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Understanding the Size of the Government Spending Multiplier: It's in the Sign[*]

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Abstract

The literature on the government spending multiplier has implicitly assumed that an increase in government spending has the same (mirror-image) effect as a decrease in government spending. We show that relaxing this assumption is important to understand the effects of fiscal policy. Regardless of whether we identify government spending shocks from (i) a narrative approach, or (ii) a timing restriction, we find that the contractionary multiplier –the multiplier associated with a negative shock to government spending– is above 1 and even larger in times of economic slack. In contrast, the expansionary multiplier –the multiplier associated with a positive shock– is substantially below 1 regardless of the state of the cycle. These results help understand seemingly conflicting results in the literature.
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1 Introduction

Understanding the impact of changes in government purchases on output is a key part of fiscal policy analysis, and the question periodically takes center stage as economies move through stages of business and political cycles. The implementation of fiscal stimulus packages across OECD countries in the early phase of the 2008-2009 crisis spurred a lot of work on the size of the government spending multiplier associated with increases in government purchases.[1] A few years later, the mirror-image question –the effect of contractionary fiscal policy– became the center of attention as the rapid rise in government debt levels led to a swift shift to fiscal consolidation, particularly in continental Europe.[2] Unfortunately, despite intense scrutiny, the range of estimates for the government spending multiplier remains wide, with estimates lying between 0.5 and 2.

Perhaps surprisingly, the literature has so far treated the effects of government intervention symmetrically: a contractionary policy is assumed to have the same (mirror-image) effect as an expansionary policy, and the size of the multiplier does not depend on the *sign* of the government spending shock. However, with occasionally binding borrowing constraints, households' marginal propensity to consume (MPC) out of temporary income changes may be asymmetric, i.e., the MPC may depend on the sign of the change in income.[3] Since the MPC is a key determinant of the size of the multiplier (e.g., Gali et al., 2007), asymmetry in the MPC raises the possibility of an asymmetric government spending multiplier.[4]

In this paper, we relax the assumption of symmetric government spending multipliers using a novel econometric procedure –Functional Approximation of Impulse Responses (FAIR)– that consists of directly modeling and estimating the economy's impulse responses to structural shocks. We find that treating separately expansionary and contractionary spending shocks is

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[1]See e.g., Hall (2009), Mountford and Uhlig (2009), Ramey (2009), Mertens and Ravn (2010), Uhlig (2010), Barro and Redlick (2011), Parker (2011), Ramey (2011, 2012a, 2012b), Auerbach and Gorodnichenko (2012, 2013), Bachmann and Sims (2012), Owyang, Ramey and Zubairy (2013) and Ramey and Zubairy (2016), Caggiano, Castelnuovo, Colombo and Nodari (2015), Caldara and Kamps (2017).

[2]See e.g., Alesina and Ardagna (2010), Guajardo, Leigh, and Pescatori (2011) and Jorda and Taylor (2016).

[3]Interestingly, Bunn et al. (2017) and Christelis et al. (2017) recently found evidence of asymmetric MPCs. For instance, Bunn et al. (2017) found evidence that the MPC out of negative income shocks is above 0.5 but that the MPC out of positive shocks is only about 0.1.

[4]This is easy to see in a basic Keynesian framework, where the spending multiplier is given by $1/(1\text{-MPC})$ if interest rates are held constant. In a model with forward-looking rational agents, if some households cannot borrow more but can save more, the economy may behave in a more Ricardian fashion (and thus feature a smaller multiplier) following an increase in government spending (where households can save more to smooth the drop in permanent income caused by future higher taxes) than following a decrease in government spending (where financially constrained households cannot borrow more to consume out of their higher permanent income).

crucial to understanding the size of the government spending multiplier. The government spending multiplier is substantially *below* 1 for expansionary shocks to government spending, but the multiplier is *above* 1 for contractionary shocks. Importantly, we reach the same conclusions regardless of whether we identify government spending shocks from –(i) a narrative identifying assumption (Ramey, 2011, Ramey and Zubairy, 2016), or (ii) a recursive identifying assumption (Blanchard and Perotti, 2002, Auerbach and Gorodnichenko, 2012)–, which have been the two main approaches to identifying government spending shocks and their effects on the economy.

Our findings of an asymmetric multiplier are also robust to using a different econometric method –Local Projections (Jorda, 2005)–, using a longer sample period with historical data over 1890-2014, or using alternative identification schemes as in the works of Jorda and Taylor (2016) and Blanchard and Leigh (2013).

We then study to what extent the expansionary and contractionary multipliers depend on the state of the cycle at the time of the shock. In doing so, we expand a recent literature that reached conflicting conclusions on the effect of slack on the size of the multiplier: while studies based on narratively-identified shocks find little evidence for state dependence,[5] VAR-based studies find that the multiplier is largest in times of slack.[6] We find that the contractionary spending multiplier is state-dependent –being largest and around 2 in recessions–, but we find no evidence of state dependence for the expansionary multiplier –being always below 1 and not larger in recessions–. Thus, while Auerbach and Gorodnichenko (2012)'s findings have sometimes been interpreted as supporting the case for fiscal stimuli in recessions, our results caution against such a conclusion. We find no evidence that increases in government spending have larger multipliers during a recession (in fact, the multiplier is consistently below 1) and thus no support for stimulus programs in times of recession. However, we find that decreases in government spending during recessions have the largest multiplier, which suggests that austerity measures during recessions can be especially harmful.

Our results provide a simple explanation for the wide range of state dependence estimates reported in the literature: the relative frequency of expansionary and contractionary shocks differs markedly across the two main identification schemes. Results obtained from a narrative identifying assumption are driven primarily by positive shocks –unexpected *increases* in government spending–, because the narratively-identified shock series (Ramey, 2011, Ramey and Zubairy, 2016) contains larger and more numerous positive shocks than negative shocks. As a result, narrative multiplier estimates mostly reflect the effects of positive shocks, which

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[5] Owyang, Ramey and Zubairy 2013, Ramey and Zubairy 2016.

[6] Auerbach and Gorodnichenko 2012, 2013, Bachmann and Sims 2012, Caggiano et al. 2015, Fazzari, Morley and Panovska 2015.

(according to our results) do not depend on the state of the business cycle. In contrast, the spending shocks identified in VARs are (by construction) evenly distributed between positive and negative values. As a result, the average multiplier is mildly state dependent, driven by the state dependence of the contractionary multiplier.

An important reason for the lack of studies on the possibly asymmetric nature of the government spending multiplier is methodological. Standard techniques are linear and make the exploration of non-linearities, in particular the asymmetric effect of spending shocks and their state dependence, difficult. In particular, structural VARs, as used by Blanchard and Perotti (2002) and Auerbach and Gorodnichenko (2012) cannot allow the impulse response function of a shock to depend on the sign of that shock.[7] The narrative approach to government spending shocks, pioneered by Ramey and Shapiro (1998) and Ramey (2011), relies on autoregressive distributed lags models (ADL) or Local Projection (LP, Jorda, 2005), and these methods can allow for some non-linearities, as illustrated by Ramey and Zubairy (2016) and Auerbach and Gorodnichenko (2013) for state dependence. However, because of their non-parametric nature, these methods are limited by efficiency considerations, and simultaneously allowing for asymmetry and state dependence is difficult.[8] Moreover, to explore the non-linear effects of shocks, ADL or LP requires a series of previously identified structural shocks, which limits their use to narrative identification schemes.

To overcome these technical challenges, we use a new method –Functional Approximation of the Impulse Responses–, which consists in (i) directly estimating a structural moving average model of the economy, i.e., directly estimating the impulse response functions to structural shocks (unlike the VAR approach, which first estimates a reduced-form VAR and thus requires the existence of a VAR representation),[9] and (ii) approximating the (high-dimensional) impulse

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[7]Regime-switching VAR models can capture certain types of non-linearities such as state dependence (whereby the value of some state variable affects the impulse response functions), but they cannot capture asymmetric effects of shocks (whereby the impulse response to a structural shock depends on the sign of that shock). With regime-switching VAR models, it is assumed that the economy can be in a finite number of regimes, and that each regime corresponds to a different set of VAR coefficients. However, if the true data generating process features impulse responses that depend on the sign of the structural shock, a new set of VAR coefficients would be necessary each period, because the (non-linear) behavior of the economy at any point in time depends on the whole history of shocks up to that point. As a result, such asymmetric data generating process cannot generally be approximated by a small number of state variables such as in threshold VARs or Markov-switching models.

[8]Riera-Crichton et al. (2015) use Jorda's local projection method to study the state dependent response of output growth to positive and negative government spending shocks. However because of efficiency considerations, they can only consider two states (expansion vs recession, whereas we use a continuous indicator of slack in the economy) and they must bring in additional information from cross-country data (30 OECD countries) while imposing that responses are the same across countries.

[9]A promising approach related to ours is Plagborg-Moller (2016), who proposes a Bayesian method to directly estimate the structural moving-average representation of the data by using prior information about the shape and the smoothness of the impulse responses.

response functions with a (small) number of Gaussian basis functions, which offers efficiency gains and allows for the exploration of a rich set of non-linearities (in contrast to the non-parametric ADL and LP approaches). While different families of basis functions are possible, Gaussian basis functions are particularly attractive for two reasons: (i) any mean-reverting impulse response function can be approximated by a sum of Gaussian basis functions, and (ii) a small number (one or two) of Gaussian functions can already capture a large variety of impulse response functions, and in fact capture the typical impulse responses found in empirical or theoretical studies. For instance, the impulse response functions of macroeconomic variables to government spending shocks are often found (or predicted) to be monotonic or hump-shaped (e.g. Ramey, 2011, Gali et al., 2007). In such cases, a single Gaussian function can already provide an excellent approximation of the impulse response function. Thanks to the small number of free parameters allowed by our functional approximation, it is possible to directly estimate the impulse response functions from the data using maximum likelihood or Bayesian methods. The parsimony of the approach in turn allows us to estimate more general non-linear models.

Our use of basis functions to approximate impulse response functions relates to a large literature in statistics that relies on basis functions (of which Gaussian functions are one example) to approximate arbitrary functions (e.g., Hastie et al., 2009). In economics, our approximation of impulse responses relates to an older literature on distributed lag models and in particular on the Almon (1965) lag specification, in which the successive weights, i.e., the impulse response function in our context, are given by a polynomial function. In Barnichon and Matthes (2017), we used FAIR to study the asymmetric effects of monetary shocks, and the present paper expands the methodology in a number of dimensions, notably by showing how to estimate FAIR models with both asymmetric and state dependent effects of shocks.

Section 2 presents the empirical model, our method to approximate impulse responses using Gaussian basis functions and the two main structural identifying restrictions used in the literature, Section 3 describes our asymmetric model and presents our results on the asymmetric effects of shocks to government spending as well as a number of robustness checks; Section 4 provides additional supporting evidence; Section 5 describes a model with asymmetry and state dependence, presents the results and discusses how the asymmetric effects of government spending shocks help reconcile the seemingly contradictory findings in the literature; Section 6 discusses how one might rationalize our empirical findings and provides avenues for future theoretical work, Section 7 concludes.

2 Empirical model

Our goal in this paper is to study how the size of the government spending multiplier, and more generally the effects of government spending on the economy, depends on the sign of the policy intervention and on the state of the business cycle at the time of the policy intervention.

To capture these possibilities, we need a model that allows the impulse response functions to depend on the sign of the shock as well as on the state of the economy at the time of the shock.[10] Our empirical model is thus a (non-linear) structural moving-average model, in which the behavior of a vector of macroeconomic variables is dictated by its response to past and present structural shocks. Specifically, denoting $\boldsymbol{y}_t$ a vector of stationary macroeconomic variables, the economy is described by

$$\boldsymbol{y}_t = \sum_{k=0}^{K} \boldsymbol{\Psi}_k(\boldsymbol{\varepsilon}_{t-k}, z_{t-k}) \boldsymbol{\varepsilon}_{t-k} \tag{1}$$

where $\boldsymbol{\varepsilon}_t$ is the vector of i.i.d. structural innovations with $E\boldsymbol{\varepsilon}_t = \boldsymbol{0}$ and $E\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' = \mathbf{I}$, K is the number of lags, which can be finite or infinite, z_t is a stationary variable that is a function of lagged values of $\boldsymbol{y}_t$ or a function of variables exogenous to $\boldsymbol{y}_t$. $\boldsymbol{\Psi}_k$ is the matrix of lag coefficients –i.e., the matrix of impulse responses at horizon k–.

Model (1) is a *non-linear* vector moving average representation of the economy, because the matrix of lag coefficients $\boldsymbol{\Psi}_k$, i.e., the impulse responses of the economy, can depend on (i) the values of the structural innovations $\boldsymbol{\varepsilon}$ and (ii) the value of the macroeconomic variable z: With $\boldsymbol{\Psi}_k$ a function of $\boldsymbol{\varepsilon}_{t-k}$, the impulse responses to a given structural shock depend on the value of that shock at the time of shock. For instance, a positive shock may trigger different impulse responses than a negative shock. With $\boldsymbol{\Psi}_k$ a function of z_{t-k}, the impulse responses to a structural shock depend on the value of z at the time of that shock. For instance, the impulse responses may be different depending on the state of the business cycle (e.g., the level of unemployment) at the time of the shock.

Importantly, our starting point is *not* a structural Vector AutoRegression (VAR). While the use of a VAR is a common way to estimate a moving-average model, it relies on the existence of a VAR representation. However, in a non-linear world where $\boldsymbol{\Psi}_k$ depends on the sign of the shocks $\boldsymbol{\varepsilon}$ as in (1), the existence of a VAR is severely compromised, because inverting (1) is generally not possible. Thus, in this paper, we work with an empirical method that side-steps the VAR and instead directly estimates the vector moving average model (1).

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[10]As we argue in two paragraphs, a VAR is ill-suited to capture such non-linearities.

2.1 Functional Approximation of Impulse Responses

Estimating moving-average models is notoriously difficult, because the number of free para-
meters $\mathbf{\Psi}_k$ in (1) is very large or infinite. To address this issue, we use a new approach
–Functional Approximation of Impulse Responses or FAIR–, which consists in representing the
impulse response functions as expansions in basis functions.

Since the intuition and benefits of our approach can be understood in a linear context, this
section introduces FAIR in a linear context, i.e., where $\mathbf{\Psi}_k(\boldsymbol{\varepsilon}_{t-k}, z_{t-k}) = \mathbf{\Psi}_k$. We postpone
non-linear models to the next sections.

Denote $\psi(k)$ an element of matrix $\mathbf{\Psi}_k$, so that $\psi(k)$ is the value of the impulse response
function ψ at horizon k. A functional approximation of ψ consists in decomposing ψ into a
sum of basis functions, i.e., in modeling $\psi(k)$ as a *basis function expansion* (see e.g., Hastie et
al., 2009), with

$$\psi(k) = \sum_{n=1}^{N} a_n g_n(k), \quad \forall k > 0 \tag{2}$$

with $g_n : \mathbb{R} \to \mathbb{R}$ the nth basis function, $n = 1, .., N$. Different families of basis functions are
possible, and in this paper we use Gaussian basis functions and posit

$$\psi(k) = \sum_{n=1}^{N} a_n e^{-(\frac{k-b_n}{c_n})^2}, \quad \forall k > 0 \tag{3}$$

with a_n, b_n, and c_n parameters to be estimated. Since model (3) uses N Gaussian basis
functions, we refer to this model as a functional approximation of order N.[11]

While other families of basis functions are possible, Gaussian basis functions are particularly
attractive for studying the (possibly non-linear) effects of shocks, because only a very small
number of Gaussian basis functions are needed to approximate a large class of impulse response
functions –in fact most impulse responses encountered in macro applications–.

