p_t} \geq \int_{\max\{f(k_t),\varepsilon\}}^{1} c_t(i)di, \quad t=1,2,\ldots. \tag{7}$$

Finally, no household is permitted to engage in Ponzi schemes through which it can borrow more than it will ever be able to repay. This requirement enters into the representative household's problem as a nonnegativity constraint for each date t on the sum of the household's current nominal asset position and the nominal value of its future endowments and government transfers, all discounted back to date 0 using the nominal interest rate:

$$W_t = \left[\prod_{s=0}^{t-1} R_s\right]^{-1}\left[M_{t+1}+B_{t+1}/R_t \right] + \sum_{j=t+1}^{\infty}\left\{\left[\prod_{s=0}^{j-1} R_s\right]^{-1}\left[p_j e_j \varepsilon + H_j\right]\right\} \geq 0, \quad (8)$$

$$t=0,1,\ldots.$$

These no-Ponzi-game conditions guarantee that the period-by-period budget constraints (5) and (6) may be combined to obtain an infinite horizon budget constraint indicating that, as of time 0, the discounted present value of the representative household's endowments and transfers must be no less than the discounted present value of its consumption and investment streams.

The representative household solves:

Problem: Maximize by choice of $\{c_t\}_{t=1}^{\infty} \in X$, nonnegative scalars $\{s_t\}_{t=1}^{\infty}$, $\{k_{t+1}\}_{t=1}^{\infty}$, and $\{M_{t+1}\}_{t=0}^{\infty}$, and scalars $\{B_{t+1}\}_{t=0}^{\infty}$ the objective function (1) subject to the constraints (4)-(8), taking B_0, k_1, and the sequences $\{p_t\}_{t=1}^{\infty}$, $\{H_t\}_{t=0}^{\infty}$, and $\{R_t\}_{t=0}^{\infty}$ as given.

A competitive equilibrium is defined by:

Definition: A *competitive equilibrium* consists of initial conditions $B_0=0$ and $k_1\geq 0$ and sequences of quantities $\{c_t, s_t, k_{t+1}, M_t, M_t^s, B_t\}_{t=1}^{\infty}$, prices $\{p_t\}_{t=1}^{\infty}$, and interest rates $\{R_t\}_{t=0}^{\infty}$ such that:

(a) The sequences $\{c_t, s_t, k_{t+1}, M_t, B_t\}_{t=1}^{\infty}$ solve the representative household's problem given B_0, k_1, $\{M_t^s\}_{t=1}^{\infty}$, $\{p_t\}_{t=1}^{\infty}$, and $\{R_t\}_{t=0}^{\infty}$.

(b) Markets clear in every period:

$$\text{(i)} \quad (e_t - s_t)\varepsilon = \int_0^1 c_t(i)di, \qquad t=1,2,\ldots,$$

$$\text{(ii)} \quad M_t = M_t^s, \qquad t=1,2,\ldots,$$

$$\text{(iii)} \quad B_t = 0, \qquad t=1,2,\ldots.$$

As is typical in general equilibrium environments, the prices and quantities consistent with a competitive equilibrium as defined above will also be consistent with competitive equilibria obtaining under a variety of different market arrangements. It could be assumed, for instance, that competitive firms rather than households have access to the financial technology. These firms, acting as financial intermediaries, rent capital to produce and sell financial services to households during each period $t \geq 1$. An argument similar to those in Stokey and Lucas with Prescott (1989, Sec. 2.3) shows that equilibrium prices and quantities under this alternative market structure are identical to those under the original market structure assumed above. Therefore, although the sections to follow characterize competitive equilibria for this model by taking households as the only type of economic agent, the equilibrium outcomes may always be thought of as being generated in an economy in which firms, acting as private financial intermediaries, supply households with financial services at cost.

III. Analytic Results

The task of characterizing competitive equilibria as defined in section II becomes considerably easier when it is assumed first that preferences are logarithmic, so that

$$U(\{c_t\}_{t=1}^{\infty}) = \sum_{t=1}^{\infty} \beta^t \left\{ \int_0^1 \ln[c_t(i)]di \right\}, \tag{9}$$

and second that the government permits the nominal money supply to vary over time as necessary to target a sequence $\{R_t\}_{t=0}^{\infty}$ of nominal interest rates with $R_t>1$ for all $t\geq0$, so that bonds will always dominate money in rate of return and the cash-in-advance constraint will always bind.[9]

Under these additional assumptions, the first order conditions for the representative household's problem are given by

$$\lambda_0 = \frac{\beta(\lambda_1+\mu_1)}{p_1}, \tag{10}$$

$$\frac{\lambda_0}{R_0} = \frac{\beta\lambda_1}{p_1}, \tag{11}$$

$$c_t(i) = \lambda_t^{-1}, \qquad i\in[0,\max\{f(k_t),\varepsilon\}),\ \forall t\geq1, \tag{12}$$

$$c_t(i) = (\lambda_t+\mu_t)^{-1}, \qquad i\in[\max\{f(k_t),\varepsilon\},1),\ \forall t\geq1, \tag{13}$$

$$\theta_t b_t \leq \lambda_t, \qquad \text{with equality if } s_t>0,\ \forall t\geq1, \tag{14}$$

$$\theta_t = \beta(1-\delta)\theta_{t+1} + \beta\chi_{t+1}\mu_{t+1}c_{t+1}[f(k_{t+1})]f'(k_{t+1}), \qquad \forall t\geq1, \tag{15}$$

$$\frac{\lambda_t}{p_t} = \frac{\beta(\lambda_{t+1}+\mu_{t+1})}{p_{t+1}}, \qquad \forall t\geq1, \tag{16}$$

$$\frac{\lambda_t}{p_t R_t} = \frac{\beta\lambda_{t+1}}{p_{t+1}}, \qquad \forall t\geq 1, \tag{17}$$

where λ_0, $\{\lambda_t\}_{t=1}^{\infty}$, $\{\mu_t\}_{t=1}^{\infty}$, and $\{\theta_t\}_{t=1}^{\infty}$ are the nonnegative Lagrange multipliers on (5), (6), (7), and (4), respectively, and the indicator functions χ_{t+1} are defined for all $t\geq 1$ by

$$\chi_{t+1} = \begin{cases} 1 \text{ if } f(k_{t+1})\geq\varepsilon \\ 0 \text{ if } f(k_{t+1})<\varepsilon. \end{cases}$$

There are, in addition, three transversality conditions among the necessary conditions for the household's problem; these are given by

$$\lim_{T\to\infty} \beta^T\theta_T k_{T+1} = \lim_{T\to\infty} \beta^T\lambda_T(M_{T+1}/p_{T+1}) = \lim_{T\to\infty} \beta^T\lambda_T(B_{T+1}/p_{T+1}) = 0. \tag{18}$$

Equations (10)-(13) and (16)-(17) imply that for all $t\geq 1$, the optimal $c_t(i)$ is a step function:

$$c_t(i) = \begin{cases} c_{1t} = \lambda_t^{-1} & \text{for } i\in[0,\max\{f(k_t),\varepsilon\}) \\ c_{2t} = (\lambda_t R_{t-1})^{-1} & \text{for } i\in[\max\{f(k_t),\varepsilon\},1). \end{cases} \tag{19}$$

As in more conventional versions of the cash-in-advance model, a positive nominal interest rate drives a wedge between the representative household's marginal utility of consuming goods that are bought on credit and its marginal utility of consuming goods that must be purchased with cash.

Equation (14) says that the marginal utility of financial capital will be equated to the marginal utility of consumption only when investment in financial innovation is positive. It retells the ratchet variable story by indicating that when the interest rate falls from a peak, so that investment in new financial innovation ceases, the household will continue to have access to the fruits of past innovation, which will no longer be as valuable as they were under higher rates. Equation (15) is analytically similar to a capital asset pricing formula, indicating that the shadow price of financial capital at time t is equal to the discounted sum of the price at time t+1 (after accounting for depreciation) and the time t+1 dividend: the value of the capital in relaxing the cash-in-advance constraint.

