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Idea Diffusion and Property Rights

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Boyan Jovanovic
New York University

Zhu Wang
Federal Reserve Bank of Richmond

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Idea Diffusion and Property Rights*

Boyan Jovanovic† and Zhu Wang‡

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Abstract

We study the innovation and diffusion of technology at the industry level. We derive the full dynamic paths of an industry's evolution, from birth to its maturity, and we characterize the impact of diffusion on the incentive to innovate. The model implies that protection of innovators should be only partial due to the congestion externality in meetings in which idea transfers take place. We fit the model to the early experiences of the automobile and personal computer industries both of which show an S-shaped growth of the number of firms.

1 Introduction

Innovation and diffusion are two fundamental drivers for technological progress and long-run growth. On the one hand, a technological innovation cannot fulfill its potential without being widely adopted, but on the other hand, rapid diffusion may reduce the incentive to innovate. In this paper, we study the interdependence between innovation and diffusion in an industry setting, and discuss welfare and policy implications.

The model features an industry in which there is a fixed downward sloping demand curve for a homogeneous product. Production requires the use of an innovation referred to as "the idea." At the outset, a group of homogeneous measure-zero agents – "firms" – decide whether to pay a sunk cost and innovate immediately or to wait and imitate later. The idea enables a firm to produce a given quantity at zero cost. As more imitators arrive and get the idea, price gradually falls and so does the value of obtaining the idea.

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†Department of Economics, New York University. Email: bj2@nyu.edu.

‡Research Department, Federal Reserve Bank of Richmond. Email: zhu.wang@rich.frb.org. The views expressed are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

Imitation occurs as ideas are copied in random pairwise meetings between those that have the idea already and those that do not. Imitation imposes no direct costs, but the imitator may have to pay a fee to the idea seller. A bargaining parameter determines the share of the idea's value that its seller receives and in shaping incentives it plays a role similar to an entry cost.

We study two regimes affecting the payment for ideas, in contrast to the benchmark where ideas are copied for free. Under regime 1, imitators *cannot* resell ideas to other imitators. A potential adopter can copy an imitator but the fee goes to an original innovator and not to the imitator – this is a scenario often seen in the world of patent licensing or franchising. Under regime 2, by contrast, imitators *can* resell ideas to other imitators and keep the proceeds, arguably leading to a faster diffusion rate. This is a scenario that sometimes arises in the cases of technology transfer or employee spin-offs. If bargaining allocates all rents to idea sellers, then under both regimes the initial innovators capture all the discounted rents from their innovation: Under regime 1 they collect all the remaining discounted rents directly from each and every imitator, and under regime 2 they capture those rents also indirectly by pricing in the resale values in their direct sales.

Our analysis yields several key findings. First, our model characterizes the growth path of a new industry and the impact of diffusion on the incentive to innovate. The results are shown to depend on the regimes (i.e., on whether imitators can resell the innovation) and on how much the innovators are compensated for their ideas (through their bargaining share).

Second, we find that the socially optimal bargain allocation depends on the diffusion process, particularly the meeting rate between idea holders and potential adopters. In a setting where the meeting rate is fixed and as a result the growth in the number of producers is constant – which we refer to as a small industry model – it is socially optimal to allocate all rents to idea sellers. However, in an alternative setting where the meeting rate is endogenous and leads to a logistic diffusion process – which we refer to as a large industry model – maximal compensation for ideas would lead to too many innovators and too few imitators. This is because of the congestion externality in meetings between innovators and imitators that innovators ignore, so that the social planner would prefer less innovation and more imitation. The distinction between the small industry model and the large industry model lies in the number of potential adopters, and we show the former is the limiting case of the latter as the number of potential adopters gets large.

Our analysis also sheds light on the debate about policy interventions on diffusion. Our findings suggest that a policy reducing the speed of diffusion encourages innovation and raises initial capacity, but that it lowers imitation and leads to a slower growth of capacity. We argue that different takes on such a policy may explain the geographic pattern of industry development (e.g., Route 128 being overtaken by the Silicon Valley), and accommodating diffusion can raise welfare.

In the large industry model we find that initial innovators need less than full protection of the revenues that their innovations generate. If the patent system guarantees that the rents that an innovation generates should all go to the original innovators, then that protection should be less than perfect. Our analysis is at the industry level, but a similar result is also found in some recent papers relating innovation and its diffusion to aggregate growth. For example, Benhabib, Perla and Tonetti (2019) show that the licensing income received by innovators becomes highly elastic with regard to the license price when innovators' bargaining power is too strong. This may lead to less licensing income which lowers the return to innovation and the aggregate growth rate. Hopenhayn and Shi (2020) add matching congestion to the analysis and show that the growth-maximizing bargaining share of innovators is also sensitive to the parameters in the matching function which they assume has constant returns.

Our analysis complements this line of macro-diffusion research in several aspects: First, unlike the existing literature focusing on the steady-state growth rate of the economy, we solve for full dynamic paths of an industry's evolution. Second, we consider two policy-relevant regimes where imitators can or cannot resell ideas. We also specify a meeting process between idea holders and potential adopters that generates the well-known logistic diffusion curves documented in the technology diffusion literature. Such a meeting process has been commonly used for studying diffusion of innovation (see Young (2009) for a review), and it corresponds to a quadratic matching function discussed recently by Lauermann, Nöldeke, and Tröger (2020). Finally, we apply our theory to match the early development of the automobile and personal computer industries, and discuss policy and welfare implications.

In our model, innovation yields a payoff that depends partly on the use of the idea in production, and partly on the value it yields when it is sold. The latter occurs in bilateral meetings and so our model relates to models in which agents search for a production partner after one has invested, such as Burdett and Coles (2001), Mailath, Samuelson, and Shaked (2000) and Nöldeke and Samuelson (2015). In these models, payoffs in a match will depend on their investments and this affects investment incentives.

In the model, owners of ideas use them to compete in the product market and thus the flow value of the idea depends on how many others are using it. Manea (2019) also assumes ideas are sold in bilateral meetings and uses bargaining to allocate rents, but in his model the flow value of an idea to its user does not depend on how many others have it.

We fit the model to the expansion path of the number of firms and of output in the automobile and the personal computer industries, both of which show the familiar S-shaped growth in the number of producers in the period before the shakeout. We thus add to the strand of literature on industry life cycles – e.g., Gort and Klepper (1982), Utterback and Suarez (1993), Jovanovic & MacDonald (1994), Klepper (1996), Filson (2001), and Wang (2017). Existing studies mainly focus on explaining the shakeout of firms, while our study explains the expansion of firm numbers prior to the shakeout.

Regarding empirical findings, in the benchmark calibration of the automobile industry we find that the socially optimal compensation share for innovators is 7 percent if imitators cannot resell ideas, or 16 percent if they can. Alternatively, the social optimum can be achieved by subsidizing 60 percent of the innovation cost. We then re-calibrate the model to the automobile industry by assuming a larger pool of potential adopters, and we also apply the model to the personal computer industry. The results are qualitatively consistent and our analysis explains the quantitative variation found across different cases.

The basic model is presented next in Section 2. Section 3 applies it to a small industry to which potential imitators arrive at a constant rate, while Section 4 deals with a large industry where the meeting rate between incumbents and outsiders depends on their numbers. Section 5 fits the model to data from the automobile and personal computer industries, and Section 6 concludes. More technical arguments are in the Appendix.

2 Model

Consider a competitive market environment in continuous time. At date 0, a measure k_0 of agents, who we call "innovators," each invest an amount c in an innovation that results in a new good. After that, the innovation spreads to other potential producers. At any date t, the measure of agents who have adopted the innovation and whom we call "incumbents" is k_t, and other agents are "outsiders." Diffusion occurs through meetings between incumbents and outsiders in which an outsider learns and imitates the innovation. At any date $t > 0$, the k_t measure of incumbents consists of both innovators and imitators.

Each incumbent produces one unit of output at zero cost, so the total output of the new good is k_t at date t. The inverse demand function for the new good is

$$p_t = Ak_t^{-\beta}, \tag{1}$$

where A is a market size parameter and $\beta > 0$ is the inverse demand elasticity. We normalize an outsider's earnings to zero.

An incoming imitator may need to pay a fee to an incumbent for transferring the innovation. We assume the transfer fee is

$$q_t = \alpha\omega_t, \tag{2}$$

where ω_t is the value of becoming an imitator at date $t > 0$, and $\alpha \in [0, 1]$ is the bargaining share of the idea seller. We shall model two regimes. Under the first one, while an imitator may have learned about the innovation from any incumbent, she has to pay an *original* innovator for officially transferring the innovation (e.g., a franchising or patent licensing fee). In other words, an imitator is *not* entitled to resell the innovation. Under the second regime, an incoming imitator can pay any incumbent, either an innovator or an imitator, for transferring the innovation, a scenario arising in some cases of technology transfer or employee spin-offs.

If each meeting results in a new incumbent, the number of meetings at date t is the measure of new adopters, denoted by dk_t/dt. As k_t rises, output of the new good rises and its price falls. At $t = 0$ an agent would take into consideration the time path of future revenues (i.e., from selling the good and earning transfer fees) to decide whether investing c to become an innovator or taking the option value of being a future imitator. We will show, at equilibrium, innovators enter only at date 0; no outsider would want to independently invent the innovation after that.

3 A small industry

We first consider a diffusion process that results in incumbents meeting potential imitators at a constant rate. This process may fit an emerging industry that is small compared with the rest of the economy, and for which the pool of potential entrants is large. Formally, the incumbents meet potential imitators at a constant rate λ and if each meeting results in a new producer, the number of incumbents grows at the rate λ so that

$$\frac{dk_t}{dt} = \lambda k_t. \tag{3}$$

Conditional on k_0, this implies that at date t industry output and the number of incumbents is

$$k_t = k_0 e^{\lambda t}. \tag{4}$$

One interpretation of this process is that at each date t, an incumbent has a chance λ of meeting an outsider to disseminate the innovation. We will show later that this emerges as a limiting case of the large industry model of Section 4 when the number of potential adopters goes to infinity. A second interpretation is that each incumbent has a chance λ to generate a spin-off, that is, an employee who leaves the employer and starts a new firm in the same industry.

3.1 Equilibrium

We now characterize equilibrium under two regimes:

1. Imitators cannot resell ideas to other adopters. A potential adopter can copy an incumbent imitator but the fee goes to an original innovator.

2. Imitators can resell ideas to other imitators and keep the proceeds.

Each meeting results in the idea being transferred so that Eq. (4) holds in both scenarios. But agents' revenues differ.

3.1.1 Imitators cannot resell ideas

If an imitator cannot resell the idea, its only revenue comes from selling the good, and its value ω_t satisfies the Hamilton–Jacobi–Bellman (HJB) equation

$$r\omega_t = p_t + \frac{d\omega_t}{dt} = A(k_0 e^{\lambda t})^{-\beta} + \frac{d\omega_t}{dt}, \tag{5}$$

where r is the interest rate. The ordinary differential equation (ODE) has the unique bounded solution

$$\omega_t = \frac{Ak_0^{-\beta}}{r+\beta\lambda} e^{-\beta\lambda t}, \tag{6}$$

which decreases at the rate $\beta\lambda$, i.e., it declines faster if the demand curve is less price elastic or if output grows faster.

Let v_t be the value of being an innovator at date t. An innovator receives revenues from selling both the good and the idea. The number of ideas sold at t is λk_t and the total date-t revenue from these sales, $\lambda k_t \alpha \omega_t$ is divided among the k_0 innovators. Thus v_t follows the HJB equation

$$rv_t = p_t + \frac{\lambda k_t}{k_0}\alpha\omega_t + \frac{dv_t}{dt} = A(k_0 e^{\lambda t})^{-\beta} + \frac{\alpha\lambda A k_0^{-\beta} e^{(1-\beta)\lambda t}}{r+\beta\lambda} + \frac{dv_t}{dt}. \tag{7}$$

Unless $\alpha = 0$, innovators receive a fraction of revenues from idea sales, we shall need to restrict the elasticity of demand to be below unity which means that $\beta \geq 1$. Imposing the boundary condition $v_{t\to\infty} < \infty$ yields the unique solution to Eq. (7):

$$v_t = \frac{Ak_0^{-\beta}e^{-\beta\lambda t}}{r+\beta\lambda} + \frac{\alpha\lambda A k_0^{-\beta}}{(r+\beta\lambda)(r+(\beta-1)\lambda)} e^{-(\beta-1)\lambda t}. \tag{8}$$

Solving for k_0.—Denote by u_t the option value of becoming a future imitator. At $t = 0$, the free entry condition reads

$$v_0 - u_0 = c. \tag{9}$$

The pool of outsiders being infinite, an outsider's chance of meeting an incumbent is zero so that $u_t = 0$ for all t, implying that $v_0 = c$. Since v_t decreases over time, no one would pay c to become an innovator at any $t > 0$. Combining $v_0 = c$ with Eq. (8) yields

$$v_0 = \frac{Ak_0^{-\beta}}{r+\beta\lambda} + \frac{\alpha\lambda A k_0^{-\beta}}{(r+\beta\lambda)(r+(\beta-1)\lambda)} = c. \tag{10}$$

Let k_0^{NOT} denote the entry of innovators that solves Eq. (10). Then

$$k_0^{NOT} = \left(\frac{A(r+(\beta-1)\lambda+\alpha\lambda)}{c\,(r+\beta\lambda)(r+(\beta-1)\lambda)}\right)^{1/\beta}, \tag{11}$$

which is valid as long as $\beta \geq 1$, and for all $\beta > 0$ if $\alpha = 0$.

3.1.2 Imitators can resell ideas

If imitators can resell the innovation, all the incumbents (be they innovators or imitators) share the same value v_t. The revenue from an idea sale is αv_t instead of $\alpha \omega_t$ and the total date-t revenue from these sales, $\lambda k_t \alpha v_t$, is shared equally among all the incumbents. Then v_t follows the HJB equation

$$r v_t = p_t + \lambda \alpha v_t + \frac{dv_t}{dt} = A(k_0 e^{\lambda t})^{-\beta} + \lambda \alpha v_t + \frac{dv_t}{dt}. \tag{12}$$

No restriction on β is needed here because innovators get no revenue from ideas sold by others and v_t must decline with t. Imposing the boundary condition $v_{t\to\infty} < \infty$ yields the solution

$$v_t = \frac{A k_0^{-\beta} e^{-\beta\lambda t}}{r + (\beta - \alpha)\lambda}, \tag{13}$$

which is unique if $r > \alpha\lambda$.

Solving for k_0.—Since v_t decreases over time and, again, innovation occurs only at $t = 0$. At $t = 0$, we again have $v_0 = c$, thus

$$v_0 = \frac{A k_0^{-\beta}}{r + (\beta - \alpha)\lambda} = c. \tag{14}$$

Let k_0^{CAN} denote the entry of innovators that solves Eq. (14). Then

$$k_0^{CAN} = \left(\frac{A}{c(r + (\beta - \alpha)\lambda)} \right)^{\frac{1}{\beta}}. \tag{15}$$

3.2 Implications of the small industry model

We assume $\beta \geq 1$ throughout this section so that the solutions for v_t^{NOT} and k_0^{NOT} always exist. Several implications then follow from Eqs. (11) and (15). We begin with

Proposition 1 *k_0^{NOT} and k_0^{CAN} both increase with A and α, and both decrease with c; moreover,*

$$\left(\frac{k_0^{NOT}}{k_0^{CAN}} \right)^{\beta} = 1 + \frac{\alpha(1-\alpha)\lambda^2}{(r + \beta\lambda)(r + (\beta - 1)\lambda)} = \begin{cases} 1 & \text{for } \alpha \in \{0, 1\} \\ > 1 & \text{for } \alpha \in (0, 1) \end{cases}. \tag{16}$$

The effect of α.—Eq. (16) states that the ratio k_0^{NOT}/k_0^{CAN} is inverted-U shaped in α and symmetric around $\alpha = 1/2$. But under free spillover ($\alpha = 0$) or full rent extraction ($\alpha = 1$), $k_0^{NOT} = k_0^{CAN}$. In the latter extreme ($\alpha = 1$), the innovators capture the entire discounted value of industry output.

The effect of λ.—The effect of diffusion speed on innovation depends on α and β. Specifically, in Eq. (15), k_0^{CAN} falls with λ if $\beta > \alpha$ because the effect of entry of competitors on reducing p_t (summarized by β) exceeds the benefit (summarized by α) that incumbents derive from selling the idea. The effect of λ on k_0^{CAN} turns positive if $\beta < \alpha$, and it vanishes if $\beta = \alpha$. Under the regime that imitators cannot resell (cf. Eq. (11)), k_0^{NOT} falls with λ as $\beta \geq 1 > \alpha$, and the effect of λ on k_0^{NOT} vanishes when $\beta = \alpha = 1$. These results are summarized in Proposition 2.

