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AVOIDING THE INFLATION TAX*

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I study the effects of inflation on the purchasing behavior of buyers in an economy where money is essential for certain transactions (as in Lagos and Wright, 2005). A long-standing intuition in this subject is that when inflation increases, agents try to spend their money holdings speedily. The standard framework fails to capture this kind of effect (see Lagos and Rocheteau, 2005). I propose a simple modification of the model that improves it in this dimension. I assume that buyers can rebalance their money holdings only sporadically (i.e., not every period). With this minimal change in the environment, I show that higher inflation induces some buyers to spend their money faster by frontloading their consumption, searching more intensively for transactions, and buying low-quality goods. In this way, the model is able to reproduce distortions in the pattern of transactions that, traditionally, have played an important role in the evaluation of the cost of inflation.

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1. INTRODUCTION

There are two main effects of expected inflation on the demand for money by individuals. First, when inflation increases, agents tend to carry less money. Inflation acts as a tax on cash transactions, and when inflation increases, agents tend to shift their consumption patterns away from cash-intensive activities. This demonetization effect has been thoroughly studied in the literature and is the basic force that determines the welfare costs of expected inflation in, for example, the canonical cash-in-advance model (see Cooley and Hansen, 1989, and Lucas, 2000).

The second effect of inflation on money demand is more subtle. When inflation increases, agents try to reduce the average time they carry a given amount of money necessary for a transaction. In the words of Irving Fisher "when depreciation is anticipated, there is a tendency among owners of money to spend it speedily" (Humphrey, 1993).[1] In this paper, I use a modern search-based model of monetary exchange to formally study this second effect of anticipated inflation.

Recently, a powerful modeling device has become common in monetary theory: A growing number of articles, following Lagos and Wright (2005), assume that agents have the ability to rebalance (at no cost) their asset portfolios at the end of every monetary-trade interaction. Combined with quasilinear preferences, this assumption results in a tractable simplification of the distribution of money holdings and its dynamics. However, not everything that results from this assumption is appealing. Counter to the intuition held by Fisher and many others, in such a model increases in inflation do not increase the willingness to trade by agents holding money.

This result is highlighted by Lagos and Rocheteau (2005) who showed that in an extended Lagos-Wright framework including a search-intensity decision, when inflation increases agents tend to *reduce* their effort to engage in (monetary) transactions.[2] In other words, agents *do not* search more or try to spend their money speedily in response to inflation. The reason for this result is clearly explained by Lagos and Rocheteau: As inflation increases, agents decide to hold less money (demonetization); lower money holdings result in smaller purchases, which are

[1] Bresciani-Turroni (1937), in his classic account of the German inflation in the interwar period, wrote (p.190): "Germans, greatly agitated by the fall of the mark, bought whatever they could at any price simply in order to exchange their money for 'material values'." For a thorough discussion of all the potential effects that high inflation can have on economic activity, see Heymann and Leijonhufvud (1995).

[2] Lagos and Rocheteau (2005) is not the only paper studying search intensity in this kind of search-based models of money. For another recent contribution see, for example, Peterson and Shi (2004).

associated with smaller benefits from finding and executing a trade; and naturally, then, agents have less incentive to search for trades.

After establishing this result, Lagos and Rocheteau (2005) modify their model and study competitive price posting (as in Rocheteau and Wright, 2005). They show that, for low inflation rates, buyers may indeed increase their search intensity when inflation increases. The reason for this effect is, again, easy to understand. Basically, the combination of search externalities and competitive search allows them to endogenize the share of the surplus obtained by buyers and, in fact, makes that share a decreasing function of the size of the trade (i.e., the quantity exchanged). As before, inflation generates a demonetization effect, which tends to decrease the size of equilibrium trades. Hence, the total surplus from a trade still decreases with inflation. However, the proportion of the surplus obtained by buyers *increases*, and, for low inflation, the actual surplus obtained by buyers also increases (even though the total surplus decreases). If buyers expect their own surplus to be higher, then they have more incentives to search for trades.

The mechanisms at work in the Lagos and Rocheteau model with competitive search fall short of capturing the logic in Fisher's statement. A clear indication of this fact is that in their model, conditional on the size of the (planned) purchase (and hence, the buyer's money holdings), a buyer makes the same effort to find a trade, irrespective of the level of inflation. In other words, conditional on the amount of money they are holding, agents make the same effort to spend it, no matter what the inflation rate is.

This situation then leaves us with an open question: Which features of the environment would induce agents to spend their money holdings "speedily" in response to inflation? This paper provides an answer to this question within the Lagos-Wright framework. In particular, we study a simple extension of the Lagos and Rocheteau (2005) model in which buyers rebalance their money holdings only sporadically (not every period, but every other period). Under this modification, buyers need to decide their spending pattern between rebalancing periods and may engage in activities (like search effort) that allow them to increase their spending early on after rebalancing.

Our minimal change in timing also creates a crucial asymmetry between buyers and sellers: Some buyers, when searching for trades, know that if they do not find a trade, they will have to wait until next period for another chance to spend their money. And, most importantly, because of inflation, that money will have depreciated by then. Sellers, on the other hand, have access every period to an "afternoon" (centralized) market where they can exchange money for goods and rebalance their money holdings.

The key factor driving the results in this paper is that, in an inflationary environment, sellers that are able to trade in the "afternoon" market value money more than buyers that do not expect to participate in such a market. This asymmetry, combined with inflation, induces an additional benefit from trade, since trade involves a reshuffling of money from buyers with low valuation to sellers with high valuation.[3]

The basic idea, then, can be recast in a simple, intuitive manner: when sellers (because of a higher degree of financial sophistication, for example) are better able than buyers to avoid the inflation tax, the resulting asymmetry in money valuation creates an incentive for buyers to transfer money holdings to sellers via trading of goods. In this way, inflation induces distortions in the pattern of transactions (and, hence, the allocation of goods in the economy) that could result in considerable welfare losses.

The rest of the article is organized as follows. The next section describes the model. Section 3 describes equilibrium conditions. Section 4 studies the effect of inflation on the trading activities of agents and the extent to which inflation induces agents to spend their money faster. The section is divided into three subsections. First, we study a consumption frontloading effect of inflation, then we combine this frontloading effect with changes in search effort, and finally, we introduce different qualities of goods and study the predisposition of agents to buy low quality goods to avoid paying the inflation tax. The last section further discusses the results and concludes.

2. THE MODEL

The model is a modified version of that in Lagos and Rocheteau (2005). Time is discrete. There are two groups of agents: buyers and sellers. All agents are infinitely lived and discount

[3]The asymmetry between buyers and sellers creates an incentive for trade that is similar to the one studied by Rubinstein and Wolinsky (1987) in their explanation of the role of middlemen. In particular, sellers are able to trade faster because they have access to the "afternoon" market every period. Inflation exacerbates the costs implied by search frictions and hence creates further incentives for (some) buyers to trade with sellers.

the future according to the discount factor $\beta \in (0, 1)$. There is a measure one of buyers and a measure one of sellers in the economy.

