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Sovereign Debt and Credit Default Swaps

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Abstract

How do credit default swaps (CDS) affect sovereign debt markets? The answer depends crucially on trading frictions, risk-sharing, arbitrage violations, and spillovers from secondary to primary markets. We propose a sovereign default model where investors trade bonds and CDS over the counter via directed search. CDS affect bond prices through several channels. First, CDS act as a synthetic bond. Second, CDS reduce bond-investing risks, allowing exposure to be unwound. Third, CDS availability increases trading profitability, which induces entry and reduces trading costs. Last, these direct effects feedback into default decisions. Our novel identification strategy exploits confidential microdata to quantify the extent of trading frictions and risk-sharing. The model generates realistic CDS-bond basis deviations, bid/ask spreads, and CDS volumes and positions. Our baseline specification predicts large effects of frictions generally but small spillovers from a naked CDS ban. These predictions hinge crucially on the identified parameters.

Keywords: sovereign debt, CDS, directed search, over-the-counter

JEL codes: F34, G12

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1 Introduction

Credit default swaps (CDS) are financial derivatives created in the 1990s that provide insurance against default risk. The volume of transactions on CDS linked to sovereign government bonds, which we refer to as sovereign CDS, has steadily increased since their inception.[1] However, very little is known about the interactions between sovereign bond and sovereign CDS markets and, specifically, how quantitatively relevant these interactions are. One possible reason for this lack of knowledge is that both of these tightly-related assets are traded in opaque over-the-counter (OTC) markets, where transactions occur bilaterally between market participants.

We propose a model of sovereign default where bonds and CDS trade over the counter, and we leverage regulatory data to quantify the impact of the CDS market on economic outcomes of Argentina. The model uses directed search to capture key liquidity properties of the data, such as bid-ask spreads, dealer CDS positions, and CDS-bond basis deviations, the last of which measures arbitrage opportunities available to CDS and bond traders. We obtain CDS positions for sovereign debt via regulatory data from the Depository Trust and Clearing House Corporation (DTCC), and large dealers' exposure to sovereign default (CDS and bond holdings) from the FR Y-14Q regulatory filings as part of the Federal Reserve's Capital Assessments and Stress Testing information collection. We expand this data with bid and ask quotes for bonds and CDS from Bloomberg. This data lets us determine how frictional bond and CDS markets are, how risk is shared among dealers and investors, and ultimately how the sovereign is affected by CDS markets.

In the model, we divide market participants into a sovereign government, dealers and investors. Dealers and investors have the same preferences, but investors have ex-ante heterogeneity in exposure to default risk, which is subject to shocks. There are three assets: a risk-free asset with a perfectly elastic supply, a sovereign bond in positive supply, and CDS on the sovereign bond in zero net supply. Dealers have access to competitive interdealer markets where they can trade bonds and CDSs without frictions. Investors have to search for dealers to trade bonds and CDS. Search is directed, and investors can choose to trade in one of a continuum of submarkets where dealers charge different intermediation fees. There is free entry of dealers into each submarket, and, in equilibrium, investors can increase their matching probability by paying higher fees to dealers. Investors first choose a submarket in bond markets and, if matched, purchase as many bonds as they want at interdealer prices. Then investor exposure potentially changes, and investors choose CDS submarkets and desired quantities of CDS at interdealer prices.[2] The risk-free asset market

[1] According to the BIS Quarterly Review of June, 2018, the volume traded of sovereign CDS by 2007 was around $1.6 trillion and representing 3.4% of the total CDS market and had more than duplicated its size to $3.3 trillion by mid-2013, accounting for 13.3% of the CDS market.

[2] While this timing of bonds first then CDS is potentially important, Section 5 shows it is not quantitatively important.

can be accessed freely by dealers and investors, which without loss of generality occurs after CDS trading.

We embed this OTC market structure into a standard sovereign default model (Arellano, 2008). The state at the beginning of the period is a country's endowment and a stock of debt. The decision to repay or default is made strategically comparing the two alternatives. In particular, the value of repaying depends on the market value of bonds, an outcome of the OTC structure of the model. If the government repays its debt, it chooses the amount of new bonds to be issued in order to maximize the lifetime utility of the representative consumer in its country. We prove that the fully liquid (zero trading frictions) version of the model nests the standard risk-neutral pricing model.

Trading frictions resulting from matching and portfolio restrictions determine equilibrium bond prices through several mechanisms. Three of these are *direct effects*. First is a classical Walrasian *demand effect*. For example, eliminating CDS induces substitution from CDS to bonds, changing aggregate bond demand. This force would be present absent search frictions. Second is an *intermediation effect*. Portfolio restrictions change dealer profits and, through free entry, trading fees. Different fees change the probability of matching and consequently aggregate demand. Third is an *entry effect*. As investors' incentives to trade change, so do the fees they are willing to pay. This induces more or less entry by dealers, and this measure matters when dealers share risk. These three direct effects are complemented by an *indirect effect*—the *default risk effect*. When one of the direct effects changes bond prices, the sovereign's borrowing and default decisions change as well, resulting in a general equilibrium effect on bond prices.

How much these effects matter for the impact of any policy change depends crucially on trading frictions. Our first quantitative contribution is in identifying these frictions using data and showing the model delivers key empirical patterns. In the data, as in the model, we divide market participants into three categories: sovereign governments; large banks active in CDS markets (dealers); and other market participants (investors). CDS buy and sell volumes and dealer bond and CDS holdings identify exposure heterogeneity, both between dealers and investors and among investors. The average bid-ask spreads of bonds and CDS identify trading fees. The elasticities of bid-ask spreads to changes in default risk identify the elasticities of the matching technology in OTC markets. Identified in this way using Argentinean data, the model replicates key untargeted patterns, including a positive correlation between CDS volume and risk and a large and positive CDS-bond basis deviation that is increasing in risk.

This last result tends to be hard to obtain because it indicates CDS are expensive relative to bonds, which normally can be arbitraged away by selling CDS protection and buying bonds (a trade permitted by the benchmark's portfolio restrictions). For instance, Oehmke and Zawadowski (2015) prove in their corporate CDS model the basis must be negative (Proposition 4). We show the model delivers a positive basis through higher gains from trade leading to higher matching

probabilities. For the identified elasticities, the higher matching probabilities boost aggregate bond demand more than CDS demand. Section 3.5 discusses in detail how this works.

Our second quantitative contribution is determining the impact of trading frictions on the sovereign bond market. We first exam the impact of trading frictions relative to the model's perfectly-liquid version and find that the direct effect of frictional trading worsens prices on average but improves them at small debt levels. The indirect effect / default risk effect amplifies these movements, resulting in a large, state-dependent impact on prices. We then turn to the bond market and allow for unbounded short positions in bond holdings. Allowing bond shorting has a strong negative impact on bond prices due to a strong demand effect amplified by the default risk effect. In contrast, closing the CDS market has only a minimal impact on bond prices. Finally, we analyze a policy actively debated during the European debt crisis and implemented by the European Union in 2012: a ban on "naked" CDS purchases—CDS purchases that exceed bond holdings and consequently generate negative exposure. We find that banning naked CDS has a small, positive effect on bond prices driven by the intermediation, demand, and default risk effects. We investigate how sensitive these results are to parameter values, and show naked CDS bans and eliminating CDS can have large effects, even when altering just a single parameter controlling matching elasticities. This result highlights the importance of our first contribution, showing results are *not* generically robust but depend crucially on how large trading frictions are. The welfare effects of trading frictions can be large, both for the sovereign and investors, and the default risk effect / general equilibrium feedback from prices to default rates proves essential for assessing them.

Relation to the Literature. Our paper contributes to two strands of the literature. First, it contributes to the sovereign default literature that followed the seminal work of Eaton and Gersovitz (1981) and the quantitative literature arising after the influential work of Arellano (2008) and Aguiar and Gopinath (2007). Our contribution is to bring into consideration how the CDS market affects sovereign bond markets and consequently a sovereign's ability to issue debt. The closest paper to ours is Salomao (2017)—to our knowledge the first paper in this literature incorporating CDS. Her work investigates how CDS affect debt restructuring outcomes and the corresponding implications for government decisions. She finds that CDS can generate uncertainty on the recovery value of defaulted bonds and such uncertainty make investors be more aggressive in debt restructuring negotiations. Our work is complementary, highlighting the risk-sharing role of CDS while taking into account the frictional nature of bond and CDS markets. In this respect, our paper is also closely related to the emergent literature on illiquidity in sovereign debt markets with random search (Passadore and Xu, 2022) and directed search (Chaumont, 2022). We extend those analyses by incorporating CDS, identifying trading frictions in a novel way, and assessing the impact of the naked CDS ban implemented during the European sovereign debt crisis.

Second, our paper contributes to the large finance literature that followed Duffie et al. (2005) and studies search frictions in OTC markets. Our model is closer to those in Lagos and Rocheteau (2009) and Lester, Rocheteau, and Weill (2015) since we allow for assets holdings to be traded in perfectly divisible quantities using directed search. Most of this literature focuses on outcomes of a single asset market, but some recent work investigates interactions between multiple assets. This is the case for recent work by Oehmke and Zawadowski (2015) and even more closely related work by Sambalaibat (2022).[3] Oehmke and Zawadowski (2015) characterize the interactions between corporate bonds and CDS and propose a theory where corporate CDS are not redundant because they are cheaper to trade. This assumption does not necessarily carry over to the sovereign CDS market, and we show how to identify the costs from the data. Additionally, CDS are not redundant in our model for any positive trading cost because there is a risk of not matching with dealers.

Sambalaibat (2022) uses a theoretical model to study the interactions between bonds and CDS in OTC markets. In her model, heterogenous investors, some of whom would like to take short positions in bonds, trade through random search. The main finding is that allowing for CDS trade improves bond prices by reducing the bond illiquidity discount. Many of the mechanisms in her model are also operative in ours. For instance, allowing CDS in her model expands the set of feasible trades and attracts more investors into the market. This is embedded in our demand effect, where investors can match with higher probability, and in our entry effect, where increased investor activity induces more dealers to actively trade. However, there are also important differences. Contrary to her main finding, the introduction of CDS in our benchmark calibration has a small negative demand and total effect on bond prices. In other calibrations, introducing CDS has a large positive effect, or a large negative effect. The disagreement with her main result might be attributable to investors inability to obtain more than one unit per individual in her model, preventing individual investor demand from playing a larger role. But there are also effects in our model not present in hers. For example, because of directed search, increases in gains from trade result in a higher probability of matching due to investors choosing higher fee submarkets.[4] This ends up being an important quantitative channel for generating a positive CDS bond-basis. One important qualitative difference between our models is our inclusion of general equilibrium feedback (the default risk effect). This amplifies the indirect effects and can change the welfare conclusions. Finally, our model is quantitative, matching key data well. We show many of the results, both qualitatively and quantitively, depend on trading frictions that our calibration strategy lets us identify from the data.

The paper is organized as follows. We present our model in Section 2. In Section 3 we calibrate the model and discuss the results. Section 4 studies the effects of different regulations and perform

[3] Additionally, Sambalaibat (2018) examines empirically the effects of the European naked CDS ban finding the permanent ban decreased bond market liquidity.

[4] She does, however, consider an extension with endogenous search intensity.

a welfare analysis. Section 5 performs robustness exercises, and Section 6 concludes.

2 The model

Our model is comprised of an OTC block and a sovereign block. The OTC block takes the issuance of bonds and default probability as given, and determines the market clearing bond and CDS prices. The sovereign block takes the price schedule for the bond as a function of the bond issuance, and determines the bond issuance and default probability.

2.1 The OTC block

We begin with the OTC block since it is the novel component of our sovereign debt model.

2.1.1 Agents, preferences, and endowments

At any given moment, there are two types of agents in action: a finite measure I of investors and an infinite measure of dealers. To ensure that prior histories do not affect current outcomes, we assume investors and dealers permanently disappear from the market after closing their trades and consuming (and are replaced with fresh ones).

Investors and dealers have quasi-linear utility functions that value consumption g this period and g' next period as $g+\beta\mathbb{E}_\delta u_i(g')$ and $g+\beta\mathbb{E}_\delta u_d(g')$, respectively. (In the calibration, investor and dealer utility will be the same up to a constant.) These preferences enable portfolio decisions to be swayed by risk while simultaneously keeping endowments from influencing anything other than the risk-free asset, which is not relevant for our purposes. Thus, we normalize dealer and investor endowments to zero in the current period. In the next period, the endowment is potentially correlated with default, paying out ω in repayment and 0 in default. Hence, investors have *exogenous exposure* to default in the amount ω.[5] Investors will also have *endogenous exposure* equal to their bonds position less any net CDS protection. Exogenous exposure is stochastic, evolving from ω^e before bonds are traded to ω after according to $\omega = (1-\rho_\omega)\mu_\omega + \rho_\omega\omega^e + \sigma_\omega\varepsilon_\omega$ with $\varepsilon_\omega \sim N(0,1)$. We assume the unconditional distribution (across investors) of ω^e and ω is the same, which requires $\omega_e \sim N(\mu_\omega, \sigma_{\omega^2}/(1-\rho_\omega^2))$.

Heterogeneity in ω^e and ω serves two purposes. First is that previous financial investments, from which our model abstracts, have ongoing implications. Second is that investors are heterogeneous in default-cost incidence. E.g., investors with local-currency debt and USD-denominated assets may gain from a default if it produces rampant inflation. Or, investors may be hedged across multiple countries, reducing idiosyncratic risk. The role of the within-period exposure shock ε_ω makes CDS useful for hedging previous bond choices, capturing its use as a tool for marginally

[5] Lemma 1 in Appendix A shows this formulation of the exogenous exposure is equivalent, due to quasi-linearity, to one with payments 0 in no-default states and $-\omega$ in default states.

increasing or decreasing exposure, which we discuss later on.

2.1.2 Financial markets and technology

There are three assets: a risk-free asset with a perfectly elastic supply, a sovereign bond b in fixed supply $B' > 0$, and CDS contracts c in zero net supply. To simplify the language, we refer to the sovereign bond as just the bond when not ambiguous. One unit of the risk-free asset a pays one unit of consumption next period and costs q_f. The supply of the risk-free asset is perfectly elastic. That is, investors and dealers can buy or sell any quantity of the risk-free asset at the price q_f.

One unit of the sovereign bond (CDS) pays one unit of consumption next period in states where the sovereign repays (defaults) and zero in states where it defaults (repays).[6] As there will be essentially only two states of uncertainty for investors and dealers, the bond is like an Arrow security paying out in repayment and the CDS an Arrow security paying out in default. We use $\delta = 1$ to denote the state of the world in which the bond defaults, and $\delta = 0$ to denote the state of the world in which it does not default. The default probability is a sufficient statistic for dealers and investors, and we denote it $\bar{\delta}$. We allow for a technological constraint on shorting bonds in the form of $b \geq \underline{b}$, where $\underline{b} \leq 0$ and $\underline{b} = -\infty$ means there is no constraint. Similarly, we allow for a constraint on *endogenous exposure* $x \equiv b - c \geq 0$ where $\underline{x} \leq 0$ and $\underline{x} = -\infty$ means there is no constraint. In a naked CDS ban, $\underline{x} = 0$, preventing an agent from benefiting financially from default. We assume that constraints have to hold with probability one at the end of the period and that $\underline{x} \leq \underline{b} \leq 0$.[7] Agents may take any short or long position in CDS as long as it satisfies the previous constraints on endogenous exposure (i.e., c is chosen from $\mathbb{R}$).

For investors to trade bonds or CDS, they must match with dealers in frictional markets. Specifically, if they wish to purchase bonds in the amount b, they choose a submarket characterized by dealer fee $f_b \in \mathbb{R}^+$. If matched, they pay an inter-dealer unit price q for a total of qb plus the fee f_b. In choosing f_b, they take as given the market tightness $\theta_b(f_b)$ in that submarket, which is the measure d of dealers active in submarket f_b relative to the measure of investors n active in that submarket. The constant returns-to-scale matching technology $M_b(n, d)$ determines investors' matching probability $\alpha_b(\theta_b(f_b))) \equiv M_b(1, \theta_b(f_b)) = M_b(n, d)/n$.[8] Likewise, to purchase or sell CDS c, they must pay f_c plus the inter-dealer cost pc if they match, which occurs with probability $\alpha_c(\theta_c(f_c))$.

Active dealers trade bonds or CDS with investors. To do so, they must pay an entry cost to

[6] We abstract from counterparty risk. In the data, CDS contracts pay an exogenous coupon when there is no default. The CDS contract here is identical to a CDS contract with a coupon κ paired with a short-position in the risk-free asset equal to κ. Since the risk-free asset is liquid, setting the coupon to zero is a normalization.

[7] The restriction $\underline{x} \leq \underline{b}$ is so that investors who matched in in bond markets and, e.g., took a position of $b = \underline{b}$ do not have to match with probability one in CDS markets in order to satisfy the exposure constraint.

[8] We assume $M_b(0, \cdot) = M_b(\cdot, 0) = 0$.

enter the respective market. Active dealers in the bond market have to pay an entry cost $\gamma_b > 0$, while active dealers in the CDS market have to pay an entry cost $\gamma_c > 0$. After entering *either* market, they can purchase any desired amount of bonds b and CDS c at inter-dealer prices in a frictionless inter-dealer market. An active dealer in the bond market chooses a submarket f_b to visit, matching at rate $\rho_b(\theta_b(f_b)) \equiv M_b(1/\theta_b(f_b), 1) = M_b(n, d)/d$.[9] Similarly, an active dealer in the CDS market chooses a submarket f_c and matches at rate $\rho_c(\theta_c(f_c))$. A useful property of the matching technology is that $\alpha_i(\theta) = \rho_i(\theta)\theta$ for $i = b, c$.

2.1.3 Timing

To define the model timing, we break the current period into three sequential sub-periods, s_1, s_2, s_3. In s_1, dealers decide to become active in bonds or CDS. Investors decide how many bonds they wish to purchase and the submarket they wish to enter.[10] Bond-market matching realizations occur at the end of s_1. At the beginning of s_2, the exposure shock ε_ω is realized. Then investors decide how much CDS they wish to purchase and the submarket they wish to enter. CDS-market matching realizations occur at the end of s_2. In s_3, investors choose their risk-free bond position. Dealers choose their bonds, CDS, and risk-free positions, and settle all their outstanding obligations (delivering b or c to investors as promised) simultaneously. At the beginning of the next period, the default shock is realized, payments are settled, and consumption occurs.[11]

2.1.4 The dealer's problem

For an active dealer, the demand for risk-free asset, sovereign bonds and CDS is independent of which markets she intermediates. This is because dealers have access to a perfectly competitive inter-dealer market. An active dealer's portfolio problem is

$$\begin{aligned} \pi \quad &= \max_{a,b,c} -q_f a - qb - pc + \beta \mathbb{E}_\delta u_d(a + \delta c + (1-\delta)b) \\ \text{s.t.} \quad & b \geq \underline{b}, b - c \geq \underline{x} \end{aligned} \tag{1}$$

In general, the optimal policies will not be unique because there is a redundant asset.

The value from becoming an active bond dealer is given by the value of being active, π, minus the entry cost to become active, γ_b, plus the expected benefits from trading with an investor in their

[9] The trading rate ρ may be greater than unity, which can be interpreted as dealers executing more than one bilateral transaction in a given period. The more transactions dealers make (in expectation) in a given submarket, the higher expected profits are. Thus, more dealers enter such submarkets to satisfy the zero expected profit condition that follows from free entry of dealers into each submarket. In submarkets where transaction fees are relatively low, the zero profit condition may imply that each dealer entering the submarket makes more than one transaction, in expectation, in order to cover the fixed entry cost.

[10] If they do not wish to trade, they can choose a zero-fee submarket where trade will occur with zero probability.

[11] Because the bond market opening before the CDS market is potentially important in driving the results, Section 5 explores a version of the model where investors access the CDS market first. We find similar quantitative results.

preferred submarket. That is,

$$\Pi_b = \pi - \gamma_b + \max_{f_b} \rho_b(\theta_b(f_b)) f_b.$$

Similarly, the value from becoming an active CDS dealer is

$$\Pi_c = \pi - \gamma_c + \max_{f_c} \rho_c(\theta_c(f_c)) f_c.$$

Becoming an active bond dealer and CDS dealer yields the following value

$$\Pi_{b,c} = \pi - \gamma_b - \gamma_c + \max_{f_b} \rho_b(\theta_b(f_b)) f_b + \max_{f_c} \rho_c(\theta_c(f_c)) f_c.$$

Finally, an inactive dealer's value is

$$\Pi_0 = \max_a -q_f a + \beta \mathbb{E}_\delta u_d(a).$$

Without loss of generality, we assume u_d, β, and q_f are such that $\Pi_0 = 0$.

We assume free-entry of dealers. Therefore, in equilibrium $\Pi_b, \Pi_c, \Pi_{b,c} \leq \Pi_0 = 0$. Given this, it is immediate that

$$\Pi_{b,c} = \Pi_b + \Pi_c - \pi \leq 0,$$

with strict inequality whenever $\pi > 0$. As we prove in the appendix, π attains a minimum of $\Pi_0 = 0$ only at risk-neutral prices.[12] Consequently, entering only one of the bond or CDS markets is always optimal, and strictly so except for one specific set of prices (which will generally not clear markets). Therefore, we will look for an equilibrium where dealers enter bond or CDS markets, but not both.