Proposition 2 *k_0^{NOT} decreases in λ for $\beta \geq 1 > \alpha$, while k_0^{CAN} decreases in λ for $\beta > \alpha$.*

Proof. Eq. (11) shows $dk_0^{NOT}/d\lambda < 0$ for $\beta \geq 1 > \alpha$, and $dk_0^{NOT}/d\lambda = 0$ for $\beta = \alpha = 1$. Eq. (15) shows $dk_0^{CAN}/d\lambda \lesseqqgtr 0$ if $\beta \gtreqqless \alpha$. ∎

The findings relate to several policy questions:

1. *Patent licensing.*—Here Eq. (11) is the relevant solution for k_0 because the initial innovators receive the licensing income. As our model illustrates, the investment on innovation highly depends on how innovators' intellectual property rights get protected (e.g., through the compensation share α and whether imitators can resell the innovation).

2. *Technology transfer via employee spin-offs.*—While our model does not include labor inputs in production, it is still relevant for explaining employee spin-offs if we interpret employees as people who have a chance to meet with innovators and learn about the innovation. For example, they could work in the same company but do not have to directly produce the new product. In this case, Eq. (15) is the relevant solution. Conventional wisdom is that spin-offs negatively affect innovation because founders of spin-off firms may copy innovations from their previous employers without pay. This is implied by Proposition 2 because in that case $dk_0^{CAN}/d\lambda < 0$ for $\beta > \alpha = 0$: The higher the spin-off rate, the fewer the entry of innovators.

3. *Restricting entry of imitators.*—In our model, if an incumbent firm cannot get sufficient compensation from the imitators, it would prefer a slower diffusion of the innovation (i.e., a lower λ). In reality, researchers and policymakers often debate whether restricting entry of imitators would be in the public interest**.** One such debate concerns whether non-compete contracts should be enforced or banned.[1] Recent research (e.g., Klepper, 2010; Samila and Sorenson, 2011; Cabral, Wang and Xu, 2018) shows that employee spin-offs lead to industry clusters and that banning non-compete contracts is an important contributing factor. According to Saxenian (1994) and Franco and Mitchell (2008), because California bans non-compete contracts while Massachusetts enforces them, Silicon Valley overtook Massachusetts' Route 128 in developing the high-tech industry.

[1] A non-compete contract requires that if an employee leaves the firm, she may not conduct business to compete against her previous employer for a period of time. Among others, Shi (2018) analyzes the effects of noncompete contracts which in her model restrict the mobility of managers and reduce welfare. In practice, the enforcement of non-compete contracts varies substantially across the 50 U.S. states (See Bishara, 2011).

Our model has a mechanism that could have led to such an overtaking pattern. Suppose that Route 128 and Silicon Valley each specialize in some high-tech subsectors, and the two locations face the same environment (same A, α, β and c) except that, because California bans non-compete contracts, diffusion would be faster so that λ is higher than in Massachusetts. Because Massachusetts enforces non-compete contracts, the Route 128 area would offer higher incentives to innovators and a resulting higher initial entry rate of firms than Silicon Valley in the early periods (a higher k_0). Then Silicon Valley's higher imitation rate would lead to its overtaking Route 128.[2]

The left and right panels of Fig. 1 show the data from Saxenian (1994) and our model simulation, respectively. In the simulation, we assume $\alpha = 0$ in two locations.[3] Eqs. (11) and (15) then suggest that $k_0 = k_0^{NOT} = k_0^{CAN} = \left(\frac{A}{c(r+\beta\lambda)}\right)^{\frac{1}{\beta}}$. We then assume $A/c = 3$, $\beta = 1$, $r = 0.05$, and plot k_t in two locations: One with a high diffusion rate ($\lambda = 0.08$) and another with a low diffusion rate ($\lambda = 0.04$). In each location,

$$k_t = k_0 e^{\lambda t} = \left(\frac{A}{c(r+\beta\lambda)}\right)^{\frac{1}{\beta}} e^{\lambda t}.$$

As a result, the location with a lower λ shows an early dominance in terms of firm entry, but its industry size later gets surpassed by the other location.

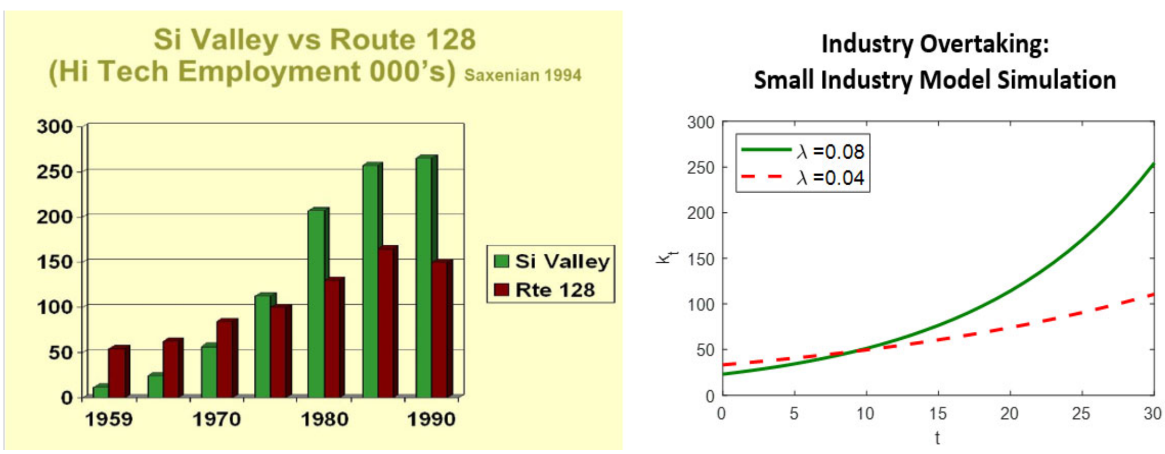

Fig. 1. Industry Overtaking: Data and Small Industry Model Simulation

[2] Enforcing non-compete contracts may also increase the bargaining share α of innovators. If that happens, the entry of initial innovators will be larger in the enforcing location and the timing of overtaking will be postponed compared with the case where both locations have the same value of α. Note that in the model, α does not affect the growth rate of k_t and the overtaking is ultimately driven by a larger diffusion rate.

[3] We could think in one location, non-compete contracts are banned so innovators do not receive any compensation from their employee spin-offs. In another location, non-compete contracts are strictly enforced and the bilateral negotiation to buy out those contracts is too costly for the parties involved, so that spin-off entrants are largely blocked. As a result, both locations have $\alpha = 0$ at equilibrium but the diffusion speed λ differs.

3.3 Welfare analysis for a small industry

We now consider the welfare problem of maximizing the discounted consumer plus producer surplus. For $\beta \in (0,1)$, aggregate utility at output k is

$$U(k) = \int_0^k A s^{-\beta} ds = \frac{A}{1-\beta} k^{1-\beta}. \tag{17}$$

Now for $\beta \geq 1$ the above integral is infinite; to ensure consumer surplus is finite, we put a maximum, $A\varepsilon^{-\beta}$, on the willingness to pay. Let $D(k) = \min\left(A\varepsilon^{-\beta}, As^{-\beta}\right)$ and define aggregate utility as $U(k) = \int_0^k D(s)\,ds = A\left(\int_0^\varepsilon \varepsilon^{-\beta} ds + \int_\varepsilon^k s^{-\beta} ds\right)$. Accordingly, for $\beta = 1$ we have

$$U(k) = A(\ln k + 1 - \ln \varepsilon), \tag{18}$$

and for $\beta > 1$, we have

$$U(k) = \frac{\beta A}{\beta - 1} \varepsilon^{1-\beta} + \frac{A}{1-\beta} k^{1-\beta}. \tag{19}$$

3.3.1 Optimal compensation share

Taking λ as given, the social planner chooses k_0 to maximize social surplus

$$W_0 = -ck_0 + \int_0^\infty e^{-rt} U(k_t)\,dt.$$

Let k_0^{SOC} denote the socially optimal value. With Eqs. (17), (18) and (19), we derive the first-order condition at any value of β that

$$k_0^{SOC} = \left(\frac{A}{(r - (1-\beta)\lambda)c}\right)^{\frac{1}{\beta}}. \tag{20}$$

Proposition 3 *In the small industry model, the socially optimal compensation share is $\alpha = 1$ for the innovators.*

Proof. Comparing (20) to the expressions in Eqs. (11) and (15), $k_0^{SOC} = k_0^{CAN} = k_0^{NOT}$ iff $\alpha = 1$, The first equality holds for all $\beta > 0$, and the second holds when $\beta \geq 1$. ■

Thus, allowing the original innovators to extract all the rents from succeeding imitators would yield the socially optimal incentive for innovation. This result holds regardless whether imitators can or cannot resell the innovation. We shall return to this question in Section 4 in which the meeting rate becomes endogenous.

3.3.2 Optimal innovation subsidy

Given a compensation share α, the planner can offer innovators a subsidy s to achieve the social optimum. The net entry cost of innovators becomes $c-s$. Under the regime that imitators cannot resell ideas, Eqs. (11) and (20) pin down the optimal subsidy s^{NOT}, so that

$$c\left[r+(\beta-1)\lambda+\alpha\lambda\right]=\left(c-s^{NOT}\right)\left(r+\beta\lambda\right),$$

which yields

$$s^{NOT}=\frac{c(1-\alpha)\lambda}{r+\beta\lambda}. \tag{21}$$

Under the regime that imitators can resell ideas, Eqs. (15) and (20) pin down the optimal subsidy s^{CAN}, so that

$$c\left[r+(\beta-1)\lambda\right]=(c-s^{CAN})[r+(\beta-\alpha)\lambda],$$

which yields

$$s^{CAN}=\frac{c(1-\alpha)\lambda}{r+(\beta-\alpha)\lambda}. \tag{22}$$

Proposition 4 *In the small industry model,* $0<s^{NOT}<s^{CAN}$ *for any* $0<\alpha<1$; $s^{NOT}=s^{CAN}=0$ *for* $\alpha=1$; *and* $s^{NOT}=s^{CAN}=\frac{c\lambda}{r+\beta\lambda}$ *for* $\alpha=0$.

Proof. Follows directly from Eqs. (21) and (22). ■

3.3.3 Optimal diffusion rate

Our model also sheds light on diffusion policies. Suppose that incumbents cannot force imitators to pay, so $\alpha=0$. From a social welfare point of view, should the planner slow down the diffusion speed λ (e.g., by restricting entry of imitators) to enhance incentives for innovation?

When $\alpha=0$, Eqs. (11), (15) and (20) imply

$$k_0^{CAN}=k_0^{NOT}=\left(\frac{A}{c(r+\beta\lambda)}\right)^{\frac{1}{\beta}}<k_0^{SOC}. \tag{23}$$

Since k_0^{CAN} and k_0^{NOT} are both decreasing in λ, one may have conjectured that the planner would prefer a lower λ.

That conjecture, however, is not true. Taking the expressions for k_0^{CAN} and k_0^{NOT} in Eq. (23) as given, the planner's problem is to choose λ to maximize social surplus

$$W_0=-ck_0+\int_0^{\infty}e^{-rt}U\left(k_t\right)dt,$$

where $U(k_t)$ is defined in (17), (18) or (19) depending on the value of β.[4]

$$W_0 = \begin{cases} A^{\frac{1}{\beta}} c^{1-\frac{1}{\beta}} \left(\frac{(r+\beta\lambda)^{1-\frac{1}{\beta}}}{(1-\beta)(r-\lambda(1-\beta))} - (r+\beta\lambda)^{-\frac{1}{\beta}} \right) & \text{if } \beta \in (0,1)\,; \\ \frac{A}{r} \ln \frac{A}{c(r+\lambda)} + \frac{A\lambda}{r^2} - \frac{A}{(r+\lambda)} + \frac{A(1-\ln\varepsilon)}{r} & \text{if } \beta = 1; \\ A^{\frac{1}{\beta}} c^{1-\frac{1}{\beta}} \left(\frac{(r+\beta\lambda)^{1-\frac{1}{\beta}}}{(1-\beta)(r-\lambda(1-\beta))} - (r+\beta\lambda)^{-\frac{1}{\beta}} \right) + \frac{\beta A}{r(\beta-1)} \varepsilon^{1-\beta} & \text{if } \beta > 1. \end{cases} \tag{24}$$

It is straightforward to show that for any $\beta > 0$

$$\frac{\partial W_0}{\partial \lambda} > 0,$$

which proves the following:

Proposition 5 *Given technology spillover is free ($\alpha = 0$), the social planner would not want to slow down diffusion.*

Surprisingly, then, even though k_0^{CAN} and k_0^{NOT} are below k_0^{SOC}, the planner does not want to slow down the diffusion – i.e., does not want a lower λ. This finding highlights the important contribution of technology diffusion to welfare. However, this result is based on the assumption that incumbents meet potential adopters at a fixed rate. We shall ask this question again in Section 4 in which the meeting rate is endogenous.

4 A large industry

Our small industry model assumes that the pool of potential entrants is unlimited, leading to a diffusion process with a constant growth of producers. Evidence shows, however, that diffusion tends to be S-shaped.[5]. The following model endogenizes the meeting rate between incumbents and outsiders and yields S-shaped diffusion.

Let N be the population of potential adopters. The model is the same as before, except that now, instead of being λk_t as in Eq. (3), the number of meetings between incumbents and outsiders at date t is $\gamma k_t (N - k_t)$,where $\gamma > 0$ is a parameter. This matching function is homogeneous of degree 2 so that an agent's meeting rate is proportional to the number of potential partners. If every meeting results in an idea transfer, the number of producers evolves as

$$\frac{dk_t}{dt} = \gamma k_t (N - k_t) \,. \tag{25}$$

[4] When $\alpha = 0$, v_t^{NOT} exists even if $\beta < 1$. So the first line in Eq. (24) is valid.

[5] Griliches (1957) documented S-shaped diffusion of hybrid corn on U.S. farms, and among many later studies, Comin and Hobijn (2004) plot S-shaped diffusion curves for major GPTs.

Accordingly, k_t follows the classic logistic diffusion curve

$$k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + b} \tag{26}$$

and b is determined by the initial condition for $k_0 \in (0, N)$ so that

$$b = \frac{N}{k_0} - 1 > 0. \tag{27}$$

We then have

$$k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + \frac{N}{k_0} - 1}. \tag{28}$$

The $N \to \infty$ limit.—The small industry model is a limit of logistic diffusion in the following sense: Define a constant $\lambda > 0$, and let

$$\gamma = \frac{\lambda}{N} \to 0 \text{ as } N \to \infty.$$

Equations (25) and (28) imply that for given k_t,

$$\frac{dk_t/dt}{k_t} = \gamma\,(N - k_t) = \lambda(1 - \frac{e^{\lambda t}}{e^{\lambda t} + \frac{N}{k_0} - 1}). \tag{29}$$

Given that the demand curve is downward slopping, k_0 has to be finite as $N \to \infty$; otherwise $p_0 \to 0$, and no innovator would enter at date 0. Therefore, Eq. (29) implies that

$$\left.\frac{dk_t/dt}{k_t}\right|_{N\to\infty} \to \lambda, \tag{30}$$

i.e., the incumbents' meeting rate converges to a constant, as seen in Eq. (3) for the small industry model.

4.1 Equilibrium

We, again, characterize the market equilibrium under two regimes: (1) imitators cannot resell ideas, and (2) imitators can resell. We assume $\beta = 1$ in our theoretical analysis to derive closed-form solutions. We will later relax the assumption in Section 5 when taking our model to data.

4.1.1 Imitators cannot resell ideas

Since an imitator cannot resell the innovation, its only revenue comes from selling the good, and its value ω_t satisfies the HJB equation

$$r\omega_t = p_t + \frac{d\omega_t}{dt} = A(\frac{Ne^{N\gamma t}}{e^{N\gamma t}+b})^{-1} + \frac{d\omega_t}{dt}.$$

Solving the ODE and imposing the boundary condition $\omega_{t\to\infty} < \infty$ yields the solution

$$\omega_t = \frac{A}{Nr} + \frac{Abe^{-N\gamma t}}{N(r+N\gamma)}. \tag{31}$$

Because an innovator receives revenues from selling both the good and the idea, the value v_t of being an innovator at date t follows the HJB equation

$$\begin{aligned} rv_t &= p_t + \frac{dk_t/dt}{k_0}\alpha\omega_t + \frac{dv_t}{dt} \\ &= A(\frac{Ne^{N\gamma t}}{e^{N\gamma t}+b})^{-1} + \frac{\gamma(1+b)\alpha Abe^{N\gamma t}}{(e^{N\gamma t}+b)^2}\left(\frac{1}{r} + \frac{b}{e^{N\gamma t}(r+N\gamma)}\right) + \frac{dv_t}{dt}. \end{aligned}$$

Solving the ODE and imposing the boundary condition $v_{t\to\infty} < \infty$ yields

$$v_t = \frac{A}{Nr} + \frac{A\left((r+b\alpha N\gamma+\alpha N\gamma)be^{N\gamma t}+b^2r\right)}{Nr(r+N\gamma)(e^{2N\gamma t}+be^{N\gamma t})}. \tag{32}$$

Accordingly, we have

$$v_0 = \frac{A}{Nr} + \frac{Ab(\alpha N\gamma + r)}{Nr(r+N\gamma)}. \tag{33}$$

With a logistic diffusion, because the outsiders' meeting rate of incumbents is endogenous and always positive, the option value of being an outsider, u_t, is not zero. Eq. (25) implies that the hazard rate of imitation for outsiders is

$$\frac{dk_t/dt}{N-k_t} = \gamma k_t. \tag{34}$$

Therefore, u_t satisfies the HJB equation

$$\begin{aligned} ru_t &= \gamma k_t\left[(1-\alpha)\omega_t - u_t\right] + \frac{du_t}{dt} \\ &= \frac{\gamma Ne^{N\gamma t}}{e^{N\gamma t}+b}\left[(1-\alpha)\left(\frac{A}{Nr} + \frac{Abe^{-N\gamma t}}{N(r+N\gamma)}\right) - u_t\right] + \frac{du_t}{dt}. \end{aligned} \tag{35}$$

The unique bounded solution for u_t is a constant:

$$u_t = \frac{A\gamma(1-\alpha)}{r^2 + N\gamma r}. \tag{36}$$

Two forces exactly offset; the meeting hazard γk_t in Eq. (34) rises at the same rate as $p_t = Ak_t^{-1}$ declines.

