Self-selection and Discrimination in Credit Markets
by Stanley D. Longhofer and Stephen R. Peters
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IN CREDIT MARKETS*

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Abstract

In this paper we make two contributions toward a better understanding of the causes and consequences of discrimination in credit markets. First, we develop an explicit theoretical model of the underwriting process in which lenders use a simple Bayesian updating process to evaluate applicant creditworthiness. Using a signal correlated with an applicant’s true creditworthiness and their prior beliefs about the distribution of credit risk in the applicant pool, lenders are able to evaluate an applicant’s expected or “inferred” creditworthiness to determine which loans to approve and which ones to deny. Second, we explicitly model the self-selection behavior of individuals to show how market frictions like bigotry can affect application decisions. Because these decisions shape banks’ prior beliefs about the distribution of credit risk, they also affect the Bayesian posterior from which banks compute an applicant’s inferred creditworthiness, implying that statistical discrimination can arise endogenously. In a market in which only some lenders have “tastes for discrimination,” we show that there are conditions under which lenders without racial animus will also discriminate.
1. Introduction

The issue of discrimination in mortgage lending has been at the forefront of policy debates in commercial banking and social economics throughout the decade. Ignited by empirical findings reported in the so-called “Boston Fed study” (Munnell et al., 1992, 1996), the debate over lending discrimination continues today. In the years following the initial Boston Fed study, academics, bankers, activists, and policy makers have struggled to agree on how best to rectify discriminatory lending practices. At the same time, no clear consensus has been reached on whether or not lenders actually do discriminate. At the heart of this conundrum is the difficulty in establishing what discrimination looks like and how it might be detected. This paper is an attempt to confront these issues by studying the loan underwriting process.

Virtually all of the research in this area has been empirical, and much of it has focused on the validity of Munnell et al.’s results and on finding ways to analyze and detect discrimination in mortgage lending data. In contrast, there has been little work on economic theories that might explain the discriminatory behavior Munnell et al. claim to have uncovered or that would provide a framework for studying the loan underwriting process. This lack of economic theory has hindered the design of appropriate empirical tests for lending discrimination.

Our paper makes two contributions toward a better understanding of the causes and consequences of discrimination in credit markets, yielding interesting insights into the behavior of both lenders and mortgage applicants. First, we develop an explicit theoretical model of the underwriting process that accounts for lenders’ efforts to ascertain applicant creditworthiness in the presence of imperfect information. In our model, lenders use a simple Bayesian updating process to evaluate applicant creditworthiness. In essence, the information a lender collects on an application is a signal that is (imperfectly) correlated with the applicant’s true credit risk. Taking into account its prior beliefs about the distribution of credit risk in the overall applicant pool, the lender is able to use this signal to calculate an applicant’s expected or “inferred”

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2 Some theoretical contributions include Calomiris et al. (1994), Calem and Stutzer (1995), Ferguson and Peters (1997a), and Han (1997).
creditworthiness; only those applicants whose inferred creditworthiness is sufficiently high are approved for loans. Thus, lenders use both the information provided by applicants and their own prior underwriting experiences to determine which loans to approve and which to deny.

Our second contribution lies in our focus on the self-selection behavior of individual applicants. We examine how the market as a whole responds when a subset of lenders have “tastes for discrimination” (Becker, 1971). Our model illustrates how minority applicants’ reactions to this discrimination alter the relative distributions of minority and white credit risk in bank applicant pools, giving non-bigoted lenders an incentive to statistically discriminate against minorities. This statistical discrimination arises endogenously as a result of applicant responses to bigotry. Furthermore, this statistical discrimination against minorities occurs in our model even though minority applicant creditworthiness and white applicant creditworthiness are assumed to be identically distributed in the general population.

An additional advantage of our model is that it allows an explicit definition of what constitutes discriminatory behavior, and can thus be used to aid in the design of empirical tests to uncover any discrimination that may exist, as well as to reveal the source of this discrimination. For example, we show that the conditional default rate of minorities is lower than that of whites at bigoted banks (as suggested by Becker, 1993). In contrast, banks that statistically discriminate against minorities will observe higher conditional default rates among their minority borrowers (consistent with the findings of Berkovec, et al., 1994). Thus, while a Becker-style test on loan default rates would be capable of identifying bigoted lenders, it would allow statistical discriminators to discriminate with impunity. To avoid this problem, we suggest a test involving applicant denial rates that can not only determine whether lenders discriminate, but also whether that discrimination arises because of lender beliefs (statistical discrimination) or lender preferences (bigotry).

The dearth of theoretical work addressing the problem of discrimination in credit markets may arise out of a belief that theoretical results developed for labor markets apply directly to credit markets as well. This, however, is often not the case. In labor markets, the employer’s problem is typically to select the most desirable applicant. In contrast, lenders generally approve all applicants that exceed a given threshold. In other words, labor market discrimination arises
from the treatment of individuals in the upper tail of the distribution of applicant characteristics, while credit market discrimination arises from the treatment of individuals who fall somewhere in the middle of this distribution. As a consequence, a given market friction can result in vastly different outcomes in labor and credit markets.³

One example of this can be seen in analyses of “cultural affinities.” Cornell and Welch (1996) consider a labor market in which employers are better able to assess the true productivity of job applicants with whom they share a common cultural background. In this setting, Cornell and Welch show that as the size of the applicant pool increases, the probability that the most qualified applicant is a member of their same group approaches one. In other words, employers will naturally tend to hire applicants with whom they have a cultural affinity, even in the absence of any tastes for discrimination. Calomiris et al. (1994) and Longhofer (1996) analyze virtually the same informational problem in the context of credit markets and show that it gives lenders an incentive to discriminate against applicants with whom they have a cultural affinity. Thus, the same informational friction provides two diametrically opposed predictions for labor and credit markets.

Given the fundamental differences between labor and credit markets, the importance of modeling discrimination incentives separately for each becomes clear. Our paper provides a major step in filling this void in the literature.

In the next section, we review several theories of discrimination and discuss how these models relate to our own. In section 3, we introduce our model of loan underwriting and show how lenders use their past underwriting experiences and beliefs in evaluating applicant creditworthiness. Then, in section 4 we complicate the model by making banks heterogeneous. In particular, we allow some banks to exhibit Beckerian tastes for discrimination, giving minority borrowers an incentive to self-select among banks. We demonstrate how this behavior can affect bank underwriting standards and other market outcomes. Because both individual application decisions and bank lending decisions are endogenous in our model, it provides a rich environment for analyzing a variety of empirical and policy questions; we discuss these in

³ Heckman (1998) discusses how precisely this kind of problem arises in “audit pair” tests for labor market discrimination. He argues these tests are flawed because they typically do not control for the qualifications of the pair relative to the qualifications necessary for acceptance.
Section 5. Section 6 concludes, while proofs of all results and propositions are found in the Appendix.

2. Discrimination in Credit Markets

Becker (1971) pioneered the economic analysis of discrimination, developing a theory based on the preference a bigot has for one group over another. As Becker argues, a taste for discrimination makes a bigot willing to expend a cost (or forgo a benefit) to associate with his preferred group. As applied to credit markets, a bigoted lender will hold applicants from his preferred group to a lower credit standard than applicants from another group, causing the lender to make loans to high-risk applicants from the preferred group, while denying equally risky applicants from other groups.\(^4\) Alternatively, bigoted lenders may charge minorities higher interest rates or fees than equally qualified whites.\(^5\)

Becker’s theory is one of preference-based discrimination. In contrast, statistical or belief-based discrimination can arise when the characteristics of an individual’s group are used to evaluate his or her personal characteristics.\(^6\) For example, if high obligation ratios are better predictors of default risk for members of one group than another, lenders may want to take group membership into consideration when making underwriting decisions. Arrow (1972a, 1972b, 1973) and, separately, Phelps (1972) were among the first to consider models of statistical discrimination, focusing on the problem of an employer with exogenously given beliefs that the average productivity of white labor is higher than that of minority labor. In the mortgage market, statistical discrimination would arise if lenders could not perfectly observe individual creditworthiness, and believed that minority applicants were less creditworthy on average.\(^7\)

\(^4\) Ferguson and Peters (1998), however, show that if credit is rationed a bigoted lender can discriminate without forgoing profits and without creating a difference in marginal default rates between minorities and non-minorities.

\(^5\) This seems less likely, as a single lender’s mortgage terms vary surprisingly little with individual applicant characteristics. See, for example, Sirmans and Benjamin (1990), Duca and Rosenthal (1994), Benjamin et al. (1995), and Avery et al. (1996). Yet a third potential for discrimination is for lenders to restrict minorities access to credit through, for example, their marketing efforts.

