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**BANK CAPITAL AND EQUITY INVESTMENT REGULATIONS**

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## **ABSTRACT**

This paper uses an intermediation model to study the efficiency and welfare implications of both banks' required capital–asset ratio and the regulation that limits, and in some countries forbids, banks' investments in equity to a certain proportion of each firm's capital. There are two sources of moral hazard in the model: one between the bank and the provider of deposit insurance, and the other between the bank and the entrepreneur who demands funds to finance an investment project. Among other things, the paper shows that capital regulation improves the bank's stability and can also be Pareto–improving. Equity regulation is never Pareto–improving and does not increase the bank's stability.

## 1 Introduction

The risk-shifting effect caused by deposit insurance, when the insurance premium does not reflect banks' risk, has long been recognized.<sup>1</sup> It is usually identified by the banks' decision to finance risky instead of safe investment projects. Nonetheless, regulators have responded not by modifying the deposit insurance contract, but instead by introducing a wide range of restrictions on banks' activities designed to limit their motivation and ability to choose very risky asset portfolios. Although these restrictions tend to vary substantially across countries, there are some common patterns, such as regulations on banks' capital and on their investments in the equity of nonfinancial firms.

The 1987 Basle Agreement on Capital Standards, reached by the G10 countries, and the 1993 introduction by the European Community of the Banks' Own Funds and the Solvency Ratio Directives, both in line with the Basle Agreement, were the main regulations that implemented the international harmonization of capital requirements.<sup>2</sup>

With respect to banks' investments in the equity of nonfinancial firms, of particular interest is the regulation that limits each of these investments to a certain percentage either of the firm's capital or of its voting rights.<sup>3</sup> For example, this limit is 50 percent in Norway; 25 percent in Portugal; 10 percent in Canada and Finland; 5 percent in Belgium, Japan, the Netherlands, and Sweden; and zero percent in the United States, because U.S. commercial banks are not allowed to invest in equity. Germany and Switzerland are examples of countries where banks' investments in equity are not limited by that form of regulation.

The objective of this paper is to study the efficiency and welfare implications of both

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<sup>1</sup>Kareken and Wallace (1978) and Dothan and Williams (1980) are examples of works that have used state preference models to prove this result; Merton (1977, 1978) pioneered the use of options to show it.

<sup>2</sup>For a presentation of the capital regulations, see Cordell and King (1992).

<sup>3</sup>In most countries, such investments are also subject to the prudential limits of the banks' capital regulation. This regulation, in general, limits these investments to a certain percentage of the bank's capital. For a characterization of these limits in the OECD countries, see Pecchioli (1987) and Schuijjer (1992).

banks' capital and equity investment regulations, and the impact of these regulations on banks' stability and on the contracts they use to finance firms.

This study is conducted in an intermediation model where banks are the only source of external funds to firms. A crucial feature of the model is the existence of two sources of moral hazard. One is in the relationship between the bank and the provider of deposit insurance, because the bank assumes that the insurance premium it pays does not reflect the risk of its assets. The other source of moral hazard is in the relationship between the bank and the entrepreneur (firm) who demands funding, because the investment's return depends on the entrepreneur's effort, which is not observable. Furthermore, because one of the regulations studied here involves banks' investments in equity, the project held by the entrepreneur is designed so that the optimal financing contract can be replicated by a (unique) combination of debt and equity.

In this model, the bank must choose its capital structure and the contract it uses to finance the entrepreneur. As a result, the risk-shifting effect due to deposit insurance is translated here not in the bank's decision to finance risky instead of safe investment projects, as is common in the literature, but in its choice of a contract that motivates the entrepreneur to adopt a riskier behavior, which in turn increases the risk of the bank's assets. Therefore, the firm's capital structure becomes dependent on the conditions under which the bank operates, namely the presence of regulations and the existence of deposit insurance.

Using this framework, the paper shows that an increase in the required capital-asset ratio improves the bank's stability, can be Pareto-improving, and in some cases, can even increase the model's efficiency, in the sense of making its solution closer to the first-best outcome. This policy has a cost, because it forces the bank to use relatively more of its most expensive source of funding (capital). However, it also has a positive effect. By increasing what the bank's equityholders have at stake in case of bankruptcy, an increase in the required capital-asset ratio decreases the bank's incentives to motivate risky behavior

by the firm to which it supplies funds. This is implemented through a reduction in the part of the contract used by the bank that is more risk-motivating (debt). As a consequence, the bank's risk of failure declines, which in turn reduces the moral hazard costs due to deposit insurance.

Some of these results have not been captured by studies of banks' capital regulation, because the approach usually adopted does not endogenize the financing contracts, and because the main focus has been to study the implications of that regulation on banks' risk. Examples of the research conducted in this area are Kahane (1977), Koehn and Santomero (1980), Kim and Santomero (1988), Furlong and Keeley (1989), Keeley and Furlong (1990), Rochet (1992), and Campbell, Chan, and Marino (1992).

With respect to the equity investment regulation, we will see that the introduction of a limit on the percentage of the firm's capital that a bank can hold has mixed effects on the efficiency of the model, does not increase the bank's stability, and is never Pareto-improving. Furthermore, we will see that it is even possible to observe an increase in the bank's risk of failure due to the introduction of this form of regulation. The intuition for these results is based on the following argument: Limiting the bank's ability to finance a firm through an equity contract forces the bank to simultaneously supply part of the funds needed by the firm through a different financial instrument that might be more risk-motivating than equity. As a result, the gains in stability that might occur from the reduction of the bank's stake in the capital of the firm are offset, and in some cases outweighed, by the risk effect of the financial instrument that the bank uses instead to finance the firm.

Regulations on banks' investments in equity have not been a prime candidate for research. In work done independently, John, John, and Saunders (1994) show that when the bank cannot control the firm's investment decisions, the efficiency of the investment is higher and the bank's risk is lower if the bank uses equity in conjunction with debt to finance the firm. However, when the bank can veto the firm's investment decisions, there

is a trade-off between the increase in investment efficiency and the increase in the bank's risk.

When the bank cannot control the firm's investment decisions, both the John et al. model and the model presented here produce the same conclusion: Limiting or forbidding the bank to finance a firm using equity in addition to debt deteriorates both the investment's efficiency and the bank's risk (John et al. rely on the variance of the bank's cash flows as their measure of risk, while here this measure is given by the bank's probability of failure).

Despite the common assumption that banks are the only source of external funds to firms, there are important differences between the two approaches. For example, the model adopted here incorporates the bank's capital structure and the existence of deposit insurance, both of which are absent from their model. This allows the study of the banks' capital and equity investment regulations in the same framework, which also happens to sustain the optimality of debt and equity contracts.

The paper proceeds as follows: Section 2 introduces the model, characterizes the first- and second-best solutions, and shows the optimality of debt and equity contracts. Section 3 studies the capital and equity investment regulations. Final remarks are presented in section 4, followed by two appendices, one containing the proofs and the other a numerical example.

## 2 The Model

The model adopted in this paper comprises four elements. First, there is an entrepreneur (firm) with an investment project, but without the necessary funds to finance it. (In subsection 3.3, I discuss the implications for the results of this model if there were many firms.) Second, there is a bank, which chooses its capital structure and the contract used to finance the project. Third, there is the deposit insurance provider, which charges the bank a premium and commits to reimburse depositors if the bank fails. Finally, there is the

public, which is willing to supply any amount of deposits as long as it receives the certainty equivalent to the risk-free interest rate.

The crucial features of the model are the nature of the relationships between the bank and the entrepreneur on the one hand, and between the bank and the provider of deposit insurance on the other hand. The relationship between the bank and the entrepreneur is characterized by a principal-agent problem, where the bank is the principal and the entrepreneur the agent. The moral hazard in this part of the model is generated by the dependence of the project's returns on the entrepreneur's effort, which is not observable. With respect to the relationship between the bank and the provider of deposit insurance, it is assumed that the bank does not take into account, *ex ante*, how its actions affect the insurance premium it must pay. A possible explanation for this behavior is that the bank views itself as a small unit of the banking sector, and it assumes the insurance premium to be determined by the risk of the whole banking sector rather than by the risk of its own assets.