At $t = 0$, the free entry condition requires $v_0 - u_0 = c$, thus Eqs. (33) and (36) yield

$$b = \frac{cNr(r + N\gamma) - AN\gamma\alpha - Ar}{A(\alpha N\gamma + r)}.$$

Since $b = \frac{N}{k_0} - 1$, we derive

$$k_0^{NOT} = \frac{A(\alpha N\gamma + r)}{cr(r + N\gamma)}. \tag{37}$$

The $N \to \infty$ limit.—Define a constant $\lambda > 0$, and let $\gamma = \frac{\lambda}{N} \to 0$ as $N \to \infty$. Eq. (37) implies that

$$k_0 = \frac{A(\alpha\lambda + r)}{cr(r + \lambda)}. \tag{38}$$

Therefore, everything else fixed, Eq. (29) applies, $N \to \infty$ results in $\frac{N}{k_0} \to \infty$ and the large industry model converges to the small industry model with $\frac{dk_t/dt}{k_t} \to \lambda$.

The $t \to \infty$ limit.—As $t \to \infty$, $k_t \to N$, and $p_t \to AN^{-1}$. Also, the incumbents' meeting rate of outsiders $\gamma(N - k_t) \to 0$ and the outsiders' meeting rate of incumbents $\gamma k_t \to \gamma N$. Eq. (31) shows that $\omega_{t\to\infty} \to \frac{A}{Nr}$. This means an outsider gets $(1-\alpha)\omega_\infty$ with the exponentially distributed waiting time t with density $\gamma N e^{-\gamma N t}$, i.e.,

$$\begin{aligned} u_\infty &= (1-\alpha)\omega_\infty \int_0^\infty e^{-rt}\gamma N e^{-\gamma N t} dt \\ &= (1-\alpha)\frac{A}{Nr}\frac{\gamma N}{(r + \gamma N)} = \frac{A\gamma(1-\alpha)}{r^2 + N\gamma r}, \end{aligned}$$

which is implied by Eq. (36).

Comparative statics.—The entry of innovators k_0 given by Eq. (37) is very similar to Eq. (11) when $\beta = 1$. Therefore, the comparative statics are similar to what we derived in Propositions 1 and 2: k_0^{NOT} increases in A and α, but decreases in c and γ (except that $\partial k_0^{NOT}/\partial\gamma = 0$ when $\alpha = 1$).

4.1.2 Imitators can resell ideas

If imitators can resell the innovation, all the incumbents (i.e., both innovators and imitators) share the same value v_t that follows the HJB equation

$$rv_t = p_t + \frac{dk_t/dt}{k_t}\alpha v_t + \frac{dv_t}{dt} = A(\frac{Ne^{\gamma Nt}}{e^{\gamma Nt}+b})^{-1} + \left(\frac{\alpha bN\gamma}{e^{\gamma Nt}+b}\right) v_t + \frac{dv_t}{dt}, \quad (39)$$

and the value of an outsider u_t solves

$$ru_t = \frac{dk_t/dt}{(N-k_t)}\left[(1-\alpha)v_t - u_t\right] + \frac{du_t}{dt} = \frac{\gamma Ne^{\gamma Nt}}{e^{\gamma Nt}+b}\left((1-\alpha)v_t - u_t\right) + \frac{du_t}{dt}, \quad (40)$$

where $b = \frac{N}{k_0} - 1$. The free entry condition $v_0 - u_0 = c$ then pins down the entry of innovators k_0 at date 0. Equations (39) and (40) in general do not have closed-form solutions, but we can solve particular cases to derive key insights.

4.2 Implications of the large industry model

In the following, we solve special cases for the regime under which imitators can resell the innovation (i.e., k_0^{CAN}), and compare them with the regime under which imitators cannot resell (i.e., k_0^{NOT}).

First, we solve k_0^{CAN} for the cases where $\alpha = 0$ and $\alpha = 1$ (see Appendix I). These two cases are the most relevant for our welfare analysis. The results show

$$k_0^{CAN} = k_0^{NOT} = \frac{A}{c(r+\gamma N)}, \quad \text{if} \quad \alpha = 0; \quad (41)$$

$$k_0^{CAN} = k_0^{NOT} = \frac{A}{rc}, \quad \text{if} \quad \alpha = 1. \quad (42)$$

These results confirm our finding from the small industry model in Proposition 1, $k_0^{NOT} = k_0^{CAN}$ for $\alpha \in \{0, 1\}$. The intuition is straightforward: Because the innovators who enter at date 0 either receive no resale revenues at all (if $\alpha = 0$) or get all the revenues (if $\alpha = 1$), whether imitators can or cannot resell the innovation becomes irrelevant.

Second, we explicitly solve k_0^{CAN} for the case where $\gamma N = r$. Appendix II shows that

$$\frac{N - k_0^{CAN}}{\frac{k_0^{CAN}}{(2-\alpha)(N-k_0^{CAN})}\left(\left(\frac{N}{k_0^{CAN}}\right)^2 - \left(\frac{N}{k_0^{CAN}}\right)^\alpha\right) - 1} = \frac{A}{rc}, \quad (43)$$

which yields comparative statics similar to Propositions 1 and 2: k_0^{CAN} increases in A and α, but decreases in c; moreover, $k_0^{NOT} > k_0^{CAN}$ for $\alpha \in (0, 1)$.

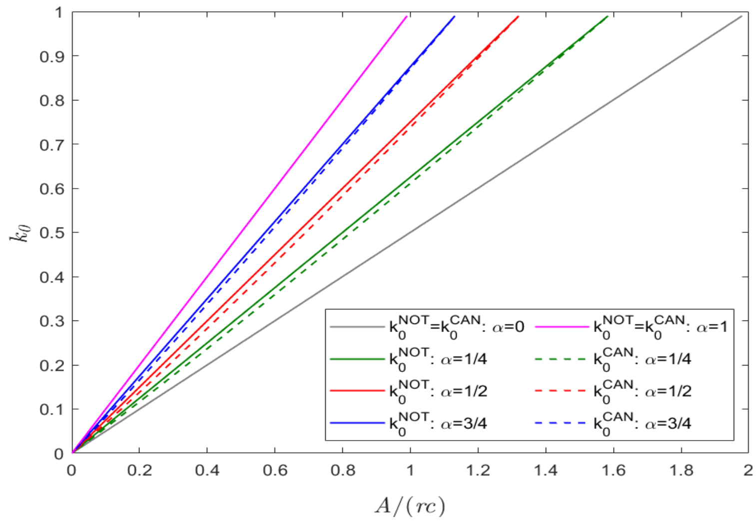

Fig. 2. Entry of Innovators: Effects of α

The comparative statics are illustrated by Fig. 2, which plots k_0^{NOT} and k_0^{CAN} according to Eqs. (37) and (43) for $\gamma N = r$ and $N = 1$. In the figure, the solid lines stand for k_0^{NOT} and the dash lines stand for k_0^{CAN}. Cases with different value of α are plotted in different colors. The figure shows that both k_0^{NOT} and k_0^{CAN} increase in A and α, but decrease in c. It also shows that, similar to the small industry model (cf. Proposition 1), $k_0^{NOT} = k_0^{CAN}$ for $\alpha \in \{0, 1\}$, and $k_0^{NOT} > k_0^{CAN}$ for $\alpha \in (0, 1)$.

Finally, the large industry model yields policy implications similar to the small industry model:

1. *Patent licensing.*—Here Eq. (37) is the relevant solution for k_0. Again, the investment on innovation highly depends on how innovators' intellectual property rights get protected (e.g., through the compensation share α and whether imitators can resell the innovation).

2. *Technology transfer via employee spin-offs.*—Here Eq. (41) is the solution when $\alpha = 0$, which confirms that spin-offs negatively affect innovation (i.e., $\partial k_0 / \partial \gamma < 0$).

3. *Restricting entry of imitators.*—The large industry model also predicts the overtaking pattern. A location restricting entry of imitators (i.e., a lower γ) tends to enjoy a higher k_0, but its industry size later gets surpassed by other places without such restrictions (i.e., a higher γ).

Figure 3 show simulation results based on the large industry model. Assuming $\alpha = 0$, $\beta = 1$, Eq. (41) shows that $k_0 = k_0^{NOT} = k_0^{CAN} = \frac{A}{c(r+\gamma N)}$. Define $\lambda = \gamma N$. We assume $A/c = 3$, $N = 1000$, $r = 0.05$, and plot k_t in two locations: One with high diffusion rate ($\lambda = 0.08$), the other with a low diffusion rate ($\lambda = 0.04$). In each location,

$$k_t = \frac{N e^{\gamma N t}}{e^{\gamma N t} + \frac{N}{k_0} - 1} = \frac{N e^{\lambda t}}{e^{\lambda t} + \frac{N c(r+\lambda)}{A} - 1}.$$

We again see an overtaking pattern, similar to the one generated by the small industry model. The location with a lower λ enjoys a higher k_t in early periods, but its industry size is later surpassed by the other location with a higher λ.

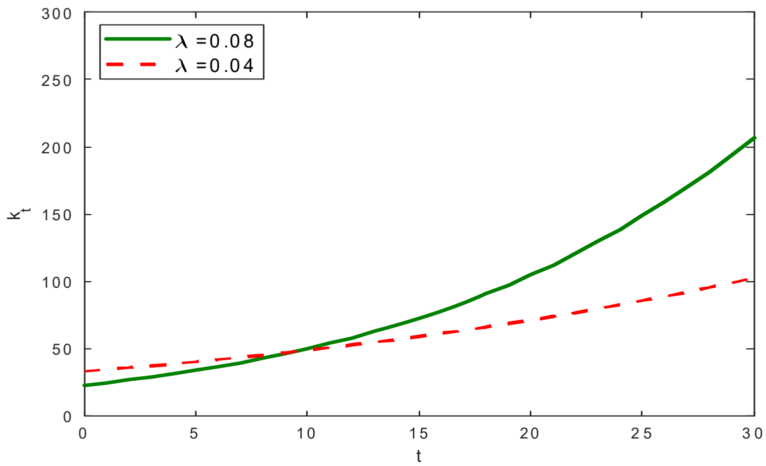

Fig. 3. Industry Overtaking: Large Industry Model Simulation

4.3 Welfare analysis for a large industry

We now consider the welfare problem from the social planner's point of view. The main difference of the large industry model is that now full rent extraction (i.e., $\alpha = 1$) is *not* socially optimal because of a congestion effect or rent-grabbing externality that innovators impose on each other. In the case of lacking intellectual property right protection (i.e., $\alpha = 0$), however, we continue to find that the planner would not want to reduce diffusion speed. To illustrate these findings, we keep the assumption $\beta = 1$. Accordingly, consumer surplus is still given by Eq. (18), i.e., $U(k) = A(\ln k + 1 - \ln \varepsilon)$. Without loss of generality, we also normalize $N = 1$ in the welfare analysis so that Eq. (28) reduces to

$$k_t = \frac{e^{\gamma t}}{e^{\gamma t} + \frac{1}{k_0} - 1}. \tag{44}$$

4.3.1 Optimal compensation share

Assume the technology diffusion parameter γ is exogenous. Given $\beta = 1$, the planner chooses k_0 to maximize social surplus

$$W_0 = -ck_0 + \int_0^\infty e^{-rt} \left[A(\ln k_t + 1 - \ln \varepsilon)\right] dt. \tag{45}$$

with k_t given by Eq. (44).

The first order condition requires

$$c = A \int_0^\infty e^{-rt} \left[k_0^2(e^{\gamma t} - 1) + k_0\right]^{-1} dt. \tag{46}$$

Let k_0^{SOC} denote the socially optimal value that solves Eq. (46). We have the following proposition.

Proposition 6 *In the large industry model, $\alpha = 1$ is not the socially optimal compensation share for the innovators.*

Proof. Eq. (42) shows $k_0^{NOT} = k_0^{CAN} = \frac{A}{rc}$ when $\alpha = 1$. We can verify that $k_0^{SOC} = \frac{A}{rc}$ is the solution for Eq. (46) when $\gamma = 0$. Because the right hand side of Eq. (46) strictly decreases in γ, k_0^{SOC} has to be smaller than $\frac{A}{rc}$ to satisfy Eq. (46) for any $\gamma > 0$. Therefore, $\alpha = 1$ is not the socially optimal compensation share. ■

When $\alpha = 1$, an innovator can appropriate all the value from the meetings that her presence generates. We have learned from the small industry model that this yields an equilibrium that is socially optimal. But why this does not hold for the large industry model?

The key difference of the large industry model is that the meeting probability is endogenous and there is a congestion effect that an innovator creates – she reduces the number of meetings that other innovators will have, and this is an effect that the innovator ignores. To see this, recall in the small industry model the probability for an incumbent to meet with an imitator is fixed (i.e., $\frac{dk_t/dt}{k_t} = \lambda$). In contrast, in the large industry model, the probability is negatively affected by the number of incumbents (i.e., $\frac{dk_t/dt}{k_t} = \gamma(N - k_t)$).

One can solve for a $\alpha^{SOC} < 1$ that generates the planner's optimum. Consider an explicit example where $\gamma = r$. In this case, Eq. (46) can be simplified as

$$\frac{\left(1 - k_0^{SOC}\right)^2}{\frac{1}{k_0^{SOC}} - 1 - \ln \frac{1}{k_0^{SOC}}} = \frac{A}{cr}. \tag{47}$$

Under the regime that imitators cannot resell the innovation, Eqs. (37) and (47) imply that

$$\alpha^{SOC} = \frac{2(1 - k_0^{SOC} + k_0^{SOC} \ln k_0^{SOC})}{(1 - k_0^{SOC})^2} - 1. \tag{48}$$

Alternatively, under the regime that imitators can resell, Eqs. (43) and (47) imply that α^{SOC} solves

$$\frac{1}{\frac{k_0^{SOC}}{(2-\alpha)(1-k_0^{SOC})}\left(\left(\frac{1}{k_0^{SOC}}\right)^2 - \left(\frac{1}{k_0^{SOC}}\right)^{\alpha}\right) - 1} = \frac{1 - k_0^{SOC}}{\frac{1}{k_0^{SOC}} - 1 - \ln \frac{1}{k_0^{SOC}}}. \tag{49}$$

Figure 4 illustrates this example where $\gamma = r$. For a given value of $A/(rc) = 0.3$, Fig. 4(I) plots the relation between W_0 and k_0, given by Eq. (45).[6] The result

[6] Equation (45) shows that c and ε are just scaling parameters and they do not affect the maximization of W_0, so without loss of generality we set $c = 1$ and $\varepsilon = 0.0001$ for plotting Figs. 4(I)-(II).

shows that welfare maximizes at $k_0^{SOC} = 0.22$. Figure 4(II) shows that this welfare maximum can be achieved by either setting $\alpha^{SOC} = 0.47$ under Regime 1 (i.e., where imitators cannot resell ideas), or setting $\alpha^{SOC} = 0.57$ under Regime 2 (i.e., where imitators can resell). Figures 4(III)-(IV) extend the results to the full domain of $A/(rc)$. Figure 4(III) plots the relation between k_0^{SOC} and $A/(rc)$ given by Eq. (47), and Fig. 4(IV) traces out the relation between k_0^{SOC} and α^{SOC} that satisfies Eqs. (48) or (49). The negative relation between two endogenous variables, k_0^{SOC} and α^{SOC}, is induced by changes in $A/(rc)$ — as $A/(rc)$ rises, so does k_0^{SOC} but α^{SOC} falls. For a given value of k_0^{SOC} (or the corresponding $A/(rc)$), the value of α^{SOC} is always smaller under Regime 1 than under Regime 2.