Each period is divided into two subperiods. In the first subperiod, (some) agents interact in a decentralized market where buyers get randomly matched with sellers and trade anonymously. When a buyer and a seller decide to trade, the buyer makes a take-it-or-leave-it offer to the seller. There are two types of goods being traded in the decentralized market: A high quality good (H) and a low quality good (L). The high quality good, when consumed, provides at least as much utility to the buyers as the low quality good but it is equally costly to produce. Let $\varepsilon_i u(q)$ be the utility for the buyer consuming q units of good i with $i = H, L$, where ε_i takes the value ε_H if the good is high quality and ε_L if it is low quality. Accordingly, we have that $\varepsilon_H \geq \varepsilon_L$ and, to simplify notations, assume that $\varepsilon_H = 1 \geq \varepsilon = \varepsilon_L$. The seller's cost of production is given by $c(q)$ where q is the quantity produced. A proportion $\sigma_H = \sigma$ of sellers produce high quality goods and the rest produces low quality goods (e.i., $\sigma_L = 1 - \sigma$).

Buyers choose a level of search intensity in the decentralized market that influences their likelihood of being matched with a seller. Let e_j be the level of search intensity chosen by buyer j and $\overline{e}$ the average search intensity in the economy. Increasing search intensity comes at a cost and we denote by $v(e)$ the utility cost of choosing search intensity e. Assume $v'(e) \geq 0$ and $v''(e) \geq 0$.

Matching in the decentralized market takes place according to the matching function $\zeta(\overline{e}, \mu_b, \mu_s)$, where μ_b is the measure of buyers in the market, μ_s is the measure of sellers, and $\overline{e}$ is the average search intensity exerted by buyers. Assume that $\zeta(\overline{e}, \mu_b, \mu_s) = \min\{\alpha \overline{e} \mu_b, \mu_b, \mu_s\}$ with $\alpha < 1$. Note that $\zeta(\overline{e}, \mu_b, \mu_s) \leq \min\{\mu_b, \mu_s\}$ for all $\overline{e} \geq 0$. This matching function limits the role of search externalities among buyers, which is inessential for the subject of this paper.[4] The probability for a buyer of being matched to a seller when choosing search intensity e_j is given by $\alpha_b(e_j) = \min\{e_j \zeta(\overline{e}, \mu_b, \mu_s)/\overline{e}\mu_b, 1\}$ and the probability for a seller of being matched to a buyer is given by $\alpha_s = \zeta(\overline{e}, \mu_b, \mu_s)/\mu_s$.

[4]Lagos and Rocheteau (2005) study the model with a matching function that exibits search externalities. These externalities are essential for the findings in the second part of their paper. The combination of competitive search and search externalities can result in increasing search effort for low levels of inflation. We concentrate in a different mechanism that is independent of the existence of search externalities. Furthermore, it can be easily verified that our results extend to a setup with competitive search.

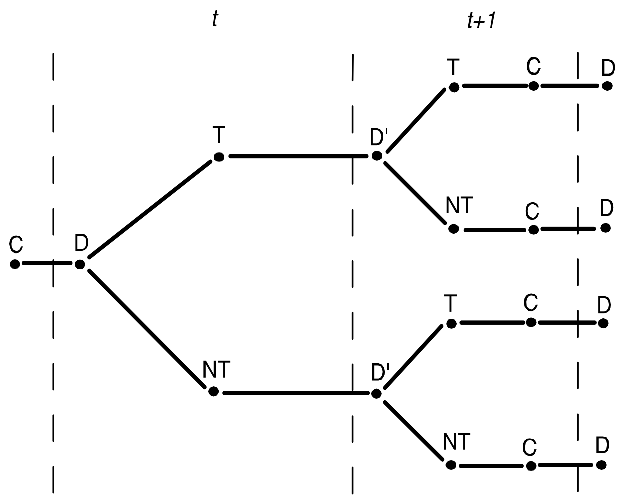

Figure 1

In the second subperiod, (some) agents interact in a centralized market and produce and consume a "general" good. Let $U(X)$ be the utility from consuming a quantity X of the general good. These goods can be produced one-to-one with labor H, from which agents experience linear disutility.

The main modification of the environment relative to that in Lagos and Rocheteau (2005) is in the timing of trade for buyers. While Lagos and Rocheteau assumed that all agents can visit the centralized market *every* period, here I assume that buyers enter the centralized market *every other* period.[5] More concretely, let us assume that half of the buyers visit the centralized market in odd periods and the other half in even periods. Sellers, on the other hand, have access to both markets every period.

Figure 1 illustrates the trading itinerary of buyers. After a buyer exits the centralized market, she enters the decentralized market in the first subperiod of the following period (node D in the figure). In this market, she may find an appropriate seller and trade (node T) or she may not find a seller (note NT). If the buyer visited the centralized market the previous period, then she does not get to visit the centralized market at the end of the current period. She just waits until the

[5]Berentsen et al. (2005) introduce a similar change in timing in the Lagos and Wright (2005) framework and study the distributional effects of inflation. This assumption should be taken as a first approximation to the case when visiting the centralized market is costly, as in Chiu and Molico (2006).

next period and goes back to the decentralized market (node D$'$). In the decentralized market, again, she may or may not get an opportunity to trade (nodes T and NT, respectively). After visiting the decentralized market for the second time, the buyer goes back to the centralized market to work and rebalance her money holdings (node C). In other words, after not visiting the centralized market for one period, the buyer has access to that market at no cost.

I maintain the standard technical assumptions on the functions u, c, and U. That is, they are twice continuously differentiable, $u(0) = c(0) = 0$, $u' > 0$, $c' > 0$, $u'' < 0$, $c'' \geq 0$, $U' > 0$, and $U'' \leq 0$. Also, there exists $q_i^* \in (0, \infty)$ such that $\varepsilon_i u'(q_i^*) = c'(q_i^*)$ for $i = H, L$. As in Lagos and Rocheteau (2005) assume that $U(X^*) = X^*$ with $U'(X^*) = 1$.

Finally, in this environment there is also an intrinsically useless, perfectly divisible and storable asset that will be called money. The stock of money in period t is given by $M_t = \gamma^t M_0 > 0$ with $t = 0, 1, ...$ We assume that $\gamma \geq \beta$ and that monetary injections take place through lump-sum transfers to buyers in the centralized market.[6]

3. EQUILIBRIUM

As it is standard in these models, the relevant monetary decision variable is real balances. Hence, define $z_t = \phi_t m_t$ as real balances, where ϕ_t is the price of money in the centralized market in period t and m_t is nominal money holdings. We denote with π_{t+1} the inflation rate between periods t and $t+1$ and we have that

$$\frac{\phi_t}{\phi_{t+1}} = 1 + \pi_{t+1}.$$

Before describing the equilibrium, we discuss in detail the problems faced by buyers and sellers and the mechanism determining the terms of trade in the decentralized market. We start with the problem of the sellers since it is simple and serves as a good introduction for the more complicated problem of the buyers.

[6] This assumption allows us to abstract from the potential distributional consequences of monetary policy, as those studied by Berentsen et al. (2005) and Williamson (2005).

3.1. The problem of the sellers

There are three types of buyers that a seller can meet: (1) a buyer that visited the centralized market last period, (2) a buyer that did not visit the centralized market last period but was able to find a trade in the decentralized market last period; and (3) a buyer that did not visit the centralized market nor found a trade last period. By the law of large numbers, and since in every period half of the buyers are due to visit the centralized market, the sellers' problem is the same regardless of whether t is even or odd.

Sellers produce for the buyer as long as trading does not reduce their welfare. The value function of a seller at the beginning of period t is given by:

$$V_t^s\left(z\right) = \alpha_{st} E\left[-c(q_t) + W_t^s(z + z_t^d)\right] + (1 - \alpha_{st}) W_t^s(z),$$

where (q_t, z_t^d) is an acceptable take-it-or-leave-it offer by a buyer and $W_t^s(z)$ is the value function from entering the centralized market in period t with z units of real balances. In the first term, the expectation is taken over the three different offers (corresponding to the different types of buyers) that the seller may receive. Finally, note that α_{st} is a function of $\overline{e}_t \mu_{bt}$ where $\overline{e}_t$ is the average search intensity chosen by buyers in the decentralized market.

