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DEFICITS AND LONG-TERM INTEREST RATES: AN EMPIRICAL NOTE

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Abstract

This note examines whether long-term nominal interest rates are cointegrated with budget deficits over the period 1959 to 1990. A key finding of this note is that long-term rates are cointegrated with deficits if a one-year ahead inflation forecast series is used to measure long-term expected inflation. However, the evidence favoring cointegration between deficits and interest rates weakens and almost disappears when inflation forecasts over longer horizons (2 to 4 years) are used. This result indicates that a one-year ahead inflation forecast series does not adequately measure long-term expected inflation. Hence, the link found between deficits and long-term rates using one-year inflation forecast series is spurious.

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1. Introduction

This note examines empirically the role of budget deficits in determining long-run movements in the long-term nominal rate of interest. In particular, the note examines whether long-term interest rates are cointegrated with budget deficits over the period 1959 to 1990. The results presented here indicate that the effect of deficits on interest rates is undetectable when long-tern inflation forecasts series are used. A key finding of this note is that long-term rates are cointegrated with deficits if a one-year ahead inflation forecast series is used to measure long-term inflationary expectations.[1] However, the evidence favoring cointegration between deficits and interest rates weakens and almost disappears when inflation forecasts over longer horizons (two to four years) are employed.

Of particular interest is the result that over the subperiod 1979 to 1990 deficits are statistically significant in an interest rate regression when one-year ahead Livingston inflation survey data are used, but they are not if ten-year ahead inflationary expectations data are used.[2] These results suggest that the long-horizon forecasts have more information about long-term expected inflation than the one-year forecasts, so that the results using the latter are spurious. Hence, the link found by Hoelscher (1986) between deficits and long-term rates using one-year survey data is tenuous. The results here accord with the conclusion reached in Plosser (1982) Mascaro and Meltzer (1983), and Evans (1985, 1987) that deficits do not matter for the behavior of long-term rates.

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[1]Hoelscher (1986) uses one-year ahead inflation forecasts from the Livingston survey and finds a significant effect of the deficit on long-term rates over the period 1954 to 1984.

[2]The long-run survey data are based on the "Decision-Makers Poll" of institutional decision-makers and are available only for this subperiod.

The plan of this note is as follows. Section 2 presents the interest rate model and the empirical methodology used to examine the link between deficits and interest rates. Section 3 presents empirical results. Concluding observations are given in Section 4.

2. Empirical Methodology

2.1 Reduced form for the Long-term Nominal Interest Rate

The interest rate equation (1) underlying the cointegration tests performed here is derived from the loanable funds model of interest rate determination discussed in Hoelscher (1986).

$$R_t^L = \alpha_o + \alpha_1 \Pi_t^L + \alpha_2 rr_t^s + \alpha_3 \Delta y_t + \alpha_4 rDEF_t + U_t; \ \alpha_1, \alpha_2, \alpha_4 > 0, \ \alpha_3 \gtrless 0; \quad (1)$$

R^L is the long-term nominal interest rate; Π^L the long-term expected inflation rate; rr^s the short-term expected real interest rate; Δy the change in real income; rDEF the real deficit; and U_t a random disturbance term. Equation 1 says that the nominal interest rate is positively related to the long-term expected inflation rate, the short-term expected real rate and the real deficit. The effect of the change in real income on the interest rate is uncertain, as higher income may raise both the flow demand for and the flow supply of loanable funds.[3]

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[3]Sargent (1969)

2.2 Testing for Cointegration

Hoelscher (1986, p. 15) asserts that the correlation between deficits and long-term interest rates is long-run in nature. He therefore estimates (1) using annual data and treats all variables appearing in (1) as stationary. I first examine the possibility that some or all of the variables included in (1) are nonstationary. I then search for the long-run deficit-interest rate link using the test for cointegration. In particular, I examine whether the long-term nominal rate is cointegrated with deficits and the other variables in (1).

The test for cointegration used here is from Engle and Granger (1987). If all the variables included in (1) are nonstationary and if the long-term nominal interest rate is cointegrated with the right hand explanatory variables, then the residual U_t in (1) is stationary. The stationarity of U_t is examined by performing a unit root test on the residuals of (1).

Following Stock and Watson (1991), I test hypotheses using the dynamic version of the cointegrating regression. Assume that the Engle-Granger test for cointegration indicates that R^L_t is cointegrated with Π^L_t, rr^s_t, and $rDEF_t$. The dynamic version of this cointegrating relationship is given below in (2).

$$R^L_t = \alpha_o + \alpha_1 \Pi^L_t + \alpha_2 rr^s_T + \alpha_3 rDEF_t + \sum_{s=-k}^{k} \alpha_{4s} \Delta\Pi^L_{t-s} + \sum_{s=-k}^{k} \alpha_{5s} \Delta rr^s_{t-s} + \sum_{s=-k}^{k} \alpha_{6s} \Delta rDEF_{t-s} + \epsilon_t \tag{2}$$

where all variables are as defined before. Δ is the first difference operator. Equation 2 includes, in addition, past, current, and future values of first differences of the right hand variables that appear in the cointegrating regression. Since ϵ_t is stationary but may be serially correlated, standard test statistics corrected for the presence of serial correlation are used to test hypotheses in (2). Thus, deficits determine long-term rates if the null hypothesis $\alpha_3 = 0$ is rejected.

2.3 Modeling Long-term Inflationary Expectations

Since data on long-term inflationary expectations (Π^L) are not available, analysts have estimated regressions like (1) employing proxies for Π^L. For example, Mascaro and Meltzer (1983) use inflation forecasts from a univariate time series model, whereas Hoelscher (1986) uses one-year ahead inflation forecasts from the Livingston survey on inflationary expectations.

