Working Paper Series

Asset Bubbles and Global Imbalances

WP 18-07

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March 6, 2018

Working Paper No. 18-07

Abstract

We analyze the relationships between bubbles, capital flows, and economic activities in a rational bubble model with two large open economies. We establish a reinforcing relationship between global imbalances and bubbles. Capital flows from South to North facilitate the emergence and the size of bubbles in the North. Bubbles in the North in turn facilitate South-to-North capital flows. The model can simultaneously explain several stylized features of recent bubble episodes.

Keywords: Rational bubbles · global imbalances · financial frictions · credit boom

JEL codes: F32 · F41 · F44

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DOI: https://doi.org/10.21144/wp18-07

1 Introduction

The recent boom and bust of asset prices in the U.S. and the subsequent financial crisis have renewed the interest among economists and policymakers in understanding the relationships between capital flows, asset bubbles, and boom-busts in economic activities. Specifically, there are three stylized features that characterize this boom-bust episode:

1. *Global imbalances:* Over the past few decades, capital has flown in large quantities from emerging economies to developed ones. In particular, the U.S. has been a net capital importer since the 1980s, especially with inflows from emerging economies in the years preceding the Great Recession. At its peak in 2006, the U.S. current account deficit exceeded $600 billion, or 6% of GDP. In contrast, emerging economies, especially China and other emerging Asian economies, have experienced expanding current account surpluses. This phenomenon of global imbalances and upstream capital flows, which coincides with a general decline in world interest rates, has been well documented (see, e.g., Bernanke, 2005; Caballero et al., 2008; Mendoza et al., 2009; Gourinchas and Rey, 2013).

2. *Boom and bust in asset prices*: The peak period, between 2002 and 2007, of capital flows from emerging economies into the U.S. was associated with a spectacular boom and bust in asset prices, especially housing prices. For instance, the S&P/Case-Shiller U.S. National Home Price Index rose by 85% between January 2000 and July 2006, before dropping 27% below its peak value by February 2012. It is difficult to explain much of these fluctuations by changes in economic fundamentals, such as demographics, construction costs, or interest rates (Case and Shiller, 2003; Mian and Sufi, 2014; Shiller, 2015).

   Furthermore, several prominent economists and policymakers have argued that the glut of savings flowing from emerging economies into the U.S. after the East Asian crisis might have caused or at least facilitated the boom in housing prices prior to the financial crisis (Bernanke, 2007; Greenspan, 2009; Yellen, 2009; Obstfeld and Rogoff, 2009; Rajan, 2011; Stiglitz, 2012; Summers, 2014).

3. *Fluctuations in economic activities*: The boom and bust in asset prices were associated with significant fluctuations in economic activities. The boom in housing prices in the 2000s was associated with a credit boom for both households and firms (Chaney et al., 2012; Mian and Sufi, 2014; Justiniano et al., 2015). On the other hand, the collapse of housing prices in 2007 was associated with contractions in aggregate economic activities

(e.g., Yellen, 2009; Mian and Sufi, 2010, 2014).

The observations above are also consistent with broader empirical regularities about bubble episodes across countries: capital inflows and credit expansion during the boom phase, but sharp economic contractions and current account readjustment during the bust phase (Mendoza and Terrones, 2008, 2012; Reinhart and Rogoff, 2008, 2009; Kindleberger and Aliber, 2011).

Motivated by these observations, we develop a tractable framework of asset bubbles, capital flows, and economic activities. We generalize the rational bubble framework in a closed economy, as pioneered by Samuelson (1958), Diamond (1965), and Tirole (1985), into a setting with two large open economies, called the North and the South. The North represents the U.S., while the South represents emerging economies such as China. Each economy consists of overlapping generations of agents who provide labor and capital.

We then introduce heterogeneous productivity and a credit friction. Agents have different levels of entrepreneurial productivity in producing capital, generating a natural motive for borrowing and lending in each economy. In the absence of financial frictions, the most productive agents would undertake all capital investment, while other agents would simply lend. This would lead to an equilibrium in which the interest rate would be equal to the return on capital investment made by the most productive agents. However, we assume that due to imperfections in the credit market, agents face a constraint on their ability to borrow. Because of this credit friction, the interest rate is determined by the return to capital investment made by *marginal investors* – agents who are indifferent between investing and lending.

Next, we introduce an asymmetry in financial development. To reflect the fact that emerging economies are less financially developed than the U.S., we assume that the credit friction is stronger in the South than in the North. Because of asymmetric frictions, the autarky interest rate is lower in the South than in the North. Then, as in the global imbalances literature, financial integration leads to upstream capital flows, as credit flows from the South to the North. As a consequence, financial integration lowers the interest rate in the North and raises it in the South. One interpretation of this result is that the gradual integration of emerging economies into the global financial market leads to a lowering of the interest rate for agents in the North.

Finally, we introduce bubbles. As in the classic Samuelson-Diamond-Tirole (SDT) framework, bubbles are assets with no fundamental value but are traded at positive prices because agents expect to be able to resell them later. Bubbles can exist either in the North or in the South. As is well-known, bubbles can naturally arise in economies with constraints against lending because they provide less productive agents with an alternative storage of wealth, crowding out less productive capital investment and raising aggregate consumption. Furthermore, we follow Martin and Ventura (2012) and assume that agents can create new bubbles. Bubble creation can be interpreted, for instance, as entrepreneurs creating new firms of building new structures. New bubbles increase the net worth of agents, hence relaxing their credit constraints and allowing more productive agents to borrow more.[1] Consequently, an asset price boom leads to a boom in credit. Through this net worth effect, bubbles can crowd in capital investment and output, but their collapse leads to sharp economic contractions. As in most of the new generation of rational bubble models, including Caballero and Krishnamurthy (2006), Kocherlakota (2009), Miao and Wang (2011), Farhi and Tirole (2012), Martin and Ventura (2012), and Hirano and Yanagawa (2014), this crowd-in mechanism allows the model to be consistent with the fact that investment and output usually contract following the collapse of a bubble episode.[2]

Our main results are as follows. First, financial integration facilitates the emergence of bubbles in the North. This is because financial integration causes capital flows from the South to the North due to the asymmetry in financial development. Capital inflows lower the interest rate in the North and hence facilitate the emergence of Northern bubbles. We also shows that capital inflows raise the size of bubbles relative to the North's economy. We interpret this result as a formalization of the claim mentioned above that the inflows of savings from developing countries contributed to a housing bubble in the U.S.. This result also has an interesting corollary. Abel et al. (1989) have argued that developed economies such as the U.S. are dynamically efficient, and thus rational bubbles are unlikely to arise in such economies. Our result implies that even if the North is dynamically efficient, the financial integration with a sufficiently dynamically inefficient South can still enable the existence of bubbles in the *open* North.

Second, bubbles in the North in turn facilitate South-to-North capital flows. The emergence of a bubble in the North raises the returns from investing in the North and hence attracts more capital from the South. Putting these results together, our theory predicts a reinforcing relationship between global imbalances and bubbles: capital flows from South to North facilitate bubbles in the North, and vice versa.

Our model also predicts a relationship between the boom and bust of a bubble episode and fluctuations in the aggregate economy. In the boom phase of a bubble episode, the North

[1] Alternative ways of modeling expansionary bubbles include assuming that entrepreneurial activities that require funding arrive at a later period in an agent's life (e.g., Caballero and Krishnamurthy, 2006 and Farhi and Tirole, 2012) or assuming that agents are infinite-lived and alternate between being in high and low productivity states (Hirano and Yanagawa, 2014).

[2] In contrast, the SDT framework predicts investment and output booms after a bubble bursts.

experiences expansions in aggregate investment, output, consumption, as well as an increase in the stock of debt and a deterioration of the current account. However, the collapse of bubbles precipitates contractions in aggregate economic activities in the North, including debt deleveraging. We interpret these predictions of the effects of large bubbles as consistent with the aforementioned stylized facts for the U.S.

Related literature: According to our knowledge, there has been a relative shortage of theoretical framework to systematically analyze the underlying relationships between asset bubbles, capital flows, and fluctuations in economic activities. Our paper is most related to the literature on rational bubbles, which has a long heritage, dating back to the original model by Samuelson (1958) and later models by Diamond (1965), Tirole (1985), and Weil (1987). Much of the literature has focused on a closed economy setting. Recently, such papers include Kocherlakota (2009), Miao and Wang (2011), Martin and Ventura (2012), Farhi and Tirole (2012), Hirano and Yanagawa (2014), Aoki and Nikolov (2015), Ikeda and Phan (2016), Bengui and Phan (2017), and Hanson and Phan (2017).[3] For a large open economy setting, however, there are only a few papers, including Kraay and Ventura (2007), Basco (2013), and Rondina (2017). While Kraay and Ventura (2007) focus on the effects of the dot-com bubble on the pattern of capital flows into the U.S., and Basco (2013) and Rondina (2017) focus on the effects of capital flows on the existence of the dot-com or the U.S. housing bubble, our paper has a more general focus of understanding the relationships between bubbles, capital flows, and economic activities.

Finally, our paper also benefits from insights from the literature on global imbalances. The mechanism in our paper where the global asymmetry in financial development causes South-to-North capital flows is similar to that in Matsuyama (2005), Caballero et al. (2008), Mendoza et al. (2009), Song et al. (2011), Gourinchas and Jeanne (2013), Gourinchas and Rey (2013), and Buera and Shin (2015). For example, using a capital wedge analysis similar to the business cycle accounting method in Chari et al. (2007), Gourinchas and Jeanne (2013) argue that the differences in domestic financial frictions, measured by wedges that distort saving and investment decisions, can help explain why capital flows from less developed to more developed countries. Chinn et al. (2014) provide some evidence for the prediction of these theories that economies with more developed financial markets have weaker current accounts. Through a panel analysis, they find that financial development (e.g., a stronger rule of law) is negatively related to the current account balance; emerging economies with less developed financial markets tend to have stronger current account surpluses, thus displaying

[3] Also, see recent surveys by Barlevy (2012), Miao (2014), and Martin and Ventura (2017).

a higher tendency for capital outflows; lagged real interest rates are negatively related to the current account balance. These findings are consistent with our theoretical prediction that economies with more developed financial markets and thus higher interest rates tend to be on the receiving end of capital flows.

The rest of the paper is organized as follows. To build intuition and establish autarky benchmarks, Section 2 develops a closed economy model. Then, Section 3 develops the full model in a world with two large open economies. Section 4 provides the main results. Section 5 provides discussions. Section 6 concludes.

2 Closed economy

It is instructive to begin with a closed economy model. We augment the classic SDT framework of rational bubbles with two features: heterogeneous productivity and a credit friction. Heterogeneous productivity gives rise to natural borrowing and lending motives, while the credit friction allows for the possibility that bubbles crowd in investment and output. We first present the bubbleless benchmark and then introduce bubbles.

2.1 Bubbleless benchmark

Time is discrete and infinite, denoted by $t = 0, 1, 2, \dots$. We abstract away from any uncertainty in the bubbleless benchmark for simplicity. There are overlapping generations, each of which lives for two periods, "young age" and "old age" (except for the old generation in $t = 0$ who live for only one period). As in Bernanke and Gertler (1989), one can interpret the generational setting as representing the entry and exit of entrepreneurial agents and interpret each period as the length of a loan contract. Each generation consists of a continuum unit mass of infinitesimal agents. For simplicity, we assume that agents consume only in old age and are risk neutral. Young agents supply one unit of labor inelastically to firms and get wage income W_t. Old agents rent capital to firms at a rental rate R_t^k. For simplicity, we assume that capital depreciates completely after one period. Firms are competitive and have a Cobb-Douglas production function, $Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$ with $0 < \alpha < 1$, where Y_t is output, K_t is capital, L_t is labor, and $A_t = (1+g)^t$ is the level of technological progress that grows at an exogenous gross growth rate $1 + g \geq 1$. As usual, it is convenient to detrend the exogenous growth component. For each equilibrium variable X_t, we define the detrended variable x_t by $x_t \equiv \frac{X_t}{A_t^{1/(1-\alpha)}} = \frac{X_t}{(1+g)^t}$. As there is no friction in factor markets, the labor market clears:

$$L_t = 1,$$

and the detrended factor prices are given by:

$$w_t = (1-\alpha)k_t^{\alpha},\ R_t^k = \alpha k_t^{\alpha-1}. \tag{2.1}$$

Heterogeneous productivity: Besides supplying labor, young agents also engage in entrepreneurial activities. A young agent can convert each unit of the consumption good into a units of capital in the subsequent period. The entrepreneurial productivity a is identically and independently distributed across agents according to a continuous distribution over a convex support $\mathbb{A} \subseteq [0, \infty)$, whose cumulative distribution function F is strictly increasing and twice differentiable.

Credit market and credit friction: Young agents can borrow or lend to each other, and the loan is repaid when they grow old. Let R_{t+1} denote the interest rate on a loan between t and $t+1$ and R_{t+1}^k denote the marginal product of capital in $t+1$. In each period t, given her net worth consisting of wage income W_t, a young agent of type a chooses her net borrowing position $D_t(a)$, where a negative position means that the agent is lending and a positive position means that the agent is borrowing, to produce capital stock $K_{t+1}(a)$:[4]

$$K_{t+1}(a) = a \cdot \underbrace{[W_t + D_t(a)]}_{I_t(a)}. \tag{2.2}$$

The consumption of the entrepreneur in period $t+1$ is simply $C_{t+1}(a) = R_{t+1}^k K_{t+1}(a) - R_{t+1}D_t(a)$.

Entrepreneurs face a leverage constraint:

$$D_t(a) \leq \lambda_t(a) \underbrace{W_t}_{\text{net worth}}, \tag{2.3}$$

which states that each entrepreneur's borrowing is limited by her net worth. The limit $\lambda_t(a)$ places a constraint on the entrepreneur's debt-over-net-worth (or leverage) ratio, where $\lambda_t(a)$ is a weakly increasing function of a. This formulation of credit market friction is sufficiently general to envelope several types of credit constraints considered in the financial friction literature. For example, if one assumes $\lambda_t(a) \equiv \frac{R_{t+1}^k \phi a}{R_{t+1} - R_{t+1}^k \phi a}$,[5] where ϕ is a positive constant, then the constraint (2.3) maps to a standard collateral constraint: $R_{t+1}D_t(a) \leq \phi R_{t+1}^k K_{t+1}(a)$, which can arise when entrepreneurs can only pledge to repay in the next

[4] As is standard, note that even though K_{t+1} has time subscript $t+1$, it is determined in period t. Similar for R_{t+1} and R_{t+1}^k.

[5] Or, equivalently, $\lambda_t(a) = \frac{\phi a}{\bar{a}_t - \phi a}$, because in equilibrium, $R_{t+1} = \bar{a}_t R_{t+1}^k$, where $\bar{a}_t$ is the productivity cutoff threshold, as to be derived in Section 2.1.1.

period at most a fraction ϕ of the value of their asset (e.g., Kiyotaki and Moore, 1997). Alternatively, if one assumes $\lambda_t(a) \equiv \lambda$, where $\lambda \geq 0$ is a constant, then the constraint (2.3) maps to an analytically convenient form of collateral constraint, which states that the amount of credit is limited by the individual's net worth and has been used extensively in the recent literature (e.g., Banerjee and Moll, 2010; Buera and Shin, 2013; Moll, 2014). In general, a larger λ can be interpreted as representing an environment with less financial friction.

