

INFLATION RISK IN THE U.S. YIELD CURVE:
THE USEFULNESS OF INDEXED BONDS

Frank F. Gong and Eli M. Remolona

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FRANK F. GONG and ELI M. REMOLONA*

*Federal Reserve Bank of New York
Capital Markets Function
New York, NY 10045
Tel.: (212) 720-6943 (Gong)
(212) 720-6328 (Remolona)
E-mail: frank.gong@frbny.sprint.com
eli.remolona@frbny.sprint.com*

Abstract

The inflation-indexed bonds the U.S. Treasury plans to issue will reduce the expected borrowing cost if the yield curve reflects a risk premium for inflation. In the United Kingdom, indexed bonds are also used to extract inflationary expectations and thus to guide monetary policy. The bonds will produce a more reliable measure of such expectations if the inflation risk premium is taken into account. We estimate such a risk premium for the United States by means of a two-factor affine-yield model of the term structure. The model allows both the inflation risk premium and real term premium to vary over time. Using monthly data on CPI inflation and on bond yields for two-year to ten-year maturities, we find both premia to have been significant for the sample period January 1984 to July 1996. We estimate that indexed bonds would have saved an average of one-fifth of the expected borrowing cost of 10-year notes.

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Inflation Risk in the U.S. Yield Curve: The Usefulness of Indexed Bonds

1. Introduction

The U.S. Treasury plans to issue inflation-indexed bonds, as a number of other governments have done.¹ We can expect the bonds to reduce the government's cost of borrowing if the nominal yield curve reflects a risk premium for inflation. In the United Kingdom, indexed bonds are also used to infer inflationary expectations and thus to guide monetary policy. In general, the bonds will produce a more reliable measure of inflationary expectations if the inflation risk premium is taken into account. In this paper, we propose a two-factor equilibrium model of the term structure that allows us to estimate such a risk premium.

Our model relies on two factors, one of which we specify as the expectation of inflation and the other we treat as a residual factor to represent other fundamentals. We use the model to derive arbitrage conditions that let us exploit information in yield movements across the term structure in extracting the inflation process perceived by market participants. In the model, heteroscedastic shocks to the factors are possible sources of risks priced by the market, allowing both inflation risk premia and real term premia to vary over time. The two factors suffice to provide a conditional decomposition of nominal

¹ Australia, Canada, and the United Kingdom are countries that have issued inflation-indexed bonds.

bond yields into four components: expectations of real rates, real term premia, expectations of inflation, and inflation risk premia.

To estimate the model, we apply a Kalman filter to monthly data on the inflation rate and on zero-coupon bond yields for two-year, five-year, and ten-year maturities. The estimation procedure allows us to exploit the conditional density of bond yields and inflation rates without requiring special assumptions about measurement error. The model's arbitrage conditions serve as over-identifying restrictions. We limit ourselves to fitting the two-year to ten-year range of the yield curve, because the effective durations of the proposed indexed bonds fall in this range. Moreover, Gong and Remolona (1996) suggest that a two-factor model works reasonably well in this range, whereas trying to fit the whole yield curve will require a more complicated model. We estimate the model for two sample periods, February 1970 to September 1979 and January 1984 to July 1996, treating them as two separate monetary regimes.

The model finds more success in capturing the perceived inflation process in the more recent period than in the earlier period. Inflation rates were much more volatile in the 1970s than in the latter part of the 1980s and the early 1990s, while bond yields were more volatile in the later period. Only for this later period do we estimate significant parameters for both the inflation process and real rate process. These estimates generate substantial variation in inflation risk and real term premia over time. We calculate that indexed bonds would have saved an average of one-fifth of the expected borrowing cost of 10-year notes issued during the period.

The paper is organized as follows: Section 2 provides background information on the U.S. and U.K. indexed bonds and on the importance of time-varying inflation risk and real term premia. Section 3 presents the two-factor equilibrium model. Section 4 explains the estimation procedure. Section 5 discusses the results. Section 6 suggests further research.

2. Indexed bonds and inflation risk

In the United States, the announced plan is to issue bonds indexed to the consumer price index (CPI), initially with a 10-year maturity.² The bonds are expected to carry lower coupon rates than nominal bonds but the principal amount will be raised by the percentage increase in the price index over each payment period. Both the coupons and the increase in principal will be taxed as interest. In principle, the difference between the coupons on the indexed bonds and those on nominal bonds will reflect not just the expected change in the price index but also the uncertainty regarding that change. The government can expect to save on borrowing costs only if removing such uncertainty serves to reduce the coupons on indexed bonds, or equivalently, if the coupons on the nominal bonds reflect a positive inflation risk premium.³ Since the present tax on nominal interest implies a tax on this risk premium, the indexed bonds will offer a tax advantage to the extent that the premium is positive.

The U.K. government has been issuing its own inflation-indexed bonds since 1981. Differences between yields on these bonds and those on nominal bonds have been used to extract estimates of inflation rates expected by market participants (e.g., Arak and Kreicher [1985], Levin and Copland [1993], Deacon and Derry [1994] and Barr and Campbell [1995]). The Bank of England has recently relied on such estimates to infer changes in inflationary expectations (King, 1995). The estimation efforts focus on accounting for the incomplete

² The Department of the Treasury (1996) calls the bonds "Treasury Inflation-Protection Securities (TIPS)." The proposed U.S. bonds are similar in structure to the Canadian indexed bonds. See also *The Wall Street Journal*, Sept. 26, 1996, p. C1.

³ One may argue that the issuer also saves money if inflation turns out to be less than expected. However, this outcome presents no savings in real terms. The holders of nominal bonds do gain, but the expectation of such gain is really a reward for holding inflation risk.

indexation arising from an eight-month lag between the index applied and time of maturity.⁴ These efforts have yet to account for a possible inflation risk premium between indexed yields and nominal yields.

In modeling the inflation risk premium, it is important to distinguish it from the real term premium and to allow it to vary over time. Fama (1990), Mishkin (1990), and Engsted (1995) find that the yield curve reflects expectations of inflation, especially at long maturities. At the same time, they find that variation in real rates or term premia obscures those expectations. Hence, Frankel and Lown (1994) extract inflationary expectations from the yield curve by letting the real interest rate vary in the short run but revert to a mean in the long run. Other studies confirm the significance of time-varying term premia in bond returns (e.g., Shiller, Campbell, and Schoenholtz [1983], Fama [1984], and Keim and Stambaugh [1986]). Until recently, the studies did not distinguish between real term premia and inflation risk premia. The distinction is important for indexed bonds, because the expected reduction in the issuer's borrowing cost comes only from inflation risk premia. Evans (1996) compares yields on U.K. nominal and indexed gilts and finds strong evidence of a time-varying inflation risk premium. Barr and Campbell (1995) point out that estimating such a risk premium requires an equilibrium model.