In order to examine whether the estimated link between the deficit and the long-term interest rate is sensitive to the proxy employed, I employ an alterative proxy based on a monetary model of inflationary expectations. Recent work by Reichenstein and Elliott (1987) and Hallman, Porter and Small (1991) indicates that models using the M2 measure of money predict inflation better than nonstructural models drawn from time series and interest rate relationships. Based on this work, I employ the following model to generate out-of-sample inflation forecasts and use them as proxy for the long-term expected inflation rate in (1).

$$\Delta \ln p_t - \Delta \ln p_{t-1} = a + \sum_{s=1}^{4} b_s (\Delta \ln p_{t-s} - \Delta \ln p_{t-s-1}) + c(\ln p_{t-1} - \ln p^*_{t-1}) + e_t \tag{3.1}$$

$$\ln p^*_t \equiv \ln(M2.\overline{V2}/y)_t \tag{3.2}$$

$$\Delta \ln y_t = c_o + \sum_{s=1}^{4} c_{1s} \Delta \ln y_{t-s} + e_{2t} \tag{3.3}$$

$$\Delta \ln M2_t = d_o + \sum_{s=1}^{8} d_{1s} \Delta \ln M2_{t-s} + e_{3t} \tag{3.4}$$

where lnp is the logarithm of the price level (measured by the implicit GNP deflator); $\ln p^*$ the logarithm of the equilibrium price level; M2 the M2 measure of money; y real GNP; and $\overline{V2}$ the long-run equilibrium M2 velocity. Equations 3.1 and 3.2 constitute an error-correction model for explaining changes in the rate of inflation. This formulation reflects the hypothesis that the velocity of M2 is stationary and that the equilibrium price level is given by the excess of M2 over real output. Equations 3.3 and 3.4 are forecasting equations used to generate predicted values for M2 and real output, which are in turn used in (3.2).[4] This inflation model is used to generate out-of-sample inflation forecasts over one to four years in the future. Inflation forecasts over horizons longer that four are not considered because they fail the test of unbiasedness described later in the paper.

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[4]This inflation model differs in some respects from the one employed in Hallman, Porter and Small (1991). The latter uses potential income, whereas the model here uses actual real income. Furthermore, the forecasting equations used to generate out-of-sample values for M2 and y are different.

In addition to using inflation forecasts from the monetary model, ten-year ahead inflationary expectations survey data are also employed in some regressions. These surveys, which are based on 'the Decision-Makers Poll' of institutional decision-makers and conducted irregularly by Drexel Burnham Lambert, are available only for the subperiod 1979 to 1990.

2.4 Data and Definition of Variables

The sample period over which the link between the deficit and the interest rate is examined is 1959 to 1990. The data over the prior period 1952 to 1958 are used to estimate the monetary inflation model, which generates out-of-sample inflation forecasts beginning in 1959. The empirical analysis is carried out using quarterly data.[5] I also present results using annual data as in Hoelscher (1986).

The long-term rate is the nominal interest rate on ten-year Treasury bonds (R10TB). The short-term real rate is defined as the nominal rate on one-year Treasury bonds (R1TB) minus the one-year ahead expected rate of inflation ($\Pi 1$). The one-year ahead forecast from the Livingston survey is denoted as $L\Pi 1_t$. The inflation forecasts from the monetary model are denoted as $m\Pi s$ (s=1,2,3,4), where s is the number of years in the forecast horizon. In the Livingston survey, the inflation rate forecast is measured by the consumer price index, whereas in the monetary inflation model the inflation rate is the implicit GNP deflator. The variable y is real GNP. The measure

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[5]The data on interest rates, the price level, real GNP, the nominal deficit and M2 are from the Citibank data base. The series on M2 was extended back to 1952 using the procedure described in Hetzel (1989). The Livingston survey data was provided by the Federal Reserve Bank of Philadelphia and the ten-year ahead inflationary expectations data were collected from various issues of Decision-Makers Poll published by Drexel Burnham Lambert Incorporated.

of the real deficit used is the national income accounts version of the federal deficit deflated by the implicit GNP deflator.

3. Empirical Results

3.1 Evaluating Inflation Forecasts

The monetary inflation model described in section 2.3 is used to generate inflation forecasts over the period 1959 to 1990.[6] These forecasts are prepared as follows. The monetary model is first estimated over an initial sample period 1953 to 1958 and then used to generate inflation forecasts for one to four years in the future. The end of the initial estimation period is then advanced one period, the model reestimated and forecasts prepared for one to four years in the future. This procedure is repeated until the estimation period uses all the data available through the end of 1990. I generated these inflation forecasts using quarterly as well as annual data.[7]

The inflation forecasts generated above are evaluated in Table 1, which present coefficients from regressions of the form

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[6]The deficit variable when included in the monetary inflation model is not statistically significant.

[7]When annual data are used, the lag structure of the estimated inflation model (3.1 - 3.4) is somewhat different. The model estimated is of the form

$$\Delta \ln p_t - \Delta \ln p_{t-1} = a + c\,(\ln p_{t-1} - \ln p^*_{t-1}) + e_{1t}$$

$$\ln p^*_t \equiv \ln(M2.\overline{V2}/y)_t$$

$$\Delta \ln y_t = c_o + e_{2t}$$

$$\Delta \ln M2_t = d_o + d_1\, \Delta \ln M2_{t-1} + e_{3t}$$

$$A_{t+s} = a + b\ P_{t+s} + U_t,\ s = 1,2,3,4 \qquad (4)$$

where A is actual inflation; P the value predicted from the model; U_t the random disturbance term; and s, the number of years in the forecast horizon. Inflation forecasts from the model are unbiased if a = 0 and b = 1. The estimated values of a and b are reported in Table 1. As can be seen, the estimated values of the coefficient b are close to unity for the monetary model. $\chi^2(2)$ is the Chi-square statistic that tests the null hypothesis (a, b) = (0, 1) and is distributed with two degrees of freedom. (The reported standard errors have been corrected for serial correlation due to the presence of the overlapping forecast horizon. Also, the test statistics that tests the null hypothesis (a,b) = (0,1) has a Chi-square, not an F, distribution.) The reported values of the χ^2 statistic are small for the inflation forecasts from the monetary model (the 5 percent critical value is 5.99).

