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Contingent Debt and Performance Pricing in an Optimal Capital Structure Model with Financial Distress and Reorganization

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Borys Grochulski
Federal Reserve Bank of Richmond

Russell Wong
Federal Reserve Bank of Richmond

FEDERAL RESERVE BANK OF RICHMOND®
Richmond ▪ Baltimore ▪ Charlotte

Contingent debt and performance pricing in an optimal capital structure model with financial distress and reorganization*

Borys Grochulski† Russell Wong‡

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Abstract

Building on the trade-off between agency costs and monitoring costs, we develop a dynamic theory of optimal capital structure with financial distress and reorganization. Costly monitoring eliminates the agency friction and thus the risk of inefficient liquidation. Our key assumption is that monitoring cannot be applied instantaneously. Rather, transitions between agency and monitoring are subject to search frictions. In the optimal contract, the firm seeks a monitoring opportunity whenever it is financially distressed, i.e., when the risk of liquidation is high. If a monitoring opportunity arrives in time, the manager is dismissed, the capital structure is reorganized as in Chapter 11 bankruptcy, and the search for a new manager begins. In agency, an optimal capital structure consists of equity, long-term debt, contingent long-term debt, and a credit line with performance pricing. In financial distress, coupon payments to contingent debt are suspended but the interest rate on the credit line is stepped-up, which gives the firm simultaneously debt relief and a steep incentive to improve its financial position. An episode of distress can end with financial recovery, transition to bankruptcy reorganization, or liquidation.

Keywords: capital structure, contingent debt, performance pricing, monitoring costs, agency costs, dynamic incentives, liquidation, financial restructuring, bankruptcy reorganization, search frictions, CEO compensation, CEO replacement

JEL codes: G32, G33, D86, D82, M52, C61

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†Federal Reserve Bank of Richmond, borys.grochulski@rich.frb.org.

‡Federal Reserve Bank of Richmond, russell.wong@rich.frb.org.

1 Introduction

In financial distress, a firm is much more likely to seek to reorganize its capital structure and continue to operate rather than to stop its operations and liquidate.[1] Modern theories of optimal capital structure of the firm, built around resolving dynamic agency problems, however, allow only for financial recovery or liquidation and do not consider the possibility of exiting financial distress by financial reorganization. In this paper, our objective is to take a step toward bridging this gap.

We build a dynamic model in which an optimal capital structure, in the spirit of Jensen (1986), is determined by the trade-off between agency costs and monitoring costs. To model agency costs, we follow DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), and Biais et al. (2007) in solving an optimal contracting problem between the firm owners and a manager who can divert funds to private use. To provide incentives, an optimal contract must credibly threaten (ex-post inefficient) liquidation. We model monitoring as an alternative to agency that eliminates the risk of liquidation but entails direct costs.

Our key assumption is that monitoring cannot be applied instantaneously. Rather, transitions between agency and monitoring are subject to search frictions. That is, agency and monitoring are states. Under an optimal contract, in the agency state, the firm seeks a monitoring opportunity whenever it is financially distressed, i.e., when the risk of liquidation is high. If a transition to monitoring arrives in time, the firm's cash flows become publicly observable, and the manager is dismissed. If such a transition does not arrive in time, the firm is forced to liquidate, as in the standard model. The monitoring costs incurred in the monitoring state are sufficiently high for the firm to seek to hire a new manager and transition to the agency state again.

In agency, an optimal capital structure consists of equity, long-term debt, contingent long-term debt, and a credit line with performance pricing. Financial distress is defined by a leverage trigger, i.e., when the balance on the credit line exceeds a threshold. When the balance remains above the distress threshold, the interest rate on the credit line is elevated but coupon payments on contingent debt are suspended. The firm issues enough contingent debt ex ante that these two interest rate adjustments imply a net relief in terms of the debt servicing costs to the firm in distress. This relief moderates the risk of liquidation and helps cover any expenses related to preparation for transition to the monitoring state. An episode of distress can end with financial recovery, a transition to monitoring, or in liquidation that takes place if the firm is unable to transition to monitoring before it exhausts its total credit limit.

[1] See, e.g., Bris et al. (2006), Jacobs Jr et al. (2012), and Corbae and D'Erasmo (2017).

In the monitoring state, as in Chapter 11 bankruptcy, the firm reorganizes its capital structure while paying the costs of monitoring its cash flow and searching for an opportunity to emerge from bankruptcy with a new capital structure and a new manager. Upon transitioning to agency again, new debt is issued and new equity is allocated to the owners and the new manager.

This optimal capital structure and the conditions under which the firm searches for an opportunity to replace management and reorganize, i.e., our definition of financial distress, are determined endogenously by the trade-off between agency and monitoring costs.

Our theory provides a new explanation of the role of performance pricing and contingent debt in the firm's capital structure, which we view as the main contribution of this paper. The optimality of performance pricing and the suspension of contingent debt coupon payments in financial distress is a consequence of the jump that the manager's continuation value experiences under the optimal contract upon a transition to monitoring. Although we allow for a severance payment to the manager, the optimal contract calls for zero severance, as this way the manager's loss of value at the transition to monitoring is maximized. Prior to the transition, this loss is compensated with higher expected growth (drift) of the manager's continuation value, which helps reduce the risk of liquidation. When leverage is high, i.e., the draw on the credit line is above the distress threshold, the firm searches for a transition to bankruptcy reorganization and the manager faces a positive probability of being instantaneously dismissed with no severance. When leverage is low, i.e., the credit balance is below the distress threshold, the firm is financially sound, does not search for bankruptcy, and the manager faces no risk of being dismissed. This difference between distress and financially sound conditions gives a role for performance pricing and contingent debt in our model.

Contingent debt that we obtain as a part of an optimal capital structure shares features with contingent convertible (CoCo) bonds, which have recently been introduced into bank capital structures in Asia and Europe, and have received a lot of attention from regulators an academics.[2] The main similarity is the idea of providing a "going concern" capital relief to a firm in financial distress. In our model, the contingent debt contract mandates a (noncumulative) suspension of coupon payments, based on an accounting trigger, similar to many CoCo bonds used in practice. Conversion to equity or a write-down at default, although feasible, are not essential features of the optimal capital structure in our model.[3]

We compute security values and comparative statics. The option to seek monitoring and

[2] See, e.g., Flannery (2005, 2014), Hanson et al. (2011), Chen et al. (2017).

[3] In non-bank capital structures, the function of contingent capital is often assumed by preferred equity. In our model, differences between contingent debt and preferred equity are relatively minor. We discuss them in the Appendix.

reorganization makes the firm's securities less risky. While the total debt capacity of the firm is increased, the optimal size of the credit line is shorter, allowing the firm to pay dividends sooner and making the risk of liquidation lower. Valuation of the firm's securities depends on their seniority structure in reorganization or liquidation. For example, if contingent debt is junior to uncontingent long-term debt, the latter form of debt becomes much less risky. Using an example, we show that the optimal contract may actually call for very little uncontingent debt to be issued, thus making it completely riskless.

Relation to the literature This paper is primarily related to the literature on agency-based theory of optimal capital structure and managerial compensation: DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), Biais et al. (2007), among others. We extend the model by allowing for agent dismissal and monitoring of the cash flow with financial restructuring, subject to search frictions.

Piskorski and Westerfield (2016) study the impact of monitoring on the optimal contract in the dynamic agency setting, but their monitoring technology is different. In their model, monitoring can generate an imperfect signal of the manager's resource diversion or shirking. Although such signals remain off-equilibrium, monitoring complements performance-based compensation in providing incentives to the manager. In our model, in contrast, monitoring directly substitutes the manager, who is dismissed when the firm transitions to the monitoring state, which does occur in equilibrium. The dismissal of the manager is an important element of our theory of financial restructuring, as in reorganization the manager's equity position in the firm is wiped out.

Tchistyi (2016) shows that a credit line with performance pricing is a part of an optimal capital structure in an agency-based model with correlated cash flows, where the agent has a stronger incentive to divert resources when the cash flow is high. Our analysis provides an alternative explanation of performance pricing based on the risk of a discrete jump in the agent's continuation value at dismissal.

In a structural model in the tradition of Leland (1994), where debt is valued for its tax advantages, Manso et al. (2010) study performance-sensitive debt and show that contingent debt instruments can be useful as a screening device allowing high-growth firms to signal their type in a separating equilibrium. In the same tradition, Antill and Grenadier (2018) study the choice of leverage and the pricing of debt allowing the firm to use a bankruptcy reorganization as an alternative to straight liquidation. Our paper complements these studies, as we obtain contingent debt and bankruptcy reorganization as a part of an optimal contract trading off agency and monitoring costs.