In summary, a young agent of type a solves:

$$\max_{\{K_{t+1}(a),\ D_t(a)\}} R^k_{t+1} K_{t+1}(a) - R_{t+1} D_t(a)$$

subject to capital production technology (2.2), the nonnegativity constraint on capital $K_{t+1}(a) \geq 0$, and credit constraint (2.3).

Equilibrium: Given an initial aggregate capital stock K_0, a *bubbleless equilibrium* consists of a set of allocations $\{D_t(a),\ K_{t+1}(a)\}_{a \in \mathbb{A}}$ and prices $\{R_{t+1},\ R^k_{t+1},\ W_t\}$ for each $t \geq 0$ such that given the prices, the set of allocation solves the problems of firms and young agents, and the credit market clears in each $t \geq 0$:

$$\int D_t(a) dF(a) = 0. \tag{2.4}$$

We focus on stationary (or balanced growth path) equilibria, where the rates of return R_{t+1} and R^k_{t+1} and detrended variables $d_t(a)$, $k_{t+1}(a)$, and w_t are time-invariant.

2.1.1 Solution

With the heterogeneity in productivity, the optimal capital accumulation decision of agents with productivity a can be summarized by:

$$\frac{K_{t+1}(a)}{W_t} \begin{cases} = 0 & \text{if } R_{t+1} > aR^k_{t+1} \\ \in [0,\ (1+\lambda_t(a))a] & \text{if } R_{t+1} = aR^k_{t+1} \\ = (1+\lambda_t(a))a & \text{if } R_{t+1} < aR^k_{t+1} \end{cases}. \tag{2.5}$$

The first line states that if the interest rate is above the return from investing in capital, then agents only engage in lending and do not accumulate capital. The second states that if they are equal, then agents are indifferent between lending and investing. Finally, the last states that if the interest rate is below the return from investing in capital, then agents invest in capital by borrowing up to the maximum amount that satisfies credit constraint (2.3).

Thus, in equilibrium, we have an endogenous segmentation of types into borrowers and lenders. In particular, there is a *productivity threshold* $\bar{a}_{nb,t} \in \mathbb{A}$ (the subscript stands for "no bubble") such that:

$$\frac{D_t(a)}{W_t} \begin{cases} = -1 & \text{if } a < \bar{a}_{nb,t} \\ \in [-1, \lambda_t(a)] & \text{if } a = \bar{a}_{nb,t} \\ = \lambda_t(a) & \text{if } a > \bar{a}_{nb,t} \end{cases}. \tag{2.6}$$

In other words, agents with productivity strictly below $\bar{a}_{nb,t}$ choose not to invest in capital and become lenders, while those with productivity strictly above $\bar{a}_{nb,t}$ optimally borrow to the maximum subject to credit constraint (2.3) to invest in capital. Those with productivity $\bar{a}_{nb,t}$ are indifferent between investing in capital and lending, and we call them *marginal investors.*

The indifference condition of the marginal investors leads to a no-arbitrage condition:

$$R_{t+1} = \bar{a}_{nb,t} R^k_{t+1}. \tag{2.7}$$

The left hand side is the return from lending, and the right hand side is the return from investing in capital for marginal investors.

Equilibrium dynamics: From equations (2.5), (2.6), and (2.7), we can completely characterize the equilibrium dynamics. By aggregating the capital accumulation equation (2.5), we get the following expression for the aggregate capital stock over net worth ratio:

$$\frac{K_{t+1}}{W_t} = \int \frac{K_{t+1}(a)}{W_t} dF(a) = \int_{a > \bar{a}_{nb,t}} (1 + \lambda_t(a)) a \; dF(a),$$

and by combining with the equilibrium wage with labor market clearing from equation (2.1), we get the following law of motion for the detrended aggregate capital stock:

$$\frac{(1+g)k_{t+1}}{(1-\alpha)k_t^\alpha} = \int_{a > \bar{a}_{nb,t}} [1 + \lambda_t(a)] \, a \; dF(a) \equiv \mathcal{K}_t(\bar{a}_{nb,t}), \tag{2.8}$$

where $\mathcal{K}_t(\bar{a}_{nb,t})$ denotes the capital accumulation rate as a function of $\bar{a}_{nb,t}$. Intuitively, capital is accumulated by agents whose productivity levels satisfy $a \geq \bar{a}_{nb,t}$, and given the binding leverage constraint, each of these agents accumulate $[1 + \lambda_t(a)] \, a$ units of capital per unit of net worth.

Similarly, by aggregating the debt equation (2.6) and combining it with the credit market clearing condition (2.4), we get the following identity that equates the aggregated investment (the left hand side) and the aggregated savings (the right hand side):

$$\underbrace{\int_{a>\bar{a}_{nb,t}} [1+\lambda_t(a)]\, dF(a)}_{\mathcal{I}_t(\bar{a}_{nb,t})} \times W_t = W_t.$$

By canceling W_t on both sides, we get an identity that equates the *investment rate* and the *savings rate*:

$$\mathcal{I}_t(\bar{a}_{nb,t}) \equiv \int_{a>\bar{a}_{nb,t}} [1+\lambda_t(a)]\, dF(a) = 1, \tag{2.9}$$

where $\mathcal{I}_t(\bar{a}_{nb,t})$ denotes the investment rate as a function of $\bar{a}_{nb,t}$. Note that both $\mathcal{K}_t(\cdot)$ and $\mathcal{I}_t(\cdot)$ are decreasing functions. Equation (2.9) implicitly and uniquely determines the threshold $\bar{a}_{nb,t}$.[6]

The interest rate is determined by the combination of the no-arbitrage equation (2.7) and the marginal product of capital from equation (2.1):

$$R_{t+1} = \bar{a}_{nb,t}\alpha k_{t+1}^{\alpha-1}. \tag{2.10}$$

Balanced growth path (BGP): We can conveniently derive the BGP of the economy from equations (2.8), (2.9), and (2.10) above. From (2.8), we get the following expression for the detrended capital stock:

$$k_{nb} = \left[\frac{1-\alpha}{1+g}\mathcal{K}(\bar{a}_{nb})\right]^{\frac{1}{1-\alpha}}. \tag{2.11}$$

From (2.9), the threshold is determined as

$$\mathcal{I}(\bar{a}_{nb}) = 1. \tag{2.12}$$

Combining (2.10) and (2.11) yields the interest rate as:

$$R_{nb} = \mathcal{R}(\bar{a}_{nb}) \equiv \frac{(1+g)\alpha}{1-\alpha}\frac{\bar{a}_{nb}}{\mathcal{K}(\bar{a}_{nb})}. \tag{2.13}$$

Equation (2.13) implies that the interest rate is increasing in the threshold. Given the

[6] The savings rate is 1 in this model because we assume that agents only consume in old age. Of course the savings rate will be different under alternative specifications of the utility function. For example, if the utility function is $\ln c_y + \beta \ln c_o$, then the savings rate would be β. The uniqueness of $\bar{a}_{nb,t}$ comes from the fact that the function $\mathcal{I}_t(x)$ is decreasing and continuous in x and that $\lim_{x\to\inf\mathbb{A}} \mathcal{I}_t(x) \geq \int dF(a) = 1$ and $\lim_{x\to\sup\mathbb{A}} \mathcal{I}_t(x) = 0$.

interest rate, equation (2.13) is written as

$$\bar{a}_{nb} = \mathcal{A}(R_{nb}) \equiv \mathcal{R}^{(-1)}(R_{nb}). \tag{2.14}$$

Then, the investment rate function can be written as a function of the interest rate as $\mathcal{I}(\mathcal{A}(R_{nb}))$. Because $\mathcal{I}(\cdot)$ is decreasing and $\mathcal{A}(\cdot)$ is increasing, the investment rate function is decreasing in the interest rate. Hence, with the fact that $\lim_{x\to\inf\mathbb{A}}\mathcal{I}_t(x) \geq 1$ and $\lim_{x\to\sup\mathbb{A}}\mathcal{I}_t(x) = 0$, equation (2.12) uniquely determines the threshold and thereby the BGP.

The following lemma summarizes our analysis above:

Lemma 1. *The bubbleless equilibrium features an endogenous segmentation of agents into borrowers and lenders. The equilibrium dynamics can be characterized by capital accumulation equation (2.5) and (2.8), debt equation (2.6), savings-equal-to-investment equation (2.9), and no-arbitrage equation (2.10).*

The corresponding equations (2.11), (2.12), and (2.13) determine the capital stock, the cutoff productivity threshold, and the interest rate on the BGP.

Proof. See the text above. □

2.1.2 Example

As the model is general, it is instructive to look at an example with a simple parametrization. Following Banerjee and Moll (2010), Buera and Shin (2013), and Moll (2014), we focus here on the most analytically tractable credit constraint, where the limit on the debt-over-net-worth ratio is a constant $\lambda \geq 0$.

Furthermore, assume that the productivity distribution is the uniform distribution over $[a_{\min}, a_{\max}] \subset [0, \infty)$. Then the productivity threshold, the interest rate, and the detrended capital stock in the bubbleless BGP have closed-form expressions:

$$\begin{aligned}
\bar{a}_{nb} &= \frac{a_{\min} + \lambda a_{\max}}{1+\lambda} \\
R_{nb} &= \frac{2(1+g)\alpha}{1-\alpha}\frac{a_{\min} + \lambda a_{\max}}{a_{\min} + (1+2\lambda)a_{\max}} \\
k_{nb} &= \frac{1-\alpha}{1+g}\frac{a_{\min} + (1+2\lambda)a_{\max}}{1+(1+2\lambda)}
\end{aligned}$$

It can be seen that $\bar{a}_{nb}$, R_{nb}, and k_{nb} are increasing in the leverage limit λ, as a larger λ allows more resources to be shifted towards more productive agents (reflected by a higher weight on $a_{\max}$ in the expressions above). Furthermore, a decrease in the distribution F in the first order stochastic dominance sense (in this case, this means a decrease in $a_{\max}$ and/or $a_{\min}$) is also associated with a decrease in $\bar{a}_{nb}$, R_{nb}, and k_{nb}. Note that the difference $a_{\max} - \bar{a}_{nb}$ can be mapped to the concept of a *capital wedge,* which reflects a wedge between the private and social returns of capital and is a reduced-form measure of imperfections in the financial market, as used in Chari et al. (2007), Gourinchas and Jeanne (2013), and Gourinchas and Rey (2013). For instance, when there is no financial friction (λ is infinite), the wedge is zero. We will revisit these comparative statics in the open economy section.

If the productivity distribution is instead lognormally distributed, then the equilibrium can only be solved numerically. Figure 1 plots the two key functions, $\mathcal{A}(R)$ and $I(\mathcal{A}(R))$, that determine the BGP. It can be seen that, as stated earlier, the threshold function $\mathcal{A}$ is increasing, while the investment function $\mathcal{I}$ is decreasing. The bottom panel shows how the interest rate R_{nb} is determined by the intersection of the downward sloping curve representing the investment rate function $I(\mathcal{A}(R))$ and the flat line representing the savings rate of 1. The panel also illustrates how the investment curve shifts downward (see the dashed line), leading to a decrease in the equilibrium interest, when there is either a decrease in λ or a decrease in F in the first-order dominance sense.

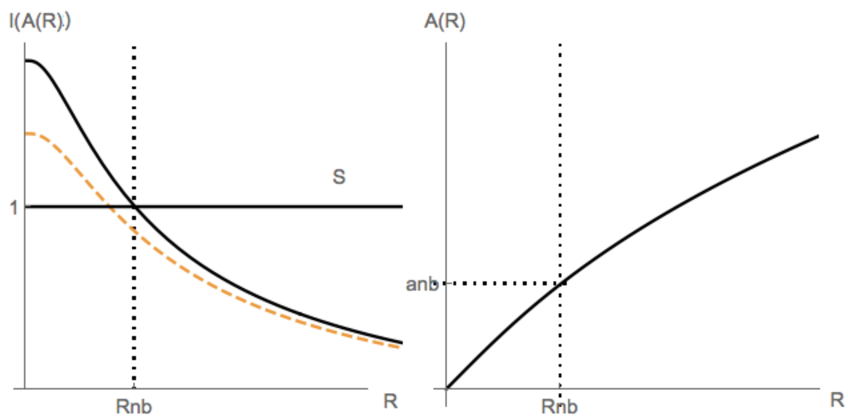


Fig. 1: Plots of threshold function $\mathcal{A}(R)$, investment rate function $\mathcal{I}(\mathcal{A}(R))$ and determination of bubbleless interest rate R_{nb} in the closed economy.

2.2 Bubbles

We now introduce asset bubbles. Following the SDT framework, we model a (pure) bubble as an asset that pays no dividend and thus has a zero fundamental value but is traded at a positive price. The only reason an individual purchases a bubble is that he or she expects to be able to resell it later.

To model the expansionary effect of bubbles on the aggregate debt and investment, we follow Martin and Ventura (2011, 2012) and introduce (exogenous) bubble creation. Each young agent is endowed with an ability to create one unit of new bubble assets, and for simplicity, we assume bubble creation is costless (bubble creation is thus effectively a wealth shock). Let B_t^O denote the value of the portfolio that contains all old bubbles and let B_t^N denote the portfolio that contains all new bubbles created in period t. By definition, the total value of all bubbles B_t is $B_t = B_t^O + B_t^N$. For simplicity, as in Martin and Ventura (2012), throughout the paper, we focus on bubble equilibria in which the relative size of new bubbles n is constant:[7]

$$B_t^N = nB_t,\ 0 \leq n < 1.$$

Consequently, the value of old bubbles is $B_t^O = (1-n)B_t$. One interpretation of bubble creation is that young agents are entrepreneurs who create new firms or structures and there are bubbles in the value of these new firms. The creation of new tech firms during the dot-com boom is an example of bubble creation. The construction of new houses during the housing bubble episode in the 2000s is another example of bubble creation.

With bubble creation, the net worth of a young agent in each period is $W_t + nB_t$, instead of W_t as in the bubbleless benchmark. The constraint (2.3) on the debt-over-net-worth ratio is thus replaced by:

$$D_t(a) \leq \lambda_t(a) \underbrace{(W_t + nB_t)}_{\text{net worth}}. \tag{2.15}$$

For convenience, we denote the aggregate bubble-over-net-worth ratio (or bubble ratio for brevity) by:

$$\beta_t \equiv \frac{B_t}{W_t + nB_t}.$$

Bubbles are fragile, and they require the coordination of expectations across generations: one would buy bubbles only if they expect someone else would buy them in the future. To model this fragility, we follow Weil (1987) and assume that in any period the value of all existing bubbles can crash to zero with an exogenous probability $p \in [0, 1)$. Agents rationally discount the risk that the bubbles crash. Once collapsed, bubbles are not expected to emerge again in the future.