One-year ahead inflation forecasts from the Livingston survey do relatively well in predicting one-year ahead actual CPI inflation ($\hat{a} = .7$, $\hat{b} = .9$, $\chi^2(2) = 3.4$). However, this performance deteriorates rapidly if these inflation forecasts are used as proxies for the actual behavior of CPI inflation over horizons longer than one year. (See estimated values of $\hat{a}$, $\hat{b}$, and $\chi^2(2)$ in Table 1.) Hence, the use of the Livingston survey data as a proxy for long-term inflationary expectations is suspect.

I also evaluate inflation forecasts from an autoregressive model, where the change in inflation is modeled as a fourth-order autoregressive process. As can be seen, inflation forecasts from this model are also biased over horizons longer than one year.

In sum, in modeling one-year ahead inflationary expectations the alternative proxies considered here do reasonably well in the sense that they are unbiased forecasts of the one-year ahead actual inflation rate. However, the monetary model does much better if we want to model inflationary expectations over horizons longer than one year.[8]

3.2 Unit Root Test Results

The interest rate regression used here is

$$R10TB_t = a_o + a_1\, \Pi^L_t + a_2\, (R1TB - \Pi1)_t + a_2\, \Delta\ln(rY)_t + a_3\, rDEF_t + U_t \qquad (5)$$

where Π^L_t is the long-term expected inflation rate; $\Pi1$ the one-year ahead expected inflation rate; and other variables have been defined as before. The alternative proxies for Π^L_t considered here are one-year ahead ($m\Pi1$), two-year ahead ($m\Pi2$), three-year ahead ($m\Pi3$) and four-year ahead ($m\Pi4$) inflation forecasts from the monetary model. The one-year ahead Livingston survey forecasts are denoted as $L\Pi1$.

In order to examine whether the long-term rate is cointegrated with the determinants suggested in (5), one first performs unit root tests on these variables. The unit root tests are performed by estimating the augmented Dickey-Fuller regression of the form

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[8]This is not to suggest that the monetary model predicts inflation well over very long horizons. When the model forecasts are evaluated over an eight-year horizon, the estimated values of $\hat{a}$ and $\hat{b}$ are 3.9 and .35, respectively. The $\chi^2(2)$ statistic is 4.9, which is significant at the ten percent level.

$$X_t = a + \rho X_{t-1} + \sum_{s=1}^{k} b_s \Delta X_{t-s} + cLT_t + \epsilon_t \tag{6}$$

where X_t is the relevant variable; LT a linear trend; Δ the first difference operator; and ϵ_t the random disturbance term. k is the number of lagged first differences of X_t necessary to make ϵ_t serially uncorrelated. If ρ=1, then X_t has a unit root. Two statistics are calculated to test the null hypothesis ρ=1. The first is the t-statistic, $t_{\hat{\rho}}$, and the second is the normalized bias statistic, $T(\hat{\rho}-1)$, where T is the number of observations. The null hypothesis is rejected if these statistics are large.

Table 2 reports the unit root tests for the long-term nominal rate ($R10TB_t$), the short-term real rate ($(R1TB - m\Pi1)_t$ or $(R1TB - L\Pi1)_t$), the alternative measures of the long-term expected inflation rate (mΠ1, mΠ2, mΠ3, mΠ4, LΠ1), levels and first differences of the logarithm of real GNP (rY_t), and the real deficit ($rDEF_t$). The t-statistic indicates that the long-term nominal rate, the short-term real rate, alternative measures of the long-term expected inflation rate, real GNP and the real deficit are nonstationary in levels. First differences of real GNP are, however, stationary.[9] The normalized bias statistic yields similar results except for the real deficit, where it indicates that the real deficit is stationary in levels. In view of the mixed results for the deficit, the link between the long-term rate and the real deficit is also examined using data in levels.

[9] The unit root tests were also performed using first differences of the other variables. These results indicate that first differences of $R10TB_t$, $(R1TB - M\Pi1)_t$, $(R1TB - L\Pi1)_t$, $m\Pi1_t$, $m\Pi2_t$, $m\Pi3_t$ $m\Pi4_t$, $L\Pi1_t$, and $rDEF_t$ are stationary, i.e., the t-statistics, respectively, are -5.2, -6.1, -5.7, -5.9, -6.8, -7.1, -6.6, -4.1, and -5.3. (The 5 percent critical value is -2.89, Table 8.5.2, Fuller (1976)). The normalized bias statistics (not reported) yielded similar conclusions.

3.3 Cointegration Test Results

Tables 3 and 4 present cointegration test results using the Engle-Granger procedure. In particular, the long-term rate is regressed on the short-term real rate, the long-term expected inflation rate, the real deficit and the level of the logarithm of real GNP. (Real GNP enters in levels, not in first differences, because the latter is stationary.) An augmented Dickey-Fuller test is then used to test for a unit root in the residuals of this regression. The test is implemented by estimating a second regression of the form

$$\Delta\hat{u}_t = \rho\ \hat{u}_{t-1} + \sum_{s=1}^{k} b_s\ \Delta\ \hat{u}_{t-s}$$

where $\hat{u}_t$ is the residual from the first regression. The pertinent variables are cointegrated if the null hypothesis $\rho=0$ is rejected.

