

Working Paper Series



This paper can be downloaded without charge from:
<http://www.richmondfed.org/publications/>



THE FEDERAL RESERVE BANK OF RICHMOND
RICHMOND ■ BALTIMORE ■ CHARLOTTE

Discretionary Monetary Policy in the Calvo Model*

Willem Van Zandweghe[†] Alexander L. Wolman[‡]

This version: May 2011

Working Paper No. 11-03

Abstract

We study discretionary equilibrium in the Calvo pricing model for a monetary authority that chooses the money supply. The steady-state inflation rate is above 8 percent for a baseline calibration, but it varies substantially with alternative structural parameter values. If the initial condition involves inflation higher than steady state, discretionary policy generates an immediate drop in inflation followed by a gradual increase to the steady state. Unlike the two-period Taylor model, discretionary policy in the Calvo model does not accommodate predetermined prices in a way that inevitably leads to multiple private-sector equilibria.

JEL codes: E31; E52

Keywords: time-consistent optimal monetary policy, relative price distortion, sticky prices, discretion

*We have benefited from discussions with Gary Anderson, Roberto Billi, Andreas Hornstein, Jinill Kim, Bob King, Per Krusell, Takushi Kurozumi, Stéphane Moyen, Pierre Sarte, Raf Wouters and Tack Yun, and from the feedback of brown bag lunch participants at the Richmond Fed, and seminar participants at the Bundesbank, Carlos III University in Madrid, the ECB, Humboldt University in Berlin, the Kansas City Fed, LMU in Munich, the National Bank of Belgium, and the Norges Bank. The views expressed in this paper are those of the authors alone. They are not the views of the Federal Reserve Bank of Kansas City, the Federal Reserve Bank of Richmond or the Federal Reserve System.

[†]Research Department, Federal Reserve Bank of Kansas City, 1 Memorial Drive, Kansas City, MO 64198. Tel: 816-881-2766. E-mail: willem.vanzandweghe@kc.frb.org.

[‡]Research Department, Federal Reserve Bank of Richmond, P.O. Box 27622, Richmond, VA 23261. Tel: 1-804-697-8262. E-mail: alexander.wolman@rich.frb.org.

1 Introduction

Over the last 15 years the New Keynesian framework has become predominant in the world of applied monetary policy analysis. This framework is commonly characterized by linear models with nominal rigidities and strong forward-looking elements, and which can be rationalized as approximations to micro-founded dynamic equilibrium models. The most common source of nominal rigidity in this framework is the Calvo (1983) pricing model as described by Yun (1996).

The fact that some prices are predetermined in these models leads to a version of Kydland and Prescott's (1977) time-inconsistency problem for monetary policy. There is a vast literature studying aspects of discretionary, i.e. time-consistent, policy in New Keynesian models with Calvo pricing. But the typical practice in the New Keynesian literature, exemplified by Clarida, Gali and Gertler (1999) and Woodford (2003), has been to work with linear models approximated around a zero-inflation steady state. The present paper studies discretionary optimal monetary policy in the underlying non-linear model.¹

The paper has two main results. First, the steady-state inflation rate can take a wide range of magnitudes for reasonable values of the structural parameters. Under a baseline calibration, discretionary equilibrium involves a steady-state inflation rate of greater than 8 percent. The steady-state inflation rate depends non-monotonically on the Calvo parameter. For low degrees of price rigidity, a small increase in rigidity is associated with higher inflation. But for high degrees of price rigidity, a small further increase in rigidity implies a lower steady-state inflation rate under discretionary policy. There is a positive relationship between the desired markup and the steady-state inflation rate, and between the labor supply elasticity and the steady-state inflation rate.

¹In Yun's (1996) version of the Calvo model there is price indexation, whereas the version in King and Wolman (1996) has no indexation. We analyze the Calvo model without indexation.

To give a sense of the magnitudes, the steady-state inflation rate varies between essentially zero for extremely high price stickiness with a desired markup of 1.11, to greater than 40 percent with a price non-adjustment probability of 0.67, a desired markup of 1.33 and an infinite labor supply elasticity. Under commitment, the long-run inflation rate is zero for all values of these parameters. Thus, the wide range of inflation rates under discretion makes it difficult to infer the degree of commitment from observed inflation rates. For instance, the model can predict a high steady-state inflation rate, which would suggest that actual monetary policymakers have access to a commitment technology. Or it can predict a low inflation rate in discretionary equilibrium, e.g. as the result of inelastic labor supply, leaving little difference between the inflation rates under commitment and discretion. Nonetheless, the results suggest that the commonly applied zero-inflation approximation is inappropriate in the absence of a fiscal scheme to eliminate the monopoly distortion.

Out of steady state, the presence of an endogenous state variable leads to a gradual transition of inflation. Specifically, if the initial condition involves inflation higher than steady state, discretionary policy generates an immediate drop in inflation followed by a gradual increase to the steady state. In contrast, if the Calvo model is approximated around the zero-inflation steady state there is no state variable, so inflation jumps immediately to zero.

Our second main result relates to an existing literature which has identified discretionary policy as a source of multiple equilibria.² Under discretionary policy, private agents make decisions such as how much to save or what prices to set, based on their expectations of future policy. Those decisions become embodied in state variables such as the capital stock or prices, and in future periods a discretionary policymaker responds to those state variables. Thus, there is the potential for a form of complementarity

²Here and throughout the paper, we restrict attention to Markov-perfect equilibria.

between future policy and expected future policy. Viewed from another angle, the fact that policy will react to endogenous state variables can be a source of complementarity among private agents' actions. Examples of such complementarity leading to multiple equilibria can be found in Glomm and Ravikumar's (1995) model of public and private education, and in the modification of Kydland and Prescott's (1977) flood control example as described by King (2006). The link between discretionary policy and multiple equilibria has been especially prominent in the monetary policy literature, among work that has studied discretionary equilibrium in full-blown nonlinear sticky-price models. Albanesi, Chari and Christiano (2003) show that multiple equilibria arise under discretionary policy in a model in which a fraction of firms have predetermined prices. Khan, King and Wolman (2001) and King and Wolman (2004) show that in Taylor-style models with prices set for three and two periods respectively, multiple equilibria arise under discretion.³ The finding of multiple equilibria in these previous studies raises the question of whether lack of commitment leads to multiple equilibria in the Calvo model as well.

We find that discretionary policy does not induce sufficiently strong complementarity to generate equilibrium multiplicity in the Calvo model. This is surprising given the basic similarity between the Calvo and Taylor models. In both models there is staggered pricing, and the policy problem of choosing the money supply to maximize welfare involves a static trade-off between the markup and a relative price distortion. What differs across the two models is the dynamic aspect of the policy problem.

In the Taylor model with two-period pricing (as studied by King and Wolman (2004)), the only intertemporal link is the nominal price set by the half of firms that adjust in the

³Siu (2008) extends King and Wolman's (2004) analysis by incorporating elements of state-dependent pricing and shows that Markov-perfect discretionary equilibrium is unique. Those papers assume that monetary policy is conducted with a money supply instrument. In contrast, Dotsey and Hornstein (2009) show that with an interest rate instrument there is a unique Markov-perfect discretionary equilibrium in a Taylor model with two-period pricing.

current period. The policymaker in the subsequent period chooses to adjust the money supply proportionally with that predetermined price, and thus the price does not affect the set of feasible outcomes for future policymakers. However, the expectation of this future policy response leads to complementarity in firms' price-setting decisions, and to multiple equilibria.

In the Calvo model, in addition to the single nominal price set by firms in the current period, the policymaker in the subsequent period inherits an entire distribution of predetermined prices. The distribution can be summarized by a statistic we will call the inherited relative price distortion. The future policymaker chooses to adjust the money supply less than proportionally with the price set by firms in the present period, for two reasons. First, firms that choose a high price have a small expenditure share in aggregate consumption, so their price has a small effect on the overall price level. Indeed, we show that if the money supply is set in proportion to the previous period's price level, and thus as a concave function of the previous period's optimal price, there is a unique private-sector equilibrium. Second, the future policymaker's policy problem is influenced by the inherited relative price distortion, which increases if adjusting firms in the present set a higher price. The larger that distortion, the less the future money supply accommodates increases in the current price level. Thus, as a result of the presence of many cohorts of predetermined prices, the high degree of complementarity necessary for generating multiple equilibria is broken.

Our paper is closely related to Anderson, Kim and Yun (2010). They study optimal allocations without commitment in the Calvo model.⁴ In contrast, our framework

⁴Yun (2005) and Adam and Billi (2007) also study optimal allocations in non-linear versions of the Calvo model. Yun includes a fiscal instrument for offsetting the markup distortion, which eliminates the time-inconsistency problem, and implies that in the steady state inflation is zero. The transition dynamics in Yun's model are affected by the state variable as they are in our analysis. Adam and Billi take into account the non-linearity arising from the zero bound on nominal interest rates in an otherwise

involves a discretionary policymaker choosing the money supply. The former approach cannot be used to investigate the possibility of multiple private-sector equilibria for a given policy action. Anderson, Kim and Yun’s solution method, like ours, is based on Chebyshev collocation. While they study a slightly different region of the parameter space, the nature of their solutions is consistent with our findings. Unlike the two-period Taylor model, where the choice of whether to study a planner’s problem or a policy problem can mean the difference between uniqueness and multiplicity, in the Calvo model this choice yields identical results for the examples we have studied.

The paper proceeds as follows. The next section relates our analysis to the early literature on discretionary monetary policy. Section 3 contains a description of the Calvo model. Section 4 defines a discretionary equilibrium in the model. Section 5 presents our numerical results, emphasizing the issue of multiplicity or lack thereof. Section 6 contains a sensitivity analysis. Section 7 concludes.

2 Relation to early literature

Although the literature on time-consistency problems for monetary policy is vast, it is comprised of two seemingly disparate branches. Much of the literature – and most of the profession’s intuition – is derived from the seminal work by Kydland and Prescott (1977) and Barro and Gordon (1983). They studied reduced-form macroeconomic models in which the frictions giving leverage to monetary policy were not precisely spelled out. In contrast, the sticky-price models popularized in the last 15 years are precise about those frictions. While there has been a great deal of work on discretionary policy in sticky-price models, the connection between that work and the seminal papers remains poorly understood. In this section we explain how our analysis of discretionary monetary policy in the Calvo model relates to Barro and Gordon (1983) (hereafter, BG), which

linear New Keynesian model.

elaborated on Kydland and Prescott's (1977) framework.