\(^6\) Statistical discrimination is sometimes distinguished from taste-based discrimination through labels such as “economic” or “rational” discrimination. Such labels can be misleading, however, since bigots are perfectly rational economic agents, maximizing their (albeit socially condemned) preference function subject to constraints.

\(^7\) See Calem and Stutzer (1995) for an alternative model of statistical discrimination in credit markets.
More recent research has suggested that cultural differences between banks and borrowers (Calomiris et al., 1994, and Longhofer, 1996) or between employers and job applicants (Cornell and Welch, 1996) can lead to a form of statistical discrimination. The intuition behind this theory is that lenders may share an affinity with members of one group due to a common ethnic background, race, religious belief, gender, education, or other social bond. Ferguson and Peters (1997) extend this idea, arguing that affinities may arise endogenously based on a bank’s experience in working with applicants from different groups. Whether cultural or experiential, however, an affinity allows lenders to more accurately evaluate the merits of applicants with whom they share this common bond.

Our model of the mortgage underwriting process is able to incorporate each of these potential sources of discrimination, and using it we can replicate the traditional empirical implications of each. In addition, however, our model allows applicants to react to the underwriting criteria established by different banks, self-selecting in an attempt to maximize their chances of obtaining loans.

Thus, our model illustrates how the choices that individuals make regarding whether or not to apply for loans—and to which bank they apply—are based on the likelihood that they will be approved and, therefore, imply a correlation between the creditworthiness of a bank’s applicant pool and race. Profit- and utility-maximizing banks have an incentive to use the information this correlation reveals in order to more accurately assess credit risk. In other words, applicant self-selection behavior can lead to endogenous differences in the average creditworthiness of different groups. As a result, statistical discrimination arises even if lenders believe that the distribution of creditworthiness is the same across groups in the general population.

It is important to note that the endogeneity of creditworthiness in our model is different from that suggested by Yezer et al. (1994), who claim that individuals adjust their application information based on hints from loan officers regarding the probability of rejection. As a result, they argue an individual’s true underlying creditworthiness is endogenous. In contrast, we assume that an individual’s creditworthiness is exogenously given. Instead, applicants in our

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8 See Coate and Loury (1993a, 1993b) for analyses of endogenous skill acquisition by job applicants.
model self-select among banks, making the distribution of credit risk endogenous in bank applicant pools.

Our model’s empirical predictions are consistent with many existing empirical studies on discrimination in mortgage markets. It is important to keep in mind, however, that the primary contribution of our paper is not simply in the specific empirical predictions it may provide, but also in the structure it provides us for thinking about how market frictions may affect the mortgage underwriting process. Our framework for analysis will provide a foundation for future researchers and policymakers to better understand the true causes of observed racial differences in credit markets, and to analyze how various policy corrections would affect the market.

3. A Model of Mortgage Underwriting

Consider a world in which individuals want to buy a house but lack sufficient funds to do so. As a result, they must obtain loans from a financial institution, which we will call a “bank.” Each individual in the population is assumed to have a true creditworthiness represented by $\theta \in [0,1]$. We interpret $\theta$ as an individual’s likelihood of repaying his loan, and assume that it captures all of the factors that might cause him to default, including disruptions to his income, changes in the value of his house, and his personal compunction about defaulting on an obligation.

Creditworthiness is assumed to be distributed throughout the population according to the probability density function $f(\theta)$, with cumulative distribution function $F(\theta) = \int_0^\theta f(t)dt$; all lenders and individuals share these prior beliefs about the distribution of true creditworthiness in the population.

Although each individual’s creditworthiness, $\theta$, is given exogenously, an individual’s application decision is endogenous.\footnote{Our model allows individuals to choose whether to apply for a loan given their creditworthiness, but it does not allow them to enhance or alter their true underlying $\theta$, in contrast to Yezer et al. (1994). Such an extension to our model would be feasible, in principle.} Therefore, we must distinguish between the distribution of creditworthiness in the population, $F(\theta)$, and the distribution of creditworthiness in the applicant pool. We will denote this latter distribution by $G(\theta)$ with corresponding density $g(\theta)$. 

Based on their costs of funds and the competitive interest rate in the market, banks determine a $\theta^*$ defining the minimum acceptable creditworthiness they are willing to approve. In a full-information world, lenders would know each individual’s true $\theta$, and would approve only those applicants with $\theta \geq \theta^*$. Anticipating this, only individuals with such a $\theta$ would apply, and lenders would approve all applicants.

Unfortunately, lenders cannot perfectly observe $\theta$. Instead, they observe a signal, $s = \theta + \epsilon$, where $\epsilon$ represents the lender’s errors in assessing credit risk. We assume that this signal aggregates all the information banks collect about an applicant’s creditworthiness (e.g., obligation ratios, credit histories, property characteristics, and other information that lenders collects in the application process). For example, $s$ would include Using a Bayesian updating process, banks use this signal to update their prior beliefs about an applicant’s true creditworthiness.

Let $p(s \mid \theta)$ be the likelihood that a lender observes signal $s$ from an applicant of type $\theta$. The usefulness of the signal will depend on the characteristics of this signal generation process. For our analysis, we assume that $p$ is characterized as follows:

**ASSUMPTION A1:** For every applicant type $\theta$,

1) $E[s \mid \theta] = \theta$;
2) $p(\theta - \delta \mid \theta) = p(\theta + \delta \mid \theta), \ \forall \delta$;
3) $p'(s \mid \theta) > 0, \forall s < \theta$ and $p'(s \mid \theta) < 0, \forall s > \theta$;
4) $\frac{\partial p(s \mid \theta)}{\partial \theta}$ is increasing in $s$; and
5) $p(s \mid \theta)$ is strictly positive on its support, $(-\infty, +\infty)$, and has continuous partial and cross-partial derivatives.

Part 1 of this assumption simply states that $s$ is an unbiased signal of an applicant’s true underlying creditworthiness, $\theta$. Part 2 is a symmetry assumption, and guarantees that applicants are equally likely to send erroneous signals in either direction. Part 3 ensures that $p$ is unimodal, or “hump-shaped.” Part 4 states that $p$ satisfies the monotone likelihood ratio property. Finally, part 5 is a technical assumption to facilitate some of the proofs that follow. All of these assumptions are reasonably standard and are satisfied by a number of common distributions,
including the normal, logistic, and non-central $T$ distributions.

Based on this signal generation process and the distribution of its applicants, the unconditional density of signals observed by a bank is

$$\omega(s) = \int_T p(s | \theta) g(\theta) d\theta, \quad (1)$$

where $T$ is the set of all types that apply in equilibrium. We then define the likelihood that an applicant sending signal $s$ has true creditworthiness $\theta$ as

$$\pi(\theta | s) = \frac{p(s | \theta) g(\theta)}{\omega(s)}, \quad \forall \theta \in T, \quad (2)$$

and 0 otherwise. Thus, $\pi$ is the lender’s Bayesian posterior beliefs on $\theta$.

Lenders are not interested in an applicant’s signal, per se. Rather, they grant credit based on an applicant’s expected creditworthiness derived from this posterior $\pi$. We will often refer to this expected creditworthiness as an applicant’s inferred “quality,” which is denoted by

$$q(s) = \int_T \theta \pi(\theta | s) d\theta; \quad (3)$$

a lender will approve an applicant if and only if $q(s) \geq q^* \equiv \theta^*$.

**RESULT 1:** *Inferred creditworthiness is increasing in the observed signal: $q'(s) > 0$.*

This result is simply a formalization of the intuition that applicants who send better signals tend to be more creditworthy. More importantly, this result assures us that there exists a unique cutoff signal determining which applicants will be approved.

**RESULT 2:** *There exists a unique $s^* = q^{-1}(\theta^*)$, such that every applicant with $s \geq s^*$ is approved.*

Given this structure of the underwriting process, we can now determine which individuals will apply for loans. Let $\alpha(\theta)$ denote the probability that a type-$\theta$ applicant is approved for a loan. Given Result 2,

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10 This Bayesian process formally models the intuitive notion that lenders use their past experiences to interpret the information on loan applications. This is true regardless of whether lenders formally subject their loan applications to scoring models.
\[ \alpha(\theta) = \int_{s}^{\infty} p(s | \theta) ds. \]  

(4)

It is straightforward to verify that the probability of being accepted for a loan is increasing in the applicant’s type.

**RESULT 3:** \[ \alpha'(\theta) > 0. \]

We assume that applying for a loan is costly, so that individuals will only do so if their chance of being approved is sufficiently high. Let \( B \) denote the gross benefit a borrower receives from a loan (the utility stream from owning a house) and \( C \) the cost of applying for a loan (both shoe-leather and direct application costs).  