For use in the investor problem, we briefly characterize active submarkets, that is, submarkets with $\theta > 0$. In an active bond submarket f_b, one must have the choice of f_b attain the maximum Π_b and, because of free-entry, have $\Pi_b = \Pi_0 = 0$; the situation is identical for active CDS submarkets. Defining *net entry costs* as $\tilde{\gamma}_b \equiv \gamma_b - \pi$ and $\tilde{\gamma}_c \equiv \gamma_c - \pi$, this requires

$$\rho_b(\theta_b) f_b = \tilde{\gamma}_b \quad \text{and} \quad \rho_c(\theta_c) f_c = \tilde{\gamma}_c,$$

or, equivalently,

$$f_b = \frac{\tilde{\gamma}_b \theta_b}{\alpha_b(\theta_b)} \quad \text{and} \quad f_c = \frac{\tilde{\gamma}_c \theta_c}{\alpha_c(\theta_c)}. \tag{2}$$

We will use this mapping from positive θ to fees in the investor problem.

[12] See Lemma 4. The "only at" requires $\underline{x} < 0$. If $\underline{x} = 0$, a range of prices generates $\pi = \Pi_0 = 0$.

2.1.5 The investor's problem

We solve the investor's problem by backward induction starting from the market for risk-free assets in sub-period s_3. In sub-period s_3, an investor with bond and CDS holdings b and c solves

$$V(\omega, b, c; s_3) \equiv \max_a -q_f a + \beta \mathbb{E}_\delta \left[u_i(a + (1-\delta)(b+\omega) + \delta c) \right]. \tag{3}$$

Let the optimal choice be denoted $a_i(\omega, b, c)$. Here, the cost of acquiring bonds and CDS does not appear because we have exploited quasi-linear utility to treat them as sunk. The first order condition of this problem is

$$q_f = \beta \mathbb{E}_\delta \left[u_i'(a + (1-\delta)(b+\omega) + \delta c) \right], \tag{4}$$

which is the Euler equation for risk-free assets. Under the assumed preferences: (i) any risk-free endowment in the next period would simply shift the optimal choice of a by a commensurate amount, but result in the same next-period consumption; and, (ii) any endowment in the current period would just scale current consumption.

In sub-period s_2, after observing ω the investor has to choose a submarket to enter and a demand for CDS in case of matching with a dealer. Using (2), which provides a map between the dealers' fees and the tightness of the market, we can write the investor's problem as choosing the tightness directly. Thus, an investor with bond holdings b solves

$$V(\omega, b; s_2) \equiv \max_{\theta_c \geq 0, c} -\tilde{\gamma}_c \theta_c + \alpha_c(\theta_c) \left[V(\omega, b, c; s_3) - pc \right] + [1 - \alpha_c(\theta_c)] V(\omega, b, 0; s_3) \tag{5}$$

$$\text{subject to } b - c \geq \underline{x}.$$

Note the fee, given implicitly by $\tilde{\gamma}_c \theta_c / \alpha(\theta_c)$, is paid conditional on matching, resulting in an expected fee of $\tilde{\gamma}_c \theta_c$. The first order conditions of this problem are

$$p \leq V_c(\omega, b, c; s_3), \text{ with equality if } b - c > \underline{x}, \text{ and} \tag{6}$$

$$\tilde{\gamma}_c = \alpha_c'(\theta_c) \left[V(\omega, b, c; s_3) - pc - V(\omega, b, 0; s_3) \right], \tag{7}$$

where $V_c(\omega, b, c; s_3)$ denotes the partial derivative of $V(\omega, b, c; s_3)$ with respect to c. The optimal choice of c conditional on matching satisfies the conventional Euler equation for CDS. Let an optimal CDS choice and optimal market tightness be denoted $c_i(\omega, b)$ and $\theta_c(\omega, b)$, respectively.

In sub-period s_1, the investor with exposure ω^e has to choose a submarket to enter and a demand for sovereign bonds upon meeting a dealer. Using equation (2) again, we can write the investor's problem as choosing the tightness directly. Thus, an investor solves

$$V(\omega^e; s_1) \equiv \max_{\theta_b \geq 0, b} -\tilde{\gamma}_b \theta_b + \mathbb{E}_{\omega|\omega^e} \left\{ \alpha_b(\theta_b) \left[V(\omega, b; s_2) - qb \right] + [1 - \alpha_b(\theta_b)] V(\omega, 0; s_2) \right\}, \tag{8}$$

$$\text{subject to } b \geq \underline{b}.$$

The first order conditions of this problem are

$$q \geq \mathbb{E}_{\omega|\omega^e} V_b(\omega, b; s_2), \quad \text{with equality if } b > \underline{b}, \text{ and} \tag{9}$$

$$\tilde{\gamma}_b = \alpha_b'(\theta_b)\mathbb{E}_{\omega|\omega^e}\left[V(\omega, b; s_2) - qb - V(\omega, 0; s_2)\right], \tag{10}$$

where $V_b(\omega, b; s_2)$ denotes the partial derivative of $V(\omega, b; s_2)$ with respect to b. Let an optimal bond choice and optimal market tightness be denoted $b_i(\omega^e)$ and $\theta_b(\omega^e)$, respectively.

2.1.6 Market clearing

There is a perfectly elastic supply of the risk-free asset at the price q_f, the market clearing price. We next provide market clearing conditions for the sovereign bond and CDS markets.

Let us introduce some notation. Define as $d_b(\omega^e)$ the measure of dealers active in the bond market in sub-period s_1 trading with investors with exposure ω^e, $d_c(\omega, b)$ the measure of dealers active in the CDS market serving investors with b bond holdings and exposure shock ω in sub-period s_2, and $D = \bar{d} + \int_{\omega^e} d_b(\omega^e)dF_\omega + \int_{\omega,b} d_c(\omega, b)dF_{\omega,b}$ is the total measure of dealers. F_ω represent the distribution of exposure shocks and $F_{\omega,b}$ is the endogenous joint distribution of investors' realized exposure shocks and bond holdings. In s_1, denote $n_b(\omega^e)$ the mass of investors with exposure ω^e. In sub-period s_2, investors who matched with a dealer have bond holdings $b_i(\omega^e)$ while those investors who did not match have bond holdings equal to 0. The exogenous mass of dealers, $\bar{d}$, is always active in the interdealer market.

The bond market clears if

$$B' = \int_{\omega^e} M_b(n_b(\omega^e), d_b(\omega^e))b_i(\omega^e)dF_\omega + Db_d. \tag{11}$$

The CDS market clears if

$$0 = \int_{\omega,b} M_c\left(n_b(\omega, b), d_c(\omega, b)\right) c_i(\omega, b)dF_{\omega,b} + Dc_d. \tag{12}$$

We assume that the following regularity conditions hold:

Assumption 1. *u_d is unbounded above.*

Assumption 2. *At least some short-selling is allowed, $\underline{b} < 0$, at least some financial exposure is allowed, $\underline{x} < 0$, and there is a strictly positive measure of exogenous dealers, $\bar{d} > 0$.*

Assumption 3. *$\alpha'(0) = \infty$, $\alpha'(\theta) > 0$, and $\alpha''(\theta) < 0$ for all $\theta > 0$.*

Assumption 1 guarantees that dealer utility from trading in interdealer markets π goes infinite when consumption next period goes infinite. This ensures that if they can make infinite profits, an infinite amount of dealers enter. Allowing some short-selling in assumption 2 ensures that investors generally have some gains from trade. If there is literally no short-selling, then one can have a situation where investors want to have negative bonds but cannot, meaning they begin with

their optimal level of bonds (zero) and have literally zero gains from trade. Zero gains from trade consequently results in no entry, and markets do not clear in a natural way.[13] However, that lack of gains from trade is okay as long as there are always some dealers, $\bar{d} > 0$. Assumption 3, which is satisfied by the matching function of section 3, ensures that θ is interior and behaves in predictable ways as net entry costs go to zero.

To see how these assumptions work to make aggregate demand move naturally, reconsider the optimal choice of θ_b given by

$$\tilde{\gamma}_b = \gamma_b - \pi = \alpha'(\theta_b)(V(b; s_2) - qb - V(0; s_2))$$

The gains from trade are $(V(b; s_2) - qb - V(0; s_2))$. If this is strictly positive, as generically it is by virtue of assumption 2, and $\tilde{\gamma}_b > 0$, then assumption 3 guarantees that (1) there is an optimal, interior $\theta_b \in \mathbb{R}^{++}$ and (2) as $\tilde{\gamma}_b \downarrow 0$, $\theta_b \uparrow \infty$ for any q. Consequently, the *aggregate* bond demand grows infinite, in absolute value, as $\tilde{\gamma}_b \downarrow 0$. By the strict quasi-convexity of π (established in lemma 4 in the appendix), $\tilde{\gamma}_b = 0$ at two points, one with high q ($\bar{q}$) and one with low q ($\underline{q}$). As one approaches either of these extremes, aggregate demand explodes, and so equilibrium is in $(\underline{q}, \overline{q})$.

2.1.7 Definition of equilibrium in OTC markets

We define *equilibrium in OTC markets given* $\bar{\delta}$ *and* B' as follows:

Definition 1. *A family* $\{V, b_i, c_i, a_i, \theta_b, \theta_c, \pi, b_d, c_d, q, p\}$ *is a symmetric equilibrium if satisfy equations* (1)-(12).

2.2 The sovereign block

We now describe the sovereign block of the model, which endogenizes the supply of bonds, B', and default decisions, δ. For any given default probability and bond supply, equilibrium in the OTC markets determines a bond price q.[14]

2.2.1 Agents, preferences, and endowments

There is a sovereign government who has a stochastic, Markov "potential" output stream Y. If the sovereign does not default and is not in autarky, this potential output stream is actual output. If the sovereign does default or is in autarky, this output stream is reduced to $h(Y) \leq Y$. The sovereign values stochastic consumption streams $\{C_t\}$ according to $\mathbb{E} \sum_t \beta_g^t u(C_t)$.

[13] They might still clear, but it would be through exploiting indifference conditions at $\theta = 0$.

[14] This part of the model is completely standard following Arellano (2008), and readers familiar with that model can probably skip to (13).

2.2.2 Financial markets

For tractability, we assume that bonds mature in one period. With one-period bonds, investors hold bonds and CDS contracts at most for one period. Because of this, the distribution of bond holdings is re-started every period. Thus, the distribution of investor types and bond holdings is not part of the aggregate state of the economy, which greatly simplifies the solution of the model.[15]

At the beginning of each period, the sovereign has some amount of existing debt obligations B. It then chooses to honor those obligations, $\delta = 0$, or default on them $\delta = 1$. If it defaults, then the sovereign enters autarky, i.e., is unable to save or borrow, from which it escapes with probability ϕ. In autarky, we say the sovereign's debt is zero.

If the sovereign is not in autarky and does not default, then it chooses an amount of debt B' to issue, taking the price schedule $q(Y, B')$ as given.

2.2.3 Timing

The timing of the sovereign block is as follows. Shocks determining the level of potential output Y and whether the sovereign leaves autarky (if applicable) are realized. The sovereign makes its default decision (if not in autarky). If the sovereign was not in autarky and repays maturing debt, the sovereign can issue new debt B'.

2.2.4 Government's problem

At the beginning of each period, the government has outstanding level of debt that it needs to repay, B, and observes the new realization of the endowment, Y. Since bonds mature in one period and the distribution of bond holdings is re-started every period, the pair (Y, B) is the relevant state of the economy. After observing the state of the economy, the government decides whether to repay the outstanding level of debt or to default.

The optimal decision is determined by weighting the costs and benefits of repaying the outstanding level of debt. The benefit of default is debt service costs can instead be used to boost current period consumption. The costs are lost output and a temporary exclusion from international credit markets. The temporary exclusion from international credit markets reduces the ability of the government to use credit as a source for consumption smoothing. The length of the exclusion is captured by an exogenous probability of regaining access to credit markets, $\phi \in (0, 1)$. When the government re-gains access to credit markets, it starts next period with no debt. The output cost is an endowment loss while the government is in default and it is given by the function $h(y) \leq y$, for

[15] Assuming long-term debt would require tracking distributions of bond and CDS holdings or assuming that investors who leave the economy after one-period somehow can re-allocate all bonds and CDS positions to new investors without being subject to trading frictions in OTC markets. This is an interesting extension with even more effects to consider, but it is worth understanding the many mechanisms present in this simpler formulation, first.

all y.

The recursive formulation of government's problem is then given by

$$\begin{aligned} W(Y,B) &= \max_{\delta\in\{0,1\}} \{\delta W^d(Y) + (1-\delta)W^r(Y,B)\}, \\ W^d(Y) &= u(h(Y)) + \beta_g \mathbb{E}\{\phi W(Y',0) + (1-\phi)W^d(Y')\}, \\ W^r(Y,B) &= \max_{C,B'\geq 0} u(C) + \beta_g \mathbb{E}\{W(Y',B')\}, \\ s.t. &: C = Y + q(Y,B')B' - B \end{aligned} \tag{13}$$

where W is the option value of repaying the debt, W^d is the value of defaulting, and W^r is the value of repaying. Whenever the government decides to repay its debt, it is allowed to choose the new debt issuances, B', taking as given the price scheduled that it faces in the market, $q(Y,B')$. Let the optimal default policy be denoted $\delta(Y,B)$ and an optimal bond policy be denoted $B'(Y,B)$.

The frictions in secondary markets for bonds and CDS play an important role in the default decision of the government. They enter the problem of the government by affecting the market price of newly issued bonds, q, and thus directly affecting the value of repaying debt, W^r.

2.2.5 Definition of equilibrium in the sovereign block

We define partial equilibrium in the sovereign block given q as follows:

Definition 2. *A partial equilibrium in the sovereign block given a price schedule q is a family $\{W, W^r, W^d, B', \delta\}$ that is a solution to the sovereign's problem.*

2.3 Combining the model blocks

To combine the two blocks, we need to impose consistency between the price schedule arising from the OTC block, and the bond issuance and default probability optimally chosen by the government. We say the price schedule $q(Y,B')$ is *consistent with OTC equilibrium* if, for every $Y, B' > 0$, there exists an equilibrium in the OTC markets, given the default probability $\bar{\delta} = \mathbb{E}\delta(Y,B')$ and debt issuance B', that results in price $q(Y,B')$. We are now ready to state equilibrium.

Definition 3. *An equilibrium is a set of functions $\{q, W, W^r, W^d, B', \delta\}$ such that $\{W, W^r, W^d, B', \delta\}$ solves the sovereign's problem taking q as given and q is consistent with OTC equilibrium.*

We characterize equilibrium in the case of $\sigma_\omega = 0$ to build intuition and provide some additional insights on the equilibrium outcomes. As the focus of our paper is quantitative, this analysis is provided in Appendix A for interested readers.

3 Calibration and benchmark results

The model is calibrated at a quarterly frequency to match Argentina.

3.1 Functional Forms

The output cost of default is

$$h(y) = y - \max\{0, d_0 y + d_1 y^2\}$$

Utility function curvature is the same for the sovereign, dealers, and investors:

$$u(c) = u_i(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} = u_d(c) + \kappa,$$

where κ is chosen to deliver $\Pi_0 = 0$. Log output follows a Gaussian AR(1) process, $\log Y = \rho_Y \log Y_{-1} + \sigma_Y \epsilon_Y$. The matching function is given by

$$M_i(n, d) = n \frac{n^{-\xi_i}}{n^{-\xi_i} + d^{-\xi_i}}$$

for $\xi_i \in (0, 1)$ and $i \in \{b, c\}$. This results in a matching probability of

$$\alpha_i(\theta) = \frac{1}{1 + \theta^{-\xi_i}}.$$

for $i \in \{b, c\}$. The parameter ξ_i controls the matching probability elasticity and can vary across bond and CDS markets.

3.2 Measurement

To bring the model to the data, a few key concepts are needed. First is that our CDS payments f_c are *upfront* but can equivalently be quoted in terms of a *running spread*. A running spread is an endogenous coupon payment in the case of non-default such that the expected, discounted, net present value of a CDS contract is zero. Second is that our bond payments f_b are also upfront but can be quoted in terms of a *Z-spread*, which is just the internal rate of return less the risk-free rate. Our measure of the CDS-bond basis breakdown follows the literature by looking at the running CDS spread less the Z-spread of sovereign debt. Appendix C describes these measures in more detail. The reason the CDS-bond basis is expected to hold is because, for small default rates, both spreads should approximately equal the expected default rate.[16]

The bid/ask spreads for bonds in our data are quoted in yield to maturity (YTM), which is equivalent to being quoted in Z-spreads. The bid/ask spreads for CDS are likewise quoted in running spreads. We define a bid/ask spread in the model by noting the following. First, CDS dealers' net profit is $\rho(\theta(f_c))f_c$. To be willing to transact with at least some positive probability, this must be positive. As any bid less than pc will never be transacted ($f_c \leq 0 \Rightarrow \rho = 0$), we think of the "bid" price as being pc. On the other hand, $f_c > 0$ will sometimes transact and sometimes not.

[16] One peculiarity of the conventional measure is that even when $q + p = q_f$, so no arbitrage holds exactly, there can still be a measured deviation in the CDS bond basis due to nonlinearities. In our model, we will have the deviations due both to this nonlinearity and the lack of arbitrage in transacted investor-dealer prices.

Hence, we say dealers ask for $pc + f_c$, in which case if that is met, they will transact for sure. (As the f_c are heterogeneous, there will also be heterogeneity here, but we will aggregate to a single number.) Consequently, a bid/ask *upfront* spread per unit of notional CDS as $(pc + f_c - pc)/c$ or f_c/c, which can be volume weighted to derive an aggregate number. However, for a bid/ask spread in terms of the *running spread*, we take the volume-weighted price $(pc + f_c)/c$ and interdealer price p, convert those both to running spreads, and take their difference. Similarly for bonds, we get the volume-weighted average bond price $(qb + f_b)/b$ and the interdealer price q, convert both to Z-spreads, and difference them to get a spread in terms of YTM. In measuring the CDS-bond basis deviation, we use the average transacted prices inclusive of fees, convert CDS to running spreads and bonds to Z-spreads, and then difference them.

A final key measurement issue is that our bonds and CDS are both short-term contracts, while in the data they are five-year contracts. For bonds, we handle this in the standard way by focusing on the debt service targets rather than debt stocks. For CDS, we do something similar, reducing our position and volume measures to one quarter's worth (5%) of a five-year contract.

3.3 Identification strategy

Table 1 provide the parameters that we set exogenously with some rationale. Most of these values are standard, but some deserve further explanation. As discussed at the end of Section 2.1.1, exogenous exposure in part captures the impact of previous financial investments. To capture legacy exposure to CDS and bonds, which our model assumes are short-term but in the data are typically 5-year contracts, we set the persistence $\rho_\omega = 0.95$ to give a 5-year half-life. To ensure market clearing, we allow a small measure of bond shorting $\underline{b} = -0.01$, which ensures gains from trade exist generically. Likewise the exogenous measure $\bar{d}$ of dealers is set to 0.001 (which will be tiny relative to the estimated measure of investors) to ensure even if there is no entry (due to no gains from trade) the bond market can still clear.

This leaves ten parameters: seven for the OTC block and three for the sovereign block. In the sovereign block we need to determine the sovereign's discount factor and the two default cost parameters. We pin these down using standard moments in the literature in the *liquid* version of the model, where investors and dealers can trade bonds without frictions. Specifically, we target the debt-service output ratio, the standard deviation of spreads (defined as the yield-to-maturity over a risk-free rate) and mean of spreads. We find $(\beta, d_0, d_1) = (0.849, -0.119, 0.128)$, and the corresponding moments are denoted in Table 3 with an asterisk. Calibrating these parameters separately prevents the estimation from distorting OTC frictions to improve model fit on these standard dimensions.

In the OTC block we need to determine the bond and CDS entry costs, the matching function elasticities for bonds and CDS, the mean and variance of the exposure shocks to investors (expo-

Table 1: Exogenously fixed parameter values with explanations

Parameter	Description	Value	Reason
Sovereign			
σ	Risk aversion	2	Standard value
ρ_Y	GDP persistence	0.949	Chatterjee and Eyigungor (2012)
σ_Y	GDP innovation std.	0.027	Chatterjee and Eyigungor (2012)
ϕ	Autarky escape prob.	0.0385	Chatterjee and Eyigungor (2012)
Dealers			
β	Discount factor	0.99	Standard value
σ	Risk aversion	2	Standard value
$\bar{d}$	Exogenous measure	0.001	Market clearing regularity
Investors			
β	Discount factor	0.99	Standard value
σ	Risk aversion	2	Standard value
ρ_ω	Exposure persistence	0.95	Five-year half life
Markets			
$\underline{b}$	Bond shorting limit	-0.001	Market clearing regularity
r	CDS coupon	0.0	Normalization
q_f	Risk-free price	0.99	Standard value

sure shocks), and the measure of investors. To identify the first four, we exploit the model's tight relationship between entry costs, matching elasticities, and bid/ask spreads. To see this relationship, consider the bid/ask spread for bonds quoted in upfront payments, f_b/b, is related to entry costs γ_b by the free entry condition (2) in a direct way:

$$\frac{f_b}{b} = \frac{\gamma_b - \pi}{b} \frac{\theta_b}{\alpha_b(\theta_b)}.$$

A similar relationship holds for CDS. We use the entry cost parameters γ_b and γ_c to match bid/ask spreads for bonds and CDS, respectively.