The optimization problem of a young agent of entrepreneurial productivity a is to choose a portfolio consisting of investment in capital $I_t(a)$, net debt position $D_t(a)$, and the expenditure on bubble investment $B_t(a)$, to maximize the expected consumption in the subsequent

[7] There could be equilibria in which the relative size of bubbles is time varying and denoted as n_t. Our analysis can be easily extended to such cases.

period:

$$\max_{\{I_t(a),D_t(a),B_t(a)\}} R_{t+1}^k K_{t+1}(a) + (1-p)\frac{B_{t+1}^O}{B_t} B_t(a) - R_{t+1} D_t(a),$$

subject to the capital production technology:

$$K_{t+1}(a) = aI_t(a),$$

the budget constraint:

$$I_t(a) + B_t(a) = W_t + nB_t + D_t(a), \tag{2.16}$$

the nonnegativity constraints on capital and bubbles, $K_{t+1}(a) \geq 0$ and $B_t(a) \geq 0$ and credit constraint (2.15). In the maximand, $R_{t+1}^k K_{t+1}(a)$ is the return from capital, $(1-p)\frac{B_{t+1}^O}{B_t}$ is the expected return from purchasing one unit of bubble in period t at price B_t and reselling at price B_{t+1}^O in the next period if bubbles do not crash, and $R_{t+1}D_t(a)$ is the repayment of debt. In the budget constraint, $I_t(a)$ is the expense required to produce $K_{t+1}(a)$ units of capital, $B_t(a)$ is the agent's expenditure on purchasing bubbles, $W_t + nB_t$ is the agent's net worth, consisting of wage income and the value of newly created bubbles, and $D_t(a)$ is the net borrowing.

Equilibrium: Given an initial aggregate capital stock $K_0 > 0$, a *(stochastic) bubble equilibrium* consists of allocations $\{B_t(a),\ D_t(a),\ K_{t+1}(a)\}_{a\in\mathcal{A}}$, prices R_t, R_t^k and W_t, and value of bubbles $B_t > 0$ for each period $t \geq 0$, such that: (i) given prices, the allocations solve the optimization problems of firms and agents, (ii) the credit market clearing condition (2.4) holds in each period, and (iii) the bubble market clears in each period:

$$\int B_t(a)dF(a) = B_t. \tag{2.17}$$

A *bubble balanced growth path* is a stochastic bubble equilibrium in which the rates of return, R_t and R_t^k, and detrended variables, b_t, $d_t(a)$, $k_{t+1}(a)$, and w_t, are time invariant.

Solution

As in the bubbleless benchmark, a bubble equilibrium is characterized by an endogenous segmentation of types into borrowers and lenders at a productivity threshold $\bar{a}_{b,t} \in \mathbb{A}$. Similar to equations (2.5) and (2.6), the capital-over-net-worth ratio across agents can be summarized by:

$$\frac{K_{t+1}(a)}{W_t + nB_t} \begin{cases} = 0 & \text{if } a < \bar{a}_{b,t} \\ \in [0, (1+\lambda_t(a))a] & \text{if } a = \bar{a}_{b,t} \\ = (1+\lambda_t(a))a & \text{if } a > \bar{a}_{b,t} \end{cases}, \tag{2.18}$$

and the leverage ratio across agents can be summarized by:

$$\frac{D_t(a)}{W_t + nB_t} \begin{cases} = -1 & \text{if } a < \bar{a}_{b,t} \\ \in [0, \lambda_t(a)] & \text{if } a = \bar{a}_{b,t} \\ = \lambda_t(a) & \text{if } a > \bar{a}_{b,t} \end{cases}. \tag{2.19}$$

Equation (2.18) shows how bubble creation increases young agents' net worth from W_t to $W_t + nB_t$, because these agents can sell their newly created bubbles at market value nB_t. As a consequence, bubbles have a crowd-in effect on capital investment, by raising the net worth of young agents and consequently raising their ability to borrow from the credit market.

The agents with type $a = \bar{a}_{b,t}$ are marginal investors, as they are indifferent among lending, investing in capital, and *investing in bubbles.* Their indifference yields no-arbitrage conditions:

$$R_{t+1} = \bar{a}_{b,t} R^k_{t+1} = \frac{(1-p)(1-n)(1+g)b_{t+1}}{b_t}. \tag{2.20}$$

The first term is the interest rate, the second term is the return from capital for marginal investors, and the last term is the expected return from bubble speculation. For agents with $a > \bar{a}_{b,t}$, the return from capital aR^k_{t+1} is greater than the return from lending and the return from bubble speculation. Thus, only those with $a \leq \bar{a}_{b,t}$ purchase bubbles.

Equilibrium dynamics: From equations (2.18), (2.19), and (2.20), the equilibrium dynamics can be characterized as follows. By aggregating the individual capital accumulation equation (2.18) and combining it with the equilibrium wage equation (2.1), we get the following law of motion for the detrended aggregate capital stock:

$$\frac{(1+g)k_{t+1}}{(1-\alpha)k_t^\alpha + nb_t} = \mathcal{K}_t(\bar{a}_{b,t}),$$

where $\mathcal{K}_t(\cdot)$ is as defined in (2.8). Note that the difference between this equation and the counterpart equation (2.21) in the bubbleless benchmark lies in the denominator on the left hand side: the (detrended) net worth of a young agent in period t is no longer $(1-\alpha)k_t^\alpha$; it is now $(1-\alpha)k_t^\alpha + nb_t$ due to bubble creation. It is more convenient to rewrite this equation by using the bubble ratio β_t:

$$\frac{(1-n\beta_t)(1+g)k_{t+1}}{(1-\alpha)k_t^\alpha} = \mathcal{K}_t(\bar{a}_{b,t}). \tag{2.21}$$

By aggregating the debt equation (2.19) and combining it with the credit market clearing condition (2.4), we get an identity that equates the aggregated investment in capital *and bubbles* (the left hand side) and the aggregated savings (the right hand side):

$$\underbrace{\mathcal{I}_t(\bar{a}_t)(W_t + nB_t)}_{\text{capital investment}} + \underbrace{B_t}_{\text{bubble investment}} = \underbrace{W_t + nB_t}_{\text{savings}},$$

where $\mathcal{I}_t(\cdot\,)$ is as defined in (2.9). By dividing the net worth $W_t + nB_t$ on both sides, we get an identity between the investment rate and the savings rate:

$$\mathcal{I}_t(\bar{a}_{b,t}) + \beta_t = 1. \tag{2.22}$$

This equation differs from the counterpart equation (2.9) in the bubbleless benchmark in an important aspect: the presence of the bubble-over-net-worth term β_t, which has to be positive in the bubble equilibrium. This captures the fact that in a bubble equilibrium, total savings can be channeled into either capital investment or bubble investment. Thus, a direct comparison between the two equations implies that $\mathcal{I}_t(\bar{a}_{b,t}) = 1 - \beta_t < 1 = \mathcal{I}_t(\bar{a}_{nb,t})$. Recall that the capital investment rate function $\mathcal{I}_t(\bar{a}) \equiv \int_{a>\bar{a}} [1 + \lambda_t(a)]\, dF(a)$ is decreasing. Therefore, an interesting implication immediately follows from equations (2.9) and (2.22): the threshold in the bubble equilibrium must be larger than the threshold in the bubbleless equilibrium:

$$\bar{a}_{b,t} > \bar{a}_{nb,t}.$$

Intuitively, as bubbles provide a new investment opportunity, some agents find it optimal to stop producing capital and instead switch to bubble speculation, causing the productivity threshold to rise from $\bar{a}_{nb,t}$ to $\bar{a}_{b,t}$.

From the no-arbitrage condition (2.20) and the marginal product of capital from equation (2.1), we have the following expression for the interest rate:

$$R_{t+1} = \bar{a}_{b,t}\alpha k_{t+1}^{\alpha-1}, \tag{2.23}$$

which is identical to equation (2.10) in the bubbleless benchmark.

Finally, the evolution of the detrended bubble value follows immediately from equation

(2.20):

$$\frac{(1+g)b_{t+1}}{b_t} = \frac{R_{t+1}}{(1-p)(1-n)}. \tag{2.24}$$

Balanced growth path: From equations (2.21) to (2.24) above, we can characterize the BGP. From (2.21), the detrended capital stock satisfies:

$$(1-n\beta)k_b^{1-\alpha} = \frac{1-\alpha}{1+g}\mathcal{K}(\bar{a}_b). \tag{2.25}$$

From equation (2.24), it immediately follows that the interest rate in the BGP is simply equal to a constant that reflects the bubble bursting risk, the bubble creation rate, and the economy's growth rate:

$$R_b = (1-p)(1-n)(1+g). \tag{2.26}$$

By combining equation (2.23) with capital equation (2.25) and the definition of the function $\mathcal{A}$, the productivity threshold is simply:

$$\bar{a}_b = \mathcal{A}\left(\frac{R_b}{1-n\beta}\right), \tag{2.27}$$

where $\mathcal{A}(\cdot)$ is as defined in (2.14). The investment-savings equation (2.22) can be rewritten as:

$$1-\mathcal{I}\left(\mathcal{A}\left(\frac{R_b}{1-n\beta}\right)\right) = \beta, \tag{2.28}$$

which is solved for the bubble ratio β.

Note that in this model, a bubble has two opposite effects on capital and output, as implied by equations (2.18) and (2.27). First, at an extensive margin, a drop in the number of investors from $1-F(\bar{a}_{nb})$ to $1-F(\bar{a}_b)$ crowds out capital and thus has a negative effect on output. Second, at an intensive margin, bubbles increase the net worth and boost the amount of investment made by investors. If the intensive margin effect dominates the extensive margin effect, bubbles become expansionary, increasing capital and output.

We summarize our characterization of the bubble equilibrium in the following lemma:

Lemma 2. *The bubble equilibrium is characterized by an endogenous segmentation of agents into borrowers and lenders at a threshold $\bar{a}_{b,t} > \bar{a}_{nb,t}$.*

The equilibrium dynamics can be characterized by the capital accumulation equation (2.21), savings-equal-to-investment equation (2.22), interest rate equation (2.23), and bubble growth equation (2.24).

The corresponding set of equations (2.25)-(2.28) determine the bubble BGP.

Proof. See the text above. □

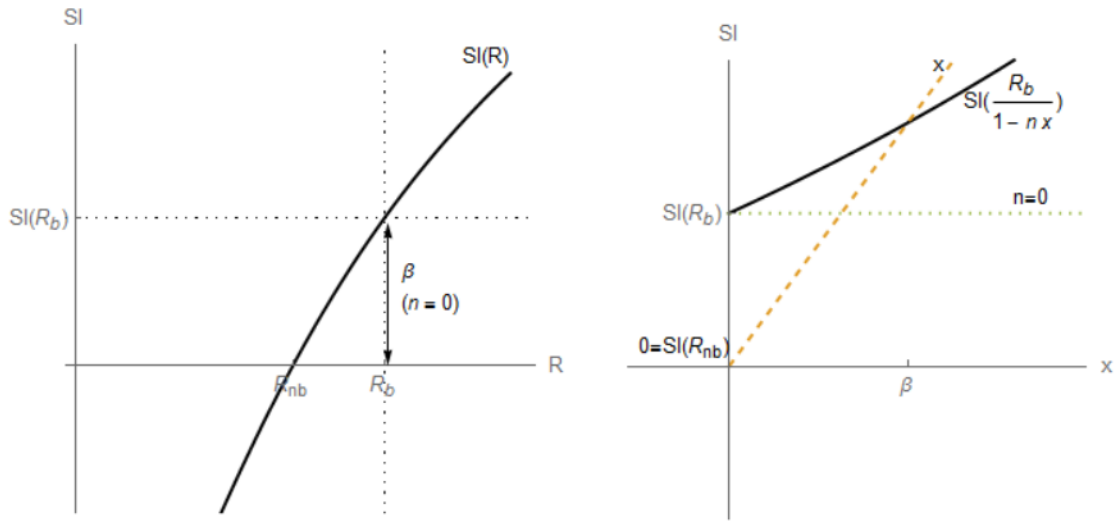


Fig. 2: Excess savings function $SI(R) \equiv 1 - \mathcal{I}(\mathcal{A}(R))$ and determination of bubble-over-net-worth ratio β.

We can use graphical analysis to gain more intuition behind the determination of the bubble BGP. First, let us consider the case without bubble creation, i.e., $n = 0$. The left panel of Figure 2 illustrates the savings-investment equation (2.28). The upward sloping solid line plots the function $SI(R) \equiv 1 - \mathcal{I}(\mathcal{A}(R))$, which is equal to the savings rate minus the capital investment rate at each interest rate R. At any interest rate R, the savings rate is inelastic at 1, while the capital investment rate is $\mathcal{I}(\mathcal{A}(R))$, as only agents whose productivity is above $\mathcal{A}(R)$ would invest in accumulating capital. The curve intersects the horizontal axis at $R = R_{nb}$, reflecting equation (2.12), which states that in the bubbleless economy, the credit market must clear at the equilibrium interest rate R_{nb}. However, in the bubble economy, due to the no-arbitrage equation between lending and investing in the risky bubble market, the interest rate is pinned down at $R = R_b$ (equation (2.26)). At this interest rate, the capital investment rate is only $\mathcal{I}(\mathcal{A}(R_b))$, leading to an "excess savings rate" of $SI(R_b)$. In equilibrium, this excess savings must be absorbed by the investment in the bubble, i.e., $\beta = 1 - \mathcal{I}(\mathcal{A}(R_b))$.

The left panel of Figure 2 also highlights an important result: when $n = 0$, the bubble ratio is positive if and only if $R_{nb} < R_b$. This corresponds to a standard result in the rational bubble literature that bubbles can only exist in a low interest rate environment, i.e., the interest rate in the bubbleless equilibrium is sufficiently low.

Now, let us consider the more general case of $n \geq 0$. The savings-investment equation (2.28) gives the bubble ratio β solution to $x = 1 - \mathcal{I}(\mathcal{A}(\frac{R_b}{1-nx}))$, or equivalently $x = SI(\frac{R_b}{1-nx})$. The right panel of Figure 2 illustrates the solution to this equation as the intersection of the dashed 45-degree line, which represents the left hand side, and the upward-sloping solid curve, which represents the right hand side. The curve is upward sloping because when $n > 0$, the function $SI(\frac{R_b}{1-nx})$ is increasing in x. (Note that when $n = 0$, the curve representing $SI(\frac{R_b}{1-nx})$ is simply a straight horizontal line, as illustrated in the figure.)

The right panel also helps us understand the existence condition of bubbles when $n \geq 0$. When $n = 0$, the excess savings curve $SI(\frac{R_b}{1-nx})$ is flat. When $n > 0$, the curve is upward sloping. Furthermore, by definition, we know that $SI(\frac{R_b}{1-nx})|_{x=1} = 1 - \mathcal{I}(\mathcal{A}(\frac{R_b}{1-n})) < 1$. Thus, the SI curve necessarily lies below the 45-degree line at $x = 1$. If the SI curve is weakly convex over $(0, 1)$, then we know that it intersects the 45-degree line at some $x = \beta \in (0, 1)$ if and only if the curve lies above the 45-degree line at $x = 0$, i.e., $SI(\frac{R_b}{1-nx})|_{x=0} > 0$ (as illustrated in the figure). This inequality is equivalent to $SI(R_b) > 0 = SI(R_{nb})$, or simply $R_b > R_{nb}$, which is exactly the existence condition established above.