The assumptions of the model are as follows:

**Assumption 1** *There is a risk-neutral entrepreneur with an investment project, but without the necessary funds to finance it. The project requires a fixed initial investment equal to  $\bar{I}$ , and produces one period later the total return  $y_i$  with probability  $p_i$ . The number of possible returns of the project is finite. In particular, I assume that it has only three possible returns:*

$$y_0 < y_1 < y_2,$$

where  $y_0 \equiv 0$  and  $y = \{y_1, y_2\}$ .

The probability distribution of the project's returns is assumed to be an endogenous variable because it depends on the entrepreneur's effort. Moreover, it is also assumed that the entrepreneur incurs a cost for each level of effort he chooses. One way to model this situation would be to take the entrepreneur's effort as the choice variable. In that case, it

would be necessary to specify a functional relationship between the probabilities and the effort, and to define a cost function with effort as an argument. Instead, I use the approach where the choice variables are the probabilities themselves. Because the entrepreneur incurs a certain cost for each probability distribution he chooses, the cost function depends on the probabilities  $(p_1, p_2)$  with the convention that  $p_0 \equiv 1 - \sum_{i=1}^2 p_i$ . The main advantage of this approach is that it avoids some of the technical difficulties that frequently appear when solving a principal–agent problem.<sup>4</sup> In addition, the choice of a cost function that is strictly convex and strictly increasing in its arguments makes it possible to use convex programming theory in solving the model.

**Assumption 2** *Let  $C(\cdot)$  denote the cost function. Then  $C(p) : p \rightarrow R^+$ , where  $p = \{p_1, p_2\}$ , and  $C(\cdot)$  is  $C^2$ , strictly increasing, and satisfies the condition  $C(0) = 0$ . In particular, I use the following cost function:*

$$C(p) = \frac{1}{2}a_1p_1^2 + \frac{1}{2}a_2p_2^2 + a_3p_2,$$

with  $a_i > 0$  for  $i = 1, 2, 3$ .

**Assumption 3** *The bank’s equityholders are risk–neutral. The opportunity cost of the bank’s capital ( $r$ ) is assumed to be larger than the risk–free interest rate ( $i$ ) because of, for example, a tax on the bank’s profits. In accordance with the actual capital regulation, the bank must satisfy a minimum capital–asset ratio, where the assets are weighted according to their risk;<sup>5</sup> that is:*

$$\frac{K}{I} \geq \theta \quad 0 < \theta < 1,$$

where  $K$  is the bank’s capital and  $\theta$  is the required capital–asset ratio.

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<sup>4</sup>For a discussion of the advantages of this approach, see Holmström (1979).

<sup>5</sup>In line with the Basle Agreement, I assume that bank’s investment in is treated, for the purpose of capital regulation, in the same way as the loan it gives to the firm. However, some countries have adopted the requirement that banks finance such investments solely with capital.



**Assumption 4** *The public in this economy is risk-averse. It is willing to supply any amount of deposits, provided it is paid the certainty equivalent to the risk-free interest rate. One way to satisfy this condition is to have the bank pay this interest rate on deposits and to hold deposit insurance, which will compensate depositors if the bank fails.*

## 2.1 The First-Best Outcome

The first-best outcome would be the solution to the model if there were no sources of moral hazard, or if the entrepreneur had enough funds to finance the project. In both cases, this outcome would be the solution to the following problem:

$$\begin{aligned} \underset{p}{Max} \quad & \Pi_E^{fb} = py - C(p) - \bar{I}(1+r) \\ \text{s.t.} \quad & pe \leq 1 \\ & p \geq 0, \end{aligned}$$

where  $\Pi_E^{fb}$  is the entrepreneur's first-best profits and  $e$  is a vector of ones.

Because the objective function is  $C^2$  and strictly concave in  $p$ , and the feasible set is convex and compact, we are in the presence of a convex programming problem. In this case, we know that there is a unique optimum and that the Kuhn-Tucker conditions are necessary and sufficient for a solution.

Assuming that the entrepreneur is not able to completely eliminate the risk of failure of the project, that is,  $p_0^{fb} > 0$ , then the first-best outcome to this model is

$$\begin{aligned} p_0^{fb} &= 1 - \left( \frac{y_1}{a_1} + \frac{y_2 - a_3}{a_2} \right), & p_1^{fb} &= \frac{y_1}{a_1}, & p_2^{fb} &= \frac{y_2 - a_3}{a_2}, \\ \Pi_E^{fb} &= \frac{H}{2} - \bar{I}(1+r), \end{aligned}$$

where  $H \equiv \left\{ \frac{y_1^2}{a_1} + \frac{(y_2 - a_3)^2}{a_2} \right\}$ .

This solution implies some restrictions for the parameters of the model. First, because we are working with probabilities, it is necessary to make sure that their values are positive

and smaller than one. Second, because of the initial assumption that the entrepreneur is not able to eliminate the risk of failure of the project, we need to have  $p_1^{fb} + p_2^{fb} < 1$ . Finally, in order for this project to be undertaken, it is necessary that its first-best profits are positive, that is,  $\Pi_E^{fb} > 0$ .

These restrictions are summarized in the following assumption:

**Assumption 5**

$$\begin{aligned}
 y_1 &< a_1, & 0 &< y_2 - a_3 < a_2, \\
 a_1(y_2 - a_3) + a_2y_1 &< a_1a_2, \\
 \frac{H}{2} - \bar{I}(1 + r) &> 0.
 \end{aligned}$$

**2.2 The Second-Best Outcome**

When the entrepreneur gets the necessary funds to finance his project from an outside source (in this model, a bank), the following question can be raised: What are the characteristics of the optimal contract that will rule their relationship? Given that the effort chosen by the entrepreneur is not observable, and that at the beginning of the period the bank will supply a fixed amount of funds equal to  $\bar{I}$  (because by assumption the entrepreneur has no funds), then the only thing left to be defined by the contract is the payment that the entrepreneur will make to the bank at the end of the period. This payment will be contingent on the observable information at that time, that is, the income of the project.

Let  $r_i$  be the payment required by the bank contingent on the return  $y_i$ . Due to the limited liability condition, we have  $r_0 \equiv 0$ , since by assumption  $y_0 \equiv 0$ , and  $r_i \leq y_i$  for  $i = 1, 2$ . Based on this definition, the contract between the two parties can be written as  $(\bar{I}, r)$ , where  $\bar{I}$  is the number of monetary units supplied by the bank and  $r$  is the vector of (non-negative) contingent payments made by the entrepreneur.

The optimal contract and the bank's optimal capital structure are given by the solution

to the following problem:

$$\begin{aligned}
 & \underset{r, K}{Max} \quad \Pi_E = \sum_{i=1}^2 p_i(y_i - r_i) - C(p) \\
 & \text{s.t.} \quad y_1 - r_1 - a_1 p_1 = 0 \\
 & \quad \quad y_2 - a_3 - r_2 - a_2 p_2 = 0 \\
 & \quad \quad 0 \leq r_i \leq y_i \quad \text{for } i = 1, 2 \\
 & \quad \quad K + B = \bar{I} \quad K \geq \theta \bar{I} \\
 & \quad \quad p_0 \text{Max} \{0, -\bar{Q}B\} + \sum_{i=1}^2 p_i \text{Max} \{0, r_i - \bar{Q}B\} - K(1 + r) \geq \Pi_B,
 \end{aligned}$$

where  $\bar{Q} \equiv [(1 + i) + \bar{q}]$ ;  $\bar{q}$  is the insurance premium, which the bank assumes independently of its actions;  $B$  are the bank's deposits; and  $\Pi_B$  are the profits demanded by the bank to finance the project, which can be any value between zero (representing the case where there is perfect competition among banks to finance this firm) and  $\Pi_B^{Max}$  (representing the case where the bank behaves like a monopolist).

