Working Paper 9504

INTEREST RATE RULES VS. MONEY GROWTH RULES: A WELFARE COMPARISON IN A CASH-IN-ADVANCE ECONOMY

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June 1995

ABSTRACT

This paper considers the welfare consequences of two particularly simple rules for monetary policy: an interest rate peg and a money growth peg. The model economy consists of a real side that is the standard real business cycle model, and a monetary side that amounts to imposing cash-in-advance constraints on certain market transactions. The paper also considers the effect of assuming a rigidity in the typical household's cash savings choice. The competitive equilibrium of the economy is not Pareto efficient, partly because of two intertemporal distortions: a distortion on the capital accumulation decision, and a distortion on portfolio choice that arises from the assumed rigidity. The principal result of the paper is that the interest rate rule (but not the money growth rule) entirely eliminates these two intertemporal distortions and is thus the benevolent central banker's policy choice.

1. Introduction

One of the oldest debates in monetary economics concerns the appropriate target for monetary policy: Should central banks target money supply growth rates or nominal interest rates? Friedman (1990) provides an introduction to this voluminous literature. Much of the early work follows Poole (1970) and Sargent and Wallace (1975) and conducts the analysis within an IS/LM-type aggregative framework. In contrast, the more recent studies, led by Sargent and Wallace (1982), address the issue in the context of general equilibrium models. The present paper belongs to this latter tradition. In the monetary economy analyzed below, the competitive equilibrium is not Pareto efficient, but is instead distorted relative to the Pareto optimum by one intratemporal distortion and two intertemporal distortions. The paper considers the welfare consequences of two simple monetary policy rules: 1) a constant money growth rate (in which case the nominal interest rate is endogenous), and 2) a constant nominal interest rate (in which case the money growth rate is endogenous). The principal result is that an interest rate rule, but not a money growth rule, entirely eliminates the two intertemporal distortions and is thus the benevolent central banker's policy choice.

Our analysis is carried out in an economy in which the real side is the standard real business cycle model. Money is introduced by imposing cash-in-advance constraints on the representative household's consumption purchases and the representative firm's wage bill. As is well known, real variables in this monetary economy generally behave quite differently from their counterparts in the corresponding real economy run by a Pareto planner. For example, the cash constraint on labor demand imposes an inflation tax on labor market activity and thus lowers equilibrium work effort (see, for example, Cooley and Hansen [1989]). In contrast to this *intra*temporal distortion, we focus on two potential *inter*temporal distortions arising in the monetary economy. First, the cash constraint on consumption imposes a distortion on the capital accumulation decision. In particular, capital accumulation is affected by the time path of the nominal rate of interest (see Fischer [1979] and Fuerst [1994a]). Second, a non-Fisherian component of interest rate determination enters into the model under the assumption that the household's cash versus bank deposit portfolio decision is made in the absence of full contemporaneous information. Models incorporating this type of portfolio rigidity are something of a growth industry, partly because they are consistent with an increase in the money growth rate (temporarily) driving down the nominal rate of interest (see Lucas [1990], Christiano and Eichenbaum [1992, 1994], Fuerst [1992, 1994b], and Carlstrom [1994]). The objective of the present paper is to show how a simple interest rate rule can eliminate both this portfolio rigidity and the capital accumulation distortion.

A common criticism of interest rate rules is their potential for giving rise to price-level indeterminacy and sunspot behavior. For example, Smith (1988) argues that one possible justification for the money growth regime in the Sargent and Wallace (1982) environment is that it precludes the possibility of sunspot equilibria. Issues of this type do arise below, but we sidestep some of them by limiting our analysis to stationary rational expectations equilibria. In particular, we ignore the possibility of self-fulfilling hyperdeflations and hyperinflations. We make this choice because: a) we have nothing new to contribute in this regard, and b) as demonstrated by Woodford (1994) in a comparable environment without capital, these equilibria are not unique to interest rate regimes, and in fact are in some sense more likely under money growth regimes. Even within the class of stationary equilibria, price-level indeterminacy does arise below. In two of the three model variants, this indeterminacy is purely nominal and would thus have no effect on real welfare comparisons. A novel result is that in the case of portfolio rigidities, this indeterminacy becomes a real indeterminacy, so that some care must be taken in defining monetary policy.

The next section lays out the basic model. Section 3 addresses the interest rate versus money growth issue in a deterministic setting, while the two sections that follow carry out the corresponding analysis in increasingly complicated economic environments. Section 4 considers the case of stochastic shocks without portfolio rigidities, while section 5 considers the case of stochastic shocks with portfolio rigidities. Section 5 also presents some computational results of a numerical welfare comparison of the two monetary policy regimes. Section 6 discusses the real indeterminacy problem mentioned above, and section 7 concludes.

2. The Model

The economy consists of numerous agents of three types: households, firms, and intermediaries. Since all behave as atomistic competitors, we will restrict our discussion to a representative agent of each type. We will first describe the optimization problem of each agent, then turn to an analysis of equilibrium behavior.