The value function $W_t^s(z)$ is given by:

$$W_t^s\left(z\right) = \max_{z_{t+1}, H_t, X_t} \left\{U\left(X_t\right) - H_t + \beta V_{t+1}^s\left(z_{t+1}\right)\right\},$$

subject to

$$z_{t+1} = z + (H_t - X_t) - \pi_{t+1} z_{t+1},$$

and the standard non-negativity conditions. It is easy to see that when $1 + \pi_{t+1} > \beta$ the solution to this problem has $z_{t+1} = 0$ and, under some mild assumptions (to avoid corner solutions), $W_t^s(z) = z + \beta V_{t+1}^s\left(0\right)$. Then, as is standard for these models, sellers have no reason to carry money to the decentralized market (Lagos and Rocheteau, 2005) and a take-it-or-leave-it offer by a buyer is acceptable for the seller if and only if

$$-c(q_t) + z_t^d \geq 0.$$

This condition is important in the determination of the equilibrium terms of trade.

3.2. The problem of the buyers

We now turn to the problem of the buyers. In the centralized market, buyers and sellers have common features. For this reason, using the same logic as that used for sellers, we can show that the value function for a buyer entering the centralized market in period t with z units of real balances is given by:

$$W_t(z) = z + \tau_t + \max_{z_{t+1}} \left\{ -(1+\pi_{t+1}) z_{t+1} + \beta V_{1,t+1}(z_{t+1}) \right\}.$$

where τ_t is the real value of the lump-sum money transfers and $V_{1,t+1}(z_{t+1})$ is the value for a buyer of entering the decentralized market in period $t+1$, with z_{t+1} units of real balances, after visiting the centralized market in the previous period.

Define $e_{k,t}$, with $k = 1, 2$, as the search intensity chosen by a buyer for whom the last time he visited the centralized market was k periods ago. Also, let $V_{2,t}(z)$ be the value for a buyer of entering the decentralized market in period t with an amount z of real balances when she expects to visit the centralized market at the end of the period.[7] This value function is given by:

$$V_{2,t}(z) = \max_{e_{2,t}} V_{2,t}(z, e_{2,t}),$$

where, using the linearity of W_t,

$$V_{2,t}(z, e_{2,t}) = \alpha_b(e_{2,t}) \left(\sum_{j=H,L} \sigma_j \left[\varepsilon_j u\left(q_{2,t}^j(z)\right) - z_{2,t}^{dj}(z) \right] \right) + W_t(z) - v(e_{2,t}).$$

The functions $q_{2,t}^j(z)$ and $z_{2,t}^{dj}(z)$, with $j = H, L$, are the quantities exchanged in the decentralized market and the corresponding payments. We study the determination of these terms of trade in the next subsection.

The value function $V_{1,t}(z)$ describing the value for a buyer of entering the decentralized market at time t holding z units of real balances, when she expects not to visit the centralized market at the end of the period, is given by:

$$V_{1,t}(z) = \max_{e_{1,t}} V_{1,t}(z, e_{1,t}),$$

[7]The subscript 2 indicates that the agent has not visited the centralized market in the previous period.

where

$$
\begin{aligned}
V_{1,t}(z, e_{1,t}) &= \alpha_b(e_{1,t}) \left(\sum_{j=H,L} \sigma_j \left[\varepsilon_j u\left(q^j_{1,t}(z)\right) + \beta V_{2,t+1}\left(\frac{z - z^{dj}_{1,t}(z)}{1+\pi_{t+1}}\right)\right]\right) \\
&\quad + \left[1 - \alpha_b(e_{1,t})\right] \beta V_{2,t+1}\left(\frac{z}{1+\pi_{t+1}}\right) - v(e_{1,t}),
\end{aligned}
$$

and the functions $q^j_{1,t}(z)$ and $z^{dj}_{1,t}(z)$, with $j = H, L$, are the terms of trade in the decentralized market (which we study in the next subsection).

The effect of inflation depreciating money holdings is captured in the argument of the function $V_{2,t+1}$: If an agent holds an amount m_t of (nominal) money in period t, then her real balances are given by $\phi_t m_t \equiv z_t$. If the agent does not spend this money in the current period and carries it to the next period, then her real balances next period are given by:

$$
\phi_{t+1} m_t = \frac{\phi_{t+1}}{\phi_t} \phi_t m_t = \frac{z_t}{1+\pi_{t+1}},
$$

which tells us that the agent's new real balances are equal to her unspent real balances from the previous period, discounted by the gross rate of inflation.

3.3. Terms of trade

When a buyer in the decentralized market holding z units of real balances meets a seller at time t, the terms of trade would depend on whether or not the buyer is visiting the centralized market later that same period. If she is, then she makes a take-it-or-leave-it offer to the seller that solves the following problem:

$$
\max_{q^j_{2,t}, z^{dj}_{2,t}} \varepsilon_j u\left(q^j_{2,t}\right) - z^{dj}_{2,t}
$$

subject to

$$
-c\left(q^j_{2,t}\right) + z^{dj}_{2,t} \geq 0, \quad z^{dj}_{2,t} \leq z, \quad q^j_{2,t} \geq 0,
$$

with $j = H, L$. Since the participation constraint of the seller always binds, we can define $S_j(z) = \varepsilon_j u\left[c^{-1}(z)\right] - z$ and restate the problem as:

$$
\max_{z^{dj}_{2,t} \leq z} S_j\left(z^{dj}_{2,t}\right).
$$

The terms of trade $\left(q_{2,t}^{j}(z), z_{2,t}^{dj}(z)\right)_{j=H,L}$ are then given by:

$$
\begin{aligned}
q_{2,t}^{j}(z) &= c^{-1}(z) \text{ and } z_{2,t}^{dj}(z) = z \quad \text{if} \quad z \leq c(q_j^*), \\
q_{2,t}^{j}(z) &= q_j^* \quad \text{ and } z_{2,t}^{dj}(z) = c(q_j^*) \quad \text{if} \quad z > c(q_j^*),
\end{aligned}
$$

where q_j^* satisfies $\varepsilon_j u'(q_j^*) = c'(q_j^*)$ for $j = H, L$. This is the usual take-it-or-leave-it offer considered by Lagos and Rocheteau (2005).

If the buyer is not visiting the centralized market later in the period, then things are more complicated. The buyer needs to take into consideration the trade-off between spending in the current match and spending in next period match (if one were formed). The corresponding take-it-or-leave-it offer, hence, solves the following problem:

$$
\max_{q_{1,t}^{j}, z_{1,t}^{dj}} \varepsilon_j u\left(q_{1,t}^{j}\right) + \beta V_{2,t+1}\left(\frac{z - z_{1,t}^{dj}}{1+\pi_{t+1}}\right)
$$

subject to

$$
-c\left(q_{1,t}^{j}\right) + z_{1,t}^{dj} \geq 0, \quad z_{1,t}^{dj} \leq z, \quad q_{1,t}^{j} \geq 0,
$$

with $j = H, L$. The solution to this problem is harder to summarize analytically. It is, however, an essential component in the determination of equilibrium outcomes. In Section 4, I study several (simpler) special cases which provide the basis for understanding the general case.