If $\chi_{t+1}=1$, equations (16), (17), and (19) imply that

$$\mu_{t+1}c_{t+1}[f(k_{t+1})] = (R_t-1)\lambda_{t+1}c_{2t+1} = (R_t-1)/R_t, \tag{20}$$

which when substituted into the asset pricing equation (15) yields[10]

$$\begin{aligned}\theta_t &= \beta(1-\delta)\theta_{t+1} + \beta\chi_{t+1}[(R_t-1)/R_t]f'(k_{t+1}) \\ &= \beta\sum_{j=0}^{\infty} [\beta(1-\delta)]^j\chi_{t+j+1}[(R_{t+j}-1)/R_{t+j}]f'(k_{t+j+1}).\end{aligned} \tag{21}$$

Using the market clearing condition for goods as well as equations (14), (19), and (21), total investment $s_t\varepsilon$ is found to be

$$\begin{aligned}s_t\varepsilon &= e_t\varepsilon - \int_0^1 c_t(i)di \\ &= e_t\varepsilon - \frac{\max\{f(k_t),\varepsilon\}}{\lambda_t} - \frac{1-\max\{f(k_t),\varepsilon\}}{\lambda_t R_{t-1}} \\ &= e_t\varepsilon + \frac{(1-R_{t-1})\max\{f(k_t),\varepsilon\}-1}{\lambda_t R_{t-1}} \\ &= \max\left\{ e_t\varepsilon + \frac{(1-R_{t-1})\max\{f(k_t),\varepsilon\}-1}{b_t\theta_t R_{t-1}}, 0\right\} \\ &= \max\{s_t^*,0\},\end{aligned} \tag{22}$$

where

$$s_t^* = e_t\varepsilon + \frac{[(1-R_{t-1})/b_tR_{t-1}]\max\{f(k_t),\varepsilon\}-1}{\beta\sum_{j=0}^{\infty}[\beta(1-\delta)]^j\chi_{t+j+1}[(R_{t+j}-1)/R_{t+j}]f'(k_{t+j+1})}. \quad (23)$$

Equations (22) and (23), indicating that investment depends on current and future values of the nominal interest rate, is a nonlinear version of the investment function emerging from the linear-quadratic environment studied by Sargent (1987, pp. 399-401). These equations show how financial innovation is a response to the joint presence of improved technology and high interest rates; neither alone is likely to be sufficient. Holding all else constant, s_t^* becomes negative (so that s_t equals zero) as b_t approaches zero. Thus, for any fixed path $\{R_t\}_{t=0}^{\infty}$ of interest rates, financial innovation will not occur at time t if b_t is too small. On the other hand, s_t^* may also become negative if, with b_t held constant, future interest rates are low enough to make the discounted sum in (23) sufficiently small. In this sense, financial innovation will not occur if interest rates too low.

Since by assumption the cash-in-advance constraint is always binding, the demand for real money balances may be written

$$\frac{M_t}{P_t} = \int_{\max\{f(k_t),\varepsilon\}}^{1} c_t(i)di$$

$$= [1-\max\{f(k_t),\varepsilon\}]c_{2t}$$

$$= \frac{1-\max\{f(k_t),\varepsilon\}}{\lambda_tR_{t-1}}$$

$$= \frac{[1-\max\{f(k_t),\varepsilon\}](e_t-s_t)\varepsilon}{(R_{t-1}-1)\max\{f(k_t),\varepsilon\}+1}, \tag{24}$$

using (19) to substitute for $c_t(i)$ and the third line of (22) to substitute for $\lambda_t R_{t-1}$. The theoretical money demand equation (24) can be used to interpret the performance of the empirical money demand equations discussed in section I. If economic conditions make financial innovation impossible or unnecessary in the model, then $k_t=k$, $s_t=0$, and hence

$$\ln\left(\frac{M_t}{p_t}\right) = \ln\left\{\frac{[1-\max\{f(k),\varepsilon\}]e_t\varepsilon}{(R_{t-1}-1)\max\{f(k),\varepsilon\}+1}\right\} \approx \gamma_0 + \gamma_1\ln(e_t) - \gamma_2 R_{t-1}$$

where $\gamma_0=\gamma_2+\ln\{[1-\max\{f(k),\varepsilon\}]\varepsilon\}$, $\gamma_1=1$, and $\gamma_2=\max\{f(k),\varepsilon\}$, so that as discovered by Goldfeld (1973), money demand will be a stable function of real income and the nominal interest rate.

When innovation is taking place, however, (24) implies that money demand changes over time with the stock of financial capital. Both Lieberman's (1977) time trend and Kimball (1980) and Dotsey's (1984, 1985) electronic funds transfer variable might be thought of as proxies for k_t in (24). Since equation (4) implies that

$$k_t = (1-\delta)^{t-1}k_1 + \sum_{j=1}^{t-1} b_j s_j \varepsilon,$$

k_t depends on $\{s_j\}_{j=1}^{t-1}$, each element of which in turn depends on $\{R_i\}_{i=j-1}^{\infty}$. Thus, if a proxy for k_t is not included in the equation, the demand for money may be found to depend on all past and future interest rates as well as the contemporaneous rate. To the extent that a past peak interest rate summarizes the entire history of interest rate behavior, a ratchet variable specification for money demand will be appropriate. More generally, however, both leads and lags of the interest rate may be needed to account for the role of past and future interest rates in determining current money demand.

In fact, equation (24) indicates that introducing a technology for financial innovation will allow the cash-in-advance model to account for a variety of unusual monetary dynamics. Since little can be said analytically about the properties of (24) under arbitrary patterns of interest rate behavior, numerical methods are applied in the next section to study the behavior of money demand in this model economy in more detail.

IV. Numerical Results

A. Computing Equilibrium Dynamics

Substituting (22)-(23) into (4) and solving the asset pricing equation (21) for θ_{t+1} yields a system of two nonlinear first order difference equations in k and θ,

$$k_{t+1} = (1-\delta)k_t + b_t \max\left\{ e_t\varepsilon + \frac{(1-R_{t-1})\max\{f(k_t),\varepsilon\}-1}{b_t\theta_t R_{t-1}} , 0 \right\} \tag{25}$$

$$\theta_{t+1} = \frac{\theta_t}{\beta(1-\delta)} + \frac{\chi_{t+1}(1-R_t)f'(k_{t+1})}{R_t(1-\delta)} . \tag{26}$$

Along with the boundary conditions $k_1 \geq 0$ given and $\lim_{T\to\infty} \beta^T\theta_T k_{T+1}=0$, (25) and (26) completely describe the dynamic behavior of the model economy given the sequences $\{R_t\}_{t=0}^{\infty}$, $\{e_t\}_{t=1}^{\infty}$, and $\{b_t\}_{t=1}^{\infty}$.

To solve the system (25)-(26) numerically, it is necessary to specify a functional form for the financial technology $f(\cdot)$ and to assign values to the parameters β, δ, and ε. In all of the examples discussed below, $f(\cdot)$ is specialized to

$$f(k_t) = \frac{k_t}{1 + k_t},$$

which, as required, maps $[0,\infty)$ into $[0,1)$ and is strictly increasing, strictly concave, and twice continuously differentiable. The discount factor β is chosen to be 0.99, so that a period in the model represents one quarter in real time.[11] The depreciation rate δ is set equal to zero, since financial capital is imagined to consist primarily of disembodied knowledge, computer hardware, and computer software, which depreciate slowly if at all. Finally, the interval $[0,\varepsilon)$ is chosen to be quite small, with $\varepsilon=0.001$, so as to make the range of goods that the representative household must acquire through trade as large as possible.