The typical household is infinitely lived, with preferences over consumption (c_t) and leisure ($1-L_t$) given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1-L_t)$$

where E_0 is the expectation operator, $\beta \in (0,1)$ is the personal discount rate, L_t denotes household labor supply, and the household's leisure endowment is normalized to unity. The household begins period t with M_t dollars and must decide how much of this cash to keep on hand for contemporaneous consumption and how much to deposit in the intermediary, where it will earn a gross nominal return of R_t. Let N_t denote the amount of cash deposited in the intermediary, a choice that we assume is fixed until the next period. An important issue below is the information the household has when making this portfolio decision. We consider two distinct possibilities: In the case of a portfolio rigidity (PR), the household selects N_t before knowing the current innovations in technology and government spending, while in the case of no portfolio rigidity (NPR), the household knows the current innovations when choosing N_t. In either case, after making its portfolio decision, the household makes its consumption and labor supply decisions with full information on the current state of the world. Consumption purchases are subject to a modified cash-in-advance constraint. In particular, households can use cash not deposited in the intermediary, as well as current labor income, to purchase consumption:

$$P_t c_t \leq M_t - N_t + W_t L_t$$

where P_t and W_t denote the price level and nominal wage, respectively. At the end of the period, the household receives a cash dividend payment from both the firm and intermediary, as well as principal plus interest on its deposits at the intermediary. Hence,

$$M_{t+1} = M_t + (R_t-1)N_t + W_t L_t + \Pi_t^f + \Pi_t^i - P_t c_t - P_t T_t$$

where Π_t^f and Π_t^i denote the profits of the representative firm and intermediary, respectively, and T_t denotes the real lump-sum taxes imposed by the fiscal authority.

The representative firm uses its accumulated capital stock (k_t) and the labor it hires from households (H_t) to produce current output via its stochastic production technology: $\theta_t f(k_t,H_t)$, where θ_t is the time t state of technology and f is a neoclassical production function. The firm keeps part of this output to augment its capital stock (I_t) and sells the rest to households (on a cash basis) for consumption. The firm also faces a cash constraint in that the current wage bill must be financed with cash loans from the intermediary. These loans are at the gross rate R_t, and are repaid at the end of the period. The firm chooses its production and investment levels to maximize the discounted value of its dividend payments:

$$E_0 \sum_{t=0}^{\infty} [\beta^{t+1} U_c(t+1)/P_{t+1}]\Pi_t^f$$

with Π_t^f and I_t given by

$$\Pi_t^f \equiv [P_t\theta_t f(k_t,H_t) - W_tH_tR_t - P_tI_t]$$

$$k_{t+1} = (1-\delta)k_t + I_t.$$

Note that in the terms of Lucas and Stokey (1987), labor is a cash good for the firm, while investment is a credit good. The technology variable is assumed to evolve according to the following stochastic process:

$$\theta_t = (1-\rho_\theta)\theta + \rho_\theta\theta_{t-1} + \varepsilon_t$$

where ρ_θ is the autocorrelation coefficient, ε_t is an i.i.d. shock, and the nonstochastic steady state of θ_t is θ.

Finally, the typical intermediary accepts deposits of N_t from households and receives the current monetary injection of $M_t^S(G_t-1)$ from the central bank, where $G_t \equiv M_{t+1}^S/M_t^S$, and M_t^S is the money supply per household. All of this cash is then loaned out to firms at the rate R_t. This implies that $\Pi_t^i \equiv R_tM_t^S(G_t-1)$.

To close our description of the model, we need to specify fiscal and monetary policy. To begin with the former, real government expenditures are exogenous and follow the stochastic process

$$g_t = (1-\rho_g)g + \rho_g g_{t-1} + \gamma_t$$

where ρ_g is the autocorrelation coefficient, γ_t is an i.i.d. shock, and the nonstochastic steady state of g_t is g. Because the model is otherwise Ricardian, we abstract from government debt by assuming that $T_t = g_t \;\forall\; t$.

We consider two schemes for the conduct of monetary policy. Under a money growth rule, $G_t = G_{ss} \;\forall\; t$ and R_t is endogenous. In contrast, under an interest rate regime, $R_t = R \;\forall\; t$ and G_t is endogenous. For ease of comparison, we set $R = G_{ss}/\beta$, so that the nonstochastic steady state of the model is unaffected by the choice of monetary regime.

