working paper

9819

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FEDERAL RESERVE BANK OF CLEVELAND

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Charles T. Carlstrom is an Economist at the Federal Reserve Bank of Cleveland.
Timothy S. Fuerst is Associate Professor of Economics at Bowling Green State University, Bowling Green, Ohio, and a visiting consultant with the Federal Reserve Bank of Cleveland.

August 1998

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This paper examines a standard sticky price monetary model. The equilibrium conditions of the model are perturbed relative to the canonical real business cycle model by two varying distortions: marginal cost and the nominal rate of interest. The paper explores the implications of two monetary policies that are frequently advocated: (1) an inflation target and (2) an interest rate target. Under an inflation rate target, marginal cost is stabilized while the nominal rate is variable. In contrast, under an interest rate target, the nominal rate is stabilized but marginal cost is (in general) variable. Both policies are subject to sunspot fluctuations arising from the endogenous movement of the money stock. These fluctuations can be avoided by eliminating the contemporaneous response of the money stock to innovations in the environment.

Price-Level and Interest-Rate Targeting

in a Model with Sticky Prices

Charles T. Carlstrom[a]

Timothy S. Fuerst[b]

[a]Federal Reserve Bank of Cleveland, Cleveland, Ohio, 44101, USA.
[b]Department of Economics, Bowling Green State University,
Bowling Green, Ohio, 43403; and Consultant, Federal
Reserve Bank of Cleveland.

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ABSTRACT: This paper examines a standard sticky price monetary model. The equilibrium conditions of the model are perturbed relative to the canonical real business cycle model by two varying distortions: marginal cost and the nominal rate of interest. The paper explores the implications of two monetary policies that are frequently advocated: (1) an inflation target and (2) an interest rate target. Under an inflation rate target, marginal cost is stabilized while the nominal rate is variable. In contrast, under an interest rate target, the nominal rate is stabilized but marginal cost is (in general) variable. Both policies are subject to sunspot fluctuations arising from the endogenous movement of the money stock. These fluctuations can be avoided by eliminating the contemporaneous response of the money stock to innovations in the environment.

Key Words: General Equilibrium; Money and Interest Rates; Monetary Policy. JEL Classification: D51; E42; E52.

1. Introduction

This paper examines two traditional monetary policies in an economy in which money is non-neutral because some nominal goods prices must be set in advance. The assumed nominal rigidity leads to a real rigidity: in the absence of central bank action, the market economy responds sluggishly to real shocks to the environment. A benevolent central banker will attempt to eliminate this real rigidity by varying money growth in response to the real shocks. However, informational restrictions make it unlikely that the central bank can vary the money supply in response to fundamental shocks. Instead, the central bank must rely on a more readily observable target. This paper considers two frequently advocated targets: a price level target, and an interest rate target.

The model's equilibrium conditions are perturbed relative to the canonical real business cycle model by two varying distortions: marginal cost and the nominal rate of interest. The first distortion arises because of sticky prices, while the latter arises from the cash-in-advance constraint. The two targets—price level and interest rates--have opposing effects on these two distortions. Under a price level target, marginal cost is stabilized but the nominal rate is variable. Under an interest rate target, the nominal rate is stabilized but marginal cost is (in general) variable. Thus choosing the target is isomorphic to choosing which distortion is variable and which is stabilized.

Both policies are subject to sunspot fluctuations arising from the endogenous movement of the money stock. That is, there is not a unique money reaction function that supports either a price level or an interest rate target, and each of these reaction functions leads to different real outcomes. These sunspot fluctuations can be avoided by specifying the money reaction function. One natural way of doing this is to limit the contemporaneous response of the money stock to innovations in the environment. The disadvantage of this approach is that it hinders the economy's response to real shocks.

There is a voluminous literature on optimal monetary targeting. A selected review of papers close in spirit to the current analysis include Carlstrom and Fuerst (1995), Ireland (1996), and Goodfriend and King (1997). Carlstrom and Fuerst (1995) examine interest-rate targeting in a limited participation environment and demonstrate that one advantage of an interest rate peg is that it enhances the private economy's ability to efficiently respond to shocks.[1] Ireland (1996) demonstrates that in a sticky price model the first-best monetary policy implies pegging the nominal rate to zero.[2] Finally, Goodfriend and King (1997) use a sticky price framework to demonstrate the advantages of price-level targeting. An interesting implication of all these analyses is that by overcoming the source of monetary non-neutrality, a welfare-enhancing monetary policy exacerbates output fluctuations arising from technology shocks.

The paper proceeds as follows. The next Section outlines the basic model. Sections 3 and 4 examine the effects of a price-level target and interest-rate target, respectively. Section 5 considers these targets under the additional constraint that the

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[1] See Lucas (1990), Fuerst (1992), and Christiano and Eichenbaum (1992), for analyses of limited participation models.

[2] The phrase "first-best" is not quite accurate. The distortion created by the assumption of imperfect competition cannot be undone by any monetary policy. Ireland (1996) demonstrates that the optimal monetary policy in this distorted world is still characterized by pegging the nominal rate to zero. Carlstrom and Fuerst (1998) demonstrate that there are some concerns with this first-best approach to policy. The foremost problem is that the combination of indeterminate real cash balances and demand-determined output opens up the possibility of sunspot equilibria that would not arise if the nominal rate were pegged above zero. The reason these equilibria do not arise in the case of a positive nominal rate is that a binding cash constraint places an additional restriction on equilibrium behavior.

money supply is predetermined. Section 6 concludes.