Below, a variety of patterns for the time varying parameters $\{R_t\}_{t=0}^{\infty}$, $\{e_t\}_{t=1}^{\infty}$, and $\{b_t\}_{t=1}^{\infty}$ are fed through the model and the effects on the income velocity of money, which using (24) is computed as

$$v_t = \frac{e_t\varepsilon}{[1-\max\{f(k_t),\varepsilon\}]c_{2t}} = \frac{e_t\varepsilon[(R_{t-1}-1)\max\{f(k_t),\varepsilon\}+1]}{[1-\max\{f(k_t),\varepsilon\}](e_t-s_t)\varepsilon}, \qquad (27)$$

are traced out. The first two numerical examples demonstrate how the model economy behaves under the simplest conditions. The next example shows how an upward spike in the nominal interest rate produces a ratchet effect on money demand. The fourth and fifth examples examine the effects of real economic growth and exogenous technological change on the demand for money. A final example generates artificial series that are compared to actual US time series data.

B. Convergence to a Steady State

When $R_{t-1}=R$, $e_t=e$, and $b_t=b$ for all $t\geq1$, (25)-(26) becomes a time-invariant system. In this case, if (k^*,θ^*) is a fixed point of (25)-(26), then $\lim_{t\to\infty} k_t=k^*$ and $\lim_{t\to\infty} v_t=v^*$ whenever k_1 is sufficiently close to k^*. As a first experiment, a constant endowment level e=1000 is chosen so that with $R_{t-1}=1.05$ and $b_t=1$ for all $t\geq1$ and with $k_1=0.05$, velocity (figure 1) converges to a stationary value of 4.41,[12] about what the income velocity of the US monetary aggregate M1-A (currency plus demand deposits) was when nominal rates were around 5% in the mid-1960's (see figure 7).[13]

Figure 2 reveals that similar dynamics are associated with a permanent increase in the nominal rate of interest. Here, the model economy at time 0 is assumed to be in the steady state reached in example 1. When R increases permanently from 1.05 to 1.15, new financial innovations help to gradually push velocity up to a new stationary value of approximately 7.6. In both examples 1 and 2, the costs of the financial innovations that permit velocity to increase are spread out over several years.

C. The Ratchet Effect

Example 3 is identical to example 2 except that the increase in interest rates is only temporary; after rising to 1.15 for five years,

R_{t-1} returns to 1.05 for all t>20. Figure 3 shows that velocity increases in response to higher rates, but does not reach the levels seen when the change in rates is permanent. Velocity remains higher after the interest rate returns to its previous level. Comparing the behavior of velocity in examples 2 and 3, therefore, demonstrates that the demand for money in any given period depends nontrivially on the entire sequence of nominal interest rates. In particular, as suggested in the ratchet variable literature, a past peak in rates has lasting effects on money demand.

D. Economic Growth and Technological Change

Equation (24) indicates that in the absence of financial innovation, the income elasticity of money demand is unity. Thus, when $s_t=0$ and $k_t=k$ for all $t\geq1$, equation (27) implies that velocity depends only on the contemporaneous nominal rate of interest. Series on velocity and income generated by the model may be consistent with the presence of economies of scale in money demand, however, because of the possibility for endogenous financial innovation.

In example 4, the nominal interest rate is held constant over time, with $R_{t-1}=1.05$ for all $t\geq0$, and the parameter b_t is held constant at unity. Real economic growth is captured by increasing the endowment level over time according to

$$e_{t+1} = (1.01)e_t, \qquad t\geq1,$$
$$e_1=1000.$$

The model economy is again assumed to be in the steady state from example 1 as of time 0.

The investment function described by (22) and (23) indicates that, given the sequences $\{R_t\}_{t=0}^{\infty}$ and $\{b_t\}_{t=1}^{\infty}$, and given an initial stock of financial capital k_1, financial innovation will take place only after the endowment exceeds some threshold level. Velocity in figure 4 remains constant until e_t exceeds this threshold level. As long as innovation continues, velocity rises along with income, so that there appear to be economies of scale in the demand for money. Decreasing returns, however, imply that innovation ceases once the stock of financial capital is sufficiently large; hence, velocity eventually levels off again even as income continues to rise.

An econometrician using the artificial series $\{v_t, e_t, R_{t-1}\}_{t=1}^{\infty}$ from example 4 to deduce the income elasticity of money demand without accounting for changes in k_t would find evidence of economies of scale in some subsamples but not in others. If k_t were included along with income and interest rates in a regression equation, however, the income elasticity would be found to be constant at unity. Similarly, using actual data Laidler (1971) and Cagan and Schwartz (1975) conclude from regression equations that do not attempt to account for the effects of financial innovation that the income elasticity of money demand has varied considerably over time in the United States, while Dotsey (1984) reports that estimates of the scale elasticity of money demand increase from 0.31 to approximately 0.90 once various proxies for financial innovation are added to his regression equation.

Since the preferences represented by the logarithmic utility function (9) are homothetic, the ratio c_{2t}/e_t is invariant to increases in e_t and hence by (27) velocity depends on the sequences $\{e_t\}_{t=1}^{\infty}$ and

$\{b_t\}_{t=1}^{\infty}$ only through their effects on the growth rate of capital. Moreover, since

$$b_t s_t = b_t \max \left\{ e_t \varepsilon + \frac{(1-R_{t-1})\max\{f(k_t),\varepsilon\}-1}{b_t \theta_t R_{t-1}} , 0 \right\}$$

$$= \max \left\{ b_t e_t \varepsilon + \frac{(1-R_{t-1})\max\{f(k_t),\varepsilon\}-1}{\theta_t R_{t-1}} , 0 \right\},$$

the growth rate of financial capital depends on the sequences $\{e_t\}_{t=1}^{\infty}$ and $\{b_t\}_{t=1}^{\infty}$ only through the evolution of the product $b_t e_t$. Two sets of sequences $\{e_t, b_t\}_{t=1}^{\infty}$ and $\{\hat{e}_t, \hat{b}_t\}_{t=1}^{\infty}$ such that $e_t b_t = \hat{e}_t \hat{b}_t$ for all $t \geq 1$, therefore, generate exactly the same time paths for velocity. This result is confirmed by example 5, which is identical to example 4 except that instead of holding b_t constant and increasing e_t by 1% per period, e_t is held constant and b_t is increased by 1% per period; the time path for velocity is the same in figure 5 as in figure 4.

Using US time series data, Lucas (1988, p. 146) notes that it is extremely difficult to distinguish the effects of income growth on velocity from those of technological change. Examples 4 and 5 show that these effects can be indistinguishable in theory as well.

E. Comparison with US Data

Money in this model economy, as in most cash-in-advance economies, is used exclusively as a means of exchange and does not bear interest. Its closest analog in the US data, therefore, is M1-A, which includes currency and demand deposits but excludes the interest-bearing checkable deposit component of the broader aggregate M1. Figure 7 compares the behavior of M1-A velocity to that of the six-month commercial paper rate from 1961 through 1991. Nominal rates have peaked on three occasions: in 1969, 1974, and 1981. Hester (1981) notes that periods of rapid innovation in US financial markets coincide with each of these peaks. Following each peak, velocity remained higher even as interest rates returned to levels seen previously; in fact, velocity marched steadily upward as rates became higher and more volatile.

In example 6, a pattern of interest rates stylized after that experienced by the US economy during the past 30 years is fed through the model economy. Rates are assumed to reach ever increasing peaks during periods 1 through 84 (the first 21 years) before declining erratically. The parameters e_t and b_t are both assumed to grow at a rate of 1% per period. The economy begins time 0 in the steady state that would obtain if R_{t-1}, e_t, and b_t were to remain constant at their time 1 levels for all $t \geq 1$.

Figure 6 shows that velocity in the model economy, like velocity in the US data, trends steadily upward, apparently responding very little to contemporaneous movements in the nominal interest rate. An econometrician using the artificial series generated in this example would report on an unstable relationship between velocity, income, and interest rates. As velocity remains permanently higher after each peak in rates, the series would be found to be consistent with ratchet variable specifications for money demand; if data (such as the series for k_t) were available to proxy for the rate of financial innovation, the proxy would be significant in a money demand regression. All of the unusual monetary dynamics associated with financial innovation in the empirical literature are captured by the model in this example.