As shown in Appendix A.1, the SI curve is weakly convex over $(0, 1)$, i.e.,

$$\frac{d^2}{dx^2} SI\left(\frac{R_b}{1-nx}\right) \geq 0, \ \forall 0 < x < 1 \tag{2.29}$$

if and only if the following holds:

$$2 + \frac{a\lambda'(a)}{1+\lambda(a)} + \frac{(1+\lambda)a^2 f(a)}{\int_{z>a}(1+\lambda)z \ dF(z)} \geq -\frac{af'(a)}{f(a)}, \ \forall \mathcal{A}\left(R_b\right) < a < \mathcal{A}\left(\frac{R_b}{1-n}\right), \tag{2.30}$$

where $f(\cdot) \equiv F'(\cdot)$ is the probability density function. Note that since the second and third terms on the left hand side are nonnegative, a sufficient condition that guarantees (2.30) is $2 \geq -\frac{af'(a)}{f(a)}$, which is a condition on the relative elasticity of the probability density function. This condition always holds if f is a uniform distribution (as the right hand side is trivially zero). In Appendix A.1, we also show that condition (2.30) always holds if f is a lognormal distribution and λ is constant.[8]

The discussion above is formalized by the following result:

Proposition 1. [Bubble existence] *Assume condition (2.30) holds. Then there exists a bubble BGP if and only if the interest rate on the bubbleless BGP R_{nb}, as given by (2.13), is sufficiently low:*

$$R_{nb} < \underbrace{(1-p)(1-n)(1+g)}_{R_b}. \tag{2.31}$$

Proof. Appendix A.2. $\square$

For example, if we assume the simple parametrization with a uniform distribution in Sec-

[8] If λ is not a constant, then a sufficient condition that guarantees (2.30) is $e^{\sigma^2+\mu} \geq \mathcal{A}((1-p)(1+g))$, where $\log a \sim N(\mu, \sigma^2)$

tion 2.1.2, then the low interest rate condition (2.31) can be written in primitive parameters:

$$\frac{2\alpha}{1-\alpha}\frac{a_{\min}+\lambda a_{\max}}{a_{\min}+(1+2\lambda)a_{\max}} < (1-p)(1-n). \tag{2.32}$$

The condition intuitively states that for a bubble to exist, the risk of bursting p and the rate of creation n cannot be too large, as otherwise bubbles would have to grow too fast to be sustainable in equilibrium. The condition also shows that a lower degree of financial development (a smaller λ) is associated with a larger existence region for bubbles.

Remark 1. Note that in the example above, if $\lambda \to \infty$ (i.e., there is no credit constraint), then the existence condition (2.32) reduces to $\frac{\alpha}{1-\alpha} < (1-p)(1-n)$, which can be mapped to a standard dynamic inefficiency condition for the existence of bubbles in an overlapping generation model without credit friction (e.g., the SDT framework), stating that the economy must exhibit sufficient dynamic inefficiency so that the bubbleless steady-state interest rate $\frac{\alpha}{1-\alpha}$ is lower than the bubble steady-state interest rate $(1-p)(1-n)$.

3 Open economies

We now extend the closed economy model to an environment with two large open economies. We first describe the model, solve the bubbleless benchmark, then analyze how financial integration affects the existence of bubbles, and finally show how bubbles affect capital flows.

Consider two open economies, called the North and the South, each having the same structure as described in the closed model. We denote variables in the South with a star ($*$) and denote corresponding variables in the closed economy with a superscript c (for example, R_t^c and R_t^{c*} represent the interest rate in the closed North and closed South, respectively). The two economies have the same TFP growth ($g = g^*$) and the same Cobb-Douglas production function ($\alpha = \alpha^*$). They can differ in the leverage constraints $\lambda_t(\cdot)$ and $\lambda_t^*(\cdot)$, which capture the extent of financial market imperfections, and they can differ in the productivity distributions F and F^*.

For simplicity, we assume that agents and firms can rent capital,[9] hire labor, and trade bubbles in the domestic markets only. However, the credit market is perfectly integrated: agents can freely borrow and lend across borders. Hence, the following interest rate parity must hold:

$$R_{t+1} = R_{t+1}^*, \tag{3.1}$$

[9] This can be interpreted as, for example, firms must rent machines and real estate offices locally.

and the world credit market must clear:

$$\int D_t(a)dF(a) + \int D_t^*(a)dF^*(a) = 0. \tag{3.2}$$

3.1 Bubbleless benchmark

We first study the bubbleless benchmark. Given initial aggregate capital stocks K_0, K_0^*, a *bubbleless equilibrium* in the open economies consists of allocations $\{D_t(a), K_{t+1}(a), D_t^*(a), K_{t+1}^*(a)\}_{a\in\mathbb{A}}$ and prices $\{R_t, R_t^*, R_t^k, R_t^{k*}, W_t, W_t^*\}$ for each period $t \geq 0$ such that: (i) given prices, the allocations solve firms' and agents' optimization problems in each country, (ii) interest rate parity condition (3.1) holds, and (iii) integrated credit market clearing condition (3.2) holds.

Equilibrium dynamics: Since the dynamics are symmetric between the two economies, it is sufficient to focus on the North. The equilibrium dynamics are similar to those in Section 2. The economy is segmented into borrowers and lenders at an endogenous productivity threshold $\bar{a}_{nb,t}$. In each period t, given the capital stock K_t as a state variable, the law of motion of capital is given by (2.8) and the no-arbitrage condition that equates the interest rate and the return from capital investment is (2.10).

However, an important difference lies in the credit market clearing condition. In the closed economy, the condition is simply $\mathcal{I}_t(\bar{a}_{nb,t}) = 1$, where the left hand side is the investment rate and the right hand side is the savings rate (equation (2.9)). In the open economy, the corresponding condition is:

$$\mathcal{I}_t(\bar{a}_{nb,t}) + \mu_t^* \mathcal{I}_t^*(\bar{a}_{nb,t}^*) = 1 + \mu_t^*,$$

where the left hand side is the weighted sum of the investment rates across both economies, and the right hand side is the weighted sum of the savings rates. The investment rates $\mathcal{I}_t(\cdot)$ and $\mathcal{I}_t^*(\cdot)$ are defined similarly as in (2.9). The weight put on the South's variables is the South's aggregated net worth (or equivalently, GDP) relative to that of the North:[10]

$$\mu_t^* \equiv \frac{w_t^*}{w_t} = \left(\frac{k_t^*}{k_t}\right)^\alpha.$$

By using the no-arbitrage equation for both economies $\bar{a}_{nb,t} R_{t+1}^k = R_{t+1} = \bar{a}_{nb,t}^* R_{t+1}^{k*}$ and the

[10] This equation is derived from the total investment equal to total savings equation:

$$\underbrace{\int_{a>\bar{a}_{nb,t}} (1+\lambda_t(a))dF(a)}_{\mathcal{I}_t(\bar{a}_{nb,t})} \times W_t + \underbrace{\int_{a>\bar{a}_{nb,t}^*} (1+\lambda_t^*(a))dF^*(a)}_{\mathcal{I}_t^*(\bar{a}_{nb,t})} \times W_t^* = W_t + W_t^*,$$

and the factor price equations: $W_t = (1-\alpha)A_t K_t^\alpha$ and $W_t^* = (1-\alpha)A_t K_t^{*\alpha}$.

factor price equation for the rental rate of capital R^k_{t+1} and R^{k*}_{t+1}, it is straightforward to show that the weight can be rewritten as:

$$\mu^*_t = \left(\frac{\bar{a}^*_{nb,t-1}}{\bar{a}_{nb,t-1}}\right)^{\frac{\alpha}{1-\alpha}}.$$

It is convenient to rewrite the credit market clearing equation above as:

$$(1 - \mathcal{I}_t(\bar{a}_{nb,t})) + \mu^*_t \left(1 - \mathcal{I}^*_t(\bar{a}^*_{nb,t})\right) = 0, \tag{3.3}$$

where the left hand side is the weighted sum of the excess savings rates (defined as the savings rate minus the investment rate).

Balanced growth path: From the equilibrium dynamics, the characterization of the BGP immediately follows. The detrended aggregate capital stocks are given by:

$$\begin{aligned} k_{nb} &= \left[\frac{1-\alpha}{1+g}\mathcal{K}(\bar{a}_{nb})\right]^{\frac{1}{1-\alpha}}, \\ k^*_{nb} &= \left[\frac{1-\alpha}{1+g}\mathcal{K}^*(\bar{a}^*_{nb})\right]^{\frac{1}{1-\alpha}}, \end{aligned} \tag{3.4}$$

as in equation (2.11) from the closed economy model, where $\mathcal{K}(\cdot)$ and $\mathcal{K}^*(\cdot)$ are defined similarly as in (2.8). The productivity threshold is given by:

$$\begin{aligned} \bar{a}_{nb} &= \mathcal{A}(R_{nb}), \\ \bar{a}^*_{nb} &= \mathcal{A}^*(R_{nb}), \end{aligned} \tag{3.5}$$

where $\mathcal{A}(\cdot)$ and $\mathcal{A}^*(\cdot)$ are defined similarly as in (2.14). From (3.3), the world interest rate R_{nb} solves the world credit market clearing condition:

$$\underbrace{(1 - \mathcal{I}(\mathcal{A}(R_{nb})))}_{SI(R_{nb})} + \underbrace{\left(\frac{\mathcal{A}^*(R_{nb})}{\mathcal{A}(R_{nb})}\right)^{\frac{\alpha}{1-\alpha}}}_{\mu^*(R_{nb})} \underbrace{(1 - \mathcal{I}^*(\mathcal{A}^*(R_{nb})))}_{SI^*(R_{nb})} = 0. \tag{3.6}$$

Note that since the excess savings functions SI and SI^* are increasing and since $SI(R^c_{nb}) = SI^*(R^{c*}_{nb}) = 0$ (recall the credit market clearing conditions in closed economies), it immediately follows from (3.6) that the world interest rate R_{nb} must lie between the closed economy interest rates R^{c*}_{nb} and R^c_{nb}.[11]

[11] To see this, suppose on the contrary that $R_{nb} < \min\{R^c_{nb}, R^{c*}_{nb}\}$. Then both $SI(R_{nb})$ and $SI^*(R_{nb})$ are negative. This contradicts (3.6). Similarly, one would get a contradiction if $R_{nb} > \max\{R^c_{nb}, R^{c*}_{nb}\}$. Thus, $\min\{R^c_{nb}, R^{c*}_{nb}\} \leq R_{nb} \leq \max\{R^c_{nb}, R^{c*}_{nbc}\}$. Note that if $R^c_{nb} \neq R^{c*}_{nb}$, then the inequalities are strict.

The following lemma summarizes our analysis:

Lemma 3. *The bubbleless equilibrium dynamics of aggregate variables in the North can be characterized by capital accumulation equation (2.8), no-arbitrage equation (2.10), and the world credit market clearing equation (3.3). The dynamics are symmetric in the South.*

The corresponding equations (3.4), (3.5), and (3.6) determine the detrended aggregate capital stock in the North, the cutoff productivity threshold in the North, and the world interest rate R_{nb} on the BGP, respectively.

The world interest rate R_{nb} lies between the closed economy interest rates R_{nb}, i.e., $\min\{R^c_{nb}, R^{c*}_{nb}\} \leq R_{nb} \leq \max\{R^c_{nb}, R^{c*}_{nbc}\}$, and the inequalities are strict if $R^c_{nb} \neq R^{c*}_{nb}$.

Proof. See text above. □

This bubbleless open economy model articulates how an asymmetry in financial frictions between two countries causes global imbalances, i.e. imbalances of trade between the North and the South. In this bubbleless benchmark, the trade balance is defined by the savings minus capital investment: $TB_t \equiv S_t - I_t = SI_t \times W_t$. Also, the ratio of trade balances to net worth is defined as $tb_t \equiv TB_t/W_t$. In the closed economy, the trade balances are zero, $TB^c_t = TB^{c*}_t = 0$, because $SI(R^c_t) = SI^*(R^{c*}_t) = 0$. The excess savings functions SI and SI^* are increasing in the interest rate, and hence a key determinant of the trade balances is the world interest rate.

Focusing on the BGP, the world credit market clearing condition (3.6) implies that the world interest rate R_{nb} must lie between the closed economy interest rates R^{c*}_{nb} and R^c_{nb}, as stated in the lemma. In particular, if the closed North interest rate R^c_{nb} is higher than the closed South interest rate R^{c*}_{nb} because the North is more financially developed than the South, then the financial integration drives the world interest rate into a range between R^{c*}_{nb} and R^c_{nb}, i.e. $R^{c*}_{nb} < R_{nb} < R^c_{nb}$. The inequalities on the interest rates, in turn, imply global imbalances, $tb < tb^*$. Financial integration between the financially developed North and the financially developing South can lead to global imbalances in which capital – excess savings – flows from the South to the North.

Figure 3 is a Metzler diagram that illustrates the determination of the world interest rate R_{nb} in the bubbleless BGP and compares it against the interest rates in the closed economy. To generate the figure, we focus on the simple case where the productivity distributions in the two economies are the uniform distributions, where the distribution in the North weakly dominates (in the first-order stochastic dominance sense) that in the South; the leverage constraint (2.3) takes the convenient form of two positive constants λ and λ^*, where we set $\lambda > \lambda^*$ to reflect an assumption that the North is more financially developed than the South.

The top panel plots the savings rate curve, which is flat at 1 for both economies, and

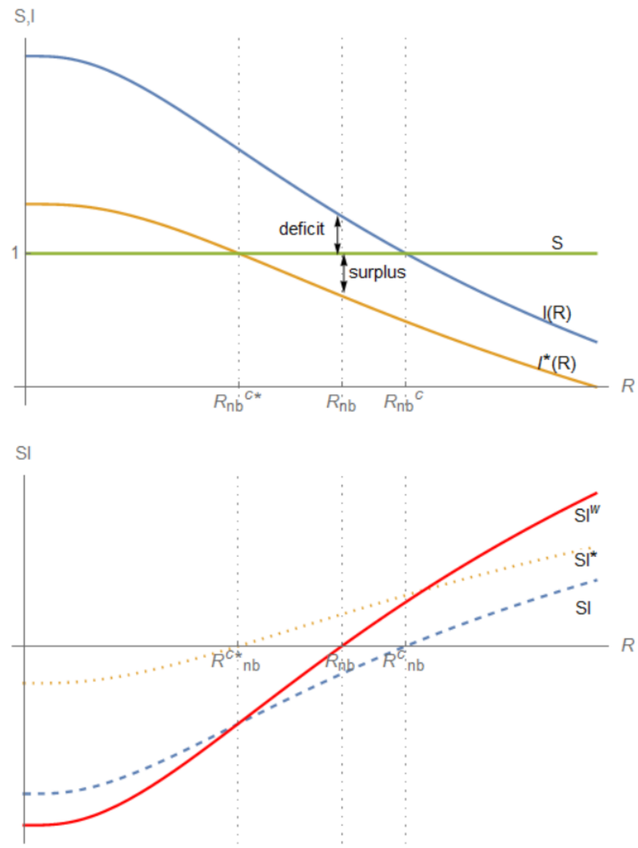


Fig. 3: Credit market clearing under financial integration

the investment rates curves $\mathcal{I}(\mathcal{A}(R))$ and $\mathcal{I}^*\mathcal{A}((R))$, both of which are downward-sloping. As we discussed in the closed economy Section 2, the intersection of the investment curve in each economy and the savings curve determines the interest rates R^c_{nb} and R^{c*}_{nb} for the closed economies of the North and of the South, respectively. As we assume the North is more financially developed, the autarky interest rate is higher in the North, i.e., $R^c_{nb} > R^{c*}_{nb}$.