There are four markets in this economy: the goods market, the labor market, the money market, and the credit market. The respective market-clearing conditions are given by

$$c_t + g_t + k_{t+1} = \theta_t f(k_t, L_t) + (1-\delta)k_t$$

$$H_t = L_t$$

$$M_t = M_t^s$$

$$W_t H_t = N_t + M_t(G_t - 1).$$

The model's equilibrium is defined by the household's and firm's optimization conditions evaluated at these equilibrium conditions. To make the model stationary, we normalize all nominal variables by M_t and define the following new variables: $p_t \equiv P_t/M_t$, $w_t \equiv W_t/M_t$, $n_t \equiv N_t/M_t$. Given the timing of the model, a more natural choice might be to normalize by M_{t+1}, since this represents the money stock available for time t transactions. However, in the PR model, this choice would not be appropriate because N_t must be chosen before M_{t+1} is known. Hence, to maintain symmetry between the two models, we will use M_t as our normalization. An equilibrium is given by the L_t, k_{t+1}, w_t, p_t, n_t, and G_t or R_t stochastic processes that satisfy the following Euler equations:

$$E_s U_c(t)/p_t = E_s \beta R_t U_c(t+1)/p_{t+1} G_t \tag{1}$$

$$U_L(t)/U_c(t) = w_t/p_t \tag{2}$$

$$p_tc_t = 1 - n_t + w_tL_t \tag{3}$$

$$\theta_tf_L(t) = R_tw_t/p_t \tag{4}$$

$$n_t+(G_t-1) = w_tL_t \tag{5}$$

$$E_t(p_t/G_tp_{t+1})U_c(t+1) = E_t\beta(p_{t+1}/G_{t+1}p_{t+2})[\theta_{t+1}f_k(t+1)+(1-\delta)]U_c(t+2) \tag{6}$$

where $E_s = E_t$ in the NPR model and $E_s = E_{t-1}$ in the PR model.

We will now turn to an analysis of the economy's behavior under the alternative monetary regimes, beginning with a deterministic version of the model and then turning to the NPR and PR cases.

3. The Deterministic Case

Suppose that $\theta_t = \theta$ and $g_t = g \ \forall \ t$, and that monetary policy is nonstochastic. Then, solving (2), (3), and (5) for p_t, w_t, and n_t, we have:

$$p_t = G_t/c_t$$

$$w_t = U_LG_t/c_tU_c$$

$$n_t = U_LG_tL_t/c_tU_c - (G_t-1).$$

Substituting these back into the remaining three equations, we are left with the following three Euler equations in L_t, k_{t+1}, and G_t <u>or</u> R_t:

$$U_c(t)c_t = R_t\beta U_c(t+1)c_{t+1}/G_{t+1} \tag{7}$$

$$R_t U_L(t)/U_c(t) = \theta f_L(t) \tag{8}$$

$$U_c(t)/R_t = \beta[\theta f_k(t+1)+(1\text{-}\delta)]U_c(t+1)/R_{t+1}. \tag{9}$$

The two distortions in this economy are apparent. First, there is an intratemporal distortion on work effort in equation (8) that arises because of the transactions constraint on the firm's wage bill. Second, notice that by substituting (1) into (6), the capital accumulation equation (9) collapses to something resembling the optimal growth equation. The difference is that the two respective marginal utilities are scaled by the corresponding nominal rates of interest. This intertemporal distortion arises because of the cash constraint on consumption. If the firm decides to increase its capital stock by one unit, then there will be p_t fewer dollars to distribute to the household at the end of period t. At the beginning of period t, the household could borrow against this expected dividend flow and finance p_t/R_t dollars of consumption. Hence, the private utility cost of increasing capital by one unit is $U_c(t)/R_t$. Next period, this capital will produce a profit flow of $p_{t+1}[\theta f_k(t+1)+(1\text{-}\delta)]$ dollars that will be paid out to households at the end of the period. At the beginning of t+1, the household could borrow against this cash flow and finance $p_{t+1}[\theta f_k(t+1)+(1\text{-}\delta)]/R_{t+1}$ dollars of consumption. Hence, the private utility gain of increasing capital by one unit is $\beta[\theta f_k(t+1)+(1\text{-}\delta)]U_c(t+1)/R_{t+1}$. The optimizing firm equates these two private margins. Note that both of these private margins are distorted relative to the social margins by the corresponding nominal rate of interest. This observation is formalized in Proposition 1 below.

Consider the economy's behavior under two different monetary regimes: i) a money growth regime in which $G_t = G_{ss}$ $\forall$ t, and ii) an interest rate regime in which $R_t = G_{ss}/\beta$ $\forall$ t. Note that the economy's unique steady-state capital stock (k_{ss}) is identical under either regime. However, the economy's behavior along the accumulation path is quite different under the two policies. Under a money growth regime, $G_t = G_{ss}\ \forall\ t$, and (7)-(9) determine the paths for L_t, k_{t+1}, and R_t. Note in particular that R_t is generally not constant along the accumulation path.[1] In contrast, under an interest rate rule, (8)-(9) determine the paths for k_{t+1} and L_t, while (7) then determines G_{t+1}. We immediately have the following:

Proposition 1: In the deterministic model, if monetary policy operates under an interest rate regime, equation (9) collapses to the accumulation equation from the optimal growth problem, that is, the intertemporal distortion on capital accumulation is entirely eliminated.

