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This paper analyzes the restrictions necessary to ensure that the policy rule used by the central bank does not introduce real indeterminacy into the economy. It conducts this analysis in a flexible price economy and a sticky price model. A robust conclusion is that to ensure determinacy the monetary authority should follow a backward-looking rule where the nominal interest rate responds aggresssively to *past* inflation rates.

Forward-Looking Versus Backward-Looking Taylor Rules*

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Abstract: This paper analyzes the restrictions necessary to ensure that the policy rule used by the central bank does not introduce real indeterminacy into the economy. It conducts this analysis in a flexible price economy and a sticky price model. A robust conclusion is that to ensure determinacy the monetary authority should follow a backward-looking rule where the nominal interest rate responds aggressively to *past* inflation rates.

I. Introduction.

The celebrated Taylor (1993) rule posits that the central bank uses a fairly simple rule when conducting monetary policy. This rule is a reaction function linking movements in the nominal interest rate to movements in endogenous variables (eg., inflation). Recently there has been a considerable amount of interest in ensuring that such rules do no harm. The problem is that by following a rule in which the central bank responds to endogenous variables, it increases the likelihood that the central bank may introduce real indeterminacy and sunspot equilibria into an otherwise determinate economy.[1] These sunspot fluctuations are welfare-reducing and can potentially be quite large.

This paper analyzes the conditions under which a simple Taylor rule, one that responds only to inflation, will be determinate and hence avoid sunspot fluctuations. In particular we ask whether the central bank should respond proactively to movements in expected future inflation, or should they look backwards and base interest rate changes on past movements in inflation. What about Taylor rules that respond to current inflation?

These questions are of more than academic interest. Several central banks around the world currently use inflation forecasts as an important part of their decision-making on policy issues.[2] For example, the official position of the Central Bank of New Zealand is that: "The Bank's inflation projections relative to the inflation target range are the

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[1] The term "sunspot" is in one sense misleading since these shocks are accommodated by monetary policy. But we use the term since the central bank introduces real indeterminacy by responding to public expectations which can be driven by sunspots.

[2] Countries with explicit inflation targets (Canada, New Zealand, and the UK, for example) all base their policy actions on inflation forecasts.

critical input in the quarter by quarter formulation of monetary policy."[3] Alan Greenspan has commented that "Implicit in any monetary policy action or inaction is an expectation of how the future will unfold, that is, a forecast."[4]

A standard argument in the literature is that to avoid real indeterminacy the central bank must respond aggressively to either expected inflation (see Bernanke and Woodford (1997) and Clarida, Gali, Gertler (1997)) or current inflation (see Kerr and King (1996)). These analyses are all reduced-form sticky price models, where the underlying structural model is a labor-only economy and money is introduced via a money-in-the-utility function (MIUF) model with a zero cross-partial between consumption and real balances. In sharp contrast to the standard result, this paper demonstrates that any forward-looking or current-looking interest rate rule will always produce real indeterminacy.

The fundamental differences between our model and the above are two-fold. First as demonstrated in Carlstrom and Fuerst (1999a), one important factor affecting the determinacy issue is the assumption about which money balances enter the utility function. The traditional MIUF assumption is that end-of-period balances enter the functional. But a direct extension of a typical cash-in-advance (CIA) economy suggests that the money the household has left after leaving the bond market and before entering the goods market is more appropriate. Remarkably this timing distinction is critical for questions of determinacy. As argued in Carlstrom and Fuerst (1999a), the CIA timing is a more natural choice so we adopt it here. We utilize a rigid CIA constraint, but

[3] See Huxford and Reddell (2000) "Implementing Monetary Policy in New Zealand."
[4] Greenspan, A. Discussion of Goodhart, C., Capie, F., and Schnadt, N., "The Development of Central Banking," in Capie, F., Goodhart, C., Fischer, S., and Schnadt, N. (eds), *The Future of Central Banking*, Cambridge University Press, 1994, quoted in Sir Alan Budd, "Economic Policy, with and without Forecasts," published in the Bank of England's *Quarterly Bulletin*, November 1998 (from a speech given at

following the arguments of our earlier paper, the results generalize to an arbitrary MIUF environment with CIA timing.

The second major difference is in the type of sticky price model analyzed. The pricing model in the earlier papers is adopted from Calvo (1983). The assumption in that model is that a fixed fraction of firms adjust their prices every period. This implies that after any arbitrarily long but finite number of periods along a deterministic path, not all prices will not have adjusted to the levels implied by a flexible price model. This pricing arrangement is problematic for issues of determinacy since an equilibrium is determinate if perturbations from the equilibrium path lead to explosive inflation dynamics. But surely this Calvo pricing arrangement would not continue to hold along these out-of-equilibrium hyperinflationary paths, so that imposing this arrangement along the path seems quite artificial.

Instead this paper makes the methodological point that for issues of determinacy the more appropriate modeling strategy is to assume that along a hyperinflationary price path the pricing mechanism becomes perfectly flexible in finite time. For example, in the model analyzed below firms preset their prices in advance for some finite but definite time period. This model has the property that for any set of initial conditions (pre-set prices) the deterministic dynamics converge to the corresponding flexible-price model in a *finite* time period. In particular, along a hyperinflationary price path the pricing mechanism becomes perfectly flexible in finite time. Remarkably, the conditions for real determinacy in a model with finite stickiness are quite different than in a model with forever stickiness (eg., Calvo). This result is analogous to the so-called “folk theorem” in

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the Sir Alec Cairncross Lecture for the Institute of Contemporary Bristish History and the St. Peter’s game theory that a game that lasts for a finite but know period of time is fundamentally different than a game that lasts forever.