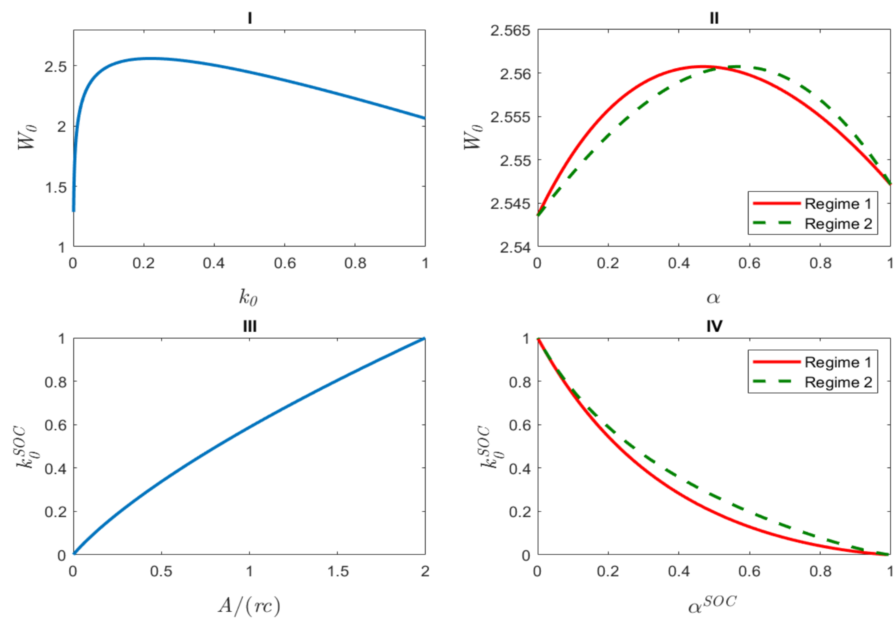

Fig. 4. Socially Optimal Compensation Share

4.3.2 Optimal subsidy/taxation

Alternatively, the planner could provide a subsidy or impose a tax to internalize externalities. In the small industry model where the meeting rate is fixed, the planner always wants to provide a subsidy to innovators for any $\alpha < 1$ (cf. Proposition 3). In the large industry model, however, the planner would want to impose an innovation tax if α is sufficiently large or provide a subsidy if α is sufficiently small. One can calculate this tax or subsidy as a function of model parameters, including α.

To see this clearly, let us again consider the explicit example where $\gamma = r$. In Fig. 5(I), we plot Eq. (47) using a black solid line and overlay it on Fig. 2 above. The figure shows that for a given level of $\frac{A}{rc}$, k_0^{SOC} always exceeds the market equilibrium level (i.e., k_0^{NOT} or k_0^{CAN}) when $\alpha = 0$, but falls short when $\alpha = 1$. Moreover, for any value of $\alpha \in (0, 1)$, the socially optimal entry k_0^{SOC} can be achieved by adding an appropriate tax or subsidy to c. In the figure, the vertical difference between a market equilibrium path (associated with a particular α and a regime whether imitators can resell the innovation or not) and the socially optimal path indicates the amount of adjustment to $\frac{A}{rc}$ (i.e., by adjusting $\frac{A}{r(c-s^{NOT})}$ or $\frac{A}{r(c-s^{CAN})}$) needed to achieve each social optimal level of k_0^{SOC}. Figure 5(II) plots the subsidy (scaled by the entry cost c) needed to achieve the social optimum. The figure shows that the scaled subsidies, s^{NOT}/c and s^{CAN}/c, both decrease in $\frac{A}{rc}$ and α, and can turn negative (i.e., becomes a tax) if $\frac{A}{rc}$ or α becomes sufficiently large. Moreover, $s^{NOT} = s^{CAN} > 0$ (i.e., a subsidy) for $\alpha = 0$, $s^{NOT} = s^{CAN} < 0$ (i.e., a tax) for $\alpha = 1$, and $s^{NOT} < s^{CAN}$ for $0 < \alpha < 1$.

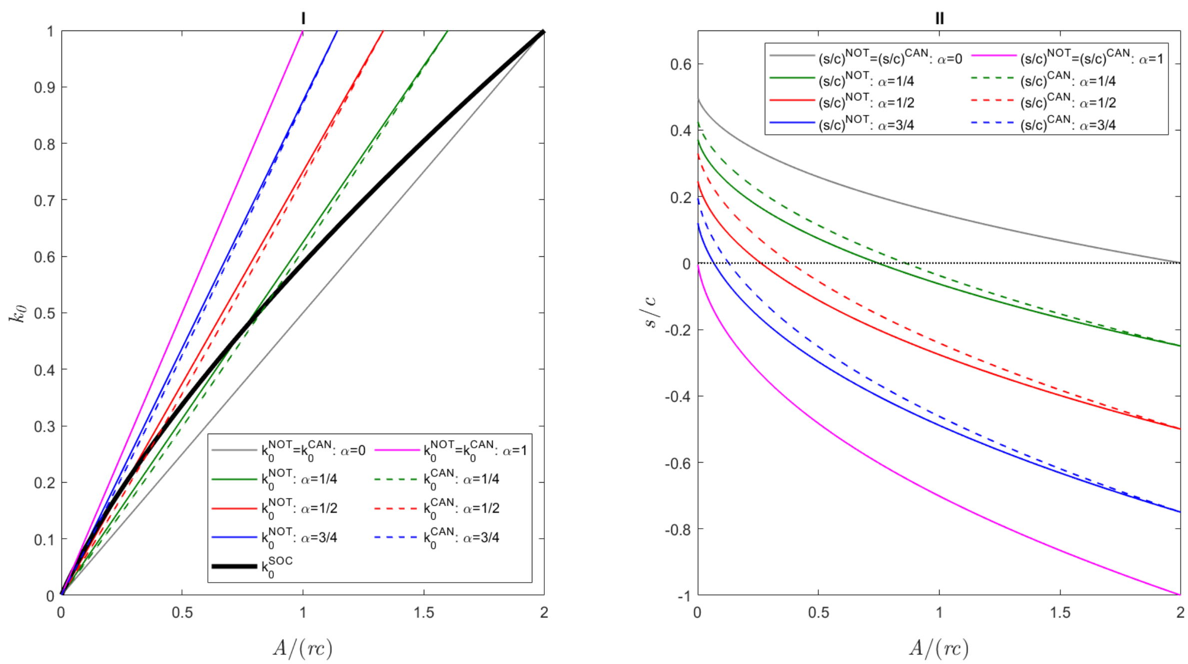

Fig. 5. Socially Optimal Subsidy/Taxation

4.3.3 Optimal diffusion rate

We now explore the implications of the large industry model on diffusion policies. Consider again the scenario where incumbents are not compensated by imitators, so $\alpha = 0$. Should the planner slow down the diffusion?

Recall that under $\alpha = 0$, Eq. (41) shows that the equilibrium entry of innovators is given by

$$k_0 = \frac{A}{c(r+\gamma)}, \tag{50}$$

regardless of whether imitators can or cannot resell the innovation.

Taking Eq. (50) as given and $\beta = 1$, the planner would choose γ to maximize social surplus:

$$W_0 = -\frac{A}{(r+\gamma)} + A\int_0^\infty e^{-rt}\left(\gamma t - \ln\left(e^{\gamma t} + \frac{c\,(r+\gamma)}{A} - 1\right)\right)dt + \text{constant}. \quad (51)$$

We can then prove $\frac{dW_0}{d\gamma} > 0$ for any $1 > k_0 > 0$, and $\frac{dW_0}{d\gamma} = 0$ when $k_0 = 1$ (see Appendix III for the proof).

This finding extends our result in Proposition 5 to the large industry model where the meeting rate between incumbents and outsiders is endogenous, and it lends further support to public policies that accommodate diffusion. Note that the finding does not rule out the possibility that policymakers can exploit the welfare gain of temporarily restricting γ. For example, policymakers could restrict γ initially to achieve the socially optimal entry of innovators k_0^{SOC}, and then free up the limitation. While this seems welfare improving, such a policy would not be time-consistent (Kydland and Prescott, 1977). Presumably policy must apply more broadly, not just to one instance, but to future products and future instances of k.

5 Empirical study

In the empirical study, we calibrate the large industry model to data and relax the assumption $\beta = 1$. We consider two historically important industries: automobile and personal computer, where idea diffusion played an important role in the industries' development.[7] Using model calibration and counterfactual exercises, we evaluate and quantify our theoretical predictions.

5.1 Automobile industry

The U.S. automobile industry started in 1890s and grew from a small infant industry to a major sector of the economy in a few decades. During the process, the industry output continued to expand, but the number of firms initially rose and later fell. Starting with 3 firms in 1895, the number of auto producers exceeded 200 around 1910. A shakeout then followed when a major process innovation, the assembly line, was introduced in the early 1910s and firm productivity rose tremendously ever since. Eventually, only 24 firms survived to 1930. Figure 6 plots the number of firms and output per firm in the U.S. auto industry from 1895-1929.

Our theoretical model describes the auto industry development very well for the pre-shakeout period, and we can also extend the model to incorporate the shakeout

[7] For example, Klepper (2010) documents in detail how the spawning of employee spin-offs and entry by firms in related industries drove the development of the automobile and semiconductor industries.

without affecting our analysis.[8] As shown in Fig. 6, the growth of the auto industry mainly relied on the extensive margin before 1910. During that period, the time path of firm numbers followed an S-shaped curve and the average output per firm stayed pretty much constant. In the following analysis, we will calibrate our model to the auto industry data for the pre-shakeout era (1895-1910) and conduct counterfactual exercises.

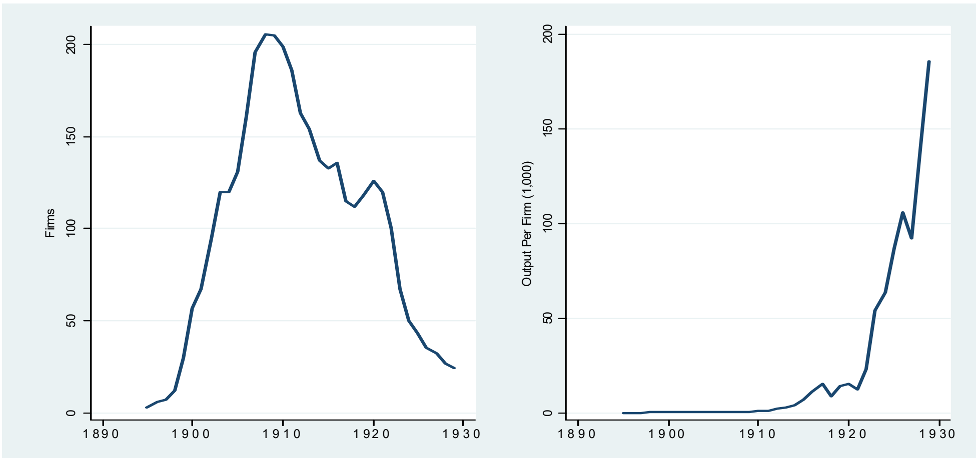

Fig. 6. Auto Firm Numbers and Output Per Firm

5.1.1 Data

The data of U.S. auto industry comes from several sources. Smith (1970) lists every make of passenger cars produced commercially in the United States from 1895 through 1969. The book records the firm that manufactured each car make, the firm's location, the years that the car make was produced. Smith's list of car makes is used to derive the number of auto firms each year. Thomas (1977) provides annual data of average car price and output from 1900-1929. Williamson (2020) provides annual data of U.S. population, real GDP, and the GDP deflator.

[8] Our model can be extended to incorporate the shakeout. Following Jovanovic and MacDonald (1994), we may assume that the industry expects a disruptive innovation to arrive at a Poisson rate ρ. This innovation would allow any incumbent firms to make an investment to transform its product and tremendously increase its output. When such an innovation arrives, a few incumbents would invest and the rest would exit. In the competitive equilibrium, the present value of an investing firm (net of its investment) is zero, and the value of an exiting firm is also zero. In the scenario where imitators can resell ideas, the HJB equation for an incumbent firm would change to $rv_t = p_t + \frac{dk_t/dt}{k_t}\alpha v_t - \rho v_t + \frac{dv_t}{dt}$, which implies that $(r+\rho)\, v_t = p_t + \frac{dk_t/dt}{k_t}\alpha v_t + \frac{dv_t}{dt}$. Similarly, we can rewrite the HJB equations for other types of firms and for the scenario where imitators cannot resell ideas. In any case, the original functional forms of our model hold, with just the discount parameter changing from r to $r+\rho$.

5.1.2 Diffusion and demand estimation

To calibrate the model, we first use the data of firm numbers to estimate the diffusion parameters.[9] With Eq. (28), we can rewrite the diffusion process of k_t as follows:

$$\ln \frac{k_t}{N - k_t} = z + \tilde{\gamma} t, \tag{52}$$

where $z = \ln \frac{k_0}{N-k_0}$, and $\tilde{\gamma} = \gamma N$.

We assume that the shakeout started after all the potential firms had entered the industry (and we will adjust this assumption later). So we assume $N = 210$ and run the regression model (52). The result shows that

$$\ln \frac{k_t}{N - k_t} = \underset{(0.26)^{***}}{-4.13} + \underset{(0.03)^{***}}{0.53}\, t, \tag{53}$$

and the standard errors are reported in the parentheses. The estimates of z and $\tilde{\gamma}$ are both statistically significant at 1% level (noted by three stars), and adjusted $R^2 = 0.96$. Based on the estimates of diffusion parameters, we calibrate $\tilde{\gamma} = 0.53$, and $k_0 = 3.31$ (i.e., $\ln \frac{k_0}{N-k_0} = -4.13$).

We then estimate an industry demand function using annual data of auto prices p_t and output Q_t from 1900–1929. Eq. (1) implies a simple log-log per capita demand function:

$$\ln(\frac{Q_t}{pop_t}) = a_t - \phi \ln(p_t).$$

In the regression, we control for log U.S. GDP per capita (as a proxy for income) in the demand intercept a_t. Both auto price and GDP per capita are in real terms.

To address potential endogeneity of the price variable, we use the output per firm (lagged by a year) as an instrumental variable to estimate the demand slope parameter ϕ. Output per firm, while is assumed fixed in our theory, did grow over time in the data due to technological progress. This would have affected the supply condition but not the demand.[10]

The first-stage regression result (adj. $R^2 = 0.89$) is given by:

$$\ln(p_t) = \underset{(1.09)^{***}}{8.56} + \underset{(0.64)^{**}}{1.66} \times \ln(\frac{GDP_t}{pop_t}) - \underset{(0.03)^{***}}{0.29} \times \ln(\textit{output per firm})_{t-1}.$$

[9]For simplicity, our model assumes that firms do not exit the industry after their entry. It is natural to extend the model to incorporate firm exits in the form of exogenous death; the analysis does not change much (See Appendix V for a detailed discussion).

[10]Cabral, Wang and Xu (2018) also estimated the auto demand function for the same sample period. They used a different instrument variable, the share of spin-off firms in the auto industry. The idea is that the founders of spin-off firms are more experienced than de novo entrants, so spin-off firms tend to perform better (Klepper, 2010). They show that their instrument variable performs well and the estimated demand slope $\phi = 3.39$, which is very close to ours.

The second-stage regression result ($R^2 = 0.83$) is in turn given by:

$$\ln(\frac{Q_t}{pop_t}) = \underset{(7.16)^{***}}{32.37} + \underset{(2.10)}{0.28} \times \ln(\frac{GDP_t}{pop_t}) - \underset{(0.38)^{***}}{3.33} \times \ln(p_t).$$

Standard errors are reported in the parentheses, with three stars, two stars and one star representing statistical significance at 1%, 5% and 10% level, respectively.

The IV estimation gives $\phi = 3.33$ and it is highly statistically significant. This suggests that we can calibrate the value of inverse demand elasticity in our model to be $\beta = \frac{1}{\phi} = 0.3$.

Figure 7 plots the estimated firm numbers and per capita auto demand, and compares them with data.

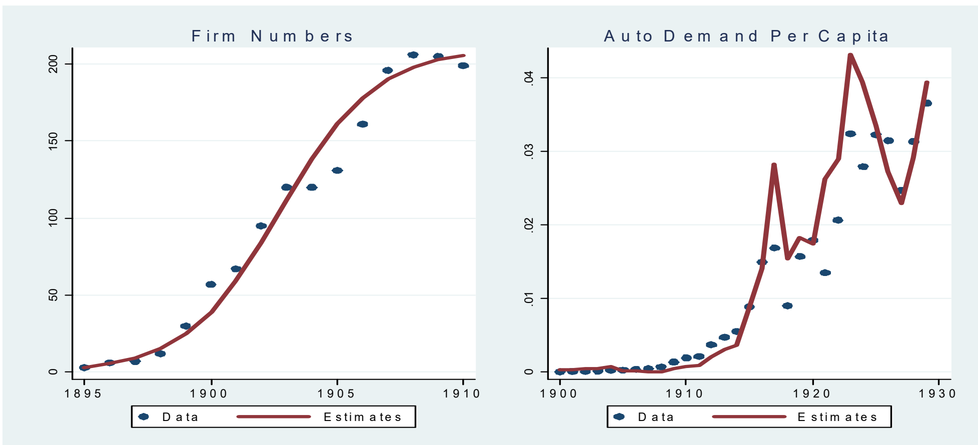

Fig. 7. Auto Diffusion and Demand Estimation

5.1.3 Model calibration

Based on the estimation results, we parameterize our model and calibrate it to the auto industry data.