The two linear constraints included in that problem are the entrepreneur's incentive constraints. They are the first-order conditions to the following problem:

$$\begin{aligned}
 & \underset{p}{Max} \quad \Pi_E = \sum_{i=1}^2 p_i(y_i - \bar{r}_i) - C(p) \\
 & \text{s.t.} \quad pe \leq 1 \\
 & \quad \quad p > 0,
 \end{aligned}$$

where  $\bar{r}_i$  are the payments demanded by the bank. The importance of these constraints results from the impossibility of observing the entrepreneur's effort, which determines  $p$ . Through them the bank motivates the entrepreneur to choose (voluntarily) the proper probability distribution.

In the process of finding the optimal contract and the bank's capital structure, the following observations are taken into account. First, the problem is solved under the assumption that the bank's probability of failure is  $p_0$ , that is,  $r_i > Q^*B^*$  for  $i = 1, 2$ . As a result, once the solution has been found, it is necessary to ensure that it satisfies this condition.

Second, since it was assumed that the bank considers the insurance premium it pays to be independent of its actions, the model is solved assuming  $q$  fixed at a certain level,  $\bar{q}$ . Once the solution has been found,  $\bar{q}$  is replaced with its fair price in order to determine the equilibrium.<sup>6</sup>

**Proposition 1** *The optimal contract to the problem defined here is  $(\bar{I}, r^*)$ , where*

$$r_0^* = 0, \quad r_1^* = (y_1 - Q^*B^*)[1 - f(\mu^*)], \quad r_2^* = (y_2 - a_3 - Q^*B^*)[1 - f(\mu^*)],$$

with: First,  $Q^* \equiv [(1+i) + q^*]$ , and  $q^*$  being the smaller root to the following equation:

$$V_2q^{*2} + V_1q^* + V_0 = 0,$$

where  $V_2 > 0$ ,  $V_1 \geq 0$  and  $V_0 > 0$ .

Second,  $f(\mu^*) = \frac{1 - \mu^*}{1 - 2\mu^*}$ , where  $\mu^*$  is the Lagrange multiplier associated with the bank's

participation constraint. It is defined as  $\mu^* = \frac{1}{2} - \frac{1}{2} \left( \frac{W(q^*)}{W(q^*) - 4[K^*(1+r) + \Pi_B]} \right)^{\frac{1}{2}}$  with

$$W(q^*) \equiv \frac{[y_1 - Q^*B^*]^2}{a_1} + \frac{[y_2 - a_3 - Q^*B^*]^2}{a_2}.$$

Finally, the bank's capital structure is

$$K^* = \theta\bar{I}, \quad B^* = (1 - \theta)\bar{I}.$$

For a sketch of the proof of this proposition, the values of  $V_i$ , and the equilibrium insurance premium ( $q^*$ ), see appendix A.

Using the results in this proposition, it is possible to compute the second-best probability

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<sup>6</sup>The fair price is the premium that the deposit insurance provider must charge in order to get zero profits. With probability  $p_0^*$ , the bank fails. In this state, the insurance provider gets nothing and must pay  $B^*(1+i)$ . With probability  $(p_1^* + p_2^*)$ , the insurance provider gets  $\bar{q}B^*$  and pays nothing. In equilibrium its profits are zero if  $p_0^*B^*(1+i) = (p_1^* + p_2^*)\bar{q}B^*$ , which implies a fair price equal to  $\frac{p_0^*}{p_1^* + p_2^*}(1+i)$ .

distribution,  $p^* = (p_0^*, p_1^*, p_2^*)$ , where

$$p_0^* = 1 - \left( \frac{y_1 - Q^* B^*}{a_1} + \frac{y_2 - a_3 - Q^* B^*}{a_2} \right) f(\mu^*),$$

$$p_1^* = \frac{y_1 - Q^* B^*}{a_1} f(\mu^*),$$

$$p_2^* = \frac{y_2 - a_3 - Q^* B^*}{a_2} f(\mu^*).$$

Comparing these results with the first-best outcome, we observe that, as expected, the entrepreneur now chooses a lower level of effort in both states where the project produces a positive outcome, that is,  $p_1^* < p_1^{fb}$  and  $p_2^* < p_2^{fb}$ . As a result, the project's probability of failure, which is also the bank's probability of failure, is now larger than the equivalent first-best value ( $p_0^* > p_0^{fb}$ ).

### 2.3 Debt and Equity as Optimal Contracts

The optimal contract found in the previous section was not characterized in terms of the financial instruments known in the corporate finance literature. However, taking into account the equityholders' limited liability condition, it is possible to prove that such a contract can be spanned by a combination of debt and equity, which shows the optimality of these financial instruments in the model adopted here.

Suppose the bank uses debt and/or equity to finance the investment project. Then the new contract can be written as  $(\bar{I}, \alpha, d)$ , where  $\bar{I}$  is the amount of funds supplied by the bank to the entrepreneur at the beginning of the period,  $\alpha$  is the proportion of the firm's equity held by the bank,  $(1 - \alpha)$  is the proportion held by the firm's entrepreneur, and  $d$  is the face value of debt borrowed by the firm. As usual, I assume that equityholders are the residual claimants and that they are protected by limited liability.

**Proposition 2** *The optimal contract for the problem presented here can be replicated by a*

unique combination of debt and equity, that is, by the contract  $(\bar{I}, \alpha^*, d^*)$ , where

$$\alpha^* = \frac{y_2 - a_3 - y_1}{y_2 - y_1} [1 - f(\mu^*)],$$

$$d^* = \frac{a_3 y_1 + (y_2 - y_1) Q^* B^*}{a_3 + (y_2 - a_3 - y_1) f(\mu^*)} [1 - f(\mu^*)],$$

and  $Q^*$  and  $B^*$  are equal to the values defined in proposition (1).

For proof of this proposition, see appendix A.

Note that, in order to make economic sense, the results in proposition (2) require the following additional assumption on the parameters of the model.

**Assumption 6**

$$y_2 - a_3 - y_1 > 0.$$

The intuition on why a combination of debt and equity spans the optimal contract is detailed in Santos (1995). It is based on the following explanation: The debt component of the contract is explained by the constant marginal cost of the entrepreneur's effort in state 2, that is,  $a_3$ . Its existence is necessary in order to avoid penalizing the entrepreneur relatively more in this state than in state 1. With respect to the equity component of the contract, its presence is justified by the difference of the project's returns across states. Its existence is important so that the entrepreneur is not relatively more penalized in state 1, which has a lower return.

Looking at the contract defined in proposition (2), we see that the financial instruments used by the bank to finance the firm (debt and equity) depend on the bank's capital structure (mix of deposits and capital). In other words, the moral hazard due to deposit insurance eliminates the known result of the separation between the bank's asset composition and its capital structure.<sup>7</sup> The nature of the relationship existing between the two sides of the

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<sup>7</sup>For a discussion about the separation of the bank's asset composition and its capital structure, see Klein (1971), Hart and Jaffee (1974), Szegö (1980), and Sealey (1985).

bank's balance sheet and its impact on the firm's capital structure will become clear in subsection 3.1.

### 3 Bank Capital and Equity Investment Regulations

As mentioned before, the efficiency costs of the two sources of moral hazard present in the model are translated in a lower level of effort chosen by the entrepreneur, implying a higher probability of failure for the project. Regulators cannot affect the efficiency costs caused by the moral hazard existing in the bank-entrepreneur relationship, because these costs are originated by an asymmetric information problem, which cannot be alleviated by regulation. But what about the efficiency costs originated by the moral hazard due to deposit insurance? Is it possible to reduce them through regulation?

