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WP 14-04R

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Does Redistribution Increase Output? The Centrality of Labor Supply[*,†]

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Abstract

The aftermath of the recent recession has seen numerous calls to use transfers to poorer households as a means to enhance aggregate activity. The goal of this paper is to study the effects of wealth redistribution from rich to poor households on consumption and output in the short run. We show that in a standard incomplete-markets model extended to allow for nominal rigidities and parametrized to match the wealth distribution in the U.S. economy, wealth redistribution does lead to a temporary boom in consumption, but *not* in output. We then show, both analytically and numerically, that the key to understanding the direction and size of the output effects of such interventions lies in labor supply decisions.

Keywords: Multipliers, Redistribution, Labor supply, Idiosyncratic Risk. **JEL Codes**: D90, E21, E25, E63

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1 Introduction

In the aftermath of the recent recession there have been numerous calls to use, as well as the actual use of, transfers to low net-wealth households as a means to enhance economic activity.[1] The conventional argument for a positive effect of transfers from rich to poor on output centers on the notion that wealth-poor households have a higher marginal propensity to consume out of wealth than wealth-rich households, so that such a transfer leads to a boom in consumption. Under a standard "Keynesian-cross" type intuition, this leads, in turn, to a boom in output. Such a view puts household heterogeneity front and center in determining the aggregate short-run response to a change in transfers. Therefore, any quantitative evaluation of the aggregate impact of wealth redistribution requires the use of macroeconomic models capable of capturing such heterogeneity in considerable detail.

Figure 1, below, shows the impact of a one-time surprise wealth redistribution from rich to poor on aggregate output and consumption within the framework most widely used to understand interactions between wealth inequality and the macroeconomy, the "standard incomplete-market (SIM)" or "Bewley-Aiyagari-Huggett" model.[2] Importantly, the version of the model we employ incorporates nominal rigidities to allow aggregate demand to play a role. The parameters relevant for the steady-state of the model are calibrated as in Castaneda et al., 2003, so that its initial conditions match the extreme wealth concentration observed in the United States. Consistent with the conventional argument, we see that a redistribution of wealth from rich to poor agents does generate a consumption boom, and one that, furthermore, is prolonged.

What is also clear from Figure 1, however, is that wealth redistribution from rich to poor does *not* result in an output boom. Given that our results arise from the benchmark model for analysis of redistributive policies that, furthermore, matches salient heterogeneity along many dimensions, they present a challenge to the conventional view that assigns short-run stimulative power to wealth redistribution. The goal of this paper is to understand the driving forces behind the short-run effects on consumption and output of wealth redistribution from rich to poor.

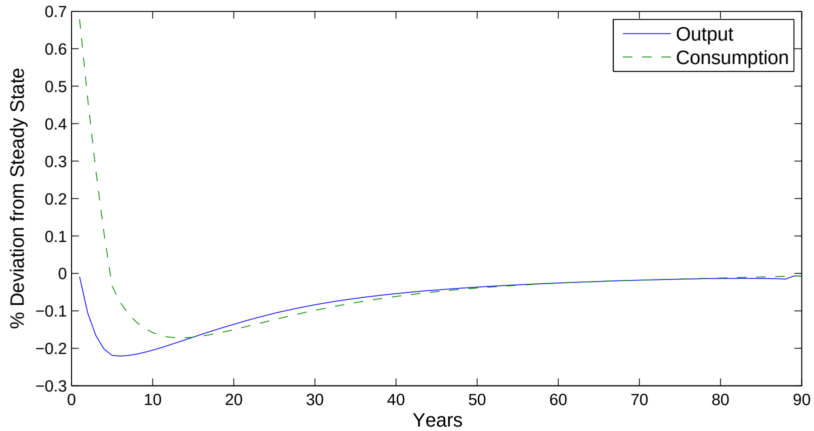
The main theme of our paper is that differences in marginal propensities to consume provide incomplete input for the evaluation of the aggregate impact of wealth redistribution. This is because these differences do not fully describe the differential impact of wealth transfers on *labor supply*. For instance, if the marginal propensity to supply labor of wealth-poor households is no different than that of wealth-rich households, aggregate labor supply will not change, all else equal.[3] The

[1]The most prominent redistributive policy implemented was the American Recovery and Reinvestment Act (ARRA), which contained a significant redistributive component in the form of extensions of transfer programs for the poor, such as unemployment insurance and food stamps (Oh and Reis, 2012). More generally, the idea that the post-crisis evolution of the wealth distribution may be suppressing aggregate output in the recent recovery, is one that has received attention. In particular, Mian and Sufi (2014) argue that, by increasing household net worth and hence aggregate consumption, debt forgiveness may be a key for accelerating the recovery. See also Dynan, 2012, and Cynamon and Fazzari, 2013.

[2]Examples of papers using models within the same class to study wealth redistribution policies include Heathcote (2005) and Conesa and Krueger (2006), among many others.

[3]As a quantitative matter, a nontrivial effect of wealth on labor supply is highlighted in work by Floden (2001) and Pijoan-Mas (2006).

Figure 1: Aggregate Impact of Wealth Redistribution



fact that the results arise in the presence of nominal rigidities highlights that labor supply behavior remains relevant even in general equilibrium models where aggregate demand can drive a wedge between the marginal product of labor and the wage rate.

A key difficulty for the view that redistribution can boost output is that the same impulse that leads wealth-poor households to consume proportionately more out of an additional dollar received than their wealth-rich counterparts also pushes them to increase their leisure proportionately more. Thus, barring countervailing forces, the same incentives that lead to a consumption boom also induce a reduction in labor supply and, hence, output. We isolate particular assumptions concerning preferences between consumption and leisure, as well as those concerning the tax system, that mitigate this effect and, in particular instances, overturn it. In particular, aggregate labor supply will tend to fall by less if, because of either preferences or the tax system, leisure is a "luxury" in the sense that the rich spend proportionately more on it. In combination, those various effects account for the fact that, while output does not rise in response to redistribution, it also does not fall immediately. In addition, we examine the potential for general equilibrium channels to act as countervailing forces, and we find that they can sometimes be relevant. The results highlight the centrality of labor supply, in particular its wealth elasticity, in order to decisively sign and measure the aggregate effect of redistributive policies.

The paper proceeds as follows. In Section 2, we present the baseline model. Section 3 presents analytical results with respect to the neutrality of aggregate outcomes to wealth redistributions. Sections 4 presents the quantitative results. Section 5 concludes.

Literature Review

Our work informs a growing quantitative literature examining the short-run aggregate effects of wealth redistribution programs. Huntley and Michelangeli (2014) and Kaplan and Violante (2011) aim to explain how wealth redistribution can lead to an increase in household consumption of the size measured in empirical work (see Johnson et al., 2006; Parker et al., 2013; Japelli and Pistaferri, 2014; and Misra and Surico, 2014), though without focusing on the behavior of labor supply. Heathcote (2005) studies the effect of a redistributive program in a heterogeneous-agent economy, but assumes that the utility function is such that labor supply is unaffected by wealth. Heathcote focuses instead on the distortionary impacts of the taxes needed to fund the transfer program. In addition, Floden (2001), Pijoan-Mas (2006), Alonso-Ortiz and Rogerson (2010), Horvath and Nolan (2013), and Zilberman and Berriel (2012) investigate the steady-state implications of redistributive programs in heterogeneous-agent economies with endogenous labor supply.

In contrast to the work above, we focus entirely on the transitional dynamics of a redistribution policy that, by design, features no steady-state effects. Thus, our paper shares the emphasis on business cycle frequency phenomena in Oh and Reis (2012), Reis and Mckay (2014), Mehrotra (2014), and Giambattista and Pennings (2014). The papers of both Mehrotra and Giambattista and Pennings analyze the effect of transfers in models with two types of agents, providing assessments of transfer multiplier given different assumptions. Oh and Reis is the most closely related, as they analyze transitional dynamics following one-off redistributive policies in a full-fledged heterogeneous-agent model. Their particular focus is on how wealth transfers ought to be targeted in order to generate output booms. By contrast, our work analyzes the impact of policies that redistribute wealth from rich to poor, as have been more typically advocated and implemented.

Lastly, our paper contributes by providing substantial analytical characterization of the effect of various policies in a model where the heterogeneity across households is potentially very rich. In existing work, analytical tractability is achieved through the use of special assumptions on income processes, preferences, or market structures (see Heathcote, Storesletten, and Violante (2014) for a recent contribution). As we will show, however, significant insight into the short-run effects of redistribution can be attained without imposing any structure on shock processes beyond what is necessary to deliver stationary aggregate outcomes.

2 Model

In what follows, we introduce a single model that nests a wide variety of Bewley-Aiyagari-Huggett style environments including extensions that allow for nominal rigidities. For notational convenience, we denote all prices in terms of units of the final consumption good, inflating them by a nominal price index whenever necessary.

2.1 Households

The economy is populated by a continuum of infinitely lived households with utility over consumption and leisure. Time is discrete and given that the short-run policy decisions that our paper hopes to inform usually have annual horizons, a model period corresponds to one calendar year. As high-frequency labor market frictions (arising, e.g., from search and matching or indivisibilities) are likely to be relatively less important at annual frequencies, households are modeled as being able to choose labor supply freely within a period.

Households differ in terms of their initial wealth and labor productivity. Labor productivity for any given household changes stochastically and purely idiosyncratically, and households cannot directly insure against those changes. Rather, households are constrained to holding only riskless bonds. Furthermore, households are credit constrained in the sense that they cannot hold bonds below a minimum threshold $\underline{a}$, which is tighter than the natural borrowing limit.

Our focus throughout will be on a single, one-off, and wholly unanticipated wealth redistribution. This generates aggregate transition dynamics upon which prices depend that, after the shock, are perfectly forecasted. We index the time since the shock by t, with $t = 0$ corresponding to the aggregate steady-state and $t = 1$ the first date after the shock. For any given date $t \geq 1$, the problem of any given household can be written recursively as:

$$
\begin{aligned}
V_t(a,s) &= \max_{b',c,l} u\left(c, \bar{l} - l\right) + \beta \sum \Pr\left[s'|s\right] V_{t+1}\left(a', s'\right) \\
s.t. &: \quad a' + c = y + a - \tau(y) + \pi_t + x_t(a) \\
& \quad\;\; y = w_t \varepsilon(s) l + r_t a + \omega(s) \\
& \quad\;\; a \geq \underline{a},\ l \geq 0
\end{aligned}
$$

The notation is as follows: $\beta < 1$ is the discount factor, c is consumption; l is labor; $\bar{l}$ is leisure endowment; u is an increasing, concave utility function that is differentiable in both its arguments; a are initial asset holdings; $\underline{a}$ is an exogenous debt limit; s is the exogenous idiosyncratic productivity state; $\varepsilon(s)$ is the labor productivity of household in idiosyncratic state s with $\varepsilon(1) = 1$; w_t is the wage per effective unit of labor; r_t is the interest rate on bonds; $\omega(s)$ are taxable government transfers; $\tau(y)$ are income taxes, with τ a (weakly) increasing function of y_t; π_t are profits distributed lump-sum to households, and $x_t(a)$ are tax exempt transfers or, if negative, non-tax-deductible payments. As usual, primed variables refer to next period values, and $\Pr[s'|s]$ is the probability of transitioning into exogenous state s' given that the household is in state s. Note that the pre-tax transfer variable $\omega(s)$ is not time indexed, whereas the post-tax variable $x_t(a)$ is. This is because the former is meant to capture permanent features of the tax code, whereas the latter will be the source of the redistribution shock whose transitional impact we will analyze.