Intuitively, impulse response functions of variables are often found to be monotonic or
hump-shaped. In such cases, one or two Gaussian functions can already provide a very good
approximate description of the impulse response. To illustrate this observation, Figure 1 plots
the impulse response functions of government spending, taxes and output to a shock to govern-
ment spending estimated from a standard VAR specification with a recursive ordering,[12] along

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[11]The functional approximation of ψ may or may not include the contemporaneous impact coefficient, that
is one may choose to use the approximation (3) for $k > 0$ or for $k \geq 0$. In this paper, we treat $\psi(0)$ as a free
parameter for additional flexibility.

[12]We describe the exact specification in the next section.

with the corresponding FAIR with only *one* Gaussian function, i.e., using the approximation

$$\psi(k) \simeq ae^{-\frac{(k-b)^2}{c^2}}, \quad \forall k > 0. \tag{4}$$

We can see that a FAIR with only one Gaussian basis function already does a good job at capturing the impulse responses implied by the VAR. With two Gaussian basis functions, the impulse responses are very close to those of the VAR (Figure 1). For illustration, Figure 2 plots the Gaussian basis functions used for each impulse response in that case.

The small number of free parameters (only three per impulse response function in the one-Gaussian case), has two important advantages. First, it allows us to directly estimate the impulse response functions from the MA representation (1).[13] Second, it will allow us to later add more degrees of freedom and allow for asymmetric or non-linear effects of shocks to government spending.

2.2   The structural identifying assumptions

Models like (1) are under-identified without additional restrictions. To identify government spending shocks, the fiscal policy literature has mainly followed two approaches:[14] (i) a recursive identification scheme, and (ii) a narrative identification scheme. In this paper, we will consider both alternatives, which is important to put our results in the context of the literature.

2.2.1   Identification from a recursive ordering

The first identification scheme was proposed by Blanchard and Perotti (2002) and consists of a short-run restriction, i.e., a restriction on $\boldsymbol{\Psi}_0$, the matrix capturing the contemporaneous impact of a shock. Government spending is assumed to react with a lag to shocks affecting macro variables, so that in a system where $\boldsymbol{y}_t$ includes government spending, taxes and output, government spending is ordered first and $\boldsymbol{\Psi}_0$ has its first row filled with 0 except for the diagonal coefficient. This identification scheme was recently challenged because of anticipation effects (Ramey, 2011), as some innovations to government spending were found to be anticipated by agents. We thus follow Auerbach and Gorodnichenko (2012), who addressed the anticipation issue by augmenting the vector $\boldsymbol{y}_t$ with a professional forecast of the growth rate of government spending in order to soak up the forecastable components of shocks to government spending.

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[13]For instance, with 4 variables, we only have $3 * 4^2 = 48$ parameters (ignoring intercepts) to estimate to capture the whole set of impulse response functions $\{\boldsymbol{\Psi}_k\}_{k=1}^{\infty}$. In comparison, a corresponding VAR with 4 lags has 64 parameters.

[14]See e.g., Perotti (2008) and Ramey (2011, 2012) for overviews of the main identification schemes used in the literature.

We now briefly describe how we use Bayesian methods to estimate multivariate FAIR models identified with a short-run restriction. More details are available in the Appendix. The key part to estimate (1) is the construction of the likelihood function $p(\boldsymbol{y}^T|\boldsymbol{\theta})$, where T is the sample size, $\boldsymbol{\theta}$ is the vector of parameters of model (1) and where a variable with a superscript denotes the sample of that variable up to the date in the superscript.

We use the prediction error decomposition to break up the density $p(\boldsymbol{y}^T|\boldsymbol{\theta})$ as follows:[15]

$$p(\boldsymbol{y}^T|\boldsymbol{\theta}) = \prod_{t=1}^{T} p(\boldsymbol{y}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1}). \tag{5}$$

Then, to calculate the one-step-ahead conditional likelihood function $p(\boldsymbol{y}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1})$, we assume that all innovations $\{\boldsymbol{\varepsilon}_t\}$ are Gaussian with mean zero and variance one, and we note that the density $p(\boldsymbol{y}_t|\boldsymbol{y}^{t-1}, \boldsymbol{\theta})$ can be re-written as $p(\boldsymbol{y}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1}) = p(\boldsymbol{\Psi}_0 \boldsymbol{\varepsilon}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1})$ since

$$\boldsymbol{y}_t = \boldsymbol{\Psi}_0 \boldsymbol{\varepsilon}_t + \sum_{k=1}^{K} \boldsymbol{\Psi}_k \boldsymbol{\varepsilon}_{t-k}. \tag{6}$$

Since the contemporaneous impact matrix is a constant, $p(\boldsymbol{\Psi}_0 \boldsymbol{\varepsilon}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1})$ is a straightforward function of the density of ε_t.

To recursively construct $\boldsymbol{\varepsilon}_t$ as a function of $\boldsymbol{\theta}$ and $\boldsymbol{y}^t$, we need to uniquely pin down the value of the components of $\boldsymbol{\varepsilon}_t$, that is we need that $\boldsymbol{\Psi}_0$ is invertible. We impose this restriction by only keeping parameter draws for which $\boldsymbol{\Psi}_0$ is invertible. It is also at this stage that we impose the identifying restriction. We order variables in $\boldsymbol{y}$ such that the professional forecast of the growth rate of government spending enters first and government spending enters second. Then, our identifying restriction is that $\boldsymbol{\Psi}_0$ has its first two rows filled with 0 except for the diagonal coefficients. Finally, to initialize the recursion, we set the first K values of $\boldsymbol{\varepsilon}$ to zero.[16,17]

To explore the posterior density, we use a Metropolis-within-Gibbs algorithm (Robert and Casella, 2004) with the blocks given by the different groups of parameters in our model; a, b, and c. The elicitation of priors is described in the Appendix.

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[15]To derive the conditional densities in decomposition (5), our parameter vector $\boldsymbol{\theta}$ thus implicitly also includes the K initial values of the shocks: $\{\boldsymbol{\varepsilon}_{-K}...\boldsymbol{\varepsilon}_0\}$. We will keep those fixed throughout the estimation and discuss the initialization below.

[16]Alternatively, we could use the first K values of the shocks recovered from a structural VAR.

[17]When K, the lag length of the moving average (1), is infinite, we truncate the model at some horizon K, large enough to ensure that the lag matrix coefficients $\boldsymbol{\Psi}_K$ are "close" to zero. Such a K exists since the variables are stationary.

2.2.2 Identification from a narrative approach

The second main identification scheme is based on a narrative approach and was proposed by Ramey (2011), building on Ramey and Shapiro (1998).

In Ramey and Shapiro (1998), wars provide exogenous variations in government spending, because the entries into war (such as World-War II or the Korean war) (i) were exogenous to domestic economic developments, and (ii) led to large increases in defense spending. Generalizing this idea, Ramey (2011) identifies unexpected changes in anticipated future defense expenditures by using news sources to measure expectations and expectation surprises.

Incorporating narratively identified shocks into FAIR models is relatively straightforward. Indeed, in that case, it is no longer necessary to specify a self-contained model capturing the relevant features of the economy (and thus a multivariate moving average model), and one can directly estimate a univariate model –a univariate FAIR– capturing the impulse response of any variable of interest to the independently identified structural shocks.

Taking y_t to be one of the variables of $\boldsymbol{y}_t$, (1) implies that

$$y_t = \sum_{k=0}^{K} \psi_y(k)\varepsilon_{t-k}^G + u_t^y \tag{7}$$

with $\{\varepsilon_t^G\}$ the (narratively-identified) shocks to government spending, $\psi_y(.)$ the impulse response function of y to shock ε^G, and u_t^y the residual.[18] We can then directly estimate the impulse response of y using a functional approximation of $\psi_y(.)$, and the multiplier is obtained by comparing the responses of Y and G.

In practice, since the multiplier is a function of two separate impulse responses (ψ_Y and ψ_G), we will jointly estimate the responses of Y and G in order to obtain a posterior distribution for the multiplier.

Specifically, we will estimate a SUR-type (Seemingly Unrelated Regression) model

$$\begin{pmatrix} Y_t \\ G_t \end{pmatrix} = \sum_{k=0}^{K} \begin{pmatrix} \psi_Y(k) \\ \psi_G(k) \end{pmatrix} \varepsilon_{t-k}^G + \begin{pmatrix} u_t^Y \\ u_t^G \end{pmatrix} \tag{8}$$

where the vector $\mathbf{u}_t = \begin{pmatrix} u_t^Y \\ u_t^G \end{pmatrix}$ follows a VAR process with

$$\mathbf{u}_t = \boldsymbol{\Upsilon}(\mathbf{L})\mathbf{u}_{t-1} + \boldsymbol{\eta}_t$$

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[18] The residual satisfies $u_t^y = \sum_j \sum_{k=0}^{K} \psi_j(k)\varepsilon_{t-k}^{(j)}$ where $\left\{\varepsilon_t^{(j)}\right\}$ are the other j shocks affecting the economy and $\psi_j(.)$ captures the impulse response function to shock $\varepsilon_t^{(j)}$.

with $\boldsymbol{\Sigma} = E\boldsymbol{\eta}_t\boldsymbol{\eta}_t'$ and $\boldsymbol{\Upsilon}(\mathbf{L})$ matrices to be estimated. The likelihood of model (1) can be constructed from the prediction error decomposition by assuming that $\boldsymbol{\eta}_t$ is i.i.d. and follows a multivariate normal distribution. More details are provided in the Appendix.[19]

While having $\mathbf{u}_t$ following a VAR process is not necessary for consistency, it will allow us to improve the efficiency of the estimation since u_t^Y and u_t^G are functions of the other structural shocks affecting the economy and are thus both serially- and cross-correlated.

2.3 Defining the government spending multiplier

We define the government spending multiplier as in Mountford and Uhlig (2009) and Ramey and Zubairy (2016), and we compute the "sum" multiplier

$$m = \sum_{k=0}^{K} \psi_Y(k) / \sum_{k=0}^{K} \psi_G(k) \tag{9}$$

where $\psi_Y(.)$ and $\psi_G(.)$ denote respectively the impulse response function of output (denoted Y from now on) and government spending (denoted G from now on) to a spending shock.

Since the multiplier captures a ratio of changes in the *levels* of Y and G, while the impulse responses are estimated for variables in logs, we need to convert the estimated impulse responses into dollar units. While a standard approach in the literature is to use an ex-post conversion based on the approximation $\frac{dY}{dG} \simeq \frac{d\ln Y}{d\ln G}\frac{\overline{Y}}{G}$ where $\frac{\overline{Y}}{G}$ is the sample average of the GDP to government spending ratio, Ramey and Zubairy (2016) argue that this approach can lead to biased multiplier estimates, because $\frac{Y}{G}$ can display large movements over the sample period. We thus use instead an ex-ante conversion approach as in Gordon and Krenn (2010) and Ramey (2016), and before estimation we re-scale all variables by "potential output", where potential output (Y^{pot}) is estimated from a quadratic trend.

3 The asymmetric government spending multiplier

We now turn to studying the asymmetric effects of government spending shocks. We first describe how to introduce asymmetry in FAIR models, and then present the estimation results using (i) a recursive identification scheme à la Auerbach and Gorodnichenko (2012), and (ii) a narrative identification scheme à la Ramey (2011). We leave a detailed description of the estimation of such models (which is a simple extension of the linear case described above) for

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[19]Note that the estimation of the multiplier from such a SUR-model is similar to an Instrument Variable (IV) regression where one regresses Y_t on lagged values of G_t using lagged values of ε_t^G as instruments. See Ramey and Zubairy (2016) for a related IV approach using Local Projections.

the appendix, and in the online appendix we show that the structural shocks can be identified in (i) or (ii) even when the shocks have asymmetric effects.

3.1 Introducing asymmetry

To allow for asymmetry, we let $\boldsymbol{\Psi}_k$ depend on the sign of the government spending shock ε^G, i.e., we let $\boldsymbol{\Psi}_k$ take two possible values: $\boldsymbol{\Psi}_k^+$ or $\boldsymbol{\Psi}_k^-$. Specifically, a general model that allows for asymmetric effects of shocks would be

$$\boldsymbol{y}_t = \sum_{k=0}^{\infty} \left[\boldsymbol{\Psi}_k^+ 1_{\varepsilon_{t-k}^G > 0} + \boldsymbol{\Psi}_k^- 1_{\varepsilon_{t-k}^G < 0} \right] \boldsymbol{\varepsilon}_{t-k} \tag{10}$$

with 1 the indicator function and $\boldsymbol{\Psi}_k^+$ and $\boldsymbol{\Psi}_k^-$ the lag matrices of coefficients for, respectively, positive and negative government spending shocks.

Denoting $\psi_i^{G+}(k)$ the impulse response of variable i at horizon k to a *positive* government spending shock (and similarly for $\psi_i^{G-}(k)$), a functional approximation of the impulse response function ψ_i^{G+} is

$$\psi_i^{G+}(k) = \sum_{n=1}^{N} a_{i,n}^+ e^{-\left(\frac{k - b_{i,n}^+}{c_{i,n}^+}\right)^2}, \quad \forall k > 0 \tag{11}$$

with $a_{i,n}^+$, $b_{i,n}^+$, $c_{i,n}^+$ some parameters to be estimated. A similar expression would hold for $\psi_i^{G-}(k)$.

We then denote m^+ the government spending multiplier (9) associated with a positive (expansionary) spending shock and m^- the multiplier associated with a negative (contractionary) shock.

3.2 Results from a recursive identification scheme

To identify innovations to government spending, we first follow Auerbach and Gorodnichenko (2012), and we consider the vector $\left(\Delta g_{t|t-1}^F, g_t, \tau_t, y_t\right)'$, where g is log real government (federal, state, and local) purchases (consumption and investment), τ is log real government receipts of direct and indirect taxes net of transfers to businesses and individuals, and y is log real gross domestic product (GDP) in chained 2000 dollars, and $\Delta g_{t|t-1}^F$ is the growth rate of government spending at time t forecasted at time $t-1$. As described in Section 2, we include the anticipated growth rate of government spending in order to soak up any forecastable changes in government spending. For $\Delta g_{t|t-1}^F$, we combine Greenbook and SPF quarterly forecasts following Auerbach and Gorodnichenko (2012) and extending their dataset so that our sample cover 1966q1-2014q4.

Figure 3 plots the impulse responses estimated using a VAR with 4 lags (dashed black line)

and a FAIR model with 1 Gaussian basis function (thick line) where we allow for a linear trend for each variable.[20] The error bands cover 90 percent of the posterior probability. The upper panel plots the impulse responses to a positive shock to G, while the lower panel plots the impulse responses to a negative shock to G.

When comparing impulse responses to positive and negative shocks, it is important to keep in mind that the impulse responses to negative shocks were multiplied by -1 in order to ease comparison across impulse responses. With this convention, when there is no asymmetry, the impulse responses are identical in the top panel (responses to an expansionary shock) and in the bottom panels (responses to a contractionary shock). Finally, the magnitude of the fiscal shock is chosen to generate a peak effect on government spending of 1 (in absolute value) in order to facilitate the interpretation of the results.