Table 3 presents test results when the Livingston inflation survey data are employed The t-statistic that tests the null hypothesis $\rho=0$ is 4.54, which is significant at the 5 percent level. (The 5 percent critical value is 4.36, Table 3, Engle and Yoo (1987).) This result means that the long-term rate is cointegrated with the pertinent variables. The dynamic version of the estimated cointegrating regression (with estimated standard errors in parentheses) is also reported in Table 3. All estimated coefficients (with the exception for real GNP) have theoretically correct signs and are statistically significant. In particular, the coefficient on the real deficit in the dynamic cointegrating regression is 1.01 and is statistically significant (the reported standard errors corrected for serial correlation). $x_c^2(1)$ is the Chi-square statistic[10] that tests the null hypothesis that rDEF is not significant in the dynamic version of the cointegrating regression, i.e., $\alpha_3 = 0$ in (2). This statistic is large 6.0 (the 5 percent critical value is 3.84) and is thus consistent with the rejection of the null hypothesis. This result implies that deficits are positively related to higher long-term interest rates.

Table 4 presents results when inflation forecasts from the monetary model are used in defining the short-term real rate and measuring long-term inflationary expectations. When one-year ahead inflation forecasts are used, cointegration test results are similar to the ones obtained using the Livingston survey data. The long-term rate is cointegrated with the pertinent variables, and the real deficit variable is statistically significant in the dynamic cointegrating regression.[11] However, when two- to four-year ahead inflation forecasts are used as proxies for the long-term expected inflation rate, the results indicate that the real deficit variable does not have a statistically significant effect upon the long-term rate. The coefficient that appears on the real deficit variable takes values 1.5 ($x_c^2(1)$:10.4), 1.74 ($x_c^2(1)$:2.4), .86($x_c^2(1)$:2), and .59 ($x_c^2(1)$:.5) when one-year ahead, two-year ahead, three-year ahead and four-year ahead inflation forecasts are alternatively used in the regression. The coefficients that appear on the short-term expected real rate and the long-term expected inflation proxy

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[10]The relevant statistic has a Chi-square, rather than a t, distribution, because the residuals in the cointegrating regression have been corrected for the presence of moving-average serial correlation. The order of the moving-average correction was determined by examining the autocorrelation function of the residuals.

[11]Regressions using four-year ahead forecasts are similar to those using three-year forecasts and are not shown in order to save space.

remain statistically significant and possess theoretically correct signs. (See the coefficients and the associated standard errors and Chi-square statistics in dynamic OLS regressions, Table 4.) These results indicate that the link between the deficit and the long-term rate using one-year ahead expected inflation data is spurious.

3.4 Annual Regressions

Table 5 presents results if the long-term interest rate regression (1) is estimated in a conventional way. All the variables included in (1) are assumed to be stationary, and the annual data are used to capture the potential long-run link between the deficit and the long-term rate. In addition to using the Livingston survey data, ten-year ahead inflationary expectations data, which is available only for the subperiod 1979 to 1990, are used to proxy for the long-term expected inflation rate.

The top part of Table 5 presents the interest rate regression estimated using the survey inflation data. As can be seen, the real deficit variable is statistically significant in the regression if one-year ahead inflation forecasts are used. However, the real deficit is no longer statistically significant if ten-year ahead inflationary expectations data are used (see the relevant t-statistics in regressions estimated for the periods 1960 to 1990 and 1979 to 1990, Table 5).

The bottom part of Table 5 presents regression results using the model-based inflation forecasts. They indicate a similar conclusion: the link between the deficit and the long-term rate disappears as inflation forecasts over a longer horizon are used. The coefficient that appears on the deficit variable is 1.3 (t-value: 8.5) if one-year ahead inflation forecasts are used and .22 (t-value:.7) if four-year ahead inflation forecasts are used (see Table 5).[12,13]

The reported standard errors in regressions that use model-based inflation forecasts have not been corrected for the bias due to the use of 'generated regressors.' But, as shown in Pagan (1984), the direction of the bias is downward, meaning that the estimated standard errors are no greater than the true standard errors. This result means that the conclusion on the statistical insignificance of the deficit can not be reversed by a correct calculation of the standard errors, as the re-computed "t- and Chi square statistics" would be lower than the ones reported in Tables 4 and 5. However, inferences concerning other variables can change.[14]

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[12]Hoelscher (1986) also considers a broader measure of government deficits that includes borrowing by state and local governments as well as federal government borrowing, measured on a national income basis. The use of this alternative measure in the regressions yield qualitatively similar results. The coefficient that appears on this measure of deficit is 1.4 (t-value: 6.3) when one-year ahead inflation forecasts are used and .6 (t-value: .44) when four-year ahead inflation forecasts are used in the annual regression. Qualitatively similar results hold if deficits are expressed as a percentage of GNP.

[13]The income variable ($\Delta lnrY_t$) used in annual regressions is generally not statistically significant (see Table 5). Annual regressions were, therefore, estimated excluding the income variable. Such regressions (not reported) yielded qualitatively similar results. In particular, the deficit variable is significant when one-year ahead forecasts are used, but not when long-horizon forecasts are used. The estimated coefficient on rDEF is 1.3 (t-value: 8.6) with the one-year forecasts and .39 (t-value:1.1) with the four-year forecasts.

[14]In order to examine further the sensitivity of inference, the standard errors were re-calculated as follows:

The standard errors reported in Table 4 and the bottom part of Table 5 are calculated using the OLS residuals from regressions of the form

$$R10TB_t = \hat{a} + \hat{b}\,(R1TB - m\Pi 1)_t + \hat{c}\; m\Pi 3_t + \hat{d}\; rDEF_t + \hat{f}\,(lnrY_t \text{ or } \Delta lnrY_t)$$

(continued...)