We denote by $\Gamma_t(a,s)$ the joint density of households with asset level a and productivity level s at the end of period t. We also denote the optimal choices of c, l, and a' at each date t during the transition by the policy functions $c_t(a,s)$, $l_t(a,s)$, and $a'_t(a,s)$. Letting $\mathcal{S}$ be the (finite) set of exogenous state and $\mathcal{A}$ the (bounded closed interval) set of wealth levels, this allows us to define aggregate consumption C_t, hours worked L_t, effective (productivity weighted) labor input N_t, end-of-period household wealth A_t, pre-tax aggregate net transfers to households Ω_t, and the post-tax aggregate net transfers X_t respectively as follows:

$$C_t = \sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} c_t(a,s)\,\Gamma_{t-1}(a,s)\,da \tag{1}$$

$$L_t = \sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} l_t(a,s)\,\Gamma_{t-1}(a,s)\,da \tag{2}$$

$$N_t = \sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} \varepsilon_t(s)\,l_t(a,s)\,\Gamma_{t-1}(a,s)\,da \tag{3}$$

$$A_t = \sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} a_t'(a,s)\,\Gamma_{t-1}(a,s)\,da \tag{4}$$

$$\Omega_t = \sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} \left(\omega(s) - \tau\left(w_t\varepsilon(s)\,l_t(a,s) + r_t a + \omega(s)\right)\right)\Gamma_{t-1}(a,s)\,da \tag{5}$$

$$X_t = \sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} x_t(a,s)\,\Gamma_{t-1}(a,s)\,da \tag{6}$$

2.2 Firms

There are two types of firms: intermediate-goods producers and final-goods producers. Final goods producers combine the intermediate goods and capital into a single final good that can be used for consumption, investment in fixed capital, or government purchases. There is a unit mass of differentiated intermediate goods producers indexed $i \in [0,1]$. Each firm i is endowed with a linear technology that allows it to transform one unit of effective labor that they use, $n_t^d(i)$, into one unit of intermediate good i that they produce, $m_t(i)$. Because intermediate goods are differentiated across firms, their producers have market power and therefore can have some discretion over the price that they charge $p_t(i)$. However, they are subject to nominal frictions, which limit their ability to choose $p_t(i)$ in a timely manner. All intermediate-goods producing firms commit to supplying the quantity $m_t^d(i)$ that final-goods producers demand at the prevailing price, ensuring that markets for all intermediate goods clear.

The production function for final-goods producers is:

$$Y_t = F(K_{t-1}, M_t),$$

where F is a neo-classical production function, K_{t-1} is the capital stock, and $M_t = \left[\int_0^1 m_t^d(i)^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}$ is an aggregate of different varieties of the intermediate good. Final-goods producing firms maximize profits, behave competitively, and use the funds they borrow from households to purchase capital. The date t value of a final-goods firm (in terms of the final good), $\Xi_t(K_{t-1})$, that enters a period with capital stock K_{t-1} is then given recursively as follows:

$$\Xi_t(K_{t-1}) = \max_{M_t,K_t} F(K_{t-1}, M_t) - \int_0^1 p_t(i)\,m_t^d(i)\,di - K_t + (1-\delta)K_{t-1} + \frac{1}{1+r_{t+1}}\Xi_{t+1}(K_t)$$

Let $P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta}\, di\right]^{\frac{1}{1-\theta}}$ denote the per-unit price of the cost-minimizing bundle of intermediate goods, and let δ denote the depreciation rate of capital.

The usual optimality conditions hold:

$$F_K(K_{t-1}, M_t) = r_t + \delta \tag{7}$$

$$F_M(K_{t-1}, M_t) = P_t \tag{8}$$

$$m_t^d(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} M_t, \tag{9}$$

Let Q_t be the nominal price level for the final good in the economy, so that $p_t(i) Q_t$ is the nominal price charged by firm i at time t. We assume that a fraction λ of the firms chooses $p_t(i) Q_t$ at the *beginning* of the period, after shocks have been realized, whereas the remaining $1-\lambda$ chooses nominal prices that they will receive in period t at the end of the *previous* period, $t-1$. Thus, a fraction $1-\lambda$ sets prices before they have had time to incorporate period t information. Firms distribute any profits $\pi_t(i) = (p_t(i) - w_t) m_t(i)$ to households.

To keep the model as comparable as possible to the benchmark competitive case, we assume that intermediate goods producers receive a subsidy that is proportional to their output and are simultaneously subject to a lump-sum tax to ensure that it is revenue neutral. Given the subsidy, firms choose to set prices as close as possible to marginal cost. That is, they set

$$p_t(i) Q_t = w_t Q_t,$$

if i belongs to the fraction λ of firms who set $p_t(i) Q_t$ at the beginning of period t. If i instead belongs to the fraction $1-\lambda$ who set $p_t(i) Q_t$ at the end of $t-1$,we have that

$$p_t(i) Q_t = E_{t-1}[w_t Q_t],$$

Note that if $\lambda = 1$, the model collapses to the standard competitive flexible price environment, since in this case all firms price at marginal cost. Furthermore, in that case, M_t is equal to effective labor supply N_t.

In steady-state there are no aggregate shocks, so all intermediate-good producing firms will have had enough time to choose their prices optimally, and prices coincide with what would have been set in the absence of nominal rigidities. At $t = 1$, a fraction $1-\lambda$ sets $p_1(i) Q_1 = p_0(i) Q_0$, whereas the remaining adjust, setting $p_1(i) Q_1 = w_1 Q_1$. From $t = 2$ onwards, the economy proceeds as in an environment without nominal rigidities since there are no more surprises.[4]

[4]This approach to modelling nominal rigidities is similar to the one adopted by Oh and Reis (2012).

2.3 Government

Finally, the government taxes and transfers resources to households. It also consumes some of the output. Government consumption has no utility advantage to households. Its budget constraint is:

$$B_t = (1 + r_t)\, B_{t-1} + G_t + T_t$$

where B_t is government debt, G_t is government consumption, and T_t is net transfers. In equilibrium, T_t is an aggregation of the transfers net of the taxes collected from all households and firms arising from taxation rules ($\tau(y)$, $\omega(s)$, $x_t(a)$). Government consumption G_t is chosen to implement a particular pre-determined, nonexplosive, and perfectly forecasted path for government debt.

Monetary policy is as follows: In steady-state, the government achieves price stability so that the price level is fixed at some level Q_0. While it seeks to achieve that price level in all periods, in $t = 1$ it does not adjust its policy immediately, so that the nominal interest rate between period 1 and 2 $(1 + r_2)\frac{Q_2}{Q_1}$ is equal to its steady state level $1 + r_0$. For $t \geq 2$ onwards it is then able to set Q_t back to its steady-state target level Q_0.[5]

2.4 Equilibrium

Equilibrium is given by sequence of prices $\{w_t, r_t, Q_t\}_{t=0}^{\infty}$, value functions $\{V_t(a,s), \Xi_t(K)\}_{t=0}^{\infty}$, choices for intermediate-goods producers $\{m_t(i), p_t(i), \pi_t(i)\}_{t=0}^{\infty}$ $\forall i \in [0,1]$, policy functions for households $\{c_t(a,s), l_t(a,s), a_t'(a,s)\}_{t=0}^{\infty}$, intermediate-goods input choices for final goods producers $\left\{m_t^d(i)\right\}_{t=0}^{\infty} \forall i \in [0,1]$, aggregate variables $\{C_t, L_t, N_t, A_t, \Omega_t, K_t, M_t, B_t, X_t, G_t, T_t, \pi_t\}_{t=0}^{\infty}$, and the joint density of assets and idiosyncratic shocks $\Gamma_t(a,s)$ such that, given the path for transfer policies $\{x_t(a)\}_{t=0}^{\infty}$, the policy functions, value functions, and sequences of K_t and M_t correspond to the optimal choices of households and final-goods producers, the choices for intermediate-goods producers are also optimal, the government follows fiscal and monetary policy rules as described in Section 2.3 and:

1) Final-goods market clears:

$$C_t + K_t = F(K_{t-1}, N_t) + (1 - \delta)\, K_{t-1}$$

2) Intermediate-goods markets clear:

$$m_t(i) = m_t^d(i)$$

3) Labor market clears:

$$N_t = \int_0^1 n_t^d(i)\, di$$

[5] Such a monetary policy rule is employed in Krugman (1998) and Krugman and Eggertson (2012) among others. In a wide range of sticky-price models, the government is able to implement policies of that kind even in a limiting cash-less economy, given suitable out-of-equilibrium commitments on its part. See, for example, Woodford (2011).

4) Capital market clears:

$$K_t + B_t = A_t$$

5) Net transfers are $T_t = \Omega_t + X_t$

6) Profits from intermediate-goods producers are rebated to households:

$$\pi_t = \int_0^1 \pi_t(i)\, di$$

7) For any interval $[a_1, a_2] \in \mathcal{A}$, the distribution of idiosyncratic states evolves as:

$$\int_{a_1}^{a_2} \Gamma_{t+1}(a, s)\, da = \sum_{\tilde{s} \in \mathcal{S}} \Pr[s|\tilde{s}] \Pr[\tilde{s}] \int_{a \in \mathcal{A}'_t(a_1, a_2, \tilde{s})} \Gamma_t(a, \tilde{s})\, da$$

Where $a \in \mathcal{A}'_t(a_1, a_2, \tilde{s})$ if $a'_t(a, \tilde{s}) \in [a_1, a_2]$.

2.5 Redistribution Experiment

We consider an experiment where at $t = 0$ the economy is in steady state, corresponding to the state approximated by the economy when $x_t(a) = 0\ \forall a$ for sufficiently long time. In the subsequent period, $t = 1$, the after-tax transfer function changes from $x_0(a) = 0\ \forall a$, to $x_1(a) \neq 0$ for some a. From $t = 2$ onward, wealth taxes revert back to $x_0(\cdot)$, i.e., $x_t(a) = x_0(a) = 0\ \forall a$ for $t \geq 2$. We furthermore impose that the redistribution be revenue neutral, that is,

$$\sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} x_1(a)\, \Gamma_0(a, s)\, da = 0$$

The surprise and one-off nature of the shock is designed to isolate the role of heterogeneity on marginal propensities that lie at the heart of conventional intuition linking redistribution to short-run aggregate outcomes. In particular, it prevents redistribution from affecting the return to savings or labor effort directly. Thus, the presence or absence of aggregate effects of redistribution are not driven by the distortionary effects of taxes on labor or capital, but by differences in marginal propensities to consume and work out of wealth. This means that the policies we consider would have no real effect whatsoever in an economy where households can be aggregated into a single representative household. Furthermore, revenue neutrality allows us to isolate the role of wealth redistribution across households in a moment in time from the role of government debt. The latter has well-known real effects given the failure of Ricardian Equivalence in models of the class we study. Thus, in our experiments, heterogeneity in the response of households to wealth transfers is the sole underlying source of the dynamics.

3 Analytics

We now proceed to present a set of analytical results which lay out various forces determining the initial impact to the equilibrium paths of an economy subjected to the one-off, unexpected redistribution experiment described in Section 2.5. Our first proposition highlights the centrality of labor supply in determining the general equilibrium impact of redistributive policies in a setting without nominal rigidities. We then discuss the extent to which the proposition is helpful in understanding environments with nominal rigidities.