So long as 0 < \( C < B \), Result 3 ensures that there exists a cutoff type \( \theta^m \) such that

\[ \alpha(\theta^m) = \int_{s}^{\infty} p(s | \theta^m) ds = \frac{C}{B} = \alpha^*, \]  

(5)

implying that all individuals with \( \theta \geq \theta^m \) will apply for loans, while those who are less creditworthy will not.  

We will often refer to \( \theta^m \) as the “marginal” applicant type. Casual observation suggests that most individuals will only apply for a loan if their likelihood of being approved is relatively high. Thus, we make the following assumption.

**ASSUMPTION A2:** \( \alpha^* > \frac{1}{2} \).

Given our assumption that \( p \) is symmetric, this implies that \( s^* < \theta^m \) in any equilibrium. Note that this in turn implies that a majority of all mortgage applications are approved, consistent with findings from Home Mortgage Disclosure Act data.

We are now able to define an equilibrium in our model as the \((s^*, \theta_m)\) pair that

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11 \( C \) is the borrower’s direct costs of applying for a loan. Lenders also bear some of the costs of originating loans; these costs are captured in the parameter \( q^* \).

12 It is possible that \( \theta^m \notin (0,1) \), in which case either all individuals apply for loans or remain out of the market. We will ignore this possibility throughout the rest of the paper.

13 This is not to suggest, however, that all applicants are acceptable to banks. In fact, it is easy to show that \( \theta^m < q^* \) in any equilibrium; otherwise banks would accept all applicants without screening, which cannot occur in equilibrium.
simultaneously solves the following two equations:

\[ q(s^*) = q^* \]
\[ \alpha(\theta^m) = \alpha^*. \]  

(6)

At this point, there is no substantive difference between this model and one in which the applicant pool is given exogenously. In order for the ability of applicants to self-select to make a qualitative difference on outcomes in the model, it is necessary for some heterogeneity to exist across lenders. We address this issue now.

4. The Impact of Self-Selection

Thus far, we have developed a model of mortgage underwriting in which applicants choose whether or not to apply for loans. This, however, is not the only decision individuals must make. In particular, differences among lenders can lead to differences in an applicant’s chances of being approved for a loan at these lenders. As a result, individuals may have an incentive to select one lender over another. It is this type of self-selection that we address in this section.

Specifically, we allow a subset of lenders to have a taste for discrimination and show how this heterogeneity can lead to differences in the distribution of credit risk across banks and between applicant groups. In doing this, we are primarily interested in how applicant self-selection affects three aspects of the equilibrium. First, we want to establish which applicants apply at each bank. This will determine the distributional characteristics of each lender’s applicant pool. Second, we are interested in how these choices by individual applicants affect lenders’ incentives to discriminate. Finally, we would like to compare conditional denial and default rates across banks as well as across groups.

To proceed, it will be useful to formally define what we mean by the word “discrimination.”

**Definition:** A bank is said to **discriminate** against members of a group if it requires them to meet a higher cutoff signal, \( s^* \), than it does members of another group.
Although the term “discrimination” is not explicitly defined in either the Equal Credit Opportunity Act or the Fair Housing Act, the two laws that directly prohibit mortgage lending discrimination, this is the definition enforced by regulators and implicitly used in policy debates.\textsuperscript{14} In addition, it has the advantage that it is based entirely on information that is observable by banks and regulators. In contrast, definitions of discrimination that depend on an applicant’s true type, $\theta$, or his inferred creditworthiness, $q(s)$, are less useful from a policy perspective, since they cannot be used for enforcement purposes.

Assume that the population of potential applicants can be divided into two groups, $A$ and $B$. For ease of exposition, we will often refer to group $A$ as the “white” group and to group $B$ as the “minority” group. Nevertheless, these groups may alternatively be thought of as distinguishing applicants along any racial, ethnic, cultural, gender, or other publicly observable characteristic that lenders can use to classify applicants.

Similarly, we assume that borrowers can apply at one of two different banks, $X$ and $Y$.\textsuperscript{15} Bank $X$ has no preference for members of either group, while bank $Y$ has a “taste for discrimination” against group $B$ applicants. Note that the assumption that bank $X$ has no taste for discrimination does not imply it will never exhibit discriminatory behavior. In fact, we will show momentarily that bank $X$ may have an incentive to discriminate against one group or the other, even though it harbors no racial animus.

In the context of our model, bank $Y$’s taste for discrimination implies that it requires group $B$ applicants to have a higher inferred quality in order to be approved for a loan.

\textbf{ASSUMPTION A3:} \[ q_A^* < q_X^* < q_Y^* . \]

Here, $q_X^*$ is the lowest inferred quality a group $B$ applicant may have and still be approved at bank $X$, $q_Y^*$ is the lowest acceptable $q$ for group $B$ applicants at bank $Y$, and $q_A^*$ is the minimum acceptable $q$ for group $A$ applicants at either bank; further uses of the subscripts $A$, $X$, and $Y$

\textsuperscript{14} See Longhofer and Peters (1999) for a discussion of the confusion that has arisen because of the lack of a precise working definition of discrimination in the literature.

\textsuperscript{15} More accurately, we assume that there are two types of bank, all of which behave competitively. To facilitate the exposition of our model, we will focus on the behavior of a representative bank of each type.
Self-selection affects the mortgage underwriting process by altering the distribution of credit risk in applicant pools across groups and across banks. To analyze this phenomenon, we assume that applicant creditworthiness is uniformly distributed across the interval $[0,1]$.

**ASSUMPTION A4**: \[ f_i(\theta) = 1 \text{ for all } \theta \in [0,1], \ i = A,B. \]

This assumption greatly simplifies our analysis of self-selection. For example, in the model of the previous section with no lender heterogeneity, \[ g(\theta) = \frac{1}{1 - \theta^n} \], so that applicant creditworthiness is uniformly distributed on the interval $[\theta^n,1]$.

Absent any applicant self-selection, a direct application of Results 1 and 2 on Assumption A3 implies that

\[ Y_{XA}^s < Y_{X}^s \] \hspace{1cm} (7)

In other words, if applicants naively choose banks irrespective of these institutions’ tastes for discrimination, bank $Y$ will discriminate against group $B$ applicants by requiring them to meet a higher cutoff signal (see Figure 1).

**RESULT 4**: *In the absence of applicant self-selection, bigoted banks have an incentive to discriminate against minority applicants by holding them to a higher cutoff signal, while non-bigoted banks hold applicants from each group to the same cutoff signal.*

The discriminatory behavior on the part of bank $Y$ will clearly affect a group $B$ applicant’s chance of receiving a loan. Indeed, because \[ \alpha(\theta) = 1 - P(s^* | \theta) \] (where \[ P(s | \theta) \equiv \int_{-	heta}^{\theta} p(t | \theta) dt \]), it must be the case that \[ \alpha_Y(\theta) < \alpha_X(\theta) \] for every $\theta$. As a result, if minority applicants could costlessly and perfectly distinguish between the bigoted and non-bigoted bank, they would always choose to apply at the non-bigoted bank. This behavior, however, would imply that minorities would never suffer from discrimination. Therefore, if discriminatory outcomes are to occur in the market, we must assume that information that distinguishes between banks is costly and/or imperfect.

\[ \text{---} \]

16 Since we have assumed that neither bank has a taste for discriminating against group $A$ applicants, the $q^*$ applied to such applicants is the same at both banks.
We assume that applicants must pay a cost $\chi$ to acquire information about bank types. This is not to imply that there exists an information store from which individuals purchase this information. Rather, individuals read local newspapers and are exposed to other media sources that can shape their beliefs about various lenders. More likely, they learn from the anecdotal experiences of friends, neighbors, and relatives, as well as from their own experiences with banks’ other products and services. Individuals may even use information, such as CRA ratings, provided by various banking regulators and community groups.\(^{17}\)

To account for the noise inherent in such information, we assume that only a fraction $\gamma > \frac{1}{2}$ of individuals, uniformly distributed across the population, succeed in applying at their preferred bank. Thus, our informational structure is such that individuals know how they will be treated by each bank, but must expend a cost to learn which bank is which. Furthermore, the information an individual receives about banks’ identities is imperfect, so that some individuals who pay this cost nevertheless end up applying for a loan at the bigoted bank.

Given this setup, group $B$ applicants will only acquire information about banks’ types if\(^{18}\)

$$B[\gamma \alpha_x(\theta) + (1 - \gamma)\alpha_y(\theta)] - C - \chi > B\left[\frac{1}{2} \alpha_x(\theta) + \frac{1}{2} \alpha_y(\theta)\right] - C$$

$$B(\gamma - \frac{1}{2})(\alpha_x(\theta) - \alpha_y(\theta)) > \chi.$$  

(8)

The left-hand side of this expression is simply a group $B$ individual’s expected net benefit from acquiring information about bank types. It incorporates the gross benefit from obtaining a loan ($B$), the increased probability of applying at bank $X$ that results from this information ($\gamma - \frac{1}{2}$), and the higher probability of being approved at bank $X$ ($\alpha_x(\theta) - \alpha_y(\theta)$). For any minority applicant with creditworthiness $\theta$, if this net benefit exceeds $\chi$, he will attempt to self-select between banks.

