Working Paper Series

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WP 05-12

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December 1, 2005

Abstract

This paper studies the equilibrium properties of asset prices in a Lucas-tree model when agents display a concave coefficient of absolute risk tolerance. The latter introduces a role for wealth inequality, even under the presence of complete markets. The paper finds evidence suggesting that the role of wealth inequality on asset prices may be non-negligible. For the baseline calibration, the equity premium in the unequal economy is half a percentage point larger than the equity premium displayed by an egalitarian economy. The difference increases to one percentage point once we allow for the fact that agents tend to hold highly concentrated portfolios.

Keywords: Asset Pricing, Wealth Inequality.

JEL Classification: *D51, G12*

*This paper has benefited from discussions with Leonardo Martinez. Any opinions expressed in this paper are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1 Introduction

The objective of this paper is to explore how robust are the implications of the standard asset pricing model once we allow for preferences that do not aggregate individual behavior into a representative agent setup. Several authors have shown that the standard asset pricing model appears to be at odds with the behavior of security prices.[1] The most notable discrepancy was pointed out by Mehra and Prescott (1985). They calibrate the Lucas tree model to the US economy and find that it is incapable of replicating the differential returns of stocks and bonds.[2] The discrepancy, known as the equity premium puzzle, has motivated an extensive literature trying to understand why agents demand such a high premium for holding stocks.

Mehra and Prescott (1985) consider a setup with complete markets and preferences that display a linear coefficient of absolute risk tolerance, or hyperbolic absolute risk aversion (HARA).[3] This justifies the representative-agent approach used in their paper. Several authors have explored how the presence of heterogenous agents could enrich the asset pricing implications of the standard model and therefore help to explain the anomalies observed in the data. Constantinides and Duffie (1996), Heaton and Lucas (1996), and Krusell and Smith (1997) are prominent examples of this literature. These papers maintain the HARA assumption, but abandon the complete markets setup. This enables the wealth distribution to play a role on asset pricing.

An alternative approach is to depart from the case of linear absolute risk tolerance. This is the avenue taken in Gollier (2001). Abandoning the assumption of a linear coefficient of absolute risk tolerance implies that neither the coefficient of absolute risk aversion nor the coefficient of relative risk aversion are constant. The first specification is widely used in the finance literature, while the second one is widely used for macroeconomic analysis. Yet, there are no strong arguments in favor of these utility representations other than the fact that they allow for a tractable framework. Gollier studies a finite-period setup and shows that when the coefficient of absolute risk tolerance is concave, the equity premium in an unequal economy is larger than the equity premium obtained in an egalitarian economy. The aim of the present paper is to quantify the analytical results provided in Gollier's paper.

The present paper considers a canonical Lucas tree model with complete markets. There is a single

[1] See Campbell (2003).

[2] The average yearly return on the Standard & Poor's 500 index was 6.98% between 1889 and 1978, while the average return on ninety day government Treasury Bills was 0.80%. In their paper, Mehra and Prescott could explain an equity premium of at most 0.35%.

[3] The coefficient of absolute risk tolerance is defined as $-\frac{u'(c)}{u''(c)}$.

risky asset in the economy, namely a tree. This asset pays either high or low dividends. The probability distribution governing the dividend process is commonly known. Agents also trade a risk-free bond. Each agent is endowed in each period with an exogenous endowment of goods, which can be interpreted as labor income. The latter varies across agents. For simplicity, we assume that a fraction of the population receives a higher endowment in every period, i.e., there is income inequality. Agents are also initially endowed with claims to the tree. The latter are unevenly distributed across agents. The last two features imply that wealth is unequally distributed. Agents share a utility function with a piece-wise linear coefficient of absolute risk tolerance.

The exercise conducted in the paper is to compare the equilibrium asset prices in an economy that features an unequal distribution of wealth with an egalitarian economy. For a concave specification of the coefficient of risk tolerance, we find evidence suggesting that the role played by the distribution of wealth on asset prices may be non-negligible. The unequal economy displays an equity premium 0.5 percent larger and a risk-free rate 0.03 percent lower than the egalitarian economy. The difference in the equity premium between the unequal and the egalitarian economy increases to one percentage point once we allow for the fact that agents tend to hold highly concentrated portfolios.