3.1 Redistribution and Aggregate Expenditures

We start by examining the impact of a redistributive policy on aggregate expenditures, defined as the sum of government consumption, household consumption, and gross fixed investment:

$$E_t \equiv G_t + C_t + K_t - (1-\delta) K_{t-1}$$

Thus, E_t corresponds to the NIPAs expenditure-side definition of GDP. We show that upon impact aggregate expenditures depend fundamentally on aggregate labor supply. In particular, we show that if aggregate labor supply remains unaffected by the wealth redistribution, then aggregate expenditures do not change on impact, so that $E_1 = E_0$, and neither do wages, so that $w_1 = w_0$.

Consider the household budget constraint at $t = 1$, with the choice variables already substituted for the corresponding policy functions.

$$a_1'(a,s) + c_1(a,s) = w_1 \varepsilon(s)\, l_1(a,s;w_1) + (1+r_1)\, a + \omega_1(s) - \tau\left(w_0 \varepsilon(s)\, l_1(a,s;w_1) + r_1 a + \omega(s)\right) + x_1 + \pi_1$$

where we make it explicit that the policy function for labor supply $l_1(\cdot)$ depends, among other things, on current wages. Changes in the function over time thus represent changes in labor supply induced by different paths for the interest rate and future wages.

Integrate across all households and apply the definitions (1) through (6) to get:

$$A_1 + C_1 = w_1 N_1(w_1) + (1+r_1) A_0 + \Omega_1 + X_1 + \pi_1,$$

where, consistent with the change in notation above, we now make explicit the dependence of aggregate labor supply, N_1, on current wages. Note that the government budget constraint implies that $X_1 + \Omega_1 = -G_1 + B_1 - (1+r) B_0$, and in equilibrium $A_1 = K_1 + B_1$. Thus, with some rearranging,

$$K_1 + C_1 + G_1 = w_1 N_1(w_1) + (1+r_1) K_0 + \pi_1.$$

We thus obtain the national accounting identity equalizing final expenditures with gross income:

$$E_1 = w_1 N_1(w_1) + (r_1 + \delta) K_0 + \pi_1.$$

3.1.1 No Nominal Rigidities

Absent nominal rigidities, $\lambda = 1$, all intermediate-goods producers choose the same price $p_1(i) = w_1$ (recall that the subsidy keeps them from charging a markup), so that $P_1 = \left[\int_0^1 p_1(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} = w_1$. It thus follows from the first order condition for the final-goods producer (8) that $w_1 = F_M(M_1, K_0)$. Also, it is the case that $M_1 = \left[\int_0^1 m_1(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} = N_1$ and therefore we can substitute M_1 for $N_1(w_1)$ in the production function so that, in equilibrium, $r_1 = F_K(N_1(w_1), K_0) - \delta$ and $\pi_1 = 0$. Thus, equilibrium aggregate expenditures are, upon impact,

$$E_1 = F_M(N_1(w_1), K_0) N_1(w_1) + F_K(N_1(w_1), K_0) K_0. \tag{10}$$

Aggregate expenditures are, therefore, entirely determined by equilibrium labor supply $N_1(w_1)$ and pre-determined variables. We can verify that, if $N_1(w_0) = N_0(w_0)$, then there is an equilibrium with $w_1 = w_0$ and hence $E_1 = E_0$. To see that, suppose the equilibrium features $w_1 = w_0$. Then, in that case, $w_1 = F_M(N_1(w_0), K_0) = F_M(N_0(w_0), K_0) = w_0$.

We state the result above in the following proposition:[6]

Proposition 1 *Consider the impact of a one-off and completely unexpected wealth redistribution. Suppose $\lambda = 1$, and the equilibrium reaction is such that aggregate labor supply does not change on impact ($N_1(w_0) = N_0(w_0)$). Then, if the equilibrium is unique, aggregate expenditures do not change on impact ($E_1 = E_0$) and neither does the wage rate ($w_1 = w_0$).*

Proposition 1 provides an important benchmark, since, as we later discuss, some commonly used assumptions about preferences and/or production opportunities yield constant labor supply as an equilibrium outcome. The proposition is significant because it shows in very stark terms the limitations of focusing on the behavior of consumption in order to understand the short-run effects of redistributive policies in the class of environments without nominal rigidities. This is because it does not require any assumption about the equilibrium behavior of consumption. In particular, in a setup without nominal frictions, knowledge about the distribution of marginal propensities to consume out of wealth is completely uninformative about the aggregate impact of the redistribution. Also, the lack of short-run reaction of output and labor input to the redistribution is true even though the equilibrium interest rate r_2 can potentially change significantly.

The proposition also implies that, absent a labor supply effect, any boom in consumption engineered by a redistributive policies translates itself into a *reduction* in other components of aggregate expenditures. This means that, absent compensating reductions in government expenditures, a redistributive policy that leads to a boom in consumption will also lead to a reduction in investment and the capital stock, thus having negative effects on aggregate output in subsequent periods.

[6]The proposition generalizes Proposition 2 in Mehrotra (2012). The key differences are that we do not require the labor supply function to depend only on current wages,.and that our proposition applies to economies with more than two types of agents.

3.1.2 Nominal Rigidities

With nominal rigidities, Proposition 1 does not hold in general. This is for two reasons. First, with nominal rigidities, profits distributed to households are not always zero. As a result, we have,

$$E_1 = w_1 N_1 (w_1) + (\delta + r_1) K_0 + \pi_1.$$

Even if $N_1 (w_0) = N_0 (w_0)$, $w_1 = w_0$ and $r_1 = r_0$, then expenditures can still change so long as profits change. Such a change can occur for example due to changes in nominal variables, which drive a wedge between the price planned by intermediate goods producers and their marginal cost.

Second, since wages can be different from the marginal product of labor, constant aggregate labor supply does not guarantee that equilibrium wages remain constant. Therefore, an equilibrium in which current wages and, therefore, labor supply respond on impact is entirely possible.

Nevertheless, we can construct an example where, given the monetary policy rule described in Section 2.3, the equilibrium with nominal rigidities is identical to the one without. The example consists of an economy where the marginal products of capital and the composite intermediate input to final goods production are invariant to quantities. In this case, $F_K (K_0, M_1) = f_k$ and $F_M (K_0, M_1) = f_M$, where f_K and f_M are constants. As a result, the real interest rate is fixed at $r_t = f_K - \delta$ and the real price index for intermediate inputs is fixed at $P_t = f_M$. From the monetary policy rule we have that:

$$1 + r_0 = (1 + r_2) \frac{Q_2}{Q_1},$$

which, substituting in $r_0 = r_1 = f_K - \delta$ and for $Q_2 = Q_0$ given by the fact that the policy rule takes the price level back to its steady-state level after $t = 1$, yields

$$(1 - \delta + f_K) = (1 - \delta + f_K) \frac{Q_0}{Q_1},$$

so that

$$Q_0 = Q_1.$$

From the definition of the price index for materials,

$$\begin{aligned} P_1 &= \left[\int_0^1 p(i)^{1-\theta} \, di \right]^{\frac{1}{1-\theta}} \\ &= \left[\int_0^1 \left(\lambda \left(p_1^{flex} \right)^{1-\theta} + (1 - \lambda) \left(p_1^{fix} \right)^{1-\theta} \right) di \right]^{\frac{1}{1-\theta}} \end{aligned}$$

where p_1^{flex} is the price set by firms not subject to nominal rigidities and p_1^{fix} is the price of those subject to nominal rigidities. As discussed above, in steady-state all firms set their price equal to their marginal cost (recall that we allow for subsidies in order to abstract from distortions induced by monopoly), so that nominal price set by firms subject to nominal rigidities is $w_0 Q_0$ and their price in terms of final goods $p_1^{fix} = w_0 \frac{Q_0}{Q_1}$. Flexible price firms set price equal to marginal cost w_1. Substituting back into the definition of the price of materials as well as $P_1 = f_M$, yields

$$f_M = \left[\int_0^1 \left(\lambda w_1^{1-\theta} + (1-\lambda)\left(w_0 \frac{Q_0}{Q_1}\right)^{1-\theta}\right) di\right]^{\frac{1}{1-\theta}},$$

which, since $Q_0 = Q_1$, yields

$$f_M = \left[\int_0^1 \left(\lambda w_1^{1-\theta} + (1-\lambda)(w_0)^{1-\theta}\right) di\right]^{\frac{1}{1-\theta}}, \tag{11}$$

It is also immediate to verify that $w_0 = f_M$, since $F_M(K_0, M_0) = f_M$ and $p_0^{fix} = p_0^{flex} = w_0$. Furthermore, rearranging equation (11) for w_1, and taking into account that $w_0 = f_M$, we find that $w_1 = f_M$. Hence, $w_1 = w_0$. Finally, profits for firm i are $\pi_1(i) = (p_1(i) - w_1)\, m_1(i)$. Since $p_1(i) = w_1 = w_0$ for all i, $\pi_1(i) = 0$ for all i, and $\pi_t = 0$. Thus, equation (10) holds. It is also straightforward to verify that $w_t = f_M$ and $r_t = f_K$ for all values of t.

More generally, given that real wages and real interest rates are exactly the same irrespective of the degree of nominal rigidities, household decisions are also identical and, thus, are all equilibrium objects. We summarize these observations in Proposition 2 below:

Proposition 2 *Consider the impact of a wealth redistribution program that is one-off and completely unexpected. Suppose $F_K(K, M) = f_K$ and $F_M(K, M) = f_M$. Then the allocation does not depend on λ. In particular, if the equilibrium reaction is such that aggregate labor supply does not change on impact ($N_1(w_0) = N_0(w_0)$) and if the equilibrium is unique, aggregate expenditures do not change on impact ($E_1 = E_0$) and neither does the wage rate ($w_1 = w_0$)*

Proposition 2 is quantitatively relevant because the short-run impact of the redistributive policy on the capital stock and, hence, on its marginal product, is likely to be very small. This is because investment is only a small fraction of the total capital stock. If, moreover, the investment technology is approximately linear in investment, then interest rates will react minimally to any reasonably sized redistribution. Therefore, in order for nominal rigidities to have quantitative impact, one needs to allow for investment adjustment costs. In the quantitative section 4, we capture similar effects in a variant of the main quantitative exercise depicted in Figure 1 by increasing the degree of concavity for capital in the production function.

3.2 Redistribution and Aggregate Labor Supply

Given the importance of aggregate labor supply movements highlighted in Section 3.1, we now examine more closely how wealth redistribution affects labor supply decisions. In particular, we examine which assumptions change aggregate labor supply, as well as the direction of the implied changes. As we will show, a key driving force is how marginal propensities to work correlate with wealth across the population. We locate conditions under which these correlations are positive, negative, or zero.

For the analysis that follows, it is convenient to separate intra-temporal decisions made by households over consumption and leisure from intertemporal decisions over wealth accumulation. We therefore start by defining z to be the sum of the household's expenditures on goods *and* leisure. Thus,

$$z \equiv c + w\varepsilon(s)\left(\bar{l} - l\right).$$

For most of this section, we examine first the case where there is no distortionary income tax, i.e., where $\tau(y) = 0$ for all y. Later in this subsection, we return to the case with distortionary taxation in order to examine its role. Without taxes, the problem of the household can be written as

$$\begin{aligned} V_t(a,s) &= \max_{b',c,l} \tilde{u}(z; w_t, s) + \beta \sum \Pr\left[s'|s\right] V_{t+1}\left(a', s'\right) \\ s.t. &: a' + z = w_t \varepsilon(s)\bar{l} + (1 + r_t)a + \omega(s) + \pi_t, \\ & \quad a \geq \underline{a}, \end{aligned}$$

with $\tilde{u}(\cdot)$ the value function obtained from the within-period problem

$$\begin{aligned} \tilde{u}(z; w, s) &= \max_{c,l} u\left(c, \bar{l} - l\right) \\ s.t. &: c + w\varepsilon(s)\left(\bar{l} - l\right) = z. \end{aligned}$$

In addition to the usual policy and value functions, the problem above also yields the policy function for z, $z_t(a,s)$. Aggregate expanded consumption expenditures therefore are given as:

$$Z_t = \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} z_t(a,s)\,\Gamma_{t-1}(a,s)\,da$$

Finally, we can write policy functions for consumption and hours coming from static sub-problem as:

$$\begin{aligned} c &= c^{static}(z, s, w) \\ l &= l^{static}(z, s, w) \end{aligned}$$

Naturally, it is the case that $c_t(a,s) = c^{static}(z_t(a,s), s, w)$ and $l_t(a,s) = l^{static}(z_t(a,s), s, w)$. The alternative, static, policy functions are useful in that they clarify that the choice of consumption and labor supply only depend on the aggregate state through wages w_t and through whatever its impact is on the choice of expanded consumption expenditures z.