This paper is also related to the large literature on optimal contracts with information frictions and costly monitoring, which goes back to Townsend (1979) and Gale and Hellwig (1985) in static settings and includes dynamic analyses of Monnet and Quintin (2005), Wang (2005), Antinolfi and Carli (2015), Chen et al. (2017), and Varas et al. (2017), among others. The innovation of our model is to add a search friction in the spirit of Mortensen and Pissarides (1994) and Duffie et al. (2005).

Organization Section 2 describes the contracting environment. Sections 3 and 4 characterize the firm value and the optimal contract. Section 5 discusses a capital structure implementation. Section 6 computes security values and derives comparative statics. Section 7 concludes.

2 Model

The contracting problem builds on DeMarzo and Sannikov (2006), hereafter DS, and extends it to two states: an agency state and a monitoring state. The firm can switch between the two states subject to search frictions. The agency state is similar to DS except for an additional choice variable, which represents the decision to search for a transition to the monitoring state. In the agency state, an agent/manager/CEO is hired to run the firm, as in DS. In the monitoring state, the firm is run by an expert, and the cash flow is public information, i.e., there are no agency frictions, but the firm is subject to additional monitoring costs. Any negative cash flows are covered by external financing, which must be secured during the process of searching for a transition to monitoring.

Cumulative cash flow up to date t, Y_t, follows

$$dY_t = \mu dt + \sigma dZ_t,$$

where Z_t is a standard Brownian motion on $(\Omega, \mathcal{F}, P)$, and $\mu > 0$ and $\sigma > 0$ are constant.

In the monitored state, an expert runs the firm and absorbs the risk of the cash flow. The owners compensate the expert and pay the costs of searching for a new manager to reenter the agency state.[4] Net of this compensation, the owners receive the flow of $\mu - \kappa_B$, which can be negative. The flow cost $\kappa_B \geq 0$ covers the expert costs. In the monitoring state, the cash flow is public information, i.e., it cannot be diverted to private use. The flow cost κ_B also covers the cost of searching for an opportunity to exit the monitoring state, back to the agency state, which arrives with intensity $\phi > 0$. When this opportunity arrives, the owners hire the manager and release the expert, thus avoiding paying the flow cost κ_B.

[4] We will make assumptions on the parameter values of the model sufficient for staying in the monitoring state permanently to be inefficient. I.e., the firm wants to exit the monitoring state as soon as possible.

In the agency state, the agent/manager runs the firm subject to the standard agency friction. The cash flow is privately observed by the agent. The agent can divert the cash flow to private use. Of each unit diverted, the agent retains $0 < \lambda \leq 1$ and $1 - \lambda$ is wasted.[5]

In the agency state, the firm can also take a costly action of searching for an opportunity to transition to monitoring. In monitoring, the firm must have financing available to cover any negative cash flows. Our key assumption is that such financing cannot be obtained instantaneously, but rather only after a search period of random duration. The cost of searching for a transition to monitoring is $\kappa \geq 0$. Conditional on searching, such transitions arrive with intensity $\rho > 0$. Upon transition, the operations are taken over by the expert, the manager is forced out, possibly with severance $S_t \geq 0$, and the firm enters the monitoring stage.[6]

The firm discounts at the marker rate $r > 0$ and the agent at $\gamma > r$. The liquidation value of the firm is $0 \leq L < \frac{\mu}{r}$. The manager's outside value, available at dismissal, is $R \geq 0$.

A contract with a manager consists of $(\tau, I, \hat{Y}, s, S)$, where τ is the manager's dismissal date, I_t is the nondecreasing process of cumulative payments to the manager, $\hat{Y}_t$ is the process of recommended reporting, $s_t \in \{0, 1\}$ is the indicator of searching for an expert to replace the agent, and S_t is severance. Because recommending $\hat{Y}_t = Y_t$ at all t is without loss of generality, we will denote a contract simply by (τ, I, s, S).

Dismissal of the agent can be due to liquidation or transition to monitoring. Let τ_L be liquidation date and τ_M be the date of switching into the monitoring state. We have $\tau = \min\{\tau_M, \tau_L\}$.

The agent chooses a reporting process $d\hat{Y}_t \leq dY_t$ to maximize

$$\mathbb{E}\left[\int_0^\tau e^{-\gamma t}\left(dI_t + \lambda(dY_t - d\hat{Y}_t)\right) + 1_{\tau=\tau_L} e^{-\gamma\tau} R + 1_{\tau=\tau_M} e^{-r\tau}(R + S_t)\right].$$

The contract is incentive compatible (IC) if the strategy of reporting $d\hat{Y}_t = dY_t$ at all t attains a maximum in the above objective.

Under an IC contract, the agency-state payoffs to the agent and the firm are

$$\mathbb{E}\left[\int_0^\tau e^{-\gamma t} dI_t + 1_{\tau=\tau_L} e^{-\gamma\tau} R + 1_{\tau=\tau_M} e^{-r\tau}(R + S_t)\right].$$

and

$$\mathbb{E}\left[\int_0^\tau e^{-rt}((\mu - s_t\kappa)dt - dI_t) + 1_{\tau=\tau_L} e^{-r\tau} L + 1_{\tau=\tau_M} e^{-r\tau_M}(M - S_t)\right],$$

[5] For brevity of exposition, we assume that all diverted cash flow is consumed by the agent on the spot. DS show that this assumption is inessential. Hidden savings can be allowed as long as the rate of return in the hidden account is not too large.

[6] We assume that the project can be run either by the manager or by the monitoring expert but not both. Thus, firing of the manager is a necessary condition for a transition to monitoring.

where M is the value of the firm upon entering the monitoring state.

Next, we study the optimal contract.

3 Firm value in the monitoring stage

We begin by solving for the value the firm obtains in the monitoring state. In this state, there is no need to provide incentives to the expert, as there is no agency friction. Let $\hat{\tau}$ be the time of arrival of an exit from monitoring (reversion to agency with a new manager). The payoff to the firm in the monitoring state is

$$M = \mathbb{E}\left[\int_0^{\hat{\tau}} e^{-rt}(\mu - \kappa_B)dt + e^{-r\hat{\tau}} b_0\right],$$

where b_0 is the value of starting with a new agent, which we can compute as

$$rM = \frac{r}{r+\phi}(\mu - \kappa_B) + \frac{\phi}{r+\phi} r b_0,$$

or

$$(r+\phi)M = \mu - \kappa_B + \phi b_0. \tag{1}$$

To ensure that monitoring forever is not efficient, we assume the monitoring costs to be high enough for the firm to generate expected losses while in monitoring, i.e., we assume $\kappa_B \geq \mu$, which implies $M < b_0$. Under this assumption, the value of the firm in the monitoring state comes exclusively from the expectation of a transition to the agency state and the flow payoff losses are covered by external financing. Next, we proceed to characterize the optimal contract in the agency state and the value it gives to the firm.

4 Optimal contract in the agency stage

We follow the standard approach of using the agent's continuation value as the state variable in the firm's problem of designing an optimal contract for the agent in the agency state.

Let $s_t \in \{0, 1\}$ be the indicator of searching for a transition out of the agency state. The agent's continuation value follows

$$dW_t = \gamma W_t dt - dI_t + \beta_t(d\hat{Y}_t - \mu dt) + s_t \Delta_t (dN_t - \rho dt),$$

where β_t is the sensitivity to the reported cash flow, Δ_t is the sensitivity to the switch in the state to the monitoring state, and N_t is a Poisson process with arrival rate $\rho > 0$.

The contract is IC if $\beta_t \geq \lambda$ at all t in the agency state. Given the concavity of the firm's value function, $b(W)$, we will have $\beta_t = \lambda$ all t in the agency state (with or without search). With this sensitivity, we have $\hat{Y}_t = Y_t$ at all t.

Denote $\Delta_t = W_t' - W_t$, where W_t' is the post-arrival value to the agent if the arrival comes when the agent's continuation value is W_t. As long as a transition to the monitoring state or liquidation do not occur, we have

$$\begin{aligned} dW_t &= \gamma W_t dt - dI_t + \lambda(dY_t - \mu dt) + s_t(W_t' - W_t)(0 - \rho dt) \\ &= ((\gamma + s_t\rho)W_t - s_t\rho W_t')dt - dI_t + \lambda\sigma dZ_t, \end{aligned}$$

where $W_t' = R + S_t$. At dismissal, the agent's continuation value is delivered by the outside value R and a lump-sum payment S_t, as there is no reason to delay payments to the agent beyond the point of dismissal.

The HJB equation for the firm's value b is

$$rb(W) = \max_{s,S} \mu - s\kappa + ((\gamma + \rho s)W - s\rho(R + S))b'(W) + \frac{1}{2}\lambda^2\sigma^2 b''(W) + s\rho(M - S - b(W)).$$

4.1 Optimal severance payment

Lemma 1 $S = 0$.