Combining the above with the submarket optimality condition (10), we obtain

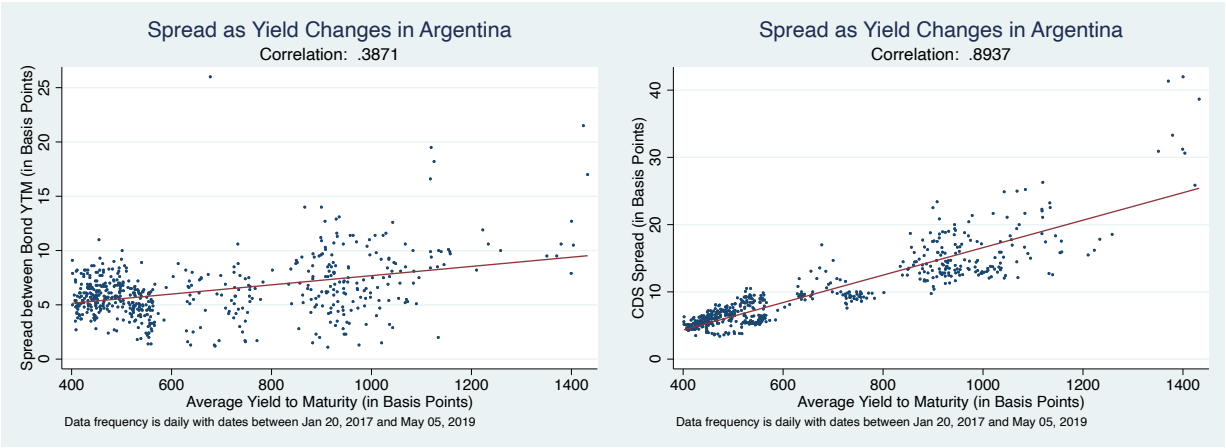
$$\frac{f_b}{b} = \frac{\alpha_b'(\theta_b)\theta_b}{\alpha_b(\theta_b)} \frac{1}{b} \frac{\gamma_b - \pi}{\alpha'(\theta_b)} = \epsilon_{\alpha_b,\theta_b}(\theta_b) \frac{\mathbb{E}_{\omega|\omega^e}\left[V(\omega, b; s_2) - qb - V(\omega, 0; s_2)\right]}{b},$$

where $\epsilon_{f,x}(x)$ denotes the elasticity of f with respect to x evaluated at x. The elasticity of the matching probability controls how much changes in gains from trade (the expectation term) translate into increases in bid/ask spreads (the lefthand side). And since default risk movements drive changes in the gains from trade, the slope in a regression of bid/ask bond spreads on default risk, which can be measured using YTM, identifies the elasticity of α_b and similarly for CDS and α_c.

In the data, there is a tight relationship between bid/ask spreads and YTM for both bonds and CDS, as Figure 1 shows. These elasticities are crucial for some of the model's predictions, and the

model's tight link between bid/ask spreads and risk is a novel way to identify these key parameters.

Figure 1: Bid/ask bond and CDS spreads against yield-to-maturity for ARG



The remaining parameters are the measure of investors and the mean and standard deviation of exogenous exposure ω (and ω^e). Since exposure shocks ω are orthogonal to default risk, the first-order effect of increases in the variance of ω is to increase the desire for risk reallocation across investors. This allows the standard deviation σ_ω to be cleanly identified by aggregate CDS volume. Due to risk-sharing, increases in the mean level of exposure μ_ω passthrough both to investors and dealers, and this is reflected (for investors who match in CDS markets) in a lower CDS position for investors and, by market clearing, a higher CDS position for dealers.

The measure of investors I is the last parameter. Note that the total amount of exposure in the economy—how many resources stand to be lost in the case of a default—is the bond supply B plus $I\mu_\omega$. For a given level of $\mu_\omega > 0$, increasing the measure I of investors increases total risk in the economy due to ω, but also decreases how much exposure needs to be allocated per agent, which is $(B + I\mu_\omega)/(I + D)$—at least ignoring the endogenous response of the dealer measure D. So as I goes infinite, the amount of exposure borne by each agent tends to decrease (with $B/(I + D)$ shrinking) and with it the amount of debt. Conversely, for I tiny, investors have little capacity to absorb debt and most of it is bought by dealers. So the exposure of dealers and bond holdings of dealers identifies the measure of investors.

3.4 Model fit

Our identification strategy and estimated fit for the OTC block are summarized in Table 2. The model reproduces the targeted moments with one exception. Specifically the bid/ask spread for CDS is too high on average. We cannot exactly pin down the average level of both bid/ask spreads, in part because of strong theoretical linkages between the bonds and CDS market (these can be seen in Proposition 5 in the appendix). However, the fit is quite good overall, and the key matching

elasticity identification strategy works as expected.

Table 2: Targeted moments from the calibration

Moment	Model	Data	Parameter	Value
Bid-ask spread for bonds mean (%)	0.0641	0.0629	Bond entry cost	0.0893
Bid-ask spread for CDS mean (%)	0.2333	0.0984	CDS entry cost	1.8232
Agg. dealer CDS position / mean GDP (%)	0.0006	0.0010	Investor exposure mean	0.1106
Agg. dealer net exposure / mean GDP (%)	0.0165	0.0170	Investor measure	3.4309
Agg. dealer bond position / mean GDP (%)	0.0171	0.0180		
Agg. dealer-investor volume / mean GDP (%)	0.1951	0.1945	Investor exposure s.d.	0.0174
Reg. coef. of YTM on bid-ask bond spreads	0.0052	0.0042	Matching function elasticity bonds	0.0901
Reg. coef. of YTM on bid-ask CDS spreads	0.0205	0.0204	Matching function elasticity CDS	0.3925

The entry costs for bonds and CDS turn out to be quite different, with the CDS entry cost about twenty times larger than for bonds. Because bonds are an inferior asset to CDS, only the latter of which can offer insurance against default, the model demands that CDS be *more* expensive than bonds to rationalize B clearing the market with a small average CDS position of dealers. The estimated elasticity parameters also appear considerably different. However, the matching probabilities α_b and α_c are not constant elasticity, and so one cannot directly map from ξ_i to $\epsilon_{\alpha_i,\theta_i}$. One important lesson from our model is that liquidity cannot simply be read off of differences in bid/ask spreads (or volume)—how those bid/ask spreads change in risk can be equally or more important. In other words, both the means and slopes in Figure 1 are necessary to properly characterize trading frictions.

The model's fit for some untargeted moments is reported in Table 3. The correlations between volumes and risk all have the correct sign, though the magnitudes are overstated. The correlation between the dealers' CDS position and risk has the wrong sign, which is natural because as risk increases, risk-sharing dictates that dealers should take on more risk, reflected in selling protection. The data's positive correlation holds in the whole sample, as can be seen in Figure 2, but evidently weakens substantially on the subsample having higher levels of risk. Additionally, over a wide swath of risk levels, the mean position of dealers stays quite flat, reflected in the small volatility of the dealers' position (small relative to volume, e.g.), which the model captures. Interestingly, the model has a large positive mean and standard deviation for CDS-bond basis deviations, like in the data. In fact, we will show the model's CDS-bond basis deviations are increasing in risk, as is the case for Argentina.

3.5 Benchmark results

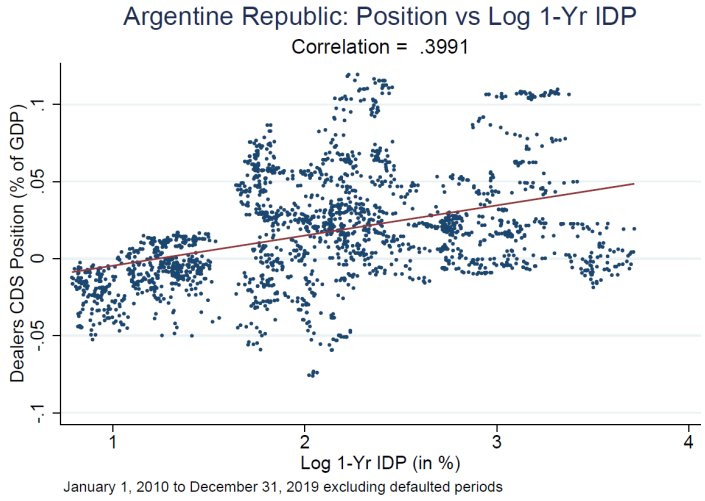
The model generates three key empirical measures of trading frictions and risk-sharing, which we depict in Figure 4. The first one is that, as yields and default probabilities increase, dealers tend

Table 3: Untargeted moments from the calibration

Moment	Model	Data
Bond spread mean (%)*	7.4446	8.1500
Bond spread std. (%)*	3.8296	4.4300
Debt service to output ratio (%)*	5.5530	5.5300
Bid-ask spread for bonds std. (%)	0.0367	0.0266
Bid-ask spread for CDS std. (%)	0.1004	0.0554
CDS-bond basis deviation mean (%)	1.3014	6.4800
CDS-bond basis deviation std. (%)	1.6148	7.1100
Agg. dealer buy volume / mean GDP (%)	0.0978	0.0975
Agg. dealer sell volume / mean GDP (%)	0.0973	0.0970
Std. of agg. dealer CDS position / mean GDP (%)	0.0312	0.0015
Default probability (%) (full sample)	1.0808	-
Corr. of log IDP and agg. dealer CDS / mean GDP	-0.6104	0.5100
Corr. of log IDP and agg. dealer-investor volume / mean GDP	0.9619	0.0900
Corr. of log IDP and agg. dealer buy volume / mean GDP	0.7826	0.0900
Corr. of log IDP and agg. dealer sell volume / mean GDP	0.9664	0.0900
Corr. of YTM spreads and bond bid-ask spreads	0.5453	0.3870
Corr. of YTM spreads and CDS bid-ask spreads	0.7840	0.8940
Reg. cons. of YTM on bid-ask bond spreads	0.0003	0.0344
Pred. bid-ask bond spreads at YTM target	0.0428	0.0690
Reg. cons. of YTM on bid-ask CDS spreads	0.0008	-0.0383
Pred. bid-ask CDS spreads at YTM target	0.1683	0.1280

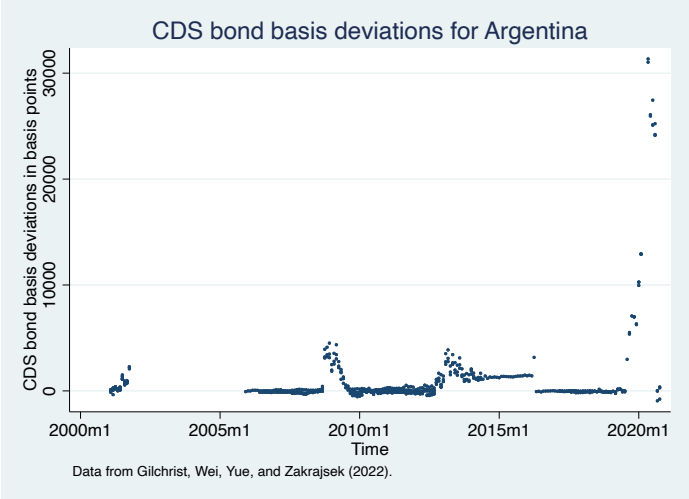
Note: the moments marked with * are targeted in a first stage calibration using the liquid version of the model; correlations are for CDS position and dealer volume are based on quarterly values.

Figure 2: Net CDS position of dealers versus yield-to-maturity for ARG



Note: data is daily, and correlation is for daily observations (which is not identical to the correlation in table 3).

Figure 3: CDS-bond basis deviations in the data

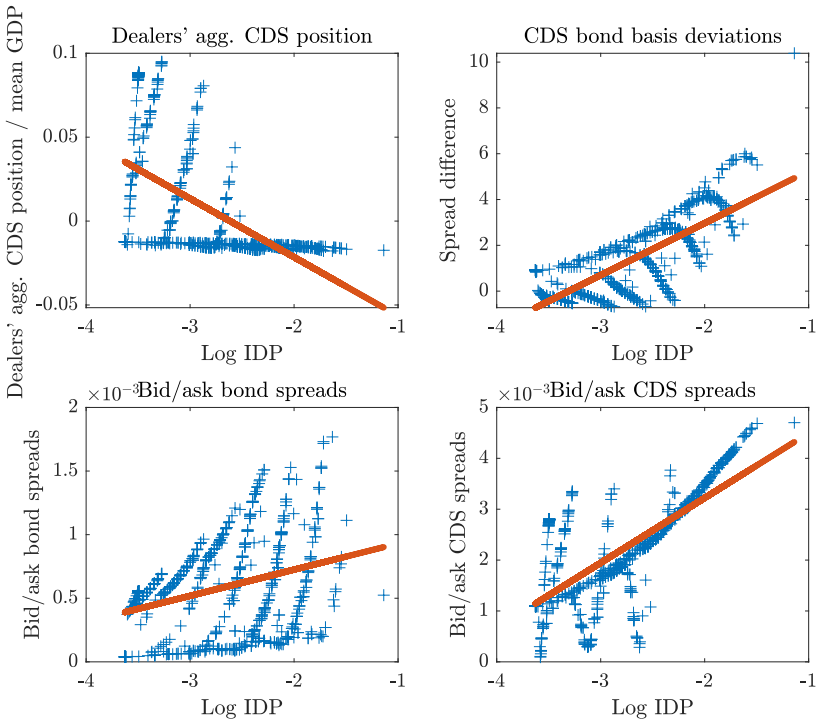


to remain more or less neutral in terms of protection (the data counterpart of this model result is Figure 2). At moderate to high levels of risk, their position in the model is tightly clustered around zero, like the data. Hence it is not the case that dealers are providing large amounts of insurance. The second key feature is that the CDS-bond basis breaks down when default risk is very high, exploding positive. Figure 3 shows the data's CDS-bond basis, which is usually close to zero but explodes up in each of Argentina's high default risk epsiodes, 2001, 2009, 2014, and 2020. The final fact is that, as yields and default probabilities increase, bid/ask spreads for both bonds and CDS increase, reflecting increased gains from trade and the correspondingly larger fees.

Figure 5 sheds light on why the model generates these patterns. It plots for a fixed debt supply some key OTC variables as the expected default rate varies. Note that only investors with low (in fact, negative) exogenous exposure ω are substantially active in the bond market (the more exposed investors are allowed to take a small short position). As default risk increases, they reduce their risk by reducing individual bond demand. At the same time, the gains from trade increase, meaning investors are willing to pay higher fees to access the market and consequently match at higher rates. Because of the model's tight connection between fees and bid/ask spreads, this generates an increase in bid/ask spreads (not pictured). It also generates an increase in matching probabilities, which means aggregate demand is decoupled from individual demand.

The dealers' small and stable CDS position is reflected in how investor demand for CDS and bonds hinges on the exogenous exposure level. Dealers, who have zero exogenous exposure, look like the moderate ω investor type, demanding almost no bonds and having little CDS demand. This behavior of dealers is targeted and so must hold in the simulation on average because dealers have a small CDS position on average, which gives buy and sell volume roughly equal—so investors are selling and buying protection from each other.

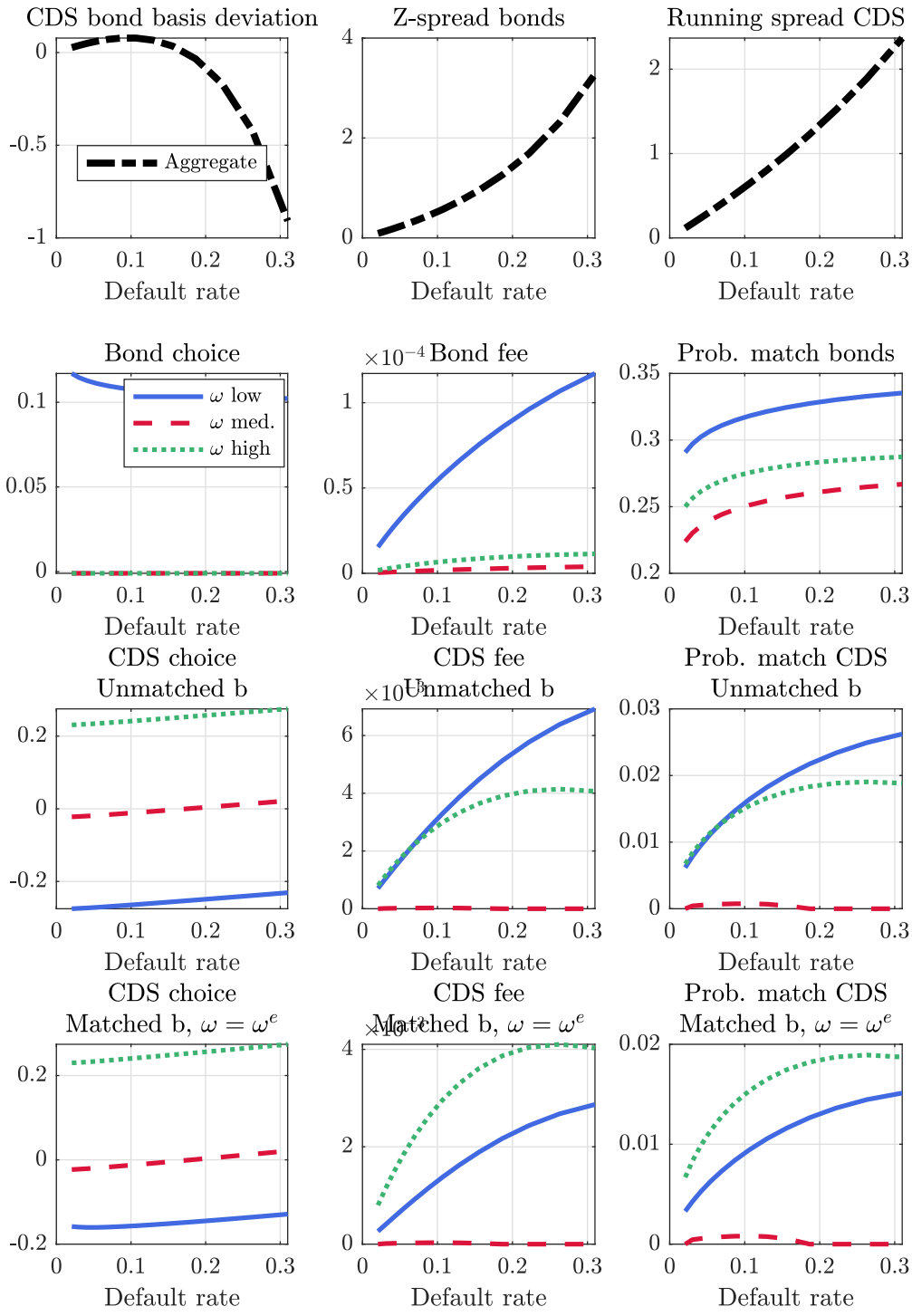
Figure 4: Simulated key variables



The most challenging pattern to understand is the CDS-bond basis deviation. It necessarily begins close to zero when the default rate is close to zero: In the limit where the default rate is zero, bond and CDS both become risk-free assets with a price of q_f and 0, respectively, implying the CDS running spread and bond YTM spreads are both equal to zero. But as default rates increase, it first rises (as the running spread on CDS rises faster than Z-spread for bonds) but subsequently falls sharply. Why?

The CDS-bond basis deviation is really a measure of how cheap bonds are relative to CDS. The more demand there is for bonds compared to demand for synthetic bonds ($-c$) via CDS, the greater the CDS-bond basis deviation will be. An increase in the basis as default risk increases from low levels reflects that aggregate demand for bonds is increasing faster/decreasing slower than aggregate demand for synthetic bonds. Here, individual demand for bonds b is falling since risk is going up. Aggregate demand is falling due to this intensive margin but rising due to the extensive margin: the probability of matching is increasing. So aggregate demand, $I \int \alpha_b(\theta_b) b dF$, necessarily falls by less in percentage terms (and could in principle increase) than the fall in b. In contrast, as the gains from matching goes up in CDS markets, the matching probability increases both for those selling protection and those buying. So these extensive margin forces partially offset. Hence, the extensive margin effect means aggregate bond demand can increase faster/fall slower than aggregate synthetic bond demand in CDS markets. This makes bonds cheap relative to CDS,

Figure 5: Investor behavior in the benchmark, varying default rates



Note: plotted for a bond supply of 0.03; "Unmatched b" ("Matched b") means investors who did not (did) match with a dealer in the bond market; this graph applies to both the benchmark and the case with liquid sovereign policies but frictional OTC markets since the bond issuance is fixed and default risk is on the horizontal axis.

resulting in a positive and increasing CDS-bond basis at low default rates.

This extensive margin effect is necessarily limited, however. Once the matching probability levels off—as strongly influenced by its elasticity—additional risk shows up as a decrease in aggregate demand for bonds. And that is where the short sale constraint on bonds really matters. In the CDS market, increases in risk are borne by all types of investors, reflected in the individual CDS protection choices varying. This is how a planner would allocate risk—in fact, for investors who match in CDS markets, markets are complete and risk is fully shared among them and dealers.[17] This efficient risk-sharing means CDS risk premia and running spreads increase at a comparatively slow rate. In contrast, in the bond market only the less-exposed investors are active because of the short-sale constraint on bonds. To bear all the risk themselves is inefficient, resulting in a greater compensation for risk reflected in a Z-spread that rises quickly in default risk (at higher levels of risk).