This leads us to

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\(^{17}\) While all of this information may be quite useful to an individual applicant, none of it on its own, or even in the aggregate, can perfectly identify bigoted banks. If it could, the justice department and other federal regulators would collect this information themselves and subsequently identify and punish bigoted lenders. Of course, such anecdotal evidence is insufficiently precise to prosecute lenders for violating fair-lending laws. Nevertheless, individuals can and do base their own application decisions on exactly this type of information.

\(^{18}\) Note that group $A$ applicants have no incentive to distinguish between banks, since they face the same cutoff signal regardless of where they apply.
PROPOSITION 1: For any fixed $\gamma$, $s^*_X$, and $s^*_Y$,

1) If $\chi$ is sufficiently small, all group B applicants will pay this cost and attempt to self-select to bank $X$;

2) If $\chi$ is sufficiently large, group B applicants will not self-select between banks, applying at each with probability $1/2$; and

3) For intermediate values of $\chi$: $\exists \theta^c \in (\theta^m_B, 1)$ such that all group B applicants with $\theta \in [\theta^m_B, \theta^c]$ will gather information on bank types and attempt to self-select to bank $X$; those with a higher $\theta$ will not gather this information and will apply at each bank with probability $1/2$.

The basic intuition behind Proposition 1 is straightforward. If gathering information about banks’ preferences is very inexpensive, all minority applicants will do so. On the other hand, if distinguishing between banks is quite costly, minorities will be better off avoiding these costs and taking their chances of ending up at bank $Y$. The third part of this proposition follows from the fact that the value of information about bank types is decreasing in $\theta$. That is, as a minority individual’s creditworthiness improves, the difference between his chances of approval at the two banks diminishes; since he is highly likely to be approved at either bank, distinguishing between them is not very important to a highly-creditworthy minority applicant.

Note that regardless of the value of $\chi$, group B applicants have a positive chance of ending up at the bigoted bank. This implies that some low-creditworthy minority individuals will not apply, even though they would in the absence of bigotry. That is, $\theta^m_B > \theta^m_A$ because group A applicants face no bigotry (this fact is proven formally in the Appendix).

In equilibrium, $s^*_X$ and $s^*_Y$ take into account not only bank $Y$’s taste for discrimination but also the impact self-selection has on bank applicant pools. To focus on the interaction of these effects, we assume that $\chi$ lies in the intermediate range required for minority self-selection between banks to occur. In this case, the distribution of applicant creditworthiness across banks and groups is:

$$g_A(\theta) = \frac{1}{1 - \theta^m_A},$$ (9)
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\[ g_X(\theta) = \begin{cases} \gamma (\theta^i - \theta_B^m) + \frac{1}{2} (1 - \theta^c), & \theta \in [\theta_B^m, \theta^c] \\ \frac{1}{2}, & \theta \in (\theta^i, 1], \end{cases} \]  
\[ g_Y(\theta) = \begin{cases} 1 - \gamma (\theta^i - \theta_B^m) + \frac{1}{2} (1 - \theta^c), & \theta \in [\theta_B^m, \theta^c] \\ \frac{1}{2}, & \theta \in (\theta^i, 1]. \end{cases} \]  

These densities are shown graphically in Figure 2. It is straightforward to verify that \( \bar{\theta}_X < \bar{\theta}_Y \) and \( \bar{\theta}_A < \bar{\theta}_Y \), where \( \bar{\theta}_j = \int_{\theta^m}^{\theta^c} g_j(\theta) d\theta \), \( j = A, X, Y \), is the average creditworthiness of a bank’s applicant pool.

This self-selection behavior on the part of group B borrowers directly affects banks’ evaluations of their inferred quality. Specifically, given any signal \( s \), the inferred quality of minority applicants at the bigoted bank will be higher than that of minorities at the non-bigoted bank and of non-minorities at either bank.

**Proposition 2:** If group B applicants self-select toward bank X, \( q_X(s) < q_Y(s) \) for all \( s \) and \( q_A(s) < q_Y(s) \) for all \( s \).

Intuitively, because minorities with relatively low \( \theta \)s try to avoid the bigoted bank, the average creditworthiness of the minority applicant pool at this bank is higher than that of whites or that of minorities at the non-bigoted bank. In Bayesian terminology, the expectation of the prior belief of a minority applicant’s creditworthiness is higher at a bigoted bank than that of a white applicant or of a minority applicant at a non-bigoted bank. Proposition 2, then, follows immediately because an applicant’s inferred quality is simply the expectation of the Bayesian posterior.

Proposition 2 implies that self-selection behavior on the part of minority applicants will act to mitigate some of the bigoted bank’s incentive to discriminate against minorities. Although bank Y has an incentive to hold group B applicants to a higher cutoff signal because of its taste for discrimination, the fact that \( q_A(s) < q_Y(s) \) gives it the opposite incentive. Nevertheless, we
can show

**Proposition 3:** $s_y^* > s_x^*$ and $s_y^* > s_A^*$.

Even when minorities adjust their application decisions in response to the effects of discrimination, it remains the case that bigoted banks will want to hold minority applicants to a higher cutoff signal than they do white applicants, and to a higher cutoff signal than do non-bigoted banks. Thus, even in the presence of self-selection, minority applicants suffer from discrimination.

The intuition behind Proposition 3 is straightforward. When low-$\theta$ minorities self-select away from bank $Y$, it raises the average creditworthiness of the minority applicant pool at that bank, and lowers it at bank $X$. This tends to bring $s_x^*$ and $s_y^*$ closer together. Nevertheless, it cannot entirely eliminate the gap between the two, since it is this gap that gives minority applicants an incentive to self-select in the first place. Similarly, $s_y^*$ is always larger than $s_A^*$, since the self-selection effect that tends to lower $s_y^*$ is second-order to the bigotry effect that raises it.

Our most striking result is that bigotry on the part of some banks can affect the treatment of minority applicants at other banks. The decision of relatively low-creditworthy minorities to apply at bank $X$ reduces the average quality of its minority applicant pool, which in turn gives that bank an incentive to statistically discriminate against minorities. Thus, the self-selection behavior of applicants can lead non-bigoted banks to discriminate against minorities, even though they have no tastes for discrimination.

**Proposition 4:** Applicant self-selection behavior can cause bank $X$ to discriminate against members of group $B$ (i.e., $s_x^* > s_A^*$).

Essentially, self-selection has two counteracting effects on the inferred quality of group $B$ applicants at bank $X$. First, the fact that low-creditworthy minorities seek to avoid the bigoted bank lowers the average creditworthiness of minority applicants at non-bigoted banks. This

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19 Bigotry has other adverse consequences as well, including the cost of distinguishing between bigoted and non-bigoted banks and the fact that fewer minority individuals apply for loans.
tends to raise $s^*_X$ above $s^*_A$. On the other hand, the fact that some particularly low-$\theta$ minorities stop applying for loans altogether (because bigotry makes it less likely that they will be approved) tends to raise the average creditworthiness of the minority applicant pool, making it possible that $s^*_X < s^*_A$. Which of these two effects dominates depends on the parameters of the problem but, in general, the higher $\theta^w_m$, the more signals for which $q_X(s)$ will be larger than $q_Y(s)$. Thus, as long as bigotry doesn’t force too many low-quality minorities out of the market, their self-selection behavior will give bank $X$ an incentive to discriminate against minorities (see Figure 3).

Finally, we would like to analyze how bigotry and self-selection interact to affect relative default rates across groups at each of the banks. To do this, note that $q(s)$ is an unbiased estimator of an individual’s true creditworthiness (i.e., his probability of repaying his loan). In the aggregate, therefore, (abusing the law of large numbers in the standard way) the function $1 - q(s)$ gives us the ex post default rate for each group at each bank, conditional on the observed signal $s$. For example, of the group $B$ borrowers ending up at bank $X$ and sending signals $s$, a fraction $1 - q_X(s)$ will default, on average.

Our previous results, then, lead immediately to the following proposition:

**Proposition 5:** Conditional on their signals:

1) Group $B$ borrowers at bank $Y$ will default less frequently than group $B$ borrowers at bank $X$ and less frequently than group $A$ borrowers at either bank;

2) If $s^*_X > s^*_A$, group $B$ borrowers at bank $X$ will default more frequently than their group $A$ counterparts; and

3) If $s^*_X < s^*_A$, group $B$ borrowers at bank $X$ with low signals will default less frequently than their group $A$ counterparts, while those who send relatively high signals will default more frequently than comparable members of group $A$.

