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# A "COALITION PROOF" EQUILIBRIUM FOR A PRIVATE INFORMATION CREDIT ECONOMY

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ABSTRACT: This paper examines an economy in which agents with private information about their own productive capabilities seek to raise capital to fund their investment projects. We employ an equilibrium concept which is closely related to Coalition Proof Nash Equilibrium. In equilibrium, all agents who succeed in raising capital (entrepreneurs) are pooled; they all receive the same contract or consumption schedule. Entrepreneurs, however, are separated from those who fail to raise capital. This separation results in productive efficiency for the economy. If the economy has no viable alternative investment opportunity (other than agents' projects) then equilibrium allocations can be supported by a (non-intermediated) securities market. If there is a viable alternative, the equilibrium allocations can only be supported through the formation of a form of financial intermedary coalition.

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## Introduction

The basic question asked in this paper has been asked many times before: how will a productive resource be allocated among potential users who are privately informed about their abilities in using that resource? Such a private information problem has been suggested by many as a source of market failure. Following Stiglitz and Weiss (1981), private information has been a key ingredient in many models of credit rationing and other types of market imperfection. Another line of research seeks to characterize efficient allocation mechanisms for private information environments and to find institutional arrangements which support efficient allocation rules. This "efficient mechanism" literature is heavily influenced by the seminal work of Hurwicz (1971).

We shall refer to the environment examined in this paper as a "private information credit economy." This environment is very similar to that examined in the "market failure" literature. We, however, examine this environment in a way which more closely follows the efficient mechanism literature. Efficient allocations are defined relative to the set of resource and incentive feasible allocations. A key question then becomes: can the economy be "decentralized" in an efficient way? Others who have addressed this question for a similar environment include Boyd and Prescott (1986), and Kahn (1987). In both of these papers, the central task is the development of a notion of equilibrium for the economy; in both cases, the chosen equilibrium concept is an adaptation of the core. The motivation for such an equilibrium concept is that, if agents in the economy are free to communicate and propose alternative arrangements, they will settle on an arrangement which is sustainable in the sense of being unblocked by any possible coalition. In Boyd and Prescott's work, the coalition structure which emerges as part of the core is interpreted in terms of the institutions which support efficient allocations.

In this paper, we follow a similar path. We propose, however, an equilibrium concept which incorporates a different notion of sustainability. We suggest that some potential deviations by coalitions may not be "credible"; if the coalition sought to form, its proposed allocation would be subject to further deviation by some of its members. Accordingly, we require that an equlibrium allocation be unblocked only by "credible" deviations.

Our focus on the credibility of deviating coalitions closely follows Bernheim, Peleg and Whinston's (1987) development of the notion of Coalition-Proof Nash Equilibrium (CPNE). For a deviating coalition to be credible, it must be immune from further credible deviations by sub-coalitions. The necessary adaptation involves the adjustment of the notion of a blocking coalition to respect the requirements of private information. We require that any individual that is not intended to be part of a blocking coalition have no incentive to gain admittance by misrepresenting his type. In addition, we do not fully articulate a game which yields the allocations we consider as CPNE. Instead, we describe the set of "Coalition-Proof" allocations. That is, the "equilibrium" concept we employ bears the same relationship to CPNE as does the core to Aumann's (1959) Strong Nash Equilibrium.

For the environment we consider, there is an (essentially) unique coalition proof allocation. This allocation divides the set of agents into those who receive capital for productive purposes and those who simply invest their endowed capital in the productive efforts of others. Agents' state contingent consumption schedules differ only according to whether an agent is allocated any capital, and the "marginal" agent is indifferent between receiving the consumption schedule of an investor and that of an "entrepreneur." Since the set of coalition proof allocations contains the set of core allocations, the core either coincides with the allocation described above or is empty. We show that for an important range of parameter values the core is empty. In doing so, we demonstrate the appeal of the credibility restriction on deviating coalitions. The coalition proof allocation can be blocked only by a coalition which is willing to allocate its capital among its members <u>inefficiently</u>.

Following Boyd and Prescott (1986), we ask whether the proposed equilibrium allocations can be achieved by a fully decentralized securities market. We find that our economy has two important cases, depending on the existence and quality of "outside" risk-free investment opportunities. In one case, securities markets "work," and in the other case they don't. When securities markets fail, we argue that equilibrium allocations can be supported by coalitional arrangements resembling financial intermediaries. These coalitions could be interpreted as organizations which contract with lenders, promising a fixed return. The capital raised is then allocated efficiently among borrowers and the risk-free investment, with borrowers making the appropriate state-contingent repayment promises. Intermediation arises without a technology for producing information; such an information technology was found essential for intermediaries to arise in the Boyd and Prescott framework. We feel that these results shed some light on the role of intermediation in private information economies.

## 1. The Environment

The economy is populated by a continuum of agents who produce and consume a single consumption good using human resources and a single physical resource. An agent is indexed by his type, $\tau \in T \equiv [0,1]$, and the population distribution of types is given by the finite, non-atomic measure $\pi$ on $\mathcal{B}(T)$, the Borel sets of T. Agents have identical preferences; they seek to maximize the expected value of their consumption of the single good.

Each agent has an endowment of one unit of the physical resource (capital), but agents have access to diverse technologies for transforming the resource into the consumption good. Specifically, each agent is endowed with a project which can produce a random output, y, per unit of capital, up to a fixed capacity, $\chi > 1$. Unit output takes one of two values, $y^g$ and $y^b < y^g$. The probability of the good outcome ($y^g$) depends on an agent's type and is denoted $p(\tau)$. We assume that the function $p(\tau)$ is continuous and strictly increasing in $\tau$ and that $0 \leq p(0)$ and $p(1) \leq 1$. To summarize, expected output of a type $\tau$ project in which $x \leq \chi$ units of capital have been invested is written

$$x\mu(\tau) \equiv x[p(\tau)y_g + (1-p(\tau))y_b]$$

In addition to the investment projects of individual agents, there is a risk free, constant returns to scale technology available to all agents. This technology delivers r units of consumption good per unit of investment. For most of what follows, it is assumed that $y_b < r < y_g$. It is worth noting, here, that a type $\tau$ individual's autarkic consumption is the maximum of r and $\mu(\tau)$.

Although all agents know the distribution of types, $\pi(\tau)$, each individual's type is her private information. There is no technology available for verifying or evaluating agents' types. The output from an individual's project is publicly observed, as are any contracts into which an agent might enter. Hence, the only possible method of signalling one's type is through one's choice of contracts. An agent seeking to raise capital can issue state contingent claims, since output is public information. An agent whose $\tau$ is high might seek to signal that fact by offering claims which imply a big difference between her good state and bad state consumptions.

Finally, a semantic note is in order. Except in the case of autarky, some agents in this economy will put capital to productive use while others will invest their capital in the projects of others or in the risk free technology. Any agent who uses capital in the operation of her own project will be called an entrepreneur. Those who do not operate projects will be called investors.

## 2. Allocations: Feasibility and Efficiency

An allocation for this economy must describe both the distribution of capital among projects and the risk free technology and the distribution of consumption goods across agents. The allocation will implicitly describe the division of agents into entrepreneurs and investors; any agent whose project is assigned a positive amount of capital is an entrepreneur. We will consider only allocations which treat all agents of a given type identically, since agents are only different in a meaningful way through their types.