A related contribution of this paper is the demonstration that in a model with finite stickiness a necessary and sufficient condition for real determinacy is for the corresponding flexible price economy to have *both* real and nominal determinacy. This implies that with sticky prices either a forward or current-looking interest rate rule will *always* be indeterminate. Finally this paper demonstrates that a necessary and sufficient condition for real determinacy is for the monetary authority to react aggressively to *past* movements in inflation.

The outline of the paper is as follows. The next section introduces a flexible price model. This model lays the groundwork for understanding indeterminacy in section III where firms preset their prices in advance. The principle conclusion is that the only way to avoid both nominal and real indeterminacy in a flexible price model, and hence real indeterminacy in a finite-lived sticky price model, is to respond aggressively to past inflation. Section IV concludes. An appendix proves the main propositions.

II. A Flexible Price Model

The economy consists of numerous households and firms each of which we will discuss in turn. Since we are concerned with issues of determinacy without loss of generality we limit the discussion to a deterministic model. As is well known, if the deterministic dynamics are not unique, then it is possible to construct sunspot equilibria in the model economy.

College Foundation, October 27, 1998).

Households are identical and infinitely-lived with preferences over consumption and leisure given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1-L_t),$$

where β is the personal discount rate, c_t is consumption, and $1-L_t$ is leisure. The utility function is given by:

$$U(c,1-L) \equiv (c^{1-\sigma}-1)/(1-\sigma) - L^{1+\gamma}/(1+\gamma).$$

For the analytical results in this paper we will be working with the Hansen-Rogerson indivisible labor formulation so that $\gamma = 0$. Although the exact quantitative results are sensitive to this assumption, numerical calculations confirm that the qualitative results are the same for alternative calibrations.

To purchase consumption goods, households are subject to the following cash-in-advance constraint:

$$P_t c_t \leq M_t + P_t w_t L_t - N_t$$

where P_t is the price level, M_t denotes beginning-of-period cash balances, w_t denotes the real wage, and N_t denotes the household's choice of one-period bank deposits. These deposits earn nominal rate R_t that is paid out at the end of the period. The household's intertemporal budget constraint is given by:

$$M_{t+1} = M_t + P_t\{w_t L_t + [r_t + (1-\delta)]K_t\} - P_t c_t - P_t K_{t+1} + \Pi_t + N_t(R_t - 1).$$

K_t denoes the households accumulated capital stock that earns rental rate r_t. Note that we are assuming that asset accumulation occurs at the household level and that cash in advance is not needed to finance its purchase. The former assumption is without loss of generality; the second assumption is quite important as it implies that the nominal interest rate acts as a consumption (but not investment) tax.[5] Π denotes the profit flow from firms and financial intermediaries.

Firms in this economy utilize a production function employing labor and capital:

$$y_t = f(K_t, H_t)$$

where H_t denotes hired labor, and f is a CRS Cobb-Douglas production function with a capital share of α and a labor share of $(1-\alpha)$.[6] To finance its wage bill the firm must acquire cash and does so by borrowing cash short term from the financial intermediary at (gross) rate R_t.

The intermediary in turn has two sources of cash, the cash deposited by households and the new cash injected into the economy by the central bank. Hence, the loan constraint is:

$$P_t w_t H_t \leq N_t + M_t^s (G_t - 1)$$

where G_t denotes the (gross) money supply growth rate, $G_t \equiv M^s_{t+1} / M^s_t$. Note that monetary injections are carried out as lump sum transfers to the financial intermediary.

We restrict our attention to equilibria with strictly positive nominal interest rates so that the two cash constraints are binding. A recursive competitive equilibrium is given by stationary decision rules that satisfy these two binding cash constraints and the following:

$$\left\{ \frac{U_c(t)}{R_t} \right\} = \beta \left\{ (f_k(t+1) + 1 - \delta) \frac{U_c(t+1)}{R_{t+1}} \right\} \qquad (1)$$

[5] This assumption is quite important for the results with flexible prices but not for the model with sticky

$$\frac{U_L(t)}{U_c(t)} = \frac{f_L(t)}{R_t} \tag{2}$$

$$\left\{\frac{U_c(t)}{P_t}\right\} = \beta\left\{R_t \frac{U_c(t+1)}{P_{t+1}}\right\} \tag{3}$$

$$c_t + K_{t+1} = f(K_t, L_t) + (1-\delta)K_t. \tag{4}$$

Equations (1)-(2) illustrate the nominal rate distortion that is so important here. Both the labor and capital margins are distorted by the nominal interest rate. The nominal rate can thus be interpreted as a consumption tax of $(1+tc_t) = R_t$. This occurs because households must acquire cash before buying the consumption good, which has an opportunity cost of R_t. A high nominal rate is equivalent to a high consumption tax, and a low nominal rate is equivalent to a low consumption tax.