Proof The optimal S maximizes $-s\rho S b'(W) - s\rho S = (-b'(W) - 1)s\rho S$, which is decreasing in S because b is strictly concave and $b'(W^1) = -1$, which means $b'(W) > -1$ at all $W < W^1$. ■

The optimal severance payment to the agent is zero in this model. The smaller the agent's post-dismissal value, the larger the loss of value to the agent upon finding a monitoring opportunity. This loss of value increases the drift in W_t in the agency state, which is valuable to the firm everywhere in the search region, as it reduces the risk of liquidation.

With this simplification, in the interior of $[R, W^1]$, the agent's continuation value process W_t has constant volatility $\lambda\sigma$ and drift $\gamma W_t + s_t\rho(W_t - R)$, which, as we see, has two components. The γW_t component is compensation for the zero payment flow to the agent at all $W < W^1$. The $\rho(W_t - R)$ component, which is only effective when the firm searches for monitoring, is compensation for the risk of losing $W_t - R$ in case the firm finds a monitoring opportunity.

With $S_t = 0$, the HJB can be written as

$$rb(W) = \max_{s \in \{0,1\}} \mu - s\kappa + ((\gamma + s\rho)W - s\rho R)b'(W) + \frac{1}{2}\lambda^2\sigma^2 b''(W) + s\rho(M - b(W)). \quad (2)$$

4.2 The region of search for monitoring

First, we derive a condition for monitoring to be used in equilibrium.

Denote by $O(W)$ the value of searching at W, i.e., the difference between searching and not searching in the HJB equation (2):

$$O(W) \equiv -\kappa + \rho(W-R)b'(W) + \rho(M - b(W)). \tag{3}$$

Let b_L denote the value function of a firm that is permanently denied the option to search for monitoring. This value function is derived in DS. Denote by $b_{L,0}$ its unique peak value. Suppose now this firm is about to liquidate at $W = R$, and at this point it unexpectedly gains the option to search for monitoring. If this firm chooses to ignore this option, then so will the firm that is always able to search; meaning the parameters κ, ρ and κ_B, ϕ are such that monitoring is just too costly relative to the liquidation value L. Conversely, if this firm chooses to search, then monitoring is useful, i.e., it will be used in equilibrium at least in a small neighborhood of R. The condition for this firm's preference to search is $O(R) = -\kappa + \rho(M - L) > 0$, where $M = \frac{\mu - \kappa_B}{r+\phi} + \frac{\phi}{r+\phi} b_{L,0}$. Thus, the necessary and sufficient condition for the monitoring technology to be used in equilibrium is

$$\frac{\kappa}{\rho} < \frac{\mu - \kappa_B}{r+\phi} + \frac{\phi}{r+\phi} b_{L,0} - L. \tag{4}$$

We will maintain this condition throughout.

Second, we partially characterize the region of the state space in which the firm searches for monitoring under the optimal contract.

Lemma 2 *The region of search is an interval* $(R, \tilde{W}]$ *with* $\tilde{W} < W_0 \equiv \arg\max b(W)$.

Proof Differentiating (3), we have $O'(W) = \rho b'(W) + \rho(W-R)b''(W) - \rho b'(W) = \rho(W - R)b''(W) < 0$ by strict concavity of b. Therefore, $O(W) > 0$ implies $O(W') > 0$ for all $W' \leq W$, i.e., the search region is an interval connected to R. By (4), this interval is nonempty. It remains to show that $\tilde{W} < W_0$. Indeed, with $M < b_0 = b(W_0)$ and with $b'(W_0) = 0$, we have $O(W_0) = -\kappa + \rho(M - b(W_0)) < 0$. ■

4.3 Verification

The verification argument showing that the solution of the HJB equation that satisfies the boundary conditions given in DS is in fact the firm's value function is standard.

Theorem 1 *The unique solution b of the HJB equation that satisfies $b'(W^1) = -1$ at a point W^1 such that $rb(W^1) + \gamma W^1 = \mu$ and $b(R) = L$ with M given in (1) is the true value function for the firm under the optimal contract.*

Proof Follows DS with minor changes related to the jump to the value M. ■

4.4 Computation

The agency HJB equation depends on the value of the monitoring stage, M, which in turn depends on b_0. Yet, thanks to Lemma 2, we can solve the HJB in a single pass going backward from the agent payment boundary.

We can write the firm's value function in the agency stage as $b(W) = \max\{b_{NS}(W), b_S(W)\}$, where b_{NS} is the value of not searching for transition to monitoring and b_S is the value of searching.

The HJB equation for the function b_{NS} is the same as in DS:

$$rb_{NS}(W) = \mu + \gamma W b'_{NS}(W) + \frac{1}{2}\lambda^2\sigma^2 b''_{NS}(W). \tag{5}$$

From Lemma 2 we have that $b(W) = b_{NS}(W)$ for all $\tilde{W} \leq W \leq W^1$. Although the $\tilde{W}$ is not known exactly, we know that $\tilde{W} < W_0$. Thus, starting from W^1 we can solve the HJB (5) backwards until the solution reaches its peak. At this point, we have found W_0 and b_0, and, hence, M. To the left of the peak, we can continue solving the HJB equation (2) allowing for both search and no search, as now, with M known, the value of searching for monitoring is known.

This computational procedure can be simplified further. At the upper boundary of the search interval, $\tilde{W}$, we have $O(\tilde{W}) = -\kappa + \rho(\tilde{W} - R)b'(\tilde{W}) + \rho(M - b(\tilde{W})) = 0$ and we also know that $b(\tilde{W}) = b_{NS}(\tilde{W})$, and $b'(\tilde{W}) = b'_{NS}(\tilde{W})$. Therefore, in order to pin down $\tilde{W}$, we can continue to solve (5) to the left of the peak of b_{NS} until the stopping condition

$$\kappa = \rho(W - R)b'_{NS}(W) + \rho(M - b_{NS}(W))$$

is met. To the left of this point, searching for a monitoring opportunity maximizes the firm's value. This value function, which we denoted by $b_S(W)$, solves the HJB equation

$$rb_S(W) = \mu - \kappa + ((\gamma + \rho)W - \rho R)b'_S(W) + \frac{1}{2}\lambda^2\sigma^2 b''_S(W) + \rho(M - b_S(W)),$$

with the boundary conditions at $\tilde{W}$ given by $b_S(\tilde{W}) = b_{NS}(\tilde{W})$, $b'_S(\tilde{W}) = b'_{NS}(\tilde{W})$. After reaching R, we update the initial agent payment threshold W^1 depending on whether $b_S(R)$ is larger or smaller than L.

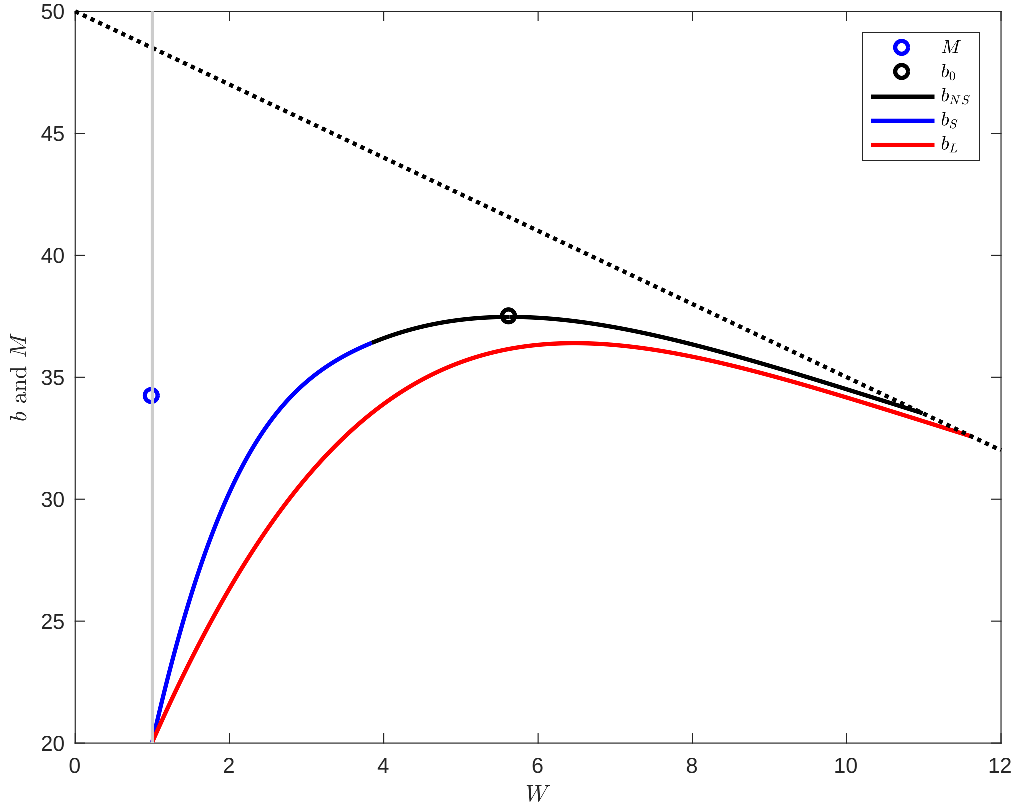

Figure 1: Value function $b(W)$ in the agency state and the value M in the monitoring state. $R > 0$.