Figure 3 illustrates the effects outlined in Proposition 5. First, as shown in Proposition 2, $q_Y(s)$ is always larger than $q_X(s)$ and $q_A(s)$. Thus, minority borrowers at bank $Y$ default less frequently than white applicants, holding constant the signal sent by each. This result is consistent with the prediction made by Becker (1993).

Second, when bank $X$ discriminates against group $B$ applicants, $q_X(s) < q_A(s)$ for every $s$ that is approved by the bank (see Panel A). As a result, minorities at that bank default more
frequently than their white counterparts, providing ex post confirmation for the beliefs that led to the statistical discrimination. Thus, if lenders statistically discriminate, the relative default rates between minorities and whites are the opposite of those that arise if lenders discriminate based on preferences.

Finally, Panel B shows what happens when bank X discriminates against group A applicants. As noted above, non-bigoted banks discriminate against group A only if the bigotry effect is so large that few minorities (only the most creditworthy) apply. In this case, because the minority applicant pool is of higher average creditworthiness, minorities sending low signals are less likely to default (i.e., are more likely to have higher $\theta$s). On the other hand, the fact that the least creditworthy minorities self-select toward bank X means that $q_X(s) < q_A(s)$ for high $s$. As a result, bank X observes minorities with very good signals defaulting more frequently than comparable whites.

It is important to recognize that the default rates we consider here are “conditional” default rates and are distinct from the “marginal” default rates often discussed in the mortgage discrimination literature. Unfortunately, the concept of a marginal default rate is often poorly defined, making it unclear as to whether it is intended to refer to the least creditworthy applicant approved for a loan, or to some larger group of high-risk applicants. In any event, statistical tests of default rates typically attempt to control for factors that might affect a borrower’s credit risk, and are thus inherently conditional default rates. As a result, the predictions outlined in Proposition 5 are more directly testable than any predictions predicated on some notion of a marginal borrower.

Finally, note that Proposition 5 makes predictions about the relative default rates of minorities and whites across banks, but does not tell us anything about their conditional default rates in the aggregate. Because minorities default more frequently than whites at some banks and less frequently at others, it is unclear whether their default rate in the market as a whole will be higher or lower than that of whites. As a result, our model does not provide firm predictions as to what one might expect to find in a study such as Berkovec et al. (1994), which focuses on default rates in an entire market. If one could control for the identity of banks, however, our model suggests a means of testing which banks may be exhibiting tastes for discrimination; we
discuss this possibility further in the next section.

To conclude this section, we reiterate our basic results when we allow borrowers to self-select in the presence of taste-based discrimination:

1) Bigoted banks will discriminate against minority applicants by holding them to a more rigorous underwriting standard (cutoff signal) than they do white applicants, and one that is more rigorous than that required by non-bigoted banks (Proposition 3).

2) Self-selection behavior by low-creditworthy minorities away from bigoted banks can cause non-bigoted banks to statistically discriminate against minority applicants (Proposition 4).

3) Conditional on their signal, minority borrowers at bigoted banks will default less frequently than white borrowers or minority borrowers at non-bigoted banks (Proposition 5 – Part 1).

4) If non-bigoted banks discriminate against minorities, conditional default rates of minority borrowers at these banks will be higher than those of white borrowers (Proposition 5 – Part 2).

5) If non-bigoted banks discriminate against whites, conditional default rates of low-creditworthy minority borrowers will be lower than those of white borrowers, while the opposite will be true for high-creditworthy borrowers (Proposition 5 – Part 3).

5. Implications for Testing and Policy

In the previous section, we demonstrated how applicant self-selection behavior affects the distribution of credit risk across banks. In this section, we explore some of the more practical applications of our model and discuss its relevance to recent policy issues.

Detecting Discrimination in Lending Markets

An ongoing debate in the mortgage discrimination literature has been on the proper method for detecting discrimination in credit markets. For example, Munnell et al. (1996) control for “every variable mentioned as important in numerous conversations with lenders, underwriters, and examiners,” and find that

[H]olding the other financial and property characteristics constant, the probability of denial [is] 8.2 percentage points [higher] for a minority applicant. (p. 33)

Implicitly, they condition on applicant signals to determine whether applicant denial decisions
are racially correlated.

In contrast, Becker (1993) argues that

The correct procedure for assessing whether or not banks discriminate...is to determine whether loans are more profitable to blacks (and other minorities) than to whites. This requires examining the default and other payback experiences of loans, the interest rates charged, and so forth. (p. 389)

In other words, in order for a bank to discriminate against minority applicants, it must forego profitable lending opportunities, thereby making (marginal) minority applicants more profitable to the bank. Berkovec et al. (1994) attempt to perform such a test using FHA default and loss data, and find that low-creditworthy minority borrowers are substantially more likely to default than comparable white borrowers.

Our model suggests that an emphasis on denial rates may be the more accurate way of detecting discrimination. This is because both bigoted banks and banks that statistically discriminate will deny minority applicants at a higher rate than white applicants, conditional on their signal. However, as outlined in Proposition 5, minorities will default more often than whites at non-bigoted banks that statistically discriminate against minorities. Thus, Becker’s test would misinterpret the behavior of non-bigoted banks, since statistical discrimination arises precisely because minority applicants are less creditworthy conditional on their signal ($q_X(s) < q_A(s)$). On the other hand, if applicant self-selection behavior is such that non-bigoted banks do not discriminate against minority applicants, then focusing on conditional default rates could lead to the inaccurate conclusion that those banks do discriminate against minorities, and do so because of racial animus.

Furthermore, and perhaps most importantly, since default-rate predictions differ based on whether a bank is a statistical discriminator or a bigot, studies focusing on a market that contains both types of discriminators are likely to be inconclusive; the lower minority default rate at

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20 Han (1997) analyzes models of bigotry and statistical discrimination, with an emphasis on the differential impact on bank profits across racial groups. He concludes that the ability to detect discrimination, whether statistical or taste-based—as well as what discrimination “looks like”—depends on the particular measure of profits employed. He analyzes several profit measures, including default rates, loan write-offs, expected profits, and realized rates of return, finding some evidence that loans to minorities are more profitable than loans to whites.

21 Although some may argue that banks should not be punished for statistically discriminating, such behavior is illegal under both the Equal Credit Opportunity Act and the Fair Housing Act.
bigoted banks may be offset by higher minority default rates at banks that statistically discriminate.\textsuperscript{22} In contrast, because denial decisions are based directly on the variable we use to measure discrimination (the applicant’s signal), tests for discrimination using conditional denial rate data do not suffer from such errors.\textsuperscript{23}

**Uncovering the Source of Discrimination**

A related issue is the ability of testing procedures to identify whether evidence of discrimination suggests statistical discrimination or bigotry. As we have argued above, both types of discrimination are illegal under current fair-lending laws, making the distinction between the two less relevant from an enforcement perspective. Nevertheless, designing effective anti-discrimination policies depends crucially on identifying the underlying source of this discrimination.\textsuperscript{24}

Our Bayesian model of the underwriting process suggests a simple approach that might be used to distinguish empirically between statistical and preference-based discrimination. Because the difference between these two forms of discrimination is located in the relationship between $q_A^*$ and $q_B^*$, the empiricist must be able to reconstruct the lender’s Bayesian updating process (the underwriting process) to arrive at each applicant’s inferred quality, $q(s)$. A finding that minority applicants are more likely to be rejected than whites, *conditional on $q(s)$*, would be consistent with preference-based discrimination. On the other hand, if the likelihood of being rejected varies across racial groups controlling for $s$, but does not vary after controlling for each applicant’s $q$, statistical discrimination is the more plausible explanation.

Munnell et al. (1996) undertake the first step in this process by looking at applicant denial rates *conditional on $s$*, and find that minority applicants are more likely to be denied loans than comparable whites. It would be premature to conclude that the source of this discrimination is lender preferences, however, because they do not consider how these signals are transformed into an inferred credit risk for each applicant. That is, they *do not control for each applicant’s*

\textsuperscript{22} This fact may explain the ambiguous nature of some of Berkovec et al.’s results.
\textsuperscript{23} Other researchers have also argued that default rates may provide a misleading picture of discrimination. See, for example, Tootell (1993), Ferguson and Peters (1998), and Han (1997).
\textsuperscript{24} See Longhofer and Peters (1999).
To do this requires knowledge of the distribution of true credit risk of each group in the lender’s applicant pool, i.e., of \( g(\theta) \). Craig et al. (1998) pursue this empirical question, using the structure provided by this model to exploit the information available in the denial-rate data used by Munnell et al.