2. A Model with Sticky Prices

We follow Chari, Kehoe, and McGrattan (1996), and utilize a model of imperfect competition in the intermediate goods market.[3] Since the model has become fairly standard, we will be quite economical in our presentation.

Households are infinitely lived, with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - L_t)$$

where E_0 denotes the expectation operator conditional on time-0 information, $\beta \in (0,1)$ is the personal discount factor, c_t is time-t consumption, L_t is time-t labor, and the leisure endowment is normalized to unity. To purchase consumption goods, households are subject to the following cash-in-advance constraint:

$$P_t c_t \leq M_t + M_t^s (G_t - 1) - B_t$$

where P_t is the price level, M_t denotes beginning-of-period cash balances, M^s_t denotes the per household money supply, and G_t denotes the (gross) money supply growth rate, $G_t \equiv M^s_{t+1}/M^s_t$. Note that monetary injections are carried out as lump sum transfers at the beginning of each period. B_t denotes the household's choice of one-period nominal bonds, each promising to pay R_t dollars at the end of time t. The existence of these bonds has no effect on the equilibrium of this representative agent economy, but simply allows us to determine the equilibrium nominal rate of interest. The household's

[3] Chari, Kehoe, and McGrattan (1996), are in turn building on the imperfect competition model of

intertemporal budget constraint is given by:

$$M_{t+1} = M_t + M_t^s(G_t - 1) + P_t(w_t L_t + r_t K_t) - P_t c_t - P_t[K_{t+1} - (1-\delta)K_t] + \Pi_t + B_t(R_t - 1)$$

where w_t and r_t are the wage and rental rates, K_t is the capital stock, δ is the rate of capital depreciation, and Π_t denotes firm profits. Note that we are assuming that capital accumulation occurs at the household level (and that investment is a credit good).

Final goods production in this economy is carried out in a perfectly competitive industry that utilizes intermediate goods in production. The CES production function is given by

$$Y_t = \{\int_0^1 [y_t(i)^{(\eta-1)/\eta}]di\}^{\eta/(\eta-1)}$$

where Y_t denotes the final good, and $y_t(i)$ denotes the continuum of intermediate goods, each indexed by $i \in [0,1]$. The implied demand for the intermediate good is thus given by

$$y_t(i) = Y_t[P_t(i)/ P_t]^{-\eta}$$

where $P_t(i)$ is the dollar price of good i, and P_t is the final goods price.

Intermediate goods firm i is a monopolist producer of intermediate good i. Fraction ν of these firms set their prices flexibly within each period, while the remainder $(1-\nu)$ must set their price one period in advance. The variable ν is thus a measure of price flexibility. Below we will refer to three natural variants of the model: (1) with $\nu = 1$, this is a flexible price model, (2) with ν between zero and one, this is a sticky price model, and (3) with $\nu = 0$, this is a model with rigid prices. Other than the difference in the timing of pricing, the firms are all symmetric, so we will henceforth

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Blanchard and Kiyotaki (1987).

drop the firm-specific notation. Let P_t^f denote the flexible price, while P_t^s will denote the pre-determined (or sticky) price. The final goods price (or aggregate price level) is given by the appropriate average of these two prices:

$$P_t = \left\{(1-\nu)P_t^{s(1-\eta)} + \nu P_t^{f(1-\eta)}\right\}^{\frac{1}{1-\eta}}. \qquad (1)$$

The flexible price is given by a constant mark-up over the marginal cost (z_t) of production:

$$P_t^f = \left[\frac{\eta}{\eta-1}\right] P_t z_t. \qquad (2)$$

The term in brackets will appear frequently below, so we define $z \equiv (\eta-1)/\eta < 1$. In a model with flexible prices, equation (2) and the assumption of symmetry implies that $z_t = z$. Combining (1)-(2), we have

$$P_t = h(z_t)P_t^s, \textit{ where } h(z_t) \equiv \left[\frac{1-\nu}{1-\nu\left(\frac{z_t}{z}\right)^{1-\eta}}\right]^{1/(1-\eta)} \qquad (3)$$

The function h is increasing so that innovations in marginal cost correspond with changes in the price level. As for the sticky price, it is given by the solution to the following maximization problem:

$$P_t^s = \arg\max E_{t-1}\left(\mu_{t+1} P_t Y_t \left[\frac{P_t^s}{P_t}\right]^{-\eta} \left\{\left[\frac{P_t^s}{P_t}\right] - z_t\right\}\right)$$

where E_{t-1} is the expectation conditional on time t-1 information. The intermediate goods firm is owned by the household, and pays its profits out to the household at the end of each period. Because of the cash-in-advance constraint on household consumption, the firm discounts its profits using $\mu_{t+1} \equiv \beta U_c(t+1)/P_{t+1}$, the marginal utility of \$1 in time t+1. The firm's optimal preset price is given by:

$$P_t^s = \left(\left[\frac{\eta}{\eta - 1} \right] \frac{E_{t-1}\{\mu_{t+1} P_t^{\eta+1} z_t Y_t\}}{E_{t-1}\{\mu_{t+1} P_t^{\eta} Y_t\}} \right).$$