How is the world interest rate R_{nb} determined? The bottom panel of Figure 3 plots the weighted sum of the excess savings rates of the two economies:

$$SI^w(R) \equiv SI(R) + \left(\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}\right)^{\frac{\alpha}{1-\alpha}} SI^*(R).$$

From world credit clearing equation (3.6), we know that R_{nb} is the intersection of this curve and the horizontal axis. As seen in the figure, the world interest rate R_{nb} lies between the autarky interest rates R^{c*}_{nb} and R^c_{nb}, i.e.,

$$R^{c*}_{nb} < R_{nb} < R^c_{nb}.$$

Furthermore, we can see that at the world interest rate R_{nb}, the North runs a trade deficit, while the South runs a trade surplus. This is because the excess savings rate is negative for the North:

$$SI(R_{nb}) < SI(R^c_{nb}) = 0$$

and positive for the South:

$$SI^*(R_{nb}) > SI^*(R^c_{nb}) = 0.$$

Illustrating this point, the top panel of Figure 3 shows that at $R = R_{nb}$, the investment rate in the North exceeds the savings rate ($\mathcal{I}(\mathcal{A}(R_{nb})) > 1$), leading to a trade deficit. Symmetrically, the savings rate in the South exceeds the investment rate, leading to a trade surplus.

The pattern of capital flows is consistent with existing theories of global imbalances (e.g., Mendoza et al., 2009): under financial integration, capital will flow from economies with relatively low expected returns on investment to economies with relatively high expected returns. In our environment, the North has a more developed financial market with less credit friction (represented by a higher leverage ratio), leading to a higher expected returns from capital investment. Hence, when the two economies integrate, agents in the South will find it attractive to lend to agents in the North in exchange for a higher interest rate. The world economy reaches an equilibrium when there are sufficient capital flows such that the interest rates equalize between the two economies.

3.2 Bubbles

We now consider the economies with bubbles. We focus on the situation where there are bubbles only in the North, as the analysis of the situation with bubbles only in the South is similar. The definition of stochastic bubble equilibria follows straightforwardly from the definitions of equilibria in Sections 2.2 and 3.1 and are omitted for brevity. As before, we define β_t as the ratio of bubbles relative to the North's net worth:

$$\beta_t \equiv \frac{B_t}{W_t + nB_t}.$$

Equilibrium dynamics: As the derivation of the equilibrium dynamics is similar to the previous sections, we omit the details for brevity. By aggregating individual capital investment decisions across agents, the detrended aggregate capital stock of the North evolves according to:

$$\frac{(1 - n\beta_t)(1 + g)k_{t+1}}{(1 - \alpha)k_t^\alpha} = \mathcal{K}_t(\bar{a}_{b,t}), \tag{3.7}$$

as in equation (2.21) from the closed economy model. As there is no bubble (and no bubble creation) in the South, the capital stock there evolves according to:

$$\frac{(1+g)k_{t+1}^*}{(1-\alpha)k_t^{*\alpha}} = \mathcal{K}_t^*(\bar{a}_{b,t}^*), \tag{3.8}$$

as in equation (2.8) from the closed bubbleless model. The indifference condition for the marginal investors in the North yields a no-arbitrage equation:

$$R_{t+1} = \bar{a}_{b,t} R_{t+1}^k = \frac{(1-p)(1-n)(1+g)b_{t+1}}{b_t},$$

as in equation (2.20). The corresponding equation in the South is:

$$R_{t+1} = \bar{a}_{b,t}^* R_{t+1}^{k*}.$$

Together, they imply a single no-arbitrage condition:

$$R_{t+1} = \bar{a}_{b,t} R_{t+1}^k = \bar{a}_{b,t}^* R_{t+1}^{k*} = \frac{(1-p)(1-n)(1+g)b_{t+1}}{b_t}. \tag{3.9}$$

However, unlike in the closed economy, the world credit market clearing condition is:

$$\mathcal{I}_t(\bar{a}_{b,t}) + \beta_t + \mu_{b,t}^* \mathcal{I}_t^*(\bar{a}_{b,t}^*) = 1 + \mu_{b,t}^*, \tag{3.10}$$

where the left hand side is the weighted sum of the investment rates *and* the bubble ratio β, while the right hand side is the weighted sum of the savings rates. The rates in the South are weighted by the relative size of its net worth:

$$\mu_{b,t}^* \equiv \frac{w_t^*}{w_t + nb_t}.$$

By the no-arbitrage equations for both economies, it is straightforward to show that the weight can be rewritten as:

$$\mu_{b,t}^* = \left(\frac{\bar{a}_{b,t-1}^*}{\bar{a}_{b,t-1}}\right)^{\frac{\alpha}{1-\alpha}} (1 - n\beta_t).$$

Credit clearing equation (3.10) can be rewritten as:

$$(1 - \mathcal{I}_t(\bar{a}_{b,t})) + \mu_{b,t}^* \left(1 - \mathcal{I}_t^*(\bar{a}_{b,t}^*)\right) = \beta_t, \tag{3.11}$$

which is the open economy counterpart to equation (2.22) in the closed economy section.

Balanced growth path: From the equilibrium dynamics above, the bubble BGP of the open economy can be summarized as follows. The detrended aggregate capital stocks are given by:

$$\begin{aligned}(1-n\beta)k_b &= \left[\frac{1-\alpha}{1+g}\mathcal{K}(\bar{a}_b)\right]^{\frac{1}{1-\alpha}} \\ k_b^* &= \left[\frac{1-\alpha}{1+g}\mathcal{K}^*(\bar{a}_b^*)\right]^{\frac{1}{1-\alpha}}.\end{aligned} \tag{3.12}$$

From the no-arbitrage equation, the world interest rate is simply:

$$R_b = (1-p)(1-n)(1+g),$$

as in (2.26), where the right hand side is the expected return rate from bubble speculation. The productivity thresholds are given by:

$$\begin{aligned}\bar{a}_b &= \mathcal{A}\left(\frac{R_b}{1-n\beta}\right), \\ \bar{a}_b^* &= \mathcal{A}^*(R_b).\end{aligned} \tag{3.13}$$

The bubble ratio is determined by the world credit market clearing condition (3.11), or equivalently:

$$\underbrace{\left(1-\mathcal{I}\left(\mathcal{A}\left(\frac{R_b}{1-n\beta}\right)\right)\right)}_{\text{excess savings } SI(\frac{R_b}{1-n\beta})} + \underbrace{\left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}\left(\frac{R_b}{1-n\beta}\right)}\right)^{\frac{\alpha}{1-\alpha}}}_{\text{relative weight } \mu_b^*}(1-n\beta)\cdot\underbrace{\left(1-\mathcal{I}^*\left(\mathcal{A}^*(R_b)\right)\right)}_{\text{excess savings } SI^*(R_b)} = \beta. \tag{3.14}$$

In summary, we have established:

Lemma 4. *Assuming bubbles in the North. The equilibrium dynamics of aggregate variables can be characterized by capital accumulation equation (3.7) for the North and (3.8) for the South, no-arbitrage equation (3.9), and world credit market clearing equation (3.11).*

On the BGP, the corresponding equations (3.12), (3.13), and (3.14) determine the detrended aggregate capital stock, the cutoff productivity thresholds, and the bubble ratio, respectively.

Proof. See text above. □

Remark 2. We have assumed for simplicity that bubbles are only traded domestically. However, the main results would not change if bubbles could be traded internationally. Specifically, if agents in the South can purchase bubbles from the North (but we maintain the assumption that only agents in the North can create Northern bubbles), then the bubble market clearing condition (2.17) will be replaced by

$$\int B_t(a)dF(a) + \int B_t^*(a)dF^*(a) = B_t,$$

where $B_t^*(a)$ is the expenditure on investing in the North's bubble market by an agent of type a in the South. Continue to define $\beta_t \equiv \frac{B_t}{W_t+nB_t}$ and $\mu_{b,t}^* \equiv \frac{W_t^*}{W_t+nB_t}$, then we have

$$\beta_t^N + \mu_{b,t}^* \beta_t^S = \beta_t,$$

where $\beta_t^N \equiv \frac{\int B_t(a)dF(a)}{W_t+nB_t}$ and $\beta_t^S \equiv \frac{\int B_t^*(a)dF^*(a)}{W_t^*}$ denote the allocations of bubble holding relative to net worth across the two economies. Then the bubble equilibrium dynamics and bubble BGP are characterized exactly as in Lemma 4, and thus Propositions 2 and 3 (which will be formalized in Section 2.17) will then naturally continue to hold. However, the disaggregated allocations of bubble holding between the North and the South (β_t^N and β_t^S) are indeterminate. As a consequence, the distributional effects of the bubble boom and bust on the consumption of agents in the North and the South are indeterminate.

4 Main results: Bubbles and capital flows feedback loop

Having established a general framework of bubbles in both closed and open economies, we are now ready to establish the main results of the paper. We will show that under general conditions, there is a reinforcing relationship between bubbles and capital flows. Throughout this section, we focus on the situation where the North is more financially developed than the South (represented by a higher autarky interest rate in the North) and focus on the possibility of bubbles in the North. We have in mind the North representing the U.S., the South representing China, and financial integration representing the integration of the Chinese economy into the global financial market. Furthermore, as our interest is in understanding the relationship between the U.S. bubble and global imbalances, we focus on the situation where bubbles arise in the North only.

To simplify our analysis, throughout this section, we again focus on the simple case where the leverage constraint (2.3) takes the convenient form of two positive constants $\lambda > \lambda^*$, and the productivity distributions F and F^* in the two economies are the uniform distributions, where the distribution in the North weakly dominates (in the first-order stochastic dominance sense) that in the South: $F \overset{fosd}{\geq} F^*$. Note that all of our analysis in this section carries through if instead we assume that the productivity distributions are lognormal. The analysis for a more general parametrization is relegated to Section 5.2.

4.1 Effects of capital flows on bubble

We begin with analyzing the effects of capital flows on bubbles. First, to get intuition, assume no bubble creation: $n = 0$. The left panel of Figure 4 plots the world's weighted sum $SI^w(R) \equiv SI(R) + \left(\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}\right)^{\frac{\alpha}{1-\alpha}} SI^*(R)$ of the excess savings rate functions (the red solid line), which corresponds to the left hand side of equation (3.14). This is the same curve as that plotted in the bottom panel of Figure 3. Furthermore, the panel also plots the excess savings rate functions $SI(R)$ and $SI^*(R)$ in each economy. All three curves are upward sloping, as the functions are increasing in R. The three curves intersect the horizontal axis at the respective interest rates R_{nb} (interest rate under financial integration), R^c_{nb} (interest rate in the closed North), and R^{c*}_{nb} (interest rate in the closed South). Because the North is more financially developed, we know from Section 3.1 that

$$R^{c*}_{nb} < R_{nb} < R^c_{nb}.$$

The interest rate in the BGP is $R_b = (1-p)(1-n)(1+g)$.

Then, as in the closed economy section, a bubble can exist under financial integration if and only if $R_{nb} < R_b$. To see this, when $R_{nb} < R_b$, the total excess savings (illustrated by the red solid line) is positive: $SI^w(R_b) > SI^w(R_{nb}) = 0$. That is, at the interest rate R_b, the total investment in capital stock cannot fully absorb the total savings in the world economy. The excess savings must be absorbed by speculative investment in the bubble asset. That is, when $n = 0$, the bubble ratio β is simply equal to $SI^w(R_b)$.

How does financial integration affect bubbles in the North? There are two effects. The first effect is on the existence condition for bubbles in the North. Recall that when $n = 0$, a bubble BGP exists in the closed North if and only if the interest rate in the closed North is low: $R^c_{nb} < R_b$. Similarly, as our analysis above suggests, bubbles exist in the open North if and only if the *world* interest rate is low: $R_{nb} < R_b$. However, as the integration of the North with the less developed South leads to a decrease in the interest rate, i.e., $R_{nb} < R^c_{nb}$, it then immediately follows that financial integration *relaxes* the existence condition for bubbles in the North. In other words, capital inflows from the rest of the world facilitate the existence of bubbles in the North.

The second effect is on the *size* of the bubble. From the figure, it is apparent that the bubble ratio in the closed economy is smaller: $\beta^c < \beta$. Formally, this inequality follows straight from the fact that $\beta^c = SI(R_b) < SI(R_b) + \mu^*(R_b)SI^*(R_b) = \beta$. Intuitively, when the North is integrated with the less developed South, savings from the South flow into the credit market of the North, lowering the interest rate, hence increasing the incentive for Northern agents to chase higher returns from bubble speculation. Increased demand for bubbles naturally leads to an increased market value of bubbles in equilibrium.

The situation is slightly different but generally similar when there is bubble creation: $n > 0$. Recall from the closed economy that β^c is implicitly defined via the closed North's savings-investment equation:

$$SI\left(\frac{R_b}{1-n\beta^c}\right) = \beta^c.$$

The right panel of Figure 4 represents β^c as the intersection of the dashed curve SI representing the left hand side and the 45-degree line representing the right hand side. On the other hand, in the open economy, β is implicitly defined via the *world* savings-investment equation (3.14):

$$SI\left(\frac{R_b}{1-n\beta}\right) + \left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(\frac{R_b}{1-n\beta})}\right)^{\frac{\alpha}{1-\alpha}} (1-n\beta)SI^*(R_b) = \beta. \tag{4.1}$$

The figure represents β as the intersection of the solid curve SI^w representing the left hand side of equation (4.1) and the 45-degree line representing the right hand side of equation (4.1). As in the closed model, as long as the SI^w is weakly convex, then the SI^w curve and the 45-degree line will intersect in the positive quadrant if and only if the SI^w curve intersects the vertical line at a positive value, i.e., $SI^w(R_b) > 0$, or equivalently, $R_{nb} < R_b$.

Furthermore, conditional on $R_{nb} < R_b$ so that bubbles exist, it is straightforward that $\beta > \beta^c$. In the right panel of the figure, this can be seen by the fact that the solid line for SI^w lies above the dashed line SI. Formally, because $R^{c*}_{nb} < R_{nb} < R_b$, it follows that $SI^*(R_b) > 0$. Therefore the $SI\left(\frac{R_b}{1-nx}\right) + \left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(\frac{R_b}{1-nx})}\right)^{\frac{\alpha}{1-\alpha}} (1-nx)SI^*(R_b) > SI\left(\frac{R_b}{1-nx}\right)$ for all x. Combined with the equations that implicitly determine β^c and β, we immediately get $\beta > \beta^c$.

