IS IMPLIED CORRELATION WORTH CALCULATING? EVIDENCE FROM FOREIGN EXCHANGE OPTIONS AND HISTORICAL DATA

Christian Walter and Jose Lopez

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Christian Walter

Economic Division
Swiss National Bank
Boersenstrasse 15
CH-8022 Zurich
Phone: (011) 41 1 631 3573
Fax: (011) 41 1 631 3980
walter.christian@snb.ch

Jose A. Lopez

Research and Market Analysis Group
Federal Reserve Bank of New York
33 Liberty Street
New York, NY 10045
Phone: (212) 720 6633
Fax: (212) 720 8363
jose.lopez@ny.frb.org

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ABSTRACT: This paper examines the performance of implied correlations in forecasting subsequently realized correlations between exchange rates. Implied correlations are derived from sets of implied volatilities on the three exchange rates in a currency trio. We compare the forecasting performance of the implied correlations from two currency trios with markedly different characteristics over two forecast horizons (one month and three months) against a set of alternative correlation forecasts based on time-series data.

For the correlations in the USD/DEM/ JPY currency trio, we find that the option-based forecasts are useful in predicting subsequently realized correlations. Specifically, they tend to be more accurate than the simple forecasts based on time-series data (i.e., historical correlations and exponentially weighted moving average correlations) and contain useful information that is not present in the other forecasts. However, since correlation forecasts based on a bivariate GARCH(1,1) model improve the performance of implied correlations, we reject the hypothesis that the implied correlations fully incorporate all the information in the price history.

For the correlations in the USD/DEM/CHF currency trio, the option-implied correlation forecasts are less useful in predicting realized correlations. For two of the three correlations, implied correlations are not as accurate as the forecasts based on time-series data and provide no additional information. For the third correlation, the implied correlations do contain useful information, but the economic benefits of using these implied correlations may be small due to this correlation's low level of variability.

Key Words: Option prices, Implied correlation, GARCH, Volatility forecasting

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I. Introduction

The concept of implied correlation as derived from option prices has to date received little attention in the literature on the information content of derivatives' prices. This is surprising since correlations play a crucial role in various fields of financial decision making, such as asset allocation, risk measurement and hedging. In addition, derivative contracts based on several financial time series have become more common in the 1990's (Mahoney, 1995). For these reasons, calculating implied correlations might be a value-enhancing activity for investors, risk-managers and treasurers alike.

The purpose of this paper is to examine the performance of implied correlation in forecasting subsequently realized correlation. Following Bodurtha and Shen (1995), Campa and Chang (1997), and Siegel (1997), we analyze the predictive ability of implied correlations between certain foreign exchange rates. We complement and extend these papers in three specific ways. First, we analyze all three implied correlations extractable from the options on the exchange rates in the currency trio consisting of the US dollar (USD), the German mark (DEM), and the Japanese yen (JPY). Second, we examine the predictive power of the three implied correlations extractable from options on the currency trio consisting of the US dollar, German mark and Swiss franc (CHF), a currency trio with markedly different characteristics. Third, we compare the implied correlations against a larger set of alternative forecasts. Fourth, we provide a detailed geometric interpretation of the relationship between the volatilities and the correlations in a currency trio.

For the three correlations in the USD/DEM/JPY currency trio, we find that implied correlations and GARCH-based correlation forecasts outperform the simple time-series forecasts, such as correlations derived from equally weighted or exponentially weighted past return observations. Although we find that implied correlations contain information not present in the time-series forecasts, we reject the hypothesis that implied correlations fully incorporate all the information in the historical data. GARCH-based correlation forecasts, but not the other forecasts, always incrementally improve the performance of implied correlations. This result suggests that the implied correlations either do not incorporate all the information in the price history or are based on a misspecified option pricing model. It also indicates that the information in the historical data that is useful in forecasting correlations is most effectively summarized by the GARCH-based correlation forecasts.

For the correlations in the USD/DEM/CHF currency trio, the economic benefits of using implied correlations in forecasting subsequently realized correlations are not substantial. In two of the three cases, the option-implied correlation forecasts do not provide additional information over the information contained in the price history. In the third case, the correlation between the DEM and the CHF measured in USD, implied correlation contains unique information that is useful in forecasting the realized correlation. However, the economic benefit of using implied correlation is small due to the low variability of this correlation.

The paper is structured as follows. Section II introduces the concept of implied correlation, reviews the literature to date, and gives an overview of possible sources for implied correlations. Section III describes the data, the method used to obtain the implied correlations, and the alternative forecasting methods used. Section IV provides evidence on the predictive power of implied correlation by comparing its performance against the alternative time series methods. Section V summarizes and concludes.

II. Implied Correlation: Concept and Sources

a. From Implied Volatility to Implied Correlation

Option pricing formulas relate the price of an option to the variables that influence its price. The famous Black-Scholes formula, for example, expresses the price of a European option on a non-dividend paying stock as a function of five variables: the option's strike price, its time to expiration, the risk-free interest rate, the underlying asset's price, and the underlying asset's volatility over the remaining life of the option. Since the first four variables and the option price are directly observable, and the option price is a monotonically increasing function of volatility, the pricing formula can be inverted to determine the underlying asset's volatility implied by the option price. This so-called implied volatility is often interpreted as the market's assessment of the underlying asset's volatility over the remaining life of the option.[1] Implied volatilities can be inferred not only from options on non-dividend paying stocks, but from options and other derivative instruments on

[1] Note that the almost universal acceptance of a pricing formula by market participants such as the Garman-Kohlhagen (1983) model for European currency options neither implies the correctness of its assumptions nor the acceptance of these assumptions by market participants. It is simply a market convention for stating the option prices. Deviations from the formula's assumptions, such as different distributional forms, are commonly incorporated by adjusting the quoted implied volatility.

other assets as well.[2] Conceptually, the procedure is the same as in the above-mentioned case.[3]

A natural question to ask then is whether volatility forecasts should be based on implied volatilities, standard forecasts from time-series models, or some combination of the two. Although this question has been addressed by numerous researchers, the debate is still open.[4] The literature currently tends to suggest that implied volatility performs better than simple time-series forecasts, such as historical volatility, in forecasting future volatility.[5] However, more recent research seems to indicate that forecasts based on GARCH models contain information that is not present in implied volatility.[6]

Implied correlation, defined as the coefficient of correlation between two variables implied by the price of a derivative or the prices of several derivatives, has not received a comparable amount of attention. This is surprising, give the significant practical benefits offered by better forecasts of future realized correlations. Specifically, financial decision making, such as structuring a portfolio, measuring its risk or hedging them, usually requires inputs describing both the individual characteristics of financial variables and measures of their comovements. For instance, an investor optimizing his portfolio in a mean-variance framework needs an *ex ante* estimate of the variance-covariance matrix of securities' returns over the relevant holding period. Clearly, better *ex ante* forecasts of this matrix should result in more efficient asset allocations from an *ex post* perspective.

The forecast of the variance-covariance matrix could be improved by using implied

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[2] Implied volatilities have been extracted from the prices of, for example, foreign exchange options (Galati and Tsatsaronis, 1995), interest rate futures and bond options (Amin and Morton, 1994), and path-dependent options (Ball, Torous and Tschoegl, 1985).

[3] Note that, in practice, extracting *the* implied volatility is not so straightforward. Often, many options with identical times to expiration are written on the same asset, and the implied volatilities extracted from these options vary according to characteristics of the individual option (strike price, the type of option, etc). Abstracting from market imperfections, such as price discretness, transactions costs, or nonsynchronous trading, this fact must be interpreted as evidence against the assumptions underlying the pricing model used. However, as a practical solution to this problem, various weighting schemes have been developed; see Mayhew (1995).

[4] For a recent review of the literature, see Mayhew (1995).

[5] In contrast, Canina and Figlewski (1993) find historical volatility to perform better than implied volatility for the S&P 100 index. Recent studies finding implied volatility to dominate historical volatility include Fleming (1994) for the S&P 100 index, and Galati and Tsatsaronis (1995) as well as Jorion (1995) for various exchange rates.

[6] GARCH is the acronym for generalized autoregressive conditional heteroscedasticity. GARCH models were introduced by Bollerslev (1986) as an extension to ARCH models developed by Engle (1982). GARCH is a general approach to modeling volatility not as a parameter, but as a stochastic process that evolves over time in a deterministic fashion. Kroner, Kneafsey and Claessens (1995) provide evidence suggesting that GARCH-based volatility forecasts contain information not present in implied volatilities.

correlations in two ways. First, an investor could construct an implied variance-covariance matrix using volatilities and correlations extracted from option prices. Such an approach would be especially useful if both implied correlations and implied volatilities have superior predictive power over other forecasting methods. Second, an investor could use a time-series model to generate the variance estimates and combine these data with the implied correlations to arrive at a variance-covariance matrix. This would make sense if the implied correlations have superior predictive power over forecasts based on time-series models, but the implied volatilities have not. Note that the market's ability to forecasts correlations could be high even though it's ability to predict volatilities is low. For this to be the case, the market's forecast for

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\ \sqrt{\mathrm{Var}(Y)}}.$$

must be more accurate than the market's forecasts for the components in the expression on the right-hand side.

Despite these potential benefits, implied correlation has been the subject of only three studies, the papers by Bodurtha and Shen (1995), by Campa and Chang (1997), and by Siegel (1997). All three papers analyze the forecasting ability of the implied correlations between certain foreign exchange rates. The market for foreign exchange options is an attractive source for implied correlations because of the existence of options on cross-rates[7]. As discussed in section III.a., implied correlations between the exchange rate pairs in a currency trio are easily extracted from the implied volatilities of the individual exchange rates.

Bodurtha and Shen (1995) use options price data from the Philadelphia Stock Exchange on the currency trio consisting of the USD, the DEM, and the JPY to estimate the implied correlation between the DEM/USD and JPY/USD exchange rates. They evaluate the forecasting ability of implied correlation by regressing the realized correlation over a one-month period on one-month implied correlation, one-month historical correlation, and exponentially weighted moving average correlation based on a decay factor of 0.97. The authors find that both historical and implied correlation provide explanatory power in explaining realized correlation. Furthermore, they

[7] The term "cross-rate"denotes any exchange rate between two non-US dollar currencies.

document that the implied correlation between the DEM/USD and JPY/USD exchange rates tends to increase when implied volatilities increase. This implies that, for an USD-based investor, the benefits of currency diversification tend to diminish when diversification is most important.

Siegel (1997) analyzes the forecasting performance of implied correlation in the context of a specific application. He examines whether using implied correlations is helpful in improving the performance of cross-currency hedges. The author uses one and a half years of options data from the Philadelphia Stock Exchange on two currency trios (the USD/DEM/JPY trio and the trio consisting of USD, DEM and British pound) to construct options-based hedge ratios for several currency positions. He then compares the volatilities of the hedged positions with the volatilities of hedged positions based on historical correlation. He finds that the hedges based on implied correlations perform significantly better in some cases and never significantly worse than the historical correlation-based hedges. Furthermore, regression results indicate that the historical correlation-based hedge ratios provide no additional information beyond the information that is already reflected in the implied correlation-based hedge ratios.

Unlike these two papers, Campa and Chang (1997) use data from the over-the-counter (OTC) market for foreign exchange options, which has three important advantages. First, since the OTC market for foreign exchange options is larger and more liquid than the market for exchange traded foreign exchange options, data from the OTC market for foreign exchange options is more informative than data from the Philadelphia Stock Exchange.[8] Second, in contrast to exchange traded options, that have a fixed expiration dates, OTC foreign exchange options are issued daily with fixed times to expiration, which eliminates the need to adjust the implied volatilities for term structure effects. Third, the OTC options are generally created with at-the-money strike prices. Since the sensitivity of options with regard to the underlying's volatility (the so-called vega) is typically highest for at-the-money options, the OTC data ensures that the most information about the expected volatility is captured. Beckers (1981) provides evidence supporting this view. He finds that the implied volatilities from at-the-money options do as well in predicting future volatilities as

[8] As has been shown by Cooper and Weston (1996), foreign exchange options are among the growing group of OTC instruments that have become the subject of an intense competition. As a consequence, terms and conditions have been standardized, and the differences in competing quote prices have become relatively small (less than one percent of the average price).

weighted averages of implied volatilities from different options. The loss of information incurred by using only at-the-money options is therefore modest. Campa and Chang (1997) analyze the forecasting ability of the implied correlation between the DEM/USD and JPY/USD exchange rates. Their study is based on six and a half years of daily data on the implied volatilities of OTC foreign exchange options with constant times-to-maturity of one month and three months. As alternative forecasts to implied correlation, they consider historical and exponentially weighted moving average correlation, as well as correlation forecasts generated by a rolling, bivariate GARCH(1,1) model. Applying a much richer econometric methodology than the two papers mentioned before, Campa and Chang (1997) find that implied correlation outperforms the other forecasts. In particular, they find that none of the time-series based forecasts is consistently capable of providing additional information to the implied correlation forecasts.

To summarize, these studies provide promising evidence on the predictive power of implied correlations. Given the potential benefits of implied correlations, this suggests that it is worthwhile to analyze the forecasting ability of implied correlations further. Specifically, it seems worthwhile to examine other correlations beside the one between the USD/DEM and USD/JPY exchange rates. Therefore, we analyze the predictive power of all the implied correlations extractable from the currency trio USD/DEM/JPY. We also examine the forecasting ability of implied correlations from a currency trio with markedly different characteristics. This currency trio consists of the USD, the DEM and the Swiss franc (CHF). Before we analyze the forecasting performance of the implied correlations, we shall give a brief overview of the instruments that allow the extraction of implied correlations.

b. Sources of Implied Correlations

A necessary condition for the extraction of implied correlation to be possible is the existence of derivatives whose prices are related to the level of correlation between two variables. There are instruments with payoffs that solely depend on the level of correlation between two variables.[9]

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[9] Such "pure correlation products" either take the form of a futures contract (whose payoff is equal to the difference between some measure of realized or implied correlation over the life of the contract and the predetermined futures price) or take the form of an option contract (for which the payoff is the maximum of zero and the difference between some measure of realized or implied correlation over the life of the contract and the predetermined strike level). Pure

However, for the most actively traded correlation-dependent instruments, the level of correlation is one price factor among others.[10]

A first group of instruments that embody information about correlations are basket options. Basket options are options with payoffs related to the cumulative performance of a basket of instruments. The underlying basket can be a selection of stocks, currencies, interest rates or commodities. Analogous to a standard option, the price of a basket option depends on the volatility of the underlying. Since the underlying of a basket option is a portfolio, the price of a basket option is a function of the return-variability of the underlying portfolio. Since the return variance of a portfolio depends on the correlation between the returns of the individual components, the prices of basket options contain information about implied correlations.

Consider, for example, a basket option written on a portfolio consisting of two stocks (A and B). Assuming that the returns of A and B are bivariate normally distributed, we know from portfolio theory that the return of the portfolio is normally distributed with variance

$$\sigma(P)^2 = w\,\sigma(A)^2 + (1-w)\,\sigma(B) + 2w(1-w)\,\rho(A,B)\,\sigma(A)\,\sigma(B),$$

where w denotes the fraction invested in stock A, $\sigma(A)$ and $\sigma(B)$ are the standard deviations of the returns of stock A and stock B, and $\rho(A,B)$ denotes the correlation between the returns of A and B. Given that options on the individual stocks and on the basket are traded, one can infer the correlation implied in the price of the basket option by a two-step procedure. In the first step, the implied values of $\sigma(P)$, $\sigma(A)$ and $\sigma(B)$ are extracted from the prices of the basket option and the options on the individual stocks by inverting an appropriate pricing formula.[11] In the second step, the implied

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correlation products allow researchers to get direct point estimates of expected future correlations (futures on correlation) or measures for the volatility of correlations (options on correlation), as well as allow investors to hedge directly against shifts in correlations. For a description of the properties of futures and options on volatility, see Grünbichler and Longstaff (1996).

[10] See also Smithson (1997) who provides a taxonomy of correlation-dependent instruments. For an analysis of the risk management issues raised by such instruments, see Mahoney (1996).

[11] Note that it would be logically inconsistent to use the same pricing formula for the extraction of the implied volatilities of the individual stocks and of the basket. The reason for that lies in the fact that a sum of log-normally distributed random variables is in general not log-normally distributed. Consider, for example, the two log-normally distributed stocks A and B. Since A and B are log-normally distributed, their values at time T (A_T and B_T) can be expressed as $A_0\,e^{\kappa T}$ and $B_0\,e^{\lambda T}$ where A_0 and B_0 are the values of A and B at time 0 and κ and λ are the normally-distributed continously compounded rates of returns of stock A and B, respectively. Consider now a basket consisting of the two stocks. The price of the basket at time T is given by $A_T + B_T$ which is equal to $A_0\,e^{\kappa T} + B_0\,e^{\lambda T}$. In general, the price of the basket can not be log-normally distributed since it is not possible to rearrange the sum $A_0\,e^{\kappa T} + B_0\,e^{\lambda T}$ in a way to arrive at an

values of $\sigma(P)$, $\sigma(A)$ and $\sigma(B)$ are plugged into the equation above and the formula is solved for $\rho(A,B)$ – the implied correlation between the returns of stock A and stock B.

In general, basket options on more than two assets (e.g., index options) do not allow the extraction of individual implied correlations. The reason is that the variance of a portfolio consisting of N assets is a function of more than one correlation, namely of $N(N\text{-}1)/2$ correlations. Therefore, given a basket option on N assets and the individual options on the N assets involved, it is only possible to extract the implied average value of the $N(N\text{-}1)/2$ correlations between the assets in the basket. This can be done by solving the following formula (see Kelly, 1994):

$$\bar{\rho} = \sigma(P)^2 - \frac{\sum_i w_i^2 \sigma(i)^2}{\sum_i \sum_{i \neq j} w_i w_j \sigma(i) \sigma(j)},$$

where w_i denotes the fraction of the basket invested in asset i (with $\sum w_i = 1$), $\sigma(i)$ is the standard deviation of the return of asset i, and $\bar{\rho}$ denotes the implied average correlation between the N assets in the basket.