3.5 An Error-Correction Model of the Long-term Rate

Even if the real deficit is not cointegrated with the long-term rate, it could still influence the long-term rate in the short run. In order to examine this possibility, an error-correction model of the long-term rate is estimated under the assumption that the long-term rate is cointegrated with the short-term real rate and the long-term expected inflation rate. In particular, the long-term rate is assumed to be determined by (7).

$$R10TB_t = a_o + a_1 \, (R1TB - m\Pi 1)_t + a_2 \, m\Pi 3_t + U_t \qquad (7.1)$$

$$\Delta R10TB_t = b_o + \lambda \, U_{t-1} + b_1(L) \, \Delta R10TB_t + b_2(L) \, \Delta(R1TB - m\Pi 1)_t + b_3(L) \, \Delta m\Pi 3_t + b_4(L) \, \Delta(\ln ry)_t + \epsilon_t \qquad (7.2)$$

where all variables are as defined before. The expression $b_i(L)$ is a finite-order polynomial in the lag operator. Equation (7.1) specifies the long-run determinants of the long-term interest rate under the assumption that U_t is stationary.[15] The residual U_t is included in (7.2), which specifies the

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[14](...continued)

$$+ \hat{u}_t \qquad (a)$$

$m\Pi 1$ and $m\Pi 3$ are 'generated regressors'; $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{f}$ are the estimated parameters; and $\hat{u}$ the OLS residual. In order to account for the uncertainty associated with the estimation of 'generated regressors,' the residuals were re-calculated replacing $m\Pi 1$ and $m\Pi 3$ in (a) by the actual, one-year and three-year ahead inflation rates. The re-computed standard errors, t- and Chi-square statistics (not reported) yielded similar results. That is, rDEF is not significant if the longer-horizon inflation forecasts are used, while other variables (with the exception of real GNP) remain statistically significant.

[15]Ordinary least squares estimation of (7.1) over 1959Q1 to 1990Q4 yielded the following regression

$$R10TB_t = .6 + .99 \, (R1TB - m\Pi 1)_t + 1.25 \, m\Pi 3_t + \hat{u}_t$$

(continued...)

short-run dynamics of the long-term rate. The error-correction equation (7.2) includes first differences of the short-term real rate, the long-term expected inflation rate, and real income.

One simple way to estimate the error-correction model (7) is to solve (7.1) for U_{t-1} and then substitute U_{t-1} into (7.2). This procedure yields the following regression

$$\begin{aligned} \Delta R10TB_t = & (b_o - \lambda a_o) + \lambda\ R10TB_{t-1} - \lambda a_1\ (R1TB - m\Pi 1)_{t-1} - \lambda a_2\ m\Pi 3_{t-1} \\ & + b_1(L)\ \Delta R10TB_t + b_2\ (L)\ \Delta(R1TB - m\Pi 1)_t + b_3(L)\ \Delta\ m\Pi 3_t \\ & + b_4(L)\ \Delta(\ln rY)_t + \epsilon_t \end{aligned} \qquad (8)$$

Both long- and short-run parameters of the interest rate model (7) appear in (8) and can be jointly estimated using a consistent estimation procedure.

Table 6 presents results of estimating (8) by ordinary least squares. The estimated regression looks reasonable.[16] All the estimated coefficients have theoretically correct signs and are statistically significant. (The reported t-values are corrected for heteroscedasticity; see

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[15](...continued)

The augmented Dickey Fuller t-statistic that tests the null hypothesis that $\hat{u}_t$ is nonstationary is -3.74 (the 5 percent critical value is -3.62, Table 3, Engle and Yoo (1987)). The null hypothesis that $\hat{u}_t$ is nonstationary is rejected.

[16]The regression presented in Table 6 passes several diagnostic checks. $x^2 1$ through $x^2 4$, presented in Table 6, are Godfrey (1978) statistics that test for first- through fourth-order serial correlation in the residuals. These statistics are small. The Ljung-Box Q statistic that tests for higher order serial correlation is also small. These results indicate that the residuals are serially uncorrelated. The Chi-Square statistic, $x^2 5(1)$, which test for a trend in the variance of the residuals, is, however, large, indicating the presence of heteroscedasticity.

footnote 16.) Thus, the long-term rate rises if the short-term real rate rises and/or if long-term inflationary expectations increase. A rise in real income appears to depress the long-term rate. Table 7 evaluates the out-of-sample performance of this regression over the subperiod 1981 to 1990. The results presented here indicate that this interest rate regression can reasonably explain the actual behavior of the long-term interest rate over the 1980s.

The statistic $x^2 6(2)$, presented in Table 6, is for the Lagrange multiplier test[17] of omitted variables. This statistic, which has a Chi-square distribution with two degrees of freedom, tests the null hypothesis that current and one past value of the change in deficit do not enter the regression presented in Table 6. The value of the statistic, $x^2 6(2)$, is small and thus indicates that the deficit does not affect the long-term interest rate in the short run.[18]

4. Concluding Observations

Most previous empirical studies of the behavior long-term rates and the deficit have found that deficits do not cause long-term rates to rise.[19] The contrary evidence shown in Hoelscher (1986) is therefore striking.

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[17]The Lagrange multiplier test for omitted variables is performed by regressing the residuals from the error-correction regression presented in Table 6 on both the original regressors and on the set of omitted variables. See Engle (1984).

[18]Allowing longer lags or using levels (as opposed to first differences) of the deficit does not change the result that the deficit does not enter the error-correction regression reported in Table 6.

[19]For example, see the papers by Plosser (1982) Mascaro and Meltzer (1983), and Evans (1985, 1987).

Hoelscher (1986, p. 15) asserts that the link between deficits and long-term rates is long run in nature and might not have been captured in previous studies, most of which have relied on quarterly or monthly data rather than annual data.

The empirical evidence presented here indicates that the inference concerning the effect of deficits on long-term rates is sensitive, not to the periodicity of the data used, but to the proxy used for long-term expected inflation. The empirical result--deficits are statistically significant in regressions when one-year ahead inflation forecasts are used, but they are not when inflation forecasts over longer-run horizons are used--indicates that one-year ahead inflation forecast series are not adequately measuring long-term expected inflation. Hence, the link found between deficits and long-term rates in such regressions is spurious.

A word of caution is in order. The results presented here simply indicate that deficits do not have an independent effect upon long-term rates once we control for effects of long-term inflationary expectations and the short-term expected real rate. Deficits may still influence long-term rates if they help determine the long-run behavior of money and/or output and thus indirectly or directly influence inflationary expectations.