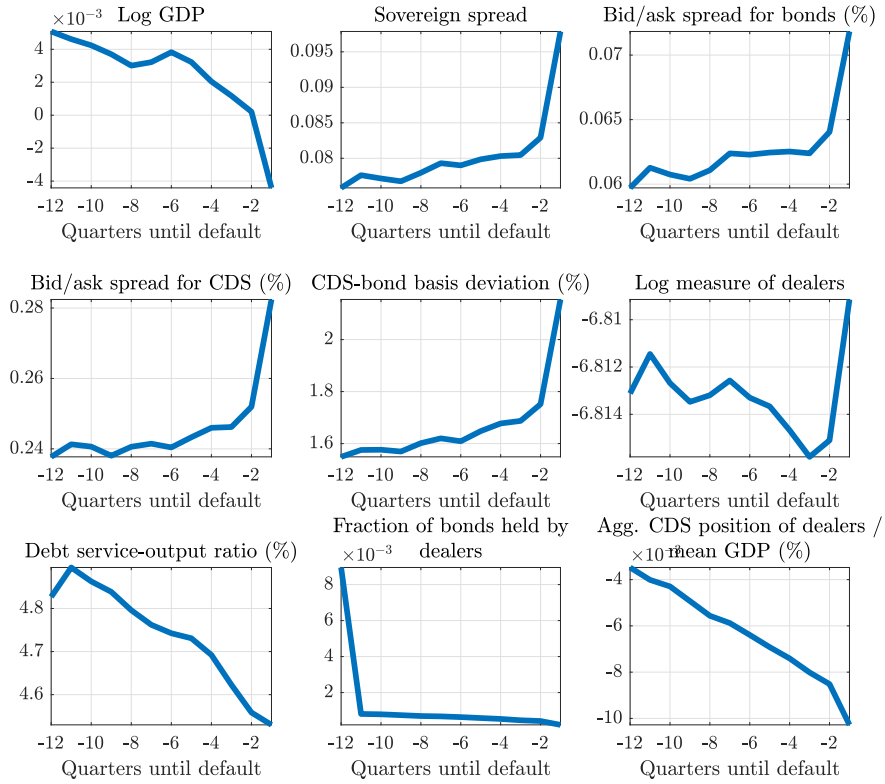
But why don't low-exposed bond investors arbitrage away the positive deviations by buying cheap bonds and buying protection in CDS markets? The reason is that frictions make this trade risky and so not pure arbitrage. If they buy bonds, there's two types of imperfectly insurable risk. First, they could fail to match in CDS markets, in which case they would be stuck with bonds and excess risk. They could mitigate this risk by paying larger fees, but that is expensive and eats into the profits of the "arbitrage" trade. Second, their exposure could change after purchasing bonds, amplifying the risk associated with not matching in CDS markets. Given these disincentives, the low-exposed investors are happy to let the CDS-bond basis deviate upwards. The negative bond basis is much harder to arbitrage for anyone—it requires selling expensive bonds and selling protection in CDS markets. But because of the short-sale constraint, it is very hard to do that. It also comes with the aforementioned risks of a suboptimal exposure level arising from not matching.

The documented patterns between bid/ask spreads, volume, dealer CDS positioning, and CDS-bond basis breakdowns all show up when considering default events, as is done in Figure 6. As output declines, default risk and spreads increase. With larger default risk, the consumption gap between matched and unmatched investors is larger and gains from trade increase, inducing investors to pay higher intermediation fees to achieve their optimal level of exposure to default risk. Larger intermediation fees are mapped into larger bid-ask spreads in the bond and CDS markets. The increases in risk and reduction in debt both generate a larger deviation of the CDS-bond basis (the first we discussed, the latter is evident in Figure 8). The greater desire to trade also results in more dealers entering the market just before default (as larger θ_b, θ_c choices result in more dealers), but this effect is very small. Dealers in the aggregate sell a little more CDS protection, but the

[17] As discussed in Section 2.1.2, there are effectively only two states of uncertainty (repayment and default) with the CDS being an Arrow security paying in the default state. The risk-free bond then spans the other state (in proper combination with the CDS).

absolute value is small relative to debt.

Figure 6: Default events



4 Counterfactuals

In this section we analyze a series of counterfactuals. Firstly, we assess the quantitative importance of trading frictions on bond prices and the response of the government by comparing our baseline model to an alternative version of the model in which bonds and CDS are liquid and can be traded in competitive markets—that is, dealers face zero entry cost (Proposition 8 in Appendix A). Secondly, we study the equilibrium responses to policy changes that modify the constraints of trade in CDS and bonds. We consider the following policies: allowing bond shorting; eliminating trading in CDS; and banning naked CDS. Finally we consider the welfare gains or losses associated with these policies.

Table 4 reports key simulation statistics for the various cases. These simulations combine the effects on prices and the optimal debt issuance response. We can isolate the effects of the changes at the same debt issuance by looking at the average price schedules. That is, $\mathbb{E}_Y[q(Y, B')]$ for differing levels of B'. These, plotted as differences from the benchmark average price schedule and smoothed, are displayed in Figure 7. (Unsmoothed values are given in Appendix D.) Even these changes have multiple effects, including not only direct effects on investors but feedback

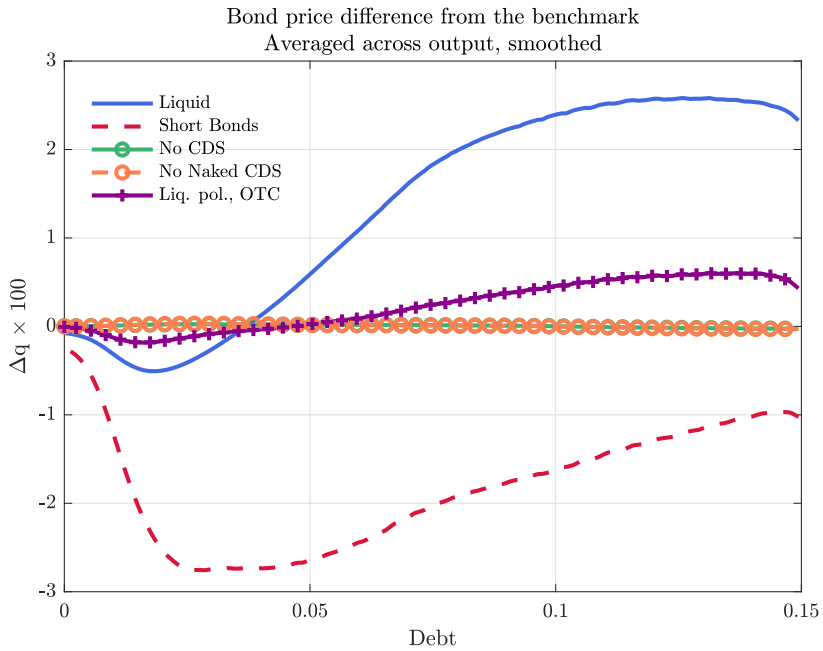
Table 4: Model comparison

Statistic	(1) Benchmark	(2) No Naked CDS	(3) No CDS	(4) Short Bonds	(5) Liquid	(6) Liq. pol., OTC
Bond spread mean (%)	7.44	7.45	7.45	7.36	7.65	8.72
Bond spread std. (%)	3.83	3.83	3.83	3.64	4.18	4.29
Debt service to output ratio (%)	5.55	5.55	5.55	5.09	5.70	5.70
Bid-ask spread for bonds mean (%)	0.06	0.06	0.06	0.02	-	0.08
Bid-ask spread for bonds std. (%)	0.04	0.04	0.04	0.02	-	0.05
Bid-ask spread for CDS mean (%)	0.23	0.98	-	0.20	-	0.26
Bid-ask spread for CDS std. (%)	0.10	0.48	-	0.13	-	0.10
CDS-bond basis deviation mean (%)	1.30	2.98	-	-0.63	-	1.20
CDS-bond basis deviation std. (%)	1.61	2.90	-	0.38	-	1.63
Aggregate dealer						
CDS position (%)	0.00	-0.02	-	0.05	-	0.00
dealer net exposure (%)	0.02	0.02	0.01	0.02	-	0.02
dealer bond position (%)	0.02	0.00	0.01	0.07	-	0.02
dealer-investor volume (%)	0.20	0.02	-	0.15	-	0.21
dealer buy volume (%)	0.10	0.00	-	0.10	-	0.11
dealer sell volume (%)	0.10	0.02	-	0.05	-	0.10
Std. of agg. dealer CDS position (%)	0.03	0.00	-	0.04	-	0.03
Default probability (%) (full sample)	1.08	1.08	1.08	0.89	1.17	1.17
Correlation of						
log IDP and agg. dealer CDS	-0.61	-0.93	-	-0.25	-	-0.56
log IDP and agg. dealer-investor volume	0.96	0.83	-	0.88	-	0.96
log IDP and agg. dealer buy volume	0.78	0.39	-	0.38	-	0.75
log IDP and agg. dealer sell volume	0.97	0.83	-	0.86	-	0.94
YTM spreads and bond bid-ask spreads	0.55	0.54	0.55	-0.11	-	0.71
YTM spreads and CDS bid-ask spreads	0.78	0.97	-	-0.09	-	0.68
Welfare gain						
Sovereign (CEV $\times 10^4$)	-	0.31	0.22	-41.69	-4.91	-
Investor (agg., money metric $\times 10^4$)	-	-0.24	-0.25	24.27	11.03	-

Note: all CDS position and volume measures are deflated by mean GDP; welfare measures are relative to the benchmark and compare average utilities along the simulated path; CEV stands for consumption equivalent variation; the "Liq. pol., OTC" case uses the policy functions from the liquid version of the model with benchmark OTC frictions; welfare for the "Liq. pol., OTC" case is not reported because prices and policies are inconsistent.

from default decisions into prices, and we will decompose these forces.

Figure 7: Price schedule differences under alternative policies



Note: bond schedules are averaged across GDP using the invariant distribution and then smoothed; the appendix reports the non-smoothed values.

4.1 Comparing the liquid and frictional markets

We assess the quantitative importance of frictional markets by taking the limit as entry costs γ_b, γ_c go to zero, recovering a version of the Arellano (2008) model. The results for a few key variables are displayed in Table 4 in columns (1) and (5). OTC frictions have two types of effects on bond prices. There is a direct effect, which changes bond prices as investor and dealer demand changes, and an indirect effect, which is how q changes as the sovereign reoptimizes default and debt issuance. To delineate these, we consider an intermediate case that uses the sovereign policies from the liquid model with pricing from the benchmark model. The results for this experiment are displayed in the column (6). The direct effect of moving from a liquid to frictional environment is captured in moving from (5) to (6), while the indirect effect is found in moving from (6) to the benchmark (1).

The direct effect of frictional trading is an adverse impact on prices, reflected in the 1pp increase in average bond spreads from 7.65% to 8.72% (Table 4). Looking beyond averages and to specific debt levels in Figure 7, the indirect effect (which is the negative of the "Liq. pol., OTC" curve) improves prices at debt levels below 0.05 and worsens them at higher debt levels. Essentially, whenever the direct effect is negative, the sovereign's value of repayment decreases (which

follows from a simple budget constraint argument).[18] This drives up default rates and creates a negative indirect effect. So the direct and indirect effect covary, with the indirect effect amplifying the direct effect. Since the direct effect of moving from the liquid case to frictional OTC markets is positive when debt levels are low, the indirect effect is also positive when debt levels are low. Generally one can expect the total effect to give the sign of both the direct and indirect effect conditional on a debt level.

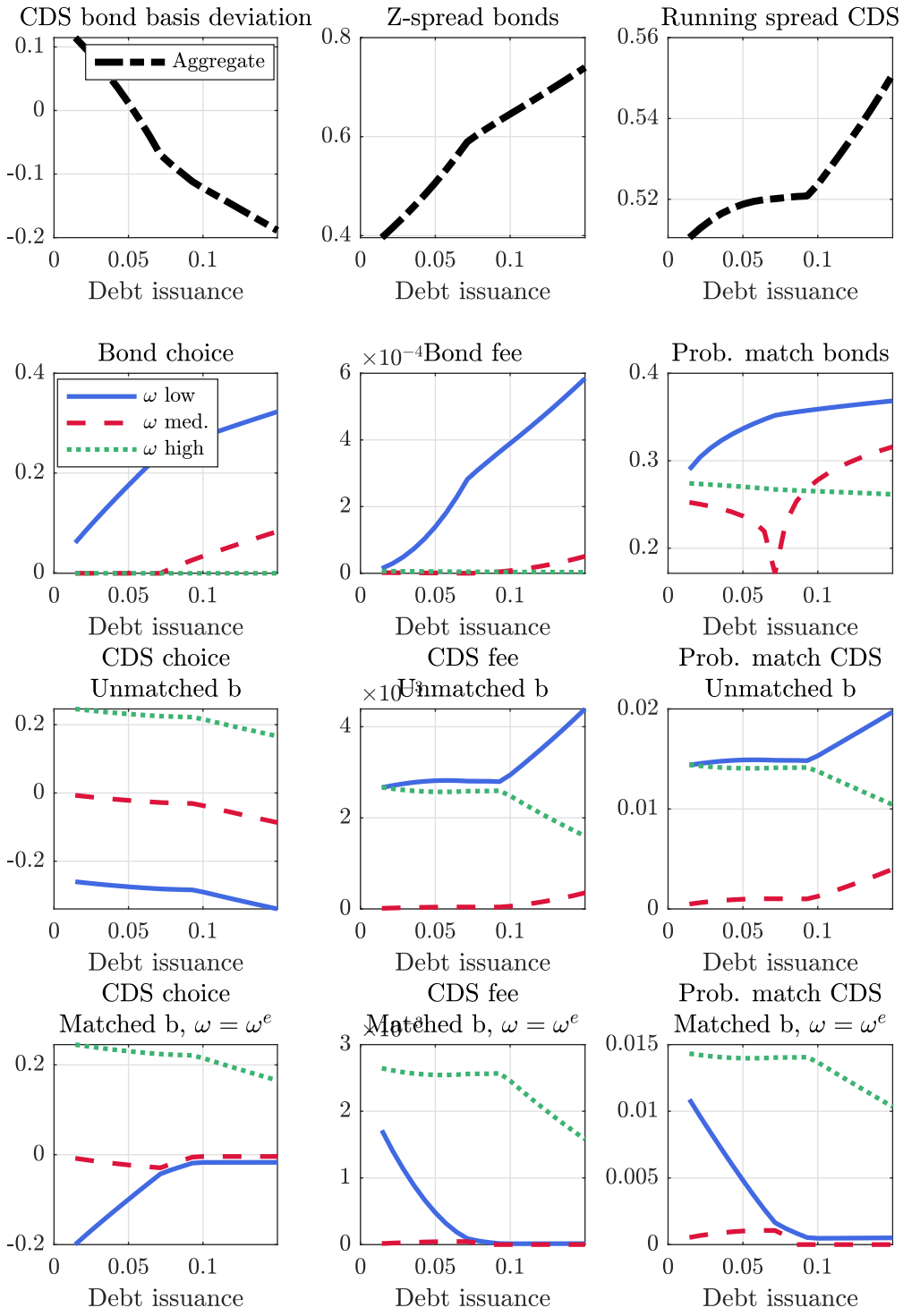
Investors with positive (negative) exposure have less (more) demand for bonds than a risk-neutral agent. So the distribution of exogenous exposure ω is key for determining whether the frictional model implies better or worse prices for the sovereign than the risk-neutral case. Based on Figure 5 where the less (in fact, negatively) exposed agents were the only bond purchasers, one would expect better pricing than the risk-neutral case. But that figure was constructed with a low amount of debt (0.03). Figure 8 is analogous to Figure 5 but varies the bond supply while holding expected default rates fixed. There, one can see that at higher levels of debt, other investors also trade the bond. Because the marginal investor shifts from less-exposed to more-exposed agents as debt increases, one should expect that at low (high) levels of debt OTC frictions improve (worsen) the bond price. And this is exactly what happens, as can be deduced from Figure 7. The "Liq. pol., OTC" curve lies above the "Liquid" curve for debt levels below 0.035, implying trading frictions increase prices when debt is small. The increase in *average* spreads reflects the average debt level is closer to 0.056 (since debt service to GDP is targeted to be 5.6% and GDP is roughly 1), which is in the higher-debt region where OTC frictions lower prices.

The mechanism through which the OTC frictions change prices is key for understanding the results of the other counterfactuals: Frictions change who the marginal investor is. In the liquid case, it is as if agents of all types can trade freely, and in the end everyone bears an infinitessimal amount of risk, and all agents price the bond. The OTC frictions in the benchmark have an asymmetric effect on risk-taking, making it harder to buy protection against default than to sell protection: Investors with positive exogenous exposure would like to short the bonds, which is not allowed, or buy CDS, which is more costly in terms of fees ($\gamma_c \gg \gamma_b$) than accessing the bond market. As a result, agents with negative exposure end up as the marginal investors at low debt levels (as seen in Figures 5 and 8), pushing the bond price up relative to risk-neutral. However, investors are also risk-averse and their capacity to absorb bonds is limited. As the total amount of debt issued increases, these negatively-exposed active bond traders are supplemented by more-exposed bond traders who become active, changing the marginal investor.

We saw already that—conditional on debt—the indirect effect amplifies the direct effect. The

[18] Given any feasible debt issuance and default policy, a larger q means that policy is still feasible but results in higher consumption; additionally, other choices may become feasible as well, resulting in a further boost from option value. Conversely, a smaller q reduces consumption in every state and can make some consumption plans infeasible.

Figure 8: Investor behavior in the benchmark



Note: plotted for a default rate of 8.7%; "Unmatched b" ("Matched b") means investors who did not (did) match with a dealer in the bond market; this graph applies to both the benchmark and the case with liquid sovereign policies but frictional OTC markets since the bond issuance is fixed and default risk is on the horizontal axis.

indirect effect averaged over the simulated path, which can be seen in moving from column (6) to column (1) in Table 4, has a more subtle interpretation. Faced with higher spreads conditional on typical debt levels, the sovereign reduces debt issuance (resulting in slightly lower average debt service costs), which brings down equilibrium spreads to a level that's even lower than in the liquid case. This behavior is driven by the sovereign's Euler equation and impatience. Loosely speaking, the sovereign will always borrow up to the point that the effective bond price equals its discount factor.[19] When prices improve, all else equal, this incentivizes borrowing that drives the price back down. When prices worsen, the reverse happens. In this respect, whatever a policy does, the sovereign will tend to undo in the simulation, which is the main reason why in columns (1) to (5) of Table 4, the spreads are remarkably similar but the debt service can vary substantially.[20]

4.2 The role of bond shorting

We turn our attention now to examine the effects of allowing investors and dealers to take negative bond positions (i.e., to short sovereign bonds), which Figure 7 shows has a large adverse impact on bond prices.

Viewed from a classic Walrasian perspective, allowing bond-shorting should always hurt bond prices, as it does in this case. The reason is that bond demand must equal bond supply. By suddenly allowing agents to take short positions, this necessarily reduces bond demand (weakly) at any given price. We label this canonical Walrasian force as the *demand effect*.

While the demand effect on prices plays an important role in our model, there are three other forces at work: an *entry effect* given by the dealers who entered or exit the market; an *intermediation effect*, which arises from changes in dealers' trading profits π which are are passed on to investors in the form of intermediation fees and matching probabilities; and a *default risk effect* given by the change in default risk due to the sovereign's response to pricing. The latter is the indirect effect already discussed, while the demand, entry, and intermediation effects are all components of the direct effect.

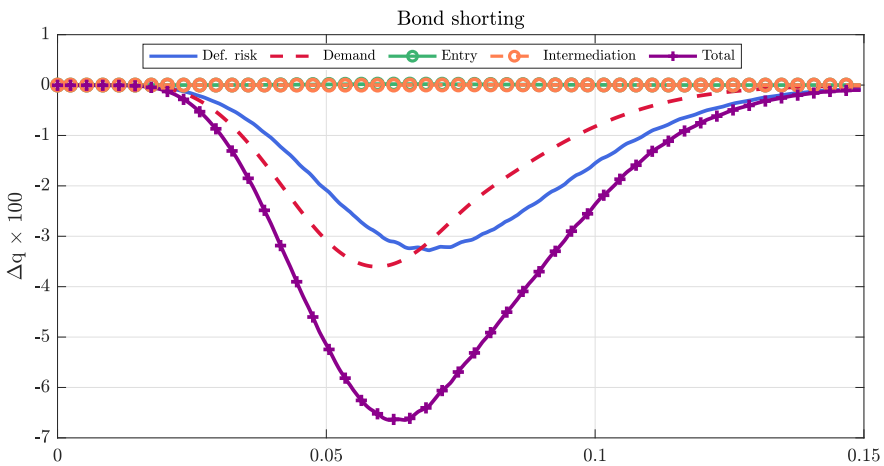
We quantify these components by allowing only one channel to operate at a time. We first maintain the benchmark policy environment but change $\mathbb{E}_{Y'|Y}[\delta(Y', B')]$ to its value in the new policy environment (bond shorting in this case). The incremental change, from $q_0(Y, B')$ to $q^{def}(Y, B')$ is our default risk effect. We then hold dealer profit π and the dealer measure D fixed but otherwise solve for a "general equilibrium" (with misspecified π, D) allowing dealer and investor b, c (and θ_b, θ_c for investors) to change. The incremental change $q^{demand}(Y, B') - q^{def}(Y, B')$ is our demand

[19] This statement ignores differentiability issues and supposes that consumption growth is approximately zero. In that case, the Euler equation approximately gives $d(q(Y, B')B')/dB' = \beta$. The effective price is not just q but also reflects how much prices move against them when issuing more debt.