In BG, under discretion the central bank takes expectations of inflation as given when choosing a policy action that directly determines actual inflation. Under commitment the policymaker would take into account the endogeneity of expectations in all but an initial period. Because surprise inflation can raise output, and because of distortions that make output inefficiently low, discretion leads to an equilibrium inflation rate that is higher than would be optimal with commitment. Modern staggered pricing models such as the Calvo model also give rise to a time-consistency problem: monopoly distortions make output inefficiently low – as in BG – and with some prices predetermined, surprise inflation can raise output – as in BG. However, an important difference between BG and analysis of discretionary monetary policy in staggered pricing models arises from the fact that staggered pricing models explicitly incorporate intertemporal choices by private agents.

At the heart of time-consistency problems for monetary policy is the notion that a discretionary policymaker takes as given private agents' expectations, but in equilibrium those expectations accurately incorporate the policymaker's optimal behavior. In BG, although the model contains multiple periods, the expectations just referred to are current expectations about current policy. The only dynamics in BG occur through serial correlation in exogenous shocks. Without other intertemporal links, the policy problem is a static one in BG: treating expectations as fixed, higher inflation is costly in its own right but brings about a beneficial reduction in unemployment. In equilibrium, private expectations are validated, and the policymaker balances the static marginal cost and marginal benefit of additional inflation.

Staggered pricing models are inherently dynamic: because prices may stay in effect for multiple periods, the optimality condition for price setting incorporates expectations of future conditions. Prices set in the past thus incorporate expectations of current

policy actions. In the current period, a discretionary policymaker chooses her action taking as given those expectations, which are embedded in the predetermined prices. Looking forward, the policymaker knows that her actions will “directly” affect prices set in the current period. Those prices in turn affect the state of the economy in the future, introducing an explicit intertemporal element into the policy problem. Whereas in BG equilibrium requires that current policy actions be consistent with current period expectations, in staggered pricing models equilibrium requires that current policy actions be consistent with expectations formed in the past.

The intertemporal nature of price setting also means that staggered pricing models generally contain one or more state variables that can be affected by a policymaker, even under discretion. While today’s policymaker takes as given past prices, today’s policy action affects current prices, which in turn affect the distribution of prices inherited by the policymaker next period. The distribution of predetermined prices affects the feasible outcomes for next period’s policymaker. Thus, the discretionary policymaker does not face a purely static tradeoff between inflation and real activity; that tradeoff is present, but it is complicated by the fact that the current policy action affects tomorrow’s state, and thus tomorrow’s value function. Regarding this intertemporal element of the policy problem, one of the main points of this paper is that different staggered pricing models have different implications for equilibrium under discretionary monetary policy.

The static output-inflation tradeoff present in staggered pricing models is similar to the one in BG, as mentioned above. However, because the Calvo model and other staggered pricing models are optimizing models, one can be explicit about the source of that tradeoff. Monopolistic competition makes output inefficiently low. How can inflation increase output? With some prices predetermined, a one-time surprise increase in the money supply that creates inflation does not fall evenly on all firms. If money demand is interest inelastic, firms that can adjust their price will not do so aggressively,

and there will be an overall reduction in the markup and an increase in output. Loosely speaking, the larger is the money surprise, the higher is the inflation and the larger is the beneficial effect on the markup and output. Because the inflation is generated by only a fraction of the firms however, higher inflation is associated with larger dispersion of relative prices. Such dispersion leads to inefficient allocation of spending across goods because, in the absence of heterogeneity among firms, it is efficient for all firms to produce the same quantities. In staggered pricing models then, the policy tradeoff involves the ability of surprise inflation to reduce the markup against the cost of surprise inflation in distorting relative prices.

3 The Calvo model

This section describes the dynamic general equilibrium model with Calvo pricing. It is characterized by a representative household that values consumption and dislikes supplying labor, a constant-velocity money demand equation, a competitive labor market, a continuum of monopolistically competitive firms producing goods for which households have constant elasticity of substitution preferences, and a monetary authority that chooses the money supply. Each firm faces a constant probability of price adjustment. We assume that the model's exogenous variables are constant; thus, there is no uncertainty about fundamentals. The money supply is an endogenous variable.

3.1 Households

There is a large number of identical, infinitely-lived households. They act as price-takers in labor and product markets, and they own shares in the economy's monopolistically competitive goods-producing firms. Households' preferences over consumption (c_t) and

labor input (n_t) are given by

$$\sum_{j=0}^{\infty} \beta^j \left[\ln(c_{t+j}) - \chi \frac{n_{t+j}^{1+\mu}}{1+\mu} \right], \quad \beta \in (0, 1), \mu \geq 0, \chi > 0,$$

where consumption is taken to be the Dixit-Stiglitz aggregate of a continuum of differentiated goods

$$c_t = \left[\int_0^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1. \quad (1)$$

The consumer's flow budget constraint is

$$P_t w_t n_t + R_{t-1} B_{t-1} + \int_0^1 d_t(z) dz \geq P_t c_t + B_t,$$

where w_t is the real wage, R_t is the one-period gross nominal interest rate, B_t is the quantity of one-period nominal bonds purchased in period t , $d_t(z)$ is the dividend paid by firm z , and P_t is the nominal price of a unit of consumption. The aggregator (1) implies the demand functions for each good,

$$c_t(z) = \left[\frac{P_t(z)}{P_t} \right]^{-\varepsilon} c_t, \quad (2)$$

where $P_t(z)$ is the price of good z . The price index is given by

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}. \quad (3)$$

From the consumer's intratemporal problem, we have the efficiency condition

$$\chi c_t n_t^\mu = w_t, \quad (4)$$

and from the intertemporal problem we have

$$\frac{c_{t+1}}{c_t} = \beta \left(\frac{R_t}{\pi_{t+1}} \right),$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate between periods $t-1$ and t . We assume that households hold money equal to the quantity of nominal consumption:

$$M_t = P_t c_t. \quad (5)$$

It will be convenient to write the money demand equation normalizing by the lagged price level:

$$m_t \equiv \frac{M_t}{P_{t-1}} = \pi_t c_t. \quad (6)$$

We will refer to m_t as the normalized money supply.

3.2 Firms

Each firm $z \in [0, 1]$ produces output $y_t(z)$ using a technology that is linear in labor $n_t(z)$, the only input, with a constant level of productivity that is normalized to unity:

$$y_t(z) = n_t(z).$$

The nominal profits in period t of a firm charging price X_t are

$$d(X_t; P_t, c_t, w_t) = X_t \left(\frac{X_t}{P_t} \right)^{-\varepsilon} c_t - P_t w_t \left(\frac{X_t}{P_t} \right)^{-\varepsilon} c_t.$$

When a firm adjusts its price, it maximizes the present discounted value of profits, which we denote V_t . Because each firm adjusts its price with constant probability $1 - \alpha$ in any period, the value of a firm upon adjustment is given by

$$V_t = \max_{X_t} \left\{ \sum_{j=0}^{\infty} Q_{t,t+j} \alpha^j d(X_t; P_{t+j}, c_{t+j}, w_{t+j}) \right\}, \quad (7)$$

where $Q_{t,t+j}$ is the j -period ahead discount factor for nominal cash flows. With households owning firms, $Q_{t,t+j}$ is determined by the sequence of one-period nominal interest rates as

$$Q_{t,t+j} = \frac{1}{\prod_{k=1}^j R_{t-1+k}} = \beta^j \left(\frac{P_t}{P_{t+j}} \right) \left(\frac{c_t}{c_{t+j}} \right),$$

where $\prod_{k=1}^0 R_{t-1+k} = 1$. The factor α^j is the probability that a price set in period t will remain in effect in period $t + j$. Note that V_t is the present value of profits associated with charging the price X_t . When the firm has an opportunity to adjust after period t , it will reoptimize, and thus those states are not relevant for determining the optimal

price in period t . The optimal price is determined by differentiating (7) with respect to X_t . We will denote the profit-maximizing value of X_t by $P_{0,t}$ and we will denote by $p_{0,t}$ the nominal price $P_{0,t}$ normalized by the previous period's price level, which serves as an index of the predetermined prices in period t :

$$p_{0,t} \equiv \frac{P_{0,t}}{P_{t-1}}.$$

Thus, we write the first order condition from (7) as,

$$\frac{P_{0,t}}{P_t} = \frac{p_{0,t}}{\pi_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{\sum_{j=0}^{\infty} (\alpha\beta)^j (P_{t+j}/P_t)^{\varepsilon} w_{t+j}}{\sum_{j=0}^{\infty} (\alpha\beta)^j (P_{t+j}/P_t)^{\varepsilon-1}}. \quad (8)$$

The real wage is equal to real marginal cost here because firm-level productivity is assumed constant and equal to one. With the constant elasticity aggregator (1) a firm's optimal markup of price over marginal cost is constant and equal to $\varepsilon/(\varepsilon - 1)$. Because the firm cannot adjust its price each period, if the real wage or the inflation rate are not constant then the firm's markup will vary over time. The optimal pricing equation (8) indicates that the firm chooses a constant markup over an appropriately defined weighted average of current and future marginal costs. Note that the economy-wide average markup is simply the inverse of the real wage.