3.2.1 No Wealth Effects on Labor Supply

One immediate implication from the decomposition above is that labor supply can only change in response to a redistribution if $l^{static}(z,s,w)$ depends on expanded consumption z. It then follows immediately from Proposition 1 that wealth redistribution upon impact keeps labor supply unchanged. The leading case where $l^{static}(z,s,w)$ does not depend on z is under preferences postulated by Greenwood et al. (1988). Under those preferences,

$$u\left(c,\bar{l}-l\right)=h\left(c+g\left(\bar{l}-l\right)\right)$$

where h is increasing and concave. At any t, the optimality condition for labor supply in the static problem satisfies

$$g'\left(\bar{l}-l_t^{static}\left(z,s;w\right)\right)=\varepsilon\left(s\right)w,$$

so labor supply depends, at the individual level, only on the current effective wage rate $\varepsilon(s)\,w$. Thus,

$$N_0\left(w_0\right)=N_1\left(w_0\right)=\sum_{s\in\mathcal{S}}\varepsilon\left(s\right)\left(\bar{l}-g'^{-1}\left(\varepsilon\left(s\right)w_0\right)\right)\Gamma_0^s\left(s\right)$$

where $\Gamma_0^s(s)$ is the marginal distribution of exogenous idiosyncratic productivity states s at time 0. Therefore, in this case, we have that $N_1(w)=N_0(w)$ for all w and, in particular, for $w=w_0$. The result highlights the essentiality of wealth effects on labor supply in determining the aggregate short-run impact of wealth redistribution.

3.2.2 Wealth Effects on Labor Supply

We now examine the effect of redistribution in the presence of wealth effects on labor supply. We start with the benchmark case of Cobb-Douglas utility within periods, and then we proceed to examine more general cases.

Cobb-Douglas Utility We will define a utility function to be "Cobb-Douglas *within the period*" if it is of the form

$$u\left(c,\bar{l}-l\right)=h\left(c^{1-\mu}\left(\bar{l}-l\right)^{\mu}\right),$$

where h is an increasing and strictly concave function. The utility function is, of course, fairly standard. In particular, it is widely used in the business cycle literature, as some of its particular cases are compatible with a balanced growth-path for the economy. Also, it is featured in heterogeneous agent models such as Krusell and Smith (1998).

Given Cobb-Douglas preferences, we have that expenditures on consumption and leisure are a constant fraction of z, so that:

$$
\begin{aligned}
c^{static}(z, s; w) &= (1-\mu) z, \\
l^{static}(z, s; w) &= \bar{l} - \frac{\mu}{w\varepsilon(s)} z.
\end{aligned}
$$

Thus, for a given wage, any shock that leads a household to increase its consumption will also lead it to decrease its labor supply. More generally, any change in the environment that leads households to reduce savings (and, therefore, increase z), will also induce them to increase consumption and reduce labor supply.

This same logic translates to aggregate quantities. To see this, first integrate $w_t\varepsilon(s)\left(\bar{l} - l_t(a,s)\right) = \mu z_t(a,s)$ across households to get

$$
\begin{aligned}
N_t &= \bar{N} - \mu \frac{Z_t}{w_t}, && (12) \\
C_t &= (1-\mu) Z_t && (13)
\end{aligned}
$$

where $\bar{N} \equiv \bar{l} \sum_{s \in \mathcal{S}} \Gamma_0^s(s) \varepsilon(s)$. Thus, given the wage rate, labor supply is *decreasing* in the amount of wealth allocated for current period consumption. Second, aggregating household budget constraints yields the following relationship between aggregate savings A_t and aggregate expanded consumption Z_t:

$$
A_t = w_t \bar{N} + (1 + r_t) A_{t-1} - Z_t.
$$

It follows that effective hours worked are increasing in aggregate savings. Hence, any change in the economic environment that leads to a reduction in aggregate savings, will, for a given wage, lead at the same time to an increase in aggregate consumption and a reduction in aggregate effective hours worked.

In particular, if prices are fully flexible, and aggregate savings do not change (for example because asset supply is fixed, as in Guerrieri and Lorenzoni (2012), or adjustment costs to capital are extremely large), Proposition 1 holds, and neither aggregate labor supply, aggregate consumption, nor aggregate expenditures change. More generally, given decreasing returns to labor input, under price flexibility any redistribution that increases aggregate consumption C_t, will also reduce effective labor supply. To see this, recall that with flexible prices, wages satisfy $w_t = F_M(N_t, K_{t-1})$ (since $M_t = N_t$) so that, holding K_0 fixed, w_1 is decreasing in N_1. Substituting out Z_t from the expression for aggregate labor supply (12), we have that

$$
\begin{aligned}
N_1 &= \bar{N} - \frac{\mu}{1-\mu}\frac{C_1}{w_1} \\
&= \bar{N} - \frac{\mu}{1-\mu}\frac{C_1}{F_M\left(N_1, K_0\right)}
\end{aligned}
$$

This induces an implicit function of N_1 on C_1 with derivative given by the implicit function theorem:

$$
\frac{dN_1}{dC_1} = -\frac{\frac{\mu}{1-\mu}\frac{1}{F_M(N_1,K_0)}}{1-\frac{\mu}{1-\mu}\frac{C_1 F_{MM}(N_1,K_0)}{(F_M(N_1,K_0))^2}}
$$

Thus, given that $F\left(M_1, K_0\right)$ is neo-classical (so that $F_{MM}\left(M_1, K_0\right) \leq 0$),any increase in C_1 leads to a reduction in N_1.

We state the result in the following Proposition:

Proposition 3 *If $\lambda = 1$, $Z_1 = Z_0$, $u\left(c, \bar{l} - l\right) = h\left(c^{\kappa}\left(\bar{l} - l\right)^{1-\kappa}\right)$, and there are no taxes and transfers, then $w_1 = w_0$, $N_1 = N_0$, and $C_1 = C_0$. If $Z_1 > Z_0$, then $C_1 > C_0$ and $N_1 < N_0$, with the inequalities reversed if $Z_1 < Z_0$.*

Note that with Cobb-Douglas preferences, it is *not* necessarily the case that wealthier households consume more leisure, since what is being kept constant are expenditures in leisure as a fraction of wealth. If wealthier households are also more productive (so that $\varepsilon\left(s\right)$ is higher), they can conceivable supply much more labor time than less wealthy households. The reason effective labor supply stays constant is that aggregate effective hours worked is the sum of individual labor supply weighted by productivity $\varepsilon\left(s\right)$.

Proposition 3 implies that Cobb-Douglas preferences yield a particular kind of aggregation in consumption and labor-supply decisions. A natural question then is: How stringent is the Cobb-Douglas requirement? In classical demand theory, homothetic preferences are typically sufficient to allow for aggregation of household choices, meaning that wealth redistribution does not change outcomes. The reason this is not sufficient in the current setting is because different households face effectively different wages. If the elasticity of substitution between consumption and leisure is not equal to 1, this implies that high-productivity households consume a different share of their expanded consumption, z, than low-productivity households, so that redistribution between them will have an impact on aggregate effective labor supply.

Small Redistributions with General Preferences We now turn to the impact of redistributive policies for general preferences. While all preceding results hold for any wealth redistribution, we now focus on redistributions that transfer from the wealthy to the poor, that is, that feature an $x_1\left(a\right)$ that is monotonically decreasing in a. In general, the impact of the redistribution on effective labor supply is

$$
N_1 - N_0 = \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} \left(\varepsilon\left(s\right)\left(l_1\left(a, s\right) - l_0\left(a, s\right)\right)\right)\Gamma\left(a, s\right) da
$$

In the case of small redistribution policies, we can clearly separate the various competing ways in which wealth redistribution can affect labor supply. Restricting attention to policy functions that are differentiable with respect to all states and prices, and defining "small" redistributions to be those for which we can approximate the policy function $\phi_t\left(a+\frac{x_1(a)}{1+r_1},s\right)$ by $\phi_t\left(a,s\right)+\frac{\partial\phi_t(a,s)}{\partial a}\frac{x(a)}{1+r_t}$, we have that

$$\begin{aligned}
& \varepsilon\left(s\right)l^{static}\left(z_0\left(a+\frac{x_1\left(a\right)}{1+r},s\right),s;w_0\right)-\varepsilon\left(s\right)l^{static}\left(z_0\left(a,s\right),s;w_0\right) \\
\cong\ & \underbrace{\frac{x_1\left(a\right)}{1+r_1}}_{(+/-)}\underbrace{\frac{\partial z_0\left(a,s\right)}{\partial a}}_{(+)}\underbrace{\frac{\partial\varepsilon\left(s\right)l^{static}\left(z\left(a,s\right),s,w_0\right)}{\partial z}}_{(-)}.
\end{aligned} \tag{14}$$

with the signs in brackets indicating the direction of change in the corresponding terms. The expression above denotes the change in labor supply for a given household, holding prices fixed, and allowing only its wealth to change. It therefore captures the partial equilibrium component of a household's reaction. We denote the expression in equation (14) by $\mathcal{P}_1\left(a,s\right)$

The expression 14 has three components. The first one is the wealth transfer itself, normalized by the interest rate, so that it is in the same units as beginning-of-period assets. Given revenue neutrality, $x_1\left(a\right)$ is positive for very wealthy households and negative for less wealthy ones. Together, the two last terms give the household's marginal propensity to work out of financial wealth. The term $\frac{\partial z_0(a,s)}{\partial a}$ denotes the effect of increased wealth on "expanded consumption" z. This is, of course, inversely related to the marginal propensity to save out of wealth. It is positive if both consumption and leisure are normal goods, since an increase in wealth relaxes the intertemporal budget constraint of the household. The final component is the static wealth effect on *effective* labor supply. Overall, the partial equilibrium component is negative for households that are on the receiving side of the redistribution policy and positive for the households that are on the contributing side.