3.4. Steady state equilibrium

The definition of equilibrium is standard for this kind of model (see Lagos and Wright, 2005). We will concentrate our attention in steady state equilibria with constant inflation. Recall that γ is the constant money-growth rate. It is easy to see that in steady state $1 + \pi_t = \gamma$ for all t. Then, given γ, we can describe a steady state monetary equilibrium by a collection of: (1) search intensity functions $e_1(z)$ and $e_2(z)$, where $e_i(z) = \arg\max V_i(z, e_i)$, with $i = 1, 2$, as described in subsection 3.2 (note that we have dropped the time subscript in the value functions since inflation is constant in steady state); (2) terms of trade for the decentralized market $\left\{\left(q_1^{j}(z), z_1^{dj}(z)\right), \left(q_2^{j}(z), z_2^{dj}(z)\right)\right\}_{j=H,L}$, as described in subsection 3.3; and (3) the individual money demand function $z_\gamma = \arg\max\{-\gamma z + \beta V_1(z)\}$, as described in subsection 3.2.

Both buyers and sellers choose to consume the optimal quantity X^* of centralized market goods and choose their work effort in that market accordingly. Since sellers do not carry money to the decentralized market, they just accept appropriate take-it-or-leave-it offers by buyers when they find a match and produce.

Finally, note that in a steady state equilibrium, there are four possible quantities of real balances that buyers might hold at the beginning of a period. Half of the buyers each period just exit the centralized market the period before and hence carry z_γ to the decentralized market. The other half of the buyers have not visited the centralized market the previous period and hence, their money holdings in the current period depend on their previous experience in the decentralized market. In particular, a fraction $\alpha_b [e_1 (z_\gamma)]$ found an appropriate match the previous period and a fraction $1 - \alpha_b [e_1 (z_\gamma)]$ did not find an appropriate match. Those that did not find an appropriate match (and, hence, did not trade) will hold real balances equal to z_γ/γ. Of the buyers that found an appropriate match, a proportion σ found a high-quality match and, hence, holds an amount $\left[z_\gamma - z_1^{dH} (z_\gamma)\right] /\gamma$ of real balances (in the current period), and a proportion $1 - \sigma$ traded low-quality goods and holds an amount $\left[z_\gamma - z_1^{dL} (z_\gamma)\right] /\gamma$ of real balances.

4. STEADY STATE INFLATION

The objective in this section is to establish how the endogenous variables depend on the level of steady state inflation. Since steady state inflation $1 + \pi$ equals the money growth rate γ for all t, we choose to index each steady state equilibrium by its corresponding value of γ.

4.1. Consumption frontloading

Consider the special case when $v(e) = 0$ for all e and $\varepsilon_H = \varepsilon_L = 1$; that is, search is costless and all goods are of the same quality. First note that all buyers will choose search intensities such that $\alpha_b = 1$. In other words, buyers find a seller in every period, at no cost. Then, the value function V_2 simplifies to $V_2 (z) = S\left[z_2^d (z)\right] + W (z)$.

When $\gamma > \beta$, it is easy to see that in equilibrium $\left[z_\gamma - z_1^d (z_\gamma)\right] /\gamma < c (q^*)$, because buyers would not choose to carry money that they do not expect to spend. Hence, $z_2^d = \left[z_\gamma - z_1^d (z_\gamma)\right] /\gamma$

in equilibrium and the terms of trade z_1^d solve the following problem:

$$\max_{0\leq z_1^d\leq z_\gamma} S\left(z_1^d\right)+\left(1-\frac{\beta}{\gamma}\right)z_1^d+\beta S\left(\frac{z_\gamma-z_1^d}{\gamma}\right).$$

Under standard Inada conditions for the utility function u, the solution to this problem is interior and solves the following first order condition:

$$S'\left(z_1^d\right)+\left(1-\frac{\beta}{\gamma}\right)-\frac{\beta}{\gamma}S'\left(\frac{z_\gamma-z_1^d}{\gamma}\right)=0.$$

It follows, then, that $z_1^d>\left(z_\gamma-z_1^d\right)/\gamma$, and consequently, that whenever $\gamma>\beta$ we have that $q_1>q_2$ in equilibrium. Note, also, that when $\gamma\to\beta^+$ we have that $q_1\to q_2^+$. These facts are the consequence of what we will call the consumption-frontloading effect of inflation: the higher the inflation rate, the higher the incentives of buyers to increase the quantity purchased (and consumed) "early on" after each monetary rebalancing (that is, after each visit to the centralized market).[8]

The fundamental feature of the model that results in this consumption frontloading effect is that some trades in the decentralized market are executed between agents that will visit the centralized market at different dates (see Appendix A). In particular, when a seller who will visit the centralized market in the current period meets a buyer who will visit the centralized market only next period, the possibility of money changing hands increases the available (private) surplus produced by the trade. The seller values money relatively more than the buyer because he will be able to use it at the end of the day (in the centralized market) and hence will not suffer the consequences of inflation. This difference in relative valuation allows the buyer to get better deals in those trades and, as a consequence, induces buyers to buy relatively more in such trades.

Of course, this difference in relative valuation of money balances in the decentralized market can arise only if some buyers visit the centralized market less frequently than every period. The change in timing proposed in this paper achieves exactly that. We could generalized the environment and have sellers visiting the centralized market every other period, some in odd and

[8]For an early discussion of consumption frontloading in an inflationary environment, see footnote 4 in Jovanovic (1982).

some in even periods. Consumption frontloading will arise only in those trades where the seller has access to the centralized market sooner than the buyer.

One way to measure the strength of the consumption frontloading effect of inflation is to track how $z_1^d(z_\gamma)/z_\gamma$ increases with inflation (across steady states with different inflation). As a benchmark example, consider the case when $u(q) = q^{1-\eta}/(1-\eta)$ with $0 < \eta < 1$ and $c(q) = q$. In this case, we have that:

$$\frac{z_1^d(z_\gamma)}{z_\gamma} = \frac{\gamma^{\frac{1-\eta}{\eta}}}{\gamma^{\frac{1-\eta}{\eta}} + \beta^{\frac{1}{\eta}}}$$

which is clearly increasing in the (gross) inflation rate, γ.

Finally, note that a (standard) demonetization effect is present in this model. In particular, it can be easily verified that in equilibrium z_1^d and z_γ satisfy the following equations:

$$S'\left(z_1^d\right) = \frac{\gamma - \beta}{\beta} \quad \text{and} \quad S'\left(\frac{z_\gamma - z_1^d}{\gamma}\right) = \frac{\gamma^2 - \beta^2}{\beta^2},$$

and, hence, both the quantities traded are decreasing with inflation.

4.2. Search intensity

In this subsection we will concentrate our attention on the effects of inflation on the choice of search intensity. To keep the presentation simple, let us maintain the assumption that $\varepsilon_H = \varepsilon_L = 1$; but now we assume that $v(e) > 0$ for $e > 0$. It is easy to see that in equilibrium $\alpha\bar{e} \leq 1$ and hence $\alpha_b(e) = \min\{e\alpha, 1\}$. In what follows, we will concentrate attention in the most interesting case in which all buyers choose levels of search effort such that $\alpha_b(e) < 1$.

