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Aggregate Demand Management with Multiple Equilibria

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Huberto M. Ennis
Federal Reserve Bank of Richmond

Todd Keister
Centro de Investigación Económica

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Aggregate Demand Management with Multiple Equilibria *

Huberto M. Ennis
Research Department, Federal Reserve Bank of Richmond

Todd Keister
Department of Economics and Centro de Investigación Económica
ITAM

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Abstract

We study optimal government policy in an economy where *(i)* search frictions create a coordination problem and generate multiple Pareto-ranked equilibria and *(ii)* the government finances the provision of a public good by taxing trade. The government must choose the tax rate before it knows which equilibrium will obtain, and therefore an important part of the problem is determining how the policy will affect the equilibrium selection process. We show that when the equilibrium selection rule is based on the concept of risk dominance, higher tax rates make coordination on the Pareto-superior outcome less likely. As a result, taking equilibrium-selection effects into account leads to a lower optimal tax rate. We also show that public-employment policies that appear to be inefficient based on a standard equilibrium analysis may be justifiable if they influence the equilibrium selection process.

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E-mail addresses: huberto.ennis@rich.frb.org (H. M. Ennis), keister@itam.mx (T. Keister)

1 Introduction

We present a model where the search and matching process creates externalities and, as a result, the market for output becomes more efficient as more people enter it. Following Diamond (1982), we refer to the intensity of participation in the market as the level of aggregate demand. There is also a government in our model that can tax market transactions and use the proceeds to provide a public good. Our interest is in finding the welfare-maximizing tax rate. If the model has a unique equilibrium for every possible policy choice, this task is straightforward. The optimal tax rate will balance the marginal benefit of the public good against the marginal cost of providing it, including the decrease in aggregate demand and hence the loss in output-market efficiency that the tax causes. However, as is common in models with trading frictions, our model has multiple equilibria for a broad range of parameter values. We assume that the government must set the tax rate before knowing the actions of private agents, which implies that the government does not know which equilibrium will obtain. It seems entirely possible in such a setting that the choice of tax rate may influence which of the equilibria the economy ends up in, an effect that should be taken into account in determining the optimal policy. In fact, Diamond (1982, p. 882) states that "... one of the goals for macro policy should be to direct the economy toward the best [equilibrium] ... after any sufficiently large macro shock." Hence it is important to try to understand the effects that government policy can have on the process of equilibrium selection and how these effects influence the government's optimal choice of policy.

In our model, there is a continuum of identical agents, each of whom must decide whether or not to produce for the market. If an agent produces, she must exert costly effort in order to find a trading partner. For a given tax rate there are two possible (symmetric, pure-strategy) equilibria: one where all agents produce and search, and one where no one produces and there is no market activity. Following Howitt and McAfee (1992), we label these the "optimistic" and "pessimistic" outcomes, respectively. The search level that agents choose in the optimistic outcome is decreasing in the tax rate, and if the tax rate is set too high the optimistic outcome is no longer an equilibrium. The pessimistic outcome is an equilibrium regardless of the tax rate; if an individual agent believes that no one else is producing for the market, then she believes that searching for a trading partner will be in vain and she will therefore choose not to produce. Whenever the optimistic outcome is an equilibrium, it Pareto dominates the pessimistic outcome.

What tax rate should the government in this economy choose? If the policy had no influence on the equilibrium selection process, then the simple structure of our model would lead to a clear answer: the tax rate should be chosen to maximize expected welfare in the optimistic equilibrium. This is because the tax policy has no effect on welfare in the pessimistic equilibrium, and therefore the same policy choice is (weakly) optimal in each of the equilibria. However, we argue that in general the likelihood of agents coordinating on a specific outcome will depend on the individual payoffs associated with each possible choice of action, which implies that the equilibrium selection process will be influenced by the tax rate. In particular, we discuss a class of equilibrium selection rules that are based on the concept of risk dominance. We show that under any of these rules, the fact that a higher tax rate decreases the benefit of producing implies that a higher tax rate also decreases the likelihood that the optimistic equilibrium will obtain. This is an additional cost of taxation that a traditional analysis that ignores equilibrium selection would miss. This cost has clear implications for policy: the optimal tax rate must be *lower* than the rate that maximizes welfare in the optimistic equilibrium. The reason for this result is simple: If the tax rate is decreased slightly from this starting point, there is a small (second order) loss in utility should the optimistic equilibrium obtain. At the same time, there is a larger (first-order) increase in the probability of reaching the optimistic equilibrium, and therefore expected utility (across equilibria) increases. Thus the effect of taking equilibrium selection into account is clear: it lowers the optimal tax rate. In other words, discouraging market activity is more costly than a standard equilibrium analysis would indicate.

When additional policy tools are introduced, taking equilibrium selection into account can also change the *composition* of the optimal policy. We demonstrate this in Section 4 by giving the government an additional policy tool: it can directly control the actions of a fraction of the agents in the economy. In other words, it can directly produce the public good by employing agents and dictating their production and search decisions. We choose parameter values so that public production is inefficient in the following sense: if government policy had no influence over the equilibrium selection process, this second policy should never be used. However, we show that when equilibrium selection effects are taken into account, the optimal policy involves a positive amount of public production. There are two key elements behind this result. First, taking equilibrium selection into account lowers the optimal tax rate, as we discussed above. This increases the marginal value of the public good and therefore makes public production more attractive. Second, unlike taxation, public production does *not* necessarily decrease the probability of the optimistic equilibrium obtaining; in our example, it actually increases that probability. This is because public production increases the level of aggregate demand and therefore makes a private agent willing to produce for a wider range of beliefs about the market environment. Therefore the cost of public production is lower than an equilibrium analysis would indicate, and at the new, higher marginal valuation of the public good, some public production is optimal. Hence, looking at equilibrium selection shows that public sector employment/production may be a useful policy for coordination purposes.

There is a large literature in which models with multiple equilibria are used to discuss policy issues without imposing an equilibrium selection mechanism as we do here. This literature is based on the idea that sometimes a meaningful comparison can be made between the sets of equilibria resulting from different policy choices, and hence a policy recommendation can be made independent of the equilibrium selection process. Suppose, for example, that one policy choice leads to a unique, Pareto optimal equilibrium while another choice generates that same equilibrium plus some other, inferior equilibria. A strong case can then be made for choosing the former policy. Early work along these lines includes Grandmont (1986), Reichlin (1986), and Woodford (1986), where the inferior equilibria involve endogenous fluctuations and the preferred policy has a natural interpretation as a stabilization policy.[1] In many situations, however, bad equilibria cannot be costlessly eliminated by the proper policy choice. Cooper and Corbae (2002) study a model with a strategic complementarity in the financial intermediation process. Monetary policy can eliminate some undesirable equilibria, but not all of them. The authors conclude that "without a theory of equilibrium selection, predictions about the effects of monetary policy are impossible to make." (p. 23) The simpler model presented in this paper contains the essential features that give rise to this problem. We provide a method for dealing with such situations, and in doing so, we show how models where undesirable equilibria cannot be easily eliminated can still be brought to bear on policy issues in interesting ways.

In the next section, we describe the model and the properties of equilibrium. In Section 3, we show how to formulate the optimal policy problem in the presence of multiple equilibria. For this, we introduce the concept of an equilibrium selection mechanism and derive the optimal policy under several different mechanisms. In Section 4, we introduce the public-

[1] The "stabilization" approach has since been applied in a wide variety of settings. See, for example, Smith (1991) and the symposium introduced by Woodford (1994). In an environment where multiple equilibria exist for all policy choices, Keister (1998) shows that meaningful policy statements can sometimes be made without imposing an equilibrium selection rule.

employment policy and show how equilibrium-selection effects can lead the government to use this policy even though it is inefficient from a standard equilibrium perspective. Finally, in Section 5 we offer some concluding remarks.