The results show that the impulses responses are strongly asymmetric:[21]

Starting with the left panels, the thick blue line depicts the response of G to a positive spending shock (an expansionary shock), while the red line depicts the shock to a negative spending shock (a contractionary shock). The responses to positive and negative shocks are similar although the response to a negative shock appears slightly more persistent.

Turning to the response of output, we can see that the impulse response of output is strong following a contractionary G shock but is not significantly different from zero following an expansionary G shock.

The strong asymmetric responses of Y imply strong asymmetries in the spending multiplier. As shown in Table 1, the "sum" multiplier to a spending contraction is $m^- = 1.25$ (cumulating the impulse responses over 20 quarters), while the expansionary multiplier is $m^+ = 0.27$. To assess the statistical significance of our result, the upper-panel of Figure 5 plots the posterior distribution of the multiplier following expansionary spending shocks (x-axis) and following contractionary shocks (y-axis). Evidence for asymmetry is strong with a 0.99 posterior probability that $m^- > m^+$.

Figure 3 shows that the response of taxes is not behind the asymmetric size of the multiplier. While the response of taxes is not different from zero following an expansionary shock, taxes declined markedly following a contractionary shock (recall that the impulse responses to contractionary shocks are multiplied by -1). Thus, the tax response should make the adverse effect of contractionary fiscal policy on output smaller, not larger.

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[20]The loose priors for the FAIR parameters are detailed in the Appendix. To determine the appropriate number of basis functions, we use posterior odds ratios to compare models with increasing number of basis functions. We select the model with the highest posterior odds ratio. This approach can be seen as analogous to the choice of the number of lags in VAR models using Bayesian Information Criteria.

[21]Another way to evaluate the significance of our results is to use Bayesian model comparison. The asymmetric FAIR model displays marginal data densities that are substantially larger than the marginal data density of a corresponding Bayesian VAR with loose, but proper, Normal-Whishart priors.

3.3 Results from a narrative identification scheme

We now turn to Ramey's narrative identification scheme, and we explore the asymmetry of the multiplier following unexpected changes in anticipated future defense expenditures. We estimate impulse response functions from a SUR-type FAIR model with 1 Gaussian basis function using quarterly data over 1939-2014.[22]

Figure 4 plots the impulse responses of government spending and output to news shocks, and Table 1 reports the corresponding sizes of the multiplier. Again the impulse responses and the size of the multiplier differ markedly between expansionary and contractionary shocks. The multiplier following a positive news shock is lower than 1 with $m^+ = 0.47$, but the multiplier following a negative shock is above one with $m^- = 1.56$. These results confirm our previous findings based on a recursive identification scheme. To assess the statistical significance of our result, the bottom-panel of Figure 5 plots the posterior distribution of the multiplier following expansionary spending shocks (x-axis) and following contractionary shocks (y-axis). Again, the evidence in favor of asymmetry is strong, as the posterior probability that $m^- > m^+$ is above 0.99.

3.4 Robustness: Using Local Projections over 1890-2014

In this section, we examine the robustness of our findings of an asymmetric spending multiplier. We consider the robustness of our results to two important aspects of our approach: (i) the sample period, and (ii) our FAIR methodology.

1. Using historical data over 1890-2014:

Our baseline results rely on two different sample sizes. In the case of the recursive identification scheme, our 1966-2014 sample is dictated by the availability of professional forecasts to soak up anticipated spending changes. In the case of the narrative identification scheme, our baseline choice of Ramey's (2011) original 1939-2014 sample instead of Ramey and Zubairy (2016)'s extended 1890-2014 sample was motivated by a number of reasons: (i) to avoid measurement issues inherent to historical macroeconomic data (e.g., Romer, 1989), (ii) to use a time period comparable with that of Auerbach-Gorodnichenko and relatedly (iii) to use a time period without large secular changes in the size and composition of government spending.[23]

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[22]The loose priors for the FAIR parameters are detailed in the Appendix. The number of Gaussian basis functions was determined by model comparison using Bayes factors.

[23]As mentioned by Gorodnichenko (2014) in his NBER Summer Institute discussion of Ramey and Zubairy (2016), the post-1939 sample has the advantage of avoiding periods with important regime changes, variation in data quality (Romer, 1986) and secular structural changes (notably a trend in the size and composition of government spending).

In this robustness section however, we make full use of the historical data available, and we use the full 1890-2014 sample with both the narrative and the recursive identification schemes (as in Ramey and Zubairy, 2016). Since data on professional forecasts are not available prior to 1966, we will resort to the original Blanchard-Perotti identification scheme (keeping in mind the caveat associated with this identification assumption (Ramey, 2011)).

2. Using Local Projections instead of FAIR:

Since our approach relies on the parametrization of the impulse response functions with Gaussian basis functions, we examine the robustness of our results to this parametrization. Specifically, we will not rely on functional approximation of the impulse responses but instead use a fully non-parametric method –Jorda's (2005) Local Projections (LP)–, which imposes no structure on the impulse response functions and is thus more robust to mis-specification (although at the expense of efficiency). There is an important drawback to such an LP-based approach however: in the case of an asymmetric data-generating process, LP requires a series of previously identified structural shocks to estimate structural impulse responses (in contrast to multivariate FAIR models).[24] While this is not a problem when using (independently identified) narrative Ramey news shocks, we cannot use a recursive identifying restriction in a LP to assess the asymmetric size of the multiplier. However, since our aim is only to assess the robustness of our findings, we will take a short-cut and use as spending shocks the structural spending shocks from a (linear) VAR a la Blanchard-Perotti (2004). While such a hybrid VAR-LP procedure is flawed (in fact, not internally consistent since the VAR shocks are identified under the assumption that the DGP is linear), we see it as a useful robustness check of our results based on FAIR.

In the interest of space, this section will present results using both historical data and Local Projections. We also estimated FAIR models over 1890-2014 or ran LP over shorter post-1939 samples and reach identical conclusions.

3.4.1 Results from a narrative identification scheme

We start by using the Ramey news shocks in Local Projections. We conduct two exercises; (i) we estimate the impulse responses of government spending and GDP to positive vs. negative news shock, and (ii) we compute the multipliers associated with these shocks.

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[24]It is possible to directly impose recursive identifying assumptions (i.e., identify structural shocks) in Local Projections by including the right set of contemporaneous and lagged observables as controls and thus to directly obtain *structural* impulse responses from the LP estimates (e.g., Barnichon and Brownlees, 2017). However, this approach does not work if the data generating process is asymmetric. In that case, the system does not have an autoregressive representation (as we discussed in Section 2), and a structural shock cannot be written as a *linear* combination of contemporaneous and lagged observables.

To have a benchmark, we first consider the linear case without asymmetry. To estimate the impulse responses, we run linear Local Projections with Instrument Variables (IV), i.e. we estimate $K + 1$ equations

$$y_{t+k} = \alpha_k + \beta_k g_t + \gamma_k' x_t + u_{t+k}, \qquad k = 0, 1, ..., K \qquad (12)$$

where y_{t+k} is the variable of interest (government spending or GDP), x_t contains lags of y_t, and g_t is government spending at time t that we instrument with the series of news shocks denoted $\left\{ \varepsilon_t^G \right\}$. As discussed in Mertens and Ravn (2013) and Ramey and Zubairy (2016), an IV approach has the advantage of allowing for measurement error in both the shock and the instrumented variable (as long as their errors are uncorrelated), which is especially important when using historical data. The impulse response of y_t is then given by $\beta_0, \beta_1, ..., \beta_K$. We use a horizon of $K = 20$ quarters, and we report Newey-West (1987) standard errors to allow for autocorrelation in the error terms.

To obtain the multiplier associated with a news shock, we follow Ramey and Zubairy (2016) and directly estimate the "sum" multiplier using again an IV approach by estimating the following $K + 1$ linear Local Projections

$$\sum_{j=0}^{k} y_{t+j} = \alpha_k + m_k \sum_{j=0}^{k} g_{t+j} + \gamma_k' x_t + u_{t+k}, \qquad k = 0, 1, ..., K \qquad (13)$$

using ε_t^G as an instrument for $\sum_{j=0}^{k} g_{t+j}$. Here, $\sum_{j=0}^{k} y_{t+j}$ is the sum of the GDP variable from t to $t + k$ and $\sum_{j=0}^{k} g_{t+j}$ is the sum of the government spending variable from t to $t + k$, and x_t includes the same set of controls as in (12). In (13), the coefficient m_k is directly the "sum" multiplier over the first k periods, and the associated standard errors can be readily estimated using Newey-West.

To allow for asymmetric effects of government spending shocks, we proceed as in the linear case except that we use as instrument either $\left\{ \varepsilon_t^G 1_{\varepsilon_t^G > 0} \right\}$ to get the effect of expansionary shocks or $\left\{ \varepsilon_t^G 1_{\varepsilon_t^G < 0} \right\}$ to get the effect of contractionary shocks.

Figure 6 shows the impulse response functions of government spending and output to expansionary and contractionary news shocks, and Table 2 reports the associated multipliers. Overall, the results are very similar to the results obtained with FAIR models over the more recent period: the multiplier is above 1 for a contractionary shock but is below 1 for an expansionary shock.[25]

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[25] Note the strong similarity between the impulse responses to a positive shock and the average impulse response estimated (Figure 6). This result owes to the fact that a few large positive shocks dominate the sample of Ramey news shocks and thus have a strong influence on the linear impulse response estimate. We come back

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3.4.2 Results from a recursive identification scheme

To use LP to estimate the asymmetric effects of spending shocks identified with a recursive identification scheme, we proceed in two steps. First, we estimate a standard structural VAR with $(g_t, \tau_t, y_t)'$ to identify structural shocks to government spending, denoted $\{\widetilde{\varepsilon}_t^G\}$. Second, we take $\{\widetilde{\varepsilon}_t^G\}$ as our measure of structural shocks, and we estimate the Local Projections (12) and (13). Again, recall that such a hybrid VAR-LP procedure is flawed but nonetheless a useful robustness check of our FAIR results.

Figure 7 shows the impulse response functions of government spending and output to expansionary and contractionary news shocks, and Table 2 reports the associated multipliers. Overall, the results are very similar to the results obtained with FAIR models: the multiplier is above 1 for a contractionary shock but is substantially below 1 for an expansionary shock.

3.5 Digging deeper: the behavior of investment and consumption

Finally, to dig deeper into the asymmetric response of the economy to government spending shocks, we study the impulse responses of investment and consumption. The response of consumption is of particular interest, since the size of the multiplier is directly related to sign of the response of consumption (e.g., Ramey, 2011).

As always, we present results for the two main identification schemes, and Figures 8 and 9 plot the impulse responses of investment (I) and consumption (C).[26] To obtain these impulse responses, we run Local Projections with IV where the instrument is the shock series $\{\varepsilon_t^G\}$ given either by Ramey news shocks series or by the series of shocks identified from a timing restriction.

When comparing impulse responses to positive and negative shocks, keep in mind that the impulse responses to negative shocks were multiplied by -1 in order to ease comparison across impulse responses. With this convention, when there is no asymmetry, the impulse responses are identical in the left-hand panels (responses to a positive shock) and in the right-hand panels (responses to a negative shock).

Regardless of the identification scheme, the response of consumption is markedly different between positive and negative shocks: the response of consumption to a contractionary shock is significantly negative (recall that the impulse responses to contractionary shocks are multiplied by -1), consistent with a contractionary multiplier above 1. In contrast, the response of consumption to an expansionary shock is weak and not significantly different from zero. The strongly asymmetric response of consumption is important since a key channel through

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to this point in Section 6.

[26]Since data on C and I are only available after 1947, all results were obtained using the 1947-2014 sample period.

which fiscal policy can have asymmetric effects is through asymmetry in the MPC to transitory shocks. We come back to this point in our theoretical discussion in Section 6.

Turning to the responses of investment, we can see that an expansionary shock leads to a decline (crowding-out) in investment, whereas a contractionary shock does not lead to any increase (crowding-in) in investment. These results are again consistent with a contractionary spending multiplier above one and an expansionary multiplier below one.

4 Additional evidence

In this section, we provide additional support for our findings of an asymmetric multiplier by considering two additional approaches to analyze the size of the multiplier: (a) Jorda and Taylor (JT, 2016) and (b) Blanchard and Leigh (BL, 2013). This additional evidence is interesting because it relies on entirely different identification schemes than the ones used by Ramey (2011) and Auerbach and Gorodnichenko (2012), and also because JT and BL's findings exploit cross-country variations in contrast to our study, which relies on time series variation in US data.

4.1 Jorda and Taylor (2016)

An influential literature studies the economic effects of fiscal consolidations by studying the effects of shocks to the cyclically adjusted primary budget balance.[27] Such shocks include (but not exclusively) shocks to government spending and thus contain information on the size of the government spending multiplier.

The latest word in that literature is Jorda and Taylor (2016), who consider two different identification schemes –an instrumental variable approach and inverse-propensity score weighting– and consistently find a multiplier that is approximately 2.

This is a larger multiplier than reported in the government spending literature, and yet we think our results are fully consistent with their finding. Indeed, the thing to notice is that Jorda and Taylor (2016) only study the effects of fiscal consolidations. As a result, their estimated multiplier is that of a contractionary multiplier, which we also find to be large. In light of our findings, it is also natural that Jorda and Taylor find larger spending multiplier than in the rest of the literature whose evidence is based on a mix of positive and negative shocks.

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[27] Alesina and Ardagna (2010), Guajardo, Leigh, and Pescatori (2011) and Jorda and Taylor (2016).

4.2 Blanchard and Leigh (2013)

Blanchard and Leigh (BL, 2013) analyze the size of the multiplier from a very different angle. Using a panel of EU countries over 2009-2012, BL regress the forecast error of real GDP growth on forecasts of fiscal adjustments. Under rational expectations, and assuming that forecasters use the correct model of forecasting, the coefficient on the fiscal adjustment forecast should be zero. However, BL find that there is a significant relation between fiscal adjustment forecasts and subsequent growth forecast errors, which indicates that the size of the multiplier was under-estimated during the last recession. Moreover, the magnitude of the under-estimation is large: if forecasters had in mind a multiplier of about 0.5, BL's estimates imply that the multiplier was 1.6 during 2009-2012.

BL's approach is an interesting testing ground for our findings. Since we find that only the contractionary multiplier is above one, BL's results should be driven by fiscal consolidations alone, and not by fiscal expansions. As we show below, this is exactly what we find.

Specifically, BL run the regression

$$\text{Forecast error of } \Delta Y_{i,t+1|t} = \alpha + \beta \left(\text{Forecast of } \Delta F_{i,t+1|t} \right) + \varepsilon_{i,t+1} \tag{14}$$

on a cross-section of European countries where $\Delta Y_{i,t+1|t}$ denotes cumulative (year-over-year) growth of real GDP in economy i and the associated forecast error is $\Delta Y_{i,t+1|t} - \widehat{\Delta Y}_{i,t+1|t}$ with $\widehat{\Delta Y}_{i,t+1|t}$ the forecast made with information available at date t, and where $\Delta F_{i,t+1|t}$ denotes the change in the general government structural fiscal balance in percent of potential GDP.