Denote by z_2 the real money holdings that a buyer takes to the second of her two visits to the decentralized market between visits to the centralized market. In equilibrium, we have that z_2 depends on the buyer's choice of money holdings in the centralized market, z_γ, her trading experience, and the inflation rate, γ. In particular, z_2 can take two possible values:

$$z_{2H}(z_\gamma, \gamma) = \frac{z_\gamma}{\gamma} \quad \text{and} \quad z_{2L}(z_\gamma, \gamma) = \frac{z_\gamma - z_1^d(z_\gamma)}{\gamma},$$

where $z_{2H}(z_\gamma, \gamma)$ corresponds to the case when the buyer *did not* find a trade in the decentralized market of the previous period, and $z_{2L}(z_\gamma, \gamma)$ corresponds to the case when the buyer *did* find a trade. Note that $z_{2H}(z_\gamma, \gamma) > z_{2L}(z_\gamma, \gamma)$ and we have used the subscripts H and L to indicate high and low money holdings, respectively. Again, it is easy to see that $z_{2L}(z_\gamma, \gamma) < c(q^*)$ and, hence, the terms of trade are such that $z_2^d = z_{2L}(z_\gamma, \gamma)$ in that case. But $z_{2H}(z_\gamma, \gamma)$ may or may not be smaller than $c(q^*)$. Hence, in some situations, when $z_{2H}(z_\gamma, \gamma) > c(q^*)$, the buyer will not spend all her money in the decentralized market before going back to the centralized market. The terms of trade are such that $z_2^d < z_{2H}(z_\gamma, \gamma)$ in that case. The reason for this fact is simple. The buyer may want to exit the centralized market carrying enough money to be able to potentially afford *two* purchases in the decentralized market before her next visit to centralized market. In the unlucky event that the buyer only finds one appropriate seller during that time, she may not want to spend all her money in a single match. The marginal utility of goods purchased in the decentralized market is decreasing and, if it becomes too low, then the buyer might prefer to wait and use the money in the centralized market.

The value function V_2 is now given by:

$$V_2(z_2, e_2) = \alpha e_2 S\left[z_2^d(z_2)\right] + W(z_2) - v(e_2),$$

and the condition for optimality of e_2 is:

$$\alpha S\left[z_2^d(z_2)\right] - v'(e_2) = 0,$$

which is a version of the condition that determines the optimal search intensity in Lagos and Rocheteau (2005). Since $S\left[z_2^d(z_{2H}(z_\gamma, \gamma))\right] > S\left[z_2^d(z_{2L}(z_\gamma, \gamma))\right]$, we have that in equilibrium the level of search intensity e_2 will take two possible values $e_{2H}(z_\gamma, \gamma)$ and $e_{2L}(z_\gamma, \gamma)$, with $e_{2H}(z_\gamma, \gamma) > e_{2L}(z_\gamma, \gamma)$. Buyers holding more real balances make larger purchases and, hence, tend to search more intensively. For the same reason, since inflation tends to reduce the equilibrium levels of z_2, higher steady state inflation implies lower levels of search intensity e_2. This is the same logic that explains the results in Lagos and Rocheteau (2005) (see Appendix B).

The terms of trade z_1^d solve the following problem:

$$\max_{0 \le z_1^d \le z_\gamma} S\left(z_1^d\right) + \left(1 - \frac{\beta}{\gamma}\right) z_1^d + \beta \alpha e_{2L} S\left(\frac{z_\gamma - z_1^d}{\gamma}\right)$$

where, by the Envelope Theorem, we can take the value of e_{2L} as given. Note that, as in the previous subsection, it is also the case here that inflation tends to induce consumption frontloading.

Finally, to determine the equilibrium values of e_1 and z_γ, note that we can write the function $V_1(z, e_1)$ as follows:

$$V_1(z, e_1) = \alpha e_1 \left[S\left[z_1^d(z)\right] + \left(1 - \frac{\beta}{\gamma}\right) z_1^d(z) - \beta\Delta_2(z,\gamma)\right] + \beta V_2\left[z_{2H}(z,\gamma)\right] - v(e_1),$$

where $\Delta_2(z,\gamma) \equiv \alpha\left(e_{2H}S\left[z_2^d(z_{2H})\right] - e_{2L}S\left[z_2^d(z_{2L})\right]\right) - \left[v(e_{2H}) - v(e_{2L})\right]$. Then, the equilibrium values of e_1 and z_γ satisfy the following two conditions:

$$\alpha\left[S\left[z_1^d(z_\gamma)\right] + \left(1 - \frac{\beta}{\gamma}\right) z_1^d(z_\gamma) - \beta\Delta_2(z_\gamma,\gamma)\right] - v'(e_1) = 0,$$

and

$$\alpha^2 e_1 e_{2L} S'(z_{2L}) + (1 - \alpha e_1)\alpha e_{2H} S'(z_{2H}) \mathbf{I}_{z_{2H}} = \frac{\gamma^2 - \beta^2}{\beta^2},$$

where $\mathbf{I}_{z_{2H}}$ is an indicator function that equals unity whenever $z_{2H} < c(q^*)$ and zero otherwise.

From the first of these conditions, we can see that there are two main forces pushing the value of e_1 to be higher for higher levels of the inflation rate. First, higher inflation results in consumption frontloading, which limits the strength of demonetization over the value of the first purchase and, hence, keeps the value of $S\left(z_1^d\right)$ relatively high. Second, as inflation increases, the second term in the square brackets tends to increase (as long as z_1^d does not decrease too much with inflation, for which frontloading clearly helps). This second term can be interpreted as the savings on the inflation tax that results from trading "early." As we will see, for low levels of inflation, the equilibrium value of e_1 can be an increasing function of inflation.

The key variables that describe a steady state equilibrium are z_γ, $z_{1\gamma}^d$, $e_{1\gamma}$, $e_{2H\gamma}$, and $e_{2L\gamma}$, where we use the subindex γ to indicate that these are the values of the variables corresponding to an equilibrium with (gross) inflation rate equal to γ. Characterizing analytically the comparative statics with respect to γ is difficult. For this reason, we turn our attention to a calibrated example that allow us to get a sense of the relative magnitudes of the three main effects present in the model: (1) the *demonetization effect*, which induces agents to choose lower money holdings z_γ for higher values of γ; (2) the *consumption-frontloading effect* that tends to associate higher values of $z_{1\gamma}^d/z_\gamma$ with higher values of γ; and (3) the *tax-savings effect*, represented by the term $\left[1 - (\beta/\gamma)\right] z_{1\gamma}^d$, which tends to increase $e_{1\gamma}$ when γ increases.

For the computation of the example we use the standard functional form for the utility function $u(q) = [(q+b)^{1-\eta} - b^{1-\eta}]/(1-\eta)$ with $0 < \eta < 1$ and $b \approx 0$, and assume that $c(q) = q$ and $v(e) = e^{\rho}$ with $\rho > 1$. We follow a simple calibration strategy. There are four (important) parameters that need to be determined: β, α, ρ, and η. First, we set the value of β to match an annualized real interest rate of 4% (i.e., $\beta = 0.96$). We then fix α and ρ at benchmark values and proceed to calibrate η.

The value of η is calibrated so that the elasticity of the inverse of the velocity of circulation of money in the decentralized market equals -0.24. This is the value for the elasticity parameter estimated by Teles and Zhou (2005) using annual U.S. data for the last hundred years. Teles and Zhou define a stable monetary aggregate that is intended to capture more accurately the transactions demand for money in the U.S. across time. We do not aim at matching the *level* of inverse velocity because the model does not make predictions about the use of money in the centralized market. In principle, some transactions in such market could be undertaken using money, but the proportion of the total is not determined in equilibrium. For this reason, in the calibration, we concentrate our attention only on the *elasticity* of inverse velocity in the decentralized market.