Formally, an allocation consists of four functions and a scalar: the function $x(\tau)$ (for all $\tau \in T$) is the capital assigned to a type $\tau$ agent; the functions $c_g(\tau)$ and $c_b(\tau)$ (for $\tau$ such that $x(\tau) > 0$) are the consumptions of a type $\tau$ entrepreneur contingent on good and bad output, respectively; a function, $c_0(\tau)$ (for $\tau$ such that $x(\tau) = 0$) is the consumption of a type $\tau$ investor; and k is the per-capita amount of capital invested in the risk free technology. For notational simplicity, we define $c(\tau) \equiv (c_g(\tau), c_b(\tau))$ for both investors and entrepreneurs and impose a consistency condition on the consumption of investors:

$$x(\tau) = 0 \Rightarrow c_g(\tau) = c_b(\tau) = c_0(\tau). \tag{2.1}$$

An allocation is denoted $a = (x,c,k)$.[1]

We will use the notation $E_\tau c(\tau')$ for the expected consumption of a type $\tau$ agent receiving the consumption vector $c(\tau')$. Hence, we have that $E_\tau c(\tau') = p(\tau)c_g(\tau') + (1-p(\tau))c_b(\tau')$. If $x(\tau') = 0$, then $E_\tau c(\tau') = c_0(\tau')$. An obvious constraint on allocations is that agents can be no worse off than under autarky:

$$E_\tau c(\tau) \geq \max[\mu(\tau), r]. \tag{2.2}$$

In what follows we will want allocations and feasibility to be defined for subsets of the population. Accordingly, we will consider arbitrary measures $\phi$ on $\mathcal{B}(T)$ such that $\phi(S) \leq \pi(S)$ for all $S \in \mathcal{B}(T)$.[2] We will define feasibility for an allocation-coalition pair $(a,\phi)$ to encompass resource and incentive feasibility. Resource feasibility, in turn, is defined by two conditions. First, capital allocated to entrepreneurs cannot exceed the aggregate capital endowment less capital invested in the alternative technology.

$$\int_T x(\tau)\phi(d\tau) \leq \phi(T) - k. \tag{2.3}$$

In addition, aggregate consumption cannot exceed aggregate production.[3]

$$\int_T E_\tau c(\tau)\phi(d\tau) \leq \int_T x(\tau)\mu(\tau)\phi(d\tau) + rk. \tag{2.4}$$

The careful reader will have noticed that the evaluation of aggregate consumption on the left hand side of (2.4) assumes that each type $\tau$ actually consumes the intended consumption: $c_0(\tau)$ if $x(\tau)=0$ and the state contingent bundle $(c_g(\tau),c_b(\tau))$ if $x(\tau)>0$. Under private information, the type $\tau$ agent could claim to be some other type, $\tau'$. In that case, type $\tau$'s expected consumption would be $E_\tau c(\tau') = p(\tau)c_g(\tau') + (1-p(\tau))c_b(\tau')$. In order to assure that the evaluation of aggregate consumption in (2.4) is valid, the consumption allocation must be incentive feasible:[4]

$$E_\tau c(\tau) \geq E_\tau c(\tau') \text{ for all } \tau, \tau' \in T.^5 \tag{2.5}$$

The set of feasible allocations $A(\phi)$ for coalition $\phi$ is the set of all allocations satisfying (2.1) through (2.5).[6]

If this were a full information economy, the matter of determining efficient allocations would be particularly simple. Consumption allocations would be a matter of (Pareto) indifference, so long as all output was consumed. The standard for the allocation of capital would simply be the maximization of aggregate output subject to the resource constraint (2.3). The solution to this problem would involve a marginal $\tau_0$. All projects with types greater than $\tau_0$ would be funded up to capacity, $\chi$, with any remaining capital going to the alternative technology.

The marginal type in a productive efficient allocation and the amount invested in the alternative technology will depend on the alternative return, r, and on the coalition's measure, $\phi$, over the types of agents. If r is large enough relative to the composition of the coalition, then the marginal entrepreneur would be type $\tau_r$ defined by $\mu(\tau_r) = r$. If r is small, then it is possible for all of a coalition's capital to be used up funding projects with $\tau > \tau_r$. Define $M(\phi)$ as the support of $\phi$. The definition of productive efficiency is complicated by the fact that $M(\phi)$ might not equal T. That is, there may be gaps in $M(\phi)$. Formally, production efficiency is defined as follows

Definition 2.1 An allocation-coalition pair, $(a,\phi)$ is production efficient if $a$ is feasible for $\phi$ and:

a) if $\chi\phi([\tau_r,1]) \geq \phi(T)$, then

$t_0 \in \{\tau \mid \chi\phi([\tau,1]) = \phi(T)\} \equiv T_0(\phi)$;

$x(\tau) = \chi$ for all $\tau \in [\tau_0,1] \cap M(\phi)$; and

$x(\tau) = 0$ for all $\tau \in [0,\tau_0) \cap M(\phi)$; and

b) if $\chi\phi([\tau_r,1]) < \phi(T)$, then

$t_0 \in \{\tau \mid \phi([\tau,1]) = \phi([\tau_r,1])\} \equiv T_0(\phi)$;

$k = \phi(T) - \chi\phi([t_0,1])$;

$x(\tau) = \chi$ for all $\tau \in [\tau_0,1] \cap M(\phi)$; and

$x(\tau) = 0$ for all $\tau$ $[0,\tau_0) \cap M(\phi)$.

For this economy, it will turn out that production efficiency is a characteristic of (some of) the Pareto optimal allocations. In private information economies, it is generally possible for incentive feasibility to be incompatible with production efficiency as defined for the full information benchmark.

An allocation $a$ is Pareto optimal if there is no other feasible allocation $a'$ such that $E_\tau c'(\tau) \geq E_\tau c(\tau)$ for all $\tau \in T$, with strict inequality on a set of positive measure. We could, extending Prescott and Townsend (1984), show that Pareto optimal allocations are solutions to problems of the form

$$\max \int_0^1 \delta(\tau) E_\tau c(\tau) d\tau$$
$$\text{subject to (2.1) through (2.5),}$$

for some arbitrary (measurable) welfare weights $\delta(\tau)$ mapping T into $\mathbb{R}_+$. Solutions to such programs correspond to what Holmstrom and Meyerson (1983) call "interim incentive efficient allocations" and is in keeping with treating all agents of a given type identically.

The set of Pareto optimal allocations is a big set. It includes a wide variety of production efficient allocations. These could include various combinations of pooling and separation among entrepreneurs of varying types. The task of the next section is to consider which, if any, of the Pareto optimal allocations we might expect to be achieved through the rational interaction of agents.

## 3. Equilibrium

How might we expect agents in this economy to solve the resource allocation problem? One can think of a number of institutional arrangements which would dictate the way in which agents interact. Capital could be exchanged for promised payments of consumption good in a decentralized, competitive securities market. Alternatively, one can imagine large intermediaries contracting separately with investors and entrepreneurs. Either one of these institutional arrangements would bring with it a natural notion of equilibrium.