**Additional Policy Implications**

Our model is also capable of shedding light on other recent developments in the mortgage market. For example, advances in the automation of mortgage processing have reduced the overall costs of applying for mortgage loans, both in terms of up-front fees and the time involved in the process. Similarly, expanded marketing efforts by lenders (like those encouraged for CRA compliance) may have lowered the shoe-leather costs associated with applying for loans, particularly for targeted groups. Our model suggests that such a reduction in the cost of applying for a mortgage should increase the number of mortgage applicants. In other words, as the cost of applying for a loan declines, previously sub-marginal individuals will find it worthwhile to apply (those with \( \theta \) just below \( \theta^m \)). This is consistent with mortgage trends through the 1990s.

At the same time, however, reductions in these costs increase borrower denial rates, both directly and indirectly. First, because more low-creditworthy individuals apply, unconditional denial rates are higher. In addition, *conditional* denial rates increase because these new applicants reduce the average creditworthiness of the applicant pool, which in turn requires lenders to establish a higher cutoff signal \( s^* \). Thus, while application or participation rates increase with a reduction in the cost of applying for a loan, reducing these costs also makes it tougher for all applicants to qualify for a loan. In other words, the increase in \( s^* \) associated with these efforts causes \( \alpha(\theta) \) to fall for all \( \theta \) (see Figure 4). This result is particularly noteworthy because application rates are argued to be one of the most reliable measures of CRA compliance (see Avery et al., 1998).

Finally, the general nature of our underwriting model makes it an ideal framework for analyzing policy initiatives in the mortgage and other credit markets. Consider, for example, the case in which non-bigoted banks statistically discriminate against minorities, and assume that

\[
q(s)\]  

More precisely, they show that \( s_\hat{\theta} - s \) differs from \( s^* - s \) for applicants sending an “average” \( s \).
regulators could impose a stigma cost on lenders that exhibit tastes for discrimination, thereby encouraging them to lower their $d_b^*$. This would have two effects on minority self-selection behavior. First, because their overall probability of being approved would rise, more minorities would apply for loans. Second, because the “bigotry effect” is now smaller, $\theta^c$ will fall. On net, this policy intervention would raise the average creditworthiness of minority applicants at non-bigoted banks relative to that of whites. Thus, combating the bigotry that exists at some lenders would also serve to eradicate statistical discrimination by non-bigoted lenders.

Although illustrative, this example by no means exhausts the possible policy uses of our model. For example, it could be used examine how improved access to information about bank lending practices through bank regulators or community groups (a reduction in $\chi$ or an increase in $\gamma$) might affect individual application decisions and the subsequent treatment of applicants by banks. Similarly, our underwriting model might also form the basis for analyzing the effects of self-selection in the presence of other market frictions such as lender affinities.\textsuperscript{26}

\textbf{6. Conclusion}

We have developed a model in which individuals choose to apply for mortgage loans based on the likelihood of being approved. Individuals self-select in such a way that the distribution of creditworthiness in the applicant pool differs across racial groups even though creditworthiness is identically distributed across groups in the general population.

This self-selection behavior on the part of applicants may give lenders an incentive to statistically discriminate against their minority applicants, \textit{even if they had no proclivity to discriminate against them in the first place}. Thus, in addition to introducing the general concept of applicant self-selection into a formal model of loan underwriting, our theory is capable of explaining how the behavior of lenders with tastes for discrimination can lead other lenders to discriminate as well.

We have also shown that if lenders statistically discriminate against minorities, conditional default rates for minorities are higher than those for whites, the opposite of the result

\footnote{\textsuperscript{26} Our preliminary efforts in this direction suggest that the results are similar to that of Longhofer (1996).}
obtained if lenders discriminate based on preferences. This fact suggests that researchers should use caution when attempting to draw conclusions about the presence of market-wide discrimination from default-rate studies, since both statistical discriminators and bigots may coexist in a single market.

It is important to note that the statistical discrimination that arises in our model is not the result of an “omitted variable problem.” That is, we assume that the signal $s$, together with an applicant’s race, is a sufficient statistic for an applicant’s true creditworthiness. Instead, discriminatory incentives arise because this signal is imperfect, even though it is unbiased.

Our model’s empirical predictions can be used to help reconcile seemingly inconsistent empirical results that have arisen in the literature. Munnell et al. (1996), Hunter and Walker (1996), and Bostic (1996) all show that minority applicants are held to a higher credit standard than are similarly situated whites. In addition, Berkovec et al. (1994) assume that their sample of Federal Housing Administration (FHA) borrowers is comprised of individuals who are “marginally” qualified for mortgage loans, and find that such borrowers tend to default at a higher rate than comparable whites. Taken together, these empirical findings on denial and default rates suggest that statistical discrimination against minorities by non-bigoted lenders is a possibility (Proposition 4).

Recent empirical work has highlighted the importance of the applicant self-selection behavior we model in this paper. Bostic and Canner (1998) find that differences in applicant pools across black-owned, Asian-owned, and white-owned peer banks account for most of their denial-rate disparities. Rosenblatt (1997) finds strong evidence that an individual’s choice between conventional and FHA mortgage products (as well as whether or not to apply at all) is based on how well their personal characteristics match the requirements of each. Finally, Avery et al. (1994) show that cross-lender variation in minority and low-income originations primarily reflects differences in home mortgage application rates, again supporting our notion that applicant decisions have a strong influence on lender behavior and other market outcomes.

Our model also provides some indirect support for concerns raised by Yezer et al. (1994) that mortgage lending involves the simultaneous determination of the availability and terms of credit. They argue that mortgage applicants can and do alter the information that ultimately ends
up on their mortgage applications (either their $s$ or their $\theta$) in response to feedback from the lender, with the ultimate goal of improving their chances of being approved. As a result, single-equation tests of conditional denial (or default) rates may provide biased tests of lending discrimination. Although we do not directly model this behavior on the part of mortgage applicants, this kind of endogeneity is comparable to that which we do model. Extending our analysis to account for applicant adjustments to $s$ or $\theta$, or to account for varying loan terms across banks or borrowers (as in Han, 1997) may be a fruitful avenue for future research.

Finally, because of the general nature of our model of loan underwriting, it provides a useful framework for analyzing policy initiatives in credit markets. As discussed in the previous section, our model is amenable to modifications that would allow future researchers to investigate the likely impact of a variety of different market frictions and policy actions. In addressing such issues, an overriding concern should be the impact on $\alpha$, the applicant’s likelihood of obtaining a loan. This, ultimately, should be the primary variable of concern.
7. Appendix

Proof of Result 1: We show that \( \pi(\theta \mid s) \) satisfies the monotone likelihood ratio property, which implies that \( q'(s) \) must be positive, since \( q(s) \) is the expectation of \( \theta \) under \( \pi \).

\[
\frac{\partial \pi(\theta \mid s)}{\partial s} = \frac{\partial p(s \mid \theta)}{\partial s} g(\theta)\omega(s) - p(s \mid \theta)g(\theta)\omega'(s)
\]

\[
\pi(\theta \mid s) = p(s \mid \theta)g(\theta)\omega(s)
\]

\[
= \frac{\partial p(s \mid \theta)}{\partial s} - \frac{\omega'(s)}{\omega(s)}
\]

Hence,

\[
\frac{\partial}{\partial \theta} \frac{\partial \pi(\theta \mid s)}{\partial s} = \frac{\partial^2 p(s \mid \theta)}{\partial s \partial \theta} p(s \mid \theta) - \frac{\partial p(s \mid \theta)}{\partial s} \frac{\partial p(s \mid \theta)}{\partial \theta}
\]

\[
[\pi(\theta \mid s)]^2
\]

\[
= \frac{\partial}{\partial s} \frac{\partial p(s \mid \theta)}{\partial \theta}
\]

which is positive since \( p \) satisfies the monotone likelihood ratio property. It is worth noting that this proof holds regardless of the distribution of credit risk in the applicant pool.

Proof of Result 2: Immediate from the monotonicity of \( q(s) \).

Proof of Result 3: By the symmetry of \( p \) (part 2 of A1), \( \alpha(\theta) \) can be rewritten as

\[
\alpha(\theta) = \int_{-\infty}^{\theta} p(t \mid s^*) dt.
\]

Differentiating with respect to \( \theta \) gives \( \alpha'(\theta) = p(\theta \mid s^*) > 0 \).

Proof of Result 4: Follows directly from the fact that \( q'(s) > 0 \) and \( q^*_A < q^*_Y \).

Proof of Propositions 1-3: In order to prove these propositions, we will need the following lemmas:

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27 This follows because the monotone likelihood ratio property implies first-order stochastic dominance; see Milgrom (1981).
**Lemma 1:** Assume there exists some $\theta^*$ such that all applicants with $\theta \in [\theta^m, \theta^c]$ attempt to self-select toward bank $j$ while those with larger $\theta$ randomize between bank $j$ and bank $k$. Then $q_j(s) > q_k(s), \forall s$.