This is the subject of the remainder of the paper. In particular, two pieces of regulation are addressed here: banks' capital-asset ratio requirement, and the regulation that limits, and in some countries forbids, banks' investments in the equity of nonfinancial firms to a certain proportion of the firms' capital.

Looking at the results in proposition (1), we see that the revenue of the contract used by the bank to finance the entrepreneur depends on the relative market power held by each of these elements in its own sector. At one extreme, we have the case where there is perfect competition among banks to finance the project, that is,  $\Pi_B = 0$ , and the entrepreneur gets all the surplus of the project. At the other extreme, we have the case where the bank behaves like a monopolist, that is,  $\Pi_B = \Pi_B^{Max}$ , and the entrepreneur gets at least the minimum he requires to undertake the project, which by assumption is zero.

Before we move to the analysis of the regulations in each market structure, it is important to take into account the following observation. Using the incentive constraints derived in subsection 2.2, it is possible to write the entrepreneur's profits as  $\Pi_E = \frac{1}{2}a_1p_1^2 + \frac{1}{2}a_2p_2^2$ . Given that it is not optimal for the bank to motivate the entrepreneur to choose  $p_1 = p_2 = 0$

(because in this case the bank would get no revenue from the entrepreneur), then when the bank behaves like a monopolist, the entrepreneur's participation constraint will not be binding, that is,  $\Pi_E > 0$ . As a result, the problem can be solved without imposing such a constraint.

Because this result simplifies the problem studied here—in particular, it allows the finding of explicit analytic solutions for all of the endogenous variables—in the next two subsections the regulations are studied for the case where the bank behaves like a monopolist. In subsection 3.3, the same regulations are studied for the case where there is perfect competition, but this time using a numerical example. As we will see, the major conclusions regarding the impact of both regulations on the bank's stability and their welfare implications do not depend on the market structure existing in the banking sector.

### 3.1 Bank Capital Regulation

When the bank behaves like a monopolist, it captures the surplus of the project and, as was explained in the previous subsection, the entrepreneur's participation constraint is not binding. In this case, it is possible to show that the equilibrium is given by the results in proposition (1) when  $\mu^* = -\infty$ , which implies in the limit  $f(\mu^*) = \frac{1}{2}$ , and<sup>8</sup>

$$p_1^* = \frac{y_1 - Q^*(1 - \theta)\bar{I}}{2a_1} \quad p_2^* = \frac{y_2 - a_3 - Q^*(1 - \theta)\bar{I}}{2a_2}$$

$$\Pi_E = \frac{W(q^*)}{8} \quad \Pi_B = \Pi_B^{Max} \equiv \frac{W(q^*)}{4} - \theta\bar{I}(1 + r)$$

As stated before, the efficiency costs due to both sources of moral hazard are translated in lower probabilities of positive outcomes ( $p_i^* < p_i^{fb}$ , for  $i = 1, 2$ ), implying a higher probability of failure for the project ( $p_0^* > p_0^{fb}$ ), which is also the bank's probability of failure.

Most of the literature that has studied the impact of mispriced deposit insurance has identified banks' risk-shifting effect with their decision to finance risky instead of safe in-

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<sup>8</sup>The same results would be found if the bank's profits were maximized subject to the entrepreneur's incentive constraints, the bank's budget constraint, and the capital-asset ratio requirement.



vestment projects. In the present model, as the results above indicate, this substitution effect occurs through a different channel. It is manifested in the bank's adjustment of the contract it uses to finance the entrepreneur in a way that motivates him to adopt a riskier behavior, which in turn implies an increase in the risk of the bank's assets. Thus, even if banks do not change their portfolio of customers, they can still take advantage of the deposit insurance subsidy by changing the way they do business with their current customers.

What is the impact of the capital regulation under these circumstances? If the bank had taken into account how the insurance premium it pays is determined, that is, if there were no moral hazard due to deposit insurance, the bank's profits would be

$$\Pi_B = \frac{H}{4} - [(1 - \theta)(1 + i) + \theta(1 + r)] \bar{I}. \quad (1)$$

Under these circumstances, since the bank's asset composition is independent from its capital structure, an increase in the required capital-asset ratio ( $\theta$ ) does not affect the contract used by the bank. As such, it does not reduce the inefficiency of the model caused by the moral hazard in the relationship between the bank and the entrepreneur, and it does not affect the entrepreneur's profits. However, since this policy forces the bank to substitute capital for deposits (that is, it forces the bank to use relatively more of its most expensive source of funding—capital), it implies a reduction in the bank's profits.

When we consider the moral hazard due to deposit insurance, the bank's profits in equilibrium are

$$\Pi_B^e = \frac{W(q^*(\theta))}{4} - \theta \bar{I}(1 + r). \quad (2)$$

Comparing (1) with (2), we see that now an increase in the required capital-asset ratio will have different implications.

**Proposition 3** *An increase in the bank's required capital-asset ratio implies*

- (a) *An improvement in the efficiency of the model, because its equilibrium gets closer to the first-best outcome.*

- (b) An improvement in the bank's stability, because its probability of failure decreases.*
- (c) An increase in the entrepreneur's profits and, within a certain range, an increase in the bank's profits, in which case the regulation is Pareto-improving.*

For proof of this proposition, see appendix A.

The results in proposition (3) can be explained in the following way: When the capital-asset ratio is increased, forcing the bank to substitute capital for deposits, there is an increase in the value of what the bank's equityholders have at stake in case of bankruptcy. Moreover, given that the bank assumes the insurance premium it pays is independent of its risky behavior, it does not internalize any potential positive effects arising from this policy. As a result, in order to minimize its costs in case of failure, the bank adjusts the contract it uses to finance the entrepreneur in order to motivate him to choose a safer behavior. This is implemented through a reduction in the importance (value) of the financial instrument that is generally more risk-motivating—debt. This is why the bank's asset composition becomes dependent on its capital structure, invalidating the separation result.

The reduction in the payments demanded to the entrepreneur explains the increase in both his profits and his effort. This, in turn, explains the reduction in the project's probability of failure and in the bank's risk of failure, which was the reason for the bank to adjust its financing contract in the first place.

Finally, the increase in the bank's stability implies a reduction in the equilibrium insurance premium. The savings to the bank of this reduction are what differentiate this situation from the case where there was no moral hazard due to deposit insurance. If they outweigh the costs imposed on the bank, because it must substitute capital for deposits, then an increase in the required capital-asset ratio also implies an increase in the bank's profits (this relation is clear in the proof to proposition [3]). Note that, because the reduction in the equilibrium insurance premium due to increases in the required capital-asset ratio occurs at a decreasing rate, eventually after a certain level of capital has been reached,

further increases in the capital requirement will imply a decrease in the bank's profits.

### 3.2 Bank Equity Investment Regulation

As explained in the introduction, countries in which banks are allowed to invest in the equity of nonfinancial firms frequently have a regulation limiting each of these investments in terms of either the firm's capital or its voting rights. In the extreme, banks in some countries are not allowed to invest in such financial instrument at all.

A frequent argument used to justify this form of regulation is that through it, the bank's involvement with each firm is limited, reducing the bank's exposure to any major disturbances caused by a firm's bankruptcy and thus improving stability. This argument is problematic, because it does not consider all the implications of this form of regulation for the bank's role as a financial intermediary. In particular, it does not take into account that by limiting the bank's ability to finance a firm through equity, it also forces the bank to use a different financial instrument to supply funds that might be even more risk-motivating than equity. As a result, the gains originated by the reduction of the bank's stake in the capital of the firm might be outweighed by the costs of using the alternative financial instrument, in which case the final outcome of the regulation would be a perverse effect.

In a similar procedure to that adopted for the study of capital regulation, this subsection studies the implications for the model's efficiency and for the bank's stability of introducing a limit on the bank's equity investment defined in terms of the firm's capital. That is, the bank is not allowed to hold more than  $\hat{\alpha}$  percent of the firm's capital, where  $\hat{\alpha} = 0$  represents the countries in which banks are not allowed to invest in equity.