Rather than imposing a particular institutional structure, we propose to allow the agents in the economy considerable lattitude in seeking out bilateral or multilateral contractual arrangements. Other authors who have taken this sort of approach to private information economies, including Boyd and Prescott (1986), Kahn (1987), Boyd, Prescott and Smith (1988), and Marimon (1988) have employed equilibrium concepts which were adaptations of the core. In these works, an equilibrium is an allocation which is resource and incentive feasible and cannot be blocked by any coalition with a feasible allocation for that coalition. These equilibria differ from the standard notion of the core by imposing additional restrictions on blocking coalitions and their allocations. These restrictions are made necessary by private information and the anonymity associated with "large" economies.

The equilibrium concept employed in this paper is similar to those in the works cited above. We wish to imagine that the agents in this economy have an unlimited ability to consider and discuss alternative arrangements. Once adopted, however, an arrangement must be "self-enforcing." Following

Boyd and Prescott (1986) and Kahn (1987), one could define an equilibrium as a feasible allocation which is not blocked by any feasible coalition-allocation pair. As has been pointed out by Bernheim, Peleg and Whinston (1987) and Kahn and Mookherjee (1988), such a definition treats the candidate equilibrium and potential blocking allocations in an asymmetric fashion. Consequently, Bernheim, Peleg and Whinston (1987) have proposed the notion of a "Coalition-Proof Nash Equilibrium." This equilibrium concept has been applied to a moral hazard economy by Kahn and Mookherjee (1988). The equilibrium concept we will develop below is closely related to the CPNE. As in the adaptations of the core to private information settings, the notion of a blocking coalition must be adjusted. Our equilibrium also differs from a CPNE in that we do not explicitly define a game to which the full definition of a CPNE can be applied. Instead, we simply seek allocations which are "coalition-proof" in the same sense as in the CPNE.[7]

The set of possible sub-coalitions to a coalition $\phi$ is

$$\Phi(\phi) \equiv \left\{ \phi':\mathcal{B}(T)\to\mathbb{R}_+ \;\middle|\; \text{for all } S \in \mathcal{B}(T),\ \phi'(S) \le \phi(S),\ \phi' \not\equiv 0 \right\}$$

For a coalition, $\phi$, let $A(\phi)$ be the set of feasible allocations for that coalition. The entire population is described by the coalition $\pi$. We will denote the set of all possible coalitions as $\Phi$ and the set of feasible allocations for the population as A.

Our definition of an equilibrium makes use of the following notion of a blocking coalition.

<u>Definition 3.1</u>: An allocation for coalition $\phi$,

$a$=(x,c,k) $\in$ A($\phi$), is <u>blocked</u> by a coalition $\phi' \in \Phi(\phi)$ and allocation $a'$ = (x',c',k') if:

(i) $a' \in A(\phi')$:

(ii) $E_\tau c'(\tau) \geq E_\tau c(\tau)$ almost everywhere with respect to $\phi'$;

(iii) there exists $S \in \mathcal{B}(M(\phi))$, with $\phi'(S) > 0$ such that $E_\tau c'(\tau) > E_\tau c(\tau)$ for all $\tau \in S$;

(iv) if $S \in \mathcal{B}(M(\phi))$ and $S' \in \mathcal{B}(M(\phi))$ are such that $\phi(S) > \phi'(S)$ and $\phi'(S') > 0$, then $E_\tau c(\tau) \geq E_\tau c'(\tau')$ for all $\tau \in S$ and $\tau' \in S'$.

The second and third conditions in the above definition are the usual requirements of a blocking coalition. The fourth condition is added to respect the constraints of private information. It states that if a deviant coalition intends to leave any agents of type $\tau$ in its complement, then those left behind cannot strictly prefer to join the coalition (including by claiming to be another type).

Using the above definition of a blocking coalition, the <u>core</u> is defined as consisting of all unblocked allocations for the entire economy. We, however, require an equilibrium allocation to be immune only to deviations by coalitions which are themselves immune to further deviations by similarly immune sub-coalitions. We will call this the requirement that an allocation be "coalition-proof." The set of coalition-proof (CP) allocations can be described by the "coalition-proof correspondence."

<u>Definition 3.2</u>: The coalition-proof correspondence (CPC) is a mapping, $\sigma:\Phi \to A$, with the following properties:

(i) $\sigma(\phi) \subseteq A(\phi)$ for all $\phi \in \Phi$; and

(ii) $a \in \sigma(\phi)$ if and only if there does not exist $\phi' \in \Phi(\phi)$ and $a' \in \sigma(\phi')$ which block $a$.

Definition 3.2 states that any allocation-coalition pair which is not coalition-proof can be blocked by a sub-coalition with an allocation which is coalition-proof. On the other hand, if an allocation is coalition-proof, then any "threat" by a coalition to deviate is deterred by a credible threat of further deviations from the deviant coalition. We can now define an equilibrium as a coalition-proof (CP) allocation for the full population.

<u>Definition 3.3</u>: An allocation $a^*$ is an equilibrium for this economy if and only if $a^* \in \sigma(\pi)$.

The requirement that an allocation be coalition-proof is weaker than the requirement that an allocation be unblocked. Accordingly, the set of equilibrium allocations will contain any core allocations. If the core is empty, as it is in some cases for the present economy, there will typically still exist coalition-proof allocations.

Except for the distinction in our definition of blocking necessitated by the information structure of our economy, Definition 3.3 corresponds to the definition of a CPNE used in Kahn and Mookherjee (1988) and follows the definition of a coalition proof correspondence given in Greenberg (1989). Greenberg shows that, given a notion of "blocking," a definition of a CPNE in terms of what we have termed the coalition-proof correspondence is fully equivalent to the recursive definition originally formulated by Bernheim, Peleg and Whinston (1987). This equivalence allows the extension of the concept to an environment with an infinite number of agents.[8]

## 4. Existence of Equilibrium

We begin proving the existence of an equilibrium by describing a candidate CPC. First, for any $\phi\in\Phi(\pi)$ select any $\tau_0\in T_0(\phi)$. Set x and k according to:

$$x(\tau) = \begin{cases} \chi & \forall\tau\in M(\phi)\cap[\tau_0,1] \\ 0 & \forall\tau\in M(\phi)\cap[0,\tau_0) \end{cases} \tag{4.1}$$

$$k = \phi(T) - \chi\phi([\tau_0,1]) \tag{4.2}$$

Note that x and k satisfy production efficiency, and that when $\tau_0 > \tau_r$, $k = 0$. Next set $c(\tau)$ to satisfy:

$$c(\tau) = \begin{cases} c_0 & \forall\tau\in M(\phi)\cap[\tau_0,1] \\ (0,c_g) & \forall\tau\in M(\phi)\cap[0,\tau_0) \end{cases} \tag{4.3}$$

where $c_0 = p(\tau_0)c_g$, and $c_g$ satisfies

$$c_g\left\{ p(\tau_0)\phi([0,\tau_0)) + \int_{\tau_0}^{1} p(\tau)\phi(d\tau) \right\} = \int_{\tau_0}^{1} \chi\mu(\tau)\phi(d\tau) + rk$$

Define $\sigma(\phi)$ as the set of allocations $a=(x,c,k)$ that satisfy (4.1), (4.2), and (4.3) for some $\tau_0 \in T_0(\phi)$. Note that $\sigma(\phi) \subseteq A(\phi)$. We can now state our central result.