2 The Model

Consider an economy with a $[0,1]$ continuum of identical agents and a single commodity that is produced using only labor. Each agent faces a binary production decision; she must either produce at a fixed scale or not at all. Producing requires labor input that gives disutility $c \geq 0$. Agents receive no utility from consuming their own output, but have a utility value F of consuming the output of someone else.[2] Finding a trading partner in the output market requires costly search effort. After observing the production decisions of all other agents, an agent who has produced chooses a search intensity. This intensity determines the probability that she will find a trading partner. If a partner is found, the two agents exchange output and consume; all exchanges are one-for-one trades. If an agent chooses not to produce, she does not enter the output market and has zero utility from trade (a normalization).

There is also a public good that can be produced using units of output that have been traded and are therefore consumable. All agents receive utility from the public good. However, because there is a continuum of agents, there is a pure free-rider problem and the public good will not be privately provided. Instead, there is a benevolent government that can tax trade at a rate $\tau \geq 0$ and use the proceeds to produce the public good. Our interest is in how the policy variable τ should be set. An important aspect of the problem we consider is the

[2]We think of this as a form of specialization in production; see Diamond (1982). One could instead allow agents to consume their own output but assume that doing so yields less utility than consuming the output of someone else. By relabelling utility values and costs, this approach is easily shown to be equivalent to the model here.

timing of decisions. We study the case where the government must set the tax rate before observing the actions of agents. In particular, the government must set τ before knowing which equilibrium will obtain. It is in such settings that our question of interest – how the policy choice affects the equilibrium selection process – arises. In addition, we assume that the government must set τ before the exact cost of production c is known. At this point the cost is a random variable with support $[c_L, c_H]$ and density function f, which we take (without any real loss of generality) to be uniform. In other words, the tax rate must be set with less information than private agents will have when they decide whether or not to produce. This uncertainty plays an important role in our formulation of the optimal policy problem; we discuss this issue in detail in Section 3 below.

Before we can study the problem of an agent deciding whether or not to produce, we must calculate the benefit of producing. For this we need to look at the workings of the output market.

2.1 Output Market

If agent i has produced, she can search for a trading partner by choosing a search intensity $\gamma_i \in [0, 1]$. In this setting, externalities naturally arise: An increase in individual search effort not only makes it more likely that the individual will find a partner, it also makes it more likely that other agents in the economy will find a partner. The fraction of agents finding a match in the output market is given by an aggregate matching function m.[3] Let $\overline{\gamma}$

[3] Assuming a matching function simplifies matters considerably, but it clearly does not come without cost. In particular, policy changes may affect the nature of the matching process, and this effect will be absent in our analysis. See Lagos (2000) for an interesting study in this direction.

denote the average level of search intensity in the economy, so that we have

$$\overline{\gamma} = \int_0^1 \gamma_i di. \tag{1}$$

Note that $\overline{\gamma}$ is also the total amount ("units") of search intensity in the market. We assume that the value of m depends only on $\overline{\gamma}$ (and not on the distribution of γ_i across agents). We assume that $m : [0,1] \to [0,1]$ is a smooth, strictly increasing bijection, so that $m(0) = 0$ and $m(1) = 1$ hold. In addition, we assume that m is strictly convex. This assumption captures the externality described above and implies that there are increasing returns in the matching process; if the number of searching agents doubles (holding search intensity constant), the number of matches formed more than doubles. Hence the efficiency of the output market is increasing in the number of searching agents and in their search intensities. Following Diamond (1982), we refer to the aggregate amount of search intensity as the level of *aggregate demand*.

Notice that the above assumptions imply that we have $m(\overline{\gamma}) \leq \overline{\gamma}$ for all possible values of $\overline{\gamma}$. Since the search intensity of each agent is less than unity, the number $\overline{\gamma}$ is necessarily lower than the fraction of agents who are searching with positive effort levels. Therefore, the fraction of agents who are matched, $m(\overline{\gamma})$, is necessarily lower than the fraction of agents who are searching with positive effort levels. This shows that the matching function always yields a feasible outcome.

While the function m gives the fraction of agents who find a partner, which agents actually get matched is random, with the probability of an individual agent being matched proportional to her search intensity. Specifically, define

$$\rho(\overline{\gamma}) = \frac{m(\overline{\gamma})}{\overline{\gamma}}. \tag{2}$$

Then ρ is the probability of being matched per unit of search intensity. An individual agent choosing intensity level γ_i will be matched with probability $\rho(\overline{\gamma})\gamma_i$. Note that our assumptions on m and the upper bound on γ_i ensure that this probability is always between zero and one.

Output that has been traded can be converted one-for-one into units of the public good. Let x denote the amount of public good produced per agent. This will be equal to the tax rate multiplied by the number of agents who make a trade (the tax base). Then all agents receive utility $v(x)$, where the function v is strictly increasing and strictly concave, and satisfies

$$v(0) = 0 \quad \text{and} \quad \lim_{x \to 0} v'(x) = \infty.$$

2.2 The Agent's Problem

Each agent first decides whether or not to produce. If she produces, she then chooses a search intensity γ_i after observing the production decisions of all other agents. We examine the latter choice first. The cost function for search intensity $d(\gamma_i)$ is smooth, strictly increasing, strictly convex, and satisfies

$$\lim_{\gamma_i \to 0} d'(\gamma_i) = 0.$$

If an agent meets a partner, they exchange output. If she does not meet a partner, she does not consume. We use the variable ϕ_i to represent the production choice; $\phi_i = 1$ corresponds to producing and $\phi_i = 0$ to not producing. If the agent produces, she will choose γ_i to maximize expected utility. The agent takes the level of provision of the public good as given and (since it enters her utility function in an additively separable way) her production and search decisions are independent of this level. She also takes as given the probability ρ of finding a match per unit of search intensity and the tax rate τ. We use $\lambda(\rho, \tau)$ to denote the expected value of producing given a particular (ρ, τ) pair, so that we have

$$\lambda(\rho, \tau) = \max_{\gamma_i \in [0,1]} \ \rho\gamma_i (1-\tau) F - d(\gamma_i). \tag{3}$$

We assume that the cost function d is such that the unique solution to this problem

$$\gamma_i = d'^{-1} (\rho (1-\tau) F)$$

is always less than unity.[4] Note that γ_i is positive as long as ρ and $(1-\tau)$ are positive. Since the expected benefit of search effort is linear in γ and the marginal cost of effort is zero when γ is zero, it is optimal to engage in a positive level of search whenever the probability of finding a partner is positive. Also note that γ_i is increasing in ρ and therefore in $\overline{\gamma}$. In other words, there is strategic complementarity in the choice of search intensity (see Cooper and John, 1988).

An agent will produce if $\lambda(\rho, \tau)$ defined in (3) is greater than the cost of producing. That is, we have the production decision rule

$$\phi_i = \left\{ \begin{array}{c} 1 \\ \in \{0,1\} \\ 0 \end{array} \right\} \text{ if } \lambda(\rho, \tau) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} c. \tag{4}$$

Note that we do not allow for mixed strategies. We can write the optimal choice of γ_i as

$$\gamma_i = \phi_i d'^{-1} (\rho (1-\tau) F). \tag{5}$$

[4]The boundary condition $\lim_{\gamma_i \to 1} d'(\gamma_i) = \infty$ is sufficient to guarantee this, but is not necessary for our results.

2.3 Private-Sector Equilibrium

In equilibrium, the decisions made by agents must generate the market conditions that each agent takes as given. The individual decisions generate functions $\gamma : [0,1] \rightarrow [0,1]$ and $\phi : [0,1] \rightarrow \{0,1\}$. Recalling the definitions of $\overline{\gamma}$ in (1) and $\rho(\gamma)$ in (2), we have the following definition of equilibrium for a given government policy.