A second group of instruments from which implied measures of correlations could be inferred are derivative securities whose payoffs are a function not of the combined performance (as in the case of basket options) but of the relative performance of two underlying variables. According to the exact specification of the relationship and the underlying risk factor (e.g., interest rates, foreign exchange rates, equity or commodity prices) such instruments are known under different names. Table 1 lists some of the better-known relative performance instruments.

To illustrate the correlation-dependency of relative performance instruments, consider an option whose payoff is related to the ratio between the prices of the two assets A and B in the following form:

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expression of the form $(A_0 + B_0)\, e^{\varepsilon T}$ and ε is a normally-distributed random variable. This implies that if one uses the Black & Scholes formula (either in its original or in a modified version) for the extraction of the implied volatilities of the individual stocks, it would be inconsistent to extract the implied volatility of the basket option by the same formula. As a solution to the problem, the distribution of a basket of log-normally distributed assets can be determined numerically (Rubinstein [1991]) or approximated analytically (Huynh [1994]).

$$\text{Max}\left(\frac{a_T}{b_T} - X,\ 0\right),$$

where a_T and b_T are the prices of the two assets at expiration of the option and X denotes the strike price of the option. Let us then define a new variable c with $c = a/b$. Differentiating the expression for c yields

$$dc = \frac{1}{b}\,da - \frac{a}{b^2}\,db,$$

so that

$$\frac{dc}{c} = \frac{1}{c}\frac{a}{b}\frac{da}{a} - \frac{1}{c}\frac{a}{b}\frac{db}{b},$$

and consequently (since $c = a/b$)

$$\frac{dc}{c} = \frac{da}{a} - \frac{db}{b}.$$

This means that the relative change of the variable c equals the difference between the relative change of the numerator a and the relative change of the denominator b (for infinitesimal small changes). Assuming that the relative changes of a and b follow a bivariate normal distribution and using the formula for the variance of a difference of random variables, we obtain the following expression for the variance of the relative change in c (which is normally distributed):

$$\sigma(c)^2 = \sigma(a)^2 + \sigma(b)^2 - 2\,\rho(a,b)\,\sigma(a)\,\sigma(b).$$

As with basket options, the variance of the asset underlying the relative performance instrument considered here is a function of the individual assets' return variances and the correlation between the returns. There is a difference between the formulas for the underlying asset's return variance in that the sign in front of the "correlation-term" is positive in the case of the basket and negative in the case of the relative performance instrument. This is intuitively clear, since the closer the prices of the assets involved move together, the smaller the variance of the relative performance instrument. That is, the higher the correlation, the larger the return variability of a basket and the smaller the variability of a relative performance instrument (for given asset return variances).

In principal, extracting implied correlations from the prices of relative performance instruments involves the same steps as in the case of basket options. In the first step, implied volatilities are extracted from the relative performance instrument and the options on the individual assets.[12] In the second step, these values are plugged into an equation linking the volatilities and the correlation between the variables, and the equations is solved for the implied correlation.

A third group of instruments with correlation-dependent payoffs are the so-called "quanto" products.[13] Quantos are derivative products denominated in a currency other than that of the underlying to which exposure is sought and with the exchange rates used to convert the returns fixed at the start of the option contracts. Unlike basket options or relative performance instruments whose prices typically depend on correlations within a certain asset class (e.g. equity prices), quanto product prices reflect the correlations between different asset classes.

The best known quanto products are equity quanto options and differential swaps. An equity quanto option is an option on a foreign stock or stock index with the exchange rate used to convert the payoff into domestic currency usually set at the spot exchange rate prevailing at the start of the option contract. Equity quanto options prices are a function of the correlation between the underlying equity instrument and the exchange rate since both variables simultaneously determine the payoff.[14] Differential (or diff) swaps are swaps in which each party pays an interest rate in one currency and receives an interest rate in another currency (plus or minus a spread) with all payments denominated in the same currency at a fixed exchange rate. As in the case of the equity quanto option, the guaranteed exchange rate feature creates the diff swap's correlation-dependency.[15]

Extracting implied correlations from quanto products involves a two-step procedure. In the first step, the values for the implied volatilities of the variables involved are extracted from plain

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[12] Note that, unlike in the basket option case, no logical inconsistency is involved by using Black & Scholes types of pricing formulas for the extraction of implied correlations for this kind of relative performance instrument. Since the ratio of two log-normally distributed random variables is also log-normally distributed, it is not inconsistent to use Black & Scholes types of pricing formulas to extract the implied volatilities of the individual variables and the implied volatility of the underlying variable of the relative performance instrument.

[13] Quanto is from the Latin word "quantun" for "how much" which stands for the implicit quantity adjusting embodied in these products. The quantity adjusting refers to the fact that the option writer, because of the stochastic nature of the underlying variable to which exposure is sought, does not know until the payment dates how much foreign currency he has to translate into domestic currency. Because of the guaranteed exchange rate feature, quanto products are often said to be currency protected or (after the quanto version of an interest rate swap) diff'd.

[14] For a closed-form pricing formula for European equity quanto options, see Reiner (1992), as well as Wei (1995).

[15] For diff swap pricing formulas, see Jamshidian (1993).

vanilla options on these products. In the case of a quanto option on the Nikkei Index denominated in US dollar, for example, the implied volatilities of the Nikkei Index and the Yen/US dollar exchange rate are extracted from Nikkei Index options and Yen/US dollar foreign exchange rate options, respectively. In the second step, the implied volatilities and the price of the quanto product observed are plugged into the appropriate pricing formula, which then can be solved for the implied correlation.

The three product groups considered so far are primary candidates for providing information about implied correlations since correlation-dependency is a common feature of all the instruments in these groups. However, there are many other derivative instruments which contain information about correlations. Since the number of such instruments has mushroomed beyond at least our ability to follow it, the following list of better-known instruments is not meant to be complete: European options on swaps (swaptions)[16], barrier options with payoffs determined upon a different variable from the barrier, binary options with payoffs related to the values of two underlying variables or Asian options with payoffs determined by a geometric average.

III. Data

Following Bodurtha and Shen (1995), Campa and Chang (1997), and Siegel (1997), we analyze the predictive ability of implied correlations extracted from sets of foreign exchange rate options. In this section, we explain how the implied correlations are derived, where our data comes from, and how the alternative correlation forecasts are generated.

a. Implied Correlations Extracted From a Set of Options on the Exchange Rates in the USD/DEM/JPY and USD/DEM/CHF Currency Trios

A foreign exchange cross-rate is a redundant variable since it is completely determined by the two underlying US dollar exchange rates. An option on a foreign exchange cross-rate, however, is not a redundant instrument. In fact, given options on the two underlying US dollar exchange rates, it allows the extraction of option-based estimates of the correlations between the three exchange rates in a currency trio.

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[16] From the prices of a series of swaptions and caps one can, at least in principle, extract the implied correlation structure between the forward rates of an interest rate curve (Rebonato [1996, p. 16-17]).

To illustrate, let us denote the US dollar prices of one unit of currency A and one unit of currency B by A^*_{USD} and B^*_{USD}. In the absence of arbitrage opportunities, the price of currency A expressed in units of currency B (denoted as A^*_B) must equal

$$A^*_B = \frac{A^*_{USD}}{B^*_{USD}}.$$

By differentiating this expression, we obtain

$$dA^*_B = \frac{1}{B^*_{USD}}\, dA^*_{USD} - \frac{A^*_{USD}}{B^{*\,2}_{USD}}\, dB^*_{USD},$$

so that

$$\frac{dA^*_B}{A^*_B} = \frac{1}{A^*_B}\frac{A^*_{USD}}{B^*_{USD}}\frac{dA^*_{USD}}{A^*_{USD}} - \frac{1}{A^*_B}\frac{A^*_{USD}}{B^*_{USD}}\frac{dB^*_{USD}}{B^*_{USD}},$$

and consequently

$$\frac{dA^*_B}{A^*_B} = \frac{dA^*_{USD}}{A^*_{USD}} - \frac{dB^*_{USD}}{B^*_{USD}}.$$

The relative change of the cross-rate equals the difference between the relative changes of the two US dollar exchange rates. Assuming that the first differences in the log of the two US dollar exchange rates are normally distributed, i.e., $\Delta\ln(A^*_{USD}) = A_{USD} \sim N\left(0, \sigma(A_{USD})^2\right)$ and $\Delta\ln(B^*_{USD}) = B_{USD} \sim N\left(0, \sigma(B_{USD})^2\right)$, it follows that

$$\Delta\ln(A^*_B) \sim N\left(0, \sigma(A_{USD})^2 + \sigma(B_{USD})^2 - 2\rho(A_{USD}, B_{USD})\,\sigma(A_{USD})\,\sigma(B_{USD})\right),$$

where $\rho(A_{USD}, B_{USD})$ is the correlation between the log returns of the two US dollar exchange rates. The variance of the log return of the cross-rate is a function of the variances of the log returns of the two underlying US dollar exchange rates and their correlation. Knowledge of the variances of the log returns of the three exchange rates in a currency trio therefore implies knowledge of the correlation between the two US dollar exchange rates.

Given that options on the three exchange rates in a currency trio are traded, one can infer the implied correlation between the two US dollar exchange rates by a two-step procedure. In the first step, the prices of the options on the three exchange rates are used to extract the implied volatilities of the three exchange rates.[17] In the second step, these implied volatilities are plugged into the equation for the variance of the log return of the cross-rate and the formula is solved for the correlation between the two US dollar exchange rates:

$$\rho(A_{USD},B_{USD})_{Imp(m),t} = \frac{\sigma(A_{USD})^2_{Imp(m),t} + \sigma(B_{USD})^2_{Imp(m),t} - \sigma(A_B)^2_{Imp(m),t}}{2\,\sigma(A_{USD})_{Imp(m),t}\,\sigma(B_{USD})_{Imp(m),t}},$$

where the subscript "$Imp(m)$, t" indicates that the values are based on the prices of options with m trading days to maturity in time t.

Note that any of the three currencies could serve as a base currency. With currency A as the base currency (in which case the exchange rate B_{USD} becomes the "cross-rate"), we obtain the following expression for the implied correlation between USD_A and B_A is:

$$\rho(USD_A,B_A)_{Imp(m),t} = \frac{\sigma(A_{USD})^2_{Imp(m),t} + \sigma(A_B)^2_{Imp(m),t} - \sigma(B_{USD})^2_{Imp(m),t}}{2\,\sigma(A_{USD})_{Imp(m),t}\,\sigma(A_B)_{Imp(m),t}}.$$

With currency B as the base currency (and A_{USD} the "cross-rate"), the implied correlation between USD_B and A_B is:

$$\rho(USD_B,A_B)_{Imp(m),t} = \frac{\sigma(B_{USD})^2_{Imp(m),t} + \sigma(A_B)^2_{Imp(m),t} - \sigma(A_{USD})^2_{Imp(m),t}}{2\,\sigma(B_{USD})_{Imp(m),t}\,\sigma(A_B)_{Imp(m),t}}.$$

Two points are worth mentioning here. First, the three implied correlations in a currency trio are not independent. This, in turn, implies some restrictions on the set of possible values that the three correlations in a currency trio can take. Second, the relationship between the volatilities and the correlations in a currency trio has an appealing geometric interpretation. Both points will be further discussed in the appendix.

The options data used in the remainder of the paper was provided by a prominent bank

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[17] The implied volatilities are usually expressed as annualized standard deviations of the log returns.

dealing in the OTC market for foreign exchange options. The options data consist of daily one, three, six and twelve month implied volatilities for the three currency pairs in the currency trio USD/DEM/JPY from October 2, 1990 through April 2, 1997 (1679 observations), and daily one, three, six and twelve month implied volatilities for the three currency pairs in the currency trio USD/DEM/CHF from September 13, 1993 through April 2, 1997 (910 observations). The comparison of the forecasting performance is conducted for horizons of one and three months, giving us twelve correlations to examine (two currency trios with three correlations each for two forecast horizons).

In the OTC market for foreign exchange options, prices are quoted in terms of implied volatility. Specifically, our study is based on quoted implied volatilities for at-the-money forward straddles. An at-the-money forward straddle is combination of a European-style call option and a European-style put option with the forward rate set as the strike price. Although prices in the OTC market for foreign exchange options are quoted in terms of implied volatility, this does not mean that these volatilities are not subject to misspecification problems. The fact that prices are quoted in implied volatilities only means that market participants have agreed to express the transaction prices of foreign exchange options in terms of the one unobservable input variable in the Garman -Kohlhagen (1983) formula. When volatility is time-varying, recovering implied volatilities from a constant volatility model such as the Garman-Kohlhagen model leads to a specification error. Since the pricing impact of stochastic volatility is very small for options that last less than one year (see Hull and White [1987]), we do not correct the implied volatilities for the presence of this misspecification error.

Figure 1A depict the movement of the term structure of the implied correlations extracted from the options data on the USD/DEM/JPY currency trio; figure 1B shows the movement of the term structure of the implied correlations for the USD/DEM/CHF trio. Tables 2A and 2B present the corresponding summary statistics. A striking feature of the data is that the two currency trios have markedly different correlation structures. The three implied correlations in the USD/DEM/JPY trio differ less in their means and standard deviations than the three implied correlations in the USD/DEM/CHF trio. Specifically, the implied correlation between the DEM and the CHF measured in USD has a much higher mean and a much lower standard deviation than the other five

correlations analyzed in this paper.[18] Another striking feature of the data is the fact that the implied correlations become less variable the longer the maturity of the underlying option contracts, as is evident from both the standard deviations and the ranges (between the maximum and the minimum values). This pattern is a consequence of the well-documented property of implied volatilities in the foreign exchange options market to exhibit mean-reversion; see Campa and Chang (1995), as well as Zhu and Avellaneda (1997).

b. Realized Correlation

The realized correlation between the two exchange rate series X_t and Y_t from time t over the next m trading days is defined as follows:

$$\rho(X,Y)_{m,t} = \frac{\sum_{i=1}^{m}\left([x_{t+i} - \bar{x}_{t+1,t+m}]\ [y_{t+i} - \bar{y}_{t+1,t+m}]\right)}{\sqrt{\sum_{i=1}^{m}\left(x_{t+i} - \bar{x}_{t+1,t+m}\right)^2}\ \sqrt{\sum_{i=1}^{m}\left(y_{t+i} - \bar{y}_{t+1,t+m}\right)^2}},$$

where $x_t = \ln(X_t / X_{t-1})$, $y_t = \ln(Y_t / Y_{t-1})$, and

$$\bar{x}_{t+1,t+m} = \frac{1}{m}\sum_{i=1}^{m} x_{t+i}, \text{ and } \bar{y}_{t+1,t+m} = \frac{1}{m}\sum_{i=1}^{m} y_{t+i}.$$

In order to minimize measurement error, we take account of the effective number of remaining trading days of the option contract used to extract the corresponding implied correlation. In the OTC market for foreign exchange options, the maturity of a contract is defined by calendar time. Specifically, a n-month contract started on the date "*X/Y/Z*" expires on the date "*X+n/Y/Z*"[19], if this day is a weekday. If the day "*X+n/Y/Z*" falls on a weekend or a holiday, the contract expires on the next workday. Hence, its time to maturity is variable, depending on the calendar. For our data set, the effective number of trading days for the one-month horizon ranges from 18 (minimum) to 23 (maximum) with a mean of 21.9 and a standard deviation of 1.0. For the three-month horizon, the effective number of trading days lies between 59 (minimum) and 66 (maximum) with a mean of 64.7

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[18] Note that Switzerland is not a member of the European Monetary System and that the Swiss franc is not linked to the German mark.

[19] Or "*V/Y/W*" if the contract does not expire in the year it was started.

and a standard deviation of 1.4.

The spot exchange rate data used to calculate the realized correlations (and the alternative correlation forecasts) is from the Swiss National Bank and consists of daily spot exchange rates from January 3, 1980 through July 2, 1997.

c. Simple Correlation Forecasts

The first group of correlation forecasts are based on rolling averages of products of past exchange rate log returns. We consider two approaches that differ only in the way the products of past log returns are weighted. Specifically, historical correlation uses equal weights, and exponentially weighted moving average (EWMA) correlation is based on weights that decline exponentially. Both methods assume that correlation forecasts are independent of the forecast horizon; i.e.

$$\rho(X,Y)_{m_1,t} = \rho(X,Y)_{m_2,t},$$

for any forecast horizon m_1 and m_2. Hence, the simple forecasts do not exhibit mean reversion in correlations.

Historical Correlation

The historical correlation forecast at time t for any forecast horizon is defined as the realized correlation over a fixed number of trading days prior to time t, i.e.,

$$\rho(X,Y)_{Hist(n),t} = \frac{\sum_{i=0}^{n-1}([x_{t-i} - \bar{x}_{t,t-n-1}]\,[y_{t-i} - \bar{y}_{t,t-n-1}])}{\sqrt{\sum_{i=0}^{n-1}(x_{t-i} - \bar{x}_{t,t-n-1})^2}\sqrt{\sum_{i=0}^{n-1}(y_{t-i} - \bar{y}_{t,t-n-1})^2}},$$

where n denotes the number of trading days used to calculate the correlation forecast (the "observation period"). Note that all n observations within the observation period are given equal weight, and all observations older than n days are given zero weight. This method is also often referred to as the simple moving average method or the equally weighted moving average method.

Since the approach gives no guidance about how to choose the length of the observation period, we examine the performance of historical correlation forecasts based on 20 days of historical data (denoted as Hist[20 days]), 60 days of historical data (Hist[60 days]), and 120 days of historical data (Hist[120 days]).