Figure 1 presents a computed solution. The firm's value function b consists of the no-search segment b_{NS}, in black, and a search segment b_S, in blue. In addition, the value function of the firm that, as in DS, can only use liquidation, b_L, is depicted in red.

5 Capital structure implementation

In the agency state, we consider a capital structure similar to DS, but with two new features. First, the credit line (short-term debt) has a performance pricing feature. Second, payments to long-term debt have a component contingent on the firm's financial position, i.e., leverage. This component can be implemented with a contingent bond.

Let x be the flow of payment to long-term debt (regular and contingent together), and let i be the interest rate on the line of credit. Let the credit limit on the credit line be C^L, and let $\tilde{C}^L < C^L$ be a financial distress threshold, i.e., the trigger of performance pricing. Let B_t denote the balance on the credit line outstanding at t. As in DS, let the agent hold fraction λ of the firm's equity.

The agent's problem under capital structure (x, i, λ) is to choose a reporting process $\hat{Y}_t$ and the dividend process Div_t to maximize their expected payoff, where the balance process follows

$$dB_t = i(B_t)B_t dt + x(B_t)dt + dDiv_t - d\hat{Y}_t. \tag{6}$$

Proposition 1 *Let* $C^L = \frac{W^1 - R}{\lambda}$ *and* $\tilde{C}^L = \frac{W^1 - \tilde{W}}{\lambda}$, *where* $\tilde{W}$ *and* W^1 *are determined in the optimal contract. Suppose*

$$i(B_t) = \begin{cases} \gamma & \text{if} \quad B_t < \tilde{C}^L, \\ \gamma + \rho & \text{if} \quad B_t \geq \tilde{C}^L, \end{cases} \tag{7}$$

and

$$x(B_t) = \begin{cases} \mu - \frac{\gamma}{\lambda}W^1 & \text{if} \quad B_t < \tilde{C}^L, \\ \mu - \frac{\gamma}{\lambda}W^1 - \frac{\rho}{\lambda}(W^1 - R) & \text{if} \quad B_t \geq \tilde{C}^L. \end{cases} \tag{8}$$

Then the dividend process

$$Div_t = \frac{I_t}{\lambda},$$

the balance process

$$B_t = \frac{W^1 - W_t}{\lambda}, \tag{9}$$

and the reporting process $\hat{Y}_t = Y_t$ *solve the agent's optimization problem under the capital structure* (x, i, λ).

Proof First, we check that if the agent follows the proposed policy, then (9) holds, where the process $(W_t; t \geq 0)$ is determined by the optimal contract.

For $B_t = 0$, we have $W_t = W^1$.

For all $0 \leq B_t < \tilde{C}^L$, from (7) and (8), we have $i = \gamma$ and $x = \mu - \frac{\gamma}{\lambda}W^1$. Using (6), we have

$$\begin{aligned} -\lambda dB_t &= -\lambda\gamma B_t dt - \lambda(\mu - \frac{\gamma}{\lambda}W^1)dt - \lambda dDiv_t + \lambda d\hat{Y}_t \\ &= \gamma\left(-\lambda B_t + W^1\right)dt - dI_t + \lambda(d\hat{Y}_t - \mu dt). \end{aligned}$$

With the identity $-\lambda B_t + W^1 = W_t$, we get back the law of motion for W_t under the optimal contract: $dW_t = \gamma W_t dt - dI_t + \lambda(d\hat{Y}_t - \mu dt)$.

For $\tilde{C}^L \leq B_t < C^L$, from (7) and (8), we have $i = \gamma + \rho$ and $x = \mu - \frac{\gamma}{\lambda}W^1 - \frac{\rho}{\lambda}(W^1 - R)$. Using (6), we have

$$\begin{aligned} -\lambda dB_t &= -\lambda(\gamma + \rho)B_t dt - \lambda(\mu - \frac{\gamma}{\lambda}W^1 - \frac{\rho}{\lambda}W^1 + \frac{\rho}{\lambda}R)dt - \lambda dDiv_t + \lambda d\hat{Y}_t \\ &= (\gamma + \rho)\left(-\lambda B_t + W^1\right)dt - \rho R dt + \lambda(d\hat{Y}_t - \mu dt). \end{aligned}$$

With the identity $-\lambda B_t + W^1 = W_t$, we get back the law of motion for W_t under the optimal contract in the search region: $dW_t = (\gamma + \rho)W_t dt - \rho R dt + \lambda(d\hat{Y}_t - \mu dt)$.

This shows that if the agent follows the proposed policy, he does as well as in the optimal contract. That he cannot do better follows by contradiction. If he could, we'd be able to construct a diversion policy $\hat{Y}$ that would give the agent a higher payoff also under the optimal contract, which contradicts the incentive compatibility property of that contract. ■

The interest rate on the credit line and the long-term debt payment are adjusted when the firm's financial position B_t crosses the distress threshold $\tilde{C}^L$. The rate i goes up, as in most performance pricing contracts. The payment to long-term debt is stepped down.

One way to implement an automatic decrease in payment x at the distress threshold is to issue two classes of long-term debt at date 0: a regular perpetuity with face value $D = r^{-1}(\mu - \frac{\gamma}{\lambda}W^1 - \frac{\rho}{\lambda}(W^1 - R))$ and the coupon rate r, resulting with a permanent payment of

$$x_d = \mu - \frac{\gamma}{\lambda}W^1 - \frac{\rho}{\lambda}(W^1 - R),$$

and a perpetual, contingent, noncumulative bond with face value $D_{cd} = r^{-1}\frac{\rho}{\lambda}(W^1 - R)$ and the coupon rate r. The contingency feature of this bond suspends (i.e., eliminates without the obligation to pay back later) its coupon payments whenever the firm is financially distressed, i.e., when $B_t > \tilde{C}^L$. Together, these two bonds replicate the structure of optimal payment to long-term debt holders, $x(B_t)$. In financially sound conditions, $B_t \leq \tilde{C}^L$, the total coupon flow, i.e., the sum of coupon payments to the two types of debt, is $\mu - \frac{\gamma}{\lambda}W^1$. In financial distress conditions, $B_t > \tilde{C}^L$, the total coupon flow is reduced by

$$x_{cd} = \frac{\rho}{\lambda}(W^1 - R),$$

as the coupons to the contingent bond become automatically suspended.

Figure 2 illustrates the joint effect of the contingency feature of debt and the performance-pricing feature of the credit line. It shows total debt service costs as a function of the draw on the credit line, B_t. In the baseline model of DS, the total debt service costs, represented by the red line, have an affine structure. The constant component is the constant payment to long-term debt, and a linear component is the interest payment on the firm's credit line, which has a constant slope of γ. In our model, total debt service costs are discontinuous at the financial distress threshold, $\tilde{C}^L$. The payment to long-term debt declines, but the slope of the variable component, i.e., the interest rate on the revolving balance B_t, increases from γ to $\gamma + \rho$. As shown in Figure 2, the joint effect of the contingency feature and the performance pricing feature is positive for the firm everywhere in the financial distress region. In fact, the optimal amount of contingent debt the firm issues is determined at the level that ensures the firm receives a debt service cost relief in the distress region. Indeed,

$$x_{cd} = \frac{\rho}{\lambda}(W^1 - R) > \rho\frac{W^1 - W_t}{\lambda} = \rho B_t \text{ for all } B_t \in [\tilde{C}^L, C^L),$$