Note that if $j = X$ and $k = Y$, this lemma is part 1 of Proposition 2.

**Proof of Lemma 1:** If applicants self-select in this way, the inferred quality of applicants at each bank can be written as

$$q_j(s) = \int_{\theta^m}^{\theta^c} \frac{p(s | \theta) \gamma}{\gamma(\theta^c - \theta^m) + \frac{1}{2}(1 - \theta^c)} \, d\theta + \int_{\theta^c}^\infty \frac{p(s | \theta) \frac{1}{2}}{\gamma(\theta^c - \theta^m) + \frac{1}{2}(1 - \theta^c)} \, d\theta$$

$$+ \int_{\theta^m}^{\theta^c} \frac{p(s | \theta) \gamma}{\gamma(\theta^c - \theta^m) + \frac{1}{2}(1 - \theta^c)} \, d\theta + \int_{\theta^c}^\infty \frac{p(s | \theta) \frac{1}{2}}{\gamma(\theta^c - \theta^m) + \frac{1}{2}(1 - \theta^c)} \, d\theta$$

(14)

and

$$q_k(s) = \int_{\theta^m}^{\theta^c} \frac{1}{p(s | \theta)(1 - \gamma) \, d\theta + \int_{\theta^c}^\infty p(s | \theta) \frac{1}{2} \, d\theta} \left[ \int_{\theta^m}^{\theta^c} \theta p(s | \theta) \gamma \, d\theta + \int_{\theta^c}^\infty \theta p(s | \theta) \frac{1}{2} \, d\theta \right]$$

(15)

Using this, we calculate

$$q_j(s) - q_k(s) \sim \int_{\theta^m}^{\theta^c} \left[ p(s | \theta)(1 - \gamma) \, d\theta + \int_{\theta^c}^\infty p(s | \theta) \frac{1}{2} \, d\theta \right] \left[ \int_{\theta^m}^{\theta^c} \theta p(s | \theta) \gamma \, d\theta + \int_{\theta^c}^\infty \theta p(s | \theta) \frac{1}{2} \, d\theta \right]$$

$$- \int_{\theta^m}^{\theta^c} \left[ p(s | \theta)(1 - \gamma) \, d\theta + \int_{\theta^c}^\infty p(s | \theta) \frac{1}{2} \, d\theta \right] \left[ \int_{\theta^m}^{\theta^c} \theta p(s | \theta) \gamma \, d\theta + \int_{\theta^c}^\infty \theta p(s | \theta) \frac{1}{2} \, d\theta \right]$$

(16)

where $\sim$ indicates that this expression is proportional to the difference $q_j(s) - q_k(s)$. Now,

$$\int_{\theta^m}^{\theta^c} \theta p(s | \theta) \, d\theta < \theta^c \int_{\theta^m}^{\theta^c} p(s | \theta) \, d\theta \int_{\theta^m}^{\theta^c} p(s | \theta) \, d\theta < \int_{\theta^m}^{\theta^c} p(s | \theta) \, d\theta \int_{\theta^m}^{\theta^c} \theta p(s | \theta) \, d\theta.$$  

(17)
Recalling that $\gamma > \gamma_2$, this implies that (16) is negative, thus proving $q_k(s) > q_j(s)$ for all $s$. □

**Lemma 2:** Define $\beta_{jk}(\theta) \equiv (\gamma - \gamma_2)[\alpha_j(\theta) - \alpha_k(\theta)] - \chi/B$. Suppose that $s^*_k > s^*_j$ and assume that $\chi > 0$ is sufficiently small such that $\beta_{jk}(\theta) > 0$ for some $\theta$. Then $\beta_{jk}(\theta)$ has exactly two roots on the real line, one of which must lie below $\theta^m$.

Note that $\beta_{jk}(\theta)$ is derived from (8) in the text, the condition determining which applicants choose to self-select toward the bank with the lower $s^*$; any applicants with $\theta$ such that $\beta_{jk}(\theta) \geq 0$ will choose to self-select toward bank $j$, while the rest will not.

**Proof of Lemma 2:** First note that because $\gamma > \gamma_2$ and $s^*_k > s^*_j$, $(\gamma - \gamma_2)[\alpha_j(\theta) - \alpha_k(\theta)] > 0$ for all $\theta$. Thus, $\exists$ some $\chi$ such that $\beta_{jk}(\theta) > 0$ for some $\theta$. Furthermore, by the symmetry of $p$, $\alpha(\theta) = P(\theta | s^*)$. As a result, $\lim_{\theta \to -\infty}[\alpha_j(\theta) - \alpha_k(\theta)] = 0$ and similarly as $\theta \to -\infty$. This implies that, given any fixed $\chi > 0$, $\beta_{jk}(\theta) < 0$ for sufficiently large and sufficiently small $\theta$. Consequently, $\beta$ must have at least two roots on the real line. Next, note that

$$\beta'_{jk}(\theta) = p(\theta | s^*_j) - p(\theta | s^*_k).$$

(18)

By the symmetry and unimodality of $p$ (parts 2 and 3 of A1), this is positive for any $\theta < (s^*_j + s^*_k)/2$ and negative for all $\theta > (s^*_j + s^*_k)/2$. Hence, $\beta_{jk}(\theta)$ is quasi-concave and bounded, reaching its maximum at $\theta = (s^*_j + s^*_k)/2$. Thus, $\beta_{jk}(\theta)$ must have exactly two roots.

It remains only to be shown that the lower root of $\beta_{jk}(\theta)$ lies below $\theta^m$. Let $\underline{\theta}$ denote this lower root, and note that $\theta < (s^*_j + s^*_k)/2$. By the symmetry of $p$ (part 2 of A1),

$$\frac{1}{2}\alpha_j\left(\frac{s^*_j + s^*_k}{2}\right) + \frac{1}{2}\alpha_k\left(\frac{s^*_j + s^*_k}{2}\right) = \frac{1}{2}.$$

(19)

Define $\widetilde{\theta}$ such that

$$\gamma\alpha_j(\widetilde{\theta}) + (1 - \gamma)\alpha_k(\widetilde{\theta}) - \chi/B = \frac{1}{2}.$$

(20)

(see Figure 5). By the continuity of $\alpha$,

$$\gamma\alpha_j(\widetilde{\theta}) + (1 - \gamma)\alpha_k(\widetilde{\theta}) - \chi/B > \frac{1}{2}\alpha_j(\widetilde{\theta}) + \frac{1}{2}\alpha_k(\widetilde{\theta}).$$

(21)

Thus, $\widetilde{\theta} > \underline{\theta}$. Recall now that $\theta^m$ is defined as the $\theta$ such that
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\[
\max\{\gamma\alpha_j(\theta^n) + (1-\gamma)\alpha_k(\theta^n) - \chi/B, \frac{1}{2}\alpha_j(\theta^n) + \frac{1}{2}\alpha_k(\theta^n)\} = \alpha^*.
\]

Since \(\alpha^* > \frac{1}{2}\) by assumption, the monotonicity of the left-hand side of (22) implies that \(\theta^m > \tilde{\theta} > \tilde{\theta}^m\). ♠

**Proof that** \(s^*_X < s^*_Y\) (Proposition 3, part 1). Assume to the contrary that \(s^*_X \geq s^*_Y\) and define

\[
\beta_{xy}(\theta) = (\gamma - \frac{1}{2})[\alpha_x(\theta) - \alpha_y(\theta)] - \chi/B
\]

as the payoff to a type-\(\theta\) applicant from attempting to self-select toward bank \(X\) (as in Lemma 2 above). This expression must be nonpositive, since \(\gamma > \frac{1}{2}\) and \(\alpha_x(\theta) \leq \alpha_y(\theta)\) for all \(\theta\) (because \(s^*_X \geq s^*_Y\)). Thus, no group \(B\) applicant will want to self-select toward bank \(X\) when \(s^*_X \geq s^*_Y\).

Now define

\[
\beta_{yx}(\theta) = (\gamma - \frac{1}{2})[\alpha_y(\theta) - \alpha_x(\theta)] - \chi/B
\]

as the payoff to a type-\(\theta\) applicant from attempting to self-select toward bank \(Y\). If \(\chi\) is sufficiently large, \(\beta_{yx}\) will be negative for all \(\theta\) and no group \(B\) applicant will attempt to self-select toward either bank. In this case, \(q_x(s) = q_y(s)\) for all \(s\) and the monotonicity of \(q\) implies that \(q^*_X > q^*_Y\), a contradiction.

Now if \(\chi\) is relatively small, Lemma 2 implies that \(\beta_{yx}\) will have two roots on the real line. Call the upper of these two roots \(\theta^c\) and note that if \(\theta^c \geq 1\), all group \(B\) applicants attempt to self-select toward bank \(Y\); once again \(q_x(s) = q_y(s)\) for all \(s\), implying the contradiction \(q^*_X > q^*_Y\).