Under the null hypothesis that fiscal multipliers used for forecasting were accurate, the coefficient β should be zero.[28] In contrast, a finding that β is negative indicates that forecasters tended to be optimistic regarding the level of growth associated with a fiscal consolidation, i.e., that they under-estimated the size of the multiplier. Using World Economic Outlook (WEO) forecast data, BL find that $\beta \approx -1.1$ for forecasts over 2009-2012, indicating that the multiplier was substantially under-estimated by forecasters, and implying that the multiplier was 1.6 (0.5+1.1) during the recession.[29]

To test our prediction that BL's findings is driven by fiscal consolidations, we re-estimate BL's baseline specification but allowing for different β coefficients depending on the sign of the fiscal adjustment –expansionary or contractionary–. Figure 14 shows the corresponding fitted

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[28] In other words, information known when the forecasts were made should be uncorrelated with subsequent forecast errors.

[29] BL conduct a number of robustness checks to argue that their non-zero β is symptomatic of an under-estimated multiplier and is not due to other confounding factors. In particular, they verify that their results hold after controlling for other factors that could trigger both planned fiscal adjustments and lower than expected growth, or that the forecast error in fiscal adjustment was not correlated with the initial fiscal adjustment forecast (which would bias β).

lines. To avoid confusion, note that a fiscal consolidation in BL's framework shows up as in *increase* in the fiscal balance and thus shows up as positive entries in Figure 14. We can see that BL's results are indeed driven by fiscal consolidations. Table 3 presents the regression results. The β associated with fiscal expansions is not different from zero –suggesting an expansionary multiplier of about 0.5 during the recession–, but the β associated with fiscal consolidations is $\beta_G^- \approx -1.2$ –suggesting a contractionary multiplier of about 1.7–. These results are close to our estimates on the size of the multiplier during recessions.[30]

A caveat in our analysis so far is that fiscal adjustments include not only changes in government purchases but also changes in revenues. To better map BL's results with ours, we follow BL and treat separately changes in spending and changes in revenues by running the regression

$$\text{Forecast error of } \Delta Y_{i,t+1|t} = \alpha + \beta_G \left(\text{Forecast of } \Delta G_{i,t+1|t} \right) + \beta_T \left(\text{Forecast of } \Delta T_{i,t+1|t} \right) + \varepsilon_{i,t+1}$$
(15)

where $\Delta G_{i,t+1|t}$ denotes the WEO forecast of the change in structural spending in 2010-11 and $\Delta T_{i,t+1|t}$ denotes the WEO forecast of the change in structural revenue in 2010-11, both in percent of potential GDP.

Column (3) of Table 3 presents the results of regression (15) where we treat separately fiscal consolidations and fiscal expansions. In line with our findings, the only significant coefficient is $\beta_G^- \approx 1.6$,[31] corresponding to a contractionary spending multiplier of about 2 in recessions (again in line with our findings), whereas β_G^+ is not significantly different from zero, consistent with an expansionary spending multiplier of about 0.5 in recessions.

5 The asymmetric and state-dependent government spending multiplier

We now explore whether the size of the multiplier depends on the state of the business cycle, and whether the magnitude of such state dependence depends on the sign of the government intervention. We first describe how we introduce state dependence into a FAIR model and then present the estimation results using both identification schemes: (i) the recursive identification scheme à la Auerbach and Gorodnichenko (2012), and (ii) the narrative identification scheme à la Ramey (2011). We leave a description of the estimation method for the appendix, and

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[30] Throughout this exercise, we follow BL in keeping the assumption that forecasters had in mind a multiplier of 0.5.

[31] The coefficient β_G^- is positive because a fiscal consolidation corresponds to a *decrease* in government spending. In contrast, in columns (1) and (2), β^- is negative because a fiscal consolidation corresponds to an *increase* in the fiscal balance.

in the online appendix we show that the structural shocks can be identified in (i) or (ii) even when the shocks have asymmetric and state dependent effects.

5.1   Introducing asymmetry and state-dependence

With asymmetry and state dependence in response to government spending shocks, the matrix $\mathbf{\Psi}_k^+$ becomes $\mathbf{\Psi}_k^+(z_{t-k})$, i.e., the impulse response to a positive shock depends on some indicator variable z_t (and similarly for $\mathbf{\Psi}_k^-$).

To construct a model that allows for both asymmetry and state dependence, we build on the asymmetric FAIR model (11) and approximate ψ_i^{G+}, the impulse response function of variable i to a positive innovation to government spending, as

$$\psi_i^{G+}(k) = (1 + \gamma_i^+ z_{t-k}) \sum_{n=1}^{N} a_{i,n}^+ e^{-\left(\frac{k-b_{i,n}^+}{c_{i,n}^+}\right)^2}, \quad \forall k > 0 \tag{16}$$

with γ_i^+, $a_{i,n}^+$, $b_{i,n}^+$ and $c_{i,n}^+$ parameters to be estimated. An identical functional form holds for ψ_i^{G-}.

In this model, the amplitude of the impulse response depends on the state of the business cycle (captured by the cyclical indicator z_t) at the time of the shock. In (16), the amplitude of the impulse response is a linear function of the indicator variable z_t. Such a specification will allow us to test whether a positive fiscal shock has a stronger effect on output in a recession than in an expansion.

Note that in specification (16), the state of the cycle is allowed to stretch/contract the impulse response, but the shape of the impulse response is fixed (because a, b and c are all independent of z_t). While one could allow for a more general model in which all variables a, b and c depend on the indicator variable, with limited sample size, it will typically be necessary to impose some structure on the data, and imposing a constant shape for the impulse response is a natural starting point.

5.2   Results from a recursive identification scheme

We estimate model (16), where we use as cyclical indicator (z_t) the unemployment rate detrended by CBO's estimate of the natural rate (available from 1949 on).[32]

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[32] We detrend the unemployment rate to make sure that our results are not driven by slow moving trends (e.g., due to demographics) in the unemployment rate, which could make the unemployment rate a poor indicator of the amount of economic slack (see e.g. Barnichon and Mesters, 2016). Using the actual unemployment rate gives similar qualitative results for state dependence, but a posterior odds ratio calculation favors a model with detrended unemployment.

As a preliminary step, and to put our results into perspective, Figure 10 plots our cyclical indicator along with the identified government spending shocks implied by the posterior mode estimates. While the cyclical indicator has zero mean, it is right-skewed, a well-known property of the unemployment rate (e.g., Neftci, 1984). As a result, fiscal shocks are observed over values of (detrended) unemployment ranging mostly from -1 to 2.

The first row of Figure 11 shows how the "sum" multiplier depends on the state of the business cycle at the time of the shock. The left column reports the multiplier following positive (expansionary) shocks, and the right column reports the multiplier following negative (contractionary) shocks. The bottom panels of Figure 11 plot the histograms of the distributions of respectively contractionary shocks and expansionary shocks over the business cycle. This information is meant to get a sense of the range of (detrended) unemployment over which we identify the coefficients capturing state dependence.

To report the effect of slack under a slightly different angle, Table 4 reports the size of the multiplier in a high unemployment state (detrended unemployment of 2) and in a low unemployment state (detrended unemployment of -1).

We can once more see a stark asymmetry between positive and negative shocks. The multiplier associated with contractionary fiscal shocks depends strongly on the state of the cycle and reaches its highest value in times on high unemployment. Specifically, the contractionary multiplier is about 0.9 around business cycle peaks but gets above to 2 around business cycle troughs, and the posterior probability that $m^{-,\text{U high}} > m^{-,\text{U low}}$ is high at 0.95 (Table 4).[33] In contrast, the multiplier associated with expansionary fiscal shocks does not depend significantly or economically on the state of the cycle ($P(m^{+,\text{U high}} > m^{+,\text{U low}}) = 0.52$). Overall, m^+ is small and not significantly different from zero regardless of the level of unemployment.

5.3   Results from a narrative identification scheme

We now perform the same exercise but using the Ramey news shocks in the SUR-type FAIR model with asymmetry and state dependence to allow the impulse responses to depend on both the sign of the news shock as well as the state of the cycle (captured by detrended unemployment) at the time of the shock. Since CBO's natural rate estimate is not available over 1939-2014 (the estimate only starts in 1949), we use as cyclical indicator the unemployment

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[33]Interestingly, these results are consistent with the recent work of Caggiano et al. (2015), who find that state dependence in the size of the multiplier comes from extreme "events", and in particular deep recessions versus strong expansionary periods. This is what one would expect if the effect of the state of the cycle on the (contractionary) multiplier was close to linear: For small business cycle fluctuations, the size of the multiplier will not vary very much (Figure 11) and its variations will be hard to detect. It is only when the unemployment reaches high levels that the contractionary multiplier can reach values close to 2 and start differing markedly from its level during expansionary times.

rate detrended with an HP-filter ($\lambda = 10^5$).

Figure 12 presents the results using the same formatting as Figure 11. The state dependence results are similar to the ones obtained with the recursive identification: positive shocks have no state dependent effects on output, –the multiplier remaining around 0.5 regardless of the state of the cycle–, while negative shocks have stronger effects on output during times of slack. And just like with the recursively identified shocks, the contractionary multiplier is about 1 around business cycle peaks but rises to about 2 around business cycle troughs with a posterior probability that $m^{-,\text{U high}} > m^{-,\text{U low}}$ of 0.98 (Table 4).

5.4 An explanation for the range of estimates in the literature

The literature has reached seemingly conflicting conclusions on the effect of slack on the size of the multiplier: while studies based on narratively-identified shocks find little evidence for state dependence, VAR-based studies finding strong evidence for state dependence.

Interestingly, the asymmetric nature of the spending multiplier offers a simple possible explanation for this set of results. To see that, note that the estimated *average* multiplier (and its state dependence) depends on the relative frequency of expansionary and contractionary shocks used for estimation. The key is then to note that this relative frequency varies markedly across the two main identification schemes. Figure 13 plots the distribution of Ramey (2011) news shocks along with the distribution of recursively-identified shocks (as in Auerbach and Gorodnichenko, 2012). Unlike with recursively-identified shocks whose distribution is (by construction) evenly distributed between positive and negative shocks, a few very large positive shocks dominate the sample of Ramey news shocks.

Our "composition effect" hypothesis is thus the following. Studies based on Ramey News shocks (Owyang, Ramey and Zubairy 2013, Ramey and Zubairy 2016) find little evidence for state dependence using news shocks to defense spending, because their results are driven predominantly by positive shocks, which (according to our results) display no detectable state dependence. In contrast, studies such as Auerbach and Gorodnichenko (2012) based on a VAR find some evidence for state dependence, because (roughly) half of the identified shocks are negative shocks, which display strong state dependence. Interestingly, and consistent with this "composition effect" hypothesis, Ramey and Zubairy (2016) do find significant evidence of state dependence when they use a recursive identification scheme (in which the distribution between positive and negative shocks is symmetric).[34]

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[34]One may wonder whether such a "composition effect" between m^+ and m^- should also imply that the *average* multiplier for government spending shocks –call it m^G (Auerbach and Gorodnichenko, 2012)– is larger than the *average* multiplier for news shocks to defense spending –call it m^{News} (Ramey and Zubairy, 2016). However, m^G and m^{News} refer to different types of government spending (defense purchases only versus all federal, state and local purchases), and news shocks to defense spending have larger multipliers (for both m^+

6 Theoretical discussion

In this final section, we discuss some theoretical channels through which a temporary, deficit-financed, increase in government spending can affect output, and we discuss two different channels that could rationalize our empirical findings of $m^- > 1$ and $m^+ < 1$: (i) the existence of financial frictions, and/or (ii) downward nominal rigidities.

To frame the discussion, we start by discussing the main (linear for now) theoretical effects of an increase in government spending financed by future higher lump-sum taxes.

In the textbook Keynesian IS/LM model, the marginal propensity to consume (MPC) out of transitory income is large, so that an increase in government spending that raises household income also raises household consumption. If the central bank does not increase the interest rate too much (so that investment does not decline too much), private spending goes up and the multiplier is above 1.

In the standard neo-classical (RBC) model however, a deficit-financed increase in government spending generates a negative wealth effect for households, who respond by saving more in anticipation of future higher tax hikes (Baxter and King, 1993). With a decline in consumption, the multiplier in RBC models is then typically less than 1.

In the New-Keynesian model, which builds a sticky-price edifice on neoclassical foundations, the multiplier can be above one provided that three assumptions are satisfied (Gali et al., 2007):[35] (i) prices are sticky, (ii) the central bank response to changes in government spending is not too strong, and (iii) household's MPC out of transitory income is large enough. With (iii), the increase in disposable income brought about by higher government spending can compensate the negative wealth effect and thus lead to an increase in household consumption and to a multiplier above one. While Gali et al. (2007) allow for a large MPC out of transitory income by assuming the existence of rule-of-thumb consumers (who behave "hand-to-mouth" by consuming all of their disposable income), their assumption is motivated by the existence of financial frictions and by the fact that a sizable fraction of households have close to zero liquid wealth and face high borrowing costs (Kaplan et al., 2014).[36]

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and m^-) than shocks to overall government spending (see Table 1 or Table 2). Thus, while the distribution of news shocks does make the average Ramey multiplier m^{News} smaller (closer to its m^+ value), this effect is not enough to compensate the fact that $m^{+,News} > m^{+,G}$ and $m^{-,News} > m^{-,G}$. As a result, we find $m^{News} > m^G$ in Table 1 and 2 (as in Ramey and Zubairy, 2016).

[35] The discussion of New-Keynesian models with government spending is not meant to be exhaustive, and other mechanisms can generate a multiplier above 1 (see e.g., Kormilitsina and Zubairy, 2016). The goal of the discussion is to highlight simple channels that can lead to an asymmetric multiplier.

[36] Wealthier individuals can also behave in an hand-to-mouth fashion –holding low liquid assets despite sizeable illiquid assets– (Kaplan, et al., 2014).

6.1 Financial frictions

The existence of financial frictions can lead to an asymmetric multiplier, because borrowing constraints can make the MPC asymmetric. Specifically, with financial constraints the MPC out of temporary income changes can be higher for negative income shocks than for positive income shocks, because borrowing constraints impede households' ability to bring future consumption forward, but do not prevent households from postponing consumption.[37] Going back to Gali et al. (2007)'s New-Keynesian framework, this would imply that the multiplier is larger following declines in government spending than following increases in spending, consistent with our findings.

Interestingly, Bunn et al. (2017) and Christelis et al. (2017) recently found such evidence of asymmetry in the MPC with households reporting changing their consumption by significantly more following temporary unanticipated falls in income than following rises of the same size. For instance, Bunn et al. (2017) estimate an average MPC of only 0.1 following a positive income shock but of about 0.6 following a negative income shock.[38] Such an asymmetric MPC is promising to explain our findings that $m^- > 1$ and $m^+ < 1$ as well as the asymmetric responses of consumption to government spending shocks: With a substantial MPC out of negative transitory income shocks but a close to zero MPC out of positive income shocks, consumption could behave asymmetrically; (i) decreasing following a contractionary government spending shock (the economy behaving in a Keynesian fashion where the fall in disposable income dominates the positive wealth effect) but (ii) not increasing (possibly even declining) following an expansionary government spending shock (the economy behaving in a neo-classical fashion where the negative wealth effect dominates). This is exactly what we found in Section 3.5.

Another channel through which financial frictions and government spending shocks can interact is the extensive margin of borrowing constraints. By affecting disposable income, a government spending shock can affect the *share* of households that are financially constrained and thus affect the *average* MPC. Interestingly, this extensive margin channel has the potential of not only generating some sign-dependence in the spending multiplier, [39] but also of explaining our finding that the contractionary multiplier is largest in recessions: In recessions, the share of households facing borrowing constraints is larger, which implies an even larger

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[37]See Christelis et al. (2017) for a quantitative discussion of how liquidity constraints (as well as prudence and a precautionary saving motive) can generate a higher MPC out of negative shocks than out of positive shocks. See also Bunn et al. (2017).