Exponentially Weighted Moving Average Correlation

The EWMA correlation forecast at time t for any forecast horizon is defined as

$$\rho(X,Y)_{EWMA(\lambda,k),t} = \frac{\sum_{i=0}^{k} \lambda^i \, x_{t-i} \, y_{t-i}}{\sqrt{\sum_{i=0}^{k} \lambda^i \, x_{t-i}^2} \sqrt{\sum_{i=0}^{k} \lambda^i \, y_{t-i}^2}},$$

where λ is the decay factor ($0 < \lambda < 1$) and k is the number of past historical observations used in the calculation. The EWMA approach, well known due to its use by J.P. Morgan's RiskMetrics™ system for forecasting variances and covariances (see J.P. Morgan [1996, p. 83]), can be seen as a constrained version of a GARCH(1,1) model (see Boudoukh, Richardson, and Whitelaw [1997]). The approach offers two advantages over the equally weighted moving average approach. First, by giving recent data more weight, the forecasts react faster to short-term movements in variances and covariances. Second, by exponentially smoothing out the effect of a shock, EWMA forecasts do not exhibit the abrupt changes that are common to the equally weighted historical forecasts once the shock falls out of the observation period. The decay factor determines the relative weights attached to the observations and the effective amount of data used. The lower the decay factor, the faster the decay in the influence of a given observation. Following Hendricks (1996), we consider three different decay factors: $\lambda = 0.94$, $\lambda = 0.97$, and $\lambda = 0.99$. We arbitrarily set $k = 1250$, such that the difference

$$\sum_{i=0}^{\infty} \lambda^i - \sum_{i=0}^{k} \lambda^i$$

is negligible in all three cases. Since we have $k = 1250$ in all three cases, the three EWMA correlation forecasts are simply denoted as EWMA(0.94), EWMA(0.97), and EWMA(0.99).

d. Correlation Forecasts Based on a Bivariate GARCH(1,1) Model

We now turn to a different modeling approach to forecast the correlation between two financial time series. In a bivariate GARCH model, correlation is specified as a stochastic process that evolves over time, and the specifications frequently used are generalizations of the methods used for the univariate GARCH models.[20] The basic structure of the bivariate GARCH model for ε_t, the vector of innovations of the two exchange rate log return series, is that the components of the conditional variance-covariance matrix H_t vary through time as functions of the squared observed bivariate innovations and past values of H_t. Specifically,

$$\mathrm{H}_t = \begin{bmatrix} h_{xx,t} & h_{xy,t} \\ h_{xy,t} & h_{yy,t} \end{bmatrix},$$

where $h_{xx,t}$ and $h_{yy,t}$ are the variances of the two series and $h_{xy,t}$ is their covariance.

A common specification of the bivariate Gaussian GARCH(p,q) process is that $\varepsilon_t \mid \Omega_{t-1} \sim N(0, \mathrm{H}_t)$, where

$$\mathrm{vech}(\mathrm{H}_t) = W + \sum_{i=1}^{q} \mathrm{A}_i \,\mathrm{vech}\left(\varepsilon_{t-i}\,\varepsilon_{t-i}'\right) + \sum_{j=1}^{p} \mathrm{B}_j \,\mathrm{vech}(\mathrm{H}_{t-j}),$$

where vech$(\cdot)$ is the vector-half operator that converts ($N \times N$) matrices into ($N(N+1)/2 \times 1$) vectors of their lower triangular elements, W is an ($N(N+1)/2 \times 1$) parameter vector, and A_i and B_j are ($N(N+1)/2 \times N(N+1)/2$) parameter matrices. For the bivariate GARCH(1,1) case

$$\begin{bmatrix} h_{xx,t} \\ h_{xy,t} \\ h_{yy,t} \end{bmatrix} = \begin{bmatrix} \omega_{xx} \\ \omega_{xy} \\ \omega_{yy} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{x,t-1}^2 \\ \varepsilon_{x,t-1}\varepsilon_{y,t-1} \\ \varepsilon_{y,t-1}^2 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} h_{x,t-1} \\ h_{xy,t-1} \\ h_{y,t-1} \end{bmatrix}.$$

In practice, this model, which has 21 parameters, is generally too cumbersome for numerical maximization of the likelihood function. Thus, to enforce parametric parsimony, we follow the

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[20] For an overview of multivariate GARCH models, see Bollerslev, Engle, and Nelson (1994), or Campbell, Lo and MacKinlay (1997). Engle and Mezrich (1996) use multivariate GARCH models to explain the comovement between several exchange rates, between interest rates and stock indices, and between individual stock prices.

work of Bollerslev, Engle and Wooldridge (1988), who assume that the A and B matrices are diagonal to reduce the number of parameters to just 9. That is,

$$\begin{bmatrix} h_{xx,t} \\ h_{xy,t} \\ h_{yy,t} \end{bmatrix} = \begin{bmatrix} \omega_{xx} \\ \omega_{xy} \\ \omega_{yy} \end{bmatrix} + \begin{bmatrix} \alpha_{xx} & 0 & 0 \\ 0 & \alpha_{xy} & 0 \\ 0 & 0 & \alpha_{yy} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{x,t-1} \\ \varepsilon_{x,t-1}\varepsilon_{y,t-1} \\ \varepsilon^2_{y,t-1} \end{bmatrix} + \begin{bmatrix} \beta_{xx} & 0 & 0 \\ 0 & \beta_{xy} & 0 \\ 0 & 0 & \beta_{yy} \end{bmatrix} \begin{bmatrix} h_{xx,t-1} \\ h_{xy,t-1} \\ h_{yy,t-1} \end{bmatrix},$$

which implies for the covariance that

$$h_{xy,t} = \omega_{xy} + \alpha_{xy}\, \varepsilon_{x,t-1}\, \varepsilon_{y,t-1} + \beta_{xy}\, h_{xy,t-1}.$$

Since we want to forecast the correlations between all the exchange rate pairs in the two currency trios analyzed, we estimate the model six times. In order not to give the GARCH-based correlation forecasts the advantage of *ex post* parametrization, we estimate the bivariate GARCH models with data up to the day the forecasting exercise begins. That is, for the correlations in the USD/DEM/JPY currency trio, the data consist of daily spot exchange rates from January 3, 1980 through October 2, 1990 (2713 observations). For the correlations in the USD/DEM/CHF currency trio, the data consist of daily spot exchange rates from January 3, 1980 through September 13, 1993 (3483 observations).

Some of the studies on the characteristics of daily foreign exchange rate changes have found significant first order autocorrelations; e.g., Hsieh (1989). In order to capture the effects of possible first order autocorrelations, we first estimate specifications with an MA(1) term in the conditional mean equation; i.e.,

$$100\,[\log(S_t) - \log(S_{t-1})] = \mu + \theta\varepsilon_{t-1} + \varepsilon_t,$$

where S_t is the vector consisting of the two exchange rates that are analyzed. We then estimate specifications without an MA term, i.e.:

$$100\,[\log(S_t) - \log(S_{t-1})] = \mu + \varepsilon_t.$$

Using the likelihood-ratio test, we find that the specifications without MA term can not be rejected

against the specifications with MA term (see the LR_{MA} figures in table 3A and 3B). Therefore, we use the specification without the MA term.

Tables 3A and 3B summarize the results of the GARCH(1,1) model estimations. The models were estimated by maximum likelihood assuming conditional normality, with presample values set to their sample means. The tables report the parameter estimates, the standard errors, and the values of the log likelihood functions. Note that the estimates suggest considerable persistence, since $\alpha_{xy} + \beta_{xy}$ is estimated to be above 0.9 in all cases.

The correlation forecasts generated from this time series model are different from the forecasts generated by the simpler models in a fundamentally important way. The previous models assume that the daily variances and covariances of the log returns of the two exchange rates x_t and y_t are constant, and thus, a forecast of the m-day correlation is exactly equal to the past observed correlation. However, for the GARCH model, forecasts of H_t change daily. Specifically, the k-step-ahead forecast of h_{xy} at time t is made with the following equation:

$$\mathrm{E}_t[h_{xy,t+k}] = \begin{cases} \omega_{xy} + \alpha_{xy}\varepsilon_{xy,t} + \beta_{xy}h_{xy,t} & \text{if } k=1 \\ \omega_{xy}\sum_{s=0}^{k-1}(\alpha_{xy}+\beta_{xy})^s + (\alpha_{xy}+\beta_{xy})^{k-1}\mathrm{E}_t[h_{xy,t+1}] & \text{if } k>1. \end{cases}$$

Since the daily innovations are not serially correlated, the forecast at time t of the variance or covariance over the subsequent m-day period is equal to

$$\mathrm{E}_t[h_{xy,t(m)}] = \sum_{s=1}^{m}\mathrm{E}_t[h_{xy,t+s}].$$

The corresponding forecast for the correlation between x and y from time t to time $t+m$ is

$$\rho(X,Y)_{GARCH(m),t} = \frac{\sum_{i=1}^{m}\mathrm{E}_t[h_{xy,t+i}]}{\sqrt{\sum_{i=1}^{m}\mathrm{E}_t[h_{xx,t+i}]}\sqrt{\sum_{i=1}^{m}\mathrm{E}_t[h_{yy,t+i}]}}.$$

We use the parameter estimates of the six GARCH specifications to forecast the twelve correlations that we are interested in (six correlations over two forecast horizons). In order to minimize the measurement errors, we take account of the effective number of trading days of the option contract that was used to extract the corresponding implied correlation when we calculate the GARCH-based

forecasts.

e. Typical Properties of the Different Forecasts

Figure 2 illustrates the various forecasts. The figures depict the realized correlation between the DEM and the JPY, as seen from the perspective of an USD investor and the corresponding values of the different forecasts. The forecasts shown are: implied correlation (Chart A), bivariate GARCH(1,1)-based correlation (Chart B), historical correlation based on 20 days (Chart C) and 120 days (Chart D) of historical data, and EWMA correlation with decay factors 0.94 (Chart E) and 0.99 (Chart F). Clearly, realized correlation is much more variable than both implied and GARCH based correlation. The other forecasts based on time-series show the typical properties documented in Hendricks (1996). The longer the observation period, the less variable the correlation forecast based on historical correlation. The higher the decay factor, the longer the effective observation period and, consequently, the less variable the EWMA correlation forecast.

IV. Evaluating the Predictive Accuracy of the Correlation Forecasts

The purpose of this section is to evaluate the predictive accuracy of the competing forecast methods presented above. We first describe the three criteria by which the forecasts are evaluated. We then report and comment on the results.

a. Forecast Evaluation Criteria

Analysis of Forecast Errors

The first set of evaluation tests is based on the analysis of the forecast errors. We evaluate the forecasting performance by calculating the mean forecast errors (MFE's) and the root mean square forecast errors (RMSFE's). We test whether the MFE's are statistically different from zero by regressing the time series of forecast errors on a constant. The statistical significance of the coefficient is evaluated using the Newey and West (1987) standard errors.[21] We then compare the RMSFE's by applying the method proposed in Diebold and Mariano (1995). Taking the RMSE loss

[21] Newey and West (1987) have proposed a covariance matrix that gives consistent estimates in the presence of both heteroskedasticity and autocorrelation.

function as the relevant loss function, we first generate the time series of the differences between the loss function values of each forecast and the best forecast available in RMSE terms, for a particular correlation. We then calculate the asymptotic Diebold-Mariano test statistics and test the null hypothesis that there is no difference in the forecasting accuracy of the competing forecasts.

Individual Predictability Regressions

The second set of evaluation criteria uses linear regressions to assess the performance of forecasts. The first group of tests in the regression based evaluation framework assesses the partial optimality of the different forecasts. Partial optimality refers to unforecastibility of forecast errors with respect to some subset of available information; see Brown and Maital (1981) as well as the discussion in Diebold and Lopez (1995). Partial optimality, for example, characterizes a situation in which a forecast is optimal with respect to the information used to construct it, but the information used was not all that was available to be used.

Following Mincer and Zarnowitz (1969), we test for the partial optimality of the different correlation forecasts by running regressions of the following type

$$\rho(\cdot)_t = \alpha + \beta\, \rho(\cdot)_{j,t} + \nu_t,$$

where $\rho(\cdot)_{j,t}$ is the forecast of the correlation at time t generated by method j. Partial optimality of a forecast corresponds to parameter estimates of α and β that are insignificantly different from zero and one, respectively. Deviation from those parameter values is evidence that the forecast does not use the information used to construct the forecasts in an optimal way.

Any partial optimal forecast should result in parameter estimates that are consistent with α=0 and β=1, independent of the information set the forecast is based on. The size of the information set, however, may have an influence on the goodness-of-fit. Consider two rationally formed forecasts based on the information sets Ω^{*}_{t-1} and Ω^{**}_{t-1}, with $\Omega^{*}_{t-1} \supset \Omega^{**}_{t-1}$. Running individual predictability regressions should in both cases result in parameter estimates consistent with α=0 and β=1. However, the forecast derived from the more inclusive Ω^{*}_{t-1} should generate a better goodness-of-fit than the forecast derived from the less inclusive information set Ω^{**}_{t}.

Encompassing Regressions

The second group of tests in the regression-based evaluation framework uses multiple regression analysis to assess the information content of different forecasts. The so-called encompassing tests enable one to determine whether a certain forecast incorporates (or encompasses) all the relevant information in competing forecasts; see Chong and Hendry (1986). To illustrate the idea, consider the case of two correlation forecasts, $\rho(\cdot)_{1,t}$ and $\rho(\cdot)_{2,t}$. If the regression

$$\rho(\cdot)_t = \alpha + \beta\, \rho(\cdot)_{1,t} + \gamma\, \rho(\cdot)_{2,t} + \nu_t,$$

results in parameter estimates (α, β, γ) that are not significantly different from (0, 1, 0), then forecast 1 is said to encompass forecast 2. Similarily, if the parameter estimates are not significantly different from (0, 0, 1); forecast 2 encompasses forecast 1. For any other values (α, β, γ), neither model encompasses the other, and both forecasts contain useful information.

In order to test for the information contribution of the various forecasts, we estimate encompassing regressions for every correlation analyzed. Since the correlation forecasts tend to be correlated, we do not include all eight forecasts as right-hand variables. Instead, we use the two sophisticated forecasts (implied and GARCH-based correlation) along with one historical correlation forecast and one EWMA correlation forecasts. Among the competing historical and EWMA correlations, we choose the variant with the maximal R^2 in the individual predictability regressions.

b. Results for the Correlations in the Currency Trio USD/DEM/JPY

Analysis of Forecast Errors

Tables 4A and 4B report the mean forecast errors (MFE's), the root mean square forecast errors (RMSFE's), and the Diebold-Mariano test statistics (D&M test) for the three correlations in the USD/DEM/JPY trio over the one-month and three-month horizon, respectively.

Regarding the MFE results, we find that the more sophisticated forecasts (i.e., implied and GARCH(1,1)-based correlation) tend to have larger forecast biases than the simple forecasts. Often, the MFE's of the sophisticated forecasts are significantly different from zero. The simple forecasts based on a short effective observation period (i.e., Hist[20 days] and EWMA[0.94]) tend to have forecast errors that are close to zero on average, which is not surprising since these simple forecasts essentially approximate the unconditional variance of the underlying process.

The reported RMSFE's and the results for D&M test indicate that the good performance of the simple forecasts in terms of MFE's does not imply that these forecasts necessarily have a close relationship to realized correlation. In fact, the simple methods tend to have higher RMSFE's than the sophisticated forecasts. The exception is the EWMA(0.97) correlation, which has the lowest RMSFE in three of the six cases and only in one case is its forecast accuracy significantly different from the accuracy of the best forecast. The RMSFE's of the implied correlation are either lower, (often significantly so) or not statistically different from the RMSFE's of the simple forecasts. The same is true for the GARCH-based correlation forecast, except for one case (the correlation between the USD and the JPY in units of DEM).

It is also interesting to note that the RMSFE's tend to be lower for the three-month correlations than for the one-month correlations. This suggest that correlation forecasts tend to be more accurate for the three-month forecast horizon than the one-month forecast horizon, possibly reflecting the mean reversion property of the correlations.

Individual Predictability Regressions

The results for the individual predictability regressions for the correlations in the currency trio USD/DEM/JPY are reported in tables 5A and 5B. Certain consistent patterns emerge from the regression results.

First, the simple forecasting methods (i.e., historical correlation and EWMA correlation) consistently violate both conditions for a partially optimal forecast (i.e., a=0, b=1). In thirty-four of the thirty-six cases, we can reject the hypothesis that the intercept coefficient a is zero. The slope coefficients b, although always positive and statistically different from zero, are generally significantly different from one. Only in one case, the hypothesis b=1 is not rejected.

Second, the sophisticated forecasting methods (i.e., implied correlation and correlation forecasts based on a bivariate GARCH(1,1) model) generally outperform the simple methods. In eight out of sixteen cases, the hypothesis a=0 is not rejected, and only for the correlation between DEM and JPY measured in USD is the hypothesis b=1 rejected. Two sets of implied correlation forecasts and two sets of GARCH-based forecasts even pass the joint efficiency test (i.e., a=0 and b=1). In terms of goodness-of-fit, however, the sophisticated forecasting methods do not appear to outperform the simple forecasts, providing the highest adjusted R^2 in only one out of six cases.

Third, the goodness-of-fit figures for the different forecasts tend to be relatively high. Specifically, the R^2's for the regressions with implied correlation go up to 0.39, a value that is much higher than any R^2 reported in the literature on the information content of implied volatilities for foreign exchange rate volatility.[22] This result is consistent with the hypothesis that at least for a subset of financial variables, correlations, since they are more stable than volatilities, tend to be more predictable than volatilities.

Encompassing Regressions

Tables 6A and 6B report the results for the encompassing regressions where the realized correlations in the currency trio USD/DEM/JPY are regressed on a constant and different sets of forecasts. These forecasts are: implied correlation, the historical correlation with the highest R^2 in the individual predictability regressions, the EWMA correlation with the highest R^2 in the individual predictability regressions, and the bivariate GARCH(1,1)-based correlation. The three main results can be summarized as follows.

First, implied correlations contain information not present in the other forecasts. The coefficients on implied correlation are always significantly positive. The Wald test rejects a zero coefficient on implied correlation in four out of six cases.

Second, implied correlations do not fully incorporate all the information extractable from historical prices. The Wald test never rejects the hypothesis that the regression coefficients on all the other forecasts is zero. This suggests that correlation forecasts based on time-series data provide significant, additional information that is useful in forecasting correlation. The fact that implied correlation does not fully incorporate all the information in the time-series data is consistent with the findings of Bodurtha and Shen (1995). It does, however, appear to be inconsistent with the results of Campa and Chang (1997), who find that time-series based correlation forecasts generally contribute no incremental information to implied correlation in forecasting the correlation between the DEM and the JPY from the USD perspective. Given that we use data from the same market (the market for OTC foreign exchange options), this result seems surprising.