Proposition 2: In the deterministic model, if labor supply is inelastic, the optimal monetary policy is an interest rate rule.

Proposition 2 cannot, in general, be extended to the case of elastic labor because then we have a second-best problem. Under an interest rate rule, there is no distortion on the capital accumulation equation and a constant distortion on the labor supply decision. In contrast, under a money growth rule, there is a varying distortion on both margins. The preferred regime will, in general, depend on preferences. However, we can state the weaker result that an interest rate policy of $R_t = 1$ dominates a money growth policy of $G_{t+1} = \beta$, since the latter does not guarantee a zero nominal interest rate along the accumulation path. (Woodford [1990] makes a similar point in a variety of

[1]The one exceptional case is separable preferences with log preferences over consumption, in which case the money growth and interest rate regimes are identical. See Fuerst (1994a) for more discussion.

models without capital.)

As an aside, note that under an interest rate regime we need an extra initial condition, that is, (7)-(9) impose no conditions on G_0 and thus none on p_0, w_0, and n_0. This is the standard result of nominal indeterminacy under an interest rate rule (see Sargent [1979] or Sargent and Wallace [1975]), which can be eliminated in the current context by specifying the initial money stock (see McCallum [1981, 1986]).[2] Given the timing of the monetary injection in the model, the money stock available for use in time 0 is M_0G_0. Because we have implicitly set $M_0 = 1$ under our normalization above, we can eliminate the nominal indeterminacy by specifying G_0. Note that in any case, there is no indeterminacy in the real variables.

4. The Stochastic Case without Portfolio Rigidities

Now, suppose that θ_t and g_t are stochastic, but that N_t is chosen after the current innovations are observed. Once again we can eliminate n_t, w_t, and p_t. Using the law of iterated expectations, we have:

$$U_c(t)c_t = R_t\beta E_t U_c(t+1)c_{t+1}/G_{t+1} \tag{10}$$

$$R_t U_L(t)/U_c(t) = \theta_t f_L(t) \tag{11}$$

$$U_c(t)/R_t = \beta E_t[\theta_{t+1}f_k(t+1)+(1-\delta)]U_c(t+1)/R_{t+1}. \tag{12}$$

Propositions 1 and 2 apply here as well: An interest rate rule eliminates the distortion on capital accumulation and thus is clearly the optimal monetary policy if we abstract

[2]Woodford (1994) demonstrates how the homogeneity property that gives rise to this nominal indeterminacy can also be eliminated by assuming that changes in the money supply are brought about through open market operations rather than through lump-sum monetary transfers.

from elastic labor supply. As in the previous section, this result does not immediately generalize to the case of elastic labor supply, because then we have a second-best problem. However, once again, a peg of $R_t = 1$ dominates a money growth policy of $G_{t+1} = \beta$.

The nominal indeterminacy (under an interest rate peg) discussed in the previous section takes on a slightly modified form here. Under a peg of R, (11)-(12) uniquely determine the behavior of L_t and k_{t+1}. This behavior is identical to that in the corresponding real business cycle economy, where the marginal utility of leisure is proportionally scaled upward by R. Given this real behavior, (10) then imposes the following restriction on the money growth process:

$$(\beta R)^{-1} = (U_c(t)c_t)^{-1} E_t U_c(t+1)c_{t+1}z_{t+1} \tag{13}$$

where $z_{t+1} \equiv (1/G_{t+1})$. The earlier nominal indeterminacy arises here in that there is no restriction on the initial G_0. However, even with such a G_0 specified, there are an infinite number of money growth processes satisfying (13). For example, if U is logarithmic, we have:

$$(\beta R)^{-1} = E_t(z_{t+1}).$$

In this economy, only the conditional mean of z_{t+1} matters; there is no restriction on the variance of z_{t+1}, nor on its covariance with the technology shocks. This is an economy in which only *expected* money growth matters. (Lucas and Stokey [1987] make a similar point in a similar context.) This indeterminacy is something of a nuisance, but has no consequence for real variables. A natural restriction on G_{t+1} is to assume that it is a time-invariant function of (k_t,g_t,θ_t), that is, $G_{t+1} \equiv G^{npr}(k_t,g_t,\theta_t)$, with $G^{npr}(k_{ss},g,\theta) = G_{ss}$.[3] A loose interpretation of this restriction is that the Fed does

[3] McCallum (1983, 1986) calls restrictions of this type the "minimal state vector solution."

not "play dice" with the money growth rate. Under this assumption, we can solve (10) for G_{t+1}:

$$G_{t+1} = G^{npr}(k_t,g_t,\theta_t) \equiv R\beta E_t[U_c(t+1)c_{t+1}/U_c(t)c_t].$$

Returning to the example of log preferences, the no-dice restriction implies $G_{t+1} = G_{ss} \ \forall \ t$.