The optimal pricing condition can be written recursively by defining two new variables, N_t and D_t , that are related to the numerator and denominator of (8), respectively:

$$N_t = \pi_t^\varepsilon (w_t + \alpha\beta N_{t+1}), \quad (9)$$

$$D_t = \pi_t^{\varepsilon-1} (1 + \alpha\beta D_{t+1}), \quad (10)$$

then,

$$p_{0,t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{N_t}{D_t}. \quad (11)$$

Because of Calvo pricing, the price index (3) is an infinite sum,

$$P_t = \left[\sum_{j=0}^{\infty} (1 - \alpha) \alpha^j P_{0,t-j}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \quad (12)$$

but it can be simplified, first writing it recursively,

$$P_t = [(1 - \alpha) P_{0,t}^{1-\varepsilon} + \alpha P_{t-1}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}},$$

and then dividing by the lagged price level:

$$\pi_t = [(1 - \alpha) p_{0,t}^{1-\varepsilon} + \alpha]^{\frac{1}{1-\varepsilon}}. \quad (13)$$

3.3 Market clearing

Goods market clearing requires that the consumption demand for each individual good is equal to the output of that good:

$$c_t(z) = y_t(z), \quad (14)$$

and labor market clearing requires that the labor input into the production of all goods equal the supply of labor by households:

$$\int_0^1 n_t(z) dz = n_t. \quad (15)$$

In the Calvo model, the labor market clearing condition is

$$n_t = \sum_{j=0}^{\infty} (1 - \alpha) \alpha^j n_{j,t}, \quad (16)$$

where $n_{j,t}$ is the labor input employed in period t by a firm that set its price in period $t-j$. Combining this expression with the goods market clearing condition (14), then using the demand curves (2) for each good, and dividing the expression by the consumption aggregator yields

$$\frac{n_t}{c_t} = \sum_{j=0}^{\infty} (1 - \alpha) \alpha^j \left(\frac{P_{0,t-j}}{P_t} \right)^{-\varepsilon}, \quad (17)$$

which can be written recursively as

$$\frac{n_t}{c_t} = (1 - \alpha) \pi_t^\varepsilon \left[p_{0,t}^{-\varepsilon} + \left(\frac{\alpha}{1 - \alpha} \right) \frac{n_{t-1}}{c_{t-1}} \right]. \quad (18)$$

This equation contains two predetermined variables, but it is only their ratio that matters. We define a new variable

$$\Delta_{t-1} \equiv \frac{n_{t-1}}{c_{t-1}}, \quad (19)$$

and this inherited relative price distortion will serve as the single state variable.⁵ The labor market clearing condition (18) can now be written as

$$\Delta_t = (1 - \alpha) \pi_t^\varepsilon \left[p_{0,t}^{-\varepsilon} + \left(\frac{\alpha}{1 - \alpha} \right) \Delta_{t-1} \right]. \quad (20)$$

We are interested in studying Markov-perfect equilibria (MPE) with discretionary monetary policy. In an MPE, outcomes depend only on payoff-relevant state variables; trigger strategies and any role for reputation are ruled out. Hence, it is important to establish what the relevant state variables are. Although there are an infinite number of predetermined nominal prices ($P_{0,t-j}$, $j = 1, 2, \dots$), for the MPE a state variable is relevant only if it affects the monetary authority's set of feasible real outcomes. It follows that in an MPE the normalized money supply will be a function of the single state variable Δ_{t-1} . Henceforth, when we refer to a "discretionary equilibrium" it should be understood that the equilibrium is Markov perfect.

3.4 Monetary authority and timing

The monetary authority chooses the money supply, M_t . In a discretionary equilibrium the money supply will be chosen each period to maximize present-value welfare. We assume the sequence of actions within a period is as follows:

1. Predetermined prices ($P_{0,t-j}$, $j > 0$) are known at the beginning of the period.
2. The monetary authority chooses the money supply.

⁵We call Δ_{t-1} the inherited relative price dispersion because from (17) it summarizes the dispersion in predetermined relative prices.

3. Firms that adjust in the current period set their prices, and simultaneously all other period- t variables are determined.

Timing assumptions are important in models with staggered price-setting. Transposing items 2 and 3 or assuming that firms and the monetary authority act simultaneously would change the nature of the policy problem and the properties of equilibrium.

4 Discretionary equilibrium in the Calvo model

In a discretionary MPE the policymaker chooses the money supply as a function of the state, taking as given the behavior of future policymakers. The policymaker also takes into account that firms adjusting in the current period will behave optimally in response to the policy action, as implied by our timing assumption. In addition, the policymaker takes into account all the other relevant private-sector equilibrium conditions. In equilibrium, the future policy that is taken as given is also the policy chosen by the current policymaker.

4.1 Equilibrium for arbitrary monetary policy

As a preliminary to studying discretionary equilibrium, it is useful to consider stationary equilibria for arbitrary monetary policy – that is, for arbitrary functions $m = \Gamma(\Delta)$. To describe equilibrium for arbitrary policy we use recursive notation, eliminating time subscripts and using a prime to denote a variable in the next period. The nine variables which need to be determined in equilibrium are N , D , p_0 , π , Δ' , w , c , m and n , and the nine equations are (i and ii) the laws of motion for N (9) and for D (10); (iii) the optimal pricing condition (11); (iv) the price index (13); (v) the labor market clearing condition or law of motion for the relative price distortion (20); (vi) the labor supply equation (4); (vii) money demand (6); (viii) the policy rule $m = \Gamma(\Delta)$; and (ix) the definition of the relative price distortion (19).

A stationary equilibrium can be expressed as two functions of the endogenous state variable Δ . The two functions $N(\Delta)$ and $D(\Delta)$ must satisfy the two functional equations

$$N(\Delta) = \pi^\varepsilon [w + \alpha\beta N(\Delta')], \quad (21)$$

$$D(\Delta) = \pi^{\varepsilon-1} [1 + \alpha\beta D(\Delta')], \quad (22)$$

where the other variables are given recursively by the following functions of Δ :

$$p_0 = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \cdot \frac{N(\Delta)}{D(\Delta)}, \quad (23)$$

$$\pi = [(1 - \alpha) p_0^{1-\varepsilon} + \alpha]^{1/(1-\varepsilon)}, \quad (24)$$

$$\Delta' = \pi^\varepsilon [(1 - \alpha) p_0^{-\varepsilon} + \alpha\Delta], \quad (25)$$

$$m = \Gamma(\Delta). \quad (26)$$

$$c = \frac{m}{\pi} \quad (27)$$

$$w = \chi cn^\mu, \quad (28)$$

$$n = \Delta' c. \quad (29)$$

Given an arbitrary policy of the form $m = \Gamma(\Delta)$, functions $N()$ and $D()$ that satisfy (21)–(29) represent a stationary equilibrium.

4.2 Discretionary equilibrium defined

A discretionary equilibrium is a particular stationary equilibrium with policy given by a mapping from the state to the money supply, $m = \Gamma^*(\Delta)$, in which the following property holds. If the current-period policymaker and current-period private agents take as given that all future periods will be described by a stationary equilibrium associated with $\Gamma^*(\Delta)$, then the current-period monetary authority maximizes welfare by choosing $m = \Gamma^*(\Delta)$ for every Δ .

More formally, a discretionary equilibrium is a policy function $\Gamma^*(\Delta)$ and a value

function $v^*(\Delta)$ that satisfy

$$\begin{aligned} v^*(\Delta) &= \max_m \left\{ \ln c - \chi \frac{n^{1+\mu}}{1+\mu} + \beta v(\Delta') \right\} \\ \Gamma^*(\Delta) &= \arg \max_m \left\{ \ln c - \chi \frac{n^{1+\mu}}{1+\mu} + \beta v(\Delta') \right\} \end{aligned} \quad (30)$$

when $v() = v^*()$. The maximization is subject to (24)–(29) and optimal pricing by adjusting firms,

$$p_0 = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \cdot \frac{\pi^\varepsilon [w + \alpha\beta N(\Delta')]}{\pi^{\varepsilon-1} [1 + \alpha\beta D(\Delta')]} , \quad (31)$$

where the functions $N()$ and $D()$ satisfy (21) and (22) in the stationary equilibrium associated with $\Gamma^*(\Delta)$. Note the subtle difference between (31) and (21)–(23): in (31), which is the constraint on the current policymaker, we have not imposed a stationary equilibrium. The policymaker takes as given that the future will be represented by a stationary equilibrium, but is constrained today only by the private-sector response to whatever money supply she chooses.

4.3 Computing a discretionary equilibrium

We approximate the value function and the expressions for $N()$ and $D()$ with Chebyshev polynomials. This computational method involves selecting a degree of approximation I , and then searching for values of v_i^* and Γ_i^* , for $i = 1 \dots I$, that solve (30) at the grid points for the state variable Δ_i defined by the Chebyshev nodes.⁶ As an initial guess for $v()$, $N()$ and $D()$ we use the discretionary equilibrium for the static model – the final period of a finite horizon model – and then solve the optimization problem (30). If the value function and policy function that solve the optimization problem are identical to the guess, then they form a discretionary equilibrium.⁷ If not, the starting values are

⁶In the example of the baseline calibration given in the next section, we use a degree of approximation $I = 10$ on the interval $[1, 1.3]$ for the state variable.

⁷Specifically, iteration j is the final iteration if $\|v^j - v^{j-1}\|_\infty$ and $\|\Gamma^j - \Gamma^{j-1}\|_\infty$ are smaller than the tolerance level 10^{-8} . To assess the accuracy of a solution, the difference between the left hand side

updated by pushing out the initial guess one period into the future, and assuming the one-period-ahead policy and value functions are the ones that solved the optimization problem.

5 Properties of discretionary equilibrium

There are three levels to a complete description of a discretionary equilibrium. At the highest level is the equilibrium transition function for the state variable, $\Delta'(\Delta)$, the associated policy function, $m = \Gamma^*(\Delta)$, the value function, $v^*(\Delta)$, and equilibrium functions for the other endogenous variables. The next level is the objective function for the policymaker: for a given value of the state variable, how does welfare vary with the policy instrument m , and what are the trade-offs that drive the shape of the objective function? Finally, for given values of the state variable and the policy instrument, what is the nature of the private-sector equilibrium? Each of these levels is discussed in turn.

Unless otherwise stated we use the following baseline calibration, interpreting a period as a quarter: $\varepsilon = 10$, $\beta = 0.99$, $\alpha = 0.67$, $\mu = 0$, $\chi = 4.5$. Some of these parameters are typical values used in the applied monetary policy literature. With $\beta = 0.99$ the annualized real interest rate is 4.1 percent. With $\varepsilon = 10$ the steady-state markup is approximately 11 percent at low rates of inflation. Prices remain fixed with probability $\alpha = 0.67$, which means that the expected duration of a price is three quarters. The calibration of μ and ε is chosen to facilitate comparison with King and Wolman (2004).

and the right hand side of (30) is calculated using that solution on a grid of 100,000 points that do not include the Chebyshev nodes. Under the baseline calibration, this residual function has a maximum absolute approximation error of 2.38^{-4} .