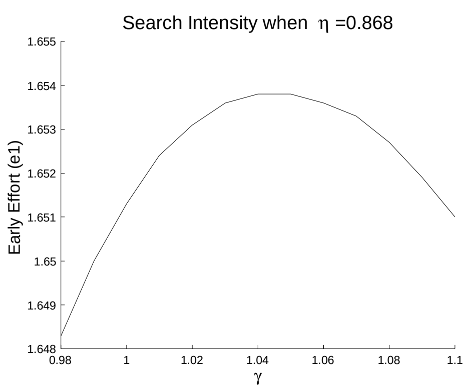

Figure 2a

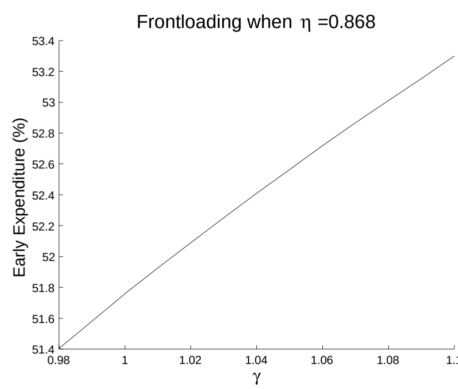

Figure 2b

As a benchmark, we set $\rho = 2$ and $\alpha = 0.5$ and obtain a calibrated $\eta = 0.868$. Fixing ρ and α at different values changes the calibrated η but not the basic results (see Appendix C). Figure 2a plots $e_{1\gamma}$ for different values of γ and Figure 2b plots $z^d_{1\gamma}/z_\gamma$ also for different values of γ. The level of search intensity $e_{1\gamma}$ is higher for higher levels of inflation when inflation stays below 5%. For levels of inflation higher than 5% the demonetization effect dominates and equilibrium search intensity decreases with inflation, as in Lagos and Rocheteau (2005).

This finding stands in sharp contrast with the result in Proposition 1 of Lagos and Rocheteau (2005), which states that in their model (with bargaining) the equilibrium level of search intensity is *always* a decreasing function of inflation (see Appendix B for an explanation). In the model presented here, buyers searching for a trade in their first visit to the decentralized market (after visiting the centralized market during the previous period) will sometimes search more when the inflation rate is higher. The reason for this pattern of behavior is that, in this model, buyers can and would like to try to spend their money speedily and, in this way, avoid paying the inflation tax.

4.3. Quality of goods

Let us now consider the general case, when goods in the decentralized market are of different qualities; that is, assume that $\varepsilon_H = 1 > \varepsilon = \varepsilon_L$. The value functions corresponding to this case were presented in Section 3. The objective here is to briefly explain how the incentives for buyers to spend their money speedily can induce them to buy bigger amounts of low quality goods in certain matches.

Consider the case of a buyer in the first round of decentralized exchange (between visits to the centralized market). Suppose the buyer is holding z_γ units of real balances and has met a low-quality seller. The amount of real balances that the buyer will spend in such a match, z_1^{dL}, satisfies the following condition:

$$S'_\varepsilon\left(z_1^{dL}\right) + \left(1 - \frac{\beta}{\gamma}\right) - \frac{\beta}{\gamma}\alpha e_{2L}\left[\sigma S'\left(\frac{z_\gamma - z_1^{dL}}{\gamma}\right) + (1-\sigma)\, S'_\varepsilon\left(\frac{z_\gamma - z_1^{dL}}{\gamma}\right)\mathbf{I}_\varepsilon\right] = 0$$

where $S_\varepsilon(z) = \varepsilon u\left[c^{-1}(z)\right] - z$ and $\mathbf{I}_\varepsilon$ is an indicator function that equals unity whenever $\left(z_\gamma - z_1^{dL}\right)/\gamma < c\left(q_L^*\right)$, and zero otherwise. This expression is a more general version of the frontloading condition studied in subsection 4.1. The second term in the left-hand side is increasing in inflation and tends to push the value of z_1^{dL} upward. Of course, as in the previous cases, inflation also induces demonetization. However, the frontloading effect present in this model limits the strength of demonetization and increases the willingness of (some) buyers to purchase low quality goods.

5. CONCLUSION AND EXTENSIONS

Faced with inflation, individuals try to minimize their exposure to the resulting tax on money holdings. One way to achieve this goal is to substitute away from those purchases that require the use of money. This reaction to inflation is the most commonly studied in the literature (see, for example, Cooley and Hansen, 1989). Another possible reaction is for agents to try to store value in real assets, preferably liquid ones, that they will exchange for money at the time they wish to make a monetary purchase. Usually, transaction costs limit the ability of agents to completely shield themselves from inflation in this manner (see, for example, Jovanovic, 1982). A third, perhaps more subtle, reaction to inflation is for some agents to distort their pattern of transactions so as to exchange with other agents who are better able to avoid the inflation tax. The basic logic is that transferring the depreciating money balances from the party that is most exposed to inflation to the party that is less exposed to inflation increases the benefit associated with trade, making more trading worthwhile. These changes in the pattern of transactions result in potentially significant misallocations; that is, some of the trades that take place may be too large, or too common, relative to what could be considered optimal. The nature of these trade distortions has been the main subject of this paper.

To the extent that sellers (say, store owners) are able to establish more effective systems than buyers (say, households) to avoid the inflation tax, the effects captured by the model presented here are likely to be representative of the reality in inflationary economies. It seems plausible that shielding from inflation involves some fixed costs that justify a certain degree of "centralization" of such activity in the hands of one set of agents. This kind of economies of scale may result on the systematic asymmetry between buyers and sellers that is needed to support the thesis of this paper. More specifically, we find that when sellers are better able than buyers to avoid the inflation tax, buyers will tend to frontload their purchases, increase their search intensity, and buy relatively more of the low quality goods in response to increases in inflation.

The ability of the model to naturally generate equilibrium search intensities that are higher when the level of inflation is higher was one of the main accomplishments of this paper. Generally, when inflation increases, agents adjust the size of their purchases. Smaller purchases result in lower search effort. However, we have described a situation where, conditional on the size of the purchase, higher inflation gives agents an incentive to try to execute purchases faster (and hence, increase their search effort). In modern economies, agents make purchases of many varying sizes. Making statements about their effort to execute trades without considering the value of those trades seems misguided. Presumably, Irving Fisher's remarks about the tendency of agents to spend their money "speedily" in an inflationary environment refers to the effort of agents to execute trades, *relative to the size and value of those trades.* The model in this paper articulates this idea precisely.

An important simplification in the model of this paper is the exogeneity in the timing of trade. A possible way to endogenize the sporadic participation of buyers in the centralized market would be to introduce a fixed cost of visiting such market, as in Molico and Chiu (2006). While such a change would complicate the distribution of money holdings and the computation of equilibrium, it would also introduce some interesting margins that are influenced by inflation. As in Jovanovic (1982), inflation will presumably change the periodicity of buyers' visits to the centralized market. In general, as long as there exists a systematic asymmetry between buyers and sellers in their ability to visit the centralized market (which is not present in Molico and Chiu's model), the effects studied in this paper would apply to such a setup.