5. The Stochastic Case with Portfolio Rigidities

The previous two sections demonstrated that under an interest rate regime, the intertemporal distortion on capital accumulation is entirely eliminated. In this section, we add another distortion to the economic environment, namely, that household portfolio allocations respond sluggishly to innovations in technology and government spending. This rigidity is of particular interest because many recent models of the monetary business cycle use it as a means of modeling monetary non-neutrality (see, for example, Carlstrom [1994], Christiano and Eichenbaum [1992, 1994], and Fuerst [1992, 1994b]). The principal result of this section is that an interest rate rule also eliminates this distortion.

In the PR case, n_t is a predetermined variable, so we must alter our solution procedure.[4] In particular, we will solve (2), (3), and (5) for p_t, w_t, and L_t:

$$p_t = G_t/c_t$$

$$w_t = G_tU_L/c_tU_c$$

[4]Blanchard and Kahn (1980) call a time t variable predetermined if it is a function only of variables known at the end of time t-1. In the present context, n_t is a function only of (k_t,θ_{t-1}), both of which are known at the end of time t-1.

$$L_t = c_t U_c(n_t+G_t-1)/G_t U_L.$$

The equilibrium is now given by the k_{t+1}, L_t, n_t, and G_t or R_t that solve:

$$L_t = c_t U_c(n_t+G_t-1)/G_t U_L \tag{14}$$

$$E_{t-1} U_c(t) c_t/G_t = E_{t-1} \beta R_t U_c(t+1) c_{t+1}/G_{t+1} G_t \tag{15}$$

$$E_t(c_{t+1}/c_t G_{t+1}) U_c(t+1) = E_t \beta (c_{t+2}/c_{t+1} G_{t+2})[\theta_{t+1} f_k(t+1)+(1-\delta)] U_c(t+2) \tag{16}$$

$$R_t U_L(t)/U_c(t) = \theta_t f_L(t). \tag{17}$$

The effect of the portfolio rigidity is most easily seen in (15). In contrast to equation (10) in the NPR case, in the PR case the nominal interest rate is equal to Fisherian fundamentals only "on average." Innovations in technology alter the shadow value of cash in the goods market (the left-hand side of [15]) and in the financial intermediary (the right-hand side of [15]). Since portfolios are rigid, these differences cannot be arbitraged away. Hence, there is a non-Fisherian component to interest rate determination. This portfolio distortion affects both the labor market and the capital market. As for the labor market, the rigidity tends to make labor less responsive to shocks.[5] For example, if U is separable and logarithmic in consumption, then (14) implies that under a money growth regime, labor is invariant to productivity and government spending shocks. The portfolio rigidity also alters the distortion on capital accumulation, since the non-Fisherian component of interest rate determination implies that (16) cannot be collapsed into (12). This latter point suggests that if an interest rate peg eliminates the portfolio distortion, then it will also eliminate the capital accumulation distortion. The goal of this section is to demonstrate this explicitly. We will begin with an observation about the portfolio rigidity.

[5]Christiano and Eichenbaum (1994) also emphasize this point.

Proposition 3: In a PR economy without capital, if monetary policy operates under an interest rate regime, then a) the real behavior of the economy is identical to the corresponding NPR economy, b) there exists a unique time-invariant central-bank reaction function, $G_t = G^{pr}(n_t,g_t,\theta_t)$, with $G^{pr}(n_{ss},g,\theta) = G_{ss}$, that supports the interest rate peg, and c) nominal interest rates are purely Fisherian. In summary, an interest rate regime eliminates the portfolio rigidity.

Proof: With no capital and a constant interest rate, (17) uniquely determines L_t as a function of g_t and θ_t, a relationship that is common to both the NPR and PR models. Substituting this L_t into (14), we can uniquely solve for the time-invariant central-bank reaction function $G_t = G^{pr}(n_t,g_t,\theta_t)$ that supports the interest rate regime, where $G^{pr}(n_{ss},g,\theta) = G_{ss}$, and n_{ss} denotes the value of n in the nonstochastic steady state. This G_t choice implies that the share of the money stock in the intermediary, $(n_t+G_t-1)/G_t$, is ultimately the same in both the NPR and PR models. This implies that an agent in the PR economy would have no desire to vary n_t in response to g_t and θ_t. Hence, nominal interest rates are purely Fisherian, and (15) is trivially satisfied. ■

Although Proposition 3 implies that an interest rate regime leads to identical real behavior in the NPR and PR models, the behavior of the current money growth rate (G_t) is quite different.[6] From (14), the key variable is the share of the time t money stock that is in the intermediary, $s_t \equiv (n_t+G_t-1)/G_t$. In the case of NPR, the previous

[6]As an aside, since $p_t = G_t/c_t$, differences in the conditional variability of G_t in the two models (NPR versus PR) imply stark differences in the variability of the price level.

section's no-dice restriction implies that G_t is predetermined, that is, $G_t = G^{npr}(g_{t-1},\theta_{t-1})$, so that the *household adjusts* n_t to ensure that s_t is at the level needed to support the response of L_t to g_t and θ_t. In contrast, in the PR model, n_t is predetermined, and the *central bank adjusts* G_t to ensure that s_t is at the level needed to support the response of L_t to g_t and θ_t, that is, $G_t = G^{pr}(n_t,g_t,\theta_t)$. It is in this precise sense that an interest rate rule enhances the ability of the PR economy to respond to real shocks.