To close the model we need to specify the central bank reaction function. In what follows we assume a reaction function where the current nominal interest rate is a function of inflation. We will consider three variations of this simple rule:

$$R_t = R_{ss}\left(\frac{\pi_{t+i}}{\pi_{ss}}\right)^{\tau}, \textit{ where } \tau \geq 0,\ R_{ss} = \frac{\pi_{ss}}{\beta}, \tag{5}$$

prices.

[6] The constant returns to scale assumption while standard is also critical to the exact quantitative results.

where i = -1 is a backward-looking rule, i = 0 is a current-looking rule, and i = 1 is a forward-looking rule.

Under any such interest rate policy the money supply responds endogenously to be consistent with the interest rate rule. It is this endogeneity of the money supply that increases the possibility of indeterminacy. If the private sector responds to sunspots, then the central bank must passively vary the money supply to keep the nominal rate on target.

There are two types of indeterminacy that may arise. First, there is *nominal indeterminacy*—are the initial values of the price level and all other nominal variables pinned down? In our notation this corresponds to the question of whether $\pi_t \equiv P_t/P_{t-1}$ is determined (where t is the initial time period). This nominal or price level determinacy is a standard occurrence under many interest rate operating procedures, the most celebrated example being an interest rate peg. This nominal indeterminacy is of no consequence in and of itself, but is important only if it leads to real indeterminacy.

By *real indeterminacy*, we mean a situation in which the behavior of one or more real variables is not pinned down by the model. This possibility is of great importance as it immediately implies the existence of sunspot equilibria which, in the present environment, are necessarily welfare reducing. In the flexible price model of this section, real indeterminacy manifests itself as an indeterminacy in expected inflation or the initial nominal interest rate. Since this rate distorts real behavior, this is a form of real indeterminacy. In the sticky price sections that follow, real indeterminacy can arise in both the initial marginal cost and nominal interest rate. Since both of these distort real behavior, this is also a form of real indeterminacy.

For simplicity, we begin the analysis with a labor-only economy, and then demonstrate how the results extend to an economy with capital. Suppose that production is a linear function of labor, f(L) = BL. Assuming preferences are linear in leisure (γ=0) then inserting the labor margin (2) into the Fisher equation (3) yields:

$$\tilde{R}_{t+1} - \tilde{\pi}_{t+1} = 0 \tag{6}$$

where the variables are expressed as log deviations from the steady-state. The generality of this result is suggested by the capital accumulation equation (1). If output is instead linear in capital then (1) also collapses to (6).

First suppose that monetary policy is forward-looking. Substituting the monetary policy rule (5) into (6) we have

$$\tilde{\pi}_{t+2} = \left(\frac{1}{\tau}\right)\tilde{\pi}_{t+1}. \tag{7}$$

Notice that (7) starts with π_{t+1}, so that the initial price level is free, π_t is free. Thus the economy always suffers from nominal indeterminacy. At this point, however, nominal indeterminacy is of no importance since it does not impact real behavior. For real determinacy we need this mapping to be explosive. Hence, there is real determinacy if and only if $0 < \tau < 1$. If this is the case then π_{t+j} is determined for $j \geq 1$. From the policy rule this pins down R_{t+j} for $j \geq 0$ so that real behavior is pinned down. At least in a flexible price model the results of Clarida, Gali, Gertler (1997) have essentially been turned on their head.

The intuition for this real indeterminacy is clear. Under this policy rule increases in expected inflation increase the nominal rate but depending on the elasticity τ these increases may or may not increase the real rate. Rewriting the above we have

$$\tilde{R}_t - \tilde{\pi}_{t+1} = (\tau - 1)\tilde{\pi}_{t+1} = \frac{(\tau - 1)}{\tau}\tilde{R}_t .$$

For aggressive policies ($\tau > 1$), nominal rate increases are associated with increases in the real rate of interest. Thus, we have an implicit consumption tax (the nominal rate) correlated tightly with expected consumption growth (the real rate). The self-fulfilling circle goes like this. A sunspot-driven increase in current consumption lowers today's real interest rate. With $\tau > 1$, the nominal rate (consumption tax) falls with this real rate movement. This increases current consumption, which completes the circle. The initial increase in consumption is therefore rational.[7]

The fact that π_t is free for all values of τ is just a manifestation of nominal indeterminacy, i.e., there is nothing to pin down the initial growth rate of money. This innocent remark has some interesting implications for a current-looking policy rule. Substituting the current-looking policy rule (5) into (6) yields

$$\tau\tilde{\pi}_{t+1} - \tilde{\pi}_{t+1} = 0 . \qquad (8)$$

Notice that π_t is free, but future inflation rates are nailed down (when $\tau \neq 1$) since $\tilde{\pi}_{t+j}$ = 0 for all $j \geq 1$. From the policy rule this pins down R_{t+j} for $j \geq 1$. But since π_t is free, this current-based policy implies that R_t is also free. Since R_t acts like a tax on consumption, real behavior is not pinned down. The nominal indeterminacy from before is now real.

[7] Schmitt-Grohe and Uribe (2000) and Carlstrom and Fuerst (1999c) report similar results for forward-looking Taylor rules in a flexible price setting.