3. A two-factor, affine-yield model with inflation

Theoretical background

Theoretical work by Vasicek (1977) and Cox, Ingersoll, and Ross (1985, hereafter CIR) showed how the term structure at a moment in time would reflect regularities in interest rate movements over time. In particular, long maturity

⁴ The proposed U.S. indexed bond itself will have a three-month lag between the index and time of payment.

yields would depend on mean reversion in the short-term interest rate and on the risk associated with the volatility of that rate. The basic results relied on an arbitrage condition imposed on continuous-time processes. Sun (1992), Backus and Zin (1994), and Campbell, Lo, and MacKinlay (1994, hereafter CLM) derive the same results for discrete-time models by means of a stochastic discount rate process called the pricing kernel. Applying the same pricing kernel to price bonds of different maturities effectively imposes an equilibrium that admits no arbitrage. Specifying a discrete-time process from the outset avoids the pitfalls of estimating a continuous-time process with discrete-time data.⁵

We start with an arbitrage condition common to intertemporal asset pricing models.⁶ The price of a zero-coupon n -period bond can be written as

$$(1) \quad P_{nt} = E_t[M_{t+1}P_{n-1,t+1}]$$

where M_{t+1} is the stochastic discount factor and $P_{n-1,t+1}$ is the price of the same bond a period later.⁷ We can solve (1) forward to get $P_{nt} = E_t[M_{t+1} \dots M_{t+n}]$, which shows that we can model P_{nt} by modeling the stochastic process for M_{t+1} . To derive an *affine-yield* model, we assume that M_{t+1} is conditionally lognormal, so that we can take logs of (1) and write it as

$$(2) \quad p_{nt} = E_t(m_{t+1} + p_{n-1,t+1}) + \frac{1}{2} \text{Var}_t(m_{t+1} + p_{n-1,t+1}),$$

where lower-case letters denote logarithms. With two factors, x_{1t} and x_{2t} , an affine-yield model is one that can be written as

⁵ As Ait-Sahalia (1996) points out, the approximation of a continuous-time process by discretization methods is difficult to justify for monthly, weekly, or daily observations. The approximation is exact only in special cases (see Wong, 1964).

⁶ Singleton (1990) surveys such models and their empirical performance.

⁷ Evans (1996) relies entirely on (1) and finds strong evidence of a time-varying inflation risk premium. The additional structure imposed by our model allows us to measure such a premium.

$$(3) \quad -p_{nt} = A_n + B_{1n}x_{1t} + B_{2n}x_{2t}.$$

Since the n -period bond yield is $y_{nt} = -p_{nt}/n$, yields will in this case also be linear in the factors. Affine-yield models are a particularly tractable class of models encompassing most of the popular equilibrium models of the term structure, including those by Vasicek (1977), CIR (1985), Longstaff and Schwartz (1992), Backus and Zin (1994), and CLM (1994).⁸ We now proceed to specify a model that leads to (3).

A pricing kernel with inflationary expectations

In this paper, we assume that the pricing kernel is driven by two factors, one of which is the expectation of inflation. Gong and Remolona (1996) suggest that a two-factor model can reproduce the basic shapes of the term structure at least for certain ranges of the yield curve. In the present model, the conditional expectation of the negative of the log stochastic discount factor is given by

$$(4) \quad -m_{t+1} = x_{1t} + x_{2t} + w_{t+1}$$

where x_{1t} and x_{2t} are the factors and w_{t+1} is a shock related to risk. Each factor follows a stationary AR(1) process with a heteroscedastic shock based on a square-root process:

$$(5) \quad x_{1,t+1} = (1 - \phi_1)\mu_1 + \phi_1 x_{1t} + x_{1t}^{1/2} u_{1,t+1}$$

$$(6) \quad x_{2,t+1} = (1 - \phi_2)\mu_2 + \phi_2 x_{2t} + x_{2t}^{1/2} u_{2,t+1},$$

where $(1 - \phi_1)$ and $(1 - \phi_2)$ are rates of mean reversion, μ_1 and μ_2 are the long-run means, and $u_{1,t+1}$ and $u_{2,t+1}$ are shocks with mean zero and volatilities σ_1 and σ_2 . We assume these shocks are uncorrelated. Up to this point, the model

⁸ Duffie and Kan (1993) establish the conditions for affine yields.

is essentially the same as the two-factor model discussed by CLM (1994). Now we explicitly specify the factor x_{1t} as representing the expectation of inflation based on the *perceived* inflation process by bond market participants. Without loss of generality, the factor x_{2t} will then represent fundamentals that drive the real discount rate such that their shocks are orthogonal to those of inflationary expectations.⁹

To model both inflation risk premia and real term premia, we specify the shock to the log stochastic discount factor as

$$(7) \quad w_{t+1} \equiv \lambda_1 x_{1t}^{1/2} u_{1,t+1} + \lambda_2 x_{2t}^{1/2} u_{2,t+1}$$

where λ_1 represents the market price of inflation risk and λ_2 the market price of real risk. We expect both λ_1 and λ_2 to be negative to produce positive inflation risk and real term premia. The model assumes that shocks to the factors are sources of risks that are priced by the bond market. Borrowing from CIR (1985), we specify the volatilities of these shocks to be proportional to the square-root of the respective factors. Such square-root processes induce time-varying risk premia while keeping yields linear in the factors.

The perceived inflation process

For our purposes, it is more important to model the market's perception of the inflation process than to model the true inflation process. Nonetheless it is also important that the perception bear some relation to reality. For this paper, we limit ourselves to a fairly simple inflation process. To derive the process specified in (5), suppose the market perceives the CPI inflation rate as a stationary AR(1) process:

⁹ Such fundamentals are likely to include real activity; see Estrella and Hardouvelis (1991) and Estrella and Mishkin (1995).

$$(8) \quad \pi_{t+1} = (1-\theta)\eta + \theta\pi_t + \varepsilon_{t+1}$$

where ε_{t+1} represents an unanticipated inflation shock with a mean of zero.¹⁰

We then have

$$(9) \quad x_{1t} \equiv E_t(\pi_{t+1}) = (1-\theta)\eta + \theta\pi_t .$$

We then write out $x_{1,t+1} \equiv E_{t+1}(\pi_{t+2})$ and substitute (8) and (9) to get