According to proposition (2), if there were no equity investment regulation, the optimal decision for the bank to finance the firm would be to choose the combination  $(\alpha^*, d^*)$ , with  $f(\mu^*) = \frac{1}{2}$ .

**Proposition 4** *The introduction of a limit on the bank's investment in equity, defined in*

*terms of the firm's capital, implies:*

- (a) A reduction in the bank's profits. As such, the regulation is not Pareto-improving.*
- (b) No improvement in the bank's stability.*

For proof of this proposition, see appendix A.

The results associated with the introduction of the limit on the bank's investment in equity are better understood if the following two effects of this form of regulation are considered: First, because in this model equity is one of the optimal financial instruments the bank uses to finance the firm, restricting its use creates in itself a distortion. Second, as was explained before, when the limit is introduced, in order to channel the same amount of funds to the firm, the bank must increase its use of another financial instrument. In this model, it increases the loan given to the firm, which is a contract that tends to be more risk-motivating than equity.

For the case where the regulation is not very restrictive (that is,  $\hat{\alpha}$  is not substantially smaller than  $\alpha^*$ ), after its introduction the bank will hold the maximum investment in equity allowed by the regulation ( $\hat{\alpha}$ ) and will demand a loan repayment  $\hat{d}$ , with  $d^* < \hat{d} < y_1$ . The impact of the equity investment regulation here is, in addition to the results in proposition (4), an increase in the entrepreneur's profits and an improvement in the efficiency of the model according to a first-order stochastic dominance criterion. There is an increase in the probability of the project's highest outcome ( $p_2$ ), a decrease in the probability of its lowest positive outcome ( $p_1$ ), and no change in its probability of failure ( $p_0$ ).

However, as the regulation becomes more restrictive (as  $\hat{\alpha}$  becomes smaller), the higher are the chances of observing the bank completely drop its use of equity and finance the firm through a loan with a face value larger than  $y_1$ , in which case the regulation not only decreases the entrepreneur's profits and the model's efficiency ( $p_2$  decreases,  $p_1$  is now zero, and  $p_0$  increases), but it also has the perverse effect of increasing the bank's risk of failure.

In sum, it is clear from this set of results that the form of equity investment regulation

studied here is never Pareto-improving and, contrary to what is usually claimed, does not improve the bank's stability.

These results are substantially different from those obtained in subsection 3.1 regarding the capital regulation. The fundamental reason for such a difference is that by increasing what the bank's equityholders have at stake in case of bankruptcy, capital regulation decreases their incentive to take advantage of the deposit insurance subsidy. However, the form of equity regulation addressed here not only lacks this effect, but it also creates a distortion against one of the optimal financial instruments used by the bank to finance the firm, which also happens to be the instrument that is less risk-motivating. As a consequence, it is difficult to justify the introduction of such a regulation on equity investments in a scenario where firms are strongly dependent on banks to raise external funds, and where the optimality of debt and equity contracts is driven by incentive effects.

### **3.3 Additional Results**

The analysis conducted here assumes that there is only one investment project, and that the bank behaves like a monopolist. This subsection studies the importance of these assumptions for the results found in subsections 3.1 and 3.2.

When we have a model with only one investment project, what results are missing compared to the case of multiple projects? Potentially, we could miss the analysis of the substitution effect, and we surely won't be able to study the scale effect of the regulations.

These effects are particularly relevant for the analysis of the capital regulation. For example, the literature that has studied this regulation in multiple-project frameworks has shown that banks substitute safe for risky investments in response to an increase in the required capital-asset ratio.

In the model adopted here, even though the bank finances only one investment, we are still able to capture the substitution effect originated by the regulations, because of the

endogeneity of the contract used by the bank to finance the firm. This effect, instead of being achieved through changes in the portfolio of the bank's investments, is implemented through the adjustment of the contract used by the bank, but the final result is identical. As observed in subsection 3.1, an increase in the capital regulation makes the bank change the financing contract in such a way that the entrepreneur is motivated to decrease the risk of his investment, implying a decline in the risk of the bank's assets.

With respect to the scale effect, the assumption of having only one project that requires a fixed investment imposes some limitations. However, it is still possible to use the results of this framework in order to understand the scale effect of the capital regulation. It seems clear that the impact on the number of projects financed by the bank due to an increase in the capital-asset ratio requirement will depend, on the one hand, on the bank's flexibility in raising additional capital and, on the other hand, on the relative size of the gains that this regulation brings to the bank (the decrease in the insurance premium it has to pay) versus its costs (the cost of having to use relatively more of its most expensive source of funding—capital). Therefore, an increase in the capital regulation will not always imply a negative scale effect. For example, it is possible to show through the model presented here that when there are many identical projects, if the cost of capital does not rise rapidly along with an increase in its demand, then we might observe an increase in the number of projects financed by the bank in response to an increase in the capital regulation.

What about the assumption of the bank's behaving like a monopolist? Will the results change if we assume that the bank behaves as if there were perfect competition in the banking sector?

The main advantage of using the monopoly assumption was the possibility of finding explicit analytic solutions for all of the endogenous variables of the model. This was possible because the entrepreneur's participation constraint was not binding and, as a result, was ignored in solving the model. When there is perfect competition among banks, this

simplification can no longer be used and, as a consequence, it becomes impossible to find explicit analytic solutions for all of the endogenous variables.<sup>9</sup>

This precludes the study of the regulations in a way identical to that adopted in subsections 3.1 and 3.2. Thus, the analysis had to be conducted through a numerical example. The parameters of this example and the effects of both regulations are in appendix B. Comparing them to the results found under a monopoly framework, we see that capital regulation improves the bank's stability and can be Pareto-improving in both market structures. In addition, when the bank behaves like a monopolist, we see that efficiency is clearly increased because of the approximation of the second-best solution to the first-best outcome. But, when there is perfect competition among banks, due to the decrease in the probability of the highest state ( $p_2^*$ ), such approximation is only partial.

Regarding the equity investment regulation, we see that in both market structures it is not Pareto-improving, it does not improve the bank's stability (in fact, when there is perfect competition, the perverse effect is dominant), and it has the same mixed effects in terms of its impact on the efficiency of the model.

This set of comparisons confirms that the main conclusions about the impact of the bank's capital and equity investment regulations hold both when there is a monopoly and when there is perfect competition in the banking sector.

## 4 Final Remarks

The risk-shifting effect due to deposit insurance has usually been identified by banks' decision to finance risky rather than safe investment projects. In the model presented here, this effect is manifested through a different channel. It occurs through the bank's adjustment of the financing contract it uses in a way that motivates the firm to adopt a riskier behavior,

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<sup>9</sup>Now we need the entrepreneur's profits so that this function can be maximized subject to, among other things, the bank's participation constraint.

which in turn increases the risk of the bank's assets. Thus, the firm's capital structure becomes dependent on the conditions under which the bank operates—namely, the existence of deposit insurance and the presence of regulations.

Under these circumstances, capital regulation decreases the bank's incentives to motivate risky behavior by the firm to which it supplies funds, because it forces the bank to hold relatively more capital—the funds it has at stake in case of bankruptcy. Hence, in addition to the cost that this policy imposes on the bank (because it requires the bank to use relatively more of its most expensive source of funding—capital), it also reduces the moral hazard costs due to deposit insurance (because the equilibrium insurance premium is reduced). As a result, an increase in the capital requirement reduces the bank's risk of failure, can be Pareto-improving, and can also have a beneficial effect on the efficiency of the model.

Introducing a limit on one of the optimal financial instruments a bank uses to finance a firm not only creates a distortion against this instrument, but it also forces the bank to use alternative contracts in order to finance the firm. This is what happens when firms depend largely on banks to raise external funds, and when the regulation limits banks' equity investments in nonfinancial firms to a certain limit, defined in terms of each firm's capital.