Returning to the model with capital, note that the proof of Proposition 3 immediately generalizes to prove a weaker result:

Proposition 4: In a PR economy with capital, if monetary policy operates under an interest rate regime, there exists a time-invariant central-bank reaction function, $G_t = G^{pr}(k_t,n_t,g_t,\theta_t)$, with $G^{pr}(k_{ss},n_{ss},g,\theta) = G_{ss}$, such that a) the real behavior of the economy is identical to the corresponding NPR economy operating under an interest rate regime, and b) nominal interest rates are purely Fisherian. Hence, an interest rate regime can eliminate both the portfolio and capital accumulation distortions.

Proof: Under an interest rate rule, the NPR economy uniquely determines the behavior of L_t and k_{t+1} in response to g_t and θ_t. Substituting these values into (14), we can solve for the unique $G_t = G^{pr}(k_t,n_t,g_t,\theta_t)$, with $G^{pr}(k_{ss},n_{ss},g,\theta) = G_{ss}$, that supports this real behavior. As in the proof of Proposition 3, s_t is ultimately the same in both the NPR and PR economies, so that nominal interest rates are purely Fisherian. This implies that (15) is trivially satisfied, and (16) collapses to (12). ■

To close this section, we will present a quantitative assessment of the welfare advantage of an interest rate policy over a money growth policy. The numerical analysis is carried out in three steps. First, the equilibrium Euler equations are linearized about the nonstochastic steady state, and the method of undetermined coefficients is used to calculate the two sets of linear decision rules characterizing the economy under the money growth regime and the interest rate regime. Second, after taking a quadratic approximation of the value function and utility function, the method of undetermined coefficients is used to find the value function under the two monetary regimes.[7] Third, and finally, the constant level of capital subsidy needed to equate the unconditional expectation of the two value functions is calculated. To be precise, let V_R and V_G denote the value functions under an interest rate and money growth regime, respectively. Then in table 1 we report the value of Δ that solves

$$E_0 V_R(k_1, \theta_1, g_1) = E_0 V_G(k_1 + \Delta k_{ss}, \theta_1, g_1)$$

where k_1, θ_1, and g_1 are integrated over their steady-state joint distribution, and Δ is expressed as the percentage increase in steady-state capital that must be given to households in the money growth regime to make them as well off as households in the interest rate regime.

Functional forms and parameter values were chosen to be consistent with the literature. Preferences are given by $U(c,1\text{-}L) = [(c^{1-\sigma}-1)/(1-\sigma) + A\ln(1-L)]$, where the constant A is chosen to imply a steady-state level of labor of .3. We experimented with

[7]To approximate the utility function, we need the equilibrium decision rule for consumption. For these calculations, we used a linear approximation of the aggregate resource constraint to determine consumption behavior. Of course, there are other possibilities, including substituting the linear decision rules for capital and labor into the actual resource constraint and backing out a nonlinear rule for consumption. In current work, we are exploring the consequences of using these alternative methods (along with the possibility of using log-linear decision rules). For a discussion of these alternatives, see Dotsey and Mao (1992).

several values of σ, all with broadly similar results. We report results for $\sigma = 1$ and $\sigma = 5$. We set $\beta = .99$ (implying a 4 percent steady-state annual real rate of interest). Technology is Cobb-Douglas, with a capital share of .36 and a capital depreciation rate of $\delta = .0175$ per quarter. We chose g to imply a steady-state g_t/Y_t ratio of .08. For the stochastic shocks, we utilized the benchmark estimates in Burnside, Eichenbaum, and Rebelo (1993): $\rho_\theta = .986$, $\sigma_\theta = .0089$, $\rho_g = .982$, $\sigma_g = .015$, and $\text{corr}(\gamma_t, \varepsilon_t) = .308$. Finally, for monetary policy, we set $G = .0075$ per quarter for the money growth rule, and $R = G/\beta$ (or about 7 percent annually) for the interest rate rule.