Using (3), this can be written as

$$E_{t-1}\left[z\mu_{t+1} Y_t [h(z_t)]^{\eta}\right] = E_{t-1}\left[z_t \mu_{t+1} Y_t [h(z_t)]^{\eta+1}\right]. \qquad (4)$$

As for production, the intermediate firm rents capital and hires labor from households and utilizes a CRS production function,

$$y_t = \theta_t f(K_t, H_t) \equiv \theta_t K_t^{\alpha} H_t^{1-\alpha}$$

where θ_t is a measure of aggregate productivity, K_t denotes capital, and H_t denotes labor. With z_t as marginal cost, we then have $r_t = z_t\theta_t f_K(K_t,H_t)$ and $w_t = z_t\theta_t f_H(K_t,H_t)$. As for the technology variable, we assume that it evolves in the standard fashion:

$$\theta_t = \exp(\zeta_t), \text{ where } \zeta_t = \rho_\theta \zeta_{t-1} + \varepsilon_{\theta t},\ E_{t-1}(\varepsilon_{\theta t}) = 0,$$

where ρ_θ is the autocorrelation coefficient, and $\varepsilon_{\theta t}$ is the innovation in technology.[4]

Monetary policy is given by a process for P_t or R_t. In either case, G_t is endogenous with the central bank varying the money supply growth rate passively to achieve the targeted price level or nominal rate. The extreme versions of these two policies are an inflation rate peg $P_{t+1} = \pi P_t$ with $\pi > \beta$, and an interest rate peg, $R_t = R > 1$.

There are four markets in this economy, the labor market, the money market, the bond market, and the goods market, and the respective market-clearing conditions are

[4] Although we restrict the analysis to technology shocks, the principle theoretical conclusions of the paper (eg., Propositions 1 and 2) are valid for a wider set of shocks including fiscal behavior.

given by $L_t = H_t$, $M_t = M^s_t$, $B_t = 0$, and

$$c_t + K_{t+1} = \theta_t f(K_t, L_t) + (1-\delta) K_t. \tag{5}$$

A recursive competitive equilibrium is given by stationary decision that satisfy (3), (4), (5), and the following:

$$U_1(t)/U_c(t) = z_t\theta_t f_L(K_t, L_t)/R_t \tag{6}$$

$$\frac{U_c(t)}{R_t} = \beta E_t\left\{\frac{U_c(t+1)[z_{t+1}\theta_{t+1} f_K(K_{t+1}, L_{t+1}) + (1-\delta)]}{R_{t+1}}\right\} \tag{7}$$

$$\frac{U_c(t)}{P_t} = \beta R_t E_t\left\{\frac{U_c(t+1)}{P_{t+1}}\right\} \tag{8}$$

$$P_t c_t = M_t G_t \tag{9}$$

Before proceeding, a few observations about the sticky price equilibrium are in order:

First, (6)-(7) are remarkably similar to the standard real business cycle (RBC) Euler equations for labor choice and capital accumulation. They differ only in that both are distorted from the Pareto optimum by the nominal rate of interest (R_t) and the marginal production cost (z_t). As neither distortion enters into the resource constraint (5), R_t and z_t can be interpreted as distortionary taxes that are rebated to the representative household. Using this public finance interpretation, $\tau^c = (1-1/R_t)$ can be viewed as a consumption tax, and $\tau^w = (1-z_t)$ as the tax rate on wage and rental income. Standard arguments from public finance suggest that constant tax rates would be preferred to variable ones. Hence, one natural goal of monetary policy is to stabilize R_t and z_t.

Second, a log-linear approximation to the system implies that we can use (4) to set $z_{t+1} = z$ in the capital accumulation equation (7). That is, fluctuations in the implicit tax on factor income are not persistent but arise only in the period of a shock. Thus fluctuations in the marginal cost distortion have an effect on current labor input, but (to a log-linear approximation) have no effect on capital accumulation. We will henceforth refer to these fluctuations as movements in the wage tax. This behavior is in contrast to the nominal interest rate distortion where fluctuations are likely to be persistent. For example, under an inflation rate target a shock that leads to persistent movements in the real rate will lead to persistent movements in the nominal rate.

Third, recall that if all prices are flexible then marginal cost is constant, $z_t = z$. Hence, the sticky price assumption affects real behavior via movements in the marginal cost distortion on factor income. Conversely, for a given interest rate path, a monetary policy that stabilizes z_t is a policy that causes the sticky price model and the corresponding flexible price model (with the same interest rate path) to be observationally equivalent in real variables.

In the next two sections, the economy's behavior under price level targeting (Section 3) and interest rate targeting (Section 4) are discussed. There are two central themes. First, either target is potentially consistent with the sticky price economy responding to shocks as would a flexible price economy. Second, both targets are susceptible to real indeterminacy and sunspot fluctuations. To avoid these bad equilibria, Section 5 analyzes the two targets under a pre-determined money supply. Such a policy avoids the sunspot equilibria, but also hinders the economy's response to real shocks.