The utility representation assumed in the paper and the choice of an exchange economy as the modeling device allow us to stress the role played by the concavity of the coefficient of risk tolerance. In this sense, our results can be interpreted as an upper bound of the impact that wealth inequality may have on asset prices once an alternative preference specification is adopted.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 discusses the assumption of a concave coefficient of absolute risk tolerance. Section 4 defines the equilibrium concept and describes how the model is solved. Section 5 outlines how the model is calibrated. Section 6 presents the results. Finally, Section 7 concludes.

2 The model

The paper analyzes a canonical Lucas-tree model. The only difference with Lucas (1978) is that our model features heterogeneous agents. We consider a pure exchange economy with complete information. There is a single risky asset in the economy: a tree. There is a unit measure of shares of the tree. The tree pays either high dividends (d_h) or low dividends (d_l). The probability that the tree pays high dividends tomorrow given that it has paid high (low) dividends today is denoted by π_h (π_l). There is a measure one of agents in the economy. Agents are initially endowed with shares of the tree and receive exogenous income y in every period. A fraction ϕ of the population is endowed in every period with high income y^r. The remaining agents receive low income y^p.[4] The exogenous income is not subject to uncertainty. The latter can be viewed as an extreme representation of the fact that labor income is less volatile than capital income. Agents trade in stocks and one-period risk-free bonds. These two assets are enough to support a complete markets allocation.

Agents share the same utility function, denoted by $u(c)$. The function governing the coefficient of absolute risk tolerance has the following structure:

$$RT(c) = -\frac{u'(c)}{u''(c)} = \begin{cases} b_0 c & \text{if } c \leq \hat{c} \\ a + b_1 c & \text{if } c > \hat{c} \end{cases},$$

where $a = (b_0 - b_1)\hat{c}$. The latter implies that the coefficient of absolute risk tolerance is continuous. It is assumed that both slope coefficients, b_0 and b_1, are strictly positive. When $b_1 < b_0$, the absolute risk tolerance is concave, and when $b_1 > b_0$, the absolute risk tolerance is convex. The standard CRRA utility function corresponds to the case where $b_1 = b_0$. This parameterization has several advantages. First, it nests the concave and convex risk tolerance cases in a simple way. Second, it enables for high degrees of curvature. Finally, it helps to provide a transparent explanation of the results.

The previous formulation implies that the utility function has the following shape:

$$u(c) = \begin{cases} K_0 \frac{c^{1-\frac{1}{b_0}}}{1-\frac{1}{b_0}} + K_1 & \text{if } c \leq \hat{c} \\ \\ \frac{(a+b_1 c)^{\frac{b_1-1}{b_1}}}{b_1-1} & \text{if } c > \hat{c} \end{cases},$$

where

$$K_0 = \frac{(a + b_1\hat{c})^{-\frac{1}{b_1}}}{\hat{c}^{-\frac{1}{b_0}}},$$

and

$$K_1 = \frac{(a + b_1\hat{c})^{\frac{b_1-1}{b_1}}}{b_1 - 1} - K_0 \frac{\hat{c}^{1-\frac{1}{b_0}}}{1 - \frac{1}{b_0}}.$$

A final comment is in order. Apart from abandoning the linear risk tolerance setup, the previous utility representation satisfies the usual assumptions: it is increasing and concave.

[4] In order to assist the reader, the subscript r stands for "rich", while the subscript p stands for "poor".

3 The concavity of the absolute risk tolerance

The results in Gollier (2001) suggest that in order for the wealth inequality to increase the equity premium, agents should display a concave coefficient of absolute risk tolerance. This section discusses to what extent this is a palatable assumption.

The results from experimental economics provide no guidance. Rabin and Thaler (2001) argue that not only is the coefficient of risk aversion an elusive parameter to estimate, but also that the entire expected utility framework seems to be at odds with individual behavior. In part, this has motivated the burst of behavioral biases models in the finance literature.[5] The landscape is different in the macro literature. The expected utility framework is still perceived as a useful tool for understanding aggregate behavior.