The numerical results are presented in table 1. Note that the welfare gain is relatively large (as welfare numbers go) for either technology shocks alone or for technology and government spending shocks. In the latter case, a value of 2 percent of the aggregate capital stock is a benchmark estimate.[8] With U.S. aggregate net worth now at approximately \$24 trillion, the welfare gain amounts to \$480 billion--a sizable free lunch. We have two comments on this result. First, these welfare numbers are quite sensitive to the variance of the shocks. For example, as ρ_θ increases and the unconditional variability of θ rises, the welfare gain of the interest rate regime grows exponentially. Second, by assumption, the portfolio rigidity disappears after one quarter. This implies that the basic difference between the two regimes is that under a money growth regime, the market economy responds to shocks with a one-period lag. To the extent that the portfolio rigidity is more long-lived, possibly because of portfolio adjustment costs as in Christiano and Eichenbaum (1992), the advantage of an interest

[8]In comparison, Lucas (1987) estimates that the welfare gain of eliminating all consumption variability is only about .008 percent of aggregate consumption into perpetuity, or (in present value) about .048 percent of the aggregate capital stock (we are using the model's steady-state real rate of interest of 4 percent, and consumption/capital ratio of .24, to make this transformation). Note, however, that Lucas' calculation is a partial equilibrium exercise and is thus not strictly comparable to the number we report.

rate rule will be even larger.

6. A Real Indeterminacy in the Case of Portfolio Rigidities

Proposition 4 demonstrates that in the PR model with capital, there exists a time-invariant central-bank reaction function that supports the interest rate peg and produces the same real dynamics as in the NPR model. However, this is not the only real behavior consistent with an interest rate peg in the PR model. We will demonstrate this by construction. To begin, linearize the system (14)-(17) about the nonstochastic steady state. For simplicity, we will set $g_t = g \ \forall \ t$. Suppose that the central bank supports the interest rate peg with the following reaction function:

$$G_{t+1} = G^{npr}(k_t, g, \theta_t) + a_1\varepsilon_t + a_2\varepsilon_{t+1} + a_3\nu_t + a_4\nu_{t+1} \tag{18}$$

where G^{npr} is the central-bank reaction function in the linearized NPR model, ε_t is the time t innovation in the technology shock, ν_t denotes an extraneous or sunspot process that is uncorrelated with the technology process, and $E_{t-1}(\nu_t) = 0$.[9] By construction, (18) satisfies (15). Substituting (18) into (16) yields the following linear equation:

$$E_t Q^{npr}(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+1}, \theta_t, \theta_{t+1}) + q(a_1\varepsilon_t + a_3\nu_t) = 0$$

where Q^{npr} is the equation that results in the corresponding NPR economy and q is a constant. Combining this equation with (17) gives us the law of motion for capital and labor:

[9]As an aside, note that in the previous two sections the choice of G_0 was entirely arbitrary, since it was an initial condition that only scaled all future nominal variables. However, in the PR case, G_0 is not an initial condition, since the choice of n_0 occurs prior to the revelation of G_0.

$$k_{t+1} = K^{npr}(k_t,\theta_t) + \alpha_1(a_1\varepsilon_t+a_3\nu_t)$$

$$L_t = L^{npr}(k_t,\theta_t) + \alpha_2(a_1\varepsilon_t+a_3\nu_t)$$

where K^{npr} and L^{npr} denote the corresponding relationships in the NPR model and the α's are constants. Note that if $a_1 = a_3 = 0$, the real behavior of this PR economy will be identical to the corresponding NPR economy. Substituting the law of motion for capital into (14) yields another linear expression for labor:

$$L_t = Q(k_t,n_t,\theta_t,a_1\varepsilon_t+a_3\nu_t,a_1\varepsilon_{t-1}+a_2\varepsilon_t+a_3\nu_{t-1}+a_4\nu_t).$$

These two expressions for L_t must, of course, agree. If $a_1 = a_3 = 0$, then since n_t is predetermined, $a_4 = 0$ and a_2 is uniquely determined. Therefore, if we restrict the money growth rule to depend only on a minimal state vector, then the real behavior of the economy is unique and identical to the NPR model, and there exists a unique reaction function to support the interest rate peg. (This is just Proposition 4.) However, this is clearly not the only reaction function that will support the interest rate peg. In particular, there is nothing to pin down either a_1 or a_3, since n_t can respond freely to past shocks. Given values for a_1 and a_3, there will exist unique values for a_2 and a_4.[10] Hence, an interest rate target can also be supported with a reaction function depending on sunspots. Since the past innovation in technology is not part of the minimal state vector that is necessary to support an interest rate target, it is also in some sense a sunspot.