3. Equilibrium with Price Level Targeting.

In a model in which some firms must pre-set prices, there are clear advantages to the central bank eliminating uncertainty about the future price level. In particular, we have the following:

Proposition 1: Consider a monetary policy given by a pre-determined price level policy $p_t = p(k_{t-1},\theta_{t-1})$, where $p_t \equiv P_t/ M_t$. Consider two economies, one with flexible prices and one with sticky prices. Assume that monetary policy is conducted according to the same pre-determined price level policy in the two economies. Then the sticky price economy and the flexible price economy are observationally equivalent. This equivalence includes the behavior of the nominal rate of interest and the money growth process.

Proof: With P_t pre-determined, (3) implies that z_t is predetermined. From (4) we then have that $z_t = z$. Thus, real behavior in the sticky price and flexible price economies are identical. The supporting money growth process is then given by the cash constraint (9). The implied nominal rate is given by the Fisher equation (8). ♦

Remark: The equivalence in Proposition 1 does not necessarily hold in the case of completely rigid prices ($\nu = 0$). With rigid prices, the relationship between marginal cost and the price level (3) is broken, so that a predetermined price level does not pre-determine marginal cost. Thus there exist equilibria in which marginal cost responds to innovations (real or sunspot). There is of course an equilibrium in which marginal cost is constant and Proposition 1 is satisfied. The rigid-price case is an interesting counterexample to the equivalence result, but is likely empirically irrelevant, so henceforth we will ignore it by assuming $\nu > 0$. ♦

The intuition for Proposition 1 is quite clear. In a flexible price economy a pre-determined price level policy implies that firms would be willing to set their prices one period in advance. Hence, such a monetary policy causes the flexible price economy to act *as if* it were a sticky price economy. Or, in other words, *in the sticky price economy operating under such a policy the "sticky" firms would not want to alter their prices even if they could do so costlessly.* This monetary policy eliminates the sticky price rigidity by making prices rigid.

Proposition 1 of course holds for a price level peg. This is the policy advocated by Goodfriend and King (1998) and King and Wolman (1997). These papers demonstrate that in a Calvo-style (1983) price-setting environment, the observational equivalence of Proposition 1 occurs only with a price level peg.[5] A price level peg is required for observational equivalence in a Calvo setting because some firms wait an arbitrarily long amount of time before having the option of adjusting their prices.

[5] The counterexample in the Remark (where no firms are contemporaneously flexible) also holds in a Calvo-style economy.

Proposition 1 is a natural extension of this result to an economy in which prices are fixed for a finite number of periods.

Despite the simplicity of a price-level peg there are problems with such a policy. Carlstrom and Fuerst (1998) demonstrate that in a flexible price economy an inflation peg causes real indeterminacy. This implies an immediate corollary to Proposition 1: in a sticky price economy, a policy of targeting inflation causes real indeterminacy.[6] This arises because under such a policy the nominal rate moves too closely with the real rate. Since the nominal rate acts as a distortionary consumption tax, this policy implies raising consumption taxes when consumption is low. This policy creates self-fulfilling behavior: by lowering current consumption, agents increase the real rate, which (under an inflation rate peg) implies that the implied consumption tax rises, thus validating the initial consumption reduction. Under an inflation rate target, marginal cost is constant, but there is nothing to pin down the real and hence nominal rate of interest.

While this indeterminacy result under inflation targeting is robust to a wide range of parameter values, it is particularly easy to see in a special case. Suppose that labor is inelastic and the shocks are iid. Under a price level peg, we can use the Fisher equation (8) to express the capital equation (7) as

$$U_c(t+1) = \beta E_t \{U_c(t+2)[z\theta_{t+1} f_K(K_{t+1},1) + (1-\delta)]\}$$

Log-linearizing this equation yields

$$E_t Q(\tilde{K}_{t+3}, \tilde{K}_{t+2}, \tilde{K}_{t+1}, \tilde{\theta}_{t+2}, \tilde{\theta}_{t+1}) = 0$$

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[6] Goodfriend and King (1997) ignore these problems by ignoring the effect of the nominal rate of interest on the work effort decision (6) and the capital accumulation decision (7).

where the ~'s denote percent deviations from the steady state. Note that neither current capital nor current productivity enter directly into this equation. For determinacy, we need <u>two</u> explosive roots. If we do have two explosive roots, then the only equilibrium path is for capital to immediately jump to the steady state and stay there forever. This equilibrium is not only peculiar, it is also unlikely. For all plausible parameter values there will be only one explosive root. In this case, the equilibrium (scrolled back to time-t) is given by

$$\tilde{K}_{t+1} = \lambda \tilde{K}_t + \gamma \tilde{\theta}_t .$$

In this case, λ is given by the stable root of the following equation:

$$\lambda^2 = \left(1 + \frac{1}{\beta}\right)\lambda + \left(\frac{\beta f'' U'}{U''}\right) - \frac{1}{\beta}.$$

The larger root exceeds one, so we are interested in the smaller root. For ease of discussion, let us assume that this root is positive.[7] The corresponding characteristic equation for the RBC model is given by

$$\lambda^2 = \left(1 + \frac{1}{\beta} + \frac{\beta f'' U'}{U''}\right)\lambda - \frac{1}{\beta}.$$

The stable root of the RBC equation will (of course) not solve the characteristic equation of the inflation peg economy. Since this RBC root is positive and less than one, casual inspection reveals that the root of the inflation peg is smaller than that of the RBC model. That is, in comparison to the optimal path, convergence to the steady-state is faster under an inflation peg. The intuition is as follows. Suppose that capital starts

[7] For all plausible calibrations, this smaller root is positive. If it is negative, there are oscillatory dynamics.

below the steady state. Along the path the real rate will be falling. Under an inflation peg this implies that the nominal rate or implicit consumption tax is falling along the path. The rational agent thus postpones consumption and increases capital accumulation thereby speeding the movement to the steady state.