An alternative is to verify whether the testable implications of a concave (or convex) absolute risk tolerance are aligned with individual savings and portfolio behavior. This is the avenue taken in Gollier (2001). He argues that the evidence is far from conclusive. He documents that even though savings and investments patterns do not seem to favor a concave absolute risk tolerance, several papers are able to explain this behavior without relying on a convex coefficient of absolute risk tolerance. More precisely, an increasing and concave absolute risk tolerance would imply that the fraction invested in risky assets is increasing with wealth, but at a decreasing rate. This is not observed in the data. However, once the complete information setup is abandoned, an alternative explanation emerges: information does not appear to be evenly distributed across market participants. This is supported by Ivkovich et al. (2004), who find evidence suggesting that wealthier investors are more likely to enjoy an informational advantage and earn higher returns on their investments.

In a model without uncertainty, a concave coefficient of risk tolerance would imply an increasing marginal propensity to consume out of wealth. The data contradict this result. But there are various plausible explanations for the increasing propensity to save. The presence of liquidity constraints is one of them. The fact that the investment set is not uniform across agents is another one.[6]

The previous arguments suggest that the data do not provide strong arguments in favor or against a concave absolute risk aversion. This validates the latter as a possible representation of individual preferences. The rest of the paper focuses on the case with concave absolute risk tolerance and gauges

[5] See Barberis and Thaler (2003).

[6] See Quadrini (2000).

how large the role of wealth inequality can be.

4 Equilibrium definition

The economy is inhabited by a large number of infinitely lived, identical agents. Agents have preferences defined over a stream of consumption goods. Preferences can be represented by a time-separable expected utility formulation, namely

$$U_0 = E\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)\right] = \sum_{t=0}^{\infty} \sum_{z^t \in Z^t} \beta^t Pr\left(z^t \mid z_0\right) u\left(c_t\left(z^t\right)\right),$$

where Z^t denotes the set of possible dividend realizations from period 0 up to period t, z^t denotes an element of such a set, $c_t(\cdot)$ denotes a consumption rule that determines the consumption level in period t for a given stream of dividend realizations, and $Pr\left(z^t \mid z_0\right)$ denotes the conditional probability of observing stream of dividend realizations z^t, given that the initial realization is z_0. Trivially, $z_0 \in \{d_l, d_h\}$.

The consumer's objective is to maximize the present value of future utility flows. Let us assume for the moment that the price of a stock is given by the function $p(\mathbf{s})$, and the price of a risk-free bond is given by the function $q(\mathbf{s})$, where $\mathbf{s}$ denotes the aggregate state. In the present framework, the latter is fully specified by the dividend realization and the distribution of wealth. Given that the price functions are time-invariant, the consumer's optimization problem can be expressed using a recursive formulation.

The timing within each period is as follows: At the beginning of the period the aggregate tree pays off and agents receive dividend income. Then, they cash in the bonds and stocks purchased in the previous period and receive the exogenous endowment (labor income). The sum of these three components define the cash-on-hand wealth available for investment and consumption. Agents trade in two markets: the market of risk-free bonds, and the market of claims to the tree. At the end of the period they consume whatever they did not invest.

The following Bellman equation captures the individual optimization problem of agent i:

$$V_i(\omega, \mathbf{s}) = \underset{a', b'}{Max} \left\{ u(c) + \beta \sum_{\mathbf{s}' \in \mathbf{S}'(\mathbf{s})} Pr\left(\mathbf{s}' \mid \mathbf{s}\right) V_i\left(\omega'\left(\mathbf{s}'\right), \mathbf{s}'\right) \right\}, \tag{1}$$

subject to

$$[p(\mathbf{s}) + d(\mathbf{s})]\, a' + q(\mathbf{s})\, b' + c = \omega,$$
$$\omega'(\mathbf{s}') = a'\,[d(\mathbf{s}') + p(\mathbf{s}')] + b' + y^i.$$