Recall the industry demand function (cf. Eq. (1))

$$p_t = Ak_t^{-\beta},$$

and the time path of firm numbers (cf. Eq. (26))

$$k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + b}, \quad \text{where} \;\; b = \frac{N}{k_0} - 1.$$

As a benchmark, we assume that the industry data is generated by the model with free spillover (i.e., $\alpha = 0$), so it makes no difference whether imitators can or cannot resell the technology.[11] Accordingly, the value of an incumbent firm in the industry

[11] Assuming $\alpha = 0$ is a natural (though not necessarily best) way to taking our model to data. One could re-do the calibration exercises by assuming a different benchmark value of α, but the method and intuition would be very similar.

at date t satisfies

$$rv_t = p_t + \frac{dv_t}{dt} = A(\frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + b})^{-\beta} + \frac{dv_t}{dt},$$

while the value of an outsider satisfies

$$ru_t = \frac{dk_t/dt}{(N - k_t)}(v_t - u_t) + \frac{du_t}{dt} = \frac{\gamma Ne^{\gamma Nt}}{e^{\gamma Nt} + b}(v_t - u_t) + \frac{du_t}{dt}.$$

Based on the diffusion and demand estimates above, we calibrate the model parameter values at $N = 210$, $\gamma N = 0.53$, $k_0 = 3.31$, $\beta = 0.3$. Because A is just a scaling parameter (i.e., only A/c matters), we normalize $A = 1$ and assume $r = 0.05$.

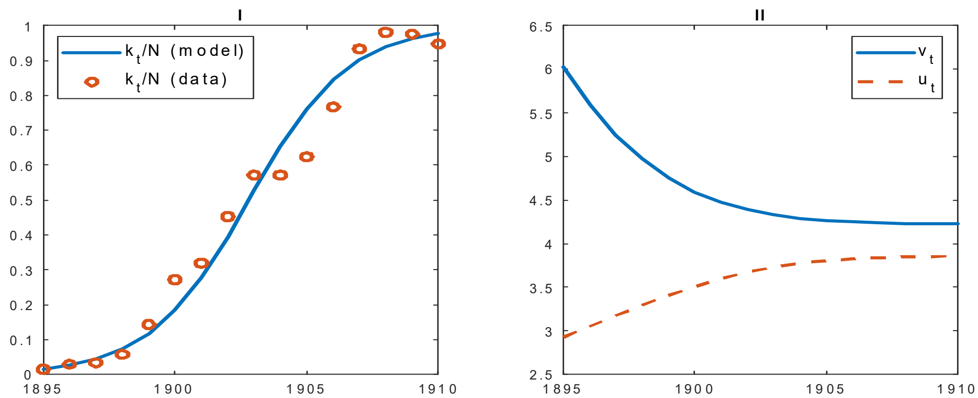

Fig. 8. Auto Industry: Model Calibration

While a closed-form solution does not exist when $\beta \neq 1$, we can numerically solve the time path (v_t, u_t) and pin down the cost of innovation c by the free entry condition $v_0 = u_0 + c$. Figure 8 plots the calibration results for the time paths of k_t/N, v_t and u_t. The number of firms k_t grows along a logistic curve. Meanwhile, v_t decreases while u_t increases over time, and the initial difference $v_0 - u_0$ pins down the innovation cost $c = 3.10$.

5.1.4 Counterfactual analysis

Given the calibrated model parameter values, we can conduct counterfactual analysis and evaluate welfare.

Optimal compensation share We first evaluate the effect of the compensation share α. We consider two regimes and label them Regime 1 (under which imitators cannot resell the innovation) and Regime 2 (under which imitators can resell), respectively.

For any value of compensation share $\alpha \in [0, 1]$, we numerically solve the equilibrium time paths for k_t, v_t and u_t (see Appendix IV for the details). Particularly, the free entry condition $v_0 = u_0 + c$ allows us to pin down the counterfactual entry of innovators k_0 at date 0. Figure 9(I) shows that k_0 strictly increases in α for $0 \leq \alpha < 0.71$ under both Regime 1 and Regime 2, and k_0 is smaller under Regime 2 than under Regime 1. Figure 9(II) shows that under the two regimes, the free entry condition holds in terms of $v_0 - u_0 = c$ when $0 \leq \alpha < 0.71$, while for $\alpha \geq 0.71$, because of the corner solution $k_0 = N$, the free entry condition holds in terms of $v_0 - u_0 > c$.

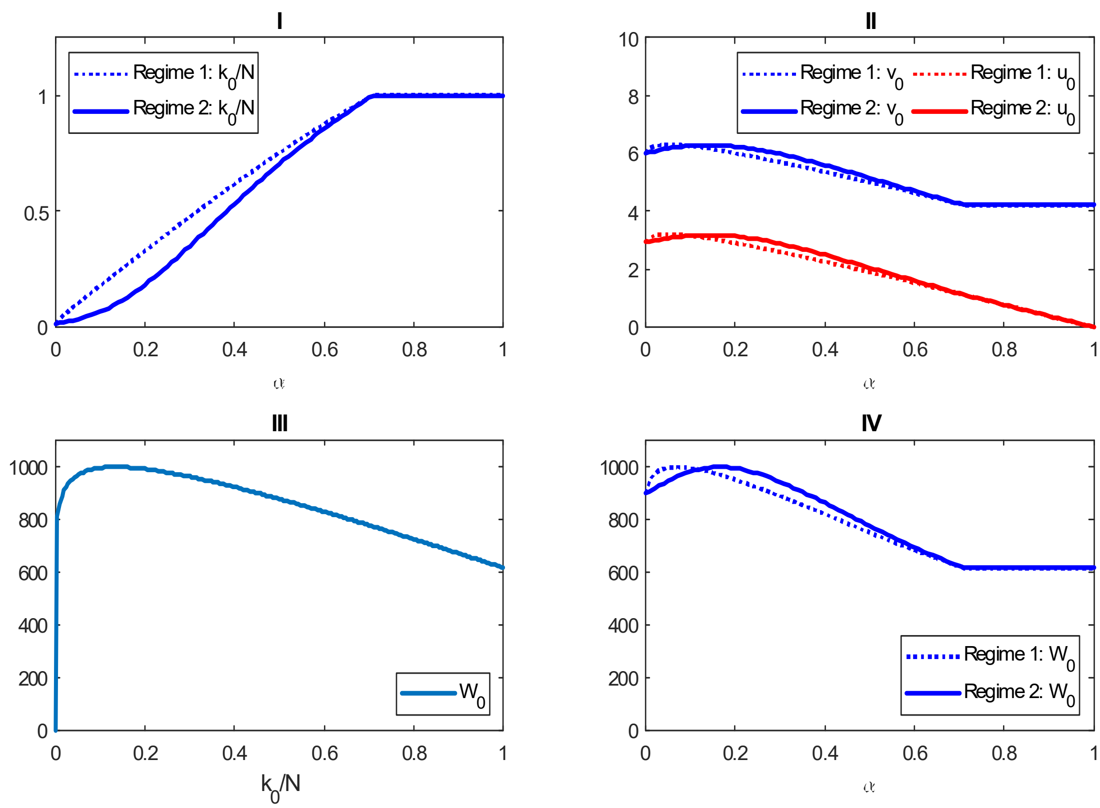

Fig. 9. Auto Industry: Effects of α

We then assess the welfare effect of changing the value of α. Figures 9(III)-(IV) plot social welfare W_0 according to different values of k_0 as well as α. The results show that social welfare W_0 is hump-shaped in k_0 and peaks at $k_0 = 28$ (i.e., $k_0/N = 0.133$). This social optimum can be achieved by choosing $\alpha^{SOC} = 0.07$ under Regime 1 or choosing $\alpha^{SOC} = 0.16$ under Regime 2. Note that under both regimes, neither $\alpha = 0$ nor $\alpha = 1$ is socially optimal: the former yields $k_0 = 3.31$ and the latter yields $k_0 = N = 210$.

The socially optimal compensation share α^{SOC} depends on the model parameters, including the inverse price elasticity β, the diffusion speed γ, the innovation cost c, and the number of potential adopters N. Figure 10 plots the comparative statics of α^{SOC} under Regimes 1 and 2. The results show the following:

- α^{SOC} increases in β.—A higher β leads to a faster decline in price and discourages k_0. This makes the congestion externality among innovators less of a concern, so the planner wants to raise the compensation share for innovators.
- α^{SOC} decreases in γ.— A higher γ leads to a faster decline in price, but the meeting rate is higher, and with it the idea-sales revenue rises fast enough to boost k_0. This worsens the congestion externality among innovators, so the planner wants to reduce the compensation share for innovators.
- α^{SOC} increases in c.—A higher c discourages k_0. This makes the congestion externality among innovators less of a concern, so the planner wants to raise the compensation share for innovators.
- α^{SOC} increases in N (holding $\gamma N = \lambda$ fixed).—A higher N leads to faster price decline that discourages k_0. This, together with a larger pool of potential adopters N, makes the congestion externality among innovators less of a concern, so the planner wants to increase the compensation share for innovators. This finding is consistent with our theoretical result that the large industry model converges to the small industry model as N gets large, suggesting that in that case α^{SOC} should converge to 1.
- Comparison of Regimes 1 and 2.—The socially optimal compensation share for innovators is higher under Regime 2 than under Regime 1, and the difference increases in β, γ, c and N.

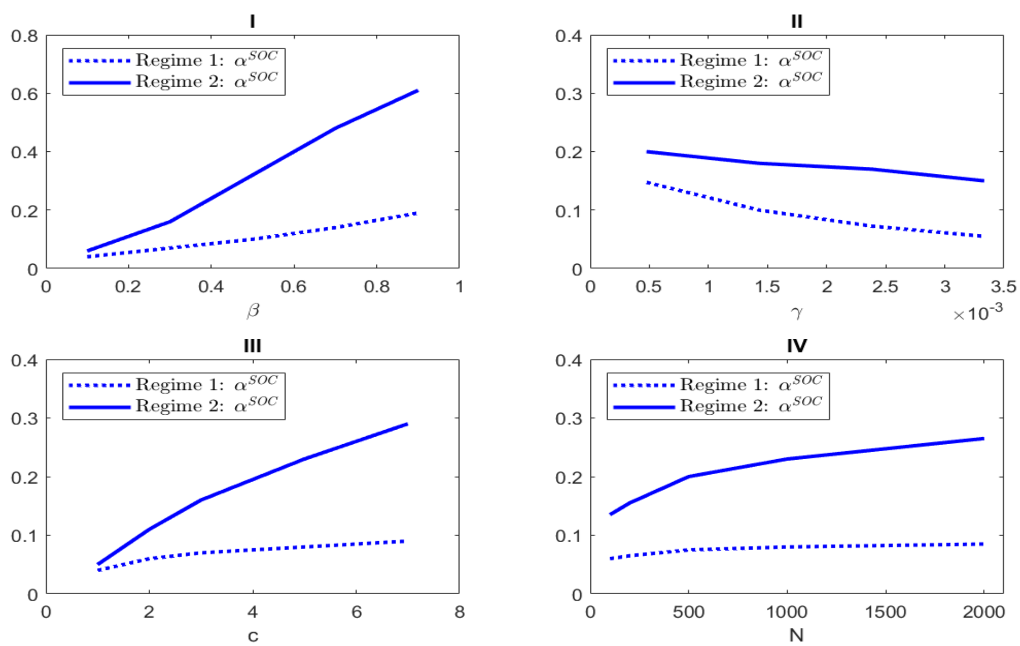

Fig. 10. Comparative Statics: α^{SOC}

Optimal innovation subsidy Given $\alpha = 0$, the entry of innovators at date 0 is lower than the socially optimal level. Providing innovators a subsidy s, instead of setting a socially optimal α, can also help achieve the social optimality. Note that with the subsidy, $c - s$ is the net entry cost for innovators. Figure 11 plots the effect of s on the entry of innovators k_0 and welfare W_0. The results show that k_0 increases in s, and the social welfare peaks at $s = 1.85$ (i.e., lowering the entry cost from 3.10 to 1.25, a 60% reduction).

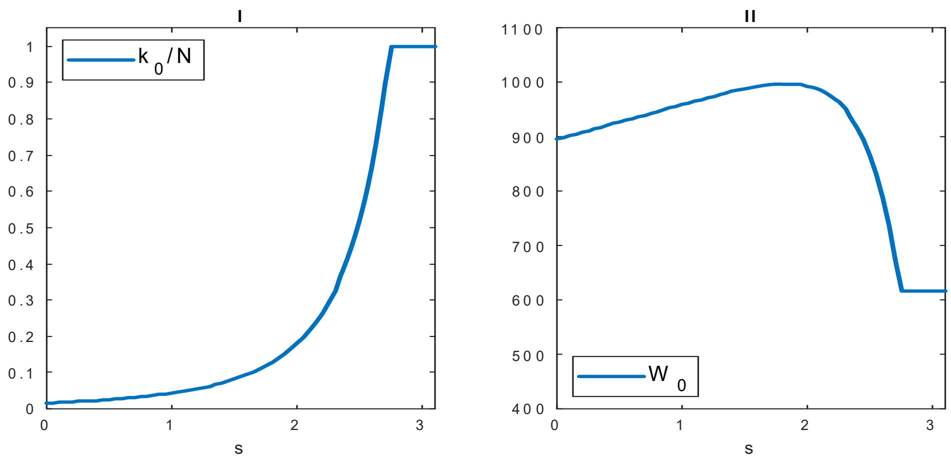

Fig. 11. Auto Industry: Effects of s

Optimal diffusion rate We can similarly evaluate the effects of varying the diffusion rate γ. Consider again the scenario where incumbents are not compensated by imitators, so $\alpha = 0$. Should the planner slow down the diffusion?

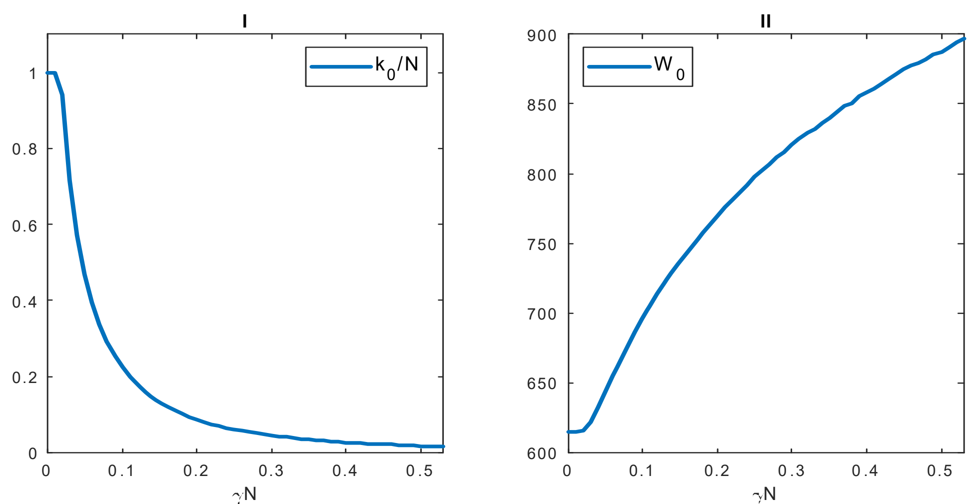

Fig. 12. Auto Industry: Effects of γ

Figure 12 shows the effects of γ on k_0 and W_0. If the planner were to push down γ from the original value where $\gamma N = 0.53$, the entry of innovators k_0 would increase. If the value of γ gets sufficiently low (i.e., $\gamma N \leq 0.01$), k_0 would reach the corner solution $k_0 = N$. However, social welfare increases in γ, which suggests that restricting diffusion would reduce welfare. The intuition is that while slowing down diffusion could encourage entry of innovators, it would forego too much free learning in the industry and the welfare effect of the latter dominates.

5.1.5 Model re-calibration: A larger pool of potential adopters

In the benchmark calibration above, we assume that the shakeout started after all the potential auto firms had entered the industry. Alternatively, we could assume that the shakeout started in the middle of the diffusion process. Below, we assume a much larger number of potential auto firms, $N = 1,000$. This implies that it would take 30 years to reach 99% adoption rate among potential producers had the shakeout not happened, doubling what was assumed in the benchmark calibration. We will see that most of our previous analysis goes through, but the optimal compensation share α increases.

We assume $N = 1,000$ and re-estimate the diffusion model (52). The result shows that

$$\ln \frac{k_t}{N - k_t} = \underset{(0.26)^{***}}{-5.00} + \underset{(0.03)^{***}}{0.30}\, t, \tag{54}$$

and the standard errors are reported in the parentheses.