As for the response to the innovation, γ is free--the equilibrium is consistent with any response of investment to the productivity shock! (And we can, of course, add an arbitrary sunspot random variable to this solution.) The intuition is similar to the previous paragraph. Suppose that we consider the case of $\gamma = 0$. In this case, investment is constant and consumption moves sharply with the technology shock. Under an inflation peg, an increase in current consumption implies that the real and nominal rates fall. Referring back to the capital equation (7), this temporarily low nominal rate stimulates consumption and deters investment to such an extent that capital does not respond to the shock.

To eliminate this real indeterminacy under inflation rate targeting, we need another real variable to be pre-determined. We take up this issue in Section 5.

4. Equilibrium with Interest Rate Targeting.

The previous discussion suggests that to avoid indeterminacy the central bank should dampen the nominal rate's response to the real rate. This suggests an interest rate peg will eliminate this form of indeterminacy. However, Carlstrom and Fuerst (1998) demonstrate that at a zero nominal rate there is real indeterminacy in a sticky price model. The next proposition extends this result to the case where nominal rates are positive. In an exact reversal of the indeterminacy under inflation rate targeting, this indeterminacy arises because marginal cost is not pinned down.

Proposition 2: Suppose that in an economy with sticky prices the central bank pegs the nominal rate $R_t = R$. Then the economy's real behavior is indeterminate. This indeterminacy arises because the money supply is endogenous, and there is no restriction on the response of marginal cost to innovations (including sunspots).

Proof: With a fixed nominal rate, real behavior is given by (4)-(7). The only restriction on marginal cost is given by (4). In particular, there is essentially no restriction on the response of z_t to innovations (either real or sunspot). This behavior is then supported by the endogenous money supply process implied by (9). ♦

This real indeterminacy and potential sunspot behavior arises because the money supply is endogenous and not uniquely determined, and fraction (1-ν) of firms are not operating on their labor demand curves but agree to supply whatever is demanded at the pre-set nominal price. These firms' hiring needs are thus determined by the monetary authority--higher money growth implies a higher nominal and, since prices are sticky, real demand for output. The firm absorbs the resulting fluctuations in costs in its mark-up: the mark-up ($1/z_t$) varies negatively with the rate of money growth.

An immediate corollary to Proposition 2 is that there exists an equilibrium in which marginal cost is constant. That is, there exists a supporting money growth process that pegs the nominal rate <u>and</u> marginal cost. Since marginal cost is constant, this supporting money growth process also predetermines the price level. Thus, a combination policy of pegging the nominal rate with a predetermined price level: (1) stabilizes the interest-rate distortion, (2) stabilizes the marginal cost distortion on factor prices, and (3) is not subject to real indeterminacy.[8] This "nirvana" policy is probably of little practical interest since it assumes a great deal of information on the part of the central bank.

5. Targeting with a Pre-Determined Money Supply.

To summarize the previous two sections, an inflation rate peg is advantageous as it stabilizes the marginal cost distortion. But such a peg is clearly inconsistent with a constant interest rate as the nominal rate moves one-for-one with the real rate. This latter fact implies that there is real indeterminacy and sunspot equilibria under an inflation rate peg. To avoid these sunspot equilibria, the variability of the nominal rate must be dampened. An interest rate peg can be consistent with a constant marginal cost and, of course, a constant nominal rate. The disadvantage of such a policy is that there are sunspot equilibria in which marginal cost fluctuates.

To avoid the sunspot equilibria under the two possible targets, we need an extra constraint on monetary policy. This Section explores one natural possibility: the central bank pre-determining the money stock, ie., eliminating the money stock's response to contemporaneous innovations. The disadvantage of this restriction is that real cash

[8] As in Goodfriend and King (1997), this policy implies that the central bank varies the money supply to achieve the desired level of real balances without movements in the price level. In particular, policy responds to the innovations in consumption spending that would arise in the corresponding flexible price economy. This is quite convenient as the flexible price economy is essentially a standard RBC economy, and we have a great deal of knowledge about consumption spending in that model. Hence, a transient (persistent) productivity increase should be matched by a relatively small (large) increase in the money supply, while a transient (persistent) government spending increase should be matched by a relatively small (large) decrease in the money supply.

balances are sticky, thus making the economy's initial response to a shock suboptimal. We begin the analysis by working out some simple examples, and then use the intuition developed to discuss the impulse response functions in a realistically calibrated business cycle model.

A Labor-Only Example:

Suppose that $U(c,1-L) = \ln(c) - L$, $f = (\theta L)^{1-\alpha}$. The most interesting difference in the economy's behavior under the two monetary targets is in the response to an innovation in the environment. Hence, for simplicity assume that the technology shocks are iid. As point of comparison, recall that RBC behavior implies that labor is constant in this well-known example.