The agent's type, i, depends on the exogenous endowment the agent receives. This means that $i \in \{r, p\}$. There are two relevant state variables for any given individual: the cash-on-hand wealth available at the beginning of the period (denoted by ω) and the aggregate state of the economy. The latter determines the current prices and the probability distribution over future prices. The state of the economy, $\mathbf{s}$, is represented by the vector (ω^r, ω^p, d). The first two components characterize the distribution of wealth, while the last component captures the current dividend realization. The amount of stocks purchased in the current period is denoted by a'. The amount of bonds purchased in the current period is denoted by b'. The next-period state realization is denoted by $\mathbf{s}'$. The set of possible aggregate state realizations in the following period is denoted by $\mathbf{S}'$. The latter may depend on the current aggregate state, $\mathbf{s}$. The function $d(\mathbf{s})$ represents the mapping from aggregate states to dividend payoffs.

A recursive competitive equilibrium consists on a set of policy functions $g_r^a(\omega, \mathbf{s})$, $g_r^b(\omega, \mathbf{s})$, $g_p^a(\omega, \mathbf{s})$, $g_p^b(\omega, \mathbf{s})$, price functions $p(\mathbf{s})$, $q(\mathbf{s})$, and an aggregate law of motion $S'(\mathbf{s})$, such that:

1. The policy functions $g_i^a(\omega, \mathbf{s})$, $g_i^b(\omega, \mathbf{s})$ solve the consumer's problem (1) for $i = r, p$.

2. Markets clear,

$$\phi g_r^a(\omega^r, \mathbf{s}) + (1-\phi)\, g_p^a(\omega^p, \mathbf{s}) = 1, \text{ and}$$
$$\phi g_r^b(\omega^r, \mathbf{s}) + (1-\phi)\, g_p^b(\omega^p, \mathbf{s}) = 0$$

   for all possible values of ω^r, ω^p, and $\mathbf{s}$.

3. The aggregate law of motion is consistent with individual behavior, i.e., $\forall\, \mathbf{s}' = (\omega^{r\prime}(d'), \omega^{p\prime}(d'), d') \in S'(\mathbf{s})$ it is the case that

$$\omega^{r\prime}(d') = g_r^a\left(\omega^h, \mathbf{s}\right)\left[p\left(\omega^{r\prime}(d'), \omega^{p\prime}(d'), d'\right) + d'\right] + g_r^b(\omega^r, \mathbf{s}) + y^r,$$
$$\omega^{p\prime}(d') = g_p^a(\omega^p, \mathbf{s})\left[p\left(\omega^{r\prime}(d'), \omega^{p\prime}(d'), d'\right) + d'\right] + g_p^b(\omega^p, \mathbf{s}) + y^p.$$

The above implies that $Pr(\mathbf{s}' \mid \mathbf{s}) = Pr(d(\mathbf{s}') \mid \mathbf{s})\ \forall\, \mathbf{s}' \in \mathbf{S}'(\mathbf{s})$.

4.1 Solving for the equilibrium

The present model features complete markets. A well known result in this setup is that, in equilibrium, the marginal rates of substitution across states and periods are equalized across agents. This implies that

$$\frac{u'\left(c_h^r\right)}{u'\left(c_h^p\right)} = \frac{u'\left(c_l^r\right)}{u'\left(c_l^p\right)} = \frac{1-\lambda}{\lambda}, \quad \text{with } \lambda \in (0,1)\,, \tag{2}$$

where c_i^j denotes the consumption of agent j in a state where the tree pays dividends d_i. The value of λ is determined in equilibrium.