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Table 1

Evaluation of Inflation Forecasts

Model/Data	One Year Ahead			Two Years Ahead			Three Years Ahead			Four Years Ahead		
	a	b	$x^2(2)$	a	b	$x^2(2)$	a	b	$x^2(2)$	a	b	$x^2(2)$
Monetary Model; Quarterly data	-.45 (.7)	1.02 (.16)	3.1	-.7 (.9)	1.07 (.18)	1.80	-.44 (1.1)	1.05 (.19)	.19	1.0 (1.8)	.80 (.35)	.29
Monetary model; Annual data	-.18 (.56)	.97 (.12)	2.4	-.4 (.55)	1.0 (.11)	2.0	-.78 (.77)	1.06 (.15)	1.56	-.6 (1.3)	1.06 (.21)	.34
One Year Ahead Inflation Forecast from the Livingston Survey Quarterly data	.69 (.5)	.99 (.17)	3.4	1.4 (.71)	.84 (.21)	3.8	2.0 (.8)	.71 (.23)	5.3**	2.5 (.9)	.62 (.25)	6.3*
Autoregressive; Quarterly data	.86 (.43)	.80 (.11)	3.9	1.3 (.51)	.70 (.12)	7.2*	1.8 (.61)	.61 (.13)	10.1*	2.1 (.7)	.54 (.13)	12.1*

Notes: The Table reports coefficients (standard errors in parentheses) from regressions of the form $A_{t+s} = a + b\, P_{t+s}$, where A is actual inflation; p inflation forecast; and s(= 1, 2, 3, 4) numbers of years in the forecast horizon. In monetary and autoregressive models inflation is measured by the implicit GNP deflator, whereas in Livingston surveys inflation is measured by the behavior of the consumer price index. Inflation forecasts from the monetary model are generated for different forecast horizons, whereas the values used for inflation forecasts in the Livingston Survey are for one year horizon only. (The monetary model and the forecasting procedure used are described in the text.) $x^2(2)$ is the Chi square statistic that tests the null hypothesis (a,b) = (0,1) and is distributed with 2 degrees of freedom. All reported standard errors are corrected for the presence of serial correlation.

** indicates significant at ten percent level

'*' indicates significant at the five percent level

Table 2

Unit Root Test Results; 1961Q2-1990Q4

x_t	$\hat{\rho}$	$t_{\hat{\rho}}$	$T(\hat{\rho}-1)$	k	Q(30)
	Augmented Dickey-Fuller Statistics				
R10TB	.94	-1.29	-6.9	8	20.3
(R1TB - $m\Pi1$)	.85	-2.05	-17.2	8	19.0
(R1TB - $L\Pi1$)	.82	-2.05	-20.1	8	24.3
$m\Pi1$	.95	-1.48	-5.3	8	31.6
$m\Pi2$	.95	-1.17	-6.1	8	37.5
$m\Pi3$	.92	-1.25	-9.7	8	37.7
$m\Pi4$	.95	-1.10	-5.6	8	40.7
$L\Pi1$	.97	-1.19	-3.2	8	17.5
ln(rY)	.92	-2.8	-8.9	8	24.7
Δln(ry)	.32	-4.5*	82.3*	4	36.4
rDEF	.82	-2.6	-21.6*	8	26.2

Notes: R10TB is the nominal rate on ten-year Treasury bonds; R1TB is the nominal rate on one-year Treasury bonds; $m\Pi1$, $m\Pi2$, $m\Pi3$, and $m\Pi4$ are respectively, one-year ahead, two-year ahead, three-year and four-year ahead inflation forecasts from the monetary model; $L\Pi1$ is one-year ahead inflation forecast from the Livingston survey; ln(ry) is the logarithm of real GNP; and rDEF is the national income accounts nominal federal deficit deflated by the implicit GNP deflator.

Augmented Dickey-Fuller statistics are from the regression $x_t = a_0 + a_1 LT + \rho X_{t-1} + \sum_{s=1}^{k} b_s \Delta X_{t-s}$, where X_t is the pertinent variable; LT a time trend; k the number of lagged first differences of x_t included to remove serial correlation in the residuals. $t_{\hat{\rho}}$ is the t-statistic; and $T(\hat{\rho}-1)$ the normalized bias statistic. Both are used in the test of the hypothesis that $\hat{\rho}=1$. T is the number of observations used in the regression. Q(30) is the Ljung-Box Q statistic, which tests for the presence of higher order serial correlation and is based on 30 autocorrelations.

'*' indicates significant at the 5 percent level.

'**' indicates significant at the 10 percent level. The 5 percent critical values for $t_{\hat{\rho}}$ and $T(\hat{\rho}-1)$statistics are -3.45 and -20.7, respectively. [see Tables 8.5.1 and 8.5.2 of Fuller (1976).]

Table 3

Cointegration Test Results: Engle-Granger Procedure; 1959Q1-1990Q4

<u>One-year Ahead Inflation Forecasts from the Livingston Survey</u>

<u>Cointegrating Regression</u>

<u>OLS</u>

$$R10TB_t = 7.1 + .74\ (R1TB - L\Pi1)_t + .83\ L\Pi1_t + 1.49\ rDEF_t - .76\ \ln(rY)_t + \hat{u}_t$$

<u>Augmented Dickey-Fuller</u> Statistics

$$\hat{\rho} = -.50 \qquad t_{\hat{\rho}} = -4.54^{*}$$

<u>Dynamic OLS</u>

$$R10TB_t = \underset{(.06)}{8.1} + .81\ (R1TB - L\Pi1)_t + \underset{(.05)}{1.00}\ L\Pi1_t + \underset{(.30)}{1.01}\ rDEF_t \underset{(.83)}{-1.03}\ \ln(rY)_t + \hat{e}_t$$

$\hat{e}_t \sim MA(3)$; $\chi^2_a(1) = 343.2$ $\qquad \chi^2_b(1) = 240.1$ $\qquad \chi^2_c(11) = 6.0$ $\quad \chi^2_d(1) = 1.54$

Notes: Regressions are estimated by ordinary least squares (OLS). Augmented Dickey -Fuller statistics are from the following regression $\Delta \hat{u}_t = \rho\, \hat{u}_{t-1} + \sum_{s=1}^{4} b_s\, \Delta\hat{u}_{t-s}$, where $\hat{u}_t$ is the residual from the cointegrating regression. $t_{\hat{\rho}}$ is the t-statistic that tests the null hypothesis that $\hat{\rho} = 0$.