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[22] Jorion (1995) reports a R^2 of 0.16 for the regression of realized volatility on a constant and implied volatility (for the USD/DEM exchange rate). Galati and Tsatsaronis (1995) report a R^2 of 0.3 in the case of the USD/Pound-Sterling volatility, and Guo (1996) reports a R^2 of 0.10 for the USD/JPY volatility.

We currently attribute these differences to three factors. First, it seems (from visual inspection of the figure 1a in Campa and Chang's paper) that implied correlation has performed especially well in the period from the summer of 1989 to the fall of 1990. This period is included in Campa and Chang's data set, but not in ours. Second, it seems to be the case that our correlation forecasts from the bivariate GARCH(1,1) specifications appear to perform better than their GARCH-based correlation forecasts. Their parameter estimates are based on a rolling GARCH model, while our estimates are based on GARCH models that are estimated over a fixed time period. Nevertheless, given that their GARCH parameter estimates are similar to ours, the difference in performance seems very large. Third, we compare implied correlation against a larger set of alternatives. Since Campa and Chang's set of alternatives is only a subset of our set of alternatives, chances are higher in our case that one of the alternatives contributes incremental information useful in predicting realized correlation, possibly only by chance and not in a consistent way.

The third main result from the encompassing regressions is that the GARCH-based correlation forecasts incorporate the information in the historical data best. In all six cases, the regression coefficient on the GARCH-based correlation forecast is significant, and the Wald test always rejects a zero coefficient on GARCH-based correlation. The other forecasts based on historical data do not consistently provide significant, additional information useful in forecasting. In fact, in only three of the six cases is a coefficient on one of these forecasts significant; specifically Hist(60 days) for $\rho(USD_{JPY}, DEM_{JPY})$ at the one-month and three-month horizons, and EWMA(0.97) for $\rho(DEM_{USD}, JPY_{USD})$.

c. Results for the Correlations in the Currency Trio USD/DEM/CHF

Analysis of Forecast Errors

In the USD/DEM/CHF case, the forecasting performance of the different forecasts does strongly depend on the correlation under consideration. In two of the three cases, the option-implied correlation forecasts perform poorly. In the third case (the correlation between DEM and CHF measured in USD), implied correlation performs better. However, since this correlation has an extremely low variability, more predictability does not imply substantial economic benefits.

Tables 4C and 4D report the mean forecast errors (MFE's), the root mean square forecast errors (RMSFE's), and the Diebold-Mariano test statistics (D&M test) for the three correlations in

the USD/DEM/CHF trio over the one-month and the three-month horizon, respectively.

The tables indicate that GARCH-based correlation performs best in forecasting the variation of the correlation between DEM and CHF measured in USD. It is unbiased and has the lowest RMSFE for both the one-month and the three-month horizon. However, differences in the forecast accuracy are not always significant. The accuracy of the GARCH-based correlation forecasts are not statistically different from the EWMA(0.94) forecast, the EWMA(0.97) forecast, and the implied correlation forecasts.

The results for the two other correlations (i.e., $\rho[USD_{DEM}, CHF_{DEM}]$ and $\rho[USD_{CHF}, DEM_{CHF}]$) are radically different from the results analyzed so far. The sophisticated forecasts, especially implied correlations, do not perform well in forecasting these two correlations. A first indication of the poor performance of the sophisticated forecasts is given by the MFE's, which are not only significant, but are quite high, reaching almost 0.3 in absolute size. In terms of RMSFE's, the implied correlation forecast generate RMSFE's that are always significantly higher than that of the best forecasts. GARCH-based correlation forecasts perform somewhat better in that their RMSFE's are significantly different from the one of the best forecast in only one case.

Note that it is not a coincidence that the implied correlation forecasts perform poorly for both $\rho(USD_{DEM}, CHF_{DEM})$ and $\rho(USD_{CHF}, DEM_{CHF})$. Since the three correlations in a currency trio are not independent and since the third correlation in the USD/DEM/CHF currency trio is very stable, the forecast errors of the two correlations are positively correlated.

Individual Predictability Regressions

The results for the individual predictability regressions, reported in tables 5C and 5D, confirm that, in the USD/DEM/CHF trio, the forecasting performance of the different forecasts does strongly depend on the correlation considered.

For $\rho(DEM_{USD}, DEM_{USD})$, we find that none of the conditions for partial optimal forecasts are met. For all the forecasts, we always reject both a=0 and b=1. The goodness-of-fit figures are generally high, implying that a large portion of the (relatively small) variability can be explained by the forecasts.

The results for the two other correlations suggest that their variation can not be predicted to the same extent as the variations of the other correlations analyzed in this paper. Except for

GARCH-based correlation, the R^2's are substantially lower than in the case of the other correlations.

Encompassing Regressions

The results for the encompassing regressions reported in tables 6C and 6D indicate that no forecast method consistently provides useful information in forecasting correlations in the currency trio USD/DEM/CHF. For $\rho(DEM_{USD}, DEM_{USD})$, the coefficients on implied correlation, on EWMA(0.94) (for the one-month horizon), and on GARCH-based correlation (for the three-month horizon) are significant. For $\rho(USD_{DEM}, CHF_{DEM})$, the parameter estimates are generally not significant, with the only exception being the EWMA(0.97) correlation forecast for the three-month horizon. For $\rho(USD_{CHF}, DEM_{CHF})$, GARCH-based correlation dominates the other forecasts.

There are two possible reasons for the poor forecasting performance of implied correlations in the currency trio USD/DEM/CHF. First, it could be that the Garman-Kohlhagen (1983) option pricing formula that was implicitly used to extract the implied volatilities is incorrect. Second, it could be that the market for the options we used to extract the implied correlations is not efficient, possibly because the trading volume is not large enough to assure proper pricing in the market.

V. Conclusion

This paper addresses the question of how well implied correlation performs in forecasting subsequent realized correlation. Using daily data on the implied volatilities of the exchange rates in the two currency trios USD/DEM/JPY and USD/DEM/CHF, respectively, we infer all the implied correlations extractable from these two currency trios. We then compare the forecasting performance of implied correlation against the a number of alternative forecasts based on time-series data. These alternatives are: historical correlations based on approximately one month, three months, and six months of historical data, exponentially weighted moving average correlations for decay factors of 0.99, 0.97, and 0.94, and correlation forecasts generated by a bivariate GARCH(1,1) model.

For the correlations in the USD/DEM/JPY currency trio, we find that implied correlation and GARCH-based correlation outperform the other forecasts in terms of accuracy. Regarding the information content of the different forecasts, our results can be summarized as follows. First, implied correlations contain useful information that is not present in the forecasts based on time

series. Second, implied correlations do not fully incorporate all the information in the price history that is useful in forecasting. Third, the relevant information in the price history not captured by implied correlation is most effectively summarized by the GARCH-based correlation forecasts. Fourth, a model that combines market information with time-series information within the GARCH framework, as in Day and Lewis (1992) for stocks, Kroner, et al. (1995) for commodities, or Amin and Ng (1997) for interest rates, may provide improved correlation forecasts.

For the correlations in the USD/DEM/CHF currency trio, the economic benefits of using option-implied correlation forecasts are not substantial. For two of the three correlations, implied correlations are not as accurate as the forecasts based on time-series and they provide no additional information to the information in the price history. For the third correlation, implied correlation contains unique information that is principally useful in forecasting. However, given the low level of variabillity of this correlation, the economic benefits of using an option-based forecast are small.

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Table 1
Relative Performance Instruments

Name	Description
Spread option	An option with a pay-off related to the spread between two assets, usually two interest rates.
Yield curve option	A spread option on different areas of the same yield curve.
Outperformance option	A call option with a pay-off related to the amount by which one of two variables outperforms the other. Also known as relative performance option.
Exchange option	An option giving the buyer the right to exchange one asset for another. Also known as Margrabe-option.
Share ratio contract	A contract paying out the ratio of an individual stock to a stock index. Share ratio contracts are traded at the Australian Stock Exchange since July 14, 1994.
Better-of-x-Assets	An option on the best (call) or worst (put) return of x ($x \geq 2$) assets. Also known as optionalternative option.

Table 2A
Descriptive Statistics: Implied Correlations in the Currency Trio USD/DEM/JPY

Means, standard deviations, coefficients of skewness and kurtosis and maximum and minimum for the implied correlations calculated from the implied volatilities of at-the-money, foreign exchange forward straddles. The observation period is October 2, 1990 through April 2,1997 (1679 observations).

	Mean	Std.Dev.	Skewness	Kurtosis	Maximum	Minimum
$\rho(DEM_{USD}, JPY_{USD})_{1\text{-}Month}$	0.5691	0.1094	-0.3726	3.0431	0.8198	0.0471
$\rho(DEM_{USD}, JPY_{USD})_{3\text{-}Months}$	0.5687	0.0858	-0.1289	2.4269	0.7737	0.3021
$\rho(DEM_{USD}, JPY_{USD})_{6\text{-}Months}$	0.5712	0.0741	-0.1392	2.5519	0.7471	0.2702
$\rho(DEM_{USD}, JPY_{USD})_{12\text{-}Months}$	0.5747	0.0646	-0.1415	2.6448	0.7234	0.3130
$\rho(USD_{DEM}, JPY_{DEM})_{1\text{-}Month}$	0.5243	0.1688	0.0036	2.4829	0.8952	-0.0480
$\rho(USD_{DEM}, JPY_{DEM})_{3\text{-}Months}$	0.5161	0.1513	-0.0521	2.3061	0.8209	0.0303
$\rho(USD_{DEM}, JPY_{DEM})_{6\text{-}Months}$	0.5085	0.1459	-0.1450	2.1083	0.8040	0.0987
$\rho(USD_{DEM}, JPY_{DEM})_{12\text{-}Months}$	0.5023	0.1416	-0.2328	1.9296	0.7682	0.1477
$\rho(USD_{JPY}, DEM_{JPY})_{1\text{-}Month}$	0.3755	0.1970	-0.6063	2.8926	0.7817	-0.4140
$\rho(USD_{JPY}, DEM_{JPY})_{3\text{-}Months}$	0.3930	0.1570	-0.6988	2.7740	0.6911	-0.1470
$\rho(USD_{JPY}, DEM_{JPY})_{6\text{-}Months}$	0.4012	0.1401	-0.6610	2.5252	0.6423	-0.1342
$\rho(USD_{JPY}, DEM_{JPY})_{12\text{-}Months}$	0.4059	0.1300	-0.5667	2.3911	0.6384	-0.0983

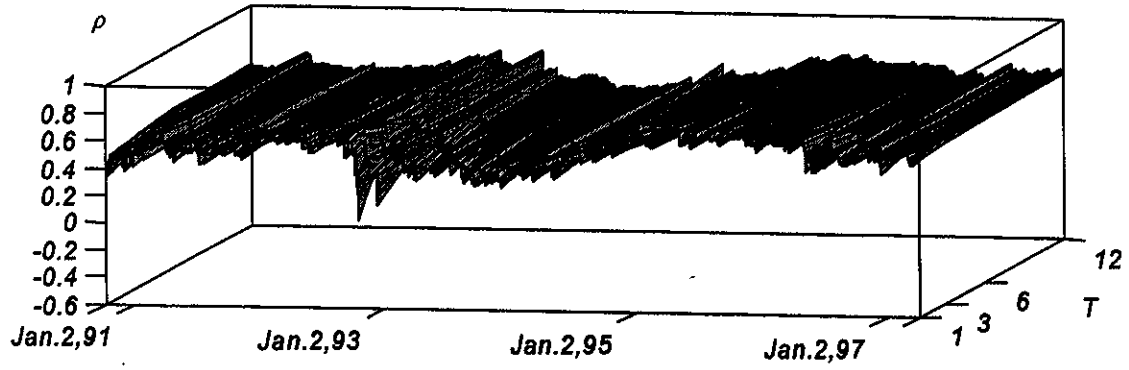
Table 2B
Descriptive Statistics: Implied Correlations in the Currency-Trio USD/DEM/CHF

Means, standard deviations, coefficients of skewness and kurtosis and maximum and minimum for the implied correlations calculated from the implied volatilities of at-the-money, foreign exchange forward straddles. The observation period is September 13, 1993 through April 2, 1997 (910 observations).

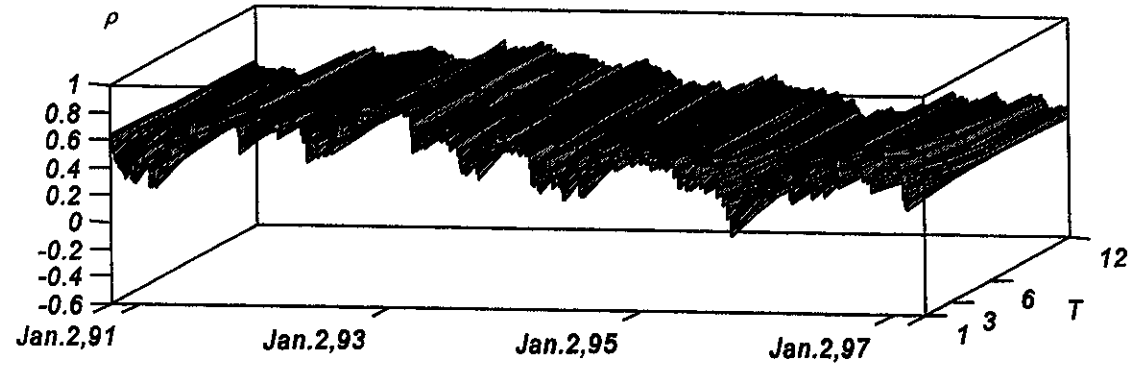
	Mean	Std.Dev.	Skewness	Kurtosis	Maximum	Minimum
$\rho(DEM_{USD}, CHF_{USD})_{1\text{-}Month}$	0.9134	0.0347	-0.1551	2.1113	0.9815	0.8310
$\rho(DEM_{USD}, CHF_{USD})_{3\text{-}Months}$	0.9169	0.0261	-0.0459	2.2126	0.9749	0.8506
$\rho(DEM_{USD}, CHF_{USD})_{6\text{-}Months}$	0.9182	0.0214	-0.0286	2.3784	0.9654	0.8557
$\rho(DEM_{USD}, CHF_{USD})_{12\text{-}Months}$	0.9188	0.0185	-0.1852	2.5328	0.9594	0.8590
$\rho(USD_{DEM}, CHF_{DEM})_{1\text{-}Month}$	-0.0216	0.1372	-0.3313	3.2780	0.4952	-0.4993
$\rho(USD_{DEM}, CHF_{DEM})_{3\text{-}Months}$	-0.0070	0.1175	-0.1516	2.6989	0.4166	-0.3968
$\rho(USD_{DEM}, CHF_{DEM})_{6\text{-}Months}$	0.0016	0.1076	-0.0398	2.0818	0.3125	-0.3074
$\rho(USD_{DEM}, CHF_{DEM})_{12\text{-}Months}$	0.0105	0.1008	-0.0357	1.9763	0.3166	-0.2226
$\rho(USD_{CHF}, DEM_{CHF})_{1\text{-}Month}$	0.4156	0.1133	0.0286	2.4365	0.6721	-0.1079
$\rho(USD_{CHF}, DEM_{CHF})_{3\text{-}Months}$	0.3979	0.1079	0.3202	2.1620	0.6301	-0.0033
$\rho(USD_{CHF}, DEM_{CHF})_{6\text{-}Months}$	0.3886	0.1053	0.4598	2.1295	0.6225	0.1047
$\rho(USD_{CHF}, DEM_{CHF})_{12\text{-}Months}$	0.3798	0.1047	0.5848	2.3047	0.6106	0.1138

Figure 1A
Term Structure of Implied Correlations in the Currency Trio USD/DEM/JPY