V. Conclusions and Implications

Conventional versions of the cash-in-advance model have recently been criticized (e.g., Christiano 1991) for failing to account for all but the simplest kinds of monetary dynamics. In fact, since as Lucas (1988) and McCallum and Goodfriend (1987) demonstrate, standard theoretical models imply that the demand for money can be expressed as a stable function of income and interest rates, these models cannot be used to understand why standard money demand functions are not found to be stable when estimated with data from the past thirty years.

The numerical work performed in section IV shows, however, that just as conventional econometric models for money demand have been modified to account for the effects of financial innovation, the conventional cash-in-advance model of money demand can be modified to capture the dynamics associated with financial innovation. The necessary theoretical modifications are suggested by the empirical literature and, in turn, provide formal support for alternative econometric models. These results suggest that introducing a transactions technology such as the one used here, which recognizes that households and firms have access to a variety of means for circumventing the use of noninterest-bearing assets in exchange, may be a useful step in developing a general equilibrium model that is consistent with enough data to be of use in evaluating policy experiments.

Certainly, acknowledging that possibilities for financial innovation exist is critical if the presence of a stable money demand function is to be relied on in policy-making. As equation (24) and the numerical work make clear, simple money demand relationships will break down when interest rates are high and volatile; instabilities will persist even after rates have settled down.

Notes

[*] Thanks go to participants in the macroeconomics seminar at the University of Michigan and in the Federal Reserve System Committee Conference on Financial Analysis at the Federal Reserve Bank of Boston as well as to Michael Dotsey, Joseph Haslag, Jeff Lacker, Milton Marquis, Kevin Reffett, Stacey Schreft, and two anonymous referees for extremely helpful comments and suggestions. The opinions expressed herein are those of the author and do not necessarily represent those of the above-mentioned individuals, the Federal Reserve Bank of Richmond, or the Federal Reserve System.

[1] Early empirical studies attributing money demand instability to financial innovation include Enzler, Johnson, and Paulus (1976) and Goldfeld (1976). Judd and Scadding (1982) and Goldfeld and Sichel (1990) survey the subsequent literature. Hester (1981) and Dotsey (1984) describe the major innovations to have occurred in US financial markets during the past 30 years.

[2] Two exceptions are Simpson and Porter (1980) and Dotsey (1985), which present versions of the classic inventory model of money demand modified to allow for endogenous changes in the intensity of agents's cash management efforts. See note 5 below.

[3] Most recently, the empirical money demand literature has focused on unusual weakness in the broader aggregate M2. Just like the earlier

episodes of instability in M1 demand, this recent episode of weakness in M2 has been associated with changes in the private financial sector, including the growth of bond mutual funds and the closing of insolvent savings and loan institutions; see Carlson and Parrott (1991) and Duca (1992).

[4]The potential magnitude of the fixed costs associated with a particular financial innovation are documented by Iida (1991), who reports that the startup cost to a commercial bank of installing software to monitor its daylight overdraft position might run as high as $700,000. Similarly, Tufano (1989) cites costs ranging from $50,000 to $5 million associated with the development of new financial instruments.

[5]Thus, Lucas and Stokey's cash-in-advance model is extended here just as the inventory model is extended in Simpson and Porter (1980) and Dotsey (1985).

[6]An economic environment similar to this one was originally described by Schreft (1992).

[7]Here and below it is assumed that $j+\varepsilon\leq 1$. When $j+\varepsilon>1$, the interval $[j,j+\varepsilon)$ should be replaced by $[j,j+\varepsilon-1)$.

[8]Thus, it is assumed here, as in Solow (1969), that technological progress must be embodied in newly-installed capital.

[9]The well-known indeterminacy of nominal quantities under interest rate targeting policies (Olivera 1970) may be eliminated here, of course, by allowing the government to choose H_0 as well as the sequence $\{R_t\}_{t=0}^{\infty}$; given the choice of H_0, the remaining transfers $\{H_t\}_{t=1}^{\infty}$ are then supplied so as to clear markets at the given interest rates $\{R_t\}_{t=0}^{\infty}$.

[10]Since $s_t \geq 0$ for all $t \geq 1$, $k_{T+1} \geq (1-\delta)^T k_1$ and hence

$$0 \leq [\beta(1-\delta)]^T \theta_T \leq \beta^T \theta_T k_{T+1}/k_1.$$

The transversality condition $\lim_{T\to\infty} \beta^T \theta_T k_{T+1} = 0$ therefore implies that $\lim_{T\to\infty} [\beta(1-\delta)]^T \theta_T = 0$ and hence that the asset pricing equation may be solved forward to obtain (21).

[11]One period in the model is both the holding period for money and the gestation period for investment in financial capital. The holding period for money suggests that one model period ought to be identified with, perhaps, one month in real time. On the other hand, the gestation period for investment suggests a longer period length, perhaps one year. One quarter is chosen, therefore, as a compromise between these two interpretations.

[12]Here and below, as well as in the figures, the quarterly interest rate and velocity series are expressed in annual terms. That is, R=1.05 means that a quarterly interest rate of approximately 1.012 is fed through the model. Similarly, v=4.4 translates into a quarterly velocity of 1.1. Reporting the artificial series in this way makes them

comparable to US data as they are most frequently reported (e.g., in figure 7).

[13] All data presented in figure 7 are taken from the DRI/McGraw-Hill database. M1-A is M1 less the OCD component. Velocity is defined as nominal GDP divided by M1-A.

References

Cagan, Phillip. "Monetary Policy and Subduing Inflation." In *Essays in Contemporary Economic Problems: Disinflation*, William Fellner, project director, pp. 21-53. Washington: American Enterprise Institute, 1984.

Cagan, Phillip and Anna J. Schwartz. "Has the Growth of Money Substitutes Hindered Monetary Policy?" *Journal of Money, Credit, and Banking* 7 (May 1975): 137-159.

Carlson, John B. and Sharon E. Parrott. "The Demand for M2, Opportunity Cost, and Financial Change." Federal Reserve Bank of Cleveland *Economic Review* 27 (1991 Quarter 2): 2-11.

Christiano, Lawrence J. "Modeling the Liquidity Effect of a Money Shock." Federal Reserve Bank of Minneapolis *Quarterly Review* 15 (Winter 1991): 3-34.

Dotsey, Michael. "An Investigation of Cash Management Practices and Their Effects on the Demand for Money." Federal Reserve Bank of Richmond *Economic Review* 70 (September/October 1984): 3-12.

________. "The Use of Electronic Funds Transfers to Capture the Effects of Cash Management Practices on the Demand for Demand Deposits: A Note." *Journal of Finance* 40 (December 1985): 1493-1503.

Duca, John V. "The Case of the Missing M2." Federal Reserve Bank of Dallas *Economic Review* (Second Quarter 1992): 1-24.

Duesenberry, James S. "The Portfolio Approach to the Demand for Money and Other Assets." *Review of Economics and Statistics* 45 (February 1963, supplement): 9-24.

Enzler, Jared, Lewis Johnson, and John Paulus. "Some Problems of Money Demand." *Brookings Papers on Economic Activity* (1976): 261-280.

Garcia, Gillian. "A Note on Bank Credit Cards' Impact on Household Money Holdings." *Journal of Economics and Business* 29 (Winter 1977): 152-154.

Goldfeld, Stephen M. "The Demand for Money Revisited." *Brookings Papers on Economic Activity* (1973): 141-189.

________. "The Case of the Missing Money." *Brookings Papers on Economic Activity* (1976): 683-730.

Goldfeld, Stephen M. and Daniel E. Sichel. "The Demand for Money." In *Handbook of Monetary Economics*, edited by Benjamin M. Friedman and Frank H. Hahn, pp. 299-356. Amsterdam: North-Holland, 1990.