Dynamic OLS is the dynamic cointegrating regression estimated including current, four past and future values of first differences of right hand explanatory variables (Stock and Watson 1991). Parentheses contain standard errors corrected for the presence of moving-average serial correlation. The order of the moving-average correction was determined by examining autocorrelations of the residuals. $\hat{e}_t \sim MA(3)$ denotes that the residuals follow a third-order moving-average process. χ^2_a, χ^2_b, χ^2_c and χ^2_d, respectively, are the Chi-square statistics that test the null hypotheses that the short-term real rate, the long-term inflation rate, the real deficit and the logarithm of real GNP are not significant in the cointegrating regression. Each is distributed with one degree of freedom (the 5 percent critical value is 3.84).

'*' indicates significant at the 5 percent level (the 5 percent critical value is 4.02; Table 3, Engle and Yoo (1987))

Table 4

Cointegration Test Results: Engle-Granger Procedure; 1959Q1-1990Q4

Inflation Forecasts from the Monetary Model

1. One-Year Ahead Inflation Forecasts

   Cointegrating Regression:

   OLS

   $$R10TB_t = 5.2 + .77\ (R1TB - m\Pi1)_t + .82\ m\Pi1_t + 1.47\ rDEF_t - .54\ \ln(rY)_t + \hat{u}_t$$

   Augmented Dickey-Fuller Statistics: $\hat{\rho} = -.49 \quad t_{\hat{\rho}} = -4.4^{*}$

   Dynamic OLS

   $$R10TB_t = 8.9 + \underset{(.05)}{.88}\ (R1TB - m\Pi1)_t + \underset{(.07)}{1.0}\ m\Pi1_t + \underset{(.39)}{1.28}\ rDEF_t \underset{(.81)}{-1.18}\ \ln(rY)_t + \hat{e}_t$$

   $\hat{e}_t \sim MA(2);\ \chi^2_a(1) = 312.9 \quad \chi^2_b(1) = 199.1 \quad \chi^2_c(1) = 10.4 \quad \chi^2_d = 2.1$

2. Two-year ahead Inflation Forecasts

   Cointegrating Regression:

   OLS

   $$R10TB_t = -.6 + .68\ (R1TB - m\Pi1)_t + .63\ m\Pi3_t + 1.74\ rDEF_t + .16\ \ln(rY)_t + \hat{u}_t$$

   Augmented Dickey-Fuller Statistics: $\hat{\rho} = -.30 \quad t_{\hat{\rho}} = -3.7$

   Dynamic OLS

   $$R10TB_t = 3.8 + \underset{(.06)}{.88}\ (R1TB - m\Pi1)_t + \underset{(.10)}{1.10}\ m\Pi3_t + \underset{(.62)}{.97}\ rDEF_t \underset{(.5)}{-.52}\ (\ln rY)_t + \hat{e}_t$$

   $\hat{e}_t \sim MA(3);\ \chi^2_a(1) = 194.8 \quad \chi^2_b(1) = 151.2 \quad \chi^2_c(1) = 2.45 \quad \chi^2_d(1) = .22$

3. Three-year ahead Inflation Forecasts

   Cointegrating Regression:

   $$R10TB_t = -9.5 + .85\ (R1TB - m\Pi1)_t + 1.06\ m\Pi4_t + .33\ rDEF_t + 1.24\ \ln(rY)_t + \hat{u}_t$$

   Augmented Dickey-Fuller Statistics: $\hat{\rho} = -.35 \quad t_{\hat{\rho}} = -3.9$

   Dynamic OLS

   $$R10TB_t = -.6 + \underset{(.08)}{.82}\ (R1TB - m\Pi1)_t + \underset{(.14)}{1.18}\ m\Pi4_t + \underset{(.74)}{.80}\ rDEF_t + \underset{(1.3)}{.07}\ \ln(rY)_t + \hat{e}_t$$

   $\hat{e}_t \sim MA(3) \quad \chi^2_a(1) = 96.5 \quad \chi^2_b(1) = 74.1 \quad \chi^2_c(1) = 1.16 \quad \chi^2_d(1) = .00$

Notes: See Notes in Table 2 and 3.

Table 5

Level Regressions; Annual Data

Inflation Survey Forecasts

1. $$R10TB_t = 1.1 + .78\,(R1TB - L\Pi 1)_t + .80\,L\Pi 1_t + 1.30\,rDEF_t + 1.9\,\Delta\ln(ry)_t$$
(3.7)(3.4) (20.6) (8.7) (.5)

Sample period: 1959-1990 $\bar{R}^2 = .97$ SER = .46 $\chi^2_{s1}(1) = 2.8$ Q(15) = 10.0

2. $$R10TB_t = -1.1 + 1.01\,(R1TB - L\Pi 1)_t + .91\,L\Pi 1_t + 1.61\,rDEF_t + 8.9\,\Delta\ln(ry)_t$$
(1.0)(10.9) (8.8) (3.6) (1.5)

Sample period: 1979-1990 $\bar{R}^2 = .96$ SER = .416 $\chi^2_{s1}(1) = 2.8$ Q(15) = 10.6

3. $$R10TB_t = .5 + .89\,(R1TB - L\Pi 1)_t + 1.01\,D\Pi 10_t - .08\,rDEF_t + 9.1\,\Delta\ln(ry)_t$$
(.4) (7.9) (7.4) (.2) (1.3)