Price changes affect labor supply directly, through current wages w_1, and indirectly, through the function $z_0\left(a,s\right)$, which, given other price changes, becomes $z_1\left(a,s\right)$. The general equilibrium component can therefore be written as

$$\begin{aligned}
& \varepsilon\left(s\right)l^{static}\left(z_1\left(a,s\right),s;w_1\right)-\varepsilon\left(s\right)l^{static}\left(z_0\left(a,s\right),s;w_0\right) \\
\cong\ & \sum_{v=1}^{\infty}\left(\frac{\partial z_0\left(a,s\right)}{\partial r_v}dr_v+\frac{\partial z_0\left(a,s\right)}{\partial w_v}dw_v\right)\frac{\partial\varepsilon\left(s\right)l^{static}\left(z_0\left(a,s\right),s,w_1\right)}{\partial z} \\
& +\frac{\partial\varepsilon\left(s\right)l^{static}\left(z_0\left(a,s\right),s,w_1\right)}{\partial w_1}dw_1.
\end{aligned} \tag{15}$$

The component in parenthesis summarizes whose savings decisions the path of prices over time affects. Those changes multiply the static wealth effect on labor supply. Additionally, current wages exert direct influence on labor supply for any given level of savings. We denote the expression in equation (15) by $\mathcal{G}_1\left(a,s\right)$. Collecting all terms, it follows that, to a first order approximation, the change in effective labor following a redistribution can be denoted by

$$N_1 - N_0 \cong \sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} \left(\mathcal{P}_1(a,s) + \mathcal{G}_1(a,s)\right)\Gamma(a,s)\, da.$$

We will from this point onward focus our analysis on the determinants of the partial equilibrium component $\sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} \mathcal{P}_1(a,s)\Gamma(a,s)\, da$. This ought to be informative about the overall impact of redistribution on labor input so long as the term is large relative to the general equilibrium component. Furthermore, in the quantitative analysis in Section 4 we verify that the intuitions derived from this analysis remain intact.

Before proceeding, we state a proposition linking the shape of the marginal propensity to work out of wealth $\frac{\partial z_0(a,s)}{\partial a}\frac{\partial \varepsilon(s) l^{static}(z(a,s),s,w_0)}{\partial z}$ to the partial equilibrium component of the change in N. Let $\Gamma_0(s|a)$ denote the proportion of households with exogenous productivity s given wealth a. Then, the following proposition holds:

Proposition 4 *Given a "small" wealth redistribution with $x_1(a)$ decreasing in a, the partial equilibrium component of its impact on aggregate effective labor supply is given by $\sum_{s\in\mathcal{S}} \int_{a\in\mathcal{A}} \mathcal{P}_1(a,s)\Gamma(a,s)\, da$. This component is positive if $\sum_{s\in\mathcal{S}} \left|\frac{\partial z_0(a,s)}{\partial a}\frac{\partial \varepsilon(s) l^{static}(z(a,s),s,w_0)}{\partial z}\right| \Gamma_0(s|a)$ is increasing in a.*

The proposition states that the partial equilibrium component of a redistribution is positive if the labor supply of high-wealth households is more sensitive to their wealth. Intuitively, if this is the case, high-wealth households will increase their effective labor supply more in response to a dollar lost than a low-wealth household will reduce theirs in response to that same dollar received. This is the analogue to the conventional intuition about how heterogeneity in marginal propensities to consume influence the aggregate consumption response to redistribution. Note that if wealth effects on labor supply are always zero, as in Greenwood-Hercowtiz-Huffman preferences, then the partial equilibrium component is also zero. We now proceed to discuss each of the terms determining the marginal propensity to work separately.

Intertemporal Determinants of Aggregate Labor Supply The first component determining the marginal propensity to work, $\frac{\partial z_0(a,s)}{\partial a}$, denotes how wealth affects the intertemporal choices made by the household. This is because, given the budget constraint, this term is symmetric to the change in wealth accumulation decision, so that $\frac{\partial z_0(a,s)}{\partial a} = 1 + r_1 - \frac{\partial a_0'(a,s)}{\partial a}$. Therefore, $\frac{\partial z_0(a,s)}{\partial a}$ is increasing in a if the policy function for asset accumulation is concave in a.

The literature on incomplete market models points to a strong presumption that $a_0'(a,s)$ ought to be convex, so that $\frac{\partial z_0(a,s)}{\partial a}$ is decreasing in a. The reason, emphasized by Gourinchas and Parker (2002) is that wealth-poor households engage in buffer-stock savings behavior, going through a lot of effort to save to self-insure against income risk. As they grow wealthier, they are able to self-insure without having to consume so little and work so hard, so they increase their expenditures and reduce their hours worked very quickly. For very wealthy households, marginal changes in their wealth do not affect their ability to self-insure. Thus, their main reason for saving is for intertemporal smoothing, saving more out of a given dollar received. To the extent that this logic carries through to a world with two goods (consumption and leisure), this implies that wealthy households will not change their consumption and labor supply much in response to a one-off, temporary change in their wealth. Thus, redistribution from wealth-rich to wealth-poor households is likely to lead to a boom in aggregate consumption and a drop in aggregate effective labor supply.

Given the consensus in the literature that $a_0'(a,s)$ is convex, proposition (4) suggests that wealth redistributions can very plausibly lead to a boom in consumption at the same time as they lead to a drop in effective hours worked. One case where this holds is when preferences are Cobb-Douglas and there are no income taxes.[7] In that case, the intratemporal component $\frac{\partial \varepsilon(s) l^{static}(z(a,s),s,w_0)}{\partial z} = -\frac{\mu}{w_0}$, so that it does not vary across households, and the partial equilibrium component of the change in labor supply is entirely determined by the shape of $\frac{\partial z_0(a,s)}{\partial a}$. We now turn to a more detailed discussion of which features of preferences and the tax-code determine the variation $\frac{\partial \varepsilon(s) l^{static}(z(a,s),s,w_0)}{\partial z}$.

Intratemporal Determinants of Aggregate Labor Supply The second component determining the marginal propensity to work, $\frac{\partial l_0^{static}(z,s;w)}{\partial z}$, denotes how changes in the amount of resources dedicated to current period expenditures, and hence to savings, affect labor supply. It, therefore, relates to the intra-temporal decision that households make about how to allocate these resources between consumption and leisure. Such intra-temporal considerations are important since, in typical calibrations, intra-temporal preferences between consumption and leisure do not satisfy homotheticity. Notably, Pijoan-Mas (2006) has emphasized the fact that departure from homotheticity is necessary to explain the low observed cross-sectional correlation between wages and hours. A similar departure is present in the calibration used by Castaneda et al. (2003) and for the experiment in Figure 1.

We characterize the shape of $\frac{\partial \varepsilon(s) l_0^{static}(z,s;w)}{\partial z}$ in terms of the share of expenditures dedicated to leisure, $\mu(z;s,w) = \frac{w\varepsilon(s)\left(\bar{l} - l^{static}(z;s,w_t)\right)}{z}$. This allows us to connect the cross-sectional variation in the static component of labor supply to classical demand theory, which emphasizes the role of variation of expenditure shares with wealth. We focus on cases in which μ is twice differentiable in z and denote by $\mu_z(z;s,w)$ and $\mu_{zz}(z;s,w)$ the first and second derivatives with respect to z. We have that:

Proposition 5 *Consider the partial equilibrium component of the impact of a wealth redistribution $x_1(a)$, with x decreasing in a. Then $\frac{\partial \varepsilon(s) l_0^{static}(z,s;w)}{\partial z}$ increases with z if $\mu_z(z;s,w_t) > 0$ and $\frac{\mu_{zz}(z;s,w_t) z}{\mu_z(z;s,w_t)} > -2$ and it decreases with z if $\mu_z(z;s,w_t) < 0$ and $\frac{\mu_{zz}(z;s,w_t) z}{|\mu_z(z;s,w_t)|} < 2$.*

The proposition states that redistribution increases labor supply if leisure is a luxury good in the intratemporal problem, i.e., if households spend an increasing fraction of their "expanded" expenditures with leisure rather than consumption goods as expanded expenditures increase. The bounds on the curvature of the share function $\mu(z;s,w)$ ensure that the intensity with which households of different wealth levels spend on leisure does not change too quickly with their wealth.

[7] This also holds for proportional income taxes. See Lemma 1.

It is important to notice that the luxury nature of a good is tied to the amount spent for given prices. Thus, it is entirely consistent with leisure being a luxury good that a wealthier household consumes less leisure than a less wealthy one, so long as they face different wages (since they have different productivities).

The following corollary shows the proposition applies to the commonly used case of separable utility:

Corollary 1 *Suppose the utility function is:*

$$u\left(c, \bar{l}-l\right) = c^{1-\sigma_C} + \phi\left(\bar{l}-l\right)^{1-\sigma_L}$$

with $\sigma_c > 0$, $\sigma_l > 0$, and $\phi > 0$. Then, $\frac{\partial \varepsilon(s) l_0^{static}(z,s;w)}{\partial z}$ increases with z if $\sigma_L < \sigma_C$ and decreases otherwise.

Thus, wealth transfers have a stimulative effect so long as the intertemporal elasticity of substitution for leisure ($\frac{1}{\sigma_L}$) is larger than for consumption ($\frac{1}{\sigma_C}$). In the intratemporal problem of the household, this implies that leisure is a luxury good and consumption is a necessity. This relation between σ_L and σ_C is exactly the one that Pijoan-Mas (2006) has argued for.

The conditions for the propositions above appear to hinge on properties of households' underlying preferences. However, it is important to note that, subject to mild regularity conditions, the tax code can induce households to behave as if leisure were a luxury good even if underlying preferences are Cobb-Douglas. More formally, the following Lemma holds:

Lemma 1 *Suppose taxes are progressive, so that $\tau''(y) > 0$ and $u\left(c, \bar{l}-l\right) = h\left(c^{\kappa}\left(\bar{l}-l\right)^{1-\kappa}\right)$, with h increasing and concave. Suppose furthermore that wealth effects on labor supply are mild enough that $ra + w\varepsilon(s) l_0(a,s)$ is increasing in a. Then the ratio $\frac{w\varepsilon(s)\left(\bar{l}-l\right)}{z}$ increases with z.*

Thus progressive taxation can be a further factor leading to a more positive impact of redistributive policies.

3.2.3 Indivisible Labor

We have thus far focused on cases in which labor supply can be smoothly varied. At higher than annual frequencies households might be unable to adjust labor in such a smooth fashion. Therefore, we conclude the analytical section by examining the short-run effect of wealth redistribution when labor supply is indivisible. The extension of the heterogeneous agent model for this case has been examined by Alonso-Ortiz and Rogerson (2010) among others. In particular, Alonso-Ortiz and Rogerson (2010) show that labor supply functions are characterized by a threshold rule, such that for each exogenous productivity level, there is a cutoff level of wealth, $\overline{a}(s)$, such that labor supply is zero if wealth exceeds the threshold and maximal otherwise. The proposition below characterizes the partial equilibrium response of such an economy to a wealth redistribution.

Proposition 6 *Consider a "small" redistributive policy $x_1(a)$, with x decreasing in a. Suppose labor supply decisions are characterized by a threshold $\bar{a}(s)$ so that $l(a,s) = 0$ if $a > \bar{a}(s)$ and $l(a,s) = \bar{l}$ if $a \leq \bar{a}(s)$. Then, the partial equilibrium component of the change in effective labor supply is positive if and only if*

$$\sum_{s \in \mathcal{S}} \varepsilon(s)\, x(\bar{a}(s))\, \Gamma(\bar{a}(s), s) > 0.$$

Therefore, output increases if the workers at threshold points are, on average, net contributors to the redistribution, where the average is weighted by how much they receive and by their labor productivities. The result highlights that the direction of the impact of redistribution on output is highly sensitive to the details of the program and of the joint distribution of wealth and productivity.

4 Quantitative Analysis

Our analytical results highlight that the short-run effect of a wealth redistribution depends fundamentally on the specific factors that influence labor supply decisions, including properties of household savings behavior. Therefore, the sign and size of the overall impact can only be resolved with the use of a quantitative model. Given our focus on the short-run reaction of the economy to a one-off redistributive shock, it is essential that the initial state accurately captures observed wealth heterogeneity in the U.S. economy. We therefore employ a baseline incomplete markets model that is specifically calibrated to match the wealth distribution in the United States along several dimensions, including the extreme concentration in its right tail.