The two equalities in equation (2), jointly with the aggregate resource constraints

$$\phi c_r^h + (1-\phi)\, c_p^h = d_h + \phi y^r + (1-\phi)\, y^p, \text{ and}$$
$$\phi c_r^l + (1-\phi)\, c_p^l = d_l + \phi y^r + (1-\phi)\, y^p,$$

fully determine the allocation of consumption as a function of λ. In turn, the consumption levels $c_i^j(\lambda)$ can be used to retrieve the market prices implied by λ. The latter must satisfy equations (3)-(6), which are derived from the first order conditions of a rich individual.[7]

$$p_h(\lambda) = \pi_h\left(d_h + p_h(\lambda)\right) + (1-\pi_h)\frac{u'\left(c_l^r(\lambda)\right)}{u'\left(c_h^r(\lambda)\right)}\left(p_l(\lambda) + d_l\right), \tag{3}$$

$$p_l(\lambda) = \pi_l\frac{u'\left(c_h^r(\lambda)\right)}{u'\left(c_l^r(\lambda)\right)}\left(d_h + p_h(\lambda)\right) + (1-\pi_l)\left(p_l(\lambda) + d_l\right), \tag{4}$$

$$q_h(\lambda) = \pi_h + (1-\pi_h)\frac{u'\left(c_l^r(\lambda)\right)}{u'\left(c_h^r(\lambda)\right)}, \tag{5}$$

$$q_l(\lambda) = \pi_l\frac{u'\left(c_h^r(\lambda)\right)}{u'\left(c_l^r(\lambda)\right)} + 1 - \pi_l, \tag{6}$$

where $p_i(\lambda)$ denotes the price of a share of the tree in a period when the latter has paid dividends d_i, and $q_i(\lambda)$ denotes the price of the risk-free bond in a period when the tree has paid dividends d_i.

Note that only the aggregate resource constraint has been used until this point. In order to pin down value of λ consistent with the equilibrium allocation, an additional market clearing condition has

[7] Given that marginal rates of substitution are equalized across agents, the same prices are obtained using the first order condition of poor individuals.

to be considered. We use the market clearing condition for stocks. An initial condition is also required. For this reason, it is assumed that the tree pays high dividends in the first period. The results are not sensitive to this. Equations (7) and (8) define two initial conditions that the demand for bonds and stocks of agent j ($a_h^{\prime j}(\lambda)$ and $b_h^{\prime j}(\lambda)$) must meet.

$$\omega_h^j - p_h(\lambda)\, a_h^{\prime j}(\lambda) - q_h(\lambda)\, b_h^{\prime j}(\lambda) = c_h^j(\lambda)\,,\ \text{and} \tag{7}$$

$$y^j + a_h^{\prime j}(\lambda)\,(p_h(\lambda) + d_h) + b_h^{\prime j}(\lambda) = \omega_h^j, \tag{8}$$

for $j = r, p$, and initial wealth levels ω_h^r, and ω_h^p. Equation (7) states that the investment decisions of an agent of type j must leave $c_h^j(\lambda)$ available for consumption in the first period. The second equation states that the cash-on-hand wealth available at the beginning of the second period in a state where the tree pays high dividends must equal the initial wealth (recall that the tree pays high dividends in the first period). The logic behind the second condition is the following. Given the stationarity of the consumption and price processes, the discounted value of future consumption flows in the first period is identical to the discounted value of future consumption flows in any period in which the tree pays high dividends. This means that the discounted value of claims to future income must also be equalized across periods with high dividend realizations, which implies that equation (8) must hold.

Thus, the value of λ consistent with the equilibrium allocation must satisfy

$$\phi a_h^{\prime r}(\lambda) + (1-\phi)\, a_h^{\prime p}(\lambda) = 1.$$

Finally, the following equation must also hold

$$y^j + a_l^{\prime j}(\lambda)\,[p_h(\lambda) + d_h] + b_l^{\prime j}(\lambda) = \omega_h^j, \tag{9}$$

for $j = r, p$. The above equality states that if the tree has paid low dividends in the current period, the cash-on-hand wealth available at the beginning of the following period in a state where the tree pays high dividends must be equal to the initial wealth of the agent. Equations (8) and (9) imply that, in equilibrium, the individual portfolio decisions are independent of the current dividend realization, i.e.,

$$a_h^{\prime j} = a_l^{\prime j} \text{ and } a_h^{\prime j} = a_l^{\prime j} \text{ for } j = r, l.$$

d_h	1.18
d_l	0.82
π_h	0.57
π_l	0.48
y^r	4.0
y^p	1.0
ϕ	0.333
$a^r_{\text{initial period}}$	1.5
b_0	0.5
b_1	0.2
$\hat{c}$	2.5