5.1 Equilibrium as a function of the inherited relative price distortion

Figure 1 plots the transition function for the state variable as well as the function mapping from the state to the inflation rate in a discretionary equilibrium in Panel A. The first thing to note is that there is a unique steady-state inflation rate of approximately 8.6 percent annually.⁸ Two natural benchmarks against which to compare the steady state of the discretionary equilibrium are the inflation rate with highest steady-state welfare and the inflation rate in the long run under optimal policy with commitment. Following King and Wolman (1999), we refer to these benchmarks as the golden rule and the modified golden rule respectively. For our baseline parameterization, the golden-rule inflation rate is just barely positive (less than one tenth of a percent) and the modified golden-rule inflation rate is zero – the latter result is parameter-independent.

In addition to showing the steady state, Panel A illustrates the dynamics of the state variable, which exhibit monotonic convergence to the steady state. This means that a policymaker inheriting a relative price distortion that is large (small) relative to steady state finds it optimal to bequeath a smaller (larger) relative price distortion to her successor. Together with the monotone downward-sloping equilibrium function for inflation, it follows that the inflation dynamics in the transition from a large (small) relative price distortion and high (low) inflation rate involve an initial discrete fall (jump) in inflation and a subsequent gradual increase (decrease) to the steady state.⁹

Panel B of Figure 1 displays the policy variable (m) and welfare (v) as functions of the state variable in the discretionary equilibrium (m is indicated on the left scale

⁸Note that in the model π is a gross quarterly inflation rate, but the figures and the text refer to annualized net inflation rates obtained as $\pi^4 - 1$.

⁹Yun's (2005) analysis of the Calvo model displays similar transition dynamics of inflation. But in his model, the steady-state inflation rate under optimal policy is zero, so the transition from a steady state with positive inflation inevitably involves a period of deflation.

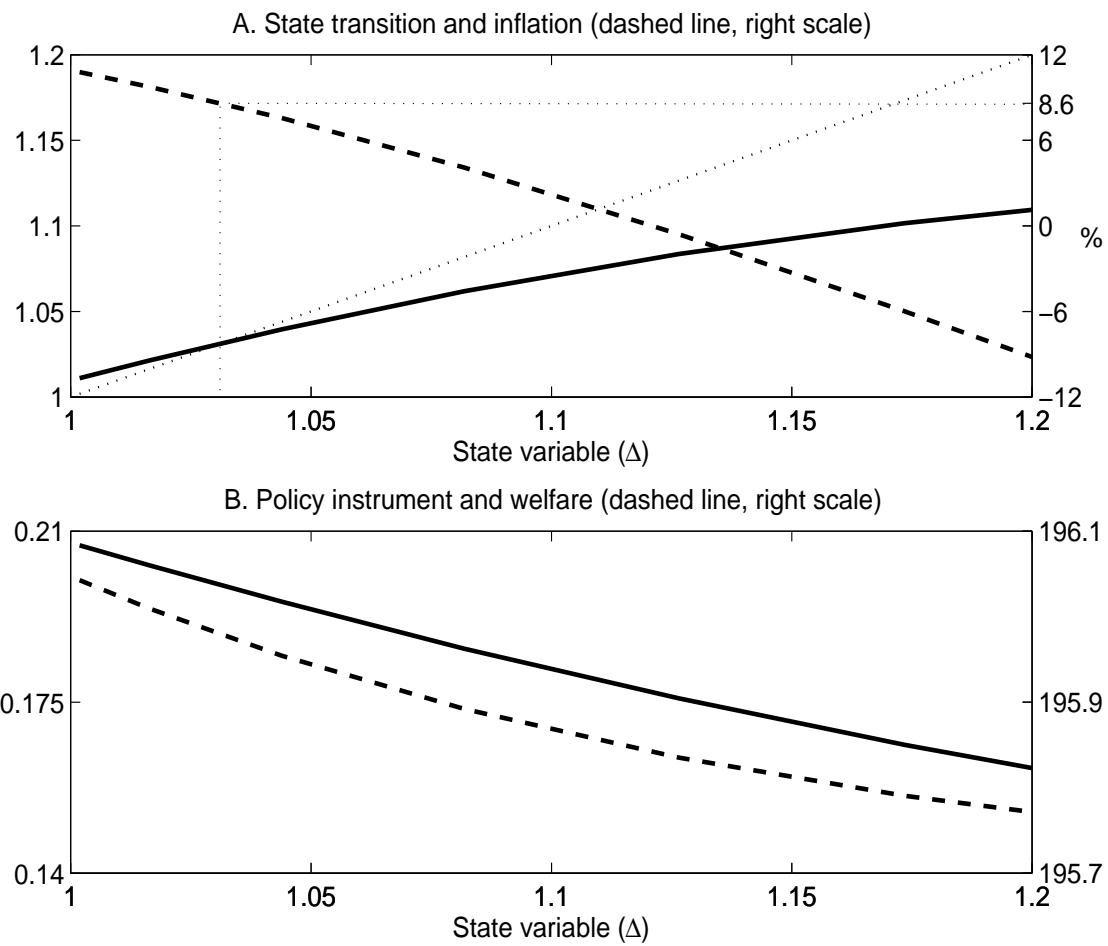


Figure 1: Equilibrium as a function of the state

and welfare on the right scale).¹⁰ Both functions are downward sloping. Intuition for the welfare function's downward slope is straightforward. Eq. (19) shows that the current relative price distortion represents the inverse of average productivity. But the current relative price distortion is also a summary statistic for the dispersion in relative prices. The higher is the inherited relative price distortion, the higher is the inherited dispersion in relative prices, and through (20) this contributes to a higher dispersion in current relative prices. Higher dispersion in current relative prices in turn reduces current productivity, reducing welfare.

It is less straightforward to understand the downward sloping policy function, $m = \Gamma^*(\Delta)$. At a superficial level, it seems consistent with the state transition function for m to be decreasing in Δ : if equilibrium involves the relative price distortion declining from a high level, then a large inherited relative price distortion ought to be met with a relatively low normalized money supply, so that newly adjusting firms do not exacerbate the relative price distortion. However, in order to develop the intuition for $\Gamma^*(\Delta)$ more fully it is necessary to examine the nature of the policy problem in equilibrium.

5.2 Policymaker's objective function

Figure 2 displays the policymaker's objective function (Panel A) and the current period component of the objective function (Panel B) for two values of the state variable (1 and 1.04). Both panels display functions that are concave, and the unique maximum is achieved with lower values of m for the higher value of the state. The future component of value, $\beta v(\Delta')$ in (30), is not plotted, but it is decreasing in m for all values of Δ . From Figure 2 then, the fact that m is a decreasing function of Δ seems to be associated with

¹⁰In Panel B of Figure 1 and in Figure 2 we have not converted welfare into more meaningful consumption-equivalent units because the magnitudes are very small. The consumption-equivalent welfare measures in these figures would vary by less than 0.02 percent.

the state variable's influence on the current-utility component of welfare. As discussed in King and Wolman (1999, 2004), real effects of monetary policy in models such as this one work through the relative price distortion (Δ') and the average markup of price over marginal cost ($1/w$ here). Thus, examining the behavior of these two distortions can help clarify why the current component of the objective function is maximized with a lower m the higher is Δ .

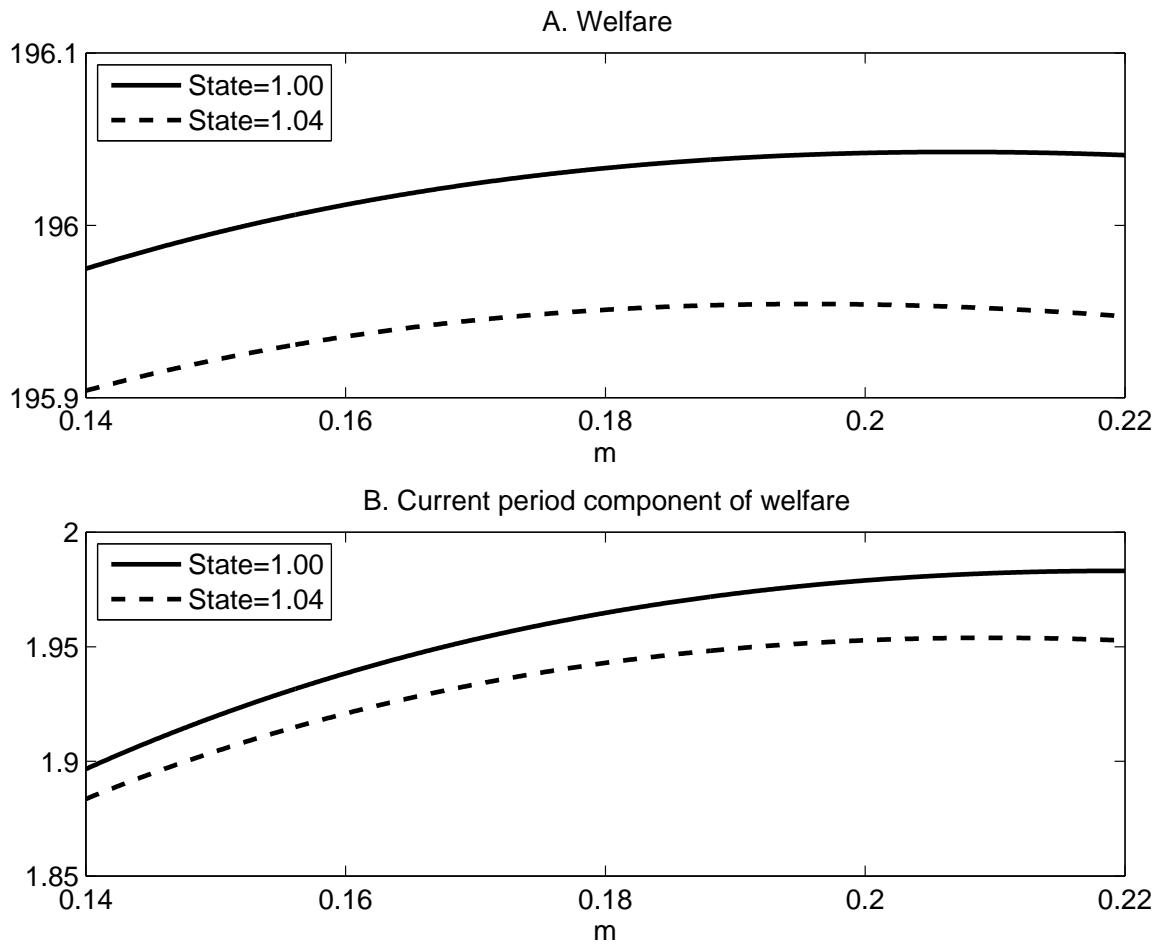


Figure 2: Policymaker's objective function

Figure 3 plots the markup distortion (Panel A) and the relative price distortion (Panel B) as a function of m for the same two values of the state variable. In both cases higher

values of m correspond to a lower markup and a higher relative price distortion. This feature is the essential short-run policy trade-off in the Calvo (or Taylor) model: a higher money supply will bring down the markup at the cost of increasing the relative price distortion. From Figures 2 and 3 it is apparent that as the state variable increases, the trade-off shifts in favor of the relative price distortion. That is, the policymaker chooses lower m at higher values of Δ because the decrease in the markup that would come from holding m fixed at higher Δ is more than offset by welfare costs of a higher relative price distortion Δ' .