Definition: A *private-sector equilibrium* is a list $\{\phi^*, \gamma^*, \rho^*, x^*\}$ such that

(i) given ρ^*, ϕ_i^* satisfies (4) and γ_i^* satisfies (5) for each private agent

(ii) $\rho^* = \rho(\overline{\gamma}^*)$

(iii) $x^* = \tau\rho^* \int_0^1 \phi_i^* \gamma_i^* di.$

The third condition is the government budget constraint: the amount of public good provided is equal to total tax revenue. We consider only symmetric private-sector equilibria, where either all agents produce or no agent produces. We label the beliefs associated with these equilibria "optimistic" and "pessimistic," respectively. We begin by looking at the former case.

2.3.1 Optimistic Beliefs

We first consider the situation where all agents produce and search. Because all agents are identical, they will choose the same search intensity, which we denote γ_H. This is then equal to the total amount of search intensity in the market $\overline{\gamma}$. Using the definition of ρ in (2), the search intensity in the optimistic equilibrium is the nonzero solution to

$$\gamma_H = d'^{-1}\left(\frac{m(\gamma_H)}{\gamma_H}(1-\tau)F\right).$$

We assume that the functions d and m are such that the right-hand side is concave in γ_H. This implies that γ_H is decreasing in τ – higher tax rates lead to lower equilibrium search intensities.[5] We use λ_H to denote the equilibrium expected value of producing under optimistic beliefs. This expected payoff is given by

$$\lambda_H(\tau) = \rho_H \gamma_H (1-\tau) F - d(\gamma_H),$$

where we have

$$\rho_H = \frac{m(\gamma_H)}{\gamma_H}.$$

Notice that the strict convexity of m implies that ρ_H is strictly increasing in γ_H and is therefore strictly decreasing in τ. This, in turn, implies that λ_H is strictly decreasing in τ. The intuition is straightforward: by taxing more, the government decreases the benefit of finding a trading partner, which leads agents to choose lower search intensities. This makes the output market less efficient, which further decreases the benefit of search effort. In other words, the tax is distortionary for two reasons; not only does it make producing and trading less valuable, it also makes finding a trading partner more difficult. Furthermore, for a given realization of the cost of production c, if τ is set high enough λ_H will fall below c and the optimistic outcome will not be an equilibrium.

In the optimistic outcome, the government budget constraint reduces to $x^* = \tau\rho_H\gamma_H$. Welfare, measured as the expected utility of each agent, is therefore given by

$$U_H(\tau, c) = \rho_H \gamma_H (1-\tau) F - d(\gamma_H) - c + v(\tau\rho_H\gamma_H) \tag{6}$$

where ρ_H and γ_H are, of course, functions of τ as discussed above. This expression gives the welfare level associated with the policy choice τ *if* the optimistic equilibrium obtains. Let

[5]The basic requirement for this to hold is that d have greater curvature than m. For example, if both functions are of the form x^α, we require that the exponent on d be larger than that on m. If the right-hand side were instead convex in γ_H, the comparative statics would be reversed, with increases in the tax rate *raising* equilibrium search levels. We focus on the more intuitive case where higher taxes discourage market activity.

$\widehat{\tau}$ denote the value of τ that maximizes this expression (notice that $\widehat{\tau}$ is greater than zero and does not depend on the value of the cost c). This would be the optimal tax rate for the government to set if it were certain that the optimistic outcome would obtain.

2.3.2 Pessimistic Beliefs

If no other agent searches, the value of searching is zero. Therefore $\lambda_L = 0$ holds, regardless of the tax rate τ. As long as the cost of production is positive, the optimal decision of an agent facing these market conditions is to not produce, and hence there is an equilibrium with $U_L = 0$. Notice that in this equilibrium, no public good is provided. The tax policy is completely ineffective if agents have pessimistic beliefs. In Section 4, we consider an additional policy tool (public production) that is more effective in this case.

2.3.3 Multiplicity

From the above discussion it should be clear that the pessimistic equilibrium exists for all values of τ and c. The optimistic equilibrium, on the other hand, only exists when the (after-tax) expected value of producing is at least as large as the cost. That is, the optimistic outcome is an equilibrium if and only if

$$\lambda_H(\tau) \geq c \tag{7}$$

holds. For a given cost c, a high enough tax rate will eliminate the optimistic equilibrium and leave the pessimistic outcome as the only equilibrium. Our interest is in situations where multiple equilibria exist for all relevant values of the policy parameter, and hence we do not want to focus on situations where the government needs to worry about the possibility of eliminating the optimistic equilibrium. We therefore assume that the following condition holds:

$$\lambda_H(\widehat{\tau}) \geq c_H. \tag{8}$$

Recalling that λ_H is decreasing in τ, this condition implies that for any $\tau \leq \widehat{\tau}$ the optimistic outcome will be an equilibrium regardless of the realization of the cost variable c. In particular, the government can set the tax rate to maximize expected utility in the optimistic outcome and be assured that this outcome will be an equilibrium once the cost is realized.

What value of τ should the government set? Our primary interest is in answering this question. In other words, we want to formulate and solve an optimal policy problem.

3 The Optimal Tax Rate

Formulating an optimal policy problem in general requires calculating the level of welfare that is generated by each possible policy choice. When there are multiple equilibria associated with at least some policy choices, it is not immediately clear how to do this. By far the most common approach in the literature is to essentially ignore the issue of multiplicity by assuming that the Pareto-best equilibrium will obtain (see, for example, Diamond, 1982). This gives a clear policy prescription: the policy should be chosen to maximize expected utility in the Pareto-best equilibrium. In our setting, the optimal policy would then be $\widehat{\tau}$. Because of the simple structure of our model, this approach might even seem to be without any loss of generality. In the pessimistic outcome, no trade takes place and hence the tax rate is irrelevant. Therefore, what could be wrong with setting the tax rate to maximize welfare in the optimistic outcome? In other words, the tax rate $\widehat{\tau}$ is (weakly) optimal regardless of which outcome obtains in equilibrium, and thus it seems like this rate must be the optimal policy. In this section, we show that this prescription is correct only if the process by which an equilibrium is selected is completely independent of the value to individual agents of producing and searching. We argue that such selection rules are intuitively unappealing. We introduce other selection rules, based on the notion of risk dominance, that allow for the selection process to depend on individual incentives, and we show that under such (more realistic) rules the optimal tax rate is *lower* than $\widehat{\tau}$.

3.1 Formulating the Optimal Policy Problem

In order to have a completely-specified optimal policy problem, the government must be able to assign a probability distribution over outcomes to each of its possible policy choices. With these probabilities, expected welfare can be calculated for each policy choice and the best policy can be determined. We refer to a complete specification of such probabilities as an equilibrium selection mechanism.

Definition: An *equilibrium selection mechanism* (ESM) is a function $\pi : \mathbb{R}_+ \times C \rightarrow [0,1]$ that assigns, for each pair (τ, c), a probability $\pi(\tau, c)$ to the optimistic outcome generated by that pair and a probability $(1 - \pi(\tau, c))$ to the pessimistic outcome. These probabilities must be consistent with equilibrium, in that:

(i) $\pi(\tau, c) = 0$ holds if the optimistic outcome is not an equilibrium for that (τ, c) pair, and

(ii) $\pi(\tau, c) = 1$ holds if the pessimistic outcome is not an equilibrium for that (τ, c) pair.

The two restrictions say that an outcome should be assigned positive probability only if it is actually an equilibrium of the economy based on that (τ, c) pair. For the moment we place no further restrictions on the function π. Suppose, for example, that the government believes that the Pareto-best equilibrium will always obtain. Then π will equal one whenever the pair (τ, c) is such that the optimistic equilibrium exists and zero otherwise. Another ESM could be generated by assuming that private agents base their beliefs (and hence their actions) on the realization of a two-state sunspot variable. Suppose that whenever τ and c are such that both equilibria exist, agents produce and search if sunspots appear and stay home if sunspots do not appear. Then, whenever τ and c are such that both equilibria exist, $\pi(\tau, c)$ would be equal to the probability of sunspots appearing. In general, any rule by which a private-sector equilibrium is selected can be represented by some function π satisfying the two restrictions given above.