> <u>Proposition 1:</u> $\sigma$ is a CPC. Thus the set of equilibrium allocations, $\sigma(\pi)$, is nonempty.

<u>Proof.</u> Since $\sigma(\phi) \subseteq A(\phi)$ $\forall \phi \in \Phi(\pi)$, we need to prove (ii) in Definition 3.2. Take an arbitrary feasible $(a,\phi)$. We will construct the set of allocation-coalition pairs $(a',\phi')$, for $\phi' \in \Phi(\phi)$, that might block $(a,\phi)$ and such that $a' \in \sigma(\phi')$. We will then prove that if $(a,\phi)$ can be blocked by such a pair, $a \notin \sigma(\phi)$, and that if $(a,\phi)$ cannot be blocked by such a pair, then $a \in \sigma(\phi)$.

We state without proof the following easily verified implications of incentive and resource feasibility: $E_\tau c(\tau)$ is nondecreasing and is strictly increasing when $c_g(\tau) \neq c_b(\tau)$; $E_\tau c(\tau)/p(\tau)$ is nonincreasing and is stricly decreasing if $c_b(\tau) \neq 0$; $c_b(\tau)$ is nonincreasing; $c_g(\tau)$ is nondecreasing; $\chi\mu(\tau)-E_\tau c(\tau)$ is nondecreasing; and $c_b(\tau) \leq c_g(\tau)$.

We begin with an arbitrary $(a,\phi)$ such that $\phi \in \Phi(\pi)$, $a \in A(\phi)$ and $\phi(T) > 0$. We will denote the set of potential blocking allocation-coalition pairs by $(a', \phi')$ with $a' \in \sigma(\phi')$. The subscript z indexes the amount (measure) of investors' capital to be used. Let $\bar{z}$ be the amount (measure) of investors in the original coalition, if it were to allocate capital according to production efficiency. If $\chi\phi([\tau_r,1]) \geq \phi(T)$, then $\bar{z} = \phi(T)(\chi-1)/\chi$. If $\chi\phi([\tau_r,1]) < \phi(T)$, then $\bar{z} = \phi(T) - \phi([\tau_r,1])$. The first case is that in which the original coalition does not have sufficient capital to fully fund all projects with expected output no less than the risk free alternative output. The second is the case in which the original coalition's endowment is sufficient to fund all type above $\tau_r$. We will call z the "investment size" of the coalition $\phi'_z$ and say that $\bar{z}$ is the efficient investment size of the original coalition $\phi$.

For each $z \in (0,\bar{z})$, define $\tau_0(z)$ and $\tau_1(z)$ as follows:

$$\tau_0(z) \equiv \{\tau \mid \phi([0,\tau]) = z\};$$

if $(\chi - 1)\phi([\tau_r,1]) < z$, then

$$\tau_1(z) \equiv \{\tau \mid \phi([\tau,1]) = \phi([\tau_r,1])\};$$

if $(\chi - 1)\phi([\tau_r,1]) \geq z$, then

$$\tau_1(z) \equiv \{\tau \mid (\chi-1)\phi([\tau,1]) = z\}.$$

In constructing production efficient allocations, one makes investors out of all $\tau$ up to some threshold. Hence $\tau_0(z)$ is the set of $\tau$ such that one can extract exactly z units of capital by making all types up to $\tau$ investors. If one seeks to make some $\tau > \sup\tau_0(z)$ an investor in a production efficient allocation, then one needs a coalition with investment size greater than z. Similarly, $\tau_1(z)$ is the set of $\tau$ such that all $\tau' \geq \tau$ can be fully invested with capital in an efficient allocation of z units of capital.

Each of the correspondences $\tau_0(z)$ and $\tau_1(z)$ is single valued except at a finite number of points in $(0,\bar{z}]$; they can be set-valued only where there are gaps in $M(\phi)$. Furthermore, $\tau_0$ is strictly increasing, and $\tau_1$ is strictly decreasing, except when $\tau_r \in \tau_1(z)$. Note that $\tau_0(\bar{z}) = \tau_1(\bar{z}) = T_0(\phi)$, as defined in the definition (2.1) of production efficiency.

The sets $\tau_0(z)$ and $\tau_1(z)$ characterize production efficient capital allocations for coalitions of investment size z. In particular, we will define $X(z) \equiv [\sup\tau_1(z),1] \cap M(\phi)$ to be the set of entrepreneurs in the coalition $\phi_z'$. Similarly, let $X^c(z) \equiv [0,\inf\tau_0(z)] \cap M(\phi)$ be the set of investors. Specification of efficient capital allocation is completed by noting that investment in the alternative is $k(z) = z - (\chi-1)\phi(X(z))$.

We now want to charaterize consumption allocations, consistent with $a'(z) \in \sigma(\phi_z')$, which might block $(a,\phi)$. Hence, all investors will be given a constant consumption, $c_0$. This consumption must be preferred to the original allocation by all types below $\tau_0(z)$ and not preferred by any type above $\tau_0(z)$. Accordingly, define $c_0(z)$ by

$$c_0(z) \equiv \{c_0 | E_\tau c(\tau) \le c_0, \text{ for } \tau \in X^c(z) \text{ and } E_\tau c(\tau) \ge c_0, \text{ for } \tau \in M(\phi) \backslash X^c(z)\}$$

for $z \in (0,\bar{z})$, and

$$c_0(\bar{z}) \equiv [E_{\underline{\tau}} c(\underline{\tau}), +\infty), \text{ where } \underline{\tau} = \inf\tau_0(\bar{z}).$$

The definition of $c_0(\bar{z})$ allows for the possibility that the original allocation "wastes" some consumption good. Once one has the efficient sets of investors and and entrepreneurs, if their consumptions don't exhaust available output, consumption can rise. Clearly, there will be some level above which $c_0(\bar{z})$ will be infeasible. It is also clear that, as $c_0(\bar{z})$ rises, the consumption given to entrepreneurs must rise. Otherwise, too many investors would be drawn to the consumption $c_0(\bar{z})$. Entrepreneurs get a consumption schedule, $(0,c_g)$, which must be preferred to the status quo by all $\tau$ above $\tau_1(z)$ and not preferred by all below $\tau_1(z)$. The set of $c_g$ which can serve this purpose can be expressed as

$$c_g(z) \equiv \{c_g | \frac{E_\tau c(\tau)}{p(\tau)} \geq c_g, \text{ for } \tau \in M(\phi)\backslash X(z), \text{ and } \frac{E_\tau c(\tau)}{p(\tau)} \leq c_g, \text{ for } \tau \in X(z)\}$$

for $z \in (0,\bar{z})$, and

$$c_g(\bar{z}) \equiv [E_{\bar{\tau}} c(\bar{\tau}), +\infty), \text{ where } \bar{\tau} = \sup \tau_1(\bar{z})$$

Let $X^c$ (with no z-index) be the set of investors in the original $(a,\phi)$. Recall that, in the original allocation, all investors received a constant consumption. Therefore, $c_0(z)$ is constant (and single valued) for $z < \phi(X^c)$. For $z \geq \phi(X^c)$, $c_0(z)$ is single valued except, possibly, at (a finite number of) points in $[\phi(X^c),\bar{z}]$ which correspond to gaps in $M(\phi)$. For $z > \phi(X^c)$, $c_0(z)$ is strictly increasing. Note that the set $\{(c_0,z) \mid c_0 \in c_0(z), z \in (0,\bar{z}]\}$ is an unbroken curve in $\mathbb{R}^2$.