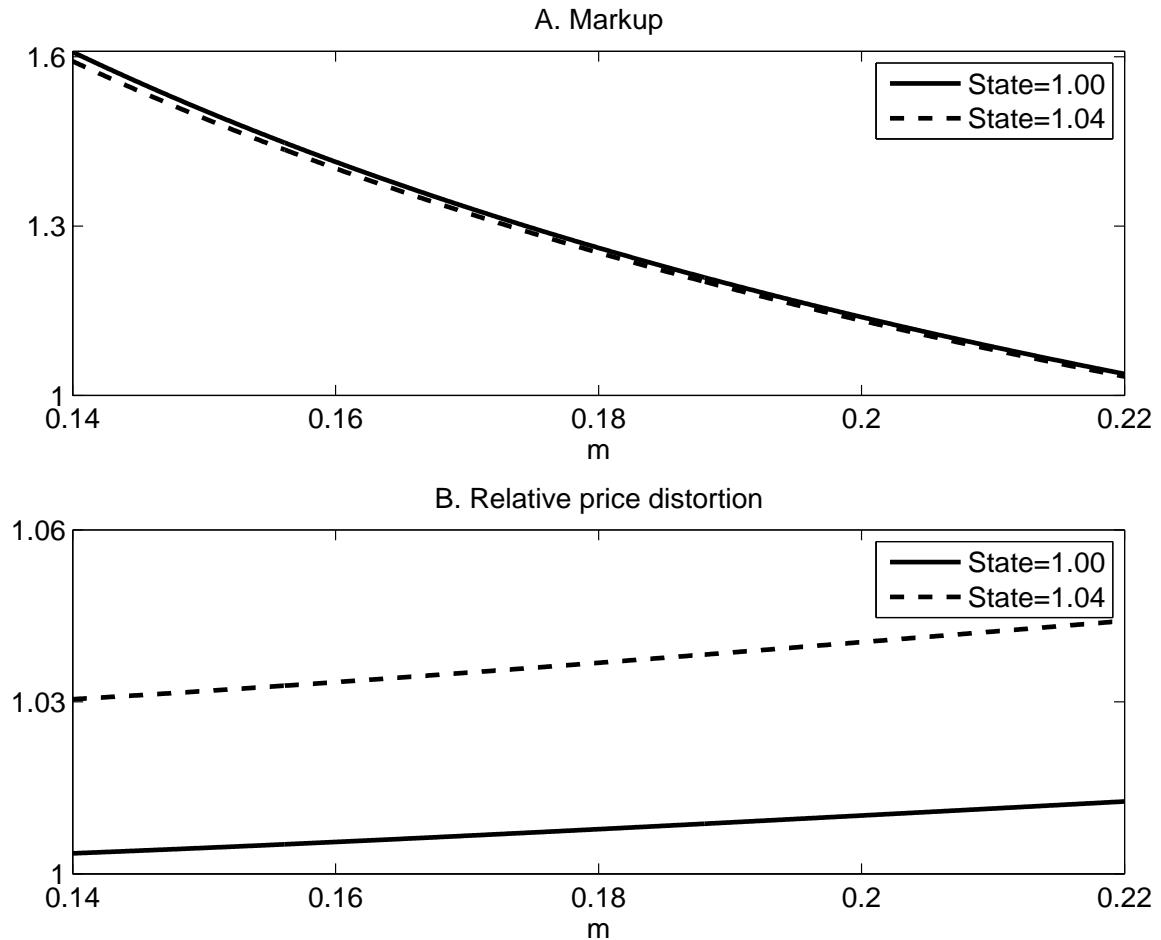


Figure 3: Distortions as functions of m

What is the intuition for increased sensitivity of the relative price distortion to m at higher levels of inherited relative price dispersion (Δ)? Although we cannot explicitly solve for the relationship between the relative price distortion and the money supply, we can study the relationship between the relative price distortion and the relative price chosen by adjusting firms. Assuming (correctly) a positive relationship between equilibrium p_0 and m , this relationship is informative for understanding why the relative price distortion can be viewed as driving the shape of the policy function.

Combining the market clearing condition (25) with the transformed price index (24) yields

$$\Delta' = \frac{(1 - \alpha) p_0^{-\varepsilon} + \alpha \Delta}{[\alpha + (1 - \alpha) p_0^{1-\varepsilon}]^{\varepsilon/(\varepsilon-1)}}. \quad (32)$$

From this expression it follows that the sensitivity of the relative price distortion to the relative price of adjusters is increasing in the state:

$$\frac{\partial^2 \Delta'}{\partial p_0 \partial \Delta} = \frac{\varepsilon \alpha (1 - \alpha) p_0^{-\varepsilon}}{[\alpha + (1 - \alpha) p_0^{1-\varepsilon}]^{1+\lceil \varepsilon/(\varepsilon-1) \rceil}} > 0. \quad (33)$$

Figure 4 illustrates the relationship between Δ' and p_0 given by (32) for $\Delta = 1$ and $\Delta = 1.04$. The current relative price distortion is a locally convex function of the relative price set by adjusting firms.¹¹ If there is no inherited relative price dispersion ($\Delta = 1$) then the relative price distortion is minimized at $p_0 = 1$, whereas for higher inherited dispersion the relative price distortion is minimized at a lower value of p_0 . As (33) states, a larger Δ also corresponds everywhere to a steeper relative price distortion with respect to p_0 . Summarizing our argument then: as the state variable increases, the current policymaker would incur increasing welfare losses due to relative price distortions if she did not react by choosing m so that price setters set a *lower* relative price. We have

¹¹The relative price distortion as a function of p_0 becomes flat and thus concave at high values of p_0 ; for high enough p_0 customers have negligible demand for the goods sold by adjusters, and additional price increases have no effect on the relative price distortion.

not plotted the relationship between m and p_0 , but in a discretionary equilibrium it is positive and nearly linear. So this reasoning leads to a policy that sets m as a decreasing function of Δ .

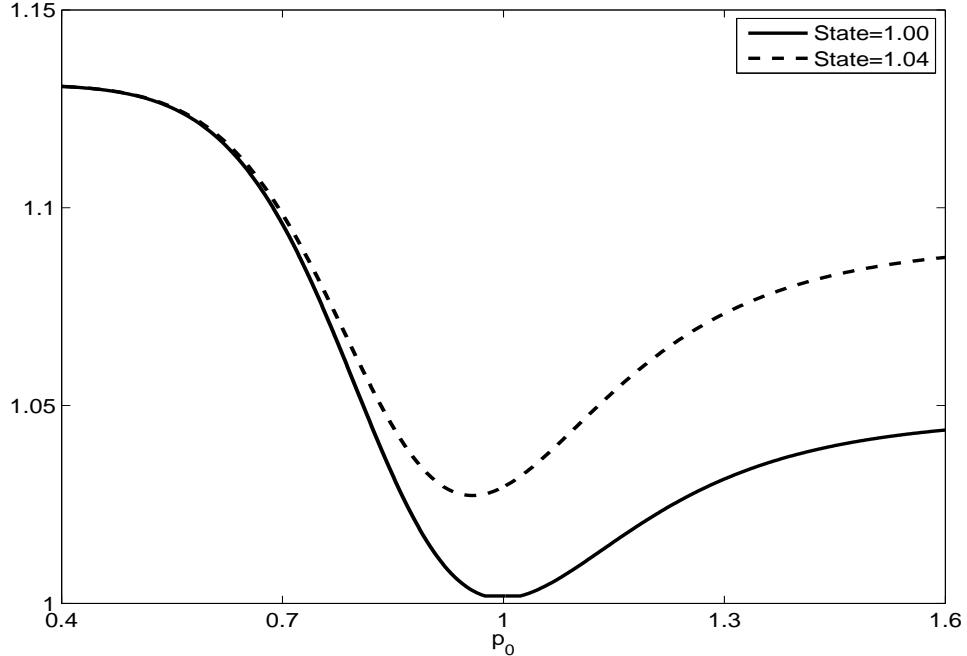


Figure 4: Relative price distortion as function of p_0

5.3 Properties of private-sector equilibrium

Our computational approach has led to finding a single discretionary equilibrium. The preceding discussion highlighted some of the properties of the equilibrium for particular parameters. Although we have not proved that the equilibrium is unique, in the many examples studied in this paper we have found no evidence of multiple equilibria.¹² This is in stark contrast to the Taylor model with two-period price setting, in which King

¹²Starting from the example of the baseline calibration, more than 40 other examples were computed, with a range of values of α , ε and μ . Details are available from the authors.

and Wolman (2004) *proved* the existence of multiple discretionary equilibria, which they traced to multiple private-sector equilibria.