Based on the estimates of diffusion parameters, we calibrate $N = 1,000$, $\gamma N = 0.3$, and $k_0 = 6.69$ (i.e., $\ln \frac{k_0}{N - k_0} = -5$). In addition, we set $\beta = 0.3$, $r = 0.05$, and normalize $A = 1$ as before. Figure 13 plots the calibration results. It shows that the entry of firms would reach $k_t = 377.44$ (i.e., $k_t/N = 0.38$) at the end of the sample period had the shakeout not occurred, and the initial difference $v_0 - u_0$ pins down the innovation cost $c = 3.94$.[12]

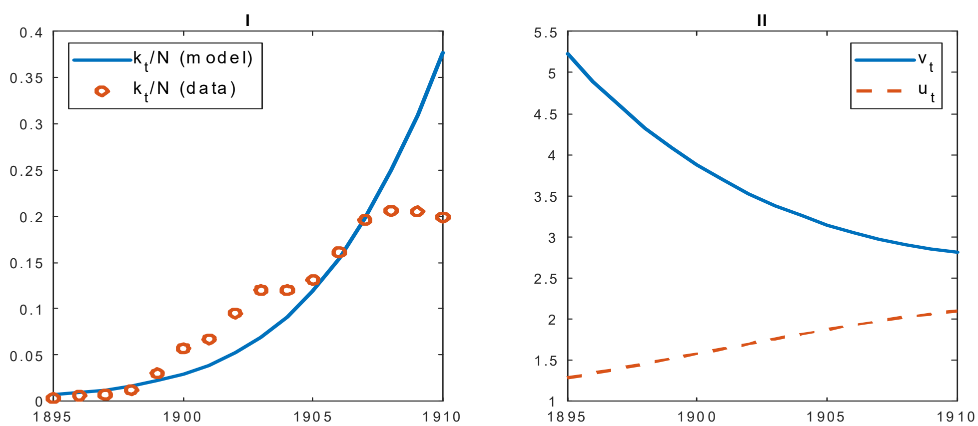

Fig. 13. Auto Industry: Model Re-Calibration

[12] Comparing Figs. 8(I) and 13(I) suggests that the benchmark calibration fits the data better. Nevertheless, we conduct the re-calibration exercise for comparison and robustness checks.

Figure 14 plots the results of varying the value of α, which reveal some differences from the benchmark calibration above. First, given a much larger pool of potential adopters, the estimated diffusion rate γN becomes smaller to match the data. As a result, k_0 would not reach the full adoption $N = 1,000$ for any $\alpha \in [0, 1]$.

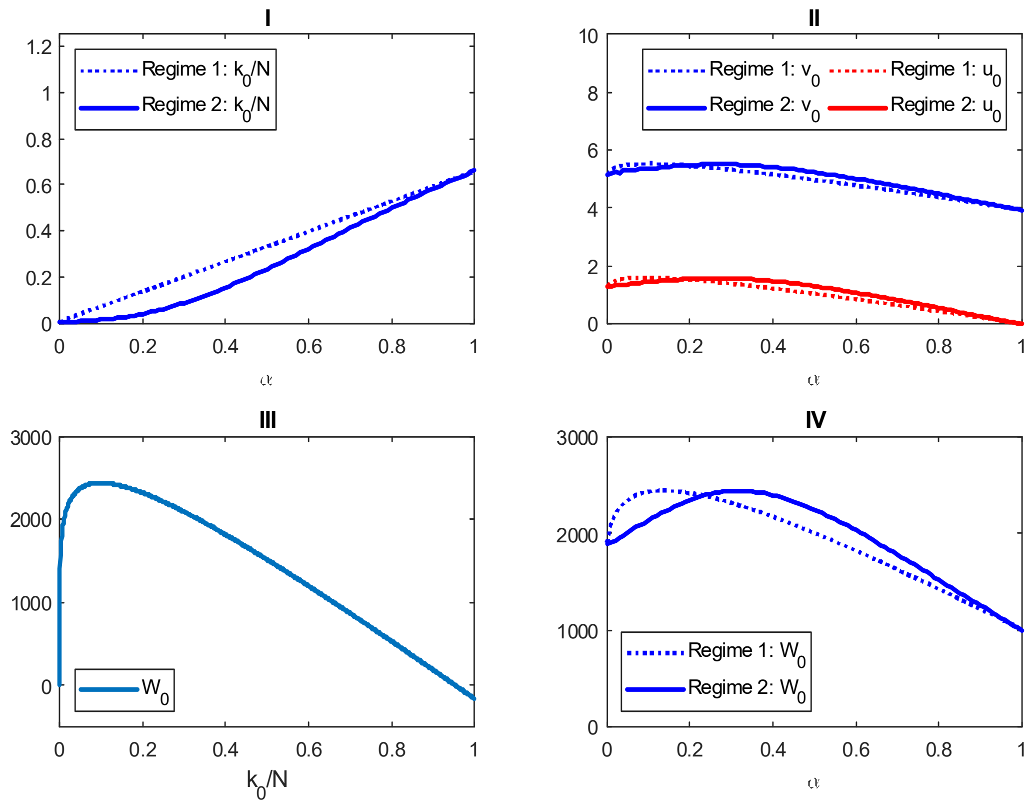

Fig. 14. Auto Industry Model Re-Calibration: Effects of α

Second, welfare is maximized at $k_0 = 100$ (i.e., $k_0/N = 0.1$), which can be achieved by choosing $\alpha^{SOC} = 0.14$ under Regime 1 and $\alpha^{SOC} = 0.32$ under Regime 2. Compared to the benchmark calibration above, the socially optimal compensation share α^{SOC} becomes larger under either regime. Our comparative-statics analysis (shown by Fig. 10) suggests that the larger α^{SOC} is due to the larger N, the lower γ, and the higher c from the re-calibrated model.

Finally, because of the larger pool of potential producers, the automobile price would fall more in the long run than it does in the benchmark calibration, which leads to a higher level of welfare W_0.

Figure 15 plots the effects of a subsidy s on the entry of innovators k_0 and welfare W_0. The results show that k_0 increases in s, and social welfare peaks at $s = 2.56$ (i.e., lowering the entry cost from 3.94 to 1.38, a 65% reduction).

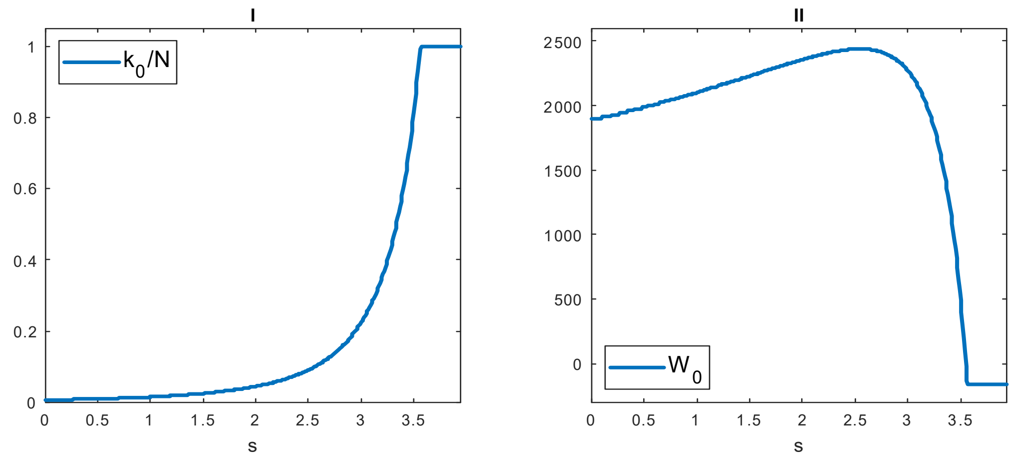

Fig. 15. Auto Industry Model Re-Calibration: Effects of s

Figure 16 plots the results of varying the value of γ, which are consistent with our findings in the benchmark calibration. The entry of innovators k_0 decreases in γ, but social welfare increases in γ.

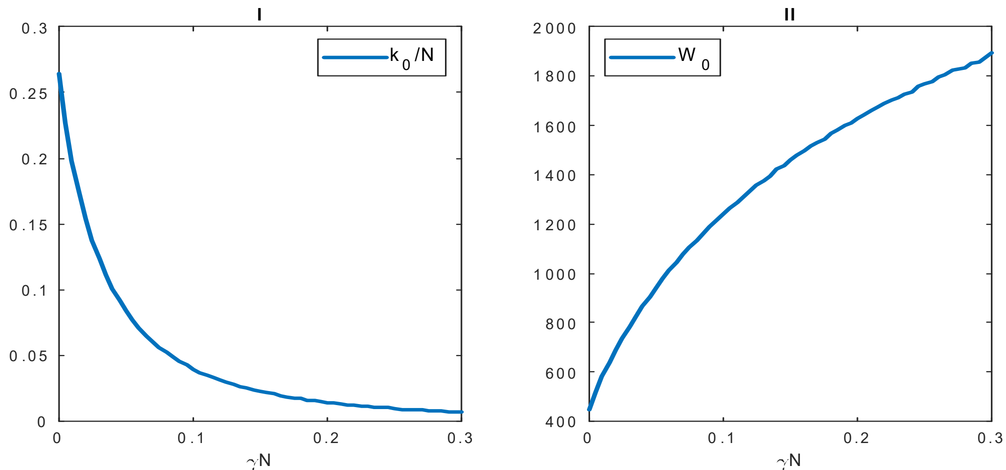

Fig. 16. Auto Industry Model Re-Calibration: Effects of γ

5.2 Personal computer industry

The personal computer (PC) industry was developed more recently than the automobile industry, but the industry evolution was not much different. Starting with two firms in 1975, the number of PC producers exceeded 430 in 1992. A shakeout then started while the industry output continued to expand, but the number of firms fell sharply. Figure 17 plots the number of firms and output per firm in the U.S. PC industry from 1975-1999.

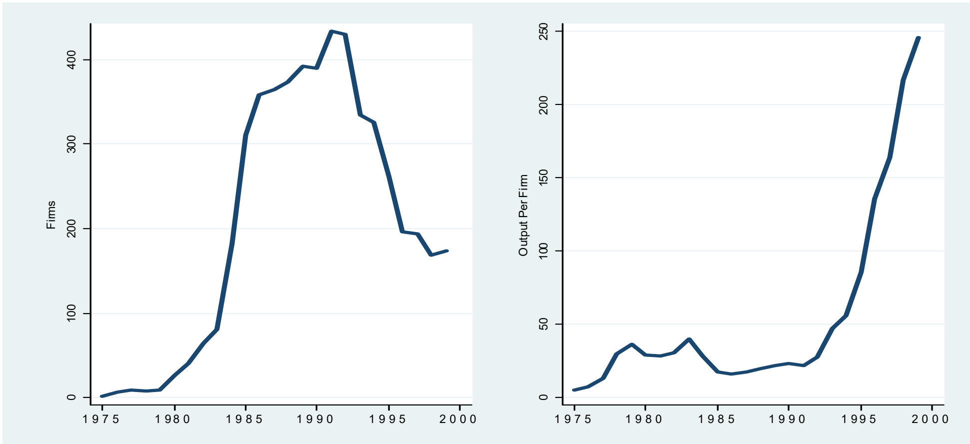

Fig. 17. PC Firm Numbers and Output Per Firm

Again, our model describes the pre-shakeout development of the PC industry well. As shown in Fig. 17, the growth of the PC industry mainly relied on the extensive margin before the shakeout. During that time, the time path of firm numbers followed an S-shaped curve and the average output per firm stayed pretty much flat. In the following analysis, we calibrate our model to the PC industry data for the pre-shakeout era (1975-1992) and conduct counterfactual exercises.

5.2.1 Data

Following Filson (2001), firm numbers in the personal computer industry are taken from Stavins (1995) and the *Thomas Register of American Manufacturers*. The data include desktop and portable computers. Price and quantity information for personal computers is from the *Information Technology Industry Data Book*. Williamson (2020) provides annual data of U.S. population, real GDP, and the GDP deflator.

5.2.2 Diffusion and demand estimation

To calibrate the model, we first use the data of firm numbers to estimate the diffusion parameters. We assume that the shakeout started after all the potential PC firms had entered the industry (and we will adjust this assumption later). Accordingly, we assume $N = 435$ and run the regression model (52). The result shows that

$$\ln \frac{k_t}{N - k_t} = \underset{(0.29)^{***}}{-5.49} + \underset{(0.03)^{***}}{0.58}\, t, \tag{55}$$

with the standard errors reported in the parentheses. All the coefficient estimates are statistically significant at 1% level (noted by three stars), and adjusted $R^2 = 0.96$. Based on the estimates of diffusion parameters, we calibrate $\gamma N = 0.58$, and $k_0 = 1.78$ (i.e., $\ln \frac{k_0}{N-k_0} = -5.49$).

We then estimate an industry demand function using annual data of PC prices p_t and output Q_t from 1975-1992. As before, in order to address potential endogeneity of the price variable, we use average output per firm (lagged by a year) as an instrumental variable to estimate the demand slope parameter ϕ. We also control for log U.S. GDP per capita as a proxy for income in the regression. Both PC price and GDP per capita are in real terms.

The first-stage regression results (adj. $R^2 = 0.95$) are

$$\ln(p_t) = \underset{(0.23)^{***}}{12.44} - \underset{(0.06)^{***}}{0.95} \times \ln(\frac{GDP_t}{pop_t}) - \underset{(0.01)^{***}}{0.07} \times \ln(\textit{output per firm})_{t-1}.$$

The second-stage regression results ($R^2 = 0.95$) in turn, are

$$\ln(\frac{Q_t}{pop_t}) = \underset{(29.00)^{***}}{143.18} - \underset{(2.63)}{2.90} \times \ln(\frac{GDP_t}{pop_t}) - \underset{(2.40)^{***}}{15.57} \times \ln(p_t).$$

Standard errors are reported in the parentheses, with three stars, two stars and one star indicating statistical significance at 1%, 5% and 10% level, respectively.

The IV estimation yields $\phi = 15.57$ and it is highly statistically significant. Accordingly, we calibrate the value of inverse demand elasticity in our model to be $\beta = \frac{1}{\phi} = 0.06$, which suggests that the PC industry is more price elastic than the auto industry.

Figure 18 plots the estimated firm numbers and per capita PC demand, and compares them with data.

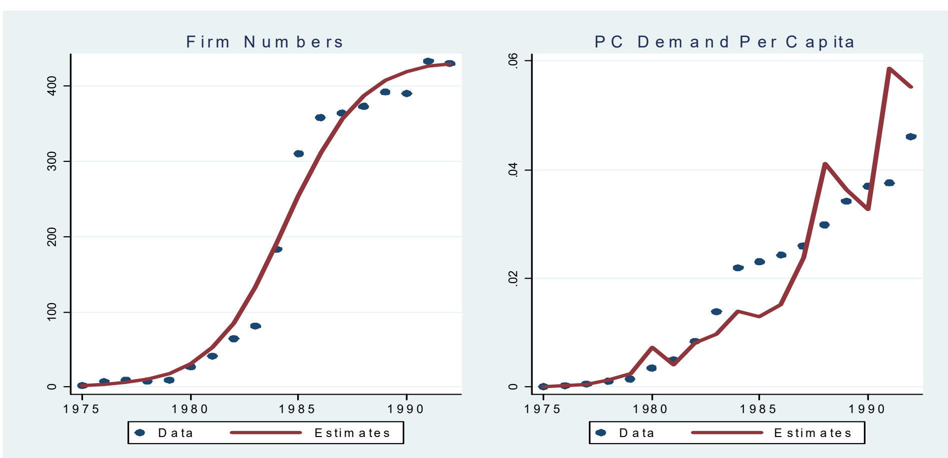

Fig. 18. PC Diffusion and Demand Estimation

5.2.3 Model calibration

Based on the estimation results, we calibrate the model parameter values $N = 435, \gamma N = 0.58$, $k_0 = 1.78$, $\beta = 0.06$, $r = 0.05$. We also normalize $A = 1$. Again, we assume that the industry data is generated by the model with free spillover (i.e., $\alpha = 0$).

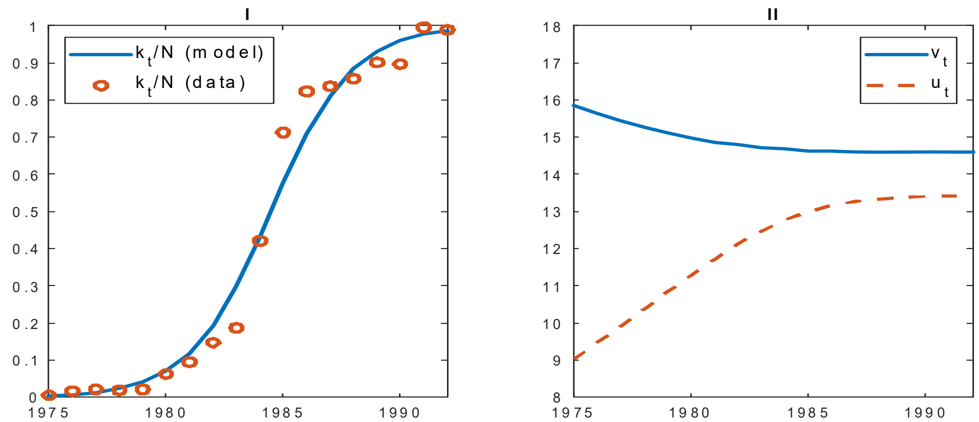

Fig. 19. PC Industry: Model Calibration

We numerically solve the time path (v_t, u_t) and pin down the cost of innovation c by the free entry condition $v_0 = u_0 + c$. Figure 19 plots the calibration results, which show the time paths of k_t/N, v_t and u_t. The number of firms k_t grows along a logistic curve. Meanwhile, v_t decreases while u_t increases over time, and the initial difference $v_0 - u_0$ pins down the innovation cost $c = 6.84$.