Table 1: **Parameter values**

5 Calibration

The baseline parameterization used in the paper is described in Table 1. The detrended index of dividends paid by stocks listed in the Standard & Poor's 500 index displayed a coefficient of variation of 17.5 percent between 1946 and 2004. The U.S. National Income and Product Accounts show that dividend income displayed a coefficient of variation of 11.5 percent over the same period. Both measures capture the volatility of a highly diversified portfolio. Several studies document that agents do not diversify as much as standard portfolio theories predict. This implies that the dividend volatility of the stocks actually held by individuals is larger than those measures. The benchmark values chosen for d_h and d_l deliver a coefficient of variation of 17.6 percent, but we also report results for higher dividend volatility.

Between 1946 and 2004 there were 31 periods in which dividends were above their trend level. We define these as periods with high dividends. The remaining ones are defined as periods with low dividends. The values of π_h and π_l –the probabilities of observing a period with high dividends following a period with high (low) dividends– were chosen to maximize the likelihood of the stream of high and low dividends observed over the sample period.

Reproducing the degree of inequality is a more difficult job. First, there have been sizable changes in the wealth distribution during the last decades. Second, the relevant measure for the purpose of this paper is the wealth inequality among stockholders, which is not readily available. The baseline parameterization assumes that a third of the population is assumed to be rich. For example, according to the Survey of Consumer Finances (SCF), 8.6 percent of American families received an annual income higher than $100,000 in 1989. The fraction of families receiving annual income between $50,000 and $100,000 in the same year was of 22.7 percent. This yields a fraction of "rich" of 27 percent. Even though the last two groups do not represent the entire population, they represent a large fraction of stockholders.[8] The exogenous endowment (labor income) received in each period by rich individuals is set equal to 4, while the exogenous endowment of poor individuals is set equal to 1. The initial endowment of stocks of rich individuals is set equal to 1.5. This implies that each of them holds twice as many stocks as poor individuals. Thus, on average rich agents receive 3 times as much income as poor individuals. According to the SCF, the ratio of mean income between rich and poor was 3.4 in 1989. Besides, the previous parameterization implies a ratio of aggregate "labor income" to capital income (dividends) equal to 2.

Finally, the preference parameter b_0 is set equal to 0.5, and b_1 is set equal to 0.2. This implies that a representative agent would display an average coefficient of relative risk aversion of 2.2, which is within the range of acceptable values. The threshold value $\hat{c}$ is set equal to 2.5. This guarantees that the consumption of poor agents is always in the region with steep absolute risk tolerance, and the consumption of rich agents is in the region of relatively flat absolute risk tolerance.

6 Results

The expected return of a tree in state i is denoted by R_i^e, where

$$R_i^e = \pi_i \frac{(p_h + d_h)}{p_i} + (1 - \pi_i) \frac{(p_l + d_l)}{p_i}.$$

The return on a risk-free bond in state i is denoted by R_i^f, where

$$R_i^f = \frac{1}{q_i}.$$

The asset pricing moments are computed using the stationary distribution. In the long run, the probability that the economy is in a state with high dividends is denoted by π, where

[8] The fact that the characteristics of stockholders may differ from the characteristics of the rest of the population was first pointed out in Mankiw and Zeldes (1991)

$$\pi = \frac{\pi_l}{1 + \pi_l - \pi_h}.$$

The average long-run return on a stock is denoted by R^e. The average long-run return on a bond is denoted by R^f. The latter are computed as follows:

$$R^e = \pi R_h^e + (1 - \pi) R_l^e, \text{ and}$$

$$R^f = \pi R_h^f + (1 - \pi) R_l^f.$$

Table 2 compares the first two moments of the equilibrium long-run risk-free rate and stock returns in two hypothetical economies. The unequal economy refers to the economy described in Section 2. In the egalitarian economy, however, every agent is initially endowed with the same amount of stocks and receive the same exogenous endowment in every period. The aggregate resources are the same as in the unequal economy.