In the model presented here, this type of regulation is not Pareto-improving, and it does not improve the bank's stability: By limiting the bank's ability to use equity to finance a firm, the regulator forces the bank to use more debt in order to channel the necessary funds to the firm. This offsets the effects of the reduction of the bank's stake in the capital of the firm and, in some cases, it might even create the perverse effect of increasing the bank's risk of failure because debt, in general, is more risk-motivating than equity.

How robust are these results? Conducting the analysis in a framework where both debt and equity are optimal contracts introduced certain limitations because of the design of the project held by the entrepreneur. Nonetheless, most of the results hold regardless of whether



there is a monopoly or perfect competition in the banking sector. In addition, the results concur both with the literature on banks' capital regulation [see, for example, Furlong and Keeley (1989)] and with the literature that has studied the risk effects associated with different financial instruments—namely, that debt financing tends to be more risk-motivating than equity financing [see, for example, Pozdena (1991)].

It remains a topic for future research to study the impact of equity investment regulations when firms have access to capital markets, particularly to the stock market.

## Appendices

### A Proofs of Propositions

#### A.1 Proof of Proposition 1

For a given value of the insurance premium ( $\bar{q}$ ), the problem defined here can be solved through the following steps. First, because of the assumption that capital is more expensive than deposits, we know that the bank chooses the minimum required capital, that is,  $K^* = \theta \bar{I}$ . Based on this and on the bank's budget constraint, we know its demand for deposits,  $B^* = (1 - \theta) \bar{I}$ . Second, using that information and the entrepreneur's incentive constraints, we can write both the bank's and the entrepreneur's profits in the probabilities. Third, from the first-order conditions of the problem, we can find the optimal probabilities ( $p_1^*, p_2^*$ ). Since this is a convex problem, there is no need to consider the second-order conditions. Fourth, using the values of  $p_i^*$ , through the entrepreneur's incentive constraints, it is possible to find  $r_i^*$ , and through the definition of the fair insurance premium, one can derive the second-degree equation in  $q^*$  presented in the proposition, where

$$\begin{aligned} V_0 &\equiv (1 + i) \left\{ a_1 a_2 - \left[ a_2 [y_1 - (1 - \theta) \bar{I}] + a_1 [y_2 - a_3 - (1 - \theta) \bar{I}] \right] f(\mu^*) \right\}, \\ V_1 &\equiv \left\{ (a_1 + a_2)(1 - \theta) \bar{I} - \left[ a_2 [y_1 - (1 - \theta) \bar{I}] + a_1 [y_2 - a_3 - (1 - \theta) \bar{I}] \right] \right\} f(\mu^*), \\ V_2 &\equiv (a_1 + a_2)(1 - \theta) \bar{I} f(\mu^*). \end{aligned}$$

Note that  $V_0 > 0$  because of the conditions imposed by the first-best solution to the model [assumption (5)],  $V_2 > 0$ , and  $V_1 \geq 0$ . In order to have an equilibrium, we need to have  $V_1 < 0$ . In this case, the equilibrium insurance premium is the smaller root to the second-degree equation referred to above, because this root Pareto dominates the larger one. Hence we have

$$q^* = \frac{-V_1 - (V_1^2 - 4V_0V_2)^{\frac{1}{2}}}{2V_2}. \quad (\text{A.1})$$

## A.2 Proof of Proposition 2

Taking into account the equityholders' limited liability condition, this proposition can be shown through a spanning argument. For a given percentage of the firm's capital held by the bank ( $\bar{\alpha}$ ), and a given face value of debt ( $\bar{d}$ ), the entrepreneur must solve the following problem:

$$\underset{p}{Max} \quad \Pi_E = \sum_{i=1}^2 p_i(1 - \bar{\alpha})(y_i - \bar{d}) - C(p)$$

$$s.t. \quad pe \leq 1$$

$$p \geq 0.$$

This is a convex programming problem, so the usual results apply. For the case where  $p_i > 0$  for  $i = 0, 1, 2$ , the incentive constraints are

$$(1 - \alpha)(y_1 - d) - a_1 p_1 = 0, \tag{A.2}$$

$$(1 - \alpha)(y_2 - d) - a_3 - a_2 p_2 = 0. \tag{A.3}$$

The idea of the proof is to show first that there exists a feasible combination of  $\alpha$  and  $d$  that motivates each entrepreneur, through his incentive constraints, to choose the probability distribution  $p^*$ , in which case the initial conditions are verified ( $p_i > 0$  for  $i = 0, 1, 2$ ), and second, that such a combination generates the same revenue to the bank as  $r^*$  does.

From the incentive constraints and the second-best probability distribution  $p^*$ , it is possible to find  $\alpha^*$  and  $d^*$ , that is, the values that the bank would have to choose in order to implement  $p^*$ . Since these values satisfy the initial conditions, the last thing left to be proved is that the combination  $(\alpha^*, d^*)$  generates the same revenue to the bank as  $r^*$  does. This is apparent immediately once we recognize that  $\alpha(y_i - d) + d = r_i$ , for  $i = 1, 2$ , and we take into account the equityholders' limited liability condition in order to explain why in state 0 (the state where the project's outcome is equal to zero), the bank in a position as debtholder receives no payment from the firm's equityholders.

### A.3 Proof of Proposition 3

Given that an increase in the bank's required capital–asset ratio implies a reduction in the equilibrium insurance premium ( $q^*$ ), it is straightforward to show that both probabilities of the project's positive outcomes rise when  $\theta$  is increased. This in turn implies an increase in the entrepreneur's profits ( $\Pi_B$ ) and a decrease in the project's probability of failure ( $p_0^*$ ). The impact, in equilibrium, of the capital regulation on the bank's profits ( $\Pi_B^e$ ) is given by

$$\frac{d\Pi_B^e(\theta)}{d\theta} = -(p_1^* + p_2^*)(1 - \theta)\bar{I}\frac{dq^*(\theta)}{d\theta} - (r - i)\bar{I}. \quad (\text{A.4})$$

From here we see what happens to the bank's profits when there is an increase in the required capital–asset ratio. This implies a cost for the bank because it must use relatively more of its most expensive source of funding—capital [note that  $r > i$  by assumption (3)]. But it also implies a positive effect for the bank—the reduction of the moral–hazard costs caused by deposit insurance, which is given by  $\frac{dq^*(\theta)}{d\theta} < 0$ . Whether the bank's profits increase with that policy depends on the relative magnitude of these two effects.

### A.4 Proof of Proposition 4

One way of showing the results in this proposition is to derive all endogenous variables as functions of  $\alpha$ , and then study the impact on these variables of a reduction in  $\alpha$ .

The problem that the bank must solve is

$$\begin{aligned} \underset{\alpha, d, K}{Max} \quad & p_0 \text{Max} \{0, -\bar{Q}B\} + \sum_{i=1}^2 p_i \text{Max} \{0, [\alpha(y_i - d) + d] - \bar{Q}B\} - K(1 + r) \\ \text{s.t.} \quad & (1 - \alpha)(y_1 - d) - a_1 p_1 = 0 \\ & (1 - \alpha)(y_2 - d) - a_3 - a_2 p_2 = 0 \\ & 0 \leq \alpha < 1 \quad 0 \leq d \leq y_1 \\ & K + B = \bar{I} \quad K \geq \theta \bar{I}. \end{aligned}$$

There is no need to consider here the entrepreneur's participation constraint because, as previously explained, this constraint is not binding when the bank behaves like a monopolist.

We already know that the bank's capital structure is  $K^* = \theta \bar{I}$  and  $B^* = (1 - \theta)\bar{I}$ . In order to find the endogenous variables as functions of  $\alpha$ , the bank's problem is solved in two steps. In the first step, the optimal value of  $d$  is found assuming that  $\alpha$  and  $\bar{q}$  are constant. In the second step, the first-order condition of the first step is substituted in the bank's problem so that the optimal value of  $\alpha$  can be computed.