Putting aside for the moment the question of where the function π would come from, notice that using an ESM does allow us to formulate an optimal policy problem. Given π, a benevolent government should choose the tax rate that maximizes the expected utility of private agents, where the expectation is both across values of the cost c *and* across the possible equilibrium outcomes. In other words, the optimal policy problem is given by

$$\max_{\tau} \int_{c_L}^{c_H} \left[\pi(\tau, c) U_H(\tau, c) + (1 - \pi(\tau, c)) U_L(\tau, c)\right] f(c)\, dc,$$

which, since $U_L = 0$ holds in this model, simplifies to

$$\max_{\tau} \int_{c_L}^{c_H} \pi(\tau, c) U_H(\tau, c) f(c)\, dc. \tag{9}$$

The best tax rate for the government to choose will clearly depend on the equilibrium selection mechanism π. For a given function π, we call the solution to (9) the π-*optimal tax rate* and denote it by τ_π^*. We provide next an expanded equilibrium concept that explicitly incorporates the equilibrium selection mechanism and the government's optimal choice of policy given this mechanism.

Definition: A *selection-based equilibrium* is a function π, a tax rate τ_π, and two lists $\{\phi_j, \gamma_j, \rho_j, x_j\}$ with $j = H, L$, such that

(i) the list $\{\phi_j, \gamma_j, \rho_j, x_j\}$ is a private-sector equilibrium for $j = H, L$,

(ii) the function π is a well-defined equilibrium selection mechanism, and (*iii*) given the function π and the lists $\{\phi_j, \gamma_j, \rho_j, x_j\}$, the tax rate τ_π solves the policy problem (9).

Note that if the model allows for a *constant* function π to be a well-defined ESM, then the above definition with such a function reduces to the definition of a sunspot equilibrium (Cass and Shell, 1983), expanded to include an optimizing government. In what follows, when the meaning is clear from the context, we will drop the prefixes *private-sector* and *selection-based,* and refer simply to an *equilibrium.* We now turn to a characterization of the optimal policy under two different classes of equilibrium selection rules.

3.2 Traditional Selection Rules

As we discussed above, the most common approach to dealing with multiple equilibria in a policy problem is to simply assume that the Pareto-best equilibrium will always obtain. This selection rule is sometimes called *payoff dominance.* We argued above that the π-optimal tax rate under this rule is given by $\widehat{\tau}$. We now verify this argument formally. The equilibrium selection mechanism under payoff dominance is given by

$$\pi(\tau; c) = \left\{ \begin{array}{cc} 1 & \text{if (7) holds} \\ 0 & \text{otherwise} \end{array} \right\}.$$

Under assumption (8), we have that $\pi(\widehat{\tau}, c) = 1$ for all c. Recall that the tax rate $\widehat{\tau}$ maximizes $U_H(\tau, c)$ for all values of c (see (6)). Therefore $\widehat{\tau}$ maximizes the integrand in (9) for every value of c, and hence maximizes the integral. This verifies that under the payoff dominance ESM, $\widehat{\tau}$ is indeed the optimal policy.

Another common approach to equilibrium selection is to assume that all agents base their actions on the realization of a sunspot variable. Suppose that after the tax rate is set and the cost of production becomes known, but before private agents choose their actions, one of two states of nature is revealed. With probability $\pi_s \in (0,1)$ sunspots appear and with probability $(1-\pi_s)$ there are no sunspots. Suppose further that all private agents have the following belief: If the pair (τ, c) is such that both equilibria exist, then the optimistic equilibrium will obtain if sunspots appear and the pessimistic equilibrium will obtain if they do not. Then it is individually optimal for each agent to produce and search if sunspots appear and to stay home if they do not. That is, there is a *sunspot equilibrium* where the optimistic outcome obtains with probability π_s whenever both equilibria exist. The equilibrium selection mechanism for the sunspots selection rule is given by

$$\pi(\tau, c)\left\{\begin{array}{cc} \pi_s & \text{if (7) holds} \\ 0 & \text{otherwise} \end{array}\right\}.$$

Notice that the government's objective function in (9) under the sunspots ESM is equal to the objective function under the payoff-dominance ESM multiplied by the constant π_s. Therefore, the policy prescription generated by the sunspots selection rule is exactly the same as that generated by payoff dominance, and we have the following result.

Proposition 1 *Under both the payoff-dominance ESM and the sunspots ESM, the* π*-optimal tax rate is given by* $\tau_\pi^* = \widehat{\tau}$.

This result can be interpreted as giving conditions under which the issue of multiplicity can be essentially ignored. If the policy only affects allocations in one of the equilibria *and* the probability assigned to that equilibrium is a constant, unaffected by the policy choice, then the policy should be chosen to maximize welfare in that equilibrium.

The sunspots selection rule is *probabilistic*, meaning that the government's policy does not completely determine the final outcome. Instead, the policy typically determines a non-degenerate probability distribution over the set of equilibria. We argue below in favor of probabilistic selection mechanisms because they are intuitively more plausible and because they tend to deliver more robust policy conclusions. For the moment, however, we focus on a less-appealing aspect that the sunspots approach shares with payoff-dominance: equilibrium selection is unaffected by the value to individual agents of producing and searching, and hence by the government's policy choice. That is, as long as both equilibria exist, the probabilities assigned to each equilibrium are the same for all values of τ. This approach to equilibrium selection seems overly simplistic to us. Imagine two scenarios, one where the tax rate is low and the value of producing λ_H is much higher than the realized cost c, and another where the tax rate is so high that λ_H is just slightly larger than c. In each of these scenarios, the optimistic outcome is an equilibrium. Thus both the payoff-dominance ESM and the sunspots ESM assert that the optimistic outcome is equally likely to obtain in each of the two scenarios. We would argue, however, that intuitively this prediction seems unreasonable. It is difficult to believe that agents will behave in the same way regardless of whether the gains from coordinating to the optimistic outcome are very large or very small. Moreover, this prediction is at odds with the experimental evidence on coordination games. Van Huyck, Battalio, and Beil (1990), (1991), for example, report that in a series of experiments the frequency with which subjects converged to each equilibrium varied systematically with the treatment variables. In other words, the frequency with which each equilibrium was selected depended on the incentives that subjects faced to choose different actions (according to the payoff matrix of the game).[6] This evidence indicates that it would be more reasonable to expect that the optimistic equilibrium would be more likely to obtain in the first scenario, when λ_H is large relative to c and the potential payoff from producing and searching is much higher. It also casts serious doubt on the optimality of the tax rate $\widehat{\tau}$. If higher taxes make coordination on the optimistic outcome less likely, then the arguments given above in favor

[6]See Cooper (1999) and Ochs (1995) for reviews of the experimental literature on coordination games.

of $\widehat{\tau}$ are no longer valid. We now examine a class of equilibrium selection mechanisms that embody the above intuition, and we show that under these rules the optimal tax rate is less than $\widehat{\tau}$.

3.3 Risk Dominance

Harsanyi and Selten (1988) have proposed using *risk dominance* as an equilibrium selection criterion.[7] In models of the type we are considering, the risk-dominance criterion has a simple interpretation. Suppose an agent believes that the optimistic and pessimistic equilibrium outcomes are equally likely to occur (that is, she has a diffuse prior that places equal weight on the possible equilibrium levels of aggregate demand). Then the optimistic equilibrium is risk dominant if her optimal action given these beliefs is to produce, while the pessimistic equilibrium is risk dominant if her optimal action is to not produce. Furthermore, if all agents assign equal probability to the possible equilibrium levels of aggregate demand, all agents will choose the risk-dominant action and the risk-dominant equilibrium will obtain. In this section, we show that applying this approach to our model generates a π function that can be used as an equilibrium selection mechanism, and we derive the implications of this mechanism for the optimal policy.