The reason for this indeterminacy is because the policy rule is responding to current inflation, which is not pinned down for standard nominal indeterminacy reasons. Because of the policy rule this nominal indeterminacy becomes real. More generally the potential for indeterminacy in both the current- and forward-looking rules arises whenever policy responds to endogenous variables. This is why an interest rate peg $(\tau = 0)$ has real determinacy.

This discussion suggests that the central bank should look backwards so that it responds only to predetermined variables. Remarkably, by looking backwards the conditions for determinacy are (almost) entirely flipped on their head from when the Taylor Rule is forward-looking. Substituting the backward-looking monetary policy rule (5) into (6) we have

$$\tilde{\pi}_{t+1} = \tau\tilde{\pi}_t \tag{9}$$

Unlike earlier we may have nominal determinacy. If the monetary authority responds aggressively to past inflation ($\tau > 1$) initial inflation and hence R_{t+1} is pinned down (the policy rule implies that R_t is predetermined by last period's inflation rate). Hence, there is real and also nominal determinacy if and only if $\tau > 1$. (As noted earlier, an interest rate peg of $\tau = 0$ yields real but not nominal determinacy.)

The intuition why an aggressive backward-looking policy can eliminate nominal and real indeterminacy is as follows. Suppose there is an increase in the current price level π_t of 1%. This implies that next period's nominal rate must rise by τ%. This increase in the future nominal rate (consumption tax) leads to an increase in current consumption. This implies that the real rate falls. The policy rule implies that the current nominal rate does not respond to π_t. Hence, the decline in the real rate must lead to an increase in π_{t+1} that is greater than the initial increase in π_t (see (9) with $\tau > 1$). This behavior is explosive, and thus eliminates this as a possible equilibrium path.

The idea that responding to a nominal variable can pin down prices is not new. This result is a general equilibrium generalization of McCallum's (1981) earlier result. He argued that because an interest rate peg suffered from nominal indeterminacy the monetary authority needed a nominal anchor, which could be accomplished by responding to a nominal variable. This analysis confirms this but shows that merely responding to a nominal variable, like past inflation, is not enough. The monetary authority has to aggressively respond to past inflation to ensure both real and nominal determinacy.

The preceding results were demonstrated in a model with linear labor. The following propositions show that the linearity exploited above to demonstrate indeterminacy is remarkably general. A CRS production function with linear leisure provides this same linearity so that the conditions for determinacy are identical for all three monetary reaction functions considered above. As noted earlier, for more general calibrations of labor supply the quantitative results differ, but the qualitative results are the same—only passive forward-looking rules and aggressive backward-looking rules are consistent with real determinacy, and only the latter eliminate nominal indeterminacy.

Proposition 1: Suppose that prices are flexible, $\gamma = 0$ (linear labor,) and that monetary policy is given by the following interest rate rule given by:

$$R_t = R_{ss}\left(\frac{\pi_{t+j}}{\pi_{ss}}\right)^{\tau}, \textit{ where } \tau \geq 0,\ R_{ss} = \frac{\pi_{ss}}{\beta}.$$

A. With a forward-looking interest rate rule (j=1) there is real determinacy if and only if $0 < \tau < 1$. In any event, there is always nominal indeterminacy as π_t is free.

B. With a current-looking interest rate rule (j=0) there is real indeterminacy for all values of $\tau \neq 0$.

C. With a backward-looking interest rate rule (j=-1) there is real determinacy if and only if $\tau = 0$ or $\tau > 1$. In the case of $\tau > 1$, there is also nominal determinacy.

Proof: See the appendix.

Although at this stage nominal indeterminacy is merely a nuisance, its presence becomes critical in the next section when we consider how nominal rigidities affect the above results. In particular we will show that if there is nominal indeterminacy in the flexible price model, then in the corresponding sticky price model there is real indeterminacy.

III. A Sticky Price Model.

In this section we consider a popular model of monetary non-neutrality—a model with sticky prices. We utilize the standard model of imperfect competition in the intermediate goods market.[8] We omit any discussion of household behavior as it is symmetric with before.

Final goods production in this economy is carried out in a perfectly competitive industry that utilizes intermediate goods in production. The CES production function is given by

$$Y_t = \{\int_0^1 [y_t(i)^{(\eta-1)/\eta}]di\}^{\eta/(\eta-1)}$$

where Y_t denotes the final good, and $y_t(i)$ denotes the continuum of intermediate goods, each indexed by $i \in [0,1]$. The implied demand for the intermediate good is thus given by

$$y_t(i) = Y_t\left[\frac{P_t(i)}{P_t}\right]^{-\eta}$$

where $P_t(i)$ is the dollar price of good i, and P_t is the final goods price.

Intermediate goods firm i is a monopolist producer of intermediate good i. Fraction ν of these firms set their prices flexibly within each period, while without loss of generality the remainder $(1-\nu)$ are assumed to set their price one period in advance. The variable ν is thus a measure of price flexibility: (1) with $\nu = 1$, this is a flexible price model, (2) with ν between zero and one, this is a sticky price model, and (3) with $\nu = 0$, this is a model with rigid prices. Other than the difference in the timing of pricing, the firms are all symmetric, so we will henceforth drop the firm-specific notation. Let P_t^f denote the flexible price, while P_t^s will denote the pre-determined (or sticky) price. The final goods price (or aggregate price level) is given by the appropriate average of these two prices:

$$P_t = \left\{(1-\nu)P_t^{s(1-\eta)} + \nu P_t^{f(1-\eta)}\right\}^{\frac{1}{1-\eta}}. \quad (10)$$

[8] See, for example, Chapter 5 of Walsh (1998).