Another important dimension that was left unexplored is the possibility that buyers respond to inflation by increasing the number of sellers that they sample every period. Specifically, we have (exogenously) restricted each buyer to meet a single seller every period. One possibility is to model more explicitly the within-period sampling decision of buyers. In this line of research, Head and Kumar (2005) study a related economy where information frictions create a nondegenerate distribution of prices and buyers can choose the number of sellers that they wish to sample per period. With this added flexibility they are able to show that the number of sellers that a "representative" buyer samples in equilibrium increases with inflation. However, their result is a consequence of the fact that inflation increases the relative price dispersion in the economy. Combining Head and Kumar's (2005) approach with the timing in this paper might reinforce their results.

In the model we have presented, sellers do not choose whether to be high- or low-quality producers. An interesting extension would be to introduce this extensive margin into the analysis.

Note, however, that to make the seller's decision over quality operative, we would need to change the pricing mechanism. Basically, if buyers have all the bargaining power, as it is the case in this paper, then sellers will always be indifferent between producing low- or high-quality goods. When buyers have all the bargaining power, sellers always get zero surplus from trade. An obvious first step to move away from this outcome would be to introduce a simple sharing rule in which sellers obtain a differential surplus from producing and trading alternative qualities (see, for example, Aruoba, Rocheteau, and Waller 2006).

Letting sellers choose the quality of the goods that they produce could allow us to capture the kind of production inefficiencies that Tommasi (1999) illustrates in his analysis of the welfare cost of inflation (see also Trejos 1997). If inflation makes buyers more willing to buy low quality goods to quickly spend their depreciating money balances, as discussed in this paper, then it is possible that some sellers anticipating the trading behavior of buyers, choose to become (inefficient) low-quality producers. Studying the welfare implication of these production inefficiencies in the Lagos-Wright framework seems a promising avenue for future research.

Appendix A: Buyer-Seller Asymmetry

Consider the following alternative timing. Assume that buyers and sellers visit the centralized market every other period, both in the same period. To keep matters as simple as possible, let us focus on the case when $v(e) = 0$ for all e and $\varepsilon_H = \varepsilon_L = 1$. The first thing to note is that when a seller meets a buyer in the first decentralized market interaction (after visiting the centralized market the previous period) the participation constraint of sellers is now given by:

$$-c(q_1) + \frac{\beta}{\gamma} z_1^d \geq 0.$$

Basically, the seller takes into account that she will only be able to use the real balances z_1^d, which she would receive as payment in the transaction, in the next period. Hence, those real balances, by the time that the seller gets to use them, would have depreciated according to the inflation rate γ. But then, the terms of trade z_1^d must solve the following problem:

$$\max_{0 \leq z_1^d \leq z_\gamma} S\left(\frac{\beta}{\gamma} z_1^d\right) + \beta S\left(\frac{z_\gamma - z_1^d}{\gamma}\right),$$

and the solution to this problem satisfy the first order condition:

$$S'\left(\frac{\beta}{\gamma} z_1^d\right) - S'\left(\frac{z_\gamma - z_1^d}{\gamma}\right) = 0,$$

which implies that $(\beta/\gamma) z_1^d = \left(z_\gamma - z_1^d\right)/\gamma$ and $q_1 = q_2$ in equilibrium. In other words, no consumption frontloading occurs in equilibrium, regardless of the level of inflation.

Sporadic visits to the centralized market are not enough to produce the inflation effects studied in this paper. The asymmetry among buyers and sellers in their ability to visit the centralized market is, indeed, essential.

Let us define z_1^d/q_1 as the relative price of an early purchase. As we saw in Section 3.3, if the seller is visiting the centralized market one period sooner than the buyer then the terms of trade satisfy $-c(q_1) + z_1^d = 0$. Comparing with the first equation in this appendix, we have that when $\beta/\gamma < 1$ the relative price of an early purchase is higher if the seller anticipates that he will only be able to spend the money one period after the sale (relative to when he anticipates spending it in the same period, as in Section 3.3). In summary, if the seller cannot better protect himself from inflation, then prices increase and no extra incentives for trade are created.

Appendix B: Discussion of Lagos and Rocheteau (2005)

Lagos and Rocheteau (2005) assume that the buyer visits the centralized market every period. For an interior solution of the agent's problem, the steady state values of z and e solve the following system of equations:

$$F_1(e,z) \equiv \alpha S(z) - v'(e) = 0,$$

and

$$F_2(e,z;\gamma) \equiv \alpha e S'(z) - \frac{\gamma-\beta}{\beta} = 0.$$

Figure A1 plots this system of equations in the (e,z)-plane. The system defines two implicit functions, $z(\gamma)$ and $e(\gamma)$, and Lagos and Rocheteau show that these two functions are both decreasing in γ. In other words, they show that higher steady state inflation is associated with lower equilibrium per capita real balances and lower search intensity (from point A to point B in the figure). The logic behind this result is simple. For higher levels of inflation the buyers have lower incentives to carry money to the next period. As a result, equilibrium consumption in a match is lower. Since the net benefit of consuming $S(z)$ is increasing in z (at the equilibrium point), higher inflation implies a lower net benefit of trading. The marginal benefit of increasing search intensity is given by $\alpha S(z)$ and hence, higher inflation levels are associated with lower marginal benefits of searching, which in turn results in lower equilibrium levels of search. A very similar logic explains the behavior of e_2 in the model in this paper.

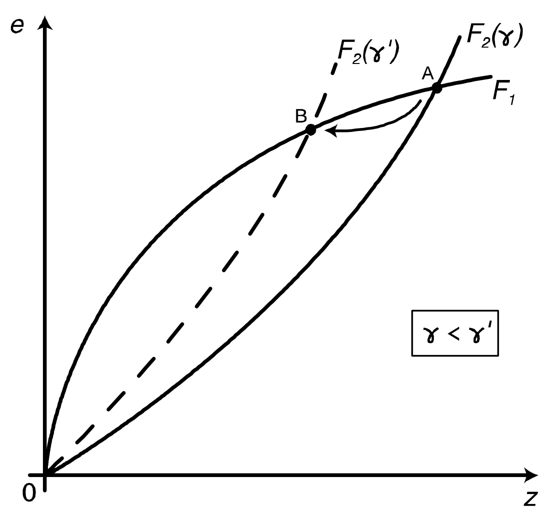

Figure A1: Lagos and Rocheteau (2005)

It is worth noticing here that there is an (implicit) "scale effect" that partly explains this result: the probability of trade does not depend on the size of the trade. If larger trades were to require more search effort, then the scale effect would be reduced and search intensity would tend to not respond to inflation. Another way of "adjusting" for this scale effect is by focusing on search effort per-unit of real balances, $e(\gamma)/z(\gamma)$, which is actually increasing with γ in the Lagos and Rocheteau model.

The result in Lagos and Rocheteau (2005) is more general than it may appear at first. For example, modifying a standard cash-in-advance model to introduce a choice of trade effort delivers the same conclusion: higher steady state inflation results in lower trading effort.

Consider a standard *cash-in-advance economy* like the one studied by Cooley and Hansen (1989). To simplify matters, assume linear production and no capital. Also, assume that agents can exert some trade effort e to increase their probability, αe, of making a trade during the period. Trade effort has a utility cost $v(e)$, with $v' > 0$ and $v'' > 0$. Just as in Cooley and Hansen (1989), let the representative agent have quasilinear preferences (i.e., linear in work effort h_t).