$x_{1,t+1} = (1-\theta)\eta + \theta x_{1t} + \theta\varepsilon_{t+1}$. Under rational expectations, $\theta = \phi_1$, $\eta = \mu_1$, and $\theta\varepsilon_{t+1} \equiv x_{1t}^{1/2} u_{1,t+1}$, so that we arrive at (5). For subsequent estimation purposes, it will be useful to also express (9) as

$$(10) \quad \pi_t = -\frac{1}{\phi_1}(1-\phi_1)\mu_1 + \frac{1}{\phi_1}x_{1t} ,$$

in which the observed inflation rate has the form of an affine yield. In the estimation procedure, (10) serves to identify x_{1t} as the expectation of inflation.¹¹

Arbitrage restrictions

The importance of the model lies in the arbitrage conditions it imposes on yield movements across the term structure. To derive these conditions, we start with the normalization $\log P_{0t} \equiv p_{0t} = 0$ to recognize that at maturity a bond trades at par. By applying (2), the short rate or one-period yield can then be shown to be

¹⁰ We are aware of studies suggesting that U.S. inflation is a nonstationary process. In particular, the standard augmented Dickey-Fuller test tends not to reject the null hypothesis of nonstationarity. However, the Johansen test tends not to reject the null hypothesis of stationarity either (see Engsted, 1995). Both tests have low power and the results are not robust to sample periods. In our model, nonstationarity leads to negative long-term yields, which is patently inconsistent with the data. Hence, we must assume that the market perceives a stationary process.

¹¹ An alternative way of identifying the factor is to use forecast data. However, Laster, Bennett, and Geoum (1996) suggest that forecasters have an incentive to make forecasts that deviate from their conditional expectation.

$$\begin{aligned}
(11) \quad y_{1t} &= -p_{1t} = -E_t(m_{t+1}) - \frac{1}{2} \text{Var}_t(m_{t+1}) \\
&= x_{1t} + x_{2t} - \frac{1}{2} \lambda_1^2 x_{1t} \sigma_1^2 - \frac{1}{2} \lambda_2^2 x_{2t} \sigma_2^2
\end{aligned}$$

which is linear in the factors, as in (3), with coefficients $A_1 = 0$, $B_{11} = 1 - \lambda_1^2 \sigma_1^2 / 2$, and $B_{21} = 1 - \lambda_2^2 \sigma_2^2 / 2$. We can next verify that in general the log price and yield of an n -period bond are also linear in the factors with the coefficients restricted by the recursive equations (see Appendix A):

$$\begin{aligned}
(12) \quad A_n &= A_{n-1} + (1 - \phi_1) \mu_1 B_{1,n-1} + (1 - \phi_2) \mu_2 B_{2,n-1} \\
B_{1n} &= 1 + \phi_1 B_{1,n-1} - \frac{1}{2} (\lambda_1 + B_{1,n-1})^2 \sigma_1^2 \\
B_{2n} &= 1 + \phi_2 B_{2,n-1} - \frac{1}{2} (\lambda_2 + B_{2,n-1})^2 \sigma_2^2
\end{aligned}$$

The coefficients B_{1n} and B_{2n} are factor loadings. The coefficient A_n represents the pull of the factors to their respective means. These coefficients can be derived by using (11) to set initial values and applying (12) recursively. The equations represent cross-sectional restrictions to be satisfied by eight parameters: the rates of mean reversion $1 - \phi_1$ and $1 - \phi_2$, the means μ_1 and μ_2 , the prices of risk λ_1 and λ_2 , and the volatilities σ_1 and σ_2 . Note that linearity in the two AR(1) factors results in yields that are ARMA(2,1) processes and conditional yield volatilities that are GARCH processes.

Inflation risk and real term premia

We can derive inflation risk premia and real term premia as components of the expected excess return on an n -period bond:

$$\begin{aligned}
(13) \quad E_t(p_{n-1,t+1}) - p_{nt} - y_{1t} &= -\lambda_1 B_{1,n-1} \sigma_1^2 x_{1t} - \frac{1}{2} B_{1,n-1}^2 \sigma_1^2 x_{1t} \\
&\quad - \lambda_2 B_{2,n-1} \sigma_2^2 x_{2t} - \frac{1}{2} B_{2,n-1}^2 \sigma_2^2 x_{2t}
\end{aligned}$$

where the terms with x_{1t} represent the inflation risk premium and the terms with x_{2t} the real term premium. The two terms *not* containing λ_1 or λ_2 represent Jensen's inequality. Since these terms are negative, the prices of risk, λ_1 and λ_2 , need to be sufficiently negative to produce positive expected excess bond returns. The expression shows that inflation risk and real term premia will depend on the bond's maturity and will vary over time with the respective factors.

4. Estimation with conditional moments

Econometric approach

To estimate the model, we apply a Kalman filter to data on CPI inflation and bond yields for three maturities. The model lends itself to this estimation procedure because the factors are not directly observable. The Kalman filter is a maximum likelihood procedure that exploits the conditional density of the observable variables to extract the unobservable variables. In our application of the procedure we impose the model's arbitrage conditions as over-identifying cross-sectional restrictions. We also include a measurement equation that links actual inflation to its expectation.

Other maximum likelihood procedures also allow the use of conditional moments to estimate term structure models but these procedures require special assumptions about measurement errors. Chen and Scott (1993) estimate a one-factor model, a two-factor model, and a three-factor model by maximizing a likelihood based on the factors' conditional moments. To derive the factors, however, their procedure requires the arbitrary assumption of zero measurement error for as many yields as they have factors. Pearson and Sun (1994) also exploit the factors' conditional density in estimating a two-factor CIR (1985) model.

They derive the factors by using two yields at a time and assume no measurement error for either yield.

The Kalman filter provides a way to exploit the conditional moments of bond yields and inflation rates while allowing measurement errors for these variables. The measurement errors may arise from bond mispricing, smoothing errors in fitting the yield curve, or poor model specification. In the case of the inflation rate, measurement error may also arise from the fact that the index represents a monthly average while the yields are based on end-of-month bond prices. Jegadeesh and Pennacchi (1996) and Gong and Remolona (1996) have implemented the Kalman filter procedure for two-factor models.

Data and summary statistics

We limit ourselves to fitting the two-year to ten-year range of the yield curve, because the effective durations of the proposed indexed bonds fall in this range. Moreover, Gong and Remolona (1996) suggest that a two-factor model works reasonably well in this range, whereas trying to fit the whole yield curve will require a more complicated model. We estimate the model for two sample periods, February 1970 to September 1979 and January 1984 to July 1996, which we consider as two separate monetary regimes.

We obtain end-of-month zero-coupon yield data from McCulloch and Kwon (1993) for 1970:2 to 1979:9 and 1984:1 to 1990:12 and from the Federal Reserve Bank of New York for 1991:1 to 1996:7. In the case of the Federal Reserve data, each zero curve is generated by fitting a cubic spline to prices and maturities of about 160 outstanding coupon-bearing U.S. Treasury securities. The securities are limited to off-the-run Treasuries to eliminate the most liquid securities and reduce the possible effect of liquidity premia. Fisher, Nychka, and Zervos (1995) explain the procedure in detail. We also collect seasonally unadjusted CPI data for 1970:1 to 1996:7 and 1983:12 to 1996:7. Inflation rates are

calculated as the change in the log index from one month to the next and are annualized by multiplying by 12.