The flexible price is given by a constant mark-up over the marginal cost (z_t) of production:

$$P_t^f = \left[\frac{\eta}{\eta-1}\right] P_t z_t . \tag{11}$$

The term in brackets will appear frequently below, so we define $z \equiv (\eta\text{-}1)/\eta < 1$. In a model with flexible prices, equation (11) and the assumption of symmetry implies that $z_t = z$. Combining (10)-(11), we have

$$P_t = h(z_t)\, P_t^s \text{, where } h(z_t) \equiv \left[\frac{1-\nu}{1-\nu\left(\frac{z_t}{z}\right)^{1-\eta}}\right]^{1/(1-\eta)} \tag{12}$$

The function h is increasing so that innovations in marginal cost correspond with changes in the price level.

The intermediate goods firm is owned by the household, and pays its profits out to the household at the end of each period. Because of the cash-in-advance constraint on household consumption, the firm discounts its profits using $\mu_{t+1} \equiv \beta U_c(t+1)/P_{t+1}$, the marginal utility of \$1 in time t+1. Therefore the sticky price is given by the solution to the following maximization problem:

$$P_t^s = \arg\max E_{t-1}\left(\mu_{t+1} P_t Y_t \left[\frac{P_t^s}{P_t}\right]^{-\eta} \left\{ \left[\frac{P_t^s}{P_t}\right] - z_t \right\} \right)$$

where E_{t-1} is the expectation conditional on time t-1 information. The firm's optimal preset price is thus given by:

$$P_t^s = \left(\left[\frac{\eta}{\eta-1}\right] \frac{E_{t-1}\left\{\mu_{t+1} P_t^{\eta+1} z_t Y_t\right\}}{E_{t-1}\left\{\mu_{t+1} P_t^{\eta} Y_t\right\}} \right). \tag{13}$$

Using (12), this can be written as

$$E_{t-1} z \mu_{t+1} Y_t [h(z_t)]^{\eta} = E_{t-1} z_t \mu_{t+1} Y_t [h(z_t)]^{\eta+1}. \qquad (14)$$

In a model without pre-set prices, this equation would hold at time-t, and thus imply that $z_t = z$.

As for production, the intermediate firm rents capital and hires labor from households and utilizing the CRS production function from before. Imperfect competition implies that factor payments are distorted. With z_t as marginal cost, we then have $r_t = z_t f_K(K_t, L_t)$ and $w_t = z_t f_L(K_t, L_t)/R_t$.

A recursive competitive equilibrium is given by stationary decision rules that satisfy (12), (13), (14), and the following:

$$\frac{U_L(t)}{U_c(t)} = \frac{z_t f_L(t)}{R_t} \qquad (15)$$

$$\frac{U_c(t)}{R_t} = \beta \left\{ \frac{U_c(t+1)[z_{t+1} f_K(K_{t+1}, L_{t+1}) + (1-\delta)]}{R_{t+1}} \right\} \qquad (16)$$

$$\frac{U_c(t)}{P_t} = \beta R_t \left\{ \frac{U_c(t+1)}{P_{t+1}} \right\} \qquad (17)$$

$$c_t + K_{t+1} = f(K_t, L_t) + (1-\delta) K_t \qquad (18)$$

The labor and capital accumulation margins are once again distorted by the nominal rate of interest (the consumption tax). The novelty is that the marginal production cost z_t also distorts behavior. Continuing with the public finance analogy, this marginal cost or imperfect competition distortion manifests itself as a tax on wage and rental income. Thus we now have two implicit taxes: the nominal interest rate or consumption tax, and marginal cost or income tax. The fact that both of these taxes are endogenous suggests that it will be more difficult to ensure determinacy.

For stability we once again turn to the deterministic model. A key insight is that since prices can adjust after one period $z_{t+j} = z$ for all $j \geq 1$, but z_t need not equal z because some prices are predetermined. From t+1 onwards, therefore, the deterministic version of the model is isomorphic to the flexible price model with a constant income tax rate of 1-z. This implies that a necessary condition for real determinacy is that the corresponding flexible price model be determinate for real variables. If the flexible price model has real indeterminacy then, obviously, the sticky price model will also suffer from real indeterminacy.

But this flexible price determinacy is only necessary. For sufficiency we also need the flexible price economy not to have *nominal* indeterminacy. This is because we need an extra condition to pin down the initial marginal cost z_t, which from (12) is a function of the initial price level P_t.