Interest Rate Peg

Suppose first that the central bank pegs the nominal rate with a predetermined money supply. Log preferences and predetermined money growth imply that money growth is constant ((8)-(9)). Approximating the function h around the steady-state of $z_t = z$, we have $d\ln(h)/d\ln(z_t) = \nu/(1-\nu)$. Expressing the cash constraint (9) in log deviations (hereafter denoted with a ~) yields:

$$\tilde{z}_t = \left[\frac{\nu - 1}{\nu}\right]\tilde{c}_t$$

where we have used the fact that with iid shocks P^s_t/M_t is a constant. Substituting this into (6) and the resource constraint yields

$$\tilde{L}_t = \left[\frac{(\nu-1)(1-\alpha)}{(1-\alpha+\alpha\nu)}\right]\tilde{\theta}_t .$$

Consider the response to a one-time shock. Contemporaneous labor input falls, with the strength of the decline determined by the fraction of firms that are sticky (1-ν). Labor input falls because marginal cost z_t falls (the wage tax rises). Output rises as long as prices are not completely rigid (ν = 0). In the next period, labor returns to its steady state as marginal cost returns to normal. Hence, relative to RBC behavior, the unusual effect of a shock arises only in the impact period.

Inflation Peg

In contrast, suppose that the central bank pegs the price level. In this case, the cash constraint implies that consumption (and thus output) is predetermined, so that labor responds sharply downward. In particular, (6), (8), and the resource constraint yield:

$$\tilde{L}_t = \left[\frac{\alpha}{(\alpha-1)}\right]\tilde{L}_{t-1} + \tilde{\theta}_{t-1} - \tilde{\theta}_t$$

Note that the contemporaneous decline in labor is equivalent to the behavior under the interest rate peg only if prices are completely rigid (ν = 0). If some firms adjust prices contemporaneously, labor responds more sharply downward under an inflation rate peg. The irony is that while an inflation peg makes the sticky price economy behave as if all prices were flexible, the economy behaves with little flexibility. The contemporaneous decline in labor occurs because the nominal rate (consumption tax) rises. With inflation pegged and current consumption fixed, the real and thus nominal rate rises because future consumption (and future labor input) rises. Thus labor responds positively with a lag to the shock, and thereafter the consumption dynamics are damped oscillations back to the steady-state. These oscillations are supported by corresponding movements in the nominal rate. Relative to the behavior under an interest rate rule, the perverse response to a transient shock is remarkably persistent.

An Example with Capital and Labor:

Suppose that U(c,1-L) = ln(c)-L, $f = K^{\alpha}(\theta L)^{1-\alpha}$, $\delta = 1$, and the technology shocks are iid. As point of comparison, recall that RBC behavior implies that consumption and investment move one-for-one with output and that labor is constant.

Interest Rate Peg

In the case of an interest rate peg, we have:

$$\tilde{L}_t = \tilde{z}_t = \left[\frac{(\nu - 1)(1-\alpha)}{(1-\alpha+\alpha\nu)}\right]\tilde{\theta}_t$$

$$\tilde{c}_t = \tilde{K}_{t+1} = \alpha\tilde{K}_t + \left[\frac{\nu(1-\alpha)}{(1-\alpha+\alpha\nu)}\right]\tilde{\theta}_t .$$

The shock leads to a fall in marginal cost z_t (an increase in the wage tax). The extent of this decline depends upon the level of price rigidity (ν). At the extreme of full price flexibility (ν=1) marginal cost and labor are constant as in the corresponding RBC model. For less than perfect flexibility, the temporary tax change produces a negative labor response and thus a dampened response of consumption and investment (relative to RBC). As before, the perverse behavior of labor in response to a shock is over after the period of the shock (once prices adjust).

Inflation Peg

In contrast, under an inflation peg, the perverse behavior is persistent. Straightforward calculations imply:

$$\tilde{L}_t = \left[\frac{(1-\alpha\beta z)(1-\alpha)}{(1-\alpha\beta z)+(1-\alpha)}\right]\tilde{K}_t - \left[\frac{(1-\alpha)(1-\alpha\beta z)^2}{\alpha^2\beta z(1-\alpha\beta z)+(1-\alpha)}\right]\tilde{\theta}_t$$

$$\tilde{c}_t = \left[\frac{(1-\alpha^2\beta z)}{(1-\alpha\beta z)+(1-\alpha)}\right]\tilde{K}_t$$

$$\tilde{K}_{t+1} = \alpha\tilde{K}_t + \left[\frac{(1-\alpha)[(1-\alpha)+(1-\alpha\beta z)]}{\alpha^2\beta z(1-\alpha\beta z)+(1-\alpha)}\right]\tilde{\theta}_t .$$

A few observations are in order.

First, the coefficient of capital-on-capital is exactly α, the RBC value. This is surprising given Section 3's demonstration that in an inelastic-labor model, convergence to the steady-state is faster under an inflation peg, that is, the capital-on-capital coefficient is less than α. The difference arises because of the effect of labor. When capital is below the steady state the nominal rate is relatively high and is expected to decline. This temporarily high nominal rate leads to two offsetting effects on capital accumulation. Since investment is a credit good, there is a stimulus to investment (the coefficient of consumption on capital is strictly greater than α). However, since leisure is also a credit good, there is a negative effect on current labor input and thus output (the coefficient of labor on capital is positive in contrast to zero for the RBC model). These two effects exactly cancel leading to a coefficient of capital-on-capital that is identical to RBC.