Summary statistics for the monthly CPI inflation rate and the zero-coupon yields for maturities of 2 years, 5 years, and 10 years for the two sample periods are reported in Table 1. The average inflation rate in the first sample period is almost double the average in the second period. The standard deviation of the inflation rate is also much higher in the earlier period. The first-order autocorrelation of monthly inflation is 0.44 in the first sample and 0.36 in the second sample. In contrast to inflation rates, bond yields are more volatile in the second period than in the first. The average yield spread between 2-year and 10-year maturities is higher in the second sample than in the first sample. The volatility curve is downward sloping and is flatter in the second period than in the first. All the yields are very persistent, with first-order monthly autocorrelations of 0.96-0.98.

Kalman filtering and maximum likelihood estimation

In our application of the Kalman filter, the yields and inflation rate as affine functions of the factors serve as the measurement equations and the factors' stochastic processes as the transition equations. Thus we write the model in linear state-space form, with the measurement equations

$$(14) \quad \begin{bmatrix} \pi_t \\ y_{lt} \\ y_{mt} \\ y_{nt} \end{bmatrix} = \begin{bmatrix} a_\pi \\ a_l \\ a_m \\ a_n \end{bmatrix} + \begin{bmatrix} b_\pi & 0 \\ b_{1l} & b_{2l} \\ b_{1m} & b_{2m} \\ b_{1n} & b_{2n} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \\ v_{3t} \\ v_{4t} \end{bmatrix}$$

where y_{lt} , y_{mt} , and y_{nt} are zero-coupon yields at time t with maturities l , m , and n for which we use 2-year, 5-year and 10-year maturities. For π_t , we use the

monthly CPI inflation rate. The coefficients are $a_\pi = -\frac{(1-\phi_1)\mu_1}{\phi_1}$, $b_\pi = \frac{1}{\phi_1}$,

$a_k = \frac{A_k}{k}$, $b_{1k} = \frac{B_{1k}}{k}$, and $b_{2k} = \frac{B_{2k}}{k}$, $k = l, m, n$. The coefficients A_k , B_{1k} , and B_{2k} are given by (12). The v 's are measurement errors distributed with zero mean and standard deviations e_1, e_2, e_3 , and e_4 .

Note that the first measurement equation is

$$(15) \quad \pi_t = -\frac{1}{\phi_1}(1-\phi_1)\mu_1 + \frac{1}{\phi_1}x_{1t} + v_{1t},$$

which links the observed inflation rate to the unobserved expectation while allowing for measurement error. Pennacchi (1991) uses survey data to identify the inflation process, while we use actual inflation data.

The transition equations correspond to (5) and (6):

$$(16) \quad \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \begin{bmatrix} (1-\phi_1)\mu_1 \\ (1-\phi_2)\mu_2 \end{bmatrix} + \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \begin{bmatrix} x_{1,t-1}^{1/2}u_{1,t} \\ x_{2,t-1}^{1/2}u_{2,t} \end{bmatrix},$$

where the shocks u_{1t} and u_{2t} are distributed normally with mean zero and standard errors σ_1 and σ_2 .

In standard linear state-space models, no restrictions link the measurement equations and transition equations. In our term-structure model, however, the arbitrage conditions serve as over-identifying restrictions that link the coefficients of the measurement equations (14) to those of the transition equations (16). These restrictions are given by (12) with initial values set by (11).

After putting the restrictions into the measurement equations the model is estimated by maximum likelihood using the Kalman filter's conditional updating procedure. Since the measurement errors prevent us from solving the measurement equations to derive the factors directly, the likelihood function is

based on the conditional density of the yields and inflation rate rather than the factors. The algorithm is discussed in Appendix B.

5. Inflation risk and real term premia

Estimates for the two sample periods

The estimates suggest that the two-factor model succeeds in capturing the perceived inflation process in the January 1984 to July 1996 period but not the process in the February 1970 to September 1979 period. Table 2 reports the parameter estimates and corresponding standard errors for the two sample periods.¹² For the earlier period, the point estimates of the parameters of the inflation process are quite plausible, but the standard errors suggest that the estimates are not reliably different from zero. The period was characterized by relatively volatile inflation rates but relatively stable bond yields. The model does seem to capture the real rate process well, with statistically significant estimates for each of the parameters λ_2 , σ_2 , μ_2 , and $1 - \phi_2$. The model finds more success with the data for the more recent period, providing significant parameter estimates for both the inflation process and real rate process. In particular, the price of risk λ_1 , the volatility σ_1 , and the mean reversion rate $1 - \phi_1$ are statistically significant, although the long-run mean μ_1 is not. The corresponding parameters for the real rate process are all significant. Hence, from here on, we will focus only on the estimates for this later period.

Mean reversion

The salient feature of the estimated inflation process is the extraordinarily slow rate of mean reversion. Our estimate for $1 - \phi_1$ implies an expected

¹² We also estimated the model using the PPI instead of CPI to measure inflation. We do not report the results based on PPI data, but the parameter estimates for the inflation process were similar.

inflation rate that reverts to its mean at the rate of 0.02 percent a month for a mean half life of 288 years.¹³ At the same time, the mean is imprecisely estimated, which tends to be the case when mean reversion is slow. This rate of mean reversion implies much more persistence than that implied by the monthly autocorrelation of the inflation rate or bond yields when taken individually (see Table 1). Nonetheless, such a slow mean reversion seems consistent with Backus and Zin's (1993) characterization of inflation as a long-memory process. Moreover, such slow mean reversion seems necessary to capture the relatively flat slope of the yield curve near the long end.¹⁴ The other factor reverts much faster to its mean. Its estimated mean reversion rate is 5.54 percent a month for a mean half life of a year. These mean reversion rates help determine the term structure of inflation risk premia and real term premia.

Time-varying premia

The significant estimates for the prices of risk λ_1 and λ_2 and the volatilities σ_1 and σ_2 induce time variation in the inflation risk and real term premia. Charts 1a and 1b plot these premia for five-year and ten-year maturities from January 1984 to July 1996. These premia are calculated from conditional estimates of x_{1t} and x_{2t} , which are backed out from the observed bond yields as well as the observed inflation rate. For five-year bonds, the inflation risk premium varies from about 3% to under 1% during the period, while the real term premium varies from nearly 2% to 0.5%. For ten-year bonds, the inflation risk premium ranges from about 1.5% to less than 0.5%, while the real term premium ranges from about 1.6% to less than 0.5%. Such time variation is substantial. The inflation risk premium in some periods can be three times the premium in other periods. The real term premium in some periods can be four

¹³ Such a slow rate of mean reversion is not unheard of: Chen and Scott (1993) estimate a mean half life of 771 years for one of the factors in a two-factor model.