[38]Moreover, households more likely to face liquidity shortages or liquidity constraints report even higher MPC to adverse income shocks.

[39]A positive government spending shock could bring some households off the constraint, leading them to behave in a Ricardian fashion and thereby making the multiplier smaller. In contrast, a negative government spending shock could push more households into the constraint and force them to reduce consumption, implying a larger multiplier.

average MPC out of negative income shocks and thus an even larger contractionary multiplier.

Taking stock, this discussion highlights how a better understanding of the effects of financial frictions on the MPC (and its asymmetry) is likely to be an important part of future research on the size of the multiplier and on the effects of fiscal policy.

6.2 Downward nominal rigidities

Another possibility for $m^- > m^+$ is the existence of downward nominal rigidities, or more generally asymmetric nominal rigidities (e.g., Kahn, 1997), where prices adjust more easily upwards than downwards. With asymmetric nominal rigidities, the economy can behave in a more "Keynesian" fashion following a decrease in government spending (implying a larger multiplier, possibly above 1) but behave in a more "classical" fashion following an increase in government spending (implying a smaller multiplier).[40]

7 Conclusion

This paper estimates the asymmetric effects of shocks to government spending by using Gaussian basis functions to approximate impulse response functions. Using either of the two main identification schemes in the literature –recursive or narrative–, we find that the multiplier is above 1 for contractionary shocks to government spending, but below 1 for expansionary shocks. The multiplier for contractionary shocks is largest in recessions, but the multiplier for expansionary shocks is always below 1 and not larger in recessions.

Our results have two interesting policy implications. First, they strongly weaken the case for fiscal packages to stimulate the economy. Second, they caution that austerity measures may have a much higher output cost than suggested by linear estimates.

One promising avenue to explain the asymmetric size of the multiplier lies in the asymmetric size of the MPC out of transitory income changes (Bunn et al., 2017, Christelis et al., 2017), and we conclude that a better understanding of the effects of financial frictions on the (asymmetric) size of the MPC should be an important part of the research agenda on the size of the multiplier and the effects of fiscal policy. While there is, as far as we know, little work on the non-linear implications of financial frictions on the size of the spending multiplier,[41] Brunnermeier and Sannikov (2014) recently showed that financial constraints can lead

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[40]An alternative mechanism involving wage rigidity (and possibly asymmetric multipliers) can be found in Alesina, Ardagna, Perotti and Schiantarelli (2002). In their model, an increase in government spending can lead to wage pressures which hurt investment by the private sector. With downward wage rigidity, a decrease in government spending would not lower wages and thus would not stimulate investment, which could lead to a larger contractionary multiplier.

[41]See Fernandez-Villaverde (2010) for an exploration of the *linear* effects of financial frictions on the multiplier.

to highly nonlinear dynamics in the economy's response to shocks, notably asymmetric and state dependent impulse-responses. The model of Brunnermeier-Sannikov does not have nominal rigidities and only explore the effects of real shocks, but such non-linear features are also likely to be present with government spending shocks. Relatedly, the recent Heterogeneous Agents New-Keynesian (HANK) models with a non-trivial share of hand-to-mouths agents (Kaplan, Moll and Violante, 2016) may offer a promising framework to explore asymmetric MPCs and asymmetric effects of fiscal shocks.

Appendix A1: Bayesian estimation of multivariate FAIR models

In this section, we describe the implementation and estimation of multivariate FAIR models, where government spending shocks are identified from a recursive ordering as in Section 2.1.1. We first describe how we construct the likelihood function by exploiting the prediction-error decomposition, discuss the estimation routine based on a multiple-block Metropolis-Hasting algorithm, prior elicitation, and finally the determination of the number of basis functions used in the basis function expansion.

Constructing the likelihood function

We now describe how to construct the likelihood function $p(\boldsymbol{y}^T|\boldsymbol{\theta}, z^T)$ of a sample of size T for the moving-average model (1) with parameter vector $\boldsymbol{\theta}$ and where a variable with a superscript denotes the sample of that variable up to the date in the superscript.

To start, we use the prediction error decomposition to break up the density $p(\boldsymbol{y}^T|\boldsymbol{\theta})$ as follows:

$$p(\boldsymbol{y}^T|\boldsymbol{\theta}) = \prod_{t=1}^{T} p(\boldsymbol{y}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1}). \tag{17}$$

To calculate the one-step-ahead conditional likelihood function needed for the prediction error decomposition, we assume that all innovations $\{\boldsymbol{\varepsilon}_t\}$ are Gaussian with mean zero and variance one,[42] and we note that the density $p(\boldsymbol{y}_t|\boldsymbol{y}^{t-1}, \boldsymbol{\theta})$ can be re-written as $p(\boldsymbol{y}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1}) = p(\boldsymbol{\Psi}_0\boldsymbol{\varepsilon}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1})$ since

$$\boldsymbol{y}_t = \boldsymbol{\Psi}_0\boldsymbol{\varepsilon}_t + \sum_{k=1}^{K} \boldsymbol{\Psi}_k\boldsymbol{\varepsilon}_{t-k}. \tag{18}$$

Since the contemporaneous impact matrix $\boldsymbol{\Psi}_0$ is a constant, $p(\boldsymbol{\Psi}_0\boldsymbol{\varepsilon}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1})$ is a straightforward function of the density of $\boldsymbol{\varepsilon}_t$.

To recursively construct $\boldsymbol{\varepsilon}_t$ as a function of $\boldsymbol{\theta}$ and $\boldsymbol{y}^t$, we need to uniquely pin down the value of the components of $\boldsymbol{\varepsilon}_t$ from (18), that is we need that $\boldsymbol{\Psi}_0$ is invertible. We impose this restriction by only keeping parameter draws for which $\boldsymbol{\Psi}_0$ is invertible.[43] It is also at this stage that we impose the identifying restriction that $\boldsymbol{\Psi}_0$ has its first two rows filled with 0 except for the diagonal coefficients. Finally, to initialize the recursion, we set the first K innovations $\{\boldsymbol{\varepsilon}_j\}_{j=-K}^{0}$ to zero.

In the non-linear case where we have $\boldsymbol{\Psi}_k = \boldsymbol{\Psi}_k(\boldsymbol{\varepsilon}_{t-k}, z_{t-k})$, we proceed similarly. However, a complication arises when one allows $\boldsymbol{\Psi}_0$ to depend on the sign of the shock *while also* imposing

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[42]The estimation could easily be generalized to allow for non-normal innovations such as t-distributed errors.
[43]Parameter restrictions (such as invertibility) are implemented by assigning a minus infinity value to the likelihood whenever the restrictions are not met.

identifying restrictions on $\boldsymbol{\Psi}_0$. The complication arises, because with asymmetry the system of equations implied by (18):

$$\boldsymbol{\Psi}_0(\boldsymbol{\varepsilon}_{t-k}, z_{t-k})\boldsymbol{\varepsilon}_t = \mathbf{u}_t \tag{19}$$

where $\mathbf{u}_t = \boldsymbol{y}_t - \sum_{k=1}^{K} \boldsymbol{\Psi}_k \boldsymbol{\varepsilon}_{t-k}$ need not have a unique solution vector $\boldsymbol{\varepsilon}_t$, because $\boldsymbol{\Psi}_0(\boldsymbol{\varepsilon}_t)$, the impact matrix, depends on the sign of the shocks, i.e., on the vector $\boldsymbol{\varepsilon}_t$. However, in the online appendix we show that this is not a problem (so that (19) has a unique solution vector $\boldsymbol{\varepsilon}_t$) in a recursive identification scheme like the one considered in this paper.

Finally, when constructing the likelihood, to write down the one-step ahead forecast density $p(\boldsymbol{y}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1})$ as a function of past observations and model parameters, we use the standard result (see e.g., Casella-Berger, 2002) that for $\boldsymbol{\Psi}_0$ a function of $\boldsymbol{\varepsilon}_t$ and z_t, we have

$$p(\boldsymbol{\Psi}_0(\boldsymbol{\varepsilon}_t, z_t)\boldsymbol{\varepsilon}_t|\boldsymbol{\theta}, \boldsymbol{y}^{t-1}) = J_t p(\boldsymbol{\varepsilon}_t)$$

where J_t is the Jacobian of the (one-to-one) mapping from $\boldsymbol{\varepsilon}_t$ to $\boldsymbol{\Psi}_0(\boldsymbol{\varepsilon}_t, z_t)\boldsymbol{\varepsilon}_t$ and where $p(\boldsymbol{\varepsilon}_t)$ is the density of $\boldsymbol{\varepsilon}_t$.[44,45]

Estimation routine and initial guess

To estimate our model, we use a Metropolis-within-Gibbs algorithm (Robert & Casella 2004) with the blocks given by the different groups of parameters in our model (there is respectively one block for the a parameters, one block for the b parameters, one block for the c parameters and one block for the constant and contemporaneous impact matrix $\boldsymbol{\Psi}_0$).

To initialize the Metropolis-Hastings algorithm in an area of the parameter space that has substantial posterior probability, we follow a two-step procedure: first, we estimate a standard VAR using OLS on our data set, calculate the moving-average representation, and we use the impulse response functions implied by the VAR as our starting point. More specifically, we calculate the parameters of our FAIR model to best fit the VAR-based impulse response functions.[46] Second, we use these parameters as a starting point for a simplex maximization routine that then gives us a starting value for the Metropolis-Hastings algorithm.

In the non-linear models, we initialize the parameters capturing asymmetry and state dependence at zero (i.e., no non-linearity). This approach is consistent with the starting point

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[44]Recall that we assume that the indicator variable z_t is a function of lagged values of $\boldsymbol{y}_t$ (so that z_t is known conditional on $\boldsymbol{y}^{t-1}$) or that z_t is a function of variables exogenous to $\boldsymbol{y}_t$ (and thus taken as given and known).

[45]In our case with asymmetry, this Jacobian is simple to calculate, but the mapping is not differentiable at $\varepsilon = 0$. Since we will never exactly observe $\varepsilon = 0$ in a finite sample, we can implicitly assume that in a small neighborhood around 0, we replace the original mapping with a smooth function.

[46]Specifically, we set the parameters of our model (the a, b and c coefficients) to minimize the discrepancy (sum of squared residuals) between the two sets of impulse responses.

(the null) of this paper: structural shocks have linear effects on the economy, and we are testing this null against the alternative that shocks have some non-linear effects. We then center the priors for these parameters at zero with loose priors, as described next.

Prior elicitation

We use (loose) Normal priors centered around the impulse response functions obtained from the benchmark (linear) VAR. Specifically, we put priors on the a, b and c coefficients that are centered on the values for a, b and c obtained by matching the impulse responses obtained from the VAR, as described in the previous paragraph. Specifically, denote $a_{ij,n}^0$, $b_{ij,n}^0$ and $c_{ij,n}^0$, $n \in \{1, N\}$ the values implied by fitting a FAIR model to the VAR-based impulse response of variable i to shock j. The priors for $a_{ij,n}$, $b_{ij,n}$ and $c_{ij,n}$ are centered on $a_{ij,n}^0$, $b_{ij,n}^0$ and $c_{ij,n}^0$, and the standard-deviations are set as follows $\sigma_{ij,a} = 10$, $\sigma_{ij,b} = K$ and $\sigma_{ij,c} = K$ (K is the maximum horizon of the impulse response function).[47] While there is clearly some arbitrariness in choosing the tightness of our priors, it is important to note that they are very loose and let us explore a large class of alternative specifications.

The use of informative priors is not critical for our approach, but we do this for a number of reason. First, since our current knowledge on the effect of government spending shocks is based to a large extent on VAR evidence, it seems natural (and consistent with the Bayesian approach) to impose priors centered on our current state of knowledge. Second, given the inherent difficulty in estimating moving-average models, the priors help discipline the estimation by keeping the parameters in a reasonable set of the parameter space. Finally, and while we could have used improper uniform prior, the use of proper priors allows us to compute posterior odds ratio, which are important to select the order of the moving-average and to compare different FAIR models.

Choosing N, the number of Gaussian basis functions

To choose N, the number of basis functions used in the functional approximation of the impulse response function, we use posterior odds ratios (assigning equal probability to any two model) to compare models with increasing number of basis functions. We select the model with the highest posterior odds ratio.

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[47]Note that these priors are very loose. This is easy to see for a and b. For c, if it easy to show that $c\sqrt{\ln 2}$ is the half-life of the effect of a shock. Thus, $c = K$ corresponds to a very persistent impulse response function, since $K\sqrt{\ln 2} = 38$ quarters.

Appendix A2: Bayesian estimation of univariate and SUR-type FAIR models

The previous section described how to estimate multivariate FAIR models when we simultaneously identify the structural shocks and estimate the impulse response functions. We now describe how to estimate models when the shocks have been previously identified (typically through a narrative approach as in Ramey, 2011). The model can be a univariate FAIR model like (7) or a SUR-type FAIR model like (8). The advantage of this approach (compared to a multivariate self-contained FAIR model as in Appendix A1) is that only the impulse responses of interest are parametrized and estimated, yielding a small parameter space and a very fast estimation procedure.

For ease of exposition, we focus on the univariate model first, since the SUR-type model is a simple extension of the univariate case. As with the multivariate FAIR model, we use Bayesian methods and the key part is the construction of the likelihood. Recall from section 3 that we have a model of the form

$$y_t = \sum_{k=0}^{K} \psi(k)\varepsilon_{t-k}^G + u_t \tag{20}$$

with

$$\psi(k) = \sum_{n=1}^{N} a_n e^{-(\frac{k-b_n}{c_n})^2}$$

where a_n, b_n and c_n can be functions of ε_{t-k}^G (in the non-linear case), and where the residual is $u_t \equiv \sum_{j} \sum_{k=0}^{K} \psi_j(k)\varepsilon_{t-k}^{(j)}$.

Since $\{u_t\}$ is serially correlated by construction, in order to improve efficiency, we allow for serial correlation in u_t by positing that u_t follows an $AR(1)$ process. That is, we posit that $u_t = \rho u_{t-1} + \eta_t$ where η_t is Normally distributed $N(0, \sigma_\eta^2)$ with σ_η a parameter to be estimated. We set η_{-1} and η_0 to zero, and from (20), it is straightforward to build the likelihood given a series of previously identified shocks $\{\varepsilon_t^G\}$. For prior elicitation, we proceed as with the multivariate FAIR, and use very loose priors with $\sigma_a = 10$, $\sigma_b = K$ and $\sigma_c = K$.

For a SUR-type model like (8), the estimation proceeds along the same lines as above, except that we take into account that the one-step forecast error u_t is now a vector that follows a VAR(1) process instead of an AR(1) process.

Estimation routine and initial guess

As with multivariate FAIR models, we use a Metropolis-within-Gibbs algorithm. Regarding the initial guess, an interesting advantage of a univariate FAIR is that it is possible to compute a good initial guess, even in non-linear models.

Obtaining a non-linear initial guess

To obtain a good (possibly non-linear) initial guess in univariate and SUR-type FAIR models, we use the following two-step method:

1. Recover the $\{a_n\}$ factors given $\{b_n, c_n\}$

Assume that the parameters of the Gaussian kernels $-\{b_n, c_n\}_{n=1}^{N}-$ are known, so that we have a "dictionary" of basis functions to decompose our impulse response. Then, estimating the coefficients $\{a_n\}_{n=1}^{N}$ in (20), a non-linear problem, can be recast into a linear problem that can estimated by OLS. In other words, compared to a direct non-linear least square of (20) that treats all three sets of parameters a_n, b_n and c_n as free parameters, our two-step approach has the advantage of exploiting the efficiency of OLS to find $\{a_n\}$ given $\{b_n, c_n\}$.