4.1 Parameterization

With the exception of the parameters we specify in this section, we follow the parameterization of Castaneda et al. (2003), since more standard parameterizations, e.g. Aiyagari (1994), normally miss the concentration present in the right tail of the distribution. Two salient features of that parameterization are as follows: First, the authors specify a labor productivity process designed to capture life-cycle movements in income. As such they include "retirement" and "rebirth" shocks, the first of which turns the labor productivity of households to zero while the second one restores it to a positive value. Second, in order to capture the extreme concentration of wealth in the U.S. economy, Castaneda et al. (2003) employ an extremely high and brief productivity state, which is reached with very low probability.

We adopt a yearly calibration of the model. Outcomes at annual frequencies are likely to be the relevant ones for macroeconomic policymakers, as opposed to monthly or quarterly frequencies. Moreover, because households are more likely to have discretion over hours worked over the course of a year than over the course of a month or a quarter, flexible labor supply is the appropriate assumption. This, in turn, confers the tractability that we exploit to derive analytics.

Since Castaneda et al. (2003) examine steady states in which nominal rigidities are irrelevant, we need to select parameters for the nominal frictions. We examine two cases: one with $\lambda = 1$, corresponding to the benchmark without nominal rigidities and one with $\lambda < 1$. For the second case, we chose $\lambda = 0.5$, so that half of the firms change prices in a given year. Given that one period corresponds to a year, this is in line with the evidence in Bils and Klenow (2004) that firms take on average close to half a year to change their prices. We also set the elasticity of substitution between different varieties of intermediate goods $\theta = 10$, as suggested by Woodford (2003).

In the baseline case, the production side of the economy is a Cobb-Douglas production function for final goods production. As we will see, in that case there is very little quantitative short-run impact on the interest rate. To understand the effects of changes in interest rate upon equilibrium, we also investigate a variant of the model in which we augment the production function with a quadratic term $-\frac{1}{2}\left(K_t - K_0\right)^2$, so that interest rates become more sensitive to changes in the capital stock.[8]

The wealth transfer at the center of our analysis is structured to move wealth, a, from richer to poorer households and takes the functional form: $x(a) \equiv \eta - \chi a$, with η and $\chi < 1$. This function gives the net change in wealth after the transfer. We will consider a tax of 2 percent, i.e. set $\chi = 0.02$, and set η so that the transfer is revenue neutral.

5 Description of the Initial Conditions

We now describe the initial state of the economy immediately following redistribution but prior to agents having time to react. It is this initial condition that determines the short-run aggregate impacts. The first column in Table 1 shows the average change in wealth implied by the policy for different wealth quantiles. The average change in wealth is positive for the first four quintiles. Indeed, 75.16 percent of the households are net recipients of wealth transfers. Consistent with the very skewed wealth distribution in the data, we find that the wealth paid out by a single household in the upper wealth percentile funds transfers to approximately 50 households in the three lowest quintiles.

Table 1 also collects, for different wealth quantiles, the conditional means of (1) the change in wealth, referred to above ($x_1(a)$), (2) the marginal propensity to save, defined as $\frac{\partial a'(a,s)}{\partial a} - 1$, and denoted by MPS, (3) the marginal propensity to consume out of wealth, defined as $\frac{\partial c(a,s)}{\partial a}$ and denoted by MPC, and (4) the marginal propensity to supply hours defined as $\frac{\partial l(a,s)}{\partial a}$ and denoted by MPL. This definition of marginal propensity to save (MPS) adjusts for the fact that, absent a behavioral response, households would mechanically increase their wealth by the amount that they receive. The second column shows clearly that the mean marginal propensity to save rises with wealth. It is a symptom of low wealth pushing households to work more and consume less in order to build up precautionary balances. Thus, the shape of the policy function for savings conforms to the general finding in the literature, discussed in Section 3.2.2. If marginal propensity to save

[8] In that case, the production function exhibits decreasing returns to scale, so that final-goods producers earn economic profits. We assume that those profits are taxed lump-sum by the government and used to finance government expenditures.

increases with wealth, then the marginal propensity to consume decreases with wealth. At the same time, the marginal propensity to work becomes less negative across all wealth quantiles, except the first. It should be noted, that, given Castaneda et al.'s (2003) calibration, the first wealth quintile includes a large fraction of retirees, for whom labor supply is perfectly inelastic.

Table 1: Average Wealth Transfer and Marginal Propensities, by Wealth Quantile

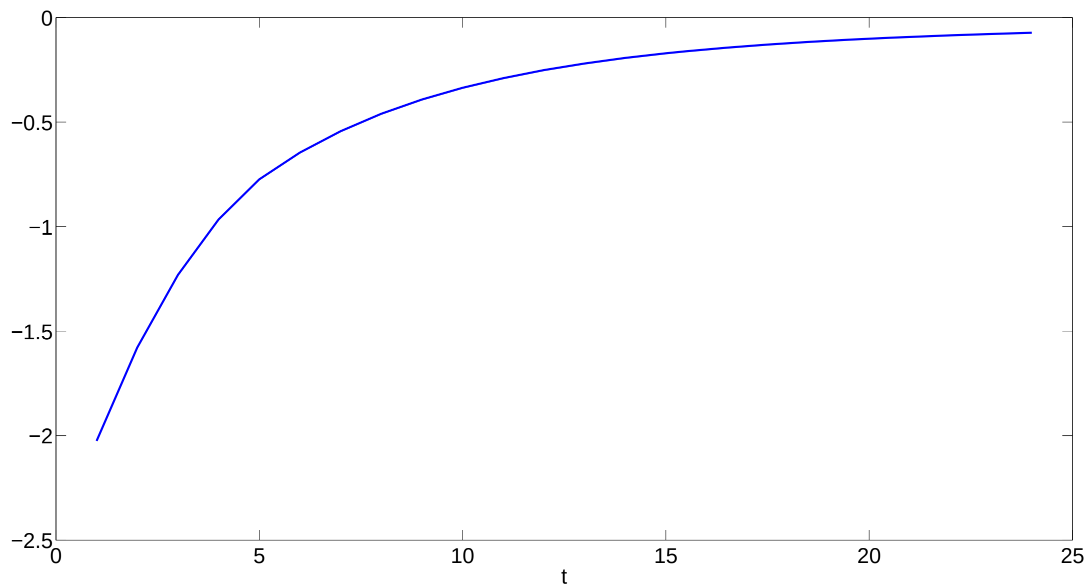
Wealth Quantiles	$x_1(a)$	MPS	MPC	MPL
First Quintile	0.2195	-0.6409	0.6399	-0.0571
Second Quintile	0.2107	-0.1934	0.0914	-0.1749
Third Quintile	0.2036	-0.1717	0.0898	-0.1497
Fourth Quintile	0.0642	-0.0911	0.057	-0.0555
Fifth Quintile	-0.699	-0.0884	0.0438	-0.0199
90-95	-0.5237	-0.0855	0.0466	-0.012
95-99	-0.7229	-0.1221	0.0355	-0.0173
99-100	-11.029	-0.0393	0.0602	-5.25E-04

5.1 Results

We now turn to the quantitative results. The state of the economy at any given point in time is given by the joint distribution of productivity and wealth. One summary of the state is wealth inequality. Figure 2 shows how inequality, as measured the Gini coefficient, evolves over the transition. We see that upon impact inequality falls by 2 percent. However, as the transition continues, we see that inequality monotonically increases throughout. The driving forces behind this mean-reversion include, most directly, the recipients' consumption of much of the transfer, as well as efforts by the donors to replenish their wealth balances. The relatively slow transition (with a half life of around five years) reflects households' attempts to smooth consumption, thus keeping wealth dynamics rather slow.

We now turn to the impact of the wealth redistribution program on macroeconomic aggregates. Figure 3 shows that the effect of the transfer program on aggregate output, consumption, investment, and hours is bounded above by around 2 percent. Output falls, as noted above, but does so over the entire transition by no more than one-quarter of 1 percent. In terms of dynamics, we see that the initial decline in output is followed by a slow increase over time as the economy returns to its pre-transfer steady-state equilibrium. If aggregate output does not change by much, its composition changes much more noticeably, with a boom in consumption and a bust in investment. There is also a large drop in hours worked, but a much more muted reduction in effective hours worked, i.e., the total amount worked weighted by individual labor productivity. This compositional change therefore generates an increase in aggregate labor productivity. As implied by proposition 1, the small drop in effective hours worked explains the small drop in output and the negative co-movement between consumption and investment. Remarkably, all aggregates have a fairly long transition back to their steady states, with half-lives of approximately five years, like

Figure 2: Gini Coefficient

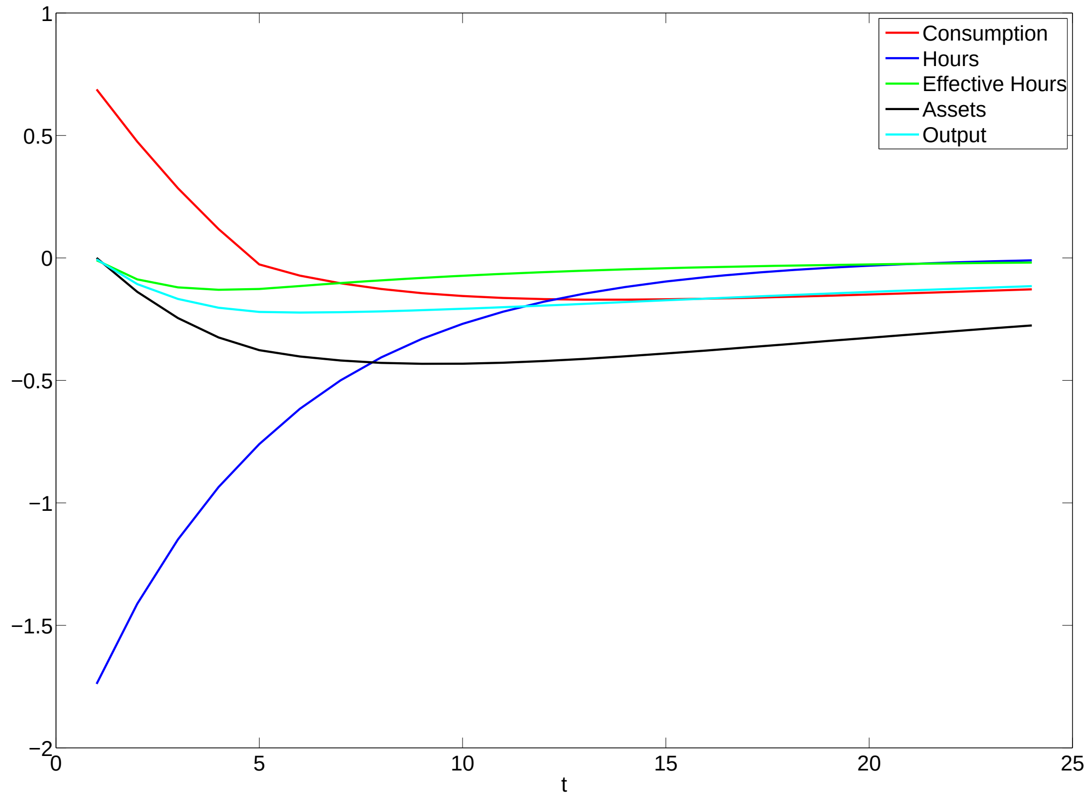


Gini coefficient.