[20] Note column (6) does not have optimal policies, and so the sovereign is off the Euler equation.

effect. We then allow the measure of dealers to change and recompute equilibrium (with misspecified π). Our entry effect is the change $q^{entry}(Y, B') - q^{demand}(Y, B')$. Last, we allow π to change, resulting in the new equilibrium prices $q_1(Y, B')$. The difference $q_1(Y, B') - q^{entry}(Y, B')$ is our intermediation effect. By construction, the individual effects sum to the total effect, $q_1(Y, B') - q_0(Y, B')$.

Figure 9: Bond-shorting decomposition



Note: bond schedules are averaged across GDP using the 40th-60th percentile conditional distribution and then smoothed; the appendix reports the non-smoothed values.

The decomposition in Figure 9 reveals a large negative demand effect, consistent with the Walrasian predictions. The magnitude of the response, however, lies in the large disparity between γ_b and γ_c. To rationalize the data, γ_b is an order of magnitude smaller than γ_c. Although trading bonds is significantly less costly in the benchmark calibration, for many investors bonds are not a great substitute for CDS. Not only can they not be shorted, but there's also the risk that exogenous exposure ω^e changes. Once bond-shorting is allowed, investors who wanted to short—but not at the "exorbitant" fees implied by γ_c—now find it attractive to short by paying the smaller γ_b-implied fees.

The large demand effect is amplified by a large default risk effect / indirect effect: Since the combined direct effects push prices lower, that drives up default rates which further depresses prices. In contrast, both the entry and intermediation effect are essentially zero. The intermediation effect *must* in fact be zero in this experiment. The reason is that dealers can attain any consumption allocation they want using a risk-free asset and CDS irrespective of whether they have access to bond-shorting. Consequently, π and the net entry fees are not directly affected in this counterfactual.

That the entry effect is close to zero is a numerical result. Bond shorting does change trading behavior, submarket tightnesses, and equilibrium fees, which implies the measure of dealers

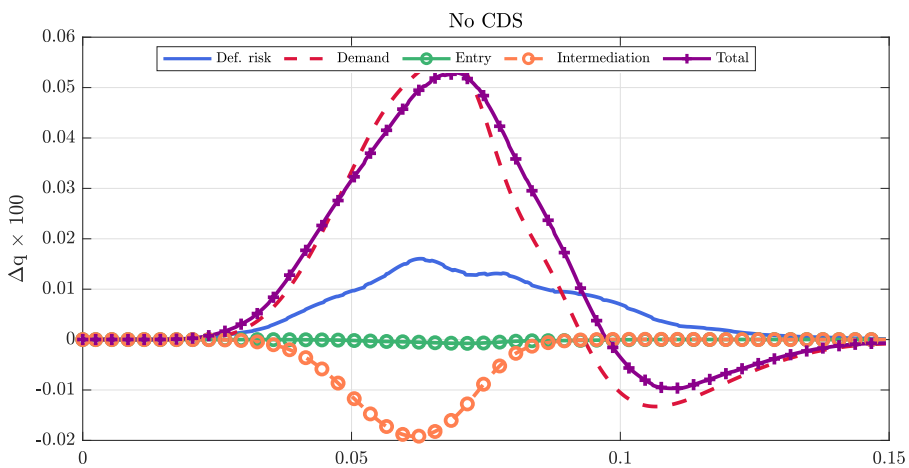
changes. However, the measure of dealers is small relative to the measure of investors (contrast Tables 2 and 5); and when dealers enter they do not bring any exogenous exposure ω with them nor do they generally have much demand for endogenous exposure, as dictated by the calibration. Consequently, the entry effect is small, which will also be true in the other counterfactuals.

4.3 Closing the CDS market

CDS are a relatively modern financial innovation that are widely used by dealers and investors. We now consider the consequences of shutting down the CDS market.

Despite their evident usefulness, the benchmark model predicts almost no spillover from the CDS market to the bond market in terms of prices (Figure 7). This is *not* a generic result, and we will show in Section 5 that some parameterizations of the model predict large spillovers. But to understand why they are estimated to be small here, we decompose the almost zero total effect into its four components in Figure 10.

Figure 10: No CDS decomposition



Note: bond schedules are averaged across GDP using the 40th-60th percentile conditional distribution and then smoothed; the appendix reports the non-smoothed values.

The figure reveals that the positive default risk effect—arising from larger q overall—and a negative intermediation effect offset one another. The negative intermediation effect arises from the ban on CDS making some allocations infeasible for dealers, which reduces their profits π and drives up net entry costs. Higher net entry costs reduce risk-sharing and result in increased risk premia / lower q. As these two forces mostly offset, the total effect looks almost identical to the demand effect.

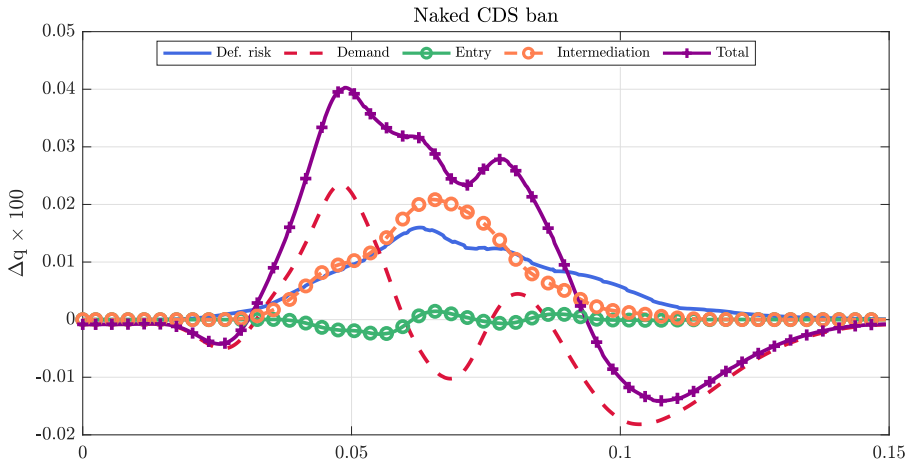
The demand effect is signed as theory would predict: By closing an opportunity to take positions in synthetic bonds, the demand curve for bonds shifts out and prices rise. However, the magnitude is much smaller than in a qualitatively similar bond-shorting-ban counterfactual (the re-

verse of allowing bond shorting). The demand effect is weaker here partly because the CDS market is estimated to be much more frictional the bond market. Technically investors can take short positions in synthetic bonds. But, fees are larger ($\gamma_c \gg \gamma_b$) and matching probabilities lower (Figures 5 and 8) than for bonds. So allowing bond-shorting has a much larger demand effect than eliminating CDS as the bond market is estimated to be much less frictional. Additionally, the volume of CDS traded is fairly small relative to debt service in our calibration. So when that market gets shut down, the demand boost from CDS-protection-selling investors substituting into bond-buying is only a small percentage of overall bond demand.

4.4 A ban on naked CDS

Motivated by the regulatory change implemented in the European Union in 2011-2012, we now use the model to investigate the consequences of a naked CDS ban—defined as a ban on the purchase of CDS protection in excess of the amount of bond exposure. As with the elimination of CDS, the benchmark calibration predicts there is almost zero response in terms of prices (Figure 10), but this is likewise not a generic result. We will show in Section 5 that a naked CDS ban can significantly improve prices, worsen prices, or leave them unchanged, depending on the calibration.

Figure 11: Naked CDS ban decomposition



Note: bond schedules are averaged across GDP using the 40th-60th percentile conditional distribution and then smoothed; the appendix reports the non-smoothed values.

To understand why the total effect is small here, we apply our decomposition strategy to create Figure 11. The demand effect begins positive but transitions to a negative value. From theory, we expect the demand effect to be positive for the following reason. If we think about a unified asset that is just endogenous exposure (i.e., $b - c$ but excluding ω), then the total supply of exposure is the bond while the total demand is bond demand plus net demand for synthetic bonds. By eliminating the ability to have negative endogenous exposure, the demand for exposure increases which

should drive up the exposure price, q. The model captures some of that, but there are additional complicating factors in the model. One pure demand force that cuts in the opposite direction arises from precautionary behavior. Investors are more willing to buy bonds if, after the realization of ω, they can undo some of that exposure. Banning naked CDS reduces the ability to reallocate risk in this way, reducing demand for the bond ex ante.

Like with all the counterfactuals, the entry effect is small and the default risk effect moves in line with the total effect for the same reasons as before. One should generally expect the intermediation effect to be negative because the portfolio choice of the dealers is being restricted, which lowers π else equal and raises net entry costs else equal. However, it is positive here for an interesting reason. As we show in Figure 15 in the appendix, the naked CDS ban effectively segments bond and CDS markets. Only the less exposed investors trade bonds, and only the more exposed trade CDS (buying protection from dealers). The consequence of this can be seen in the substantial increase in the CDS-bond basis deviation in Table 4, which indicates bonds are cheaper relative to synthetic bonds ($c < 0$) than in the benchmark. Dealers benefit as they can obtain the exposure they want by selling CDS protection to the very exposed investors, increasing their profits. This behavior is perhaps most clearly seen in Table 5. The naked CDS ban has dealers drop almost all bond holdings but increase exposure by selling protection to highly exposed investors.

Table 5: Dealer behavior along the simulated path

Statistic	Benchmark	No Naked CDS	No CDS	Short Bonds
Measure of dealers	0.0011	0.0011	0.0011	0.0014
Individual dealer bond	0.1452	0.0187	0.0831	0.5055
Individual dealer CDS	-0.0048	-0.1572	0.0011	0.3364
Individual dealer exposure	0.1500	0.1759	0.0820	0.1691
Individual dealer profit	0.0005	0.0008	0.0002	0.0004
Individual dealer buy vol	0.9004	0.0000	0.0000	0.6902
Individual dealer sell vol	0.9058	0.1640	0.0000	0.3539

4.5 Welfare

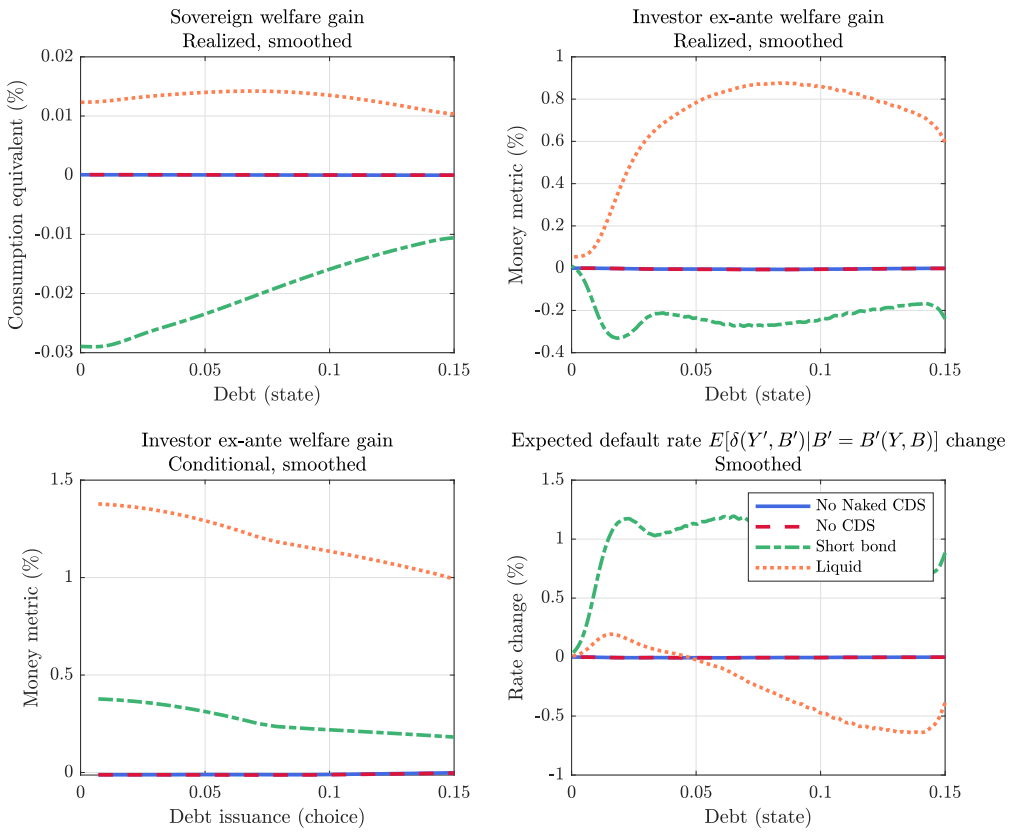
In this section we consider the welfare impact of each of the counterfactual policies.

To measure welfare for the sovereign, we use the standard consumption equivalent variation (CEV) measure, which scale the consumption policy in the benchmark up or down until the value associated with that policy equals the value under the new regime. Dealers' welfare is always zero by free entry. To measure welfare for investors, we use a money metric. Specifically, given indirect utility U_1 from a new regime and U_0 from the benchmark, the welfare measure is simply $U_1 - U_0$ times the measure of investors I. Because of quasi-linear utility, this gain is effectively measured

in terms of the consumption good; and, since GDP is close to one, the money metric gain can be thought of as a share of GDP. Similarly, because GDP is close to one and the sovereign's consumption roughly equals GDP, the CEV measure is also in terms of a share of GDP (approximately). The measures therefore are roughly comparable.

Figure 12 depicts the changes in welfare relative to the benchmark for different levels of debt issuance. Consistent with the previous findings that the ban on naked CDS and eliminating the CDS market had little effect on prices, we find virtually no impact of such policies on welfare. Since the effects are small, Figure 16 in the appendix plots only those two policies, which reveals that the sovereign benefits from them while investors end up worse off.

Figure 12: Welfare analysis



Note: the top panels and the bottom right panel are functions of (Y, B) and have been averaged using the invariant distribution and then smoothed; the panel labeled "Conditional" is a function of $B', \bar{\delta}$ and has had a numerical average taken across a grid of default rates; all series have been smoothed, the appendix reports the non-smoothed values.

We observe, however, that introducing sovereign bond shorting or perfect liquidity has a significant welfare effect on investors and sovereign governments. With sovereign bond shorting, both the sovereign government and investors welfare decrease. The sovereign loses about 1-3 bps of GDP (top left panel), while investors lose around 30 bps of GDP (top right panel). That investors lose is actually a general equilibrium effect induced by higher default rates (bottom right panel). In partial equilibrium with the default rate fixed, welfare for investors is higher by 20bps-40bps

(bottom left panel). So it is welfare-improving for both the sovereign and investors for investors to limit their ability to short. This perhaps counterintuitive result requires the combination of the OTC and sovereign debt model blocks, as our model does.

Eliminating OTC frictions entirely has a significant positive effect on the sovereign government and investors. The sovereign gains about 1.5-2 bps of GDP while investors can gain close to 100 bps of GDP (top right panel) and even more in partial equilibrium (bottom left panel).

5 Robustness

This section examines the robustness of the quantitative results to alternative parameterizations and an alternative timing.

5.1 Naked CDS and CDS ban large effects, bond shorting small effects

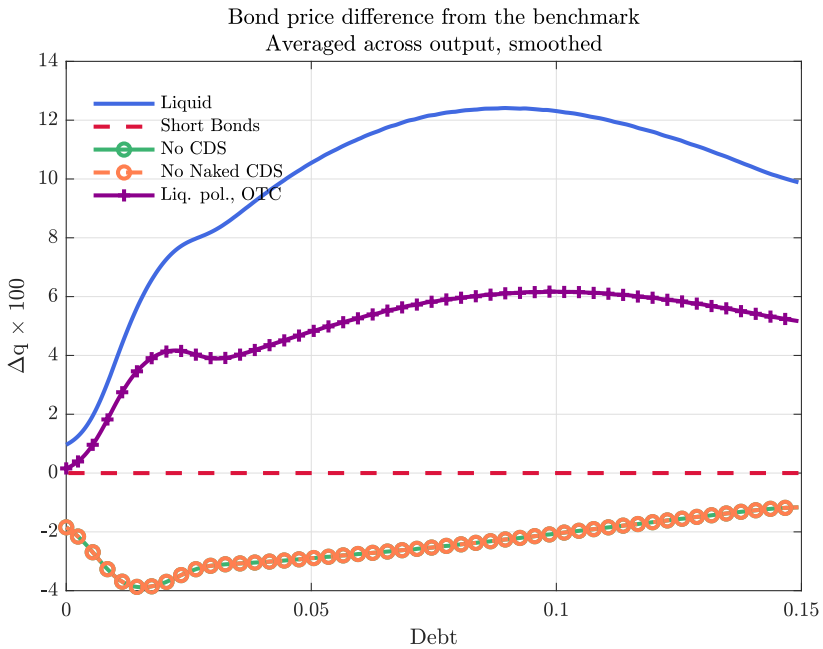
A ban on naked CDS trading or a total ban on CDS trading has a positive, but very small, impact on prices and welfare. This is not a generic result coming from the model but rather a result of the calibration. To illustrate this point, we consider variation in a single parameter, ξ_b, which changes the elasticity of bond matching probabilities. Figure 13 shows the bond price effects when we increase it from the benchmark's value of 0.09 to 0.7. With a more elastic bond supply, a naked CDS ban or eliminating CDS has a large negative impact on the bond price schedule. Similarly, the effect of bond shorting goes from a large negative effect in the benchmark parameterization to zero. Clearly then the matching elasticity parameters ξ_b and ξ_c are key, and we identified these carefully by using how bid/ask spreads varied in default risk (see Section 3.3).

5.2 Alternative parameterizations

We now consider more general variation of the key matching and risk-sharing parameters identified in Section 3.3 and reassess the counterfactuals' impacts on bond prices. Table 6 provides a summary of the decompositions from (an unsmoothed version of) Figure 7 for a number of parameter specifications. The maximum change in q relative to the benchmark appears in the first of the five numeric columns, while the minimum change in q appears in the last five. A finding that holds in virtually every parameter specification is that allowing for bond shorting reduces bond prices, often by a substantial amount but sometimes only weakly. As we showed in the decomposition exercise of Section 4.2, bond-shorting is special in that every effect works the same way: demand, intermediation, entry, and default risk are all weakly negative. So it makes sense that this finding is robust.

The effects of naked CDS bans and eliminating CDS are usually close to zero, but can be large and negative for some matching elasticities and sometimes large and positive. That these results are

Figure 13: Price schedule differences for an elasticity parameter of $\xi_b = 0.7$



Note: bond schedules are averaged across GDP using the invariant distribution and then smoothed; the appendix reports the non-smoothed values.

Table 6: Robustness of counterfactuals to various parameters

	Maximum $\Delta q \times 100$					Minimum $\Delta q \times 100$				
	Liquid	Short	No CDS	No Naked	Liq. pol., OTC	Liquid	Short	No CDS	No Naked	Liq. pol., OTC
Benchmark	4.18	0.00	0.10	0.09	2.31	-1.00	-3.77	-0.03	-0.03	-0.66
$\xi_b = 0.7$	13.98	0.00	0.00	0.00	7.80	0.00	-0.00	-4.82	-4.82	0.00
$\xi_b = 0.7, \xi_c = 0.1$	5.18	0.07	0.00	0.00	2.37	0.00	-0.00	-12.03	-11.91	0.00
$\xi_b = 0.1, \xi_c = 0.7$	4.24	0.00	0.00	0.00	2.32	-0.97	-3.74	-0.00	-0.00	-0.62
$\xi_b = 0.4, \xi_c = 0.4$	8.65	0.00	0.00	0.00	4.68	0.00	-1.18	-2.23	-2.23	0.00
$\gamma_b = 4, \gamma_c = 1/4$	4.49	0.00	0.27	0.26	2.37	-0.33	-3.35	-0.34	-0.36	-0.12
$\gamma_b = 1, \gamma_c = 1$	4.28	0.00	0.06	0.04	2.32	-0.95	-3.69	-0.02	-0.03	-0.61
$\gamma_b = 1/4, \gamma_c = 4$	4.40	0.00	0.12	0.10	2.36	-0.78	-3.55	-0.09	-0.10	-0.50
$\mu_\omega = 0.2$	5.43	0.00	0.14	0.11	2.37	0.00	-3.88	-0.03	-0.03	0.00
$\mu_\omega = 0.0$	0.82	0.00	0.07	0.77	0.00	-3.14	-3.65	-0.03	-0.03	-2.37
$\mu_\omega = -0.2$	0.00	0.00	0.07	1.97	0.00	-7.40	-3.48	-0.03	-0.07	-4.05
$\sigma_\omega = 0.01$	4.70	0.00	0.02	0.01	2.37	0.00	-2.71	-0.07	-0.08	0.00
$\sigma_\omega = 0.03$	1.28	0.00	1.08	1.15	0.00	-3.81	-7.71	0.00	0.00	-2.37
$\sigma_\omega = 0.05$	0.00	0.00	2.65	1.40	0.00	-9.39	-13.87	0.00	-1.42	-4.69
$I = 0.1$	24.83	0.00	0.00	0.00	12.28	0.00	-0.15	-1.87	-1.87	0.00
$I = 1$	8.44	0.00	0.01	0.01	3.48	-0.01	-2.68	-0.52	-0.54	0.00
$I = 10$	0.46	0.00	0.28	0.31	0.00	-2.98	-4.85	0.00	0.00	-2.37

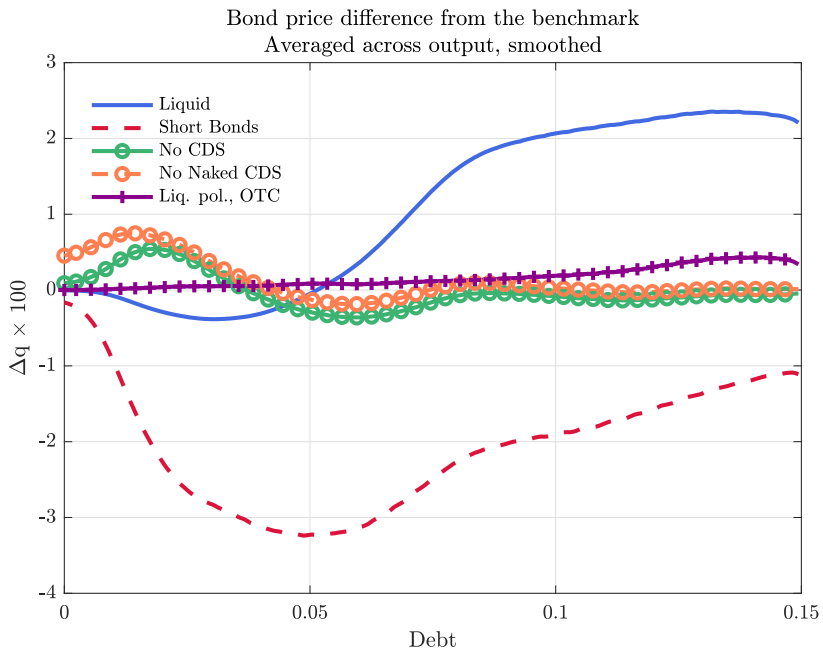
Note: maximums and minimums are taken after averaging across output but using unsmoothed values.

sensitive reflects the conflicting forces in the decomposition exercise. Eliminating OTC frictions does not necessarily improve prices, like in the benchmark, but can sometimes reduce them by a large amount, particularly when there are at least some agents with negative exogenous exposure. The key there is who the marginal investor is. That is why the average exposure μ_ω and the measure of investors I play such a crucial role for that counterfactual.