Hester, Donald D. "Innovations and Monetary Control." *Brookings Papers on Economic Activity* (1981): 141-189.

Iida, Jeanne. "Fed Nears Decision on Plan to Charge Overdraft Fees." *American Banker* 156 (5 November 1991): 1 and 3.

Judd, John P. and John L. Scadding. "The Search for a Stable Money Demand Function: A Survey of the Post-1973 Literature." *Journal of Economic Literature* 20 (September 1982): 993-1023.

Kimball, Ralph C. "Wire Transfer and the Demand for Money." *New England Economic Review* (March/April 1980): 5-22.

Laidler, David E.W. "The Influence of Money on Economic Activity--A Survey of Some Current Problems." In *Monetary Theory and Monetary Policy in the 1970s*, edited by G. Clayton, J.C. Gilbert, and R. Sedgwick, pp. 75-135. London: Oxford University Press, 1971.

Lieberman, Charles. "The Transactions Demand for Money and Technological Change." *Review of Economics and Statistics* 59 (August 1977): 307-317.

Lucas, Robert E., Jr. "Money Demand in the United States: A Quantitative Review." *Carnegie-Rochester Conference Series on Public Policy* 29 (Autumn 1988): 137-168.

Lucas, Robert E., Jr. and Nancy L. Stokey. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics* 12 (July 1983): 55-93.

McCallum, Bennett T. and Marvin S. Goodfriend. "Demand for Money: Theoretical Studies." In *The New Palgrave Dictionary of Economics*, edited by John Eatwell, Murray Milgate, and Peter Newman, v. 1, pp. 775-781. London: The Macmillan Press Limited, 1987.

Olivera, Julio H.G. "On Passive Money." *Journal of Political Economy* 78 (July/August 1970): 805-814.

Sargent, Thomas J. *Macroeconomic Theory*, 2nd ed. New York: Academic Press, 1987.

Schreft, S.L. "Transaction Costs and the Use of Cash and Credit." *Economic Theory* 2 (April 1992): 283-296.

Simpson, Thomas D. and Richard D. Porter. "Some Issues Involving the Definition and Interpretation of the Monetary Aggregates." *Federal Reserve Bank of Boston Conference Series* 23 (October 1980): 161-234.

Solow, Robert M. "Investment and Technical Progress." In *Readings in the Modern Theory of Economic Growth*, edited by Joseph E. Stiglitz and Hirofumi Uzawa, pp. 156-171. Cambridge: MIT Press, 1969.

Stokey, Nancy L. and Robert E. Lucas, Jr. with Edward C. Prescott. *Recursive Methods in Economic Dynamics*. Cambridge: Harvard University Press, 1989.

Tufano, Peter. "Financial Innovation and First-Mover Advantages." *Journal of Financial Economics* 25 (December 1989): 213-240.

White, Kenneth J. "The Effect of Bank Credit Cards On the Household Transactions Demand for Money." *Journal of Money, Credit, and Banking* 8 (February 1976): 51-61.