In order to understand the link between the aggregate results and the intuition developed through analytics in Section 3, it is useful to observe the average sensitivity of agents' behavior within a given wealth category to changes in their wealth. We construct two related decompositions in Tables 2 and 3. Each element in these two tables reports the contribution – via wealth effects alone – of individuals in a given wealth (Table 2) or productivity (Table 3) category to the total change in a given aggregate. More precisely, the elements of the first column of Table 2 are given by $\left[\sum_{s\in\mathcal{S}}\sum_{a\in A_k}\left(\frac{\partial a_0'(a,s)}{\partial a}-1\right)x(a)\Gamma\left(a,s\right)da\right]/A_0$, with A_k denoting different wealth quantiles, and the elements in the first column of Table 3 are given by $\frac{\left[\sum_{s\in S_k}\sum_{a\in\mathcal{A}}\left(\frac{\partial a_0'(a,s)}{\partial a}-1\right)x(a)\Gamma(a,s)da\right]}{A_0}$, where S_k are different productivity groups. Other columns in the two tables are defined similarly. This decomposition is accurate when that wealth redistribution is small relative to individual wealth holdings. Furthermore, it provides an accurate decomposition of total changes in the various aggregates to the extent that changes in the policy function induced by general equilibrium effects of prices are relatively small.

The decomposition by wealth is depicted in Table 2. Recall that each of these numbers give the total change within a given group. We see immediately that, in response to the transfer, low-wealth households contribute negatively to the change in aggregate savings rate, whereas wealth-rich households contribute positively. Low-wealth households achieve lower savings both by increasing their consumption and by decreasing their labor supply. In particular, the consumption boom is

Figure 3: Aggregate Variables



disproportionately driven by those in the first quintile, who by themselves generate a 1.40 percent change in total consumption from the steady state. We also see that the lower-wealth households reduce their hours while those with higher wealth increase effort slightly. The more than proportional reduction in hours by low-wealth households explains the substantial drop in hours worked. However, once weighted by productivity, higher-wealth groups' aggregate effort rises substantially more, since they are more likely to be also more productive. The net result is that wealth-rich households nearly offset the decline in effective hours of wealth-poor ones.[9]

The correlation between wealth and productivity becomes apparent in Table 3, which organizes households by their idiosyncratic state $s \in \{1, 2, .., 8\}$. Before proceeding, we note that our parameterization, following Castaneda et al. (2003), reserves states $s = \{5, ..., 8\}$ to capture the presence of retirees, who are not endowed with labor time, and differ only in the probability distributions that govern their return or rebirth into working life. The table shows that more productive

[9]In effect, in the benchmark case of Cobb-Douglas preferences, one would expect higher productivity households to change their leisure by proportionately less as compared to consumption, since, given their higher wages, a smaller change in hours worked delivers the same change in expenditures with leisure.

Table 2: Decomposition of Changes in Aggregates by Wealth Quantile

Wealth Quantiles	$S_1 - S_0$	$C_1 - C_0$	$L_1 - L_0$	$N_1 - N_0$
First Quintile	-0.262	1.4	-0.3108	-0.1393
Second Quintile	-0.0824	0.2082	-1.0116	-0.452
Third Quintile	-0.0541	0.1506	-0.6418	-0.2887
Fourth Quintile	-0.012	0.0417	-0.1134	-0.0683
Fifth Quintile	0.105	-0.3344	0.1414	0.7413
90-95	0.0206	-0.0581	0.0388	0.1548
95-99	0.0349	-0.0541	0.067	0.2887
99-100	0.0406	-0.1912	8.32E-04	0.2334
Total	-0.3055	1.4661	-1.9361	-0.207

households are more likely to reduce their consumption and increase their hours worked by more than less productive households increase their consumption and decrease their hours worked. This is consistent with the notion that productive households are being taxed whereas less productive households are receiving transfers. We see the sharp drop in hours overall, as already noted, but also the concentration of that drop among the relatively unproductive ($s = 1$), explaining the increase in aggregate labor productivity. As for consumption, the bulk of the boom is concentrated in groups $s = \{5, .., 8\}$, encompassing the retirees, with their contribution to consumption growth equaling 1.24 percent, roughly 85 percent of the total increase. This occurs because retirees effectively discount the future more heavily than other agents – they face a positive probability of being replaced by a descendant they care for with a discount rate β, and who will be better off than them. Furthermore, since they do not supply labor, all their dissaving takes place through increased consumption rather than through reduced labor supply. As a result, this group has relatively diminished intertemporal smoothing motives and a higher marginal propensity to consume than other groups, all else equal.[10]

Table 4 compares the effects upon impact of wealth redistribution with and without nominal rigidities. The first two lines compare the results in the baseline case with standard Cobb-Douglas production function. The changes in consumption and output in both cases is virtually identical. This reflects the fact that the interest rate and wages change very little in the case without nominal rigidities. Hence, general equilibrium effects play a very small role. For the case in which interest rate is sensitive to changes in savings, both models with and without nominal rigidities generate a boom in consumption and output, although the boom in consumption is smaller than in the first two rows. It is notable that in the presence of nominal rigidities output rises by only half as much as it does in its absence. Thus, it is not the case that nominal rigidities necessarily amplify the output boom induced by redistributive shocks.

The analytical results in Section 3 imply that the aggregate impact of the redistribution shock

[10]Note that the overall change in consumption predicted by the decomposition is about twice as large as the one implied by the model. This is in large part an artifact of the linearization and, specifically, because the marginal propensity to consume of the retirees falls very rapidly as their wealth increases.

Table 3: Decomposition in Changes in Aggregates by Exogenous State

exogenous state (s)	$S_1 - S_0$	$C_1 - C_0$	$L_1 - L_0$	$N_1 - N_0$
1	-0.1517	0.2732	-2.0471	-0.9006
2	0.0015	-0.0215	0.0047	0.0065
3	0.0572	-0.1113	0.1057	0.455
4	0.0272	-0.0073	4.97E-04	0.2322
5	-0.2212	1.2477	0	0
6	-0.0181	0.1053	0	0
7	0	-0.0135	0	0
8	0	-0.0065	0	0
Total	-0.3055	1.4661	-1.9361	-0.207

Table 4: The Role of Nominal Rigidities

		$C_1 - C_0$	$N_1 - N_0$	$Y_1 - Y_0$
Baseline	$\lambda = 1$	0.68	-0.01	-0.00
	$\lambda = 0.5$	0.69	-0.01	-0.00
Sensitive Interest Rates	$\lambda = 1$	0.31	0.27	0.10
	$\lambda = 0.5$	0.27	0.14	0.08

on output is a function of preferences, the tax system and the incentives for precautionary savings. To demonstrate the extent to which the intuitions developed in the analytical section inform the quantitative results, we now examine alternative specifications. Table 5 shows how aggregate responses upon impact change if we alter the specification of the model. The first line of the table restates the results described in this section. Rows 2 and 3 in Table 5 show how removing the progressivity of the tax code and making preferences homothetic both lead to a progressively larger drop in effective labor supply and output on impact. This is in line with the discussion in Section 3.2.2 and more specifically in Proposition 5 and Lemma 1, regarding how, respectively, non-homotheticity in preferences and a progressive tax-code lead wealthy households to choose to spend proportionately more on leisure than on consumption. Once we make preferences homothetic and taxes linear, the wealthy reduce their expenditure on leisure by less than in the baseline case, thus also increasing their labor supply by less. Row 4 shows that increasing the sensitivity of interest rates to savings (by introducing more concavity to capital in the production function) suppresses the increase in consumption and leads to an output boom. This highlights the role of inter-temporal choices in keeping aggregate labor supply from rising in the baseline model. With sensitive interest rates, aggregate savings change by less in equilibrium, and intra-temporal choices dominate.

The last two lines of Table 5 examine whether some of the special assumptions made in Castaneda et al. (2003) to allow the model to match the extremely skewed wealth distribution could be driving the results. Rows 4 and 5 present the results when the model is calibrated as in Floden (2001) and Alonso-Ortiz and Rogerson (2010), respectively. In both these cases there is a clear decline in hours and effective hours. The parametrization in Alonso–Ortiz and Rogerson (2009) presumes indivisble labor. As highlighted by Proposition 6, the fact that output falls in that case suggests that given the proposed redistributive policy, recipients are more likely to be close to the margin between working and not working.

Table 5: Alternative Parameterizations

	$C_1 - C_0$	$L_1 - L_0$	$N_1 - N_0$	$Y_1 - Y_0$	% of Households better off
Baseline	0.69	-1.72	-0.01	-0.01	75.16
Flat tax	0.74	-2.04	-0.08	-0.05	75.57
Flat tax, homothetic	0.74	-2.26	-0.32	-0.2	77.11
Sensitive Interest Rates	0.31	-1.18	0.43	0.27	-
Floden (PE)	0.44	-0.83	-0.34	-	69.93
Alonso-Ortiz and Rogerson	0.31	-6.49	-1.85	-1.19	68.46

6 Conclusion

The recent recession has brought attention to the possibility that wealth redistribution can stimulate output. In this paper, we have taken a step, both analytically and quantitatively, in evaluating the aggregate impact of short-term redistributive economic policy that transfers wealth from rich to poor households. Our benchmark is a standard incomplete-markets model of consumption and labor supply that incorporates nominal rigidities and, in its quantitative version, also accurately captures the U.S. wealth distribution. We show that the conventional intuition with respect to the stimulative effect of wealth redistribution indeed holds for the behavior of aggregate consumption. However, we show that while redistributive policies can have a stimulative impact on consumption, their effect on aggregate output depends, potentially quite importantly, on the nature of household labor supply. In particular, we highlight the role of wealth effects on labor that, in our quantitative model, are strong enough to overturn the output effects of the consumption boom altogether.

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8 Proofs of Propositions

Proofs of Propositions 1 through 3:

See text.

■

Proof of Proposition 4:

Let $\mathcal{A}^- = \{a | x(a) < 0\}$, $\mathcal{A}^+ = \{a | x(a) > 0\}$. Then

$$\left| \int_{a \in A^-} x(a) \Gamma(a) \, da \right| = \int_{a \in A^+} x(a) \Gamma(a) \, da = M$$

For notational simplicity, let $m(a) \equiv \frac{\partial z_0(a,s)}{\partial a} \frac{\partial \varepsilon(s) l^{static}(z(a,s),s,w_0)}{\partial z}$ denote the marginal propensity to work out of wealth.

Since $x(a)$ is decreasing in a, it follows that if $a \in \mathcal{A}^+$ and $a' \in \mathcal{A}^-$, then $a < a'$. Moreover, by assumption, $|m(a)|$ is increasing in a. It follows from both of these observations that, for all s, $\sup_{a \in \mathcal{A}^+} |m(a)| \leq \inf_{a \in \mathcal{A}^-} |m(a)|$. Thus,

$$\begin{aligned}
\int_{a \in \mathcal{A}^+} x(a) |m(a)| \Gamma(a) \, da &< M \sup_{a \in \mathcal{A}^+} |m(a)| \leq \\
&\leq M \inf_{a \in \mathcal{A}^-} |m(a)| \\
&< \left| \int_{a \in \mathcal{A}^-} x(a) |m(a)| \Gamma(a) \, da \right|.
\end{aligned}$$

Therefore

$$\int_{a\in\mathcal{A}^+} x(a)\,|m(a)|\,\Gamma(a)\,da + \int_{a\in\mathcal{A}^-} x(a)\,|m(a)|\,\Gamma(a)\,da < 0,$$

and

$$\int_{a\in\mathcal{A}^+} x(a)\,m(a)\,\Gamma(a)\,da > 0.$$

The proof for the converse case in which $\left|\frac{\partial z_0(a,s)}{\partial a}\frac{\partial \varepsilon(s) l^{static}(z(a,s),s,w_0)}{\partial z}\right|$ is decreasing in a is analogous.