5.3 Reverse Bond-CDS timing

In our benchmark model, agents first trade bonds and then, after the exposure shocks ε_ω are realized, they trade in the CDS market. In order to assess the importance of this timing assumption, in this section we reverse the timing.[21] With the reverse timing assumption, a naked CDS ban effectively eliminates CDS: investors in the first period do not have access to bonds yet, and so they cannot take out positive amounts of CDS. There can still be some CDS trade, but it is exclusively between dealers buying protection and investors selling protection (dealer sell volume is zero).

Figure 14: Price schedule differences under alternative policies with reverse Bond/CDS timing



Note: bond schedules are averaged across GDP using the invariant distribution and then smoothed; the appendix reports the non-smoothed values.

Figure 14 depicts the average difference in bond prices across debt issuance for different policies relative to the benchmark under the reverse timing convention. When comparing the impact of the policies with the reverse timing (Figure 14) and without it (Figure 7), one can see that prices move more for the no CDS and no-naked CDS cases with the reverse timing. However, the general message is still the same—such policies have a small impact. This is confirmed by the welfare analysis in Figure 17 in Appendix D. The analysis of the impact of bond shorting and market liquidity also reveals that the reverse timing does not impact the qualitative outcomes for prices. The welfare results are also robust, with liquidity tending to improve welfare for both the sovereign and investors and bond shorting tending to decrease welfare for both. The magnitudes of the effects on prices and welfare are also similar.

[21] We thank our discussants for emphasizing this issue.

There are two main reasons for the results to be similar across timing conventions. The first one is that, in our calibration, the CDS market is very frictional relative to the bond market. In particular, the probability of matching in bond markets is much higher than in CDS markets (see, e.g., Figure 8), and increasing that probability through higher fees is expensive. This implies that, in the benchmark timing, investors do not rely on matching in CDS markets later because it is a low probability event (and costly to do). So individual investor bond demand is mostly unchanged when the probability of subsequent access to the CDS market goes from a small positive value (in the benchmark timing) to zero (in the reverse timing). And, in the reverse timing, the measure of investors who access the CDS market is small irrespective of investor type. This makes the pool of potential bond investors in the reverse timing similar to the pool in the benchmark timing.

The second reason for similar reverse timing results is that the information premium associated with buying bonds after the exogenous exposure shock, as opposed to before, is small. The variance of the exposure shock σ_ω^2 is small relative to the variance of overall exposure $\sigma_\omega^2/(1-\rho_\omega^2)$ (recall $\rho_\omega = 0.95$). This makes ex-post exogenous exposure ω highly predictable using the ex-ante state ω^e. As a result, bond demand before and after the information is revealed are roughly the same, and so the information premium embedded in bond prices is small.

6 Conclusions

The market for sovereign debt CDS is relatively new and not well understood. At the same time, policy is being implemented regulating it. To understand the role of CDS, it is essential to incorporate and understand issues of liquidity, risk-sharing, violations of arbitrage, and relationships between primary and secondary markets. In this paper, we have proposed a model that addresses many of these issues in an attempt to understand and quantify the market as it is and as it could be. We show how to identify the model's trading frictions, and show the model reproduces key risk-sharing and trading-friction patterns in the data. The model's counterfactuals indicate that CDS regulation *can* have substantial effects on sovereign debt markets. However, our identification strategy forces us to conclude that eliminating CDS or implementing a naked CDS bans has few spillover effects to sovereign bonds. In contrast, liquidity can help or hurt depending on debt levels while bond shorting almost always hurts prices. While there is much more to study and understand in these markets, our data-disciplined model provides novel insights into existing policy proposals, both for those that have been enacted and those that may be in the future.

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Appendix for
"Sovereign Debt and Credit Default Swaps"

Gaston Chaumont, Grey Gordon, Bruno Sultanum and Elliot Tobin

A Theoretical results

It is worth highlighting here that, because of quasi-linearity, having ω paying in no-default states is equivalent to having payments 0 in no-default states and $-\omega$ in default states for all the model's predictions except risk-free borrowing, which is not of interest.

Lemma 1. *Consider an alternative formulation for the exogenous exposure with payment 0 in no-default states and $-\omega$ in default states. Then the investors choice of θ_b, b, θ_c and c are the same as in the original model.*

Proof of lemma 1. To see this note that

$$\begin{aligned}
V(\omega, b, c; s_3) &= \max_a -q_f a + \beta \mathbb{E}_\delta \left[u_i(a + (1-\delta)(b+\omega) + \delta c)\right] \\
&= \max_a -q_f a + \beta \mathbb{E}_\delta \left[u_i(a + \omega + (1-\delta)b + \delta(c-\omega)\right] \\
&= \underbrace{\max_{\tilde{a}} -q_f \tilde{a} + \beta \mathbb{E}_\delta \left[u_i(\tilde{a} + (1-\delta)b + \delta(c-\omega)\right]}_{\text{value function of alternative problem}} + q_f \omega \\
&\equiv \tilde{V}(\omega, b, c; s_3) + q_f \omega,
\end{aligned}$$

where we performed the substitution $\tilde{a} = a + \omega$, and defined $\tilde{V}(\omega, b, c; s_3)$ as the value function of the alternative problem with payments 0 in no-default states and $-\omega$ in default states. We can see that if we replace $V(\omega, b, c; s_3) = \tilde{V}(\omega, b, c; s_3) + q_f\omega$ in problem (5), we obtain the same problem up to a constant $q_f\omega$. Therefore, we obtain the same choices of θ_c and c in the alternative model and that $V(\omega, b; s_2) = \tilde{V}(\omega, b; s_2) + q_f\omega$. Similarly, if we replace $V(\omega, b; s_2) = \tilde{V}(\omega, b; s_2) + q_f\omega$ in problem (8), we obtain the same problem up to a constant $q_f\mathbb{E}[\omega|\omega^e]$; resulting in the same choice of θ_b and b in the alternative model and $V(\omega^e; s_1) = \tilde{V}(\omega^e; s_1) + q_f\mathbb{E}[\omega|\omega^e]$. □

Now we turn our attention to the CDS-bond basis, which is an no-arbitrage condition that implies bonds, CDS, and the risk-free asset must satisfy a certain relationship. Here, we must make distinction between measurement and theory. In measurement, the CDS-bond basis is measured as the CDS running spread minus the Z-spread of the bond. Theoretically, that does *not* have to be zero, even when no-arbitrage holds. (It must hold approximately though for small default rates). We say the CDS-bond basis holds if the actual no arbitrage relationship, $q + p = q_f$, holds. With one caveat, we can show the basis holds for *inter-dealer* prices:

Proposition 1. *In any OTC equilibrium, $q + p \leq q_f$. If $b_d > \underline{b}$, then $q + p = q_f$, i.e., CDS-bond basis holds for inter-dealer prices.*

Proof of proposition 1. The dealer can always purchase one unit of bonds and one unit of CDS at a cost $q + p$ and sell one unit of the risk-free asset at a cost q_f. The resulting next period consumption allocation is unaltered. Consequently, for an optimal portfolio choice to exist, profits must be bounded requiring $q + p - q_f \geq 0$.

Additionally, if $b_d > \underline{b}$, then the dealer can implement the reverse trade, implying $q+p-q_f \leq 0$. Consequently, $q + p = q_f$. □

We can characterize the equilibrium allocations further by focusing on the limiting case in which there is no uncertainty or heterogeneity over the value of ω, i.e., $\sigma_\omega = 0$. The remaining characterizations assume that dealer's optimal portfolio choice is not constrained and the CDS-bond basis holds for inter-dealer prices. We also assume Inada conditions for u_d and u_i.

The first results shows that, when the CDS-bond basis holds for inter-dealer prices, there is no point in entering the bond market and then entering the CDS market. The reason is any desired consumption allocation can be achieved with the risk-free asset and bonds or the risk-free asset and CDS, so there's no point in paying fees twice. In other words, the only reason why investors hold both bonds and CDS is because there is uncertainty about the optimal exposure to default risk, and, once the uncertainty is resolved, investors trade CDS to adjust their exposure.

Proposition 2. *Suppose investors are unconstrained at their optimal choices. When the CDS-bond basis holds, b_i is such that $\theta_c(b_i) = c_i(b_i) = 0$.*

Proposition 3. *Suppose investors are unconstrained at their optimal choices.*

The CDS-bond basis must hold if $\alpha(\theta_c(b_i)) = 1$. In general, investors bound the CDS-bond basis

$$\alpha(\theta_c(b_i))(q + p - q_f) = (1 - \alpha(\theta_c(b_i)))(\beta(1 - \bar{\delta})u'(a(b_i, 0) + b_i) - q).$$

Below, we combine the proof of proposition 2 and 3.

Proof of proposition 2 and proposition 3. Let $q^* \equiv \beta(1 - \bar{\delta})$ and $p^* \equiv \beta\bar{\delta}$ be the risk-neutral bond and CDS prices.

The first order condition (FOC) for risk-free assets solves

$$q_f = q^* u'(a(b, c) + b + w) + p^* u'(a(b, c) + c).$$

For use below, we note that the implicit function theorem gives

$$a_c(b, c) = \frac{-p^* u''(a(b, c) + c)}{q^* u''(a(b, c) + b + w) + p^* u''(a(b, c) + c)} < 0$$

When the naked CDS constraint doesn't bind, $p = V_c = p^* u'(a(b, c) + c)$ and this is

$$q_f = q^* u'(a(b, c) + b + w) + p,$$

giving

$$\frac{q_f - p}{q^*} = u'(a(b, c) + b + w),$$

which holds when matched with CDS.

When the short-sale constraint doesn't bind $b_i > \underline{b}$, the FOC is satisfied

$$q = V_b(b; s_2)$$

By the envelope theorem, this is

$$\begin{aligned}
q &= \alpha(\theta_c(b))V_b(b,c;s_3) + (1-\alpha(\theta_c(b)))V_b(b,0;s_3) \\
&= \alpha(\theta_c(b))q^* u'(a(b,c)+b+w) + (1-\alpha(\theta_c(b)))q^* u'(a(b,0)+b+w) \\
&= \alpha(\theta_c(b))q^* \frac{q_f - p}{q^*} + (1-\alpha(\theta_c(b)))q^* u'(a(b,0)+b+w) \\
&= \alpha(\theta_c(b))(q_f - p) + (1-\alpha(\theta_c(b)))q^* u'(a(b,0)+b+w)
\end{aligned}$$

Then we can write

$$(1-\alpha(\theta_c(b)))q + \alpha(\theta_c(b))(q+p-q_f) = (1-\alpha(\theta_c(b)))q^* u'(a(b,0)+b+w)$$

or

$$\alpha(\theta_c(b))(q+p-q_f) = (1-\alpha(\theta_c(b)))(q^* u'(a(b,0)+b+w) - q)$$

The LHS is the expected gain from entering the CDS market and arbitraging, and the RHS is the expected cost, which is the failure to meet and hence have suboptimal consumption.

Note the following:

- Whenever $\alpha(\theta_c) = 1$, the CDS-bond basis must hold.

Now consider when the CDS-bond basis holds. Then if $\alpha(\theta_c) < 1$,

$$q = q^* u'(a(b,0)+b+w)$$

We already established that when matched in CDS markets,

$$\frac{q_f - p}{q^*} = u'(a(b,c)+b+w),$$

When CDS-bond basis holds, this then gives

$$u'(a(b,0)+b+w) = \frac{q}{q^*} = u'(a(b,c)+b+w),$$

which implies $a(b,0) = a(b,c)$. We noted above that $a_c < 0$, so this implies $c = 0$. At $c = 0$, $\theta_c = 0$ since there is no gain from trade.

Therefore,

- whenever the CDS-bond basis holds and $\alpha(\theta_c) \neq 1$, then $c = 0$ and $\theta_c = 0 = \alpha(\theta_c)$.

Now we must rule out $\alpha(\theta_c) = 1$. Suppose $\alpha(\theta_c) = 1$. Then the CDS-bond basis holds. A choice of b_i portfolio with $c_i(b_i) \neq 0$ results in consumption of $a(b_i, c_i(b_i)) + b_i + w = u'^{-1}(q/q^*)$ in repayment and $a(b_i, c_i(b_i)) + c_i(b_i) = u'^{-1}(p/p^*)$ in default. Consider now $\tilde{c}_i(\tilde{b}_i) = 0 = \theta_c(\tilde{b}_i)$ by choosing $\tilde{b}_i$ such to satisfy

$$a(\tilde{b}_i, \tilde{c}_i(\tilde{b}_i)) + \tilde{b}_i + w = a(\tilde{b}_i, 0) + \tilde{b}_i + w = u'^{-1}(q/q^*)$$

Then because consumption is the same in repayment for $(\tilde{b}_i, \tilde{c}_i)$ and (b_i, c_i), the FOC for assets ensures that consumption is the same in default. Consequently, the two portfolios result in the same lottery over consumption. Now consider the price of the portfolios as viewed from an investor already matched in the bond market. The cost of $(\tilde{b}_i, \tilde{c}_i)$ is just $q\tilde{b}_i + q_f a(\tilde{b}_i, 0)$. The cost of (b_i, c_i)

is $qb_i + \tilde{\gamma}_c\theta_c(b_i) + pc_i + q_f a(b_i, c_i)$. Comparing the cost, we find

$$\begin{aligned}
&qb_i + \tilde{\gamma}_c\theta_c(b_i) + pc_i + q_f a(b_i, c_i) \\
&= (q - q_f)b_i + \tilde{\gamma}_c\theta_c(b_i) + pc_i + q_f(a(b_i, c_i) + b_i) \\
&= (q - q_f)b_i + \tilde{\gamma}_c\theta_c(b_i) + pc_i + q_f(a(\tilde{b}_i, \tilde{c}_i) + \tilde{b}_i) \\
&= (q - q_f)(b_i + a_i - a_i) + p(c_i + a_i - a_i) + \tilde{\gamma}_c\theta_c(b_i) + q_f(a(\tilde{b}_i, 0) + \tilde{b}_i) \\
&= (q - q_f)(\tilde{b}_i + \tilde{a}_i - a_i) + p(\tilde{c}_i + \tilde{a}_i - a_i) + \tilde{\gamma}_c\theta_c(b_i) + q_f(a(\tilde{b}_i, 0) + \tilde{b}_i) \\
&= (q - q_f)(\tilde{b}_i + \tilde{a}_i - a_i) + p(\tilde{c}_i + \tilde{a}_i - a_i) + \tilde{\gamma}_c\theta_c(b_i) + q_f(a(\tilde{b}_i, 0) + \tilde{b}_i) \\
&= (q - q_f)(\tilde{b}_i + \tilde{a}_i) + p(\tilde{c}_i + \tilde{a}_i) - a_i(q + p - q_f) + \tilde{\gamma}_c\theta_c(b_i) + q_f(a(\tilde{b}_i, 0) + \tilde{b}_i) \\
&= (q - q_f)(\tilde{b}_i + \tilde{a}_i) + p(\tilde{c}_i + \tilde{a}_i) + \tilde{\gamma}_c\theta_c(b_i) + q_f(a(\tilde{b}_i, 0) + \tilde{b}_i) \\
&= q(\tilde{b}_i + \tilde{a}_i) + p(\tilde{c}_i + \tilde{a}_i) + \tilde{\gamma}_c\theta_c(b_i) \\
&= q(\tilde{b}_i + \tilde{a}_i) + p(\tilde{a}_i) + \tilde{\gamma}_c\theta_c(b_i) \\
&= q\tilde{b}_i + q_f\tilde{a}_i + \tilde{\gamma}_c\theta_c(b_i)
\end{aligned}$$

So we see the cost is strictly greater with $\theta_c > 0$, as it must be to have $\alpha(\theta_c) = 1$ provided that $\tilde{\gamma}_c > 0$. Even with $\tilde{\gamma}_c = 0$, it is still weakly better to have $\theta_c = 0$. □

Almost as a corollary of the preceding result, we next show that consumption allocations are the same conditional on matching in the CDS or bond markets.

Proposition 4. *Let investors be unconstrained. When the CDS-bond basis holds, consumption allocations are the same if matched in the CDS or the bond market. Moreover,*

$$-c_i(0) = b_i.$$

Proof of proposition 4. Let g_δ denote consumption in state δ.

Consider an investor who matches in the bond market. Since allocations are unconstrained, q pins down $q = q^* u'(g_0)$, and $q_f = q^* u'(g_0) + p^* u'(g_1) = q + p^* u'(g_1)$ so $p^* u'(g_1) = q_f - q$, with $q^* \equiv \beta(1 - \bar{\delta})$ and $p^* \equiv \beta\bar{\delta}$. Because of the CDS-bond basis, that's the same as $p^* u'(g_1) = p$, so consumption allocation for g_1 is the same as if matching in CDS. The converse, for an investor who matches in the CDS market, follows an analogous argument and results in the same g_0 as if matched in bonds. So the allocations are the same.

So we have that the allocation g_1 is the same if trading in bonds and CDS markets, and the same happens for g_0. So the allocation for the case with $\delta = 1$ is $a(b_i, 0) + 0 = a(0, c_i(0)) + c_i(0))$ and for the case with $\delta = 0$ the allocation is $a(b_i, 0) + b_i + w = a(0, c_i(0)) + w \implies a(b_i, 0) + b_i = a(0, c_i(0))$. Combining these two equations we have that $a(0, c_i(0)) + c_i(0)) + b_i = a(0, c_i(0)) \implies b_i = -c_i(0)$.

The prices of the portfolios must also be the same. So current and next period consumption allocations are the same. □

Since the consumption allocations are the same conditional on matching, the value of matching must be as well. This leads to a tight relationship between tightness in the two markets that is independent of the gains from trade (except through submarket choices). In particular, when choosing to match in the bond market, investors weigh the possibility that they fail to gain access through bonds and then can gain access through CDS. Increasing the fee paid in bonds increases

the probability of matching, which reduces the need to match in CDS markets in sub-period two. How optimal that is depends on the matching probability elasticity.

Proposition 5. *Suppose investors are unconstrained at their optimal choices. When the CDS-bond basis holds and θ choices are interior,*

$$\frac{\gamma_b - \pi}{\gamma_c - \pi} = \frac{\alpha'(\theta_b)}{\alpha'(\theta_c(0))}(1 + (\epsilon_{\alpha,\theta} - 1)\alpha(\theta_c(0))),$$

where $\epsilon_{\alpha,\theta} \leq 1$ is the elasticity of α evaluated at $\theta_c(0)$. When $\gamma_b = \gamma_c$, θ choices are interior, and α is strictly concave,

$$\theta_b < \theta_c(0).$$

Lemma 2. *For a weakly increasing and weakly (strictly) concave function $f : \mathbb{R}^+ \to \mathbb{R}$ having $f(0) = 0$, the elasticity of f is weakly (strictly) less than one.*

Proof of lemma 2. Consider strict concavity. Since $f(0) = 0$, $f(x) = \int_0^x f'(t)dt$. By concavity, f' is decreasing, and so $f(x) > \int_0^x f'(x)dt = xf'(x)$. Hence, $\frac{xf'(x)}{f(x)} < 1$, which says the elasticity of f is strictly less than one. The case of weak concavity obviously follows. □

Proof of proposition 5. An investor matching in the bond market gets $V(b_i; s_2) - qb_i$. And an investor matching in the CDS market, get $V(0, c_i(0); s_3) - pc_i(0)$. Proposition 4 shows those alternative matches result in the same consumption bundles. Hence, define $V^* \equiv V(0, c_i(0); s_3) - pc_i(0) = V(b_i; s_2) - qb_i$. Also, define the utility of being unmatched in either market as $\underline{V} = V(0, 0; s_3)$.