The following result summarizes the argument above:

Proposition 2. [Financial integration facilitates bubbles] *Assume leverage constraints $\lambda > \lambda^*$ and uniform productivity distributions $F \overset{fosd}{\geq} F^*$. Then:*

1. *There exists a BGP with a bubble in the North if and only if the* world *interest rate is low: $R_{nb} < R_b$. As a corollary, financial integration expands the parameter region in which a bubble can exist in the North: $\{R^c_{nb} < R_b\} \subset \{R_{nb} < R_b\}$.*

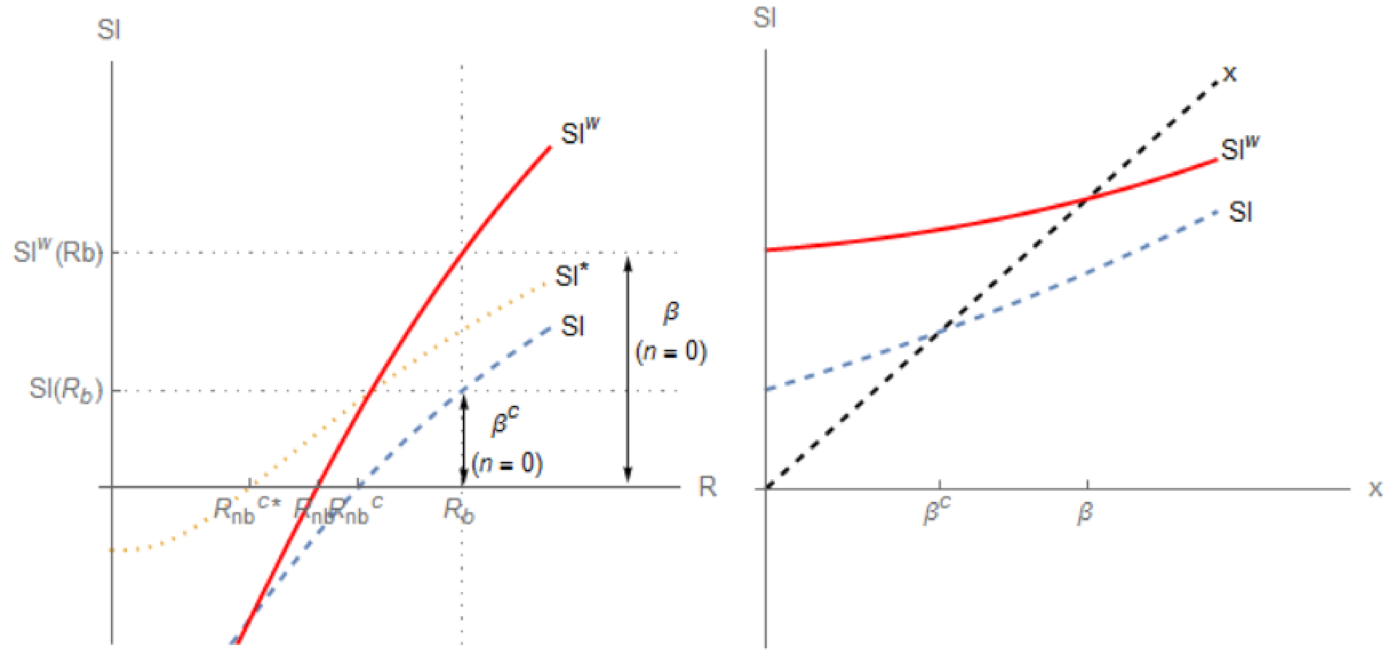


Fig. 4: Determination of bubble ratio in open economy

2. *Financial integration enlarges the bubble ratio for the North:* $\beta^c < \beta$.

Proof. Appendix A.3. □

There is an interesting corollary of Proposition 2. Abel et al. (1989) have argued that developed economies such as the U.S. are dynamically efficient and thus it is unlikely that the conditions for the existence of rational bubbles are satisfied in such economies.[12] Our result points out that even if the North is dynamically efficient, the financial integration with a sufficiently dynamically inefficient South can still enable the existence of bubbles in the North.

Remark 3. An immediate corollary of the first claim in Proposition 2 is that financial integration *reduces* the parameter region in which a bubble can exist in the South, i.e., $\{R_{nb} < R_b\} \subset \{R^{c*}_{nb} < R_b\}$. The intuition is simple: financial integration with the more financially developed North raises the interest rate in the South, and as we already know, it is harder for bubbles to arise in higher interest rate environments.

4.2 Effects of bubble on capital flows

The previous section has shown that capital flows from the South to the North, driven by an asymmetry in financial frictions between the two countries, and facilitates bubbles in the

[12] For a more recent assessment of dynamic efficiency, see Geerolf (2017).

North by lowering the interest rate the North faces. We now show a feedback effect: a bubble in the North, in turn, facilitates further South-to-North capital flows.

To account for capital flows, the trade balance is defined as savings minus total investment (in capital and in bubbles):

$$TB_t = S_t - (I_t + B_t).$$

Furthermore, we define the ratio of trade balance to net worth as

$$tb_t \equiv \frac{TB_t}{W_t + nB_t}$$

On the bubbleless BGP, the trade balance ratio is simply the excess savings rate evaluated at the equilibrium world interest rate R_{nb}:

$$tb_{nb} = SI(R_{nb}),$$

and symmetrically for the South:

$$tb^*_{nb} = SI(R^*_{nb}).$$

On the BGP with a bubble in the North, the corresponding expression for the North is:

$$tb_b = SI\left(\frac{R_b}{1 - n\beta}\right) - \beta,$$

and the expression for the South is:

$$tb^*_b = SI^*(R_b).$$

On the one hand, bubbles increase the trade balance ratio of the South. As the excess savings function $SI(R)$ is increasing in R, and as $R_{nb} < R_b$ (the condition for the existence of bubbles), it immediately follows that $0 < tb^*_{nb} = SI^*(R_{nb}) < SI^*(R_b) = tb^*_b$. Hence, bubbles reinforce global imbalances that originally facilitated bubbles to emerge in the North. Initially, financial integration allows capital to flow from the South to the North, decreasing the interest rate the North faces. This relaxes the bubble existence condition of the North. Once a bubble emerges, it absorbs savings in the North and raises the interest rate, which, in turn, attracts capital from the South, leading to further global imbalances.

On the other hand, bubbles also reduce the trade balance ratio of the North. To see this, focusing on a special case of $n = 0$, from the world credit market clearing condition (4.1) the trade balance ratio of the North in the bubble BGP can be rewritten as the negative of the weighted trade balance ratio of the South:

$$-tb_b = \left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(R_b)}\right)^{\frac{\alpha}{1-\alpha}} tb_b^*.$$

Given the uniform distribution, it is straightforward to verify that the ratio $\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}$ is increasing in R. Thus, $\left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(R_b)}\right)^{\frac{\alpha}{1-\alpha}} > \left(\frac{\mathcal{A}^*(R_{nb})}{\mathcal{A}(R_{nb})}\right)^{\frac{\alpha}{1-\alpha}}$. Combined with $tb_b^* > tb_{nb}^* > 0$, we get:

$$\left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(R_b)}\right)^{\frac{\alpha}{1-\alpha}} tb_b^* > \left(\frac{\mathcal{A}^*(R_{nb})}{\mathcal{A}(R_{nb})}\right)^{\frac{\alpha}{1-\alpha}} tb_{nb}^*.$$

That is, not only do bubbles raise the trade balance ratio of the South, they also raise the *weighted* trade balance ratio of the South. By using the world credit market clearing condition (4.1) again, the right hand side of the inequality above is exactly equal to $-t_{nb}$, the negative of the trade balance ratio of the North in the bubbleless BGP. Therefore, $-tb_b > -tb_{nb}$, or equivalently,

$$tb_b < tb_{nb}.$$

The argument is similar (with a bit more algebra) when $n > 0$.

The following proposition generalizes the analysis above and formalizes the notion that bubbles in the North enhance the capital flows from South to North:

Proposition 3. [Bubble facilitates capital flows] *Assume leverage constraints $\lambda > \lambda^*$ and uniform productivity distributions $F \overset{fosd}{\geq} F^*$. Assume $R_{nb} < R_b$ so that the bubble BGP exists. Then bubbles increase the trade deficit ratio of the North and the trade surplus ratio of the South:*

$$tb_b < tb_{nb}$$
$$tb_b^* > tb_{nb}^*.$$

Proof. Appendix A.3. □

Note that an immediate corollary of the proposition is that bubbles reduce the trade-balance-over-GDP ratio of the North and increases that of the South. This is because, for the South, the inequality $tb_b^* > tb_{nb}^*$ is equivalent to $\frac{TB_b^*}{Y_b^*} > \frac{TB_{nb}^*}{Y_{nb}^*}$, as $\frac{Y_b^*}{W_b^*} = \frac{1}{1-\alpha} = \frac{Y_{nb}}{W_{nb}}$. On the other hand, in the North, the inequality $tb_b < tb_{nb}$ implies a weaker inequality $\frac{TB_b}{Y_b} < \frac{TB_{nb}}{Y_{nb}}$. This is because $\frac{Y_b}{W_b+nB} < \frac{Y_b}{W_b} = \frac{1}{1-\alpha} = \frac{Y_{nb}}{W_{nb}}$.

5 Discussions

5.1 Interpretation of recent events through the model

The recent boom-bust episode in asset prices in the U.S. could be interpreted through the lens of the theory we have developed. As mentioned in the introduction, in the decades leading to the Great Recession, the U.S. had been a net importer of capital, especially from China and other emerging market economies. The peak period of capital inflows was the early 2000s, with the current account deficit exceeding 6% of U.S. GDP in 2006 (Figure 6a). This period coincided with a boom in housing and stock prices, with the average housing price index rising by 85% between 2000 and 2006 (Figure 6b).

From the lens of our theory, the correlation between the capital inflows and asset prices is not just a mere coincidence, but a possible consequence of a feedback loop. As formalized in the previous section, Proposition 2 predicts that capital inflows facilitate the emergence and the size of a bubble in the U.S. This result thus formalizes the "savings glut hypothesis" that capital inflows from emerging countries, especially China, helped fuel the boom in asset prices in the U.S. (e.g., Bernanke, 2007; Greenspan, 2009; Yellen, 2009; Obstfeld and Rogoff, 2009; Rajan, 2011; Stiglitz, 2012; Summers, 2014). Stewen and Hoffmann (2015) provide some evidence for this view by using state-level data to document that housing prices were more sensitive to capital inflows in states that had opened their banking markets to out-of-state banks earlier. In addition, Proposition 3 predicts that the bubbly boom in asset prices in turn facilitates more capital inflows. An immediate corollary is that the collapse of bubbles would precipitate a downward adjustment in the quantity of capital inflows to the U.S., which is also consistent with the observation that the U.S. current account deficit improved when U.S. asset prices fell in 2007.

Furthermore, the boom in asset prices in the 2000s was associated with a boom in credit in the U.S. (Chaney et al., 2012; Mian and Sufi, 2014; Justiniano et al., 2015). For instance, Chaney et al. (2012) estimated that between 1993 and 2007, a \$1 increase in the value of the real estate that firms own led to a \$0.06 increase in investment by a representative U.S. corporation. On the other hand, the collapse in asset prices in 2007 precipitated the Great Recession, with sharp contractions in aggregate economic activities (e.g., Mian and Sufi, 2010, 2014; also see Figure 6c). These observations are also consistent with our theory. In our model, bubbles in asset prices have a crowd-in effect on borrowing, and as a consequence, an expansionary bubble episode is associated with a boom in credit. When bubbles collapse, the boom turns into a bust, as aggregate economic activities are adjusted from the expansionary bubble steady state to the bubbleless steady state.

Figure 5 further illustrates the impacts of bubbles on the North's economy through a

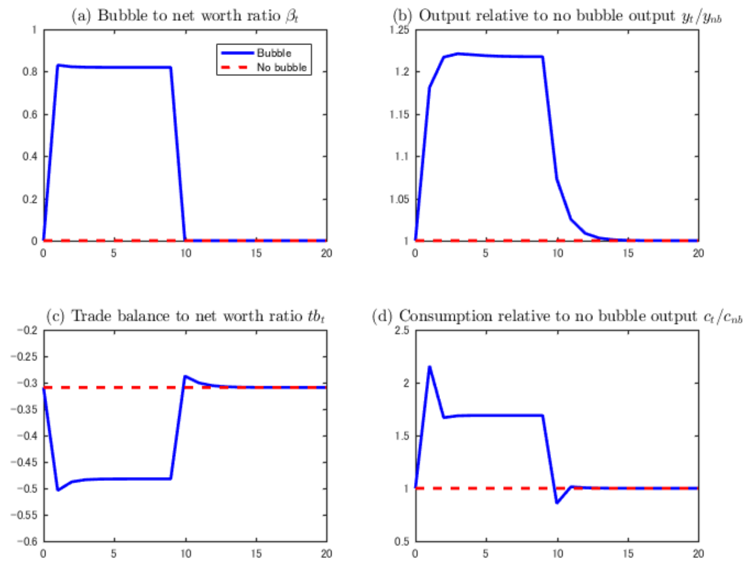


Fig. 5: Effects of a bubble episode on business cycles in the open North.

simple numerical simulation of the open economy model presented in Section 3.[13] In this simulation, the global economy is in the bubbleless steady state in period $t = 0$. Expansionary bubbles unexpectedly emerge in the North in period $t = 1$ and eventually collapse in period $t = 10$. The figure plots the bubble ratio β_t, the aggregate output and aggregate consumption relative to their bubbleless steady-state value in the North y_t/y_{nb} and c_t/c_{nb}, and the North's trade balance tb_t. As the figure shows, bubbles crowd in both output and consumption. The expansionary bubble exacerbates the trade balance, increasing the trade deficit $(-tb_t)$ further. Note that the increase in the trade deficit and consumption is much more pronounced than the increase in output, which is qualitatively consistent with the U.S. experience during the 2000s: a large boom in housing prices that led to a big increase in trade deficits and consumption, but a relatively mild increase in output.

After bubbles collapse, the economy reverts to the bubbleless steady state. Figure 5 also shows that when bubbles collapse, aggregate consumption "overshoots," i.e., drops below the bubbleless steady state value: $c_{10} < c_{nb}$. Intuitively, this occurs if bubbles sufficiently increase the indebtedness of domestic agents, and its collapse then causes indebted old agents to deleverage by cutting down consumption. The contraction in consumption is related to

[13] We stress that this exercise should not be interpreted as a quantitative analysis, but rather as a qualitative illustrative example. The simulated model assumes constant λ and λ^* and a lognormal distribution with mean unity. The parameter values used in the example are: $\alpha = 0.4$ (capital share), $n = 0.5$ (bubble creation), $p = 0.01$ (bubble burst probability), $g = 0$ (exogenous growth rate), $\lambda = 0.6$ (financial friction in the North), $\lambda^* = 0.3$ (financial friction in the South), $\sigma = 1$ (lognormal distribution parameter).