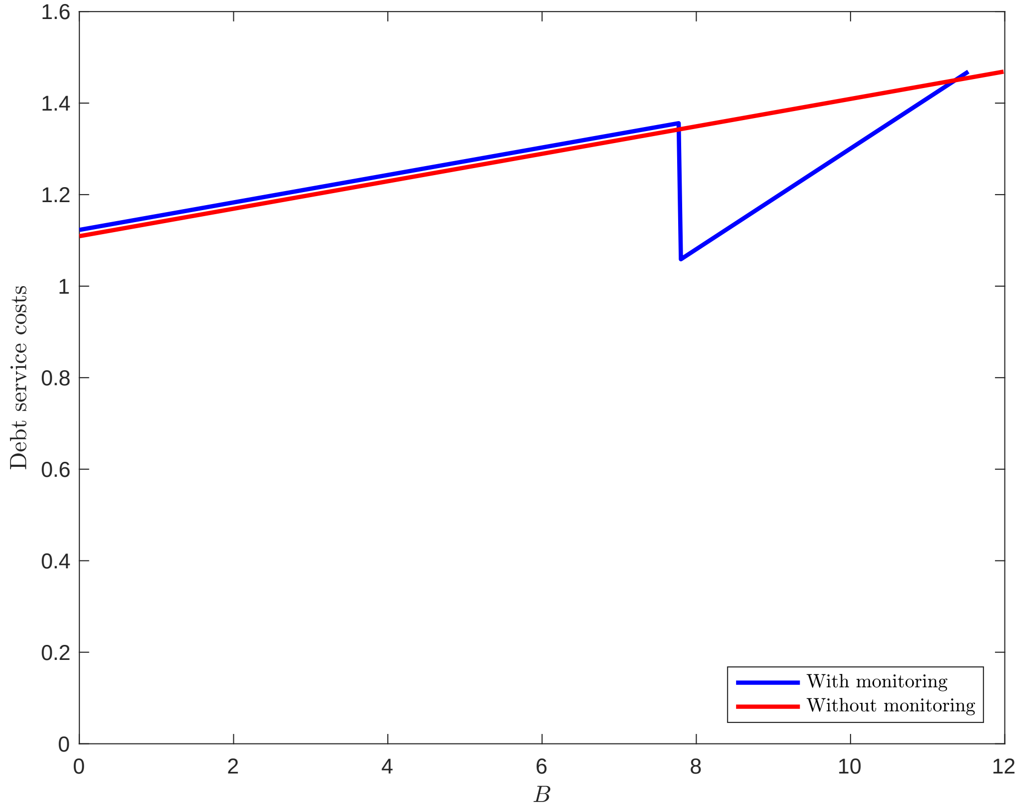

Figure 2: Total debt service costs.

i.e., the suspension of payments to contingent debt outweighs the additional interest charged on the credit line under performance pricing.

In the distress region, the firm also searches for an opportunity to reorganize. The search costs, κ, represent preparation costs for bankruptcy reorganization. In particular, the firm must line up financing to cover negative cash flows during the bankruptcy reorganization stage.

In bankruptcy itself, the monitoring and reorganization flow costs are κ_B. Upon transition to bankruptcy, the manager is dismissed with a severance payment of zero. The value of the manager's equity stake in the firm is zero, as equity is extinguished in bankruptcy and debt holders become firm owners, which gives the manager the total continuation value of just R. The value of the firm in bankruptcy is M, which is allocated to long-term debt holders, contingent debt holders, and the revolving debt holders in the order of seniority. Operating losses are covered by external, prearranged financing. This financing problem is free of the agency friction, thanks to the costly monitoring activities exerted during bankruptcy. As soon as the firm has an opportunity to transition back to the agency stage, a new manager is hired and a new capital structure is set up with the initial value b_0 to the owners, and the value of W_0

to the new manager. The time spent in bankruptcy captures the delay in creditor negotiations and search for a new manager.

5.1 Alternative capital structures

In this capital structure, contingent debt is similar to (noncumulative) preferred equity. The distinction between the two comes from the ex ante commitment by the firm to make payments to this liability class everywhere in the non-distress region, $B_t \in [0, \tilde{C}^L)$, even when common equity dividends are zero. This distinction, however, is not essential. In the Appendix, we provide an alternative capital structure under which the manager prefers to make payments to this liability class whenever not distressed even if the only consequence for not paying is an onset of search for restructuring.

Provision of financial relief to the firm in distress combined with an increased incentive to pay revolving debt off are essential features of our model. As we have shown, these can be implemented with performance pricing on revolving debt combined with a contingent liability similar to noncumulative contingent debt or preferred equity. Our model, therefore, gives an efficiency-based explanation for why these features are observed in corporate capital structures.

6 Security values and comparative statics

In order to compute security values and obtain comparative statics results, we follow the approach of DS. The following lemma adapts their method to our model by allowing for the possibility of a jump to a state M.

Lemma 3 *Let W_t follow the equilibrium law of motion for the manager's continuation value until a stopping time $\tau = \min\{\tau_L, \tau_M\}$. Let g be a flow return function defined on $[R, W^1]$ and k and F_M be real numbers. Then the same function G defined on $[R, W^1]$ solves both*

$$rG(W) = g(W) + (\gamma W + 1_{W \leq \tilde{W}} \rho(W - R))G'(W) + \frac{1}{2}\lambda^2\sigma^2 G''(W) + 1_{W \leq \tilde{W}} \rho(F_M - G(W))$$

with boundary conditions $G(R) = F_L$ and $G'(W^1) = -k$, and

$$G(W_0) = \mathbb{E}\left[\int_0^\tau e^{-rt} g(W_t)dt - k\int_0^\tau e^{-rt} dI_t + e^{-r\tau}\left(1_{\tau=\tau_L} F_L + 1_{\tau=\tau_M} F_M\right)\right].$$

Proof Follows the martingale argument of DS. ■

6.1 Market value of securities

Let us denote market value of security $\theta \in \{e, d, cd, cl\}$ conditional on the current draw on the credit line B by $V_\theta(B)$, where e stands for equity, d for long-term debt, cd for contingent debt, and cl for the credit line. Let us fix the liquidation payoffs at some nonnegative constants $F_{L,\theta}$ such that $\sum_\theta F_{L,\theta} = L$, and the reorganization payoffs at some non-negative constants $F_{M,\theta}$ such that $\sum_\theta F_{M,\theta} = M$, where the constants respect some specific seniority structure of claims. With these terminal payoffs, the values of securities are as follows:

$$\begin{aligned}
V_e(B_t) &= \mathbb{E}_t\left[\int_0^\tau e^{-rt} dDiv_t + e^{-r\tau}\left(1_{\tau=\tau_L} F_{L,e} + 1_{\tau=\tau_M} F_{M,e}\right)\right], \\
V_d(B_t) &= \mathbb{E}_t\left[\int_0^\tau e^{-rt} x_d dt + e^{-r\tau}\left(1_{\tau=\tau_L} F_{L,d} + 1_{\tau=\tau_M} F_{M,d}\right)\right], \\
V_{cd}(B_t) &= \mathbb{E}_t\left[\int_0^\tau e^{-rt} 1_{B\le \tilde{C}^L} x_{cd} dt + e^{-r\tau}\left(1_{\tau=\tau_L} F_{L,cd} + 1_{\tau=\tau_M} F_{M,cd}\right)\right], \\
V_{cl}(B_t) &= \mathbb{E}_t\Bigg[\int_0^\tau e^{-rt} d(Y_t - Div_t) - \int_0^\tau e^{-rt}(x_d + 1_{B_t\le \tilde{C}^L} x_{cd} \\
&\qquad + 1_{B_t > \tilde{C}^L}\kappa) dt + e^{-r\tau}\left(1_{\tau=\tau_L} F_{L,cl} + 1_{\tau=\tau_M} F_{M,cl}\right)\Bigg].
\end{aligned}$$

These values can be computed using the general formula in Lemma 3.

In particular, to compute the value of long-term debt, we use formula of Lemma 3 with the flow return function $g(W) = x_d$ at all W, the value of $k = 0$, and terminal payoffs $M_d = \min\{D, M\}$ and $F_d = \min\{D, L\}$, where $D = \frac{x_d}{r}$. For contingent debt, we use $g(W) = 1_{W>\tilde{W}} x_{cd}$, and $k = 0$. Because we have assumed contingent debt to be junior to regular long-term debt, the terminal values for contingent debt are $M_{cd} = \min\{D_{cd}, M_c\}$ and $F_{cd} = \min\{D_{cd}, F_d\}$, where, we recall, $D_{cd} = \frac{x_{cd}}{r}$. Having verified $M_{cd} < D_{cd}$ and $F_{cd} < D_{cd}$, the terminal payoffs for both security classes junior to contingent debt, i.e., the unsecured credit line and equity, are zero. To compute the value of the credit line, we use $g(W) = \mu - x_d - 1_{W>\tilde{W}} x_{cd} - 1_{W\le\tilde{W}}\kappa$, and $k = 1$. For equity we use $g(W) = 0$ at all W and $k = -\frac{1}{\lambda}$.

Figure 3 presents a computed example. In this example, the seniority structure is as follows: long-term debt, contingent debt, credit line, equity. Under the parameter values of this example, long-term debt becomes riskless, as its face value D is below both M and L. Contingent debt is risky, with face value D_{cd} higher than M (and hence also L). In both liquidation and reorganization, the short-term unsecured line of credit recovers zero, as does equity.

Figure 4 uses the same parametrized example to compute security values in the DS model. In that model, the optimal capital structure has only one kind of long-term debt. In this example, this debt is risky. The introduction of the possibility of monitoring increases the total debt capacity of the firm, as $D + D_{cd} > D_{DS}$, and the optimal credit line is shorter.