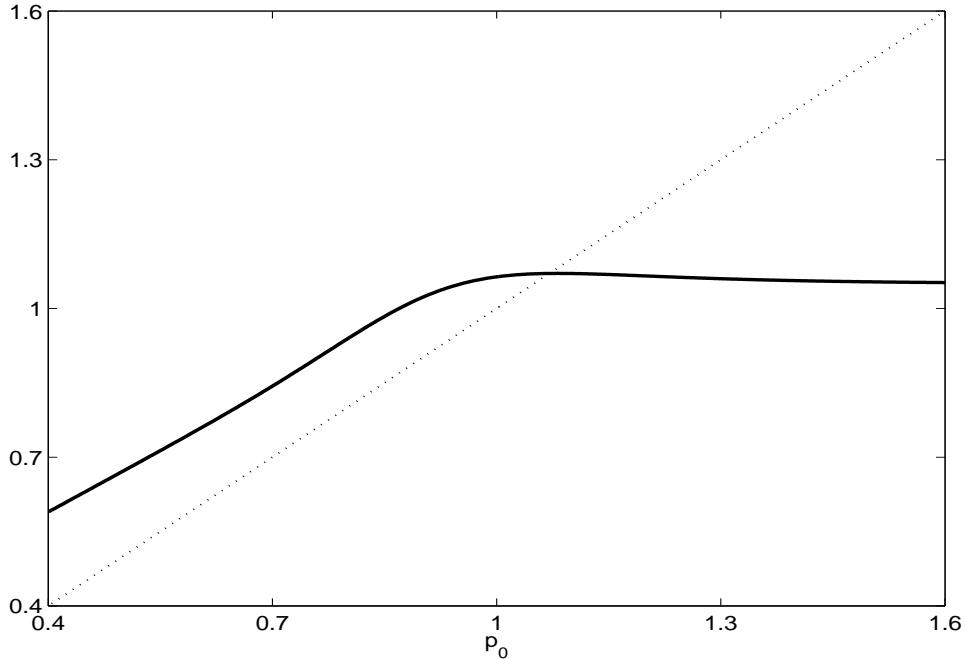


Figure 5: Pricing best response function: State = 1.04, $m = 0.20$

To help explain why multiplicity of private-sector equilibrium is less prevalent in the Calvo model, we turn to the best-response function for price-setting firms. The best-response function describes an individual firm's optimal price as a function of the price set by other adjusting firms. Figure 5 plots a typical best-response function in a discretionary equilibrium of the Calvo model, using the baseline calibration. It has a unique fixed point, and is concave in a neighborhood of the fixed point.¹³ In contrast, the

¹³Our computations have not revealed multiple fixed points in equilibrium. However, we have encountered rare instances of multiple fixed points for sub-optimal values of m . In Figure 5 there is a convex region of the best-response function to the left of the fixed point. In the case of multiple fixed points, the convex region of the best-response function intersects the 45-degree line twice, with a third fixed point located on the concave portion.

best-response function in the two-period Taylor pricing model is upward sloping, strictly convex and generically has either two fixed points or no fixed points (see Appendix A for more details of the Taylor model).

The different shape of the best response function under Calvo pricing is associated with a different relationship between firms' current optimal price and the future nominal money supply. This relationship is nonlinear, unlike in the Taylor model, for two reasons. First, the relationship between the optimal price and the future index of predetermined prices is nonlinear. Second, the optimal price determines the real future state variable to which future policy responds. We consider in turn how both these reasons weaken the complementarity between the price of optimizing firms.

First, suppose that the future policymaker were to set a constant m , raising the nominal money supply in proportion to the index of predetermined prices. In the Taylor model, where such a policy is optimal, the price set by adjusting firms *is* the index of predetermined prices, so the future nominal money supply rises linearly with the price set by adjusting firms. Understanding that this future policy response will occur, and that the price it sets today will also be in effect in the future, an individual firm's best response is to choose a higher price when all other adjusting firms choose a higher price.

In the Calvo model, in contrast, next period's index of predetermined prices comprises an infinite number of lagged prices, of which the price set by adjusting firms today is just one element. Under a constant m policy, the effect of an increase in prices set today on next period's nominal money supply depends on the effect of such an increase on next period's index of preset prices. That index of preset prices – which is just today's price index – is highly sensitive to low levels of the price set by firms today, and relatively insensitive to high levels of the price set by firms today. That is because goods with high (low) prices have a low (high) expenditure share and thus receive a low (high) weight in the price index. As the price set by firms goes to infinity, it has no effect on the index

of preset prices and no effect on tomorrow's nominal money supply.

Thus, in the Calvo model a constant m policy would lead to a nominal money supply that is increasing and concave in the price set by adjusting firms. Because a higher future money supply leads firms to set a higher price today, concavity of the future money supply corresponds to decreasing complementarity between the prices set by adjusters. This intuition is confirmed by the following result for the infinite labor supply elasticity case, which is the baseline calibration.

Proposition 1 *Suppose the money supply is always set according to a constant m policy, regardless of the state, and let $\mu = 0$. Then the Calvo model has a unique private-sector equilibrium.*

Proof. See Appendix B. ■

The second reason for weaker complementarity in the Calvo model is that the relationship between the current optimal price and the future nominal money supply depends on the future state variable. Indeed, the policy maker does not hold m constant, instead lowering it with the state (see Figure 1.B). The response of next period's normalized money supply to the price set by adjusting firms today therefore depends on the relationship between p_0 and Δ' . Equation (32) implies that for high (low) values of p_0 the future state is increasing (decreasing) in p_0 , holding fixed the current state:

$$\frac{\partial \Delta'}{\partial p_0} = \frac{\varepsilon\alpha(1-\alpha)p_0^{-\varepsilon-1}}{[\alpha + (1-\alpha)p_0^{1-\varepsilon}]^{1+[\varepsilon/(\varepsilon-1)]}} (\Delta p_0 - 1). \quad (34)$$

Given that equilibrium m is decreasing in Δ , future m is decreasing in p_0 for high values of p_0 and increasing in p_0 for low values of p_0 . That is, a higher price set by adjusting firms— if it is greater than $1/\Delta$ — translates into a higher value of the future state, and thus a lower value of the future normalized money supply. At low values of p_0 this relationship is reversed: increases in p_0 reduce the future state, and the policymaker would respond by raising future m .

Summarizing the argument: in the Taylor model the normalized money supply is constant in equilibrium, and this results in an increasing convex best-response function with multiple fixed points. In the Calvo model, if policy kept the normalized money supply constant there would be a unique equilibrium: complementarity would be weaker at high p_0 than in the Taylor model, because next period's index of predetermined prices responds only weakly to p_0 at high levels of p_0 . Because the normalized money supply is not constant in the Calvo model, the complementarity is weakened even further; m is decreasing in the state, and future m is decreasing in p_0 for high p_0 .

Both parts of this argument rely on the fact that there are many cohorts of firms with predetermined prices in the Calvo model. In the first part, the effect of prices set by adjusting firms on tomorrow's index of predetermined prices depends on the level of those prices set today, because consumers can shift their expenditures to the cohorts that set prices in previous periods. In the second part, the presence of many predetermined prices gives rise to the state variable, through which the future policymaker is dissuaded from accommodating large increases in the current price level. Thus, the existence of many cohorts of prices seems to be key to explaining why the Calvo model does not have the same tendency toward multiple discretionary equilibria as the Taylor model with two-period pricing. This reasoning suggests however that a Taylor model with longer duration pricing might not have multiplicity, because the same opportunities to substitute would be present. Unfortunately, it is computationally infeasible to study discretionary equilibrium in a Taylor model with long-duration pricing, unless one uses linear approximation methods as in Dotsey and Hornstein (2003).

Although our computations have found only one equilibrium in every case, it is important to note that we have not proved uniqueness of equilibrium. However, Proposition 1 gives us some confidence that the numerical results do generalize: the constant m policy, which is key to proving that there are multiple private sector equilibria in the

Taylor model, implies a unique private sector equilibrium in the Calvo model. If, as we suppose, Markov Perfect equilibrium is unique, the nature of equilibrium ought to be invariant to (i) the policy instrument and (ii) whether we solve a planner's problem, in which the planner picks allocations directly as in Anderson, Kim and Yun (2010). For our baseline parameterization we have confirmed that the same steady-state inflation rate obtains whether the policy instrument is the money supply or the nominal interest rate. In addition, we have replicated the steady-state inflation rate of 2.2 percent for the baseline case with $\alpha = 0.75$, $\varepsilon = 11$, and $\mu = 1$ reported in Anderson, Kim and Yun (2010), for both interest rate and money supply instruments.

6 Inflation sensitivity to structural parameters

Steady-state inflation under the baseline calibration exceeds 8 percent, as mentioned before. Since there is not widespread agreement about the proper values for the price non-adjustment probability α , the desired markup $\varepsilon/(\varepsilon-1)$, or the labor supply elasticity $1/\mu$, Figure 6 displays the steady-state inflation rate as a function of α (Panel A), $\varepsilon/(\varepsilon-1)$ (Panel B) and $1/\mu$ (Panel C).

As the figure shows, the steady-state inflation rate in a discretionary equilibrium varies between essentially zero for extremely high price stickiness with a desired markup of 1.11, to greater than 40 percent with a price non-adjustment probability of 0.67, a desired markup of 1.33 and an infinite labor supply elasticity. These results show that the Calvo model does not provide a clear-cut answer to the question, how big is the inflation bias?

The steady-state inflation rate is increasing in the price non-adjustment probability for low α and decreasing for high α , with a maximum inflation rate of 10.1 percent reached when $\alpha = 0.71$. The steady-state inflation rate is monotonically increasing in the desired markup and the labor supply elasticity.