Eggertsson and Krugman (2012)'s result that deleveraging can depress aggregate demand and is qualitatively consistent with what happened during the Great Recession (e.g., Mian and Sufi, 2010, 2014). Overall, the numerical example illustrates that our model is qualitatively consistent with the observed stylized features of bubble episodes, namely the boom and bust in output, consumption, and capital flows.

5.2 Results under more general parametrization

Section 4 focused on the simple case where the productivity distributions in the two economies are the uniform distributions, and the leverage constraint (2.3) takes the convenient form of two positive constants $\lambda > \lambda^*$. However, as aforementioned, these assumptions are not restrictive. As shown below, our main results – Propositions 2 and 3 – hold for more general continuous productivity distributions and credit constraints,[14] as in the general setup of Sections 2 and 3, as long as we impose the elasticity condition (2.30), the assumptions that $R_{nb}^{c*} < R_{nb}^{c}$ (so that capital will flow from the South to the North under financial integration) and $\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}$ is increasing in R.

The following is the generalization of Proposition 2:

Proposition 4. [Financial integration facilitates bubbles] *Assume parameters such that condition (2.30) holds, $R_{nb}^{c*} < R_{nb}^{c}$, and $\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}$ is increasing in R. Then:*

1. *There exists a BGP with a bubble in the North if and only if the* world *interest rate is low: $R_{nb} < R_b$. As a corollary, financial integration expands the parameter region in which a bubble can exist in the North: $\{R_{nb}^{c} < R_b\} \subset \{R_{nb} < R_b\}$.*

2. *Financial integration enlarges the bubble ratio: $\beta^c < \beta$.*

Proof. Appendix A.4.

□

And the following is the generalization of Proposition 3:

Proposition 5. [Bubble facilitates capital flows] *Assume parameters such that condition (2.30) holds, that $R_{nb}^{c*} < R_{nb}^{c}$, and $\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}$ is increasing in R. Furthermore, assume $R_{nb} < R_b$ so that the bubble BGP exists. Then bubbles increase trade deficit ratio of the North and the*

[14] Including the Kiyotaki and Moore (1997) type of credit constraint with $\lambda_t(a) \equiv \frac{R_{t+1}^k \phi a}{R_{t+1} - R_{t+1}^k \phi a}$ that was discussed after the specification of leverage constraint (2.3).

trade surplus ratio of the South:

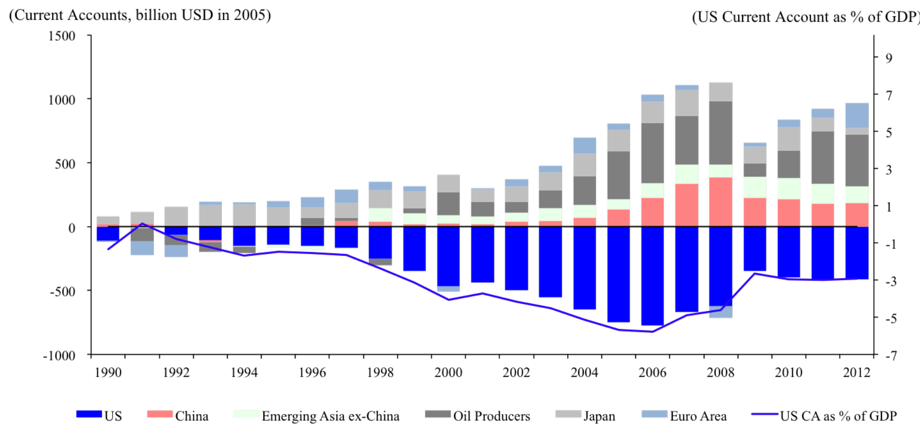
$$tb_b < tb_{nb}$$
$$tb_b^* > tb_{nb}^*.$$

Proof. Appendix A.5. $\square$

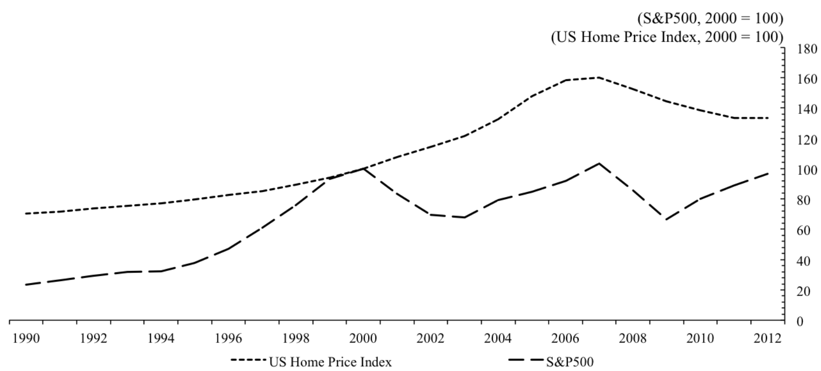
The assumptions made in Propositions 4 and 5 are relatively mild. As shown in Appendix A.1, the elasticity condition (2.30) holds for uniform or lognormal distributions when the leverage constraint functions $\lambda(\cdot)$ and $\lambda^*(\cdot)$ are constant. The assumption on $\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}$ ensures that the world excess saving rate function $SI^w(R)$ is increasing in R. Because the saving rate is unity for each country as agents consume when they grow old, the increasing excess saving rate function implies that the net investment is decreasing in the interest rate, which is a plausible assumption. The assumption $R_{nb}^{c*} < R_{nb}^c$ is easily satisfied if the degree of asymmetries in financial frictions and/or productivity distributions is large enough. Finally, these assumptions restrict only parameters that pertain to an asymmetry between the North and the South, namely, borrowing constraints and productivity distributions. It is such an asymmetry that leads to our main results on bubbles and global imbalances.

6 Conclusion

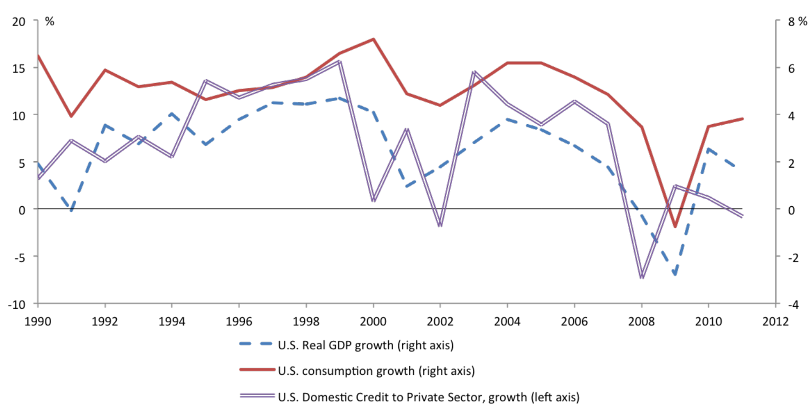
We have built a two-country open economy model with asset bubbles and global imbalances. The model provides an analysis of the underlying relationship between bubbles, capital flows, and boom-busts in economic activities. Our results predict a close and reinforcing relationship between capital flows and asset bubbles. Specifically, the financial integration of the South with the North leads capital to flow into the North. Capital inflows in turn facilitate the emergence of large bubbles in the North, which further exacerbate global imbalances. Several predictions of the model are qualitatively consistent with stylized features of recent boom-bust episodes.



(a) Current accounts (bars) and U.S. current account as percentage of GDP (line). Data sources: IMF and World Development Index.



(b) S&P 500 and U.S. aggregate house price index. Data sources: Federal Housing Finance Agency and S&P 500.



(c) Percentage changes in real GDP, consumption, and domestic credit to private sector of the U.S. Data sources: Federal Reserve Economic Data and World Development Indicators.

Fig. 6: Global imbalances, U.S. asset prices, and U.S. economic activities.

References

Abel, A. B., Mankiw, N. G., Summers, L. H., and Zeckhauser, R. J. (1989). Assessing dynamic efficiency: Theory and evidence. *The Review of Economic Studies*, 56(1):1–19.

Aoki, K. and Nikolov, K. (2015). Bubbles, banks and financial stability. *Journal of Monetary Economics*, 74:33–51.

Banerjee, A. V. and Moll, B. (2010). Why does misallocation persist? *American Economic Journal: Macroeconomics*, 2(1):189–206.

Barlevy, G. (2012). Rethinking theoretical models of bubbles. *New Perspectives on Asset Price Bubbles.*

Basco, S. (2013). Globalization and financial development: a model of the dot-com and the housing bubbles. *Journal of International Economics.*

Bengui, J. and Phan, T. (2017). Asset pledgeability and endogenously leveraged bubbles. *Working paper.*

Bernanke, B. (2007). Global imbalances: Recent developments and prospects. *Speech at the Bundesbank Lecture, Berlin Germany.*

Bernanke, B. and Gertler, M. (1989). Agency costs, net worth, and business fluctuations. *The American Economic Review*, 79:14–31.

Bernanke, B. S. (2005). The global saving glut and the U.S. current account deficit. *Board of Governors of the Federal Reserve System (US) Speech*, (Mar 10).

Buera, F. J. and Shin, Y. (2013). Financial frictions and the persistence of history: A quantitative exploration. *Journal of Political Economy*, 121(2):221–272.

Buera, F. J. and Shin, Y. (2015). Productivity growth and capital flows: The dynamics of reforms. *Working paper.*

Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2008). An equilibrium model of global imbalances and low interest rates. *American Economic Review*, 98(1):358–393.

Caballero, R. J. and Krishnamurthy, A. (2006). Bubbles and capital flow volatility: Causes and risk management. *Journal of Monetary Economics*, 53(1):35–53.

Case, K. E. and Shiller, R. J. (2003). Is there a bubble in the housing market? *Brookings Papers on Economic Activity*, 2003(2):299–362.

Chaney, T., Sraer, D., and Thesmar, D. (2012). The collateral channel: How real estate shocks affect corporate investment. *American Economic Review*, 102(6):2381–2409.

Chari, V., Kehoe, P., and McGrattan, E. R. (2007). Business cycle accounting. *Econometrica*, 75(3):781–836.

Chinn, M. D., Eichengreen, B., and Ito, H. (2014). A forensic analysis of global imbalances. *Oxford Economic Papers*, 66(2):465–490.

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5):1126–1150.

Eggertsson, G. B. and Krugman, P. (2012). Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach. *The Quarterly Journal of Economics*, page qjs023.

Farhi, E. and Tirole, J. (2012). Bubbly liquidity. *The Review of Economic Studies*, 79(2):678–706.

Geerolf, F. (2017). Reassessing dynamic efficiency. *Working Paper*.

Gourinchas, P.-O. and Jeanne, O. (2013). Capital flows to developing countries: The allocation puzzle. *The Review of Economic Studies*, 80(4):1484–1515.

Gourinchas, P.-O. and Rey, H. (2013). External adjustment, global imbalances, valuation effects. *Handbook of International Economics*, 4.

Greenspan, A. (2009). The fed didn't cause the housing bubble. *Wall Street Journal*, 11:A15.

Hanson, A. and Phan, T. (2017). Bubbles, wage rigidity, and persistent slumps. *Economics Letters*, 151:66–70.

Hirano, T. and Yanagawa, N. (2014). Asset bubbles, endogenous growth, and financial frictions. *Working Paper*.

Ikeda, D. and Phan, T. (2016). Toxic asset bubbles. *Economic Theory*, 61(2):241–271.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2015). Household leveraging and deleveraging. *Review of Economic Dynamics*, 18(1):3–20.

Kindleberger, C. P. and Aliber, R. Z. (2011). *Manias, panics and crashes: a history of financial crises*. Palgrave Macmillan.

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of Political Economy*, 105(2):211–248.

Kocherlakota, N. (2009). Bursting bubbles: Consequences and cures. *Unpublished manuscript, Federal Reserve Bank of Minneapolis*.

Kraay, A. and Ventura, J. (2007). The dot-com bubble, the bush deficits, and the U.S. current account. In *G7 Current Account Imbalances: Sustainability and Adjustment*, pages 457–496. University of Chicago Press.

Martin, A. and Ventura, J. (2011). Theoretical notes on bubbles and the current crisis. *IMF Economic Review*, 59(1):6–40.

Martin, A. and Ventura, J. (2012). Economic growth with bubbles. *American Economic Review*, 102(6):3033–3058.

Martin, A. and Ventura, J. (2017). The macroeconomics of rational bubbles: a user's guide. Technical report.

Matsuyama, K. (2005). Credit market imperfections and patterns of international trade and capital flows. *Journal of the European Economic Association*, 3(2-3):714–723.

Mendoza, E. G., Quadrini, V., and Rios-Rull, J.-V. (2009). Financial integration, financial development, and global imbalances. *Journal of Political economy*, 117(3):371–416.

Mendoza, E. G. and Terrones, M. E. (2008). An anatomy of credit booms: evidence from macro aggregates and micro data. *NBER Working Paper*.

Mendoza, E. G. and Terrones, M. E. (2012). An anatomy of credit booms and their demise. *NBER Working Paper*.

Mian, A. and Sufi, A. (2010). The great recession: Lessons from microeconomic data. *The American Economic Review: Papers & Proceedings*, pages 51–56.

Mian, A. and Sufi, A. (2014). *House of Debt: How They (and You) Caused the Great Recession, and How We Can Prevent It from Happening Again*. University of Chicago Press.

Miao, J. (2014). Introduction to economic theory of bubbles. *Journal of Mathematical Economics*, 53:130–136.

Miao, J. and Wang, P. (2011). Bubbles and credit constraints. *Working Paper*.

Moll, B. (2014). Productivity losses from financial frictions: can self-financing undo capital misallocation? *The American Economic Review*, 104(10):3186–3221.

Obstfeld, M. and Rogoff, K. (2009). Global imbalances and the financial crisis: Products of common causes. *CEPR Discussion Papers*.

Rajan, R. G. (2011). *Fault Lines: How hidden fractures still threaten the world economy*. Princeton University Press.

Reinhart, C. M. and Rogoff, K. (2009). *This Time is Different: Eight Centuries of Financial Folly*. Princeton University Press.

Reinhart, C. M. and Rogoff, K. S. (2008). Is the 2007 us sub-prime financial crisis so different? an international historical comparison. *The American Economic Review*, 98(2):339.

Rondina, G. (2017). Bubbly financial globalization. *Working Paper*.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *The Journal of Political Economy*, 66(6):467–482.

Shiller, R. J. (2015). *Irrational exuberance*. Princeton University Press.

Song, Z., Storesletten, K., and Zilibotti, F. (2011). Growing like China. *The American Economic Review*, 101(1):196–233.

Stewen, I. and Hoffmann, M. (2015). Holes in the dike: the global savings glut, U.S. house prices and the long shadow of banking deregulation. *ZBW Working Paper*.

Stiglitz, J. E. (2012). *The price of inequality: How today's divided society endangers our future*. W. W. Norton.

Summers, L. H. (2014). Us economic prospects: Secular stagnation, hysteresis, and the zero lower bound. *Business Economics*, 49(2):65–73.

Tirole, J. (1985). Asset bubbles and overlapping generations. *Econometrica*, 53(6):1499–1528.