By Lemma 2, the only remaining case to consider is \(\theta^c \in (\theta^n, 1)\). In this case, Lemma 1 implies that \(q_x(s) > q_y(s)\) for all \(s\). This, together with our assumption that \(s^*_X > s^*_Y\) would again imply the contradiction \(q^*_X > q^*_Y\) (see Figure 6). ♠

**Proof that** \(\theta^m_B > \theta^n_A\): Suppose to the contrary that \(\theta^m_A > \theta^n_B\). Following the same logic used in the proof Lemma 1, this would imply that \(q_X(s) < q_A(s)\) for all \(s\).\(^{29}\) Because \(q^*_X = q^*_A\), this would in turn imply that \(s^*_X > s^*_A\). Above, however, we proved that \(s^*_Y > s^*_X\), implying that

\(^{28}\) It is worth noting that this fact rules out the possibility of a perverse signaling equilibrium, in which high-\(\theta\) minority applicants attempt to self-select toward the bigoted bank in order to signal their quality.

\(^{29}\) Proof of this fact is omitted for brevity; a formal derivation can be obtained from the authors upon request.
\[ \alpha_A(\theta) > \gamma \alpha_X(\theta) + (1 - \gamma) \alpha_Y(\theta) - \chi / B, \quad \forall \theta. \]

This, however, would imply that \( \theta^m_B < \theta^m_A \), a contradiction. ♠

**Proof that** \( q_Y(s) > q_A(s) \) **for all** \( s \) **(Proposition 2, part 2):** The proof follows using bounding techniques like those employed in the proof of Lemma 1, applying the fact that \( \theta^m_B > \theta^m_A \), and is omitted for brevity.\(^{30} \) ♠

**Proof that** \( s_Y^* > s_A^* \) **(Proposition 3, part 2):** Suppose to the contrary that \( s_Y^* > s_A^* \). In this case, the fact that \( s_X^* < s_Y^* \) implies that \( \alpha_X(\theta) < \frac{\gamma}{2} \alpha_X(\theta) + \frac{\gamma}{2} \alpha_Y(\theta) < \gamma \alpha_X(\theta) + (1 - \gamma) \alpha_Y(\theta) \) for all \( \theta \) (i.e., for any given \( \theta \), a group-B applicant is more likely to be approved than a group-A applicant, regardless of whether he attempts to self-select toward bank \( X \)). This, however, implies that \( \theta^m_B < \theta^m_A \), a contradiction. ♠

**Proof of Proposition 1:** Define once again
\[
\beta_{XY}(\theta) \equiv (\gamma - \gamma) [\alpha_X(\theta) - \alpha_Y(\theta)] - \chi / B
\]
as the payoff to a type-\( \theta \) applicant from attempting to self-select toward bank \( X \) and note that the first term in this expression must be positive, since \( \gamma > \frac{1}{2} \) and \( \alpha_X(\theta) < \alpha_Y(\theta) \) for all \( \theta \) \((s_Y^* > s_X^*)\). Thus, for sufficiently large \( \chi \), \( \beta_{XY} \) will be negative for all \( \theta \in [\theta^m_B, 1] \), proving the second part of the proposition. For smaller values of \( \chi \), Lemma 2 implies that \( \beta_{XY} \) will have two roots on the real line, one of which is lower than \( \theta^m_B \). Call the upper of these two roots \( \theta^c \) and note that if \( \theta^c \geq 1 \), all group \( B \) applicants attempt to self-select toward bank \( Y \), proving the first part of the proposition. Finally, if \( \theta^c \in (\theta^m_B, 1) \), Lemma 2 implies \( \beta_{XY}(\theta) > 0 \) only for \( \theta \in [\theta^m_B, \theta^c] \), proving the third part of the proposition. ♠

**Proof of Proposition 4:** We prove existence by means of a numerical example. Equilibrium in our model with self-selection is given by the \((s_X^*, s_Y^*, \theta^m_B, \theta^c)\) combination that solves the following system of equations:

\[^{30}\] Once again, a formal derivation can be obtained from the authors upon request.
\[
q_X(s^*_X) \equiv \frac{\int_{\theta_0}^{\theta^*} \theta p(s^*_X | \theta) d\theta + \int_{\theta_0}^{\theta^*} \theta p(s^*_X | \theta) \frac{d\theta}{2}}{\int_{\theta_0}^{\theta^*} p(s^*_X | \theta) d\theta + \int_{\theta_0}^{\theta^*} p(s^*_X | \theta) \frac{d\theta}{2}} = q^*_X
\]
\[
q_Y(s^*_Y) \equiv \frac{\int_{\theta_0}^{\theta^*} \theta p(s^*_Y | \theta) (1 - \gamma) d\theta + \int_{\theta_0}^{\theta^*} \theta p(s^*_Y | \theta) \frac{d\theta}{2}}{\int_{\theta_0}^{\theta^*} p(s^*_Y | \theta) (1 - \gamma) d\theta + \int_{\theta_0}^{\theta^*} p(s^*_Y | \theta) \frac{d\theta}{2}} = q^*_Y
\]
\[
\hat{\alpha}(\theta^*_B) \equiv \gamma \alpha_X(\theta^*_B) + (1 - \gamma) \alpha_Y(\theta^*_B) - \frac{\chi}{B} = \alpha^*
\]
\[
\beta(\theta^*_c) \equiv (\gamma - \frac{\gamma}{2}) \left[ \alpha_X(\theta^*_c) - \alpha_Y(\theta^*_c) \right] - \frac{\chi}{B} = 0;
\]
as before, \(s^*_A\) and \(\theta^*_B\) are determined by (6) in the text.

Let \(p(s | \theta)\) be a normal density with mean \(\theta\) and variance \(.1\), and assume that \(q^*_A = q^*_X = .575\), \(q^*_Y = .6\), \(\alpha^* = .6\), \(\chi = .01\), \(B = 1\), and \(\gamma = .85\). Numerical calculation of (27) and (6) using these parameter values gives us the following solution: \(s^*_Y = .493\), \(s^*_X = .485\), \(s^*_A = .478\), \(\theta^*_A = .504\), \(\theta^*_B = .514\), and \(\theta^*_c = .544\). ♠

Proof of Proposition 5: Follows immediately from Proposition 2 and the discussion in the text. ♠
8. Figures

Figure 1

Effect of bigotry on the required cutoff signal with no self-selection.

In the absence of minority applicant self-selection behavior, the higher $q^*$ required by a bigoted lender leads it to discriminate against minority applicants by holding them to a higher cutoff signal.
Figure 2

Distributions of applicant creditworthiness across banks and groups.
Minority self-selection behavior lowers the average creditworthiness in bank $X$’s minority applicant pool (i.e., shifts $q_X(s)$ down), giving bank $X$ an incentive to statistically discriminate against members of group $B$. Because $1 - q(s)$ is also a measure of the ex post conditional default rate, bank $X$ observes higher conditional default rates from group $B$ borrowers than it does from its group $A$ borrowers.
Figure 3
Panel B

Bank X statistically discriminates against group A.

If enough group B applicants drop out of the applicant pool, it raises this group’s average creditworthiness at bank X, giving it an incentive to statistically discriminate against members of group A. In this case, group B borrowers that send relatively low signals default more frequently than their white counterparts, while group B borrowers that send very good signals default less frequently than comparable whites.
Reducing the cost of applying for a mortgage has the effect of reducing $\alpha^*$ to $\alpha_2^*$. This, in turn, lowers the creditworthiness of the marginal applicant to $\theta_2^m$. The resulting lower average creditworthiness in the mortgage applicant pool causes lenders to raise their required cutoff signal to $s_2^* > s_1^*$, reducing all applicants’ probabilities of being approved for a loan.
Figure 5

Proof of Lemma 2.

\( \frac{1}{2} \alpha_j(\theta) + \frac{1}{2} \alpha_k(\theta) \)

\( \gamma \alpha_j(\theta) + (1 - \gamma) \alpha_k(\theta) - \gamma/B \)

\( \theta \) must lie below \( \theta^m \) because, by the monotonicity of \( \alpha \), it must lie below \( \tilde{\theta} \), which is smaller than \( \theta^m \) by the continuity of \( \alpha \).
Proof that $s_x^* < s_y^*$.

If $s_y^* \leq s_x^*$, this would imply that $q_x(s) \geq q_y(s)$ for all $s$. But then the monotonicity of $q$ would imply that $q_x^* \geq q_y^*$, a contradiction.
9. References


Han, Song. “Credit Risk, Discrimination, and Regulation in Credit Markets.” University of Chicago, unpublished manuscript, November 1997.