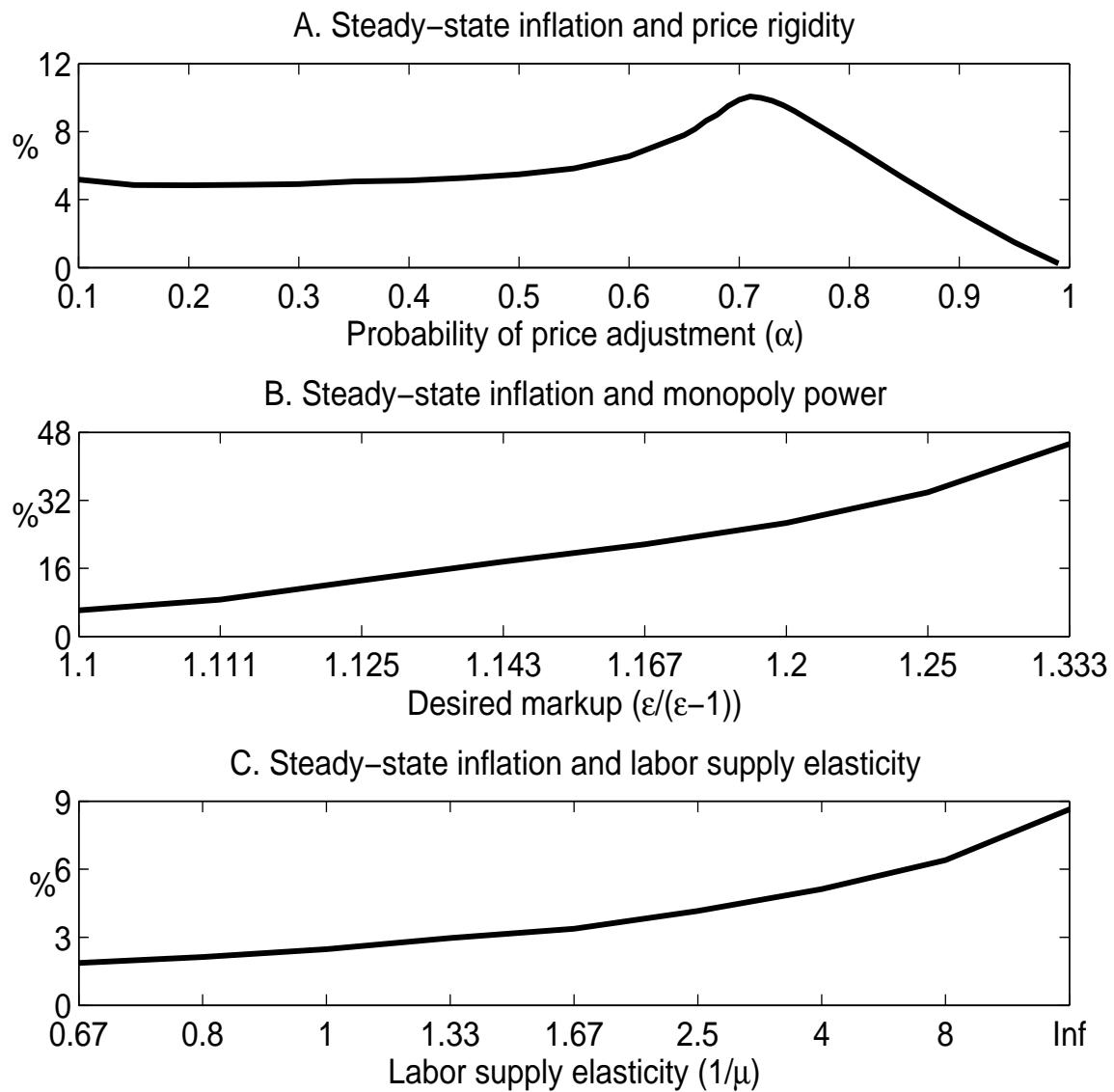


Figure 6: Steady-state inflation rate

In interpreting these figures, the policymaker’s trade-off between the relative price distortion and the markup is central. In a steady-state equilibrium, the policymaker is optimizing. Thus, the marginal benefit from decreasing the markup through a higher inflation rate is offset by the marginal cost associated with a larger relative price distortion. At higher degrees of price stickiness, the policymaker has greater leverage over the markup. Thus, as we move to the right in Panel A, the marginal benefit of higher current inflation at a given steady-state inflation rate is increasing. In order to counteract this larger marginal benefit, there must be a larger marginal cost through the relative price distortion. At low levels of price stickiness this larger marginal relative-price-distortion cost requires a higher inflation rate. But as we move to very high levels of price stickiness, the higher price stickiness itself accomplishes the required increase in the marginal cost of inflation. Thus, equilibrium occurs at lower inflation rates.

Panels B and C of Figure 6 display a monotonically increasing relationship between inflation and the desired markup and between inflation and the labor supply elasticity. The same reasoning applies to these relationships. As the desired markup or the labor supply elasticity increases, the monetary authority has more leverage over the markup, so the marginal benefit of higher inflation that arises from its ability to reduce the markup is increasing. In order to balance this larger marginal benefit, there must be a larger marginal relative-price-distortion cost of inflation. That in turn requires a higher steady-state inflation rate.

7 Concluding remarks

The Calvo model linearized around a zero-inflation steady state yields the New Keynesian Phillips curve, which has become the leading framework for applied monetary policy analysis. While there have been numerous analyses of discretionary monetary policy using the New Keynesian Phillips curve, little attention has been devoted to understanding discre-

tionary equilibrium in the underlying (non-linear) Calvo model. This paper has aimed to further such understanding. Discretionary equilibrium involves a positive steady-state inflation rate, and the steady-state inflation rate varies non-monotonically with the degree of price stickiness; together these results suggest that the zero-inflation approximation is inappropriate in the absence of a fiscal scheme to eliminate the monopoly distortion.

We also compared discretionary equilibrium in the Calvo model to the Taylor model with two-period pricing. The complementarity inherent in the Taylor model (King and Wolman, 2004) is substantially weakened in the Calvo model, typically leading to a unique private-sector equilibrium. The choice between superficially similar models (Calvo and Taylor) can thus have important implications for policy analysis, here for the nature of equilibrium when the policymaker cannot commit to future plans.

Our finding of large variation in the steady-state inflation rate across parameter values raises the question of whether exogenous price adjustment is a reasonable assumption. One could argue that the Calvo model should only be studied local to a steady state – only there is it reasonable to assume fixed price adjustment probabilities – and thus that LQ approaches to discretionary equilibrium are appropriate. Another view, which we favor, is that the very nature of the discretionary policymaker’s problem merits a global analysis. If the results cast doubt on the Calvo pricing assumption, the proper response is not to change the analysis from global to LQ, but to use a different model such as one where firms’ likelihood of price adjustment is allowed to vary with economic conditions. It seems inevitable that expanding our analysis to an environment with state-dependent pricing would add state variables to the problem, but it is surely a worthwhile topic for future research.¹⁴

¹⁴Siu (2008) studies discretionary policy in a model with some state-dependence, limiting the state space by allowing firms to adjust costlessly after two periods.

Appendix A: Background results from the Taylor model

In the Taylor model each firm sets its price for two periods. The description of the representative household remains unchanged, but the value of a firm upon adjustment is given by

$$\tilde{V}_t = \max_{X_t} \{d(X_t; P_t, c_t, w_t) + Q_{t,t+1} d(X_t; P_{t+1}, c_{t+1}, w_{t+1})\},$$

and the optimal price satisfies the first order condition,

$$\frac{P_{0,t}}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \cdot \frac{w_t + \beta (P_{t+1}/P_t)^\varepsilon w_{t+1}}{1 + \beta (P_{t+1}/P_t)^{\varepsilon-1}}.$$

Whereas in the Calvo model the index of predetermined prices was given by P_{t-1} , in the Taylor model there is just one predetermined price, $P_{0,t-1}$. Normalizing the optimal price and the price index by $P_{0,t-1}$ and using the definitions $\tilde{p}_{0,t} \equiv P_{0,t}/P_{0,t-1}$ and $p_t \equiv P_t/P_{0,t-1}$, we have

$$\frac{\tilde{p}_{0,t}}{p_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \cdot \frac{w_t + \beta (\tilde{p}_{0,t} p_{t+1}/p_t)^\varepsilon w_{t+1}}{1 + \beta (\tilde{p}_{0,t} p_{t+1}/p_t)^{\varepsilon-1}}. \quad (35)$$

As in the Calvo model, this condition indicates that the firm chooses a constant markup over a weighted average of current and future marginal costs.

Labor supply is given by equation (4). Money demand (5) is normalized by the lagged optimal price instead of the lagged price level

$$\tilde{m}_t \equiv \frac{M_t}{P_{0,t-1}} = p_t c_t. \quad (36)$$

We eliminate the predetermined variable from the price index,

$$P_t = \left(\frac{1}{2} P_{0,t}^{1-\varepsilon} + \frac{1}{2} P_{0,t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}},$$

by dividing by the lagged optimal price:

$$p_t = \left(\frac{1}{2} \tilde{p}_{0,t}^{1-\varepsilon} + \frac{1}{2} \right)^{\frac{1}{1-\varepsilon}}. \quad (37)$$

The labor market clearing condition yields

$$\begin{aligned} n_t &= \frac{1}{2} \sum_{j=0}^1 n_{j,t} \\ \frac{n_t}{c_t} &= \frac{1}{2} p_t^\varepsilon [(\tilde{p}_{0,t})^{-\varepsilon} + 1]. \end{aligned} \quad (38)$$

There is one predetermined nominal price ($P_{0,t-1}$), but there are no state variables in the labor market clearing condition.

In the Taylor model, the five equations (4) and (35)–(38), together with the behavior of future policymakers, implicitly define the set of feasible values for w_t , c_t , n_t , p_t and $\tilde{p}_{0,t}$ attainable by the current-period monetary authority. The current-period monetary authority chooses the money supply, or equivalently \tilde{m}_t , the money supply normalized by the predetermined price. Unlike the Calvo model, no state variables constrain the monetary authority in an MPE. The lagged optimal price $P_{0,t-1}$ matters for the levels of nominal variables, but is irrelevant for the determination of real allocations.

King and Wolman (2004) use a price-setting firm's best-response function to study discretionary equilibria in the Taylor model. That function is the optimal pricing condition (35) rewritten so that the right hand side is in terms of current and future \tilde{m} and current and future \tilde{p}_0 :

$$\tilde{p}_0 = \left(\frac{\varepsilon \chi}{\varepsilon - 1} \right) \cdot [(1 - \theta') \tilde{m} + \theta' \tilde{m}' \tilde{p}_0],$$

where

$$\theta' \equiv \frac{\beta [\tilde{p}_0 p(\tilde{p}'_0)/p(\tilde{p}_0)]^{\varepsilon-1}}{1 + \beta [\tilde{p}_0 p(\tilde{p}'_0)/p(\tilde{p}_0)]^{\varepsilon-1}}.$$

For any value of \tilde{m} , King and Wolman show that for fixed \tilde{m}' and \tilde{p}'_0 the best-response function is monotonically increasing and strictly convex with two fixed points or no fixed points (there is a knife-edge case with a unique fixed point). The presence of two fixed points for arbitrary \tilde{m} means that there are multiple discretionary equilibria, indexed by the distribution over the two fixed points of the best-response function (these fixed

points vary with the distribution). In a discretionary equilibrium there are endogenous fluctuations over the two fixed points.