Weil, P. (1987). Confidence and the real value of money in an overlapping generations economy. *The Quarterly Journal of Economics*, 102(1):1–22.

Yellen, J. (2009). A Minsky meltdown: Lessons for central bankers. In *Speech at the 18th Annual Hyman P. Minsky Conference, New York*.

A Appendix

A.1 Derivation of condition (2.30)

For convenience, let $s(x) \equiv SI(\frac{R_b}{1-nx}) = 1 - \mathcal{I}\left(\mathcal{A}\left(\frac{R_b}{1-nx}\right)\right)$. We show that $s''(x) \geq 0$ for $x \in (0,1)$ if and only if (2.30) holds. By chain rule, the derivative of s is:

$$s'(x) = -\mathcal{I}'\left(\mathcal{A}\left(\frac{R_b}{1-nx}\right)\right)\frac{d\mathcal{A}}{dx}\left(\frac{R_b}{1-nx}\right).$$

By the definition of $\mathcal{I}$, we have:

$$\mathcal{I}'(a) = \frac{d\int_{z>a}(1+\lambda(z))dF(z)}{da} = -(1+\lambda(a))f(a).$$

To derive $d\mathcal{A}\left(\frac{R_b}{1-nx}\right)/dx$, recall that $\bar{a}_b = \mathcal{A}\left(\frac{R_b}{1-nx}\right)$ is implicitly defined by:

$$\frac{R_b}{1-nx} = \frac{(1+g)\alpha}{1-\alpha}\frac{\bar{a}_b}{\mathcal{K}(\bar{a}_b)}.$$

Totally differentiating this with respect to $\bar{a}_b$ and x yields

$$\frac{d\mathcal{A}}{dx}\left(\frac{R_b}{1-nx}\right) = \frac{\frac{(1+g)\alpha}{1-\alpha}\bar{a}_b n}{R_b(1+\lambda(\bar{a}_b))\bar{a}_b f(\bar{a}_b) + \frac{(1+g)\alpha}{1-\alpha}(1-nx)}. \tag{A.1}$$

Combining the three equations above yields:

$$s'(x) = \frac{n}{\frac{1-\alpha}{(1+g)\alpha}R_b + \frac{(1-nx)}{(1+\lambda(\bar{a}_b))\bar{a}_b f(\bar{a}_b)}}.$$

For $n > 0$, the above equation for $s'(x)$ implies that s' is weakly decreasing in $x \in (0,1)$ if and only if $\frac{1-nx}{[1+\lambda(\bar{a}_b)]\bar{a}_b f(\bar{a}_b)}$ is weakly decreasing in $x \in (0,1)$. By taking the derivative of this ratio, we get that $s''(x) \geq 0$ for $0 < x < 1$ if and only if

$$n + (1 - nx)\frac{d\bar{a}_b}{dx} \times \left\{ \frac{\lambda'(\bar{a}_b)}{1 + \lambda(\bar{a}_b)} + \frac{[f(\bar{a}_b) + \bar{a}_b f'(\bar{a}_b)]}{\bar{a}_b f(\bar{a}_b)} \right\} \geq 0,$$

for $0 < x < 1$. By substituting (A.1) for $\frac{d\bar{a}_b}{dx} = \frac{d\mathcal{A}}{dx}\left(\frac{R_b}{1-nx}\right)$, and by manipulating the algebra, we get an equivalent inequality:

$$1 + \frac{1 + \frac{\bar{a}_b \lambda'(\bar{a}_b)}{1+\lambda(\bar{a}_b)} + \frac{\bar{a}_b f'(\bar{a}_b)}{f(\bar{a}_b)}}{1 + \frac{1-\alpha}{(1+g)\alpha}\frac{R_b}{1-nx}[1 + \lambda(\bar{a}_b)]\bar{a}_b f(\bar{a}_b)} \geq 0,$$

for $0 < x < 1$. Substituting the definitions for $\mathcal{A}$ and $\mathcal{R}$, algebraic manipulations show that this inequality is equivalent to

$$2 + \frac{a\lambda'(a)}{1 + \lambda(a)} + \frac{(1 + \lambda)a^2 f(a)}{\int_{z>a}(1 + \lambda)z \; dF(z)} \geq -\frac{af'(a)}{f(a)},$$

for $\mathcal{A}(R_b) < a < \mathcal{A}\left(\frac{R_b}{1-n}\right)$, which is exactly condition (2.30).

Note that all the terms on the left hand side of (2.30) are non-negative (recall $\lambda' \geq 0$ by assumption). Thus, a sufficient condition is

$$2 \geq -\frac{af'(a)}{f(a)}, \tag{A.2}$$

for $\mathcal{A}(R_b) < a < \mathcal{A}\left(\frac{R_b}{1-n}\right)$. If F is a uniform distribution, then f is a constant, and thus (2.30) is automatically satisfied. If instead $\log a \sim N(\mu, \sigma)$ (lognormal distribution), then the elasticity on the right hand side is equal to $1 + \frac{\log a - \mu}{\sigma^2}$. Since $\mathcal{A}$ is increasing, (A.2) is then equivalently reduced to $e^{\sigma^2+\mu} \geq \mathcal{A}(R_b/(1-n)) = \mathcal{A}((1-p)(1+g))$, as stated in Footnote 8.

Finally, note that if λ is a constant, then (2.30) reduces to

$$2 + \frac{a^2 f(a)}{\int_{z>a} z dF(z)} \geq -a\frac{f'(a)}{f(a)},$$

for $\mathcal{A}(R_b) < a < \mathcal{A}\left(\frac{R_b}{1-n}\right)$. Note that, for a distribution with no upper bound on the support (such as the lognormal distribution), this inequality converges to an equality as $a \to \infty$, because

$$\lim_{a\to\infty}\left[2 + \frac{af'(a)}{f(a)} + \frac{a^2 f(a)}{\int_{z>a} z dF(z)}\right] = \lim_{a\to\infty}\left[2 + \frac{af'(a)}{f(a)} + \frac{2af(a) + a^2 f'(a)}{-af(a)}\right] = 0.$$

With $\log a \sim N(\mu, \sigma)$, this inequality becomes

$$\mu + \sigma^2 + \frac{\sigma^2}{e^{\mu+\frac{1}{2}\sigma^2}} \frac{a\phi(\log a)}{\Phi\left(\frac{\mu+\sigma^2-\log(a)}{\sigma}\right)} - \log(a) \geq 0, \tag{A.3}$$

where ϕ and Φ are the probability density function and the cumulative distribution function of the standard normal distribution. The left hand side can be shown to be decreasing in a. Furthermore, we know that the inequality converges to an equality as $a \to \infty$. Thus, (A.3) holds for all $a > 0$. In other words, with λ being a constant, (2.30) always holds for a lognormal distribution.

A.2 Proof of Proposition 1

As established in the main text, the bubble ratio on the BGP β is equal to the excess savings rate:

$$\beta = SI(\frac{R_b}{1-n\beta}).$$

Note that from the definition of $SI(R) \equiv 1 - I(\mathcal{A}(R))$, it immediately follows that $SI < 1$, and therefore $\beta < 1$. Thus, there exists a BGP with a bubble if and only if the equation $s(x) = x$ has a solution $x = \beta \in (0, 1)$, where $s(x) \equiv SI(\frac{R_b}{1-nx})$.

From the derivation in Section A.1, we know that condition (2.30) guarantees that the SI function is weakly convex over $(0, 1)$, i.e., $s''(x) \geq 0$ for all $0 < x < 1$. Furthermore, $s(1) < 1$. Therefore, due to the continuity and convexity of s, the equation $s(x) = x$ has a solution in (0,1) if and only if $s(0) > 0$, or equivalently $SI(R_b) > 0$. Recall that $SI(R_{nb}) = 0$ and SI is a strictly increasing function. Therefore, an equivalent inequality is $R_b > R_{nb}$. In other words, there exists a bubble on the BGP if and only if $R_b > R_{nb}$. □

A.3 Proof of Propositions 2 and 3

Proposition 2 is a direct corollary of the more general Proposition 4, whose proof we provide below in Section A.4. Proposition 2 considers the open economy model with constant λ and λ^* and uniform distributions for productivity distributions. This special case satisfies the conditions of Proposition 4. Similarly, Proposition 2 is a direct corollary of Proposition 5, whose proof we provide below in Section A.5.

A.4 Proof of Proposition 4

Proof of part 1. As established in equation (4.1) in the main text, the North's bubble ratio on the BGP β in the open economy is equal to the world's excess savings rate:

$$\beta = SI\left(\frac{R_b}{1-n\beta}\right) + \left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(\frac{R_b}{1-n\beta})}\right)^{\frac{\alpha}{1-\alpha}} (1-n\beta)SI^*(R_b).$$

As $\beta > 0$, $SI, SI^* \leq 1$ and $\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(\frac{R_b}{1-n\beta})} < \frac{\mathcal{A}^*(R_b)}{\mathcal{A}(R_b)}$, it follows that $\beta < \frac{1+(\mathcal{A}^*(R_b)/\mathcal{A}(R_b))^{\frac{\alpha}{1-\alpha}}}{1+n} \equiv \overline{\beta}(R_b)$. Thus, there exists a BGP with a bubble if and only if the equation $s^w(x;R_b) = 0$ has a solution $x = \beta \in (0, \overline{\beta}(R_b))$, where $s^w(x;R_b) \equiv SI\left(\frac{R_b}{1-nx}\right) + \left(\mathcal{A}^*(R_b)/\mathcal{A}(\frac{R_b}{1-nx})\right)^{\frac{\alpha}{1-\alpha}} (1-nx)SI^*(R_b) - x$.

Proposition 4 states that there exists a BGP with a bubble if and only of $R_{nb} < R_b$, given that condition (2.30) holds and $\mathcal{A}^*(R)/\mathcal{A}(R)$ is increasing in R. We first show the if part. Suppose $R_{nb} < R_b$. Under the assumptions, $s^w(0;R)$ is increasing in R. This is because both $SI(R)$ and $SI^*(R)$ are increasing in R, and by assumption, $\mathcal{A}^*(R)/\mathcal{A}(R)$ is increasing in R. Then, $R_{nb} < R_b$ implies that $0 = s^w(0;R_{nb}) < s^w(0;R_b)$. By construction, $s^w(\overline{\beta}(R_b);R_b) < 0$. Hence, because function $s^w(x;R_b)$ is continuous in x, there exists a solution β such that $s^w(\beta;R_b) = 0$. This completes the proof of the if part.

Next, we show the only if part by contradiction. Suppose $R_{nb} \geq R_b$. Then, because $s^w(0;R)$ is increasing in R as we showed above, we have $0 = s^w(0;R_{nb}) \geq s^w(0;R_b)$. We already know that $s^w(\overline{\beta}(R_b);R_b) < 0$. We are going to show that there is no $\beta \in (0, \overline{\beta}(R_b))$ such that $s^w(\beta;R_{nb}) = 0$. It is useful to observe:

$$\begin{aligned} s^w(x;R_b) =& SI\left(\frac{R_b}{1-nx}\right) + \left(\frac{A^*(R_b)}{\mathcal{A}(\frac{R_b}{1-nx})}\right)^{\frac{\alpha}{1-\alpha}} (1-nx)SI^*(R_b) - x \\ \leq& SI\left(\frac{R_b}{1-nx}\right) + \left(\frac{A^*(R_b)}{\mathcal{A}(R_b)}\right)^{\frac{\alpha}{1-\alpha}} SI^*(R_b) - x \equiv \tilde{s}^w(x;R_b). \end{aligned}$$

By construction, $\tilde{s}^w(0;R_b) = s^w(0;R_b) \leq 0$ and $s^w(\overline{\beta}(R_b);R_b) < 0$. If $\tilde{s}^w(x;R_b)$ is decreasing in x, there is no β such that $\tilde{s}^w(\beta;R_b) = 0$, as we have desired. So, consider the other case that $\tilde{s}^w(x;R_b)$ is increasing for some x. As shown in Appendix A.1 the condition (2.30) implies that $\partial^2\tilde{s}^w(x;R_b)/\partial x^2 \geq 0$. Hence, $\tilde{s}^w(x;R_b)$ is convex. This implies that there is no β such that $\tilde{s}^w(\beta;R_b) = 0$, because the function $\tilde{s}^w(x;R_b)$ has to be concave at least for some x for the solution to exist. Because $s^w(x;R_b) \leq \tilde{s}^w(x;R_b)$, it is straightforward that there is also no β such that $s^w(\beta;R_b) = 0$. This completes the proof of the only if part.

Proof of part 2. Recall that β^c of the closed economy is the solution to $s(x;R_b) = 0$, where $s(x;R_b) \equiv SI(\frac{R_b}{1-nx}) - x$. On the other hand, β of the open economy is the solution to

$s^w(x; R_b) = 0$, where $s^w(x; R_b) = s(x; R_b) + \left(\mathcal{A}^*(R_b)/\mathcal{A}(\frac{R_b}{1-nx})\right)^{\frac{\alpha}{1-\alpha}} (1-nx)SI^*(R_b)$. Under the condition $R_b > R_{nb}$ for the existence of bubbles in the open economy, we know that $SI^*(R_b) > SI^*(R_{nb}) > SI^*(R_{nb}^{c*}) = 0$, i.e., the South must have a trade surplus (capital outflows). Thus, $s^w(x; R_b) > s(x; R_b)$. It thus immediately follows that $\beta > \beta^c$. □

A.5 Proof of Proposition 5

The fact that $tb_b^* > tb_{nb}^*$ (bubbles raise the South's trade balance) has been proved in the main text. It remains to show that $tb_b < tb_{nb}$ (bubbles reduce the North's trade balance). Due to the world credit market clearing condition, we have

$$-tb_b = \left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(\frac{R_b}{1-n\beta})}\right)^{\frac{\alpha}{1-\alpha}} (1-n\beta)tb_b^*.$$

Under the assumption of the elasticity condition (2.30), the assumption that $\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}$ is increasing in R, and the existence condition $R_b > R_{nb}$, it follows that $\left(\frac{\mathcal{A}^*(R_b)}{\mathcal{A}(\frac{R_b}{1-n\beta})}\right)^{\frac{\alpha}{1-\alpha}} (1-n\beta) > \left(\frac{\mathcal{A}^*(R_{nb})}{\mathcal{A}(R_{nb})}\right)^{\frac{\alpha}{1-\alpha}}$. Thus:

$$-tb_b > \left(\frac{\mathcal{A}^*(R_{nb})}{\mathcal{A}(R_{nb})}\right)^{\frac{\alpha}{1-\alpha}} tb_b^*.$$

Combined with the fact that $tb_b^* > tb_{nb}^*$, we have

$$-tb_b > \left(\frac{\mathcal{A}^*(R_{nb})}{\mathcal{A}(R_{nb})}\right)^{\frac{\alpha}{1-\alpha}} tb_{nb}^*.$$

By using the credit market clearing condition one more time, the right hand side of the inequality above is exactly equal to $-tb_{nb}$. Thus

$$-tb_b > -tb_{nb}$$

or equivalently

$$tb_b < tb_{nb}.$$

□