Second, the response of labor to an innovation is negative, but not as sharply negative as in the inflation-peg case without capital. The investment channel is critical. Although consumption is predetermined, since investment can respond output is not predetermined. Hence, output responds positively to the shock. Since consumption is predetermined movements in output must be entirely absorbed by movements in investment. For example, notice that the coefficient of investment on the shock is strictly greater than the RBC value of (1-α), and that output responds less strongly than in the RBC economy.

Finally, in the previous model without capital, the labor response is always larger (less negative) in the interest-rate-peg economy than in the inflation-peg economy. However, in a model with capital, the initial labor responses in the two economies are identical at the degree of price rigidity given by

$$(1-\nu) = \frac{(1-\alpha\beta z)^2}{(1-\alpha^2\beta z)}.$$

With β = .99, α = 1/3, z = 10/11, we have a bound of (1-ν) = .54. If prices are more sticky than this, then the initial labor response is smaller (more negative) in the interest-rate peg economy than in the inflation-peg economy.

Impulse Response Functions in a Calibrated Model

The previous examples illustrate the basic mechanisms at work in the numerical simulations. A positive technology shock will cause the marginal cost distortion (wage tax) to rise under an interest rate peg, and the nominal rate distortion (consumption tax) to rise under an inflation peg. These tax movements will alter the responses of employment and investment to productivity shocks relative to the RBC responses. The simulations put some quantitative detail on this qualitative frame.

The model is calibrated as follows. Preferences are given by U(c,1-L) = ln(c) – AL, where A is chosen to imply that steady-state work effort is 1/3. Assuming the time period to be one quarter, we set $\beta = 0.99$ implying a 4% annual real rate of interest. The capital coefficient is set at $\alpha = 0.34$, and the quarterly rate of depreciation is $\delta = 0.02$. The steady-state level of quarterly money growth is G = 1.01 in all simulations, implying a steady-state annual nominal rate of 8%. The steady-state mark-up is set at 10% implying $\eta = 11$. As for the degree of stickiness in the interest rate peg economies, figure 1 reports results for $\nu = 0.20$, $\nu = 0.80$, and $\nu = 1$ (the RBC economy). Figure 1 also shows the impulse response for the inflation peg economy. The shock is a 1% productivity shock with $\rho = .95$.

A key result is that the two monetary policies lead to differing movements of the implicit taxes. Depending on the degree of stickiness, the tax movement under an inflation peg can be higher or lower than the tax movement under an interest rate peg. With ν=1 the wage tax is constant. With $\nu = 0$, in order to keep consumption constant the wage tax must increase from 9.1% to 12.5%! In contrast, under an inflation peg, to keep consumption constant the consumption tax need only rise from 1.98% to 2.3%. This demonstrates clearly the difference between a consumption tax and a wage tax in a model with investment.

The intermediate values of ν illustrate an important non-linearity. A little flexibility goes a long way. With ν=0 the wage tax rises by 3.4%, vs. 0.9% for ν=0.2 and 0.1% for ν=0.8. This non-linearity also appears in the employment response. With

$\nu = 0$, employment falls by 9.1%. At the other extreme, with $\nu = 1$, employment rises by 0.97%. With $\nu = 0.2$, the employment response is –1.6% (3/4 of the way between –9.1% and 0.97%) while the employment response is 0.76% (99% of the way between –9.1% and 0.97%) with ν=0.8.

In the case of an inflation peg, the temporary consumption tax increase discourages work effort but causes an investment surge (relative to an RBC economy) which tends to stimulate work effort. Taken together, the inflation peg's employment response is only slightly below that of the RBC model, while investment is amplified considerably relative to RBC (6.2% vs. 5.1%). Consumption does not respond contemporaneously, but then jumps up in the second period.

Comparing the different employment responses for the $\nu = 0.80$ case and the inflation peg case reveals the differing distortions at work here. The shock causes the implicit consumption tax (in the inflation peg economy) to rise by more than the implicit wage tax (in the interest rate peg economy). But since one is a wage tax, and the other is a consumption tax, the effect on employment is much different because of the possibility of altering investment behavior.

The earlier 100% depreciation example emphasized that under an inflation peg a shock leads to a persistent deviation of the nominal rate (or implicit consumption tax) from the steady state. In contrast, under an interest rate peg the movement of marginal cost from steady state ends after one period (as soon as all prices can adjust). With plausible calibration, however, this qualitative difference in the two monetary policies seems to be quantitatively unimportant. The deviation of the nominal rate from steady state is trivial in the periods after the shock. Thus, in the period after a shock, the two economies behave essentially as an RBC economy with a constant wage tax and a constant consumption tax. For example the capital-on-capital coefficients are identical while the consumption-on-capital coefficient rises from 0.5 in the interest rate peg economy to 0.52 in the inflation peg economy.