Fig. 1. Example 1, Convergence to a Steady State

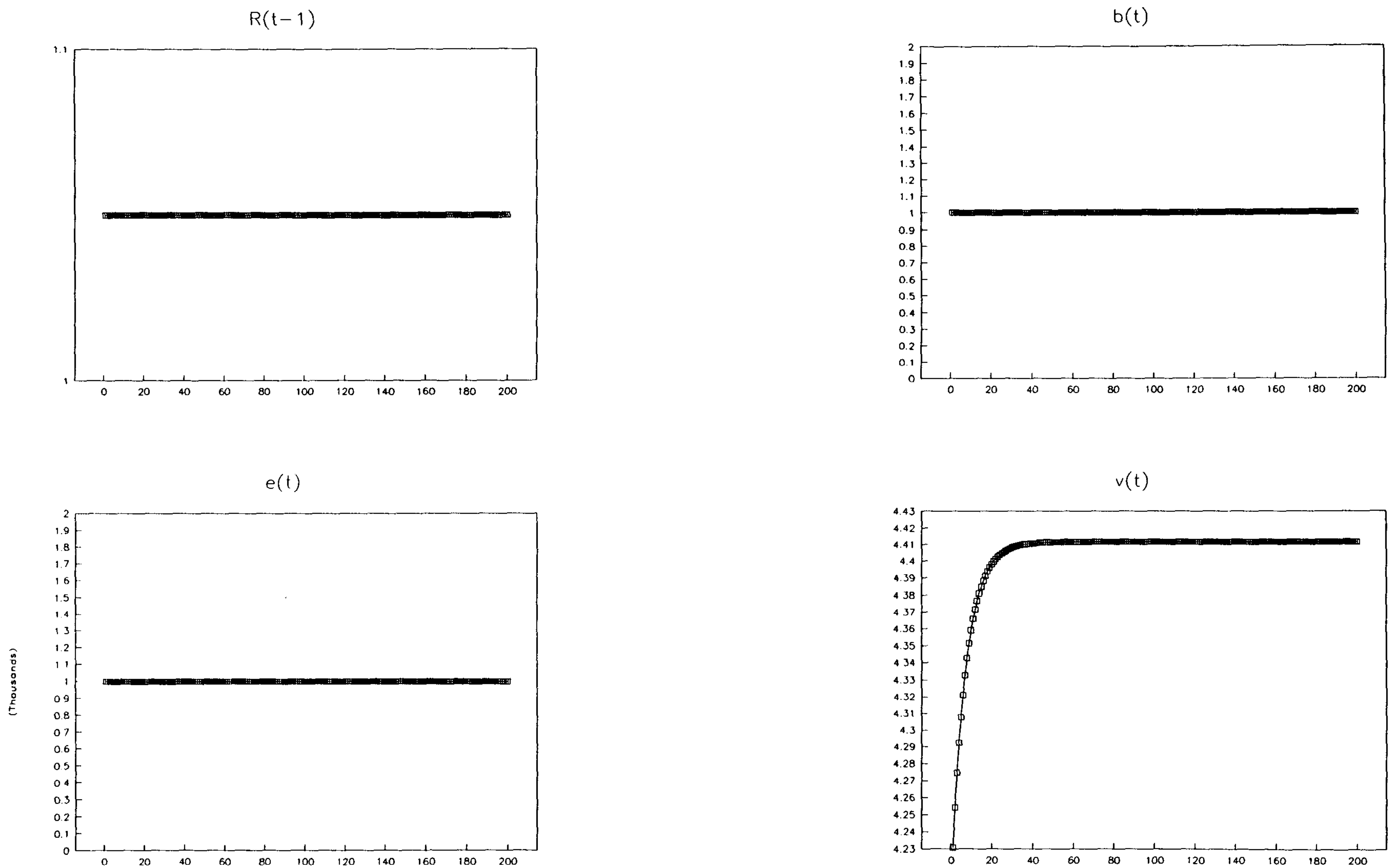


Fig. 2. Example 2, Permanent Increase in Interest Rates

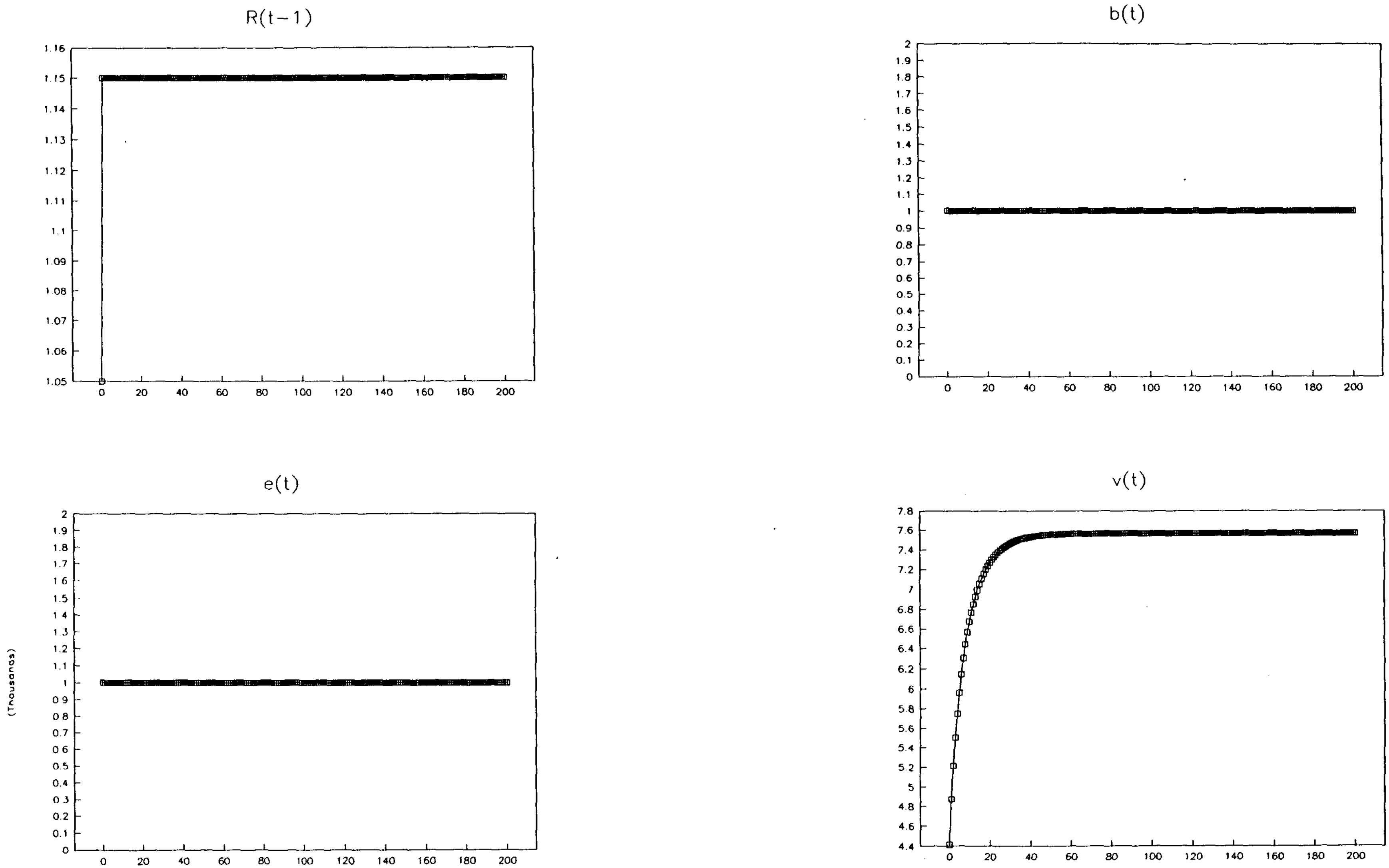


Fig. 3. Example 3, Temporary Increase in Interest Rates

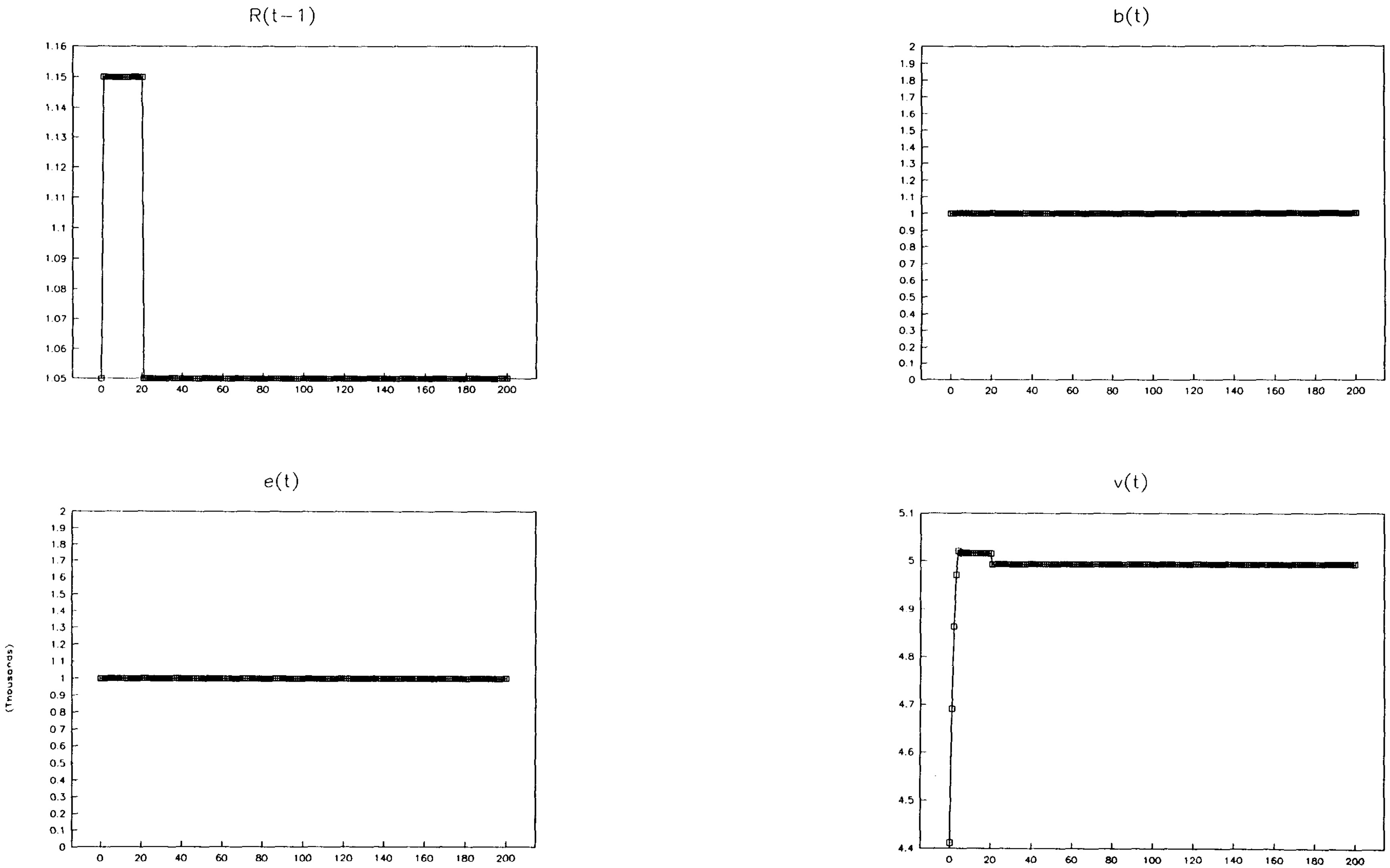