To help understand the intuition of this result, we return to the special case in which production is a linear function of labor, f(L) = BL. Assuming preferences are linear in leisure (γ=0) then inserting the labor margin (15) into the Fisher equation (17) yields:

$$\tilde{z}_t + \left[\tilde{R}_{t+1} - \tilde{\pi}_{t+1}\right] = 0 \quad \text{and} \tag{19a}$$

$$\tilde{R}_{t+j} - \tilde{\pi}_{t+j} = 0 \quad \text{for all } j \geq 2. \tag{19b}$$

where $\tilde{z}_{t+j} = 0$ for all j ≥ 1 (because prices are fixed for only one period). Notice that because prices are flexible from period 2 on we have split the conditions into their sticky (19a) and flexible price (19b) periods. Equation (19a) represents one restriction on the initial z_t. The second restriction comes from (12) which implies that z_t is a function of $\pi_t = P_t / P_{t-1}$.

Is z_t pinned down? Consider first the flexible price part (19b). If the flexible price economy has *both* real and nominal determinacy, then (19b) pins down π_{t+1}. If this occurs then (12) and (19a) imply that z_t and π_t are both determined. But if (19b) suffers from nominal indeterminacy so that π_{t+1} is not pinned down, then (19a) implies that z_t and π_t will also be free. Remarkably the presence of nominal indeterminacy in the flexible price economy necessarily implies that expected inflation (and thus real variables) are indeterminate in a sticky price economy.

Why does the forward-looking policy rule always produce real indeterminacy? Consider a sunspot increase in expected inflation. The monetary policy rule implies that today's funds rate must increase in response. To achieve this the monetary authority lowers today's money growth. This temporarily lowers output and hence consumption thus increasing the real interest rate. Given this monetary contraction, when tomorrow comes, firms' pre-set prices will be too high. The monetary authority, therefore, must increase tomorrow's money growth to keep the nominal rate in a neutral position. This increases today's expected inflation thus completing the circle.

This analysis suggests that, perhaps, indeterminacy could be avoided if the monetary policy rule completely stabilized expected inflation ($\tau=\infty$). Such a rule would make the economy isomorphic to a flexible price economy (see Carlstrom and Fuerst (1998)). From the flexible price model developed earlier we know that such a rule will always suffer from indeterminacy since the real rate and the nominal rate would move in tandem.

As in the previous section, the results of this simple example extend to a wider environment with a CRS production function. In particular, we have the following:

Proposition 2: Suppose that monetary policy is given by either a forward-looking or a current-looking Taylor rule. Then in the sticky price model there is real indeterminacy for all values of τ*.*

Proof: see the appendix.

Proposition 3: Suppose that monetary policy is given by a backward-looking Taylor rule. Then in the sticky price model there is real determinacy if and only if $\tau > 1$*.*

Proof: see the appendix.

There are of course many convex combinations of possible interest rate rules that we do not address above. We will conclude with two interesting cases. First, suppose that policy is both forward- and backward-looking:

$$\tilde{R}_t = \tau[\varepsilon\tilde{\pi}_{t-1} + (1-\varepsilon)\tilde{\pi}_{t+1}].$$

In this case, a necessary and sufficient condition for real determinacy in the sticky price model is that $\tau > 1$ and $\varepsilon > 1/2$. That is, policy can look forward if and only if the weight of the policy rule is on the past.

Second, consider policy rules that contain an inertial component:

$$\tilde{R}_t = \rho\,\tilde{R}_{t-1} + \tau\tilde{\pi}_{t+i}.$$

Under such a rule, forward-looking rules (j = 1) are always subject to sunspots. Current-looking rules (j = 0) are determinate if and only if $1 < \tau < (1+\rho)/(1-\rho)$. Backward-looking rules (j = -1) are determinate if and only if $\tau > 1$.[9]

The results in this paper also generalize to other pricing arrangements. The basic argument applies to any longer-lived but finite stickiness. All that is necessary is that for any set of initial conditions (P_{t-1}), the deterministic dynamics converge to the flexible-price model in a finite time period. In particular, along a hyperinflationary price path, the aggregate pricing mechanism becomes perfectly flexible in finite time. This is clearly not the case for Calvo (1983) pricing, nor is it generally true for Taylor (1980) pricing. However, it is the case for Fischer (1977) pricing.

While both Fischer and Taylor employ staggered contracting, the difference between Fischer and Taylor pricing is that in the former firms set their prices some finite number of periods in advance but they can choose different prices in each period. With Taylor pricing they are constrained to choose the same nominal price in every period. We argue that in much the same way that Calvo pricing is nonsensical along these hyperinflationary paths so too is Taylor pricing. In both cases firms are selecting a price

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[9] See Carlstrom and Fuerst (1999b) for the proof of these results in a small open economy. These proofs extend to a closed economy using the methods outlined in the appendix.

that is wildly out of line with the general price level even though these movements in the general price level are deterministic. Why should we utilize such an assumption for issues of real determinacy?

IV. Conclusion.

The central issue of this paper is to identify the restrictions on the Taylor interest rate rule needed to ensure real determinacy. A standard result in the literature is that an aggressive response to either forward or current inflation is necessary and sufficient for determinacy. This standard result derives from reduced form MIUF models. The essential point of this paper is that this result does not stand up to more careful structural modeling.

In our view, these models ignore two central issues: (1) the appropriate timing for money demand modeling, and (2) the instability of the Phillips curve along a hyperinflationary path. This paper considers both of these issues and overturns the standard result.

We believe that both of these issues are of the first importance. Basing policy advice on a model that does not include these issues seems quite premature. We have argued elsewhere (Carlstrom and Fuerst (1999a)) for a reassessment of the timing conventions used in monetary models. We will not review those arguments here.