¹⁴ See Gong and Remolona (1996).

times the premium in other periods. These premia also tend to be smaller for longer maturities, reflecting the risk dampening effect of mean reversion.

The savings from indexed bonds

We can calculate how much the U.S. Treasury would have saved had it been issuing the proposed indexed bonds. In this calculation, we need to adjust for the fact that the model is specified in terms of zero-coupon bonds while actual bonds bear coupons that serve to shorten the bonds' durations. Chart 2 illustrates how we calculate the inflation risk premium on a coupon-bearing 10-year note. First, we take the note's yield at issuance and find the effective maturity for such a yield on the zero-coupon yield curve for that period. We then use the model to estimate the inflation risk premium at that maturity. Chart 3 shows the coupons on the 10-year notes issued from February 1984 to November 1995 and the estimated savings in inflation risk premia had the notes been issued in the form of indexed bonds. The savings vary depending on the month of issuance, but in general they are substantial. On average, indexed bonds would have saved the U.S. Treasury about 20 percent of the expected borrowing cost.

6. Conclusion

We believe the two-factor model we propose is the bare minimum needed to estimate a time-varying inflation risk premium. Such a model is useful because it allows us to exploit conditional information from bond yields across the term structure to extract the parameters of the inflation process perceived by market participants. These parameters lead to estimates of inflation risk premia and thus of the savings in borrowing cost that can be expected from the issuance of inflation-indexed bonds. Our estimates for the period from February 1984 to

July 1996 provide a sense of how inflation risk premia and real term premia may vary over time. These premia also suggest that had the U.S. Treasury been issuing indexed bonds during the period, the savings would have averaged about a fifth of the borrowing cost of the 10-year notes issued.

The major limitation of the proposed two-factor model may be the assumption of a fixed long-run mean for the inflation process. Hence, the model would be inappropriate for gauging the possible effects of monetary policy on long-run inflationary expectations. Indeed the model did not work so well for the 1970s, when changing inflationary expectations were probably more consequential than in the latter part of the 1980s or the early 1990s. If such changes in expectations have an important bearing on inflation risk, the model may also provide misleading estimates of inflation risk premia. In these conditions, a model with a time-varying mean for long-run inflation may be more appropriate. Such a mean would represent an extra factor, a more complicated model, and a much more involved estimation procedure. We think, however, that the possible gains would be worth the effort.

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Table 1: Summary Statistics**Sample February 1970 to September 1979**

Variable	Mean	Standard Deviation	First Order Autocorrelation
CPI Inflation Rate	7.03%	4.08%	0.44
2-Year Bond Yield	6.92%	1.29%	0.96
5-Year Bond Yield	7.19%	0.95%	0.96
10-Year Bond Yield	7.40%	0.84%	0.98

Sample January 1984 to July 1996

Variable	Mean	Standard Deviation	First Order Autocorrelation
CPI Inflation Rate	3.48%	2.50%	0.36
2-Year Bond Yield	7.11%	2.00%	0.97
5-Year Bond Yield	7.77%	1.87%	0.97
10-Year Bond Yield	8.24%	1.71%	0.97

Table 2: Parameter Estimates

Parameters	Sample 70:2-79:9	Sample 84:1-96:7
ϕ_1	0.992 (0.010)	0.9998* (0.0001)
ϕ_2	0.907* (0.006)	0.9446* (0.0078)
μ_1	4.528 (2.336)	3.211 (7.841)
μ_2	7.300* (0.758)	11.783* (1.147)
λ_1	-7.283 (9.335)	-11.380* (1.232)
λ_2	-8.070* (0.365)	-8.169* (0.001)
σ_1	0.180 (0.262)	0.095* (0.013)
σ_2	0.174* (0.004)	0.173* (0.000)
Standard Deviation of Measurement Errors		
e_1	4.16	5.31
e_2	0.016	0.011
e_3	0.114	0.084
e_4	0.451	0.869
Mean Log Likelihood	-1.30	-2.35

Note: Asterisks indicate statistical significance at the 5% level. In the case of ϕ_1 and ϕ_2 , we indicate significant difference from one instead of zero.