Suppose that the government has set a particular tax rate τ. The crucial question is then: For what values of the cost c will the optimistic outcome be risk dominant (and hence obtain)? This will happen if an agent who assigns equal probability to the two possible

[7] See Harsanyi and Selten (1988, Section 3.9) and the discussion in Young (1998). Carlsson and van Damme (1993) provide additional justification for this criterion. They study global games (where the payoff structure is determined by a random draw) and show that as the noise vanishes, iterated elimination of dominated strategies leads to the risk dominant equilibrium as the unique outcome.

equilibrium levels of aggregate demand is willing to produce; that is, if we have

$$\frac{1}{2}\lambda_H(\tau) + \frac{1}{2}\lambda_L(\tau) \geq c.$$

Using the fact that $\lambda_L = 0$ holds for all values of τ, we can rewrite this condition as

$$\frac{1}{2}\lambda_H(\tau) \geq c. \tag{10}$$

Recall that λ_H is strictly decreasing in τ. Therefore, a higher tax rate implies that an even lower draw for the cost of production will be required to make the optimistic equilibrium risk dominant. The risk-dominance equilibrium selection mechanism is given by

$$\pi(\tau; c) = \left\{ \begin{array}{cc} 1 & \text{if (10) holds} \\ 0 & \text{otherwise} \end{array} \right\}. \tag{11}$$

Notice that condition (10) is clearly stronger than the condition for the existence of the optimistic equilibrium (7). If λ_H is only slightly above c, then the optimistic equilibrium exists but is not risk dominant, and hence under this mechanism the pessimistic equilibrium will obtain. Only if the incentive to produce is "large enough" does this mechanism predict that the optimistic equilibrium will obtain.

Before returning to the optimal policy problem, we note that the (unconditional) probability of the optimistic outcome is given by

$$\Pi(\tau) \equiv \int_{c_L}^{c_H} \pi(\tau; c) f(c)\, dc = \text{Prob}\left[c < \frac{1}{2}\lambda_H(\tau)\right].$$

If $\frac{1}{2}\lambda_H(\tau) \in (c_L, c_H)$ holds, then using the fact that the distribution of c is uniform allows us to rewrite this expression as

$$\Pi(\tau) = \frac{\frac{1}{2}\lambda_H(\tau) - c_L}{c_H - c_L}.$$

This expression is clearly strictly decreasing in τ. In other words, setting a higher tax rate makes it less likely that the optimistic equilibrium will be risk dominant and hence makes it more likely that the pessimistic equilibrium will obtain.

We now impose two conditions on the support of the cost variable that make the equilibrium selection problem nontrivial. Specifically, we assume that

$$\lambda_H(0) > 2c_L \quad \text{and} \quad \lambda_H(\widehat{\tau}) < 2c_H \tag{12}$$

hold. The first condition says that if the tax rate is set to zero, then it is possible for the optimistic equilibrium to be risk dominant. If this condition did not hold, the pessimistic equilibrium would be risk-dominant for all (τ, c) pairs, and hence the choice of policy would be irrelevant. Recall that λ_H is independent of the distribution of the cost variable, and hence this condition can always be made to hold by setting c_L close enough to zero. The second condition says that when the government chooses the tax rate $\widehat{\tau}$, it is possible for the pessimistic equilibrium to be risk-dominant. If this did not hold, the optimistic equilibrium would always be risk dominant under $\widehat{\tau}$, and hence using risk dominance would essentially reduce to using payoff dominance. This condition can always be made to hold by setting c_H high enough.

We now show that the equilibrium-selection effect of taxation implies that the government should set the tax rate lower than $\widehat{\tau}$.

Proposition 2 *If (12) holds, the π-optimal tax rate under the risk-dominance ESM satisfies $\tau_\pi^* < \widehat{\tau}$.*

Proof: We need to consider two cases. First suppose that $\lambda_H(\widehat{\tau}) < 2c_L$ holds. Then if the government chooses $\widehat{\tau}$ the pessimistic equilibrium will be risk dominant for all values of c. In this case, the optimal policy clearly satisfies $\tau_\pi^* < \widehat{\tau}$, since choosing $\widehat{\tau}$ gives an expected utility of zero while the first condition in (12) implies that a positive expected utility is possible.

The second, and perhaps more interesting, case is where $\lambda_H(\widehat{\tau}) > 2c_L$ holds. This condition implies that under the policy $\widehat{\tau}$ the optimistic equilibrium is risk dominant for some values of c. Then define

$$\Lambda_H(\tau) = \lambda_H(\tau) + \upsilon(\tau\rho_H\gamma_H),$$

so that we have

$$U_H(\tau, c) = \Lambda_H(\tau) - c.$$

Note that the definition of $\widehat{\tau}$ implies that $\Lambda'_H(\widehat{\tau}) = 0$ holds. For values of τ close enough to $\widehat{\tau}$, so that $\frac{1}{2}\lambda_H(\tau) \in (c_L, c_H)$ holds, we can use (11) to write the objective function of the government as

$$\int_{c_L}^{\lambda_H(\tau)/2} [\Lambda_H(\tau) - c] f(c)\, dc. \tag{13}$$

From this expression it is apparent that the optimal policy τ^*_π cannot be greater than $\widehat{\tau}$. Moving the tax rate above $\widehat{\tau}$ lowers both Λ_H and λ_H. Such a move would lead to integrating a strictly smaller function over a strictly smaller domain, which would clearly yield a lower level of expected utility. Therefore taking equilibrium selection into account must either lower the optimal tax rate or leave it unchanged. Since the distribution of c is uniform, (13) is easily integrated to yield

$$\left(\frac{1}{2}\lambda_H(\tau) - c_L\right)\Lambda_H(\tau) - \frac{1}{2}\left(\frac{1}{4}\lambda_H(\tau)^2 - c_L^2\right).$$

The derivative of this expression is given by

$$\left(\frac{1}{2}\lambda_H(\tau) - c_L\right)\Lambda'_H(\tau) + \frac{1}{2}\lambda'_H(\tau)\left[\Lambda_H(\tau) - \frac{1}{2}\lambda_H(\tau)\right].$$

At an interior solution, this derivative must be zero. The first term in parentheses is positive for all $\tau \leq \widehat{\tau}$. The term in square brackets is always positive (this follows from the definition of Λ_H). We saw above that $\lambda'_H(\tau)$ is always negative; therefore at an interior solution $\Lambda'_H(\tau^*_\pi)$ must be positive. This shows that $\widehat{\tau}$ cannot be the solution, and therefore that $\tau^*_\pi < \widehat{\tau}$ must hold. ■

Another way of seeing this result is to note that decreasing τ slightly from $\widehat{\tau}$ will cause only a small (second order) decrease in the utility value of the optimistic outcome, and will bring a larger (first order) increase in the probability of attaining that outcome. Therefore expected utility must increase, and the effect of equilibrium selection on the optimal policy is clear: it *lowers* the optimal tax rate. The tax on trade decreases the benefit of producing when aggregate demand is high, which makes producing a riskier (or less attractive) activity when an agent is uncertain about the level of aggregate demand. This makes coordination on the optimistic outcome less likely. This proposition demonstrates the central idea of the paper: when equilibrium selection depends on the incentives faced by individual agents, the optimal tax rate is lower than $\widehat{\tau}$. In such situations, ignoring the issue of multiplicity will lead to the wrong policy conclusion.