Fig. 4. Example 4, Real Economic Growth

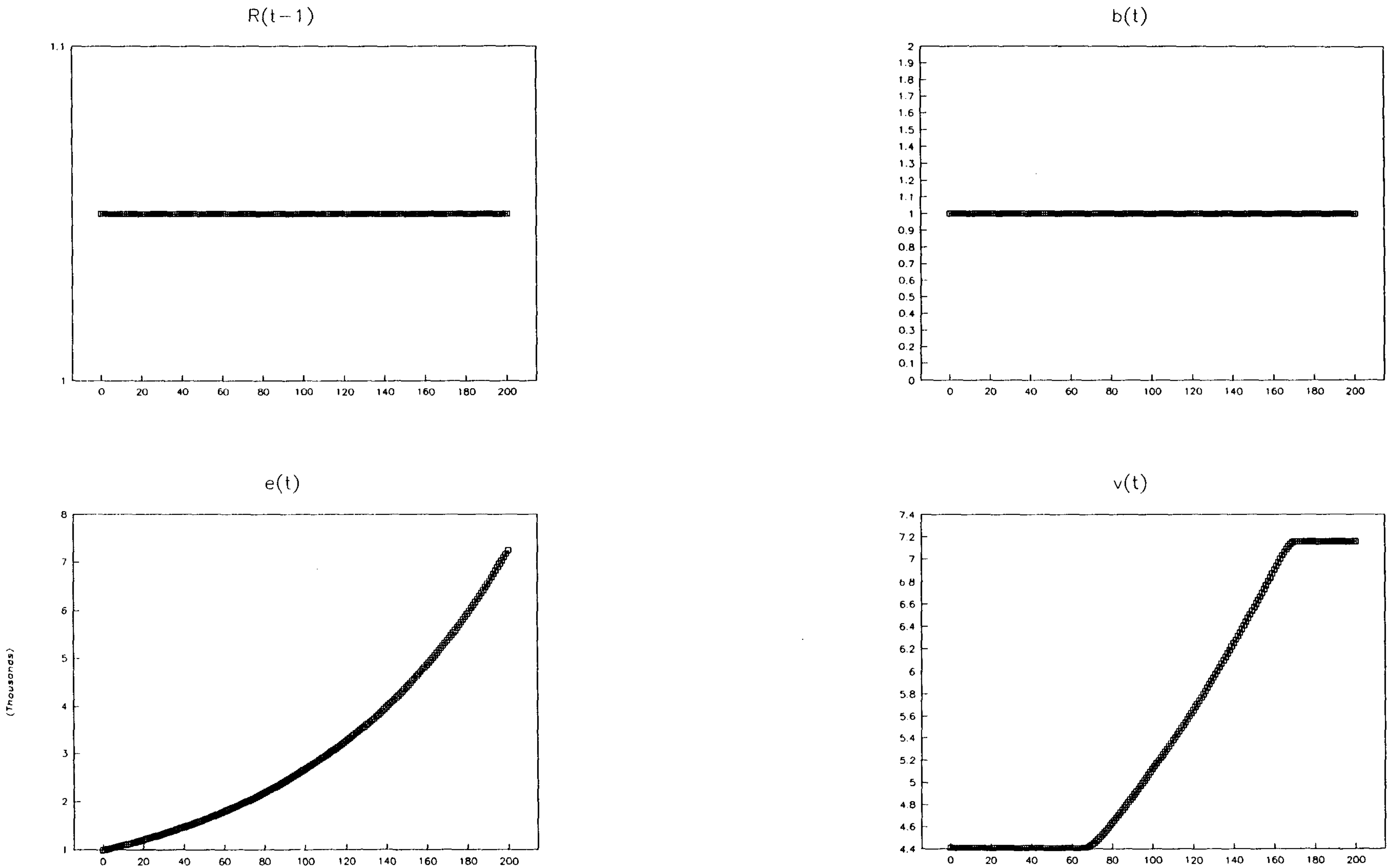


Fig. 5. Example 5, Technological Change

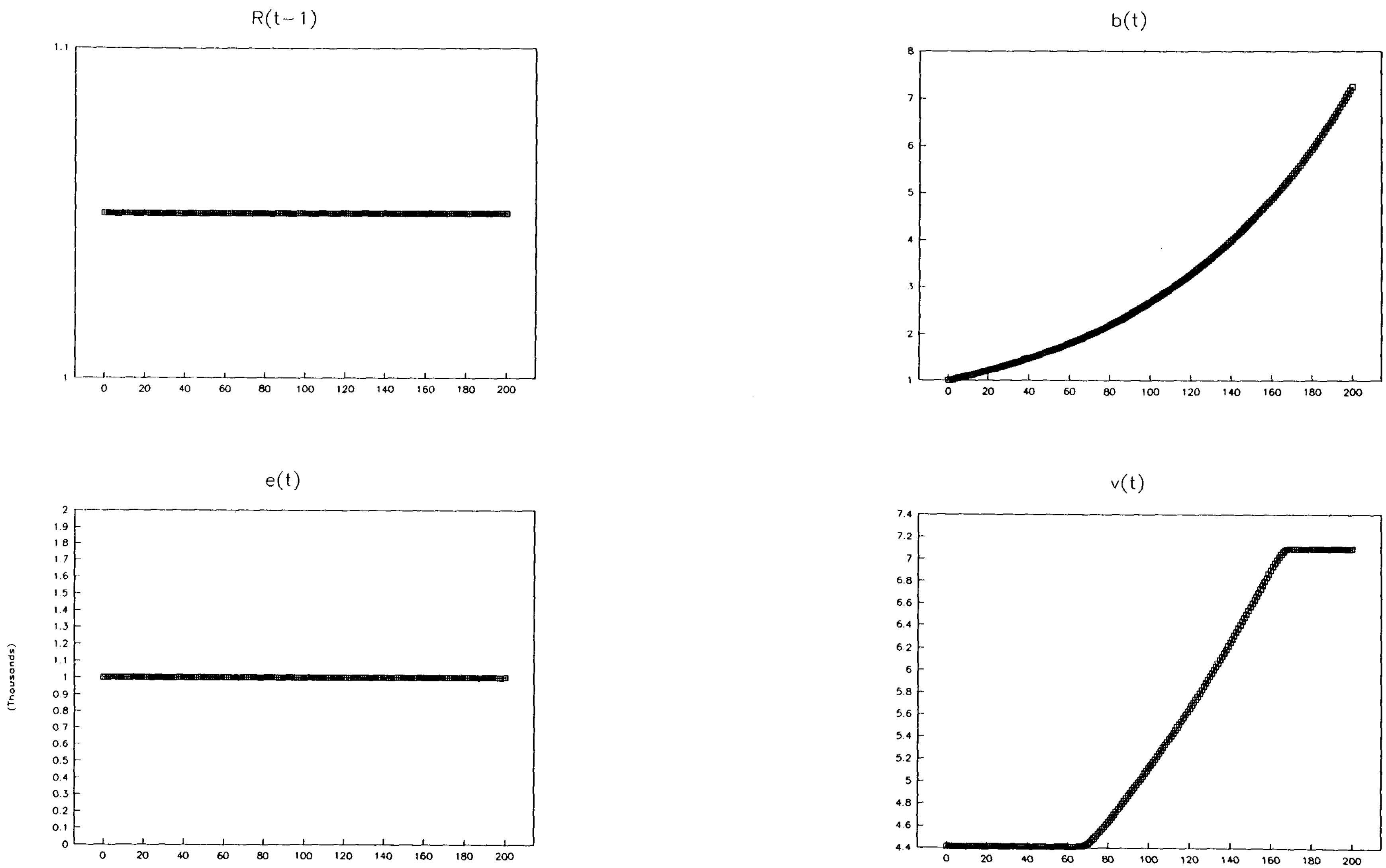


Fig. 6. Example 6, Comparison with US Data

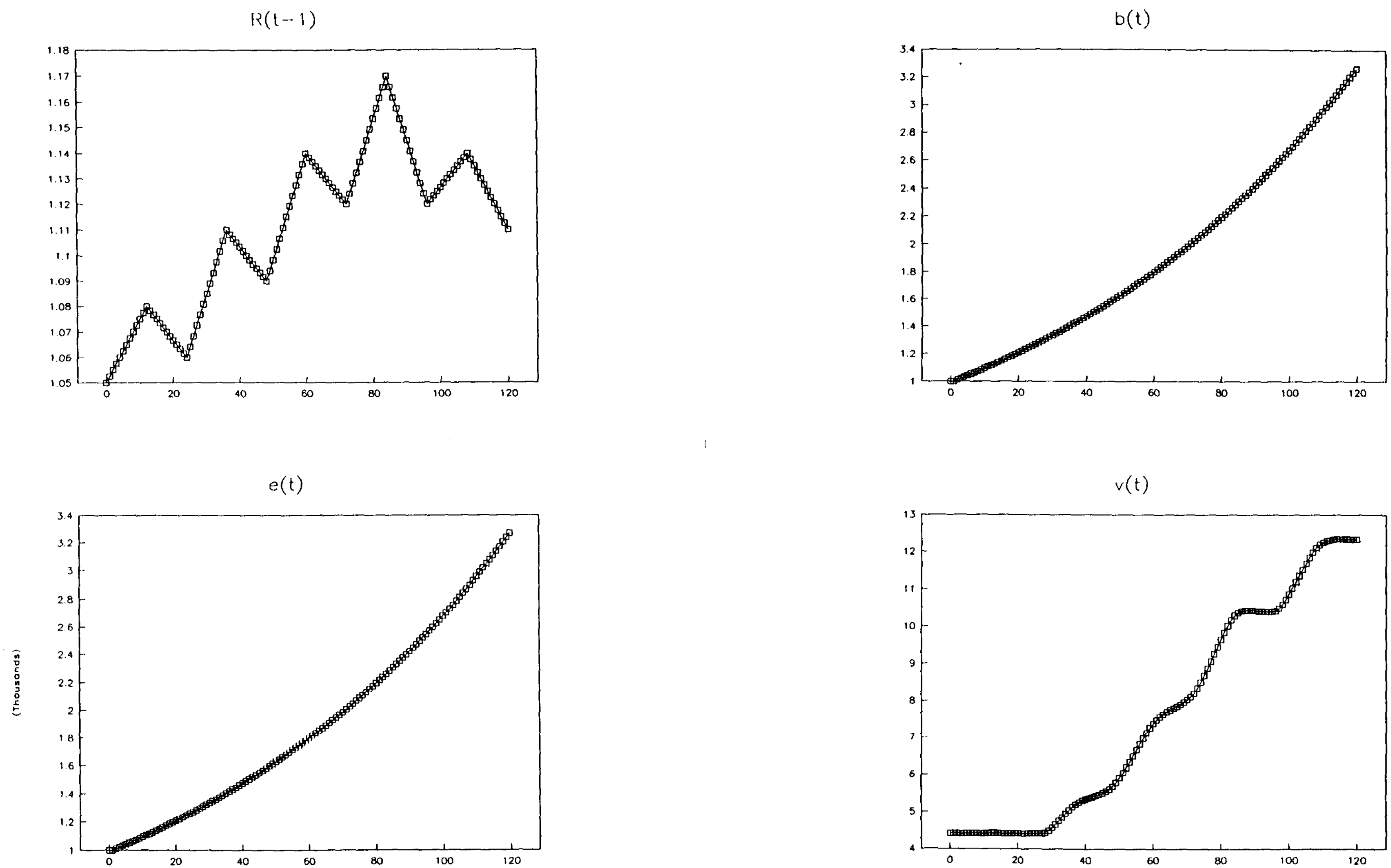
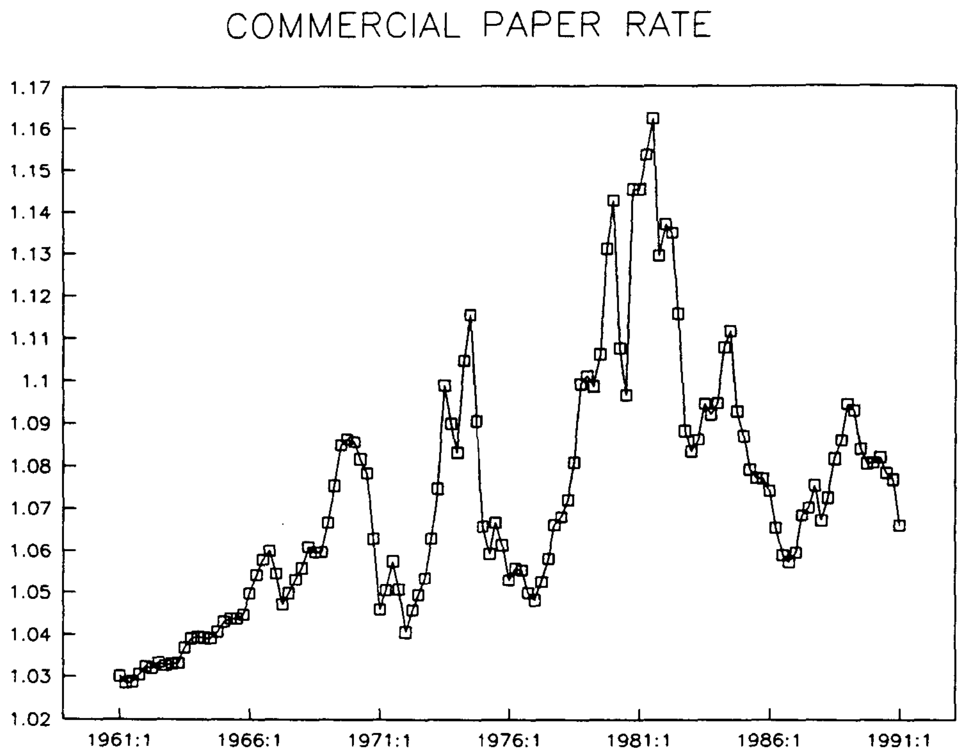