Recall, now, that feasibility of $(a,\phi)$ implies that $E_\tau c(\tau)/p(\tau)$ is nonincreasing, and strictly decreasing when $c_b(\tau) > 0$. Since $c_b(\tau)$ is nonincreasing, $E_\tau c(\tau)/p(\tau)$ is constant on at most a right hand interval of $[0,1]$, and is strictly decreasing elsewhere. Discontinuities can only occur at gaps in $M(\phi)$. Hence, $c_g(z)$ is set valued only at gaps in $M(\phi)$. Note that, as we raise z, we draw in entrepreneurs from the top down. Therefore, $c_g(z)$ is constant on, at most, a left-hand interval in $[0,\bar{z}]$ (corresponding to the right-hand interval in [0,1] over which $E_\tau c(\tau)/p(\tau)$ is constant). Elsewhere, it is strictly increasing.

The sets $X(z)$, $X^c(z)$, $c_0(z)$, and $c_g(z)$ describe the set of all allocations for coalitions of investment size z which: make all members at least as well off as in the original allocation; give nonmembers ($\tau \in (\sup\tau_0(z),\inf\tau_1(z))$ no incentive to join, even by misrepresenting their types; and have (internally) incentive feasible consumption schedules which have the "right form" (some $c_0$ for investors and some $(0,c_g)$ for entrepreneurs). To complete our specification of potential blocking $(s,\phi)$, we need to find investment sizes for which the allocations given by these correspondences are resource feasible. To do so, we begin by defining the correspondence $\Gamma(z)$ by:

$$\Gamma(z,c_g) \equiv \frac{1}{z}\left\{ \int_{X(z)} [\chi\mu(\tau)-p(\tau)c_g]\phi(d\tau) + rk(z) \right\}$$

and

$$\Gamma(z) \equiv \{\Gamma(z,c_g) \mid c_g \in c_g(z)\}$$

Note that $\chi\mu(\tau) - p(\tau)c_g$ is strictly increasing in $\tau$. This fact is most clearly seen by observing that $\chi y_b<r$, and that, for an investment to be preferred to the alternative, $\chi\mu(\tau)-p(\tau)c_g\geq r$. As z increases, X(z) grows by

including lower types ($\tau_1(z)$ nonincreasing). Therefore, the average residual paid to investors, $\Gamma(z,c_g)$, is nonincreasing in z. Further, the average residual can only be constant in z if X(z) is empty. This would occur if all members of the original coalition had types below $\tau_r$. In that case, the average residual would be identically equal to r, for all z in $[0,\bar{z}]$. As a result, we have that $\Gamma(z)$ is either constant everywhere, or strictly decreasing everywhere. Note that the set $\{(\gamma,z) \mid \gamma\in\Gamma(z),\ z\in(0,\bar{z}]\}$ is an unbroken curve in $\mathbb{R}^2$. Given the definition of $c_g(\bar{z})$, $\Gamma(\bar{z})$ extends to $-\infty$.

For any $z \in (0,\bar{z}]$, define $\phi_z'$ as follows: $\phi_z'(S) = \phi(S)$ for $S\in\mathcal{B}(X(z)\cup X^c(z))$; and $\phi_z'(S) = 0$ for $S\in\mathcal{B}[T|(X(z)\cup X^c(z))]$. Let $\alpha(z)\subseteq\sigma(z)$ be the set of allocations that satisfy: if $\tau \in X^c(z)$, then $x(\tau) = 0$ and $c(\tau) = (c_0,c_0)$, $c_0 \in c_0(z)$; if $\tau \in X(z)$, then $x(\tau) = \chi$ and $c(\tau) = (0,c_g)$, $c_g \in c_g(z)$. The set of feasible $(a',\phi')$ which satisfy $\phi'\in\Phi(\phi)$ and $a'\in\sigma(\phi')$ and which potentially block $(a,\phi)$ can be derived from the set $Z^*$:

$$Z^* \equiv \{z \mid c_0(z) \cap \Gamma(z) \text{ is nonempty}\}.$$

To see that $Z^*$ is nonempty, we need to show that $\sup\Gamma(z)\geq\inf c_0(z)$ for some $z>0$ (since $\Gamma(z)$ is decreasing and $c_0(z)$ is nondecreasing). To find such z, note, first, that for all $z \leq \phi(X^c)$, $c_0(z)=c_0$, a constant. Now consider the limit of $\sup\Gamma(z)$ as z goes to zero. If X (the set of investors in the original $(a,\phi)$) is nonempty, then this limit is $[\chi\mu(\bar{\tau}) - E_{\bar{\tau}}c(\bar{\tau})]/(\chi-1)$, where, $\bar{\tau} = \sup M(\phi)$. If X is empty, $\Gamma$ is identically r everywhere, including in the limit. From the feasibility of the original allocation, this limit is greater than $c_0$ when X is nonempty and greater than or equal to $c_0$ when X is empty. The existence of a z such that $\Gamma(z) \geq c_0(z)$ implies that there is some z such that $\Gamma(z)=c_0(z)$.

We have constructed a set of allocation-coalition pairs, $D^* \equiv \{(a',\phi'_z) \mid z \in Z^*$, and $a' \in \alpha(z)\}$. Pairs in $D^*$ satisfy conditions (i), (ii) and (iv) of definition 3.1 of a blocking allocation-coalition. The remaining condition for blocking (condition (iii) of Definition 3.1) is the condition that some types be made strictly better off for a successful block. Hence, we can complete our proof of the proposition by proving that $a \in \sigma(\phi)$ (where $(a,\phi)$ is the original coalition-allocation pair) if and only if $E_\tau c'(\tau) = E_\tau c(\tau)$ for all $\tau \in M(\phi')$, for all $(a',\phi') \in D^*$.

First, suppose that $a \in \sigma(\phi)$. Then, for all $z \in (0,\bar{z}]$, $c_0(z) = c_0$, and $c_g(z) = c_g$ (where $c_0$ and $c_g$ are investors' and entrepreneurs' consumptions from the original allocation). Therefore, $E_\tau c'(\tau) = E_\tau c(\tau)$ for all $\tau \in M(\phi'_z)$, for all allocation-coalition pairs in $D^*$.

Now, suppose that $E_\tau c'(\tau) = E_\tau c(\tau)$ for all $\tau \in M(\phi')$, for all $(a',\phi') \in D^*$. We will prove that $a \in \sigma(\phi)$ for three cases: 1) $\bar{\tau} \leq \tau_r$, so X(z) is empty for all z; 2) X(z) is nonempty ($\bar{\tau} > \tau_r$) and $Z^* = \{\bar{z}\}$; and 3) X(z) is nonempty and $Z^* = \{z^*\}$, $z^* < \bar{z}$ (recall that, when X(z) is nonempty, $\Gamma$ is strictly decreasing, so that, in both of the last two cases, $Z^*$ is a singleton).