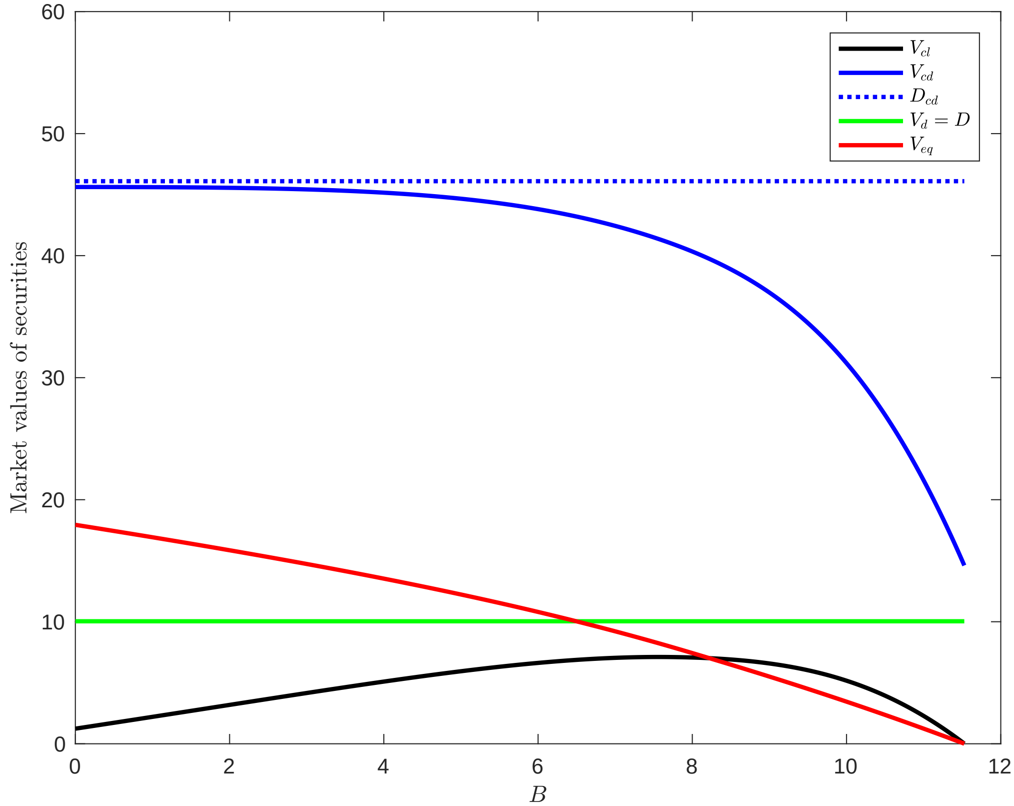

Figure 3: Security values.

Security values are highly sensitive to the order of seniority in bankruptcy. In our model, in particular, it is easy to see that if both forms of long-term debt are pari-passu to each other, then regular long-term debt is no longer risk-free. The total firm value, however, as determined by the optimal contract, is independent of the order of seniority of debt claims.

6.2 Comparative statics

In this section, we use the approach of Lemma 3 to compute comparative statics of the model. We present the derivation of the comparative statics with respect to the search friction parameter ρ. The comparative statics with respect to other parameters of the model, presented in Table 1 below, are obtained using the same methodology.

The effect of the search friction parameter, ρ, on the firm's profit can be found by differentiating the HJB equation and its boundary conditions. Starting in the no-search region, we have

$$r \frac{\partial b_{NS,\rho}(W)}{\partial \rho} = \gamma W \frac{\partial}{\partial W} \frac{\partial b_{NS,\rho}(W)}{\partial \rho} + \frac{1}{2} \lambda^2 \sigma^2 \frac{\partial^2}{\partial W^2} \frac{\partial b_{NS,\rho}(W)}{\partial \rho}.$$

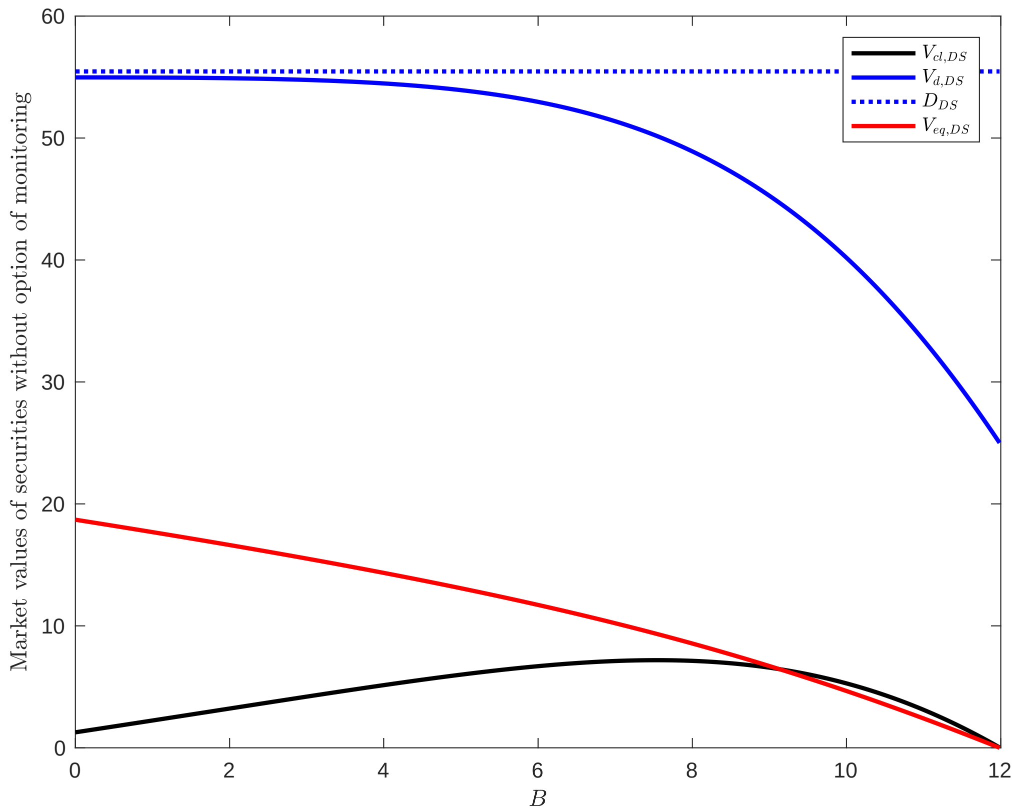

Figure 4: Security values in the model without monitoring.

Using the Feynman-Kac formula of Lemma 3, this can be simplified to

$$\frac{\partial b_{NS,\rho}(W)}{\partial \rho} = \frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} G_S(W) \text{ for all } W \in [\tilde{W}, W^1],$$

where

$$G_S(W) \equiv \mathbb{E}\left[e^{-r\tau_S} | W_0 = W\right] \text{ for all } W \in [\tilde{W}, W^1],$$

and τ_S is the stopping time indicating the firm's first entrance into the search region, i.e., $\tau_S = \min\{t : W_t = \tilde{W}\}$.

Similarly, in the search region we have for all $W \in [R, \tilde{W}]$

$$\frac{\partial b_{S,\rho}(W)}{\partial \rho} = G_\rho(W) + \left(\frac{1}{r+\rho} M_\rho + \frac{\rho}{r+\rho} \frac{\partial M_\rho}{\partial \rho}\right)(1 - G_\tau(W)) + \frac{\partial M_\rho}{\partial \rho} G_M(W) + \frac{\partial b_{NS,\rho}(\tilde{W})}{\partial \rho} G_{NS}(W), \tag{10}$$

where M_ρ is the firm value in the monitoring stage with parameter ρ, and, for all $W \in [R, \tilde{W}]$,

$$\begin{aligned} G_\rho(W) &\equiv \mathbb{E}[\int_0^\tau e^{-(r+\rho)t} (W_t - R) b'_{S,\rho}(W_t) dt | W_0 = W], \\ G_\tau(W) &\equiv \mathbb{E}[1 - e^{-(r+\rho)\tau} | W_0 = W], \end{aligned}$$

$$
\begin{aligned}
G_M(W) &\equiv \mathbb{E}\left[e^{-r\tau}1_{\tau=\tau_M}|W_0=W\right], \\
G_{NS}(W) &\equiv \mathbb{E}\left[e^{-r\tau}1_{\tau=\tau_{NS}}|W_0=W\right], \\
G_L(W) &\equiv \mathbb{E}\left[e^{-r\tau}1_{\tau=\tau_L}|W_0=W\right],
\end{aligned}
$$

where $\tau = \min\{\tau_{NS}, \tau_M, \tau_L\}$ is the stopping time indicating the firm's first exit from the search region due to either financial recovery ($\tau = \tau_{NS}$), or transition to monitoring ($\tau = \tau_M$), or liquidation ($\tau = \tau_L$). Thus, we have

$$
G_\tau(W) = G_{NS}(W) + G_M(W) + G_L(W). \tag{11}
$$

Finally, from the definition of the value in monitoring, M, we have

$$
\frac{\partial M_\rho}{\partial \rho} = \frac{\phi}{r+\phi}\frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} G_S(W^*).
$$

The following lemma signs the comparative statics for the value functions and the boundary conditions.