Sample period: 1979-1990 $\bar{R}^2 = .94$ SER = .489 $\chi^2_{s1}(1) = 2.6$ Q (6) = 10.0

Model based Inflation Forecasts

4. $$R10TB_t = .96 + .78\,(R1TB - m\Pi 1)_t + .81\,m\Pi 1_t + 1.33\,rDEF_t + 2.7\,\Delta\ln(ry)_t$$
(2.9)(8.9) (9.0) (8.5) (.7)

Sample period: 1959-1990 $\bar{R}^2 = .97$ SER = .442 $\chi^2_{s1}(1) = 2.3$ Q(15) = 11.0

5. $$R10TB_t = .52 + .85\,(R1TB - m\Pi 1)_t + .92\,m\Pi 2_t + 1.04\,rDEF_t - 2.7\,\Delta\ln(ry)_t$$
(1.5)(20.8) (19.7) (6.8) (.7)

Sample period: 1959-1990 $\bar{R}^2 = .97$ SER = .426 $\chi^2_{s1}(1) = 1.0$ Q15) = 12.3

6. $$R10TB_t = .16 + .92\,(R1TB - m\Pi 1)_t + 1.1\,m\Pi 3_t + .65\,rDEF_t - 11.7\,\Delta\ln(ry)_t$$
(.4)(17.8) (16.1) (3.4) (2.7)

Sample period: 1959-1990 $\bar{R}^2 = .92$ SER = .514 $\chi^2_{s1}(1) = 2.4$ Q(15) = 11.1

7. $$R10TB_t = .2 + .95\,R1TB - m\Pi 1)_t + 1.2\,m\Pi 4_t + .22\,rDEF - 24.9\,\Delta\ln(rY)_t$$
(.3)(12.3) (10.3) (.7) (3.9)

Sample Period 1959-1990 $\bar{R}^2 = .93$ SER = .753 $\chi^2_{s}(1) = .21$ Q(15) = 11.5

Notes: All Variables are as defined before except DII10, which is ten-year ahead inflationary expectations based on 'the Decision-Makers Poll' of institutional decision-markers conducted by Drexel Burnham Lambert. All regressions are estimated by ordinary least squares. Parentheses contain t-values. $\chi_s^2(1)$ is the Lagrange mutiplier test for first-order serial correlation and is distributed Chi-square with one degree of freedom (the 5 percent critical value is 3.84). Q(l) is the Ljung-Box Q statistic based on the l number of autocorrelations.

Table 6

An Error-Correction Model for the Long-term Interest Rate; 1960Q1 - 1990Q4

$$\Delta R10TB_t = \underset{(1.9)}{-1.9} - \underset{(5.5)}{.20}\, R10TB_{t-1} + \underset{(5.52)}{.19}\,(R1TB - m\Pi 1)_{t-1}$$

$$+ \underset{(5.9)}{.28}\, m\Pi 3_{t-1} + \underset{(2.0)}{.15}\, \Delta R10TB_{t-1} + \underset{(3.1)}{.26}\, \Delta R10TB_{t-1}$$

$$+ \underset{(13.9)}{.37}\, \Delta(R1TB - m\Pi 1)_t + \underset{(1.4)}{.04}\, \Delta(R1TB - m\Pi 1)_{t-1} - \underset{(2.4)}{.07}\, \Delta(R1TB - m\Pi 1)_{t-2}$$

$$- \underset{(2.1)}{.05}\, \Delta(R1YTB - m\Pi 1)_{t-3} + \underset{(5.6)}{.25}\, \Delta m\Pi 3_t - \underset{(-2.7)}{8.9}\, \Delta(\ln ry)_t$$

CRSQ = .69 SER = 3.07 DW = 1.97 Q(33) = 32.59

$x^2 1(1) = .02$ $x^2 2(2) = 1.9$ $x^2 3(3) = 2.7$ $x^2 4(4) = 2.98$

$x^2 5(1) = 37.9^*$ $x^2 6(2) = 2.1$

Notes: All variables are as defined before. CRSQ is the corrected R-squared; SER standard error of the regression; DW the Durbin-Watson statistic; and Q(33) the Ljung-Box Q statistic based on 33 autocorrelations of the residuals.

$x^2 1$, $x^2 2$, $x^2 3$, and $x^2 4$, respectively, are Chi-square statistics (degrees of freedom in parentheses) that test for the first-order, second-order, third-order and fourth-order serial correlation in the residuals. $x^2 5$ is the Chi-square statistic that tests for linear trend in the variance of the residuals. $x^2 6$ is the Chi-square statistic that tests the null hypothesis that current and one-period lagged value of (change in) real deficit do not enter the regression.

'*' indicates significant at the 5 percent level.

Table 7

Out-of-Sample Forecasts; 1981-1990

Year	Actual (R10TB)	Predicted ($\widehat{R10TB}$)	Error
1981	13.9	13.7	.1
1982	13.0	14.8	-1.8
1983	11.1	9.8	1.3
1984	12.4	11.9	.5
1985	10.6	10.3	.3
1986	7.7	8.6	-.9
1987	8.4	8.1	.3
1988	8.8	8.8	.0
1989	8.5	8.7	-.2
1990	8.5	7.8	.7
Mean Error			.04
Mean Absolute Error			.63
RMSE			.84

Notes: The values presented are annual averages, actual and predicted, of the interest rate on ten-year Treasury bonds (R10TB). The predicted values are generated using the regression given in Table 6. The regression is first estimated over 1960Q1 to 1980Q4 and then dynamically simulated out-of-sample over the next four quarters. The mean values, calculated using four quarters data, generate actual and predicted values for 1981. The end of the initial estimation period is then advanced four quarters to 1960Q1 to 1981Q4, the regression reestimated and forecasts prepared as above. The procedure is repeated until the final estimation period 1960Q1 to 1989Q4 is reached.