<u>Case 1</u>: Since $\bar{\tau} \leq \tau_r$, production efficiency requires no entrepreneurs (all investment goes to the alternative technology). For all z, $\Gamma(z) = r = c_0'$. The assumption that $E_\tau c'(\tau) = E_\tau c(\tau)$ implies that, for all $\tau \in X^c$, $c_0 = c_0' = r$. Feasibility of $(a,\phi)$ requires

$$c_0 \leq \frac{1}{\phi(X^c)}\left\{ \int_X [x(\tau)\mu(\tau) - E_\tau c(\tau)]\phi(d\tau) + rk \right\}.$$

The right hand side of the above inequality is less than or equal to r, with equality only if $\phi(X) = 0$. Thus feasibility requires that $(a,\phi)$ is production efficient, in order for $c_0 = r$ to be satisfied. For this case, production efficiency and investors' consumption equal to r yield $a \in \sigma(\phi)$.

Case 2: If $Z^* = \{\bar{z}\}$, then the only possible blocking coalition is the coalition of the whole. Since $E_\tau c(\tau) = E_\tau c(\tau)$ for all $\tau \in M(\phi) = M(\phi')$, we know that: $c(\tau) = (c_0', c_0')$ for all $\tau \in X^c(\bar{z})$; and $c(\tau) = (0, c_g')$ for all $\tau \in X(\bar{z})$. Therefore, $X(\bar{z}) \subseteq X$; only entrepreneurs can have output-contingent consumption. Feasibility of $(a,\phi)$ implies

$$c_0\phi(X^c) + \int_X E_\tau c(\tau)\phi(d\tau) \leq \int_X x(\tau)\mu(\tau)\phi(d\tau) + rk.$$

The left hand side of the above can be written

$$\begin{aligned} &c_0\phi(X^c) + c_0\phi(X^c(\bar{z})\backslash X^c) + \int_{X(\bar{z})} p(\tau)c_g\phi(d\tau) \\ &= c_0\phi(X^c(\bar{z})) + \int_{X(\bar{z})} p(\tau)c_g'\phi(d\tau) \\ &= \int_{X(\bar{z})} \chi\mu(\tau)\phi(d\tau) + rk. \end{aligned}$$

Therefore, resource feasibility of $(a,\phi)$ requires that $a$ be production efficient and, thus, $X=X(\bar{z})$, $x(\tau) = \chi$ for $\tau \in X$, and $k = k(\bar{z})$. These equalities imply that $a = a'$, and, therefore, that $a' \in \sigma(\phi)$ by construction.

Case 3: In this case, the potential blocking coalition has investment size $z^*$ less than $\bar{z}$, so that $(X(z^*) \cup X^c(z^*)) \subset (X \cup X^c)$. As in case 2, we have that $c(\tau) = (0, c_g')$ for all $\tau \in X(z^*)$. This equality implies that $X(z^*) \subseteq X$. For all $\tau \in X^c(z^*)$, we have $c(\tau) = (c_0', c_0')$. This, in turn, implies that $c(\tau) = (c_0', c_0')$ for all $\tau \in X^c$. Define $\bar{X}$, $\bar{X}^c$, $\bar{k}$ as the production efficient capital allocation for $\phi$ (the entire original coalition). From the fact that $\phi$ is strictly bigger (in investment size) than $\phi'$, together with the fact that $\chi\mu(\tau) - E_\tau c(\tau)$ is increasing in $\tau$ (from incentive feasibility), we have:

$$c_0 = c_0' \;=\; \frac{1}{\phi(X^c(z^*))}\left\{ \int_{X(z^*)} [\chi\mu(\tau) - E_\tau c(\tau)]\phi(d\tau) + rk \right\}$$

$$> \frac{1}{\phi(X^c)}\left\{ \int_{\bar{X}} [\chi\mu(\tau) - E_\tau c(\tau)]\phi(d\tau) + r\bar{k} \right\}.$$

The last inequality follows from the fact that the left hand side is $\Gamma(z^*)$, while the right hand side is $\Gamma(\bar{z})$. From this inequality, we have:

$$\int_X x(\tau)\mu(\tau)\phi(d\tau) + rk \;\le\; \int_{\bar{X}} \chi\mu(\tau)\phi(d\tau) + r\bar{k}$$

$$< \; c_0\phi(\bar{X}^c) + \int_{\bar{X}} E_\tau c(\tau)\phi(d\tau)$$

$$\le \; c_0 + \int_X E_\tau c(\tau)\phi(d\tau).$$

The last inequality, here, follows from the fact that $E_\tau c(\tau) \geq c_0$ for all $\tau \in M(\phi)$, and the fact that $\bar{X} \subseteq X$. This chain of inequalities implies that $(a,\phi)$, the original allocation-coalition pair, was not feasible, a contradiction. Hence, case 3) is not possible. That is, if $\bar{\tau} > \tau_r$ (so that $X(z)$ is nonempty for all $z$), and if $E_\tau c'(\tau) = E_\tau c(\tau)$, then $z^* = \bar{z}$. Then, by case 2), $a \in \sigma(\phi)$. Q.E.D.

Equilibrium allocations, $\sigma(\pi)$, have a straightforward interpretation. Take a production efficient allocation of capital and support it with a feasible consumption allocation which: does not waste any consumption good; pools all entrepreneurs at a single consumption bundle $(0,c_g)$; and pools all investors at a single consumption point, $c_0$, such that the marginal type $\tau_0$ is just indifferent between the investor's and the entrepreneur's consumption. The marginal type, $\tau_0$, is determined by the selection of a production efficient allocation of capital. For any coalition, including the coalition of the whole, it is possible for there to be more than one possible marginal type (that is, for $T_0(\phi)$, as defined in Definition 2.1, to contain more than a single point). For any given efficient marginal type, the procedure described by (4.1), (4.2) and (4.3) yields a unique allocation. Hence, a sufficient condition for the uniqueness of equilibrium is that the support of $\pi$ be the entire interval $[0,1]$.

It will be useful, here, to examine the relationship between the coalition proof allocations derived above and the core of the economy. The core can be defined, using definition 3.1, as any allocation on the entire economy which is unblocked. Clearly, any unblocked allocation is, in particular, unblocked by coalition proof deviations. Hence, our set of equilibrium allocations contains the core. Since our equilibrium is unique (except when $T_0$ contains more than a single element), it follows that either the core is the same single allocation, or the core is empty. We show below that, if production efficiency requires that some capital go to the alternative investment, the core is empty.

> <u>Proposition 2:</u> Let $k^*$ denote the allocation of capital to the alternative technology in an allocation $a \in \sigma(\pi)$. Let $X^*$ denote the set of entrepreneurs in $a$. If $k^* > 0$, and $\pi(X^*) > 0$, then the core is empty.

<u>Proof:</u> The inclusion of the core in $\sigma(\pi)$ means that to prove that the core is empty we need only construct, for any CP allocation $a$, a blocking $(a',\phi')$. All other allocations are blocked by something in the CPC. Note that in the CP allocation $a$, with $k^* > 0$ and $\pi(X^*) > 0$, two types of "investments" are being made; some capital is being allocated to the alternative, and the rest is being allocated to entrepreneurs in the form of a pooled contract $(0,c_g)$. Since all entrepreneurs have $\tau \geq \tau_r$, the expected return on this pooled contract is strictly greater than r. Consider a coalition which takes all of the entrepreneurs from the original allocation and "some" of the investors. Suppose that this coalition invests all of its capital in entrepreneur's projects (none in the alternative). That is, the coalition's composition can be given by: $\phi'(X) = \pi(X)$; and $\phi'(X^c) = \pi(X^c) - k^*$ (where X and $X^c$ are the entrepreneurs and investors, respectively, in the CP allocation).