Lemma 4 *$\frac{\partial b_{NS,\rho}(W)}{\partial \rho} > 0$, $\frac{\partial b_{S,\rho}(W)}{\partial \rho} > 0$, $\frac{\partial M_\rho}{\partial \rho} > 0$ and $\frac{\partial W^1}{\partial \rho} < 0$. Further, $\frac{\partial \tilde{W}}{\partial \rho} < 0$ if $\frac{\kappa}{\rho}$ is sufficiently small.*

Proof From the definition, obviously, we have $G_S(W) \geq 0$, thus the sign of $\frac{\partial b_{NS,\rho}(W)}{\partial \rho}$ is the same as the sign of $\frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho}$. Suppose $\frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} \leq 0$. Again from the definition of $G_S(W)$, we have $G_S(\tilde{W}) = 1$ and $G'_S(\tilde{W}) < 0$. At the super-contact point $W = \tilde{W}$, we have $\frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} = \frac{\partial b_{NS,\rho}(\tilde{W})}{\partial \rho}$ and $\frac{\partial^2 b_{S,\rho}(W)}{\partial W \partial \rho} = \frac{\partial^2 b_{NS,\rho}(\tilde{W})}{\partial W \partial \rho} = \frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} G'_S(\tilde{W}) \geq 0$. So for an arbitrarily small $\varepsilon > 0$ we have $\frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} \geq \frac{\partial b_{S,\rho}(\tilde{W}-\varepsilon)}{\partial \rho}$ and hence

$$
\begin{aligned}
\frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} \geq\ & G_\rho(\tilde{W}-\varepsilon) + \frac{1}{r+\rho} M_\rho \left(1 - G_\tau(\tilde{W}-\varepsilon)\right) + \frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} G_{NS}(\tilde{W}-\varepsilon) \\
& + \frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} G_S(W^*) \frac{\phi}{r+\phi}\left(\frac{\rho}{r+\rho}\left(1 - G_\tau(\tilde{W}-\varepsilon)\right) + G_M(\tilde{W}-\varepsilon)\right).
\end{aligned}
$$

Rearranging and using (11), we obtain

$$
\begin{aligned}
& \frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho}\left(G_L(\tilde{W}-\varepsilon) + \left(1 - \frac{\phi}{r+\phi}\frac{\rho}{r+\rho} G_S(W^*)\right)\left(1 - G_\tau(\tilde{W}-\varepsilon)\right) + \left(1 - \frac{\phi}{r+\phi} G_S(W^*)\right) G_M(\tilde{W}-\varepsilon)\right) \\
& \geq G_\rho(\tilde{W}-\varepsilon) + \frac{1}{r+\rho} M_\rho \left(1 - G_\tau(\tilde{W}-\varepsilon)\right).
\end{aligned} \tag{12}
$$

The left side of (12) is negative under the premise $\frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} \leq 0$. By Lemma 2, $b'_{S,\rho}(W) > 0$ for all $W \in [R, \tilde{W}]$, which implies $G_\rho(\tilde{W}-\varepsilon) > 0$. Thus, the right side of (12) is strictly positive, which is a contradiction.

Table 1: Comparative statics for the captial structure.

	dC^L	dD	dD_{cd}	dW^*	db_0
$d\rho$	$-$	$+$	$+$	$-$	$+$
$d\kappa$	$+$	$-$	$-$	$+$	$-$
$d\phi$	$-$	$+$	$+$	$-$	$+$
$d\kappa_B$	$+$	$-$	$-$	$+$	$-$
dL	$-$	$+$	$+$	$-$	$+$
dR	$-$	$-$	$-$	$+$	$-$
$d\gamma$	$-$	$\pm$	$\pm$	$-$	$-$
$d\sigma^2$	$+$	$-$	$-$	$\pm$	$-$

Also, since we have $\frac{\partial b_{NS,\rho}(W)}{\partial \rho} > 0$, we must have $\frac{\partial M_\rho}{\partial \rho} = \frac{\phi}{r+\phi}\frac{\partial b_{NS,\rho}(W^*)}{\partial \rho} > 0$ and $\frac{\partial b_{S,\rho}(W)}{\partial \rho} > 0$ from (10).

Differentiating the boundary condition $rb_{NS,\rho}(W^1) + \gamma W^1 = \mu$, we have

$$\frac{\partial W^1}{\partial \rho} = -\frac{r}{\gamma - r}\frac{\partial b_{S,\rho}(\tilde{W})}{\partial \rho} G_S(W^1) < 0.$$

Differentiating the indifference condition $\kappa = \rho(\tilde{W} - R)b'_{NS}(\tilde{W}) + \rho(\tilde{W} - b_{NS}(\tilde{W}))$, we have

$$\frac{\partial \tilde{W}}{\partial \rho} = \frac{-1}{\rho \tilde{W} b''_{NS,\rho}(\tilde{W})}\left(\frac{\kappa}{\rho} + \rho(\tilde{W} - R)\underbrace{\frac{\partial b'_{NS,\rho}(\tilde{W})}{\partial \rho}}_{<0} + \rho\underbrace{\left(\frac{\phi}{r+\phi}G_S\left(W^*\right) - 1\right)}_{<0}\frac{\partial b_{NS,\rho}(\tilde{W})}{\partial \rho}\right),$$

where $G_S\left(W^*\right) \leq 1$ since $W^* > \tilde{W}$ (by Lemma 2 again), $G_S(\tilde{W}) = 1$ and $G_S\left(W\right)$ is decreasing. Thus, we have $\frac{\partial \tilde{W}}{\partial \rho} < 0$ if $\frac{\kappa}{\rho}$ is sufficiently small. ■

Lemma 4 can now be used to derive comparative statics for the optimal capital structure. Recall that the credit line satisfies $C^L = \frac{1}{\lambda}(W^1 - R)$ and the distress threshold is $\tilde{C}^L = \frac{1}{\lambda}(W^1 - \tilde{W})$. Long-term debt is $D = \frac{1}{r}\left(\mu - \frac{\gamma}{\lambda}W^1 - \frac{\rho}{\lambda}(W^1 - R)\right)$ and the contingent bond is $D_{cd} = \frac{\rho}{r\lambda}(W^1 - R)$. Applying the lemma to these expressions, we obtain comparative statics for the capital structure. Table 1 summarizes these results for the credit line, long-term debt, contingent debt, the value for a new manager, and the value of the firm in the agency stage.

The comparative statics for the distress threshold $\tilde{C}^L$ are nonmonotone. When search frictions are very severe, for example when κ/ρ is very large, the firm will only search in a small neighborhood of liquidation, which means $\tilde{C}^L$ is close to C^L and the firm's value function b is close to the DS value function b_L over almost all of the domain $[R, W^1]$ with a boundary condition at R approaching L. When search friction are very light, for example when ρ is very high, the firm does not search much either because it can find transition to monitoring quickly. In this case, $\tilde{C}^L$ is also close to C^L and value function is represented by b_{NS} over almost all of the domain $[R, W^1]$ with a boundary condition at R approaching $M > L$. At intermediate levels of search costs, the firm commences the search for a bankruptcy opportunity early, i.e., the ratio $\tilde{C}^L/C^L$ is lowest.

7 Conclusion

Asquith et al. (2005) and Manso et al. (2010) document that, broadly defined, performance pricing on debt instruments is a feature used commonly in the practice of corporate finance. In this paper, we show that performance pricing arises as a part of an optimal credit arrangement when firms can use a financial reorganization procedure similar to Chapter 11 bankruptcy. In an optimal contract, management is dismissed with no severance and the value of equity is wiped out as the firm enters bankruptcy reorganization. This discrete loss of value is compensated by giving the firm a relief in its debt service costs during financial distress, when the arrival rate of bankruptcy is positive. The role of performance pricing on the firm's debt obligations is to implement this relief.

Appendix

Capital structure with preferred stock

In this Appendix, we discuss a modified capital structure under which the firm is not committed ex ante to the payment of contingent debt coupons in the non-distress region. Rather, payments to the contingent liability class are optional, at the discretion of the manager, with a covenant attached dictating that the firm searches for a bankruptcy reorganization whenever the optional payment to this liability class is skipped. This payment optionality feature makes this contingent liability class very similar to preferred equity.