To see that, consider first a linear model where $\psi(k)$ is independent of ε_{t-k}^{G}. We then re-arrange (20) as follows:

$$
\begin{aligned}
\sum_{k=0}^{K} \psi(k)\varepsilon_{t-k}^{G} &= \sum_{k=0}^{K}\sum_{n=1}^{N} a_n e^{-(\frac{k-b_n}{c_n})^2}\varepsilon_{t-k}^{G} \\
&= \sum_{n=1}^{N} a_n \sum_{k=0}^{K} e^{-(\frac{k-b_n}{c_n})^2}\varepsilon_{t-k}^{G}.
\end{aligned}
$$

Defining

$$
X_{n,t} = \sum_{k=0}^{K} e^{-(\frac{k-b_n}{c_n})^2}\varepsilon_{t-k}^{G},
$$

our estimation problem becomes a linear problem (conditional on knowing $\{b_n, c_n\}_{n=1}^{N}$):

$$
y_t = \sum_{n=1}^{N} a_n X_{n,t} + \alpha + \beta u_t \tag{21}
$$

where the $\{a_n\}$ parameters can be recovered instantaneously by OLS. Assuming that u_t follows an AR(1), we can estimate the $\{a_n\}$ with a NLS procedure.

The method described above is straightforward to apply to a case with asymmetry and state dependence. Consider for instance the case with asymmetry

$$a_n(\varepsilon_{t-k}^G) = a_n^+ 1_{\varepsilon^G \geq 0} + a_n^- 1_{\varepsilon^G < 0}.$$

Then, we can proceed as in the previous section and define the following right-hand side variables

$$\begin{cases} X_{n,t}^+ = \sum_{k=0}^{K} h_n(k)\varepsilon_{t-k}^G 1_{\varepsilon \geq 0} \\ X_{n,t}^- = \sum_{k=0}^{K} h_n(k)\varepsilon_{t-k}^G 1_{\varepsilon < 0} \end{cases}$$

and use OLS to recover a_n^+ and a_n^-.

2. Choose $\{b_n, c_n\}$

To estimate $\{b_n, c_n\}_{n=1}^{N}$ (and therefore $\{a_n\}_{n=1}^{N}$ from the OLS regression), we minimize the sum of squared residuals of (21) using a simplex algorithm.