As for the Phillips curve, we argue that for issues of determinacy the appropriate modeling strategy is a model with finite stickiness. While the Calvo setup may be more appropriate for positive analysis, clearly the assumption that prices have to be forever sticky is especially troublesome for issues of determinacy. This can be understood by remembering that an equilibrium is determinate if perturbations from the equilibrium path lead to explosive inflation dynamics. But surely the Calvo pricing arrangement would not continue to hold along such a path. If instead we assume that for any set of initial conditions, the deterministic dynamics converge to the flexible-price model in a finite time we are back to a world with finite stickiness. This argument is in the spirit of McCallum (1994) who questions the robustness of any model that violates the natural rate hypothesis.

Even if one remained wed to a Calvo pricing arrangement for these determinacy issues, there are other problems. In a companion piece, Carlstrom and Fuerst (2000) add investment spending to the Clarida, Gali, Gertler (1997) environment. The role of investment across the business cycle has a long tradition in monetary economics so that adding this to the environment seems like a good idea. The end result of Carlstrom and Fuerst (2000) is that for anything but the most extreme parameter values, adding investment spending to the Clarida, Gali, Gertler (1997) environment implies that monetary policy must respond aggressively to past inflation to generate determinacy.

Yet whichever model is used the essential point of this paper remains. To avoid doing harm, the central bank should place the most weight on past movements in the inflation rate. As long as this link between current interest rates and past inflation is aggressive enough, the central bank can eliminate the possibility of self-fulfilling behavior. An immediate implication of this analysis is that inflation targeting over short horizons, which necessarily implies the use of forecasts, is potentially a dangerous policy as it is prone to sunspots.

Appendix

Proposition 1: Suppose that prices are flexible, γ = 0 (linear labor,) and that monetary policy is given by the following interest rate rule given by:

$$R_t = R_{ss}\left(\frac{\pi_{t+i}}{\pi_{ss}}\right)^{\tau}, \text{ where } \tau \geq 0,\ R_{ss} = \frac{\pi_{ss}}{\beta}.$$

A. With a forward-looking interest rate rule (i=1) there is real determinacy if and only if

0 < τ < 1. In any event, there is always nominal indeterminacy as π_t is free.

B. With a current-looking interest rate rule (i=0) there is real indeterminacy for all values of $\tau \neq 0$.

C. With a backward-looking interest rate rule (i=-1) there is real determinacy if and only if τ = 0 or τ > 1. In the case of τ > 1, there is also nominal determinacy.

A. Proof: The relevant equilibrium conditions are

$$\left\{\frac{P_t U_c(t+1)}{P_{t+1}}\right\} = \beta\left\{(f_k(t+1)+1-\delta)\frac{P_{t+1}U_c(t+2)}{P_{t+2}}\right\} \tag{1}$$

$$\frac{U_L(t)}{U_c(t)} = \frac{\theta_t f_L(t)}{R_t} \tag{2}$$

$$\left\{\frac{U_c(t)}{P_t}\right\} = \beta\left\{R_t \frac{U_c(t+1)}{P_{t+1}}\right\} \tag{3}$$

$$c_t + K_{t+1} = f(K_t, L_t) + (1-\delta)K_t \tag{4}$$

The outline of the proof is that we are going to collapse these four equations into three by substituting out the labor equation (2).

Given the assumption that utility is linear in leisure (γ = 0) the first order conditions (1) and (2) can be written as

$$\beta\left(\frac{U_1(t)}{R_t}\right) = \frac{x_t^{\alpha}}{(1-\alpha)}, where\ x_t = \frac{L_t}{K_t} \tag{A1}$$

$$\left(\frac{U_1(t)}{R_t}\right) = \beta\ \left(\frac{U_1(t+1)}{R_{t+1}}\left\{\alpha x_{t+1}^{1-\alpha} + (1-\delta)\right\}\right) \tag{A2}$$

where (A1) is the revised labor equation and (A2) is the revised capital accumulation equation (1). Substituting (A1) and equation (A1) scrolled forward one period into (A2) yields:

$$x_t^{\alpha} = \ \left(\alpha\beta\ x_{t+1} + \beta(1-\delta)x_{t+1}^{\alpha}\right) \tag{A3}$$

or the newly rewritten capital accumulation equation. The resource constraint provides another equation:

$$K_{t+1} = K_t x_t^{1-\alpha} + (1-\delta)K_t - c_t. \tag{A4}$$

We wish to substitute (A1) into both (A4) and the Fisher equation (3). Note that (A1) implies that c_t depends only on R_t and x_t. Inverting the monetary policy rule yields:

$$\frac{\pi_{t+1}}{\pi_{ss}} = \left(\frac{R_t}{R_{ss}}\right)^{\frac{1}{\tau}}$$

Using this and (A1), we can express the Fisher equation in terms of R_{t+1}, x_t, x_{t+1}, and R_t.