Now consider the following allocation for $\phi'$: $X' = X$, and $X^{c\prime} = X^c$; $c(\tau) = (0,c_g)$ for $\tau \in X$; and $c(\tau) = (c_0,c_0)$ for $\tau$ in $X^c$. This is not a blocking allocation, since all types have the same consumption as in $a$. Note, however, that average output will be strictly greater than in the original allocation. None of this surplus output can be given to investors without drawing in all investors (in violation of $\phi'(X^c) < \pi(X^c)$ ). Give some of the surplus to entrepreneurs by setting $c_g' = c_g + \varepsilon$, for $\varepsilon$ close to zero. This increase makes entrepreneurs strictly better off. Note, however, that the increase in $c_g$ also makes some investors near the margin ($\tau$ just below $\tau_r$) switch to being entrepreneurs. This switch lowers average output. For small $\varepsilon$, though, average output will stay high enough so that the remaining investors can be paid the original $c_0$. In fact, for $\varepsilon$ small enough, there will still be surplus output left after paying the investors $c_0$ and the entrepreneurs $c_g + \varepsilon$.

For some $\varepsilon$ near zero, then, $(a',\phi')$, as constructed above, successfully blocks $(\sigma(\pi),\pi)$. Since any allocation $a \neq \sigma(\pi)$ can be blocked by $(a',\phi')$ for some $\phi' \in \Phi(\pi)$ and $a' \in \sigma(\phi')$, the core is empty. Q.E.D.

We feel the proof is suggestive of the value of the credibility restriction on potential deviations. All of the allocations which can be used to block allocations in $\sigma(\pi)$ are inefficient. They necessarily attract "sub-marginal" entrepreneurs. In order to avoid investing in the alternative technology, a blocking coalition must invest some capital in projects which are even worse than the alternative technology. In addition, a blocking allocation may have to throw away some surplus output. The less it throws away, the more "inferior" entrepreneurs are attracted. One can easily imagine that the coalition described above, if it formed, would realize that it was using its capital inefficiently. If one so imagines, however, it is hard to imagine that such a coalition would ever form as an effective blocking coalition.

If the coalition proof allocation does set k=0 (because r is "low"), then even an inefficient (non-credible) coalition cannot effectively deviate. In this case, there is only one "type" of investment being made. All capital is invested in the single, pooling, entrepreneur's contract. Hence there is no gain to entrepreneurs from forming a coalition which invests nothing in the alternative; nothing is already invested in the alternative. In this case, it seems as though the CP allocation is also the (unique) core allocation.

Note that the comparison between CP allocations and the core depends on the value of the return to the alternative investment. In the next section, we argue that the value of this return is also crucial in determining whether CP allocations can be supported as the competitive equilirium of a decentralized securities market. We might suggest, here, that the case in which r is "high" (and k > 0) is the more "natural" case. If we wish to imagine an economy with an "entrepreneurial sector" and a sector of more well established productive activity (the alternative investment), then the typical case would seem to be that in which some capital goes to each of the sectors. Such reasoning is somewhat self-serving, since it tends to strengthen the case for the coalition proof equilibrium concept we employ. Even though the core is empty, we are able to describe an arrangement with desirable sustainability properties. The remaining task is to search for institutions which may support this arrangement.

5. Alternative institutional arrangements

The equilibrium concept employed above imposes very little structure on the way in which agents interact in this economy. A natural question to ask is whether the essentially unique coalition-proof allocation described in section 4 can be supported by a competitive securities market. If not, what other institutional arrangements might allow the economy to achieve the proposed allocation? This line of questioning follows in the spirit of Boyd and Prescott (1986). For a somewhat different private information credit economy, they show cases in which a securities market "works" and others in which such an institution "fails." In the latter cases, they argue that one should expect alternative institutions to arise. In the case of their economy, the alternative institution which does the trick is a form of "large" financial intermediary. What follows is an informal discussion of the effectiveness of and alternatives to securities markets in our economy.

For this economy, we propose to view a securities market as one in which entrepreneurs offer contingent claims contracts in order to raise capital from investors. Given their beliefs about what types of entrepreneurs are offering what contracts, investors will purchase claims paying the highest rate of return. Taking the beliefs of investors and the "market rate of return" as given, entrepreneurs will seek to offer contracts which maximize their own expected consumption. Clearly, in equilibrium, all projects which receive financing must pay the same rate of return under investors' beliefs. In addition, it cannot be possible for any entrepreneur (or would be entrepreneur) to offer a contract which increases her own expected consumption and which is expected by investors to pay a greater than market rate of return.

A securities market equilibrium will typically exist in our economy. [9] Whether a securities market equilibrium fails or succeeds to support the coalition-proof allocation seems to depend on the return to the alternative technology. Consider, first, the case in which this return, r, is small enough so that production efficiency implies no investment in the alternative (this is case (a) of Definition 2.1). In this case $c(\tau) = (0, c_g)$ for all $\tau \geq \tau_0$. This allocation corresponds to a market in which each entrepreneur offers output contingent payments r(y) such that $r(y_g) = \chi y_g - c_g$ and $r(y_b) = \chi y_b$. Since all entrepreneurs are offering the same (pooling) contract, investors' expected return is averaged over $[\tau_0, 1]$. This average return is, of course, exactly the consumption $c_0$ assigned to investors by the coalition-proof allocation. With the pooled contract, higher $\tau$ entrepreneurs will find themselves paying a higher than average return while lower $\tau$ entrepreneurs pay lower than average (as evaluated by the entrepreneurs themselves, not by investors). Hence, high $\tau$ entrepreneurs would like to be able to separate themselves by offering a contract with a higher $c_g$ and a lower $c_b$. This is not possible, however, since $c_b = 0$. Any feasible deviation which a high type would find attractive would also be attractive to lower type entrepreneurs. The coalition-proof allocation is supported by a securities market equilibrium in this case.

The case of low r in our model corresponds to one of the cases discussed in Boyd and Prescott (1986). Their general model allows for the costly production of publicly observed signals of agents' types. One special case arises when the signal is uncorrelated with the true type. This is equivalent to there being no technology for information production, as in our model. As in our low r case, there is no alternative investment technology. For this case, Boyd and Prescott show that a securities market equilibrium supports their core allocation. Except for the pooling of many types of entrepreneurs on one consumption point (in the Boyd and Prescott model there are but two types of agent), our coalition-proof allocation is very similar to their core allocation. In both cases, those agents who become investors are paid enough to keep them from trying to mimic better types (entrepreneurs). They earn rents in that they earn a rate of return which is strictly greater than their autarkic consumption.