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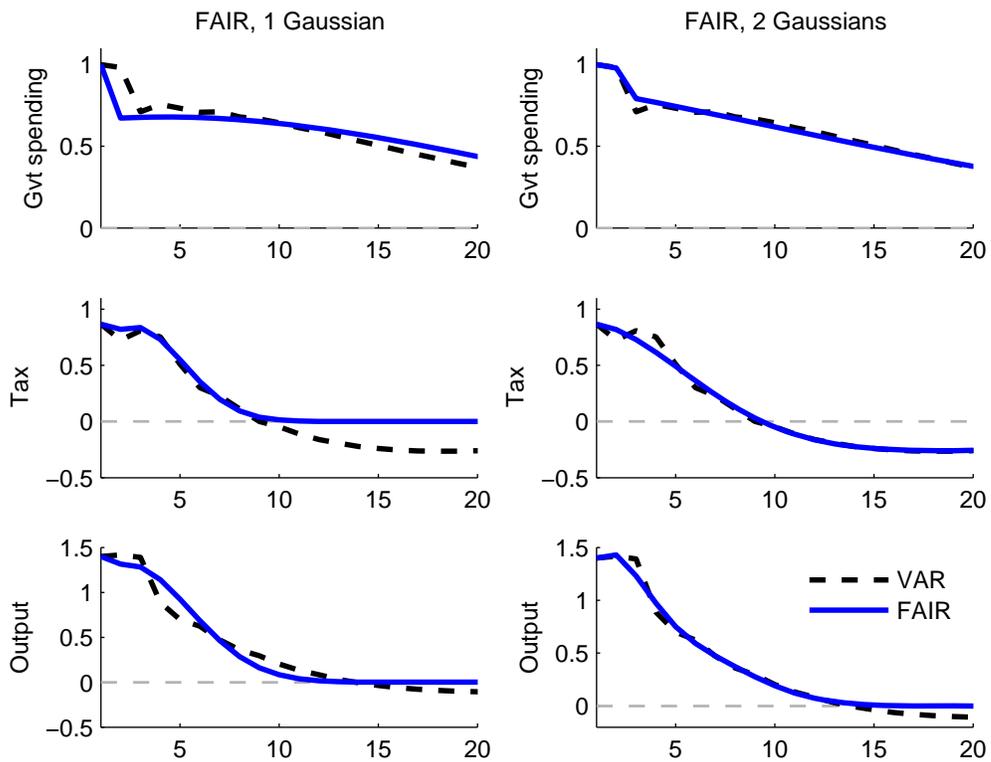
Figure 1: Impulse response functions (in percent) of government spending, government revenue ("Tax") and output to a one standard-deviation government spending shock. Impulse responses estimated with a VAR (dashed-line) or approximated with one Gaussian basis function (FAIR, left-panel, thick line) or two Gaussian basis functions (FAIR, right panel thick line). Estimation using quarterly data covering 1966-2014.
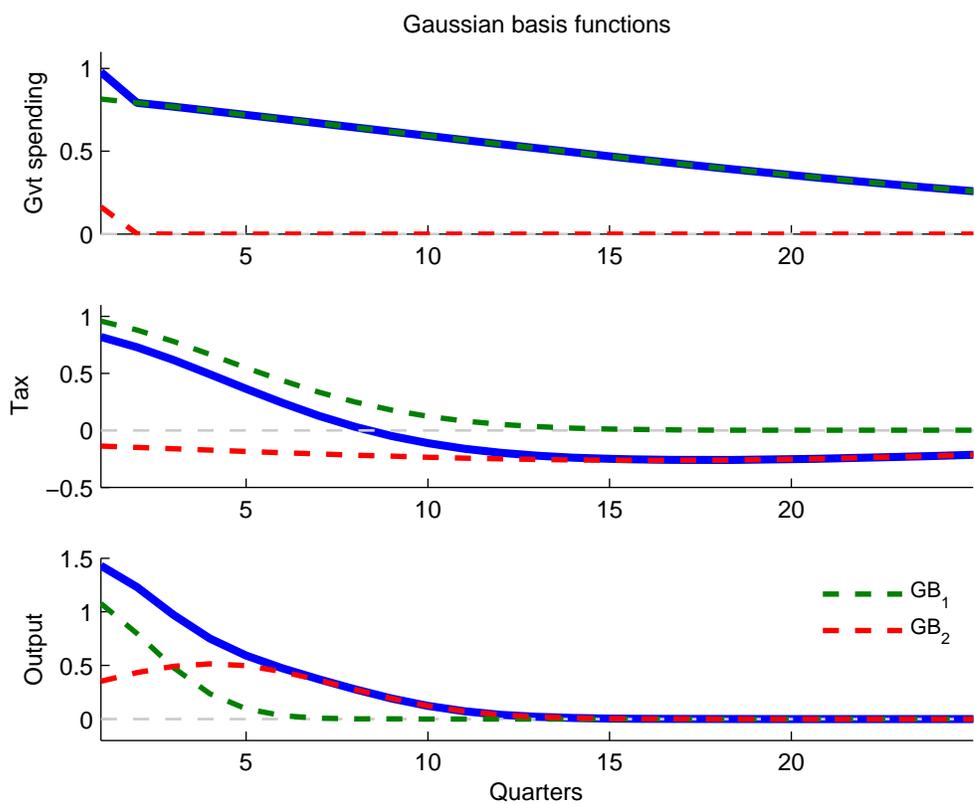
Figure 2: Gaussian basis functions (dashed lines) used by a FAIR with two Gaussian basis functions for the responses of government spending, government revenue ("Tax") and output to a government spending shock. The basis functions are appropriately weighted so that their sum gives the functional approximation of the impulse response functions (solid lines) reported in the right-panels of Figure 1.
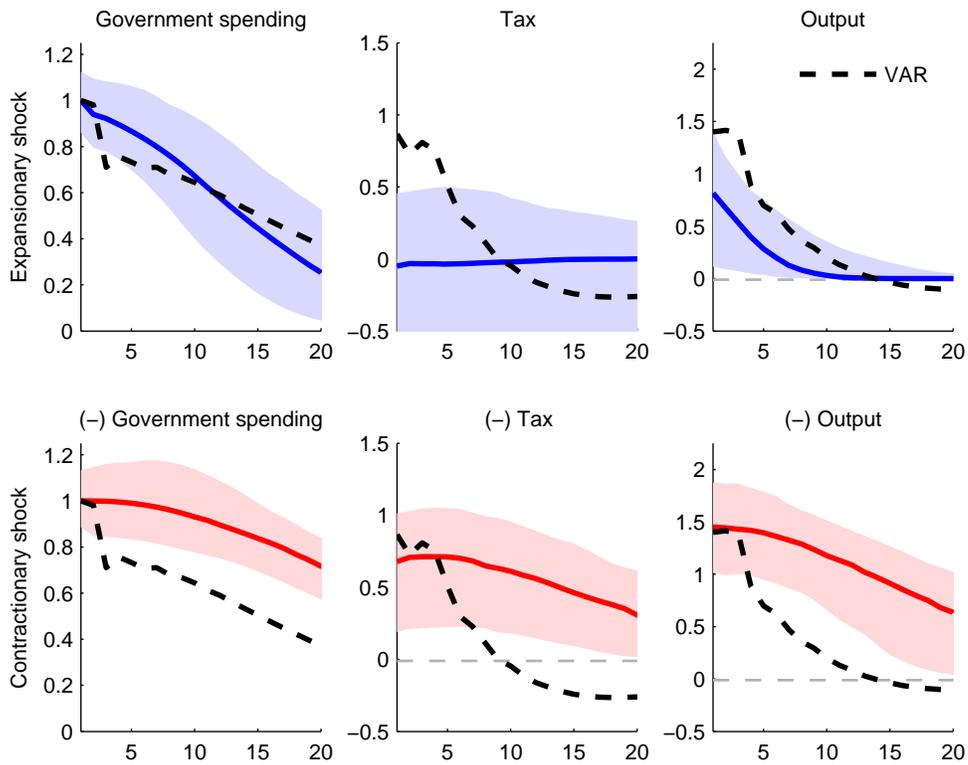
Figure 3: **Recursive identification scheme, FAIR, 1966-2014**: Impulse response functions (in percent) of government spending, government revenue ("Tax") and output to a government spending shock identified from a timing restriction. Estimation from a standard VAR (dashed-line) or from a FAIR with one Gaussian basis function (plain line). The thin lines cover 90% of the posterior probability. For ease of comparison between the top and bottom panels, the responses to a contractionary shock are multiplied by -1 in the bottom panels.
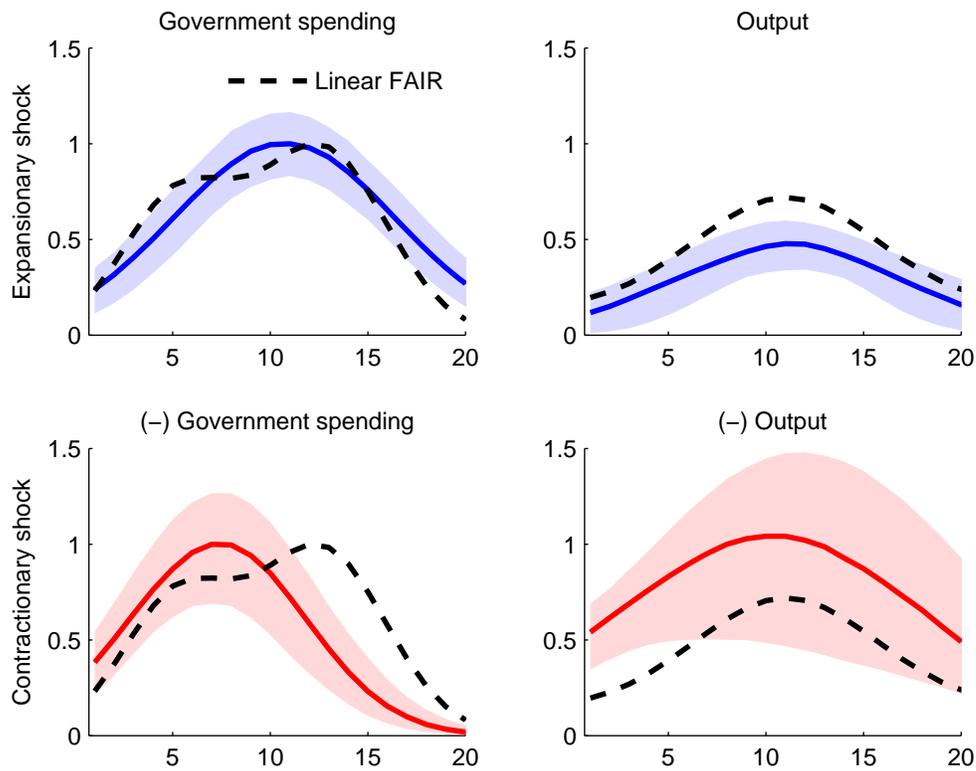
Figure 4: **Narrative identification scheme, FAIR, 1939-2014**: Impulse response functions (in percent) of Government spending and Output to a Ramey news shock identified from a narrative approach. Estimation from a linear (i.e., symmetric) FAIR model (dashed-line) or from an asymmetric FAIR model with one Gaussian basis function (plain line). The thin lines cover 90% of the posterior probability. For ease of comparison between the top and bottom panels, the responses to a contractionary shock are multiplied by -1 in the bottom panels.
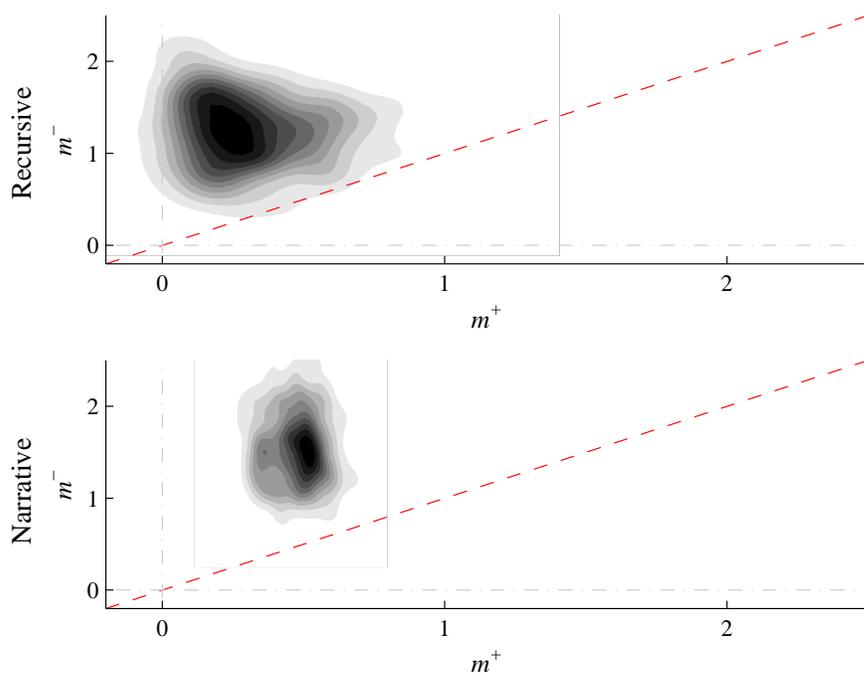
Figure 5: Posterior distribution of the "sum" government spending multiplier for expansionary spending shocks (m^+, x-axis) and contractionary spending shocks (m^-, y-axis). Estimation using a recursive identification scheme over 1966-2014 (top panel) or a narrative identification scheme with Ramey news shocks over 1939-2014 (bottom panel). The dashed red line marks the symmetric case; with equal multipliers for expansionary and contractionary shocks. The "sum" multiplier is calculated over the first 20 quarters after the shock.
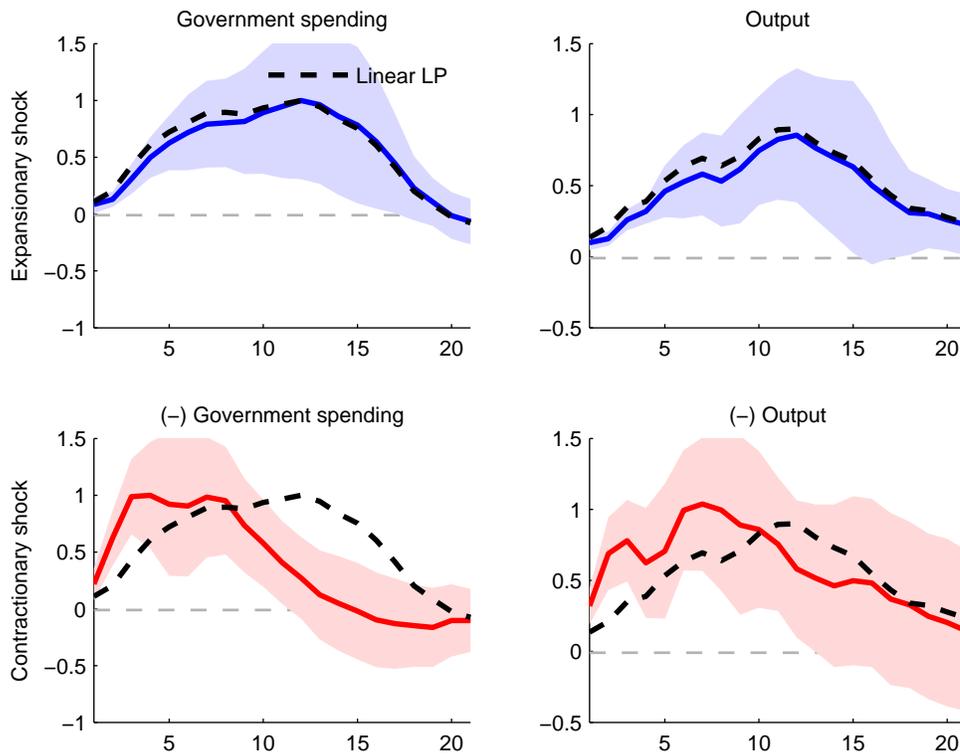
Figure 6: **Narrative identification scheme, Local Projections, 1890-2014**: Impulse response functions (in percent) of Government spending and Output to a Ramey news shock. Estimates from Local Projections. The dashed lines report the point estimates from a linear (i.e., symmetric) model. For ease of comparison between the top and bottom panels, the responses to a contractionary shock are multiplied by -1 in the bottom panels. The shaded areas are the 90 percent confidence bands calculated using Newey-West standard errors.
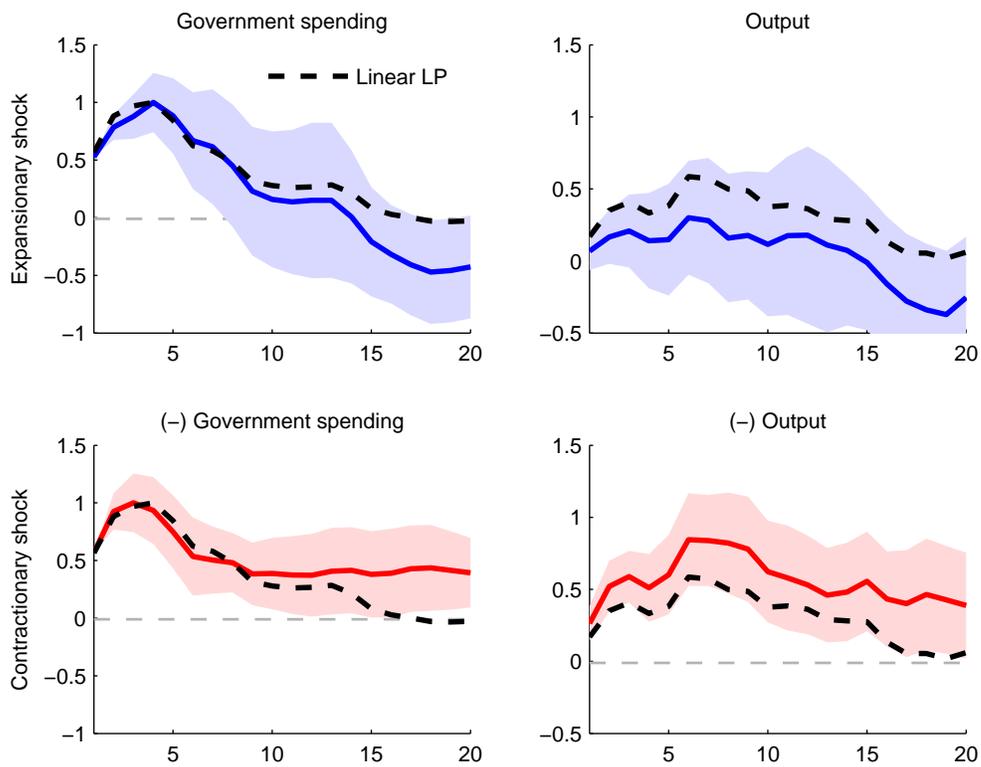
Figure 7: **Recursive identification scheme, Local Projections, 1890-2014**: Impulse response functions (in percent) of Government spending and Output to a government spending shock. Estimates from Local Projections. The dashed lines report the point estimates from a linear (i.e., symmetric) model. For ease of comparison between the top and bottom panels, the responses to a contractionary shock are multiplied by -1 in the bottom panels. The shaded areas are the 90 percent confidence bands calculated using Newey-West standard errors.
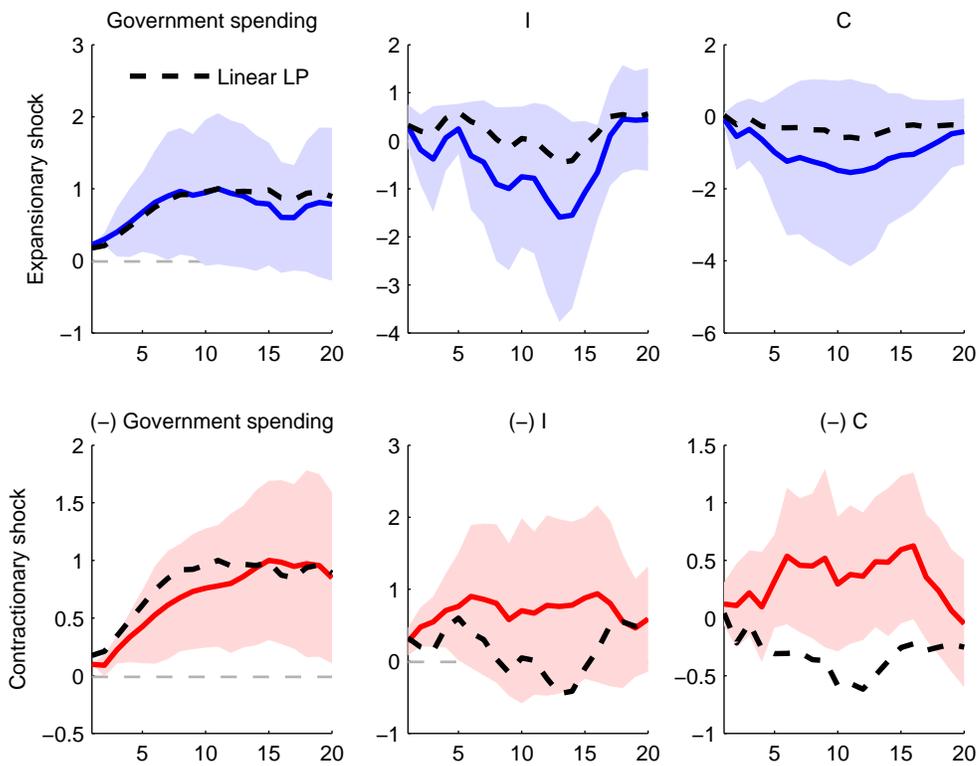
Figure 8: **Narrative identification scheme, Local Projections, 1947-2014**: Impulse response functions (in percent) of Government spending, Investment (I), and Consumption (C) following an expansionary Ramey news shock (left panel) and a contractionary Ramey news shock (right panel). Estimates from Local Projections. The dashed lines report the point estimates from a linear (i.e., symmetric) model. For ease of comparison between the left and right panels, the responses to a contractionary shock are multiplied by -1 in the right panels. The shaded areas are the 90 percent confidence bands calculated using Newey-West standard errors.
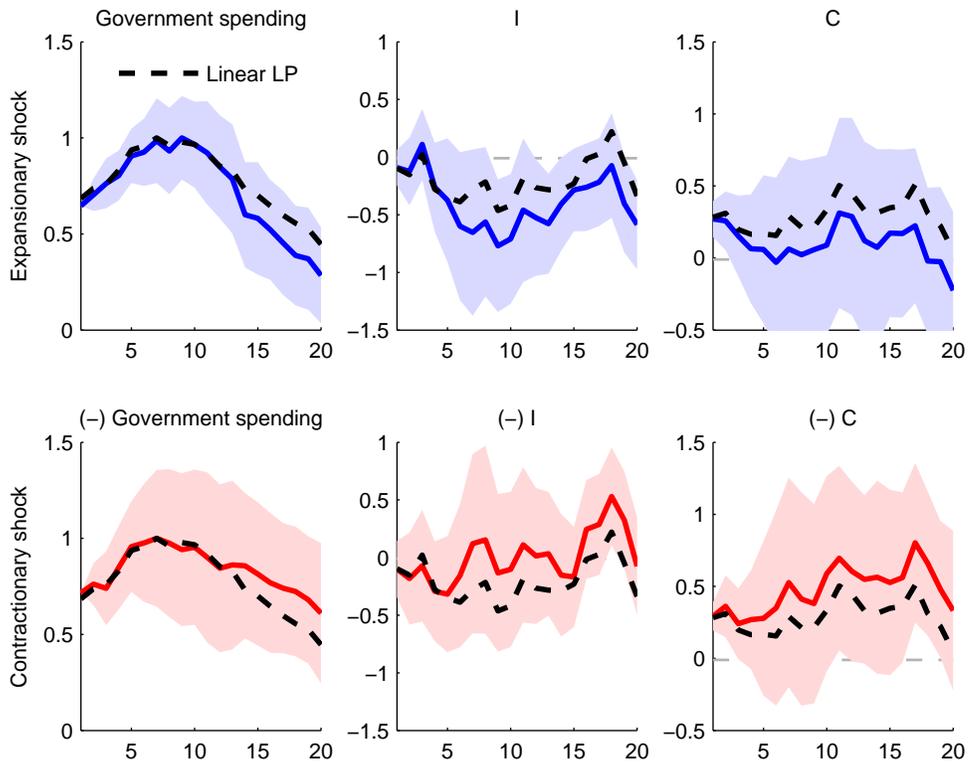
Figure 9: **Recursive identification scheme, Local Projections, 1947-2014**: Impulse response functions (in percent) of Government spending, Investment (I) and Consumption (C) following an expansionary government spending shock (left panel) and a contractionary shock (right panel). The dashed lines report the point estimates from a linear (i.e., symmetric) model. For ease of comparison between the left and right panels, the responses to a contractionary shock are multiplied by -1 in the right panels. The shaded areas are the 90 percent confidence bands calculated using Newey-West standard errors.
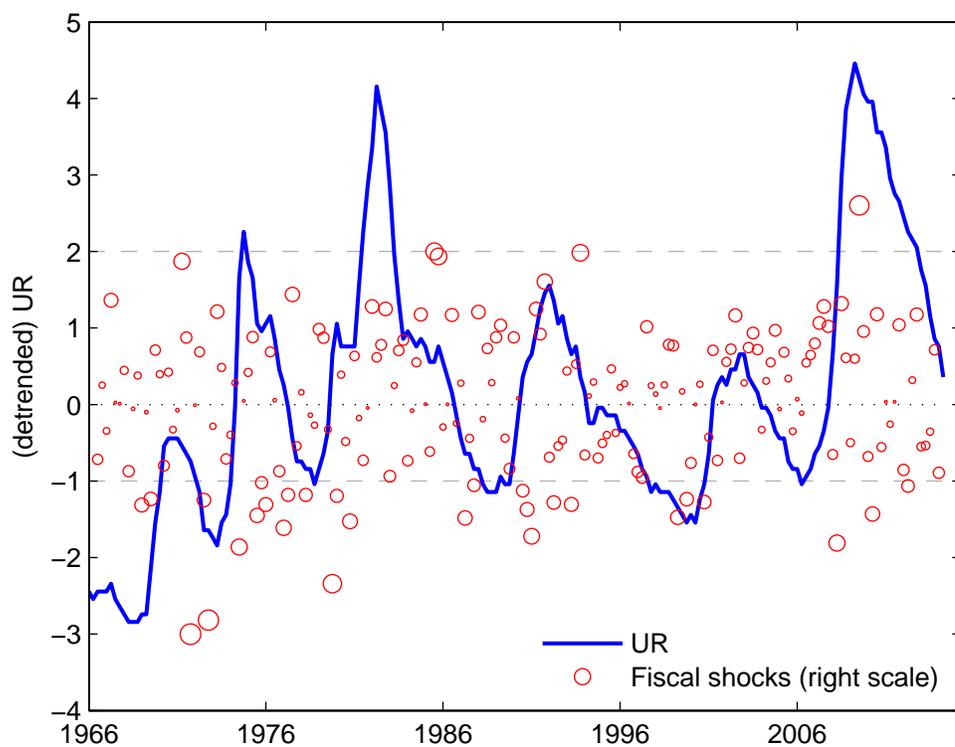
Figure 10: The business cycle indicator –the unemployment rate detrended with CBO's natural rate estimate– (solid line, left scale)–, and government spending shocks identified from a recursive ordering (circles, right scale) with larger circles indicating larger shocks.
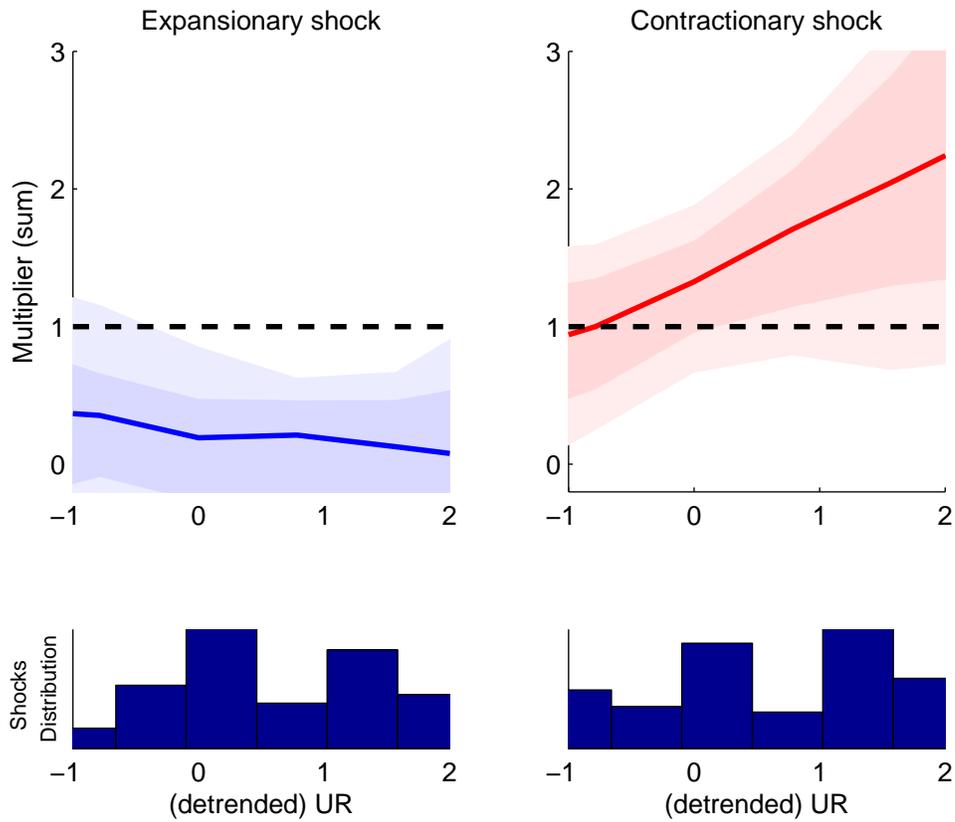
Figure 11: **Recursive identification scheme, FAIR, 1966-2014**: Size of the "sum" multiplier as a function of the state of the business cycle (measured with detrended unemployment) for expansionary government spending shocks (left panel) and contractionary government spending shocks (right panel). The shaded areas respectively cover 68 and 90 percent of the posterior probability. The bottom panels plot the distributions of (respectively) contractionary shocks and expansionary shocks over the business cycle.
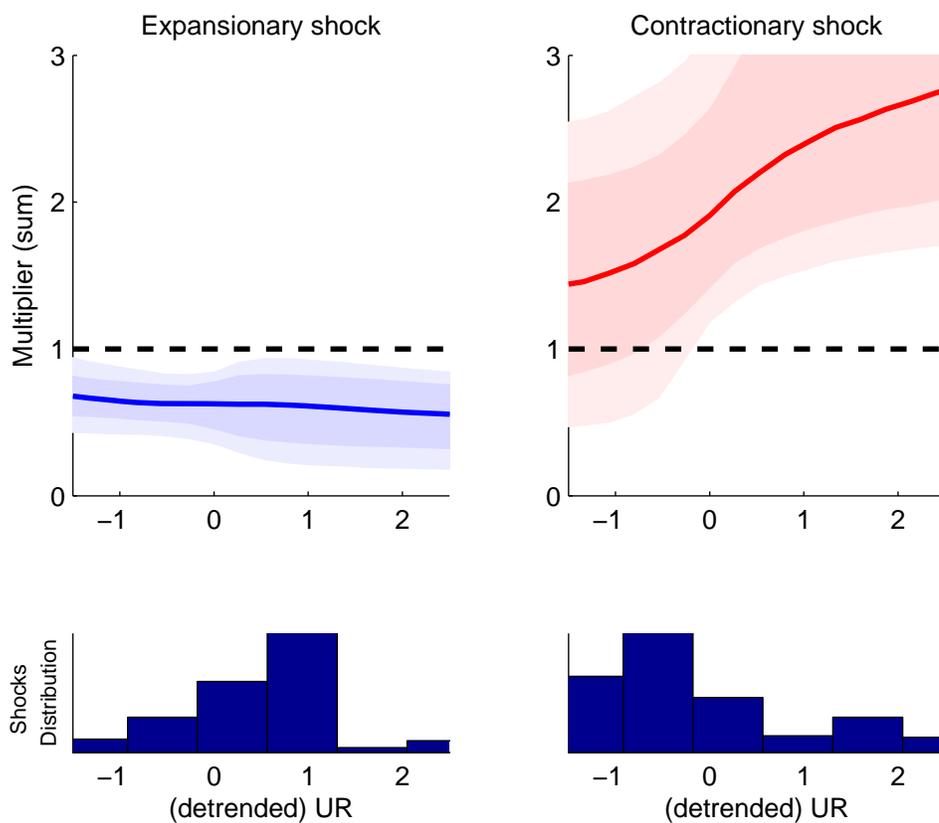
Figure 12: **Narrative identification scheme, FAIR, 1939-2014**: Size of the "sum" multiplier as a function of the state of the business cycle (measured with detrended unemployment) for expansionary Ramey news shocks (left panel) and contractionary Ramey news shocks (right panel). The shaded areas respectively cover 68 and 90 percent of the posterior probability. The bottom panel plots the distribution of (respectively) contractionary shocks and expansionary shocks over the business cycle.
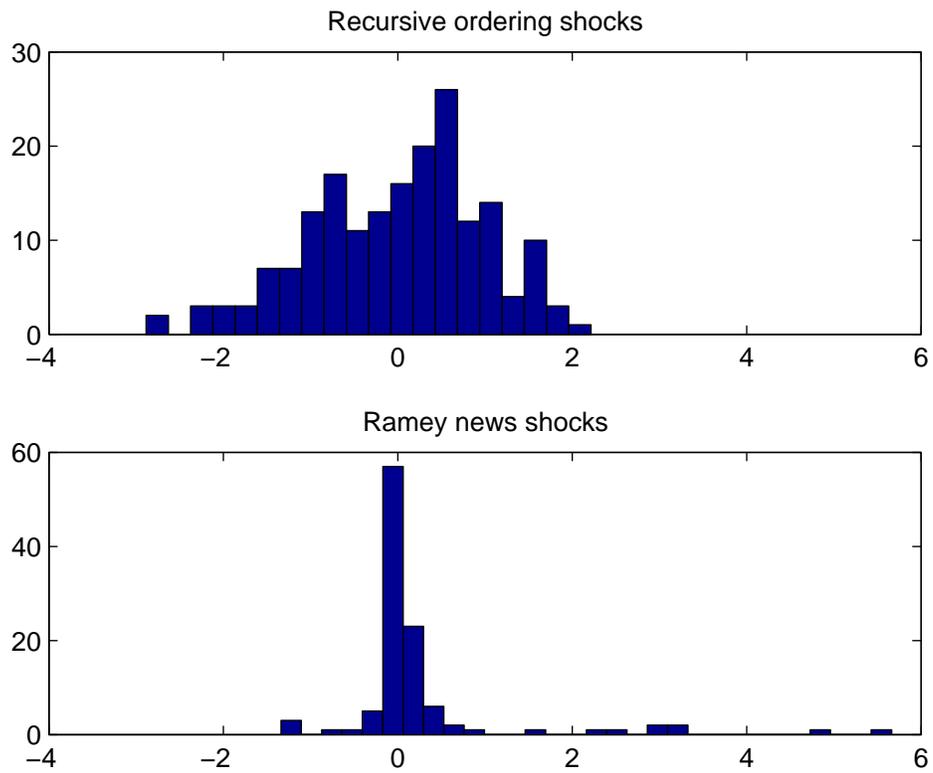
Figure 13: Histograms of the distributions of government spending shocks (rescaled by their standard-deviation). The upper-panel depicts the distribution of shocks recovered from a recursive ordering (1966-2014), the bottom-panel depicts the distribution of Ramey news shocks (1939-2014).
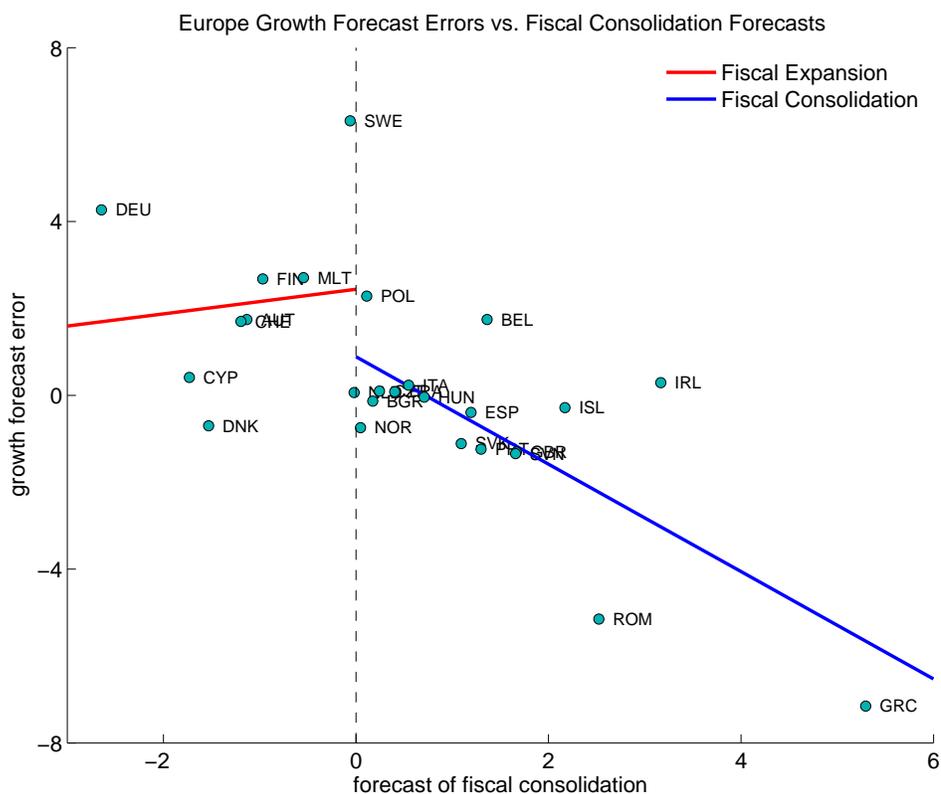
Figure 14: **Blanchard and Leigh (2013) approach**: Regression of forecast error for real GDP growth in 2010 and 2011 relative to forecasts made in the spring of 2010 on forecasts of fiscal consolidation for 2010 and 2011 made in spring of year 2010. The lines depict the regression lines for respectively fiscal consolidation (increase in budget surplus, blue line) and fiscal expansion (decrease in budget surplus, red line). Note that a fiscal consolidation corresponds to an *increase* in the fiscal balance and thus enters as a positive entry on the x-axis.