These sunspots are reminiscent of our discussion of the NPR model. To uniquely determine nominal variables, a no-dice restriction had to be imposed. In general, money

[10]Note that there is nothing special here about the technology shock and the indeterminacy of a_1 and a_2. A similar situation would arise for the case of government spending shocks.

growth in the NPR economy could depend in an arbitrary way on a sunspot term. Similarly, in the PR economy, money growth could depend on sunspots, but unlike the NPR case, these sunspots will have real consequences. Because of these real consequences, money growth will need to depend on past sunspots (those that portfolios can react to), and on current sunspots as well, in order to support an interest rate target.[11]

An intuitive explanation may be helpful. A positive technology innovation increases the demand for labor and indirectly raises the demand for loanable funds. The latter effect will tend to increase the nominal interest rate. One natural way of preventing this is for the central bank to increase G_t by exactly the amount needed to support NPR behavior. However, we have just argued that this is not the only method. One alternative is to keep G_t the same but to reduce labor supply so that the implied increase in real wages will eliminate the increased demand for loanable funds. To reduce labor supply, the central bank needs to stimulate current consumption by lowering capital accumulation. The desired effect can be achieved by varying G_{t+1} and thus altering expected inflation.[12]

At a more basic level, the real indeterminacy under an interest rate peg arises here because the standard nominal indeterminacy conflicts with the model's assumption of a nominal rigidity (that is, n_t is predetermined). In the previous two sections, the standard nominal indeterminacy is easily eliminated by specifying the initial money stock

[11]Note that the real indeterminacy problem we are highlighting is quite different from the indeterminacy problem discussed in Blanchard and Kahn (1980), who provide restrictions on the eigenvalues of the matrix governing deterministic dynamics that ensure the existence of a unique path to the nonstochastic steady state. In contrast, under an interest rate rule, the deterministic dynamics of the present model are unique (because the model is identical to the corresponding real business cycle economy, with the marginal utility of leisure proportionally increased by the nominal rate of interest). Instead, the indeterminacy problem that arises here concerns the impulse response to a technology, fiscal, and/or sunspot innovation.

[12]This discussion highlights why real indeterminacy is not a problem in the model without capital.

and assuming that the Fed does not play dice. In the PR case, the issue is a bit more complicated, since n_t is chosen before G_t is observed, so that G_t potentially alters real activity. Our approach to resolving this problem is to restrict G_t to be a time-invariant function of the state variables--what Proposition 4 calls the central bank's reaction function, $G_t = G^{pr}(k_t, n_t, g_t, \theta_t)$. (This is the assumption we used in our numerical calculations at the end of section 5.) This assumption of a stationary reaction function is analogous to the no-dice restriction in the NPR case. Hence, to fully articulate an interest rate policy in the PR model, one must specify both R and the reaction function the central bank uses to support R. (McCallum [1986, p. 148] analyzes a nonoptimizing model and comes to a similar conclusion.)

7. Conclusion

Poole's (1970) classic analysis of the targeting debate concluded that, in an environment with numerous money demand shocks, an interest rate rule is preferred because it lowers the volatility of output. This observation raises three issues: a) What is the nature of money demand in our model? b) Are there money demand shocks in our model? and c) How do our conclusions relate to Poole's? We will address each of these issues in turn.

The typical criticism of the cash-in-advance constraint (relative to a more general transactions-cost technology) is that it does not allow for endogenous fluctuations in velocity in response to movements in the nominal interest rate. However, this criticism seems unwarranted in the current context. It obviously does not apply to the interest rate regime where, by assumption, the nominal rate of interest is constant. It also does not alter our negative conclusion on money growth rules unless one makes the heroic assumption that endogenous movements in velocity can replicate the welfare-improving role of a constant nominal rate of interest.

Are there money demand shocks in this model? Our cash-in-advance assumption implies that there are no shocks to the payments technology--one dollar of cash is always needed to conduct one dollar of transactions. However, there are shocks to the demand for transactions. Positive technology innovations increase the firm's demand for workers, and thus their demand for cash. Similarly, positive government spending innovations drive down the real wage by increasing labor supply, and thus once again increase the firm's demand for cash. Although in a general equilibrium environment it is difficult (if not impossible) to cleanly demarcate IS from LM shocks, it is clear that the shocks in this model do have money demand consequences.

This leads us back to Poole (1970). If we follow the previous discussion and interpret the model's shocks as money demand shocks, our conclusion is similar to Poole's. We find this quite remarkable, since our modeling strategy and welfare criteria could not be more different. The differences in welfare criteria illustrate a central point of the paper. Poole advocates an interest rate rule (in the stochastic money demand environment) because it reduces the variability of output. This paper advocates an interest rate rule because it increases the typical household's expected lifetime utility by providing more flexibility in responding to real shocks. For example, in the NPR economy operating under a money growth rule, a technology shock will generally cause the nominal rate to deviate from the steady state, and a time-varying path for the nominal rate of interest distorts the capital accumulation decision (see equation [12]). Similarly, in the PR economy operating under a money growth rule, the response of labor input to a technology shock is greatly muted (see equation [14]). The remarkable fact is that a simple interest rate rule entirely eliminates both of these distortions and allows the household to respond more efficiently to technology and government spending shocks. In sharp contrast to Poole, this increased flexibility improves welfare by actually increasing output variability.

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Table 1

Welfare Gain of an Interest Rate Rule

(expressed as percentage of steady-state capital stock)

σ	θ shocks	g and θ shocks
1	1.44	2.11
5	.92	1.80

Source: Authors' calculations.