Chart 1a

Inflation Risk and Real Term Premia for 5-year Bonds

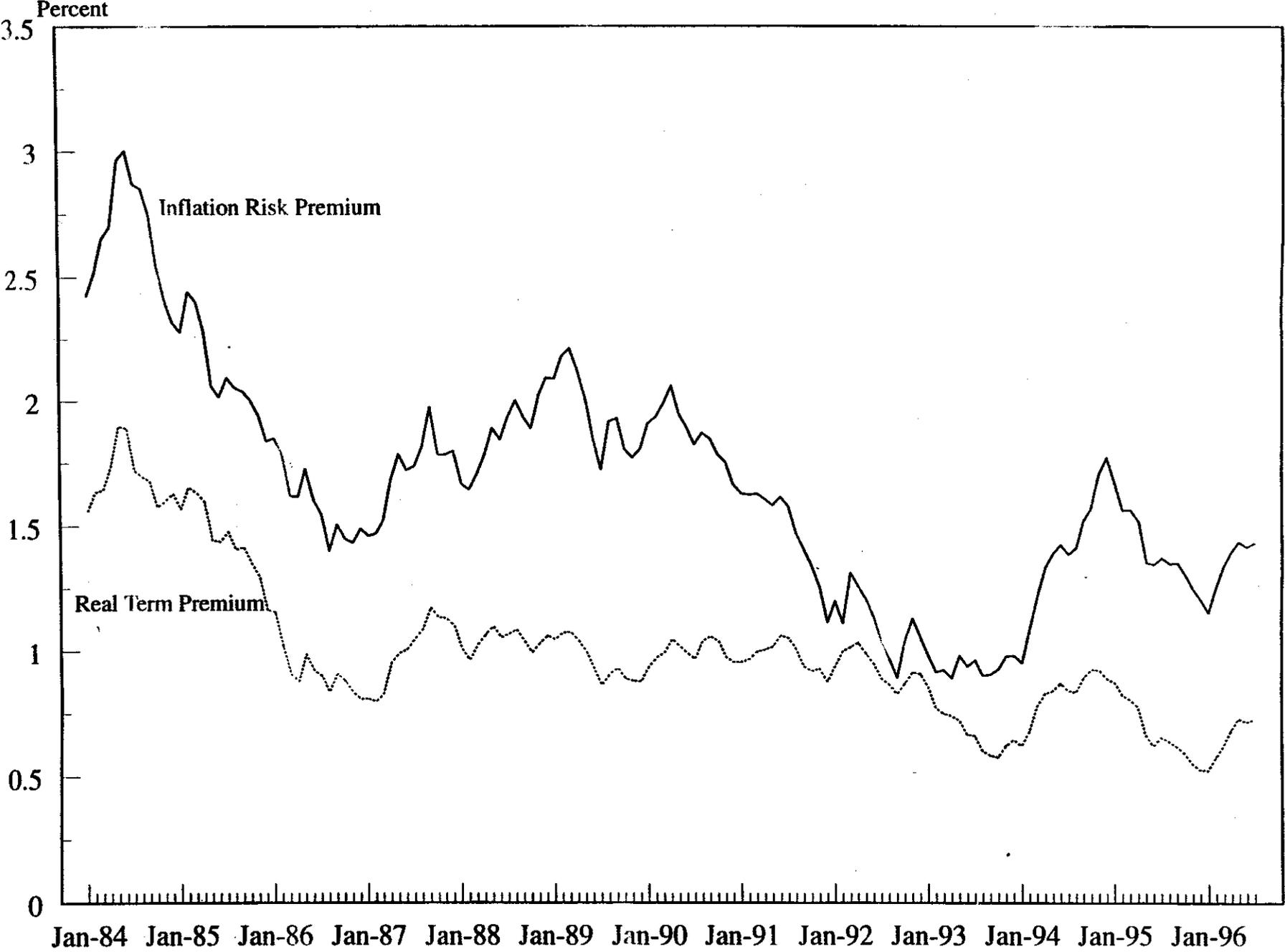


Chart1b

Inflation Risk and Real Term Premia for 10-year Bonds