Table 1: Asymmetric government spending multipliers, FAIR estimates

	Linear	**Expansionary shock**	**Contractionary shock**
		"Sum" multiplier, 5 year integral	
AG shocks **1966-2014**	0.58 (0.2--1.0)	0.27 (0.0--0.6)	1.25 (0.7--1.8)
$P(m^->m^+)$		P=0.99***	
Ramey News shocks **1939-2014**	0.73 (0.6--0.8)	0.47 (0.3--0.6)	1.56 (1.0--2.3)
$P(m^->m^+)$		P>0.99***	

Note: The "sum" multiplier is calculated by cumulating the impulse responses over the first 20 quarters. Estimates from FAIR models with 1 Gaussian basis function. Numbers in parenthesis cover 90% of the marginal posterior probability. AG shocks refer to shocks obtained as in Auerbach and Gorodnichenlo (2012) from a Blanchard-Perotti recursive identification scheme augmented with professional forecasts of government spending. Ramey news shocks are the unexpected changes in anticipated future expenditures constructed by Ramey (2011). $P(m^->m^+)$ reports the posterior probability that the contractionary multiplier m^- is larger than the expansionary multiplier m^+. "***" denote a posterior probability above 0.99.

Table 2: Robustness check:
Asymmetric government spending multipliers, Local Projections, 1890-2014

	Linear	**Expansionary shock**	**Contractionary shock**
		"Sum" multiplier, 5 year integral	
BP shocks **1890-2014**	0.60 (0.08)	0.15 (0.22)	1.00 (0.17)
Ramey News shocks **1890-2014**	0.92 (0.08)	0.88 (0.07)	1.50 (0.38)

Note: Numbers in parenthesis denote the Newey-West standard errors. BP shocks refer to shocks obtained as in Blanchard and Perotti (2002). Ramey news shocks are the unexpected changes in anticipated future expenditures constructed by Ramey and Zubairy (2016). The "sum" multiplier is calculated by taking the integrals of the impulse responses over the first 20 quarters.

Table 3: Blanchard-Leigh-type regressions

Forecast error of $\Delta Y_{i,t|t+1} = \alpha + \beta \text{Forecast of } \Delta F_{i,t|t+1} + \varepsilon_{i,t|t+1}$

Specification	(1)	(2)	(3)
β	-1.09*** (.25)		
β^+		0.28 (.81)	
β^-		-1.23*** (.34)	
β_G^+			-0.46 (.78)
β_G^-			1.66*** (.80)
β_T^+			-0.15 (.58)
β_T^-			-0.95 (.67)
Obs	26	26	26
R^2	.49	.56	.60

Note: Data from Blanchard-Leigh (2013). The table reports estimates and heteroskedasticity-robust standard errors in parentheses. ***, **, and * denote statistical significance at the 1, 5, and 10 level, respectively. Equation (1) reports the results of estimating the baseline specification of Blanchard-Leigh (equation (15) in the main text), equation (2) allows for separate βs depending on the sign of the planned fiscal adjustment ΔF : β^+ is the coefficient for fiscal expansions (decrease in budget surplus) ; β^- is the coefficient for fiscal consolidations (increase in budget surplus); equation (3) reports the results of estimating equation (16) in the main text but allowing for separate βs depending on the sign of the planned spending adjustment ΔG or planned revenue adjustment ΔT : β_G^+ is the coefficient for fiscal expansions (increase in government spending) ; β_G^- is the coefficient for fiscal consolidations (decrease in government spending) and similarly for β_T^- and β_T^+ but for revenues.

Table 4: Asymmetric multipliers and labor market slack, FAIR estimates

"Sum" multiplier 5 year integral	Expansionary shock		Contractionary shock	
	U low	U high	U low	U high
AG shocks 1966-2014	0.3	0.1	0.9	2.1
$P(m^{U\,high}>m^{U\,low})$	P=0.52		P=0.95**	
Ramey News shocks 1939-2014	0.6	0.5	1.5	2.7
$P(m^{U\,high}>m^{U\,low})$	P=0.35		P=0.98**	

Note: The "sum" multiplier is calculated by cumulating the impulse responses over the first 20 quarters. Estimates from FAIR models with 1 Gaussian basis function. AG shocks refer to shocks obtained as in Auerbach and Gorodnichenlo (2012) from a Blanchard-Perotti recursive identification scheme augmented with professional forecasts of government spending. Ramey news shocks are the unexpected changes in anticipated future expenditures constructed by Ramey (2011). $P(m^{U\,high}>m^{U\,low})$ reports the posterior probability that the multiplier *m* is larger in state of high unemployment (detrended unemployment of +2) than in a state of low unemployment (detrended unemployment of -1). "**" denotes a posterior probability above 0.95.