A. $\rho(DEM_{USD}, JPY_{USD})$



B. $\rho(USD_{DEM}, JPY_{DEM})$



C. $\rho(USD_{JPY}, DEM_{JPY})$

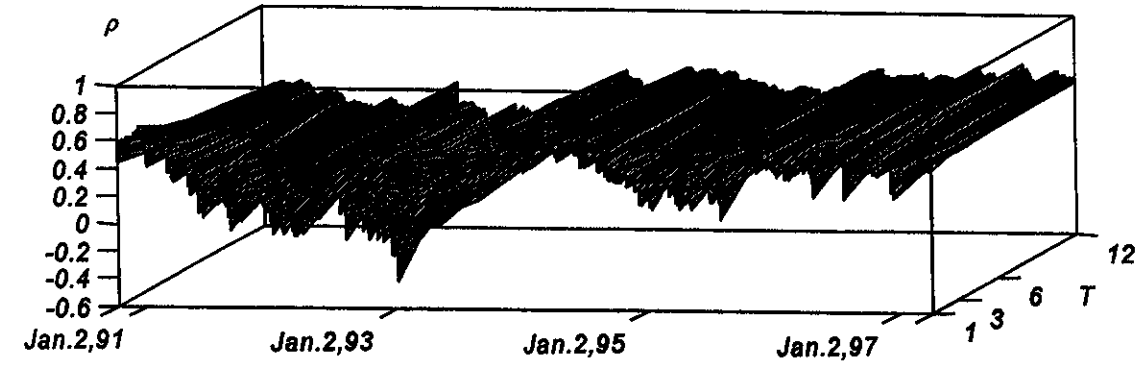
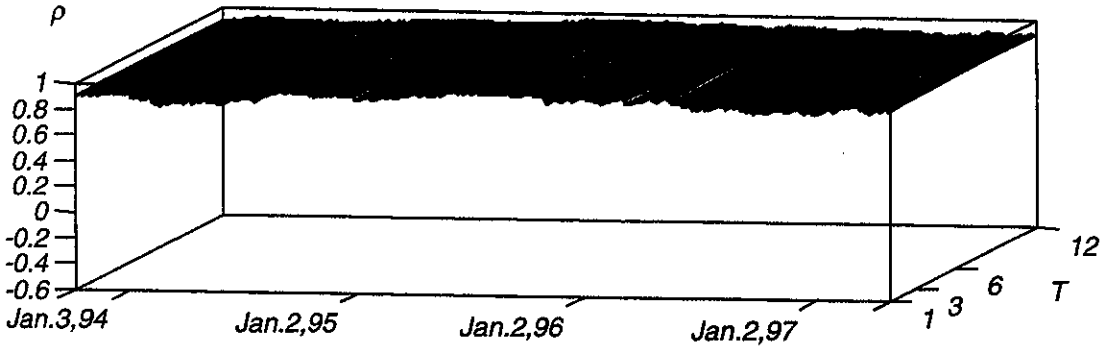
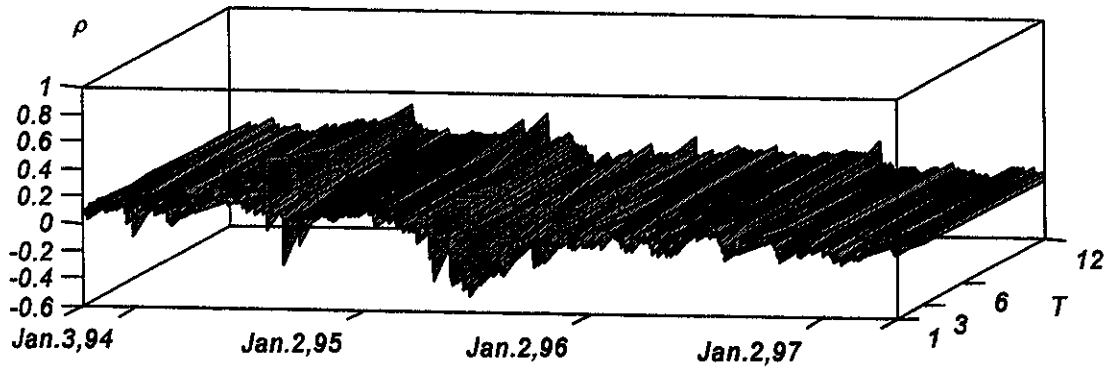


Figure 1B
Term Structure of Implied Correlations in the Currency Trio USD/DEM/JPY

A. $\rho(DEM_{USD}, CHF_{USD})$



B. $\rho(USD_{DEM}, CHF_{DEM})$



C. $\rho(USD_{CHF}, DEM_{CHF})$

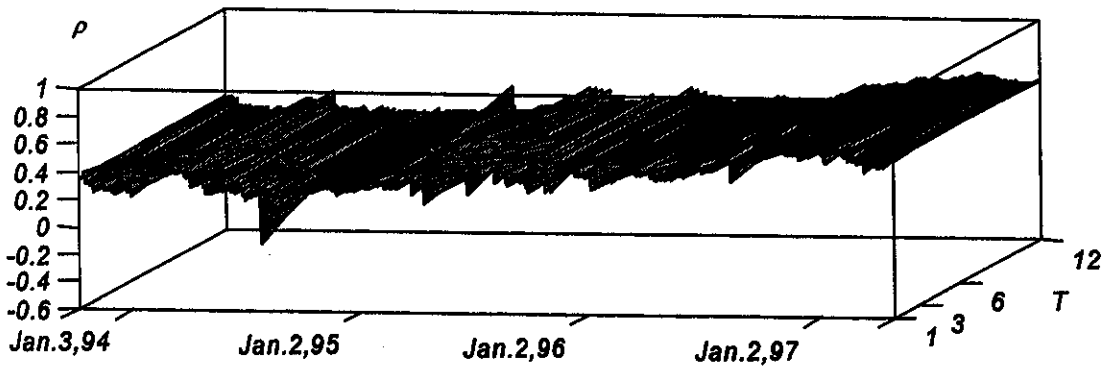


Table 3A
Bivariate GARCH(1,1) Parameter Estimates for the Exchange Rate Pairs in the Currency Trio USD/DEM/JPY

This table reports the estimation results of the following bivariate GARCH(1,1) model for the three exchange rate pairs in the currency trio USD/DEM/JPY:

$$S_t = [X_t, Y_t]$$
$$100\left[\log(S_t) - \log(S_{t-1})\right] = \mu + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, H_t)$$
$$h_{ij,t} = \omega_{ij} + \alpha_{ij}\,\varepsilon_{i,t-1}\,\varepsilon_{j,t-1} + \beta_{ij}\,h_{ij,t-1}$$

where X and Y are the two exchange rates analyzed. The observation period is January 3, 1980 through October 2, 1990 (2714 observations). LR_{MA} is the likelihood ratio test for the null of $\theta = 0$ in the specification where the conditional mean equation has the following structure:

$$100\left[\log(S_t) - \log(S_{t-1})\right] = \mu + \theta\varepsilon_{t-1} + \varepsilon_t.$$

	$X = DEM_{USD}$ & $Y = JPY_{USD}$		$X = USD_{DEM}$ & $Y = JPY_{DEM}$		$X = USD_{JPY}$ & $Y = DEM_{JPY}$	
	Parameter Estimates	Standard Errors	Parameter Estimates	Standard Errors	Parameter Estimates	Standard Errors
μ_1	-0.0065	0.0121	0.0083	0.0125	-0.0155	0.0115
μ_2	0.0118	0.0113	0.0208**	0.0082	-0.0186**	0.0075
ω_{11}	0.0185**	0.0027	0.0217**	0.0041	0.0276**	0.0033
ω_{12}	0.0149**	0.0019	0.0043**	0.0013	0.0048**	0.0007
ω_{22}	0.0244**	0.0027	0.0095**	0.0019	0.0074**	0.0013
α_{11}	0.1353**	0.0100	0.1184**	0.0107	0.1276**	0.0092
α_{22}	0.1180**	0.0087	0.1202**	0.0100	0.1320**	0.0085
α_{33}	0.1196**	0.0098	0.1628**	0.0157	0.1512**	0.0117
β_{11}	0.8441**	0.0105	0.8470**	0.0125	0.8285**	0.0122
β_{22}	0.8551**	0.0094	0.8512**	0.0106	0.8421**	0.0099
β_{33}	0.8397**	0.0113	0.8159**	0.0157	0.8376**	0.0122
Log Likelihood	-436.192		-436.481		-437.386	
LR_{MA}	0.018		0.326		0.576	

** Indicates statistical significance at the 1 percent level.
* Indicates statistical significance at the 5 percent level.

Table 3B
Bivariate GARCH(1,1) Parameter Estimates
for the Exchange Rate Pairs in the Currency Trio USD/DEM/CHF

This table reports the estimation results of the following bivariate GARCH(1,1) model for the three exchange rate pairs in the currency trio USD/DEM/CHF:

$$S_t = [X_t, Y_t]$$
$$100\left[\log(S_t) - \log(S_{t-1})\right] = \mu + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, H_t)$$
$$h_{ij,t} = \omega_{ij} + \alpha_{ij}\,\varepsilon_{i,t-1}\,\varepsilon_{j,t-1} + \beta_{ij}\,h_{ij,t-1}$$

where X and Y are the two exchange rates analyzed. The observation period is January 3, 1980 through April 2,1997 (3483 observations). LR_{MA} is the likelihood ratio test for the null of $\theta = 0$ in the specification where the conditional mean equation has the following structure:

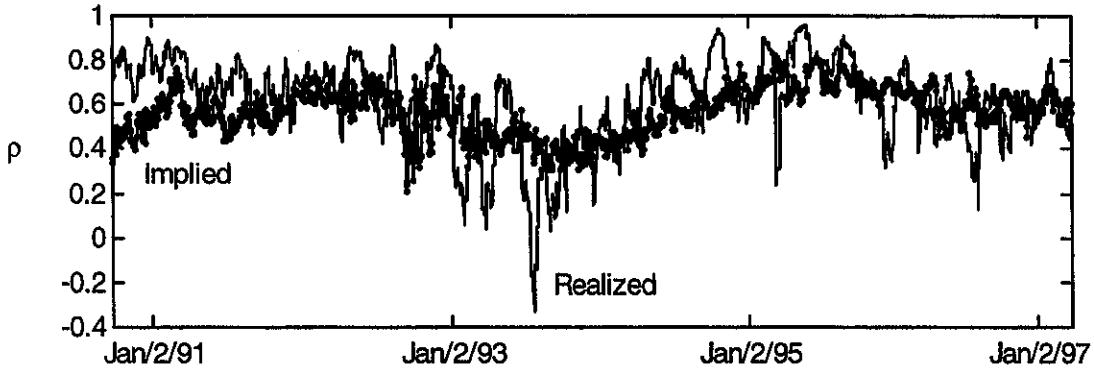
$$100\left[\log(S_t) - \log(S_{t-1})\right] = \mu + \theta\varepsilon_{t-1} + \varepsilon_t.$$

	$X = DEM_{USD}$ & $Y = CHF_{USD}$		$X = USD_{DEM}$ & $Y = CHF_{DEM}$		$X = USD_{CHF}$ & $Y = DEM_{CHF}$	
	Parameter Estimates	Standard Errors	Parameter Estimates	Standard Errors	Parameter Estimates	Standard Errors
μ_1	0.0003	0.0113	0.0034	0.0115	0.0049	0.0127
μ_2	-0.0025	0.0123	-0.0031	0.0045	0.0031	0.0046
ω_{11}	0.0163**	0.0027	0.0139**	0.0031	0.0171**	0.0040
ω_{12}	0.0153**	0.0025	-0.0004**	0.0002	0.0030**	0.0007
ω_{22}	0.0172**	0.0029	0.0023**	0.0006	0.0026**	0.0006
α_{11}	0.0874**	0.0066	0.1032**	0.0102	0.0965**	0.0098
α_{22}	0.0802**	0.0062	0.0584**	0.0092	0.0657**	0.0095
α_{33}	0.0765**	0.0062	0.0845**	0.0139	0.0957**	0.0127
β_{11}	0.8883**	0.0084	0.8799**	0.0106	0.8841**	0.0114
β_{22}	0.8967**	0.0079	0.9194**	0.0115	0.9050**	0.0138
β_{33}	0.9011**	0.0083	0.8923**	0.0188	0.8798**	0.0171
Log Likelihood	-417.859		-418.811		-418.171	
LR_{MA}	0.442		0.206		0.460	

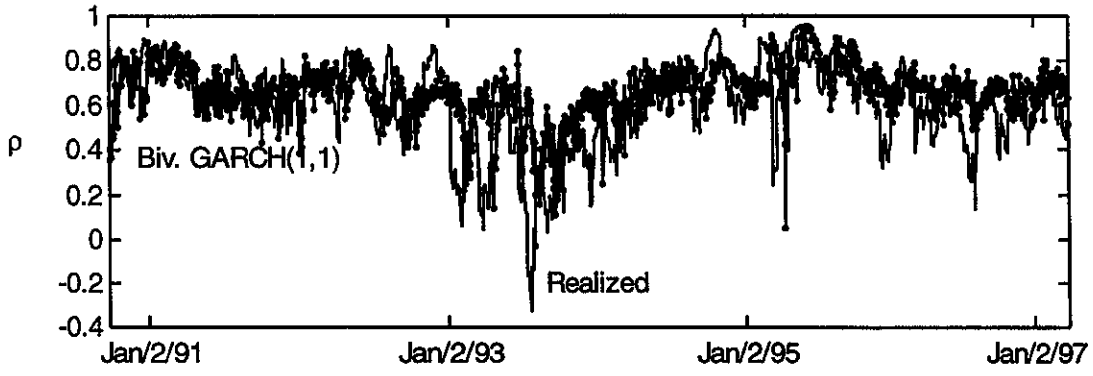
** Indicates statistical significance at the 1 percent level.
* Indicates statistical significance at the 5 percent level.

Figure 2
Realized and Forecasted One-Month Correlation between the DEM and the JPY in USD between October 2, 1990 and April 2, 1997

A. Realized and Implied Correlation



B. Realized and Biv. GARCH(1,1) based Correlation



C. Realized and 20 days Historical Correlation

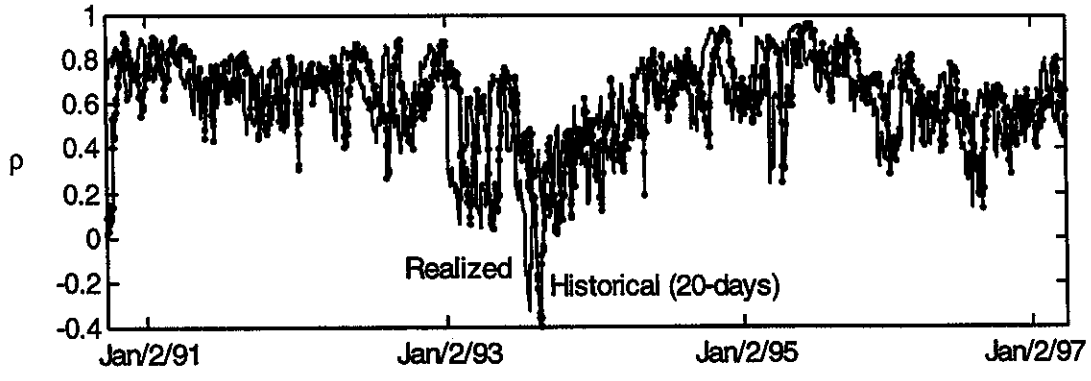
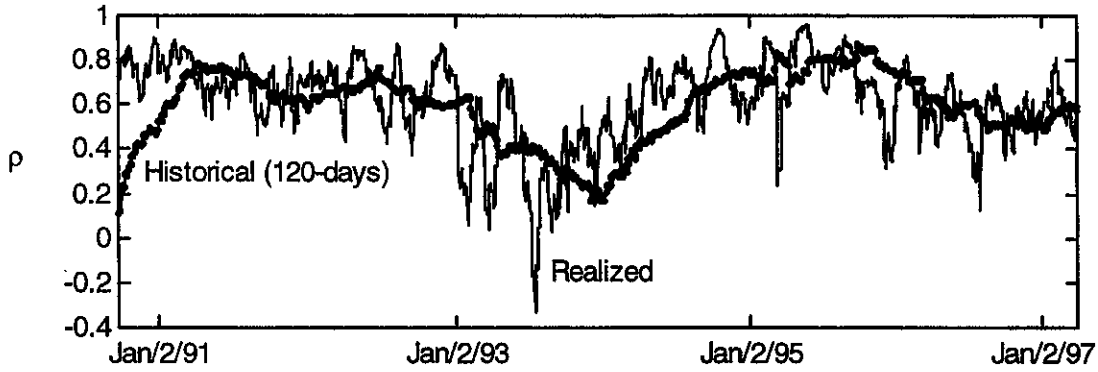
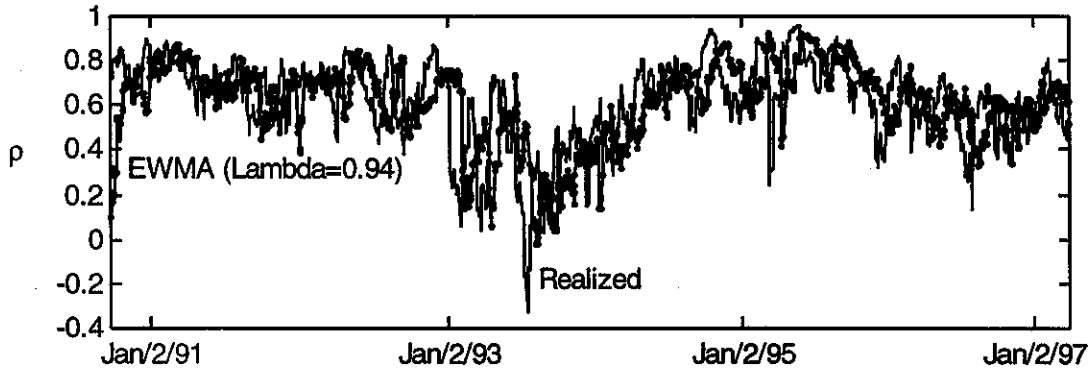


Figure 2 - Continued
Realized and Forecasted 1-Month Correlations between the DEM and the JPY in USD between October 2, 1990 and April 2, 1997

D. Realized and 120 days Historical Correlation



E. Realized and EWMA (λ=0.94) Correlation



F. Realized and EWMA (λ=0.99) Correlation

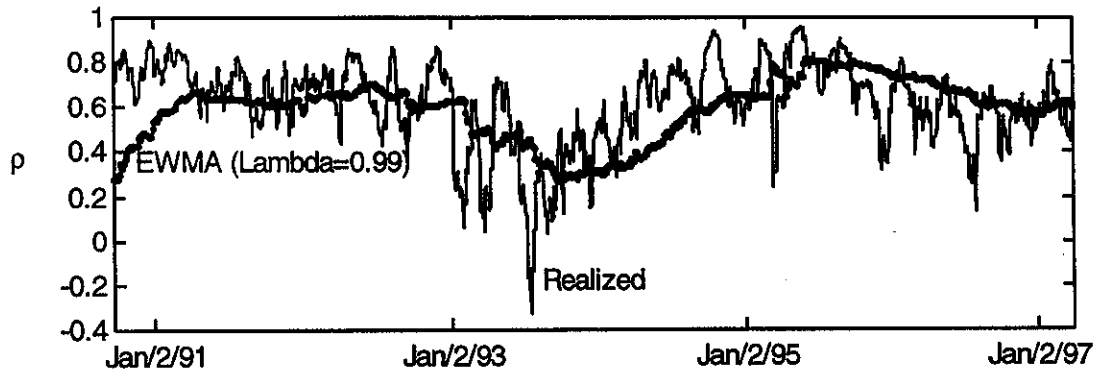


Table 4A
One-Month Correlations in the Currency Trio USD/DEM/JPY: Analysis of Forecast Errors

This table reports the mean forecast errors (MFE's), the root mean square forecast errors (RMSFE's), and the Diebold and Mariano test statistic (D&M test) for the null hypothesis of no difference in the forecast accuracy (loss function: squared errors) for the correlations analyzed here. The MFE's and RMSFE's are defined as follows:

$$\text{MFE} = \frac{1}{n}\sum_{t=1}^{n}\left[\rho(\bullet)_t - \rho(\bullet)_{j,t}\right], \text{ and } \text{RMSFE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left[\rho(\bullet)_t - \rho(\bullet)_{j,t}\right]^2},$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The observation period is October 2, 1990 through April 2, 1997 (1679 daily observations). Statistical significance of the MFE's is assessed by running regressions of the forecast errors on a constant; the standard errors of the estimated parameters are corrected for the induced heteroskedasticity and autocorrelation using the Newey and West (1987) procedure.