3.4 Other Approaches

Using the risk-dominance criterion is a fairly simple and well-accepted way of studying how changes in payoffs (caused here by changes in the tax rate) affect the process of equilibrium selection. However, the risk-dominance ESM does have a substantial drawback: it generates a large discontinuity in the government's objective function. Suppose that the government knew the value of the cost of production c when it chose τ. Then the optimal policy would be to set τ as close to $\widehat{\tau}$ as possible, subject to the constraint that the optimistic equilibrium be risk dominant. This is intuitively rather unappealing, particularly in cases where the constraint is binding, *i.e.*, when the pessimistic equilibrium is risk dominant for $\tau = \widehat{\tau}$. The government would then be setting a policy that makes the optimistic outcome barely risk dominant, with arbitrarily close policies leading to the pessimistic outcome. In our model, the fact that the government faces some uncertainty about the cost of production eliminates this sharp discontinuity in the government's objective function, leading to more robust policy conclusions.

However, in general it seems more realistic to assume that even for a given value of c, the government faces some uncertainty about which equilibrium will be selected. In discussing the sunspots ESM above, we referred to mechanisms that have this property as being *probabilistic*. We showed that while the sunspots ESM has the advantage of being probabilistic, is has the disadvantage of assuming that equilibrium selection is completely unaffected by the value of producing. At this point it seems natural to ask how one could combine the probabilistic aspect of the sunspots approach with the payoff-sensitive properties of risk dominance. In order to do this, we build on the intuition discussed above in which the likelihood of an equilibrium occurring is related to the strength of the incentives that agents have to choose that action. Following Young (1998), we define the *risk factor of the optimistic equilibrium* to be the smallest probability p such that if an agent believes that with probability p the likelihood of finding a match in the output market is given by ρ_H (the level in the optimistic equilibrium), and with probability $(1-p)$ this likelihood is zero (its level in the pessimistic equilibrium), then she (weakly) prefers producing (and searching with effort level γ_H) to not producing.[8] The risk factor of the pessimistic equilibrium is then $(1-p)$. The risk factor measures how willing an agent is to choose an equilibrium action

[8] Young's definition applies to 2×2 games, but the extension to our setting is straightforward. Morris, Rob, and Shin (1995) introduce the related notion of p-dominance for two-player, multi-action games. In the binary-choice environment, the risk factor of an equilibrium is equal to the smallest number p such that the action profile where both agents choose that equilibrium action is p-dominant.

when she is unsure of what the market conditions will be. When an equilibrium has a low risk factor, agents are willing to choose that action for a wide range of beliefs, and therefore we should expect that the economy is very likely to coordinate on that outcome. Note that risk-dominance is a particular equilibrium selection rule that is based on risk factors; it says that the optimistic equilibrium will be selected whenever its risk factor is less than one-half. Our intuitive argument in favor of probabilistic ESMs can be restated as saying that when both equilibria exist, the probability of each equilibrium should be strictly between zero and one and should be a decreasing function of its risk factor.[9] In our model, the risk factor of the optimistic equilibrium is given by

$$p = \frac{c - \lambda_L(\tau)}{\lambda_H(\tau) - \lambda_L(\tau)} = \frac{c}{\lambda_H(\tau)}.$$

Here we see that the risk factor is increasing in both the tax rate and the cost of production. A higher tax rate or a higher cost will imply that an agent needs to be more optimistic about the output market in order to be willing to produce. If the probability assigned to the optimistic outcome is a decreasing function of the risk factor, as we have argued that it should be, then it will also be a decreasing function of the tax rate and of the cost. We now state two conditions that summarize these desirable properties that an ESM in our model should possess:

$$\begin{array}{l} \text{If } (\tau, c) \text{ is such that both (private-sector) equilibria exist, then} \\ \quad (i)\ \ 0 < \pi(\tau, c) < 1 \text{ holds} \\ \quad (ii)\ \ \pi \text{ is strictly decreasing in both } \tau \text{ and } c. \end{array} \tag{14}$$

We have shown elsewhere (Ennis and Keister, 2003b, Section 4) how applying the stochastic learning process of Howitt and McAfee (1992) to a binary-choice economy with identical

[9] Ennis and Keister (2003a) derives the implications of risk-factor-based equilibrium selection in a model of bank runs. It also provides a detailed discussion of the relationship between the risk-factor-based approach and the standard sunspots approach.

agents like the one we study here generates an equilibrium selection mechanism with exactly these two properties. In the learning approach, information about the true value of the cost c arrives gradually, and agents take actions and learn about market conditions as this information is arriving. The coevolution of agents' beliefs about c and about market conditions determines where the economy converges. In other words, which equilibrium agents end up coordinating on depends on the actual path taken by the beliefs about c en route to the true value. Equilibrium selection thus depends on the realization of a sequence of random variables, and is therefore random. Rather than presenting the details of the learning process here, we will derive the policy implications of any equilibrium selection mechanism that satisfies the two conditions given in (14). To keep things simple, we will assume that π is everywhere differentiable in τ. This is stronger than is necessary, but allows us the prove the proposition in a simple and transparent way.

Proposition 3 *Assume condition (8) holds. For any equilibrium selection mechanism π satisfying (14), the π-optimal tax rate satisfies $\tau_\pi^* < \widehat{\tau}$.*

Proof: The optimal tax rate maximizes

$$\int_{c_L}^{c_H} \pi(\tau, c) [\Lambda_H(\tau) - c] f(c)\, dc.$$

Differentiating with respect to τ yields

$$\int_{c_L}^{c_H} \left(\frac{\partial \pi}{\partial \tau}(\tau, c) [\Lambda_H(\tau) - c] + \pi(\tau, c) \Lambda_H'(\tau) \right) f(c)\, dc.$$

Evaluating the derivative at $\widehat{\tau}$ gives us

$$\int_{c_L}^{c_H} \frac{\partial \pi}{\partial \tau}(\widehat{\tau}, c) [\Lambda_H(\widehat{\tau}) - c] f(c)\, dc.$$

Condition 8 tells us that the optimistic equilibrium exists at $(\widehat{\tau}, c)$ for all values of c. This implies that (i) the term in square brackets is positive for all c and (ii) the $\partial\pi/\partial\tau$ term is negative for all c. Therefore, the entire derivative is negative and the optimal tax rate must be less than $\widehat{\tau}$. ■

The point of Propositions 2 and 3 can be summarized as follows. When higher tax rates adversely affect the process of equilibrium selection, ignoring the issue of multiplicity of equilibrium will lead to the wrong policy conclusion. In such cases, there is an additional cost of taxation that a standard equilibrium analysis will not recognize. Even though the tax rate $\widehat{\tau}$ is (weakly) optimal in each of the private-sector equilibria separately, the propositions show that the true optimal tax rate must be less than $\widehat{\tau}$.

4 A Public Employment Policy

In previous sections, we showed how taking equilibrium selection effects into account can change the optimal level of a policy variable. In this section, we show how these effects also tend to influence the *types* of instruments that are used as part of the optimal policy. To demonstrate this, we give the government a second policy tool: it can produce the public good by employing a fraction of the agents in the economy and directing their production and search decisions. We use ψ to denote this fraction, and we refer to these agents as the *public sector*. We assume that the government must choose ψ at the same time that it sets the tax rate τ, before the cost of production c is known and before private agents make their production decisions. However, the government chooses the search intensity of public sector agents γ_G later, at the time that private agents set their search intensity. In other words, while τ and ψ are policies that must be set in advance, the level of γ_G can be adjusted to the ex-post situation in the economy. We think that this treatment of the decision-making process captures an important feature of the actual situations faced by policy makers: some policy instruments are more flexible while others must be set in advance and hence require making predictions about future market outcomes.