■

Proof of Proposition 5:

Let $\mu(z,s,w_t) = \frac{w_t\varepsilon(s)\left(\bar{l}-l_t^{static}(z,s,w_t)\right)}{z}$, so that $l_t^{static}(z,s,w_t) = \bar{l} - \frac{\mu(z,s,w_t)z}{w_t\varepsilon(s)}$ We have that, for all t,

$$\frac{\partial \varepsilon(s)\, l_0^{static}(z,s;w)}{\partial a} = -\frac{1}{w_t}\left[\mu_z(z_0(a,s),s,w_0)\, z_0(a,s) + \mu(z_0(a,s),s,w_0)\right],$$

Thus, $\frac{\partial \varepsilon(s) l_0^{static}(z,s;w)}{\partial a}$ increases with z if $\mu_z(z,s,w_t)\,z + \mu(z,s,w_t)$ to increase with z. It is increasing in z if

$$\mu_{zz}(z,s,w_0)\,z + 2\mu_z(z,s,w_0) > 0$$

This will be true if $\mu_z(z,s,w_0) > 0$ (leisure is a luxury) and $\frac{\mu_{zz}(z,s,w_0)z}{\mu_z(z,s,w_0)} > -2$. Conversely, $N_1 - N_0 < 0$ if $\mu_z(z,s,w_t)\,z + \mu(z,s,w_t)$ decreases with z. This will be the case if

$$\mu_{zz}(z,s,w_0)\,z + 2\mu_z(z,s,w_0) < 0$$

Which is the case if $\mu_z(z,s,w_0) < 0$ (leisure is a necessity) and $\frac{\mu_{zz}(z,s,w_0)z}{|\mu_z(z,s,w_0)|} < 2$

■

Proof of Corollary 1

In the case of separable utility, we have that

$$w\varepsilon(s)\,c^{-\sigma_C} = \chi\left(\bar{l}-l\right)^{-\sigma_L}$$

The budget constraint is:

$$w\varepsilon(s)\left(\bar{l}-l\right) + c = z$$

Bringing the two together defines $\mu(z,s,w)$ implicitely as

$$\mu(z,s,w) + \chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}}\,\mu(z,s,w)^{\frac{\sigma_L}{\sigma_C}}\, z^{\frac{\sigma_L}{\sigma_C}-1} = 1$$

From the implicit function theorem, we have that

$$\mu_z(z,s,w) = \left(1 - \frac{\sigma_L}{\sigma_C}\right) \frac{\chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}} \mu(z,s,w)^{\frac{\sigma_L}{\sigma_C}} z^{\frac{\sigma_L}{\sigma_C}-2}}{1 + \frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}} (\mu(z,s,w) z)^{\frac{\sigma_L}{\sigma_C}-1}}$$

Thus, $\mu_z(z,s,w) > 0$ if $\sigma_L < \sigma_C$ and $\mu_z(z,s,w) < 0$ otherwise. Also, taking natural logarithms on both sides and differentiating with respect to $\ln z$ yields

$$\begin{aligned}\frac{\mu_{zz}(z,s,w) z}{\mu_z(z,s,w)} &= \frac{\sigma_L}{\sigma_C}\frac{\mu_z(z,s,w) z}{\mu(z,s,w)} + \frac{\sigma_L}{\sigma_C} - 2 \\ &\quad - \frac{\frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}} (\mu(z,s,w) z)^{\frac{\sigma_L}{\sigma_C}-1}}{1 + \frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}} (\mu(z,s,w) z)^{\frac{\sigma_L}{\sigma_C}-1}} \left(\frac{\sigma_L}{\sigma_C} - 1\right)\left(\frac{\mu_z(z,s,w) z}{\mu(z,s,w)} + 1\right)\end{aligned}$$

Note that $\frac{\frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}} (\mu(z,s,w) z)^{\frac{\sigma_L}{\sigma_C}-1}}{1 + \frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}} (\mu(z,s,w) z)^{\frac{\sigma_L}{\sigma_C}-1}} = \frac{1}{1-\frac{\sigma_L}{\sigma_C}}\frac{\sigma_L}{\sigma_C}\frac{\mu_z(z,s,w) z}{\mu(z,s,w)}$, so that

$$\frac{\mu_{zz}(z,s,w) z}{\mu_z(z,s,w)} = \frac{\sigma_L}{\sigma_C}\left(\frac{\mu_z(z,s,w) z}{\mu(z,s,w)} + 2\right)\frac{\mu_z(z,s,w) z}{\mu(z,s,w)} + \frac{\sigma_L}{\sigma_C} - 2$$

So that if $\mu_z(z,s,w) > 0$, $\frac{\mu_{zz}(z,s,w) z}{\mu_z(z,s,w)} > -2$. If $\mu_z(z,s,w) < 0$, we have that

$$\frac{\mu_{zz}(z,s,w) z}{|\mu_z(z,s,w)|} = \frac{\sigma_L}{\sigma_C}\left(2 - \frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)}\right)\frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)} - \frac{\sigma_L}{\sigma_C} + 2$$

So that $\frac{\mu_{zz}(z,s,w) z}{|\mu_z(z,s,w)|} < 2$ if

$$\frac{\sigma_L}{\sigma_C}\left(2 - \frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)}\right)\frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)} < \frac{\sigma_L}{\sigma_C}$$

Simplifying and rearranging,

$$\left(\frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)}\right)^2 - 2\frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)} + 1 > 0$$

Note that the LHS is equal to zero if $\frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)} = 1$. Also, if we take the first derivative of the LHS and set $\frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)} = 1$, we see that this is a local minimum. Thus, unless $\frac{|\mu_z(z,s,w)| z}{\mu(z,s,w)} = 1$, the condition is satisfied. This would require

$$\left(1 - \frac{\sigma_C}{\sigma_L}\right) \frac{\frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}} (\mu(z,s,w) z)^{\frac{\sigma_L}{\sigma_C}-1}}{1 + \frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}} w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}} (\mu(z,s,w) z)^{\frac{\sigma_L}{\sigma_C}-1}} = 1$$

Under the assumption that $\sigma_L > \sigma_C$ (so that $\mu_z(z,s,w) < 0$), we have that $0 < \frac{\sigma_C}{\sigma_L} < 1$, so that

$1-\frac{\sigma_C}{\sigma_L}<1$. Also, $\frac{\frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}}w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}}(\mu(z,s,w)z)^{\frac{\sigma_L}{\sigma_C}-1}}{1+\frac{\sigma_L}{\sigma_C}\chi^{-\frac{1}{\sigma_C}}w\varepsilon(s)^{\frac{1-\sigma_L}{\sigma_C}}(\mu(z,s,w)z)^{\frac{\sigma_L}{\sigma_C}-1}}<1$, so that the equality cannot hold. Thus, $\frac{\mu_{zz}(z,s,w)z}{|\mu_z(z,s,w)|}<2$.

■

Proof of Lemma 1

The intra-temporal optimality condition is

$$c=\frac{\kappa}{1-\kappa}w\varepsilon\left(s\right)\left(1-\tau'\left(ra+w\varepsilon\left(s\right)l+\omega\left(s\right)\right)\right)\left(\bar{l}-l\right)$$

Given the definition

$$\mu_0\left(a,s\right)=\frac{w\varepsilon\left(s\right)\left(\bar{l}-l_0\left(a,s\right)\right)}{c_0\left(a,s\right)+w\varepsilon\left(s\right)\left(\bar{l}-l_0\left(a,s\right)\right)}$$

we have that

$$\mu_0\left(a,s\right)=\frac{1}{\frac{\kappa}{1-\kappa}\left(1-\tau'\left(ra+w\varepsilon\left(s\right)l\left(a,s\right)+\omega\left(s\right)\right)\right)+1}$$

Thus,

$$\frac{\partial\mu_0\left(a,s\right)}{\partial a}=\frac{\kappa}{1-\kappa}\frac{\tau''\left(ra+w\varepsilon\left(s\right)l+\omega\left(s\right)\right)}{\left(\frac{\kappa}{1-\kappa}\left(1-\tau'\left(ra+w\varepsilon\left(s\right)l\left(a,s\right)+\omega\left(s\right)\right)\right)+1\right)^2}\frac{\partial\left(ra+w\varepsilon\left(s\right)l\left(a,s\right)\right)}{\partial a}$$

If taxes are progressive, so that $\tau''\left(ra+w\varepsilon\left(s\right)l+\omega\left(s\right)\right)$, then $\frac{\partial\mu_0(a,s)}{\partial a}>0$.■

Proof of Proposition 6

We want to consider the impact of an infinitesimally small redistribution program. We consider a sequence of vanishingly small programs with transfers to hosueholds with wealth level a given by $\upsilon x_1\left(a\right)$, where υ is a perturbation parameter. Under the indivisible labor case, effective labor supply at $t=0$ and $t=1$ is, in partial equilibrium, given by

$$N_0=\bar{l}\sum_{s\in\mathcal{S}}\varepsilon\left(s\right)\int_{a\in A}\mathbf{1}\left(a\leq\bar{a}\left(s\right)\right)\Gamma_0\left(a,s\right)da$$
$$N_1=\bar{l}\sum_{s\in\mathcal{S}}\varepsilon\left(s\right)\int_{a\in A}\mathbf{1}\left(a+\frac{\upsilon x_1\left(a\right)}{1+r}\leq\bar{a}\left(s\right)\right)\Gamma_0\left(a,s\right)da$$

To characterize the change in effective labor supply given the program, we take the limit

$$
\begin{aligned}
\lim_{\upsilon\to 0}\frac{N_1-N_0}{\upsilon} &= \lim_{\upsilon\to 0}\bar{l}\sum_{s\in\mathcal{S}}\varepsilon(s)\int_{a\in A}\left[\frac{\mathbf{1}(a+\upsilon x_1(a)\leq\bar{a}(s))-\mathbf{1}(a\leq\bar{a}(s))}{\upsilon}\right]\Gamma_0(a,s)\,da \\
&= \lim_{\upsilon\to 0}\bar{l}\sum_{s\in\mathcal{S}}\varepsilon(s)\int_{a\in A}\left[\frac{\mathbf{1}(a+\upsilon x_1(a)\leq\bar{a}(s))-\mathbf{1}(a\leq\bar{a}(s))}{x_1(a)\,\upsilon}\right]x_1(a)\,\Gamma_0(a,s)\,da \\
&= \bar{l}\sum_{s\in\mathcal{S}}\varepsilon(s)\lim_{u\to 0}\frac{\int_{\bar{a}(s)-u}^{\bar{a}(s)}x_1(a)\,\Gamma_0(a,s)\,da}{u}
\end{aligned}
$$

where in the last line we substitute $x_1(a)\,\upsilon\equiv u$. Using L'Hospital's rule and Leibniz rule,

$$
\begin{aligned}
\lim_{\upsilon\to 0}\frac{N_1-N_0}{\upsilon} &= \bar{l}\sum_{s\in\mathcal{S}}\varepsilon(s)\lim_{u\to 0}\frac{\partial\frac{\int_{\bar{a}(s)-u}^{\bar{a}(s)}x_1(a)\Gamma_0(a,s)da}{\partial u}}{\frac{\partial u}{\partial u}} \\
&= \bar{l}\sum_{s\in\mathcal{S}}\varepsilon(s)\lim_{u\to 0}\frac{x_1(\bar{a}(s)-u)\,\Gamma_0(a,s)\,da}{1} \\
&= \bar{l}\sum_{s\in\mathcal{S}}\varepsilon(s)\,x_1(\bar{a}(s))\,\Gamma_0(a,s)\,da
\end{aligned}
$$

■