	$\rho(DEM_{USD}, JPY_{USD})$			$\rho(USD_{DEM}, JPY_{DEM})$			$\rho(USD_{JPY}, DEM_{JPY})$		
	MFE	RMSFE	D&M Test	MFE	RMSFE	D&M Test	MFE	RMSFE	D&M Test
Forecast Method									
Imp	0.047**	0.182	0.49	-0.024**	0.233	–	-0.079**	0.297	0.85
Hist (20 days)	0.003	0.218	4.02**	0.002	0.281	4.23**	-0.001	0.323	3.65**
Hist (60 days)	0.016	0.199	1.69*	-0.008	0.262	2.32*	-0.028*	0.295	2.13*
Hist (120 days)	0.026**	0.189	1.19	-0.013	0.280	3.13**	-0.043**	0.310	1.85*
EWMA (λ=0.94)	0.009	0.188	1.99*	-0.005	0.243	0.97	-0.016	0.288	1.12
EWMA (λ=0.97)	0.018	0.180	–	-0.009	0.241	0.87	-0.034*	0.279	–
EWMA (λ=0.99)	0.031**	0.190	1.27	-0.009	0.257	2.03*	-0.068**	0.319	1.66*
Biv. GARCH(1,1)	-0.068**	0.189	0.49	0.093**	0.250	0.91	-0.016**	0.283	0.59

** Indicates statistical significance at the 1 percent level.
* Indicates statistical significance at the 5 percent level.

Table 4B
Three-Month Correlations in the Currency Trio USD/DEM/JPY: Analysis of Forecast Errors

This table reports the mean forecast errors (MFE's), the root mean square forecast errors (RMSFE's), and the Diebold and Mariano test statistic (D&M test) for the null hypothesis of no difference in the forecast accuracy (loss function: squared errors) for the correlations analyzed here. The MFE's and RMSFE's are defined as follows:

$$\text{MFE} = \frac{1}{n}\sum_{t=1}^{n}\left[\rho(\bullet)_t - \rho(\bullet)_{j,t}\right], \text{ and } \text{RMSFE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left[\rho(\bullet)_t - \rho(\bullet)_{j,t}\right]^2},$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The observation period is October 2, 1990 through April 2, 1997 (1679 daily observations). Statistical significance of the MFE's is assessed by running regressions of the forecast errors on a constant; the standard errors of the estimated parameters are corrected for the induced heteroskedasticity and autocorrelation using the Newey and West (1987) procedure.

	$\rho(DEM_{USD}, JPY_{USD})$			$\rho(USD_{DEM}, JPY_{DEM})$			$\rho(USD_{JPY}, DEM_{JPY})$		
	MFE	RMSFE	D&M Test	MFE	RMSFE	D&M Test	MFE	RMSFE	D&M Test
Forecast Method									
Imp	0.036**	0.145	0.17	-0.007	0.194	–	-0.072**	0.268	0.55
Hist (20 days)	-0.009	0.190	3.74**	0.011	0.281	4.31**	0.025	0.332	1.70*
Hist (60 days)	0.004	0.151	1.34	0.001	0.250	3.09**	-0.002	0.279	0.60
Hist (120 days)	0.014	0.158	1.22	-0.005	0.254	3.20**	-0.018	0.300	1.06
EWMA (λ=0.94)	-0.003	0.156	1.78*	0.004	0.240	2.92**	0.010	0.289	0.84
EWMA (λ=0.97)	0.005	0.142	–	-0.001	0.229	2.52**	-0.008	0.289	0.42
EWMA (λ=0.99)	0.019	0.162	1.54	-0.001	0.223	2.24*	-0.043*	0.272	1.19
Biv. GARCH(1,1)	-0.110**	0.180	1.51	0.146**	0.248	2.22*	0.013	0.258	–

** Indicates statistical significance at the 1 percent level.
* Indicates statistical significance at the 5 percent level.

Table 4C
One-Month Correlations in the Currency Trio USD/DEM/CHF: Analysis of Forecast Errors

This table reports the mean forecast errors (MFE's), the root mean square forecast errors (RMSFE's), and the Diebold and Mariano test statistic (D&M test) for the null hypothesis of no difference in the forecast accuracy (loss function: squared errors) for the correlations analyzed here. The MFE's and RMSFE's are defined as follows:

$$\text{MFE} = \frac{1}{n}\sum_{t=1}^{n}\left[\rho(\bullet)_t - \rho(\bullet)_{j,t}\right], \text{ and } \text{RMSFE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left[\rho(\bullet)_t - \rho(\bullet)_{j,t}\right]^2},$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The observation period is September 13, 1993 through April 2, 1997 (910 daily observations). Statistical significance of the MFE's is assessed by running regressions of the forecast errors on a constant; the standard errors of the estimated parameters are corrected for the induced heteroskedasticity and autocorrelation using the Newey and West (1987) procedure.

	$\rho(DEM_{USD}, CHF_{USD})$			$\rho(USD_{DEM}, CHF_{DEM})$			$\rho(USD_{CHF}, DEM_{CHF})$		
	MFE	RMSFE	D&M Test	MFE	RMSFE	D&M Test	MFE	RMSFE	D&M Test
Forecast Method									
Imp	0.017**	0.041	1.52	-0.232**	0.362	2.43**	0.151**	0.260	2.68**
Hist (20 days)	-0.001	0.040	2.08*	-0.018	0.332	3.04**	0.020	0.260	3.24**
Hist (60 days)	-0.002	0.041	1.78*	-0.006	0.293	0.79	0.003	0.230	1.17
Hist (120 days)	-0.004	0.046	1.79*	-0.002	0.294	1.67*	0.001	0.237	1.88*
EWMA (λ=0.94)	-0.001	0.035	0.37	-0.010	0.293	1.02	0.008	0.226	1.93*
EWMA (λ=0.97)	-0.003	0.037	0.75	-0.005	0.279	0.05	0.004	0.215	0.94
EWMA (λ=0.99)	-0.008	0.042	2.04*	-0.011	0.278	–	0.014	0.207	–
Biv. GARCH(1,1)	0.001	0.034	–	-0.139**	0.299	0.27	0.079**	0.213	0.69

** Indicates statistical significance at the 1 percent level.
* Indicates statistical significance at the 5 percent level.

Table 4D
Three-Month Correlations in the Currency Trio USD/DEM/CHF: Analysis of Forecast Errors

This table reports the mean forecast errors (MFE's), the root mean square forecast errors (RMSFE's), and the Diebold and Mariano test statistic (D&M test) for the null hypothesis of no difference in the forecast accuracy (loss function: squared errors) for the correlations analyzed here. The MFE's and RMSFE's are defined as follows:

$$\text{MFE} = \frac{1}{n}\sum_{t=1}^{n}\left[\rho(\bullet)_t - \rho(\bullet)_{j,t}\right], \text{ and } \text{RMSFE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left[\rho(\bullet)_t - \rho(\bullet)_{j,t}\right]^2},$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The observation period is September 13, 1993 through April 2, 1997 (910 daily observations). Statistical significance of the MFE's is assessed by running regressions of the forecast errors on a constant; the standard errors of the estimated parameters are corrected for the induced heteroskedasticity and autocorrelation using the Newey and West (1987) procedure.

	$\rho(DEM_{USD}, CHF_{USD})$			$\rho(USD_{DEM}, CHF_{DEM})$			$\rho(USD_{CHF}, DEM_{CHF})$		
	MFE	RMSFE	D&M Test	MFE	RMSFE	D&M Test	MFE	RMSFE	D&M Test
Forecast Method									
Imp	0.013**	0.038	1.31	-0.265**	0.348	2.43**	0.193**	0.260	3.57**
Hist (20 days)	-0.001	0.043	2.00*	-0.037	0.315	4.41**	0.044	0.252	2.56**
Hist (60 days)	-0.002	0.044	1.34	-0.024	0.257	1.09	0.027	0.200	1.48
Hist (120 days)	-0.004	0.047	1.41	-0.021	0.245	0.91	0.025	0.175	1.61
EWMA (λ=0.94)	-0.001	0.039	0.87	-0.029	0.260	1.39	0.038	0.206	1.73**
EWMA (λ=0.97)	-0.003	0.039	0.91	-0.024	0.239	0.45	0.028	0.182	1.25
EWMA (λ=0.99)	-0.009	0.041	1.53	-0.029	0.231	–	0.032	0.156	–
Biv. GARCH(1,1)	0.001	0.034	–	-0.194**	0.278	0.68	0.174**	0.215	2.41**

** Indicates statistical significance at the 1 percent level.
* Indicates statistical significance at the 5 percent level.

Table 5A
One-Month Correlations in the Currency Trio USD/DEM/JPY: Individual Predictability Regressions

This table reports the results of a series of regressions of realized correlations on a constant and an individual forecasts. The following equation was estimated separately for all the forecasts generated for the three correlations in the currency trio USD/DEM/JPY:

$$\rho(\bullet)_t = a + b\,\rho(\bullet)_{j,t} + \varepsilon_t,$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The observation period is October 2, 1990 through April 2,1997 (1679 daily observations). Since we use daily observations, we apply the Newey and West (1987) procedure to correct the standard errors for the induced heteroskedasticity and autocorrelation.

	$\rho(DEM_{USD}, JPY_{USD})$			$\rho(USD_{DEM}, JPY_{DEM})$			$\rho(USD_{JPY}, DEM_{JPY})$		
	a	b	R^2	a	b	R^2	a	b	R^2
Forecast Method									
Imp	0.195**	0.741“	0.18	-0.067	1.084‡	0.39	-0.122	1.112	0.37
Hist (20 days)	0.392**	0.367“	0.15	0.229**	0.544“	0.31	0.120**	0.590“	0.36
Hist (60 days)	0.297**	0.532“	0.22	0.169**	0.652“	0.29	0.061**	0.727“	0.39
Hist (120 days)	0.304**	0.529“	0.20	0.190**	0.604“	0.19	0.054**	0.714“	0.32
EWMA (λ=0.94)	0.302**	0.519“	0.24	0.149**	0.697“	0.40	0.075*	0.708“	0.43
EWMA (λ=0.97)	0.257**	0.600“	0.26	0.110**	0.767“	0.37	0.034	0.794“	0.43
EWMA (λ=0.99)	0.302**	0.537“	0.15	0.089*	0.806“	0.26	-0.005	0.826	0.26
Biv. GARCH(1,1)	0.136*	0.701“	0.18	0.112**	0.956	0.38	-0.029	1.020‡	0.36

** Indicates that a is significantly different from 0 at the 1 percent level, given the Newey and West (1987) standard errors.
* Indicates that a is significantly different from 0 at the 5 percent level, given the Newey and West (1987) standard errors.
“ Indicates that b is significantly different from 1 at the 1 percent level, given the Newey and West (1987) standard errors.
‘ Indicates that b is significantly different from 1 at the 5 percent level, given the Newey and West (1987) standard errors.
‡ Indicates that the Wald-test for joint hypothesis a=0 and b=1 can not be rejected at the 1 percent level.
† Indicates that the Wald-test for joint hypothesis a=0 and b=1 can not be rejected at the 5 percent level.

Table 5B
Three-Month Correlations in the Currency Trio USD/DEM/JPY: Individual Predictability Regressions

This table reports the results of a series of regressions of realized correlations on a constant and an individual forecasts. The following equation was estimated separately for all the forecasts generated for the three correlations in the currency trio USD/DEM/JPY:

$$\rho(\bullet)_t = a + b\ \rho(\bullet)_{j,t} + \varepsilon_t,$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The observation period is October 2, 1990 through April 2,1997 (1679 daily observations). Since we use daily observations, we apply the Newey and West (1987) procedure to correct the standard errors for the induced heteroskedasticity and autocorrelation.

	$\rho(DEM_{USD}, JPY_{USD})$			$\rho(USD_{DEM}, JPY_{DEM})$			$\rho(USD_{JPY}, DEM_{JPY})$		
	a	b	R^2	a	b	R^2	a	b	R^2
Forecast Method									
Imp	0.067	0.946	0.25	0.018	0.952‡	0.36	-0.052	0.950	0.25
Hist (20 days)	0.372**	0.378“	0.34	0.314**	0.392“	0.25	0.195**	0.426“	0.28
Hist (60 days)	0.268**	0.561“	0.27	0.273**	0.465“	0.22	0.140**	0.561“	0.33
Hist (120 days)	0.294**	0.527“	0.18	0.287**	0.433“	0.15	0.152**	0.500“	0.23
EWMA (λ=0.94)	0.283**	0.530“	0.34	0.255**	0.504“	0.31	0.168**	0.524“	0.34
EWMA (λ=0.97)	0.238**	0.612“	0.37	0.226**	0.555“	0.28	0.128**	0.588“	0.34
EWMA (λ=0.99)	0.310**	0.504“	0.17	0.202**	0.602“	0.21	0.136**	0.509“	0.14
Biv. GARCH(1,1)	-0.112	1.003	0.22	0.124*	1.060	0.31	-0.041	1.134‡	0.26

** Indicates that a is significantly different from 0 at the 1 percent level, given the Newey and West (1987) standard errors.
* Indicates that a is significantly different from 0 at the 5 percent level, given the Newey and West (1987) standard errors.
“ Indicates that b is significantly different from 1 at the 1 percent level, given the Newey and West (1987) standard errors.
‘ Indicates that b is significantly different from 1 at the 5 percent level, given the Newey and West (1987) standard errors.
‡ Indicates that the Wald-test for joint hypothesis a=0 and b=1 can not be rejected at the 1 percent level.
† Indicates that the Wald-test for joint hypothesis a=0 and b=1 can not be rejected at the 5 percent level.

Table 5C
One-Month Correlations in the Currency Trio USD/DEM/CHF: Individual Predictability Regressions

This table reports the results of a series of regressions of realized correlations on a constant and an individual forecasts. The following equation was estimated separately for all the forecasts generated for the three correlations in the currency trio USD/DEM/CHF:

$$\rho(\bullet)_t = a + b\,\rho(\bullet)_{j,t} + \varepsilon_t,$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The observation period is September 13, 1993 through April 2, 1997 (910 daily observations). Since we use daily observations, we apply the Newey and West (1987) procedure to correct the standard errors for the induced heteroskedasticity and autocorrelation.

	$\rho(DEM_{USD}, CHF_{USD})$			$\rho(USD_{DEM}, CHF_{DEM})$			$\rho(USD_{CHF}, DEM_{CHF})$		
	a	*b*	R^2	*a*	*b*	R^2	*a*	*b*	R^2
Forecast Method									
Imp	0.206**	0.793“	0.37	-0.240**	0.633‘	0.09	0.392**	0.420“	0.06
Hist (20 days)	0.354**	0.619“	0.40	-0.169**	0.357“	0.14	0.408**	0.290“	0.11
Hist (60 days)	0.329**	0.645“	0.34	-0.135**	0.474“	0.15	0.385**	0.322“	0.07
Hist (120 days)	0.474**	0.489“	0.15	-0.139**	0.456“	0.12	0.418**	0.263“	0.03
EWMA (λ=0.94)	0.254**	0.726“	0.48	-0.137**	0.474“	0.18	0.353**	0.382“	0.12
EWMA (λ=0.97)	0.225**	0.756“	0.41	-0.118**	0.545“	0.19	0.326**	0.427“	0.10
EWMA (λ=0.99)	0.217**	0.759“	0.21	-0.113**	0.581“	0.14	0.286**	0.508“	0.06
Biv. GARCH(1,1)	-0.248**	1.281‘	0.46	-0.128**	0.880‘	0.16	0.155**	0.897	0.14

** Indicates that *a* is significantly different from 0 at the 1 percent level, given the Newey and West (1987) standard errors.
* Indicates that *a* is significantly different from 0 at the 5 percent level, given the Newey and West (1987) standard errors.
“ Indicates that *b* is significantly different from 1 at the 1 percent level, given the Newey and West (1987) standard errors.
‘ Indicates that *b* is significantly different from 1 at the 5 percent level, given the Newey and West (1987) standard errors.
‡ Indicates that the Wald-test for joint hypothesis *a*=0 and *b*=1 can not be rejected at the 1 percent level.
† Indicates that the Wald-test for joint hypothesis *a*=0 and *b*=1 can not be rejected at the 5 percent level.

Table 5D
Three-Month Correlations in the Currency Trio USD/DEM/CHF: Individual Predicability Regressions

This table reports the results of a series of regressions of realized correlations on a constant and an individual forecasts. The following equation was estimated separately for all the forecasts generated for the three correlations in the currency trio USD/DEM/JPY:

$$\rho(\bullet)_t = a + b\ \rho(\bullet)_{j,t} + \varepsilon_t,$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The observation period is September 13, 1993 through April 2, 1997 (910 daily observations). Since we use daily observations, we apply the Newey and West (1987) procedure to correct the standard errors for the induced heteroskedasticity and autocorrelation.