If the public sector were assumed to be as efficient as the private sector, the optimal policy would involve having a large public sector (thereby reducing the fraction of the economy subject to coordination failures) and not using taxation. This would not be very interesting. Our goal is to show that even a very inefficient public employment/production policy – one that would never be used based on a standard equilibrium analysis – *can* be part of the optimal policy when equilibrium selection effects are considered. We therefore assume that public sector agents are relatively inefficient in the production of the public good. In particular, when a public sector agent is matched in the output market, only a fraction $\sigma < 1$ of the output she receives in trade is transformed into the public good; the rest is lost. In our example below, we will set σ at a fairly low level so that, in general, taxation is the most efficient way of providing the public good.

An important difference between taxation and public production is the effect these policies have on the value of producing for private agents. As we have shown above, taxation decreases the value of producing. Public production, on the other hand, can increase this value, especially in the case where no other private agents produce. When an agent is deciding whether or not to produce, she may be unsure about the actions of other private agents, but she knows that at least the ψ public agents will be in the market. Thus the public employment policy can achieve something that the tax policy cannot: it can decrease the potential loss associated with having produced when no one else has. Because these losses never happen in equilibrium, this benefit of the policy is not captured in a simple equilibrium analysis or when equilibrium selection follows a payoff-insensitive rule. However, when equilibrium selection is sensitive to individual incentives (as reflected in the risk factors), this benefit implies that public production increases rather than decreases the probability of reaching the optimistic equilibrium outcome. Combined with the lower optimal tax rate (see the previous section) and the resulting higher marginal utility of the public good, this property makes public production much more attractive, and in our example makes the optimal level of such production positive. Hence the set of policy instruments used when equilibrium selection issues are taken into consideration can be different than those prescribed by an analysis that ignores the selection problem.

For the sake of simplicity, we assume that the government randomly chooses the agents who will comprise the public sector and that it does not compensate them. In a more realistic situation the government would compensate public sector agents to make them indifferent between public-sector and private-sector work. We work with the former case because it is notationally much simpler, and does not change the main message. In either case, a natural objective for the government is to maximize the integral of all agents' utilities (where all agents are equally weighted), which in our setting is the same as maximizing the expected utility of an agent before she is assigned to the public or private sector.

It is worth pointing out here that the public production policy makes it possible for the government to eliminate the pessimistic outcome as an equilibrium. If the public sector is made large enough, there will be enough output-market activity to induce a private agent to produce and search regardless of the actions of other private agents. However, having a large public sector is costly, both because public production is inefficient and because there is diminishing marginal utility of the public good. For most parameter values, eliminating the pessimistic equilibrium is not optimal; expected utility is higher when the Pareto-inferior outcome is allowed to occur with positive probability.[10]

[10]This point seems relevant for the type of research undertaken by Chatterjee and Corbae (2000). They study the potential gains of eliminating the possibility of events like the Great Depression in the U.S. The authors correctly point out that their calculation does not take into account the cost of implementing the policies that would eliminate these Depression-like episodes. We show that when the choice of policies is studied explicitly, it may not be

The timing of events is as follows. The government first chooses the pair (ψ, τ). Then the cost of production c is revealed and agents decide whether to produce or not. Finally both the government and the agents chose their search intensities. If the economy has a unique private-sector equilibrium, this is equivalent to saying that the government chooses ψ and τ before uncertainty about c is revealed and then chooses γ_G as a Stackelberg leader in the determination of the public and private levels of search intensity. If there is more than one such equilibrium, it also means that the government must choose the policies (ψ, τ) before it knows what the level of aggregate demand will be for each possible value of c.

In the Appendix, we describe how to compute the two equilibrium outcomes for given values of (ψ, τ) and c. We denote the welfare level in the optimistic outcome by $U_H(\psi, \tau; c)$ and that in the pessimistic outcome by $U_L(\psi; c)$. These functions reflect the optimal decision rules of: (i) private agents, who choose whether to produce or not, and if producing, the optimal level of search intensity, and (ii) the government, which chooses the search intensity γ_G to maximize welfare in a given private-sector equilibrium.

We now discuss how the government determines the optimal policy pair (ψ, τ). We concentrate on this stage because it is where the equilibrium selection issues arise. Suppose that the government completely ignores the question of equilibrium selection and simply assumes that the optimistic outcome will always obtain. Then the government chooses ψ and τ to maximize

$$\int_{c_L}^{c_H} U_H(\psi, \tau; c) f(c) dc.$$

Let $(\widehat{\psi}, \widehat{\tau})$ be the solution to this problem.[11] Alternatively, when the government believes

optimal to fully eliminate the possibility of these types of events. See Dagsvik and Jovanovic (1994) for a study where a coordination failure of the type we examine here is considered to represent a Depression-like episode.

[11]Later, we will choose parameter values such that $\lambda_H(\widehat{\psi}, \widehat{\tau}) > c_H$ holds, and therefore the

that equilibrium selection is driven by sunspots, the optimal policy problem is

$$\max_{\psi,\tau} \int_{c_L}^{c_H} \left[\pi_s U_H(\psi,\tau;c) + (1-\pi_s) U_L(\psi;c)\right] f(c)dc, \tag{15}$$

where π_s is the fixed probability of the optimistic outcome. Let (ψ_s, τ_s) be the solution to this problem.

We want to compare the solutions to these two problems with the optimal policy when risk dominance determines equilibrium selection.[12] The cut-off value of c under the risk-dominance equilibrium selection mechanism is now given by:

$$\widetilde{c}(\psi,\tau) \equiv \frac{1}{2}\left[\lambda_H(\psi,\tau) + \lambda_L(\psi,\tau)\right],$$

and the optimal policy problem is similar to (15) but where the constant π_s is now replaced by the function

$$\pi(\psi,\tau;c) = \left\{ \begin{array}{ll} 1 & \text{if } c < \widetilde{c}(\psi,\tau) \text{ holds} \\ 0 & \text{otherwise} \end{array} \right\}.$$

Note that this policy problem can be re-written as

$$\max_{\psi,\tau} \int_{c_L}^{\widetilde{c}(\psi,\tau)} U_H(\psi,\tau;c) f(c)dc + \int_{\widetilde{c}(\psi,\tau)}^{c_H} U_L(\psi;c) f(c)dc.$$

Let (ψ^*, τ^*) be the solution to this problem. We now provide an example where $\widehat{\psi} = \psi_s = 0$ and $\psi^* > 0$ hold. In other words, in the following example if the government ignores the influence of individual-level incentives on the equilibrium selection process, it chooses not to run a public sector. However, when it takes risk-dominance-based equilibrium selection into account, the government chooses a positive level of public employment.

optimistic outcome is indeed an equilibrium for all values of c.

[12] Similar results can be obtained with a probabilistic ESM based on the risk factors of the equilibria, as presented in Section 3.4

Example. There is a quadratic matching function $m(x) = x^2$, a cubic cost function $d(x) = x^3$, and the public-good utility function is given by $v(x) = x^{\frac{1}{2}}$. The cost of production is uniformly distributed with support $[0, 0.15]$. We use parameter values $F = 1.5$ and $\sigma = 0.1$.