We are thus left with three equations:

$x_{t+1} = F(x_t)$

$K_{t+1} = G(x_t,R_t,K_t)$

$R_{t+1} = H(x_t,x_{t+1},R_t)$

The characteristic matrix is

$$\begin{bmatrix} F_x - e & 0 & 0 \\ G_x & G_K - e & G_R \\ H_{x_{t+1}} F_{x_t} + H_{x_t} & 0 & H_R - e \end{bmatrix}$$

The three eigenvalues are

$$e_1 = \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)} < 1,\ \ e_2 = \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta} > 1,\ e_3 = \frac{1}{\tau}.$$

Since there is only one predetermined variable, K_t, for the economy to be determinate two eigenvalues need to lie outside the unit circle. This is satisfied if and only if $\tau < 1$. This, however, only pins down π_{t+1}, π_t is free. QED

B. Proof:

Suppose $\tau > 0$. The proof proceeds exactly as in Proposition 1. The only difference arises because

$$\frac{\pi_{t+1}}{\pi_{ss}} = \left(\frac{R_{t+1}}{R_{ss}}\right)^{\frac{1}{\tau}}.$$

Therefore plugging (A1) into (3) and using the above relationship yields the following revised Fisher equation

$R_{t+1} = H(x_t,x_{t+1})$.

The absence of R_t from the above relationship coupled with the absence of R_{t+1} from the x_{t+1} and K_{t+1} equations imply that $e_3 = 0$.

Since there is only one predetermined variable, K_t, for the economy to be determinate two eigenvalues need to lie outside the unit circle. This is clearly never satisfied.

Suppose $\tau = 0$. With the nominal interest rate given the real economy is given by (1), (2), and (4). This is just the Canonical RBC model perturbed by a constant consumption tax and is thus unique. QED

C. Proof:

Without loss of generality, assume $\tau > 0$. Proceeding as in Proposition 1, we have

$x_{t+1} = F(x_t)$

$K_{t+1} = G(x_t, R_t, K_t)$

$R_{t+2} = H(x_t, x_{t+1}, R_{t+1})$

where the last equation is of a different form because the inverted monetary policy rule is

$$\frac{\pi_t}{\pi_{ss}} = \left(\frac{R_{t+1}}{R_{ss}}\right)^{\frac{1}{\tau}}$$

which we have scrolled forward one period to solve for π_{t+1}. The characteristic matrix is

$$\begin{bmatrix} F_x - e_1 & 0 & 0 & 0 \\ G_x & G_{K_t} - e_2 & 0 & G_{R_t} \\ H_{x_{t+1}} F_{x_t} + H_{x_t} & 0 & H_{R_{t+1}} - e_3 & 0 \\ 0 & 0 & 1 & e_4 \end{bmatrix}$$

The four eigenvalues are

$$e_1 = \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)} < 1, \quad e_2 = \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta} > 1, \; e_3 = \tau, \; e_4 = 0.$$

Now there are two pre-determined variables, K_t and R_t, so that we only need two explosive roots. Hence, we have real determinacy if and only if $\tau > 1$. In this case, R_{t+1} is pinned down implying that π_t is determined. QED

Proposition 2: Suppose that monetary policy is given by either a forward-looking or a current-looking Taylor rule. Then in the sticky price model there is real indeterminacy for all values of τ.

Proof: Consider the forward-looking rule (the other case is symmetric). After the initial period, the deterministic dynamics of the sticky price model are identical to that of the flexible price model. Hence, a necessary condition for the sticky-price model to be determinate is for the flexible price model to be determinate. Assume that τ is chosen so that this is the case. The flexible price determinacy implies that x_{t+1} and R_{t+1} are unique functions of K_{t+1}. For sufficiency, we must turn to the initial period. The initial period Euler equations are:

$$x_{t+1} = F(x_t,z_t)$$

$$K_{t+1} = G(x_t,R_t,K_t,z_t)$$

$$R_{t+1} = H(x_t,x_{t+1,}R_t)$$

Since x_{t+1} and R_{t+1} are unique functions of K_{t+1}, we have three equations in four unknowns x_t, R_t, K_{t+1}, and z_t. The only other restriction is that z_t is a function only of $\pi_t \equiv P_t / P_{t-1}$. Hence, we have real indeterminacy. QED

Proposition 3: Suppose that monetary policy is given by a backward-looking Taylor rule. Then in the sticky price model there is real determinacy if and only if $\tau > 1$.

Proof: After the initial period's price stickiness, the model's dynamics are exactly that of the flexible price economy ($z_t = z$ after the initial period). Hence, under the conditions in Proposition 3 ($\tau > 1$), there is real determinacy from time t+1 onwards. This implies that x_{t+1} and R_{t+2} are unique functions of K_{t+1} and R_{t+1} (R_{t+1} is predetermined since the nominal rate is backward-looking). We can now turn to the initial period. The initial Euler equations are:

$$x_{t+1} = F(x_t, z_t)$$

$$K_{t+1} = G(x_t, R_t, K_t, z_t)$$

$$R_{t+2} = H(x_t, x_{t+1}, R_{t+1})$$

Since x_{t+1} and R_{t+2} are unique functions of K_{t+1} and R_{t+1}, these represent three equations in K_{t+1}, R_{t+1}, x_t, and z_t. But R_{t+1} and z_t are <u>both</u> functions only of $\pi_t \equiv P_t / P_{t-1}$. Hence, we have three independent linear equations in three unknowns K_{t+1}, x_t, and π_t. QED

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