King and Wolman stress that the complementarity necessary for multiple fixed points is associated with the fact that under discretion, the policymaker in the next period is certain to raise the nominal money supply proportionally with the price set by firms in the current period. An individual firm in the current period responds positively to the price set by other firms in order to avoid being stuck next period with high demand and a nominal price that is low relative to nominal costs. In the Taylor model, this effect is relatively weak at low values of p_0 and relatively strong at high values of p_0 . Another way to view the complementarity is between future policy and expected future policy: if firms expect a higher nominal money supply in the future, they will set a higher price today, and the future policymaker will accommodate with a higher money supply.

Appendix B: Proof of Proposition 1

This appendix presents the proof of Proposition 1. Recall from equation (13) that inflation is the following function of the optimal reset price

$$\pi(p_{0,t}) = [(1 - \alpha)p_{0,t}^{1-\varepsilon} + \alpha]^{\frac{1}{1-\varepsilon}},$$

which has the following properties:

$$\pi'(p_{0,t}) = (1 - \alpha) [(1 - \alpha) + \alpha p_{0,t}^{\varepsilon-1}]^{-\frac{\varepsilon}{\varepsilon-1}} = (1 - \alpha) \left[\frac{\pi(p_{0,t})}{p_{0,t}} \right]^\varepsilon > 0 \quad (39)$$

$$\pi''(p_{0,t}) = -\alpha(1 - \alpha)\varepsilon\pi(p_{0,t})^{2\varepsilon-1}p_{0,t}^{3(\varepsilon-1)} < 0. \quad (40)$$

Let T denote the final period, so $N_{T+1} = D_{T+1} = 0$. Then:

$$N_T = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \pi(p_{0,T})^{\varepsilon-1} [\chi m_T + \alpha\beta N_{T+1}\pi(p_{0,T})] = \left(\frac{\varepsilon\chi}{\varepsilon - 1} \right) \pi(p_{0,T})^{\varepsilon-1} m_T, \quad (41)$$

$$D_T = \pi(p_{0,T})^{\varepsilon-1} [1 + \alpha\beta D_{T+1}] = \pi(p_{0,T})^{\varepsilon-1}, \quad (42)$$

and the pricing best-response function is

$$\hat{p}_{0,T} = \frac{N_T}{D_T} = \left(\frac{\varepsilon\chi}{\varepsilon - 1} \right) m_T. \quad (43)$$

The outcomes $p_{0,T}$, N_T , and D_T do not depend on the state because monetary policy does not depend on the state. Moreover, there can be no complementarity in price setting in period T , because the pricing best-response function (43) of any given firm does not depend on other firms' price decisions.

Note from (41) that $N_T = N_T(m_T, p_{0,T})$ and from (42) that $D_T = D_T(p_{0,T})$. We can now analyze the period $T - 1$ pricing best-response function to determine whether there is a unique fixed point.

$$\begin{aligned} N_{T-1} &= \left(\frac{\varepsilon}{\varepsilon - 1} \right) \pi(p_{0,T-1})^{\varepsilon-1} [\chi m_{T-1} + \alpha\beta N_T(m_T, p_{0,T})\pi(p_{0,T-1})] \\ D_{T-1} &= \pi(p_{0,T-1})^{\varepsilon-1} [1 + \alpha\beta D_T(p_{0,T})]. \end{aligned}$$

Hence, the period $T - 1$ best response function is

$$\hat{p}_{0,T-1} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \left[\frac{\chi m_{T-1}}{1 + \alpha\beta D_T(p_{0,T})} + \frac{\alpha\beta N_T(m_T, p_{0,T})\pi(p_{0,T-1})}{1 + \alpha\beta D_T(p_{0,T})} \right] \quad (44)$$

The optimal price does not depend on the state because the monetary policy function and the functions N_T and D_T do not depend on the state. To see that the best response function has a unique fixed point, first write it as

$$\hat{p}_{0,T-1} = A_{T-1}(p_{0,T})m_{T-1} + B_{T-1}(m_T, p_{0,T})\pi(p_{0,T-1}),$$

where $A_{T-1}(p_{0,T}) > 0$ and $B_{T-1}(m_T, p_{0,T}) > 0$ because $m_T > 0$. It follows from (39) and (40) that

$$\begin{aligned} \frac{\partial \hat{p}_{0,T-1}}{\partial p_{0,T-1}} &= B_{T-1}\pi'(p_{0,T-1}) > 0 \\ \frac{\partial^2 \hat{p}_{0,T-1}}{\partial p_{0,T-1}^2} &= B_{T-1}\pi''(p_{0,T-1}) < 0 \end{aligned}$$

Because the best response function is always positive and concave it has a unique fixed point. Therefore, there exists a unique private-sector equilibrium in period $T - 1$.

Write $N_{T-1} = N_{T-1}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T})$ and $D_{T-1} = D_{T-1}(p_{0,T-1}, p_{0,T})$. In period $T - 2$ we obtain

$$\begin{aligned} N_{T-2} &= \left(\frac{\varepsilon}{\varepsilon - 1} \right) \pi(p_{0,T-2})^{\varepsilon-1} [\chi m_{T-2} + \alpha \beta N_{T-1}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T}) \pi(p_{0,T-2})] \\ D_{T-2} &= \pi(p_{0,T-2})^{\varepsilon-1} [1 + \alpha \beta D_{T-1}(p_{0,T-1}, p_{0,T})]. \end{aligned}$$

Hence the period $T - 2$ best response function can be written as

$$\hat{p}_{0,T-2} = A_{T-2}(p_{0,T-1}, p_{0,T})m_{T-2} + B_{T-2}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T})\pi(p_{0,T-2}),$$

where $A_{T-2} > 0$ and $B_{T-2} > 0$ because $m_{T-1}, m_T > 0$. By the same arguments as above there is a unique fixed point in period $T - 2$.

Repeating the same steps, we can show that for period t ,

$$\begin{aligned} N_t &= \left(\frac{\varepsilon}{\varepsilon - 1} \right) \pi(p_{0,t})^{\varepsilon-1} [\chi m_t + \alpha \beta N(m_{t+1}, m_{t+2}, \dots, p_{0,t+1}, p_{0,t+2}, \dots) \pi(p_{0,t})] \\ D_t &= \pi(p_{0,t})^{\varepsilon-1} [1 + \alpha \beta D(p_{0,t+1}, p_{0,t+2}, \dots)]. \end{aligned}$$

The period t best-response function therefore can be written as

$$\hat{p}_{0,t} = A_t(p_{0,t+1}, p_{0,t+2}, \dots)m_t + B_t(m_{t+1}, m_{t+2}, \dots, p_{0,t+1}, p_{0,t+2}, \dots)\pi(p_{0,t}),$$

where $A_t > 0$ and $B_t > 0$ because $m_{t+1}, m_{t+2}, \dots > 0$.

Therefore, by backward induction, there is a unique private-sector equilibrium associated with the arbitrary constant m policy.

References

- [1] Adam, K. and R.M. Billi (2007), “Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates,” *Journal of Monetary Economics* 54, 728-752.
- [2] Albanesi, S., V.V. Chari, and L.J. Christiano (2003), “Expectation Traps and Monetary Policy,” *Review of Economic Studies* 70, 715-741.
- [3] Anderson, G., J. Kim and T. Yun (2010), “Using a Projection Method to Analyze Inflation Bias in a Micro-Founded Model,” *Journal of Economic Dynamics and Control* 34, 1572-1581.
- [4] Calvo, G.A. (1983), “Staggered Prices in a Utility-Maximizing Framework”, *Journal of Monetary Economics* 12, 383-398.
- [5] Clarida, R., J. Gali and M. Gertler (1999), “The Science of Monetary Policy: A New Keynesian Perspective,” *Journal of Economic Literature* 37, 1661-1707.
- [6] Dotsey, M. and A. Hornstein (2009), “On the Implementation of Markov-Perfect Interest Rate and Money Supply Rules: Global and Local Uniqueness,” Working Paper 09-06, Federal Reserve Bank of Richmond.
- [7] Dotsey, M. and A. Hornstein (2003), “Should a Monetary Policymaker Look at Money?” *Journal of Monetary Economics* 50, 547-579.
- [8] Glomm, G. and B. Ravikumar (1995), “Endogeneous Public Policy and Multiple Equilibria,” *European Journal of Political Economy* 11, 653–662.
- [9] Khan, A., R.G. King and A.L. Wolman (2001), “The Pitfalls of Discretionary Monetary Policy,” Working Paper 01-16, Federal Reserve Bank of Philadelphia.
- [10] King (2006), “Discretionary Policy and Multiple Equilibria,” Federal Reserve Bank of Richmond *Economic Quarterly* 92, 1-15.

- [11] King, R.G. and A.L. Wolman (2004), “Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria,” *Quarterly Journal of Economics* 119, 1513-1553.
- [12] King, R.G. and A.L. Wolman (1999), “What Should the Monetary Authority Do When Prices Are Sticky?” in *Monetary Policy Rules*, J.B. Taylor ed., University of Chicago Press, 1999, 349-398.
- [13] King, R.G. and A.L. Wolman (1996), “Inflation Targeting in a St. Louis Model of the 21st Century,” Proceedings, Federal Reserve Bank of St. Louis, 83-107.
- [14] Kydland, F.E. and E.C. Prescott (1977), “Rules Rather than Discretion: the Inconsistency of Optimal Plans,” *Journal of Political Economy* 85, 473-491.
- [15] Siu, H.E. (2008), “Time Consistent Monetary Policy with Endogenous Price Rigidity,” *Journal of Economic Theory* 138, 184-210.
- [16] Woodford, M., *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.
- [17] Yun, T. (2005), “Optimal Monetary Policy with Relative Price Distortions,” *American Economic Review* 95, 89-108
- [18] Yun, T. (1996), “Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles,” *Journal of Monetary Economics* 37, 345-370.