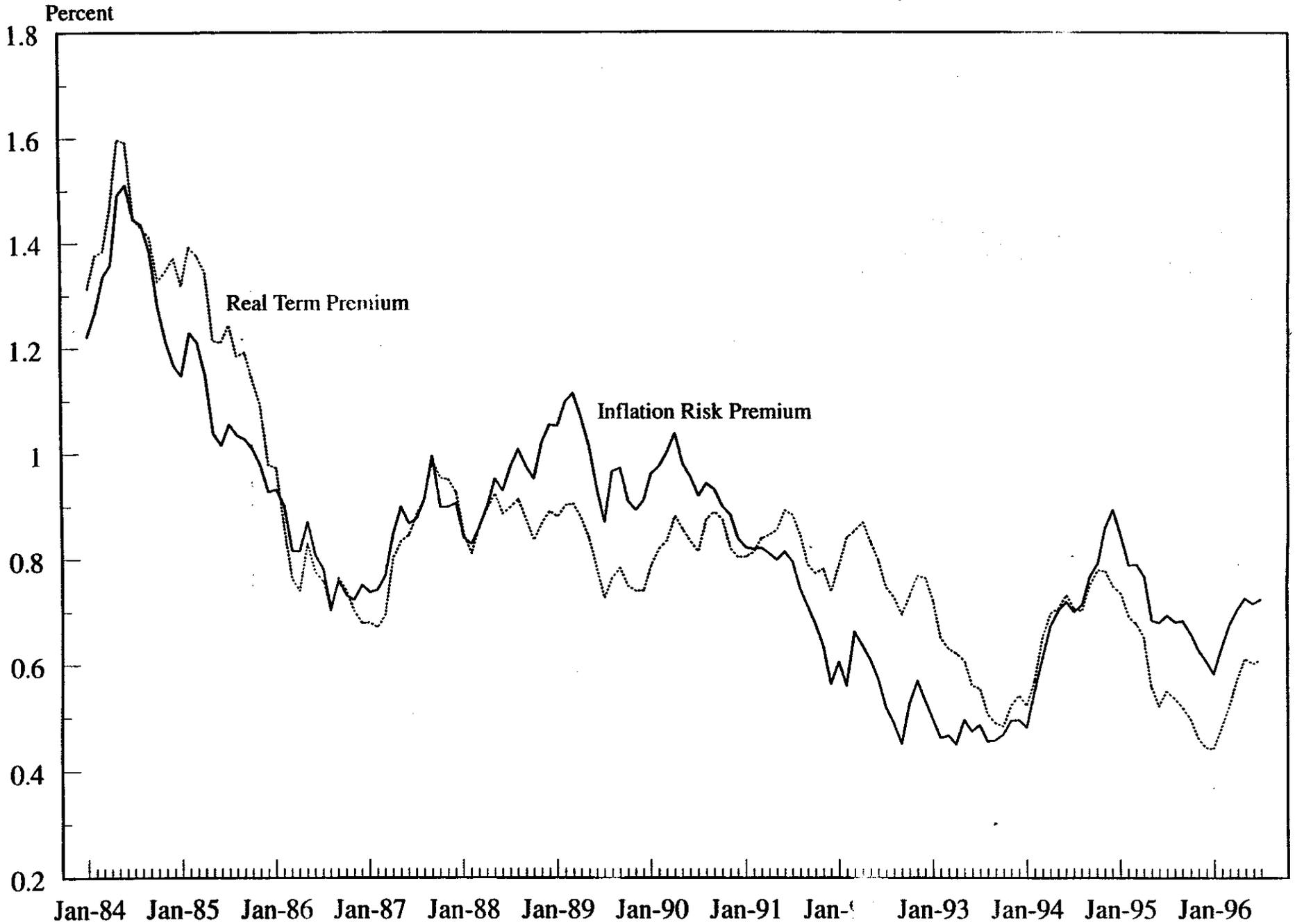


Chart 2

Calculating the Inflation Risk Premium for a 10-year Note

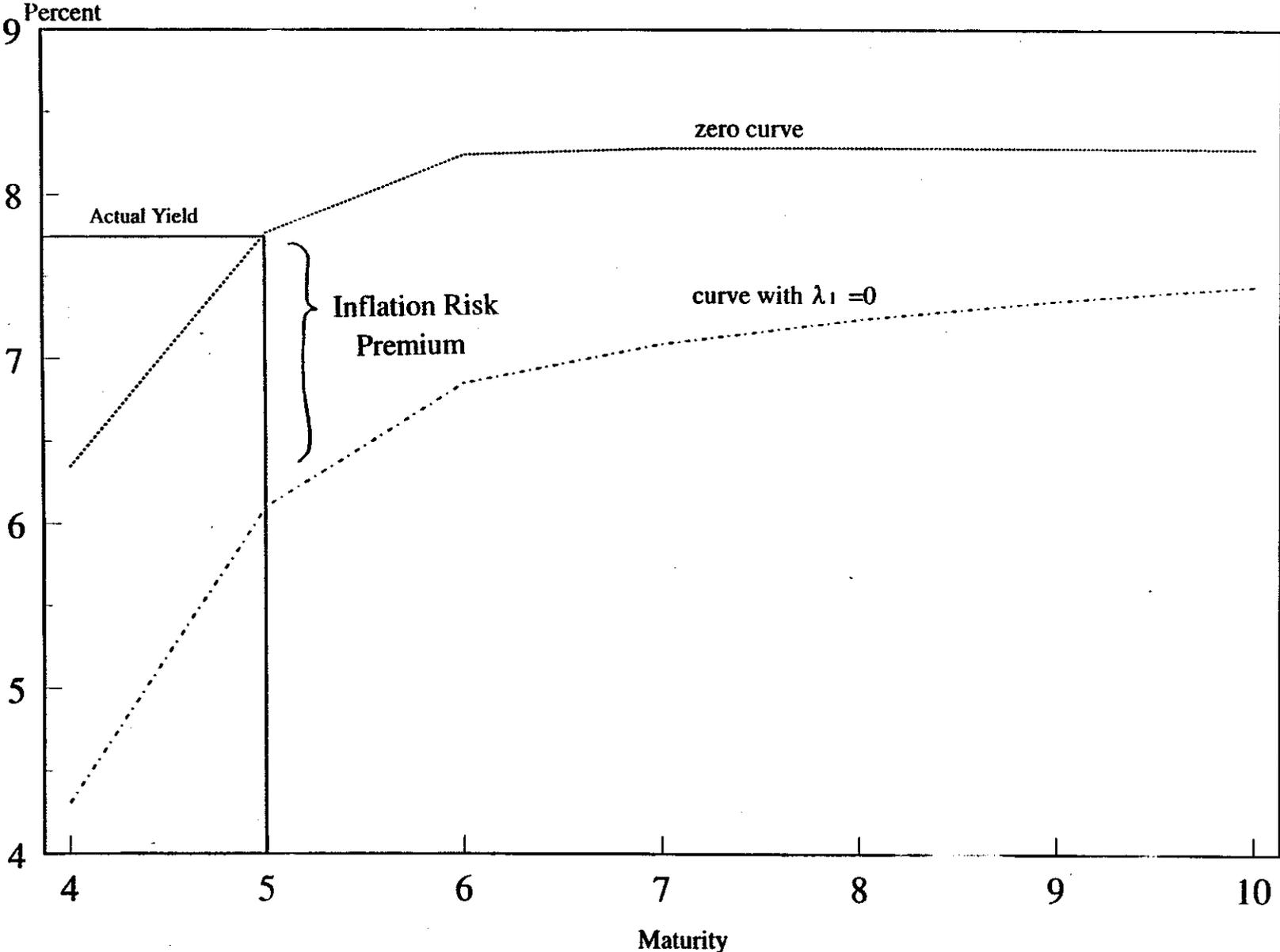
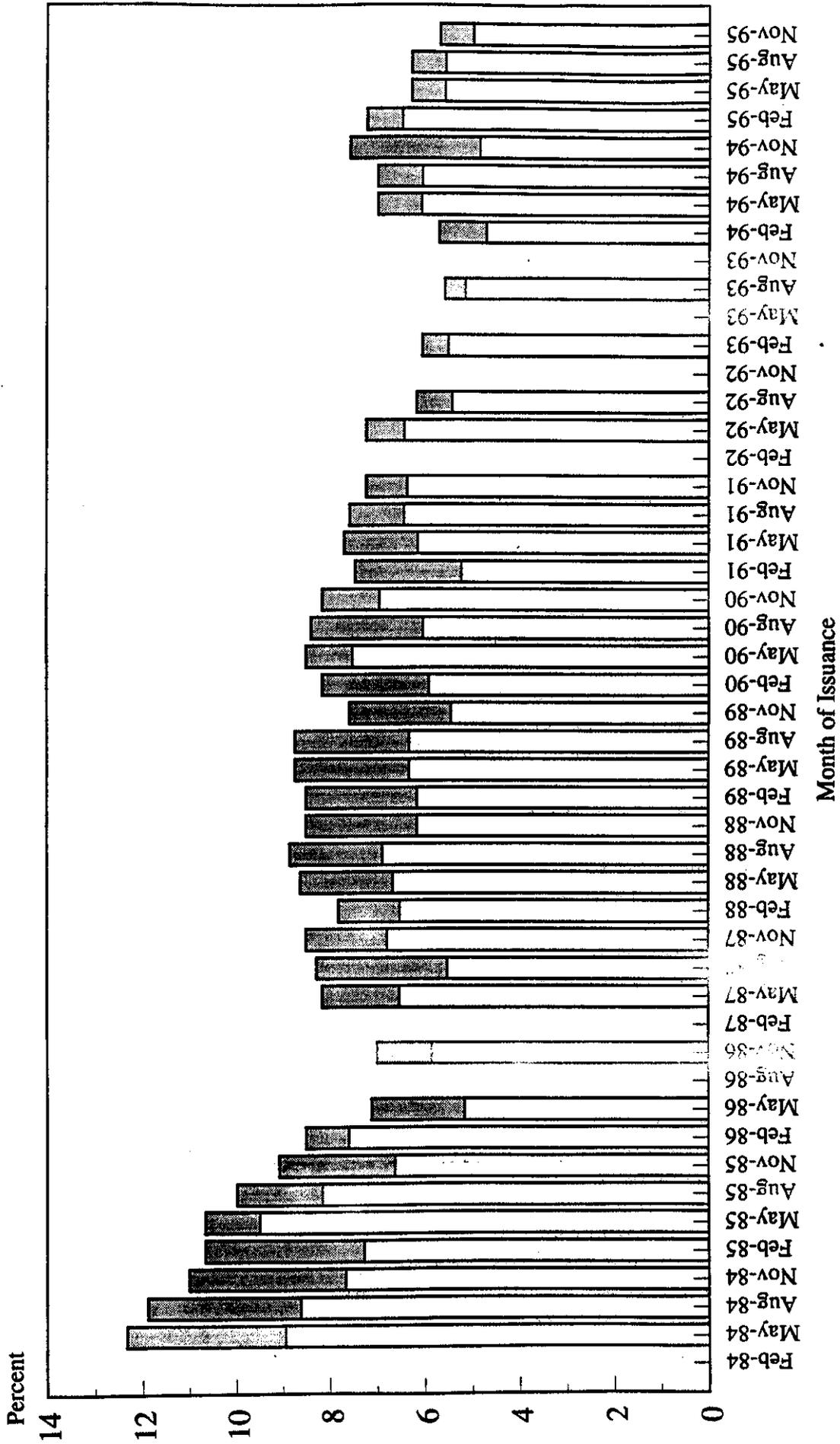