Denote with θ_t a binomial random variable that takes the value one if the agent finds a trade in period t, and takes the value zero otherwise. Hence, $\theta_t = 1$ with probability αe_t and $\theta_t = 0$ with probability $1 - \alpha e_t$. The problem of the agent is

$$\max E \sum_{t=0}^{\infty} \beta^t \left[\theta_t u(c_t) - h_t - v(e_t)\right]$$

subject to the budget constraint:

$$\theta_t c_t + (1 + \pi_{t+1}) z_{t+1} + b_{t+1} = h_t + z_t + R_t b_t + \tau_t,$$

where b_t is a real bond and τ_t is a transfer; the cash-in-advance constraint $c_t \leq z_t$; and the standard non-negativity constraints. Assume that $1 + \pi_t = \gamma$, constant for all t and concentrate in steady state equilibria. Assume also that γ is high enough that the cash-in-advance constraint is binding. In particular, $\gamma > \beta$. Then, the following system of equations characterizes the equilibrium levels of trade effort (e) and consumption ($c = z$):

$$\begin{aligned} \alpha\left[u(z) - z\right] - v'(e) &= 0, \\ \alpha e\left[u'(z) - 1\right] - \frac{\gamma - \beta}{\beta} &= 0. \end{aligned}$$

Denote by $[\widehat{e}(\gamma), \widehat{z}(\gamma)]$ the solution of this system. Then, it is easy to see that $d\widehat{e}(\gamma)/d\gamma < 0$. The logic is similar to the one explaining the result in Lagos and Rocheteau (2005): Since $\gamma > \beta$, we have that $u'(\widehat{z}) > 1$ and hence $u(\widehat{z}) - \widehat{z}$ is increasing in $\widehat{z}$. An increase in γ tends to reduce $\widehat{z}$ and hence it reduces the marginal benefit of exerting trade effort given by $\alpha\,[u(\widehat{z}) - \widehat{z}]$. As a consequence, agents exert less trade effort.

By increasing the trade effort in period t the agent will increase the likelihood of spending z_t. However, this effort does not reduce the inflation tax $\pi_t z_t$ paid by the agent, which in the standard cash-in-advance setup applies to cash balances at the beginning of the period (independently of whether those balances are later used during the period).

In this standard cash-in-advance model the inflation tax applies to money holdings at the beginning of the period, and it does not depend on whether the agents spend their money during the period. Changing the timing so that agents can use in transactions the cash obtained in the same period (but have to hold any money not spent until the next period) will change the result.[9] It is interesting to note, however, that in the Lagos and Wright (2005) framework the same change in timing does not revert the Lagos and Rocheteau (2005) result. This is the case because the buyers have all the bargaining power and, when the sellers find themselves holding cash in an inflationary environment, all the inflation costs associated with it are passed through to the buyers during price negotiations. In the end, the buyer bears the entire inflation tax and cannot avoid it by searching more.[10]

[9] McCallum and Goodfriend (1987) set up a model where time devoted to transactions and money are substitutes (see also the discussion in Lucas, 2000). For a given level of consumption, by holding more money, the agent can reduce the required amount of time devoted to transaction. Higher inflation will increase transaction time in such a model. The result, though, hinges on a reduce-form transactions function, which may be regarded as somewhat arbitrary.

[10] This is yet another instance where explicitly modeling transactions as in Lagos and Wright (2005) implies a different result from that in the reduce-form cash-in-advance model.

Appendix C: Calibration

For the purpose of calibration, we define the velocity of circulation of money in the decentralized market as the ratio of total quantities traded and total real money holdings. Since we assume that $c(q) = q$ and the seller's participation constraint always holds with equality, the real value of money spent is equal to the quantities traded. Then, we have that the value of total quantities traded Y is given by the following expression:

$$Y = \alpha e_1 z_1^d + \alpha^2 e_1 e_{2L} \frac{z_\gamma - z_1^d}{\gamma} + (1 - \alpha e_1)\, \alpha e_{2H} \left(\min \left\{ \frac{z_\gamma}{\gamma}, q^* \right\} \right).$$

The value of total real money holdings M/P is given by the following expression:

$$\frac{M}{P} = z_\gamma + \alpha e_1 \frac{z_\gamma - z_1^d}{\gamma} + (1 - \alpha e_1) \frac{z_\gamma}{\gamma};$$

and the velocity of circulation of money is the ratio of Y over M/P. We use the standard utility function $u(q) = \left[(q+b)^{1-\eta} - b^{1-\eta} \right] / (1-\eta)$ with $0 < \eta < 1$ and $b \approx 0$, and the search intensity cost function $v(e) = e^\rho$ with $\rho > 1$. We set the value of β to match an annualized real interest rate of 4%. There are three other important parameters that need to be determine: α, ρ, and η. We fix α and ρ at (reasonable) arbitrary numbers and we calibrate η to match the observed elasticity of inverse velocity in the data.

We calibrate the model using annual data. In particular, we use the value of the elasticity of inverse velocity estimated by Teles and Zhou (2005) based on U.S. data for the last hundred years. This value is equal to -0.24. We proceed as follows. We fix the values of α and ρ at arbitrary numbers and then calibrate the value of η so that the elasticity of inverse velocity that results from the model is (approximately) equal to that in the data. In this appendix, we present two new calibrated examples where we change the value of α and ρ relative to the example presented in the body of the paper and show that in general, for the calibrated parameters, search intensity e_1 is increasing in inflation as long as inflation is not too high.

Figure A1 presents the equilibrium e_1 as a function of γ when we lower the value of ρ and recalibrate the value of η to match the observed elasticity of inverse velocity -0.24. The resulting parameters are: $\beta = 0.96$, $\alpha = 0.5$, $\rho = 1.8$, $\eta = 0.843$.

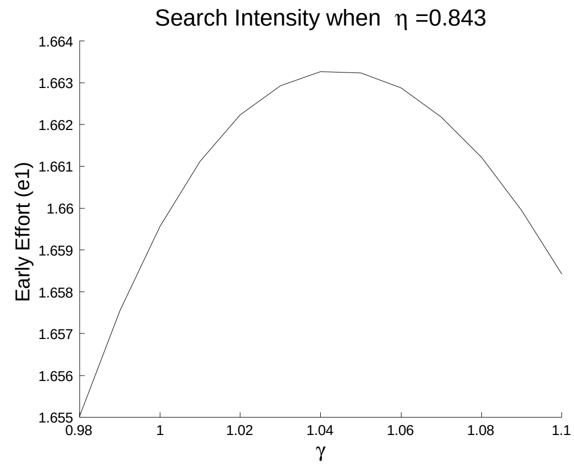

Figure A1

To produce Figure A2 we reset the value of ρ to 2 and lower the value of the parameter α in the matching function. This change represent an increase in the degree of search frictions. After recalibrating, the parameters are: $\beta = 0.96$, $\alpha = 0.4$, $\rho = 2$, $\eta = 0.8$.

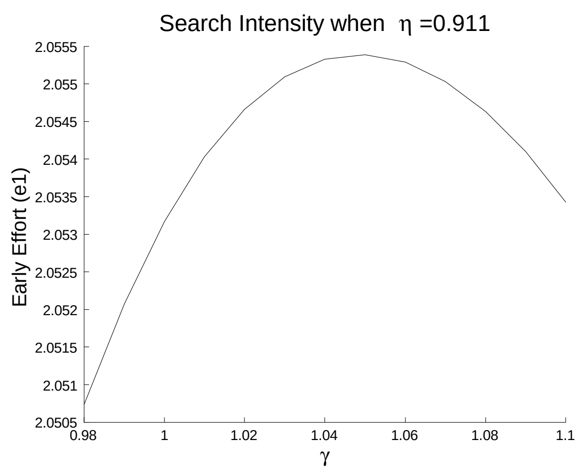

Figure A2

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