Table 2 suggests that, once the standard preference specification is abandoned, wealth inequality may play a sizable role in the determination of asset prices.[9] The risk-free interest rate in the egalitarian economy is similar to the risk-free rate in the unequal economy, but the premium for holding stocks is half a percentage point larger in the unequal economy. This is not negligible. As the distribution of wealth becomes more unequal, the gap in the equity premium increases. For example, when each "rich" agent is initially endowed with 2 stocks, instead of 1.5, the premium for holding stocks is 0.7 percent higher in the unequal economy compared to the egalitarian economy.[10] On the other hand, if the standard deviation of equity returns is increased to 21.6 percent in the egalitarian economy, the equity premium increases by one percent, and the risk-free rate decreases by 0.5 percent in the unequal economy. This scenario corresponds to a coefficient of variation of dividends of 24.3 percent. The latter is not such a large number once we internalize the fact that investors do not diversify as much as standard portfolio theories predict.[11]

[9] The actual data reported in Table 2 differ from Mehra and Prescott (1985) for two reasons. First, the sample period is different. The present paper uses post-war data, which displays less disruptions than the sample period used in Mehra and Prescott (1985). Second, a period in the model corresponds to a year. Thus, the variable chosen as a proxy for the risk-free rate is in the real interest rate on one-year Treasury bills. Mehra and Prescott (1985) use 90-day Treasury bills.

[10] In this case, the ratio of financial wealth between "rich" and "poor" agents is equal to 4. The ratio equals 2 in our benchmark parameterization.

[11] See Ivkovich et al. (2004).

Variable	Egalitarian economy	Unequal economy	Post-war Data [12]
Mean returns on equity	5.94	6.41	4.17
Mean risk-free rate	4.08	4.05	1.79
Equity premium	1.80	2.27	2.33
Std. dev. of equity returns	15.2	13.5	15.5
Std. dev. of risk-free rate	14.9	13.2	1.9

Table 2: **Average returns and volatility.**

At the same time, it must be said that allowing for a concave coefficient of risk tolerance does not seem to help to account for the differential volatility between the risk-free interest rate and equity returns. The unequal economy displays lower standard deviations of bond and stock returns compared to the egalitarian economy, though the volatility gap between the two assets is unaltered.

6.1 Interpretation of the results

Gollier (2001) shows that in an economy with wealth inequality, the absolute risk tolerance of the hypothetical representative agent consists of the mean absolute risk tolerance of the market participants. Thus, when the coefficient of risk tolerance is concave, Jensen's inequality implies that the absolute risk tolerance of a hypothetical representative agent in an economy with wealth inequality is below the absolute risk tolerance of the representative agent in an economy with an egalitarian distribution of wealth. In turn, he shows that the latter implies that the equity premium in an economy with an unequal distribution of wealth is higher than the equity premium in an economy with an egalitarian distribution of wealth. This result holds regardless of whether the absolute risk tolerance is increasing or decreasing with consumption. The baseline parameterization used in this paper considers the first case, which appears to be in line with the data. It implies that, in equilibrium, wealthier agents bear more aggregate risk.

Even though Gollier (2001) relies on a two-period model, the results in this section suggest that his results also hold in an infinite horizon setup. An intuitive explanation is provided in Figure 1. The graph describes how consumption of rich and poor agents is affected by the nonlinearity of the coefficient

[12] The equity returns correspond to the real returns of the stocks listed in the Standard & Poor's 500 index. The risk-free interest rate corresponds to the one-year Treasury bill.