Table 1: The Equilibrium Selection Effect on Policy

	Ignoring Eqm. Selection	Risk Dominance ESM
ψ	0.000	0.096
τ	0.087	0.026
λ_H	0.190	0.263
λ_L	0.000	0.004

In this example, when the government ignores equilibrium selection and believes that the optimistic outcome will always obtain, it chooses not to run an active public sector. The public sector is relatively inefficient and taxation is a lower-cost way of providing the public good. (Note that $\lambda_H > c_H$ holds, so that the optimistic outcome is always an equilibrium.) Similarly, for all values of π_s greater than 0.35 a government solving problem (15) would choose not to run a public sector, i.e., $\psi_s = 0$.[13] It is easy to see that in such a case the optimal tax rate under sunspots τ_s will also equal $\widehat{\tau}$. However, when the government is aware of equilibrium selection effects that are driven by risk dominance, the optimal policy involves an active public sector that employs 9.6% of the population and produces 37% of

[13] One way to get a sense of what would be a reasonable value for π_s is to note that in this example, when the government sets the policy at (ψ^*, τ^*), the probability of observing the optimistic equilibrium under risk-dominance selection is about 0.89. Furthermore, the threshold value for π_s depends on the value of σ, the efficiency of public employment, and with a small enough value of σ the government would choose not to run a public sector for any value of the probability π_s.

the public good in the optimistic equilibrium. This demonstrates how taking into account (risk-factor-based) incentives in the equilibrium selection process can have a significant effect on the types of policies that are optimal for a government to use. □

5 Concluding Remarks

The above analysis has illustrated what we think is a fairly general property of equilibrium selection in environments where aggregate demand matters: Policies that reduce the value to an individual agent of participating in the market will make it less likely that agents coordinate on a high-market-activity equilibrium. We have shown that risk-factor-based equilibrium selection mechanisms have clear policy implications: Taking equilibrium selection into account reveals policies like taxation to be more costly than a simple equilibrium analysis would indicate, and therefore such policies should be used less. If the government has a variety of tools available, taking selection effects into account will lead it to substitute away from the ones that decrease the value of producing and toward the ones that do not, even if the latter are inefficient from the standard equilibrium viewpoint. We also intend for the analysis to make a larger point: By taking a stand on the properties of the equilibrium selection process, one can derive meaningful predictions and policy prescriptions from models with multiple equilibria.

Our result in Section 4 with respect to public employment is reminiscent of a colorful suggestion by Keynes (1936, p. 129).

> If the Treasury were to fill old bottles with banknotes, bury them at suitable depths in disused coal mines which are then filled up to the surface with town rubbish, and leave it to private enterprise on well-tried principles of laissez-faire to dig the notes up again (the right to do so being obtained, of course, by tendering for leases of the note-bearing territory), there need be no more unemployment and, with the help of the repercussions, the real income of the community, and its capital wealth also, would

probably become a good deal greater than it actually is. (Emphasis added.)[14]

In other words, Keynes suggests that the government undertake a costly activity solely in order to increase the benefit to private agents of engaging in market activity. This seems difficult to justify from a traditional equilibrium point of view. While encouraging agents to choose higher search intensities does improve the efficiency of the output market, it is hard to believe that the multiplier effect could possibly be large enough to justify the initial (entirely wasteful) outlay. It seems likely to us that Keynes instead believed that government policy could be used to direct the economy toward a good equilibrium (as in the quote from Diamond (1982) in our introduction). Viewed in this light, such a policy recommendation seems less absurd. Perhaps it is just an exaggerated way of making our same point: policies that encourage market activity are more likely to bring about coordination on outcomes with high levels of market activity.

Appendix: Computing Equilibria with Public Employment

In this appendix, we describe how to compute an equilibrium of the economy for a given policy (ψ, τ) and a cost of production c. The economy's average level of search intensity is given by

$$\overline{\gamma} = \psi\gamma_G + \int_0^{1-\psi} \phi_i \gamma_i di,$$

and the total amount of the public good provided in equilibrium is equal to

$$x = \psi\rho\gamma_G\sigma + \tau \int_0^{1-\psi} \rho\phi_i\gamma_i di.$$

[14]We thank Tom Humphrey for bringing this passage to our attention.

The optimal decision rules of a private agent are still given by (4) and (5). We again only consider symmetric outcomes, so that either all private agents produce and search at intensity γ_H or no private agent produces.

In the optimistic outcome, when private agents produce, the total amount of search intensity in the market is given by $\overline{\gamma} = (1-\psi)\,\gamma_H + \psi\gamma_{GH}$, and the probability of being matched per unit of search intensity is given by

$$\rho(\gamma_H, \gamma_{GH}, \psi) = \frac{m\left((1-\psi)\,\gamma_H + \psi\gamma_{GH}\right)}{(1-\psi)\,\gamma_H + \psi\gamma_{GH}}.$$

We can find the equilibrium search intensity by inserting $\rho(\gamma_H, \gamma_{GH}, \psi)$ into (5) and solving for γ_H. This intensity is a function of the policy parameters, and we denote it $\gamma_H(\gamma_G, \psi, \tau)$. Define also the function $\rho_H(\gamma_G, \psi, \tau) \equiv \rho(\gamma_H(\gamma_G, \psi, \tau), \gamma_G, \psi)$. Welfare in this case is given by

$$U_H = (1-\psi)\left[\lambda_H - c\right] + \psi\left[-d\left(\gamma_G\right) - c\right] + v\left(x_H\right),$$

where we have $x_H = (1-\psi)\,\tau\rho_H\gamma_H + \psi\sigma\rho_H\gamma_G$, and $\lambda_H = \rho_H\gamma_H(1-\tau)F - d(\gamma_H)$. Using the definitions of $\gamma_H(\gamma_G, \psi, \tau)$ and $\rho_H(\gamma_G, \psi, \tau)$ given above, we can obtain the function $U_H(\gamma_G, \psi, \tau; c)$ that gives agents' expected utility as a function of the policy parameters. The government then chooses the public sector's search intensity in the optimistic outcome to maximize the function $U_H(\gamma_G, \psi, \tau; c)$. The solution is given by

$$\gamma_{GH}(\psi, \tau) = \arg\max U_H(\gamma_G, \psi, \tau; c).$$

The optimistic outcome is an equilibrium as long as $\lambda_H\left(\psi, \tau\right) > c$ holds.

In the pessimistic outcome, when no private agent produces, the total level of search intensity is given by $\overline{\gamma} = \psi\gamma_G$ and the average probability of being matched is given by

$$\rho_L\left(\gamma_G, \psi\right) = \frac{m\left(\psi\gamma_G\right)}{\psi\gamma_G}.$$

Welfare is then given by $U_L(\gamma_G, \psi; \overline{p}) = \psi(-d(\gamma_G) - c) + v\left(\rho_L\left(\gamma_{G,}\psi\right)\psi\gamma_G\sigma\right)$. Since private agents do not produce, welfare is equal to the utility provided by the public good minus the production and search costs of the public sector agents. The government chooses the search intensity γ_G according to

$$\gamma_{GL}(\psi) = \arg\max U_L(\gamma_G, \psi; c).$$

Note that γ_{GL} is only a function of the size of the public sector ψ. Since private agents are not producing, the tax policy is irrelevant in this outcome.

For given values of $(\psi,\ \tau)$ and c, the pessimistic outcome is an equilibrium whenever a private agent who believes that the average probability of being matched is equal to $\rho_L(\psi) \equiv \rho_L(\gamma_{GL}(\psi), \psi)$ decides not to produce. If she were to produce, she would set her search intensity to

$$\gamma_L(\psi, \tau) = d'^{-1}\left(\rho_L(\psi)(1-\tau)F\right),$$

and her expected benefit from producing would be given by

$$\lambda_L(\psi, \tau) = \rho_L(\psi)\gamma_L(\psi, \tau)(1-\tau)F - d\left(\gamma_L(\psi, \tau)\right).$$

If $\lambda_L(\tau, \psi) < \ c$ holds, then the expected benefit of producing is lower than the cost, she would choose not to produce, and the pessimistic outcome is an equilibrium. However, if the government sets (τ, ψ) so that $\lambda_L(\tau, \psi) > c$ holds for a given c, then even an agent with pessimistic beliefs would choose to produce and therefore such beliefs are not self-fulfilling. In other words, the policy has eliminated the pessimistic outcome as an equilibrium and only the optimistic outcome is possible. However, notice that even if the government recruits all of the agents in the economy (so that $\psi = 1$), the value of ρ only reaches $\frac{m(\gamma_G)}{\gamma_G} < 1$. Hence there is a bound on the ability of the government to increase λ_L, and it may be the case that no level of ψ can eliminate the pessimistic equilibrium for some realizations of c.

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