5.2.4 Counterfactual analysis

Given the calibrated model parameter values, we then conduct counterfactual analysis and evaluate welfare.

Optimal compensation share We first evaluate the effect of the compensation share α. Again, we consider two regimes: Regime 1 (under which imitators cannot resell the innovation) and Regime 2 (under which imitators can resell).

Figure 20(I) shows k_0 strictly increases in α for $0 \leq \alpha < 0.43$ under both regimes, and k_0 is smaller under Regime 2 than under Regime 1. Figure 20(II) shows that under both regimes, the free entry condition holds in terms of $v_0 - u_0 = c$ for $0 \leq \alpha < 0.43$. For $\alpha \geq 0.43$, because of the corner solution $k_0 = N$, the free entry condition holds in terms of $v_0 - u_0 > c$.

Figures 20(III)-(IV) plot social welfare W_0 according to different values of k_0 and α. The results show that social welfare W_0 is hump-shaped in k_0 and peaks at $k_0 = 71$ (i.e., $k_0/N = 0.16$). Accordingly, the social optimum can be achieved by choosing $\alpha^{SOC} = 0.05$ under Regime 1 or choosing $\alpha^{SOC} = 0.12$ under Regime 2. Note that neither $\alpha = 0$ nor $\alpha = 1$ would be socially optimal: the former yields $k_0 = 1.78$ and the latter yields $k_0 = N = 425$.

The finding that α^{SOC} is smaller for the PC industry than for the auto industry can be explained by the comparative-statics analysis shown in Fig. 10. Compared to the benchmark calibration of the auto industry, the PC industry has a smaller γ, a higher c, and a larger N. All these should have led to a larger value of α^{SOC} for the PC industry than for the auto industry. Therefore, the only reason that the PC industry has a smaller α^{SOC} is because the demand for PCs is more price elastic than for autos: According to our estimates, $\beta = 0.06$ for the PC industry while $\beta = 0.3$ for the auto industry.

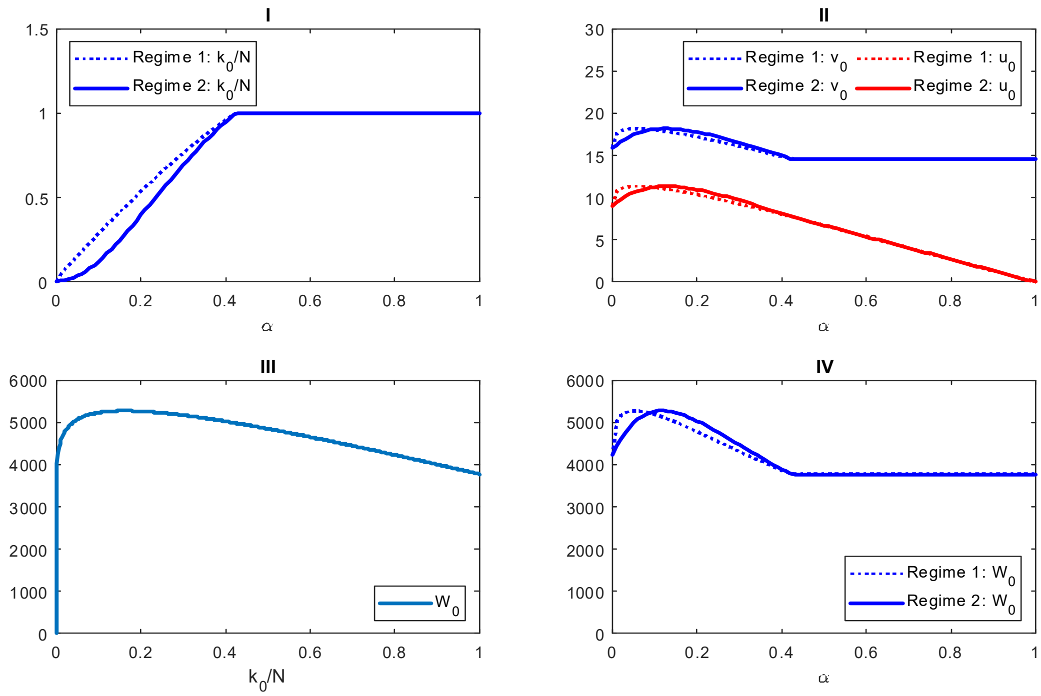

Fig. 20. PC Industry: Effects of α

Optimal innovation subsidy Figure 21 plots the effects of a subsidy s on the entry of innovators k_0 and welfare W_0. The results show that k_0 increases in s and the social welfare peaks at $s = 5.8$ (i.e., lowering the entry cost from 9.88 to 4.08, a 59% reduction).

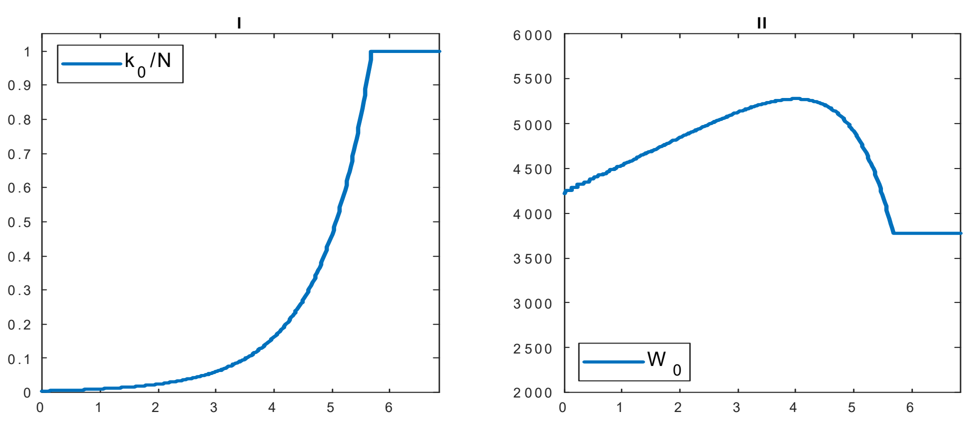

Fig. 21. PC Industry: Effects of s

Optimal diffusion rate We can similarly evaluate the effects of varying the diffusion rate γ. Figure 22 shows that if the planner were to push down γ from the original value where $\gamma N = 0.58$, the entry of innovators k_0 would increase. Eventually, k_0 would reach the corner solution $k_0 = N$ if the value of γ gets sufficiently low (i.e., $\gamma N \leq 0.05$). However, social welfare W_0 increases in γ, so the social planner would not want to slow down diffusion.

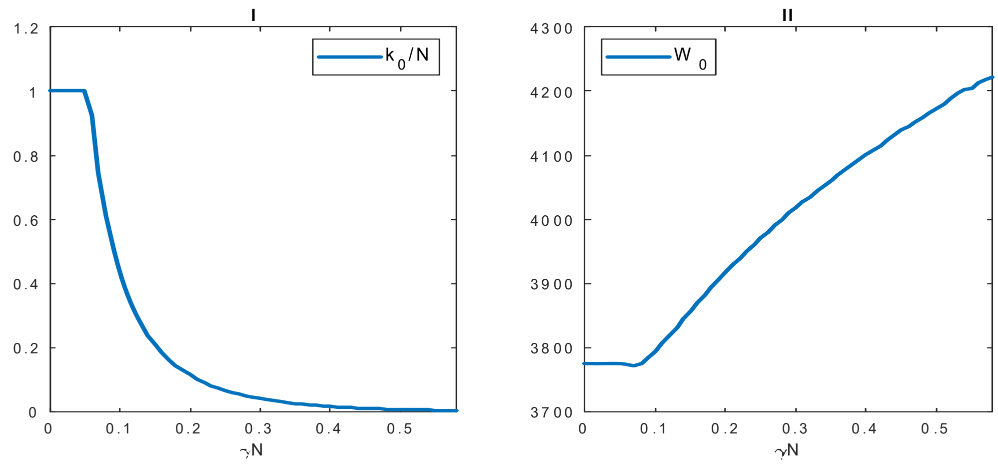

Fig. 22. PC Industry: Effects of γ

5.2.5 Model re-calibration: A larger pool of potential adopters

We can repeat the exercise by assuming a larger pool of potential adopters. Assume that the shakeout started in the middle of the diffusion and the number potential PC firms $N = 2,000$. In this scenario, it would have taken 30 years to reach 99% adoption rate among potential producers had the shakeout not happened, doubling what was assumed in the benchmark calibration. The exercise delivers very similar results as what we found in the automobile case (See Appendix VI for the details).

6 Conclusion

We modeled an innovation and its diffusion in one industry and discussed policy and welfare. Capacity constraints imply that licensing raises the revenues of innovators and that licensing is also socially beneficial to a degree. We showed that the welfare outcome depends on whether imitators can resell the innovation, and on how much the innovators are compensated for transferring the innovation.

We found that the socially optimal bargain allocation hinges on the diffusion process, particularly the congestion externality in meetings between innovators and imitators. Our analysis also showed that slowing down diffusion encourages innovation and raises initial capacity, but it lowers imitation so that capacity grows more slowly. We argued that this may help explain the overtaking of Route 128 by the Silicon Valley.

We calibrated the model to data of the U.S. automobile and personal computer industries. Our empirical findings match well the expansion of firm numbers prior to the shakeout in each industry and quantify the theoretical predictions of the model.

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Appendix I.

Note that with $\alpha = 0$, we can rewrite Eq. (39) as

$$rv_t = A(\frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + b})^{-1} + \frac{dv_t}{dt},$$

which has an unique bounded solution

$$v_t = \frac{A}{Nr} + \frac{Ab}{Ne^{\gamma Nt}\,(r + N\gamma)}. \tag{56}$$

We can also rewrite Eq. (40) as

$$ru_t = \frac{\gamma Ne^{\gamma Nt}}{e^{\gamma Nt} + b}\left(\frac{A}{Nr} + \frac{Ab}{Ne^{\gamma Nt}\,(r + \gamma N)} - u_t\right) + \frac{du_t}{dt}, \tag{57}$$

which has an unique bounded solution that is constant over time

$$u_t = \frac{A\gamma}{r(r + \gamma N)}.$$

At $t = 0$, the free entry condition requires $v_0 - u_0 = c$, thus Eqs. (56) and (57) yield

$$Ar(1 + b) = cNr\,(r + \gamma N)\,.$$

Given Eq. (27) that $b = \frac{N}{k_0} - 1$, we can solve the entry of innovators k_0^{CAN} at date 0:

$$k_0^{CAN} = \frac{A}{c(r + \gamma N)}.$$

Similarly, we can solve the case for $\alpha = 1$. We rewrite Eq. (39) as

$$rv_t = A(\frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + b})^{-1} + \left(\frac{bN\gamma}{e^{\gamma Nt} + b}\right) v_t + \frac{dv_t}{dt},$$

which has an unique bounded solution :

$$v_t = \frac{A}{Nr} + \frac{Ab}{Nre^{N\gamma t}}.$$

We can also rewrite Eq. (40) as

$$ru_t = -\frac{\gamma Ne^{\gamma Nt}}{e^{\gamma Nt} + b}u_t + \frac{du_t}{dt},$$

which has an unique bounded solution: $u_t = 0$.

At $t = 0$, the free entry condition requires $v_0 - u_0 = c$, thus Eqs. (56) and (57) yield

$$\frac{A(1+b)}{Nr} = c.$$

Given Eq. (27) that $b = \frac{N}{k_0} - 1$, we can solve the entry of innovators k_0^{CAN} at date 0:

$$k_0^{CAN} = \frac{A}{rc}.$$

Appendix II.

With $r = \gamma N$, we can rewrite Eq. (39) as

$$\frac{dv_t}{dt} + \left(\frac{\alpha b}{e^{rt} + b} - 1\right) r v_t + \frac{A}{N}(1 + be^{-rt}) = 0, \tag{58}$$

which has a general solution

$$v_t = \frac{A\left(b + e^{rt}\right)^2}{N(2-\alpha)bre^{rt}} + \Psi e^{(1-\alpha)rt}\left(b + e^{rt}\right)^{\alpha}. \tag{59}$$

Using the boundary condition that $v_{t\to\infty} < \infty$, we can pin down $\Psi = \frac{-A}{N(2-\alpha)br}$. To see this, note that Eq. (59) suggests

$$v_t = \frac{Ab^2 + 2Abe^{rt} + Ae^{2rt}}{N(2-\alpha)bre^{rt}} + \Psi e^{rt}\left(1 + be^{-rt}\right)^{\alpha}.$$

Using Taylor expansion, we have

$$\begin{aligned} v_t &= \frac{Ab}{N(2-\alpha)re^{rt}} + \frac{2A}{N(2-\alpha)r} + \frac{Ae^{rt}}{N(2-\alpha)br} \\ &\quad + \Psi e^{rt}\left(1 + \alpha be^{-rt} + \frac{1}{2}\alpha(\alpha-1)b^2e^{-2rt} + \ldots\right) \\ &= \frac{Ab}{N(2-\alpha)re^{rt}} + \frac{2A}{N(2-\alpha)r} + \left(\frac{A}{N(2-\alpha)br} + \Psi\right)e^{rt} \\ &\quad + \alpha b\Psi + \frac{\Psi}{2}\alpha(\alpha-1)b^2e^{-rt} + \ldots \end{aligned}$$

To satisfy the boundary condition $v_{t\to\infty} < \infty$, we must have $\Psi = \frac{-A}{N(2-\alpha)br}$, so the terms involving e^{rt} sum to 0 and

$$v_{t\to\infty} = \frac{2A}{N(2-\alpha)r} - \frac{\alpha A}{N(2-\alpha)r} = \frac{A}{Nr}.$$

Hence, we can derive the particular solution:

$$v_t = \frac{A\left(b+e^{rt}\right)^2}{N(2-\alpha)bre^{rt}}\left(1-e^{(2-\alpha)rt}\left(b+e^{rt}\right)^{\alpha-2}\right). \tag{60}$$

Accordingly, we have

$$v_0 = \frac{A}{N(2-\alpha)br}\left((b+1)^2-(b+1)^{\alpha}\right). \tag{61}$$

We can then solve closed-form solution for u_0 without involving v_t. Note that total discounted revenue is

$$R_0 = \int_0^{\infty} e^{-rt}Adt = \frac{A}{r} \tag{62}$$

This is shared by the two groups – the initial incumbents k_0 and the outsiders $N-k_0$:

$$\begin{aligned} R_0 &= k_0v_0+(N-k_0)\,u_0 \\ &= Nu_0+k_0\,(v_0-u_0) \\ &= Nu_0+ck_0, \end{aligned}$$

or equivalently,

$$R_0 = Nv_0 - c\,(N-k_0)\,.$$

Then substituting for R_0 from Eq. (62),

$$Nv_0 - c\,(N-k_0) = \frac{A}{r}$$

and substituting for v_0 from Eq. (61), we get

$$\frac{A}{(2-\alpha)br}\left((b+1)^2-(b+1)^{\alpha}\right) - c\,(N-k_0) = \frac{A}{r}.$$

Since $b = \frac{N}{k_0}-1$, this yields

$$\frac{N-k_0}{\frac{k_0}{(2-\alpha)(N-k_0)}\left(\left(\frac{N}{k_0}\right)^2-\left(\frac{N}{k_0}\right)^{\alpha}\right)-1} = \frac{A}{rc}.$$

Appendix III.

Eq. (51) suggests that

$$\frac{dW_0}{d\gamma} = \frac{A}{(r+\gamma)^2} + A\int_0^{\infty} e^{-rt}tdt - A\int_0^{\infty} e^{-rt}\frac{te^{\gamma t}+\frac{c}{A}}{e^{\gamma t}+\frac{c(r+\gamma)}{A}-1}dt. \tag{63}$$

We first verify that given $\gamma > 0$ and $r > 0$,

$$\frac{d\left(\frac{te^{\gamma t}+\frac{c}{A}}{e^{\gamma t}+\frac{c(r+\gamma)}{A}-1}\right)}{dc} = \frac{e^{\gamma t}-1-te^{\gamma t}(r+\gamma)}{A[e^{\gamma t}+\frac{c(r+\gamma)}{A}-1]^2} < 0$$

for any $t > 0$ (note that $e^{\gamma t}-1-te^{\gamma t}(r+\gamma) = 0$ for $t = 0$ and $\frac{\partial(e^{\gamma t}-1-te^{\gamma t}(r+\gamma))}{\partial t} < 0$). Recall Eq. (50) that $k_0 = \frac{A}{c(r+\gamma)}$. For any $1 > k_0 > 0$, this requires $c > \frac{A}{r+\gamma}$, so we have

$$A\int_0^\infty e^{-rt}\frac{te^{\gamma t}+\frac{c}{A}}{e^{\gamma t}+\frac{c(r+\gamma)}{A}-1}dt < A\int_0^\infty e^{-rt}\frac{te^{\gamma t}+\frac{1}{r+\gamma}}{e^{\gamma t}}dt. \tag{64}$$

Therefore, Eqs. (63) and (64) yield that

$$\begin{aligned}\frac{dW_0}{d\gamma} &> \frac{A}{(r+\gamma)^2}+A\int_0^\infty e^{-rt}tdt - A\int_0^\infty e^{-rt}\frac{te^{\gamma t}+\frac{1}{r+\gamma}}{e^{\gamma t}}dt \\ &= \frac{A}{(r+\gamma)^2}+A\int_0^\infty e^{-rt}tdt - A\int_0^\infty e^{-rt}tdt - \frac{A}{(r+\gamma)^2} = 0.\end{aligned}$$

This also implies that for $k_0 = 1$, we have $c = \frac{A}{r+\gamma}$, so $\frac{dW_0}{d\gamma} = 0$.