The first-order condition of first step is

$$d = \frac{a_2 y_1 (1 - 2\alpha) + a_1 y_2 (1 - 2\alpha) - a_1 a_3 + (a_1 + a_2) \bar{Q} (1 - \theta) \bar{I}}{2(a_1 + a_2)(1 - \alpha)}. \quad (\text{A.5})$$

Using (A.5) and the entrepreneur's incentive constraints, one can compute the project's probabilities as functions of  $\alpha$ . They are

$$p_1^*(\alpha) = \frac{1}{2a_1(a_1 + a_2)} [P_{1,1} + P_{1,2}\alpha], \quad (\text{A.6})$$

$$p_2^*(\alpha) = \frac{1}{2a_2(a_1 + a_2)} [P_{2,1} - P_{2,2}\alpha], \quad (\text{A.7})$$

$$p_0^* = \frac{1}{2a_1 a_2} [a_2 y_1 + a_1 (y_2 - a_3) - \bar{Q} (1 - \theta) \bar{I}], \quad (\text{A.8})$$

where

$$P_{1,1} \equiv (a_1 + a_2)[y_1 - \bar{Q}(1 - \theta)\bar{I}] - a_1(y_2 - a_3 - y_1),$$

$$P_{1,2} \equiv 2a_1(y_2 - y_1),$$

$$P_{2,1} \equiv (a_1 + a_2)[y_2 - a_3 - \bar{Q}(1 - \theta)\bar{I}] + a_2(y_2 - a_3 - y_1),$$

$$P_{2,2} \equiv 2a_2(y_2 - y_1).$$

Since  $P_{1,2} > 0$  and  $P_{2,2} > 0$ , the equity regulation implies an increase in  $p_2$  and a decrease in  $p_1$ . Furthermore, since  $p_0$  does not depend on  $\alpha$ , this regulation does not affect the bank's probability of failure and, as a result, has no impact on the equilibrium insurance premium.

Through the same procedure, one can determine both the entrepreneur's and the bank's profits as functions of  $\alpha$ . They are

$$\Pi_E(\alpha) = \frac{1}{8a_1 a_2 (a_1 + a_2)} [E_0 - E_1 \alpha + E_2 \alpha^2], \quad (\text{A.9})$$

$$\Pi_B(\alpha) = \frac{1}{4a_1a_2(a_1 + a_2)} [B_0 + B_1\alpha - B_2\alpha^2] - \theta\bar{I}(1 + r), \quad (\text{A.10})$$

where

$$E_0 \equiv \left\{ a_2[y_1 - \bar{Q}(1 - \theta)\bar{I}] + a_1[y_2 - a_3 - \bar{Q}(1 - \theta)\bar{I}] \right\}^2 + 4a_1a_2(y_2 - a_3 - y_1)^2,$$

$$E_1 = 8a_1a_2(y_2 - y_1)(y_2 - a_3 - y_1),$$

$$E_2 = 4a_1a_2(y_2 - y_1)^2,$$

and

$$B_0 \equiv \left\{ a_2[y_1 - \bar{Q}(1 - \theta)\bar{I}] + a_1[y_2 - a_3 - \bar{Q}(1 - \theta)\bar{I}] \right\}^2,$$

$$B_1 \equiv 4a_1a_2(y_2 - y_1)(y_2 - a_3 - y_1),$$

$$B_2 \equiv 4a_1a_2(y_2 - y_1)^2.$$

Because the profits of the entrepreneur are a decreasing, strictly convex function of  $\alpha$  in the relevant range, the equity investment regulation implies an increase in the value of this function. With respect to the bank's profits, note that they are an increasing, strictly concave function of  $\alpha$  with its maximum, as expected, at the point where  $\alpha = \alpha^*$  with  $f(\mu^*) = \frac{1}{2}$ . Thus, the introduction of the equity limit implies a reduction in the bank's profits.

The results derived so far in this proof assume that the optimal value of  $d$ , determined by (A.5) for a given maximum equity that the bank can hold ( $\hat{\alpha}$ ), is smaller than  $y_1$ . However, because there is an inverse relationship between  $d$  and  $\hat{\alpha}$ , the smaller the value of  $\hat{\alpha}$  (the more restrictive the regulation), the higher the chances that  $d$  becomes larger than  $y_1$ . In this case, depending on the parameters of the model, it may be optimal for the bank to completely drop its use of equity and begin financing the entrepreneur using only debt with a face value higher than  $y_1$ . Under these conditions, the entrepreneur chooses to put forth no effort in state 1, which implies an increase in the bank's probability of failure and, as a result, an increase in the equilibrium insurance premium. Note, however, that the bank does not take this into account, because it assumes that the insurance premium it pays is independent of the risk of its assets.

## B Numerical Example

The set of parameters chosen for the numerical example was

$$\begin{array}{lll}
 y_1 = 3.4 & a_1 = 10 & i = 0.06 \\
 y_2 = 6.8 & a_2 = 10 & r = 0.10 \\
 & a_3 = 0.4 & \bar{I} = 1
 \end{array}$$

The first-best solution to the model with these parameters is

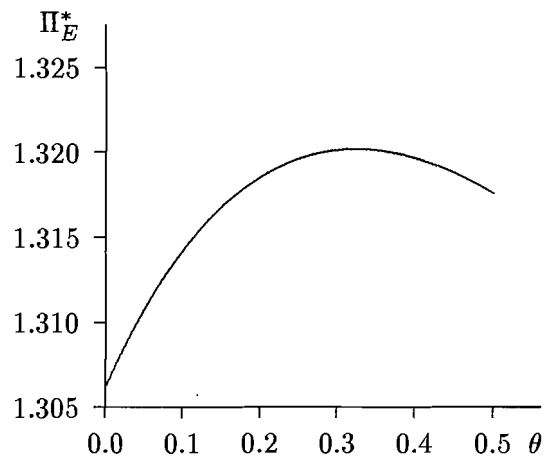
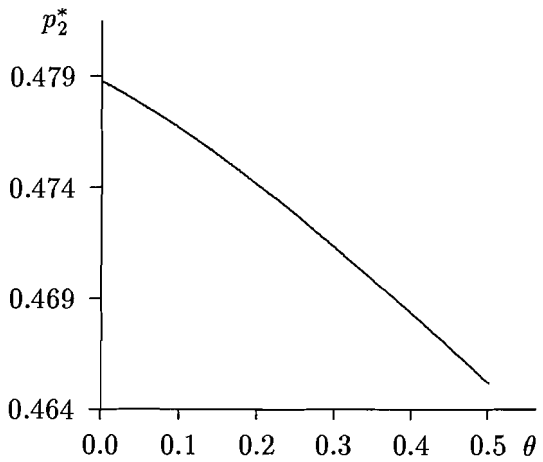
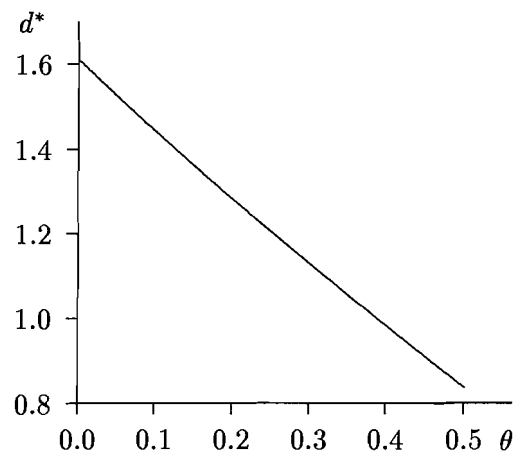
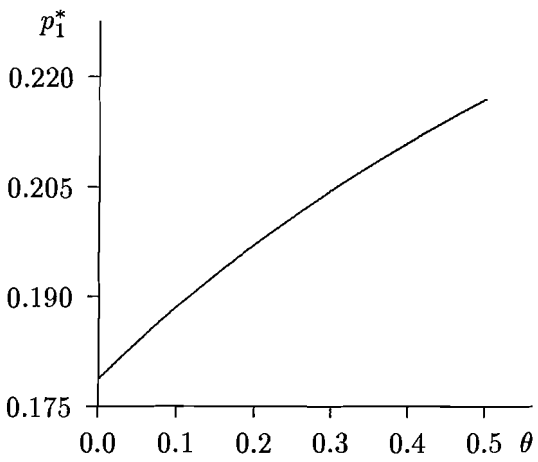
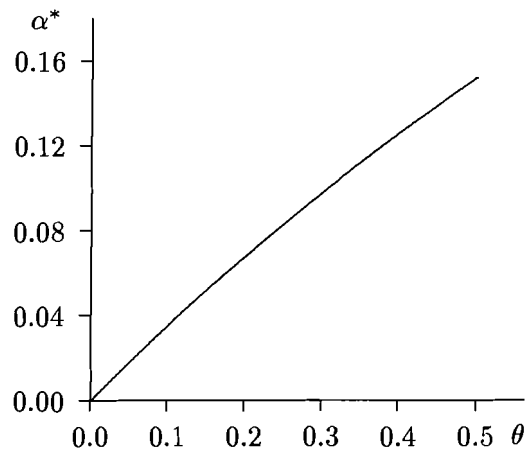
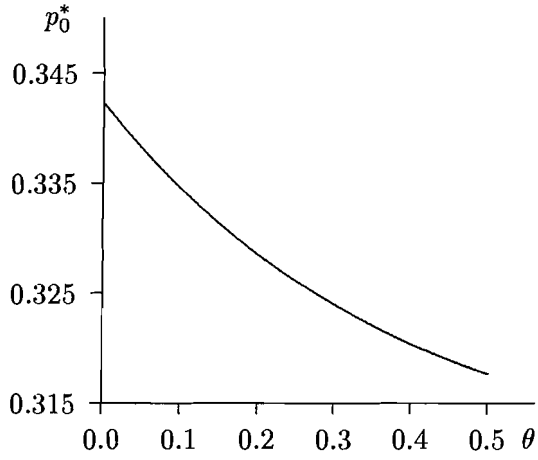
$$\begin{array}{lll}
 p_0^{fb} = 0.02 & p_1^{fb} = 0.34 & p_2^{fb} = 0.64 \\
 \Pi_E^{fb} = 3.13 & & 
 \end{array}$$