Consider the following capital structure consisting of a long-term bond, preferred equity, common equity, a primary revolving credit line, and a secondary revolving credit line.

Long-term debt pays a constant coupon rate x_d. The preferred equity dividend rate is p. The primary revolving credit line charges the interest rate γ on the balance outstanding, which is denoted by A_t, and has the credit limit $\bar{A}$. The secondary revolving credit line charges the interest rate $\gamma + \rho$ on the outstanding balance B_t, and has the limit $\bar{B}$.

The covenant structure attached to the above liabilities is as follows. Missing a coupon payment to long-term debt triggers liquidation. Preferred equity dividends p are non-cumulative, i.e., can be paid out at the manager's discretion. Although missing a payment p does not cause default, it triggers the search for reorganization at all times at which p is not paid out. Resumption of preferred dividend payments stops the search. Likewise, a positive balance B_t on the secondary credit line triggers the search for reorganization. The primary credit line can be used at the manager's discretion, up to the limit $\bar{A}$, with no triggers attached.

The manager holds the fraction λ of common equity. The manager chooses the common equity cumulative dividend process, Div_t, the preferred equity cumulative dividend process, Div_t^p, a process of drawing upon the secondary credit line, b_t, and the cash flow reporting process $\hat{Y}_t$.

The manager maximizes

$$\mathbb{E}\left[\int_0^\tau e^{-\gamma t}(\lambda d(Y_t - \hat{Y}_t) + \lambda dDiv_t) + e^{-r\tau}R\right]$$

subject to

$$\begin{aligned} dA_t &= \gamma A_t dt + x_d dt + dDiv_t + dDiv_t^p - d\hat{Y}_t - db_t, & (13) \\ dB_t &= (\gamma + \rho)B_t dt + db_t, & (14) \\ \hat{Y}_t &\leq Y_t, \end{aligned}$$

where $\tau = \min\{\tau_L, \tau_M\}$ is the stopping time indicating liquidation or transition to reorganization.

Proposition 2 *Suppose*

$$\begin{aligned} p &= \frac{\rho}{\lambda}(\tilde{W} - R), & (15) \\ x_d &= \mu - \frac{\gamma}{\lambda}W^1 - p, \\ \bar{A} &= \frac{W^1 - \tilde{W}}{\lambda}, & (16) \\ \bar{B} &= \frac{\tilde{W} - R}{\lambda}. & (17) \end{aligned}$$

Then the strategy

$$
\begin{aligned}
\hat{Y}_t &= Y_t, \\
dDiv_t^p &= \begin{cases} pdt \ \textit{if } A_t < \bar{A} \textit{ and } B_t = 0, \\ 0 \ \textit{otherwise}, \end{cases} \\
db_t &= \begin{cases} 0 \ \textit{if } A_t < \bar{A} \textit{ and } B_t = 0, \\ \gamma \bar{A} dt + x_d dt - dY_t \ \textit{otherwise}, \end{cases} \\
Div_t &= \frac{1}{\lambda} I_t,
\end{aligned}
$$

solves the manager's problem, and the implied revolving credit balance processes A_t and B_t satisfy

$$
W_t = \begin{cases} W^1 - \lambda A_t, \ \textit{if } A_t < \bar{A} \textit{ and } B_t = 0, \\ \tilde{W} - \lambda B_t, \ \textit{otherwise}, \end{cases} \tag{18}
$$

where $(W_t; t \geq 0)$ is the manager's value process obtained as the solution to the optimal contracting problem.

Proof First, we check that if the manager follows the proposed strategy, then (18) holds.

With the credit limits (16) and (17), we have the following boundary conditions. With $A_t = B_t = 0$, we have $W_t = W^1$. With $A_t = \bar{A}$ and $B_t = 0$, we have $W_t = \tilde{W}$. With $A_t = \bar{A}$ and $B_t = \bar{B}$, we have $W_t = R$.

For all $A_t \in [0, \bar{A})$ and $B_t = 0$, the proposed strategy implies $\hat{Y}_t = Y_t$, $dDiv_t^p = pdt$ and $db_t = 0$. Hence (14) implies $dB_t = 0$ and (13) implies $dA_t = \gamma A_t dt + \mu dt - \frac{\gamma}{\lambda} W^1 dt - dY_t$. Under this law of motion, the balance processes satisfy (18) at all $W_t \in (\tilde{W}, W^1]$ because

$$
\begin{aligned}
d(W^1 - \lambda A_t) &= -\lambda dA_t \\
&= -\lambda \gamma A_t dt + \lambda \frac{\gamma}{\lambda} W^1 dt + \lambda (dY_t - \mu dt) \\
&= \gamma(-\lambda A_t + W^1) dt + \lambda \sigma dZ_t \\
&= \gamma W_t dt + \lambda \sigma dZ_t,
\end{aligned}
$$

which replicates the law of motion for W_t determined by the optimal contract in the no-search region $W_t \in (\tilde{W}, W^1]$ and the boundary condition for $A_t = B_t = 0$ matches the boundary condition $W_t = W^1$.

For $A_t = \bar{A}$ and $B_t \in [0, \bar{B})$, the proposed strategy implies $\hat{Y}_t = Y_t$, $dDiv_t^p = 0$ (so the firm searches for a transition to monitoring), $db_t = \gamma \bar{A} dt + x_d dt - dY_t$, from which we obtain $dA_t = 0$ (with A_t remaining at $\bar{A}$ as long as B_t is positive), and, using, (14), $dB_t = (\gamma + \rho) B_t dt + \gamma \bar{A} dt +$

$(\mu - \frac{\gamma}{\lambda}W^1 - \frac{\rho}{\lambda}(\tilde{W} - R))dt - dY_t$. Under this law of motion, the balance processes satisfy (18) at all $W_t \in (R, \tilde{W}]$ because

$$
\begin{aligned}
d(\tilde{W} - \lambda B_t) &= -\lambda dB_t \\
&= -\lambda \left((\gamma + \rho) B_t dt + \gamma \frac{W^1 - \tilde{W}}{\lambda} dt + (\mu - \frac{\gamma}{\lambda} W^1 - \frac{\rho}{\lambda}(\tilde{W} - R))dt - dY_t \right) \\
&= -\lambda \left((\gamma + \rho) B_t dt + \gamma \frac{-\tilde{W}}{\lambda} dt + (-\frac{\rho}{\lambda}(\tilde{W} - R))dt - (dY_t - \mu dt) \right) \\
&= -\lambda (\gamma + \rho) B_t dt + \gamma \tilde{W} dt + (\rho(\tilde{W} - R))dt + \lambda(dY_t - \mu dt) \\
&= (\gamma + \rho) \left(-\lambda B_t dt + \tilde{W} \right) dt - \rho R dt + \lambda(dY_t - \mu dt) \\
&= (\gamma + \rho) W_t dt - \rho R dt + \lambda \sigma dZ_t
\end{aligned}
$$

which replicates the law of motion for W_t determined by the optimal contract in the search region $W_t \in (R, \tilde{W}]$ and the boundary condition for $A_t = \bar{A}$, $B_t = 0$ matches the boundary condition $W_t = \tilde{W}$.

Now we show that the proposed strategy is optimal to the manager. Due to the higher interest rate and the search trigger associated with the secondary line of credit, it is clearly optimal for the manager to not use it unless $A_t = \bar{A}$. Further, since the maxing out the primary credit line triggers search already, the manager has no longer an incentive to pay preferred dividends p when $A_t = 0$ and $B_t > 0$. As in DS, the manager has no incentive to pay common dividends when balance on revolving lines is positive because he owns fraction λ of both the dividends and the slack on the credit lines. For the same reason, the manager has no incentive to divert the cash flow.

Further, we need to check that the manager prefers to pay the noncumulative preferred dividend p whenever $A_t < \bar{A}$. Suspending the payment of pdt triggers search in dt, which costs the manager the expected loss of $\rho dt(W_t - R)$, but, on the other hand, it allows the manager to reduce the balance A_t at the rate pdt, which increases the manager's value W_t at the rate λpdt. We need to check that the loss is weakly larger than the gain whenever $A < \bar{A}$. Since the manager's loss is monotone in W_t, it is sufficient to check the case with the smallest W_t in the preferred dividend payment region, i.e., at $W_t = \tilde{W}$, which takes place when $A_t = \bar{A}$. Thus, we need to check that

$$\rho(\tilde{W} - R) \geq \lambda p = \lambda \frac{\rho}{\lambda}(\tilde{W} - R),$$

which is true, by (15).

This shows that if the manager follows the proposed policy, he does as well as in the optimal contract. That he cannot do better follows by contradiction. If he could, we'd be able to construct a diversion policy $\hat{Y}$ that would give the manager a higher payoff also under the optimal contract, which contradicts the incentive compatibility property of that contract. ■

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