A Welfare Analysis

We will exploit this approximate RBC-equivalence (in the period following the shock) in conducting a simple welfare experiment. Consider two economies: (1) an economy operating under an inflation-peg, and (2) an economy operating under an interest-rate peg. Suppose both economies begin at the non-stochastic steady-state and are hit by a one-time productivity innovation (with $\rho=0.95$) in, say, period 1. The previous discussion implies that from period 2 onwards the economies (essentially) follow RBC dynamics. To solve for the initial response to the shock, we will use the period 1 Euler equations and the log-linear decision rule for RBC consumption in period 2.[9] We can then solve this nonlinear system that describes period 1 behavior without resorting to any further linear approximations. This is particularly useful as the fluctuating taxes have important nonlinear effects.[10] Once we have calculated period 1 consumption, employment, and investment, we can then compare utility across the two economies by turning period 2 capital into utility units using period 2's RBC marginal utility of consumption. Since the period 1 responses are non-linear, we perform this

[9] In the case of the interest rate peg, we also need an expression for period 2 employment and thus add period 2's (nonlinear) employment Euler equation to the system.

[10] For example, suppose that utility were log-log and we used the log-linear decision rules for period 1. Then the productivity shock would enter the welfare comparison linearly so that the expected welfare

calculation for a positive and a negative shock, and then average the two together. That is, we are interested in the welfare consequences of either a positive or negative productivity innovation, each of which occurs with equal probability.

The results of the welfare experiments are reported in Figures 2 and 3 (measured in utility differences). For very low levels of price flexibility ($\nu < 0.18$), the inflation peg dominates the interest rate peg. However, if the economy is more flexible than this ($\nu > 0.18$), an interest rate peg dominates although the welfare gain is relatively small.

This basic result is largely insensitive to alternative values for the substitution elasticity (η) and money growth rate (G). These values do reveal an interesting nonlinearity. Under an interest rate rule, for large values of η ($\eta = 51$), stickiness is preferred to flexibility. That is, welfare is higher for $0.09 < \nu < 1$ than an economy with perfect price flexibility ($\nu = 1$). This implies that a variable wage tax is preferred to a constant wage tax. This surprising result occurs because in the former case the average value of z is larger than the non-stochastic steady-state value of z (recall that z = 1 is the first-best).

This nonlinearity can best be understood by recalling the pricing equation (3) and the binding cash constraint (9). With a predetermined money stock and sticky prices, movements in consumption can occur only if marginal cost (z) moves in the opposite direction. This implicit relationship between consumption and marginal cost is convex. Hence, fluctuating consumption will lead to an average level of z that is higher than the steady-state z. For benchmark parameters ($G = 1.01$, and $\eta = 11$), this effect

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differences would all be zero.

peaks at $\nu = 0.05$, where the average wage tax (1-z) is 9.03% vs. a steady-state wage tax of 9.09%. With greater convexity, this effect is magnified: with $\eta = 51$ (a 2% mark-up), the average wage tax is 1.71%, while the steady-state is 2.0%.

There are a few caveats to this welfare comparison. First, by using the log-linear decision rule for period 2 consumption we ignore a potential nonlinearity. It is unclear which way this biases the results. However, given that we start the economy at the steady-state capital stock and consider a small productivity shock, this bias is likely to be small. Second, as already noted, the estimate assumes that under the inflation rate peg the nominal rate is back to the steady-state for period 2 and onwards. This assumption probably biases the results in favor of an inflation peg.

Third, and most obviously, these are welfare comparisons along an impulse response from a one-time shock, and not lifetime utility measures in a stochastic economy. In particular, we are ignoring all future nonlinearities that would arise in an economy subject to repeated shocks. For example, as noted above the expected future value of marginal cost is greater than the non-stochastic steady state. A similar condition holds for the consumption tax in the inflation peg case. This discrepancy is very small: for the baseline calibration, the average consumption tax (1-1/R) is 1.9805% vs. 1.9801% for the non-stochastic steady state. These omissions tend to bias the results in favor of the inflation peg.

6. Conclusion

This paper is a tale of distortions. The model economy is distorted relative to RBC behavior by marginal cost and the nominal rate of interest. These distortions are fundamental, arising from the assumption of sticky prices and the need to use money to facilitate transactions.

The monetary policy rule in place alters the economy's response to shocks by altering the endogenous movement of these distortions. A price level peg stabilizes marginal cost, but allows the nominal rate to vary. An interest rate peg has the exact opposite effect.

As a result, these two different monetary policies have different positive implications for the real economy's response to a shock. Under an interest rate peg, the employment and investment responses are "too small" (relative to RBC). Under an inflation peg, the investment response is "too large" while the consumption response is "too small".

On the normative side, these very different responses lead to a ranking of policies. For small degrees of price flexibility, the marginal cost distortion is potentially quite variable so that an inflation peg dominates. If more than 20% of firms have price flexibility, this dominance is reversed and the interest rate peg is preferred. However, the welfare differences of the two policies are quite small.

This near-welfare-invariance to cyclical monetary policy is similar to the results of Dow (1995), Carlstrom and Fuerst (1995), and Ireland (1998). All of these analyses ignore the potential welfare cost of a monetary policy that introduces sunspot equilibria into the economy. These costs could be quite large. From this perspective, the most useful policy suggestion is to do no harm: be somewhat agnostic in the choice of cyclical monetary policy, but make sure that one chooses a policy rule that does not introduce sunspot equilibria. This latter goal is achieved by using a policy rule that severely restricts the immediate policy response to innovations in the real environment.

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