Chart 3

Coupons on 10-year U.S. Treasury Notes and Implied Inflation Risk Premia



Note: Coupons are expressed in terms of a continuously compounded rate. Shaded portions represent implied inflation risk premia. The average premium as a percent of the coupon rate is 20.1%

Appendix A: Recursive Restrictions

We start with the general pricing equation:

$$p_{nt} = E_t(m_{t+1} + p_{n-1,t+1}) + \frac{1}{2} \text{Var}_t(m_{t+1} + p_{n-1,t+1})$$

The short rate is derived by setting $p_{0,t} = 0$:

$$\begin{aligned} y_{1t} = -p_{1t} &= -E_t(m_{t+1}) - \frac{1}{2} \text{Var}_t(m_{t+1}) \\ &= x_{1,t} + x_{2,t} - \frac{1}{2} \lambda_1^2 \sigma_1^2 x_{1,t} - \frac{1}{2} \lambda_2^2 \sigma_2^2 x_{2,t}, \end{aligned}$$

showing the short rate to be linear in the factors.

Now we guess that the price of an n -period bond is affine:

$$-p_{n,t} = A_n + B_{1,n}x_{1,t} + B_{2,n}x_{2,t}$$

We verify that there exist A_n , $B_{1,n}$, and $B_{2,n}$ that satisfy the general pricing equation:

$$\begin{aligned} -p_{n,t} &= -E_t(m_{t+1} + p_{n-1,t+1}) - \frac{1}{2} \text{Var}_t(m_{t+1} + p_{n-1,t+1}) \\ &= (A_{n-1} + (1 - \phi_1)\mu_1 B_{1,n-1} + (1 - \phi_2)\mu_2 B_{2,n-1}) \\ &\quad + (1 + \phi_1 B_{1,n-1} - \frac{1}{2}(\lambda_1 + B_{1,n-1})^2 \sigma_1^2)x_{1,t} \\ &\quad + (1 + \phi_2 B_{2,n-1} - \frac{1}{2}(\lambda_2 + B_{2,n-1})^2 \sigma_2^2)x_{2,t} \end{aligned}$$

Now by matching coefficients we have

$$\begin{aligned} A_n &= A_{n-1} + (1 - \phi_1)\mu_1 B_{1,n-1} + (1 - \phi_2)\mu_2 B_{2,n-1} \\ B_{1,n} &= 1 + \phi_1 B_{1,n-1} - \frac{1}{2}(\lambda_1 + B_{1,n-1})^2 \sigma_1^2 \\ B_{2,n} &= 1 + \phi_2 B_{2,n-1} - \frac{1}{2}(\lambda_2 + B_{2,n-1})^2 \sigma_2^2 \end{aligned}$$

Appendix B: The Kalman Filter Algorithm

For the state-space models in section 4, the measurement and transition equations can be written in the following matrix form:

Measurement Equation:

$$y_t = A + BX_t + v_t$$

where $v_t \sim N(0, R)$.

Transition Equation:

$$X_{t+1} = C + FX_t + u_{t+1}$$

where $u_{t+1|t} \sim N(0, Q_t)$.

The Kalman filter algorithm of this state-space model is the following:

1. Initialize the state-vector S_t :

The recursion begins with a guess $S_{1|0}$, usually given by

$$\hat{S}_{1|0} = E(S_1).$$

The associated *MSE* is

$$\begin{aligned} P_{1|0} &\equiv E[(S_1 - \hat{S}_{1|0})(S_1 - \hat{S}_{1|0})'] \\ &= \text{Var}(S_1). \end{aligned}$$

The initial state S_1 is assumed to be $N(\hat{S}_{1|0}, P_{1|0})$.

2. Forecast y_t :

Let I_t denote the information set at time t . Then

$$\begin{aligned} \hat{y}_{t|t-1} &= A + BE[S_t | I_{t-1}] \\ &= A + B\hat{S}_{t|t-1}. \end{aligned}$$

The forecasting *MSE* is

$$E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})'] = BP_{t|t-1}B' + R$$

3. Update the inference about S_t given I_t :

Knowing y_t helps to update $S_{t|t-1}$ by the following:

Write

$$\begin{aligned} S_t &= \hat{S}_{t|t-1} + (S_t - \hat{S}_{t|t-1}) \\ y_t &= A + B\hat{S}_{t|t-1} + B(S_t - \hat{S}_{t|t-1}) + v_t \end{aligned}$$

We have the following joint distribution:

$$\begin{bmatrix} S_t \\ y_t \end{bmatrix} | I_{t-1} \sim N \left(\begin{bmatrix} \hat{S}_{t|t-1} \\ A + B\hat{S}_{t|t-1} \end{bmatrix}, \begin{bmatrix} P_{t|t-1} & P_{t|t-1}B' \\ BP_{t|t-1} & BP_{t|t-1}B' + R \end{bmatrix} \right),$$

Thus,

$$\begin{aligned} \hat{S}_{t|t} &\equiv E[S_t | y_t, I_{t-1}] \\ &= \hat{S}_{t|t-1} + P_{t|t-1}B'(BP_{t|t-1}B' + R)^{-1}(y_t - BS_{t|t-1} - A) \\ P_{t|t} &\equiv E[(S_t - \hat{S}_{t|t})(S_t - \hat{S}_{t|t})'] \\ &= P_{t|t-1} - P_{t|t-1}B'(BP_{t|t-1}B' + R)^{-1}BP_{t|t-1} \end{aligned}$$

4. Forecast S_{t+1} given I_t :

$$\begin{aligned} \hat{S}_{t+1|t} &= E[S_{t+1} | I_t] = F\hat{S}_{t|t} \\ P_{t+1|t} &= E[(S_{t+1} - \hat{S}_{t+1|t})(S_{t+1} - \hat{S}_{t+1|t})'] \\ &= FP_{t|t}F' + Q_t \end{aligned}$$

5. Maximum Likelihood Estimation of Parameters

The likelihood function can be built up recursively

$$\log L(Y_T) = \sum_{t=1}^T \log f(y_t | I_{t-1}),$$

where

$$\begin{aligned} f(y_t | I_{t-1}) &= (2\pi)^{-1/2} |H'P_{t|t-1}H + R|^{-1/2} \\ &\quad * \exp\left\{-\frac{1}{2}(y_t - A - B\hat{S}_{t|t-1})'(B'P_{t|t-1}B + R)^{-1}(y_t - A - B\hat{S}_{t|t-1})\right\} \\ \text{for } t &= 1, 2, \dots, T \end{aligned}$$

Parameter estimates can then be based on the numerical maximization of the likelihood function.

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