	$\rho(DEM_{USD}, CHF_{USD})$			$\rho(USD_{DEM}, CHF_{DEM})$			$\rho(USD_{CHF}, DEM_{CHF})$		
	a	b	R^2	a	b	R^2	a	b	R^2
Forecast Method									
Imp	0.172*	0.827‘	0.27	-0.270**	0.348“	0.04	0.551**	0.102“	0.01
Hist (20 days)	0.496**	0.467“	0.27	-0.217**	0.237“	0.12	0.532**	0.108“	0.04
Hist (60 days)	0.520**	0.440“	0.19	-0.187**	0.344“	0.15	0.526**	0.116“	0.02
Hist (120 days)	0.626**	0.326“	0.08	-0.183**	0.356“	0.13	0.514**	0.136“	0.02
EWMA (λ=0.94)	0.411**	0.557“	0.33	-0.187**	0.353“	0.18	0.504**	0.155“	0.05
EWMA (λ=0.97)	0.415**	0.552“	0.26	-0.172**	0.406“	0.19	0.496**	0.168“	0.04
EWMA (λ=0.99)	0.397**	0.568“	0.14	-0.174**	0.406“	0.11	0.469**	0.219“	0.03
Biv. GARCH(1,1)	-0.464**	1.500“	0.36	-0.166**	0.934	0.15	0.316**	0.660“	0.07

** Indicates that a is significantly different from 0 at the 1 percent level, given the Newey and West (1987) standard errors.
* Indicates that a is significantly different from 0 at the 5 percent level, given the Newey and West (1987) standard errors.
“ Indicates that b is significantly different from 1 at the 1 percent level, given the Newey and West (1987) standard errors.
‘ Indicates that b is significantly different from 1 at the 5 percent level, given the Newey and West (1987) standard errors.
‡ Indicates that the Wald-test for joint hypothesis a=0 and b=1 can not be rejected at the 1 percent level.
† Indicates that the Wald-test for joint hypothesis a=0 and b=1 can not be rejected at the 5 percent level.

Table 6A
One-Month Correlations in the Currency Trio USD/DEM/JPY: Encompassing Regressions

This table reports the results of a series of regressions of realized correlations on a constant and a combination of forecasts. The following equation was estimated separately for the three correlations analyzed here:

$$\rho(\bullet)_t = a + \sum_{j=1}^{4} b_j \rho(\bullet)_{j,t} + \varepsilon_t,$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The combination of forecasts used as right-hand variables consists of implied correlation, bivariate GARCH(1,1) based correlation, and the historical and EWMA correlation with the highest R^2 in the individual predictability regressions. The observation period is October 2, 1990 through April 2,1997 (1679 daily observations). Regressions use daily observations, and standard errors are corrected for the induced heteroskedasticity and autocorrelation using the Newey and West (1987) procedure. χ^2 (Implied = 0) denotes the Wald test on the restriction that the coefficient on implied correlation equals zero, χ^2 (Other = 0) denotes the Wald test on the restriction that the coefficients on all other forecasts are zero, and χ^2 (Biv. GARCH[1,1] = 0) denotes the Wald test on the restriction that the coefficient on the forecast from the bivariate GARCH(1,1) model equals zero.

	$\rho(DEM_{USD}, JPY_{USD})$	$\rho(USD_{DEM}, JPY_{DEM})$	$\rho(USD_{JPY}, DEM_{JPY})$
a	0.050 *	-0.035	-0.059
Forecast Method (b_j's)			
Imp	0.308**	0.523**	0.322*
Hist (20 days)		-0.145	
Hist (60 days)	0.056		0.220*
Hist (120 days)			
EWMA (λ=0.94)		0.282	0.183
EWMA (λ=0.97)	0.132*		
EWMA (λ=0.99)			
Biv. GARCH(1,1)	0.499**	0.468**	0.332*
R^2	0.29	0.45	0.46
χ^2 (Implied = 0)	4.9*	17.6**	4.55*
χ^2 (Other = 0)	30.6**	31.2**	51.6**
χ^2 (Biv. GARCH[1,1] = 0)	12.4**	11.8**	6.3*

** Indicates statistical significance at the 1 percent level, given the Newey and West (1987) standard errors.
* Indicates statistical significance at the 5 percent level, given the Newey and West (1987) standard errors.

Table 6B
Three-Month Correlations in the Currency Trio USD/DEM/JPY: Encompassing Regressions

This table reports the results of a series of regressions of realized correlations on a constant and a combination of forecasts. The following equation was estimated separately for the three correlations analyzed here:

$$\rho(\bullet)_t = a + \sum_{j=1}^{4} b_j \, \rho(\bullet)_{j,t} + \varepsilon_t,$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The combination of forecasts used as right-hand variables consists of implied correlation, bivariate GARCH(1,1) based correlation, and the historical and EWMA correlation with the highest R^2 in the individual predictability regressions. The observation period is October 2, 1990 through April 2,1997 (1679 daily observations). Regressions use daily observations, and standard errors are corrected for the induced heteroskedasticity and autocorrelation using the Newey and West (1987) procedure. χ^2 (Implied = 0) denotes the Wald test on the restriction that the coefficient on implied correlation equals zero, χ^2 (Other = 0) denotes the Wald test on the restriction that the coefficients on all other forecasts are zero, and χ^2 (Biv. GARCH[1,1] = 0) denotes the Wald test on the restriction that the coefficient on the forecast from the bivariate GARCH(1,1) model equals zero.

	$\rho(DEM_{USD}, JPY_{USD})$	$\rho(USD_{DEM}, JPY_{DEM})$	$\rho(USD_{JPY}, DEM_{JPY})$
a	0.026	-0.054	0.048
Forecast Method (b_j's)			
Imp	0.102	0.698**	0.010*
Hist (20 days)		-0.013	
Hist (60 days)	0.109		0.341*
Hist (120 days)			
EWMA (λ=0.94)		-0.061	0.096
EWMA (λ=0.97)	0.366		
EWMA (λ=0.99)			
Biv. GARCH(1,1)	0.331*	0.660**	0.405*
R^2	0.39	0.41	0.36
χ^2 (Implied = 0)	0.6	31.8**	0.1
χ^2 (Other = 0)	59.4**	38.1**	52.9**
χ^2 (Biv. GARCH[1,1] = 0)	5.7*	23.6**	5.0*

** Indicates statistical significance at the 1 percent level, given the Newey and West (1987) standard errors.
* Indicates statistical significance at the 5 percent level, given the Newey and West (1987) standard errors.

Table 6C
One-Month Correlations in the Currency Trio USD/DEM/CHF: Encompassing Regressions

This table reports the results of a series of regressions of realized correlations on a constant and a combination of forecasts. The following equation was estimated separately for the three correlations analyzed here:

$$\rho(\bullet)_t = a + \sum_{j=1}^{4} b_j \, \rho(\bullet)_{j,t} + \varepsilon_t,$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The combination of forecasts used as right-hand variables consists of implied correlation, bivariate GARCH(1,1) based correlation, and the historical and EWMA correlation with the highest R^2 in the individual predictability regressions. The observation period is September 13, 1993 through April 2, 1997 (910 daily observations). Regressions use daily observations, and standard errors are corrected for the induced heteroskedasticity and autocorrelation using the Newey and West (1987) procedure. χ^2 (Implied = 0) denotes the Wald test on the restriction that the coefficient on implied correlation equals zero, χ^2 (Other = 0) denotes the Wald test on the restriction that the coefficients on all other forecasts are zero, and χ^2 (Biv. GARCH[1,1] = 0) denotes the Wald test on the restriction that the coefficient on the forecast from the bivariate GARCH(1,1) model equals zero.

	$\rho(DEM_{USD}, CHF_{USD})$	$\rho(USD_{DEM}, CHF_{DEM})$	$\rho(USD_{CHF}, DEM_{CHF})$
a	-0.011	-0.118	0.054
Forecast Method (b_j's)			
Imp	0.240**	0.074	0.245
Hist (20 days)	-0.061		0.107
Hist (60 days)		-0.007	
Hist (120 days)			
EWMA (λ=0.94)	0.459*		-0.325
EWMA (λ=0.97)		0.381	
EWMA (λ=0.99)			
Biv. GARCH(1,1)	0.377	0.285	1.164**
R^2	0.51	0.20	0.17
χ^2 (Implied = 0)	7.1**	0.1	0.3
χ^2 (Other = 0)	63.4**	31.9**	30.7**
χ^2 (Biv. GARCH[1,1] = 0)	3.3	1.2	14.2**

** Indicates statistical significance at the 1 percent level, given the Newey and West (1987) standard errors.
* Indicates statistical significance at the 5 percent level, given the Newey and West (1987) standard errors.

Table 6D
Three-Month Correlations in the Currency Trio USD/DEM/CHF: Encompassing Regressions

This table reports the results of a series of regressions of realized correlations on a constant and a combination of forecasts. The following equation was estimated separately for the three correlations analyzed here:

$$\rho(\bullet)_t = a + \sum_{j=1}^{4} b_j \, \rho(\bullet)_{j,t} + \varepsilon_t,$$

where $\rho(\bullet)$ denotes the realized correlation and $\rho(\bullet)_j$ denotes the correlation forecast according to method j. The combination of forecasts used as right-hand variables consists of implied correlation, bivariate GARCH(1,1) based correlation, and the historical and EWMA correlation with the highest R^2 in the individual predictability regressions. The observation period is September 13, 1993 through April 2, 1997 (910 daily observations). Regressions use daily observations, and standard errors are corrected for the induced heteroskedasticity and autocorrelation using the Newey and West (1987) procedure. χ^2 (Implied = 0) denotes the Wald test on the restriction that the coefficient on implied correlation equals zero, χ^2 (Other = 0) denotes the Wald test on the restriction that the coefficients on all other forecasts are zero, and χ^2 (Biv. GARCH[1,1] = 0) denotes the Wald test on the restriction that the coefficient on the forecast from the bivariate GARCH(1,1) model equals zero.

	$\rho(DEM_{USD}, CHF_{USD})$	$\rho(USD_{DEM}, CHF_{DEM})$	$\rho(USD_{CHF}, DEM_{CHF})$
a	-0.240	-0.156	0.262**
Forecast Method (b_j's)			
Imp	0.206*	-0.295	0.018
Hist (20 days)	-0.139		-0.027
Hist (60 days)		-0.269	
Hist (120 days)			
EWMA (λ=0.94)	0.315		-0.030
EWMA (λ=0.97)		0.834*	
EWMA (λ=0.99)			
Biv. GARCH(1,1)	0.879**	-0.191	0.847**
R^2	0.39	0.20	0.08
χ^2 (Implied = 0)	2.7	3.7	0.1
χ^2 (Other = 0)	37.5**	56.7**	20.0**
χ^2 (Biv. GARCH[1,1] = 0)	9.7**	0.3	8.4**

** Indicates statistical significance at the 1 percent level, given the Newey and West (1987) standard errors.
* Indicates statistical significance at the 5 percent level, given the Newey and West (1987) standard errors.

Appendix: The Relationship between Volatilities and Correlations in a Currency Trio

It has been shown in section IIIa. that the variance of the log return of a cross-rate is a function of the variances of the log returns of the two underlying US dollar exchange rates and their correlation:

$$\sigma(A_B)^2 = \sigma(A_{USD})^2 + \sigma(B_{USD})^2 - 2\rho(A_{USD}, B_{USD})\,\sigma(A_{USD})\,\sigma(B_{USD}),$$

where A_{USD} and B_{USD} denote the US dollar exchange rates of currencies A and B, respectively, A_B is the cross-rate, $\sigma(X_Y)$ denotes the standard deviation of the log return of exchange rate between currencies X and Y, and $\rho(X_Y, Z_Y)$ is the correlation between the log returns of exchange rates X_Y and Z_Y.[1] By applying the law of cosine in trigonometry we are able to represent the relationship between the volatilities and correlations in a currency trio in an intuitively appealing way.[2]

The law of cosines states that if the angles of a triangle are lettered α, β, and γ, respectively, and the lengths of the sides opposite the angles are a, b and c lettered, respectively, then the following expression holds (analog expressions for $\cos(\alpha)$ and $\cos(\beta)$ are obtained by cyclic permutation of the letters)

$$\cos(\gamma) = \frac{(a^2 + b^2 - c^2)}{2ab},$$

or by solving for c^2

$$c^2 = a^2 + b^2 - 2\cos(\gamma)ab.$$

This representation of the law of cosines has the same structure as the equation for the variance of the return of the cross-rate. Therefore, the two equations can be transformed into each other by making the following substitutions

$$\sigma(A_{USD}) \leftrightarrow a;\ \sigma(B_{USD}) \leftrightarrow b;\ \sigma(A_B) \leftrightarrow c;\ \rho(A_{USD}, B_{USD}) \leftrightarrow \cos(\gamma).$$

Hence, the volatilities and correlations in a currency trio can be represented by a triangle with the lengths of the sides being the volatilities and the cosines of the angles being the correlations. To illustrate this, consider the currency trio US dollar (USD), Japanese yen (JPY) and German mark

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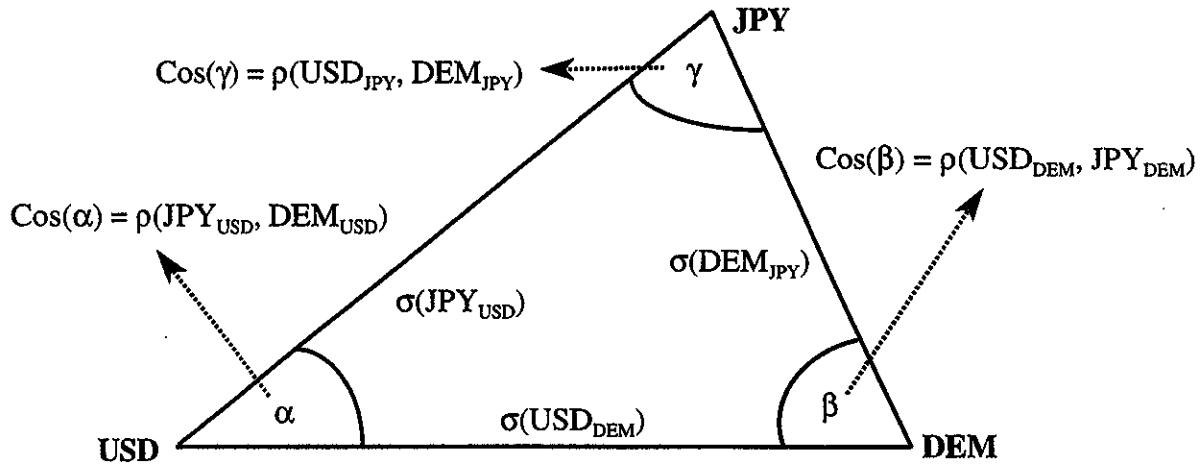
[1] Note that we assume $\sigma(A_{USD})$, $\sigma(B_{USD})$, $\sigma(A_B) > 0$.

[2] See Zerolis (1995) who also presents the geometry of volatilities and correlations in a currency-quartet.

(DEM). Exhibit 1 shows how the volatilities of and the correlations between the three exchange rates defined by this currency trio are related.

Exhibit 1:
Volatility and Correlation Triangle for the Currency Trio USD/DEM/JPY



To make the notation clear, consider the corner of the triangle labeled USD. In this corner, the two sides involved stand for the volatilities of the JPY/USD exchange rate (side between the USD and the JPY corner) and the DEM/USD exchange rate (side between the USD and the DEM corner), respectively. The involved angle in the USD corner is labeled α, with its cosines being the correlation between the JPY/USD and the DEM/USD exchange rates.

The next two exhibits show two volatility and correlation triangles constructed from real data: The first triangle is the 1 month implied volatility and correlation triangle for the currency trio USD/DEM/JPY as of January 2, 1997. The triangle is an acute triangle. This reflects the fact that the three implied correlations are positive and that the arcus cosines of positive numbers are between 0 and 90 degrees (in degree measure) or 0 and $\pi/2$ (in radian measure). Although the triangle is not an equilateral triangle, its sides and its angles are of roughly similar size. Therefore, the implied correlations are also of similar size. The highest implied correlation is 0.56 (between the DEM measured in USD and the JPY measured in USD) and the lowest correlation is 0.42 (between the JPY and the USD from the point of view of a DEM-investor). The implied volatility and correlation triangle for the currency trio USD/DEM/CHF as of January 2, 1997 has a markedly different shape.

Its shape is characterized by the low implied volatiliy of the DEM/CHF exchange rate (4.8). In view of the two other implied volatilities, the low DEM/CHF volatility implies a high correlation between the DEM and the CHF from the viewpoint of a USD-based investor (0.9). Hence, for a USD-based investor with DEM's in the portfolio, the diversification benefit by acquiring CHF's is relatively low. For a DEM-based investor, on the other hand, the CHF offers substantial diversification benefits. Its correlation with the USD is negative (-0.23). Finally, for a CHF-based investor, the correlation between the two foreign currencies is somewhere in the middle of the two extremes.