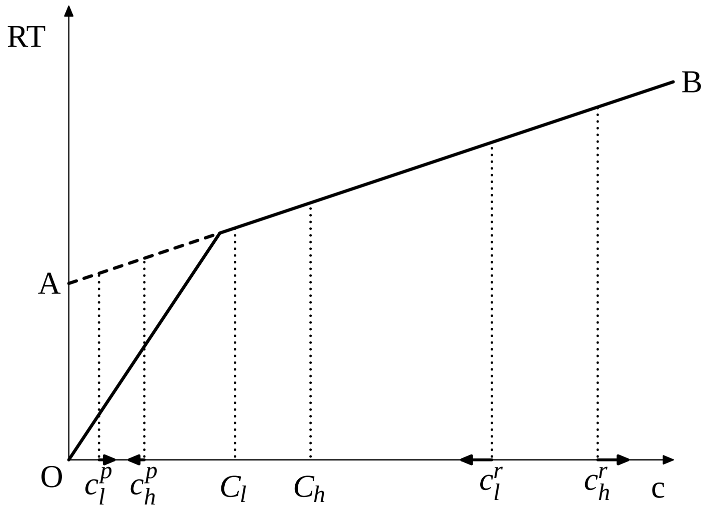

Figure 1: C_i **denotes average consumption in state** i**,** c_i^R **denotes consumption of a rich agent in state** i **when the coefficient of risk tolerance is represented by AB, and** c_i^P **denotes consumption of a poor agent in state** i **when the coefficient of risk tolerance is represented by AB. The arrows illustrate how the consumption of rich and poor agents move when the risk tolerance is given by the curve OB, instead of AB.**

of absolute risk tolerance. The solid line described the coefficient of absolute risk tolerance. If the latter was linear, and represented by the line AB, the economy would behave as if there was a representative agent. In this case, the consumption levels of rich and poor agents in state i would corresponds to points like c_i^r and c_i^p, respectively. C_i denotes the average per capita consumption in state i. In equilibrium, the marginal rates of substitution are equalized across agents:

$$\frac{u'(c_l^r)}{u'(c_h^r)} = \frac{u'(C_l)}{u'(C_h)} = \frac{u'(c_l^p)}{u'(c_h^p)}.$$

When the absolute risk tolerance is represented by the curve OB, instead of AB, poor agents are more risk averse. This means that at the prices prevailing when the economy behaves as if there was a representative agent, poor individuals are willing to consume less than c_h^p in the high dividend state and more than c_l^p in the low dividend state. Thus, the "new" equilibrium consumption levels of rich and poor agents must move in the direction of the arrows. Note that the marginal rate of substitution for rich agents $(u'(c_l^r)/u'(c_h^r))$ is higher in the economy with concave risk tolerance, compared to the economy with linear risk tolerance (curve OB versus curve AB). From the perspective of a rich individual, the mean price of stocks must therefore decrease. The reason is that the tree is paying low returns in states that have now become more valuable (low consumption) and high returns in states that have become less valuable (high consumption). Since markets are complete, the marginal rate of substitution are equalized across agents, so poor agents agree with their rich counterparts. As a consequence, the average equity premium asked to hold stocks is larger than in the economy with a representative agent.[13]

7 Conclusions

The objective of this paper is to explore how robust are the asset pricing implications of the standard model once alternative preference specifications are considered. The latter is motivated on the grounds that there is no strong evidence in favor of the CARA or CRRA utility representations. Following Gollier (2001), the paper focuses on a case with a concave coefficient of absolute risk tolerance. In the economy analyzed in this paper, the heterogeneity of individual behavior does not get washed out in the aggregate. This introduces a role for the wealth distribution. The model is parameterized based on the post-war performance of US stocks, and to approach the salient features of the wealth and income inequality among stockholders. For the baseline parameterization, the equity premium is 0.5 percent larger in the unequal economy, compared to the economy in which the wealth inequality is eliminated. This increases once we allow for the fact that agents typically hold portfolios that are more concentrated than the market portfolio. For example, if the tree displayed a coefficient variation of dividends of 24.3 percent (40 percent larger than in our baseline calibration), the increase in the equity premium in the unequal economy increases to one percentage point. All this suggests that the role played by the distribution of wealth on asset prices may be non-negligible.

[13]Note that the ranking of consumption in Figure 1 respects the ranking of consumption given by the baseline parameterization. In particular, the average consumption level is always above the threshold value $\hat{c}$.

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