Our case of high r does not correspond to any case considered by Boyd and Prescott. A number of other authors, however, have produced market failure results for private information credit markets with non-trivial alternative investment opportunities. Stiglitz and Weiss (1981), among others, have suggested the existence of credit rationing in such environments while Meza and Webb (1987) have produced an over-investment result for an environment very similar to our own. The notion of equilibrium used in these models is very similar to the securities market equilibrium sketched above. It should not be surprising, then, that, in our case of high r, a securities market cannot support the coalition-proof allocation. The reason is quite intuitive. There are two types of investments that investors can make; they can invest in the pooled contract offered by the entrepreneurs or they can invest in the alternative technology. Since, in the coalition-proof allocation, all entrepreneurs are "infra-marginal" ($\mu(\tau) \geq r$), the expected rate of return from entrepreneurs' projects is greater than r. This is incompatible with a securities market equilibrium. An equilibrium must have the return averaged over entrepreneurs' projects equal to r. This is achieved by having investors' consumption smaller than in the coalition-proof allocation and entrepreneurs' good state consumption ($c_g$) greater than in the coalition-proof allocation. These consumptions draw "sub-marginal" ($\mu(\tau) < r$) types out of investing and into entrepreneurship. This yields an "over-investment" result analogous to that of de Meza and Webb; too much is invested in entrepreneurial projects and not enough in the alternative technology.

If a securities market fails to be efficient in the case of high r, then is there some other form of institution which succeeds? A natural possibility is some form of intermediated financial market. One can imagine some agents acting as intermediaries (one might wish to assume here that acting as an intermediary does not interfere with an agent's other role as an investor or entrepreneur); an intermediary would try to attract agents into its organization or coalition by offering contracts for both entrepreneurs and investors which imply an allocation for that coalition. Once a coalition is formed, its members might be free to renegotiate contracts; if so, the intermediary's initial proposal must be coalition proof for the coalition it is seeking to attract. If there is "free entry" into intermediation, then intermediaries must make zero profit.

The intermediaries in this scenario have four of the five characteristics of financial intermediaries highlighted by Boyd and Prescott; they borrow from one subset of agents (investors) and lend to another (entrepreneurs); both subsets are "large"; would-be borrowers have private information concerning their own credit risk; and claims issued by the intermediaries (payments to investors) have different state-contingent payoffs from the claims issued by the ultimate borrowers. The one characteristic missing is that intermediaries do not spend resources producing information on the attributes of would-be borrowers. While expenditures on evaluation (and monitoring) are certainly an important component of the activities of real-world intermediaries, it is interesting that we find a role for intermediaries in an environment where no such expenditure is possible. In the Boyd and Prescott environment, the existence of an informative, costly signal was essential for intermediaries to have an important role in resource allocation.

Although the environments are different, the reason why intermediaries may be needed is essentially the same in the present model as in Boyd and Prescott; intermediaries allow the economy to avoid the "excessive signalling" that occurs in a securities market. In Boyd and Prescott, excessive signalling takes the form of over-investment in evaluation. In our case of high r, signalling amounts to claiming to have a high enough $\tau$ to be an entrepreneur; this claim is backed up by a willingness to accept the state-contingent entrepreneur's consumption schedule. When agents with $\mu(\tau) < r$ seek to be entrepreneurs (as occurs in the securities market if r is high), then excessive signalling is occuring. Hence, the need for intermediaries is brought about not by the existence of a costly evaluation technology but by the existence of any signalling behavior which might be over-utilized in a securities market setting.

## 6. Conclusion

Our understanding of the allocation of resources under private information is still in its infancy as compared, for instance, to our understanding of "classical" (Arrow - Debreu) environments. Key questions deal with how and why observed institutions arise and with the role of institutions in solving resource allocation problems. We have contributed to the effort to answer these questions by applying an equilibrium concept which puts relatively few restrictions on the way in which economic agents interact. There may be conditions under which more restrictions are natural. For instance, limited communication may restrict agents' abilities to propose and discuss alternative arrangements. Similarly, limited commitment or *ex post* private information can restrict agents' abilities to make state-contingent arrangements. One goal of the theory of mechanism design should be to map out how various combinations of such frictions affect the types of bilateral or multilateral arrangements into which economic decision makers will enter. This paper has attempted a modest step in that direction.

## NOTES

1. Actually, we will regard as equivalent any two functions on T that differ only on sets of measure zero. Thus, an allocation is an equivalenc class of functions.

2. Note that, since $\pi$ is nonatomic, all measures $\phi$ on $\mathcal{B}(T)$ such that $\phi(S) \leq \pi(S)$ for all $S \in \mathcal{B}(T)$ are also nonatomic.

3. The resource constraint (2.4) is, as written, requires that <u>expected</u> aggregate consumption not exceed <u>expected</u> aggregate production. However, with a continuum of independent random variables, realized aggregate values are not necessarily equal to expected aggregate values. We, however, do not need the law of large numbers. All we need is for entrepreneurs to get their state-contingent $(c_b(\tau), c_g(\tau))$, and for the realized residual to be distributed among investors in some (possibly random) way so that expected consumption is $c_0(\tau)$ for investors. That is, given risk neutrality, $c_0$ can be an expected value of a consumption lottery. It is, nevertheless, convenient to speak of $c_0$ as if it were deterministic (that is, as if a law of large numbers did apply). Nothing in our analysis would change if we treated $c_0$ explicitly as a random variable.

4. In order to avoid cumbersome statements of conditions regarding $\tau$, we adopt the following semantic convention: if $g(\tau,\tau')$ is some measurable expression (such as $E_\tau c(\tau) - E_\tau c(\tau')$), then the statement, "$g(\tau,\tau') \geq 0 \ \forall \tau \in \hat{S}$,

$\forall\tau'\in\hat{S}'$," means that, for all $S\in\mathcal{B}(\hat{S})$ such that $\phi(S)>0$ and for all $S'\in\mathcal{B}(\hat{S}')$ such that $\phi(S')>0$,

$$[\phi(S)\phi(S')]^{-1}\int_S\int_{S'} g(\tau,\tau')\phi(d\tau')\phi(d\tau) \;\geq\; 0.$$

Equalities are defined similarly.

5. Imposing incentive compatibility as a feasibility constraint amounts to invoking the revelation principal in some form. In a multilateral setting, this requires showing that any attainable allocation can be achieved by a direct mechanism that indices truthful revelation as a (Nash) equilibium. With an infinite number of agents, and a fixed "number" (measure) of agents of each type, an individual agent's message has no effect on the "aggregate" nature of messages sent. This allows a straightforward application of the revelation principal. Note that, although we are allowing for some degree of "cooperation" in the choice of allocations, the implementation of an allocation must satisfy the noncooperative requirement of incentive compatibility.

6. We also require that $x$ and $c$ be $\phi$-measurable. Recall, from note 1, that $x$ and $c$ are equivalence classes of functions which differ only on a set of measure zero with respect to $\phi$.

7. We could, following Kahn and Mookherjee (1988), define a contracting game whose CPNE outcome is an allocation satisfying our definition of an equilibrium. Our equilibrium bears the same relationship to CPNE as do the core-type equilibria to the Strong Nash Equilibrium concept of Aumann (1959).

8. Kahn and Mookherjee (1990) show that, with infinite sets of agents and infinite strategy spaces, what they call a consistent set, analagous to our CPC, may fail to exist even though CPNE, by the recursive definition, do exist. If, however, a consistent set does exist, then it exactly corresponds to the set of CPNE.

9. Because of the dependence on investors' (off-equilibrium) beliefs, there may in fact be multiple equilibria for the securities market institution. We are confident that the equilibria on which we focus would survive the usual type of refinements based on beliefs (as, for instance, in Cho and Kreps (1987)).

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