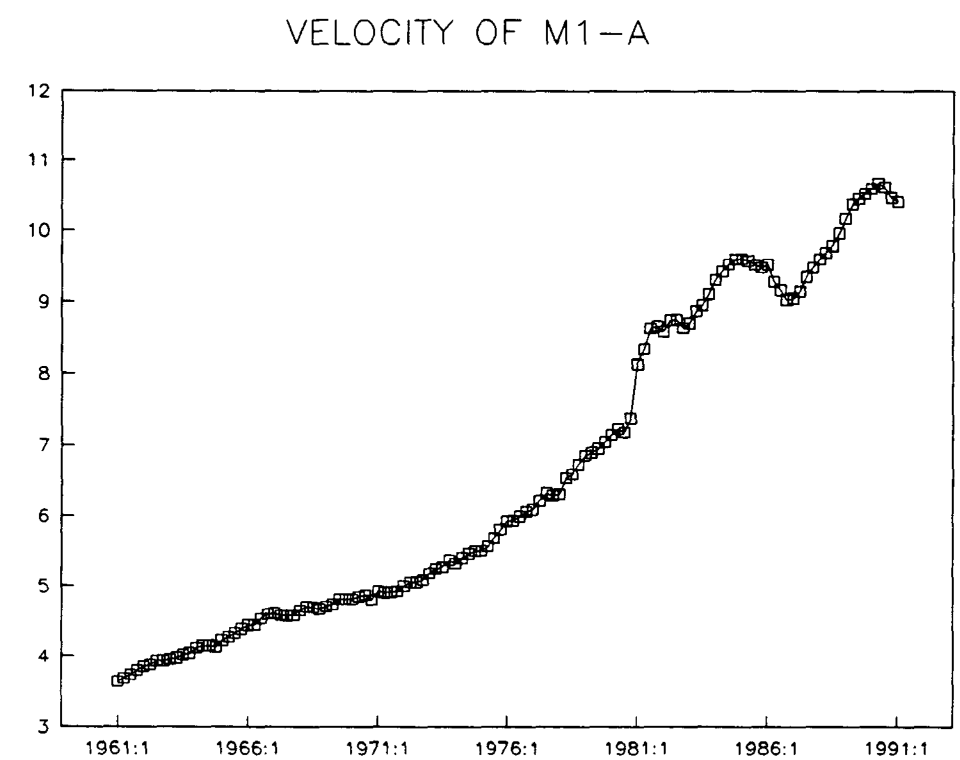


Fig. 7. US Data


COMMERCIAL PAPER RATE
1.17
1.16
1.15
1.14
1.13
1.12
1.11
1.1
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1961:1
1966:1
1971:1
1976:1
1981:1
1986:1
1991:1


VELOCITY OF M1-A
12
11
10
9
8
7
6
5
4
3
1961:1
1966:1
1971:1
1976:1
1981:1
1986:1
1991:1