The optimal choice of θ_c satisfies

$$\tilde{\gamma}_c = \alpha'(\theta_c)(V^* - \underline{V}).$$

The optimal choice of θ_b satisfies

$$\begin{aligned}
\tilde{\gamma}_b &= \alpha'(\theta_b)(V^* - V(0; s_2)) \\
&= \alpha'(\theta_b)(V^* - [\alpha(\theta_c(0))V^* + (1 - \alpha(\theta_c(0)))\underline{V} - \tilde{\gamma}_c\theta_c(0)]) \\
&= \alpha'(\theta_b)((1 - \alpha(\theta_c(0)))(V^* - \underline{V}) + \tilde{\gamma}_c\theta_c(0))
\end{aligned}$$

Replacing $V^* - \underline{V}$ with $\tilde{\gamma}_c/\alpha'(\theta_c(0))$, we have

$$\begin{aligned}
\tilde{\gamma}_b &= \alpha'(\theta_b)\left\{(1 - \alpha(\theta_c(0)))\frac{\tilde{\gamma}_c}{\alpha'(\theta_c(0))} + \tilde{\gamma}_c\theta_c(0))\right\} \\
\Leftrightarrow \frac{\tilde{\gamma}_b}{\tilde{\gamma}_c} &= \frac{\alpha'(\theta_b)}{\alpha'(\theta_c(0))}\{1 - \alpha(\theta_c(0)) + \theta_c(0)\alpha'(\theta_c(0))\} \\
&= \frac{\alpha'(\theta_b)}{\alpha'(\theta_c(0))}\left\{1 - \alpha(\theta_c(0))\left[1 - \frac{\theta_c(0)\alpha'(\theta_c(0))}{\alpha(\theta_c)}\right]\right\} \\
&= \frac{\alpha'(\theta_b)}{\alpha'(\theta_c(0))}\left[1 + \alpha(\theta_c(0))(\epsilon_{\alpha,\theta} - 1)\right].
\end{aligned}$$

The observations for $\gamma_b = \gamma_c$ follow from the LHS being equal to 1, which imposes requirements on $\alpha'(\theta_b)/\alpha'(\theta_c(0))$ being greater, less than, or equal to one depending on $\epsilon_{\alpha,\theta}$. Then those $\alpha'(\theta_b)/\alpha'(\theta_c(0))$ requirements are mapped to θ_b orderings of θ_c noting that α'^{-1} is a decreasing function for $\alpha'' \leq 0$.

For the claim that $\epsilon_{\alpha,\theta} \leq 1$, note that the second order condition for $\theta_c(0)$ requires that $\alpha''(\theta_c) \leq 0$. Then concavity implies the elasticity is less than one.

If α is strictly concave, then the elasticity is strictly less than one, by Lemma 2. □

The result says investors pay lower fees in the bond market and face a lower market tightness than in the CDS market ceteris paribus. This is because investors can achieve exactly the same exposure and consumption allocation by trading in the CDS market or the bond market. At the time they are choosing the submarket to purchase bonds, investors know that if they fail they can still achieve the desired exposure by trading in the CDS market. However, investors who are trading at the CDS market are those that failed to trade bonds, and thus they are willing to pay relatively larger fees to trade because there are no further opportunities to do so after the CDS market closes.

The following proposition shows that the equilibrium q must be less than the risk-free price when there is no exogenous exposure, $\mu_\omega = 0$. The proof works by first establishing that at risk-neutral prices, investors and dealers want no exposure—they are risk-neutral at the margin, and if $\mu_\omega = 0$, then risk-neutral prices result in no trade. Without any trade, markets can't clear because no one is willing to hold bonds. So, the bond price must fall to induce trade and holding of bonds in equilibrium.

Proposition 6. *In equilibrium with $\mu_\omega = 0$, q is strictly less than the risk-neutral price.*

Lemma 3. *If $\mu_\omega = 0$, then when q and p are given by the risk-neutral prices (in partial equilibrium), investors and dealers hold zero bonds and CDS, and market tightnesses are zero.*

Proof of lemma 3. By definition, at risk-neutral prices, the CDS bond basis holds, since $\beta(1-\bar{\delta}) + \beta\bar{\delta} = \beta = q_f$. Let q^* denote the risk-neutral price of the bond, which is $\beta(1-\bar{\delta})$.

Let us guess the solution to the investor's problem has $a = u'^{-1}(1)$ and $b_i = c_i(b_i) = c_i(0) = \theta_b = \theta_c(b_i) = \theta_c(0) = 0$ is optimal. To confirm this, note that $q = V_b$ is satisfied:

$$q = \beta(1-\bar{\delta})u'(a+b) = q^* u'(u'^{-1}(1)) = q^*.$$

Likewise, $p = V_c$ holds

$$\begin{aligned} p &= \beta\bar{\delta}u'(a+c) \\ &= \beta\bar{\delta}u'(a) + \beta u'(a) - \beta u'(a) \\ &= -\beta(1-\bar{\delta})u'(a) + \beta u'(a) \\ &= -\beta(1-\bar{\delta})u'(u'^{-1}(1)) + \beta u'(u'^{-1}(1)) \\ &= -\beta(1-\bar{\delta}) + \beta \\ &= -q^* + q_f, \end{aligned}$$

which holds from the risk-neutral CDS-bond basis. Then, with $b = c = 0$ optimal, there are no gains from trade. Therefore $\theta_b = \theta_c = 0$ is optimal.

For the dealer's problem, it is likewise trivial to show that $b_d = c_d = 0$ is optimal. □

Proof of proposition 6. Suppose the CDS-bond basis holds. From Lemma 3, at $q = q^*$, investors and dealers both demand zero exposure. Moreover, an increase in q causes both dealer and investor's to hold fewer bonds. Therefore, to induce the bonds to be held in equilibrium, one must have $q < q^*$ to incentivize strictly positive bond demand. □

This reasoning also means that investors and dealers both want to have exposure, which immediately implies investor bond positions are positive and the short-selling constraint is not binding. It also implies that investors' CDS positions are negative, implying dealers own CDS protection from market clearing. But dealers also want exposure, which means they must own bonds too, implying the short-sale constraint cannot bind for them either.

Proposition 7. *In equilibrium with $\mu_\omega = 0$, the short-selling constraint on bonds is not binding.*

Proof of proposition 7. Note that at q given by the risk-neutral price, Lemma 3 gives optimal bond demand as zero for dealers and investors. In equilibrium, q must therefore be lower to induce someone to hold the bonds. Lower q thus increases demand for bonds by both dealers and investors. Therefore, in equilibrium, both dealers and investors with a match in the bond market demand a strictly positive amount of bonds and the short-selling constraint does not bind. □

The remaining results hinge on dealer entry. However, entry hinges not only on the matching function and entry costs, but also on gains from trade both from the investor and dealer perspective. Proposition 8 uses that $\underline{q}$ and $\overline{q}$ converge to the risk-neutral prices as entry costs shrink to zero, meaning the equilibrium price must converge to the risk-neutral one.

Proposition 8. *Assume 1 and 2. If the CDS-bond basis holds (such as when $\underline{b} = -\infty$ or when $\mu_\omega = 0$) and $\underline{x} < 0$, the model limit as γ_b goes to zero for fixed γ_b/γ_c has q given by the risk-neutral price.*

Lemma 4. *Assume 1 and 2. If the CDS-bond basis holds and $\underline{x} < 0$, then π is strictly quasi-convex, attains a minimum of zero when q is the risk-neutral price, and goes to ∞ as $q \downarrow 0$ or $q \uparrow \infty$.*

Proof of lemma 4. Let the risk-neutral prices for bonds and CDS be denoted $q^* = \beta\bar{\delta}$ and $p^* = \beta(1-\bar{\delta})$, and note $q^* + p^* = q_f$. Then the dealer profit problem can be written

$$\pi = \max_{a,b,c} -q_f a - qb - pc + q^* u(a+b) + p^* u(a+c)$$

$$\underline{b} - b \leq 0 \qquad (\theta_b)$$

$$c + \underline{x} - b \leq 0. \qquad (\theta_x)$$

$$a + b \geq 0$$

$$a + c \geq 0$$

Because the CDS-bond basis holds, and using the envelope theorem, the total derivative of π with respect to q is

$$\begin{aligned}\frac{d\pi}{dq} &= \frac{\partial \pi}{\partial q} - \frac{\partial p}{\partial q}\frac{\partial \pi}{\partial p} \\ &= -b + c.\end{aligned}$$

Totally differentiating again,

$$\frac{d^2\pi}{dq^2} = \frac{d(c-b)}{dq}.$$

The first order conditions (FOCs) of the problem are

$$\begin{aligned}-q + q^* u'(a+b) &= -\theta_b - \theta_x \\ -p + p^* u'(a+c) &= \theta_x \\ -q_f + q^* u'(a+b) + p^* u'(a+c) &= 0\end{aligned}$$

(where we ignore the last two feasibility constraints because of an Inada condition). Summing the first two of these, and using $q + p = q_f$, one has $0 = -\theta_b$. Consequently, the short-constraint on bonds cannot bind for dealers when the CDS-bond basis holds.

Solving for $a + c$ and $a + b$ from the second and first FOC, and then differencing, gives

$$c - b = u'^{-1}\left(\frac{p + \theta_x}{p^*}\right) - u'^{-1}\left(\frac{q - \theta_x}{q^*}\right). \tag{14}$$

Now consider if $\theta_x > 0$. In that case, $c - b = -\underline{x}$ and so $d(c-b)/dq = 0$ in that case. If $\theta_x = 0$, then totally differentiating gives

$$\begin{aligned}\frac{d(c-b)}{dq} &= \frac{1}{u'' \circ u^{-1}\left(\frac{p}{p^*}\right)}\frac{dp/p^*}{dq} - \frac{1}{u'' \circ u^{-1}\left(\frac{q}{q^*}\right)}\frac{dq/q^*}{dq} \\ &= -\frac{1}{u'' \circ u^{-1}\left(\frac{p}{p^*}\right)}\frac{1}{p^*} - \frac{1}{u'' \circ u^{-1}\left(\frac{q}{q^*}\right)}\frac{1}{q^*} \\ &> 0\end{aligned}$$

So,

$$\frac{d^2\pi}{dq^2} = \frac{d(c-b)}{dq} \geq 0.$$

This proves π is necessarily quasi-convex, but not strictly quasi-convex because there could be a flat spot (where $d\pi/dq = 0$ for a range of values).

To establish strict quasi-convexity of π, we will show that $d\pi/dq = 0$ at a single point, which is risk-neutral pricing. This will then give risk-neutral pricing as the minimizer of dealer profits.

With $\underline{x} < 0$, the case of $\theta_x > 0$ means $\frac{d\pi}{dq} = -b + c = -\underline{x} > 0$. So the function is strictly increasing when the constraint is binding, and so not flat. With $\theta_x = 0$, $\frac{d\pi}{dq} = -b + c$ is only zero if $-b + c = 0$. Since $\frac{d(c-b)}{dq} > 0$ for $\theta_x = 0$, that can occur only at one point, which is where $c = b$. For $\theta_x = 0$ and $c = b$, (14) requires

$$\frac{p}{p^*} = \frac{q}{q^*} \Leftrightarrow \frac{q_f - q}{q_f - q^*} = \frac{q}{q^*} \Leftrightarrow q = q^*.$$

That is, only at risk neutral prices does the sovereign demand zero exposure. Since this is where $d\pi/dq = 0$ and the function is strictly quasi-convex, this point must be the profit minimizer.

For q less than risk-neutral, consider the portfolio $a = 1$ and $c = 0$ with b such that qb is a positive constant. This plan is feasible. And as q goes to 0, b goes infinite, as does $u(a + b)$. So, if u is unbounded above, π goes infinite.

Similarly, if there is no exposure constraint ($\underline{x} = -\infty$), then a similar argument with $a = 1, b = 0$ and pc constant gives π infinite as q goes to q_f, implying p goes to 0.

When there is an exposure constraint that binds for $q \in (q^*, q_f)$, the exposure constraint also binds at all higher q. And in this region profit increases linearly at a constant rate $-\underline{x} > 0$ since we already established $\frac{d\pi}{dq} = -b + c = -\underline{x} > 0$. So as q goes infinite, so does π.

□

Proof of proposition 8. In any equilibrium, the net entry costs must be non-negative, $\tilde{\gamma}_b, \tilde{\gamma}_c \geq 0$. As γ_b and γ_c go to zero, this requires that π goes to zero.

By lemma 4, profits π are strictly quasi-convex and attain a minimum of zero at the risk-neutral

prices. Each level of $\pi = \min\{\gamma_b, \gamma_c\}$ is associated with two values of q, say $\underline{q} \leq \overline{q}$ corresponding to $\min\{\tilde{\gamma}_b, \tilde{\gamma}_c\} = 0$. The equilibrium q must be in $[\underline{q}, \overline{q}]$. Both of these bounds move continuously towards the risk-neutral price.

□

B Data Description

We construct a database containing information about bond and CDS prices, bid/ask spreads for bonds and CDS, CDS and bond holdings of dealers, and standard aggregate macro variables for Argentina. In this appendix, we describe the data sources and some additional details.

B.1 Data Sources

We obtain a given bank's CDS position on a sovereign's CDS via regulatory data from the Depository Trust and Clearing House Corporation (DTCC). The Dodd-Frank Wall Street Reform and Consumer Protection Act (2010) requires real-time reporting of all swap contracts to a registered swap data repository (SDR), which the DTCC operates in the CDS market. The Dodd-Frank Act also requires SDRs to make all reported data available to appropriate prudential regulators.[22] As a prudential regulator, the Federal Reserve has access to the transactions and positions involving individual parties, counter-parties, or reference entities that are regulated by the Federal Reserve.

The DTCC data contains every US-regulated CDS trade. We drop any trades where the reference entity is not a country's government. This means that in addition to dropping any CDS trades where the reference entity is a non-governmental organization, we also remove CDS trades with city or state level reference entities.

In addition to CDS Positions, we calculate the quarter-end net sovereign bond exposure (CDS and bond holdings) of U.S.-headquartered banks classified as dealers in the DTCC database. We obtain this information from the banks' FR Y-14Q regulatory filings as part of the Federal Reserve's Capital Assessments and Stress Testing information collection.

Our definition of dealers in the data consist of CDS dealers as classified by the DTCC and banks for whom we have data in Y14Q. Ultimately, this gives us a list of five banks. In our sample, every dealer actively trades CDS in every quarter.

We obtain bond prices and CDS prices from Bloomberg.

B.2 CDS Positions

We create a CDS position for each bank at each point in time. We observe the initial position for each bank as of January 1st, 2010. Every time a bank buys (sells) protection in the entity during the month, its position increases (decreases). We also subtract any expiring contracts from dealer's position in that reference entity. Thus, we assume a dealer d has CDS position at time t relative to the end of the previous period $(t-1)$ as follows:

$$Position_{dt} = Position_{d(t-1)} + CDSBought_{dt} - CDSSold_{dt} - ExpiredContracts_{dt}.$$

After calculating the position for each bank, we aggregate the positions of dealers. The volume measures likewise reflect aggregate volumes.

[22] See Sections 727 and 728 of The Dodd-Frank Wall Street Reform and Consumer Protection Act.

B.3 Bond and exposure position of dealers

In Y14Q, we have the notional quarter-end Sovereign Debt Securities and CDS net exposure to Argentinean sovereign debt as reported on the Securities Main and Hedging schedule and Trading Sovereign schedule. Combining this bonds-less-net-CDS protection information with the quarter-end net position on CDS we get from DTCC allows us to infer the quarter-end bond position of dealers.

B.4 CDS and Bond Prices

CDS and bond price data was collected from Bloomberg by downloading data for generic 5-year CDS and bond. The 5 year CDS was chosen because it is the most commonly traded CDS contract. We also collected generic 5-year bond yield data from Bloomberg.

C CDS-bond basis deviation measurement

To measure the CDS-bond basis deviations, we use the Z-spread approach, consistent with our data that comes from Gilchrist et al. (2022).[23] This compares the CDS running spread with the usual spread from sovereign debt. We now describe the measurement of these in the model and the data.

The running spread is the endogenous coupon s^{cds} amount paid at predetermined intervals such that—assuming a constant Poisson arrival rate λ for default and some recovery rate in the case of default—the net present value of the CDS contract is zero. In the model, default intensity λ is given implicitly by the solution to

$$\underbrace{\frac{\lambda}{\rho+\lambda}(1-e^{-(\rho+\lambda)/4})}_{\text{“Floating leg” value } \int_0^{1/4} e^{-\rho\tau}\lambda e^{-\lambda\tau} d\tau} = \underbrace{F}_{\text{“Fixed leg” value}}$$

where $e^{-\rho} = (1+r^*)^{-4}$ gives the discount rate ρ and F is the upfront payment per unit of notional. (The IDP associated with λ is $1-e^{-\lambda}$.) The running spread is then the s that solves

$$\frac{\lambda}{\rho+\lambda}(1-e^{-(\rho+\lambda)/4}) = \underbrace{se^{-(\rho+\lambda)/4}}_{\text{Expected coupon value}}$$

$$\Leftrightarrow s = \frac{\lambda}{\rho+\lambda}(e^{(\rho+\lambda)/4}-1)$$

The annualized spread is $s^{cds} = 4s$. Small default rates and discount rates imply $s^{cds} \approx 1-(1-\mathbb{E}[d])^4$, i.e., the running spread is approximately the default rate. However, it should be kept in mind this is an approximation. In particular, as λ goes large, s^{cds} can become very large.

The Z-spread is the usual spread in sovereign debt. Specifically, it is the constant Z such that the net present value of the bond, discounted by $1+r^*+Z$, is zero. Annualized then, this spread is

$$s^{bond} = (1/q)^4 - (1+r^*)^4.$$

For small default rates, $s^{bond} \approx (1+r^*)^4\mathbb{E}[d]^4$.

[23] We thank the authors for providing us with their CDS-bond basis deviations data.

The CDS bond basis deviation is defined as

$$s^{cds} - s^{bond}.$$

Consequently, for small default rates and small risk-free rates, the deviations should be roughly $4r^*$ times the annual default probability, or just a few basis points. In our model, the deviations do not occur simply because the approximations fail to hold but also because the fees investors pay in CDS and bond markets break the no-arbitrage relationship.

D Additional quantitative results

This section provides additional quantitative results.

D.1 Naked CDS ban OTC equilibrium behavior

Figure 15 is identical to Figure 8 but with a naked CDS ban.

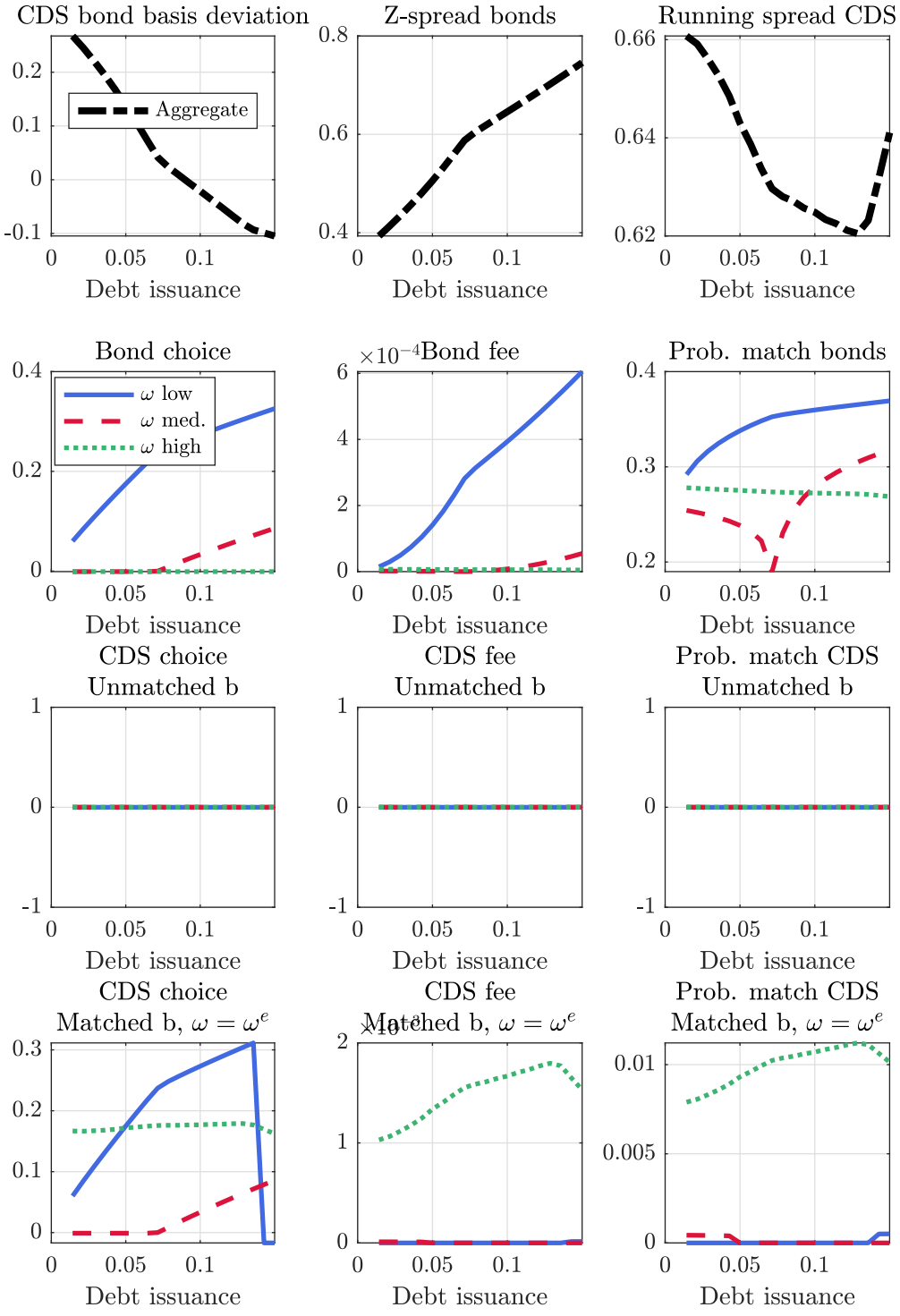
D.2 More welfare analysis

Figure 16 reports welfare just for the naked CDS and CDS bans. Figure 17 gives the welfare breakdown when the timing convention is reversed.