The results for the bank's capital and equity investment regulations, when there is perfect competition among banks, are presented in the next two subsections. For the case of the capital regulation, subsection B.1, the equilibrium is given by the results in propositions (1) and (2) with  $\Pi_B = 0$ . The values of the endogenous variables are plotted as functions of  $\theta$ , the required capital-asset ratio. The impact of an increase in the capital regulation is given by the variation in those variables when  $\theta$  is increased.

With respect to the equity investment regulation, subsection B.2, the equilibrium for each given value of  $\hat{\alpha}$  (the maximum stake of the firm's capital that the bank can hold) is determined by the face value of debt that the bank has to charge the firm, given the bank's zero-profit condition, and its behavior with respect to deposit insurance.

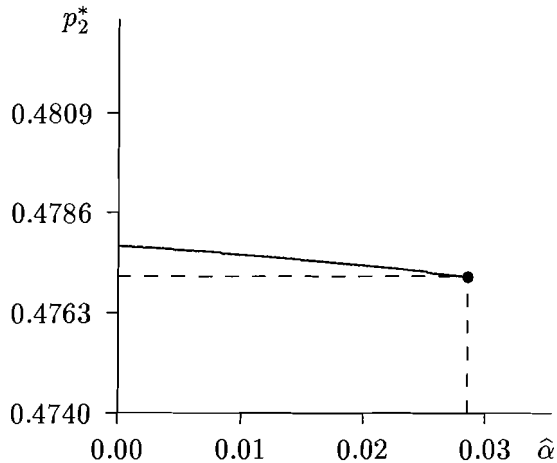
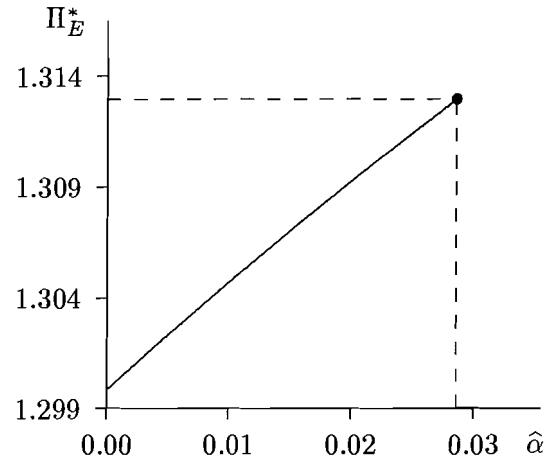
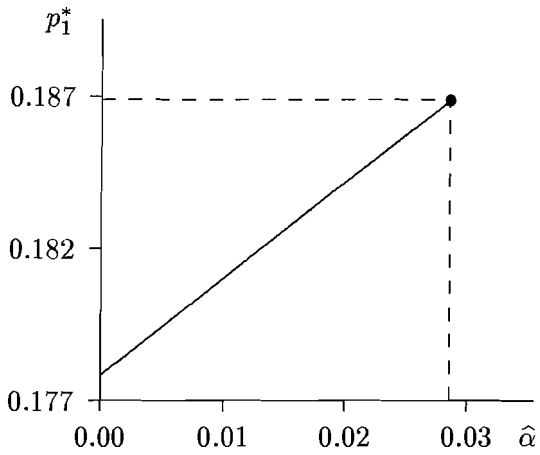
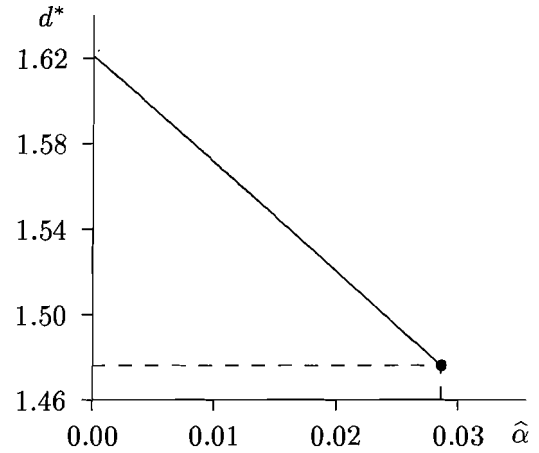
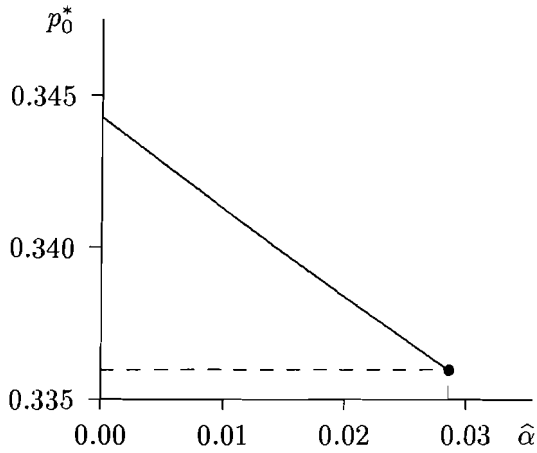
The values of the endogenous variables are plotted as functions of  $\hat{\alpha}$ , for the range of the numerical example where this limit is binding, that is, for  $\hat{\alpha} < \alpha^*$ . The impact of the equity investment regulation is given by comparing the second-best solution to the model with the values of these variables associated with a given value of  $\hat{\alpha}$ .

### B.1 Results of the Capital Regulation





## B.2 Results of the Equity Investment Regulation



For  $\theta = 0.08$  :

• Represents the second-best solution to the model.

— Represents the equilibrium for each maximum stake of the firm's capital that the bank is allowed to hold ( $\hat{\alpha}$ ).

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