Appendix IV.

We rewrite the model into the discrete-time version and then solve it numerically.

Model calibration:

$$k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt}+b} \quad \text{where} \quad b = \frac{N}{k_0}-1 > 0;$$

$$p_t = Ak_t^{-\beta};$$

$$v_t = p_t + \frac{1}{1+r}v_{t+1};$$

$$u_t = \frac{1}{1+r}\left(\gamma k_t v_{t+1} + (1-\gamma k_t)u_{t+1}\right).$$

Welfare:

$$W_0 = -ck_0 + \sum_{t=0}^{\infty}\frac{1}{(1+r)^t}\left(\frac{A}{1-\beta}k_t^{1-\beta}\right).$$

Counterfactual analysis:

Regime 1. Imitators cannot resell ideas.

Denote ω_t the value of an imitator, v_t the value of an innovator, and u_t the value of an outsider at date t, respectively.

$$k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + b} \quad \text{where} \quad b = \frac{N}{k_0} - 1 > 0;$$

$$p_t = Ak_t^{-\beta};$$

$$\omega_t = p_t + \frac{1}{1+r}\omega_{t+1} \quad \text{for} \quad t \geq 1;$$

$$v_t = \begin{cases} p_t + \frac{1}{1+r}v_{t+1} & \text{for} \quad t = 0, \\ p_t + \frac{\gamma k_{t-1}(N-k_{t-1})}{k_0}\alpha\omega_t + \frac{1}{1+r}v_{t+1} & \text{for} \quad t \geq 1; \end{cases}$$

$$u_t = \frac{1}{1+r}\left(\gamma k_t(1-\alpha)\omega_{t+1} + (1-\gamma k_t)u_{t+1}\right).$$

Regime 2. Imitators can resell ideas.

Denote ω_t the value of a new imitator, v_t the value of an incumbent, and u_t the value of an outsider at date t, respectively.

$$k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + b} \quad \text{where} \quad b = \frac{N}{k_0} - 1 > 0;$$

$$p_t = Ak_t^{-\beta};$$

$$\omega_t = p_t + \frac{1}{1+r}v_{t+1} \quad \text{for} \quad t \geq 1;$$

$$v_t = \begin{cases} p_t + \frac{1}{1+r}v_{t+1} & \text{for} \quad t = 0, \\ \begin{array}{l} p_t + \gamma(N-k_{t-1})\alpha\omega_t + \frac{1}{1+r}v_{t+1} \\ = [1+\gamma(N-k_{t-1})\alpha](p_t + \frac{1}{1+r}v_{t+1}) \end{array} & \text{for} \quad t \geq 1; \end{cases}$$

$$u_t = \frac{1}{1+r}\left(\gamma k_t(1-\alpha)\omega_{t+1} + (1-\gamma k_t)u_{t+1}\right).$$

Appendix V.

We extend the large industry model to allow firms to exit at a constant rate σ every period, and the analysis is very similar.

Assume that an innovation takes a logistic diffusion process among N potential adopters. Each period, a fraction σ of firms which have adopted the innovation exit the industry (i.e., unadopt the innovation). Once an incumbent firm leaves the industry, it will be replaced by a new entity entering the pool of potential adopters, so that the total number of potential adopters remains N.

The law of motion for firm numbers is

$$\frac{dk_t}{dt} = \gamma k_t(N - k_t) - \sigma k_t.$$

Solving the ODE yields the solution of k_t:

$$k_t = \frac{N - \frac{\sigma}{\gamma}}{1 + \left((\frac{N-\sigma/\gamma}{k_0} - 1) \right) \exp\left(-\left(N - \frac{\sigma}{\gamma} \right) \gamma t \right)}, \tag{65}$$

and there exists a steady state $k_{t\to\infty} \to k_{ss}$ where

$$k_{ss} = N - \frac{\sigma}{\gamma}.$$

Substituting $N = k_{ss} + \frac{\sigma}{\gamma}$ into Eq. (65) we obtain

$$k_t = \frac{k_{ss}}{1 + \left(\frac{k_{ss}}{k_0} - 1 \right) \exp\left(-k_{ss}\gamma t \right)}, \tag{66}$$

which is the same as the diffusion equation in our original model except that N is replaced by k_{ss}.

Diffusion estimation.—To estimate the extended model, we can rewrite Eq. (66) as

$$\ln\left(\frac{k_t}{k_{ss} - k_t} \right) = -\ln\left(\frac{k_0}{k_{ss} - k_0} \right) + \gamma k_{ss} t. \tag{67}$$

Assume that the industry has achieved the steady-state number of firms before the shakeout, we assign $k_{ss} = 210$ (i.e., the observed peak number of firms). As a result, we are back to the exactly same regression model as before, and all the estimation results remain unchanged: $\gamma k_{ss} = 0.53$, and $k_0 = 3.31$ (i.e., $\ln \frac{k_0}{k_{ss}-k_0} = -4.13$).

Model calibration.—Denote v_t the value of an incumbent, and u_t the value of an outsider at date t, respectively. As before, we assume that the industry data are generated by the model with free spillover (i.e., $\alpha = 0$), so it makes no difference whether imitators can or cannot resell the technology.

The model is now represented by the following equations:

$$k_t = \frac{k_{ss} e^{k_{ss}\gamma t}}{e^{k_{ss}\gamma t} + b}, \quad \text{where} \quad b = \frac{k_{ss}}{k_0} - 1 > 0;$$

$$p_t = Ak_t^{-\beta};$$

$$v_t = p_t + \frac{1-\sigma}{1+r} v_{t+1};$$

$$u_t = \frac{1}{1+r} \left(\gamma k_t v_{t+1} + (1 - \gamma k_t) u_{t+1} \right).$$

We calibrate the model parameter values as before: $k_{ss} = 210$, $\gamma k_{ss} = 0.53$, $k_0 = 3.31$, $\beta = 0.3$. We assume the exit rate $\sigma = 0.15$. Again, we normalize $A = 1$ and assume the discount parameter $r = 0.05$.

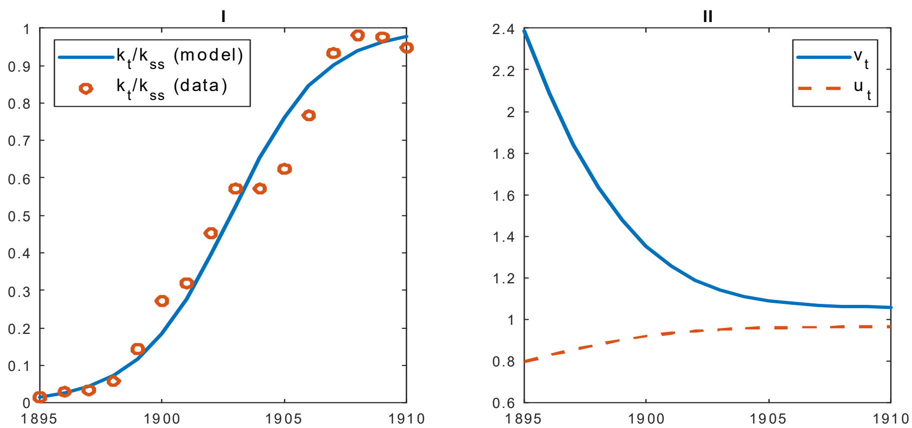

Fig A1. Auto Industry Calibration: Model Extension

Figure A1 plots the calibration results. The difference of $v_0 - u_0$ pins down the innovation cost $c = 1.59$. The estimated value of c is smaller than what we got in the original model calibration because firms take into account the exit rate (i.e., firms have zero scrape value when they exit).

Welfare.—The welfare equation remains the same as our original model.

Appendix VI.

We assume a larger pool of potential PC firms. The exercise delivers very similar results as what we found in the automobile case.

Assuming $N = 2,000$, we re-run the regression model (52). The result shows that

$$\ln \frac{k_t}{N - k_t} = \underset{(0.27)^{***}}{-6.09} + \underset{(0.03)^{***}}{0.34}\ t, \tag{68}$$

and the standard errors are reported in the parentheses.

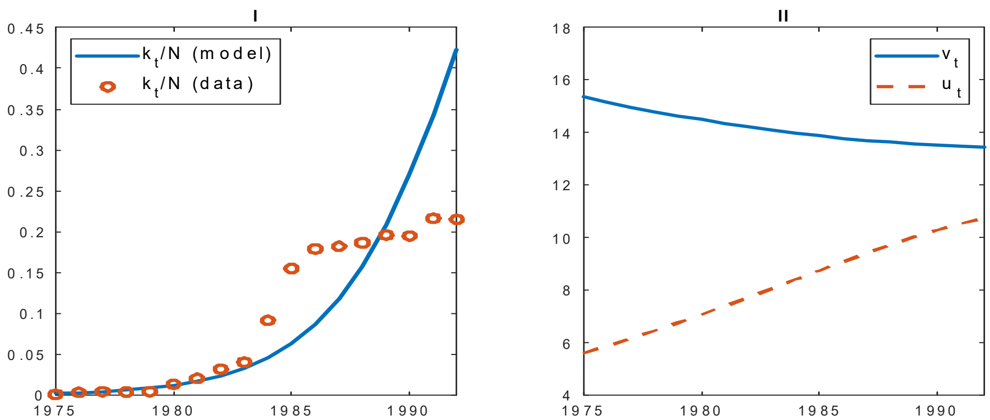

Fig. A2. PC Industry: Model Re-Calibration

Based on the estimates of diffusion parameters, we calibrate $N = 2000$, $\gamma N = 0.34$, and $k_0 = 4.51$ (i.e., $\ln \frac{k_0}{N-k_0} = -6.09$). In addition, we set $\beta = 0.06$, $r = 0.05$, and normalize $A = 1$ as before. Figure A2 plots the calibrated time paths of k_t, v_t and u_t. The initial difference $v_0 - u_0$ pins down the innovation cost $c = 9.75$.

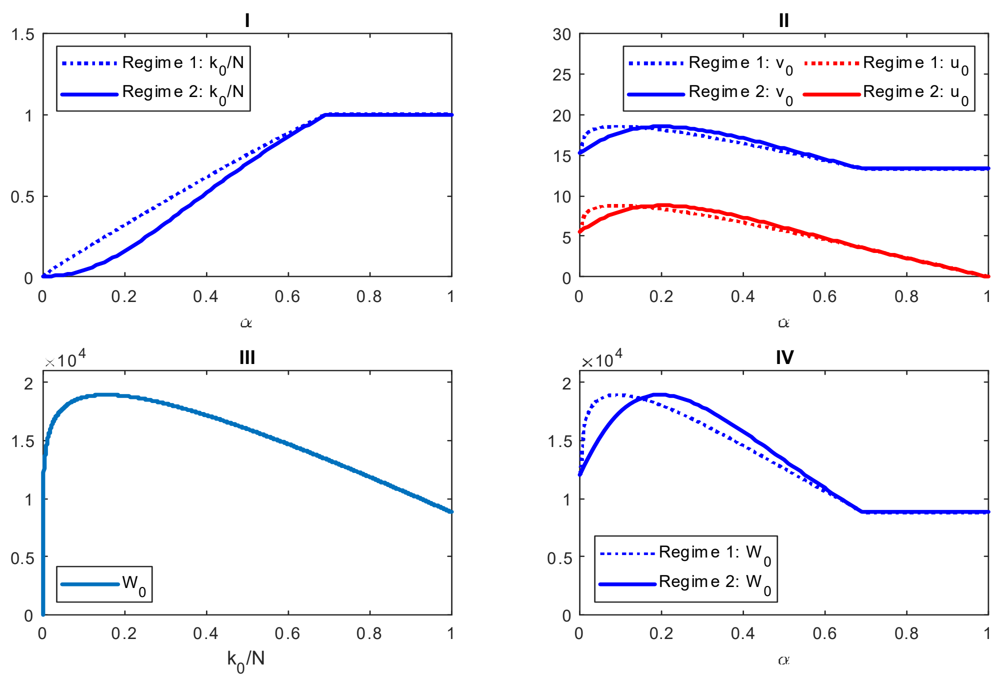

Fig. A3. PC Industry Model Re-Calibration: Effects of α

Figure A3 plots the results of varying the value of α, which is consistent with our findings from the auto case. First, given a slower diffusion speed γ, the entry of innovators k_0 does not reach the full adoption unless $\alpha \geq 0.7$. Second, welfare maximizes at $k_0 = 300$ (i.e., $k_0/N = 0.15$), which can be achieved by choosing

$\alpha^{SOC} = 0.09$ under Regime 1 or $\alpha^{SOC} = 0.20$ under Regime 2. Compared to the benchmark calibration, the optimal compensation share α^{SOC} becomes larger in either regime, which is driven by the larger N, the lower γ, and the higher c from the re-calibrated model.

Figure A4 plots the effects of a subsidy s on the entry of innovators k_0 and welfare W_0. The results show that k_0 increases in s, and the social welfare peaks at $s = 5.8$ (i.e., lowering the entry cost from 9.75 to 3.95, a 59% reduction).

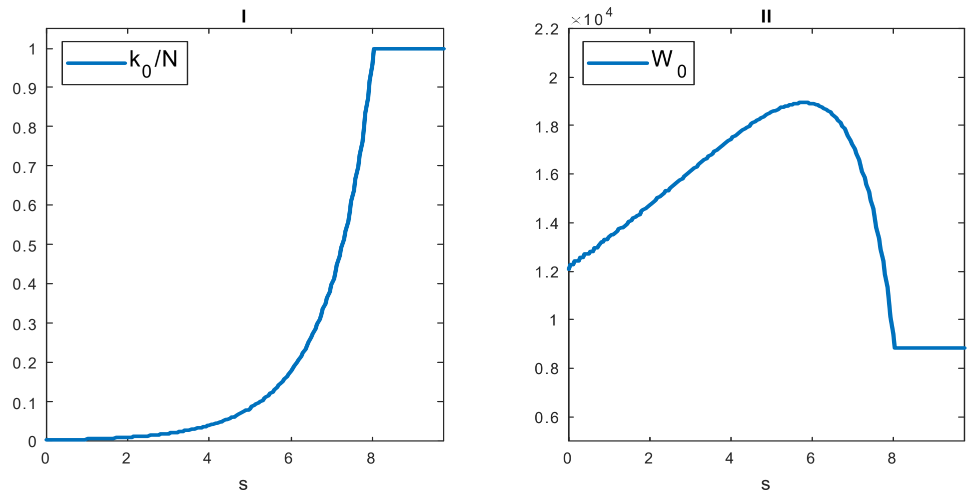

Fig. A4. PC Industry Model Re-Calibration: Effects of s

Finally, Fig. A5 plots the results of varying the value of γ. As before, we find that the entry of innovators k_0 decreases in γ, while social welfare increases in γ.

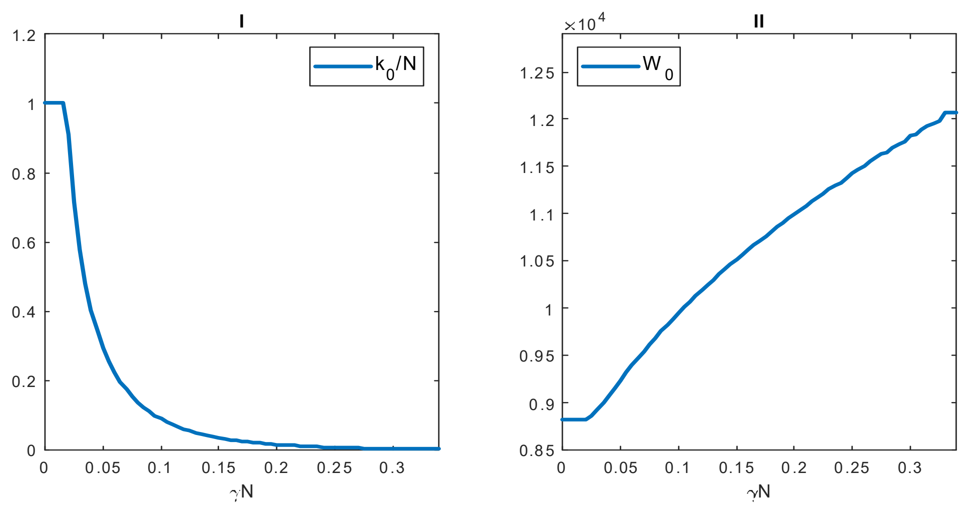

Fig. A5. PC Industry Model Re-Calibration: Effects of γ