D.3 Unsmoothed versions of figures

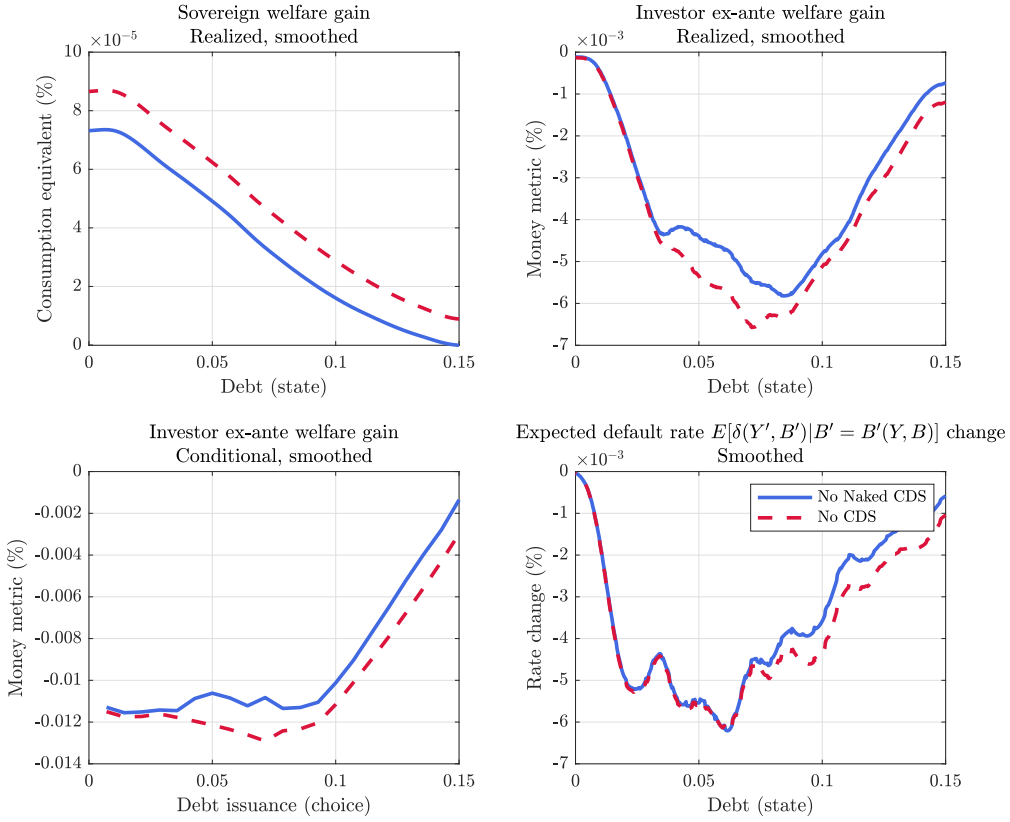
The unsmoothed versions of Figures 7, 13, and 14 are in Figures 18, 19, and 20, respectively. Unsmoothed versions of Figures 9, 10, and 11 are in the top, middle, and bottom panel of Figure 21, respectively. The unsmoothed version of Figure 12 appears in Figure 22.

Figure 15: Investor behavior with a naked CDS ban



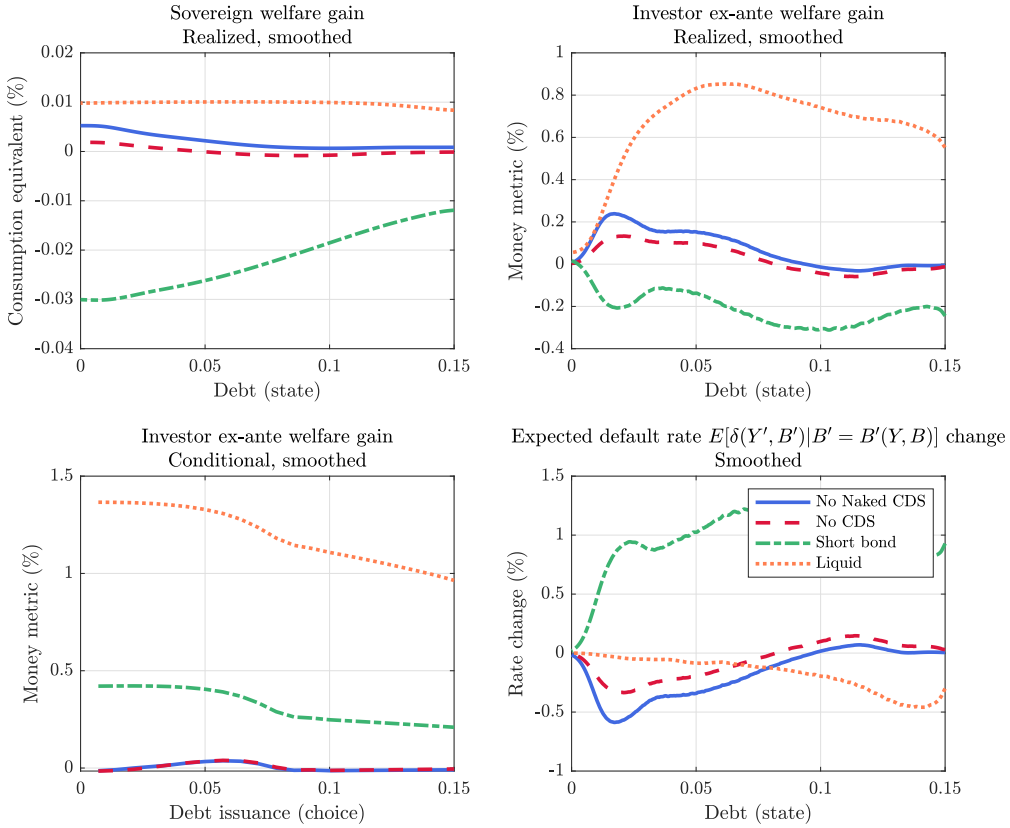
Note: plotted for a default rate of 8.7%; "Unmatched b" ("Matched b") means investors who did not (did) match with a dealer in the bond market.

Figure 16: Welfare analysis (just naked CDS and no CDS cases)



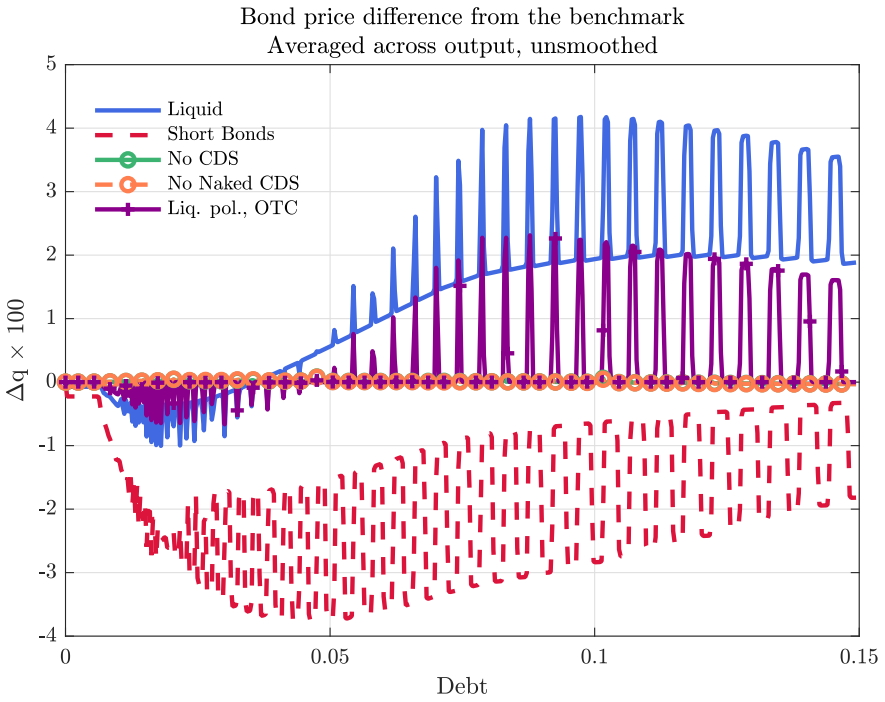
Note: panels labeled "Realized" have been averaged using the invariant distribution and then smoothed; the panel labeled "Conditional" has had a numerical average taken across a grid of default rates; all series have been smoothed, the appendix reports the non-smoothed values.

Figure 17: Welfare with reverse Bond/CDS timing



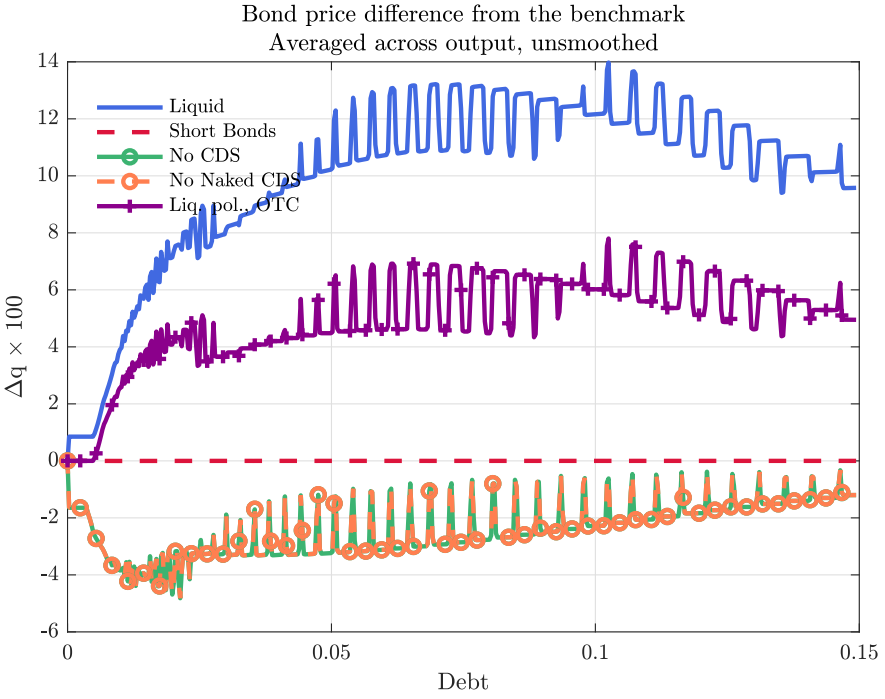
Note: panels labeled "averaged across output" have been averaged using the invariant distribution and then smoothed; panels labeled "averaged across default rates" have had a numerical average taken across a grid of default rates; the appendix reports the non-smoothed values.

Figure 18: Price schedule differences under alternative policies (unsmoothed)



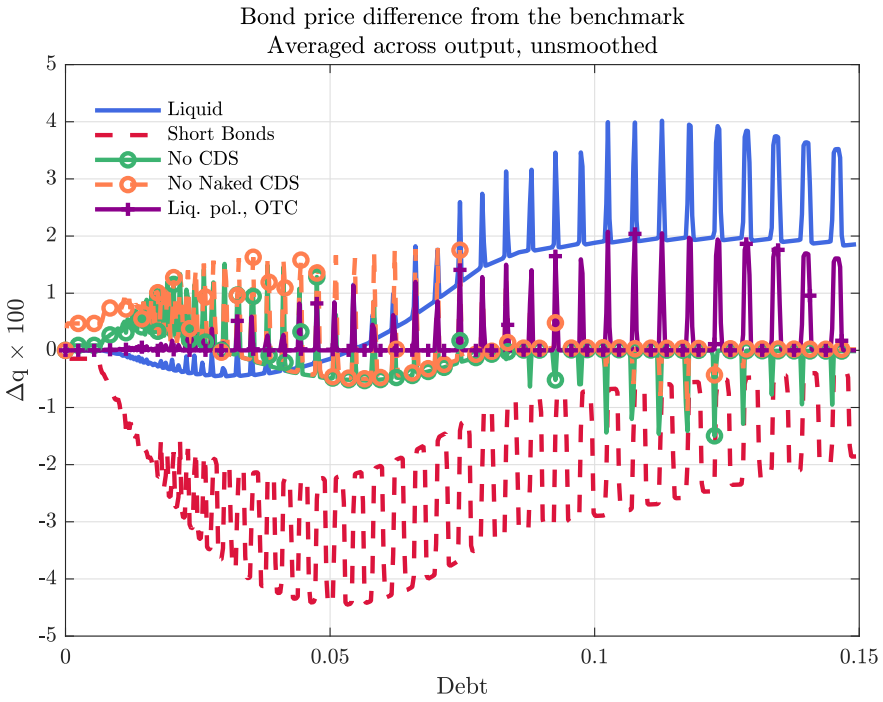
Note: bond schedules are averaged across GDP using the invariant distribution.

Figure 19: Price schedule differences for an elasticity parameter of $\xi_b = 0.7$ (unsmoothed)



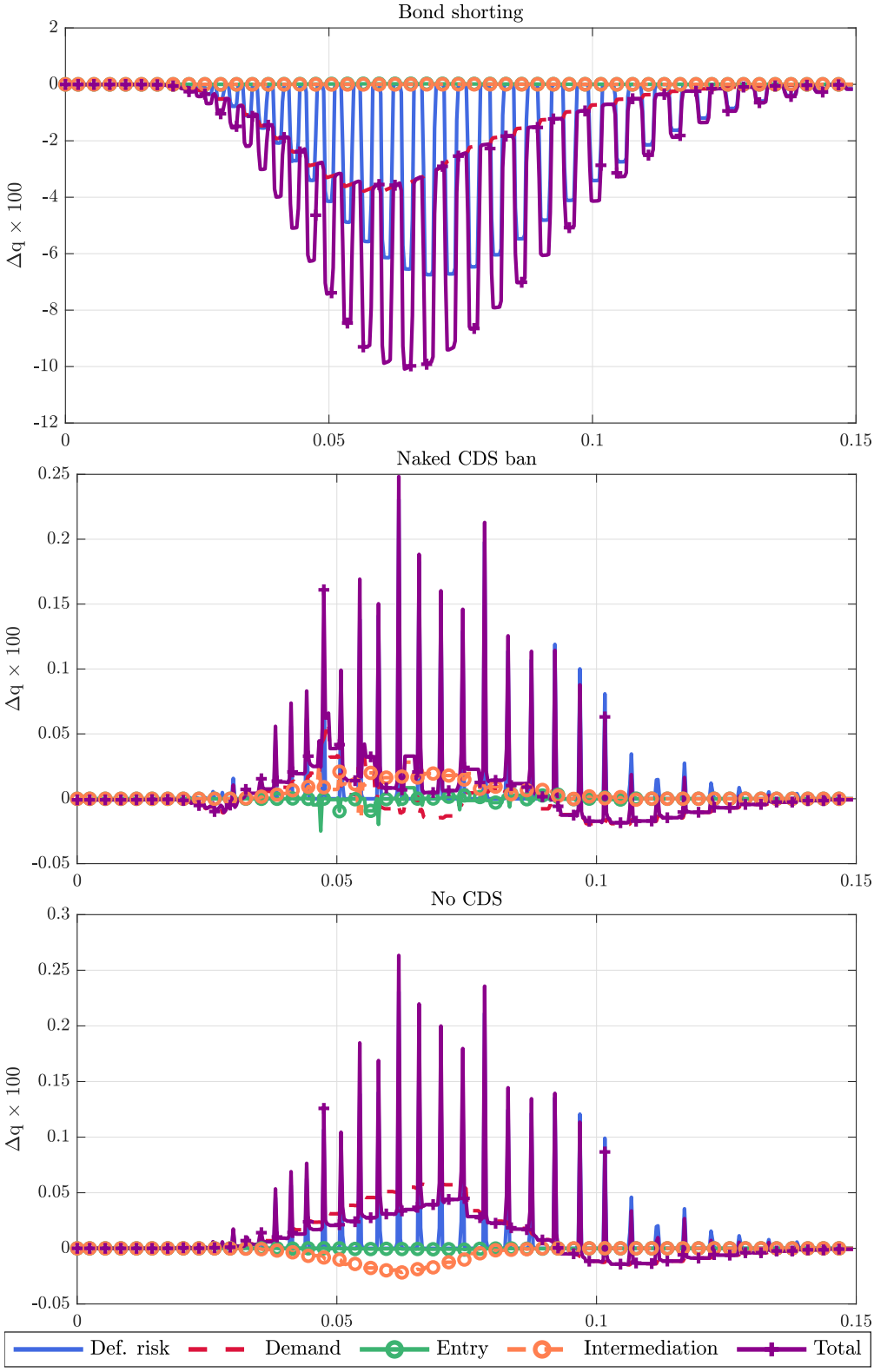
Note: bond schedules are averaged across GDP using the invariant distribution.

Figure 20: Price schedule differences under reversed timing (unsmoothed)



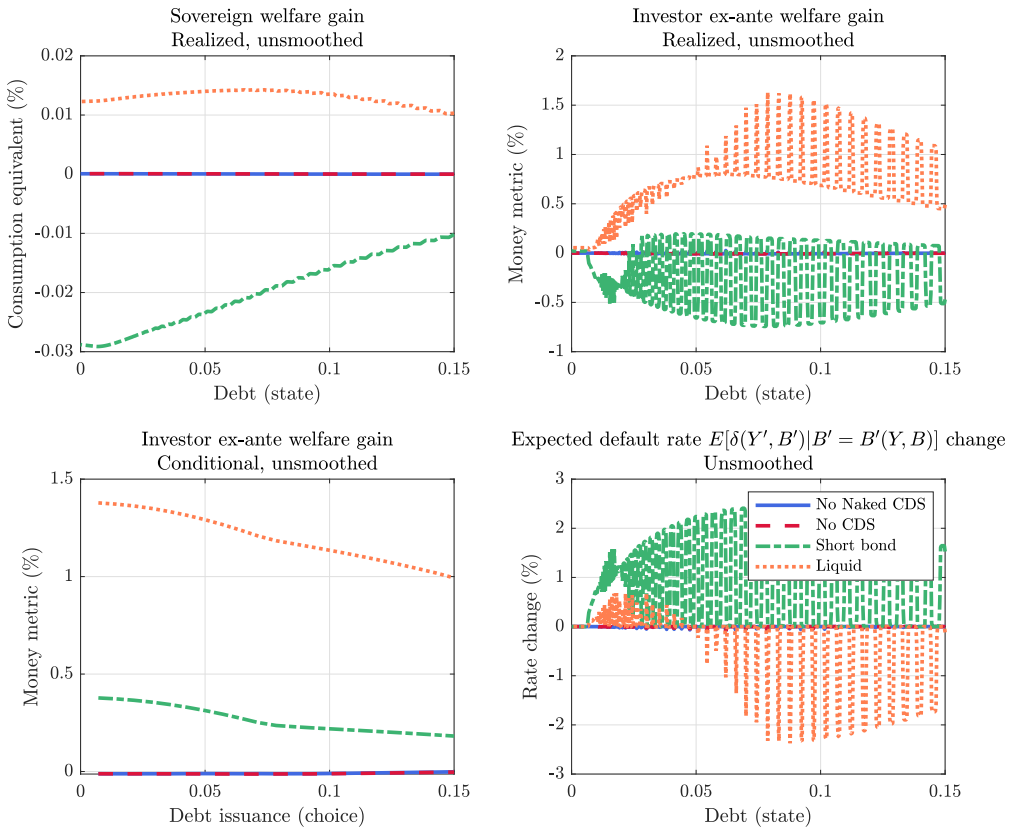
Note: bond schedules are averaged across GDP using the invariant distribution.

Figure 21: Decompositions (unsmoothed)



Note: bond schedules are averaged across GDP using the 40th-60th percentile conditional distribution.

Figure 22: Welfare evaluations (unsmoothed)



Note: the top panels and the bottom right panel are functions of (Y, B) and have been averaged using the invariant distribution and then smoothed; the panel labeled "Conditional" is a function of $B', \bar{\delta}$ and has had a numerical average taken across a grid of default rates.