Exhibit 2:
1 Month Implied Volatility and Correlation Triangle USD/DEM/JPY as of January 2, 1997

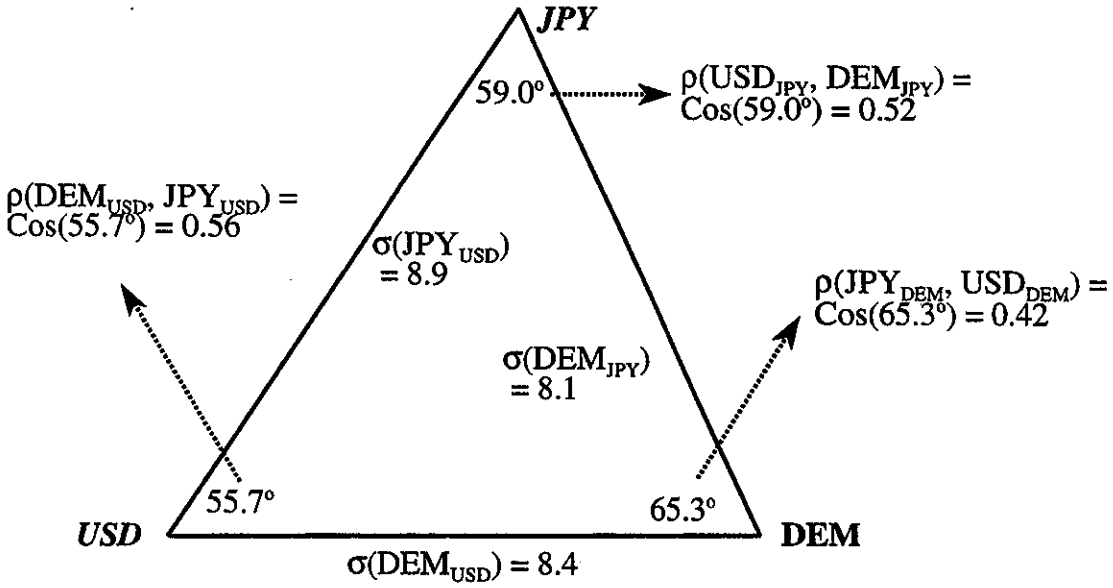
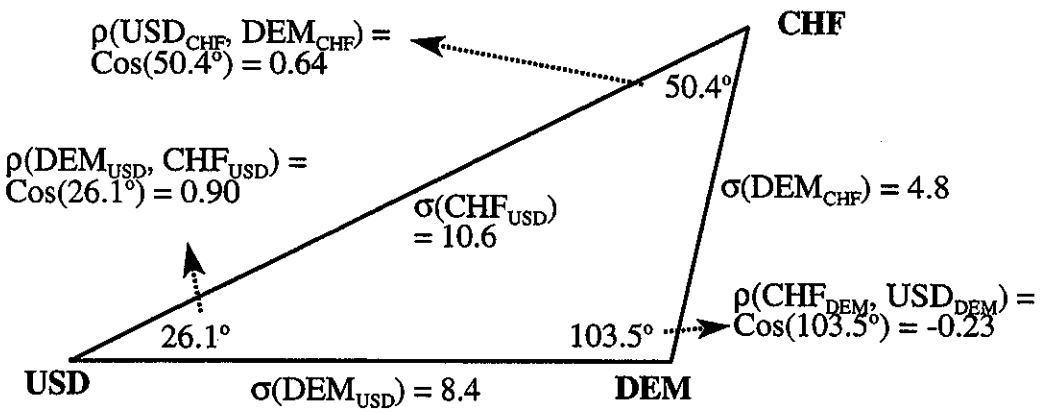


Exhibit 3:
1 Month Implied Volatility and Correlation Triangle USD/DEM/CHF as of January 2, 1997



Given this representation of the relationship between the volatilities and correlations within a currency trio and the assumption that $\sigma(A_{USD})$, $\sigma(B_{USD})$, $\sigma(A_B) > 0$, it is obvious that the following propositions on the relationships between volatilities and correlations in a currency trio hold:

PROPOSITION 1: For a currency trio, the sum of the arcus cosines of the correlations must equal 180 degrees (in degree measure) or π (in radian measure).

PROPOSITION 2: For a currency trio, the sum of any two volatilities must be larger than the third volatility.

PROPOSITION 3: Knowledge of the volatilities (implied or realized) in a currency trio implies knowledge of the correlations (implied or realized) in a currency trio.The opposite, however, is not true: Knowledge of the correlations (implied or realized) does not imply knowledge of the volatilities (implied or realized). This implies that if the predictive power of implied volatility for future realized volatility is high, then the predictive power of implied correlation is also high; the reverse, however, is not true.[3]

Since the sum of the arcus cosines of the three correlations in a currency trio equals π, that is

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[3] This statement does not only hold for implied correlation but also for the two simple forecast methods based on rolling averages of past returns (historical correlation and exponentially weighted moving average correlation).

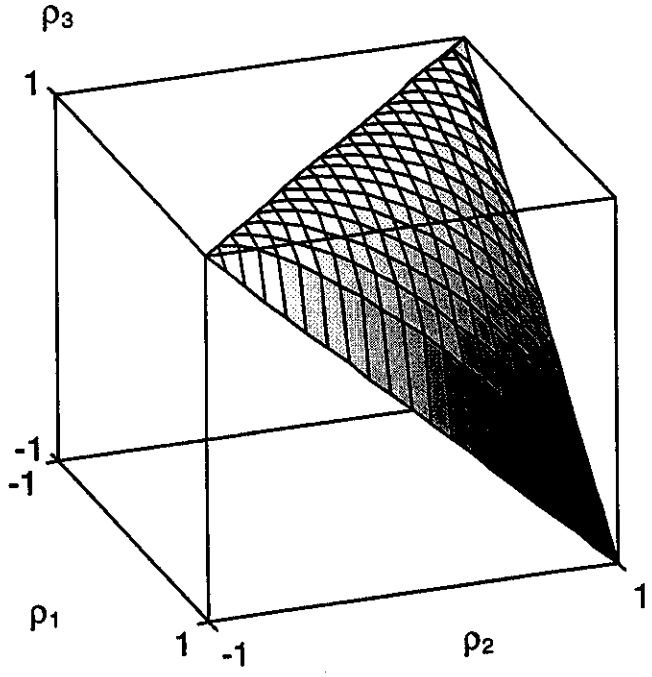
$$\arccos(\rho_1) + \arccos(\rho_2) + \arccos(\rho_3) = \pi,$$

we can express one of the three correlations as a function of the two other correlations. The third correlation, for example, equals

$$\rho_3 = \cos(\pi - \arccos[\rho_1] - \arccos[\rho_2]).$$

We can picture the set of possible combinations of correlations in a currency trio. Each combination can be seen as a point in the cube $[-1,1]^3$ in three-dimensional space. Exhibit 4 shows the set of possible combinations. It consists of the correlation combinations that fulfill the condition that the sum of the arcus cosines equals π and that the number of negative correlations can not be higher than one.

Exhibit 4:
Possible Combinations of Correlations in a Currency Trio



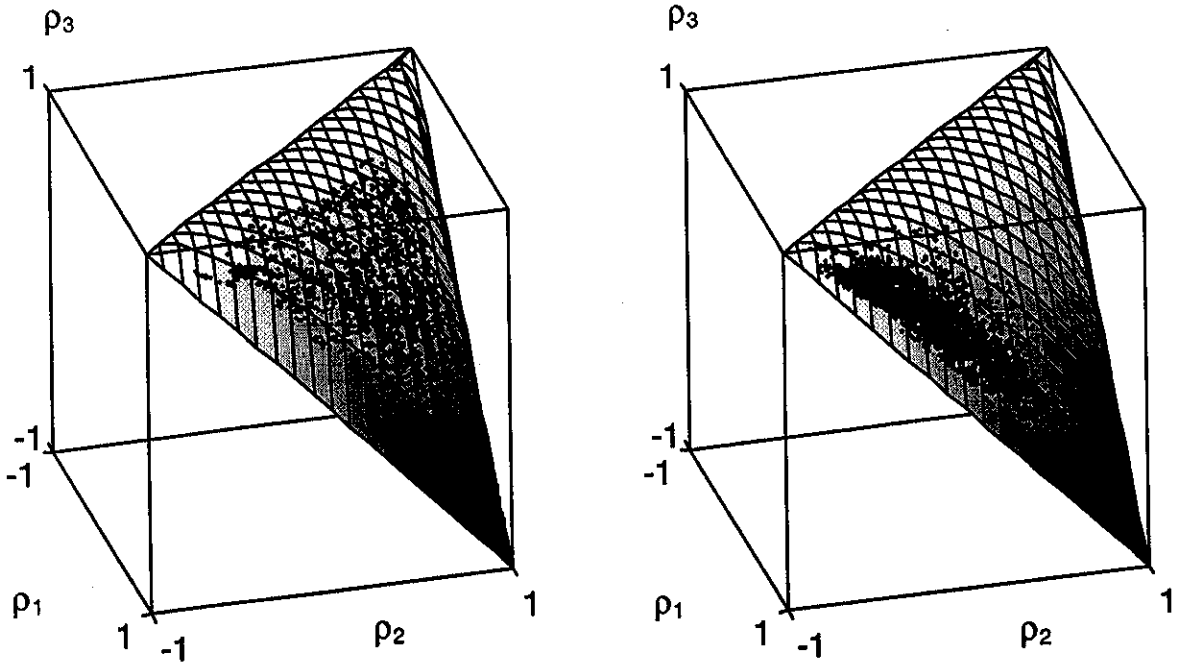
The following two exhibits show the locations of two sets of combinations of correlations on this surface. The first picture shows the one-month realized correlations in the currency trio USD/DEM/JPY from January 2, 1980 through February 21, 1997, where

$$\rho_1 = \rho(DEM_{USD}, JPY_{USD}),$$
$$\rho_2 = \rho(USD_{DEM}, JPY_{DEM}),$$
$$\rho_3 = \rho(USD_{JPY}, DEM_{JPY}).$$

The second picture shows the one-month implied correlations in the currency trio USD/DEM/CHF over the same period, where

$$\rho_1 = \rho(DEM_{USD}, CHF_{USD}),$$
$$\rho_2 = \rho(USD_{DEM}, CHF_{DEM}),$$
$$\rho_3 = \rho(USD_{CHF}, DEM_{CHF}).$$

Exhibit 5:
1-Month Realized Correlations in Currency Trios USD/DEM/JPY and USD/DEM/CHF



The interpretation of the correlations in a currency trio can be taken yet one step further. The above shown surface can be seen as a boundary of the set of all possible combinations of correlations in a valid 3-by-3 correlation matrix. To see this consider three times series $\{X_t, Y_t, Z_t\}$.

Their correlation matrix A has the form:

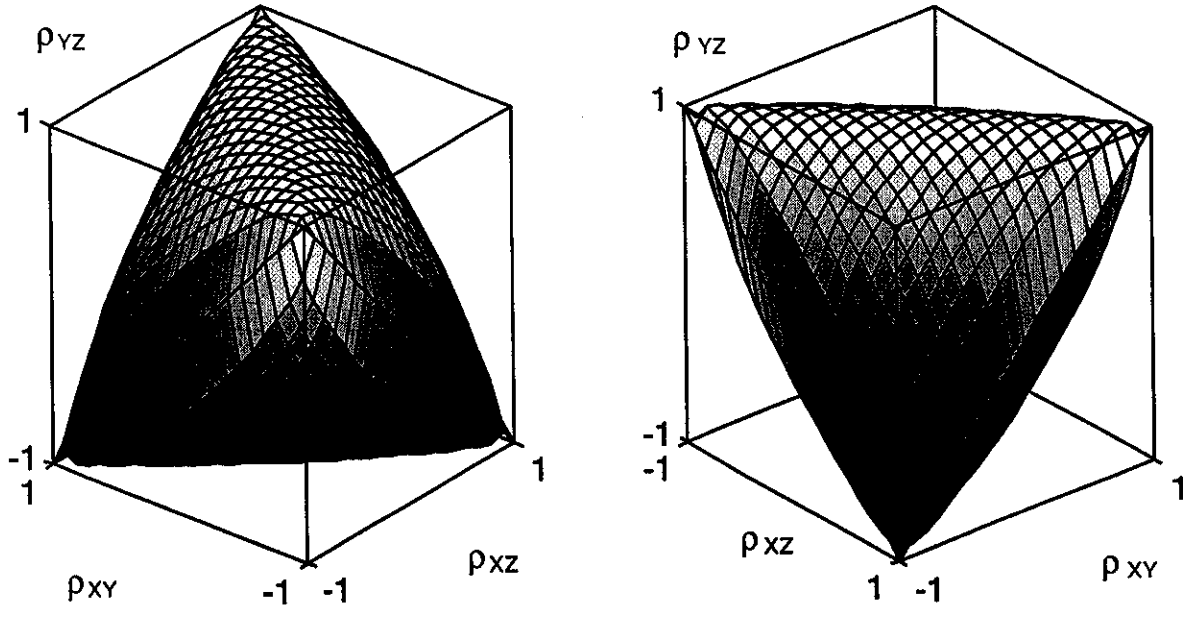
$$A = \begin{pmatrix} 1 & \rho_{XY} & \rho_{XZ} \\ \rho_{XY} & 1 & \rho_{YZ} \\ \rho_{XZ} & \rho_{YZ} & 1 \end{pmatrix}.$$

A is a correlation matrix if and only if it is positive semidefinite, or:

$$|A| = 1 + 2\,\rho_{XY}\,\rho_{XZ}\,\rho_{YZ} - \rho_{XY}^2 - \rho_{XZ}^2 - \rho_{YZ}^2 \geq 0.$$

That is, we cannot take any three numbers in [-1, 1] and expect that these numbers form a correlation matrix. The set of possible combinations that fulfill the above-stated condition can be represented graphically. Exhibit 6 shows the set of all possible 3-by-3 correlation matrices seen from two different perspectives.[4] Note that the surface of the convex body in three-dimensional space consists of the correlation matrices with a zero determinant and that strictly positive definite correlation matrices are represented by points inside the body.

Exhibit 6:
Possible Correlation Combinations in a 3-by-3 Correlation Matrix



[4] See Rousseeuw and Molenberghs (1994).

To see how the correlations in a currency trio relate to this surface, consider the following three time series:

X_t: log return series of the USD exchange rate of currency A (A_{USD}),

Y_t: log return series of the USD exchange rate of currency B (B_{USD}),

Z_t: log return series of the exchange rate A measured in units of currency B (A_B).

Since the return of a cross-rate is a linear combination of the returns of the two underlying exchange rates ($Z_t = X_t - Y_t$), the correlation matrix for he three series $\{X_t, Y_t, Z_t\}$ has a zero determinant, i.e.

$$\begin{vmatrix} 1 & \rho(A_{USD},B_{USD}) & \rho(A_{USD},A_B) \\ \rho(A_{USD},B_{USD}) & 1 & \rho(B_{USD},A_B) \\ \rho(A_{USD},A_B) & \rho(B_{USD},A_B) & 1 \end{vmatrix} = 0.$$

This implies that

$$\rho(A_{USD},B_{USD})^2 + \rho(A_{USD},A_B)^2 + \rho(B_{USD},A_B)^2 - 2\rho(A_{USD},B_{USD})\rho(A_{USD},A_B)\rho(B_{USD},A_B) = 1.$$

We can now locate the set of possible correlation combinations for three time series where $Z_t = X_t - Y_t$ on the surface of the convex body representing the set of possible correlation combinations for three variables. It corresponds to the part of the surface labeled "Area IV" in exhibit 7. The other areas correspond to the following linear relationships between X_t, Y_t, and Z_t:

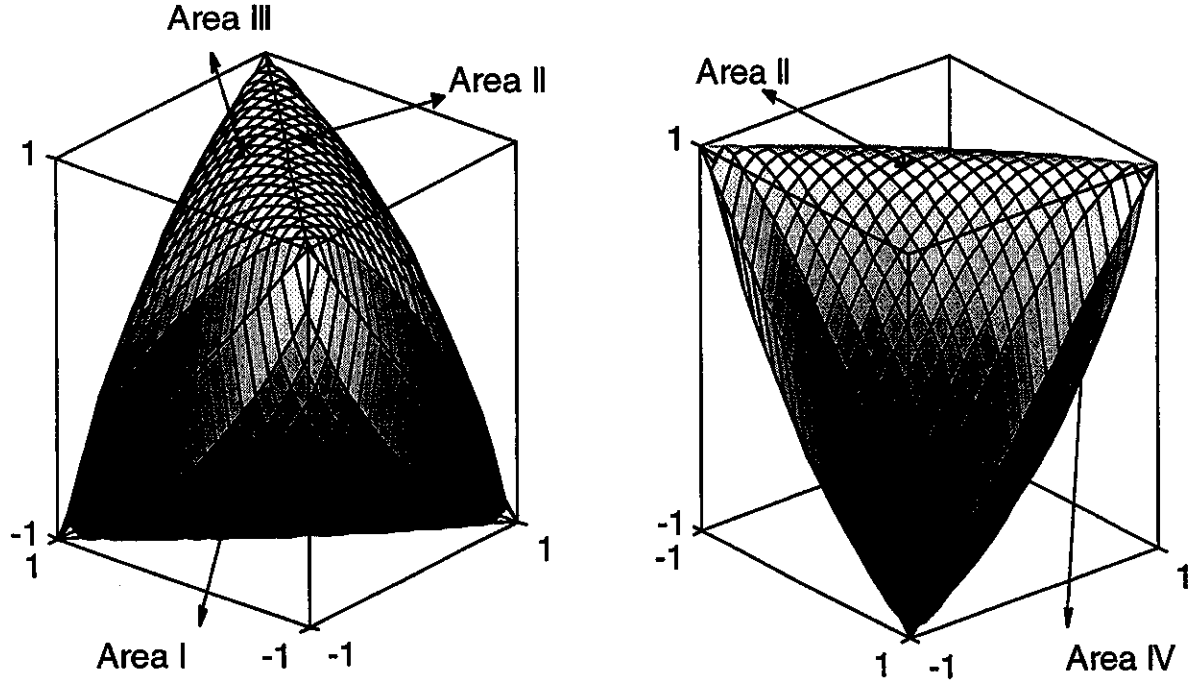
AREA I: $Z_t = -aX_t - bY_t$,

AREA II: $Z_t = -aX_t + bY_t$,

AREA III: $Z_t = aX_t + bY_t$,

where a and b are rational and positive numbers.

Exhibit 7:
Four Different Areas



Note, that the correlations in terms of the three underlying series $\{X_t, Y_t, Z_t\}$ are not fully consistent with our usual definition of the correlations in a currency trio. Our usual definition is based on the exchange rates between two of the three currencies measured in units of the third currency. The two definitions are equivalent for the correlation between currencies A and B measured in USD and the correlation between the USD and currency B measured in A [since $\rho(A_{USD}, A_B)=\rho(USD_A, B_A)$]. However, the third correlation used above [$\rho(B_{USD}, A_B)$] does not confirm to our definition of the correlation between the USD and currency A measured in B [$\rho(USD_B, A_B)$]. Since, in general, $\rho(X_Y, Z_Y) = -\rho(X_Y, Y_Z)$, the former equals the latter times minus one. Replacing $\rho(B_{USD}, A_B)$ with $-\rho(USD_B, A_B)$ gives the following condition.

$$\rho(A_{USD},B_{USD})^2 + \rho(USD_A,B_A)^2 + \rho(USD_B,A_B)^2$$
$$+ 2\rho(A_{USD},B_{USD})\rho(USD_A,B_A)\rho(USD_B,A_B) = 1.$$

This condition implies the set of possible combinations of correlations that is depicted in exhibits 4 and 5.

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